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TESTS ON REINFORCED CONCRETE BEAMS.

By E. Brown (Associate Member, Can. Soc. C. E.)

(To be read before the General Section, October 15, 1908.)

This paper embodies the results of tests made in the Testing Laboratories, McGill University, during the session of 1906-07, under the author's direction. Some of the tests described formed the laboratory work of the graduating class in Civil Engineering, while the remainder was carried out independently. The work was arranged so as to include different methods of reinforcement, and it is intended to carry on further investigations, the results herein described having particular reference to reinforcement by Kahn and Johnson bars. Some reference is also made to the Ransome bars, but the tests on beams reinforced with this bar were limited in number owing to lack of time, and to the interruption of work resulting from the fire in the Engineering Building, early in April, 1907.

Two sets of beams were used, the moulds being 6" x 8" x 6' 4½" long, and 8" x 12" x 10' 6" long, respectively. These moulds were utilized to give beams to be tested on 6' 0" and 10' 0" centres, and different depths could be obtained by finishing the concrete below the level of the top of the mould. The moulds were built of heavy board, thoroughly boiled in oil. The sides were stiffened at intervals by vertical iron tubes running through the wood. The moulds were built in halves, lined with galvanized sheet iron, and hinged along the base so as to be easily parted. For removal of beams loose ends were provided, and the halves of the moulds were held together across the top by iron clamps, the ends of which were forced into the open tubes used as side stiffeners. These moulds gave every satisfaction. The beams turned out were of good shape and the surfaces were smooth. The beams were removed from the moulds after about four days, stored in tiers in the laboratory, and sprayed with water from day to day until required for test.

In all the tests the beams were supported at the ends and loaded at the third points, a condition which gives no shear between the loading points, but which gives a bending moment diagram approximating towards the parabolic form due to uniform loading. All the small beams (6-ft. centres) were tested in an Emery Testing Machine, and the large beams in a Buckton Machine. Plate XXIII shows a large beam in position in the latter machine, with the extensometers attached. Plate XXIV shows the extensometers attached to a wooden beam.

The portion of the beam between the loading points is subjected to simple bending, and extensometer measurements were made on it to ascertain to what extent the ordinary laws of bending are true in a reinforced beam. In most previous tests with which the author is familiar two extensometers only have been used, one being placed along the line of the steel reinforcement, and the other near the uppermost compression layer of the concrete. The position of the neutral axis has been then determined by *assuming* a linear law of straining to hold between these layers. In the tests here described, exact measurements of the strain were made at *five* horizontal layers of the beam, *viz.*, the two above mentioned and three intermediate ones. The strain curve is then obtained from five actual readings, and not from an assumed law applied to two extreme readings. In many cases the curves have been practically straight, in most cases they are slightly concave, while in one or two instances the concavity is very marked, especially on the tension side. These points will be evident from an inspection of the plates accompanying the paper, and will be noted as occasion requires. The movement of the neutral axis during test can also be seen clearly in the plates. The extensometers used were of the reflecting type, and were such that an alteration of length of 1/1000″ between the gauge points, which were 10″ apart, gave a movement of 2 cms. on the scale. By reading to millimetres only, a strain of 1/200000 could be measured. From the strain at the reinforcement line the stress in the steel can be found, on the assumption that the elongation of the steel and concrete is the same. The moment of resistance of the beam as determined by experiment may then be computed. An example of the method of calculation will be found later (p. 20).

The central deflection of each beam was determined by stretching a fine wire half way down the beam, over pulleys clamped to the beam over the points of support, and reading the movement of

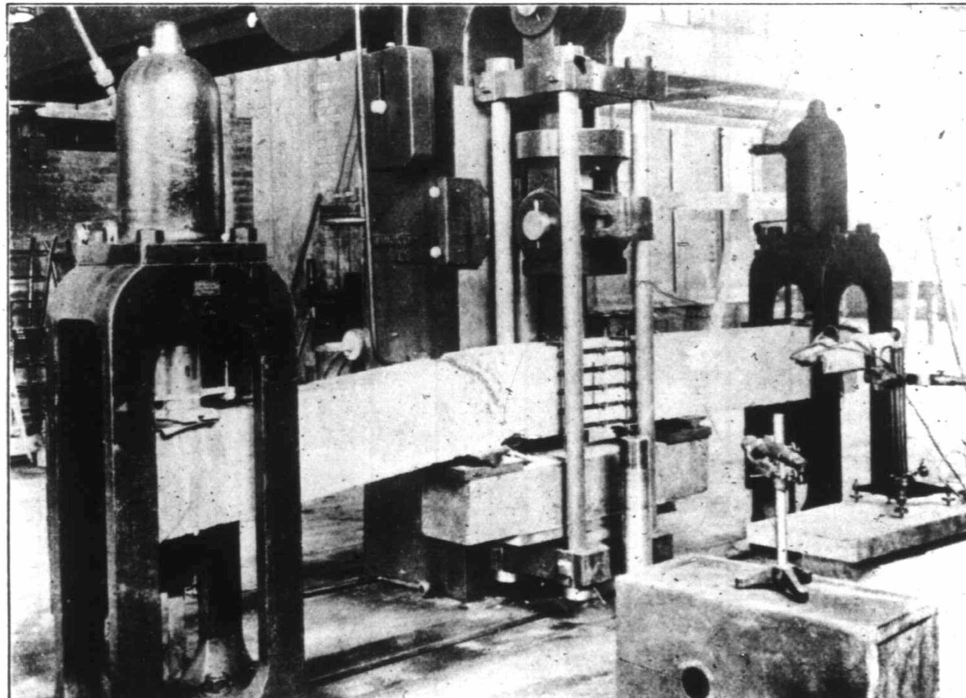

Plate XXIII.

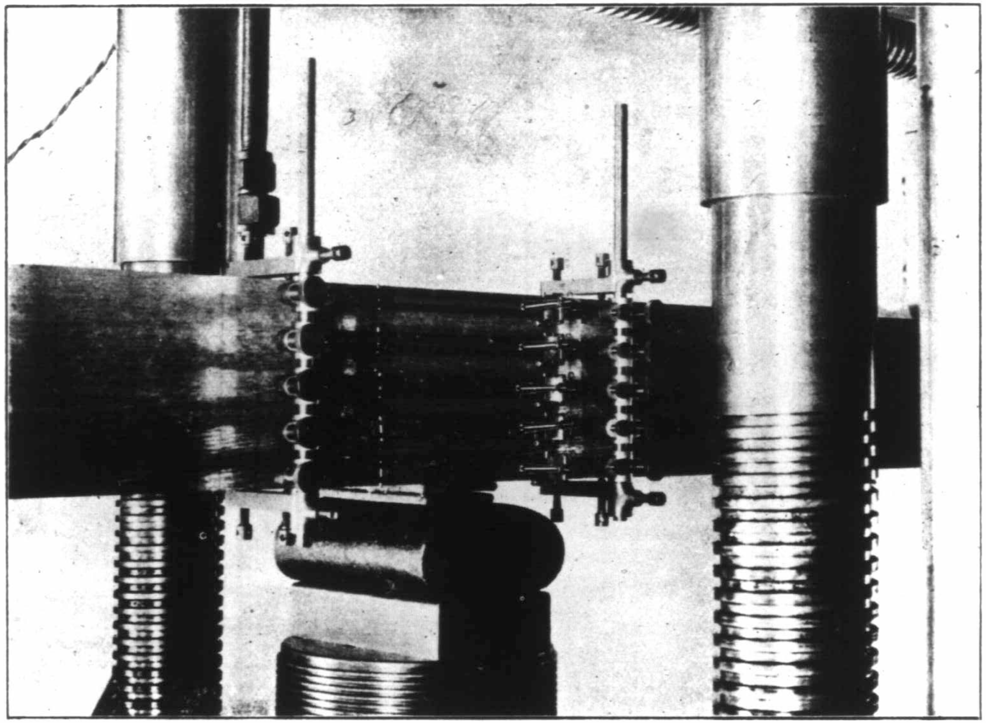

Plate XXIV.

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this wire over a scale attached to the face of the beam. The scale was graduated to 1/100″, and was read by telescope to 1/1000″

Careful observations were made to trace the beginning and progress of cracking of the beams. The surfaces were kept moist by the use of wet cloths; and, by using mirrors and magnifiers, together with oblique artificial illumination, it was possible to detect very minute cracks.

Concrete Used.

All the concrete consisted of a 1-2-4 mixture by weight of dry materials. The stone was trap rock, from the quarries at Delorimier Avenue, Montreal. It was ¾″ ring, weighed 157.8 lbs. to the cubic foot, and the voids were about 46%. Tests of this stone were made, and it gave an ultimate crushing strength (average of four tests) of 26,050 lbs. per square inch, which shows it to be a remarkably strong stone.

International Portland Cement was used throughout the work. It was intended to make complete tests of samples of the cement used, at the termination of the session's work, but the stock of cement was ruined by water during the fire in the Engineering Building, early in April, 1907.

The sand was local river bank sand.

To the dry mixture was added about 10% of water by weight. The concrete was thoroughly worked by hand, and tamped in the moulds. This was found to give a reasonably stiff mixture, not too wet before tamping, and the resulting beams had a smooth surface facilitating observations.

Compression cubes were made from the same mixture as the beams in all cases. The results of the tests of these are separately given in a later part of the paper (p. 11), together with some remarks on the tests made to ascertain the modulus of elasticity of the concrete in compression.

Steel Used.

Kahn, Johnson, and Ransome bars were used.

The steel bars were arranged in all cases so that the centre line of the reinforcement was as nearly as possible ¾″ above the bottom of the beams. The dimensions of each beam, together with the amount and percentage of reinforcement, are given on the diagrams showing the results of the tests, and need not be stated here.

Tensile Tests.

Johnson Bars—Half-inch bars (old style) were used having a net section of 0.18 square inch. The average of five tests showed the yield point to be 45,320 lbs. per square inch, and the breaking stress 71,240 lbs. per square inch. The elongation on 8″ was 23%, and the reduction of area was 42%. The fractures in all cases were of good appearance.

Extensometer tests gave Young's modulus 29.9×10^6 lbs. per sq. in.

Kahn Bars—Both half-inch and three-quarter inch bars were used. Unsheared bars were supplied for the tensile tests.

Tests of Half-inch Bars—These bars have a section 0.38 square inches unsheared. The yield point was 45,275 lbs. per square inch, and the breaking stress 68,500 lbs. per square inch. The elongation on a length of 8 inches was 28.5%. Owing to the peculiar section of the bar it was not possible to measure accurately the reduction of area, and it was estimated at 60%.

Three-quarter-inch Bars—The net unsheared section was 0.78 sq. in. The breaking stress was 63,930 lbs. per sq. in., and stress at yield point 38,300 lbs. per sq. in. The elongation on 8″ was 27.5%.

Young's modulus was 29.7×10^6 lbs. per sq. in.

Ransome Bars—Half-inch bars were used, and samples of the plain bar gave a yield point 42,000 lbs. per sq. in., and breaking strength 61,600 lbs. per sq. in. The elongation on 8″ was 27%, and the reduction of area 61%.

Samples of the twisted bar supplied gave yield point 78,820 lbs. per square inch, and breaking strength 86,240 lbs. per sq. in.

A sample of the plain bar supplied was twisted cold in the laboratory to the same extent as the Ransome bar, *viz.*, one twist in 2¾″ length. When tested the yield point was found to be 76,800 lbs. per sq. in., and the breaking strength 78,400 lbs. per sq. in. The fracture was near the grip. The yield point was therefore raised by cold twisting from about 42,000 lbs. per sq. in. to 78,000 lbs. per sq. in.—a rise of 85%.

A sample of the twisted bar supplied was annealed by heating to about 900° F., and cooling in ashes. It then gave a yield point 42,000 lbs. per sq. in., and breaking strength 60,480 lbs. per sq. in., results practically identical with those for the plain bar.

These results indicate the great increase in yield point stress due to cold twisting.

Disposition of Reinforcing Bars.

The beams were tested by supporting at the ends, and loading equally at the third points. The bending moment and shearing force diagrams are as shown in Fig. 1. Over the central third there is no shear, and no diagonal reinforcement need be provided. In the case of the Ransome and Johnson rods the following procedure was adopted. Suppose that five rods are used to reinforce over the central third. Outside the loading point towards the supports the bending moment diminishes, and after a certain distance four rods will be sufficient for reinforcement. This section will be at a distance $\frac{1}{5}$ of $\frac{l}{3}$, *i.e.* $\frac{l}{15}$ from the load, at which section the bending moment is four-fifths of the bending moment at the load. At this

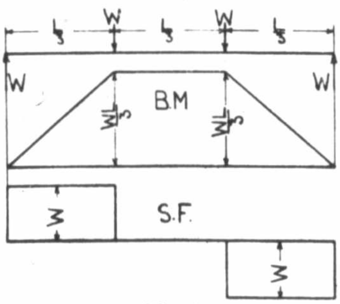


Fig. 1.

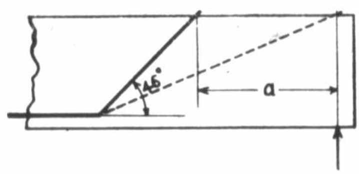


Fig. 2.

section one of the five rods, say the central one, can be dispensed with as tension reinforcement, and can be bent up at an angle to act as a diagonal reinforcement to resist the shearing. There are now four reinforcing rods, and when the remaining distance to the support is halved, two of these may be dispensed with as tension reinforcement, and be bent up to act as diagonal reinforcement. The two remaining rods would run the entire length of the beam. If the rods are bent up at the theoretical angle of 45°, there will be some horizontal length of beam over which there is no diagonal reinforcement, such as *a* in Fig. 2. But if the rod is bent so as to come out of the upper surface of the beam at a point over the support (see dotted line in Fig. 2) diagonal reinforcement is provided over the whole length from the bend to the support. Both these methods were tried, and the latter gave the better results.

The Kahn bars could only be obtained in the usual form, *i.e.*, with wings sheared over the entire length. It was not possible, therefore, to make any such disposition of material as described above in the case of the other bars, and the diagonal reinforcement ran the whole length of the beam. The tension area was also constant.

Percentage Reinforcement.

This is given by the expression $\frac{100 A}{bd}$ (see Fig. 3).

In most of the tests made, the percentage reinforcement varied from 1% to 1.30%. In one case, that of a beam with Ransome rods, it was 1.72%.

In the case of beams reinforced with Kahn bars, the percentage reinforcement is calculated on the net sheared section of the bars. But as the gross section of these bars is so much greater than the net section (being 40% to 50% in excess) it seems necessary in making a comparison of different methods of reinforcement to consider the total *weight* of reinforcing metal in each beam. In the Kahn bar, quite one-third of the total weight of metal is in the wings, whereas the straight bars can be bent up when required, as previously explained, and the proportion of material placed diagonally is much less than in the case of a Kahn bar. For example, if a bar like the Johnson or Ransome is bent at an angle θ so as to cover a horizontal length l, the diagonal length is l sec. θ. For $\theta = 30°$, this is only $1.16l$, and since some rods run straight through the whole length of the beams, and the others are only bent up over portions of it, the additional weight of steel in a beam so reinforced, over the weight in one in which straight rods run the entire length, is but small. Thus, consider a beam 6 ft. long supported at its ends and loaded at the third points. If reinforced with three bars, one of these might be dispensed with as tension reinforcement outside the load point at a distance of $\frac{2}{3} \times 24'' = 16''$ from the support. If the beam is 7″ deep, the centre line of reinforcement $\frac{3}{4}''$ above base, and the rod bent so as to come out of the beam surface over the support, the diagonal length would be $\sqrt{16.0^2 + 6.25^2} = 17.5''$.

The length of bent bar is then $40 + 2 \times 17.5 = 75''$, or only 3″ in excess of the length of a straight rod. Total length of rods is then $144 + 75 = 219''$. The additional weight due to the diagonal reinforcement is only $\frac{3}{216} \times 100 = 1.4\%$.

In $\frac{1}{2}''$ Kahn bar the gross section is 0.38 sq. in. Hence the section of the wings is 0.13 sq. in. The additional weight due to diagonal metal is therefore $\frac{13}{25} \times 100 = 52\%$, whereas in the case of straight rods (corrugated, twisted, or plain) the additional weight due to the bending up is almost negligibly small. If the latter method gives sufficient diagonal reinforcement it is evidently much more economical of material than the method of using heavy wings.

Method of Presentation of Results.

In all cases curves have been plotted showing the measured deformations at the various layers of the beams. From these the position of the neutral axis at various stages of the loading can be seen at a glance. It has been stated already that some of these curves show slight concavity, but in nearly all cases the strain curve for the compression layers is very approximately a straight line. If it be assumed that the modulus of elasticity of concrete in compression is constant, the stresses in the concrete will also be represented by the straight line, and the centre of the compression forces will be at a distance from the neutral line, equal to two-thirds of the distance of that line from the compression face of the beam, which distance can be measured from the diagram. By thus measuring the distance of the centre of the compressive forces from the centre line of reinforcement, the length of the arm of the resisting couple can be found. The total tensile force in the steel can be estimated by determining the stress intensity in the steel from the deformation at the reinforcement line, and multiplying by the net area of the reinforcing bars. Neglecting the tensile force on the concrete, the product of the force in the steel and the arm of the couple, gives the resisting moment of the beam as calculated from the measured deformations, and the result can be compared with the actual moment due to the applied loads. The author is well aware that this method may be regarded by some as altogether too simple, in that it ignores the variable co-efficient of elasticity of concrete in compression. Before work was commenced, the author had always felt that many of the formulae which have been proposed and developed from tests on beams, are far more elaborate than is justifiable in view of the variable quality both of the component parts of concrete, and of the resulting mixtures. Reinforced concrete work has always been ahead of our experimental knowledge of its properties, and there is perhaps some ground for the belief that many of the formulae are also somewhat ahead of our experimental data. The tests made on the concrete cubes have shown surprising differences in the compressive strains, even when the ultimate strengths have been about the same, and no definite law of variation of the co-efficient of elasticity of concrete in compression could be found from the tests. These points are referred to more fully when considering the tests on the cubes. In the author's opinion the simple form of treatment adopted in estimating the moment of resistance is justifiable. There are so many variable quantities in connection with the making of a reinforced beam—the qualities of sand, stone, and cement are all variable, the concrete itself may be more variable than any of its components, the sections of the rods are liable to fluctuations of 5% either way on the nominal values, their physical properties are not absolutely constant, and the accuracy of setting of the rods is probably not very great even when much care is taken. It is uncertain in what direction these and other variable conditions, such as the efficiency of the labour employed, are operating in the finished beam. It is unlikely that they are all favourable or all unfavourable at the same time. But a recognition of their existence should, in the author's opinion, tend towards some moderation in

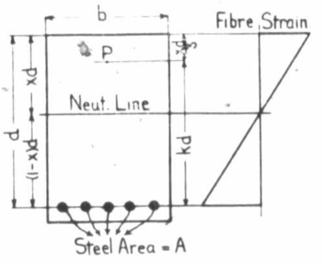


Fig. 3.

the methods of interpreting experimental results, and in the making of formulae in the absence of any such knowledge.

Position of Neutral Axis.

The following simple analysis has been used as the basis of the work, in accordance with the views expressed in describing the method of presentation of results. In Fig. 3

let b = breadth of beam.

d = depth of beam to centre line of reinforcement,

xd = depth of neutral axis from outermost compression layer,

A = net sectional area of reinforcing steel,

f_s = intensity of stress in steel,

f_c = intensity of compressive stress in concrete at outermost layer,

E_c = modulus of elasticity of concrete in compression,

E_s = modulus of elasticity of steel in tension.

$$e = \frac{E_s}{E_c}$$

Then assuming linear law for variation of compression stress

$\frac{bxd\ f_c}{2}$ = total compressive force.

$\therefore \frac{bxd\ f_c}{2} = Af_s =$ total tensile force, neglecting tension in concrete.

But if the strain at horizontal layers follows the linear law,

$$\frac{f_c}{f_s} = \frac{E_c\ xd}{E_s(1-x)d}.$$

Hence, $\frac{f_c}{f_s} = \frac{2A}{bxd} = \frac{E_c\ x}{E_s(1-x)} = \frac{x}{c(1-x)}$

$$\therefore \quad \frac{x^2}{1-x} = \frac{2Ac}{bd} = 2pc \quad (1)$$

where p is the ratio $\frac{A}{bd}$, *i.e.* ratio of section of metal to section of concrete. In this equation c is the ratio $\frac{E_s}{E_c}$, and the equation gives the position of the neutral axis for any assigned values on the right-hand side of the equation.

The equation gives

$$x = \sqrt{p^2c^2 + 2pc} - pc.*$$

Let $E_c = 2 \times 10^6$ lbs. per sq. in. and

$E_s = 30 \times 10^6$ lbs. per sq. in. Then $c = 15$ and

the equation gives

$c =$	p	x
15	0.007	0.365
15	0.010	0.417
15	0.015	0.483
15	0.020	0.530

showing that the neutral axis is lower as the amount of reinforcement is greater, and is below the half depth when the reinforcement exceeds 2%.

Corresponding values for $c = 10$, *i.e.*, $E_c = 3 \times 10^6$ lbs. per sq. in. are

$c =$	$p =$	$x =$
10	0.007	0.311
10	0.010	0.358
10	0.015	0.418
10	0.020	0.463

* At ¼ ultimate deformation, Talbot, using a variable modulus of elasticity for concrete in compression, gives

$$x = \sqrt{\frac{144}{121}p^2c^2 + \frac{24}{11}pc} - \frac{12}{11}pc$$

$c = 10$

$p = .01$ gives $x = 0.39$ against 0.358 by the linear analysis.

The centre of compressive stresses is at P, and the moment of resistance of the steel stresses in the beam is given by

$$M = \text{steel force} \left(d - \frac{xd}{3}\right)$$

$$= Af_s d \left(1 - \frac{x}{3}\right)$$

$= k\ Af_s d$ where k is some co-efficient to be determined experimentally, and which varies with x. In the tabulated results of the tests for each beam the value of k is given. It is found to vary from 0.82 to 0.88, and is usually about 0.85. This method of analysis is more applicable during the later stages of the loading, than when the concrete is uncracked and carrying some tension. In the earlier stages of loading the above moment is always much less than the load moment, owing to tension in the concrete. The formula $M = 0.85\ Afd$, in which f is the yield point of the steel, was first suggested by Capt. Sewell, and gives a very close approximation to the ultimate moment of resistance of beams. Some remarks on the methods of determining safe loads will be found later (p. 62).

Compression Tests.

Cubes for compression tests were made from the mixtures used in the making of the various beams. These were tested at different ages, and measurements were taken to determine the value of the modulus of elasticity of the concrete in compression. The tests showed some large variations in the rate of yielding of the concrete under load, although the compressive strength varied but slightly, the latter ranging from 2080 lbs. per sq. in. to 2486 lbs. per sq. in. in seven tests out of nine. The lowest value recorded was 1545 lbs. per sq. in. in the case of a cube which was poorly rammed, one face of which was very porous at the start, and showed decided weakness all through the test. The results for this cube are not tabulated below. The ultimate load for the remaining cube was beyond the capacity of the machine. The cubes were moulded in open boxes. In some tests the load was applied on the top and bottom faces of the cubes, *i.e.*, to the horizontal faces as moulded. In other tests the load was applied to the side faces, this being the direction of the compressive stress in a loaded beam. As will be seen, the manner of application of load has no apparent effect on the ultimate strength. All the cubes were sound, and failure was general on all faces.

The results are shown below:

Cube No.	Dimensions inches.	Cracking noticeable at lbs.	Max. load lbs.	Max. load lbs. sq. in	Age in days.	Load applied.
0	9.8 × 8.8 × 9.0	170,000	190,000	2,400	29	Top & bottom
1	8.75 × 9.0 × 9.0	150,000	195,300	2,480	28	Top & bottom
4	8.75 × 9.0 × 9.07	100,000	163,800	2,080	31	Sides
5	8.80 × 9.0 × 9.05	100,000	188,100	2,380	32	Sides
2	7.99 × 7.97 × 8.0	100,000	143,400	2,250	58	Top & bottom
3	8.8 × 9.10 × 9.30	140,000	180,000	2,250	53	Sides
6	9.1 × 9.0 × 9.0	160,000	203,400	2,486	56	Top & bottom
7	8.85 × 9.1 × 9.0	200,000	(215,400) (not max.)	(2,670) (not max.)	54	Sides

No. 4 }
No. 5 } Distinct lime smell on fracturing.

No. 5—Same mixture as No. 12 beam.

No. 4—Same mixture as No. 10 beam.

Modulus of Elasticity of Concrete in Compression.

In the making of formulae for the strength of reinforced concrete structures, much emphasis has been laid upon the variations

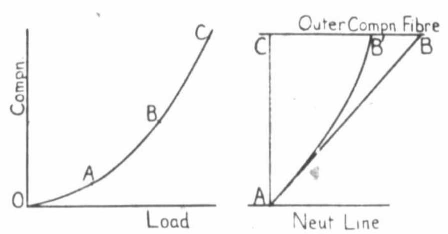


Fig. 4.

of the modulus of elasticity of concrete in compression. It is an undoubted fact that the relation between stress and strain in such a case is not constant over any considerable range of stress, and that in the case of a cube having an ultimate strength of 200,000 lbs. the total yielding for each 50,000 lbs. of the load will, in general, not be constant. The rate of yielding increases as the load increases, as shown in Fig. 4.

If such a curve is considered as being made up of a series of straight lines, say, OA, AB, BC, an approximate value of Young's modulus can be found for each range of load considered, and some idea can be formed of its variation. Many writers have emphasized the variations in the modulus of elasticity to a very great extent, and the curve of distribution of compression stress has been modified from the straight line form assumed in working out the results of the tests described in this paper. Values V of the modulus calculated from a curve such as that in Fig. 4 will decrease as the load increases.

Hence, if the yielding of the various compression layers of a beam is represented by the ordinates from a line AC to a line AB (Fig. 4) the actual compression *stresses* would be represented by the ordinates to some curve such as AB′, the form of AB′ being dependent upon the variations of the decreasing compression modulus. The centre of compression stresses would then be through the centre of gravity of the Fig. ACB′, instead of through the centre of gravity of ACB. It would, therefore, pass closer to the neutral line in the former than in the latter case, and the lever arm of the forces causing the resisting couple would be reduced. Furthermore, the position of the neutral axis of the beam depends on the relation between the moduli of elasticity of steel and concrete, and is, therefore, affected by changes in the latter.

Many formulae have been advanced for the calculation of the position of the neutral axis, making allowance for variations in the modulus, and the theory of computation of the strength of a reinforced beam is thereby rendered much more cumbersome than the one used by the author in this paper, and by many other writers.

It may, for example, be assumed that the compression modulus varies according to some parabolic law. (See curve AB′, Fig. 4.) Some writers place the vertex of the parabola at the outermost compression layer, while others place it outside the beam above the compression face. If xd is the distance of the outer compression layer from the neutral axis, the distance z $(= \frac{xd}{3}$ in Fig. 3), of the centre of compression stresses from the outermost layer, will alter as the intensity of the stress in the concrete changes. Assuming that the vertex of the parabola is at the outermost compression layer, and that the ultimate value of the modulus is two-thirds of the initial value, a theoretical investigation along the above lines shows that when the concrete at the outermost layer reaches $\frac{1}{4}$ of its ultimate deformation $z = 0.34xd$; at $\frac{1}{2}$ ultimate deformation $z = 0.35xd$; at $\frac{3}{4}$ ultimate deformation $z = 0.36xd$. If a constant modulus is assumed z is always $\frac{xd}{3}$. It will be seen that the result of such deviations from the simple linear law is to alter the position of the centre of stresses very slightly indeed. With the above notation the ultimate moment of resistance of the beam is $A\,f\,(d\text{-}z)$. It is shown by actual tests that for average conditions x may be about 0.4. Thus for a beam in which $d = 10''$, the value of z at ¾ ultimate deformation of concrete is $0.36 \times 0.4\ d = 1.44''$. Taking $z = 0.33xd$, the result would be 1.33", a difference of 1/10". Thus by making allowance for the variation in modulus of concrete, there is a difference of the order of 1/10" in the value $(d\text{-}z)$, the arm of the resisting couple. The theoretical value of x is slightly different in the two cases, but the difference is not so great as to affect the order of the change in $(d\text{-}z)$. In deciding the practical value of a formula, the degree of accuracy with which the conditions of practice approach the assumed conditions of the analysis must be carefully considered. In this connection the author contends that the conditions under which rods are placed do not allow of the depth to the centre line of the reinforcement being gauged to an accuracy approaching 1/10" in a 10" beam, or indeed in any other beam or structure. It is very doubtful whether, with the utmost care in a laboratory, such a degree of accuracy of setting could be obtained. The important quantity is $(d\text{-}z)$, and a process which involves an extreme degree of refinement in the determination of the smaller term z, and ignores the practical conditions which govern the larger term d cannot well be defended as being a necessary part of the work of calculation of the strength of an actual reinforced beam. The author is further inclined to doubt whether the law of variation of the modulus of elasticity for concrete in compression, and the actual values of this modulus, are sufficiently definite to completely justify the refined calculations to which reference has been made. The value of the modulus of elasticity for different concretes may be fairly well established, the law of its variation is less certain, and in view of the variations in quality of the component parts of concrete and the conditions of practice under which it is made and laid, there is reasonable room for doubting whether the ultimate strength, modulus of elasticity, and the law of its variation can be known to within perhaps 20%. In the author's opinion such considerations should tend towards a simplification of the formulae for use under practical conditions. The investigation of the stresses in a reinforced beam under certain assumptions regarding the physical properties of the materials and the disposition of those materials in the beam, is an interesting exercise in the beam theory, but care should be taken lest the results of such analysis should convey the impression that the properties of a reinforced concrete beam are as certain, and as accurately known as an inspection of the formulae so deduced would seem to imply. In saying this the author does not wish to suggest that there is a large element of uncertainty in reinforced concrete construction. Our knowledge of its properties is sufficiently definite to enable a safe design to be executed under any normal conditions, if reasonable care be taken in the application of simple principles. The author believes that the conditions of practice offer a far more suitable field for the adaptation of such principles than for the use of many of the elaborate formulae which have been proposed. As a matter of fact the differences introduced into the strength formulae by the more complex analysis referred to are not of a large order of magnitude, and the author believes that any one of these differences might be completely discounted by an inaccuracy in the setting of the rods, or by a change in the properties of the concrete due to some alteration in the conditions of mixing and laying. In other words, there is a tendency to strain at the gnat and swallow the camel. The results of such straining may be right, but it cannot be known, because the conditions are so complex. And, after all, what does a considerable variation in the value of the modulus for concrete amount to?

The ratio $\frac{E_c}{E_s}$ is principally of interest in the theoretical determination of the position of the neutral axis.

Take the formula $\frac{x^2}{1-x} = \frac{2Ac}{bd}$ (p. 9) for the position of the neutral axis, on the assumption of linearity of stress across the section. Put $\frac{A}{bd} = \frac{1}{100}$, *i.e.*, 1% reinforcement. Suppose $E_c = 2 \times 10^6$ lbs. per sq. in. and $E_s = 30 \times 10^6$ lbs. per sq. in. so that $c = 15$; the equation gives $x = 0.42$. If $\frac{E_c}{E_s} = \frac{1}{10}$, *i.e.*, $E = 3 \times 10^6$ lbs. per sq. in. then $x = 0.36$. Hence in a beam having a depth of 10″ to the centre line of the reinforcement, the neutral axis is 4.2″ from the outer compression layer, if $E = 2 \times 10^6$ lbs. per sq. in.; and 3.6″ from that layer, if $E = 3 \times 10^6$ lbs. per sq. in.

The arm of the resisting couple is therefore $(10 - \frac{4.2}{3} = 8.6''$ in the former case, and $(10 - \frac{3.6}{3} = 8.8''$ in the latter case, a difference of some 2½%, so small as to be completely discounted by an error in setting of the rods. The more complex theories give differences of the same order of magnitude. Can it be seriously contended that the actual value of the modulus, and the law of its variation, if there be a definite law, are of such overwhelming importance in the design of reinforced concrete beams, as would be implied by an inspection of some of the formulae for such designs? And are we not in danger of allowing our formulae to reach a state of apparent theoretical accuracy, altogether beyond what is justifiable in view of the physical properties of the materials concerned, and of the practical conditions under which these materials are used?

Modulus of Elasticity from Tests.

The cubes made from the beam mixtures were intended primarily for the determination of the compressive strength of the concrete. The author is well aware that in the determination of the compression modulus it is desirable to have prismatic specimens of a length equal to about three times their transverse dimensions, and such specimens probably give a more accurate measure of the compressive strength than cubes. The use of the cubes to determine the strength is, however, more common in practice, and the results of the tests can be compared with existing data. The measurements of the rate of yielding of the concrete were made only as an auxiliary to the compression tests, but the results were of such a character as to make them of some interest. In the case of the first cube tested, overlapping scales graduated to 1/100 inch were used and read to 1/1000 inch, using a magnifier, but such readings are necessarily only approximate. In the other tests two Martens Extensometers were used, one on each of two opposite faces of the cube. The gauge length was usually 4 inches. In one case (No. 1 cube) it was 6 inches. The reflecting mirror gave a movement on the scale such that 1mm. corresponded to 1/10000″ change on the specimen. Fractions of millimeters could be read easily, and, therefore, compressions of 1/100000″ were measurable. The compressive strengths tabulated (p. 11) show the cubes to be of reasonable uniformity in this respect. The rate of yielding was noticeably different on the two sides of the cubes in all cases—in some cases very remarkably so. Still greater variations were observed between the rates of yielding of the different cubes, and so far as a determination of the modulus for the concrete and the law of its variation are concerned, the results were negative. The modulus always diminished as the load increased, but the initial and final values were quite irregular.

It may be objected that such extensometer measurements give only a localized value of the compressive strain and could not be expected to give consistent values for the modulus of the block as a whole. The author would remind any to whom this thought may occur that all extensometer measurements made on the faces of the beams themselves are likewise localized measurements. If consistent results are obtainable for the value of the modulus, by some method of measurement giving a different value for the *average* compression, from that obtained by localized measurements such as the above, it would not seem reasonable to apply such results to merely local measurements made on the face of beams, and the author's contention respecting the variable properties of a concrete mixture would be amply justified. His own results are consistent as regards each face of each individual cube, but as between different cubes, there is a remarkable variation, although values of the compressive strength were very reasonably consistent. The results are shown in Plate XX, and the values of the modulus as determined by regarding the curves as being represented approximately by a series of straight lines are tabulated below. These values were obtained from curves plotted to a larger scale than that in the Plate. Extensometer readings were taken at every 10,000 lbs. of load, the intermediate readings to those shown on the Plate lying equally as well on the curves as the plotted readings.

Cube No.	Dimensions, inches	Load Range lbs.	Stress Range lbs. per sq. in.	E_c in millions of lbs. per sq. in.		
				One face	Other face	On mean readings between two faces
1	8.75 × 9.0 × 9.0	10,000- 50,000	129-645	2.66	1.200	1.650
		50,000-100,000	645-1290	2.03	0.953	1.296
2	7.99 × 7.97 × 8.0	10,000-100,000	157-1570	0.632	..	..
		10,000- 40,000	157-630	..	0.539	..
		40,000- 80,000	630-1260	..	0.326	..
3	8.8 × 9.1 × 9.3	10,000- 50,000	125-625	4.33	3.07	3.63
		50,000- 80,000	625-1000	2.635	2.00	2.274
		80,000-110,000	1000-1375	1.65	1.21	1.390
4	8.75 × 9.0 × 9.07	20,000- 50,000	260-645	2.875	6.05	3.89
		50,000- 70,000	645-910	1.360	6.05	2.22
		70,000-100,000	910-1290	1.000	2.18	1.37
		100,000-150,000	1290-1935	0.567	..	..
5	8.8 × 9.0 × 9.05	10,000- 70,000	126-885	1.98	..	..
		20,000- 60,000	253-760	1.98	1.84	..
		70,000-100,000	885-1265	Irregular	1.01	
6	9.1 × 9.0 × 9.0	10,000- 50,000	122-610	2.370	(from average line for whole range) 3.53	..
		50,000- 70,000	610-855	1.240		..
		70,000-150,000	855-1830	0.777		..

In testing cube No. 0 overlapping scales were used. The rate of yielding was about the same on the two faces, and over the middle range was about 1/1000" for each 10,000 lbs. load increment. This corresponds to $E_c = 1.14 \times 10^6$ lbs. per sq. in.

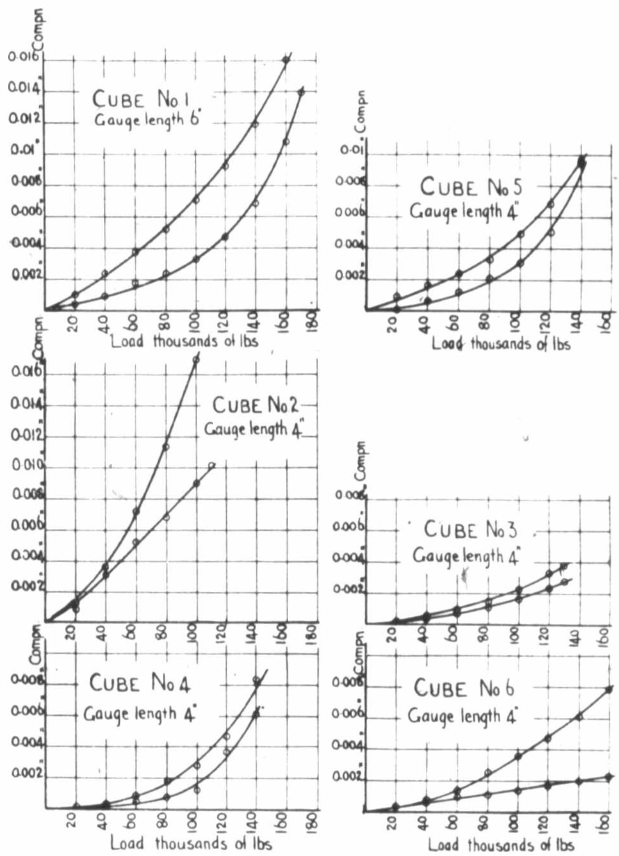


Plate XX.

A study of these results shows some remarkable variations. Especially noticeable is the large yielding of No. 2, and the small yielding of No. 3. There is undoubtedly a diminution in the value of the modulus as the stress increases, but there is no uniformity in either the value of the modulus or in its variations. From the compression tests it would be said that good, sound, uniform concrete was used. In no case was there marked failure on any one face or of any one cube, such as would account for the variations shown above. The results do not give any encouragement to the view that the value of the modulus of elasticity can be assigned with any great degree of accuracy, and if this be so, a method of analysis involving a law of variation of a quantity which in itself of very uncertain value, cannot be altogether justified especially in view of the fact that in its results it modifies but slightly the quantities which it is important to estimate in a design. The results of actual tests will agree reasonably well with either the simple theory used in this paper, or with a more elaborate theory taking account of rather uncertain laws of variation of physical properties of the materials used, which variations do not very materially affect the vital points in a design. It is extremely probable that neither theory is correct. To justify the elaborate theories which have become so fashionable, the author considers:

(1) That the physical properties of the materials used should be of such a degree of constancy as would correspond with the degree of refinement of the calculations employed;

(2) That the results of actual tests on beams should agree more closely with the theories so deduced than with the simple theory used in this paper.

The author does not believe that either of these points can be sufficiently established.

Actual Beam Test Records.

Records were made by an observer facing the side of the beam carrying the extensometers. The loading point on his right is termed the *right load point*, and that on his left, the *left load point*. The side of the beam carrying the extensometers is called the front, and the other side, carrying the deflection scale, is called the back. The beams 6′ 0″ *c.* to *c.* were tested with the compression face uppermost, while in the case of the larger beams 10′ 0″ *c.* to *c.* the tension face was uppermost. Load increments of 500 lbs. were used for the 6′ 0″ beams, and increments of 1000 lbs. for the 10′ 0″ beams. The tables submitted show the increments of stress, deflection, etc., due to the machine load, the zero readings of the instruments being taken under the dead load of the beam itself. The bending moment due to the weight of the beam and loading arrangements is included in the ultimate moment used in determining the constant in the relation—Ult. moment = const. × bd^2. All beams were loaded at the third points. The upper and lower extensometers were each ¾″ from the edge of the beam, the three intermediate ones being then uniformly spaced. The positions are sufficiently indicated in the plates. A complete record of one test is given below, together with the method of working out results. The complete tests are then summarized.

Beam No. 3. Johnson Beam—Concrete, 1-2-4. Beam, 6″ × 8″ × 6′ 0″ c. to c. Reinforcement, 3 — ½″ rods = 0.54 sq. in. One rod bent

Machine load lbs.	No. 1		No. 2		No. 3		No. 4		No. 5		Deflect'n scale inches
	Read.	Diff.	Read.	Diff.	Read.	Diff.	Read.	Diff.	Read.	Diff.	
0.0	+165.6	0.0	+152.0	0,0	−21.2	0.0	−187.3	0.0	−184.2	0.0	2.860
500.	161.1	−4.5	149.6	−2.4	22.0	−0.8	186.7	+0.6	180.8	+3.4	2.860
1,000.	155.4	10.2	146.3	5.7	22.0	0.8	184.5	2.8	176.6	7.6	2.862
1,500.	150.0	15.6	143.8	8.2	22.0	0.8	182.0	5.3	170.9	13.3	2.867
2,000.	144.0	21.6	140.2	11.8	22.0	0.8	178.2	9.1	164.2	20.0	2.873
2,500.	133.1	32.5	137.8	14.2	21.2	0.0	173.2	14.1	154.8	29.4	2.878
3,000.	130.8	34.8	134.7	17.3	19.2	+2.0	167.3	20.0	145.8	38.4	2.885
3,500.	124.1	41.5	132.2	19.8	17.8	3.4	159.9	27.4	132.8	51.4	2.892
4,000.	117.2	48.4	129.9	22.1	14.1	7.1	152.2	35.1	121.8	62.4	2.900
4,500.	110.8	54.8	128.0	24.0	11.5	9.7	144.1	43.0	107.6	76.6	2.910
5,000.	102.1	63.5	125.2	26.8	8.5	12.7	135.3	52.0	94.0	90.2	2.920
5,500.	95.1	70.5	122.8	29.2	6.9	14.3	126.3	61.0	79.5	104.7	2.930
6,000.	85.8	79.8	120.1	31.9	3.2	18.0	118.0	69.3	65.8	118.4	2.940
6,500.	78.5	87.1	117.1	34.9	0.8	20.4	109.0	78.3	50.1	134.1	2.950
7,000.	66.8	98.8	112.1	39.9	+0.8	22.0	101.0	86.3	36.5	147.7	2.962
7,500.	57.0	108.6	108.0	44.0	2.8	24.0	91.2	96.1	21.1	163.1	2.975
8,000.	47.8	117.8	104.5	47.5	5.1	26.3	83.2	104.1	8.0	192.2	2.985
8,500.	33.8	131.8	97,6	54.4	6.8	28.0	71.1	116.2	+12.2	196.4	3.000
9,000.	22.1	144.5	92.8	59.2	8.2	29.4	62.3	125.0	27.8	212.0	3.016
9,500.	12.1	154.5	88.1	63.9	9.7	30.9	53.8	133.5	43.5	227.7	3.027
10,000.	−7.2	172.8	78.8	73.2	10.7	31.9	43.2	144.1	63.0	247.2	3.045
10,500.	24.1	189.7	69.8	82.2	10,2	31.4	33.3	154.0	84.5	268.7	3.070
11,000.	41.1	206.7	62.1	89.9	11.2	32.4	23.5	163.8	98.5	282.7	3.082

up at 45° outside load points. Centre of rods, ¾″ above base. % reinforcement = 1.25. Load at third points. Age, 49 days. Total weight, 314 lbs. Steel, 11.5 lbs. Extensometer gauge length, 10″. Scale readings in mms such that 20 mms = 0.001″ movement on face of beam. Extensometers in order 1 to 5 from compression to tension face.

Notes—6500 lbs. load vertical crack 6″ to right of centre on face

7000 lbs.—Slow movements on extensometers indicating breakdown of material. First crack extends above c. l. of steel and across base. New crack 2" outside right load point on back.

8500 lbs.—Small vertical crack on face, 1" outside left load point, running 2" across base. New crack at 7000 lbs. runs half way across base.

8800 lbs.—Crack on back. 5" to left of centre, 1½" up face. Similar crack 2" inside left load point. Both run partly across base.

9000 lbs.—Vertical crack (small) 2" outside left load point on back.

9500 lbs.—Diagonal crack on back. Starts 1" above base and 9" from left support. Runs beyond half depth and reaches as far as diagonal bar.

11,000 lbs.—Crack at 9500 lbs. further developed, but new crack at other end caused failure. Runs from support diagonally towards load point over end 15" of beam. This portion is not diagonally reinforced, as the central bar is bent up at 45°, starting 8" outside load point.

Calculated Results for Above Test.

The deformation curves shown in Plate III are straight lines, showing that plane sections before bending have remained plane.

7500 lbs. load—The plotted results show that the extensometer on the steel centre line (No. 5) moved 163 mms. This corresponds to $\frac{163}{20} \times \frac{1}{1000} = 0.00815$ inches on the face of the beam. The gauge length $= 10''$.

$\therefore$ Strain $= \frac{0.00815}{10} = 0.000815.$

$\therefore$ Stress in steel $= 30 \times 10^6 \times 0.000815.$

$= 24450$ lbs. per sq. in.

Total steel force $= 0.54 \times 24450.$

$= 13200$ lbs.

At this load the neutral axis was 3.4" from the compression face of the beam. The arm of the resisting couple $= 7.25'' - \frac{3.4''}{3} = 6.12'' = kd$ (Fig. 3).

Moment of resistance of steel force $= 13,200 \times 6.12.$

$= 80,700$ lb. inches.

Load moment $= \frac{7500}{2} \times 24 = 90,000$ lb. inches.

The co-efficient $k = \frac{6.12}{7.25} = 0.845.$

The difference between actual load moment and moment of steel force indicates the existence of tensile stress in the concrete.

10,000 lbs. load—Similar calculations show that the steel stress was 36,700 lbs. per sq. in., and the steel force 19,830 lbs. Arm of couple = 6.08″.

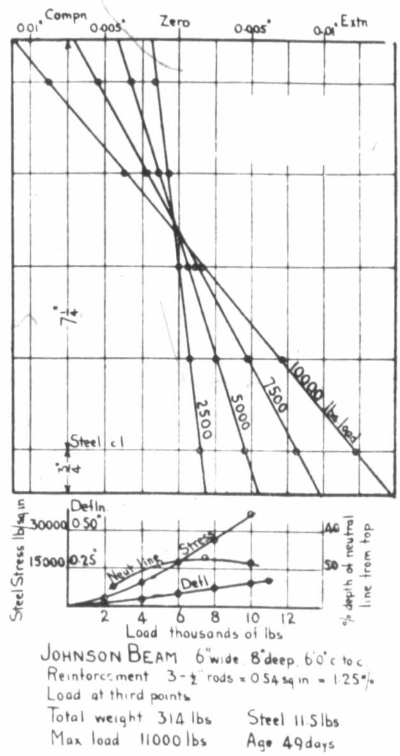


Plate III.

Moment of steel force = 19,830 × 6.08 = 120,600 lb. inches.

Moment of load $= \frac{10,000}{2} \times 24 = 120,000$ lb. inches.

The closer agreement between load and steel moment, as the load approaches its ultimate value, indicates the complete breakdown of the tension value of the concrete.

An estimate of the compressive stress in the concrete may be made from the above result. The total tension = 19,830 lbs. since the concrete tension is negligible. Hence total compression = 19,830 lbs. Depth to neutral plane = 3.5″. Area in compression = 6 × 3.5 = 21.0 sq. in. Assuming linear variation of stress, the

maximum compressive stress = $\frac{19,830 \cdot 2}{21}$ = 1890 lbs. per sq. in., a value within the ultimate strength of the concrete. The beam failed by diagonal cracking. The yield point of the steel was not reached, the stress at 11,000 lbs. load being 42,400 lbs. per sq. in.

5000 lbs. load—The beam was not noticeably cracked on the tension face at this load, and an estimate may be made of the tensile stress in the concrete.

Load moment = 60,000 lb. inches.

Steel moment = 44,200 lb. inches, (found as above).

Tensile concrete moment = 15,800 lb. inches.

If the tensile stress is assumed to vary according to the linear law, the arm of the tensile force will be two-thirds of the depth of the beam. If the depth is measured to the steel centre line, the arm = ⅔ × 7.25 = 4.83″.

$\therefore$ Concrete force (tensile) = $\frac{15,800}{4.83}$

= 3280 lbs.

Area in tension = (7.25 — 3.51) × 6.

= 22.44 sq. in.

$\therefore$ Max. tensile stress = $\frac{2 \cdot 3280}{22.44}$ = 292 lbs. per sq. in., a reasonable value, at a load approaching that at which definite cracks were observed.

The total compression force at this load = steel force + concrete force = (7280 + 3280) = 10,560 lbs. Area in compression = 3.51 × 6 sq. in. Max. compression stress = $\frac{2 \cdot 10,560}{3.51 \cdot 6}$ = 1000 lbs. per sq. in.

The deflection at 5000 lbs. load was 0.06″, being $\frac{1}{1200}$ of the span.

Ultimate moment:

Load moment	= $\frac{11,000}{2}$	× 24 =	132,000
Weight "	= $\frac{314}{8}$	× 72 =	2,826
Loading rail	= $\frac{100}{2}$	× 24 =	1,200
			136,026

Hence 136,026 = const. bd^2.
= const. $\times 6 \times 7.25^2$.
∴ Constant = 432.

The discrepancy between steel and load moments at the lower loads is due to the neglected tensile stress on the concrete, and at the higher loads there is a close agreement. The neutral axis rises during the early stages of loading, and falls as the breaking load is approached. The detail readings for No. 3 extensometer indicate that there was a small amount of compression at that layer of the beam until the load reached 2500 lbs. Beyond that load, tensile readings are indicated. The neutral line remains close to No. 3 instrument during the early stages of the loading. Similar effects were noted frequently in other tests.

The results are summarized briefly below, and corresponding tables are then given for all the beams tested, together with brief notes of the behaviour during test.

Machine load, lbs.	Depth of neut. axis from comp'n face	Centre of comp'n to c. l. of steel = k. depth	Steel stress, lbs. per sq. in.	Steel force, lbs.	Steel moment, lb. ins.	Load moment, lb. ins.	% of Load moment on concrete.
2,500	4.0 ins.	5.92 = 0.82d	4,400	2,380	14,100	30,000	53.0
5,000	3.51	6.08 = 0.84d	13,500	7,280	44,200	60,000	26.3
7,500	3.40	6.12 = 0.845d	24,420	13,200	80,700	90,000	10.3
10,000	3.51	6.08 = 0.84d	36,700	19,830	120,600	120,000	0.0

SUMMARY OF TESTS.

Beam No. 1. Johnson Beam—Concrete, 1-2-4. Beam, 6″ × 8″ × 6′ 0″ c. to c. Reinforcement, 3 — ½″ Johnson bars = 0.54 sq. in. One rod bent up at 45°, 8″ outside load points. Centre line of reinforcement, ¾″ above base. % reinforcement, 1.25. Total weight of beam, 311 lbs. Weight of steel, 11.5 lbs. Age, 28 days. Loaded at third points.

The results are shown in Plate I, the deformation curves having a slight concavity. This beam was the first one tested, and the observations for initial cracking were not so careful as in the later tests when the beam surface was kept moistened and was viewed by oblique illumination. At 9000 lbs. load the extensometer on the steel line showed marked creeping along the scale, suggesting that the steel stress was near the yield point. The indicated stress was 44,300 lbs. per sq. in., the average yield point given by tests on the bars (see p. 4) being 45,320 lbs. per sq. in., the greatest value being 48,100 lbs. per sq. in. This observation is of interest as the steel moments at the higher loads figure in excess of the actual load moments, if the linear law for increase of stress is assumed. If the yield point was passed the stresses indicated in the table below would be in excess of the actual values, and the excess of

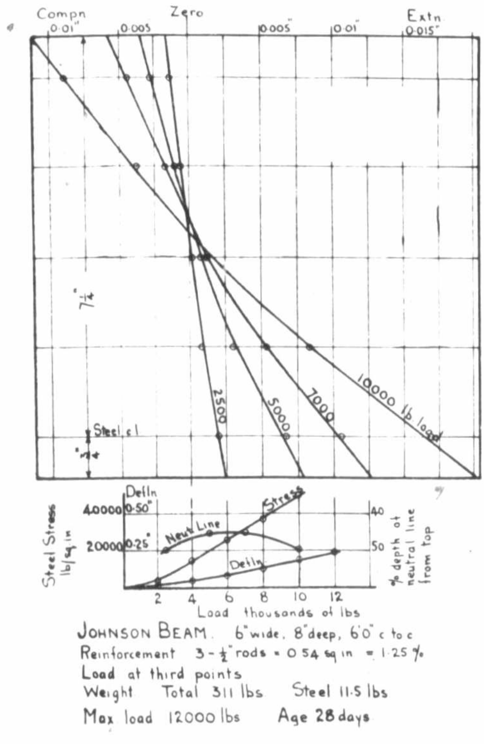


Plate I.

steel moment over load moment would be largely accounted for. At 10,000 lbs. load many cracks extended across the base, and ran one to two inches up the face of the beam. The failure occurred at 12,000 lbs. load by a diagonal crack running from the support to the bent bar over that portion of the beam not having diagonal reinforcement. This failure was similar to that for Beam No. 3, already considered in detail. The deflection was, however, much greater in the present instance. This would account for the high steel stresses. For this beam $M = 470\ bd^2$.

Machine load, lbs.	Depth of neut. axis from comp'n face	Centre of comp'n to c. l. of steel k. depth	Steel stress lbs. per sq. in.	Steel force, lbs.	Steel moment lb. ins.	Load moment lb. ins.	% of load moment on concrete
2,500	3.7 ins.	$6.02 = 0.83d$	6,150	3,320	19,860	30,000	33.8
5,000	3.4	$6.12 = 0.845d$	19,500	10,520	64,300	60,000	?
7,000	3.4	$6.12 = 0.845d$	31,600	17,060	104,200	84,000	?
10,000	3.6	$6.05 = 0.835d$	50,850*	27,450*	163,800*	120,000	?

* Beyond yield point.

Beam No. 2. Johnson Beam—Concrete, 1-2-4. Beam, 6″ × 8″ × 6′ 0″ c. to c. Reinforcement, 3 — ½″ Johnson bars = 0.54 sq. in. One bar bent up at 24°, 8″ outside load point so as to come out of beam over support. Centre line of reinforcement ¾″ above base. % reinforcement = 1.25. Total weight of beam, 313 lbs. Steel, 12.2 lbs. Age, 29 days. Loaded at third points.

The results are shown in Plate II. The deformation curves are less concave than in the case of Beam No. 1, and are practically straight lines. At 8000 lbs. load the extensometers showed signs of creeping, the steel stress being 34,700 lbs. per sq. in. At 9500 lbs. a small diagonal crack was noted 2″ outside left load point, and half way down face; at 10,000 lbs. a similar crack 6″ outside right load point, and a crack across base 1″ deep 4″ outside right load point. At 11,000 lbs. several base cracks 1″ deep between load points, and under right load point. At 11,500 lbs. small diagonal cracks general at ends outside load points, but not developing on account of diagonal bar running as far as supports. At 12,270 lbs. beam failed by opening of vertical crack 4″ to left of centre and compression failure above. The deflection at 12,200 lbs. was 0.44″, and was increasing up to the maximum load. The extensometer on the steel line indicated a stress of 56,800 lbs. per sq. in. at load of 11,500 lbs. The steel was probably beyond yield point, as this stress is beyond that found by tests of the bars. The steel moments at the higher loads are also in excess of the load moments as was the case in Beam No. 1. The steel stress at failure being beyond the yield point would probably be in the neighbourhood of 50,000 lbs. per sq. in., corresponding to a tension force of 27,000 lbs. The area in compression at 11,500 lbs. was 22.2 sq. in., giving a maximum compression stress of $\frac{2 \times 27000}{22.2} = 2430$ lbs. per sq. in., which would agree with compression failure.

For this beam $M = 480\ bd^2$.

The diagonal reinforcement seemed to be more effective than that in Beams No. 1 and No. 2, in which it was at 45°. The diagonal cracks did not develop, and the deflection before failure was noticeably much greater.

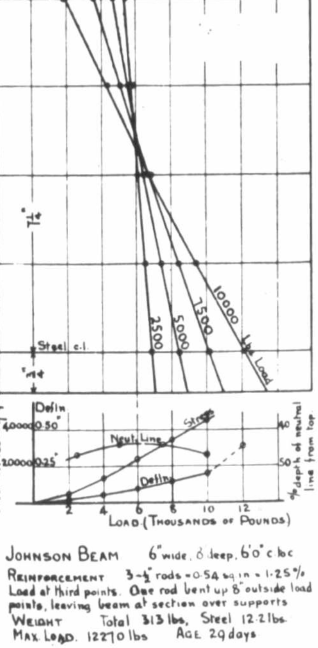


Plate II.

Machine load lbs.	Depth of neut. axis from comp'n face.	Centre of compn. to c. l. of steel = k. depth.	Steel stress lbs. per sq. in.	Steel force lbs.	Steel moment lb. ins.	Load moment lb. ins.	load moment on concrete.
2,500	3.40 ins.	$6.12 = 0.845d$	6,000	3,240	19,820	30,000	33.9
5,000	3.21	$6.18 = 0.853d$	18,720	10,100	62,400	60,000	?
7,500	3.21	$6.18 = 0.853d$	31,460	16,980	105,000	90,000	?
10,000	3.40	$6.12 = 0.845d$	46,400*	25,050*	153,100*	120,000	?

* Beyond or very near yield point.

Beam No. 3. Johnson Beam—Fully described (p. 20).

Beam No. 4. Johnson Beam—Concrete, 1-2-4. Beam, 8″ × 12″ × 10′ 0″ c. to c. Reinforcement, 4 — ½″ Johnson bars = 0.72 sq. in. Two bars bent up at 45° half way between load points and supports. Centre line of bars ¾″ above base. % reinforcement = 0.80. Total weight of beam, 1009 lbs. Steel, 25.5 lbs. Age, 55 days. Tested with tension face uppermost. Loaded at third points.

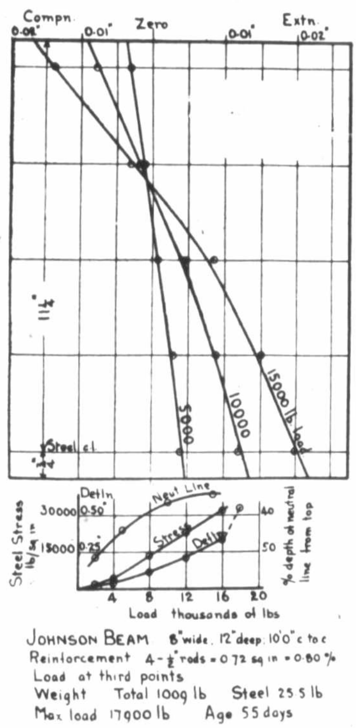


Plate IV.

The results are shown in Plate IV. The deformation curves are noticeably concave downwards. At 9000 lbs. load two base cracks appeared, one 6″ outside right load point, the other 6″ inside left load point. At 11,000 lbs. crack over right load point and also 4″ inside it. At 12,000 lbs. crack over left load point and 12″ to right of right load point. At 14,000 lbs. base crack 3″ to right of centre line; and at 15,000 lbs. base crack 18″ outside right load point.

First crack at 9000 lbs. now extends 1″ down side. At 16,000 lbs. base crack 8″ outside right load point. At 17,000 lbs. diagonal cracks opened at each end about half way down beam but closed as beam failed at 17,900 lbs., by opening of cracks between the load points, over left load point, 12″ to right of it and 6″ inside right load point. As the ends of the beam were further depressed the concrete crushed in compression above the cracks, but not simultaneously with the tension failure in the concrete. The yield point of the steel and the compressive strength of the concrete were not developed fully at the instant of failure.

The ultimate moment = 361 bd^2, the percentage reinforcement being less than in Beams 1, 2, and 3.

Machine load lbs.	Depth of neut. axis from comp'n face.	Centre of comp'n to c. l. of steel = k. depth.	Steel stress lbs. per sq. in.	Steel force lbs.	Steel moment lb. ins.	Load moment lb. ins.	% load moment on concrete.
5,000	4.95 ins.	$9.60 = 0.817d$	6,440	4,640	44,500	100,000	55.5
10,000	4.11 ,,	$9.88 = 0.842d$	18,730	13,500	133,400	200,000	33.3
15,000	3.96 ,,	$9.93 = 0.846d$	30,000	21,600	214,500	300,000	28.5

A fair proportion of load moment was carried by the concrete at 15,000 lbs. load, the tension failure in the concrete not occurring until near the maximum load of 17,900 lbs.

Beam No. 5. Johnson Beam—Concrete, 1-2-4. Beam, 8″ × 11¾″ × 10′ 0″ c. to c. Reinforcement, 5 — ½″ Johnson bars = 0.90 sq. in. Three bars bent up at 45°, one 8″ from load point, and two at 24″ from load point (see p. 5). Centre line of bars ¾″ above base. % reinforcement = 1.02. Total weight of beam, 1008.5 lbs. Steel, 31 lbs. Age, 56 days. Tested with tension face uppermost. Loaded at third points.

The results are shown in Plate V, the deformation curves being straight lines. At 6000 lbs. load numerous minute cracks appeared on surface of base. At 9000 lbs. crack 14″ to right of centre goes 1″ down side. At 10,000 lbs. crack at centre 3″ down side, and 9000 lb. crack also 3″ down. At 14,000 lbs. crack 3″ outside right load point runs 5″ down side. At 15,000 lbs. diagonal crack 15″ outside left load point extending over 3″ of depth but not running to base (shear crack). At 16,000 lbs. base crack joins diagonal crack, and new crack appears 12″ outside right load point on both faces. At 17,000 lbs. diagonal crack extends. Crack 3″ to right of centre runs down to neutral axis. At 20,000 lbs. crack at left load point 6″ down face, and one 2″ inside load point runs to within 2″ of compression face. Deflection then $0.44'' = \frac{\text{Span}}{262}$. Maximum load, 20,500 lbs. Cracks developed 3″ inside right load point, and concrete crushed above.

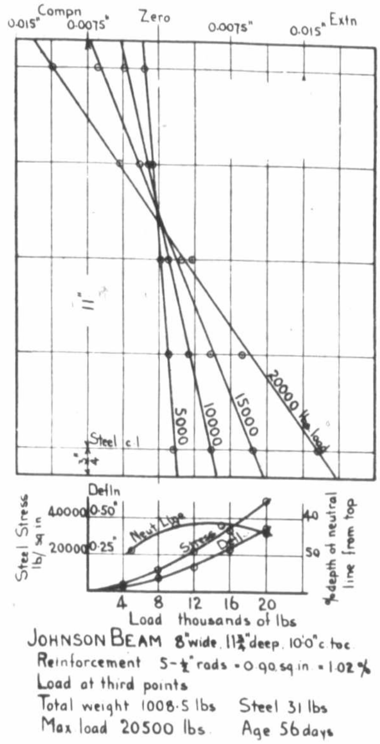


Plate V.

Machine load lbs.	Depth of neut. axis from comp'n face.	Centre of comp'n to c.l. of steel = k. depth.	Steel stress lb. per sq. in.	Steel force lbs.	Steel moment lbs. in.	Load moment lbs. in.	% load moment on concrete.
5,000	5.40 ins.	$9.2 = 0.837d$	4,800	4,320	39,700	100,000	60.3
10,000	4.59	$9.47 = 0.862d$	16,500	14,850	140,500	200,000	29.7
15,000	4.57	$9.48 = 0.863d$	30,000	27,000	256,000	300,000	14.7
20,000	4.80	$9.4 = 0.855d$	50,000*	45,000*	423,000*	400,000	0.0

* Beyond or very near yield point.

Ultimate moment $= 431\ bd^2$.

The total compression at 20,000 lbs. load was 45,000 lbs. Area in compression $= 8 \times 4.8 = 38.4$ sq. in. Maximum compression stress $= \frac{2 \times 45000}{38.4} = 2345$ lbs. per sq. in., a reasonable value in view of the crushing of the concrete at maximum load. The full value of the steel seems to have been developed, and concrete and steel reached ultimate and yield point stresses simultaneously.

Beam No. 6. Kahn Beam—Concrete, 1-2-4. Beam, 6″ × 8″ × 6′ 0″ c. to c. Reinforcement, 2 — ½″ Kahn bars = 0.76 sq. in. gross; 0.50 sq. in. net. Centre line of bars, ¾″ above base. % reinforcement = 1.15 (net). Total weight of beam, 314 lbs. Weight of steel, 17.8 lbs. Age, 28 days. Loaded at third points.

The results are shown in Plate VI. The deformation curves are slightly concave downwards, the concavity increasing as the load increases. At 6000 lbs. cracks across base under each load point, and 4″ to left of right load point. At 7000 lbs. crack at left load point runs 3½″ up side on back. At 7500 lbs. crack under right load point 1″ up front. At 8500 lbs. base cracks at centre and 4″ to right. At 10,000 lbs. crack under left load point runs 3½″ up face, and crack 7″ outside right load point across base and up sides 1″ on front, 3″ on back. At 11,000 lbs. small diagonal cracks traced for 1½″, at about 2″ above base, 7″ outside left load point. At 11,000 lbs. base cracks 6″ to left and 4″ to right of centre run 1″ up sides. At 12,000 lbs. (maximum load) crack 6″ to left of centre opened and the concrete then crushed above.

Ultimate moment $= 470\ bd^2$.

Machine load lbs.	Depth of neut. axis from comp'n face.	Centre of comp'n to c. l. of steel = k. depth.	Steel stress lb. per sq. in.	Steel force (net) lbs.	Steel moment (net) lb. in.	Load moment lb. in.	% load moment on concrete.
2,500	4.0 ins.	$5.92 = 0.817d$	4,500	2,250	13,320	30,000	55.6
5,000	3.3	$6.14 = 0.847d$	12,000	6,000	36,840	60,000	38.6
7,500	3.3	6.14 = ,,	20,210	10,105	62,000	90,000	31.0
10,000	3.3	6.14 = ,,	31,000	15,500	92,000	120,000	23.3

The above table shows that the steel moment figured on the net area was appreciably less than the load moment even at 10,000 lbs. load. The extensometers were removed from the beam at 11,000 lbs. load when tne steel stress was 35,700 lbs. per sq. in. It is therefore probable that at maximum load the stress in the steel was beyond 40,000 lbs. per sq. in. Tests showed a yield point of about 45,000 lbs. per sq. in. It may be noted that the wings of the bar were sheared according to the standard practice of the Kahn bar manufacturers, so that the wings covered a depth of 4¼ ins. The bars were ¾" above the base, and hence extended to about 3 ins. from the compression face. In Beams No. 10 and 11 the depth

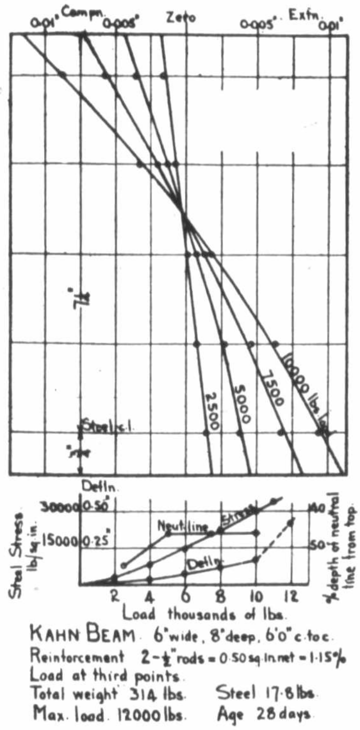


Plate VI.

was about 7", and a greater proportion of the depth was covered by the wings. The 8" beam may be considered by some to be rather deep for wings sheared as above, but the wings cover that portion of the depth over which the shearing stresses are greatest. The tension area, 0.50 sq. in., is slightly less than that in the Johnson beams Nos. 1, 2, and 3 of the same size, but allowing for the wings the weight of steel in the present beam is considerably in excess of that in the beams reinforced with the corrugated bar. The beam does not, however, carry any greater load. The bending of the Johnson bars diagonally in Beam No. 2 resulted in a concrete tension failure in the central third, as in the case of this beam, and the diagonal reinforcement seems to have been sufficient. The wing metal in the Kahn beam does not seem to produce advantages comparable with the amount present. The author wished to test the Kahn bar with the central third unsheared since there is no shearing force over that portion of the beam, but the bars were only obtainable sheared in the standard form. He believes that if the end bond was sufficient to develop the tensile value of the unsheared section over the central third, a much better result might have been obtained.

Beam No. 7. Kahn Beam—Concrete, 1-2-4. Beam, 6″ × 8″ × 6′ 0″ c. to c. Reinforcement, 2 — ½″ Kahn bars = 0.76 sq. in. gross; 0.50 sq. in. net. Centre of bars, ¾″ above base. % reinforcement = 1.15 net. Total weight of beam, 301 lbs. Weight of steel, 17.8 lbs. Age, 55 days. Loaded at third points.

This beam is similar to No. 6, but of twice the age. The results are shown in Plate VII. The curves are only very slightly concave, and the concavity is upwards. At 6000 lbs. cracks appeared across base 4″ to left, and 5″ to right of centre, and 1″ inside left load point. At 6500 lbs. the second crack noted at 6000 lbs. runs 2″ up back face. At 8000 lbs. a crack 2½″ inside right load point across part of base and 1″ up front. At 9000 lbs. cracks 3″ and 6″ outside right load point across part of base and 1″ up front. At 9500 lbs. crack across base at centre, also 6″ outside left load point. At 10,000 lbs. second crack noted at 6000 lbs. runs 3″ up face. At 10,500 lbs. diagonal crack extending over 3″ depth starts from base 8″ outside left load point, on back of beam. Cracks over central 12″ of beam run 2″ to 3″ up sides. At 12,950 lbs. (maximum load) these cracks 5″ either side of centre developed and concrete failed above.

Ultimate moment = 505 bd^2 against 470 bd^2 for Beam No. 6. This result is better than that for the Johnson Beams Nos. 1, 2, and 3, in which the tension area was nearly the same, but there is a very considerable weight of metal in the wings of the Kahn bar, and this must be remembered in making comparisons. (See p. 56.) The deflection at maximum load was 0.66″.

The maximum compressive stress in the concrete near failure was $\frac{24700 \times 2}{6 \times 3.7} = 2230$ lbs. per sq. in., a value in close agreement with its ultimate strength. The gradual transfer of moment from concrete to steel is shown in the table.

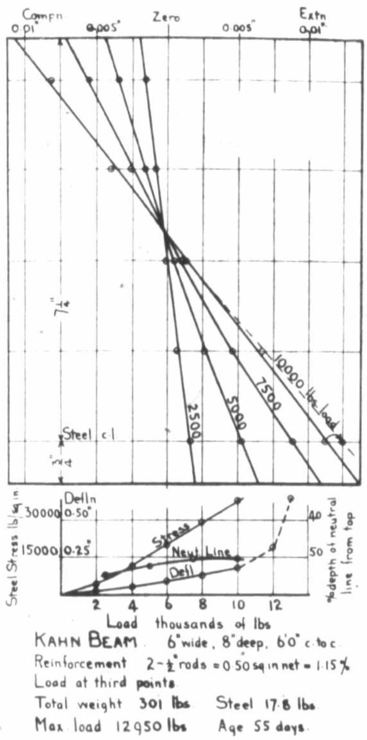


Plate VII.

Machine load lbs.	Depth of neut. axis from comp'n face.	Centre of comp'n to c. l. of steel = k. depth.	Steel stress lb. per sq. in.	Steel force (net) lbs.	Steel moment (net) lb. in.	Load moment lb. in.	% load moment on concrete.
2,500	4.0 ins.	$5.92 = 0.817d$	4,600	2,300	13,600	30,000	54.7
5,000	3.81	$5.98 = 0.826d$	15,000	7,500	44,800	60,000	25.3
7,500	3.66	$6.03 = 0.832d$	26,200	13,100	78,800	90,000	12.4
10,000	3.66	$6.03 = 0.832d$	37,450	18,725	112,900	120,000	6.0
11,500	3.70	$6.02 = 0.832d$	49,400*	24,700*	148,600*	138,000	0.0

* Beyond yield point.

Beam No. 8. Kahn Beam—Concrete, 1-2-4. Beam, 6″ × 8″ × 6′ 0″ c. to c. Reinforcement, 1 — ¾″ Kahn bar = 0.78 sq. in. gross;

0.56 sq. in. net. %, reinforcement = 1.3. Centre line of bar ¾" above base. Weight of beam, 303 lbs. Weight of steel, 17.0 lbs. Age, 28 days. Loaded at third points.

The percentage net reinforcement is practically the same as in Johnson beams Nos. 1, 2, and 3, and rather greater than in Kahn beams Nos. 6 and 7. Owing to the heavy wings there is

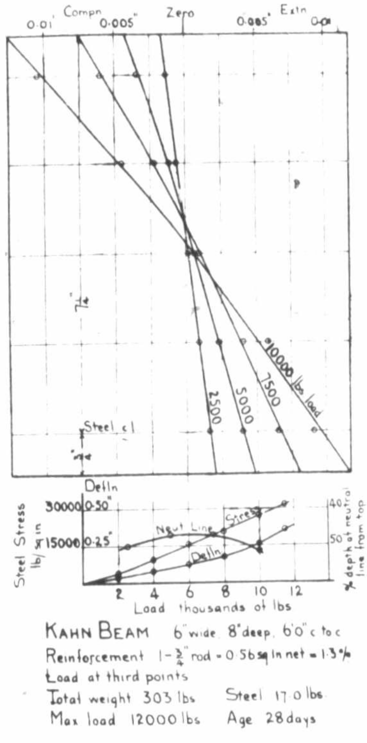


Plate VIII.

much more steel in the beam than in the Johnson beams 1, 2 and 3. A single bar would scarcely be expected to distribute the stresses through the concrete as effectively as a number of smaller bars, but the depth of wings was sufficient to cause them to project a little above the beam surface.

The results are shown in Plate VIII. The deformation curves are slightly concave downwards. At 6000 lbs. load base crack 3" inside right load point, and 2″ outside left load point, developing 1″ up sides at 6500 lbs. At 7500 lbs. base crack 10″ outside left load point. At 9000 lbs. crack 5″ to right of centre across base. Previous cracks developing. At 9500 lbs. crack across middle of base and 4″ inside left load point. At 10,000 lbs. base crack 3″ to left of centre. Base cracks outside load points running a little diagonally. At 11,000 lbs. diagonal crack half way down beam 6″ outside left load point, not extending to base. Beam failed at 12,000 lbs. by development of diagonal crack noted at 11,000 lbs. This ran down to base and crack extended across base 9″ outside load point, the concrete along base being split along the rod over lengths of 10″ towards the support and 6″ towards load point. The deflection at 11,500 lbs. was 0.36″. The yield point of the steel was not reached, and the indications were that the single rod did not distribute the stress altogether satisfactorily. The load carried was practically the same as for the other 8″ beams already noted.

Ultimate moment $= 470\, bd^2$.

Machine load lbs.	Depth of neut. axis from comp'n face.	Centre of comp'n to c. l. of steel = k. depth.	Steel stress lb. per sq. in.	Steel force (net) lbs.	Steel moment (net) lb. ins.	Load moment lb. ins.	% load moment on concrete.
2,500	3.67 ins.	6.03 = 0.832d	5,200	2,910	17,550	30,000	41.5
5,000	3.40	6.12 = 0.845d	12,080	6,760	41,360	60,000	31.0
7,500	3.45	6.10 = 0.842d	19,680	11,020	67,200	90,000	25.3
10,000	3.75	6.00 = 0.828d	29,050	16,270	97,500	120,000	18.7

The maximum compressive stress in concrete at 11,500 lbs. load calculated from steel tension only was 1580 lbs. per sq. in., a value well within its ultimate strength.

Beam No. 9. Kahn Beam—Concrete, 1-2-4. Beam, 6″ × 8″ × 6′ 0″ c. to c. Reinforcement, 1 — ¾″ Kahn bar = 0.78 sq. in. gross; 0.56 sq. in. net. Centre of bar ¾″ above base. % reinforcement = 1.3. Weight of beam, 3.15 lbs. Weight of steel, 17.0 lbs. Age, 55 days. Loaded at third points.

This beam is similar to No. 8 but twice the age. The results are shown in Plate IX, the deformation curves being either straight lines or slightly concave upwards. The maximum load was 14,000 lbs. against 12,000 lbs. in the case of Beam No. 8, the failure being by opening of a vertical crack between the load points and the crushing of the concrete above. The yield point of the steel was passed, the stress at 13,500 lbs. load being about 47,000 lbs. per sq. in. assuming it to be proportional to deflection. The greatest recorded yield point stress during tests on the bars was 41,200 lbs. per sq. in.

At 4000 lbs. load cracks across base 1″ inside right load point, and 4″ inside left load point. Former runs 1″ up side at 4500 lbs., and latter $2\frac{1}{2}$″ up on back at 6000 lbs. At 6500 lbs. crack 3″ inside

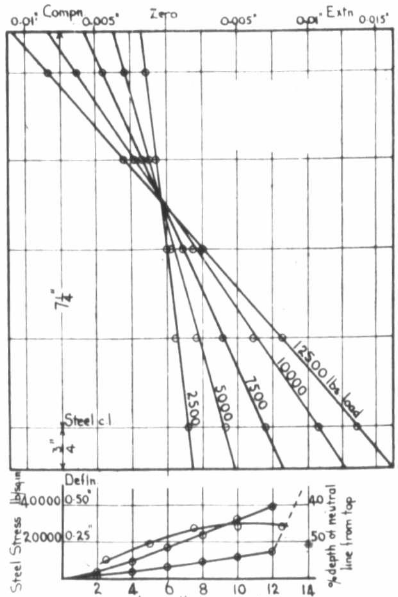


Plate IX.

right load point 3″ up side. At 7500 lbs. base crack 2″ to right of centre, and base cracks general outside load points at 8500 lbs. At 9000 lbs. crack 3″ to left of right load point runs 5″ up side. At 9500 lbs. crack 3″ to 4″ outside left load point runs vertically for 2″, then diagonally towards load point, about 4″ deep. At 12,000 lbs. crack 9″ to right of right load point goes 3″ deep vertically. At 12,500 lbs. crack 10″ to left of left load point on back runs 2″

vertically and then diagonally. At 14,000 lbs. vertical crack $2\frac{1}{2}''$ inside right load point opened, and concrete crushed above.

Ultimate moment $= 547\ bd^2$. This is a better result than for beam No. 8, the steel stress being much better developed.

Machine load lbs.	Depth of neut. axis from comp'n face.	Centre of comp'n to c. l. of steel k. depth.	Steel stress lb. per sq. in.	Steel force (net) lbs.	Steel moment (net) lb. ins.	Load moment lb. ins.	load moment on concrete.
2,500	4.1 ins.	$5.88 = 0.812d$	4,500	2,520	14,820	30,000	50.6
5,000	3.66	$6.03 = 0.833d$	12,600	7,050	42,500	60,000	29.1
7,500	3.36	$6.13 = 0.846d$	21,280	11,920	73,000	90,000	18.9
10,000	3.30	$6.15 = 0.848d$	30,480	17,080	105,000	120,000	12.5
12,500	3.24	$6.17 = 0.852d$	40,850*	22,900	141,300	150,000	5.8

* Very near yield point.

The estimated compression stress in concrete at 12,500 lbs. load was $\frac{2 \times 22900}{6 \times 3.24} = 2360$ lbs. per sq. in., a value in close agreement with its ultimate strength.

The deflection at maximum load was 0.713″.

Beam No. 10. Kahn Beam—Concrete, 1-2-4. Beam, $6'' \times 7\frac{1}{2}'' \times 6'\ 0''$ c. to c. Reinforcement, $2 - \frac{1}{2}''$ Kahn bars $= 0.76$ sq. in. gross; 0.50 sq. in. net. Centre of bars, $\frac{3}{4}''$ above base. % reinforcement 1.24 (net). Weight of beam, 292 lbs. Weight of steel, 17.8 lbs., Age, 28 days. Loaded at third points.

The results are shown in Plate X. The deformation curves are remarkable for the concavity shown on the tension side. In no other case was such a form of curve obtained. It appears to be due to small readings at the steel line, as the other four points might well lie upon a very flat curve. There was no reason to doubt the readings, however, and the extensometer at the time of test was apparently in proper working order. The curve of deformation at 2500 lbs. load is quite flat.

At 5000 lbs. base cracks 1″ outside both load points, under left load point and 7″ to right of centre. At 7000 lbs. base crack 2″ to right of centre, over half base and 1″ deep. Crack on base 8″ to left of left load point. At 8000 lbs. vertical cracks 2″ and 9″ outside left load point run in diagonal direction towards centre. At 9000 lbs. base crack under right load point and also $5\frac{1}{2}''$ to right of centre. Extensometers removed at 9500 lbs., and beam failed at 10,000 lbs. Cracks 4″ and 7″ inside left load point opened vertically and concrete crushed above. Accepting the curves as shown in the plate, the maximum steel stress does not seem to have been up to yield point. Ultimate moment $= 452\ bd^2$.

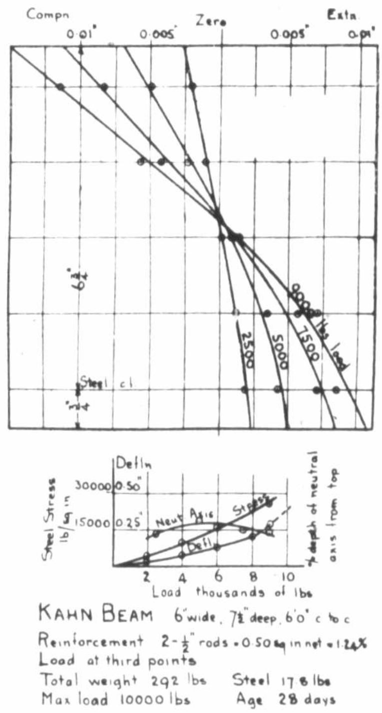


Plate X.

Machine load lbs.	Depth of neut. axis from comp'n face.	Centre of comp'n to c. l. of steel = k. depth.	Steel stress lb. per sq. in.	Steel force (net) lbs.	Steel moment (net) lb. ins.	Load moment lb. ins.	% load moment on concrete.
2,500	3.45 ins.	5.6 $=0.83d$	5,900	2,950	16,500	30,000	45.0
5,000	3.21	5.68 $=0.84d$	13,720	6,860	38,950	60,000	35.0
7,500	3.36	5.63 $=0.835d$	22,300	11,150	62,700	90,000	21.4
9,000	3.42	5.61 $=0.832d$	27,300	13,650	76,500	108,000	*29.2

* The small steel moment at 9000 lbs. load, and the check in the diminution of the percentage of load moment carried on concrete, seems to indicate some probable error in the reading of the extensometers on the tension side. No source of error could be detected, however, during the test, but the results, differing as they do from those for all other beams, render highly probable the existence of some disturbing factor.

Beam No. 11. Kahn Beam—Concrete, 1-2-4. Beam, 6″ × 7.15″ × 6′ 0″ c. to c. Reinforcement, 2 — ½″ Kahn bars = 0.76 sq. in. gross; 0.50 sq. in. net. Centre of bars, ¾″ above base. % reinforcement, 1.3 net. Weight of beam, 282.5 lbs. Weight of steel, 17.8 lbs. Age, 56 days. Loaded at third points.

This beam is almost the same as No. 10, being slightly shallower,

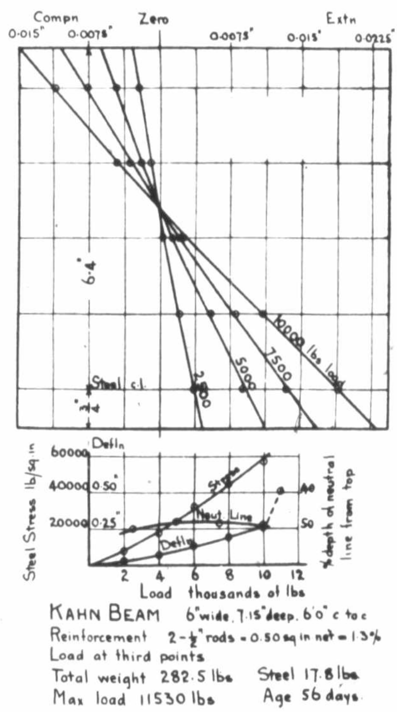


Plate XI.

and therefore having a little more reinforcement. The results are shown in Plate XI. The deformation curves are very nearly straight lines, and no such abnormal results were obtained as in Beam No. 10.

At 5000 lbs. base cracks appeared 2″ inside, and just outside right load point, 4″ to left of centre, under left load point, and 7″ outside it. Fine hair-like lines had been noted at 3500 lbs. At 5500 lbs. two cracks about 7″ outside left load point. At 6000 lbs.

crack $8\frac{1}{2}''$ to left of centre runs 2″ up side, and at 6500 lbs. crack under right load point runs 2″ up side. All initial base cracks developing. At 7000 lbs. base crack $5\frac{1}{2}''$ to right of centre, turning towards centre and reaching 4″ up face at 10,500 lbs. At 9000 lbs. diagonal crack on face 9″ outside right load point. At 11,200 lbs. compression failure started over the crack noted at 7000 lbs. 5″ to right of centre, the deflection being then 0.514″. Failure resulted at 11,530 lbs. by development of these conditions, the deflection being then 1.164″. The diagonal crack noted at 9000 lbs. was closed after the beam failed.

Ultimate moment $= 580\ bd^2$.

Machine load lbs.	Depth of neut. axis from comp'n face.	Centre of comp'n to c. l. of steel = k. depth.	Steel stress lb. per sq. in.	Steel force (net) lbs.	Steel moment lb. ins.	Load moment lb. ins.	% load moment on concrete.
2,500	3.21 ins.	5.33 = 0.832d	11,100	5,550	29,600	30,000	0.0
5,000	3.06	5.38 = 0.842d	27,000	13,500	72,500	60,000	..
7,500	3.09	5.37 = 0.84d	40,400	20,200	108,400	90,000	..
10,000	3.12	5.36 = 0.838d	57,000*	28,500*	152,500*	120,000	..

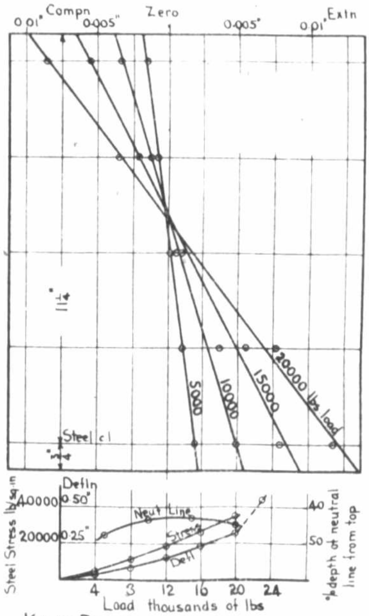
* Beyond yield point.

It will be seen that the steel stresses, when assumed to be proportional to deformation, give a value well beyond the yield point at a load of 10,000 lbs., and that the load moment so figured is in excess of the actual moment. At loads of 5000 lbs. and 7500 lbs the calculated steel moment is 20% in excess of the load moment. The concrete compressive stress at 10,000 lbs. estimated from the figures in the table would be $\frac{28500 \times 2}{6 \times 3.12} = 3040$ lbs. per sq. in., a value in agreement with the compression failure. There is no doubt but that the full values of concrete and steel were developed in this test.

Beam No. 12. Kahn Beam—Concrete, 1-2-4. Beam, 8″ × 12″ × 10′ 0″ c. to c. Reinforcement, 2 — $\frac{3}{4}''$ Kahn bars = 1.56 sq. in gross; 1.12 sq. in. net. Centre line of reinforcement, $\frac{3}{4}''$ above base. % reinforcement = 1.25 (net). Weight of beam (not recorded), about 1050 lbs. Weight of steel, 56.5 lbs. Age, 28 days. Loaded at third points, tension face uppermost.

The results shown in Plate XII indicate straight line deformation diagrams. At 9000 lbs. several small base cracks inside and outside load points, one 10″ inside left load point across base. At

11,000 lbs. crack 1″ inside right load point runs 2″ up face, and cracks noted at 9000 lbs. develop. At 15,000 lbs. crack 8″ to right of centre runs across base. At 17,000 lbs. a crack over left load point runs half way through depth. At 18,000 lbs. crack across base 2″ to left of centre. At 22,000 lbs. considerable movement on com-



KAHN BEAM 8″ wide, 12″ deep, 10′0″ c to c
Reinforcement 2-¾″ rods = 1.12 sq in net = 1.25 %
Load at third points
Total weight 1050 lbs Steel 56.5 lbs
Max load 23600 lbs Age 28 days

Plate XII.

pression extensometers foreshadowing failure. Instruments removed at 23,000 lbs. load. Failure at 23,600 lbs. by crushing of concrete over a length of 10″ above a crack starting 3″ inside left load point, and also above a crack 6″ inside right load point. The deflection just before failure was 0.95″.

Ultimate moment = 475 bd^2.

Machine load lbs.	Depth of neut. axis from comp'n face.	Centre of comp'n to c. l. of steel = k. depth	Steel stress lb. per sq. in.	Steel force (net) lbs.	Steel moment (net) lb. ins.	Load moment lb. ins.	% load moment on concrete.
5,000	5.4 ins.	9.45 = 0.842d	6,290	7,030	66,400	100,000	33.6
10,000	4.9	9.62 = 0.856d	15,300	17,110	164,500	200,000	17.8
15,000	4.83	9.64 = 0.858d	25,700	28,740	277,000	300,000	7.7
20,000	5.1	9.55 = 0.850d	36,200	40,450	386,000	400,000	3.5

The steel stress at 22,000 lbs. load was about 40,000 lbs. per sq. in., a value very close to the yield point of the bars. The corresponding maximum compressive stress would be about 2200 lbs. per sq. in., in agreement with compression failure.

Beam No. 13. Kahn Beam—Concrete, 1-2-4. Beam, 8″ × 11¾″ × 10′ 0″ c. to c. Reinforcement, 2-¾″ Kahn bars = 1.56 sq. in. gross; 1.12 sq. in. net. Centre of bars, ¾″ above base. % reinforcement = 1.27 net. Weight of beam, 1043 lbs. Weight of steel, 56.5 lbs. Age, 56 days. Loaded at third points; tension face uppermost.

The curves shown in Plate XIII indicate that the deformation was linear. The beam is slightly shallower than No. 12, being 11¾″ deep against 12″. The maximum load carried was 25,100 lbs. against 23,600 lbs. in the case of No. 12, the respective ages being 56 and 28 days. At 7000 lbs. incipient cracks on base visible when wet, but not open so far as eye could see. At 8000 lbs. base cracks 5″ inside and 8″ outside left load point, and 4″ outside right load point extending a little down side. At 9000 lbs. crack 7″ inside right load point 2″ down side; 5″ inside left load point 1″ down, and over right load point ½″ down. At 11,000 lbs. base cracks over left load point and 6″ to left of centre. At 13,000 lbs. base crack 11″ inside left load point running 4″ down side. At 17,000 lbs. crack over right load point 4″ down, and cracks at left end 2″ to 4″ down. At 21,000 lbs. all vertical cracks developing, and at 22,000 lbs. extensometers indicated approaching failure. At 23,000 lbs. crack 8″ inside right load point opened vertically down 7″, compression failure commenced above it at 24,700 lbs., and the maximum load was 25,100 lbs. Compression failure also commenced inside left load point. The deflection at 25,000 lbs. was 0.49″.

Ultimate moment = 526 bd^2, against 475 bd^2 for beam No. 12.

Machine load lbs.	Depth of neut axis from comp'n face.	Centre of comp'n to c. l. of steel k depth.	Steel stress lb. per sq. in.	Steel force (net) lbs. *	Steel moment (net) lb. ins.	Load moment lb. ins.	% load moment on concrete.
5,000	5.58 ins.	9.14 = 0.83d	4,800	5,370	49,100	100,000	50.9
10,000	5.07	9.31 0.847d	13,950	15,610	145,400	200,000	27.3
15,000	5.18	9.27 = 0.843a	23,820	26,650	247,000	300,000	17.7
20,000	5.28	9.24 0.84d	34,420	38,500	355,500	400,000	11.1

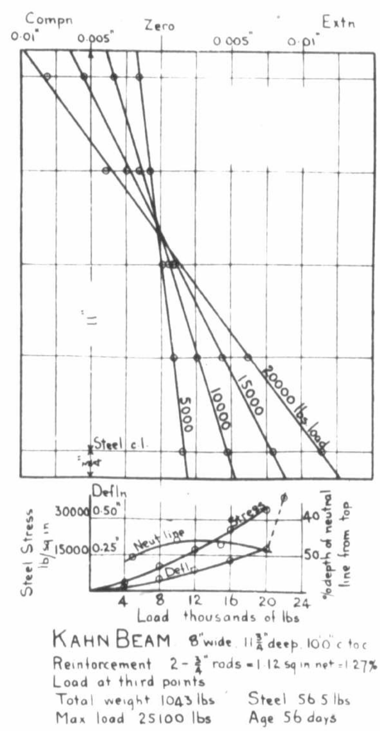


Plate XIII.

The steel stress at 23,000 lbs. load was 39,000 lbs. per sq. in., corresponding to a maximum compression in the concrete of about 2070 lbs. per sq. in., indicating that yield point and ultimate stress in the concrete were reached almost simultaneously.

Beam No. 14. Ransome Beam—Concrete, 1-2-4. Beam, $6\frac{3}{4}'' \times 9\frac{1}{2}'' \times 6' \, 0''$ c. to c. Reinforcement, $3 - \frac{1}{2}''$ Ransome twisted bars = 0.75 sq. in. One bar bent up 8″ outside load point so as to come out of beam just over support and give diagonal reinforcement over whole length outside the bend. Centre of bars, $\frac{3}{4}''$ above base. % reinforcement = 1.27. Weight of beam, 431 lbs. Weight of steel, 17.5 lbs. Age, 29 days. Loaded at third points.

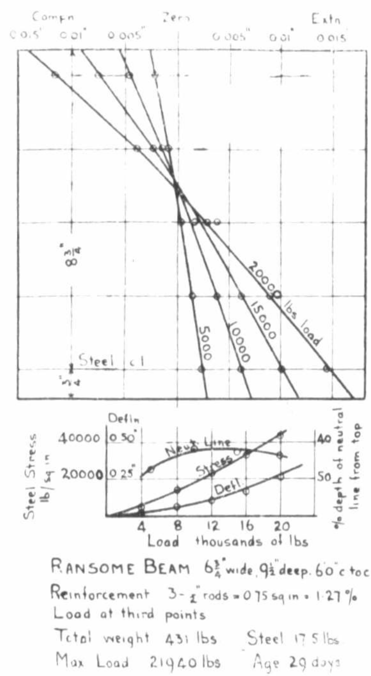


Plate XIV.

The beam was rather short as compared with its depth, and diagonal end failure occurred. The results are given in Plate XIV, the deformation curves being slightly concave downwards. At 11,000 lbs. general cracking over base; crack 3″ outside right load point runs $\frac{3}{4}''$ up face. At 13,000 lbs. crack across base 2″ outside left load point. This, and the one noted at 11,000 lbs., run up face and turn diagonally towards the centre. At 15,000 lbs. fine base cracks every few inches. At 16,000 lbs. crack 6″ outside left load point runs diagonally to about half depth of beam, and almost across base. Two similar cracks 17″ and 7″ outside right load point. At 17,000 lbs. several small diagonal cracks at right end. At 18,000 lbs. diagonal crack 18″ outside left load point. Diagonal cracks developed at 19,000 lbs. running to within 2″ of top at each end. Load well sustained. Diagonal failure occurred at left end at load of 21,940 lbs. Concrete cracked on base along reinforcing line towards load point. Rod (diagonal) pulled in at top and concrete split longitudinally, the crack turning sideways towards load point and joining the diagonal crack on the side. The deflection at maximum load was 0.395″.

Ultimate moment $= 518\ bd^2$.

Machine load lbs.	Depth of neut. axis from comp'n face.	Centre of comp'n to c. l. of steel = k. depth.	Steel stress lb. per sq. in.	Steel force lbs.	Steel moment lb. ins.	Load moment lb. ins.	% load moment on concrete.
5,000	4.15 ins.	$7.37 = 0.843d$	7,040	5,280	38,900	60,000	35.1
10,000	3.65	$7.53 = 0.865d$	18,700	14,030	105,100	120,000	12.4
15,000	3.75	$7.50 = 0.860d$	30,000	22,500	168,800	180,000	6.2
20,000	3.81	$7.48 = 0.855d$	43,120	32,400	242,000	240,000	0.0

The table above shows that the high yield point of the steel (about 78,000 lbs. per sq. in. See p. 4) was not reached. The maximum compressive stress in the concrete at 20,000 lbs. load, assuming linear variation, $= \frac{2 \times 32400}{6.75 \times 3.81} = 2520$ lbs. per sq. in. There was no sign of compression failure, apart from slow creeping of the compression extensometer, and this value is higher than that recorded in any of the previous tests.

Beam No. 15. Ransome Beam—Concrete, 1-2-4. Beam 6″ × 8″ × 6′ 0″ c. to c. Reinforcement, 3 — ½″ Ransome twisted bars = 0.75 sq. in. Centre of bars ¾″ above base. One bar bent up 8″ outside load point to come out over support giving diagonal reinforcement between bend and support. % reinforcement = 1.72. Weight of beam, 318 lbs. Weight of steel, 16 lbs. Age, 29 days. Loaded at third points.

This beam is heavily reinforced, and it was not expected that the full tension value of the high yield point steel would be developed before failure. The test was made principally as a contrast with the other 6″ × 8″ beams the percentage reinforcement of which averaged about 1.25. The results are shown in Plate XV, the deformation curves being practically straight lines. The beam was accidentally upset in handling before testing, and a crack 2″

inside the right load point ran vertically through the beam to about 2″ above the base. A considerable portion of the tension value of the concrete was therefore lost, and the steel took up the greater proportion of the load moment from the beginning of the test. At 4500 lbs. the above crack extended to the base and half across it at the back. At 5500 lbs. base crack 3″ inside left load point at back,

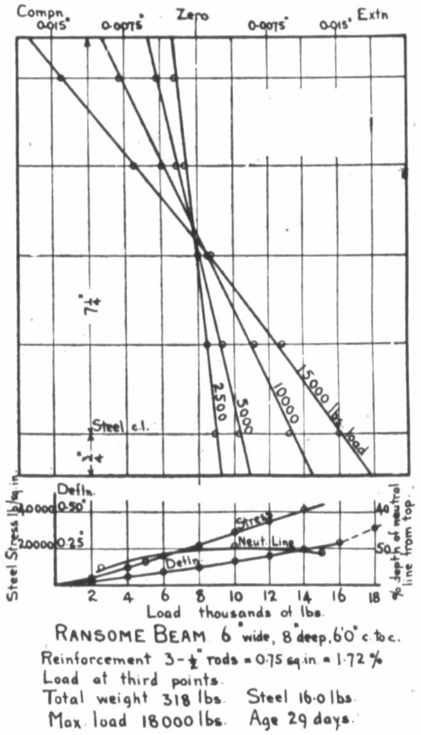


Plate XV.

and at 6000 lbs. crack 4″ to left of centre, both running 1″ up side. Also a crack 2″ to right of centre 4″ up back and across base. At 7500 lbs. initial crack extends across base, the opening on the compression side gradually closing, until at 8500 lbs. the line was visible but the crack apparently closed. At 9000 lbs. a crack 6″ outside right load point, 3″ up back. At 11,000 lbs. crack 16″ outside right load point runs 2″ up side and then diagonally. Also similar crack 6″ outside left load point. At 12,500 lbs. the diagonal crack at right end runs past rod. At 13,500 lbs. the diagonal cracks run to more than half depth, and the crack at 9000 lbs. is developing. The above cracks extended, especially the diagonal cracking at right end, and at 18,000 lbs. (maximum load) the main crack ran from 8″ inside support to a point 3″ below load point, several minor diagonal cracks covering the neighbouring surface below it. The concrete crushed over a length of 8″ inside the load point, as the beam deflected at failure, the rods pulling into the beam both at the right end base, and on top over the support. Failure was due to the diagonal cracking at the end. The deflection at the moment of failure was 0.392″.

Ultimate moment $= 700\ bd^2$, the high value of the co-efficient corresponding with the heavy reinforcement.

Machine load lbs.	Depth of neut. axis from comp'n face.	Centre of comp'n to c. l. of steel = k. depth	Steel stress lb. per sq. in.	Steel force lbs.	Steel moment lb. ins.	Load moment lb. ins.	% load moment on concrete.
5,000	3.9 ins.	$5.95 = 0.821d$	12,900	9,670	57,500	60,000	4.1
10,000	3.6	$6.05 = 0.835d$	29,220	21,930	132,700	120,000	0.0
15,000	3.7	$6.02 = 0.831d$	45,200	33,920	204,000	180,000	—

The beam behaved abnormally owing to initial cracking, and practically the whole tension force came on the steel at an early stage of the loading. At 15,000 lbs. the steel moment is in excess of load moment, but a close agreement can scarcely be expected in a beam initially cracked.

Tests of Beams and Cubes Roasted in Furnace.

Four beams and four cubes were placed in a furnace of the metallurgical laboratories of the University, and gradually heated from the ordinary room temperature to 1250° F., the record being as shown:

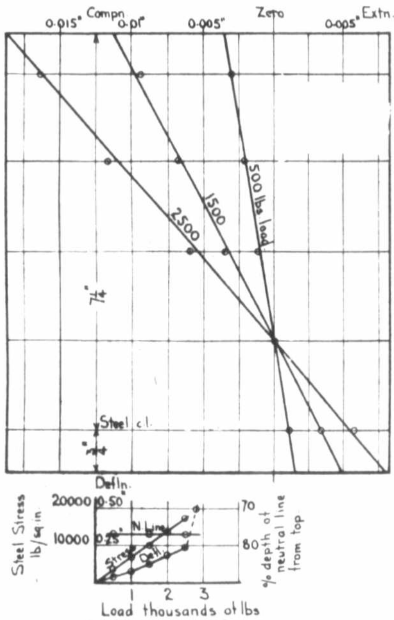
Time	8.30 a.m.	11 a.m.	Noon.	1 p.m.	2 p.m.	2.40 p.m.	3 p.m.
Temp. °F.	60	750	1090	1150	1170	1210	1245

The furnace was then allowed to cool slowly and the beams were removed on the following day when the furnace could be entered. The beams were supported on firebricks so as to be inclined upwards from the fire end of the furnace. The gases were thus equally distriouted on their way to the flue. The reinforcement was uppermost and the firebrick supports about 2 ft. from the ends of the beams, which were, for test, on 6′ 0″ centres. On removal from the furnace it was evident that there was initial strain in the beams for they were appreciably cambered, so as to be concave on the reinforcement side. The amount of this camber, and the general appearance of each beam, is given below with the summary of tests.

Beam No. 16. Kahn Beam—Concrete, 1-2-4. Beam, 6″ × 8″ × 6′ 0″ c. to c. Reinforcement, 1 — ¾″ Kahn bar = 0.78 sq. in. gross; 0.56 sq. in net. Centre of bar, ¾″ above base. % reinforcement = 1.3 (net). Weight of beam, 283 lbs. Age when roasted, 55 days. Loaded at third points.

Owing to an oversight, the beams were not weighed before roasting. The weights of the corresponding beams, Nos. 8 and 9, were 303 and 315 lbs.—average, 309. The loss of weight was probably about 25 lbs. The general appearance of the beam was fair. One end was rather damaged, and was patched with plaster of paris to give a good bearing. The diagonal wings projected through the surface, which was cracked round the projections, the wings being slightly loose. Cracks ran from the compression face downwards, as follows: 4½″ to left of right load point 4″ down; 13″ to right of same load point 3″ down; 4″ to left of left load point 3″ down; 7″ to left of it, fine crack traceable to middle of depth. Small cracks 3″ and 8″ inside left load point on back. Minor cracks within 6″ of right support. Crack along central base longitudinally from damaged end inwards for 15″. Indications in places of longitudinal cracks along sides on line of reinforcement, but cracks only slight. The camber at the centre was 0.17″.

The behaviour of the beam is shown in Plate XVI. The deformation curves are practically straight lines, and the neutral surface is very close to the reinforcement line, a point which will be referred to when describing tests on the concrete cubes which were similarly roasted. When the load reached 2000 lbs. (increments of 500 lbs.) considerable cracking was heard, being due to the crushing of the very dry concrete. The extensometers moved in a jerky manner, about ½ m.m. at a time, and the indicator of the Emery machine made a corresponding movement, showing how sensitive the machine is to any slight fluctuations in the condition of the loaded beam. At 2000 lbs. the deflection was 0.17″, indicating that the camber had disappeared. When the load reached 3000 lbs. the initial crack along the base from the damaged end extended to 4″ from the load point and then turned to the side, up which it extended. The concrete had by this time closed tightly round the wings which projected through the surface, and although a diagonal crack opened at the damaged end at 3000 lbs., the load increased to 3200 lbs. before the concrete fell away from the bar at the damaged end and the failure took place. The deflection was then 0.99″ from the cambered position. The neutral axis was about 5.6″



KAHN BEAM (roasted) 6″ wide, 8″ deep, 6′0″ c to c
Reinforcement 1 - ¾″ rod = 0.56 sq in net = 1.3%
Load at third points Age 55 days
M... load 3200 lbs Compare Plates VIII and IX

Plate XVI.

from the compression face at all stages, giving 5.38″ as the distance of the centre of compression from centre line of steel. The steel stress at 2500 lbs. was 17,300 lbs. per sq. in. Hence the steel moment = 17,300 × 0.56 × 5.38 = 52,100 lb. ins. The load moment = 30,000 lb. ins. Such a calculation can have little value, as the initial straining actions are probably somewhat complex, and the beam was damaged quite appreciably in places. It is probable that the projecting wings of the bar conducted a considerable amount of heat to the interior of the beam and to the tension rod, and that in consequence the internal conditions were more severe than in the case of the beams in which the rods were completely embedded.

Beam No. 17. Kahn Beam—Concrete, 1-2-4. Beam, 6″ × 8″ × 6′ 0″ c. to c. Reinforcement, 2 — ½″ Kahn bars = 0.76 sq. in. gross; 0.50 sq. in. net. Centre of bars, ¾″ above base. % reinforcement = 1.15 (net). Weight of beam, 289 lbs. Estimated loss of weight by

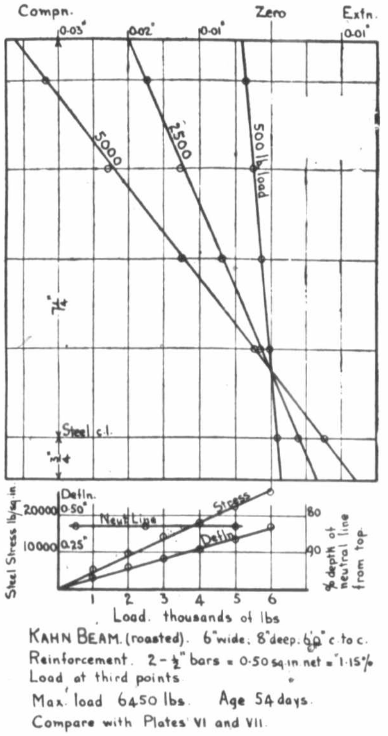


Plate XVII.

roasting, 18 lbs. Age when roasted, 54 days. Loaded at third points.

The results are given in Plate XVII, the deformation curves being straight lines, and the neutral surface close to the steel reinforcement as in Beam No. 16. The beam was in good condition after roasting. Outside the left load point the following cracks were noted on the compression side: 3″ from support, 2″ down side; at 2″, 6″, and 15″ from load point 1″ down side; at 6″ inside load point small crack 1½" across top. At right end a crack 3" from support runs 2" to 3" across top and 2" down side. Camber at centre, 0.08".

At 4000 lbs. crack on end face, starts 4" from support at about 2" above base and runs diagonally to about 4" from top. This crack showed on both sides at 4500 lbs., and also a similar one 2" from it. At 5000 lbs. the first of the above cracks extends to the support. Failure occurred at load 6450 lbs. by development of above, the crack extending from the support to the load point. Deflection at instant of failure = 0.441". A small diagonal crack showed at other end at 4500 lbs., but did not develop. At 2500 lbs. load the steel stress = 12,000 lbs. per sq. in; steel moment (net) = 31,500 lb. ins.; load moment = 30,000 lb. ins. At 5000 lbs. load steel stress = 22,500 lbs. per sq. in.; steel moment (net) = 59,000 lb. ins.; load moment 60,000 lb. ins. These results are in much better agreement than in the case of No. 16. The rods were much better protected from the furnace heat, being completely embedded.

The maximum load of 6450 lbs. is about half that carried by the corresponding unroasted beam No. 7, viz., 12,950 lbs. The camber was removed when the load reached about 1100 lbs.

Beam No. 18. Kahn Beam—Concrete, 1-2-4. Beam, 6" × 7" × 6' 0" c. to c. Reinforcement, 2 — ½" Kahn bars = 0.76 sq. in. gross; 0.50 sq. in. net. Centre of bars, ¾" above base. % reinforcement = 1.33 net. Weight of beam, 255 lbs. Age when roasted, 54 days. Loaded at third points.

The beam was in good condition after roasting. There was one notable crack 7" to left of left load point on compression face running half-way down beam on one side and to within 2" of base on other side. Also a crack on compression face 3" outside right load point running 3" down the side and about 1" across top. The camber was 0.08".

The results shown in Plate XVIII indicate that the deformations were not quite according to the linear law, the curves being concave upwards. The neutral surface is close to the steel as in Beams Nos. 16 and 17. At 4000 lbs. small base cracks appeared, and at 5000 lbs. a crack 1" inside left load point extended across the base. At 6000 lbs. several small diagonal cracks showing on both sides of beam to left of left load point. These developed gradually, and failure resulted from this cause at 7000 lbs. load, the deflection being then 0.73" from the cambered position. The initial camber was removed when the load reached about 700 lbs.

At 5000 lbs. steel stress = 12,200 lbs. per sq. in.; steel moment (net) = 26,400 lb. ins; load moment = 60,000 lb. ins. This is a very rough estimate, as the points on the deformation curves are not so regular as in the case of Beam No. 17, in which a much better agreement was obtained. If the actual reading of the steel extensometer at 5000 lbs. is taken, instead of the point from the smoothed curve the steel moment would be 32,000 lb. ins., a value still far from agreement.

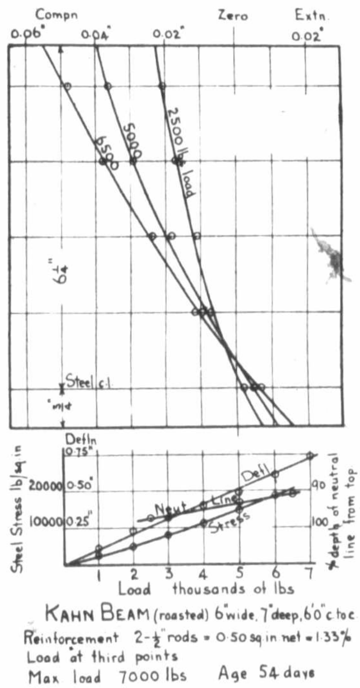
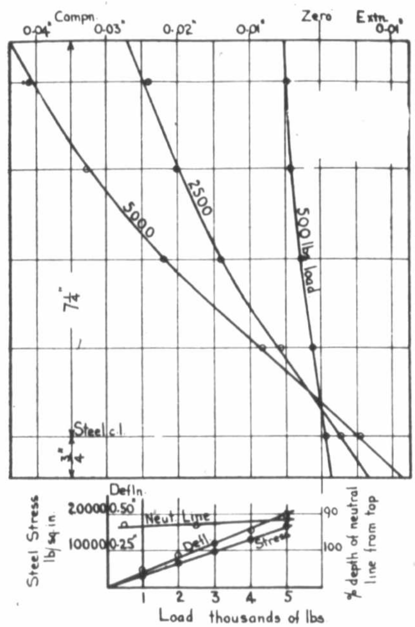


Plate XVIII.

Beam No. 19. Johnson Beam—Concrete, 1-2-4. Beam, 6" × 8" × 6' 0" c. to c. Reinforcement, 3 — ½" Johnson bars. One bent up at 45°, at 8" outside load point. Net area, 0.54 sq. in. Centre of bars, ¾" above base. % reinforcement = 1.25. Age when roasted, 48 days. Weight, 290 lbs. Estimated loss of weight, 23 lbs. Loaded at third points.

The general appearance of the beam after fire treatment was good. The left end was slightly damaged, and patched up with plaster of paris for testing. Cracks on the compression side were noted as follows: 6″ to left of centre running 4″ to 5″ down; 3″ to right of centre running 3″ down; crack at centre (small) running 3″ down; small cracks 4″ and 7″ outside left load point run a litt e down sides. Minor cracks near left end crack 4″ outside right load point 2″ to 3″ down side; small cracks at right end near point at which the diagonal bar comes out of beam. Camber, 0.28″.



JOHNSON BEAM 6″ wide. 8″ deep. 6′0″ c. to c.
Reinforcement 3-½″ rods = 0.54 sq in = 1.25 %
Load at third points Total weight of beam 290 lbs.
Max load 5500 lbs. Age 48 days. Beam roasted.
Compare Plates I, II and III

Plate XIX.

The results are plotted in Plate XIX, the deformation curves being slightly concave upwards. At 2500 lbs. a diagonal crack appeared on face 4″ to right of right load point; and at 3500 lbs. showed on back also. Other smaller diagonal cracks showed nearer the support. Failure occurred at 5500 lbs. by diagonal crack running from the left support (patched end) towards the diagonal rod, over the portion of the beam not diagonally reinforced. Failure was similar to that in Beams Nos. 1 and 3, which were similarly reinforced, the loads carried being 12,000 lbs. and 11,000 lbs. respectively. The maximum load in the case of the roasted beam was about half that for the beam under normal conditions.

Tests of Roasted Cubes.

Four cubes roasted in the furnace with the beams described above were tested for compressive strength with the following results:

Cube dimensions ins.	Weight lbs.	Age when roasted days	Ultimate strength lbs. per sq. in.	Remarks
(*a*) 8″ × 8″ × 8.1″ deep	40.0	47	1340	Uniform failure
(*b*) 9.0″ × 9.1″ × 9.6″ deep	56.5	54	1873	Uniform failure
(*c*) 9.05″ × 9.1″ × 9.25″ deep	56.0	48	1465	Uniform failure
(*d*) 9″ × 9″ × 9.2″ deep	53.0	54	1190	Failed on two faces and one corner

The average of the ultimate strengths of the cubes (*a*), (*b*), (*c*) which failed uniformly, was 1560 lbs. per sq. in. Cube (*d*) was unsatisfactory, and showed decided weakness on two faces from the outset.

Extensometers were attached to the faces of the cubes (*a*), (*c*), and (*d*). In the case of (*a*), the first cube tested, surface cracking was mistaken for indications of coming failure, and the extensometers were removed at an unnecessarily early load. The value of E for the range of stress considered was 114,000 lbs. per sq. in.

In the case of cube (*c*) the yielding of the opposite faces was unequal, especially at the higher loads. Taking the mean yield of the two faces, the value of the modulus in compression was as follows: From 10,000-20,000 lbs. load, $E = 99{,}100$ lbs. per sq. in.; from 20,000-30,000 lbs. load, $E = 126{,}000$ lbs. per sq. in; from 30,000-40,000 lbs. load, $E = 150{,}000$ lbs. per sq. in.; from 40,000-50,000 lbs. load, $E = 165{,}000$ lbs. sq. in. The average of these values is 135,000 lbs. per sq. in. The value seems to increase with increase of load, which is the reverse of what occurred with the normal concrete. It is probable that the removal of all moisture from the cubes may account for this. Initially there was evidence of internal change by the noise accompanying the increase of load, the concrete being in some measure rather porous and brittle. At later stages of the loading the material would be more packed together, and the rate of yielding lower than initially.

Cube (*d*) failed irregularly. Taking the mean value of the yield from 0-50,000 lbs., the value of $E = 114{,}000$ lbs. per sq. in.

The above results indicate a marked reduction in the value of the compression modulus, an average value being about 125,000 lbs. per sq. in., against 2 to 3 million lbs. per sq. in. for normal concrete. The average compressive strength, 1560 lbs. per sq. in., is about 70% of that of normal concrete.

The reduction in concrete modulus shows some reason for the fact that the neutral surface is so near the steel reinforcement. The position of the neutral surface is given by

$$k = \sqrt{p^2c^2 + 2pc} - pc. \qquad \text{(See p. 9.)}$$

on the assumption that the strain follows the linear law.

$$c = \text{ratio } \frac{E_s}{E_c} = \frac{30 \times 10^6}{125{,}000} = 240$$

$100\ p =$ percentage reinforcement.

The average reinforcement in beams Nos. 16, 17, 19, all of which were 8″ deep, was 1.23%.

These values give

$$\begin{aligned} k &= \sqrt{\frac{1{\cdot}23^2}{1000^2} \times 240^2 + \frac{2 \times 1{\cdot}23}{1000} \times 240} - \frac{1{\cdot}23}{1000} \times 240 \\ &= \sqrt{870 + 59} - 29{\cdot}48 \\ &= 30.48 - 29.48 \\ &= 1.0 \end{aligned}$$

This would mean that the neutral surface was at the steel line, but the above values of E_c are necessarily only very approximate, and the calculation shows that the actual results of the beam tests check reasonably with those of the tests on the cubes. It is quite reasonable that the properties of the concrete in the cubes should differ somewhat from those of the concrete in the beams. In the latter case, heat must have been conducted to the interior of the beam by the metal rods, and the effect of the high temperature in a beam 6″ × 8″, with reinforcement, may well be different from the effect of the same conditions on 9″ concrete cubes. A slight change of the ratio $\frac{E_s}{E_c}$ would bring the value of *k* into agreement with the experiments.

Comparative Table of Results.

The preceding results are tabulated below, so that the principal points may be seen at a glance

Beams	Size	% reinf.	Age days	Max. load lbs.	Const. in $M = cbd^2$	Wgt. of beam lbs.	Wgt. of steel lbs.	Failure
Johnson No. 1	6″ × 8″	1.25	28	12,000	470	311	11.5	Diagonal end
" " 2	"	1.25	29	12,270	480	313	12.2	Tension central third; comp'n above
" " 3	"	1.25	49	11,000	432	314	11.5	Diagonal end
" " 4	8″ × 12″	0.80	55	17,900	361	1009	25.5	Tension central third
" " 5	8″ × 11¾″	1.02	56	20,500	431	1008.5	31.0	Tension central third; comp'n above
Kahn No. 6	6″ × 8″	net 1.15	28	12,000	470	314	17.8	Tension central third; comp'n above
" " 7	6″ × 8″	1.15	55	12,950	505	301	17.8	" "
" " 8	6″ × 8″	1.30	28	12,000	470	303	17.0	Diagonal end
" " 9	6″ × 8″	1.30	55	14,000	547	315	17.0	Tension central third; comp'n above
" " 10	6″ × 7.5″	1.24	28	10,000	452	292	17.8	" "
" " 11	6″ × 7.15″	1.30	56	11,530	580	282.5	17.8	" "
" " 12	8″ × 12″	1.25	28	23,600	475	1050	56.5	" "
" " 13	8″ × 11¾″	1.27	56	25,100	526	1043	56.5	" "
Ransome No. 14	6¾″ × 9½″	1.27	29	21,940	518	431	17.5	Diagonal end
" " 15	6″ × 8″	1.72	29	18,000	700	318	16.0	" "

Note—Kahn Beams 6, 7, 10, and 11 contained 2 — ½″ rods, wings sheared 6″ long.

Kahn Beams 8 and 9 contained 1 — ¾″ rod, wings sheared 12″ long.

In the latter case (¾″ rods) the wings projected beyond the compression face. In the former case (½″ rods) the shorter wings reached to about 2½″ below the compression face.

A perusal of the table shows that two of the Johnson beams failed by diagonal cracking at the ends. In both these the centre bar of the three was bent up at 45°, leaving some portion of the beam near the supports without diagonal reinforcement. (Length *a* in Fig. 2, p. 5.) Beam No. 2 had a diagonal rod running to the support, and failed in the central third, at the highest load of the three 6″ × 8″ beams. Of the Kahn beams, only one failed by diagonal cracking at the ends. Both Ransome beams failed by diagonal cracking. In the case of No. 14 the span, 6′ 0″, was short compared with the depth of 9½″, and the conditions were such as to induce excessive diagonal tension due to end shear. Ransome No. 15 was much more heavily reinforced than the other 6″ × 8″ beams, and carried a much higher load. Owing to the high yield point of the steel, and the heavy reinforcement, it was not surprising that diagonal end failure resulted. Both Ransome beams therefore may be regarded as being of proportions liable to result in diagonal end failure. The author does not suggest that beams reinforced with this rod are liable to such failure. In fact, a test made this year (March 12) on a Ransome beam, 6″ × 8″ × 6′ 0″ c. to c., reinforced with two ½″ rods, and having *no diagonal reinforcement whatsoever*, carried a maximum load of 16,000 lbs. at the third points. The concrete was nearly thirteen months old, the beam having been made on February 22, 1907. The failure of the above beam occurred by tension cracking *within* the load points, and compression failure above. The ends, although having no diagonal reinforcement, remained perfectly sound. The percentage reinforcement was 1.15. The total weight of the beam was 312 lbs. Steel, 10.8 lbs. It may be said therefore that Johnson beams Nos. 1 and 3, and Ransome beams, Nos 14 and 15 were liable to end failure for special reasons. Of the eight Kahn beams tested only one failed diagonally at the end.

The following table shows the summarized results of tests made in the early part of this year on beams similar to those described in the preceding pages. The beams were made at the same time as those already described, and were stored carefully after the fire in the Engineering Building in April, 1907. At the time of test they were about twelve months older than the beams tested in the previous year. The results may be compared readily with those in the preceding table.

Beam	Size	% reinf.	Age days	Max. load lbs.	Const. in $M = cbd^2$	Wgt. of beam lbs.	Wgt. of steel lbs.	Failure
Johnson 5-½″ rods 3 at 45°	7.9″ × 11.7″ × 10′0″ c. to c.	1.04	392	22,000	473	1001	31.0	Tension in central third, and compression above.
Ransome 3-½″ rods One bar bent up to come out over support	6¾″ × 9¾″ × 6′ 0″ c. to c.	1.25	363	36,400	800	450	17.5	End face cracked diagonally. Cracks joining base cracks. Rods pulled at end. Compression failure showing at one support
Kahn 2-½″ rods	6″ × 7″ × 6′ 0″ c. to c.	1.33 net	405	12,500	645	279	17.8	Tension in central third, compression in concrete above
Kahn 1-¾″ rod	6″ × 8″ × 6′ 0″ c. to c.	1.30 net	413	15,100	578	310 est.	17.0	Tension in central third, compression in concrete above
Ransome 2-½″ rods No diagonal bars	6″ × 8″ × 6′ 0″ c. to c.	1.15	386	16,000	612	312	11.0	Tension in central third, compression in concrete above

In all cases where comparisons are possible it will be seen that there was a decided gain of strength during the twelve months which elapsed between the two sets of tests. This was specially noticeable in the case of the Ransome beam, for which the constant c in $M = cbd^2$ reached the value of 800, against 518 at 29 days. The beam was slightly deeper in the twelve months' test than in the one month test, but the gain is remarkable, and must be attributed to the maturing of the concrete to such an extent that it was able to develop more fully the very high yield point of the Ransome bar (see p. 4). The gain was not so noticeable in the other cases, but was, however, quite appreciable. Two concrete cubes were likewise tested after the twelve months' interval. One, 392 days old, failed at 2620 lbs. per sq. in., while the other, 370 days old, was quite sound after being subjected to a stress of 2730 lbs. per sq. in., which represented the full capacity of the testing machine. These results show a very appreciable gain in strength of the concrete during the year.

The last test summarized in the table is that for the beam already mentioned, containing only two half-inch Ransome bars, with no diagonal reinforcement whatever.

The tests described in the preceding pages cover considerable variation as regards method and percentage of reinforcement, including beams with no diagonal reinforcement, beams with diagonal reinforcement secured by bending up of tension bars when no longer required as tension reinforcement, and beams with wing bars having a very considerable weight of diagonal reinforcing metal over the entire length. The results have been presented in sufficient detail to enable the reader to follow the make-up of each beam, and its behaviour under test. It would be idle to attempt to make any detailed comparisons between the carrying capacity of the various beams and the weights of steel employed, since such comparisons must inevitably be affected by a variety of conditions, the exact influence of which would be in all probability uncertain, and in any case would be the subject of dispute. The wing bars gave diagonal reinforcement over the entire length of the beams, irrespective of the central third not being under shearing action. The diagonal wings on this portion would appear, therefore, to have mainly assisted the bond. The bars were only obtainable sheared over the whole length. None of the diagonal metal is figured in the percentage net reinforcement, which was constant throughout the length. On the other hand, in the case of beams reinforced with straight rods, corrugated or twisted, the only diagonal reinforcing provided was that obtained by bending up tension bars when the conditions permitted. In this way the tension reinforcement was diminished towards the free ends, and the total weight of steel was very little in excess of what it would have been for a uniform tension reinforcement alone throughout the beam. The number of rods used to obtain the required degree of reinforcement is of importance in making comparisons, as it affects the distribution of stress between steel and concrete. The nature of the loading, the ratio of depth to span of the beam, are also factors to be borne in mind in considering the advantages of any particular type of reinforcement. Roughly speaking, in the case of the wing bars, about one-third of the gross section appears as diagonal reinforcement, leaving two-thirds to be used in figuring the net tension reinforcement. By using the same weight of steel in the form of straight bars, the gross section can be utilized as tension reinforcement and a much lighter diagonal reinforcement be provided by the bending up of bars at intervals. The exact effects of thus equalizing the *gross* weights of steel used, in reinforcing by methods differing so essentially as as those discussed above, cannot be inferred with certainty from the results of tests in which the percentage reinforcements figured on the net sections are much more nearly equal than the gross weights. The nature and extent of such influences would ever be a matter of personal opinion. A close observation of the behaviour of the beams under test, and a study of the results of the tests has, however, led the author to the opinion that under ordinary circumstances a sufficient reinforcement can be provided without using so much material as is involved in a bar having heavy wings spaced uniformly along its length irrespective of the form of shear diagram, and providing the same net tension reinforcement throughout irrespective of the variable bending moment. Beams in practice may have to take up their loads at earlier periods after manufacture than beams under laboratory test, and additional precautions may be necessary. But the author believes that all adequate reinforcement can be provided by a careful disposition of straight rods, due regard being paid to the form of the bending moment and shearing force diagrams. The results of a test on a beam reinforced with two Ransome bars and having no diagonal reinforcement whatever (see Table, p. 57) have been referred to already. The beam was loaded at the third points, just outside which the conditions were severe, as the shear attained its full value, and the bending moment was sensibly the same as over the central third. The beam failed, not by end shear, but in the central third. It is true that the concrete was rich and mature, the beam being thirteen months old. But bearing these facts in mind, the result is a striking one, and taken in conjunction with the evidences of the tests, as to the efficacy of a bent bar in resisting end shearing, it has suggested the thought that diagonal reinforcing may possibly be a somewhat over-estimated factor in the proportioning of reinforced concrete beams. It is necessary beyond any doubt, but the means by which the necessary amount may be obtained with the minimum expenditure of steel will probably be determined by experience in practice, and by comparative tests outside the scope of this paper, rather than by theoretical investigations on assumed conditions, imperfectly realized in practice.

Experimental Position of Neutral Axis.

The position of the neutral axis is shown clearly in the foregoing plates, and the following table is appended to enable a rough comparison to be made between the experimental position, and that indicated by the analysis based on linear straining action and constant concrete modulus as given on p. 9. The plates show that the position of the neutral axis varies considerably. The value tabulated below is an average one. Detail comparison at various stages of loading may be made by reference to the plates. The concrete moduli given by tests on cubes were somewhat variable, and for that reason the table includes theoretical positions of the neutral axis for $E = 2 \times 10^6$ lbs. per sq. inch and $E = 3 \times 10^6$ lbs. per sq. inch. In most cases there is a rough agreement between the average experimental value, and the theoretical value corresponding to $E = 2 \times 10^6$ lbs. per sq. inch. The results, however, indicate appreciable differences in the position of the neutral axis in the case of beams having the same net reinforcement, and support the view that since the actual position of the neutral axis is liable to such variations, any elaborate theoretical calculation of its position is out of place.

Beam	% net reinforcement	Depth of neutral axis from compression layer expressed as percentage of depth of beam.		
		Average of experimental result	Theoretical $E = 2 \times 10^6$ lbs. per sq. in.	Theoretical $E = 3 \times 10^6$ lbs. per sq. in.
No. 1	1.25	47.0	45.5	39.2
2	1.25	44.0	45.5	39.2
3	1.25	48.5	45.5	39.2
4	0.80	43.5	38.3	32.8
5	1.02	42.0	41.8	36.0
6	1.15	45.5	44.0	38.0
7	1.15	51.0	44.0	38.0
8	1.30	47.5	46.0	39.8
9	1.30	48.0	46.0	39.8
10	1.24	49.0	45.2	39.0
11	1.30	48.5	46.0	39.8
12	1.25	43.5	45.5	39.2
13	1.27	47.0	45.7	39.5
14	1.27	42.5	45.7	39.5
15	1.72	49.5	50.5	44.0

The author now presents some notes on the methods of design of reinforced concrete beams, together with a detail study of typical examples of the gradual breaking down of beams under test.

Determination of Safe Working Loads.

Opinions are divided as to the best method of determining working loads for reinforced beams. Some designers first calculate the ultimate load, basing their calculation on the fundamental principle that the beam is so reinforced that the concrete reaches its ultimate compressive strength at the same time as the steel reaches its yield point. Some fraction of the ultimate load, say, one-third or one-fourth, is then taken as the safe load. Others, however, base their calculations for the safe load on assumed safe stresses for steel and concrete, say, 16,000 to 20,000 lbs. per sq. in. in tension for the former, and 600 to 800 lbs. per sq. in. in compression for the latter. These stresses alone are supposed to exist in the beam, i.e., it is assumed that there is no tension in the concrete.

In considering these methods of design, it is of interest to study carefully the gradual breaking down process of a beam tested to destruction. The case chosen for illustration is that of Johnson Beam No. 5 (p. 28), which was the first worked out in detail. The results are shown fully in Plate XXI. Space forbids the presentation of all the calculations in this paper, but they follow strictly along the lines indicated in presenting the curves for the various beams tested.

(a) Neutral Axis—By plotting the extensometer readings at each stage of the loading, the position of the neutral axis was determined, and is seen to have been initially at 52% of the depth from the outer compression layer, until a load of 3000 lbs. was reached. The ultimate load was 20,500 lbs. Between loads of 3000 to 7000 lbs., the neutral axis rises gradually to about 41% of the depth from the compression layer, and during the subsequent stages it remains in practically the same position. The extensometer readings at the load of 20,000 lbs. are not to be relied upon to the same extent as at the lower loads, for the beam was then nearly at its ultimate load, and as it yielded, there was a continuous movement of extensometers which could not be read simultaneously. At lower loads there were no such difficulties. A reference to the record of the behaviour of this beam (p. 29) will show that surface cracks were detected at 6,000 lbs. load, and that the cracks extended about 1″ up the sides of the beam at 9000 lbs. load. The rise of the neutral axis appears, therefore, to be intimately associated with the gradual breaking down of the concrete in tension, and this was indicated by the extensometers before it was evident by careful outside inspection.

(b) Stress in Steel—The extensometer readings give the steel stresses directly, and inspection of the curve in Plate XXI. shows

that these increase gradually, and at an increasing rate up to a load of about 10,000 lbs. Beyond that load the rate of increase of steel stress is practically uniform, the only irregularity in the curve

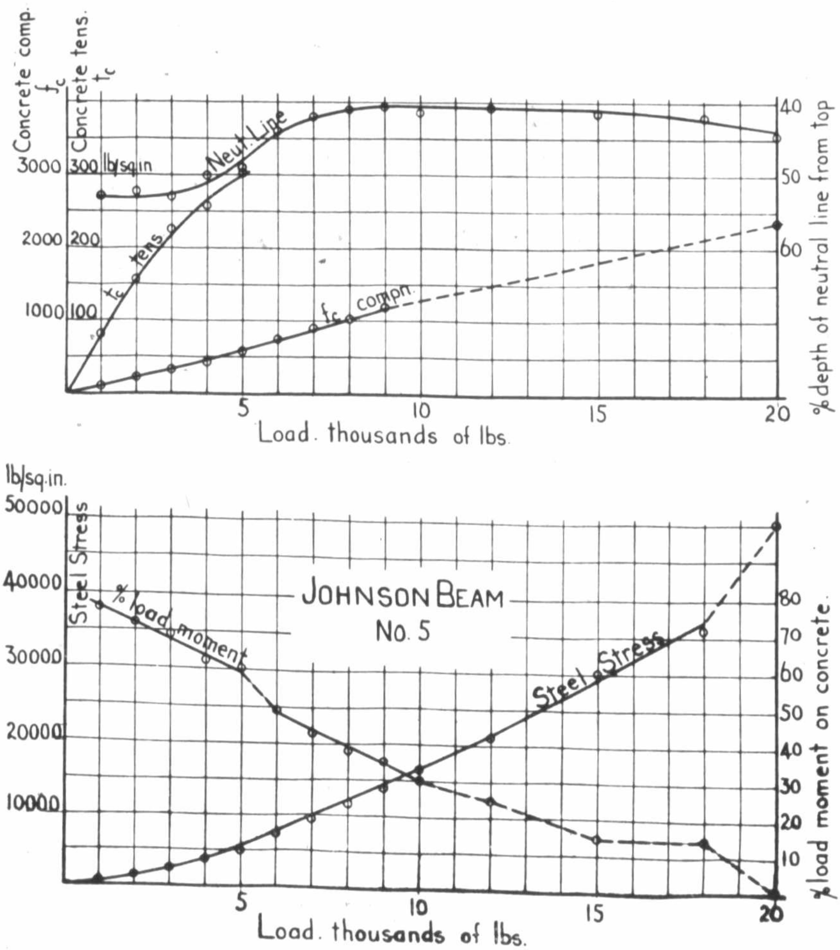


Plate XXI.

being at loads near the ultimate load, at which extensometer reading are a little uncertain. At the above load, 10,000 lbs., the beam was decidedly cracked, and the steel stress was about 16,000 lbs.

per sq. in. The stresses at the lower loads can be seen from the curve. The steel stress at 20,000 lbs. load was about 45,000 lbs. per sq. in., a value in close agreement with the yield point of the steel. (See p. 30.)

(c) Proportion of Load Moment carried by Concrete—It has been shown already that at loads of the order of one-fourth of the ultimate load, a very considerable proportion of the load moment is carried by the concrete. This proportion is shown in Plate XXI., the percentage scale being on the right of the figure. At a load of 1000 lbs., 75% of the moment is carried by the concrete. This percentage diminished steadily to a value of 60, at a load of 5000 lbs. At 6000 lbs. the beam was observed to be cracked. The percentage was then under 50, diminished steadily to 30 at a load of 10,000 lbs., and then somewhat irregularly to practically zero at a load of 20,000 lbs. The marked break in the curve between 5000 lbs. and 6000 lbs. corresponds with the appearance of surface cracks and with a very considerable rise in the position of the neutral axis, the curve for the latter being steepest in the vicinity of these loads.

(d) Estimate of Tensile Stress in Concrete—On the assumption that both tensile and compressive stresses in the concrete follow the linear law (not necessarily the same line however), the resultant tensile force in the concrete must act at a distance ⅔d from the centre of the compressive stresses, *d*, being the depth of the beam to the reinforcement line. Knowing the moment carried on the concrete in tension, the total tensile force is at once determined, and from the known position of the neutral axis, the greatest intensity of tensile stress in the concrete is easily obtained. Such calculations cannot be carried beyond the load at which the first cracking is noted, for it cannot be known how far a crack really extends. The curve, Plate XXI., shows a steadily increasing tensile stress up to a value of 300 lbs. per sq. in., which was reached when the first cracks were noted. An ultimate tensile stress of 300 lbs. per sq. in. for concrete seems reasonable.

(e) Estimate of Compressive Strength in Concrete—The total compressive force at any instant is equal to the total tensile force at that instant. Hence, since the area in compression is known at all stages of the loading, the greatest compressive stress is easily obtained. It will be seen to have reached a value of 750 lbs. per sq. in., at a load of 6000 lbs. (first observable crack). The points between 6000 lbs. and 9000 lbs. load were obtained on

the assumption that the concrete was good in tension until the cracks extended about 1″ up the sides. Owing to the gradual disintegration of the beam, no reliable calculation can be made for the later stages, at which neither the tensile force in concrete nor its lever arm are known, and all that can reasonably be done is to estimate the ultimate concrete compressive stress at 20,000 lbs. load, when practically all the load moment was carried by the steel. This is found to have been about 2345 lbs. per sq. in., a reasonable value, as the concrete failed in compression at a little higher load.

It is interesting to note the result of the transfer of tensile stress existing in the concrete when uncracked at 5000 lbs. load, to the steel at the load of 20,000 lbs., the concrete being then badly cracked.

Estimated tension in concrete at 5000 lbs. load	= 7680 lbs.
Lever arm of concrete force at 5000 lbs.	= 7.83 ins.
At 20,000 lbs. the lever arm of steel force	= 9.40 ins.
∴ Force in steel transferred from concrete	$= \frac{7,680 \times 7.83}{9.40}$
	= 6,400 lbs.
Force in steel at 5000 lbs.	= 4,320 lbs.

Force in steel at 9.40 ins. leverage due to load increase from 5000 lbs. to 20,000 lbs.

$$= \left(\frac{20,000-5,000}{2}\right) \cdot \left(\frac{49 \text{ ins., } i.e. \ \frac{1}{3} \text{ length}}{9.40}\right) = 31,900 \text{ lbs.}$$

∴ Total force in steel at 20,000 lbs = 31,900 + 6400 + 4320 = 42,620 lbs.

$$\text{Steel stress} = \frac{42,620}{0.90} = 47,400 \text{ lbs. per sq. in.}$$

Extensometer readings gave 50,000 lbs. per sq. in.—a reasonable agreement, 50,000 lbs. per sq. in. being beyond yield point of any of the bars tested.

Similar calculations for other beams show the characteristic features described above, modified as would be expected, by the particular conditions accompanying the breakdown.

The curves for Ransome Beam No. 14, p. 44, are shown in Plate XXII. The concrete tensile stress appears to have been approximately constant at 180 lbs. per sq. in., after a load of 4000 lbs. was reached. The record of behaviour shows that the extensometers were creeping slowly at 7000 lbs., indicating that at this load and beyond, a little time had to elapse after the increase of load before the readings became steady, and probably signifying some breaking up not observable by eye, the neglect of which in the calculations

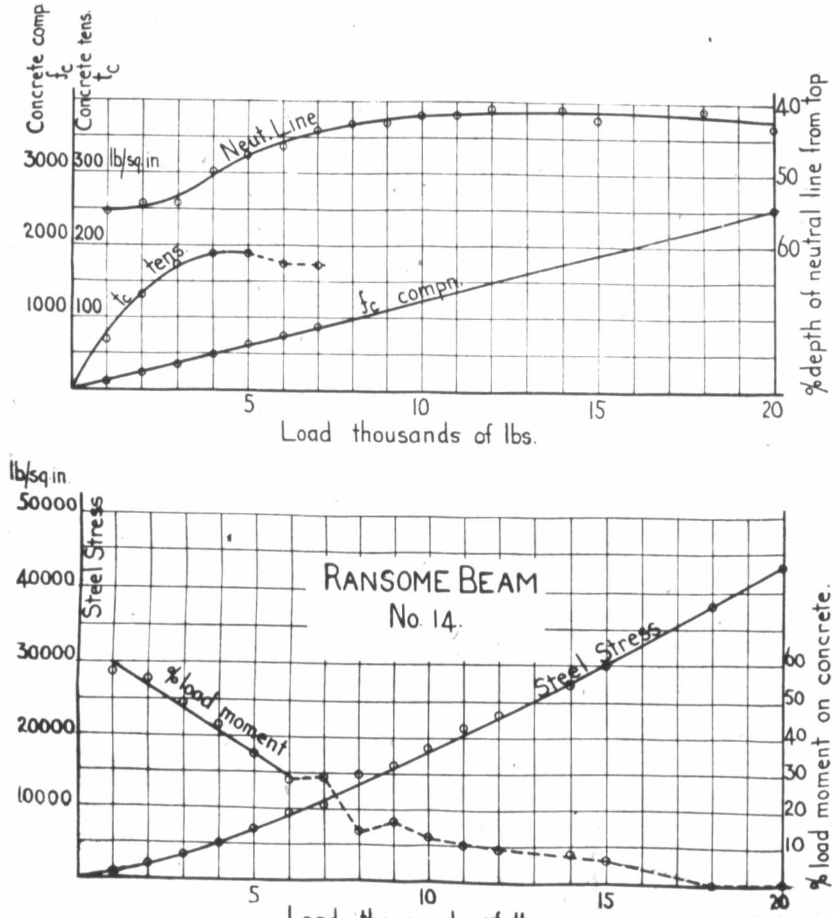


Plate XXII.

may account for the values of tensile stress shown. There is the same marked rise in the position of the neutral axis, preceding and following the observed surface cracking.

The bearing of these results on the two methods of design outlined above may now be considered. In the case of each beam the marked rise in the position of the neutral axis takes place between loads of say, 3000 lbs. to 8000 lbs. The Johnson beam failed at 20,500 lbs., and the Ransome at 21,940 lbs. The above limits, therefore, include all loads less than one-third of the ultimate loads, *i.e.*, all reasonable working loads, and it is precisely over this range of practicable loading that there is such a considerable movement in the position of the neutral axis. The author considers that it is impossible to know *precisely* where the neutral line is, under a working load, whether the beam be designed on a basis of ultimate loads, or on a basis of safe stresses. In any sound uncracked beam of average proportions it is probably somewhere between 40% and 50% of the depth of the beam from the compression layer, and to assert its position to any very close degree of accuracy does not seem to be justifiable.

The methods of design will now be considered.

Design Based on Assigned Safe Stresses—It is assumed that the steel carries all the tension and a safe tensile stress is assigned. The safe compression stress in the concrete is also assigned. It is impossible to say from surface inspection how far a crack really extends, and the only safe assumption is that it destroys entirely the tensile resistance of the concrete. But at loads less than the load at cracking there is a very appreciable amount of tension on the concrete, and the assumption that the steel carries *all* the tension is incorrect. It leads to a design which is amply safe as regards steel tension, for some of the stress for which the steel is designed is carried by the concrete. The estimated compressive stress in the concrete, based on an *assumed* stress in the steel, which is reached only when the concrete is destroyed in tension, cannot be known to be correct when the tension value of the concrete is not so destroyed, and such a calculated stress may be fallacious. The real point to be considered is this:—*If a load which is insufficient to crack the concrete in tension, what proportion of the bending moment is carried by the steel and what by the concrete in tension?* In Fig. 5 (*a*) the *assumed* conditions of design are represented. The centre of compressive stresses is at C.

Moment of resistance = moment of actual loads $= T\left(d - \frac{xd}{3}\right)$ where T is the total tension in the steel. In Fig. 5 (*b*) are represented the *actual* conditions, if the concrete is not cracked in tension. The neutral line may now be $x_1 d$, from the *compression layer*. For simplicity assume the tensile stress in concrete to extend only to the reinforcement line, although if known to be good at all in tension, the concrete is really wholly good to the base of the beam. There is now some tensile force in the concrete at distance $\frac{2}{3}d$ from the centre of compressive stresses. The tension in the steel is now t. Moment of resistance = moment of actual loads

$$= t\left(d - \frac{x_1 d}{3}\right) + t_c \frac{2}{3} d.$$

Any diminution in steel moment must be made up by concrete moment, since the total moment is unchanged. The arm of the concrete tensile force t_c, is appreciably less than that of the steel force. Hence (assuming at present that $xd = x_1 d$), to make up the diminution of steel moment, the tensile force set up in the concrete must be greater than the reduction in the steel force. The total tension $(t + t_c)$ therefore exceeds T, and an estimate of the compressive stresses based on the conditions represented in Fig. 5

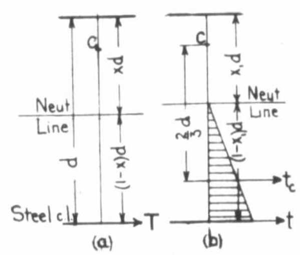


Fig. 5.

(*a*), may be fallacious. Any difference in the position of the neutral line is an important factor. The author's experiments go to prove that in an uncracked beam the neutral axis is well below its position in a beam in which the concrete is so cracked as to have no tensile value. Hence it is probable that $x_1 d$ in the actual beam is greater than xd in the assumed design, and the arm of the steel force is thereby reduced. In fact, if the actual steel stress when there is concrete tension, is half the assumed steel stress when there is no concrete tension, the actual steel moment will be less than half the moment equivalent to the assumed steel stress, on account of the reduced lever arm of the steel force.

Take the following numerical example in illustration of this method of design:

A beam of 15 ft. 4 ins. span c. to c. of bearings is required to carry a uniformly distributed load of 24,000 lbs. inclusive of weight of beam. The stress in the steel is not to exceed 16,000 lbs. per sq. in., and in the concrete 600 lbs. per sq. in. compression. The breadth of the beam is 14 inches. Find the sectional area of metal and the depth required.

$$\text{Bending moment} = \frac{24,000 \times 184}{8} = 552,000 \text{ lb. ins.}$$

Then using the notation on p. and taking $e = 15$

$$\frac{15 \times 600}{16000} = \frac{x}{1 - x}$$

giving $x = 0.36$

If d = depth of beam $\left(d - \frac{0.36d}{3}\right) = 0.88d$ is the lever arm of the steel force. From the equality of steel force and concrete compression force

$$p = \frac{0.36 \times 600}{2 \times 16,000} = 0.00675,\ i.e.,\ 0.675\% \text{ reinforcement.}$$

$$\text{Then } f_s = \frac{552,000}{.00675 \times 14 \times d^2 \times 0.88} = 16000$$

giving $d = 20.4$ inches.

Sectional area of metal $= 0.00675 \times 20.4 \times 14$

$= 1.92$ sq. ins.

Five ⅝" sq. rods, would give 1.95 sq. ins.

Now consider the possible *actual* conditions under which this beam would operate. From what has been said already, a considerable percentage of the load moment will actually be carried on the concrete, probably quite 50%. The author does not see how this amount can be known very exactly. Assume that the above percentage of load moment is carried on the concrete, *and that the neutral axis is in the position calculated in the design.* The steel stress will be halved. Concrete moment $= \frac{552,000}{2} = 276,000$ lb. ins.

Lever arm of tension in concrete $= \frac{2}{3} \times 20.4 = 13.6$ inches.

Tensile force on concrete $= \frac{276,000}{13.6} = 20,300$ lbs.

Tension force in steel $= \frac{1.92 \times 16,000}{2} = 15,360$ lbs.

Total tension $= 20,300 + 15,360 = 35,660$ lbs.

Area in compression $= 14 \times 0.36 \times 20.4 = 102.5$ sq. ins.

Compressive stress at outer layer $= \frac{35,660 \times 2}{102.5}$

$= 697$ lbs. per sq. in.

This is 16% greater than the assumed value of 600 lbs. per sq. in. If allowance be made for a probable change in the position of the neutral axis, from that given by the assumed stresses, the calculation must be modified. In studying the breakdown of a beam it was seen that the neutral axis changed its position by some 10% of the depth during the stages in which the concrete is being destroyed in tension. This is probably a variable amount, difficult to foretell, and dependent on a variety of conditions. In this example a 10% variation would amount to a change of 2.0 inches. Under working conditions it is reasonable to assume about half this amount, say 1.0 inch. Hence the neutral axis is likely to be $(0.36 \times 20.4 + 1)$, say 8.40 inches below the outermost layer. The arm of the steel force is now $\left(20.4 - \frac{8.4}{3}\right) = 17.6$ ins. Hence, if half the load moment is carried by the steel, the steel force will be $\frac{276,000}{17.6} = 15,700$ lbs.

and the steel stress $\frac{15,700}{1.92} = 8176$ lbs. per sq. in.

i.e., slightly more than half the stress assumed in the design.

The force in the concrete necessary to contribute the remaining half of the moment will be 20,300 lbs. as before, making the total tension $20,300 + 15,700 = 36,000$ lbs.

The maximum compressive stress is then $\frac{36,000 \times 2}{14 \times 8.4} = 613$ lbs. per sq. in. This is practically the same as the assumed value, its amount depending on the uncertain position of the neutral axis. The increase of area in compression above that figured in the design, approximately compensates for the increase in total compression, and gives the same intensity of stress.

It appears then, that while the steel stress under the probable actual conditions will be considerably less than that assumed in the design, the compressive stress in the concrete may be fairly close to its assumed value.

The above considerations suggest this query:—The actual conditions in a reinforced concrete beam, being very different from those assumed in a design based on its ultimate strength, can the extent of the divergence be estimated in practice to such a degree of accuracy as to justify the application of the somewhat complex formulae for design, which have come into use? The author's opinion is emphatically in the negative.

Design Based on Ultimate Strength of Materials— Consider the same problem as that presented in the preceding method of design, and suppose that the ultimate strength of the concrete is 2250 lbs.

per sq. in., and the yield point of the steel 50,000 lbs. per sq. in. Allow a factor of safety of four, and neglect concrete tension.

Moment to be designed for $= 4 \times 552{,}000$

$= 2{,}208{,}000$ lb. ins.

Assume that the ratio of moduli is 15.

Then, as before

or $\dfrac{15 \times 2250}{50{,}000} = \dfrac{x}{1-x}$, giving $x = 0.40$

Then if $d =$ depth, it follows from the equality of tensile and compressive stresses that $\dfrac{2250bxd}{2} = 50{,}000 \times$ steel area.

$$\frac{\text{Steel area}}{bd} = \frac{2250 \times 0.4}{2 \times 50{,}000}$$

$$= 0.009$$

$$= 0.9\% \text{ reinforcement.}$$

Then, since lever arm of steel force $= \left(d - \dfrac{0.4d}{3}\right)$

$$= 0.867\ d$$

$$50{,}000\left(\frac{9}{1000} \times 14 \times d\right) 0.867\ d = 2{,}208{,}000$$

giving $d^2 = 404.5$

or $d = 20$ inches, against 20.4 ins. by the other method of design.

Sectional area of metal $= \left(\dfrac{9}{1000} \times 14 \times 20\right) = 2.52$ sq. ins.

Six $\frac{11}{16}''$ sq. rods would give 2.83 sq. ins.

If the width of beam were 12 inches, d would be 21.75 ins., and the area of metal 2.34 sq. ins., for which six $\frac{5}{8}''$ sq. rods would suffice.

Taking a yield point of 55,000 lbs. per sq. in., factor of safety of four, breadth 14 inches, the following would be obtained:—Percentage reinforcement, 0.776; depth, 20.6 inches; area of metal required, 2.235 sq. ins., for which five $\frac{11}{16}''$ sq. rods would suffice. (2.36 sq. ins.)

Any change in width of beam, ultimate stresses, factor of safety, or ratio of moduli, will modify the design in directions which can be ascertained readily by inspection of the preceding calculation.

The question now arises as to the exact meaning of the factor of safety. The existence of a considerable amount of tension in the concrete at working loads is admitted, and allowed for in the formulae used by designers following this method of design. The steel stress differs appreciably from that given by dividing the yield point stress by the factor of safety as measured by the beam load. The question to be considered is this:—Is it known definitely whether the concrete reaches its ultimate stress at a uniform rate? In other words, is the concrete stress at one-fourth the ultimate load equal to one-fourth of its ultimate stress? If not, what is the concrete stress, and what is the real margin of safety.

Following the same methods of analysis used in this paper, the following tables have been prepared showing the estimated stresses at loads approximately one-fourth of the ultimate loads. In all cases the beams were uncracked at these loads, but cracked at slightly higher loads.

STRESSES AT ULTIMATE AND AT FRACTION OF ULTIMATE LOAD

Beam	Ultimate Load lbs.	Load, lbs.	Steel stress lbs. per sq. in.	Concrete tension, lbs per sq. in	Concrete comp'n lbs. per sq. in.
Johnson No. 5	20,500	20,000 5,000	*50,000 4,800	0.0 303.0	*2,345 556
Johnson No. 4	17,900	15,000 5,000	30,000 6,440	not destroyed 250.0	1,365(+?) 590
Johnson No. 3	11,000	10,000 2,500	36,700 4,400	0.0 250.0	1,890 448
Ransome No. 14	21,940	20,000 5,000	43,120 5,760	0.0 185.0	2,515 614
Kahn No. 12	23,600	20,000 5,000	36,200 6,290	0.0 156.0	2,000 520

* Steel stress beyond yield point.

From these results it will be seen that the ratio of concrete compressive stresses is in all cases almost the same as the ratio of the actual loads. Hence, it appears that it is reasonable to expect the working compressive stress to be about the same fraction of the ultimate stress, as the working load is of the ultimate load. The ratio of steel stress at working load to steel stress at ultimate load is always much less than the ratio of those loads, and therefore this method of design seems to lead to a satisfactory beam.

Conclusion.

The author does not claim to have put forward any original theory with regard to the strength of reinforced concrete beams. The paper is in the main a record of more exact and more detailed measurements of the actual deformation of concrete beams than have hitherto been made, and it embodies a plea for a simpler treatment of the question from its theoretical side, than has been usual in much of the recent work on the subject. This point has been kept in evidence throughout, and needs little recapitulation here. All existing theories of the strength of reinforced concrete beams are based on the assumption that the deformation of the various layers of the beams follows the same law as that for an ideal homogeneous substance, *i.e.*, that it is proportional to the distance of the layers from the neutral surface. This law is not *absolutely* true for all steel sections. Exact extensometer measurements will indicate slight discrepancies, not of such a magnitude however, as to mar the practical accuracy of calculations based on the law of linear strains. Concrete does not possess the same degree of homogeneity as steel, and the localization of a large proportion of the internal tensile stress of the beam in the isolated steel rods must set up in the surrounding concrete conditions of stress not absolutely determinate, and in any case differing from those of an ideal beam in which the steel is supposed to be distributed through the entire width of beam. The extensometer measurements made at five layers of the beams tested, show that the actual deformation curve may be (*a*) linear, as assumed in the theory (*b*) concave towards the compression side (*c*) concave towards the tension side. In no case can the exact form of the curves be known without actual testing. The concavity, when it appears, is quite distinct. The fundamental assumption of linear deformation is therefore inexact in many cases. Reference to the curves will show that if a straight line be drawn joining the points representing the compression at the outer layer of the concrete, and the extension at the reinforcement line, it would locate a layer of zero strain, *i.e.*, the neutral surface, in a position differing materially from that obtained by considering the five actual observations. This difference is a very appreciable fraction of the effective depth of the beam in many cases, and is quite comparable with, even if it does not exceed, the difference in the position of the neutral surface which results from comparative calculations based (*a*) on a constant modulus of elasticity of concrete (*b*) on a variable modulus of elasticity of concrete. The justification of the introduction of the latter theory can therefore scarcely be tested by comparing its results with an experimental location of the neutral surface given by arbitrarily drawing a straight line between two points representing extreme tension and compression deformations. While believing firmly in the adaptability of reinforced concrete construction to a large variety of engineering problems, the author considers:

(1) That it is, in general, replacing materials of which our knowledge is more exact, having been gained by prolonged experience, and by careful experiment under conditions capable of being closely specified.

(2) That there are present in some applications of the newer form of construction, conditions which make for greater variations in the properties of the finished structure, than occur in the form of construction superseded. (Compare, for example, a reinforced concrete beam with a steel I beam of similar capacity.) These variations are inherent in the materials themselves, and in the methods and conditions of use.

(3) That the exact conditions of experimental investigation cannot be known as accurately in the case of reinforced concrete construction, as in the case of some of the types of construction which it is replacing, and that the degree of accuracy to which the results of the most careful experiments are applicable to practical conditions is, for similar reasons, less certain.

These conditions do not in any way militate against the successful application of reinforced construction to designs of a varied character, provided that a rational and conservative view be taken of the knowledge already gained by practical experience and experimental investigation. Their existence can scarcely be questioned, although the extent of the effects may be a matter of opinion. The apparent exactness and the complexity of many recent formulae suggest that they express closely the results of the most delicate physical experiments, rather than the results of tests of concrete beams. In the author's opinion these formulae have been built up on an inadequate experimental basis, and it is his belief that a study of the results of careful measurements of the actual strains throughout a beam section, such as have been described, should form the starting point for our theoretical considerations. When this is done, the law of linear straining will be found to be only approximately true. The retention of the law of linear strain as the basis of any theory should then render unnecessary any elaborate modifications which produce changes comparable only with the degree of divergence of the actual from the assumed strains. Neither the conditions of laboratory test nor of practice can be specified to a degree of accuracy even approximating to that of some of the formulae used, and the author trusts that a realization of this fact may result in the general adoption of simpler formulae, more appropriate to the actual conditions.

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