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THE SOLAR ROTATION

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General.

1. A paper published by the authors in last year's Transactions bearing a similar title gave a brief historical summary of the previous work in the determination of the Solar Rotation by the Doppler displacement of the spectral lines at opposite limbs of the sun. It described the instruments and methods employed in obtaining the spectra, the difficulties encountered, and the precautions required for accurate work. It also gave some preliminary measures of the velocity at the solar equator, but refrained from discussing, except very slightly, these results. The present paper contains the results of the measures of the three series of rotation plates made during the year 1911, and a discussion of the various points of interest and value arising from these results. It has not been thought necessary to again describe the instruments and methods as reference can be made to the previous paper.* It may, however, be well to state here that, although the determination of the rotation of the sun by the spectrographic method was, as early as 1905, planned as one of the investigations to be undertaken at the Dominion Observatory, delays in the construction of the shelter for the coelestat telescope and especially the long delay in obtaining a suitable grating prevented much work being done until last year.

2. The whole plan was placed upon a much more definite basis at the Mount Wilson meeting of the International Union for Co-operation in Solar Research in 1910, where the regions of spectrum to be investigated were allotted to the different members of the Rotation Committee, a general region to be observed by all was selected (centre at $\lambda 4250$) and the various questions to be determined were laid down. It may be useful to summarize here the principal points.

(*A*) The region to be observed at the Dominion Observatory is in the yellow green, $\lambda 5500$—$\lambda 5700$.

(*B*) The general region to be observed by all is from $\lambda 4220$ to $\lambda 4280$ in the violet.

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* Transactions Royal Society of Canada, 1911, Sec. III, p. 107.

(*C*) The latitudes to be observed in the special region are—0°, 15°, 30°, 45°, 60°, 75° and if possible 80° and 85°. The latitudes to be observed in the general region are—0°, 30° and 60°.

(*D*) 15 or 20 lines are to be measured in the special regions, these to be selected to include as many elements as possible especially those of high or low atomic weight; about 10 lines, selected by the Secretary of the Committee after consultation, are to be measured in the general region.

3. The principal objects of a study of the sun's rotation by the spectroscopic method are:—

(*a*) The accurate determination of the velocity of rotation at various latitudes and the derivation of a formula representing the variation of velocity with latitude.

(*b*) A definite conclusion in regard to the existence of variations in the rate of rotation.

(*c*) The investigation of the rate of rotation, as shown by the lines of different elements and of the arc and enhanced lines of the same element, to determine whether either the absolute rate of rotation or the law of variation with latitude differs for different elements.

(*d*) The detection of possible systematic proper motions or drifts in the sun's reversing layer.

4. In accordance with the above plan three series of plates were made during 1911, two in the special region at λ5600 and one in the general region at λ4250. With a solar diameter of, on the average, 227 mm., the distance of the observed points from the limb in the first series, at λ5600, varied from 3.0-4.5 mm.; in the second series, also at λ5600, was nearly 10 mm.; and in the third series at λ4250 was about 6.5 mm. The distance was varied in order to see if any difference in the rotational value was obtained, and also to see if much change in the definition occurred as the distance from the limb was increased. As will be seen later, the difference, if any, is slight both in the velocity and the definition. Owing to the considerably larger corrections required to reduce the measured to the actual values of the rotation as the distance from the limb increases, it is not deemed desirable to, in future, make the spectra from points at a greater distance than 5 mm. from the limb.

PRECAUTIONS.

5. In all these plates particular care was taken to guard against every known cause of instrumental and other error tending to introduce spurious displacements of the lines, and the experience of one of the writers in stellar radial velocity determinations was of great value in this similar work. Temperature changes and flexure, the chief

difficulties in stellar spectroscopy, are not however of much moment here for, owing to the short and simultaneous exposures on opposite limbs, temperature changes will have no appreciable effect, and there can be no flexure when the spectrograph is stationary during the exposure. It may not be amiss to repeat here the four essential precautions for accurate observations given in the previous paper.

(*a*) The emulsion on the photographic plate must be exactly in the focus of the spectrum.

(*b*) The illumination of the grating from the opposite limbs of the sun must be similar and uniform.

(*c*) The solar definition must be good, the image steady, and the sky free from haze.

(*d*) Care must be taken that the reflecting prisms receive light from the desired latitudes.

6. Precautions *a* and *b*, conditions inside the spectrograph, to which may be added the avoidance of undue heating of the slit jaws, are very necessary to prevent systematic displacements of the lines as a whole introducing corresponding errors in the velocity values. If either *a* or *b* are exactly fulfilled an approximate realization of the other should be sufficient; but, as it is practically impossible either to get or keep the plate at the exact focus or to have absolutely equal and uniform illumination of the lens and grating from the opposite limbs, the only safe procedure is to fulfil both conditions as closely as possible. Consequently the plate focus was determined frequently both by the definition test and, as a check, by the Hartmann method of extra-focal exposures. It was found that the field both in the λ5600 and in the λ4250 region was curved, concave to the lens, about 2.5 mm. longer at the centre than at the ends of a plate 30 cm. long and inclined about 1°, in opposite directions for the two regions, to the normal to the axis. The illumination of lens and grating was tested before and after each plate, which consisted usually of seven spectra one of each of the six latitudes from 0° to 75°, and one of the pole. This was done by opening the slit wide enough to allow a visible image of the illuminated concave mirror to be projected on the diaphragmed front surface of the collimating objective. If this image was not central for both systems of prisms it was easily made so by the adjusting screws provided. It was found frequently that a slight change in position of the overlapping images occurred during the time the seven exposures were made, but never sufficient (since the image is considerably larger than the used portion of the grating) to prevent uniform illumination. This change of adjustment of the prisms must be due to the heating produced by the sun's rays and to minimise this effect, the heating of the slit jaws, and the distortion of the coelostat, secondary

and concave mirrors, the coelostat mirror, and consequently the whole system is kept shaded by a blind except during the actual exposures, which occupy from 30 to 60 seconds each.

7. Precautions *c* and *d*, conditions external to the spectrograph, were always carefully looked after. The solar definition during the summer months, on the clear and bright days which only were employed, is usually fairly good and, as undue heating of the mirrors was prevented by keeping them shaded for suitable intervals between the exposures, the definition did not much deteriorate. It is essential that there be fair definition to ensure that the light reaching the slit may be confined to a small region around the desired portion on the sun's disc. Great care was taken in the relative adjustment of guide plate and prisms, so that when the image was kept central and the spectrograph rotated to the desired and previously calculated position angle from the E. W. line (determined by the drift of the solar image when the coelostat clock was stopped), the positions of the points on the disc from which the light was taken were accurately known. This is rendered much easier and more certain by the large size of the solar image (about 227 mm.), and consequently it is improbable that any errors can have arisen either in this regard or due to poor definition. The only effect of the latter would be to introduce a small amount of light at slightly higher and lower latitudes or at greater and less distances from the limb, and the effects thereby produced would practically compensate one another. The necessity of observing only when the sky is free from haze will be evident when it is realized that the effect of the superposed sky spectrum, which is a blend of the spectrum from the whole disc of the sun is, to diminish the displacement and give too low a value of the velocity. DeLury made some experiments on this effect, and found a measurable influence on the equatorial displacement only when the ratio of intensity of sky spectrum to limb spectrum reached about 1 to 20. As on a clear day this ratio is 1 to 100 or less it is evident that no error can thereby be introduced.

Observational Data.

8. The plates were made by the authors jointly, as to make the focus and illumination tests and to guide the sun's image carefully can be much more easily and satisfactorily done by two than one. The dates of the plates used will be given in the tables of measures to follow to save space.

As stated above, in the λ5600 region, rotation spectra of each of the six latitudes to be observed, 0°, 15°, 30°, 45°, 60°, 75°, with one of the pole, 90°, for check purposes were made on each plate, but in the higher latitudes 80° and 85°, three of each with one of the pole were made on each plate. In the λ4250 region, two spectra each of the latitudes 0°, 30°, 60°, and one at 90° were made on each plate. If any of the plates showed a greater displacement in the spectrum at the pole than about .03 km., they were rejected on the assumption that some instrumental displacement had occurred, and that possibly the other latitudes were affected.

9. The plates of series I and III were measured by Plaskett on the Repsold Measuring Engine with an eyepiece micrometer, while those of series II were measured by DeLury on the Toepfer Measuring Machine with 300 mm. screw. The lines measured in series I and II at λ5600 and in series III at λ4250 are given with intensities, velocity, constants, etc., in the following tables. Four settings are made on the line in the centre strip and two each on the outside strips, and after measurement of all the lines the plate is reversed on the machine and remeasured. This diminishes the danger of systematic errors and also, as the lines are viewed in the opposite direction in the two cases and the number of settings doubled, the accidental errors.

TABLE I.—LINES IN λ5600 REGION

No.	Wave Length	Ele.	Int.	Velocity Constant	No.	Wave Length	Ele.	Int.	Velocity Constant
1	5506·095	Mn	1	19·336	11	5598·524	Fe	1	18·801
2	5514·563	Ti	2	19·289	12	5601·505	Ca	3	18·788
3	5514·753	Ti	2	19·287	13	5624·769	Fe	3	18·653
4	5528·641	Mg	8	19·207	14	5638·488	Fe	3	18·575
5	5544·157	Fe	2	19·118	15	5658·097	Y	2	18·461
6	5560·434	Fe	2	19·024	16	5682·869	Na	5	18·320
7	5562·933	Fe	2	19·010	17	5684·710	Si	3	18·309
8	5578·946	Ni	1	18·919	18	5686·757	Fe	3	18·297
9	5582·198	Ca	4	18·899	19	5688·436	Na	6	18·288
10	5590·343	Ca	3	18·852					

TABLE II—LINES IN λ4250 REGION

No.	Wave Length	Ele.	Int.	Velocity Constant	No.	Wave Length	Ele.	Int.	Velocity Constant
1	4196·699	La	2	26·906	9	4257·815	Mn	2	26·400
2	4197·257	C	2	26·902	10	4258·477	Fe	2	26·394
3	4216·136	C	1	26·745	11	4266·081	Mn	2	26·331
4	4220·509	Fe	3	26·710	12	4268·915	Fe	2	26·296
5	4225·619	Fe	3	26·666	13	4276·836	Zr	2	26·243
6	4232·887	Fe	2	26·606	14	4290·377	Ti	2	26·133
7	4241·285	Fe-Zr	2	26·502	15	4291·630	Fe	2	26·122
8	4246·996	Sc	5	26·490					

The lines in the yellow green region were selected to include as many elements as possible among the limited number of measurable lines in the region. Some, such as the lines of Mn, Ti, Si, are not of very good quality for measurement, but were included in order to give evidence in regard to question *c.*, Section 3 above. In the violet region No. 4 to No. 13 inclusive, are the ten lines selected to be measured by all observatories co-operating in this work and the other five are lines which Adams and Lasby* found gave systematically higher or lower values of the rotation than the general reversing layer. The column "Velocity Constant" gives the half value of the multiplier required to reduce the millimetre displacement to kilometres per second, and will evidently give the observed velocity of the sun's limb. These multipliers are readily determined, in the well known way, when the linear dispersion at the region is known. As the grating gives practically a normal spectrum over the narrow limits used, it is sufficient to determine this dispersion, which is about 0.70° Å. per millimetre at λ5600 and 0.75 Å at λ4250, for five or six lines over the region used. When these values and the multipliers are plotted on cross section paper they are found to lie within the errors of observation on a straight line, and the constants for all the lines measured can be at once read off.

Reduction of Measures.

10. The observed or measured velocities are the radial components of the actual velocities at certain points on the sun's disc whose latitudes can be readily computed, and it is hence necessary to know the angle of inclination between the radius vector and the direction of motion at the point in order to apply the necessary corrections, the further correction for the motion of the earth in its orbit being made to obtain the sidereal rate. In the early observations, by Dunér and Halm, of the rotation of the sun by the spectroscopic method, the measurements were made at the limb and the computations and corrections were straightforward. When, however, as in Adams' work and our own the observed points are some distance within the limb, the matter is not quite so simple. Adams' method of reduction** depends upon projecting the observed points radially to the limb and obtaining the corrections by Dunér's methods and tables, but this assumes the rotation of the sun to be that of a solid body, which is of course not the case. A further correction is therefore necessary for the difference in angular velocity at the observed and computed points.

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* Adams and Lasby—An investigation of the Rotation Period of the Sun by Spectroscopic Methods, p. 119.

** Adams and Lasby, p. 13.

Nearly all of Adams' plates were made with the observed points close to the limb, and this final correction is in the majority of cases inappreciable and only reaches in a few plates, around latitudes 45° and 60°, 0.01 km. per second. Nevertheless, as it is always in the same direction, it should be applied. This is especially necessary in our own observations where the distance from the sun's limb is frequently much greater and where the value of the correction may reach 0.03 km. per second. Two methods have been followed here in reducing the observed to the actual velocity. The first consists in applying a correction to Adams' method for the change in angular velocity, thus obtaining the sidereal rate at the radially projected point on the limb, while the second determines the corrections to be applied to obtain the sidereal velocity at the observed points. In order to make the methods clearly understood it will be desirable to give a brief summary of the formulæ used.

Let R = Radius of sun's disc.
r = Distance of observed points from centre of disc.
χ = Position angle of observed point.
φ = Heliographic latitude of observed point.
λ = Difference of heliographic longitude between the observed point and the earth.
D = Heliographic latitude of the earth.
i = Inclination of sun's equator to ecliptic = 7° 15′.
Ω = Longitude of ascending node of sun's equator on ecliptic = 74° 31′.*
$\odot$ = Longitude of the sun.
ρ = Angular distance of observed point from centre of apparent disc as viewed from sun's centre.
η = Angle between direction of motion and line of sight.
s = Sidereal correction at limb (Dunér's Tables).
v = Measured velocity (linear).
V = Corrected velocity.
ξ = Daily angular sidereal velocity.

11. First Method—Projection to Limb.

Latitude at limb. $\quad \operatorname{Sin} \varphi = \cos \chi \sin D$

Angle at limb $\quad \operatorname{Sin} \eta = \dfrac{\sin i \sin (\odot - \Omega)}{\cos \varphi}$

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* At the time of writing new values obtained by the Maunders for i and Ω have appeared but these corrections would introduce only quite inappreciable changes in our computed values.

$$\text{Synodic radial Compt. at limb} = v.\ \frac{R}{r}$$

$$\text{Sidereal " " "} = v.\ \frac{R}{r} + s$$

$$\text{Sidereal velocity of rotation } V = \frac{\xi}{\xi'-}\left(v.\ \frac{R}{r} + s\right) \sec \eta$$

Where ξ and $\xi'-$ are the angular velocities at the limb and at the mean latitude † of the observed points φ_1 and φ_2 obtained from the second method

$$\xi \text{ (Adams)} = 11^\circ.04 + 3^\circ.5 \cos^2 \varphi.$$

12. Second Method—Corrections at observed points.

(*a*) Determine the heliographic latitudes φ_1 and φ_2 of the observed points by the Greenwich method

$$\sin \varphi = \cos \rho \sin D + \sin \rho \cos D \cos \chi$$

$$\left(\sin \rho \text{ and } \cos \rho \text{ obtained from De LaRue's tables argument } \frac{R}{r}\right)$$

also the differences of longitude λ_1 and λ_2

$$\sin \lambda = \sin \chi \sin \rho \sec \varphi.$$

(*b*) Determine the angles η_1, η_2 at the two observed points

$$\cos \eta = \cos D \cos \left(\frac{\pi}{2} - \lambda\right)$$

(*c*) Divide the total sidereal radial velocity into the two following parts proportional to the angular velocities at the latitudes φ_1, φ_2 (obtained closely enough from Adams' formula $\xi = 11^\circ.04 + 3^\circ.5 \cos^2 \varphi$)

$$2\left(v + \frac{r}{R} s\right) \frac{\xi_1}{\xi_1 + \xi_2} \quad , \quad 2\left(v + \frac{r}{R} s\right) \frac{\xi_2}{\xi_1 + \xi_2}$$

(*d*) Sidereal Velocities of Rotation:—

$$V_1 = 2\left(v + \frac{r}{R} s\right) \frac{\xi_1}{\xi_1 + \xi_2} \sec \eta_1$$

$$V_2 = 2\left(v + \frac{r}{R} s\right) \frac{\xi_2}{\xi_1 + \xi_2} \sec \eta_2$$

For *c* and *d* may preferably be substituted the following practically identical but simpler method.

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† Instead of taking the mean latitude $\frac{\varphi_1 + \varphi_2}{2}$ it is more correct to take the angle φ^1 such that $11^\circ.04 + 3.5 \cos^2 \varphi^1 = \frac{1}{2}(11^\circ.04 + 3.5 \cos^2 \varphi_1 + 11^\circ.04 + 3.5 \cos^2 \varphi_2)$. This was not necessary in Series I and III but in Series II this difference in one case reaches 23′ which changes the correction slightly.

(c') Obtain the ratio of V_2 to V_1 from the formula of Adams, or more simply from the curve representing the change of linear velocity with latitude.

(d') The final velocities V_1 and V_2 can then be obtained from the formula

$$V_1 \cos \eta_1 + V_2 \cos \eta_2 = 2\left(v + \frac{r}{R}\, s\right)$$

It may be seen by comparing the residuals in Table IX, Section 19, that they are practically the same for the three reduced values of each observed value obtained by the two methods of reduction, and it is therefore immaterial so far as accuracy is concerned which is employed. Both have been carried through in this investigation for the sake of comparison and to determine which is the more suitable.

Summary of Measures.

13. It is impossible within the limits of this paper to give the separate measures for each spectrum, and so in the succeeding tables a summary of the measures and other necessary data are given. In series I the 19 lines given in the preceding tables were measured on 14 of the 19 plates. On the remaining 5 plates, 8 of the best defined lines only were measured. This number was reduced to diminish the great labor of measurement and because the measures of the 14 plates had shown that, as will be seen later, any differences in rotational value for different elements were accidental in character. Furthermore, even with the reduced number of lines, the probable error of a plate as determined from the internal agreement among the lines was on the average less than half the probable error obtained from the measures of different plates. In series II, however, owing to the much higher probable error of measurement all the lines were measured throughout and in series III also on account of the systematic differences previously found for the different lines by Adams.

In these summaries φ_1 φ_2 and V_1 V_2 represent as above the latitudes and velocities at the observed points on the disc of the sun, while φ and V are the latitudes and velocities at the points radially projected through the observed points to the limb.

Table III—Summary of Measures.

Series I—Measured by Plaskett — λ 5600.

0°

Plate	Date G.M.T. 1911	Measured Velocity	1st Correction Method		2nd Correction Method			
			φ	V at φ	φ_1	φ_2	V_1	V_2
772a	June 15·22	1·812	0° 0′	1·991	0° 15′	0° 15′	1·990	1·990
777a	" 17·13	1·840	0 0	2·022	0 18	0 18	2·025	2·025
779b	" 17·20	1·825	0 0	2·003	0 18	0 18	2·004	2·004
782a	" 19·15	1·854	0 0	2·031	0 18	0 18	2·030	2·030
784d	" 19·20	1·818	0 0	1·995	0 20	0 20	1·997	1·997
787a	" 20·15	1·848	0 0	2·026	0 22	0 22	2·028	2·028
789g	" 21·30	1·824	0 0	1·998	0 23	0 23	1·999	1·999
796a	" 30·23	1·841	0 30	2·036	1 12	0 14	2·035	2·037
804a	July 8·15	1·833	0 0	2·030	0 57	0 57	2·033	2·033
813a	" 20·32	1·858	0 0	2·058	1 13	1 13	2·059	2·059
814g	" 20·35	1·801	0 0	1·999	1 13	1 13	1·999	1·999
817a	" 22·20	1·839	0 4	2·040	1 20	1 13	2·042	2·042
819a	" 27·15	1·809	0 0	2·009	1 22	1 22	2·011	2·011
820a	" 27·19	1·799	0 0	2·000	1 22	1 22	2·002	2·002
821a	" 27·22	1·806	0 0	2·007	1 22	1 22	2·009	2·009
822a	" 27·35	1·800	0 0	2·003	1 24	1 24	2·005	2·005
826a	Aug. 1·15	1·800	0 0	2·004	1 30	1 30	2·006	2·006
827a	" 1·18	1·815	0 0	2·019	1 29	1 29	2·021	2·021
831a	" 1·36	1·840	0 4	2·045	1 32	1 25	2·046	2·046
Means	(Linear)		0 2	2·017	1 1	0 54	2·018	2·018
Means	(Angular)			14°·32			14°·33	14°·33

Probable Error Single Plate........ = ± ·013 km. per second
" " Mean.............. = ± ·003 km. per second

15°

Plate	Date G.M.T. 1911	Measured Velocity	φ	V at φ	φ_1	φ_2	V_1	V_2
772c	June 15·22	1·680	15° 0′	1·844	14° 52′	14° 21′	1·850	1·856
777c	" 17·13	1·732	15 0	1·901	14 54	14 16	1·905	1·912
779c	" 17·20	1·718	15 0	1·891	14 56	14 19	1·892	1·898
782b	" 19·15	1·761	15 0	1·933	15 0	14 18	1·935	1·943
784b	" 19·20	1·672	15 0	1·841	15 0	14 18	1·842	1·850
787b	" 20·15	1·704	15 0	1·868	15 0	14 14	1·868	1·876
789b	" 21·30	1·702	15 0	1·861	15 3	14 16	1·861	1·870
796b	" 30·23	1·751	15 29	1·940	15 43	14 13	1·938	1·958
799b	July 4·17	1·709	14 58	1·902	15 20	13 33	1·901	1·923
804b	" 8·15	1·677	14 58	1·866	15 26	13 28	1·865	1·887
813b	" 20·32	1·645	14 57	1·828	15 43	13 12	1·822	1·852
814b	" 20·35	1·670	14 57	1·855	15 43	13 14	1·845	1·875
817b	" 22·20	1·651	15 0	1·842	15 50	13 11	1·835	1·865
819b	" 27·15	1·722	14 56	1·915	15 51	13 2	1·906	1·941
820b	" 27·19	1·702	14 56	1·896	15 52	13 1	1·887	1·922
821b	" 27·22	1·710	14 56	1·904	15 52	13 1	1·895	1·930
822b	" 27·35	1·760	14 56	1·953	15 52	12 59	1·941	1·977
826b	Aug. 1·15	1·641	14 55	1·837	15 57	12 52	1·826	1·863
827b	" 1·18	1·766	14 55	1·960	15 58	12 53	1·947	1·986
Means	(Linear)		15 0	1·886	15 28	13 37	1·882	1·907
Means	(Angular)			13°·86			13°·86	13°·93

Probable Error Single Plate...................... = ± ·027
" " Mean.......................... = ± ·006

TABLE III.—SUMMARY OF MEASURES.

Series I—Measured by Plaskett— λ5600.

30°

Plate	Date G.M.T. 1911	Measured Velocity	1st Correction Method φ	1st Correction Method V at φ	2nd Correction Method φ_1	2nd Correction Method φ_2	2nd Correction Method V_1	2nd Correction Method V_2
772d	June 15·22	1·493	30° 0′	1·633	29° 26′	28° 52′	1·647	1·658
777d	" 17·13	1·496	29 59	1·639	29 27	28 44	1·651	1·667
779d	" 17·20	1·545	29 59	1·701	29 32	28 50	1·715	1·730
782c	" 19·15	1·483	29 59	1·639	29 37	28 50	1·646	1·662
784c	" 19·20	1·546	29 59	1·699	29 38	28 52	1·706	1·729
787c	" 20·15	1·447	29 59	1·583	29 23	28 45	1·600	1·611
789c	" 21·30	1·487	29 59	1·622	29 42	28 49	1·630	1·649
796c	" 30·23	1·498	30 27	1·665	30 12	28 33	1·672	1·707
804c	July 8·15	1·455	29 56	1·622	29 55	27 44	1·628	1·673
813c	" 20·32	1·468	29 53	1·627	30 13	27 25	1·620	1·680
814c	" 20·35	1·455	29 53	1·614	30 15	27 30	1·605	1·665
817c	" 22·20	1·520	29 56	1·692	30 20	27 15	1·683	1·751
819c	" 27·15	1·440	29 51	1·611	30 23	27 16	1·602	1·669
820c	" 27·19	1·504	29 51	1·678	30 22	27 13	1·670	1·740
821c	" 27·22	1·555	29 51	1·731	30 22	27 13	1·721	1·793
822c	" 27·35	1·517	29 51	1·680	30 22	27 11	1·671	1·741
826c	Aug. 1·15	1·467	30 20	1·642	30 56	27 31	1·630	1·707
827c	" 1·18	1·491	29 50	1·657	30 27	27 3	1·642	1·716
831c	" 1·36	1·494	29 46	1·660	30 24	27 2	1·645	1·719
Means	(Linear)		29 58	1·652	30 5	27 56	1·652	1·698
Means	(Angular)			13°·54			13°·55	13°·64

Probable Error Single Plate .... = ± ·025
" " Mean........................... = ± ·006

45°

Plate	Date G.M.T. 1911	Measured Velocity	1st Correction Method φ	1st Correction Method V at φ	2nd Correction Method φ_1	2nd Correction Method φ_2	2nd Correction Method V_1	2nd Correction Method V_2
772e	June 15·22	1·133	44° 59′	1·238	43° 54′	43° 13′	1·269	1·286
777e	" 17·13	1·063	44 59	1·168	43 51	43 0	1·197	1·220
779e	" 17·20	1·172	44 59	1·271	44 1	43 10	1·300	1·323
782d	" 19·15	1·081	44 59	1·205	44 4	43 5	1·231	1·256
784[1]a	" 19·20	1·163	44 59	1·288	44 9	43 12	1·312	1·336
787d	" 20·15	1·048	44 59	1·151	44 5	43 1	1·176	1·200
789d	" 21·30	1·126	44 58	1·227	44 12	43 8	1·249	1·280
796d	" 30·23	1·181	45 26	1·316	44 34	42 34	1·350	1·394
804d	July 8·15	1·142	44 53	1·275	44 17	41 39	1·295	1·370
813d	" 20·32	1·193	44 48	1·317	44 40	41 16	1·321	1·416
814d	" 20·35	1·171	44 48	1·294	44 42	41 23	1·296	1·389
817d	" 22·20	1·195	44 43	1·336	44 47	41 17	1·340	1·440
819d	" 27·15	1·134	44 44	1·274	44 51	41 6	1·275	1·375
820d	" 27·19	1·125	44 44	1·266	44 50	41 3	1·266	1·368
821d	" 27·22	1·083	44 44	1·221	44 50	41 3	1·221	1·319
822d	" 27·35	1·162	44 44	1·288	44 49	40 59	1·268	1·394
826d	Aug. 1·15	1·227	44 42	1·372	44 55	40 50	1·370	1·487
827d	" 1·18	1·274	44 42	1·406	44 55	40 51	1·402	1·522
831d	" 1·36	1·152	44 39	1·279	44 52	40 51	1·276	1·384
Means	(Lingear)		44 52	1·273	44 29	41 56	1·286	1·356
Means	(Angular)			12°·75			12°·80	12°·94

Probable Error Single Plate........................ = ± ·042
" " Mean............................ = ± ·010

TABLE III.—SUMMARY OF MEASURES—*Continued.*

Series I—Measured by Plaskett— λ 5600.

60°

Plate	Date G.M.T. 1911	Measured Velocity	1st Correction Method		2nd Correction Method			
			φ	V at φ	φ_1	φ_2	V_1	V_2
772f	June 15·22	·643	59° 59′	·703	58° 1′	57° 4′	·754	·778
777f	" 17·13	·700	59 58	·764	57 52	56 41	·821	·852
779f	" 17·20	·791	59 58	·883	58 8	57 0	·940	·974
782e	" 19·15	·724	59 58	·814	58 14	56 54	·865	·901
784^{1}b	" 19·20	·795	59 58	·886	58 15	56 59	·940	·978
787e	" 20·15	·800	59 57	·867	58 16	56 50	·918	·958
789e	" 21·30	·703	59 57	·765	58 29	57 1	·810	·851
796e	" 30·23	·621	60 22	·715	58 34	55 55	·761	·827
804e	July 8·15	·850	59 48	·950	58 22	54 55	·999	1·112
813e	" 20·32	·643	59 38	·721	58 52	54 21	·742	·852
814e	" 20·35	·692	59 38	·772	58 58	54 32	·789	·908
817e	" 22·20	.771	59 41	·871	58 49	54 11	·900	1·036
819e	" 27·15	·689	59 33	·789	59 9	54 9	·799	·931
820e	" 27·19	·652	59 33	·754	59 4	54 1	·762	·892
821e	" 27·22	·703	59 33	·803	59 4	54 1	·821	·957
822e	" 27·35	·702	59 33	·785	59 3	53 59	·802	·936
826e	Aug. 1·15	·723	59 29	·826	59 10	53 45	·840	·992
827e	" 1·18	·731	59 29	·819	59 12	53 48	·830	·977
831e	" 1·36	·796	59 26	·886	59 10	53 49	·898	1·056
Means	(Linear)		59 46	·809	58 40	55 16	·842	·935
Means	(Angular)			11°·41			11°·50	11°·65

Probable Error Single Plate........................ = ± ·042
" " Mean.......................... = ± ·010

75°

Plate	Date G.M.T. 1911	Measured Velocity	φ	V at φ	φ_1	φ_2	V_1	V_2
772g	June 15·22	·379	74° 57′	·403	71° 0′	69° 30′	·512	·554
777g	" 17·13	·329	74 56	·353	70 44	68 52	·455	·500
779g	" 17·20	·408	74 56	·463	71 11	69 23	·583	·638
782g	" 19·15	·314	74 55	·367	71 22	69 16	·457	·509
784^{1}c	" 19·20	·383	74 55	·438	71 28	69 22	·544	·607
787f	" 20·15	·389	74 54	·414	71 21	69 5	·513	·576
789f	" 21·30	·342	74 53	·366	71 49	69 28	·444	·502
796f	" 30·23	·329	75 14	·390	71 27	67 20	·493	·601
804f	July 8·15	·294	74 33	·355	71 24	66 9	·429	·493
813f	" 20·32	·351	74 15	·400	72 13	65 21	·453	·630
814f	" 20·35	·370	74 15	·418	72 18	65 33	·473	·658
817f	" 22·20	·373	74 15	·445	72 26	65 21	·500	·704
819f	" 27·15	·304	74 3	·374	72 44	65 4	·407	·590
820f	" 27·19	·415	74 3	·494	72 32	64 52	·544	·788
821f	" 27·22	·407	74 3	·485	72 32	64 52	·535	·774
822f	" 27·35	·344	74 3	·398	72 30	64 47	·439	·638
826f	Aug. 1·15	·395	73 56	·474	72 42	64 30	·515	·759
827f	" 1·18	·371	73 56	·431	72 46	64 34	·467	·691
831f	" 1·36	·393	73 52	·456	72 46	64 37	·490	·722
Means	(Linear)		74 28	·417	71 58	66 44	·487	·628
Means	(Angular)			11°·06			11°·17	11°·29

Probable Error Single Plate........................ = ± ·026
" " Mean.......................... = ± ·006

TABLE III.—SUMMARY OF MEASURES.

Series I—Measured by Plaskett— λ 5600. 80°

Plate	Date G.M,T. 1911	Measured Velocity	1st Correction Method φ	1st Correction Method V at φ	2nd Correction Method φ_1	2nd Correction Method φ_2	2nd Correction Method V_1	2nd Correction Method V_2
780a	June. 17·23	·221	79° 55	·231	74° 57′	72° 48′	·349	·395
c	" 17·23	·178	79 55	·189	74 57	72 48	·283	·324
e	" 17·23	·206	79 55	·217	74 57	72 48	·326	·373
781a	" 17·24	·241	79 55	·284	74 53	72 43	·429	·490
c	" 17·24	·216	80 54	·257	75 25	73 10	·421	·486
e	" 17·24	·260	79 55	·303	74 44	72 32	·463	·529
783a	" 19·17	·171	79 53	·181	75 16	72 46	·266	·311
c	" 19·17	·201	79 53	·213	75 16	72 46	·311	·364
e	" 19·17	·191	79 53	·202	75 16	72 46	·296	·346
785a	" 19·18	·244	79 53	·289	75 30	73 0	·417	·487
c	" 19·18	·226	79 53	·270	75 30	73 0	·391	·457
e	" 19·18	·226	79 53	·270	75 30	73 0	·391	·457
788a	" 20·16	·210.	79 52	·222	75 20	72 35	·323	·384
c	" 20·16	·217	79 52	·236	75 20	72 35	·334	·397
e	" 20·16	·190	79 52	·202	75 20	72 35	·294	·349
790a	" 21·31	·300	79 50	·316	75 23	72 31	·455	·543
c	" 21·31	·260	79 50	·273	75 19	72 25	·399	·478
e	" 21·31	·269	79 50	·285	75 9	72 15	·417	·499
Means	(Linear)		79 53	·247	75 13	72 43	·365	·426
Means	(Angular)			9°·98			10°·16	10°·18

Probable Error Single Plate........................ = ± ·027
" " Mean............................ = ± ·006

85°

Plate	Date G.M,T. 1911	Measured Velocity	1st Correction Method φ	1st Correction Method V at φ	2nd Correction Method φ_1	2nd Correction Method φ_2	2nd Correction Method V_1	2nd Correction Method V_2
780b	June 17·23	·092	84° 49′	·100	77° 38′	75° 6′	·245	·291
d	" 17·23	·119	84 49	·129	77 38	75 6	·313	·372
f	" 17·23	·135	84 49	·145	77 38	75 6	·353	·419
781b	" 17·24	·109	84 49	·142	77 38	75 3	·345	·411
d	" 17·24	·136	84 49	·168	77 27	74 54	·420	·495
f	" 17·24	·133	84 49	·165	77 22	74 48	·415	·490
783b	" 19·17	·107	84 45	·118	78 1	75 3	·272	·334
d	" 19·17	·098	84 45	·107	78 1	75 3	·250	·307
f	" 19·17	·084	84 45	·092	78 1	75 3	·216	·266
785b	" 19·18	·129	84 45	·165	78 17	75 20	·379	·463
d	" 19·18	·161	84 45	·199	78 17	75 20	·458	·559
f	" 19·18	·145	84 45	·182	78 17	75 20	·418	·510
788b	" 20·16	·103	84 43	·113	78 5	74 53	·259	·328
d	" 20·16	·088	84 43	·097	78 5	74 53	.223	·283
f	" 20·16	·104	84 43	·114	78 5	74 53	·262	·332
790b	" 21·31	·098	84 50	·108	78 5	74 40	·255	·322
d	" 21·31	·084	84 50	·094	78 0	74 33	·217	·274
f	" 21·31	·111	84 50	·123	77 48	74 22	·288	·364
Means	(Linear)		84 47	·131	77 55	74 58	·310	·379
Means	(Angular)			10°·23			10°·51	10°·37

Probable Error Single Plate........................ = ± ·020 per km.
" " Mean............................ = ± ·005 per km.

TABLE IV.—SUMMARY OF MEASURES.

Series II.—Measured by DeLury—λ5600.

0°

Plate	Date G.M.T. 1911	Measured Velocity	1st Correction Method		2nd Correction Method			
			φ	V at φ	φ_1	φ_2	V_1	V_2
L 833	Aug. 10·18	1·715	0° 0′	1·994	2° 23′	2° 23′	1·993	1·993
834	" 10·21	1·771	0 0	2·054	2 23	2 23	2·052	2·052
836	" 30·15	1·565	0 0	1·844	2 46	2 46	1·844	1·844
837	" 30·18	1·615	0 0	1·898	2 45	2 45	1·897	1·897
838	" 30·20	1·664	0 0	1·949	2 45	2 45	1·950	1.950
839	" 30.22	1·682	0 0	1·970	2 45	2 45	1.970	1·970
842	Sep. 8·13	1·651	0 0	1.949	2 52	2 52	1·947	1.947
843	" 8·16	1·763	0 0	2·068	2 51	2 51	2·068	2·068
844	" 8·18	1·671	0 0	1·966	2 50	2 50	1·966	1·966
845	" 8·21	1·633	0 0	1·925	2 50	2 50	1·924	1·924
846	" 8·22	1·626	0 0	1·920	2 52	2 52	1·920	1·920
847	" 8·30	1·605	0 0	1·896	2 52	2 52	1·896	1·896
848	" 8·31	1·643	0 0	1·938	2 51	2 51	1·936	1·936
849	" 8·33	1·619	0 0	1·910	2 51	2 51	1·910	1·910
851	" 11·20	1·705	0 0	2·005	2 50	2 50	2·004	2·004
L 852	" 11·21	1·616	0 0	1·908	2 51	2 51	1·908	1·908
Means	(Linear)		0 0	1·950	2 46	2 46	1·949	1·949
Means	(Angular)			13°·84			13°·85	13°·85

Probable Error Single Plate........................= ± ·038
" " Mean.............................. ± ·009

15°

Plate	Date G.M.T. 1911	Measured Velocity	φ	V at φ	φ_1	φ_2	V_1	V_2
L 833	Aug. 10·18	1·526	14° 54′	1·780	16° 17′	11° 23′	1·768	1·831
834	" 10·21	1·526	14 54	1·779	16 17	11 24	1·763	1·826
836	" 30·15	1·533	14 53	1·801	16 34	10 52	1·780	1·852
837	" 30·18	1·406	14 53	1·661	16 36	10 56	1·641	1·709
838	" 30·20	1·461	14 53	1·720	16 34	10 55	1·670	1·738
839	" 30·42	1·529	14 53	1·795	16 36	10 56	1·773	1·846
842	Sept. 8·13	1·454	14 41	1·722	16 25	10 32	1·701	1·770
843	" 8·16	1.580	15 5	1·859	16 47	10 55	1·834	1·912
844	" 8·18	1·568	15 5	1·845	16 47	10 56	1·821	1·898
845	" 8·21	1·574	15 5	1·851	16 47	10 57	1·827	1·903
846	" 8·22	1·667	15 5	1·956	16 47	10 53	1·930	2·012
847	" 8·30	1·639	15 5	1·925	16 47	10 54	1·900	1·981
848	" 8·31	1·657	15 5	1·943	16 47	10 55	1·918	1·999
849	" 8·33	1·590	15 5	1·870	16 47	10 55	1·845	1·923
851	" 11·20	1·614	15 5	1·896	16 47	10 56	1·871	1·950
L 852	" 11·21	1·649	15 5	1·935	16 47	10 54	1·910	1·991
Means	(Linear)		14 59	1·834	16 39	10 57	1·810	1·884
Means	(Angular)			13°·48			13°·41	13°·62

Probable error single plate........................= ± ·058
" " Mean.............................. ± ·014

TABLE IV.—SUMMARY OF MEASURES.

Series II.—Measured by DeLury.—λ 5600.

30°

Plate	Date G.M.T. 1911	Measured Velocity	1st Correction Method		2nd Correction Method			
			Z	V at Z	Z_1	Z_2	V_1	V_2
L 833	Aug. 10·18	1·394	29° 48′	1·617	30° 11′	24° 48′	1·618	1·722
834	" 10·21	1·402	29 48	1·626	30 11	24 49	1·626	1·731
836	" 30·15	1·379	29 44	1·613	30 24	24 10	1·606	1·726
837	" 30·18	1·443	29 44	1·680	30 28	24 17	1·672	1·799
838	" 30·21	1·390	29 44	1·622	30 24	24 13	1·633	1·755
839	" 30·22	1·320	29 44	1·547	30 28	24 17	1·540	1·656
842	Sep. 8·14	1·482	29 26	1·731	30 5	23 41	1·727	1·859
843	" 8·17	1·369	30 2	1·606	30 39	24 14	1·599	1·726
844	" 8·18	1·371	30 2	1·608	30 38	24 11	1·604	1·730
845	" 8·21	1·411	30 2	1·650	30 40	24 16	1·644	1·772
846	" 8·22	1·425	30 2	1·668	30 38	24 11	1·662	1·793
847	" 8·30	1·384	30 2	1·623	30 38	24 12	1·618	1·746
848	" 8·31	1·510	30 2	1·758	30 39	24 14	1·752	1·890
849	" 8.33	1·472	30 2	1·718	30 39	24 14	1·711	1·846
831	" 11·20	1·495	30 2	1·743	30 39	24 14	1·739	1·876
L 852	" 11·21	1·409	30 2	1·650	30 38	24 12	1·644	1·774
Means	(Linear)		29 53	1·654	30 30	24 16	1·650	1·775
Means	(Angular)			13°·54			13°·60	13°·82

Probable Error Single Plate........................= ± ·039
" " Mean " ....= ± ·010

45°

Plate	Date G.M.T. 1911	Measured Velocity	Z	V at Z	Z_1	Z_2	V_1	V_2
L 833	Aug. 10·19	1·143	44° 39′	1·322	43° 55′	37° 37′	1·353	1·517
834	" 10·30	1·003	44 39	1·172	43 56	37 39	1·201	1·344
836	" 30·15	1·122	44 33	1·306	44 5	36 48	1·328	1·516
837	" 30·19	1·168	44 33	1·354	44 12	36 58	1·376	1·568
838	" 30·21	1·098	44 33	1·278	44 5	36 53	1·292	1·472
839	" 30·22	1·126	44 33	1·309	44 12	36 58	1·330	1·515
842	Sep. 8·14	1·065	44 3	1·248	43 31	36 4	1·271	1·499
843	" 8·17	1·050	45 2	1·231	44 26	36 56	1·255	1·439
844	" 8·19	1·064	45 2	1·246	44 24	36 52	1·270	1·455
845	" 8·22	1·017	45 2	1·195	44 24	36 52	1·221	1·399
846	" 8·23	·981	45 2	1·057	44 24	36 52	1·178	1·350
847	" 8·30	1·028	45 2	1·207	44 25	36 53	1·232	1·412
848	" 8·32	1·060	45 2	1·241	44 26	36 56	1·266	1·451
849	" 8·33	1·106	45 2	1·291	44 26	36 56	1·317	1·510
851	" 11·20	1·006	45 2	1·183	44 26	36 56	1·208	1·384
L 852	" 11·22	1·086	45 2	1·270	44 24	36 53	1·298	1·487
Means	(Linear)		44 58	1·251	44 14	36 56	1·274	1·457
Means	(Angular)			12°·52			12°·62	12°·94

Probable Error Single Plate........................= ± ·052
" " Mean " ....= ± ·013

Table IV.—Summary of Measures.

Series II.—Measured by DeLury.—λ5600

60°

Plate	Date G.M.T. 1911	Measured Velocity	1st Correction Method		2nd Correction Method			
			φ	V at φ	φ_1	φ_2	V_1	V_2
L 833	Aug. 10·19	·717	59° 23′	·839	57° 10′	49° 11′	·897	1·145
834	" 10·30	·699	59 23	·820	57 12	49 14	·875	1·117
836	" 30·16	·678	59 14	·801	57 15	48 7	·847	1·116
837	" 30·19	·624	59 14	·740	57 22	48 21	·782	1·029
838	" 30·21	·634	59 14	·751	57 17	48 13	·795	1·047
839	" 30·23	·681	59 14	·802	57 22	48 21	·847	1·115
842	Sep. 8·15	·682	58 15	·805	56 12	47 9	·853	1·114
843	" 8·18	·595	60 11	·708	57 45	48 25	·766	1·020
844	" 8·19	·588	60 11	·706	57 52	48 19	·755	1·012
845	" 8·22	·569	60 11	·684	57 52	48 19	·733	·983
846	" 8·23	·603	60 11	·722	57 52	48 19	·773	1·036
847	" 8·30	·563	60 11	·679	57 53	48 21	·726	·973
848	" 8·32	·630	60 11	·750	57 45	48 25	·806	1·075
849	" 8·33	·563	60 11	·678	57 45	48 25	·728	·971
851	" 11·20	·719	60 11	·847	57 55	48 26	·906	1·212
852	" 11·22	·584	60 11	·701	57 52	48 21	·751	1·005
Means	(Linear)		59 47	·752	57 31	48 16	·809	1·061
Means	(Angular)		10°·61	10°·61			10°·70	11°·08

Probable Error Single Plate........................= ± ·039
" " Mean " ....= ± ·010

75°

Plate	Date G.M.T. 1911	Measured Velocity	φ	V at φ	φ_1	φ_2	V_1	V_2
L 833	Aug. 10·19	·319	73° 43′	·397	68° 48′	58° 6′	·524	·796
834	" 10·30	·332	73 43	·412	68 51	58 10	·543	·832
836	" 30·16	·344	73 24	·431	68 50	56 43	·554	·895
837	" 30·19	·320	73 24	·403	69 12	57 2	·510	·821
838	" 30·22	·384	73 24	·476	68 54	56 51	·611	·986
839	" 30·23	·419	73 24	·516	69 12	57 2	·654	1·063
842	Sep. 8·15	·246	75 8	·325	69 33	56 51	·534	·888
843	" 8·18	·270	75 8	·354	69 46	57 2	·488	·818
844	" 8·19	·246	75 8	·325	69 39	56 54	·452	·755
845	" 8·22	·213	75 8	·285	69 39	56 54	·398	·665
846	" 8·23	·273	75 8	·357	69 39	56 54	·496	·829
847	" 8·30	·332	75 8	·427	69 41	56 56	·592	·991
848	" 8·32	·252	75 8	·331	69 46	57 2	·459	·768
849	" 8·33	·319	75 8	·412	69 46	57 2	·568	·952
851	" 11·20	·244	75 8	·323	69 45	57 3	·448	·748
852	" 11·22	·310	75 8	·402	69 40	56 57	·558	·933
Means	(Linear)		74 31	·386	69 25	57 6	·525	·859
Means	(Angular)			10°·37			10°·61	11°·23

Probable Error Single Plate........................= ± ·041
" " Mean " ....= ± ·010

Table V.—Summary of Measures.

Series III.—Measured by Plaskett—λ 4250 0°

Plate	Date G.M.T. 1911	Measured Velocity	1st Correction Method		2nd Correction Method			
			φ	V at φ	φ_1	φ_2	V_1	V_2
859a	Oct. 3·13	1·764	0° 0′	2·022	2° 11	2° 11′	2·025	2·025
b	" 3·13	1·746	0 0	2·005	2 11	2 11	2·006	2·006
860a	" 3·17	1·733	0 0	1·992	2 14	2 14	1·994	1·994
b	" 3·17	1·779	0 0	2·042	2 14	2 14	2·043	2·043
861a	" 3·18	1·780	0 0	2·043	2 14	2 14	2·044	2·044
b	" 3·18	1·758	0 0	2·019	2 14	2 14	2·021	2·021
865a	" 5·28	1·710	0 0	1·962	2 8	2 8	1·962	1·962
b	" 5·28	1·784	0 0	2·040	2 8	2 8	2·041	2·041
866a	" 5·30	1·774	0 0	2·030	2 9	2 9	2·031	2·031
b	" 5·30	1·799	0 0	2·056	2 9	2 9	2·058	2·058
867a	" 5·32	1·766	0 0	2·022	2 9	2 9	2·023	2·023
b	" 5·32	1·817	0 0	2·076	2 9	2 9	2·077	2·077
869a	" 7·13	1·719	0 0	1·976	2 10	2 10	1·978	1·978
b	" 7·13	1·770	0 0	2·031	2 10	2 10	2·033	2·033
870a	" 7·15	1·735	0 0	1·992	2 9	2 9	1·994	1·994
b	" 7·15	1·742	0 0	1·999	2 9	2 9	2·002	2·002
871a	" 7·18	1·751	0 0	2·009	2 9	2 9	2·011	2·011
b	" 7·18	1·725	0 0	1·982	2 9	2 9	1·983	1·983
872a	" 7·19	1·749	0 0	2·007	2 9	2 9	2·009	2·009
b	" 7·19	1·758	0 0	2·016	2 9	2 9	2·019	2·019
873a	" 9·12	1·713	0 0	1·968	2 6	2 6	1·969	1·969
b	" 9·12	1·727	0 0	1·982	2 6	2 6	1·984	1·984
874a	" 9·14	1·749	0 0	2·006	2 6	2 6	2·008	2·008
b	" 9·14	1·749	0 0	2·006	2 6	2 6	2·008	2·008
Means	Linear		0 0	2·012	2 10	2 10	2·013	2·013
Means	Angular			14°·28			14°·30	14°·30

Probable Error Single Plate........................ = ± ·018
" " Mean " .... = ± ·004

TABLE V.—SUMMARY OF MEASURES.

Series I Measured by Plaskett— λ 4250. 30°

Plate	Date G.M.T. 1911	Measured Velocity	1st Correction Method		2nd Correction Method			
			φ	V at φ	φ_1	$_2\varphi$	V_1	V_2
859c	Oct. 3·13	1·410	29° 59′	1·611	30° 33′	25° 36′	1·606	1·711
860c	" 3·17	1·450	29 59	1·664	30 35	25 31	1·653	1·767
d	" 3·17	1·450	29 59	1·658	30 35	25 31	1·645	1·753
861c	" 3·18	1·429	29 59	1·632	30 35	25 31	1·623	1·735
d	" 3·18	1·419	29 59	1·622	30 35	25 31	1·612	1·723
863c	" 5·13	1·416	29 59	1·619	30 35	25 39	1·608	1·713
865c	" 5·28	1·376	29 59	1·572	30 35	25 43	1·561	1·666
d	" 5·28	1·374	29 59	1·570	30 35	25 43	1·559	1·665
866c	" 5·30	1·408	29 59	1·606	30 35	25 43	1·594	1·703
d	" 5·30	1·400	29 59	1·597	30 35	25 43	1·585	1·695
867c	" 5·32	1·391	29 59	1·587	30 35	25 43	1·577	1·683
d	" 5·32	1·445	29 59	1·645	30 35	25 43	1·634	1·746
869c	" 7·13	1·428	30 0	1·630	30 31	25 39	1·620	1·731
d	" 7·13	1·444	30 0	1·648	30 31	25 39	1·637	1·748
870c	" 7·15	1·453	30 0	1·657	30 31	25 39	1·646	1·759
d	" 7·15	1·460	30 0	1·665	30 31	25 39	1·654	1·767
871c	" 7·18	1·469	30 0	1·674	30 31	25 39	1·664	1·777
d	" 7·18	1·390	30 0	1·590	30 31	25 39	1·581	1·687
872c	" 7·19	1·480	30 0	1·686	30 31	25 39	1·674	1·791
d	" 7·19	1·425	30 0	1·626	30 31	25 39	1·617	1·727
873c	" 9·12	1·415	30 0	1·815	30 29	25 43	1·608	1·712
d	" 9·12	1·416	30 0	1·616	30 29	25 43	1·609	1·713
874c	" 9·14	1·412	30 0	1·612	30 29	25 43	1·605	1·709
d	" 9·14	1·414	30 0	1·614	30 29	25 43	1·606	1·711
Means	Linear		29 59	1·625	30 33	25 39	1·616	1·725
Means	Angular			13°·32			13°·32	13°·59

Probable Error Single Plate ....= ± ·020
" " Mean " ....= ± ·004

TABLE V.—SUMMARY OF MEASURES.

Series III.—Measured by Plaskett.—λ 4250. 60°

Plate	Date G.M.T. 1911	Measured Velocity	1st Correction Method		2nd Correction Method			
			φ	V at φ	φ_1	φ_2	V_1	V_2
859f	Oct. 3·13	·730	59° 50′	·834	58° 24′	50° 51′	·887	1·100
860e	" 3·17	·748	59 50	·854	58 25	50 42	·901	1·123
f	" 3·17	·725	59 50	·830	58 25	50 42	·875	1·096
861e	" 3·18	·723	59 50	·829	58 25	50 42	·873	1·094
f	" 3·18	·705	59 50	·810	58 25	50 42	·853	1·069
863e	" 5·13	·649	59 50	·739	58 25	50 42	·790	·988
f	" 5·13	·645	59 50	·735	58 25	50 42	·786	·983
865e	" 5·28	·600	59 52	·695	58 33	51 6	·732	·910
f	" 5·28	·620	59 52	·715	58 33	51 6	·753	·938
866e	" 5·30	·617	59 52	·711	58 33	51 6	·749	·935
f	" 5·30	·620	59 52	·714	58 33	51 6	·753	·939
867e	" 5·32	·597	59 52	·692	58 33	51 6	·728	·906
f	" 5·32	·586	59 53	·679	58 33	51 6	·716	·892
869e	" 7·13	·733	59 53	·836	58 21	50 55	·885	1·099
f	" 7·13	·712	59 53	·813	58 21	50 55	·861	1·070
870e	" 7·15	·724	59 53	·825	58 21	50 55	·875	1·086
f	" 7·15	·726	59 53	·828	58 21	50 55	·877	1·089
871e	" 7·18	·733	59 53	·836	58 21	50 55	·885	1·098
f	" 7·18	·742	59 55	·846	58 21	50 55	·894	1·111
872e	" 7·19	·734	59 55	·836	58 21	50 55	·886	1·099
873e	" 9·12	·713	59 55	·813	58 19	51 55	·864	1·070
f	" 9·12	·714	59 55	·814	58 19	51 2	·865	1·071
874e	" 9·14	·709	59 55	·809	58 19	51 2	·859	1·064
f	" 9·14	·709	59 55	·809	58 19	51 2	·859	1·064
Means	Linear		59 53	·788	58 25	50 55	·834	1·037
Means	Angular			11°·15			11°·30	11°·68

Probable Error Single Plate .... = ± ·039
" " Mean " .... = ± ·008

Comparison of Measures.

14. Before discussing the velocity and the law of its variation with the latitude, it is desirable to attempt an explanation of the systematic difference between the values of the velocity obtained by Plaskett and De Lury. Although the plates used are not the same, the difference persists when the same plates are measured by the two observers, as will be seen later. In the early measures of rotation plates the fields of the measuring microscopes were left unmasked, but later, as the quantity of light getting through was very fatiguing to the eye, diaphragms were arranged to cut out part of the illumination. This was effected in the case of the Repsold measuring machine used by Plaskett by placing a thin disc with three slots cut in it in the eyepiece of the micrometer just above the focal plane. By this means only the three strips of spectra were visible, the light coming through on the outside and between the strips being occulted by the disc. In the case of the Toepfer machine used by De Lury, a single slit of the width of one of the spectral strips was cut in a brass plate which was held by a movable arm attached to the rigid microscope carriage close to the plate and which could be, by a convenient screw, readily moved transversely, so that in measuring, only one of the three strips of spectrum could be seen at a time. This latter arrangement was devised by De Lury to keep the configuration of the spectrum lines the same for each measurement, because he found that his measures were affected by the configuration of the lines in the three strips;* and, further, to keep himself in ignorance of the magnitude and direction of the displacement so that his measures could in no way be affected by prejudice. For this latter reason also he postponed his computations of the velocities until all the measurements of his series were completed. On the other hand, as only part of the line in the strip being measured can be seen distinctly at one time, as the eye has to move up and down to make the best placing of the wire, it was felt by Plaskett, that as the other strips could not be seen while the setting was being made, they could not influence his measures in any way. Consequently the simpler expedient of a fixed diaphragm occulting only the extraneous light was deemed sufficient by him. This is corroborated by the fact that the difference between Plaskett and De Lury is practically constant at all latitudes (except the pole), although the relative displacement of the lines in the spectra varies widely. On the other hand, the measures of 15 equatorial spectra by De Lury both with and without the mask gave a systematically greater value for the former of 0.012

* Jour. Roy. Astron. Soc. Can. 5, 384–407.

km. on the average. It is an open question, as these measures were made at different epochs, whether this difference is to be ascribed to the use of the mask or to a change in the habit of measurement. The measures were made with great care by both observers and in precisely the same way:—four settings on the line in the centre strip and two on each of the outside strips with the screw moving alternately forward and back, and, after all the lines were measured, repeated with the plate reversed on the carriage. Moreover, as the measures are purely differential, the displacement of one absorption line with respect to another precisely similar absorption line, the presence of this comparatively large systematic difference between the two observers is not readily explainable. Different methods of measurement and various comparisons were made in an attempt to explain or overcome the difficulty, but the difference still persisted practically unchanged in magnitude and sign throughout. It is proposed * by De Lury in order to obtain absolute values of the displacement, which are uncertain under present circumstances, to impress upon the spectra, in addition to the rotation displacement, an arbitrary displacement of say the order of a millimetre in magnitude. This would be effected by using a double or broken slit, the central section (of the width of one of the spectral strips) being displaced laterally any desired distance with respect to the body of the slit. If a rotation spectrum be made through such a slit the displacement will be s + r, where r is the rotation displacement and s the displacement due to the slit. If a spectrum of the limb at the pole where there is no rotational displacement be made through this slit the displacement will be s. The measured value of these displacements will be $s + r + e$ and $s + e$ where e is the error of measurement, varying with different observers, yet which should (for each observer) have the same value *in the mean* of a large series of measures, as the two displacements are relatively of nearly the same magnitude. The true value of the rotational displacement will then be

$$s + r + e - (s + e) = r.$$

and in this result personal habits of measurements should be eliminated. Besides the mechanical and observational difficulties in the way of this proposal, however, there is the further one that the accidental error of measurement would be increased and the amount of measuring required doubled. Furthermore, as these spectra could not be taken under identical conditions, the possibility of instrumental errors affecting the results is rather a serious one. Even with rotation spectra

* Jour. Roy. Astron. Soc. Can. 5, 405.

made directly following one another on the same plate, and under apparently identical conditions, such errors creep in, as for example in Table V in the equator plates of series III. In plates 860, 865, 867, 869, the difference in the displacements of successive exposures are 0.066, .074, .051, .051 km. per second, a greater difference than the one in question. Consequently although the method will be tried later it was not deemed desirable to delay further the publication of the obtained values, but to determine if possible, the probable corrections to be applied to the above given velocities.

15. For this purpose all the equator spectra of Series I and 7 of Series III were measured by De Lury and all of Series II by Plaskett to determine systematic differences at the equator. In addition, to see how this difference varied with the latitude, 5 complete plates (7 latitudes on each) of Series I were measured by De Lury and 5 complete pates of Series II by Plaskett. Two representative plates of Series I, Nos. 813 and 820 were sent to Mt. Wilson and were kindly measured by Mr. Adams and Miss Lasby in order to compare Ottawa and Mt. Wilson measures. All these comparisons are tabulated below and serve to show not only the difference in velocity obtained by different measures from the same plates which appear to be generally systematic in character, but indicate also the accidental errors of measurement to be looked for. The detailed measures for plates 813 and 820 show the great differences in accuracy of setting, for the probable error of setting on a single line (given below the means) varies on the average from 0.008 by Miss Lasby to 0.019 by Adams and Plaskett to 0.052 km. per sec. by De Lury equivalent, in linear values, to 0.0004, .001, and .003 mm.

TABLE VI.—COMPARISONS OF MEASURES.

Plates at Equator.

Series I.

Plate	Plaskett	De Lury (Mask)	Diff. P-D
772	1·812	1·770	·042
777	1·840	1·814	26
779	1·850	1·839	11
782	1·854	1·832	22
784	1·818	1·740	78
787	1·848	1·786	62
789	1·776	1·770	6
796	1·841	1·805	36
804	1·833	1·748	85
813	1·858	1·845	13
814	1·801	1·794	7
817	1·839	1·823	16
819	1·809	1·784	25
820	1·792*	1·750*	42
821	1·806	1·744	62
822	1·800	1·734	66
826	1·800	1·709	91
827	1·815	1·713	102
831	1·840	1·823	17
Means	1·823	1·780	·043

Series II.

Plate	Plaskett	De Lury (No Mask)	De Lury (Mask)	Diff. D (Mask–No Mask)	Diff. P-D (Mask)
833	1·770	....	1·715	....	·055
834	1·807	1·765	1·771	6	36
836	....	1·549	1·565	16	....
837	1·660	1·608	1·615	7	45
838	1·694	1·651	1·664	7	30
839	1·703	1·651	1·682	31	21
842	1·731	1·644	1·651	7	80
843	1·786	1·732	1·763	31	23
844	1·709	1·690	1·671	–19	38
845	1·695*	1·611*	1·620	9	75
846	1·669	1·639	1·626	–13	43
847	1·694	1·574	1·605	31	89
848	1·705	1·632	1·643	11	62
849	1·638	1·580	1·619	39	19
851	1·772	1·691	1·705	14	67
852	1·679	1·607	1·616	9	63
Means	1·711	1·642	1·658	12	·050

Series III.

Plate	Plaskett	De Lury	Diff. P-D
859	1·764	1·791	–·027
860	1·733	1·721	12
861	1·780	1·776	4
865	1·710	1·674	36
866	1·774	1·768	6
867	1·766	1·716	50
869	1·719	1·752	– 33
Means	1·749	1·742	+·007

* Mean Values.

Table VII.—Comparisons of Measures.

Plates with all Latitudes.

Series I.

		0°		15°		30°		45°		60°		75°		90°	
Plate	Observer	Measures	Diff.	Meas.	Diff.	Meas.	Diff.	Meas.	Diff.	Meas.	Diff.	Meas.	Diff.	Meas.	Diff.
813	P	1·858	....	1·645	....	1·468	....	1·193	....	·643	....	·351	....	—·001	....
813	D	1·827	+ 31	1·627	+ 18	1·456	+ 12	1·170	+ 23	·635	+ 8	·331	+ 20	+·034	— 35
814	P	1·801	....	1·670	....	1·455	....	1·171	....	·692	....	·370	....	+·029	
814	D	1·805	— 4	1·599	+ 71	1·402	+ 53	1·117	+ 54	·670	+ 22	·343	+ 27	+·063	— 34
817	P	1·839	....	1·651	....	1·520	....	1·195	....	·771	....	·373	....	—·006	
817	D	1·828	+ 11	1·578	+ 73	1·458	+ 62	1·138	+ 57	·743	+ 28	·362	+ 11	+·017	— 23
819	P	1·809	....	1·722	....	1·440	....	1·134	....	·689	....	·304	....	—·044	
819	D	1·787	+ 22	1·767	— 45	1·452	— 12	1·118	+ 16	·678	+ 11	·318	— 14	—·081	+ 37
820	P	1·799	....	1·702	....	1·504	....	1·125	....	·652	....	·415	....	+·001	
820	D	1·757	+ 42	1·675	+ 27	1·404	+100	1·055	+ 70	·676	— 24	·345	+ 70	+·006	— 5
Mean Diffs. P.—D.			+ 20		+ 29		+ 43		+ 44		+ 9		+ 23		— 12

Table VII.—Comparisons of Measures.—*Continued.*

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Table VII.—Comparisons of Measures.—*Continued.*

Plates with all Latitudes.

Series II.

		0°		15°		30°		45°		60°		75°		90°	
Plate	Observer	Measures	Diff.	Meas.	Diff.	Meas.	Diff.	Meas.	Diff.	Meas.	Diff.	Meas.	Diff.	Meas.	Diff.
839	P	1·683	....	1·501	....	1·372	....	1·156	....	·696	....	·421	....	+ ·056	
839	D	1·682	+ 1	1·529	— 28	1·320	+ 52	1·126	+ 30	·681	+ 15	·419	+ 2	+ ·015	+ 41
842	P	1·717	....	1·519	....	1·499	....	1·132	....	·723	....	·247	....	—·007	
842	D	1·651	+ 66	1·454	+ 65	1·482	+ 17	1·065	+ 67	·682	+ 41	·246	+ 1	—·028	+ 21
843	P	1·786	......	1·593	....	1·398	....	1·087	....	·624	....	·315	....	+ ·025	
843	D	1·763	+ 23	1·580	+ 13	1·369	+ 29	1·050	+ 37	·595	+ 29	·270	+ 45	+ ·043	— 18
844	P	1·709	....	1·595	....	1·425	....	1·059	....	·630	....	·261	....	+ ·001	
844	D	1·671	+ 38	1·568	+ 27	1·371	+ 54	1·064	— 5	·588	+ 42	·246	+ 15	—·020	+ 21
845	P	1·679	....	1·601	....	1·410	....	1·048	....	·619	....	·230	....	+ ·023	
845	D	1·633	+ 46	1·574	+ 27	1·411	— 1	1·017	+ 31	·569	+ 50	·213	+ 17	+ ·011	+ 12
Mean Diffs. P.–D.			+ 35		+ 21		+ 30		+ 32		+ 35		+ 16		+ 15
Mean Diffs. Series I and	II		+ 27		+ 25		+ 36		+ 38		+ 22		+ 20		+ 1

TABLE VIII.—COMPARISON OF OTTAWA & MT. WILSON MEASURES.

Plate 813.

No. of Line	0°			15°			30°		
	Plaskett	De Lury	Lasby	Plaskett	De Lury	Lasby	Plaskett	De Lury	Lasby
1	1·843	1·935	1·861	1·667	1·508	1·746	1·435	1·322	1·471
2	1·864	1·874	1·870	1·551	1·512	1·717	1·478	1·514	1·503
3	1·872	1·846	1·872	1·636	1·543	1·719	1·484	1·519	1·510
4	1·842	1·778	1·858	1·659	1·730	1·719	1·463	1·408	1·515
5	1·833	1·759	1·858	1·631	1·585	1·718	1·450	1·428	1·489
6	1·866	1·814	1·849	1·730	1·846	1·688	1·476	1·497	1·500
7	1·853	1·882	1·858	1·651	1·654	1·716	1·446	1·365	1·464
8	1·819	1·689	1·841	1·655	1·617	1·726	1·481	1·562	1·493
9	1·872	1·827	1·844	1·658	1·727	1·715	1·452	1·358	1·453
10	1·878	1·893	1·856	1·652	1·630	1·705	1·438	1·494	1·467
11	1·870	1·855	1·850	1·637	1·466	1·711	1·512	1·457	1·458
12	1·865	1·796	1·849	1·620	1·593	1·710	1·452	1·533	1·476
13	1·846	1·917	1·857	1·644	1·675	1·710	1·483	1·405	1·474
14	1·866	1·840	1·844	1·711	1·580	1·702	1·482	1·491	1·488
15	1·861	1·761	1·841	1·633	1·812	1·691	1·440	1·373	1·462
16	1·845	1·813	1·862	1·594	1·594	1·691	1·463	1·432	1·461
17	1·871	1·882	1·831	1·656	1·580	1·701	1·490	1·518	1·462
18	1·870	1·731	1·856	1·621	1·612	1·691	1·489	1·522	1·479
19	1·860	1·812	1·863	1·657	1·656	1·702	1·470	1·470	1·476
Means	1·858	1·827	1·854	1·645	1·627	1·710	1·468	1·456	1·479
P. Error single line	±·010	·043	·007	·021	·068	·010	·015	·046	·012

TABLE VIII.—COMPARISON OF OTTAWA & MT. WILSON MEASURES.

TABLE VIII.—COMPARISON OF OTTAWA & MT. WILSON MEASURES.

Plate 813.

No. of Line	45°			60°			75°		
	Plaskett	De Lury	Lasby	Plaskett	De Lury	Lasby	Plaskett	De Lury	Lasby
1	1·193	1·040	1·126	0·742	0·738	0·640	0·369	0·411	0·333
2	1·214	1·261	1·132	·615	·473	·642	·345	·440	·329
3	1·235	1·236	1·140	·642	·549	·646	·276	·165	·325
4	1·191	1·202	1·126	·677	·773	·658	·281	·324	·344
5	1·182	1·171	1·145	·666	·529	·637	·363	·301	·327
6	1·174	1·027	1·148	·689	·677	·641	·324	·345	·351
7	1·165	1·079	1·138	·640	·712	·645	·350	·331	·330
8	1·200	1·284	1·123	·686	·788	656	·356	·359	·333
9	1·177	1·241	1·118	·615	·619	·646	·395	·360	·338
10	1·234	1·191	1·128	·651	·628	·651	·369	·354	·338
11	1·200	1·208	1·140	·652	·686	683	·380	·338	·333
12	1·225	1·224	1·154	·648	·747	·648	·348	·257	·350
13	1·229	1·318	1·118	·596	·600	·636	·304	·283	·338
14	1·158	1·119	1·101	·640	·659	·643	·409	·434	·340
15	1·147	1·107	1·113	·580	·552	·633	·388	·443	·337
16	1·142	1·100	1·124	·603	·644	·638	·356	·184	·323
17	1·195	1·201	1·132	·610	·556	·646	·337	·349	·341
18	1·219	1·127	1·127	·635	·560	·653	·309	·250	·319
19	1·182	1·093	1·133	·630	·581	·647	·406	·355	·326
Means	1·193	1·176	1·130	0·643	0·635	0·644	0·351	0·331	0·334
P. Error single line	± ·019	·060	·009	·023	·059	·005	·025	·050	·006

Table VIII.—Comparison of Ottawa & Mt Wilson Measures.

Plate 820.

No. of Line	0°						15°		
	Plaskett 1.	Plaskett 2.	De Lury 1.	De Lury 2.	Lasby	Adams	Plaskett	De Lury	Lasby
1	1·852	1·823	1·818	1·869	1·850	1·797	1·725	1·784	1·758
2	1·849	1·814	1·721	1·622	1·847	1·821	1·729	1·635	1·746
3	1·790	1·854	1·711	1·836	1·851	1·876	1·739	1·790	1·778
4	1·753	1·783	1·823	1·823	1·854	1·817	1·687	1·508	1·761
5	1·824	1·847	1·812	1·699	1·841	1·785	1·746	1·719	1·790
6	1·826	1·776	1·729	1·725	1·868	1·779	1·688	1·655	1·751
7	1·783	1·744	1·808	1·762	1·861	1·806	1·696	1·692	1·759
8	1·810	1·765	1·733	1·757	1·861	1·774	1·696	1·697	1·767
9	1·795	1·782	1·671	1·682	1·855	1·800	1·696	1·695	1·742
10	1·780	1·770	1·727	1·854	1·854	1·770	1.696	1·642	1·758
11	1·764	1·754	1·628	1·693	1·855	1·776	1·713	1·676	1·765
12	1·830	1·780	1·707	1·687	1·871	1·793	1·712	1·621	1·762
13	1·787	1·749	1·746	1·805	1·839	1·786	1·721	1·635	1·755
14	1·823	1·805	1·804	1·779	1·851	1·808	1·699	1·745	1·765
15	1·806	1·824	1·787	1·818	1·848	1·855	1·705	1·759	1·749
16	1·756	1·746	1·663	1·709	1·841	1·767	1·650	1·606	1·763
17	1·778	1·781	1·677	1·788	1·835	1·767	1·653	1·613	1·762
18	1·760	1·751	1·735	1·745	1·856	1·785	1·706	1·769	1·743
19	1·808	1·749	1·827	1·721	1·829	1·802	1·685	1·576	1·746
Means	1·799	1·784	1·744	1·757	1·851	1·798	1·702	1·675	1·758
P. Error Single line	±·020	·023	·043	·047	·007	·019	·017	·052	·008

16. The comparison of plates at the equator shows a systematic difference for measures of the same plates of 0.046 km. per second. When the 5 complete plates of Series I and II are compared it is found that in these plates the average difference at the equator is smaller about 0.027, and that this remains unchanged practically for all latitudes except the pole. This shows that the difference is evidently not due to any effect of the magnitude of the displacement of the lines of one strip with respect to the other, else it should vary with the latter which changes from 0.1 mm. at equator to about 0.017 mm. at 75°. It may be said therefore that Plaskett measures the displacements from 0.03 to 0.05 km. per second higher than De Lury in the region at λ 5600. The peculiar nature of the difference P — D at the pole should not pass without comment. The mean value of this difference is $+0{\cdot}001$. Although it is of the same sign as the other differences in the Series II plates it is of the opposite sign in Series I, and is hence not systematic as at the other latitudes, and it might therefore be regarded as evidence that the magnitude or sense of the displacement influences the measures of one or both of the observers. Owing to the method of measurement used by De Lury he would seem to be less likely to be influenced in this way. When we compare the measures in the λ4250 region we find that the difference found in the λ 5600 region nearly vanishes, being only 0.007 km., scarcely large enough considering the few plates measured by De Lury to be deemed systematic. The spectra in the λ4250 region are much more easily measurable than at λ 5600. Not only is the grain of the plate finer, but the lines themselves are much more uniform in character and better defined. Consequently it seems likely that the large difference between the two measures in the λ 5600 region depends in some way upon the character of the lines for measurement. Although the probable error of measurement of a single line, given for plates 813 and 820 above, for Plaskett is only about a third of that for De Lury, ± 0.019 and ± 0.052 km. per sec., and hence the former's measures should be considered of greater weight, yet that does not settle the question of the correct value of the velocity. Possibly some information may be obtained from the Mt. Wilson measures.

17. Mr. Adams and Miss Lasby have had greater experience than anyone else in the measurement of photographic rotation spectra, and their measurements should be given great weight. Yet when we come to make comparisons, Table VIII, plates 813 and 820 we find practically the same difficulties and the same differences between them as between the writers. For example, in plate 820 at the equator we have Miss Lasby's value 1.851, Mr. Adams 1,798, Plaskett's 1,799 and 1,784, De Lury's 1,757 and 1,744. Indeed in several cases Miss Lasby's value is as much higher than Plaskett's, as his is than De Lury's. On the

other hand, in 813, 45° it is lower than both, and in 813, 60° and 75° all three are practically the same. When we compare these differences with the probable error of measurement of the plates, less than one quarter of the numbers given at the bottom of the tables varying from .002 to .015, we are forced to the conclusion that they are systematic and personal in nature, but are at a loss to account for their cause.

It is unfortunate that Mr. Adams was unable to measure more than one spectrum, but the close agreement of his result with Plaskett's and the generally higher values of Miss Lasby and lower of De Lury would naturally, from the law of averages, lead to the acceptance of Adams' and of Plaskett's measures as probably being nearest to the true values. If such a conclusion be accepted then it would be necessary to apply a positive correction to De Lury's measures in the λ 5600 region, which, when all the comparisons are taken into account, should be about 0.040 km. at the equator and possibly slightly less at the higher latitudes. A further evidence that this is probably the proper course is given by the practical agreement of Plaskett's and De Lury's measures in Series III at λ 4250. As the velocities of rotation obtained by Plaskett from the measures of Series I, II, and III are all practically the same, while those obtained by De Lury are about 3 per cent. lower for Series I and II, but the same for Series III, the inference is that, in the poorer quality lines in the yellow green, some personal effect causes the difference and that this disappears when the lines become better defined, as is the case in the violet. On the other hand, if there be no systematic differences in the measuring of the line displacements by De Lury at the two regions λ 4250 and λ 5600, this would imply a difference in the rates of rotation as determined from lines of different wave length, a thing which though in itself not impossible is perhaps not very probable.

Absolute Value of Velocity.

Variation of Velocity with Latitude.

18. The above discussion and comparison of measures have shown that it is hardly possible to state exactly the absolute velocity of the rotation of the sun and furthermore if, as seems likely, earlier determinations were affected in the same way, they are also uncertain to to the same extent, that of the "personal equation" of measurement.

19. In order to place the preceding summaries of measures in a more convenient form for discussion and comparison, the following tables containing the observed mean linear velocities at the mean latitudes have been compiled. From these linear velocities, the observed angular velocities have been directly computed, while the other columns will be explained below:—

TABLE IX.—SUMMARY

Series I.

Latitude	Linear Velocities.			Angular Velocities.		
	Observed	Computed	Residual (0—C)	Observed	Computed	Residual (0—C)
0° 2′	2·017	2·014	+ 0·003	14° ·32	14° ·40	— 0°·08
0 54	2·018	2·014	+ ·004	14 ·33	14 ·40	— ·07
1 1	2·018	2·014	+ ·004	14 ·33	14 ·40	— ·07
13 37	1·907	1·928	— ·021	13 ·93	14 ·17	— ·24
15 0	1·886	1·910	— ·024	13 ·86	14· 12	— ·26
15 28	1·882	1·905	— ·023	13 ·86	14 ·10	— ·24
27 56	1·698	1·679	+ ·019	13 ·64	13 ·50	+ ·14
29 58	1·652	1·632	+ ·020	13 ·54	13 ·38	+ ·16
30 5	1·652	1·630	+ ·022	13 ·55	13· 37	+ ·18
41 56	1·356	1·328	+ ·028	12 ·94	12 ·58	+ ·36
44 29	1·286	1·258	+ ·028	12 ·80	12 ·40	+ ·40
44 52	1·273	1·246	+ ·027	12 ·75	12 ·38	+ ·37
55 16	·935	·951	— ·016	11 ·65	11 ·66	— ·01
58 40	·842	·854	— ·012	11 ·50	11 ·44	+ ·06
59 46	·809	·823	— ·014	11 ·41	11 ·37	+ ·04
66 44	·628	·625	+ ·003	11 ·29	10 ·97	+ ·32
71 58	·487	·481	+ 006	11 ·17	10 ·73	+ ·44
72 43	·426	·460	— ·034	10 ·18	10 ·70	— ·52
74 28	·417	·413	+ ·004	11 ·06	10 ·66	+ ·40
74 58	·379	·399	— ·020	10 ·37	10 ·62	— ·25
75 13	·365	·392	— ·027	10 ·16	10 ·61	— ·45
77 55	·310	·320	— 010	10 ·51	10 ·53	— ·02
79 53	·247	·267	— ·020	9 ·98	10 ·46	— ·48
84 47	·131	·137	— ·006	10 ·23	10 ·37	— ·14

TABLE IX.—SUMMARY.

Series II

Latitude	Linear Velocities.			Angular Velocities.		
	Observed	Computed	Residual (0—C)	Observed	Computed	Residual (0—C)
0° 0′	1·950	1·971	— 0·021	13°·84	14°·04	— 0°·20
2 45	1·949	1·969	— ·020	13 ·85	14 ·03	— ·18
10 56	1·884	1·917	— ·033	13 ·62	13 ·89	— ·27
14 59	1·834	1·870	— ·036	13 ·48	13 ·78	— ·30
16 40	1·810	1·847	— ·037	13 ·41	13 ·72	— ·31
24 16	1·775	1·716	+ ·059	13 ·82	13 ·37	+ ·45
29 53	1·654	1·596	+ ·058	13 ·54	13 ·04	+ ·50
30 30	1·650	1·583	+ ·067	13 ·60	13 .01	+ ·59
36 56	1·457	1·424	+ ·033	12 ·94	12 ·60	+ ·34
44 14	1·274	1·230	+ ·044	12 ·62	12 ·10	+ ·52
44 48	1·251	1·215	+ ·036	12 ·52	12 ·06	+ ·46
48 23	1·061	1·115	— ·054	11 ·08	11 ·81	— ·73
57 6	·859	·871	— ·012	11 ·23	11 ·23	0
57 32	·809	·859	— ·050	10 ·70	11 ·20	— ·50
59 47	·752	·796	— ·044	10 ·61	11 ·06	— ·55
69 26	·525	·532	— ·007	10 ·61	10 ·54	+ ·07
74 31	·386	·397	— ·011	10 ·37	10 ·33	+ ·04

Series III.

Latitude	Linear Velocities.			Angular Velocities.		
	Observed	Computed	Residual (0—C)	Observed	Computed	Residual (0—C)
0° 0′	2·012	2·020	— 0·008	14 ·28	14 ·33	— ·05
2 10	2·013	2·018	— ·005	14 ·30	14 ·33	— ·03
25 39	1·725	1·720	+ ·005	13 ·59	13 ·54	+ ·05
29 59	1·625	1·619	+ ·006	13 ·32	13 ·27	+ ·05
30 33	1·616	1·605	+ ·011	13 ·32	13 ·23	+ ·09
50 55	1·037	1·046	— ·009	11 ·68	11 ·78	— ·10
58 25	·834	·830	+ ·004	11 ·30	11 ·26	+ ·04
59 53	·788	·789	— ·001	11 ·15	11 ·16	— · 01

20. From these mean values about one-third of which are due to Method I of reduction and two-thirds to Method II, the law of variation of latitude has to be obtained. Many different forms containing both sine and cosine terms of the latitude in different powers were tried and, although some gave close agreement, none, on the whole, were as good as the simple Faye formulæ

$$V = (a + b \cos^2 \varphi) \cos \varphi$$
$$\xi = a' + b' \cos^2 \varphi.$$

Using the method of least squares to determine the constants the following formulæ were obtained.

$$\text{Series I} \begin{cases} V = (1.504 + .509 \cos^2 \varphi) \cos \varphi \\ \xi = 10^\circ.34 + 4^\circ.06 \cos^2 \varphi \end{cases}$$

$$\text{Series II} \begin{cases} V = (1.448 + .523 \cos^2 \varphi) \cos \varphi \\ \xi = 10^\circ.04 + 4^\circ.00 \cos^2 \varphi \end{cases}$$

$$\text{Series III} \begin{cases} V = (1.421 + .599 \cos^2 \varphi) \cos \varphi \\ \xi = 10^\circ.10 + 4^\circ.23 \cos^2 \varphi \end{cases}$$

From these formulæ the values in columns headed "Computed" and "Residual" in the preceding tables (Table IX) were obtained. The residuals in Series I and III are satisfactorily small and show no tendency to systematic arrangement of sign. In Series II, however, they are considerably larger and systematically grouped as to sign, indicating the necessity of an additional term in the Faye formula.

If the observations of Series I and III are grouped together we get formulæ which represent the observations in both series nearly as well as the separate formulæ. The difference between the formulæ for Series I and III above is probably due to the small number of latitudes observed (only three) in Series III, in which case a small deviation of one of the values would make a large change in the co-efficients. The formulæ from both Series

$$\text{Series I and III (combd.)} \begin{cases} V = (1.483 + .532 \cos^2 \varphi) \cos \varphi \\ \xi = 10^\circ.32 + 4^\circ.05 \cos^2 \varphi \end{cases}$$

may therefore be considered as the formulæ obtained from Plaskett's measurements. Series II is not included in this on account of the systematic difference and because another term would be necessary to obtain reasonable agreement between the observed and computed values. However, if we compare the co-efficients from Series II with those from Series I and III combined we find them practically the same except for the difference in the first terms which is in line with what has been found by comparison of the measures. Moreover, this difference, when the necessary allowance is made for the difference of the co-efficients of the second terms, is 0.044 km. or 0°.33 which is not far from the assumed 0.040 km.

21. For convenience of comparison the previously obtained formulæ are tabulated beside those just given and we at once notice a remarkable similarity between the Ottawa and Mt. Wilson co-efficients.

TABLE X.—FORMULAE FOR SOLAR ROTATION.

Observer.	V, Linear Velocities.	ξ, Angular Velocities.
Dunér............	....	$10°{\cdot}60 + 4°{\cdot}21 \cos^2\varphi$
Halm............	....	$12{\cdot}03 + 2{\cdot}50 \cos^2\varphi$
Adams (1906–7)....	$(1{\cdot}575 + 0{\cdot}480 \cos^2\varphi) \cos\varphi$	
" (1908)......	$(1{\cdot}507 + 0{\cdot}546 \cos^2\varphi) \cos\varphi$	$10{\cdot}57 + 4{\cdot}04 \cos^2\varphi$
Adams (Mean).....	$(1{\cdot}550 + 0{\cdot}501 \cos^2\varphi) \cos\varphi$	$11{\cdot}04 + 3{\cdot}50 \cos^2\varphi$
Plaskett (1911).....	$(1{\cdot}483 + 0{\cdot}532 \cos^2\varphi) \cos\varphi$	$10{\cdot}32 + 4{\cdot}05 \cos^2\varphi$
De Lury (1911)....	$(1{\cdot}448 + 0{\cdot}523 \cos^2\varphi) \cos\varphi$	$10{\cdot}04 + 4{\cdot}00 \cos^2\varphi$

This is especially the case with the 1908 Mt. Wilson determination and the mean formulæ from Series I and III where, in the angular form, the difference is only in the constant term. In the linear form also they are quite similar, and their agreement in both forms is so marked as compared with the widely different co-efficients obtained from the 1906-7 Mt. Wilson observations as to confirm the presence of some systematic error in the latter, suspected by Adams, and to indicate the substantial accuracy of the law of variation obtained.

22. For convenience of comparison the daily angular value of the rotational velocity has been computed from the empirical formulæ given in the preceding table for the latitudes from the equator to the pole by intervals of 5°. A column containing the results of Storey and Wilson* at Edinburgh is added and a column for the velocities of sunspots, the means from three formulæ given in Adams' † work. Further the linear velocities from Adams, 1908, and Plaskett's formulæ have been computed and are given in the two last columns.

* M. N. LXII, p. 674.

† Adams & Lasby, p. 118.

TABLE XI.—VELOCITIES OF ROTATION.

Latitude	Daily Angular Velocities.								Linear Velocities.	
	Sun Spots	Duner	Halm	1906–7 Adams	1908 Adams	1908–10 Storey and Wilson	1911 Plaskett	1911 DeLury	1908 Adams	1911 Plaskett
0°	14°·40	14°·81	14°·53	14°·63	14°·61	14°·81	14°·37	14°·05	2·053	2·015
5	14 ·38	14 ·78	14 ·50	14 ·59	14 ·58	14 ·72	14 ·34	14 ·02	2·041	2·003
10	14 ·31	14 ·68	14 ·46	14 ·50	14 ·49	14 ·59	14 ·25	13 ·93	2·007	1·968
15	14 ·20	14 ·53	14 ·37	14 ·37	14 ·34	14 ·46	14 ·10	13 ·78	1·948	1·912
20	14 ·06	14 ·32	14 ·24	14 ·17	14 ·13	14 ·32	13 ·89	13 ·58	1·869	1·855
25	13 ·89	14 ·06	14 ·09	13 ·94	13 ·89	14 ·15	13 ·65	13 ·34	1·772	1·740
30	13 ·69	13 ·76	13 ·90	13 ·67	13 ·60	13 ·97	13 ·36	13 ·05	1·659	1·630
35	13 ·47	13 ·42	13 ·70	13 ·39	13 ·28	13 ·74	13 ·04	12 ·73	1·535	1·508
40	....	13 ·07	13 ·50	13 ·09	12 ·94	13 ·52	12 ·70	12 ·40	1·400	1·375
45	....	12 ·70	13 ·28	12 ·81	12 ·58	13 ·26	12 ·34	12 ·05	1·259	1·237
50	....	12 ·34	13 ·07	12 ·54	12 ·24	13 ·01	11 ·99	11 ·70	1·113	1·093
55	....	11 ·99	12 ·86	12 ·30	11 ·91	12 ·71	11 ·65	11 ·37	·967	·950
60	....	11 ·65	12 ·66	12 ·11	11 ·58	12 ·43	11 ·33	11 ·05	·821	·807
65	....	11 ·35	12 ·48	11 ·97	11 ·29	12 ·04	11 ·04	10 ·76	·677	·666
70	....	11 ·09	12 ·32	11 ·91	11 ·05	11 ·64	10 ·80	10 ·52	·538	·528
75	....	10 ·88	12 ·20	11 ·91	10 ·84	11 ·24	10 ·59	10 ·32	·399	·392
80	....	10 ·74	12 ·11	12 ·00	10 ·69	....	10 ·44	10 ·17	·265	·260
85	....	10 ·63	12 ·05	12 ·17	10 ·60	....	10 ·35	10 ·08	·130	·128
90	....	10 ·60	12 ·03	12 ·43	10 ·57	....	10 ·32	10 ·05	0	0

The agreement of Dunér's, Adams', 1908, and the Ottawa values, except for small and nearly constant differences, is quite striking, and gives good grounds for the belief that the law of variation with latitude is represented to a high degree of accuracy by a Faye formula with coefficients approximately the same as those given in these three formulæ.

23. In regard to the absolute value of the rotational velocity the question can not be regarded as by any means settled. Considering the velocity values at the lower latitudes we find that Halm and Adams get nearly the same values, Dunér and Storey and Wilson are about 1 per cent. higher, Plaskett about 2 per cent. lower and De Lury about 4 per cent. lower. But at the higher latitudes Dunér and Adams (1908) agree, Plaskett is 2 per cent. lower as before, De Lury about 5 per cent. lower, Storey and Wilson are 5 per cent. higher, while Halm and Adams (1906-7) are some 15 or 20 per cent. higher. At the equator Plaskett's values are in practical agreement with the motion of sun spots. As it is generally considered that the reversing layer and sunspots are at the same level from the practical identity of their spectra, this so far as it goes gives weight to the lower value of 14°.4 at the equator. On the other hand as the latitude increases the sun spot velocities agree better with the higher values of the reversing layer such as those of Halm and of Adams', 1906-7 observations.

24. These differences in values may be due to one or more of three causes:—*a*. A variation in the rate of rotation of the Sun. *b*. Instrumental errors. *c*. Personal errors of measurement.

(*a*) The question of a change in the rotational velocity of the Sun, which was raised by Halm *, was quite fully discussed by Adams,† who reached the conclusion that the evidence to date was against variation. The later values by Storey and Wilson and those obtained here, of which the former is higher and the latter lower than Adams' results, would indicate a variation in the rate of rotation were it not for the possibility of small instrumental and the probability of personal measurement errors (Sections 15-17). As it is, until the latter are eliminated, it will be impossible to make any definite statement in regard to either the variation or constancy of the rate. Certainly the possibility of a variation must, until further evidence is available, be taken into account in considering the difference obtained.

(*b*) So far as instrumental errors are concerned although every known precaution was taken to avoid them, it is possible that some small systematic effects may be present in these results. The only

* A. N. 173 p. 294.

† Adams & Lasby, p. 115.

means of detecting such an error would be by the comparison of spectra made at the same epoch by different instruments and methods and measured by the same observer, but such is not easy to arrange. The differences in value for successive plates taken under so far as known identical conditions (previously referred to in Section 16) is most likely due to some sort of instrumental error unless rapid changes in local motions in the reversing layer are responsible. Although these differences are apparently quite accidental they may nevertheless contain a small systematic deviation.

(*c*) Personal Errors of Measurement.—It has been shown (Sections 15-17) that it is possible, even probable, for such differences as those in question to be obtained on measurement of the same plate by different observers, and it seems useless to consider other sources of error until it is possible to eliminate this. Although the difference between Plaskett and De Lury is fairly well determined at λ 5600 as at present about 0.040 km. per second, sufficient plates in common have not yet been measured to determine the difference between Miss Lasby, by whom most of the Mt. Wilson plates were measured, and the writers. Her measures appear to be somewhat higher on the whole (Section 17) than Plaskett's, and the same tendency was shown even more markedly during a visit of the latter to Mt. Wilson in 1910, where comparisons of the measured displacements of several lines on rotation plates at the equator showed that Miss Lasby's measures were always two or three per cent. higher than Plaskett's. If there is this difference, then the actual velocity displacements on the Mt. Wilson and Ottawa plates are approximately the same, and it only remains to determine whose measurement is the most nearly correct. At present, however, we will have to be satisfied with recognizing the presence of personal differences of measurement, as accounting for part at any rate of the differences in velocity obtained.

25. In view of these actual differences of velocity obtained by the different observers and after the discussion of the probable causes of these differences, we can only state that the velocity of the solar rotation as determined from Plaskett's measurements is represented by the formulæ

$$V = (\ 1.483 \ + 0.532 \ \cos^2 \varphi) \cos \varphi$$
$$\xi = \ 10^\circ.32 \ + 4^\circ.05 \ \cos_2 \varphi$$

and that this angular formula differs from Adams' 1908 formula practically only in the constant term and is also in good agreement with Dunér's, and that hence it probably represents very closely the relative velocities at the different latitudes, although the absolute values may be uncertain by, say, 2 per cent.

PROBABLE ERRORS.

26. As Adams * has already compared his errors of measurement with those of Dunér and Halm, showing the marked advantage of the photographic method, it will suffice here to give the Ottawa values and compare them with Adams.

The mean probable error of measurement of the velocity from a single line determined by the use of *all* the lines on *all* the plates is

Series I = ± 0.024 km. per sec.
Series III = ± 0.015 ,, ,, ,,

The probable errors in Series I vary for the different plates from 0·010 to 0.040 and in Series III from 0.006 to 0.023. As the number of lines measured in each plate in the two series have been 19 and 15 respectively the probable error of an average plate as determined from the internal agreement of the measure is

Series I = ± 0.0055 km. per sec.
Series III = ± 0.0038 „ „ „

The average probable error of a plate determined from comparisons of the velocities of all plates at the same latitudes and for all the latitudes is for

Series I = ± 0.028 km. per sec.
Series III = ± 0.026 „ „ „

or 5 and 7 times the probable error as determined from the internal agreement of the lines.

These somewhat anomalous results are however not unusual as about the same ratio of probable errors is obtained in stellar radial velocity work and in many other astrophotographic methods, but the cause of this comparatively high ratio can not be satisfactorily explained.

One can imagine that changing instrumental conditions might cause differences in displacement in plates taken on different dates but where, as in the example previously cited, differences of from 0.05 to 0.07 km. were found on exposures taken one immediately after the other on the same plate on the same region of the sun and under, so far as known, identical conditions, no explanation, except that of rapidly changing proper motions on the sun, can be assigned.

27. In comparing these probable errors with those of Mt. Wilson, only series III which is in the same region, λ 4250, as the Mt. Wilson plates must be considered for, as the relative probable errors indicate,

* Adams & Lasby, p. 117.

the lines are of much better quality for measurement than at λ 5600. When the probable errors (in kilometres) are reduced to linear measure they become more than twice as great at λ 5600 as at λ 4250. The probable errors for a single line obtained at Mt. Wilson are

p. e. = ± 0.015 km. per sec. (1906-7).
p. e. = ± 0.009 " " " (1908)

The Ottawa value as above stated is ± 0.015. It must not be forgotten, however, that the Mt. Wilson values are from one or two plates, the Ottawa from the mean of all the plates; On the Mt. Wilson plates the lines giving, systematically, velocities differing from the mean were excluded, on the Ottawa plates these and all lines were included; and lastly that the Mt. Wilson linear dispersion was in 1906-7, 10 per cent. and in 1908 30 per cent. greater than the Ottawa. Hence it is evident that the probable error of measurement is about the same at the two places. Although the probable error of a plate determined from the agreement among the plates is not given, it is readily computed, and for the equator (1908) is ± 0.011 km. per sec. as compared with ± 0.018 here. This is considerably smaller, but yet about 5 times that obtained from agreement among the lines.

28. It is evident from the ratios of the probable errors that a great many more lines than necessary for the actual determination of the rotation have been measured, and that it would be preferable to measure four or five times as many plates with only one fourth or fifth the number of lines, and that even then the probable error obtained from comparison of the plates would be twice that deduced from the internal agreement of the lines. However, in this investigation a larger number of lines was measured for the purpose of determining whether different elements and different lines of the same element give different velocities of rotation.

Systematic Differences of Velocity for Different Elements.

29. Considerable attention has been devoted to this phase of the investigation which is of importance not only because of its interest in the theory of the sun, but also because it was one of the questions proposed by the Rotation Committee, and because Adams has found some small systematic differences for different elements and his results should be confirmed.

As previously mentioned in the λ 5600 region the lines were chosen particularly with this point in view and include as large a number of elements as is possible among the limited number available for measurement. Similarly in the λ 4250 region besides the 10 lines selected for measurement by the committee 5 other lines, embracing those found by Adams to give systematic deviations were included.

30. The following table contains the mean residuals in metres per second grouped according to latitude, obtained from Plaskett's measures of about 14 plates and from DeLury's measures of 16 plates at λ 5600. The first three columns contain the wave-length and the source and intensity of the lines measured. The next seven columns contain the mean residuals, taking account of the signs, at the seven different latitudes observed. Each of these is the mean of the residuals from 14 or 16 plates. The separate residuals are not given on account of the space that would be required. The last column but one is the mean of all the residuals without regard to sign or the average residual, while in the last column the sign is taken into account, and we have the mean algebraic residual. At the foot of the columns the mean probable errors of measurement of a single line at each latitude are given.

TABLE XII.—RESIDUALS IN REGION λ5500—λ5700—Plaskett.

Wave Length	Element	Intensity	Mean Algebraic Residuals at Latitudes.							No. of Measures	Mean Numi. Residual	Mean Algebc. Residual
			0°	15	30°	45°	60°	75°	90°			
5506·095	Mn	1	+ 0	— 2	+ 4	+ 21	— 13	0	+ 14	97	32	+ 3
5514·563	Ti	2	+ 22	+ 14	+ 11	— 3	— 24	— 1	— 9	81	36	+ 1
5514·753	Ti	2	+ 22	+ 21	+ 19	+ 4	— 13	— 19	+ 28	97	41	+ 9
5528·641	Mg	8	— 16	— 7	— 16	+ 2	— 5	— 5	— 18	97	27	— 9
5544·157	Fe	2	+ 7	+ 5	— 31	+ 9	— 7	0	— 20	97	28	— 5
5560·434	Fe	2	+ 6	+ 12	0	+ 12	+ 9	— 3	+ 5	97	32	+ 6
5562·933	Fe	2	+ 12	— 7	— 7	— 9	+ 3	— 1	+ 2	97	25	— 1
5578·946	Ni	1	+ 6	+ 1	+ 5	— 6	— 8	+ 10	+ 21	97	31	+ 3
5582·198	Ca	4	+ 2	— 3	— 7	— 14	+ 2	— 4	+ 4	97	25	— 3
5590·343	Ca	3'	— 3	— 9	— 5	+ 9	+ 3	— 8	+ 5	97	24	— 1
5598·524	Fe	1	— 18	+ 5	+ 11	— 11	0	+ 2	— 1	81	26	— 1
5601·505	Ca	3	— 2	+ 6	— 13	+ 6	— 4	+ 2	+ 14	96	22	+ 1
5624·769	Fe.-V.	4	+ 6	+ 8	+ 14	+ 13	+ 8	+ 5	+ 8	97	28	+ 9
5638·488	Fe	3	— 2	— 5	+ 11	— 10	+ 6	+ 21	+ 13	97	28	+ 5
5658·097	Y	2	+ 6	+ 4	+ 2	— 5	+ 7	+ 17	— 22	97	25	— 1
5682·869	Na	5	— 17	— 24	— 6	— 17	+ 3	— 17	— 21	97	28	— 14
5684·710	Si	3	— 1	+ 4	+ 12	— 14	+ 6	— 1	+ 3	97	27	+ 1
5686·757	Fe	3	— 17	— 13	0	+ 10	+ 27	+ 3	— 10	81	28	0
5688·436	Na	6	— 4	— 8	+ 4	+ 1	+ 5	+ 1	— 7	97	25	— 1
Prob. Error	Single Line		21	25	23	25	25	23	26			

TABLE XII.—RESIDUALS IN REGION λ5500—λ5700—DE LURY.

Wave Length	Element	Intensity	Mean Algebraic Residuals at Latitudes.							No. of Measures	Mean Numl. Residual	Mean Algebc. Residual
			0°	15	30°	45°	60°	75°	90°			
5506·095	Mn	1	+ 37	+ 42	+ 12	— 11	+ 24	— 24	+ 23	112	85	+ 20
5514·563	Ti	2	+ 28	— 15	+ 50	— 18	+ 12	0	— 32	"	82	+ 3
5514·753	Ti	2	— 8	+ 22	— 30	+ 37	— 30	— 32	+ 21	"	71	— 3
5528·641	Mg	8	— 27	+ 5	— 10	0	— 5	+ 3	— 9	"	83	— 6
5544·157	Fe	2	+ 5	— 24	+ 9	— 15	— 5	+ 2	— 17	"	66	— 6
5560·434	Fe	2	+ 35	0	— 34	— 18	— 2	— 2	+ 6	"	71	— 2
5562·933	Fe	2	— 4	+ 2	+ 4	+ 55	— 6	+ 2	+ 26	"	59	+ 11
5578·946	Ni	1	— 13	0	+ 38	+ 15	+ 53	+ 28	+ 3	"	66	+ 18
5582·198	Ca	4	— 7	+ 3	+ 6	+ 27	— 1	+ 13	— 2	"	48	+ 6
5590·343	Ca	3	+ 18	— 34	+ 4	+ 18	+ 12	+ 22	+ 3	"	65	+ 6
5598·524	Fe	1	+ 23	+ 24	— 4	— 14	— 33	— 15	+ 7	"	65	— 2
5601·505	Ca	3	— 4	+ 5	— 6	— 9	— 3	+ 6	0	"	57	— 2
5624·769	Fe–V	4	— 9	— 19	— 11	+ 5	— 35	— 6	+ 3	"	59	— 10
5638·488	Fe	3	— 5	+ 7	+ 27	— 31	+ 14	+ 21	— 25	"	59	+ 1
5658·097	Y	2	— 11	+ 19	+ 11	— 12	+ 54	+ 24	— 14	"	69	+ 10
5682·869	Na	5	— 4	— 58	— 23	0	— 4	— 24	— 13	"	65	— 18
5684·710	Si	3	— 51	— 21	— 28	— 30	— 22	+ 8	— 3	"	69	— 21
5686·757	Fe	3	— 1	+ 17	— 4	+ 32	— 25	+ 8	+ 8	"	63	+ 5
5688·436	Na	6	— 1	+ 26	+ 7	— 35	— 4	+ 6	— 13	"	62	— 2
Probable error single line.			56	61	61	56	55	50	54			

The trend and magnitude of these mean residuals in Plaskett's measures for the different latitudes and the ratio of the mean algebraic to the mean numerical residual, which is except in one case less than one-third, do not indicate any systematic differences for the different lines. If any lines or elements gave a different velocity to the mean reversing layer, then the mean residuals for the different latitudes should be of the same sign, should diminish as the latitude increased and should vanish at the pole; but we find, on the contrary, that none of the lines fulfils this condition, but that the residuals bear the appearance of being quite accidental in character. Even take the case of the Na line 5682.869 which gives a strong negative residual, we find no decrease with higher latitudes and the mean residual for the pole is much higher than the average, showing that the difference is probably due to something in the line. Again, if this sodium line did give a lower value of the velocity, the other sodium line, the last on the list, should also give a negative residual, whereas we see its residuals are entirely accidental. The same condition of affairs is shown by the tabulated residuals from De Lury's measures of Series II in which the mean algebraic is always less than one-fourth the mean numerical residual, although the numbers are higher owing to his higher probable error of measurement.

These considerations form sufficient grounds for the statement that in the region around λ 5600 none of these lines or elements shows any differences of velocity from that of the reversing layer other than can readily be accounted for by accidental errors of measurement.

31. The same thing appears to be the case in the λ 4250 region. The following table contains the residuals in metres per second from the 15 lines measured on 24 plates at the equator.

TABLE XIII.—RESIDUALS IN REGION NEAR λ4250

Wave Lengths	4196·699	4197·257	4416·136	4220·509	4225·619	4232·887	4241·885	4246·996	4257·815	4258·477	4266·081	4268·915	4276·836	4290·377	4291·630
Element	La	C	C	Fe	Fe	Fe	Fe–Zr	Sc	Mn	Fe	Mn	Fe	Zr	Ti	Fe
Intensity	2	2	2	3	3	2	2	5	2	2	2	2	2	2	2
859a	— 39	— 32	+ 4	+ 9	+ 1	— 3	+ 33	— 4	+ 23	+ 12	— 8	+ 12	— 30	+ 23	— 4
b	+ 19	— 6	— 16	— 15	+ 27	+ 18	— 24	— 6	+ 4	— 6	— 4	+ 4	+ 11	0	— 1
860a	— 13	+ 25	— 12	+ 8	— 6	— 33	+ 27	+ 24	+ 20	— 29	+ 18	+ 2	+ 2	— 10	— 32
b	+ 16	+ 26	+ 28	+ 24	— 17	— 9	+ 27	— 3	— 3	— 36	— 23	— 19	— 8	— 14	+ 16
861a	+ 31	+ 1	— 5	+ 10	— 3	— 3	+ 32	— 17	+ 2	— 30	— 9	+ 17	+ 5	— 27	— 1
b	+ 12	+ 17	+ 23	+ 18	+ 28	— 50	— 12	— 11	+ 22	— 20	+ 22	— 36	+ 5	— 10	— 3
865a	+ 51	— 10	+ 5	+ 5	+ 4	+ 1	— 25	+ 10	— 8	— 40	+ 30	— 13	+ 25	— 35	— 7
b	— 29	— 19	+ 13	— 12	— 27	— 6	+ 32	+ 3	+ 3	— 24	+ 31	+ 26	+ 11	+ 17	+ 15
866a	+ 8	— 24	— 1	— 10	+ 7	— 7	+ 26	— 29	+ 21	— 9	+ 41	+ 3	+ 11	— 38	— 1
b	+ 26	— 20	+ 36	— 9	+ 11	— 8	+ 1	— 9	— 31	— 15	— 9	— 1	+ 11	+ 10	+ 11
867a	+ 20	+ 15	— 36	+ 18	— 41	+ 17	+ 29	— 16	+ 4	— 20	— 29	+ 3	— 5	— 20	— 10
b	+ 14	+ 19	— 56	+ 33	+ 33	+ 10	+ 3	— 41	+ 4	— 18	— 22	+ 26	— 32	+ 12	+ 19
869a	— 36	+ 35	+ 2	+ 7	— 18	+ 23	— 13	+ 17	— 24	— 4	+ 8	+ 16	0	+ 2	— 15
b	+ 8	+ 21	— 5	+ 5	+ 2	— 9	+ 40	— 24	— 34	— 19	— 24	— 35	+ 15	+ 37	+ 15
870a	+ 3	— 10	+ 5	— 4	+ 5	— 5	+ 14	— 4	— 2	— 15	+ 16	+ 5	— 6	— 15	+ 15
b	+ 39	+ 19	+ 19	+ 20	— 23	— 32	— 17	— 32	— 27	— 16	+ 8	+ 16	+ 11	+ 7	+ 13
871a	0	+ 3	— 11	— 5	— 16	+ 14	+ 9	— 5	+ 7	— 34	+ 5	+ 9	+ 1	+ 4	+ 14
b	+ 21	— 9	+ 23	+ 14	+ 10	— 19	+ 10	+ 17	— 24	0	— 10	— 17	— 28	— 9	+ 15
872a	+ 16	+ 2	— 1	+ 23	— 38	— 7	+ 18	— 18	+ 27	— 8	— 19	— 14	+ 21	+ 7	— 13
b	+ 25	+ 6	+ 11	— 7	— 4	— 1	— 21	— 18	+ 26	— 10	+ 37	— 28	— 6	+ 5	— 18
873a	— 7	— 13	+ 23	+ 36	— 9	— 10	+ 15	+ 8	— 10	— 31	+ 20	— 4	+ 5	— 1	+ 30
b	+ 8	— 33	+ 13	— 42	+ 43	— 1	— 16	+ 1	+ 24	+ 3	+ 13	— 12	— 27	+ 20	— 2
874a	— 9	+ 36	— 5	+ 26	— 19	— 2	+ 2	— 7	— 3	— 5	— 19	— 38	+ 9	+ 17	+ 19
b	+ 22	— 6	+ 13	+ 3	+ 6	+ 22	— 23	— 39	— 6	+ 1	— 27	— 9	— 9	— 2	+ 6
Mean Numerical	20	17	15	15	17	13	19	15	15	17	19	16	12	14	12
Mean Algebraic	+ 9	+ 2	+ 3	+ 6	— 2	— 4	+ 7	— 8	+ 1	— 16	+ 2	— 4	0	— 1	— 2

There are of course similar tables of residuals for the 24 plates measured at each of the latitudes 30° and 60°; but it is only necessary to give the mean numerical and algebraic residuals for each of these latitudes the same as has been done at the foot of the preceding table.

The following table contains a summary of the mean residuals for each line at each of these latitudes and then the final mean residuals for the 72 measures of each line.

TABLE XIV.—MEAN RESIDUALS AT DIFFERENT LATITUDES—REGION λ 4250

Number of Lines	1	2	3	4	5	6	7	8	9	10	11	12	13	14	5
Element	La	C	C	Fe	Fe	Fe	Fe-Zr	Sc	Mn	Fe	Mn	Fe	Zr	Ti —	Fe
Intensity	2	2	2	3	3	2	2	5	2	2	2	2	2	2	2
Numerical															
0°......	20	17	15	15	17	13	19	15	15	17	19	16	12	14	12
30°......	21	22	26	16	24	15	17	19	19	17	19	15	20	16	22
60°......	26	25	22	17	21	15	14	21	15	20	11	20	17	17	18
Mean...........	22	21	21	16	21	14	17	18	16	18	16	17	16	16	17
Algebraic															
0°......	+ 9	+ 2	+ 3	+ 6	— 2	— 4	+ 7	— 8	+ 1	— 16	+ 2	— 4	0	— 1	+ 2
30°......	+ 3	— 2	+ 4	— 8	+ 18	— 2	0	+ 6	0	— 2	— 2	— 7	— 14	— 1	— 3
60°......	+ 8	+ 8	0	— 8	0	+ 1	— 6	+ 9	0	— 3	— 6	— 5	0	— 2	+ 1
Mean...........	+ 7	+ 3	+ 2	— 3	+ 5	— 2	0	+ 2	0	— 7	— 2	— 5	— 5	— 2	+ 1
Adams..........	L — 14	L — 11	L — 11	+ 4		+ 2			H + 9	+ 5	+ 7	+ 7	+ 2	L — 4	+ 6

Again, it will be noticed that in the final values no mean algebraic is one-third as large as the mean numerical residual, even though in three cases the algebraic mean for one of the latitudes is nearly as large as the corresponding numerical mean. At the foot of the table are given the mean residuals obtained from Adams' 1908 values and indicated directly above by the letters L and H those lines which Adams claimed gave lower or higher values than the general reversing layer. It will at once be seen that the results obtained from the 72 plates measured by Plaskett do not agree with those of Adams, but are generally of the opposite sign. It seems to us therefore that the only safe conclusion to be drawn from the evidence at hand is that any differences found in both Adams' and Plaskett's values are not real differences of velocity but are, if not wholly accidental, rather some personal effect in the measurement due possibly to the character of the line. It is unfortunate that no plates containing H_a and Ca λ4227 were obtained here in order to compare the rotational values obtained from these lines with the general reversing layer, as was done by Adams; but it seems likely that personal differences, at least as high as those occurring in the general reversing layer, would be present in the measures of these broad and difficult lines.

Summary.

32. The principal conclusions reached from this investigation may be briefly summarized as follow:—

(*a*) The Ottawa values of the solar rotation may be represented by the formulæ

$$\left.\begin{aligned} V &= (1.483 + .532 \cos^2 \varphi) \cos \varphi \\ \xi &= 10^\circ.32 + 4^\circ.05 \cos^2 \varphi \end{aligned}\right\} \text{Plaskett}$$

$$\left.\begin{aligned} V &= (1.448 + .532 \cos^2 \varphi) \cos \varphi \\ \xi &= 10^\circ.04 + 4^\circ.00 \cos^2 \varphi \end{aligned}\right\} \text{De Lury}$$

which are in remarklaby good agreement with Dunér's and Adams' 1908 results except for small and nearly constant differences, and which probably represent very closely the law of variation with latitude.

(*b*) The absolute velocity of the solar rotation seems to be uncertain by the small differences above referred to, of the order of two or three per cent. which is apparently due to personal differences in the habit of measurement of the rotational displacements on the plates.

(*c*) The tabulation and discussion of about 3,000 residuals from different lines and elements in the regions measured, show that no systematic difference of velocity for different elements is present in the Ottawa plates. The frequently opposite signs of the mean residuals at Ottawa and Mt. Wilson from the same lines, (those found at the latter place to give systematically higher or lower velocities) would point to the conclusion that the deviations previously found might have been either accidental, or more probably personal and due to the character of the lines.

It gives us much pleasure to record here our appreciation of the interest the Director, Dr. W. F. King, has taken in this work, of the help he has afforded, and of his willingness to meet the many needs in the matter of apparatus arising in the course of the work.

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