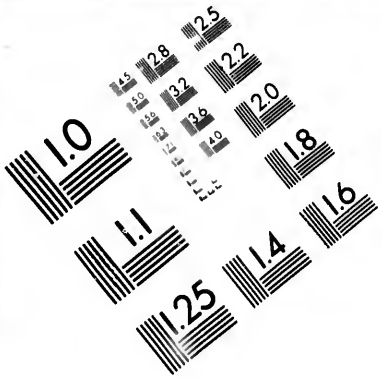

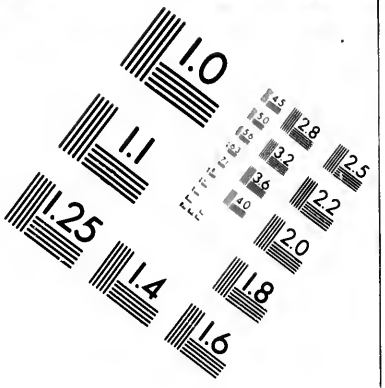

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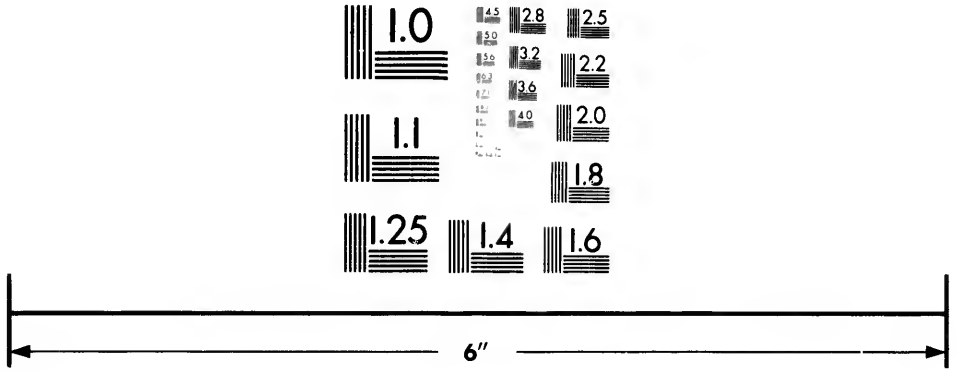

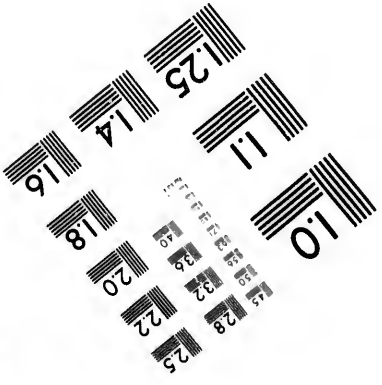

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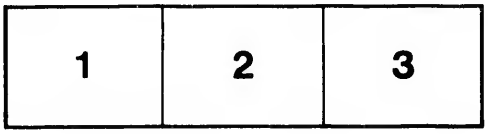

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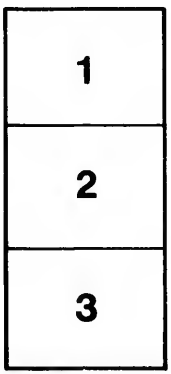

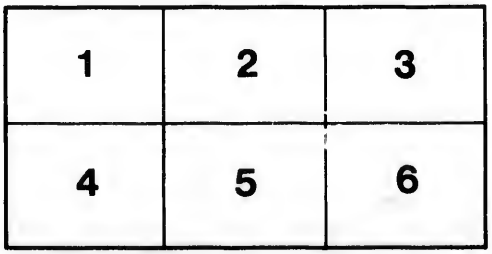

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LONDON COLONIAL EXHIBITION, 1886.

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SOLID GEOMETRY

MADE EASY.

THE

STEREOMETRICON

CROWNED

IN FRANCE, BELGIUM,
ITALY, RUSSIA, JAPAN,
BRAZIL, CANADA AND THE
UNITED STATES OF AMERICA,
13 MEDALS OF HONOR,
17 DIPLOMAS.

One and the same short rule (the prismoidal formula) applied to all solid forms.

INTRODUCTION OF THE NEW SYSTEM INTO ALL PRIMARY AND POLYTECHNIC SCHOOLS OF THE RUSSIAN EMPIRE.

The only rule which can be taught in the primary and elementary schools of the world, to artisans and others who most often require to apply it.

Cubic contents, capacities worked out by the new rule in from one tenth to one hundreth part of the time required by the old system.

THE

STEREOMETRICON

SOLID GEOMETRY:

STEREOMETRY, STEREOTOMY.

STUDY OF SOLID FORMS:

Their bases, lateral faces, conic and other sections, offer all plane figures, all figures of single and double curvature : cylindrical, conical, spherical, prismoïdal, conoïdal, spheroïdal, etc., which it is possible to conceive.

Developement of their surfaces; their measurement. Measurement of their volumes, capacities or solid contents.

The models in relief in the hands of the pupil, interest him, facilitate his study of solid forms, render it more attractive, more expeditious, more practical.

The 200 solids on the board, or as setforth in the accompanying pamphlet, (a key to the "The Stereometricon") represent all the elementary forms which it is possible to meet with in nature, in the Art and Trades, in Engineering, Architecture and every species of construction.

The varied forms in the day light, or with the help of a lighted candle or taper which can be moved into different positions, allow of the study of their shades and shadows ; of those which they project on the board or on any other horizontal,

vertical or oblique surface; of those which they are capable of projecting on each other or on any surface of single or double curvature.

The putting together and varied superposition and juxta position of the several forms, suggests the idea of the nature of their lines of intersection and penetration, as of the thousands of complex forms of which the elementary solids are the components.

Necessity of the models in relief as asserted by Walter Smith, to learn to draw their horizontal, vertical and other projections, ere one can attempt industrial design or to draw from nature.

The only system which will allow of teaching solid mensuration in the most elementary schools of all countries as now done in Russia.

Solid mensuration, much of it requiring the higher mathematics, is taught for the most part in Colleges and Universities only, and precisely to those who can never need it: as professional men, lawyers, notaries, doctors, men of literary persuits, ministers of the gospel and others.

The machinist, artisan, mechanic, practical builder and measurer, the ship and boat builder, the merchant and manufacturer, the architect and engineer, the brewer, farmer, remain untaught because that in primary and elementary schools, they can not be, by the present system of multifarious formulae, most of them beyond the possiblity of comprehension by the pupil.

The proposed new system does away with the difficulty and will enable every one who has been taught the mensuration of plane areas and the first four rules in arithmetic, to take hold of the most abstruse or apparently complicated solid, that of which the measurement is the most difficult by the old or ordinary rules, and obtain its contents with ease and accuracy, in one tenth of the time required by the old methods; nay very often in one hundreth part of the time required, as in getting the true contents of a cask of any variety of form.

THE NEW RULE:

" To the sum of the parallel end areas (of the solid or vessel whose cubical " content or capacity is required) add four times the middle area (half way between " the ends or bases) and multiply the whole by one sixth part of the length or " height or diameter of the body, perpendicular to the ends or bases. "

Application of the formula to a vessel having the form of the frustum of a cone, one of the most ordinary forms to be met with in every part of the civilized world; as, on a large scale, in distilleries, breweries, soap and potash factories, the manufacture of wines, liquors and every species of liquid substance; on a smaller scale, in the domestic circle: the salting tub, butter firkin, pail, hamper, pan, porringer and washtub, &c., &c.; in architecture, the shaft of the grecian column, a tower, roof, component part of a spire, &c.; in engineering, the quai, pier, reservoir, connecting link or reducer between two water, drainage or other pipes or conduits of different diameters, &c., &c.; see No. 82 of Stereometricon.

EXAMPLE

Let the given diameters, top and bottom be 6 and 10. The middle diameter will be half their sum or 8, and let the height be 9.

For areas of circles to eighths, tenths, or twelfths of any unity, see tables II, III and IV of the Stereometricon.

BY THE NEW RULE.

Diameter 6 corresponds to an area.....	28.2744
Diameter 10 " to "	78.54
Diameter 8 gives 50.2656 which × 4 = ...	201.0624
Sum of areas = ...	307.8768
Multiplying this by one sixth of 9	1½
We get the solid content = ...	461.8152

This is an operation of barely 3 minutes duration and brings out the true result in cubical units of the same name as the lineal units of the diameters or square units of the areas: feet, inches, meters, yards, or any other unit, as the case may be.

If the dimensions in this case be in english feet lineal, the result will be in cubic ft. which being multiplied by the proper number will give gallons. If in inches, the result will give cubic inches and the proper divisor being applied will give gallons, or if divided by 1728 will give cubic ft.

The great advantage of the formula in this case is that the factor 8 for the middle area is an arithmetic mean between those of the bases; which allows of it being computed in a second or two, and very often mentally, without even putting down a figure on paper as: 6 and 10 are 16 and the half of 16 is 8; whereas by the old rule followed out in schools (Legendre's formula, elegant though it be) the supplementary area required is a geometrical mean between those of the end areas, which is quite another thing to work out, as witness the following computation:

BY THE OLD RULE.

Diam. 10 as before.....	= area	78.54
Diam. 6	=	28.2744
Geometrical mean area.....	=	47.1240
Sum of areas.....	=	153.9384
This multiplied by one third (⅓) the height.....		3
Gives as before.....		461.8152

but to get the geometrical mean area herein above made use of, the two end areas must first be multiplied into each other :

28.2744
78.54
1130976
1413720
2261952
1979208
2220.671376

And now there must be extracted the square root of this product :

	22,20.67,13,76	47.1240
	16	
87	62,6	
	609	
941	116,7	
	941	
9422	2261,3	
	1884 4	
94244	37697,6	
	37697 6	

An operation of at least 15 minutes or 5 times the duration of the other, which did not require 3 minutes to work out ; to say nothing of the fact that while the new system requires only a knowledge of the four first rules of arithmetic, which no one can forget as having to make use of them every day, the old rule on the contrary requires a knowledge of the mode of extracting the square root of a number, which every one forgets in a very short time

It has just been said that the duration of the latter operation is about 15 minutes, but this is only because the number of which the root is to be extracted happens to be an exact square, giving rise to a finished decimal ; whereas, had such not been the case, the operation, to secure sufficient accuracy or prove the identity of the results, would have had to be carried out to at least 3 additional decimal places, which would have made the duration of the process likely 20 or 25 minutes instead of 15.

NOW

we will apply the new system to another form of vessel to be met with on various scales of size, throughout the whole world, and in which millions are involved : sellers, buyers, consumers, francs, dollars, pounds sterling, &c.

Let it be required to obtain the contents of a cask of which the inside dimensions are : length 40 inches, bung diameter 36 inches, head or end diameters each 16 inches and the diameter half way between the head and bung 31.8 inches.

THE NEW RULE.

Bung diameter 36 gives area=.....	1017.8784
Head or end diameter " =.....	201.0624
Area to diam 31.8=794.2278 which × 4 — =.....	3176.911[2]
	4395.8520
Multiplying by one sixth (⅙) length 40.....	40
	6) 175834.0800
	29305.68

The true content as worked out below by Bonnycastle's rule or series of rules is 29257.29. The difference being .00048 or say half a thousandth or $\frac{1}{20}$ of one per cent in excess.

THE OLD RULE.

By Bonnycastle's mensuration, page 142 of his edition of 1844 :

" To find the solidity of the middle frustum of a circular spindle ; the length " of the frustum, its middle diameter and that of either of the ends being given.

RULE.

I. " Divide the square of half the length of the frustum by half the difference " between the middle diameter and that of either of the two ends; and half this " quotient added to $\frac{1}{4}$ of the said difference will give the radius of the circle. "

II. " Find the central distance and the revolving area, as in the last " problem. "

III. " From the square of the radius, take the square of the central distance, " and the square root of the remainder will give half the length of the spindle."

IV " From the square of half the length of the spindle take $\frac{1}{3}$ of the square " of half the length of the frustum and multiply the remainder into the said half " length."

V " From this product take that of the generating area and central distance " and the remainder multiplied by 6.2832 will give the content of the frustum."

Applying this rule to the aforesaid example where the diameter of the frustum at its centre is CD=36 inches, its end diameter Nn=16 inches and length Ee=40 inches, with letter O at centre of spindle ; in fact copying the whole operation as worked out at page 143 of Bonnycastle, we have :

$$\frac{CD-Nn}{2} = \frac{36-16}{2} = \frac{20}{2} = 10$$

$$\frac{20^2}{10} + 10 = \frac{400}{10} + 10 = 40 + 10 = 50 = \text{diameter.}$$

Hence radius of the circle =25.

Also 25—CO=25—18=7=central distance.

CO—NE=18—8=10=versed sine of arc NC. Hence, Prob. XV. Rule II.

$\frac{10}{50} = .2$ = tabular versed sine.

.111823 = tabular area
2500

559115
223646

279.557500 = area of segment NCP.
$40 \times 8 = 320$ = area NEeP
599.5575 = generating area NEePC
7 = central distance

4196.9025

$AO = \sqrt{25^2 - 7^2} = \sqrt{576} = 24 = \frac{1}{2}$ length of spindle

$24^2 = 576$

$\frac{1}{3} \times 20^2 = 133.3333$

442.6667
20

8853.3340
4196.9025

4656.4315
6.2832

93128630
139692945
372514520
93128630
279385890

29257.29040080 = solidity in cubic inches.

ONE MORE EXAMPLE

must prove amply conclusive of what I advance in favor of the new system "Only one simple rule to charge the memory with" and let me repeat it, as it can not be too often recited.

"To the sum of the end areas, add 4 times the middle area and multiply "for solid content, by the sixth part of the length."

SEE NOW

how this contrasts with the following rule from Bonnycastle, page 147 of his Mensuration of 1844.

THE OLD RULE.

"To find the solidity of the middle frustum of an elliptic spindle; its length, "its diameters at the middle and end being given; also the diameter

"which is half way between the middle and end diameter being "known.

I. From the sum of three times the square of the middle diameter, and the square of the end diameter, take four times the square of the diameter between the middle and end, and from four times the last diameter take the sum of the least diameter and three times that of the middle, and $\frac{1}{4}$ of the quotient arising from dividing the former difference by the latter will give the central distance.

II. Find the axes of the ellipse by Problem II and the area of the elliptical segment whose cord is the length of the frustum by Prob. V.

III. Divide three times the area thus found by the length of the frustum and from the quotient subtract the difference between the middle diameter and that of the end and multiply the remainder by eight times the central distance.

IV. Then from the sum of the square of the least diameter and twice the square of that in the middle, take the product last found and this difference multiplied by the length and the product again by .261799 &c, will give the solidity required.

EXAMPLE.

What is the solidity of the frustum Nnsm, whose length is 28 inches, the middle diameter CD of the frustum 24 inches, the end diameter Nn 21.6 inches and the diameter PQ half way between CD and Nn equal to 23.40909 inches.

here $CD^2 \quad 24^2 = 576$

therefore $3\ CD^2 = 1728$

$Nn^2 = (21.6)^2 = 466.56$

$3\ CD^2 + Nn^2 = 2194.56$

$4\ PQ^2$ —— $= 2191.9419785124$

difference $= \quad 2.6180214876$

again $4\ PQ = 93.63636$

$Nn + 3\ CD \quad = 93.6$

difference $= \quad .03636$

hence $\frac{2.6180214876}{.03636} = 72$ very nearly and $\frac{72}{4} = 18 =$ central distance

Now $18 + 12 = 30 =$ semi-conjugate axis. Also $CM = 12 - 10.8 = 1.2$.

And the remainder of the conjugate axis $= 58.8$. Hence from the nature of the ellipse $58.8 \times 1.2 : 14^2 :: 30^3 :$ (semi transverse)^2 or $8.4 : 14 :: 30 :$ semi-transverse; therefore the semi transverse axis $= \frac{420}{8.4} = 50$ inches.

Again $\frac{1.2}{60} = .02 =$ tabular versed sine.

And .003748 = tabular area corresponding to it.
100 = transverse axis.

3748
60 = conjugate axis.

22.4880 = elliptic area of which NGm is the arc.
3

28) 67.464 = 3 times the area.
2.40942
2.4 = diff. of middle and end diameters.

.00942
144 = 8 times the central distance.

3768
3768
942

1.35648
1618.56 = $Nn^2 + 2\,CD^2$

diff. = 1617.20352
28 = the length EF

1293762816
323440704

45281.69816 and this product being multiplied by .261799 or .2618 will give 11854.748683008 cubic inches equal to the required solidity.

Such a calculation is absolutely appalling, nor would any one resort to it unless, may be, it were a solid of some of the precious metals or a cask of diamonds and even then look at the chances of error in so many successive operations.

Now once more compare this with the mode of getting at the same thing in about 3 minutes—instead of as many hours—by the prismoidal formula and first—using only the middle diameter of the spindle (or bung diameter of the cask) and its end diameter.

THE NEW RULE.

Area to diameter.... 24=452.3904×4=1809.5616
Area end diam. 21.6 .....366.4362×2= 732.8724

Sum of Areas..... 2542.4340
Multiply by ⅔ of length..... 28

20339.472
50848.68

6 | 71188.152

Cubic contents in inches..... = 11864.692
Deducting true content..... 11854.749

Difference..... ..... 9.943
Equal to about ½ of one per cent.

If we again compute this frustum by taking in the half way diameter between the centre or bung and head or end, which of course affords the nearest approximation to the truth, the calculation will stand thus:

End diam. 21.6 gives area = ...	366.4362
Middle or bung diam. 24, area = ...	452.3904
Area to diam. 23.40909 or say 23.41 = 430.4 × 4 = .	1721.6
Sum of areas.....	2540.4266
Multiplying by 28÷6.....	28
	203234128
	50808532
	6)71131.9448
Cubic contents in inches = ...	11855.3241
Deducting true content.....	11854.7487
Difference = ...	.5754

or about $\frac{1}{20{,}000}$ or $\frac{1}{200}$ of one per cent.

And this latter mode is the one which should always be followed; for if the cask or frustum be portion of an ellipsoid, then will either of the above processes bring out the true content; and if it be portion of a spindle to which the formula does not strickly apply, the half way diameter between the centre and end of the figure, taking in as it does the very element or factor which causes the content to vary while the centre and end diameters remain the same, will always bring out the result with almost absolute accuracy; the result never differing from the true result by more than a small fraction of one per cent: say one pint or less on a 60 gallon keg.

Of course it will be said that for certain forms, as that of the prism or cylinder, the pyramid or cone, the old or ordinary rule is the more simple, and so it is; but it has the disadvantage of being a separate and additional rule and of introducing a second and a third formulae into the process.

And this is altogether unnecessary; for suppose it be required to cube a cylinder, the pupil or artisan knows that in this solid as in all prisms, the parallel bases are equal and all sections parallel thereto also equal, and ere he has had time to put a figure down on paper, the process of reasoning goes on in his mind, to the effect that the two end bases and 4 times the middle section being all equal quantities, it is the same thing to multiply the sum of these six areas by one sixth the height, or to multiply once the area, once the base, that is the base by the height or whole height; and the pupil or artisan arrives at or deduces the more simple rule from the general formula, without the necessity of this separate formula being stored in his mind as a separate process to be remembered.

Again with the Cone or Pyramid, its diameter half way up is exactly half that of the base and as $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ and as four times $\frac{1}{4} = 1$ and as 1 and $1 = 2$, the student or measurer again sees that twice the base into one sixth the altitude is equivalent

to once the base, that is the base into $\frac{1}{3}$ the altitude and again comes he back to the conviction that it is useless for him to charge his memory with additional formulae.

With regard to the Sphere, the old rule is: the cubic content equal to 4 great circles or to the spherical area of the sphere into $\frac{1}{3}$ the radius; but we see immediately that $\frac{1}{3}$ the radius is $\frac{1}{6}$ the diam.; and that the end areas in this case being null, the formula reduces to: 4 times the middle area of the sphere, that is of a section through its centre, into $\frac{1}{6}$ of the diameter.

CONCLUSION.

Again let it be repeated that the *New Rule* applies with absolute accuracy to every variety of Geometrical form, including all Segments thereof and all Frusta thereof, lateral, central or excentric between parallel bases inclined or not to the axes of the solids.

And where the rule does not apply exactly as with the hoofs and ungulae of cones and cylinders, the middle frusta of spindles (casks) it brings out results so very near the truth (from $\frac{1}{4}$ to $\frac{1}{10}$ and $\frac{1}{20}$ of one per cent) and so easy and quick to work out, that it can not but prove the only reliable practical rule, that can and should be made use of in mensuration, cask gauging, and

HOW TO PROCEED.

in each of the 200 cases set forth in the Key to the "Stercometricon": a description of the solid, its name, what it is representative of in every day life, the nature of its bases and middle section and all other necessary information is given in a way to render it intelligible to all.

CHS. BAILLAIRGÉ, M. A.

Fellow of the Royal Society of Canada; member of several learned Societies at home and abroad; Recipient of 17 diplomas and 13 medals of honor, from France, Italy, Belgium, Russia, Japa[illegible]zil, Canada and the United States of A.

Quebec, Canada, April 1886.