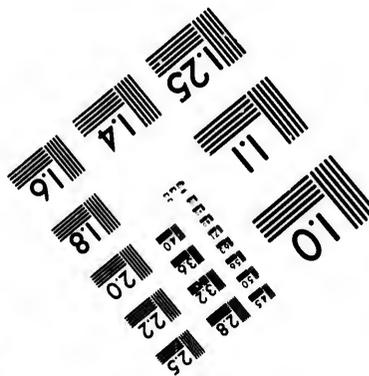
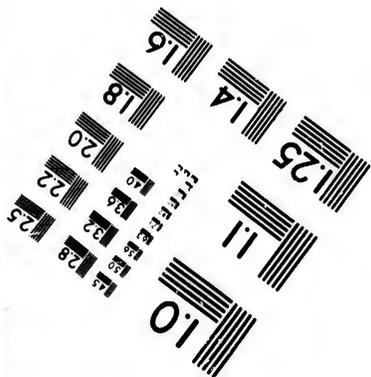
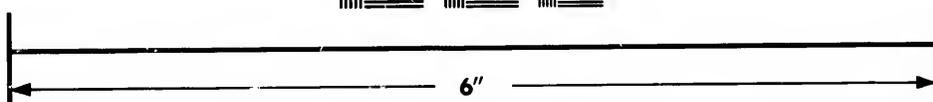
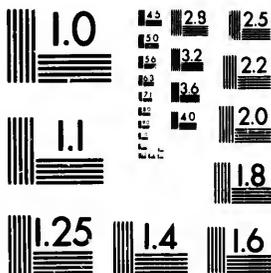


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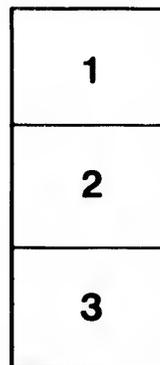
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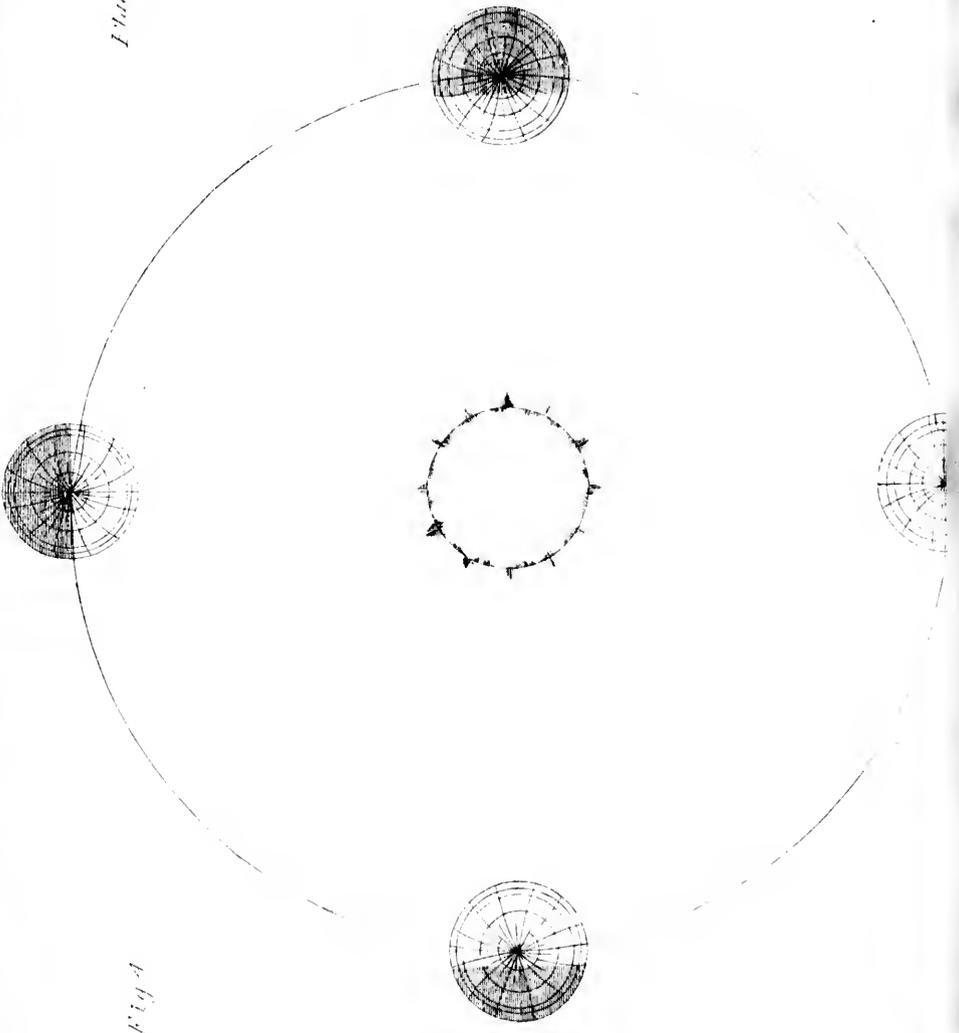
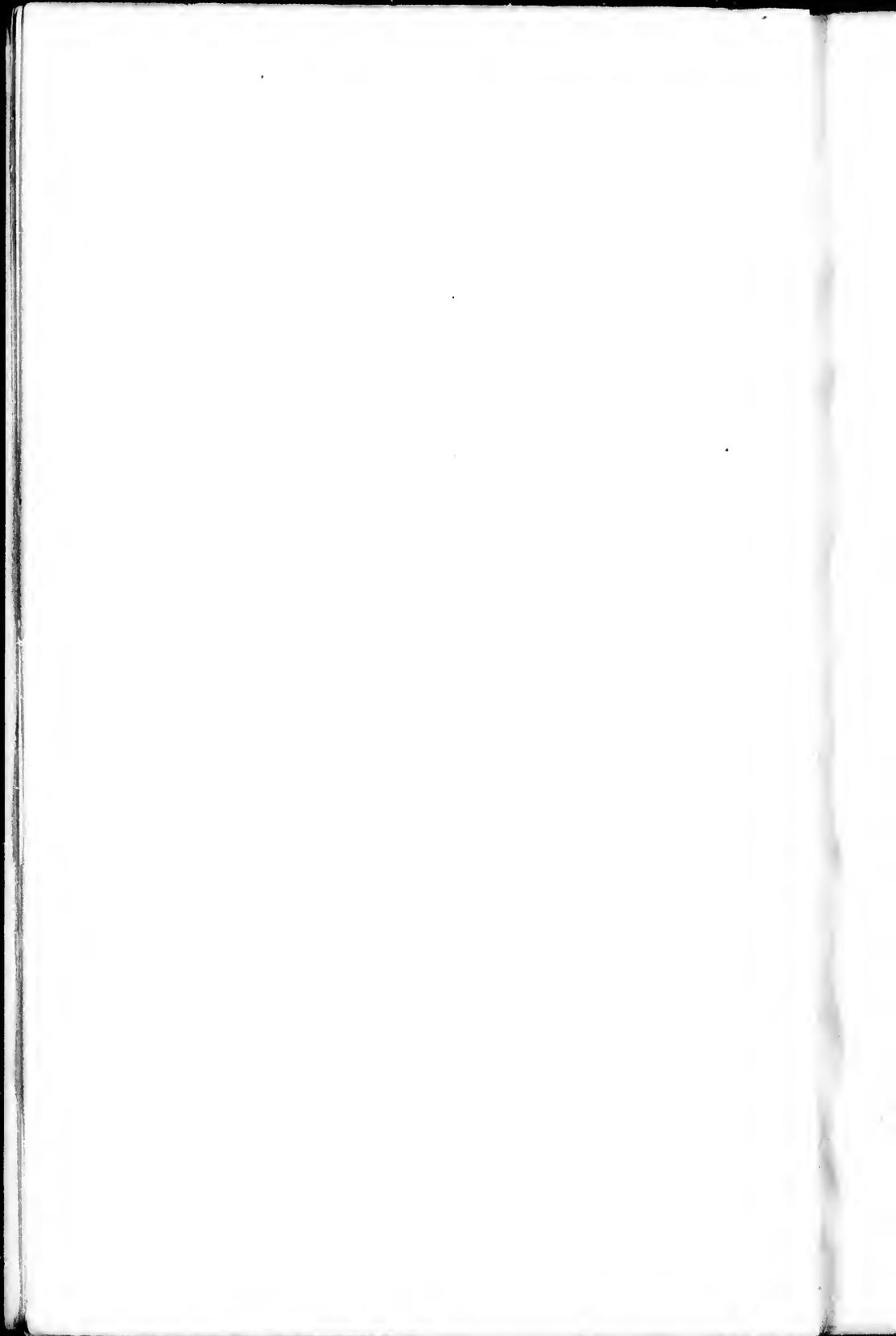


Fig. 4









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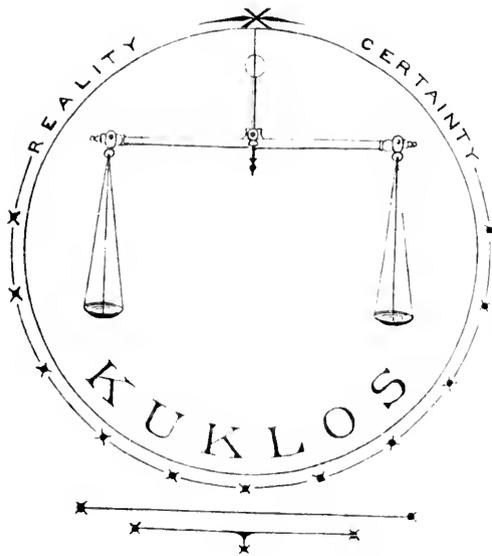
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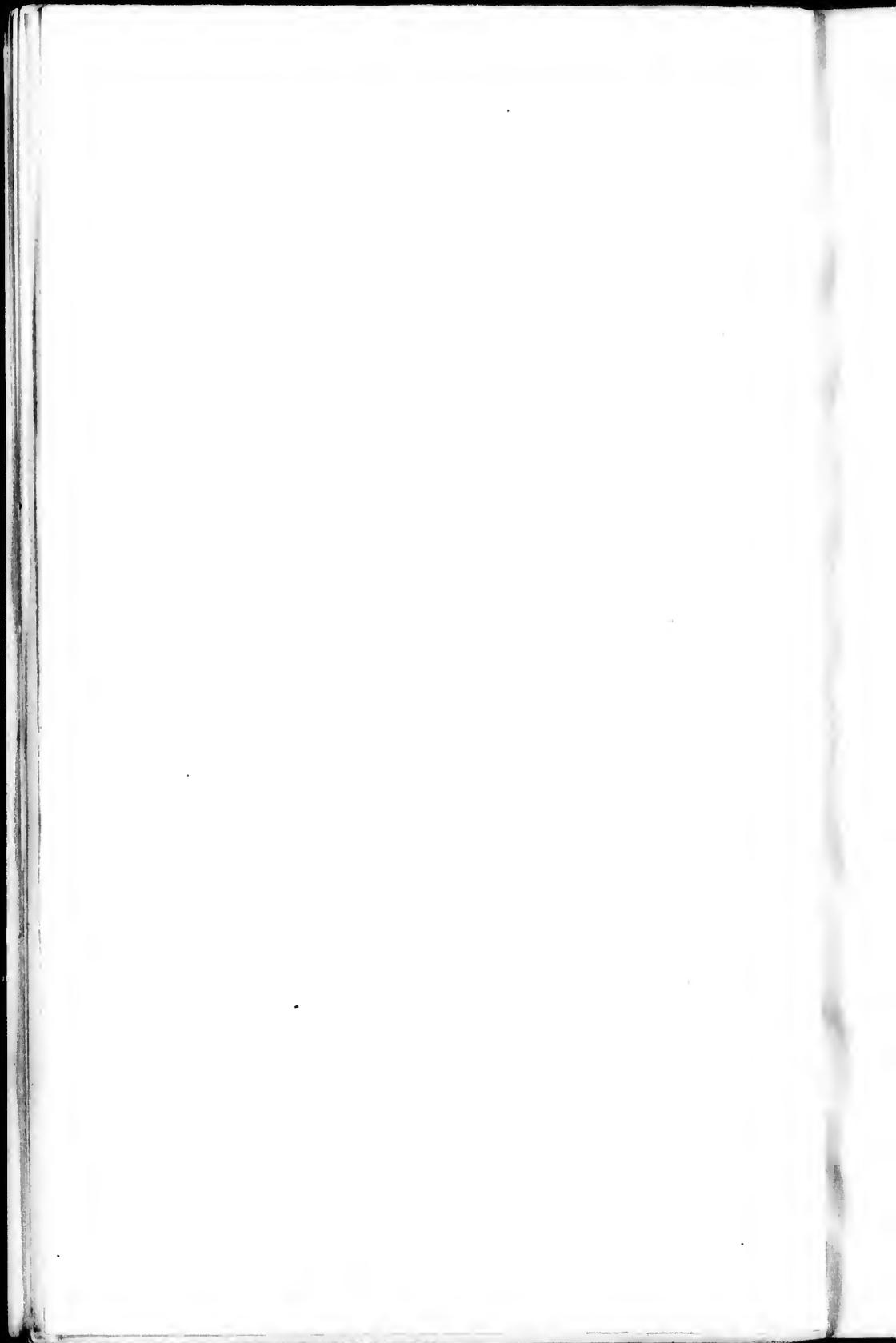
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—

BY
JOHN HARRIS.

—
MONTREAL:
JOHN LOVELL, ST. NICHOLAS STREET.

—
AUGUST, 1873.

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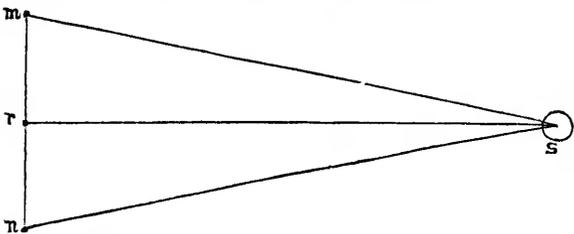
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THE SOLAR SYSTEM.

In considering the revolutions of the earth round the sun, we have seen that the distance of the moving body (the earth) from the centre of gravitation (the sun), does not remain the same throughout the orbit but increases and diminishes continually. From that place in the orbit at the greatest distance from the sun (called the aphelion) the earth continually approaches the sun until the place of least distance (the perihelion) is reached, and then commences to recede and continues to recede until the aphelion is again arrived at. So far, however, we have confined the consideration of the case to a revolution assumed to take place wholly in a plane passing through the centres of the sun and the earth, and at right angles to the vertical axis (or pole) of the sun; or, in other words, we have supposed the radius-vector connecting the centres of the sun and the earth to constantly form a right-angle with an axis perpendicular to the orbital plane of revolution.

We have seen that, an oscillating motion, having been communicated to the revolving body, causing it, whilst travelling in the orbit of revolution, to alternately approach and recede from the centre of gravitation, such motion becomes continuous, occasioning a constant deviation from the circular path of revolution and producing a greater or lesser amount of eccentricity in the elliptical orbit. Since an oscillating effect, whether caused by a primary influence or by a perturbing force acting in a direction at right-angles to the vertical axis, becoming permanent, occasions a constant deviation from the circular orbit; may we not expect to find that a like effect is also operative in the direction parallel to the vertical axis of the (sun) centre of gravitation; that is, perpendicular to the plane of revolution. It would at first appear that, unless we suppose some additional (unknown) force to operate

in restraining the motion, a deviation in this direction (i. e. the vertical, considering the plane of revolution as horizontal) will be caused by an influence of proportionally much less magnitude, because, even when the deviation has become considerable, it will be only the force represented by the vertical distance to which the deviation from the horizontal plane extends, compared to the horizontal distance from the centre of gravitation, which will be effective in bringing back the moving body to the horizontal plane of revolution; and the amount of the force so represented can only be, even in an extreme case, a fraction of the whole gravitating influence which is exerted at right angles to the vertical axis of the central sphere. In the accompanying figure, [fig 1.] where S.



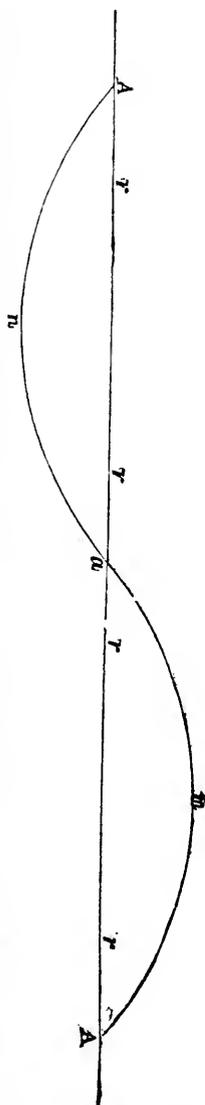
is the sun, E. the earth, and S. r. the horizontal plane of orbital revolution. Let m. n. represent the extreme vertical deviation (i. e. above and below the horizontal plane) which is supposed to determine the actual path of the revolving body E. throughout the entire orbit. If E. is supposed to have arrived at m. the place of maximum elevation; the proportion of the sun's gravitating influence [represented by S. m.] effective in the direction r. m., to that in the horizontal direction S. r. will be [by the rule for the resolution of forces] as the respective lengths of those lines; viz. as m. r. : S. r. Now, since in the actual case of the earth and sun, S. r. is known to be about 95,000,000 miles, it is evident that the distance r. m. would have to become considerable before any very effective proportion of the sun's attractive force could act upon

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the earth in the direction $m. r.$ Hence it might be hastily inferred that, because an external perturbing force acting in the horizontal direction in opposition to the whole gravitating influence (represented by $S. r.$) causes a deviation of a certain amount in that direction, therefore a like perturbing force acting in the vertical direction in opposition to only the small fraction of $S. r.$, represented by $r. m.$ would produce a much greater amount of deviation in the vertical direction; but so to infer would be to some extent erroneous, because it would be leaving out of consideration the centrifugal force which counterbalances the gravitating influence in the former (the horizontal), and which is (almost) absent in the latter (the vertical) direction; consequently it appears that a limit will be soon reached when the vertical motion derived from the perturbing force or other impulse being overpowered by that portion of the gravitating force represented by $r. m.$, the revolving body will be restrained from deviating further in that direction and will be then compelled to return towards $r.$ in the horizontal plane $S. r.$ The motion in the direction of $r.$ will not cease, however, when the revolving body arrives at that plane, but the acquired momentum will cause the motion to continue in the direction $r. n.$ The point $n.$ being reached,



the amount of gravitating force acting in the direction *r. n.* and opposing the motion (*r. n.* being equal to *r. m.*) will overpower the motion, as in the case of the vertical deviation above the plane, causing the body again to return towards *r.*, and so on continuously, the revolving body *E.* having thus a vertical motion of oscillation (vibration) between the points *m.* and *n.* whilst travelling through the circular (elliptical) orbital path of revolution. The ascending and descending path of the body thence resulting may be more clearly understood by reference to the accompanying figure (fig 2) in which *r. r. r. r.* represents the outer edge of the horizontal orbital plane, and *A. m. a. n. A.* the path of the revolving body; the elliptical figure having been separated at *A.* and unbent into the figure contained between the [separated] points *A.A.*

The present teaching on this subject is that the axis of the earth is inclined to the ecliptic at an angle of about $23^{\circ} 28'$ and that the earth travels throughout the orbit retaining the same absolute position: or, as it is described in some astronomical works, 'the earth's axis remains parallel to itself in all positions'; the meaning of which is defined by illustration to be that . . . if the axis of the sun perpendicular to the ecliptic is supposed to be extended north and south, then . . . at the summer solstice, the north pole of the earth inclines towards the perpendicular axis of the sun, and the south pole of the earth away from it; . . . at the winter solstice, the south pole of the earth inclines towards the perpendicular solar axis and the north pole of the earth away from it; . . . and at the vernal and autumnal equinoxes, the polar axis of the earth is inclined, at the same angle, in the direction of the earth's orbital path; in the one case the south pole being in advance, and in the other the north pole being in advance. Fig. 3, (Pl. 1. & 2.) illustrates the supposed position of the earth relatively to that of the sun in the four places of the orbital revolution, namely—the equinoxes and solstitial points.

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The first part of the theory of the inclined axis is concerned with the determination of the position of the axis of symmetry of a body. This is done by finding the center of mass of the body, and then the axis of symmetry is the line passing through the center of mass and perpendicular to the plane of the body.

The second part of the theory is concerned with the determination of the moment of inertia of a body about an axis. This is done by finding the moment of inertia of the body about a parallel axis through the center of mass, and then using the parallel axis theorem to find the moment of inertia about the axis of symmetry.

The third part of the theory is concerned with the determination of the angular velocity of a body about an axis. This is done by finding the angular velocity of the body about a parallel axis through the center of mass, and then using the parallel axis theorem to find the angular velocity about the axis of symmetry.

The fourth part of the theory is concerned with the determination of the angular momentum of a body about an axis. This is done by finding the angular momentum of the body about a parallel axis through the center of mass, and then using the parallel axis theorem to find the angular momentum about the axis of symmetry.

The fifth part of the theory is concerned with the determination of the kinetic energy of a body about an axis. This is done by finding the kinetic energy of the body about a parallel axis through the center of mass, and then using the parallel axis theorem to find the kinetic energy about the axis of symmetry.

The sixth part of the theory is concerned with the determination of the potential energy of a body about an axis. This is done by finding the potential energy of the body about a parallel axis through the center of mass, and then using the parallel axis theorem to find the potential energy about the axis of symmetry.

The seventh part of the theory is concerned with the determination of the work done by a force on a body about an axis. This is done by finding the work done by the force on the body about a parallel axis through the center of mass, and then using the parallel axis theorem to find the work done about the axis of symmetry.

The eighth part of the theory is concerned with the determination of the power of a force on a body about an axis. This is done by finding the power of the force on the body about a parallel axis through the center of mass, and then using the parallel axis theorem to find the power about the axis of symmetry.

The ninth part of the theory is concerned with the determination of the impulse of a force on a body about an axis. This is done by finding the impulse of the force on the body about a parallel axis through the center of mass, and then using the parallel axis theorem to find the impulse about the axis of symmetry.

The tenth part of the theory is concerned with the determination of the change in angular momentum of a body about an axis. This is done by finding the change in angular momentum of the body about a parallel axis through the center of mass, and then using the parallel axis theorem to find the change in angular momentum about the axis of symmetry.

Plate I



THE SEASONS
FROM DREWS ASTRONOMY

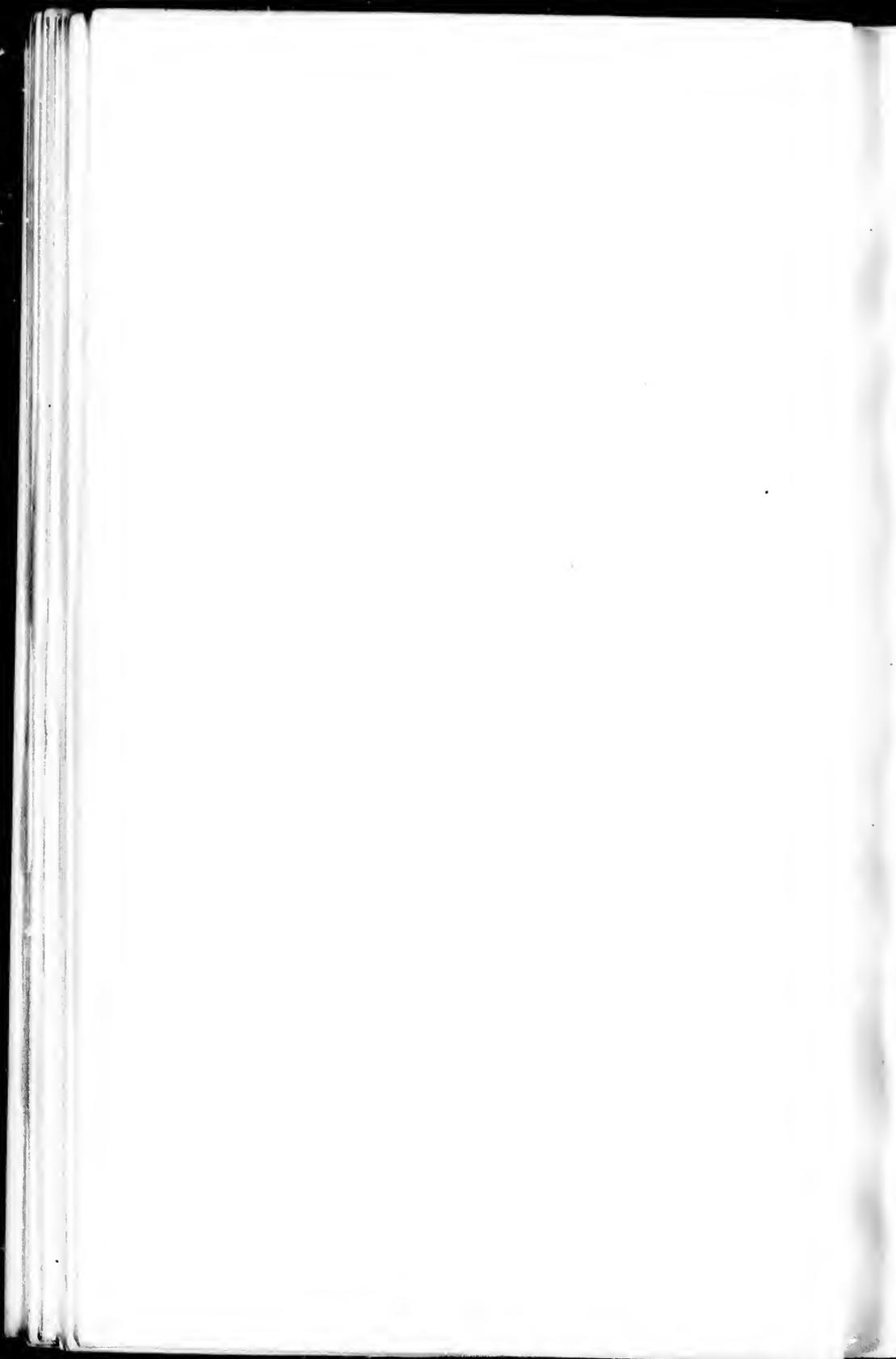
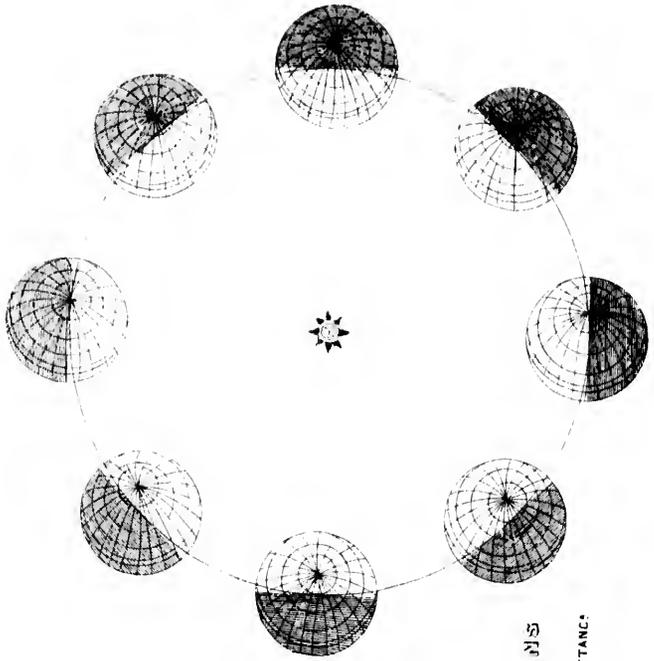




Plate 2



THE SEASONS
FROM THE ENCYCLOPEDIA BRITANNICA



PLATE 10
FIG. 10. 10. 10. 10.

to a distance of about 190,000,000 miles, that is—to the opposite extremity of its orbit—and yet still have the same star almost directly over its north pole as before; such, however, is the case, and when the comparatively enormous distance of the star is taken into consideration it is readily understood that the diameter of the earth's orbit, great as a distance of 190,000,000 miles appears to be in a merely terrestrial sense, is not sufficient to cause an appreciable alteration in the angle at which the light from the star arrives at the eye of the observer. This may be shown by illustration as in Fig. 3 (Pl. 3.) where (B) represents the sun and the earth, and (A) the relative place of the pole star. *

In this figure we assume the position of the earth's axis to be that which we assert that it actually is: namely, parallel to the axis of the earth's orbit or perpendicular to the plane of the ecliptic. Now if we take the theory of the inclined axis, it is true that the apparent situation of the pole star relative to the north pole (and other parts) of the earth will be the same and will remain unchanged during the progress of the earth throughout its orbit of revolution, but it is also evident that, relatively to other stars, the locality of the pole star will have shifted westwards to a distance of about $23\frac{1}{2}^{\circ}$ and the pole of the ecliptic, remaining unmoved, would then necessarily be at a distance of $23\frac{1}{2}^{\circ}$ from the pole star; and moreover the locality of the pole of the ecliptic would be always in the same direction from the pole star, in whatever part of its orbit the earth might be: but this is negated by the observed facts and the teaching of Astronomy, namely, that the locality of the pole of the ecliptic very nearly

* In addition to the sun and earth, the other principal members of the solar system, with exception of Neptune, are also shown (the inferior planets underneath to prevent crowding). The arrows crossing the earth illustrate the direction in which the axis of the earth inclines according to the present astronomical theory. The distance of the pole star from the sun, as shown in this figure, is simply for illustration and is much less than the supposed actual distance.

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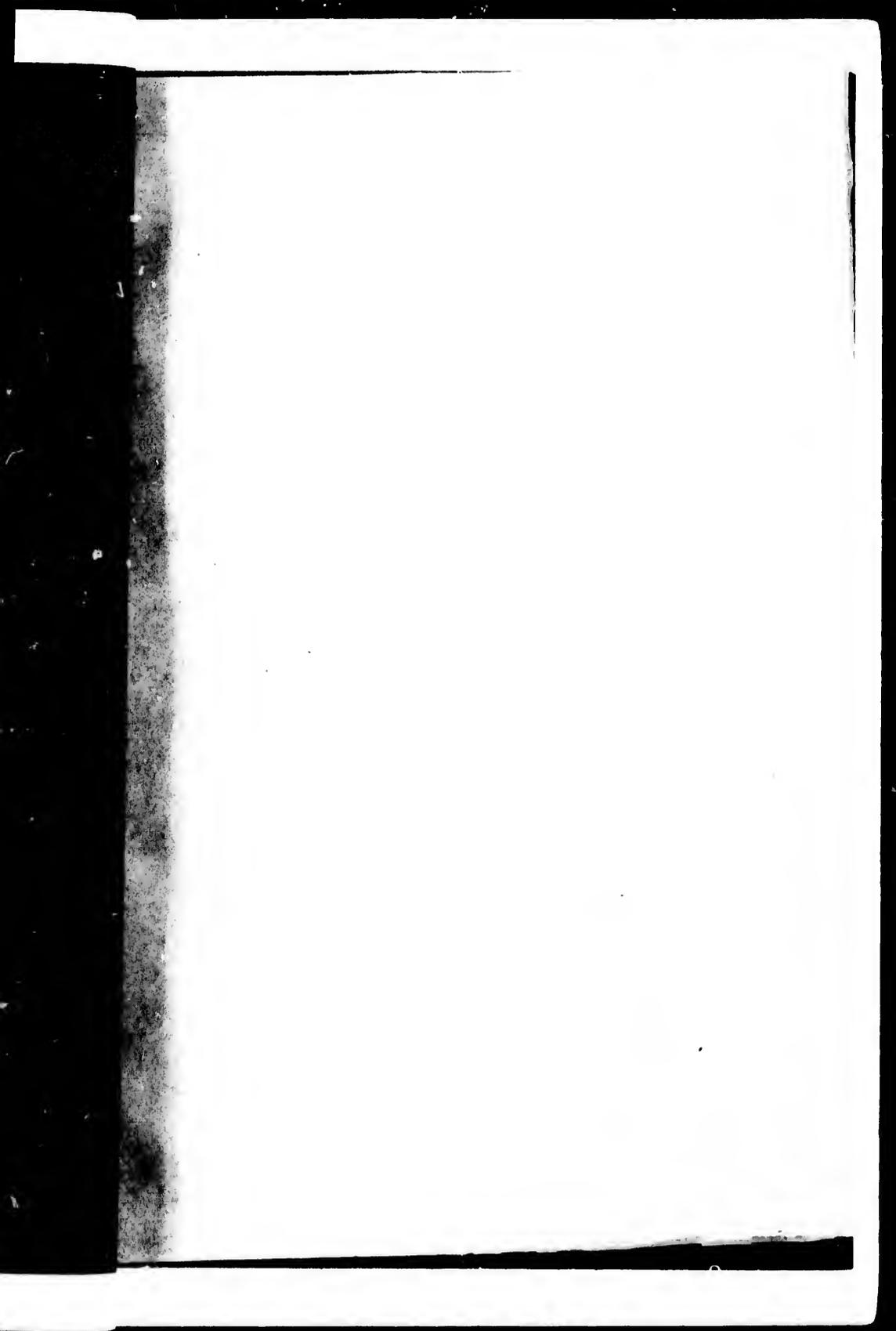
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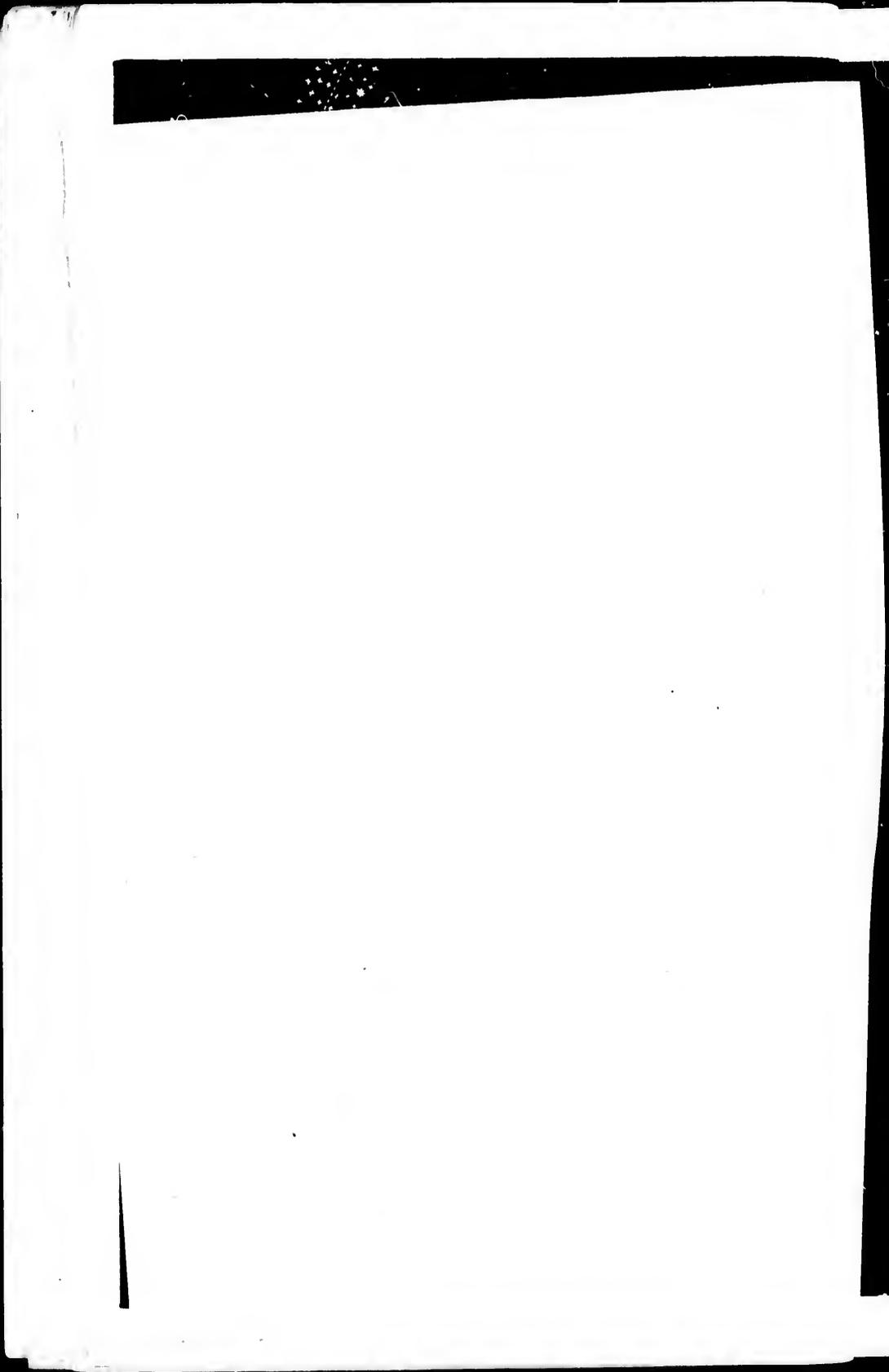


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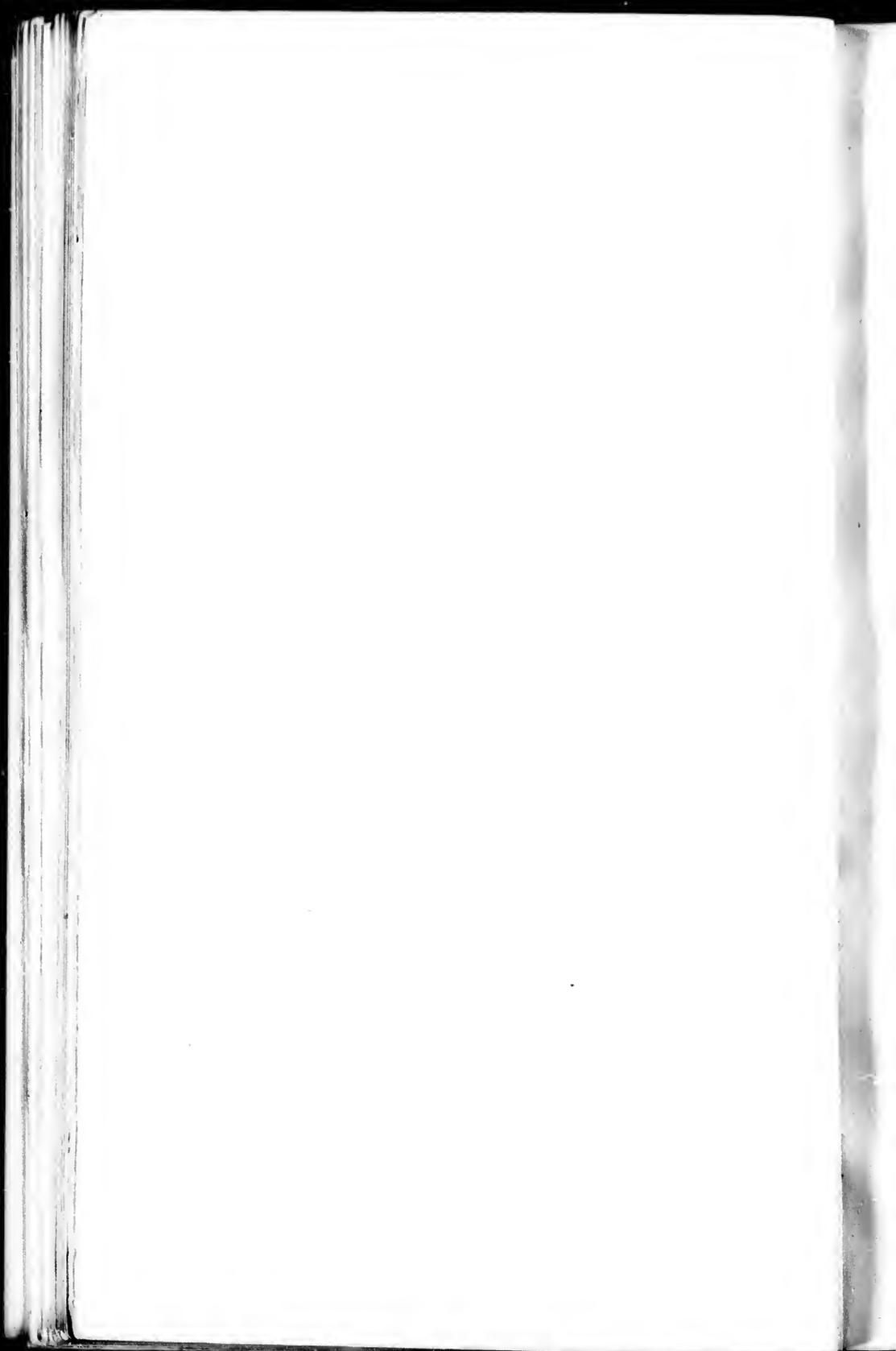
Fig 3











corresponds with the place of the pole star and that the pole of the earth extended to the heavens revolves around the pole of the ecliptic instead of being away at a distance of about $23\frac{1}{2}^{\circ}$ on the one (same) side of it, as required by the theory: wherefore, it at once appears that, the theory in question is untenable and must fall. Assuming that the sun itself rotates, which is not perhaps quite established as an observed fact, we can scarcely hesitate to infer that the sun's axis of rotation or pole is parallel to that of the earth, and that it represents and coincides with the pole of the ecliptic; in other words, that the sun's axis of rotation being extended becomes the pole of the ecliptic. At present we are taught that the polar axis of the sun is inclined at an angle of about $7^{\circ} 15'$ to the pole of the ecliptic. This teaching is a consequent of the assumption as to the inclination of the earth's axis, and belongs to the same theory; it may be however considered in some degree an independent hypothesis, being particularly based on a direct observation of the solar spots. It is therefore desirable briefly to consider this instance in order to show that the observed fact is more simply and conclusively, and therefore more satisfactorily, accounted for on the assumption that the axis of rotation both of the sun and of the earth are perpendicular to the plane of the ecliptic.

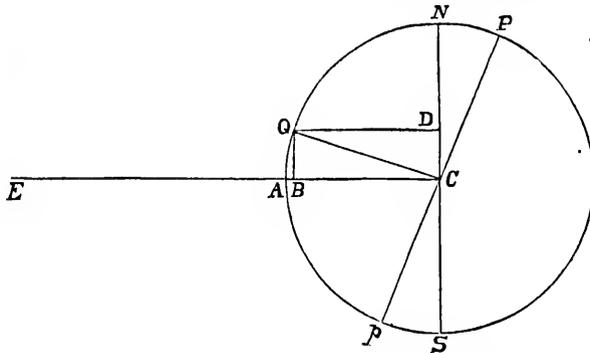
Herschels outlines of astronomy, (page 248 to 251).

(390) "when the spots are attentively watched, their situation on the disc of the sun is observed to change. They advance regularly towards its western limb or border, where they disappear, and are replaced by others which enter at the eastern limb, and which, pursuing their respective courses, in their turn disappear at the western. The apparent rapidity of this movement is not uniform, as it would be were the spots dark bodies passing, by an independent motion of their own, between the earth and the sun; but is swiftest in the middle of

their paths across the disc, and very slow at the borders. This is precisely what would be the case supposing them to appertain to and make part of the visible surface of the sun's globe, and to be carried round by a uniform rotation of that globe on its axis, so that each spot should describe a circle parallel to the sun's equator, rendered elliptic by the effect of perspective. Their apparent paths also across the disc conform to this view of their nature, being generally speaking, ellipses, much elongated, concentric with the sun's disc, each having one of its chords for its longer axis, and all these axes parallel to each other. At two periods of the year only do these spots appear to describe straight lines, viz., on and near to the 4th of June and 5th of December, on which days, therefore, the plane of the circle, which a spot on the sun's equator describes (and consequently, the plane of that equator itself), passes through the earth. Hence it is obvious, that the plane of the sun's equator is inclined to that of the ecliptic, and intersects it in a line which passes through the place of the earth on these days. The situation of this line, or *the line of the nodes of the sun's equator* as it is called, is, therefore, defined by the longitudes of the earth as seen from the sun at those epochs, which, according to Mr. Carrington, are respectively $73^{\circ} 40'$ and $253^{\circ} 40'$ ($= 73^{\circ} 40' + 180^{\circ}$) for 1850, being, of course, diametrically opposite in direction.

(391). The inclination of the sun's axis (that of the plane of its equator) to the ecliptic is determined by ascertaining the proportion of the longer and the shorter diameter of the apparent ellipse described by any remarkable, well-defined spot; in order to do which, its apparent place on the sun's disc must be very precisely ascertained by micrometric measures, repeated from day to day so long as it continues visible, (usually about 12 or 13 days), according to the magnitude of the spots which always vanish by the effect of foreshortening before they attain the actual bor-

der of the disc — but the larger spots being traceable closer to the limb than the smaller.) The *reduction* of such observations, or the conclusion from them of the element in question, is complicated with the effect of the earth's motion in the interval of the observations, and with its situation in the ecliptic, with respect to the line of nodes. For simplicity, we will suppose the earth situated as it is on the 4th of March, in a line at right angles to that of the nodes, i.e., in the heliocentric longitude $163^{\circ} 40'$, and to remain there stationary during the whole passage of a spot across the disc. In this case the axis of rotation of the sun will be situated in a plane passing through the earth and at right angles to the plane of the ecliptic.



Suppose C, to represent the sun's centre, P C p its axis, E C the line of sight, P N Q A p S, a section of the sun passing through the earth, and Q a spot situated on its equator, and in that plane, and consequently in the middle of its apparent path across the disc. If the axis of rotation were perpendicular to the ecliptic, as N S. this spot would be at A, and would be seen projected on C, the centre of the sun. It is actually at Q, projected

* This sentence by itself appears to be rather equivocal; the meaning is, however, defined by what follows, to be, that the plane of section, and not the axis of the sun, is perpendicular to the plane of the ecliptic.

on D, at an apparent distance C D to the north of the centre, which is the apparent smaller semi-axis of the ellipse described by the spot, which being known by micrometric measurement, the value of $\frac{CD}{CN}$ or the cosine of Q C N, the inclination of the sun's equator becomes known, C N being the apparent semi-diameter of the sun at that time. At this epoch, moreover, the northern half of the circle described by the spot is visible (the southern passing behind the body of the sun), and the south pole p of the sun is within the visible hemisphere. This is the case in the whole interval from December 5th to June 4th, during which the visual ray falls upon the southern side of the sun's equator.

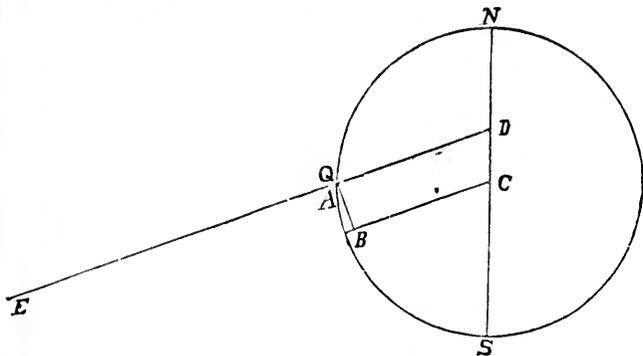
The contrary happens in the other half year, from June 4th to December 5th, and this is what is understood when we say that the *ascending* node (denoted) of the sun's equator lies in $73^{\circ} 40'$ longitude—a spot on the equator passing that node being then in the act of ascending from the southern to the northern side of the plane of the ecliptic—such being the conventional language of astronomers in speaking of these matters.

(392) If the observations are made at other seasons (which, however, are the less favourable for the purpose the more remote they are from the epochs here assigned); when, moreover, as in strictness is necessary, the motion of the earth in the interval of the measures is allowed for (as for a change of the point of sight); the calculations requisite to deduce the situation of the axis in space, and the duration of the revolution around it, become much more intricate, and it would be beyond the scope of this work to enter into them. According to Mr. Carrington's determination, the inclination of the sun's equator to the ecliptic is about $7^{\circ} 15'$ (its nodes being as above stated),

† “These periods are those of a spot in heliographic latitude 15° N or S of the sun's equator. Owing to solar atmospheric drift, the periods of rotation deduced from observations of spots in high or low heliographic latitudes differ considerably.”

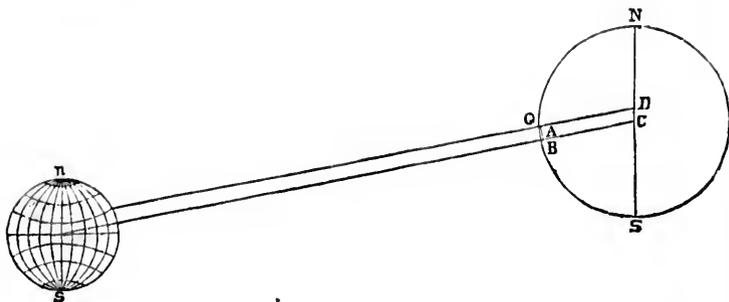
and the period of rotation 25 days 9 hours 7 minutes; the corresponding synodic period 27 days 6 hours 36 minutes."

The foregoing explanation is evidently based on an assumption that the centres of the earth and the sun are always in the same horizontal plane; or, in other words, that both of them are in the plane of the ecliptic. It is hence concluded that "if the axis of rotation were perpendicular to the ecliptic, as N S. (fig 6) this spot would be at A, and would be seen projected on C, the centre of the sun. It is actually at Q, projected upon D &c." But if we repeat Fig 6, and in Fig 7 (a) we suppose the earth,



having its polar axis perpendicular to the plane of the ecliptic, (which plane we also assert to be the plane of the sun's equator and at right angles to the sun's axis of rotation,) to have its centre so much below that plane that a line joining its centre to that of the sun will form an angle (of $7^{\circ} 15'$) with that plane—the above statements will obviously no longer hold good; but, on the contrary, the sun's axis of rotation being now perpendicular to the ecliptic, the spot seen at A, is also actually at Q, (because A, and Q, now coincide,) and is projected upon D, as observed. It is evident that S, the south pole of the sun, will now occupy precisely the same position relatively to the place of the earth which was occupied

by p. the supposed south pole in Fig 6. This is more clearly illustrated by Fig 7 (b), in which the axes of the earth and of the sun are parallel to each other and perpendicular to the plane of the ecliptic; S. the south pole of the sun is within the visible hemisphere, and it now appears that the point B. mistaken in fig 6 for the sun's equator is, in fact, below the equator by the distance Q D. *



The ascending and descending vertical deviation of the earth's orbit from the plane of the ecliptic.

This is ascertained by daily observations of the (declination, right ascension, and longitude,) place of the sun; from which observations a knowledge of the actual situation of the earth as it progresses throughout its orbital revolution around the sun is obtained. In consequence of the hypothesis of an inclined axis of rotation having been long since accepted as a demonstrated fact, the apparently oblique path of the sun has been attributed to the oblique position of the earth combined with the orbital motion around the sun: the observed result is therefore denoted by such terms as 'the obliquity of the equator';

* The difficulty and complication introduced by the supposition of the inclined axis is indicated in the next section of Herschel's work (392), from which it appears that the calculation if applied to (based on) observations made at other seasons becomes extremely intricate.

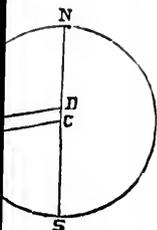
'the ascending and descending nodes of the ecliptic'; the teaching being that the plane of the ecliptic (*i. e.* the orbital plane of the earth) and the plane of the earth's equator are inclined to each other at an angle of about $23\frac{1}{2}^{\circ}$ (and which is equivalent to a statement that the polar axis of the earth is inclined to the pole of the ecliptic at an angle of about $23\frac{1}{2}^{\circ}$.) As to the astronomical observations there is no reason to call in question their correctness, and we shall accordingly consider the apparent path of the sun and the angle of apparent obliquity (now recognized) as observed facts; and by their aid proceed to explain briefly the actual path of the earth in its revolution around the sun.

Fig. 8, Pl. 5. We will commence with the winter solstice, the earth being at A the extremity of the major diameter of the ellipse at the least distance from the sun. The earth is now at the maximum elevation above the plane of the ecliptic (or the plane of the sun's equator), and a line joining the centre of the earth and the centre of the sun, at this place forms with that plane an angle of about $23\frac{1}{2}$ degrees. Since the distance of the earth from the sun is known, we can easily obtain the approximate value of the $23\frac{1}{2}^{\circ}$ of vertical deviation at that distance. Taking the distance roughly at 95 million miles we have $95,000,000 \times \frac{6}{164} =$ (about) 37,255,000 miles* as the maximum elevation of the earth's centre above the plane of the ecliptic.

The theoretical condition of the earth, so situated in respect to solar illumination and its consequences, will be, as illustrated by the figure, precisely that which is known to be the actual condition. The arctic circle will be unilluminated; the antarctic circle constantly

* Since the tangent of the arc and not the arc itself, should be taken, the vertical distance from the plane would be a little greater, but as this is the place of least horizontal distance from the sun the vertical elevation must be diminished accordingly. The figures given above are merely a rough approximation to illustrate the case.

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illuminated. To the southern regions it will be summer ; because the days will be longer, the rays of the sun will strike the earth almost vertically and those parts of the earth will also be the nearest to the sun. To the northern regions it will be winter ;—the days being shorter, and the rays of the sun falling very obliquely on those parts of the earth which will also be the most distant from the sun. When the longitude of the sun has increased 30° that is when the earth has advanced 30° in its orbit, (to the first point of Leo) the elevation of the earth above the plane of the ecliptic will have decreased by one-third, and will be therefore $(37,255,000 - 12,418,333) = 24,836,667$ miles. The days, in the northern regions of the earth, will have become proportionately longer, and in the southern regions shorter. Another advance of 30° in its orbit again reduces the earth's elevation to $(24,836,667 - 12,418,333) = 12,418,333$ miles; and when it has progressed through the quadrant, the earth's centre will be in the plane of the ecliptic ; or, in other words, since the polar axis of the sun and earth are parallel, the equatorial plane of the earth and the plane of the ecliptic will coincide : consequently the days and nights will be of equal duration in both hemispheres of the earth. The earth has arrived at the vernal equinox. The earth (the equatorial plane of the earth) continuing to descend as the orbital motion proceeds, passes the plane of the ecliptic and when an advance of another 30° has been made, is at a distance below that plane of 12,418,333 miles. When the second quadrant has been completed, the descent of the earth below the plane of the ecliptic will be a distance of 37,255,000 miles, equal to the elevation above that plane at the opposite extremity of the orbit. The conditions of solar illumination on the earth's surface compared to those first considered, are now reversed. The arctic regions experience a continual day ; the antarctic a continual night : to the northern parts of the earth it is now the season of summer ; to the southern parts the season

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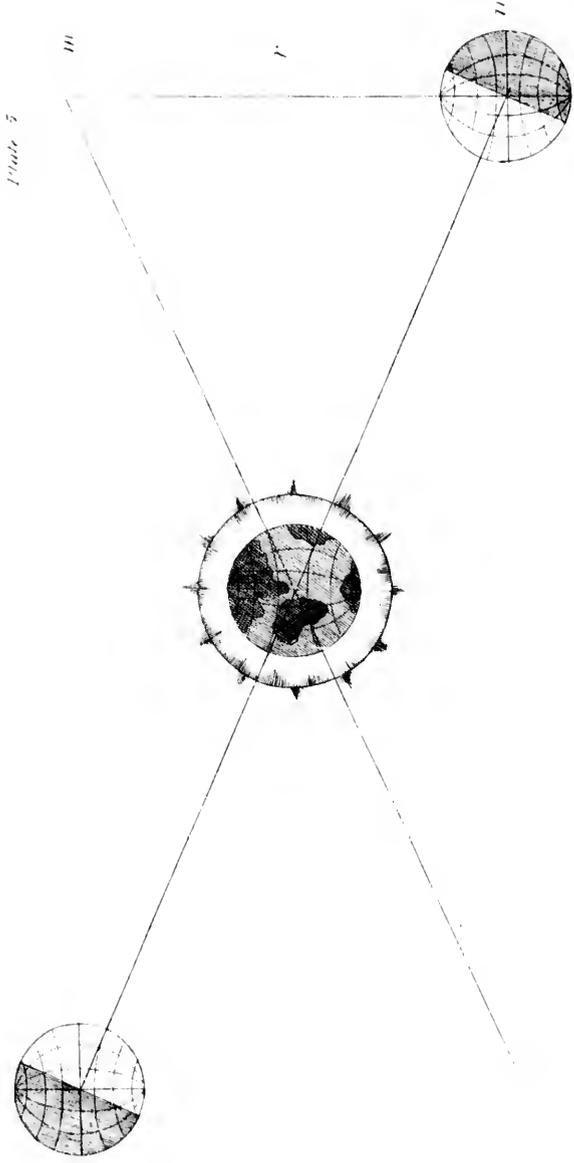


Table 5

Fig. 3

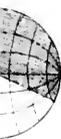
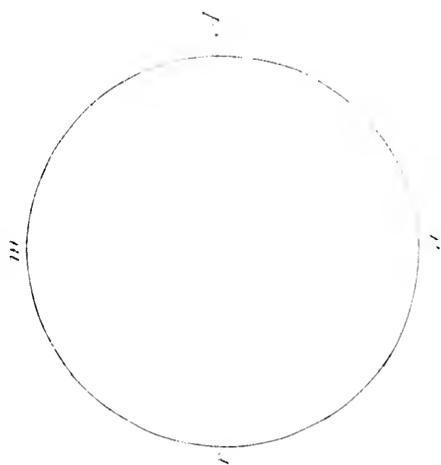






Fig. 9





of winter. The maximum depression of the earth's equatorial plane below the ecliptic has been now attained and the earth commences again to ascend; on the completion of the third quadrant the centres of the earth and the sun are again in the same horizontal plane, and the autumnal equinox has been arrived at. The ascent of the earth still continuing, the plane of the ecliptic is again passed and when the orbital revolution has been completed and the earth has again arrived at the plane first considered (viz, the winter solstice) the maximum elevation has been attained of 37,255,000 miles above the plane of the ecliptic. To avoid unnecessary complication we have here considered the earth's orbit of revolution as a circle. When we take also into consideration the ellipticity of the orbit, the probability at once suggests itself that the elevation and depression of the earth at various parts of the orbit will be proportional (or very nearly so) to the horizontal distance of the earth from the sun; that is to say, the amount of maximum elevation for the one semi-orbit and the amount of maximum depression for the other semi-orbit will be proportional to the comparative lengths of the horizontal radius-vector measured at each extremity of the major diameter of the ellipse respectively; because the extent of the vertical vibration is evidently dependent upon the horizontal distance from the sun; and, moreover, the vertical vibration (or oscillation) being of precisely the same nature as the horizontal vibration to which the ellipticity of the orbit is correctly attributable, it appears to be at least probable that the same impulse, whether imparted in the first instance or derived from some perturbing influence subsequently, which caused the one, will have also caused the other; and hence that the arcs of vibration, or the spaces through which the oscillating motions extend, in the two directions, viz. the vertical and the horizontal, will be proportional to each other. A correction will therefore have to be made accordingly—.....taking the distance

at aphelion.....96,595,000 miles,
 and at perihelion.....93,405,000 miles,
 we have $37,255,000 + 625,500 = 37,880,500$ miles for the
 maximum depression (at the northern summer solstice);
 and $37,255,000 - 625,500 = 36,629,500$ miles for the
 maximum elevation (at the northern winter solstice). It
 is not desirable for the present to enter into a particular
 consideration of the phenomenon known as the precession
 of the equinoxes; we may, however, here suggest that
 the assumption of an orbital motion of the sun itself relative-
 y to other stellar systems [and which theoretically appears
 extremely probable] would, if established in fact, afford
 a readily intelligible and satisfactory explanation.

In taking leave of this particular subject for the
 present, we will call attention to an apparent difficulty
 in the explanation now put before the reader; the diffi-
 culty is one undoubtedly requiring close attention and
 careful consideration, and is of such a kind that when for
 the first time apprehended, and looked at from one point
 of view only, it may appear to be almost fatal to the cor-
 rectness and truth of the (theory) explanation. Referring
 to Fig. 8. The earth E., is, according to the explanation,
 ascending from n. in the direction n.r.m.; having passed
 r. it continues to ascend with a velocity of nearly three
 million miles in a week. Why does the ascending
 motion cease on arriving at m.? Since the earth is
 (nearly) spherical, and the direction of the attractive
 force is at an angle with the direction of motion, approach-
 ing to a right angle; why does not the earth continue to
 move on, in the same direction, in a vertical orbit of
 revolution? Or, by combining this motion with that of
 the horizontal orbit, move in an oblique path around the
 sun? Having stated the difficulty, we will in the first
 place observe that the presently accepted teaching of the
 inclined axis and inclined planes exhibits a difficulty of
 the same kind; and which, although not perhaps so
 apparently startling for the moment, will be found on close

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inspection to be really more formidable—viz., there is no assignable cause of restraint and no reason has been shown why the earth's orbit (or the orbit of any other planet) should be and continue to be in an undeviating horizontal plane. If there was no apparent cause for deviation it would be incumbent upon us to show positive cause, but, with so many centres of attractive force above, below, and in every angular direction vertical and oblique to the orbital plane—it is almost inconceivable that the earth (or other planet) throughout its entire orbit should not be subjected to some vertical attractive force: and to counteract such an influence however slight, no restraining force whatever has been shown, nor is there any known force or influence which can be assigned as operating to prevent such a motion of vertical deviation above or below the orbital plane. Even if we put aside the supposition of a vertical or oblique perturbing influence, the difficulty is not much diminished; for it is scarcely conceivable, for instance, that the earth can revolve around the sun at a distance of 95 million miles in an orbital plane perfectly coincident with the equatorial plane of the sun, deviating neither above nor below that plane in any degree without any positive cause to prevent its ascending or descending: and yet it must be supposed to keep constantly in this horizontal path notwithstanding that its axis is inclined to the pole of the ecliptic, that its form is not perfectly spherical but ellipsoidal, that its hemispheres are unequally heated and cooled during the progress of its revolution, and that internal changes are continually taking place which may, at least temporarily, disturb in some slight degree the equality of gravitating effect above and below the horizon. This however must be not only conceived to be possible but must be admitted as established fact before we can accept the theory of the equatorial plane inclined to the plane of the ecliptic, and of the planetary revolution in a perfectly horizontal orbit without vertical deviation

therefrom. We will now briefly consider the actual conditions of the case in order to trace that actual motion which results from the vertical motion being compounded with the horizontal motion of the earth in its orbit of revolution. In Fig S, Pl. 5, The earth at n, is at the place of maximum deviation below the plane of the ecliptic, and is advancing along the orbital path in the direction n. r. m; and the velocity or rate of advance in that direction is about 50 million miles a month. Now since the axis of the earth is vertical to the direction of the motion, the centrifugal force which counteracts the attractive force of the sun operates from the centre of the orbit at right angles to the axis: but the whole attractive force of the sun is now acting on the earth from the higher position at an angle of $23\frac{1}{2}$ degrees; therefore, if this force is resolved, about three-fourths thereof acting in the direction horizontal to the axis is counteracted by the centrifugal force which is directly opposed to it; and nearly the one fourth is effective in attracting the earth vertically upwards in the direction r. m.; since this vertical attractive force is entirely unopposed by any counteracting influence or force it is immediately effective in producing an ascending motion of the earth; the motion so produced is a continually accelerated motion until the plane of the ecliptic is reached, but it is not a uniformly accelerated motion, because that fraction of the sun's gravitating influence, the originating and accelerating force to which this vertical motion is subject—continually decreases as the angle diminishes, and when the plane of the ecliptic is reached its influence in the vertical direction upwards entirely ceases; the earth by its acquired momentum passes the horizontal plane of orbital revolution (the ecliptic) at r. and as it ascends above that plane is again subjected to the vertical influence of a part of the sun's attractive force which is now exerted in the reverse direction, namely, from below upwards, opposing and counteracting the upward vertical motion

of the earth. As the angle (of the line joining the centre of the earth and sun with the plane of the ecliptic) increases, the fraction of the sun's gravitating influence which is thus effective in the vertical direction becomes greater and when the place of maximum elevation at *m*. has been reached, the resisting force then again amounting to nearly one fourth of the whole gravitating influence of the sun upon the earth, is sufficient to overcome the upward motion and continues to operate by causing a vertical motion of descent. The undulating path of the earth's revolution which results from the combination of the vertically oscillating with the horizontal motion in the plane of the orbit, is illustrated at Fig 9, where the circular orbit (a) is supposed to be separated at A, (one of the nodes) and to be straightened out into an undulating line (b) terminated by the points of separation A, and A.

(Note)—As this case may not improbably appear to present considerable difficulty when the attention is first directed to it, we will again revert to the explanation already given—in order to point out more particularly the influence of the centrifugal force in preserving perpendicularity of the earth in its vertical ascent and descent. Taking the position of the earth again as illustrated at n. fig 8, (the place of maximum depression) the centrifugal force is acting equally on the whole mass of the earth, impelling it in the horizontal direction outwards; as this force, the centrifugal is sufficient to counterbalance (neutralize) all the gravitating force acting in the horizontal direction inwards, no motion can take place in that direction; consequently that part of the oblique force capable of acting in the vertical direction, being unopposed, is effective in producing motion in that direction, i.e. the directly vertical.

