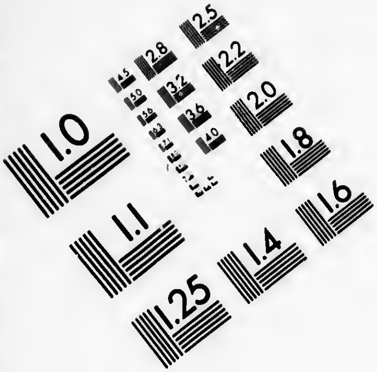
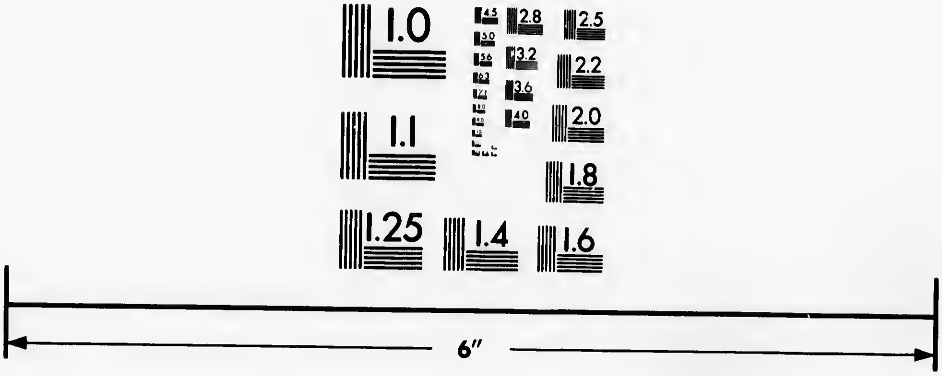
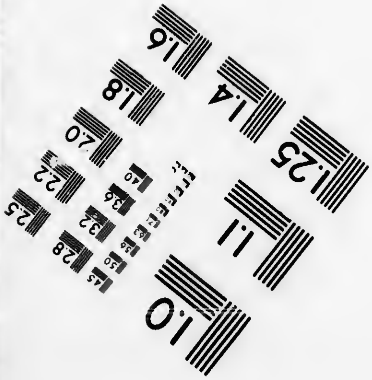


IMAGE EVALUATION
TEST TARGET (MT-3)





Photographic
Sciences
Corporation

23 WEST MAIN STREET
WEBSTER, N.Y. 14580
(716) 872-4503

Technical and Bibliographic Notes / Notes te

The Institute has attempted to obtain the best original copy available for filming. Features of this copy which may be bibliographically unique, which may alter any of the images in the reproduction, or which may significantly change the usual method of filming, are checked below.

☑ Coloured covers/
Couverture de couleur

☐ Covers damaged/
Couverture endommagée

☐ Covers restored and/or laminated/
Couverture restaurée et/ou pelliculée

☐ Cover title missing/
Le titre de couverture manque

☐ Coloured maps/
Cartes géographiques en couleur

☐ Coloured ink (i.e. other than blue or black)/
Encre de couleur (i.e. autre que bleue ou noire)

☑ Coloured plates and/or illustrations/
Planches et/ou illustrations en couleur

☐ Bound with other material/
Relié avec d'autres documents

☑ Tight binding may cause shadows or distortion along interior margin/
La reliure serrée peut causer de l'ombre ou de la distorsion le long de la marge intérieure

☐ Blank leaves added during restoration may appear within the text. Whenever possible, these have been omitted from filming/
Il se peut que certaines pages blanches ajoutées lors d'une restauration apparaissent dans le texte, mais, lorsque cela était possible, ces pages n'ont pas été filmées.

☐ Additional comments:/
Commentaires supplémentaires:

This item is filmed at the reduction ratio checked below/
Ce document est filmé au taux de réduction indiqué ci-dessous.

10X 14X 18X 22X

☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☑ ☐ ☐

12X 16X 20X

L'I
lui
exe
bibl
repr
dan
ci-d

☐
☐
☐
☑
☐
☑
☑
☐
☐
☐
☐
☐

d Bibliographic Notes / Notes techniques et bibliographiques

est original
copy which
alter any
may
ning, are

lack)/
ou noire)

stortion

ou de la

nay appear
se have

ajoutées
s le texte,
es n'ont

ked below/
diqué ci-dessous.

L'Institut a microfilmé le meilleur exemplaire qu'il lui a été possible de se procurer. Les détails de cet exemplaire qui sont peut-être uniques du point de vue bibliographique, qui peuvent modifier une image reproduite, ou qui peuvent exiger une modification dans la méthode normale de filmage sont indiqués ci-dessous.

☐ Coloured pages/
Pages de couleur

☐ Pages damaged/
Pages endommagées

☐ Pages restored and/or laminated/
Pages restaurées et/ou pelliculées

☑ Pages discoloured, stained or foxed/
Pages décolorées, tachetées ou piquées

☐ Pages detached/
Pages détachées

☑ Showthrough/
Transparence

☑ Quality of print varies/
Qualité inégale de l'impression

☐ Continuous pagination/
Pagination continue

☐ Includes index(es)/
Comprend un (des) index

Title on header taken from:/
Le titre de l'en-tête provient:

☐ Title page of issue/
Page de titre de la livraison

☐ Caption of issue/
Titre de départ de la livraison

☐ Masthead/
Générique (périodiques) de la livraison

18X 22X 26X 30X

			✓											

20X 24X 28X 32X

copy filmed here has been reproduced thanks
he generosity of:

National Library of Canada

images appearing here are the best quality
sible considering the condition and legibility
he original copy and in keeping with the
ing contract specifications.

inal copies in printed paper covers are filmed
inning with the front cover and ending on
last page with a printed or illustrated impres-
, or the back cover when appropriate. All
er original copies are filmed beginning on the
page with a printed or illustrated impres-
, and ending on the last page with a printed
lustrated impression.

last recorded frame on each microfiche
contain the symbol → (meaning "CON-
JED"), or the symbol ▼ (meaning "END"),
:hever applies.

s, plates, charts, etc., may be filmed at
rent reduction ratios. Those too large to be
ely included in one exposure are filmed
nning in the upper left hand corner, left to
and top to bottom, as many frames as
ired. The following diagrams illustrate the
od:

L'exemplaire filmé fut reproduit grâce à la générosité de:

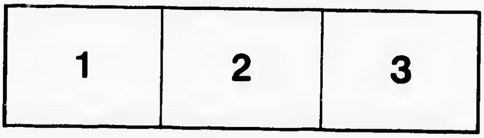
Bibliothèque nationale du Canada

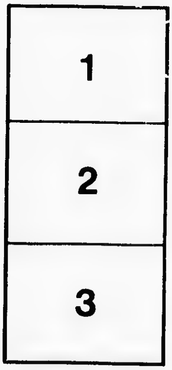
Les images suivantes ont été reproduites avec le plus grand soin, compte tenu de la condition et de la netteté de l'exemplaire filmé, et en conformité avec les conditions du contrat de filmage.

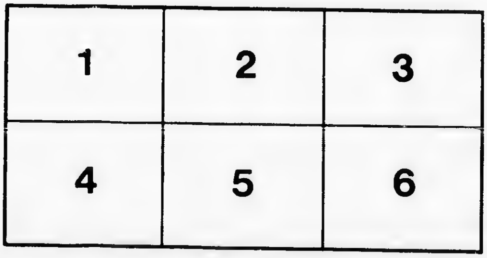
Les exemplaires originaux dont la couverture en papier est imprimée sont filmés en commençant par le premier plat et en terminant soit par la dernière page qui comporte une empreinte d'impression ou d'illustration, soit par le second plat, selon le cas. Tous les autres exemplaires originaux sont filmés en commençant par la première page qui comporte une empreinte d'impression ou d'illustration et en terminant par la dernière page qui comporte une telle empreinte.

Un des symboles suivants apparaîtra sur la dernière image de chaque microfiche, selon le cas: le symbole → signifie "A SUIVRE", le symbole ▼ signifie "FIN".

Les cartes, planches, tableaux, etc., peuvent être filmés à des taux de réduction différents. Lorsque le document est trop grand pour être reproduit en un seul cliché, il est filmé à partir de l'angle supérieur gauche, de gauche à droite, et de haut en bas, en prenant le nombre d'images nécessaire. Les diagrammes suivants illustrent la méthode.








A MARI
USQUE
AD MARE

4

AN ELEMENTARY TREATISE

ON

ARITHMETIC

FOR USE IN THE

Public and Model Schools of Ontario.

BY

WILSON TAYLOR, B.A.,

MATHEMATICAL MASTER OF CHATHAM COLLEGIATE INSTITUTE, FORMERLY OF INGERSOLL
AND STRATFORD COLLEGIATE INSTITUTES.

"'Tis plain that number is the object of Arithmetick."
—JOHN WILSON, 1741.

TORONTO:
WILLIAM BRIGGS, WESLEY BUILDINGS,
29 TO 33 RICHMOND STREET WEST.
1898.

PREFACE.

Probably no subject of study in our schools is of more educational value than Arithmetic. More than any other it requires the student to form an accurate idea as to the magnitude of a quantity; to conceive the exact relation between the magnitudes of two quantities; by comparing two relations among three quantities, to obtain the necessary third relation by an act of the mind; and, finally, by repeating these processes under varying conditions, to bring a number of seemingly disconnected facts into relations, all tending towards a certain desired end. This on a small scale is just what the pupil must do on a large scale when he enters in earnest upon the practical affairs of life.

The failure of Arithmetic, if there be any, to secure this object is due to no fault of the subject itself, but to causes which may be easily removed.

(*a*) Too much time is spent in trying to teach "number analysis and synthesis" to young pupils, whose minds have not sufficiently matured to form the "concept" for which the symbol 5 stands. It may even be doubted that this concept ever is formed, that the pure number 13 is anything but a mere word or symbol, and that operations upon pure numbers are anything more than mere combinations of names, of symbols, or of sounds.

It cannot be claimed that the teaching of mere words contributes much to mental growth. A large part of the time so spent in the primary classes would be better employed in teaching how to read and write the English language.

(*b*) There is much misconception resulting from the want of a clear distinction between quantity and number. When our present text-books call "5" a number, and, at the same time, call "5 apples" a number, they violate the first principles of speech. And, further, when they attempt to explain the rules of Arithmetic, it is no wonder that they are led by this confusion of ideas and terms into many absurdities, that the teacher is dissatisfied with his own explanations, and that the pupil is bewildered on every side.

(*c*) Based upon the misconception that numbers are *nouns*, which are the names of things, is the further absurdity that numbers can be made to express thought. Thus it happens that the proper use of the English language in Arithmetic is neglected. To conceive and to express thought accurately is the mark of intellectual power; but Arithmetic, laboring under misconception, and ignoring the use of language, is made to teach all kinds of inaccuracy.

The object of the present book is to clear up these misconceptions, by inquiring into the origin and use of numbers; to lay down those principles which are at the foundation of all knowledge of number; and to build upon these fundamental principles the beautiful and useful science of Arithmetic.

To attain this object many meaningless and technical words are left out, and a few have had their application extended and

rds contributes
me so spent in
eaching how to

m the want of
r. When our
same time, call
ples of speech.
ules of Arith-
s confusion of
the teacher is
t the pupil is

rs are *nouns*,
bsurdity that
t happens that
metic is neg-
urately is the
boring under
, is made to

se misconcep-
mbers; to lay
of all know-
nental princi-

hnical words
extended and

---

explained; as, for instance, the words "rate," "order" and "rent." The divisions of the subject have been brought into more logical order. The usages and customs of commercial life are explained in simple straightforward language, and brought into harmony with the principles laid down in the preceding chapters.

The book is written both for the teacher and for the pupil. In the earlier stages of the subject, while the pupil is too young to read, it is written for the teacher. In the later stages, when the pupil is sufficiently advanced, it is written for the pupil. In either case, we trust that we have presented the subject in such a way that he who reads will understand.

The limits to which the book is restricted do not permit the full treatment of many of the divisions of the subject; but the work will be found complete in itself, and sufficient to meet the requirements of the Public School Leaving or Form I. Examination. It is the intention of the author to proceed, later on, with a complete treatise of the subject, for the use of those who are preparing for Public School and High School teachers' certificates.

WILSON TAYLOR.

CHATHAM, January, 1898.

CONTENTS.

CHAPTER I.

CHAPTER II.

CHAPTER III.

CHAPTER IV.

CHAPTER V.

CHAPTER VI.

CHAPTER VII.

CHAPTER VIII.

CHAPTER IX.

CHAPTER X.

CHAPTER XI.

CHAPTER XII.

CHAPTER XIII.

CHAPTER XIV.

CHAPTER XV.

CHAPTER XVI.

ELEMENTARY ARITHMETIC.

CHAPTER I.

THE ORIGIN AND USE OF NUMBERS.

1. Of the things about us many are called by the same name. The persons in the school-room are called pupils; there are trees in the orchard, leaves in this book, houses in a town, wheels on a waggon and legs to a table.

Let us now group some of the things which are of the same name—for instance, the matches in a box. This we will do as follows:

Tie the matches into bundles, each containing ten matches; there will be left a few matches less than ten, which we shall lay to one side. Next, tie these bundles into larger bundles, so that each contains ten of the small bundles; there will be left a few small bundles less than ten, which we shall also lay to one side. Again, tie these larger bundles into still larger bundles, so that each contains ten of the larger bundles; there will be left a few larger bundles less than ten, which we shall also lay to one side. Continue this process till all the matches are laid aside.

Each match is called a *unit.* Each bundle which contains ten matches is called a *multiple unit of the 1st order.* Each bundle which contains ten of these bundles is called a *multiple unit of the 2nd order.* Each bundle which contains ten multiple units of the 2nd order is called a *multiple unit of the 3rd order*; and so on.

In a similar way all things which are called by the same name may be grouped into units, multiple units of the 1st order, multiple units of the 2nd order, and so on. Any one of the individual things is called a unit.

2. The symbols which are used instead of the words one, two, three, four, five, six, seven, eight, nine, and nought, are 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. Each of these, except 0, is called a figure, a digit, or a simple number.

3. Suppose, now, in Article 1, the number of matches less than ten which were laid to one side is 7; the number of bundles each containing ten matches which were laid to one side is 5; the number of larger bundles, each containing ten small bundles, which were laid to one side is 3; and the number of the still larger bundles is 4. In other words, suppose the number of units laid aside is 7; the number of multiple units of the 1st order laid aside is 5; the number of multiple units of the 2nd order laid aside is 3; and the number of multiple units of the 3rd order is 4.

It is agreed by all that the *number* of multiple units of the 1st order shall be written to the left of, and next to, the *number* of units; that the *number* of multiple units of the 2nd order shall be written to the left of, and next to, the *number* of multiple units of the 1st order; and so on.

Therefore we should write these numbers thus: 4357. We have now counted the matches in the box, and find the box contains 4357 matches.

4. When a number consists of only one of the nine symbols—1, 2, 3, 4, 5, 6, 7, 8, and 9—it is called a simple number, but a simple number may be with one or more 0's; and when it consists of more than one of these symbols, it is called a compound number. Thus, 700 is a simple number, but 4357 is a compound number.

5. How the operation of counting is represented.—Let it be agreed by all that the operation of grouping and counting, which we have described in the preceding articles, shall be represented by a horizontal line drawn in this manner:

$$\frac{\text{the matches in the box}}{\text{one match}};$$

and that when we have performed the operation we may write,

$$\frac{\text{the matches in the box}}{\text{one match}} = 4357.$$

This written statement may be read: "The number of matches in the box when counted by one match is 4357."

In the same way, the operation of counting other things which are of the same name may be represented. For instance,

$$\frac{\text{the pupils in this room}}{\text{one pupil}} = 32;$$

that is, the number of pupils in this room when counted by one pupil is 32.

6. Quantity and Number.—When we have a collection of things of the same name, we call the whole collection a quantity, and each individual thing we call a unit. Thus we say, "the quantity of matches" in the box, when one match is the unit by which the quantity is counted. We may also say, "the quantity of pupils" in this room, where one pupil is the unit by which this quantity is counted.

When we say the *quantity* of matches we do not mean the same as when we say the *number* of matches. When we say the quantity of matches, we mean the matches which we can feel and see; but when we say the number of matches, we mean, not the matches, but the number 4357, which at present is not much more than a name. Thus we say,

the quantity of matches = 4357 matches,
but the number of matches = 4357.

We are now able to express a quantity in terms of a unit and a number, thus:

the population of Toronto = 216153 individuals,
the chairs in the room = 49 chairs,
the value of my horse = 95 dollars,

and so on.

7. The Use of Numbers.—Having now shown how numbers are obtained, we shall next show what their use is. Consider the length of a field, which we are told is 4352 feet. The number 4352 tells us how the foot is used to make up the length of the field. That is, the number tells us

(*a*) To lay down a foot length ten times in the same straight line, without missing or overlapping. This makes a ten-foot length, or a multiple unit of the 1st order.

(*b*) To lay down a ten-foot length ten times, as before. This makes a hundred-foot length, or a multiple unit of the 2nd order.

(*c*) To lay down a hundred-foot length ten times, as before. This makes a thousand-foot length, or a multiple unit of the 3rd order.

(*d*) And lastly, to lay down 2 one-foot lengths, 5 ten-foot lengths, 3 hundred-foot lengths, and 4 thousand-foot lengths in the same straight line, without missing or overlapping. This makes up the length of the field. In other words, a foot being the unit, the length of the field is made up by putting together 2 units, 5 multiple units of the 1st order, 3 multiple units of the 2nd order, and 4 multiple units of the 3rd order.

Thus, the length of the field is made up of, or is derived from, a foot length; and the number 4352 tells us the precise way in which it is derived.

8. How the Operation of Deriving a Quantity from the unit is represented. Let it be agreed by all that the operation of deriving a quantity from the unit, which we have described above, shall be represented by the sign "×" written between the unit and number, thus:

the length of the field = a foot × 4352.

This written statement we shall read, "the length of the field is derived from a foot by the number 4352," and we shall call the sign "×" the sign of derivation.

Accordingly, we shall represent that a quantity is derived from itself, thus:

a yard = a yard × 1.

It thus seems that when a number as 4352 is written alone it has no meaning, but when it is written in connection with a unit, as a foot, it has a meaning which can always be explained. Thus, when we are told that

a tub of butter = a pound of butter × 35,

we mean that 5 one-pound rolls and 3 ten-pound rolls together make up the whole quantity of butter.

9. Number Defined.—When we think of a number as being obtained by counting the units which make up a quantity, we call the number *the measure of the quantity*; but when we think of it as telling us how a quantity is derived from a unit we call the number *a rate*. We therefore define a number to be:

(1) The measure of a quantity of units.
(2) The rate which tells how a quantity is derived from a unit.

10. NOTE.—(*a*) It will be seen later on that there is a peculiar advantage in writing the number after the unit, and not before it, as in ordinary language. Besides, it is the natural order of thought, first to regard the quantity as a whole, then to think of some known unit with which to measure it, and lastly to conceive the exact relation between the quantity and the unit, which is expressed by the number. It is worthy of note, also, that the word rate is here used in the same sense as it is used in commerce, and that it is synonymous with ratio.

Further, to read "a yard × 329," we may say in full, "the length (quantity) which is derived from a yard by the rate 329." But for brevity, we may say "a yard by 329." Some, however, would prefer to read it, "329 times a yard."

(*b*) It is assumed, at the first of this chapter, that the young pupil knows the number names—one, two, three, four, five, six, seven, eight, nine and ten—in their proper order, and that he can count things of the same name up to, but no farther than, ten things. This is all that is necessary. The analysis of "abstract" numbers should be postponed till after he has learned the Addition and Multiplication Tables.

It is suggested, also, that the pupil should be able to read and write ordinary language with some degree of accuracy before he learns the more difficult language of number; for it is through the ordinary language, either spoken or written, that he must learn any subject.

EXERCISE I.

1. Describe or perform the operation which is represented by the line in each of the following:

(*a*) $\dfrac{\text{A handful of beans}}{\text{one bean.}}$ (*c*) $\dfrac{\text{The length of the desk}}{\text{one inch.}}$

(*b*) $\dfrac{\text{A basket of marbles}}{\text{one marble.}}$ (*d*) $\dfrac{\text{An armful of wood}}{\text{one stick.}}$

2. Describe the operation which is represented by the sign "×" in each of the following:

(*a*) A load of wheat = a bushel of wheat × 25.
(*b*) The length of a rope = a yard × 253.
(*c*) The value of a farm = a dollar × 5326.
(*d*) A box of marbles = a marble × 144.

3. Read each statement in 2.

4. Complete the statements in 1, and read them.

CHAPTER II.

READING AND WRITING NUMBERS.

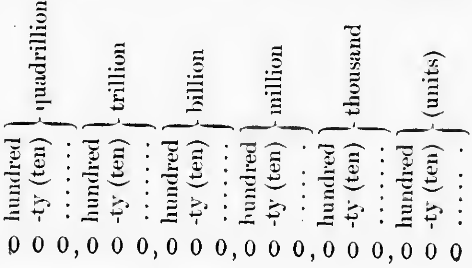
11. In the preceding chapter we have supposed that numbers are read by giving the digits in their order; thus, 49503 would be read four, nine, five, nought, three; and nearly every purpose in Arithmetic would be served by reading them in this way. But ordinarily numbers are read as follows:

10 is ten
11 is eleven
12 is twelve
13 is thirteen
14 is fourteen
15 is fifteen
16 is sixteen
17 is seventeen
18 is eighteen

19 is nineteen
20 is twenty
30 is thirty
40 is forty
50 is fifty
60 is sixty
70 is seventy
80 is eighty
90 is ninety

100 is one hundred
300 is three hundred
28 is twenty-eight
64 is sixty-four
73 is seventy-three
96 is ninety-six
348 is three hundred and forty-eight
and so on.

For numbers with more than three figures, the following scheme shows the manner of reading and writing them:



According to this, we read the number 42,156,321,983, forty two *billion*, one hundred and fifty-six *million*, three hundred and twenty-one *thousand*, nine hundred and eighty-three. Again, we should write in symbols the number which reads three hundred and nine *million*, five hundred and sixty-five, thus, 309,000,565. The student should learn this language, so that he can correctly read in words or write in symbols any number proposed.

EXERCISE II.

1. Read the following numbers:
   (*a*) 532; 429; 680; 903; 207.
   (*b*) 182314; 516812; 500304; 821693.
   (*c*) 1234567890; 3204030001; 58214000004.
   (*d*) 1000000020004; 5003000472031.

2. Write in symbols the following numbers:
   (*a*) Two hundred and sixty-eight.
   (*b*) Nine hundred and thirty-six.
   (*c*) Nine hundred and thirty-six thousand, two hundred and sixty-eight.
   (*d*) Three hundred million, four thousand and two.
   (*e*) Two hundred and fifty-nine million, two hundred and thirty-four thousand, five hundred and thirteen.

56,321,983, forty
hree hundred and
hree. Again, we
ds three hundred
hus, 309,000,565.
he can correctly
proposed.

004.

l, two hundred

and two.
o hundred and
and thirteen.

CHAPTER III.

ADDITION.

12. Suppose there are 5 chairs in the kitchen and 8 chairs in the dining-room. If now we put all the chairs in these two rooms into one room and count them, we shall find that in all there are 13 chairs. Thus we are able to say that the number 5 and the number 8 together make the number 13. We may write this, "$5 + 8 = 13$," which we read, "5 and 8 are 13," or "5 plus 8 is 13." Now in order to avoid the tedious process of counting, the student should learn the following results, so that he may give them accurately and rapidly in any order, thus: $5 + 8 = 13$, $8 + 5 = 13$, $13 = 8 + 5$, $13 = 5 + 8$. When we say $8 + 5 = 13$ we add 8 and 5.

13. The Addition Table.

$2 + 2 = 4$
$2 + 3 = 5$
$2 + 4 = 6$
$2 + 5 = 7$
$2 + 6 = 8$
$2 + 7 = 9$
$2 + 8 = 10$
$2 + 9 = 11$
$3 + 3 = 6$

$3 + 4 = 7$
$3 + 5 = 8$
$3 + 6 = 9$
$3 + 7 = 10$
$8 = 11$
$3 + 9 = 12$
$4 + 4 = 8$
$4 + 5 = 9$
$4 + 6 = 10$

$4 + 7 = 11$
$4 + 8 = 12$
$4 + 9 = 13$
$5 + 5 = 10$
$5 + 6 = 11$
$5 + 7 = 12$
$5 + 8 = 13$
$5 + 9 = 14$
$6 + 6 = 12$

$6 + 7 = 13$
$6 + 8 = 14$
$6 + 9 = 15$
$7 + 7 = 14$
$7 + 8 = 15$
$7 + 9 = 16$
$8 + 8 = 16$
$8 + 9 = 17$
$9 + 9 = 18$

14. The next step in Addition.—Suppose now we group and count the apples in a basket, and find that the apples in the basket = an apple × 76. This means that, in the basket, there are 6 apples, and 7 piles each containing ten apples. Let us now put with these 8 apples more, then there will be in the basket 14 apples and 7 piles. But the 14 apples consist of 4 apples and 1 pile. Therefore all the apples will consist of 4 apples and 8 piles, or as we have agreed to write it,

all the apples = an apple × 84.

Now, we obtain 84 by adding 76 and 8 by memory of the addition table, in two steps, as follows: $8+6=14$, and $1+7=8$. Here we call 84 the *sum* of 76 and 8. The student must now learn to give accurately all such results as the following:

$$43+4=47, \quad 59+6=65, \quad 27+3=30, \quad 84+8=92.$$

15. Addition of Compound Numbers.—Suppose that

the matches in one box = a match × 824,
the matches in a second box = a match × 596,
the matches in a third box = a match × 859.

If now we put into one lot all these matches, we shall find how many there are, not by grouping and counting them, but by *adding* the numbers 824, 596 and 859, which we do by our memory of the addition table. Beginning at the right, the manner of adding the numbers of units is:

$$9+6=15, \quad 15+4=19.$$

The manner of adding the numbers of multiple units of the 1st order is: $1+5=6$, $6+9=15$, $15+2=17$.
Also the manner of adding the numbers of multiple units of the 2nd order is: $1+8=9$, $9+5=14$, $14+8=22$.
Thus we are able to say, all the matches = a match × 2279.

In practice the numbers are set down as below, and the addition is performed mentally, thus:

$$\begin{array}{r}824\\596\\859\\\hline 2279\end{array}$$

16. The Use of Addition.—There are thus two operations:

(1) Putting quantities together, which operation we may perform with our hands; and

ere will be in the
apples consist of
es will consist of
rite it,
4.

y memory of the
14, and 1 + 7 = 8.
tudent must now
ollowing:
84 + 8 = 92.

.—Suppose that
h × 824,
h × 596,
h × 859.
e shall find how
g them, but by
h we do by our
the right, the

units of the 1st
7.
ple units of the

ch × 2279.
, and the addi-

two operations:
n we may per-

---

(2) Adding the numbers which are the measures of the quantities put together, which operation we perform with our minds by memory of the addition table.

We perform the 2nd operation so that we may know the result of the 1st, without actually performing it.

The examples in the following exercise may be easily increased, until the student can add with accuracy.

EXERCISE III.

1. Set down in columns and add:

(*a*) 27 and 38; 93 and 27; 143 and 571; 2965 and 2186.
(*b*) 123, 421, 561, 329, 244, 652, 531 and 508.
(*c*) 32475, 2196, 821, 599, 23 and 7.
(*d*) 44, 9999, 53216, 28, 214 and 9162043.

2. Find the total quantity of apples when the following quantities are put together: an apple × 97, an apple × 532, an apple × 963, an apple × 301, and an apple × 129.

3. Set down in columns and add upwards and downwards the numbers: 52176, 8043, 32145, 219765, 321, 5837, 50096, 21430 and 521643.

4. In the following scheme add the numbers in each column, and set the sum in the space below. Add the numbers in each row, and set the sum in the space to the right. Next, add the numbers in the spaces below, and add the numbers in the spaces to the right, and, if these two sums agree, set the result in the corner space.

4715	3281	9763	1258	2196	
7143	2108	9768	9596	8753	
2178	5963	2146	2158	9635	

5. Make a scheme similar to that in 4, fill it in with any numbers, add and test, as in 4.

CHAPTER IV.

SUBTRACTION.

17. Having now learned all the combinations of numbers in the addition table, the student next learns them in a different order. Thus, since $7+8=15$, he must learn to give this in the order $15-7=8$, which he may read, "15 less 7 is 8," or "7 from 15 leaves 8." So also $15-8=7$, and so on.

When we say $15-8=7$, we subtract 8 from 15; and we call 7 the difference between 15 and 8.

18. The First Method of Subtraction.—Suppose that in a pail there was a known quantity of wheat, and that a known part was taken out, we shall now see how to find out, without counting, the quantity of wheat left. That is, suppose

the wheat in the pail at first = a grain × 72,
and the wheat taken out = a grain × 25.

Here, a grain of wheat being the unit, the number 72 tells us that the wheat in the pail at first consists of 2 units, and 7 multiple units of the 1st order. If, therefore, we change 1 multiple unit of the 1st order into 10 units, we see that the wheat consists of 12 units, and 6 multiple units of the 1st order.

Now, the wheat taken away consists of 5 units, and 2 multiple units of the 1st order. Therefore the wheat left in the pail consists of 7 units, and 4 multiple units of the 1st order; that is,

the wheat left in the pail = a grain × 47.

Again, suppose

the wheat in the pail at first = a grain × 4072,
and the wheat taken out = a grain × 2785.

ns of numbers in
em in a different
give this in the
s 7 is 8," or "7
1.
15; and we call

.—Suppose that
eat, and that a
now to find out,
That is, suppose
× 72,
× 25.
number 72 tells
2 units, and 7
, we change 1
ve see that the
f the 1st order.
and 2 multiple
in the pail con-
der; that is,
7.

× 4072,
× 2785.

Here, the number 4072 tells us that the wheat in the pail at first consists of

4 multiple units of the 3rd order,
7 multiple units of the 1st order,
and 2 units.

These can be changed, so that the whole wheat consists of

3 multiple units of the 3rd order,
9 multiple units of the 2nd order,
16 multiple units of the 1st order,
and 12 units.

Now, the wheat taken away is to consist of

2 multiple units of the 3rd order,
7 multiple units of the 2nd order,
8 multiple units of the 1st order,
and 5 units.

Therefore, the wheat left in the pail consists of

1 multiple unit of the 3rd order,
2 multiple units of the 2nd order,
8 multiple units of the 1st order,
and 7 units.

That is, the wheat left in the pail = a grain × 1287.

In practice, the numbers are set down, the less under the greater, thus:

$$\begin{array}{r}4072\\2785\\\hline 1287\end{array}$$

and mentally the student performs the operations as follows: $12 - 5 = 7$, $16 - 8 = 8$, $9 - 7 = 2$, and $3 - 2 = 1$.

19. The Use of Subtraction.—We see here, as in addition, that there are two operations:

(1) Taking a part of a quantity away from the whole quantity, which operation we perform with our hands; and

(2) Subtracting the measure of the part from the measure of

the whole, which operation we perform with our minds, by memory of the addition table.

We perform the second operation that we may know the result of the first without actually performing it.

20. The Second Method of Subtraction.—There is another method of subtraction in common use, which leaves the digits of the larger number unchanged, but changes those in the smaller number. This we shall now explain.

The wheat in the pail at first = a grain × 4072,
and the wheat taken out = a grain × 2785.

Let us increase the wheat in the pail at first, by putting with it

10 units,
10 multiple units of the 1st order,
and 10 of the 2nd order;

so that it will then consist of

12 units,
17 multiple units of the 1st order,
10 of the 2nd order,
and 4 of the 3rd order.

Let us also increase the wheat to be taken away by the *same* quantity, namely:

1 multiple unit of the 1st order,
1 of the 2nd order,
and 1 of the 3rd order;

so that the quantity of wheat taken away consists of

5 units,
9 multiple units of the 1st order,
8 of the 2nd order,
and 3 of the 3rd order.

Therefore the quantity of wheat left is the same as before, and consists of

7 units,
8 multiple units of the 1st order,
2 of the 2nd order,
and 1 of the 3rd order;
that is, the wheat left = a grain × 1287.

As before, we set the numbers down thus:

$$\begin{array}{r} 4072 \\ 2785 \\ \hline 1287 \end{array}$$

and perform mentally the following operations:

$$12-5=7,\ \ 17-9=8,\ \ 10-8=2 \text{ and } 4-3=1.$$

EXERCISE IV.

1. Subtract from 100 each of the following numbers: 80, 60, 70, 20, 25, 50, 75, 64, 21, 19, 36, 42, 85, 99, 79, 88 and 11.

2. Find the difference between the following pairs of numbers:
   (*a*) 2315 and 6913.
   (*b*) 2008 and 1963.
   (*c*) 83143 and 9406.
   (*d*) 100000 and 12345.
   (*e*) 135791113 and 24681012.
   (*f*) 1003005 and 300105.
   (*g*) 32145 and 9614835.
   (*h*) 214 and 100000.

3. Find what is left when
   (*a*) A foot × 532 is taken from a foot × 934.
   (*b*) A dollar × 1035 is taken from a dollar × 2110.
   (*c*) A book × 29 is taken from a book × 53.
   (*d*) A grain of sand × 1934876 is taken from a grain of sand × 2043798.

4. What quantity of apples must be put with an apple × 203 to make an apple × 501?

5. Subtract 43972 from 307804 as many times as you can. How many times?

6. How often can 837496 be subtracted from 4096382? What number will be left?

7. On Monday I had a cent × 4325; on Tuesday I spent a cent × 1320; on Wednesday I spent a cent × 931; on Thursday I spent a cent × 239; on Friday I spent a cent × 96; and on Saturday I spent a cent × 466. How much money had I then left?

21. The Roman System of Writing Numbers, which is used in numbering the chapters of a book and the hours on a clock, and for a few other purposes, employs the letters I, V, X, L, C, D and M, which stand for the numbers 1, 5, 10, 50, 100, 500 and 1000, respectively. In this system I may be written before V or X, and X may be written before L or C, and C may be written before D or M. Then the combination of the two letters stands for the difference between the numbers indicated by the separate letters. Thus, IV = 5 − 1 = 4, and XL = 50 − 10 = 40, and so on.

In all other cases the combination of letters stands for the sum of the numbers indicated by the separate letters, thus, XXV = 10 + 10 + 5 = 25, XLIX = 40 + 9 = 49. No letter is written in succession more than three times. A dash over a letter increases the number a thousandfold.

EXERCISE V.

1. Write all the numbers from 1 to 100 in Roman symbols.

2. Write the numbers: (*a*) 329, (*b*) 148, (*c*) 693, (*d*) 1437, (*e*) 2109, (*f*) 3016, (*g*) 1009, (*h*) 999, (*i*) 888, (*j*) 777, (*k*) 358, (*l*) 421 in Roman symbols.

3. What are the following numbers: (*a*) MDCLXVI, (*b*) XCIV, (*c*) CML, (*d*) CXIX and (*e*) CXLIV?

sday I spent a cent
31 ; on Thursday I
× 96 ; and on Satur-
had I then left?

Numbers, which
nd the hours on a
he letters I, V, X,
s 1, 5, 10, 50, 100,
I may be written
L or C, and C may
nation of the two
numbers indicated
and XL = 50 − 10

rs stands for the
ate letters, thus,
9. No letter is
A dash over a

oman symbols.
) 693, (*d*) 1437,
j) 777, (*k*) 358,

MDCLXVI, (*b*)

CHAPTER V.

MULTIPLICATION.

22. Suppose that a pile of marbles = a marble × 7.

Here the sign "×" represents the operation of putting marbles together, and 7 tells how many are put together to make up the pile. Suppose, also, that in a box there are 6 of these piles of marbles, so that

∴ the box of marbles = a pile of marbles × 6.

Here the sign "×" represents the operation of putting piles together, and 6 tells how many piles are put together to make up the box of marbles. We may, therefore, represent both these operations in one statement, thus:

the box of marbles = a marble × 7 × 6.

If now we perform both operations and count, or find out by addition, the number of marbles in the box, we shall be able to say,

the box of marbles = a marble × 42.

We say, then, that the single rate (or number) 42 will derive the same quantity from the unit as the two rates 7 and 6, which are used in succession. This is what we mean when we say,

$$7 \times 6 = 42,$$

which is read, "6 times 7 is 42."

In the same way we may show that

$$6 \times 7 = 42,$$

which is read, "7 times 6 is 42."

The single rate (or number) which will derive the same quantity from the unit as the two rates 7 and 6 which are used in succession, is called the *product* of these two rates, and the process of finding it is called multiplication.

23. Use of Multiplication.—It thus appears that the sign "×" has two meanings:

(1) When it is written between the unit and number, as a marble × 7, the sign "×" denotes the operation of putting single marbles together, which operation we may perform with our hands. It is then called the sign of derivation.

(2) When it is written between two numbers, as 7 × 6, the sign "×" denotes the operation of multiplying 7 by 6, which operation we perform with our minds, by memory of the table given in the next article. Here it is called the sign of multiplication.

Now we perform the second of these operations, so that we shall not have to perform two operations of the first kind, but only one. It is consistent then to read the statement,

the box of marbles = a marble × 7 × 6,

thus: "the box of marbles is derived from a marble by the rate 7 multiplied by 6."

24. The Multiplication Table.

$2 \times 2 = 4$
$2 \times 3 = 6$
$2 \times 4 = 8$
$2 \times 5 = 10$
$2 \times 6 = 12$
$2 \times 7 = 14$
$2 \times 8 = 16$
$2 \times 9 = 18$
$2 \times 10 = 20$
$2 \times 11 = 22$
$2 \times 12 = 24$
$3 \times 3 = 9$
$3 \times 4 = 12$
$3 \times 5 = 15$
$3 \times 6 = 18$
$3 \times 7 = 21$
$3 \times 8 = 24$
$3 \times 9 = 27$
$3 \times 10 = 30$
$3 \times 11 = 33$
$3 \times 12 = 36$
$4 \times 4 = 16$
$4 \times 5 = 20$
$4 \times 6 = 24$
$4 \times 7 = 28$
$4 \times 8 = 32$
$4 \times 9 = 36$
$4 \times 10 = 40$
$4 \times 11 = 44$
$4 \times 12 = 48$
$5 \times 5 = 25$
$5 \times 6 = 30$
$5 \times 7 = 35$
$5 \times 8 = 40$
$5 \times 9 = 45$
$5 \times 10 = 50$
$5 \times 11 = 55$
$5 \times 12 = 60$
$6 \times 6 = 36$
$6 \times 7 = 42$
$6 \times 8 = 48$
$6 \times 9 = 54$
$6 \times 10 = 60$
$6 \times 11 = 66$
$6 \times 12 = 72$
$7 \times 7 = 49$
$7 \times 8 = 56$
$7 \times 9 = 63$
$7 \times 10 = 70$
$7 \times 11 = 77$
$7 \times 12 = 84$
$8 \times 8 = 64$
$8 \times 9 = 72$
$8 \times 10 = 80$
$8 \times 11 = 88$
$8 \times 12 = 96$
$9 \times 9 = 81$
$9 \times 10 = 90$
$9 \times 11 = 99$
$9 \times 12 = 108$
$10 \times 10 = 100$
$10 \times 11 = 110$
$10 \times 12 = 120$
$11 \times 11 = 121$
$11 \times 12 = 132$
$12 \times 12 = 144$

ric.

appears that the sign

it and number, as
peration of puttin
e may perform with
ivation.

ers, as 7 × 6, the sign
by 6, which opera
y of the table given
n of multiplication.
rations, so that we
the first kind, but
tatement,
7 × 6,
marble by the rate

7 × 12 = 84
8 × 8 = 64
8 × 9 = 72
8 × 10 = 80
8 × 11 = 88
8 × 12 = 96
9 × 9 = 81
9 × 10 = 90
9 × 11 = 99
9 × 12 = 108
10 × 10 = 100
10 × 11 = 110
10 × 12 = 120
11 × 11 = 121
11 × 12 = 132
12 × 12 = 144

This table includes the results of multiplying two simple numbers together in the same way as in Article 22. These the student must be able to give accurately, rapidly and in all orders, thus: "7 times 6 is 42," "6 times 7 is 42," "42 is 6 times 7," and "42 is 7 times 6."

Note.—No suggestion is here offered as to the best means of learning this table, but each teacher or pupil will adopt the method best suited to himself.

25. The Order of the Digits of a Compound Number.—Suppose the matches in a box = a match × 8763. In the number 8763 we say that, since 6 is the number of multiple units of the 1st order, the order of the 6 is the 1st place to the left of 3; the order of 7 is the 2nd place to the left of 3; and the order of 8 is the 3rd place to the left of 3. In other words, for shortness, let us say that

the order of 6 is + 1,
the order of 7 is + 2,
and the order of 8 is + 3,

where we let "+" stand for the words, "to the left of the units figure." Then "+ 4" means "the 4th place to the left of the units figure." So also the order of the units figure is 0.

Caution.—Many in finding the order of 8 in such a number as 8000, count the 0's which follow the 8. This leads to error. They should count the figures and 0's before the last 0, and should count from right to left, in the following manner:

(7)(6)(5)(4)(3)(2)(1)(0)
8 3 2 0 0 0 0 0

Thus the order of 8 in 80000 is + 4, and the order of 7 in 700 is + 2; while 6 written in the order + 3 is 6000, 4 written in the order + 5 is 400000, and 300 written in the order + 4 is 3000000.

26. The Rule of Order in Multiplication.

Since, a match being the unit,

A multiple unit of the 5th order

$$= \text{a multiple unit of the 2nd order} \times 1000;$$
$$= \text{a match} \times 100 \times 1000.$$

But also a multiple unit of the 5th order

$$= \text{a match} \times 100000.$$

Hence, $\overset{(2)}{1}\,\overset{(1)}{0}\,0 \times \overset{(3)}{1}\,\overset{(2)}{0}\,\overset{(1)}{0}\,0 = \overset{(5)}{1}\,\overset{(4)}{0}\,\overset{(3)}{0}\,\overset{(2)}{0}\,\overset{(1)}{0}\,0$

where the orders are indicated above the numbers.

Now the order of 1 in the 1st number is $+2$,
the order of 1 in the 2nd number is $+3$,
and the order of 1×1, or 1, in the product is $+5$,
which is obtained by adding $+2$ and $+3$.

Therefore, *the order of the product of two simple numbers is found by adding the orders of the simple numbers.* We shall show how to use this rule in the next article.

27. To Multiply two Simple Numbers.—For instance, to multiply together 3000 and 20000. From the Multiplication Table we know that $2 \times 3 = 6$.

Again, since the order of 3 is $+3$,
and the order of 2 is $+4$,
therefore the order of 6 is $+7$,
we must then write 6 in order $+7$. This we shall do by setting down the units 0 first, and count to the left as follows:

$$\overset{(7)}{6}\,\overset{(6)}{0}\,\overset{(5)}{0}\,\overset{(4)}{0}\,\overset{(3)}{0}\,\overset{(2)}{0}\,\overset{(1)}{0}\,0.$$

Hence $3000 \times 20000 = 60000000.$

28. To Multiply a Compound Number by a Simple Number.—For instance, to multiply 4923 by 600. The process of the preceding article is repeated.

Thus, $9 \times 6 = 54$; and since the order of 9 in 4923 is $+2$, and

of 6 in 600 is $+2$, therefore the order of 54 is $+4$. If then we set down the numbers thus, 4923
we perform the operation as follows, 600

$$\begin{array}{lr} 3\times 6=18 \text{ in order } +2= & 1800\\ 2\times 6=12 \text{ in order } +3= & 12000\\ 9\times 6=54 \text{ in order } +4= & 540000\\ 4\times 6=24 \text{ in order } +5= & 2400000\\ \hline \text{The total product then} \quad = & 2953800 \end{array}$$

The student in practice does this in one line, thus:

$$\begin{array}{r} 4923\\ 600\\ \hline 2953800 \end{array}$$

performing in his mind the following:

$3\times 6=18$, $2\times 6+1=13$, $9\times 6+1=55$, and $4\times 6+5=29$.

29. To Multiply a Compound Number by a Compound Number.—For instance, to multiply 4976 by 537. Here we shall have to multiply each simple number in 4976 by each in 537, write each product in its proper order, and add the products. As before, the numbers are set down:

$$\begin{array}{llr} & & 4976\\ & & 537\\ \cline{3-3} \text{Then} & 4976\times 7 = & 34832\\ & 4976\times 3 = & 14928\\ & 4976\times 5 = & 24880\\ \cline{3-3} \text{therefore} & 4976\times 537 = & 2672112 \end{array}$$

Further, the Rule of Order enables us to perform the operations in any order.

Thus: to multiply 70361 by 3075, we may proceed as below:

$$\begin{array}{lrl} & 70361 & \\ & 3075 & \\ \cline{2-2} 70361\times 3 = & 211083 & \text{in the order } +3\\ 70361\times 5 = & 351805 & \text{in the order } 0\\ 70361\times 7 = & 492527 & \text{in the order } +1\\ \cline{2-2} \text{therefore the product} = & 216360075 & \end{array}$$

30. If the length of a line CD = the length of $AB \times 4371$, and the length of AB = an inch × 893, then the length of CD = an inch × 893 × 4371.

Here the number 893 tells us how to derive the length of AB from an inch, and the number 4371 tells us how to derive the length of CD from the length of AB.

Now, when we find by multiplying that 893 × 4371 = 3903303, the number 3903303 tells us how to derive the length of CD from an inch without using, or thinking of, the length of AB.

EXERCISE VI.

A.

1. Tell the order of all the digits in the numbers (*a*) 4200, (*b*) 5321761 and (*c*) 200960304.

2. In the number 200000 what is the order of 2? of 20? of 200? of 20000?

3. Write down:

(*a*) 6 in the order + 2.
(*b*) 5 in the order + 4.
(*c*) 7 in the order + 1.
(*d*) 9 in the order + 6.
(*e*) 1 in the order + 5.
(*f*) 4 in the order 0.
(*g*) 13 in the order 0.
(*h*) 28 in the order + 3.
(*i*) 149 in the order + 5.
(*j*) 200 in the order + 4.
(*k*) 120 in the order + 2.
(*l*) 56 in the order + 1.

B.

4. What is the order of:

(*a*) 2 × 3 in 200 × 30.
(*b*) 4 × 3 in 4000 × 300.
(*c*) 5 × 5 in 50 × 50.
(*d*) 8 × 9 in 8000 × 90.
(*e*) 6 × 5 in 60 × 500.
(*f*) 7 × 7 in 7 × 7.
(*g*) 8 × 1 in 800000 × 10000.
(*h*) 3 × 8 in 300 × 800000.

5. Find the product in each of the following:

(*a*) 400 × 80.
(*b*) 300 × 300.
(*c*) 2000 × 70.
(*d*) 60 × 50000.
(*e*) 10000 × 100.
(*f*) 9000 × 8000.
(*g*) 500 × 40000.
(*h*) 2000 × 80.

6. By what reasoning did you obtain the results in 4 and 5?

C.

7. Multiply the following pairs of numbers together: 12 × 3, 2431 × 2, 10321 × 3 and 401 × 4.

8. Find single rates equivalent to 132 × 4, 516 × 4, 321 × 4, 19643 × 4 and 52176 × 4.

9. Multiply 1234567 by 5, by 4 and by 3.

10. Multiply 23476 by 6 and the product by 6.

11. Multiply 3215987 by 7, by 6.

12. Multiply 28342 by 9 and the product by 9.

13. Multiply 80357 by 8 and the product by 8.

These examples should be continued until the Multiplication Table is learned accurately.

D.

14. Multiply the following pairs of numbers together:
    (*a*) 512 and 23.
    (*b*) 1234 and 21.
    (*c*) 476 and 34.
    (*d*) 2030 and 504.
    (*e*) 5176 and 214.
    (*f*) 835 and 2965.
    (*g*) 246 and 8409.
    (*h*) 3245 and 809.

15. What is the order of 2 × 3 in the product of 35417 and 296? Why? How many figures will there be in the product of 35296 and 2473? How do you tell without multiplying?

16. Find the product of:
    (*a*) 9876 and 3987.
    (*b*) 99893 and 976.
    (*c*) 87969 and 9596.
    (*d*) 793, 257 and 578.
    (*e*) 314159 and 27828.
    (*f*) 95329 × 49867.

17. A mile = a yard × 1760; and a yard = an inch × 36; what is the number of inches in a mile?

18. If a pile of wood = an armful × 35, and an armful = a stick × 14; how many sticks will make up the pile of wood?

19. If one row of squares = a square × 43, and a field = a row of squares × 26; how many squares will make up the field?

20. If the apple-trees in an orchard = a row × 38, and a row of trees = a tree × 26; how many trees are there in the orchard?

21. If a large box of matches = a small box of matches × 27, and a small box of matches = a match × 144; how many matches does a large box contain?

22. If the distance to the moon = a mile × 237125, and the distance to the sun = the moon's distance × 391; find in miles the distance to the sun.

23. Reduce to one number each of the following:

(*a*) $792 \times 38 + 421 \times 69 - 803 \times 50$.

(*b*) $532 \times 693 - 216 \times 257 + 125 \times 160$.

(*c*) $2395 \times 999 - 2396 \times 998$.

24. Find the product of:

(*a*) $5 \times 5 \times 5 \times 5 \times 5$.

(*b*) $6 \times 6 \times 6 \times 6 \times 6 \times 6$.

(*c*) $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$.

(*d*) $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$.

(*e*) $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$.

25. Find the sum of all the numbers from 1 to 199 inclusive.

26. Find the sum of all the numbers from 35 to 69 inclusive.

27. Find the sum of all the numbers between 321 and 563.

28. Find the sum of all the numbers between 1893 and 3753.

29. A man promised to pay to a hospital as follows: on January 1st 1 cent, on January 2nd 2 cents, on January 3rd 3 cents, and so on for a year. How much in all did he promise?

30. A small box of matches contains 63 matches, a large box contains 27 small boxes, a case contains 36 large boxes, and a car load contains 360 cases. How many single matches are in the car load?

31. If you are told that a bushel of wheat contains 193472 grains, what does the figure 9 in this number tell you? Explain in detail the information given you by the number 193472.

CHAPTER VI.

DIVISION.

31. We have shown how to group and count things which are of the same name. We shall now show how such things as the length of the table, the flat surface of the table, the water in a barrel, and the time from now till noon are measured.

(*a*) *To measure the length of the table.*—Place a pencil along the long edge as many times as will fill up the whole length of the table. By so doing, the length of the table is cut into parts, each as long as the pencil. Having counted the parts, let us suppose there are 13 parts. We have now measured the length of the table, and we represent the act or operation of measuring by a horizontal line, thus:

$$\frac{\text{the length of the table}}{\text{the length of the pencil}} = 13.$$

This statement we read, "The measure of the length of the table, when the length of the pencil is the unit, is 13."

Here the *quantity* measured is the length of the table; the *unit*, by means of which it is measured, is the length of the pencil; the *measure* of the quantity is the number 13; the *line* denotes the operation of measuring; and the sign "=" indicates that the measuring is completed.

(*b*) *To measure the time from now till noon.*—To do this we use a watch or clock, which is specially made, not only to divide the time into equal parts, but to count the parts. All we have to do is to read the number of parts indicated on its face. The time it takes the second-hand to go once around is the unit of time, which is called a minute. Suppose the number of these units required to make up the time from now till noon is 96; then, as before, we represent the operation of measuring by a horizontal line, thus:

$$\frac{\text{the time from now till noon}}{\text{a minute}} = 96.$$

(*c*) In the same way any quantity may be measured. A part of the quantity is chosen as a unit, and it is found out how many times this unit occurs to make up the quantity. Then always

$$\frac{\text{the quantity}}{\text{the unit}} = \text{a number}.$$

32. Division Defined.—By multiplying we find that

$$\text{a yard} \times 13 \times 28 = \text{a yard} \times 364,$$

which we may write backwards, thus,

$$\text{a yard} \times 364 = \text{a yard} \times 13 \times 28.$$

If now we regard a yard × 364 as the quantity to be measured, and a yard × 13 as the unit by which it is measured, we shall have,

$$\frac{\text{a yard} \times 364}{\text{a yard} \times 13} = 28;$$

where the horizontal line represents the operation of measuring described in the preceding article.

Again, speaking only of numbers, since 364 was obtained by multiplying 28 by 13; let us agree that,

$$\frac{364}{13} = 28.$$

Then the horizontal line represents the operation which *undoes* the result of multiplication; that is, which reverses the operation of multiplying. This operation is called *Division;* the product 364 is called the *Dividend;* the number 13 below the line is called the *Divisor;* and the whole combination $\frac{364}{13}$ is called the

Quotient, which in this case is 28. The statement $\frac{364}{13} = 28$ may be read, "the quotient of 364 by 13 is 28."

33. The Use of Division.—It thus appears that the horizontal line has two meanings:

(1) When it is drawn between two quantities of the same kind, thus,

$$\frac{\text{the surface of a board}}{\text{a square foot}},$$

it represents the operation of measuring the quantity above it, by means of the quantity below it as a unit; which operation we perform with our hands, assisted by instruments. The line, then, is called the *sign of measuring*.

(2) When the line is drawn between two numbers, thus:

$$\frac{364}{13},$$

it represents the operation of dividing the number above it by the number below it, which operation we perform with our minds by memory of the tables. Here the line is called the *sign of division*.

Now, by measuring, we find that

$$\frac{\text{a yard} \times 364}{\text{a yard} \times 13} = 28,$$

and, by dividing, that $\frac{364}{13} = 28$.

Therefore, since the first operation, that is, measuring, is usually difficult to perform, and often impossible, we may avoid it by performing the second operation, that is, division. So that the measure of one quantity, by another as a unit, is the quotient, when the measure of the first is divided by the measure of the second.

Note.—Instead of the horizontal line, the sign "÷" is sometimes used. Thus, in "172 feet ÷ 4 feet," or "a foot × 172

÷ a foot × 4," the sign "÷" denotes the operation of measuring 172 feet by 4 feet as a unit; while in "172 ÷ 4," the sign "÷" denotes the operation of dividing 172 by 4.

34. Rule of Order in Division.—Since the order of the product of two simple numbers is found by adding the orders of the simple numbers, therefore the order of one of the simple numbers is found by subtracting the order of the other simple number from the order of the product. In other words, *the order of a simple quotient by a simple number is found by subtracting the order of the Divisor from the order of the Dividend.*

35. To Divide, by a Simple Number, such numbers as are found in the Multiplication Table. For instance, to divide 3500000 by 700.

From the table, the student knows that $\frac{35}{7} = 5$.

Again, since the order of 35 is + 5,
and the order of 7 is + 2,
therefore the order of the quotient 5 is + 3. Article 34.

So that

$$\frac{3500000}{700} = 5000.$$

36. To Divide a Compound Number by a Simple Number.—For instance, to divide 58416 by 6. The student knows, from the Multiplication Table, the numbers which 6 will divide, namely, 6, 12, 18, 24, 30, 36, 42, 48 and 54; and the number, 58416, is made up of these, as follows:

54 in the order + 3, that is, 54000,
42 in the order + 2, that is, 4200,
18 in the order + 1, that is, 180,
and 36 in the order 0, that is, 36.

Now the divisor 6 is in the order 0. Therefore, by repeating the process of Article 35, the quotient will consist of

9 in the order + 3,
7 in the order + 2,
3 in the order + 1,
and 6 in the order 0,

that is, the quotient is 9736.

In practice the student sets down the number thus:

$$\begin{array}{r} 6\,)\,\underline{58416} \\ 9736 \end{array}$$

performing mentally the operations as follows:

$$\frac{54}{6}=9,\quad 58-54=4,\quad \frac{42}{6}=7,\quad 44-42=2,\quad \frac{18}{6}=3,$$

$$21-18=3,\quad \frac{36}{6}=6.$$

37. Inexact Division.—To divide 473 by 7. As before, 473 is made up of 42 in the order + 1, 49 in the order 0, and 4 in the order 0. Then the divisor 7, being in the order 0, the quotient consists of 6 in the order + 1, 7 in the order 0, that is, 67; but the remaining 4 is not divisible by 7 at present. We shall show later on how 4 can be divided by 7, but now, we shall only indicate that it is to be done, thus:

$$\frac{473}{7}=67+\frac{4}{7}\quad\text{or}\quad 67\frac{4}{7}.$$

38. *To measure* 8325 *pounds of pork by the unit* 5 *pounds of pork.*—To *actually* measure this we should have to cut the pork into pieces each containing 5 pounds; and then group and count the pieces. This operation is indicated by the horizontal line when we write

$$\frac{\text{8325 pounds of pork}}{\text{5 pounds of pork}},\ \text{or}\ \frac{\text{a pound of pork}\times 8325}{\text{a pound of pork}\times 5}.$$

It is not desirable to perform this operation, and it is shown in Article 33 how we may avoid doing it, by performing in our minds another operation quite different, namely, by dividing th number 8325 by the number 5.

This operation is indicated by the line in $\frac{8325}{5}$,

and having performed it, we may say, $\frac{8325}{5} = 1665$.

Therefore, the measure of 8325 pounds of pork when the unit i 5 pounds of pork is 1665; that is,

$$\frac{\text{a pound of pork} \times 8325}{\text{a pound of pork} \times 5} = 1665.$$

Further, we may say, 8325 pounds of pork = 5 pounds of por × 1665, that is, "8325 pounds of pork is derived from 5 pound of pork by the rate 1665."

EXERCISE VII.

1. Divide: (*a*) 560000 by 80. (*c*) 210000 by 7000.
   (*b*) 8100 by 90. (*d*) 36000 by 600.
   (*e*) 420000000 by 700000.
   (*f*) Three hundred and fifty million by seventy thousand
   (*g*) Three hundred trillion by sixty million.

Give the reason for the order of the simple number in th quotient in each case.

2. Show how the number 435 is made up of the products of found in the Multiplication Table, written in their proper order Hence, write down the quotient when 435 is divided by 5.

3. Show how the number 439821 is made up of the product of 9, found in the table. Hence, find the quotient when th number is divided by 9.

4. Divide: (*a*) 2139216 by 2, by 3, by 4, by 8.
   (*b*) 31425 by 5 and the quotient by 5.
   (*c*) 593621 by 7.
   (*d*) 6168960 by 2, the quotient by 3, the second quotien by 4, and so on, by 5, 6, 7, 8 and 9.

(*e*) 6168960 by 9, the quotient by 8, and so on, by 7, 6, 5, 4 and 3.

5. A mile = a foot × 5280, and a yard = a foot × 3. Find the result of performing the operation $\frac{\text{a mile}}{\text{a yard}}$.

6. A wheat field contains 420 shocks of grain, each shock contains 10 sheaves. How many sheaves are in the field, and how many loads, each consisting of 300 sheaves? How did you find out?

7. In a box are 2384 matches. If these are tied in bundles each containing 70 matches, how many matches are left which are not enough to make a bundle?

8. What is the measure of a pound × 891, when the unit is a pound × 9?

9. Perform the following operations as far as possible:

(*a*) $\frac{3246}{5}$. (*b*) $\frac{76103}{8}$. (*c*) $\frac{10000}{7}$. (*d*) $\frac{83000}{9}$.

(*e*) $\frac{2143}{2}$. (*f*) $\frac{4713}{6}$. (*g*) $\frac{2103}{3}$.

10. Find the result of performing the operations:

(*a*) $\frac{\$816}{\$4}$. (*b*) $\frac{\text{5191 cents}}{\text{7 cents}}$. (*c*) $\frac{\text{a yard} \times 8321}{\text{a yard} \times 8}$.

(*d*) $\frac{\text{18321 men}}{\text{6 men}}$. (*e*) $\frac{\text{a match} \times 8347}{\text{a match} \times 5}$.

(*f*) $\frac{\text{a minute} \times 480000}{\text{an hour}}$.

11. Distinguish between the operations indicated in No. 10 and those in No. 9.

12. Divide 47585 by 4, by 40, by 400 and by 4000.

13. Divide 6038071 by 60, by 6000, by 800, by 90000 and by 1000.

39. To Divide by a Compound Number.—For instance, to divide 14324 by 593.

It will be sufficient to find the order of the first figure of the quotient, thus:

Since the order of 14 in the dividend is $+3$,
and the order of 5 in the divisor is $+2$;
therefore the order of the first figure in the quotient is $+1$; that is, the quotient will consist of two figures. We next show how to find these figures.

By trial we find that 593×3 is more than 1432,
and that 593×2 is less.

Then the first partial dividend $= 1432$,
and $593 \times 2 = 1186$,
therefore the remainder $= 246$, which is less than 593.

Again, by trial, we find that 593×5 is more than 2464,
and that 593×4 is less.

Then the second partial dividend $= 2464$,
and $593 \times 4 = 2372$,
therefore the second remainder $= 92$,
which is less than 593. Hence, $\dfrac{14324}{593} = 24\dfrac{92}{593}$.

In practice these operations are performed as follows:

$$\begin{array}{rr} 593\,) & 14324\,(\,24 \\ & \underline{1186} \\ & 2464 \\ & \underline{2372} \\ & 92 \end{array}$$

It will be shown, later on, how 92 may be divided by 593.

40. Inexact Measurement of Quantities.—Let us measure the quantity 4325 inches by the unit, a yard, which is 36 inches. By Article 33, we find the required measure by dividing 4325 by 36. When this is done, we find that $\frac{4325}{36} = 120\frac{5}{36}$. Now, we are not as yet able to divide 5 by 36. This we might have foreseen, had we actually measured the distance 4325 inches by the yard length. For we should then have found that the yard would have been placed down 120 times, but that a small portion, 5 inches, would have been left, which it would have been impossible to measure with a yard as unit. We shall show, in the next chapter, how we may proceed both with the division and with the measuring in such cases.

EXERCISE VIII.

1. Divide:

(*a*) 144 by 24.
(*b*) 1728 by 36.
(*c*) 2448 by 17.
(*d*) 83376 by 18.
(*e*) 543125 by 125.
(*f*) 31416 by 24.

2. Divide:

(*a*) 139748 by 629.
(*b*) 82143 by 5389.
(*c*) 7218356 by 84162.
(*d*) 8000000 by 725.
(*e*) 835129 by 61723.
(*f*) 800405 by 90301.

3. State the result of the following operations, after you have performed them:

(*a*) $\frac{1234567}{4321}$. (*b*) $\frac{203527}{777}$.

4. Find, without dividing, the number of figures in the quotients:

(*a*) $\frac{123476}{241}$. (*b*) $\frac{596213}{4132}$. (*c*) $\frac{100000000}{56042}$.

5. If the sun's distance is 92445600 miles, and the moon's distance is 237040 miles; what is the measure of the sun's distance when the moon's distance is the unit?

6. How many 25 cent pieces will make up 29450 cents?

7. A mile = a yard × 1760. How many miles are there in a yard × 1283040?

8. A car is loaded with 19992 lbs. of flour, put up in barrels each containing 196 lbs. of flour. Required the number of barrels.

9. Divide:

(*a*) 249493596792 by 427964.
(*b*) 248155914760 by 620072.
(*c*) 609435012763918 by 870120506.

10. Simplify as far as possible:

(*a*) $\frac{183 \times 2971}{4307}$. (*b*) $\frac{100004}{301 \times 17}$.

(*c*) $\frac{93214 \times 6128}{133 \times 506}$. (*d*) $\frac{32571692}{999 \times 999}$.

11. Measure 192 feet with 6 feet as a unit. Express this operation of measuring. Need we perform it? How may we avoid measuring this distance?

12. How many times will 7321 inches be contained in 397645 inches? How much will be left?

13. What is the quotient when 52763 is divided by 732? Write it down both before and after you divide.

14. Explain how you tell the number of figures there will be in the quotient before you divide. State the rule by which you tell.

15. Write a description of how you would obtain the numbe 376 by measuring the length of a field with a yard as a unit.

16. What is the meaning of the horizontal lines in each of the following:

(*a*) $\frac{\text{a yard length}}{16}$. (*b*) $\frac{16}{\text{a yard length}}$.

(*c*) $\frac{\text{the length of a field}}{\text{the weight of a ball}}$. (*d*) $\frac{\text{a dollar}}{\text{an hour}}$.

(*e*) $\frac{\text{the surface of a field}}{\text{the length of the field}}$. (*f*) $\frac{\text{72 pounds}}{\text{9 ounces}}$.

Why?

CHAPTER VII.

REGULAR SUBDIVISION OF THE UNIT—DECIMALS.

41. In the preceding chapters, we supposed that the unit occurred an exact number of times to make up the quantity. Let us now see what is done when this is not the case.

Let the length of a line drawn on the table be the quantity to be measured. Let any unit be chosen. Beginning at one end of the line, repeat the unit until the other end is reached. It will be found that the last time the unit of length is placed down its end reaches beyond the end of the line. There is, therefore, a portion of the line which cannot be measured by the unit chosen. The greater part of it is measured, but a small part less than the unit remains unmeasured.

To measure this small part, we proceed to choose a more convenient unit. Let the original unit be cut into 10 equal parts, each of which we shall call a *sub-unit of the 1st order*. Proceeding with this sub-unit to measure the part of the line still unmeasured, we find, as before, that the greater part of it is measured, but a small part less than this sub-unit is still unmeasured.

Again, to measure this small part, let us cut this sub-unit of the 1st order into 10 equal parts, each of which we shall call a *sub-unit of the 2nd order*. Proceeding with this sub-unit to measure the part still unmeasured, we find, as before, that the greater part of it is measured, but a small part less than this sub-unit is still unmeasured.

By continuing this process, we may measure the line until the part unmeasured is so small that it may be neglected. It may happen, however, that a sub-unit exactly measures the part left, in which case the line is accurately measured.

42. Suppose, now, just before the end of the line was reached, the original unit was placed down 7 times, the sub-unit of the 1st order 5 times, the sub-unit of the 2nd order 8 times, the sub-unit of the 3rd order 4 times, and that we neglect the part still unmeasured.

According to what was agreed upon in Article 3, we write these simple numbers in the order 7584; and when we have distinguished the figure which is the number of original units from the other figures, we shall have completely expressed the measure of the quantity. This is usually done by writing a point after the 7 thus, 7·584. But this way of writing the point is somewhat misleading, and we shall, in this chapter, mark the units figure by writing the point above it, thus, $\dot{7}584$. In either case we read the number, "Seven point, five, eight, four."

We are now able to say,

$$\frac{\text{the length of the line}}{\text{the unit of length}} = \dot{7}584,$$

which is read: "The measure of the length of the line by the unit of length is $\dot{7}584$."

As before, the line indicates the operation of measuring which we have just described.

43. All Numbers are Rates.

—This number $\dot{7}584$ also tells us how to use the unit in order to make up the quantity, that is, how to derive the quantity from the unit, and thus it is called a *Rate.*

When we regard the number in this light, we say,

$$\text{the quantity} = \text{the unit} \times \dot{7}584.$$

For example, if we are told that the weight of an iron ball = a pound × $7\dot{3}24$, the number tells us that 7 ten-pound weights, 3 one-pound weights, 2 weights of which 10 make a pound, and

---

4 weights

make up t

the weigh

as before,

derivation

Further

are put to

44. In

sary to re

be written

Thus, $1\dot{2}3$

1. Desc

the unit i

(*a*)

(*b*)

(*c*)

(*d*)

(*e*)

(*f*)

2. Find

a pound ×

pound × $\dot{2}$

3. A me

a $ × $8\dot{9}3$

$8\dot{2}79$; on

What is th

isures the part
l.
ne was reached,
sub-unit of the
times, the sub
ct the part still

cle 3, we write
when we have
f original units
expressed the
by writing a
riting the point
pter, mark the
584. In either
t, four."

ne line by the

easuring which

ber $\dot{7}584$ also
the quantity,
and thus it is

y,

n iron ball = a
ound weights,
a pound, and

---

4 weights of which 10 make one of the preceding, all put together make up the weight of the iron ball. This operation of deriving the weight of the iron ball from the pound weight is denoted, as before, by the sign "×," which is here called the sign of derivation.

Further, when we write a unit × $8\dot{3}402$, we mean that:

8 multiple units of the 1st order,
3 units,
4 sub-units of the 1st order,
and 2 sub-units of the 3rd order,

are put together to make up a certain quantity.

44. In working the following examples, it is sometimes necessary to remember that, when the units digit is marked, 0's may be written either before or after a number without changing it. Thus, $1\dot{2}3 = 001\dot{2}3000$, but 72 is not the same as 7200.

EXERCISE IX.

1. Describe the process by which the quantity is derived from the unit in each of the following:

(*a*) A rod = a yard × $\dot{5}5$.
(*b*) The length of a field = a rod × $3\dot{2}6$.
(*c*) A roll of butter = a pound of butter × $\dot{5}25$.
(*d*) The surface of the table = a square foot × $5\dot{9}34$.
(*e*) The cost of a yard of cloth = a dollar × $\dot{1}875$.
(*f*) The cost of a bicycle = a dollar × $6\dot{4}5$.

2. Find the total of the following quantities: a pound × $\dot{3}62$, a pound × $8\dot{9}75$, a pound × $8\dot{0}213$, a pound × $51\dot{2}$, and a pound × $\dot{2}53$.

3. A merchant received, on Monday, a $ × $3\dot{4}56$; on Tuesday, a $ × $8\dot{9}31$; on Wednesday, a $ × $8\dot{2}35$; on Thursday, a $ × $8\dot{2}79$; on Friday, a $ × $1\dot{2}67$; and on Saturday, a $ × $19\dot{3}89$. What is the total sum he received during the week?

4. Add together the rates $4\dot{2}396$, $\dot{5}31$, $\dot{0}4197$, $80\dot{0}1$, $560\dot{2}13$, $\dot{0}00008$, $50\dot{0}$, $8\dot{9}604$, $\dot{3}21$ and $\dot{0}3$.

5. Find the sum of $\dot{4}321596$, $30\dot{0}97$, $\dot{0}00146$, $39\dot{6}2$, $8009\dot{3}21$, $50\dot{3}7968$ and $\dot{9}87654$.

6. Find the difference of the following quantities:

(*a*) A pound × $8\dot{7}239$ and a pound × $\dot{9}36$.

(*b*) A minute × $72\dot{3}8$ and a minute × $81\dot{6}27$.

(*c*) A yard × $1823\dot{7}14$ and a yard × $897\dot{1}384$.

(*d*) A dollar × $176\dot{3}42$ and a dollar × $31\dot{8}62$.

(*e*) A cubic foot × $39\dot{0}4$ and a cubic foot × $4\dot{3}72$.

7. Subtract:

(*a*) $\dot{0}5$ from $\dot{1}$.

(*b*) $\dot{8}3$ from $1\dot{0}$.

(*c*) $\dot{1}$ from $\dot{3}2$.

(*d*) $\dot{0}32$ from $\dot{0}41$.

(*e*) $\dot{0}062$ from $\dot{0}532$.

(*f*) $\dot{0}9999$ from $\dot{1}$.

(*g*) $\dot{9}8765$ from $1\dot{2}376$.

(*h*) $\dot{0}005$ from $\dot{0}0061$.

(*i*) $2003\dot{0}42$ from $3901\dot{6}214$.

(*j*) $123\dot{4}56$ from $1234\dot{5}6$.

8. If out of a barrel of water which consists of a gallon × $3\dot{2}06$, there be taken a pailful which contains a gallon × $\dot{3}71$, as many times as possible; how much will there be left?

45. The Order of the Digits of a Number.—In any number, such as $135\dot{7}98642$, as in Article 25, we say that the order of 9 is the 1st place to the right of the units digit, the order of 8 is the 2nd place to the right of the units digit, and so on. In other words, for shortness, let us say that,

the order of 9 is − 1,
the order of 8 is − 2,
the order of 6 is − 3, and so on,

where we let the sign "−" stand for the words "to the right of the units digit;" then − 5 denotes the 5th place to the right of the units digit. The orders of the digits are shown in the scheme:

+(6)(5)(4)(3)(2)(1) . (1)(2)(3)(4)(5)(6)(7)−
0 0 0 0 0 0 0 0 0 0 0 0 0 0

46. To Find the Sum of Two Orders.—For instance, to add -5 and -3. Beginning *from* the units place we count to the right 5 places, arriving at the fifth place; then, *from* the 5th place we count to the right three places, arriving at the eighth place. Thus we are able to say:

$$-5-3=-8.$$

To add -5 and $+3$. As before, we arrive at the 5th place; then, from the 5th place we count to the left 3 places, arriving at the 2nd place to the right. Thus also we can say:

$$-5+3=-2.$$

In the same way we may show that $+5+$ [illegible] $+8$,
and that $+5-3=+2$.

We have, therefore, the following rules for finding the sum of two orders:

(*a*) "When the signs are alike, add the numbers, and write before the sum the same sign."

(*b*) "When the signs are unlike, subtract the less number from the greater, and write before the difference the sign of the greater."

47. To Subtract one Order from another.—For instance, to subtract -3 from -5. Beginning *from* the units place, we count to the right 5 places, arriving at the fifth place; then *from* the fifth place we count to the left (not to the right, as in adding orders) 3 places, arriving at the second place to the left.

Again, to subtract $+3$ from -5. As before, we arrive at the fifth place to the right; then, *from* the fifth place we count to the right (not to the left, as in adding orders) 3 places, arriving at the eighth place to the right.

Hence we have the rule for subtracting one order from another:

"Change the sign of the order to be subtracted, and use the Addition Rule."

Thus, to subtract -5 from $+2$, we have $+2+5=+7$.

EXERCISE X.

1. What is the order of each digit in the numbers $100\dot{0}304$, $\dot{0}300082$, $1800\dot{0}01$, $\dot{7}$, $1\dot{7}08$ and $8000\dot{0}10000007$?

2. Write 3 in the orders + 3, − 3, − 5, + 1, 0, − 1 and + 2.

3. Write 72 in the orders + 2, − 2, + 1, − 1, − 5, + 4 and 0.

4. Write 30 in the orders + 2, + 5, − 2, 0, − 8, + 1, − 1 and − 3.

5. Write 293 in the orders + 4, − 4, + 2, − 2, + 1, − 1 and 0.

6. Write 8200 in the orders − 7, + 1, − 8, + 4, − 3, − 1, 0 and + 2.

7. Find the sum of

(*a*)	− 3 and + 2.	(*f*)	+ 2 and + 3.	(*k*)	+ 8 and − 8.
(*b*)	− 8 and + 6.	(*g*)	+ 3 and − 7.	(*l*)	− 3 and − 4.
(*c*)	+ 5 and − 3.	(*h*)	− 5 and + 7.	(*m*)	− 348 and + 962.
(*d*)	− 8 and − 1.	(*i*)	+ 7 and + 5.	(*n*)	− 1203 and − 481.
(*e*)	0 and − 2.	(*j*)	+ 1 and − 1.		

8. Subtract the 2nd order from the 1st in each of the above pairs.

9. Subtract the 1st order from the 2nd in each pair of No. 7.

48. The Rule of Order in Multiplication.—Since a sub-unit of the 3rd order

$$= \text{a multiple unit of the 2nd order} \times \dot{0}00001$$
$$= \text{the unit} \times 10\dot{0} \times \dot{0}00001,$$

and a sub-unit of the 3rd order = the unit × $\dot{0}001$.

Therefore $\overset{(2)}{1}\,\overset{(1)}{0}\,\overset{.}{0} \times \overset{.}{0}\,\overset{(1)}{0}\,\overset{(2)}{0}\,\overset{(3)}{0}\,\overset{(4)}{0}\,\overset{(5)}{1} = \overset{.}{0}\,\overset{(1)}{0}\,\overset{(2)}{0}\,\overset{(3)}{1}$.

Now, the order of 1 in the 1st number is + 2,
the order of 1 in the 2nd number is − 5,
and the order of 1 × 1 or 1 in the product is − 3,
which is the sum of + 2 and − 5.

Therefore, in this case, as in Article 26, *the order of the product of two simple numbers is found by adding the orders of the simple numbers.*

Similarly every case, which we may examine as above, is found to be comprehended in this rule. This rule governs all the operations of compound numbers, and the student should master it.

49. To Multiply $4\dot{3}052$ by $8\dot{0}076$.

Since the order of 2 in $4\dot{3}052$ is -3,
and the order of 6 in $8\dot{0}076$ is -3,
therefore the order of 12 in the product is -6.

Again, the order of 4 in $4\dot{3}052$ is $+1$,
and the order of 6 in $8\dot{0}076$ is -3,
therefore the order of 24 in the product is -2, and so on.

Now, each simple number in $4\dot{3}052$ is multiplied by each simple number in $8\dot{0}076$, the orders of the products are found as above, the products are written in these orders and added. This is done conveniently, as follows:

$$\begin{array}{lcr}
 & & 4\dot{3}052 \\
 & & 8\dot{0}076 \\
\hline
\text{The product by 6} & = & \dot{0}258312 \\
\text{the product by 7} & = & \dot{3}01364 \\
\text{the product by 8} & = & 344\dot{4}16 \\
\hline
\text{therefore } 4\dot{3}052 \times 8\dot{0}076 & = & 344\dot{7}431952
\end{array}$$

EXERCISE XI.

1. Write down the product of each of the following pairs of numbers, and give the reasoning by which you find the order:

(*a*) $30\dot{0} \times \dot{0}04$.	(*e*) $700\dot{0} \times \dot{0}00008$.
(*b*) $\dot{0}7 \times \dot{0}08$.	(*f*) $4000 \times \dot{0}004$.
(*c*) $\dot{0}003 \times 40\dot{0}$.	(*g*) $\dot{0}1 \times \dot{0}1$.
(*d*) $800\dot{0} \times \dot{0}02$.	(*h*) $\dot{0}02 \times \dot{0}02$.

2. Multiply together

(*a*) $10\dot{0}4$ and $\dot{0}3$.
(*b*) $\dot{0}703$ and $\dot{1}02$.
(*c*) $\dot{7}29 \times \dot{5}31$.
(*d*) $1\dot{3}82 \times 2\dot{9}76$.
(*e*) $148\dot{9}6 \times \dot{0}0342$.
(*f*) $\dot{7}698 \times \dot{3}72$.
(*g*) $219\dot{0}$ and $\dot{8}27$.
(*h*) $\dot{7}8912$ and $\dot{0}0397$.
(*i*) $94\dot{3}7$ and $8\dot{6}47$.
(*j*) $1\dot{0}97083$ and $40\dot{7}301$.

3. A rod = a foot × $1\dot{6}5$, and a foot = an inch × 12. What rate will derive a rod from an inch?

4. A barrel of water = a gallon × $3\dot{1}5$, and a gallon = a cubic inch × $27\dot{7}274$. What is the capacity of the barrel in cubic inches?

5. The circumference of a circle = its diameter × $\dot{3}1416$, and the diameter = a foot × $2\dot{5}75$. How many feet does its circumference consist of.

6. Simplify $\dot{0}125 \times \dot{6}12 \times 2\dot{1}3 - \dot{0}375 \times \dot{5}04 \times \dot{3}12$.

50. The Rule of Order in Division.—Since Division is the operation which reverses the operation of Multiplication; that is, when the product of two rates and one of these rates are given, Division is the process by which the other rate is found.

Moreover, since the order of the product of two simple numbers is found by adding the orders of the simple numbers; therefore, the order of one of the simple numbers is found by subtracting the order of the other simple number from the order of the product. In other words:

The order of the quotient by a simple number is found by subtracting the order of the Divisor from the order of the Dividend.

51. Examples.—1. *To divide* $\dot{0}042$ *by* $7\dot{0}$.

From the tables, $\frac{42}{7} = 6$,

Now, since the order of 42 in $\dot{0}042$ is -3,
and the order of 7 in $\dot{7}0$ is $+1$,
therefore the order of 6 in the quotient is -4, which is $-3-1$.
So that the quotient $= \dot{0}0006$.

2. *To divide* $\dot{4}768$ *by* $\dot{0}08$.

As in Article 36, the Dividend $\dot{4}768$ is made up of the products of 8 found in the Table, as follows:

40 in the order -1, that is, $\dot{4}0$,
72 in the order -2, that is, $\dot{0}72$,
and 48 in the order -3, that is, $\dot{0}048$.

And since the order of 8 in the divisor is -2; therefore the quotient consists of 5 in the order $+1$, 9 in the order 0, and 6 in the order -1. The quotient then is $5\dot{9}6$.

In practice the operation may be set down thus:

$$\dot{0}08\ )\ \frac{\dot{4}768}{5\dot{9}6}\quad\text{or}\quad\frac{\dot{4}768}{\dot{0}08}=5\dot{9}6.$$

3. *To divide* $147\dot{2}036$ *by* $\dot{0}562$.

It will be sufficient to determine the order of the 1st figure of the quotient, thus:

Since the order of 14 in the dividend is $+2$,
and the order of 5 in the divisor is -1,
therefore the order of the 1st figure of the quotient is $+3$; that is, the units figure is the 4th figure of the quotient.

Next, we divide, as in Article 39, as follows:

$$\begin{array}{lr}
\dot{0}562\ )\ 147\dot{2}036\ (\ 261\dot{9}28 & \\
\phantom{\dot{0}562\ )\ }\underline{1124} & \\
\phantom{\dot{0}562\ )\ }\ \ 3480 & \\
\phantom{\dot{0}562\ )\ }\ \ \underline{3372} & \\
\phantom{\dot{0}562\ )\ }\ \ \ 1083 &
\end{array}
\qquad
\begin{array}{r}
1083\\
\underline{562}\\
5216\\
\underline{5058}\\
158
\end{array}
\qquad
\begin{array}{r}
1580\\
\underline{1124}\\
4560\\
\underline{4496}\\
64
\end{array}$$

Hence $\dfrac{147\dot{2}036}{\dot{0}562} = 261\dot{9}28.$

We may continue the division as far as we wish, but in practical results we seldom require more than six figures in the quotient.

4. *To continue the division in Article 39*, all we have to do is to mark the units figures, affix 0's to the dividend, and proceed as above, thus:

$$\begin{array}{r l}
59\dot{3}\ )\ 1432\dot{4}0\ (\ \dot{2}415 & \\
\underline{1186} & \\
2464 & \\
\underline{2372} & \\
920 & \\
\underline{593} & \\
3270 & \\
\underline{2965} & \\
305 &
\end{array}$$

NOTE.—In the following chapters we shall mark the units figure by writing the point after it, as is the custom, thus:

$$13\dot{5}7 = 135{\cdot}7.$$
$$\dot{0}032 = 0{\cdot}032 = {\cdot}032.$$

EXERCISE XII.

1. Obtain the quotient in each of the following, giving the reasoning by which its order is known.

(*a*) $\dfrac{0\dot{3}2}{\dot{0}8}$. (*d*) $\dfrac{4200}{\dot{0}7}$. (*g*) $\dfrac{360}{\dot{0}009}$. (*j*) $\dfrac{200}{\dot{0}05}$.

(*b*) $\dfrac{\dot{0}0072}{\dot{0}09}$. (*e*) $\dfrac{63}{\dot{0}009}$. (*h*) $\dfrac{\dot{0}36}{9000}$. (*k*) $\dfrac{100}{5}$.

(*c*) $\dfrac{240}{\dot{0}04}$. (*f*) $\dfrac{\dot{6}4}{800}$. (*i*) $\dfrac{2\dot{8}0}{7000}$. (*l*) $\dfrac{1}{200}$.

2. Reduce the following to single rates:

(*a*) $\frac{5}{16}$. (*c*) $\frac{9}{80}$. (*e*) $\frac{41}{64}$. (*g*) $\frac{1}{64}$.
(*b*) $\frac{7}{32}$. (*d*) $\frac{18}{25}$. (*f*) $\frac{200}{625}$. (*h*) $\frac{1237}{512}$.

3. Find the order of the first figures in the quotients when
(*a*) $7\dot{2}3$ is divided by $\dot{6}24$.
(*b*) $1\dot{0}83$ is divided by $70\dot{9}3$.
(*c*) $\dot{0}814$ is divided by $5\dot{6}02$.
(*d*) $\dot{0}01379$ is divided by $\dot{0}0000835$.

4. Obtain the quotients, each to six figures, in

(*a*) $\frac{6\dot{2}4}{7\dot{2}3}$. (*c*) $\frac{189\dot{6}4312}{\dot{5}0372}$. (*e*) $\frac{234\dot{5}6}{2\dot{3}4}$.
(*b*) $\frac{9\dot{5}3}{\dot{0}037}$. (*d*) $\frac{100000}{15\dot{6}25}$. (*f*) $\frac{2\dot{3}45}{123\dot{4}}$.

5. Obtain the following quotients, each to 5 figures:

(*a*) $\frac{1}{2}$. (*d*) $\frac{1}{7}$. (*g*) $\frac{1}{13}$. (*j*) $\frac{1}{117}$.
(*b*) $\frac{1}{3}$. (*e*) $\frac{1}{9}$. (*h*) $\frac{1}{17}$. (*k*) $\frac{1}{1111}$.
(*c*) $\frac{1}{5}$. (*f*) $\frac{1}{11}$. (*i*) $\frac{1}{19}$. (*l*) $\frac{1}{999}$.

6. A metre = an inch × $3\dot{9}37$, and a yard = an inch × 36. Find to 6 figures the number of metres in a yard.

7. A mile = a yard × 1760. How many miles in $40\dot{7}821$ yards (6 figures)?

8. A rod = a foot × $1\dot{6}5$. What rate will derive $43\dot{5}6$ feet from a rod?

9. The length of a desk = an inch × $4\dot{5}7$, and its width = an inch × $1\dot{8}3$. How will its length be obtained from its width?

10. By division and addition simplify

$$\frac{3}{4}+\frac{3}{16}+\frac{9}{32}+\frac{5}{64}+\frac{23}{128}.$$

11. Find a single rate of 5 figures equivalent to

$$\frac{431}{213}+\frac{5\dot{9}2}{\dot{7}13}+\frac{\dot{2}17}{1\dot{2}7}.$$

12. Find the single rate to the fourth place after the units figure, which is equivalent to

$$\frac{17\dot{6}21\times 79\times \dot{0}07}{365}.$$

13. Find five figures of $\dfrac{32\cdot947\times\cdot009143\times21\cdot57}{37\cdot214\times\cdot01576\times213}$.

14. If the length $AB=$ the length $BC\times3\cdot225$,
the length $BC=$ the length $CD\times\cdot205$,
the length $CD=$ the length $DE\times640$,
and the length $DE=$ the length $PQ\times\cdot00932$,
find the single rate which tells how to derive the length AB from the length PQ.

15. Find to the order -5 each of the following:

(*a*) $\dfrac{800\times11\dot{1}256}{138\dot{5}8-2\dot{5}764}$.

(*b*) $\dfrac{3\dot{0}27\times\dot{5}8235}{\dot{4}96\times\dot{5}25}$.

(*c*) $\dfrac{\dot{1}05\times\dot{1}05\times\dot{1}05}{\dot{1}05\times\dot{1}05\times\dot{1}05-1}$.

16. What is the use of the point above a digit? What objection is there to writing it after the digit?

CHAPTER VIII.

THE UNIT UNDIVIDED—MULTIPLES AND SUB-MULTIPLES.

52. When a quantity can be measured *exactly* by the unit without cutting the unit into equal parts, as we did in Chapter VII., we call the quantity a *Multiple* of the unit, and the unit a *Sub-multiple* of the quantity, thus:

Since a yard = an inch × 36, we call the yard a multiple of the inch, and the inch a sub-multiple of the yard.

It is also evident from Article 33 that, when one quantity as a unit *measures exactly* another quantity, the measure of the first *divides exactly* the measure of the second. Hence also, when one number divides exactly another number, we call the less number a *divisor* (or *factor*) of the larger, and the larger a *multiple* of the less. Thus:

a yard = an inch × 36,
and a foot = an inch × 12.

Now, a foot *measures* a yard exactly, and 12 *divides* 36 exactly; and the measure in the one case is the quotient in the other. Therefore we find a sub-multiple of a quantity by finding a divisor (or factor) of its measure.

53. Prime Factors.—The student, from the Multiplication Table, knows the factors of many numbers from 144 down. The factors of other small numbers will be learned as he goes on. A number is prime when it has no factors other than 1 and itself: thus, 2, 3, 5, 7, 11, 13, 17, 19, etc., are prime numbers. A number is said to be the product of its factors. The prime factors of a large number are found by trying whether the prime numbers will divide it or not; but this is rarely necessary. An *even* number is one which has 2 for a factor, and all other natural numbers are odd numbers.

It is convenient sometimes to write 4^3 for 4 × 4 × 4, 3^2 for 3 × 3, and so on.

EXERCISE XIII.

1. Write as the product of prime factors the numbers 9, 16, 24, 48, 60, 96, 84, 108, 42, 90, 125 and 144.

2. Write the prime factors of 46, 76, 92, 91, 39, 51, 68, 111, 112, 117, 94, 95, 85, 68, 67, 47, 58, 126 and 135.

3. Express as the product of prime factors (*a*) 2560, (*b*) 2880, (*c*) 15625, (*d*) 21600, (*e*) 108, (*f*) 1024, (*g*) 1001.

4. Factor into prime factors 50609 and 28105.

5. Factor the numbers in each set, each into two factors, of which one is the greatest divisor of both numbers:

(*a*) 8, 12.
(*b*) 36, 24.
(*c*) 30, 40.
(*d*) 24, 32.
(*e*) 68, 51.
(*f*) 21, 63.
(*g*) 16, 40.
(*h*) 28, 91.
(*i*) 111, 75.
(*j*) 24, 42.
(*k*) 56, 63.
(*l*) 38, 76.
(*m*) 42, 66.
(*n*) 55, 77.
(*o*) 250, 150.
(*p*) 91, 52.
(*q*) 135, 75.
(*r*) 32, 52.
(*s*) 1200, 1800.
(*t*) 135, 105.
(*u*) 10000, 15625.

6. Write all the prime numbers below 100.

7. Write all the prime numbers between 100 and 200.

8. Write down all the sub-multiples of 210 feet.

9. Write down all the sub-multiples of a pound × 132.

10. Ascertain whether or not a yard × 34·56 will measure exactly a yard × 241·92. If so, what is the measure?

11. Will a gallon × 2·93 exactly measure a gallon × 23·54? Why?

12. What quantity must be taken from a yard × 7321 in

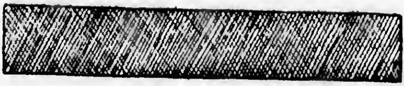
order that the remainder may be measured exactly by a yard × 236?

13. What is the smallest number which must be added to 73401 to make the sum a multiple of 834?

14. Find sub-multiples of the following quantities, the length (*a*) and the surface (*b*):

(*a*)

(*b*)



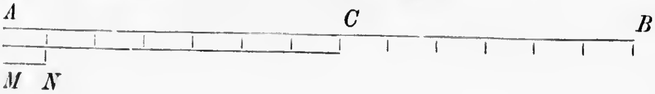
54. The Greatest Common Divisor of Two Numbers.—The greatest quantity which will measure exactly two or more others is called the *Greatest Common Sub-multiple* of them; or for shortness, the G. C. S. of them. It is evident, then, that we shall find this quantity by finding the greatest number which will exactly divide the measures of the quantities. This number we shall call the *Greatest Common Divisor* of their measures; or for shortness, the G. C. D. of their measures.

55. The G. C. D. of Small Numbers is easily found if the student knows their factors.

Thus, since $91 = 7 \times 13$, and $65 = 5 \times 13$, therefore the G. C. D. of 91 and 65 is 13.

So, also, let the length $AB = CD \times 91$, and the length $MN = CD \times 65$, where 91 and 65 are the measures of AB and MN with CD as unit; then the greatest length which will exactly measure both AB and MN is $CD \times 13$, which we call the G. C. S. of AB and MN.

56. Let AB and AC be two quantities (lengths), each of which is measured exactly by the unit MN;



then it is evident that the difference BC is also measured exactly by MN. In a similar manner we may see that MN will also measure the sum of AB and AC, the sum of $AB \times 3$ and $AC \times 5$, and the difference of $AB \times 11$ and $AC \times 7$.

Therefore, if one quantity is a sub-multiple of each of two others, it is also a sub-multiple of the sum, the difference, the sum of any multiples, or the difference of any multiple of these two quantities.

Therefore, also, if one number is a divisor (factor) of each of two numbers, it is also a divisor of the sum, the difference, the sum of any multiples, or the difference of any multiples of these two numbers.

Thus, since 7 divides 42 and also 63, therefore 7 divides $42 \times 9 - 63 \times 4$; that is, 126. So, also, since an inch × 7 measures exactly an inch × 42 and also an inch × 63, therefore an inch × 7 measures exactly an inch × $\{42 \times 9 - 63 \times 4\}$; that is, an inch × 126.

57. To find the G. C. D. of Two Large Numbers.— For instance, of 1752 and 2701. For brevity, we will denote their G. C. D. by G. Now, since G divides 1752 and also 2701, therefore G divides $1752 \times 2 - 2701$, which is 803, a number less than either 1752 or 2701.

Again, since G divides 1752 and also 803, therefore G divides $1752 - 803 \times 2$, which is 146, a number less than 803.

Again, since G divides 803 and also 146, therefore G divides $803 - 146 \times 5$, which is 73. Now, 73 divides 146.

We shall now prove that 73 divides both 1752 and 2701.

Since 73 divides both 146 and 73, therefore 73 divides $146 \times 5 + 73$, which is 803.

Since 73 divides both 803 and 146, therefore 73 divides $803 \times 2 + 146$, which is 1752, one of the numbers.

Finally, since 73 divides both 803 and 1752, therefore 73 divides $1752 \times 2 - 803$, which is 2701, the other number.

Hence the G. C. D. of 1752 and 2701 is 73.

In practice these operations are conveniently performed thus:

2	1752	2701	
	1606	3504	
5	146	803	2
	146	730	
		73	2

The numbers outside the lines are the multipliers of the numbers next them on the inside, and they are so chosen that the products are as nearly as possible equal to the numbers on the other side under which the products are placed.

58. *To find the G. C. S. of* 159531 *inches and* 70479 *inches.*

We find the G. C. D. of 159531 and 70479 thus:

	159531	70479	2
	140958	74292	
4	18573	3813	5
	19065	3936	
8	492	123	4
	492		

Hence the G. C. D. of the measures is 123. Therefore the G. C. S. of the quantities is 123 inches.

EXERCISE XIV.

1. Find the G. C. D. of the following pairs of numbers:

(*a*) 36 and 48.
(*b*) 32 and 54.
(*c*) 120 and 64.
(*d*) 81 and 108.
(*e*) 16539 and 27417.
(*f*) 835125 and 69375.
(*g*) 3378183 and 3052575.
(*h*) 790123456 and 999999999.

2. Find the G. C. S. of an inch × 169037 and an inch × 66429.

3. A field 546 feet wide and 686 feet long is to be fenced with boards as long as possible without cutting them. How long are the boards, and how many will it take if the fence is 5 boards high?

4. Find the G. C. D. of 650935, 530620 and 947095.

5. A field 91 rods long by 65 rods wide is to be divided off into the largest possible squares. What is the size of each square, and how many will there be?

59. The Least Common Multiple of two or more quantities is the least quantity which each of these quantities will measure exactly. It is evident, as before, that to find it, we shall find the least number which the measures of these quantities will divide exactly. Thus, since 24 is the least number which 3, 4 and 8 will divide, therefore 24 feet is the least quantity which 3 feet. 4 feet and 8 feet will measure exactly. The measures are, of course, 8, 6 and 3, respectively.

60. To find the L. C. M. of Two Numbers.—For instance, to find the L. C. M. of 621 and 989. By the method of Article 57, we find two factors of each of these numbers, one of which is their G. C. D.

Thus, $621 = 23 \times 27,$

and $989 = 23 \times 43.$

Since 621 divides exactly the L. C. M., which we are finding, therefore the L. C. M. must have 23 and 27 for two of its factors.

And since 989 also divides the L. C. M., the L. C. M. must have a third factor, 43. These three factors are necessary, and they are sufficient; therefore, the L. C. M. of the numbers $= 23 \times 27 \times 43$.

Now, $23 \times 27 = 621,$

and $43 = \frac{989}{23},$

therefore the L. C. M. $= 621 \times \frac{989}{23}$. Hence we have the rule:

The L. C. M. of two numbers is found by multiplying one of the numbers by the quotient when the other is divided by their G. C. D.

Example 1.—Find the L. C. M. of 96 and 144.

Since the G. C. D. of 96 and 144 is 48,
and the quotient of 96 by 48 is 2,
therefore, by the rule, the L. C. M. is 144 × 2, or 288.

61. The L. C. M. of more than Two Numbers is found by repeating the above process, as follows:

To find the L. C. M. of 96, 42, 70 and 60.

The G. C. D. of 96 and 42 is 6, the quotient of 42 by 6 is 7; therefore the L. C. M. of 96 and 42 is 96 × 7.

Again, the G. C. D. of 96 and 70 is 2, and the quotient of 70 by 2 is 35, that is, 7 × 5; but 7 is already written down in 96 × 7; therefore the L. C. M. of 96, 42 and 70 is 96 × 7 × 5.

Further, it is seen that 60 has its factors, 12 and 5 included in this, so that the L. C. M. of 96, 42, 70 and 60 is 96 × 7 × 5.

For small numbers, this is set down as follows, where the reasoning is mentally performed.

Thus, the L. C. M. of 24, 36, 40, 48, 60 = 60 × 2 × 3 × 2 = 720.

62. The L. C. M. of Large Numbers is found in the same way.

Example 1.—To find the L. C. M. of 621, 4209 and 2024.

The G. C. D. of 621 and 4209 is found as usual, thus:

7	621	4209	
	552	4347	
2	69	138	4
		138	

Then the quotient of 621 by 69 is 9.

Therefore the L. C. M. of 621 and 4209 is 4209 × 9.

Again, the G. C. D. of 4209 and 2024 is found :

```
2 | 2024 | 4209 |    \     Also we divide thus :
  | 1932 | 4048 |     |        23 ) 2024 ( 88
  |------|------|     |              184
2 |   92 |  161 | 12   >             ----
  |   92 |  184 |     |               184
  |------|------|     |               184
  |      |   23 | 4  /
```

Therefore the L. C. M. of the three numbers
$= 4209 \times 9 \times 88 = 3333528.$

EXERCISE XV.

1. Find the L. C. M. of the numbers in each set :

(*a*) 40, 50.	(*f*) 81, 54.	(*k*) 1287, 6281.
(*b*) 72, 45	(*g*) 121, 99.	(*l*) 132288, 107328.
(*c*) 83, 47.	(*h*) 117, 65.	(*m*) 94605, 96509.
(*d*) 28, 42.	(*i*) 250, 300.	(*n*) 9534, 15663.
(*e*) 16, 34.	(*j*) 1000, 325.	(*o*) 138448323, 468695032.

2. Find the L. C. M. of the numbers in each set :
   (*a*) 10, 12, 16, 18, 24, 28, 30, 40, 42 and 48.
   (*b*) 24, 28, 36, 44, 55 and 60.
   (*c*) 32, 48, 64, 96, 80 and 108.
   (*d*) 96, 120, 144, 84 and 90.
   (*e*) 120, 200, 240, 300 and 320.

3. Find the L. C. M. of :
   (*a*) 5607, 8165 and 4347. (*b*) 42240, 56210 and 23360.
   (*c*) 545, 26487, 1853 and 11421.

4. Find the least quantity which each of the following quantities will measure exactly.

A foot × 54, a foot × 72 and a foot × 84.

5. The hind and fore wheels of a waggon are 16 feet and 12 feet in circumference. How far will the waggon go so that each

wheel may make an exact number of turns? How many turns will each make?

6. A square field is of such a size that it can be fenced off into the smallest number of lots, each 66 feet wide and 72 feet long. How long is one side of the field, and how many lots will there be?

7. The L. C. M. of 391 and another number is 12121, and their G. C. D. is 23; find the other number.

8. The L. C. M. of two numbers is 634938944494, and their G. C. D. is 9187; one of the numbers is 85044059; find the other.

9. *A* can go around a race-course in 54 seconds, *B* in 63 seconds, *C* in 84 seconds, and *D* in 91 seconds. If they all start from the same line, how long time will elapse before they are all on the line together? How many rounds will each make?

10. Find the G. C. S. of the following quantities, the lengths (*a*) and (*b*):

(*a*)____

(*b*)____

11. Find the L. C. M. of (*a*) and (*b*) in No. 10.

12. Find the smallest distance which will contain 15939 inches and 9933 inches exactly.

13. The length, width and height of a block are 28 inches, 18 inches and 13 inches. What is the greatest length that will measure each exactly?

14. The hind and fore wheels of a waggon are 14 feet and 12 feet around. How far will the waggon go until each wheel makes an exact number of turns? How many times will this occur in going a mile?

15. A rectangular field is 27264 inches long and 16512 inches wide. It is marked off into square plots as large as possible. What is the size of each plot, and how many are there?

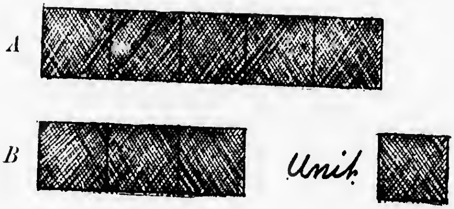
16. Find the least quantity which will contain exactly 14 lbs., 18 lbs., 21 lbs., 44 lbs., 33 lbs., 28 lbs. and 210 lbs.

CHAPTER IX.

IRREGULAR DIVISION OF THE UNIT. FRACTIONS.

63. Let us now consider two quantities of the same kind, namely, the surfaces *A* and *B*; so that

$$\text{the surface } A = \text{the unit surface} \times 3.$$
$$\text{and the surface } B = \text{the unit surface} \times 5.$$



Now, by Article 33, we know that the measure of the surface *A* by means of the surface *B* as a unit is the quotient when 3 is divided by 5, these numbers being the measures of *A* and *B*;

that is, $$\frac{\text{the surface } A}{\text{the surface } B} = \frac{3}{5}.$$

This $\frac{3}{5}$ is called a Fraction, which may be read, "the quotient of 3 by 5," or "three fifths." We may, therefore, define a fraction to be an indicated quotient which represents the measure of a quantity.

64. Fractions are Rates.—On referring to the quant in the preceding article, we see that we may derive the surface A from the surface B as follows:

"Cut the surface B into 5 equal parts, and put 3 of these parts together." These will make up the surface A. As before, we denote this manner of deriving the one quantity from the other, thus:

$$\text{The surface } A = \text{the surface } B \times \frac{3}{5},$$

where "×" is the sign of derivation.

Hence, a *fraction* is a rate which tells how one quantity is derived from another as a unit, by cutting the unit into equal parts other than ten; while a *decimal* (such as those in Chapter VII.) is a rate which tells how a quantity is derived from another as a unit, by dividing and subdividing the unit into ten equal parts.

65. The Quantity and the Unit Interchanged.—Since, when we say the surface A = the surface $B \times \frac{3}{5}$, we mean that the surface A consists of 3 parts, made by cutting the surface B into 5 equal parts; therefore the surface B consists of 5 parts, made by cutting the surface A into 3 equal parts; that is,

$$\text{the surface } B = \text{the surface } A \times \frac{5}{3}.$$

So also, since the surface A = the unit surface × 3,

$$\text{therefore the unit surface} = \text{the surface } A \times \frac{1}{3}.$$

In general, then, since the quantity = the unit × the rate,

$$\text{therefore the unit} = \text{the quantity} \times \frac{1}{\text{the rate}}.$$

That is, if a quantity is derived from the unit by any rate, then the unit is derived from the quantity by 1 ÷ that rate.

Thus, since a yard = an inch × 36,

$$\text{therefore an inch} = \text{a yard} \times \frac{1}{36}.$$

66. Note on the Words Numerator and Denominator.—It would seem, then, that when speaking of $\frac{4}{7}$, it is misleading to call 4 the number or "numerator," and 7 the name or "denominator." These words have arisen from the incomplete expression of an idea resulting in misconception as to the meaning and use of the $\frac{4}{7}$. When $\frac{4}{7}$ is spoken of alone, we mean nothing more than the quotient when 4 is divided by 7 (Article 63); and when it is written or used as a rate, thus, "an inch × $\frac{4}{7}$" or "$\frac{4}{7}$ of an inch," 7 is the *number* of equal parts into which the inch is divided, and 4 is the *number* of these parts put together to make the quantity referred to by the above expressions. The denominator is not 7, but "an inch × $\frac{1}{7}$," or "$\frac{1}{7}$ of an inch." We shall, therefore, not use these words, but use instead the shorter and more expressive words *Dividend* and *Divisor*.

EXERCISE XVI.

1. Explain the meaning of the fractions in the following statements:

(*a*) A foot = a yard $\times \frac{1}{3}$.

(*b*) A yard = a rod $\times \frac{2}{11}$.

(*c*) A square yard = a square rod $\times \frac{4}{121}$.

(*d*) The price of a pound of butter = a dollar $\times \frac{4}{25}$.

(*e*) A roll of butter = a pound of butter $\times \frac{43}{8}$.

2. Find the decimal rates equivalent to the fractional rates: $\frac{3}{4}$, $\frac{5}{16}$, $\frac{9}{80}$, $\frac{17}{125}$ and $\frac{49}{3200}$.

3. Find the aggregate of the following quantities by reducing

the fractional rates to decimal rates: a pound × $\frac{5}{8}$, a pound × $\frac{3}{40}$, a pound × $\frac{9}{80}$, a pound × $\frac{4}{32}$, and a pound × $\frac{7}{16}$.

4. Which rate, $\frac{7}{9}$ or $\frac{3}{4}$, will derive the greater quantity from the same unit?

5. If the volume of a block = the volume of a sphere × $\frac{16}{23}$; explain fully how the volume of the sphere is derived from the volume of the block.

6. If my money = my brother's money × 2·56; what decimal rate will derive my brother's money from mine?

7. If the line AB = the line $CD \times \frac{13}{17}$; explain why the line CD = the line $AB \times \frac{17}{13}$.

8. A's farm = B's farm × ·625; express by a compound number B's farm in terms of A's farm.

67. Reduction of a Fraction.—Let a quantity = the unit $\times \dfrac{5}{12}$; then the quantity consists of 5 parts, made by cutting the unit into 12 equal parts. If now each of these parts be subdivided into 7 equal parts, then the unit will be cut into 84 equal parts, and the quantity will consist of 35 of them; that is, the quantity = the unit $\times \dfrac{35}{84}$.

Hence, we say $\dfrac{5}{12} = \dfrac{5 \times 7}{12 \times 7}$.

Therefore, (1) The dividend and divisor of any fractional rate may be multiplied by the same number without changing the quantity derived by the rate.

(2) The dividend and divisor of a fractional rate may be divided by the same number without affecting the rate.

(3) A fraction is reduced to its simplest form by dividing its dividend and divisor by their G. C. D.

(4) Any integral (whole) number may be changed into a fraction with any number as divisor.

Thus, $13 = \frac{13}{1} = \frac{13 \times 8}{1 \times 8} = \frac{104}{8}$.

(5) A mixed number may be changed into a fraction.

Thus, $13\frac{4}{7} = 13 + \frac{4}{7} = \frac{13 \times 7}{7} + \frac{4}{7} = \frac{91}{7} + \frac{4}{7} = \frac{95}{7}$.

EXERCISE XVII.

1. Reduce the fractions to their simplest form:

(*a*) $\frac{12}{16}$. (*d*) $\frac{72}{96}$. (*g*) $\frac{9}{36}$. (*j*) $\frac{96}{112}$.

(*b*) $\frac{42}{63}$. (*e*) $\frac{120}{144}$. (*h*) $\frac{84}{21}$. (*k*) $\frac{78}{68}$.

(*c*) $\frac{140}{240}$. (*f*) $\frac{91}{65}$. (*i*) $\frac{32}{48}$. (*l*) $\frac{21}{35}$.

2. Reduce to their simplest forms:

(*a*) $\frac{1440}{1728}$. (*b*) $\frac{221221}{370370}$. (*c*) $\frac{15625}{100000}$. (*d*) $\frac{2389}{4576}$.

3. Change the following mixed numbers into fractions:

(*a*) $4\frac{3}{7}$. (*d*) $820\frac{1}{30}$. (*g*) $1\frac{1496}{8214}$. (*j*) $374\frac{216}{517}$.

(*b*) $5\frac{1}{9}$. (*e*) $4196\frac{5}{8}$. (*h*) $3\frac{873}{2196}$. (*k*) $6493\frac{85}{99}$.

(*c*) $1421\frac{3}{11}$. (*f*) $821\frac{3}{7}$. (*i*) $8\frac{48763}{193214}$. (*l*) $8437\frac{1285}{3571}$.

4. Reduce to mixed numbers the following:

(*a*) $\frac{423}{7}$. (*d*) $\frac{21740}{1351}$. (*g*) $\frac{2176184}{12345}$.

(*b*) $\frac{96}{23}$. (*e*) $\frac{7926}{400}$. (*h*) $\frac{7964}{7963}$.

(*c*) $\frac{4396}{144}$. (*f*) $\frac{532176}{843}$. (*i*) $\frac{99999}{33332}$.

5. Prove that $\frac{4}{5} = \frac{32}{40}$.

6. Find a decimal rate to the 4th order to the right equivalent to $\frac{3}{8} + \frac{4}{9} + \frac{3}{16} + \frac{21}{5}$.

68. Addition and Subtraction of Fractional Rates.—

Let the weight of one block = a pound × $\frac{5}{7}$,

and the weight of another block = a pound × $\frac{3}{8}$.

When these two blocks are put together, we wish to find a fractional rate which will derive the whole weight from a pound.

Now, by Article 67,

the weight of the 1st block = a pound × $\frac{40}{56}$.

and the weight of the 2nd block = a pound × $\frac{21}{56}$.

In each case, therefore, the pound has been divided into the same number of equal parts, and hence the parts are of the same size. The first quantity consists of 40, and the second quantity of 21 of these parts; so that the total quantity consists of 61 of the parts.

Therefore the total weight of the blocks = a pound × $\frac{61}{56}$.

We have, then, the Rule for finding the sum of two or more fractional rates:

Change the rates into equivalent rates having the same divisor, and write above this divisor the sum of the resulting dividends.

69. Examples solved.—(1) Add together $\frac{7}{24}$, $\frac{5}{36}$, $\frac{11}{48}$ and $\frac{13}{54}$.

The common divisor is the L. C. M. of 24, 36, 48 and 54, which, by the method of Article 60, is found to be 24 × 3 × 2 × 3, or 24 × 18. The multipliers of the dividends, therefore, are 18, 12, 9 and 8; so that the resulting dividends are,

$$7 \times 18,\ 5 \times 12,\ 11 \times 9 \text{ and } 13 \times 8,$$

$$\text{that is, } 126,\quad 60,\quad 99 \text{ and } 104.$$

The sum of these rates, then, is $\frac{389}{432}$.

In practice the operations are conveniently arranged, thus:

$$\frac{7}{24}+\frac{5}{36}+\frac{11}{48}+\frac{13}{54},$$

$$=\frac{7\times 18+5\times 12+11\times 9+13\times 8}{24\times 18},$$

$$=\frac{126+60+99+104}{24\times 18}=\frac{389}{432}.$$

(2) To subtract $\frac{9}{32}$ from $\frac{7}{24}$. It is evident that a similar process must be employed here, thus:

$$\frac{7}{24}-\frac{9}{32}=\frac{7\times 4-9\times 3}{24\times 4}=\frac{28-27}{96}=\frac{1}{96}.$$

EXERCISE XVIII.

1. Add together:

(*a*) $\frac{1}{2}$ and $\frac{1}{3}$.

(*b*) $\frac{1}{3}$ and $\frac{3}{4}$.

(*c*) $\frac{2}{3}$ and $\frac{4}{5}$.

(*d*) $\frac{1}{3}$ and $\frac{5}{6}$.

(*e*) $\frac{4}{9}$ and $\frac{5}{12}$.

(*f*) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$.

(*g*) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$.

(*h*) $\frac{7}{12}$, $\frac{5}{16}$ and $\frac{11}{36}$.

(*i*) $\frac{2}{\text{[illegible]}}$ and $\frac{5}{91}$.

(*j*) $\frac{\text{[illegible]}}{64}$, $\frac{1\text{[illegible]}}{96}$ and $\frac{41}{144}$.

2. Simplify

(a) $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$.

(b) $4\frac{1}{3}+5\frac{1}{4}+6\frac{2}{3}+9\frac{5}{6}$.

(c) $132\frac{3}{8}+291\frac{1}{4}+312\frac{3}{8}$.

(d) $\frac{4}{9}-\frac{5}{12}$.

(e) $\frac{3}{40}-\frac{7}{120}+\frac{13}{160}$.

(f) $\frac{153}{117}-\frac{96}{91}$.

(g) $432\frac{3}{7}-231\frac{1}{9}$.

(h) $82\frac{1}{2}-36\frac{2}{3}$.

(i) $196\frac{2}{3}-43\frac{3}{4}-21\frac{1}{6}+18\frac{5}{6}$.

(j) $28\frac{4}{9}-13\frac{7}{11}$.

(k) $145\frac{19}{36}-136\frac{21}{32}$.

3. Find the aggregate of the following quantities: A metre × $\frac{2}{3}$, a metre × $\frac{5}{9}$, a metre × $\frac{11}{12}$, a metre × $\frac{13}{18}$, and a metre × $\frac{17}{24}$.

4. In a certain town during one year the money required for schools = the assessment × $\frac{1}{200}$, that for roads and sidewalks = the assessment × $\frac{3}{800}$, that for the interest on the debt = the assessment × $\frac{11}{4000}$, and that for other purposes = the assessment × $\frac{13}{8000}$. Find what rate will derive the whole money needed.

5. What is left when a yard × $\frac{4}{11}$ is taken from a yard.

6. A man willed his property to his two sons. The eldest received the property × $\frac{17}{24}$; what did the younger receive?

7. *A*, *B* and *C* perform a certain work. *A* did $\frac{2}{3}$ of the work, *B* did $\frac{1}{15}$ of the work; what did *C* do?

8. Find the difference between

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7} \text{ and } \frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}.$$

9. What must be added to $24\frac{7}{12}-16\frac{9}{16}$ to make the sum $12\frac{5}{24}-1\frac{17}{48}$.

10. Add $\frac{1043}{7209}$, $\frac{265}{621}$ and $\frac{196}{2047}$.

70. Multiplication of Fractions.

Suppose the length $AB = CD \times \frac{4}{7}$,

and the length $CD = MN \times \frac{5}{9}$,

therefore the length $AB = MN \times \frac{5}{9} \times \frac{4}{7}$,

(Read thus: "The line AB is derived from CD by the rate $\frac{4}{7}$")

where $\frac{5}{9}$ is the rate which tells how CD is derived from MN,

and $\frac{4}{7}$ is the rate which tells how AB is derived from CD.

As in Article 22, the process, or operation, of finding a single fractional rate which will derive the same quantity as two fractional rates used in succession is called multiplication of fractions, and the single rate is called the product of the other two.

Suppose, now, the length MN is cut into 63 equal parts;

then $MN \times \frac{1}{9} =$ 7 of these parts of MN;

therefore $MN \times \frac{5}{9} =$ 35 of these parts of MN;

therefore $MN \times \frac{5}{9} \times \frac{1}{7} =$ 5 of these parts of MN;

therefore $MN \times \frac{5}{9} \times \frac{4}{7} =$ 20 of these parts of MN

$= MN \times \frac{20}{63}$, as we have agreed to write it.

Hence, $\frac{5}{9} \times \frac{4}{7}$ derives the same quantity from the unit as $\frac{20}{63}$;

which is what we mean when we say,

$$\frac{5}{9} \times \frac{4}{7} = \frac{20}{63} = \frac{5 \times 4}{9 \times 7}.$$

Hence, we have the following rule:

The dividend of the product of two fractions is the product of their dividends; and the divisor of their product is the product of their divisors.

71. Examples solved.—(1) Simplify $\frac{32}{63} \times \frac{35}{48}$.

Here $\frac{32}{63} \times \frac{35}{48} = \frac{32 \times 35}{63 \times 48}$, by the rule, Article 70,

$= \frac{16 \times 2 \times 5 \times 7}{9 \times 7 \times 16 \times 3}$, by factoring,

$= \frac{10}{27}$, by reduction. Article 67.

(2) To multiply together $8\frac{1}{8}$, $4\frac{4}{5}$ and $\frac{15}{52}$.

The product $= 8\frac{1}{8} \times 4\frac{4}{5} \times \frac{15}{52}$;

$= \frac{65}{8} \times \frac{24}{5} \times \frac{15}{52}$, by (5); Article 67.

$= \frac{13 \times 5 \times 8 \times 3 \times 5 \times 3}{8 \times 5 \times 4 \times 13}$, by the rule,

$= \frac{45}{4}$, by reduction, Article 67 (1).

$= 11\frac{1}{4}$, by division.

72. Division of Fractions.—Since division is the operation which undoes the result of multiplying; that is, when the product of two fractional rates and one of the rates are given, division is the operation by which the other is found. Let us suppose that the product is $\frac{7}{11}$, and that one of the rates is $\frac{5}{9}$;

then the other rate $\times \frac{5}{9} = \frac{7}{11}$;

that is, the quotient $\times \frac{5}{9} = \frac{7}{11}$.

Multiply these equal numbers by $\frac{9}{5}$;

therefore the quotient $\times \frac{5}{9} \times \frac{9}{5} = \frac{7}{11} \times \frac{9}{5}$;

that is, the quotient $= \frac{7}{11} \times \frac{9}{5}$, since $\frac{5}{9} \times \frac{9}{5} = 1$.

Hence we have the rule:

Division is turned into multiplication by inverting the divisor.

73. Example in Division solved.

1. Simplify $7\frac{13}{16} \div 2\frac{19}{28}$.

The quotient $= 7\frac{13}{16} \div 2\frac{19}{28}$;

$= \frac{125}{16} \div \frac{75}{28}$, by Article 67 (5);

$= \frac{125}{16} \times \frac{28}{75}$, by the rule;

$= \frac{25 \times 5 \times 4 \times 7}{4 \times 4 \times 25 \times 3}$, by multiplication;

$= \frac{35}{12}$, by reduction;

$= 2\frac{11}{12}$, by division.

2. If Tom's money is $\frac{7}{8}$ of Henry's money, and Fred's money is $\frac{5}{12}$ of Henry's money; compare Tom's money with Fred's.

Since Tom's money = Henry's money $\times \frac{7}{8}$, and Fred's money $= \text{Henry's money} \times \frac{5}{12}$,

therefore $\dfrac{\text{Tom's money}}{\text{Fred's money}} = \frac{7}{8} \div \frac{5}{12}$, Article 33.

$= \frac{7}{8} \times \frac{12}{5}$, by the rule, $= \frac{21}{10}$.

Therefore, also, Tom's money $=$ Fred's money $\times \frac{21}{10}$.

Thus Tom's money is compared with Fred's money.

EXERCISE XIX.

1. Simplify

(a) $\frac{4}{5} \times \frac{15}{24}$.

(b) $\frac{1}{2} \times \frac{6}{7}$.

(c) $\frac{8}{9} \times \frac{5}{12}$.

(d) $\frac{32}{35} \times \frac{42}{80}$.

(e) $\frac{21}{56} \times \frac{84}{91}$.

(f) $3\frac{1}{3} \times 4\frac{2}{3}$.

(g) $5\frac{1}{2} \times 8\frac{2}{11}$.

(h) $2\frac{11}{12} \times 4\frac{3}{14}$.

(i) $\frac{2}{3} \times \frac{5}{6} \times \frac{12}{25}$.

(j) $1\frac{4}{5} \times 2\frac{2}{9} \times 5\frac{1}{2}$.

(k) $12\frac{1}{2} \times \frac{8}{25} \times 3\frac{1}{7}$.

(l) $219\frac{1}{8} \times \frac{5}{7}$.

(m) $13\frac{4}{5} \times 11\frac{18}{23}$.

(n) $\frac{1225}{480} \times \frac{1728}{1001}$.

(o) $10\frac{10}{1001} \times \frac{2}{5}$.

2. Simplify into one fraction :

(a) $\frac{240}{117} \div \frac{520}{39}$.

(b) $126\frac{1}{4} \div 3\frac{5}{32}$.

(c) $\left(\frac{1}{2} \times \frac{1}{3} \div \frac{1}{4}\right) \div \frac{1}{5}$.

(d) $\left(1\frac{4}{9} \text{ of } \frac{9}{13}\right) + \left(\frac{3}{8} \text{ of } 1\frac{11}{12}\right) + \left(\frac{16}{6} \div \frac{22}{9}\right)$.

(e) $\left(7\frac{13}{16} \times 5\frac{3}{5}\right) - \left(1\frac{1}{35} \div \frac{24}{42}\right)$.

(f) $\dfrac{\frac{17}{10} + \frac{7}{11}}{\frac{17}{10} - \frac{7}{11}} - \dfrac{9\frac{14}{27}}{4\frac{1}{3}}$.

3. Prove that a pound $\times \frac{2}{3} \times \frac{5}{9} =$ a pound $\times \frac{10}{27}$.

4. A man gave $\frac{2}{5}$ of his money to *A*, and $\frac{1}{4}$ of the remainder to *B*. How much had he left?

5. A merchant sold $\frac{2}{9}$ of his goods to *A*, $\frac{3}{7}$ of the remainder to *B*, and $\frac{3}{8}$ of what then remained to *C*. How much did he have left?

6. *A*, *B* and *C* reaped a field; *A* reaped the field × $\frac{2}{7}$, *B* reaped the field × $\frac{2}{5}$. How much did *C* reap more than *A*?

7. A boy spent for nuts, his money × $\frac{1}{7}$; for marbles, his money × $\frac{1}{15}$; for oranges, his money × $\frac{1}{9}$; and for fireworks, his money × $\frac{5}{21}$. He had 139 cents left. What money did he have at first?

8. A farmer had in pasture $\frac{1}{12}$ of his farm; in corn, $\frac{1}{3}$ of his farm; in wheat, $\frac{2}{9}$ of his farm; in oats, $\frac{1}{24}$ of his farm; in orchard, $\frac{1}{8}$ of his farm. The rest, $19\frac{4}{9}$ acres, was in wood. How large was his farm?

9. If my money = Henry's money × $\frac{5}{13}$, and Henry's money = my brother's money × $\frac{3}{8}$; find what rate will derive my money from my brother's money.

10. If Tom's marbles = Dick's marbles × $\frac{9}{11}$, and Harry's marbles = Dick's marbles × $\frac{15}{22}$; what is the measure of Tom's marbles, when Harry's marbles is the unit?

11. If *A's* farm = *B's* farm × $\frac{2}{3}$, and *C's* farm = *B's* farm × $\frac{3}{4}$, show the relation between *A's* farm and *C's* farm.

12. One gallon = a cubic inch × $277\frac{1}{4}$, and a cubic foot = a cubic inch × 1728. How many gallons are in a cubic foot?

13. Divide $44 between *A* and *B*, so that what *B* gets = what *A* gets × $\frac{4}{7}$.

14. A man left his estate, valued at $10245, to be divided between his two sons, so that the younger would receive $\frac{5}{7}$ of what the elder received. Find the share of each.

15. Divide 250 lbs. of flour between two families, consisting of 9 persons and 7 persons, respectively, in such a way that each person will receive the same amount of flour.

16. Apportion \$1463 among *A*, *B* and *C*, so that *A's* share = *B's* share × $\frac{5}{9}$, and *C's* share = *B's* share × $\frac{7}{12}$.

17. Apportion \$3420 among *A*, *B* and *C*, so that *A's* share = *B's* share × $\frac{5}{8}$, and *B's* share = *C's* share × $\frac{7}{10}$.

18. Distribute \$120 among the members of a family, consisting of father, mother, two sons and three daughters, in such a way that a son's share = a daughter's share × $\frac{3}{2}$, the mother's share = a son's share × $\frac{5}{3}$, and the father's share = the mother's share × $\frac{7}{5}$.

19. Distribute \$777·50 among *A*, *B*, *C* and *D*, so that *A's* share = *B's* share × 1·28, *B's* share = *C's* share × 1·25, and *C's* share = *D's* share × ·75.

74. Decimals as Fractions.—Since, if we have

$$\text{a quantity} = \text{the unit} \times \dot{0}3,$$

we mean that the quantity consists of 3 parts made by cutting the unit into 10 equal parts; and since we have shown that this is expressed thus,

$$\text{the quantity} = \text{the unit} \times \frac{3}{10};$$

therefore $\dot{0}3$ or $0{\cdot}3 = \frac{3}{10}$.

Similarly we may show that

$$\begin{aligned} {\cdot}275 &= \frac{2}{10} + \frac{7}{100} + \frac{5}{1000}; \\ &= \frac{275}{1000} \text{ by addition;} \\ &= \frac{9}{40}, \text{ by reduction.} \end{aligned}$$

75. Miscellaneous Examples.

EXERCISE XX.

1. Simplify $\dfrac{3\frac{1}{2} - {\cdot}04}{5 - {\cdot}0625} \div \dfrac{{\cdot}015 + 2{\cdot}1}{{\cdot}035}$.

2. Simplify $(4\frac{1}{5} - 2\frac{7}{8}) \times (\frac{3}{4} - \frac{4}{33}) \times (9\frac{1}{11} - 4\frac{2}{13})$.

3. Reduce to one fraction $\dfrac{5\frac{5}{8} \times \frac{3}{2}}{1\frac{1}{5} \times \frac{5}{9} \div 10\frac{1}{3}} \times \dfrac{2}{5} \times \dfrac{1\frac{1}{2} \times 4\frac{1}{9}}{13\frac{7}{8} \times 5\frac{1}{3}}$.

4. Reduce $\left\{\dfrac{2\frac{1}{4} - (\frac{2}{3} \text{ of } 1\frac{5}{6})}{(\frac{1}{5} \text{ of } 3\frac{1}{3}) + \frac{13}{36}} - \dfrac{1}{2\frac{1}{2}}\right\} \div \dfrac{1}{1\frac{2}{3}}$.

5. Reduce

$$\text{a hundredweight} \times \frac{3\frac{1}{7}}{5\frac{4}{8} \times 3\frac{3}{7}} \times \frac{16\frac{5}{12} - 5\frac{11}{12}}{5\frac{1}{4} - (3\frac{3}{4} \times 2\frac{1}{2} \times \frac{3}{10})} \times \frac{1}{10}.$$

6. Simplify $1 - \dfrac{1}{6} + \dfrac{1}{24} - \dfrac{61}{5040} + \dfrac{277}{72576}$.

7. Explain and reduce

$$\text{a pound} \times \frac{\text{a dollar} \times 6{\cdot}25}{\text{a dollar} \times 7{\cdot}5} \times \frac{\text{a rod} \times \frac{33}{18}}{\text{a rod} \times \frac{25}{9}}.$$

8. Find the aggregate of

$$\text{a £} \times \tfrac{2}{5} \times (3\tfrac{1}{3} + 1\tfrac{1}{4})\,;\ \text{a £} \times \tfrac{1}{4} \times {\cdot}475 \times \frac{1\frac{1}{8} - (\frac{1}{3} \text{ of } 1\frac{5}{6})}{(\frac{1}{20} \text{ of } 3\frac{1}{3}) + \frac{13}{144}}\,;$$

$$\text{and a £} \times \frac{4{\cdot}2}{{\cdot}012 \times 240}.$$

9. Simplify $\dfrac{{\cdot}1234 \times {\cdot}4321 - {\cdot}01}{{\cdot}00481346}$.

10. Simplify $\dfrac{1\frac{7}{9} + 8\frac{1}{4}}{7\frac{5}{6} - 4\frac{2}{9}} \times \dfrac{45\frac{1}{8}}{21\frac{1}{4}} \div \dfrac{2\frac{1}{19} + \frac{7}{38}}{\frac{7}{38} - \frac{7}{76}}$.

11. A man invests $\frac{1}{2}$ of his fortune in land, $\frac{1}{5}$ of it in bank stock, $\frac{1}{6}$ of it in railroad stock, and loses the remainder, $8000, in speculation. What was his fortune at first?

12. Multiply ·01019 by 23·04, and explain why the partial products are placed where you write them.

13. Divide ·01342 by ·0055, and explain how you find the order of the 1st figure of the quotient.

14. Reduce to its simplest form $\dfrac{692307}{999999}$.

15. Find the G. C. D. of 94605 and 96509, and explain why your method gives the correct number.

16. Find the L. C. M. of 11, 7, 21, 28, 22, 27, 81, 243 and 216, and explain fully your method.

17. What purposes are served by addition, subtraction, multiplication, division, finding the G. C. D. and finding the L. C. M.?

18. Explain fully how to subtract $1650\frac{431}{432}$ from $1761\frac{5}{18}$.

19. Give two definitions of a fraction, and from one of these definitions prove that $\frac{7}{5} \times \frac{3}{8} = \frac{21}{40}$.

20. Prove that $8 \times \frac{1}{9} = \frac{8}{9}$.

21. Divide 2 quadrillion, 18 million, 760 thousand, 681, by sixty-three million, two hundred and forty-five thousand, five hundred and fifty-three.

22. Find a decimal rate to the 7th order to the right equivalent to $1 + \dfrac{1}{5} + \dfrac{1}{5 \times 5} + \dfrac{1}{5 \times 5 \times 5} + \dfrac{1}{5 \times 5 \times 5 \times 5} +$ etc.

23. A man died, leaving 5 sons, *A*, *B*, *C*, *D* and *E*. He willed his property, valued at $10000, to them in such a way that *A* would get $200 more than *B*, *B* $250 more than *C*, *C* $300 more than *D*, and *D* $350 more than *E*. Find the sums they get.

24. Gunpowder is composed of saltpetre, charcoal and sulphur in the proportion of 15, 3 and 2. A certain quantity of gunpowder is known to contain 325 lbs. of charcoal; find its weight and also the weights of the saltpetre and sulphur.

25. In finding the value of an article from $1350 by the rate $\frac{79}{96}$, a boy used instead the rate $\frac{96}{79}$ and a girl the rate $\frac{79}{69}$. Which made the greater error, and by how much?

26. A man gave $\frac{2}{15}$ of his money to *A*, $\frac{3}{8}$ of the remainder to *B*, $\frac{4}{15}$ of what then remained to *C*, and divided the rest equally between *P*, *Q* and *R*. If *R* received 143 cents, what did *A*, *B* and *C* each receive?

CHAPTER X.

QUANTITIES IN PROPORTION.

76. In the preceding Chapters we have shown that a number is the measure of one quantity when another of the *same kind* is the unit. We have also shown how a number is used to tell how one quantity is derived from another of the *same kind*. We shall now examine how numbers are used in connection with some quantities of *different kinds*.

77. Suppose a farmer is taking a load of wheat to market, and that he knows that the load of wheat = a bushel × 45. The number 45 tells him how his load is made up of, or is derived from, a bushel of wheat. Now, the farmer is thinking about the price of a bushel and the price of his load of wheat—two other quantities of a kind different from the wheat he has with him; and the connection between these two kinds of quantities is such, that whatever rate is used to derive the load of wheat from a bushel of wheat, the same rate is used to find the price of the load from the price of a bushel. Thus,

since the load of wheat = a bushel × 45,
therefore the price of the load = the price of a bushel × 45.

When quantities of two different kinds are connected in this way, we say that quantity of one kind *is proportional* to quantity of the other kind; or that the quantity of the one kind *varies as* the quantity of the other kind.

78. Some Quantities which are in Proportion.—There are many quantities of different kinds connected with each other, as in Article 76. Thus:

(*a*) The amount of a commodity varies as its price.

(*b*) The rent of a farm varies as the time it is rented.

(*c*) The distance a man travels varies as the time he is travelling.

(*d*) The work done by a man varies as the time he is working.

(*e*) The work done in a day varies as the (quantity of) men employed.

(*f*) The rent for the use of money varies as the money in use.

(*g*) The distance a train goes in a given time varies as its *speed* or velocity.

79. Examples solved.—The following examples illustrate the use of numbers:

Ex. 1. Find the cost of a load of wheat consisting of 45 bushels at 85 cents a bushel.

Solution.—Since the load of wheat = a bushel × 45;
therefore the cost of the load = the cost of a bushel × 45.
But the cost of a bushel = a cent × 85;
therefore the cost of the load = a cent × 85 × 45;
= a cent × 3725 = $37·25.

Ex. 2. Find the weight of 30 yards of wire, if 5 yards of it weigh 13 ounces.

Solution.—Since $\dfrac{\text{a yard of wire} \times 30}{\text{a yard of wire} \times 5} = 6$; Article 33.

therefore 30 yards of wire = 5 yards of wire × 6.
Therefore the weight of the wire = the weight of 5 yards of it × 6.
But the weight of 5 yards of wire = an ounce × 13.
Therefore the weight of the wire = an ounce × 13 × 6,
= an ounce × 78.

Ex. 3. If A can do a work in 14 days, and B in 21 days; how long will it take both to do it working together?

Solution.—Since A's time to do the work $=$ a day $\times 14$,

Therefore a day $= A$'s time to do the work $\times \frac{1}{14}$. Article 65.

Therefore a day's work for A $=$ the whole work $\times \frac{1}{14}$. Art. 77.

Similarly a day's work for B $=$ the whole work $\times \frac{1}{21}$.

Therefore a day's work for both $=$ the whole work $\times \left(\frac{1}{14}+\frac{1}{21}\right)$;

$=$ the whole work $\times \frac{5}{42}$, by adding.

Therefore the whole work

$=$ a day's work for both $\times \frac{42}{5}$. Article 65.

Therefore the time to do the work

$=$ a day $\times \frac{42}{5} = 8\frac{2}{5}$ days. Article 77.

Ex. 4. If 3 men can reap 25 acres in 7 days, how long will it take 7 men to reap 63 acres?

Solution.—Here the work of reaping an acre $\times 25$

$=$ a day's work for a man $\times 3 \times 7$. [Given.

Therefore the work of reaping an acre

$=$ a day's work for a man $\times \frac{21}{25}$. [$\times \frac{1}{25}$.

Therefore the work of reaping 63 acres

$=$ a day's work for a man $\times \frac{21 \times 63}{25}$; [$\times 63$.

$=$ a day's work for 7 men $\times \frac{1}{7} \times \frac{21 \times 63}{25}$;

$$= \text{a day's work for 7 men} \times \frac{189}{25}.$$

Therefore the time for 7 men to do it = a day $\times \frac{189}{25} = 7\frac{14}{25}$ days.

80. Note on Solutions.—Of course, if only practical results are required, the student may contract the above solutions to very narrow limits. But it is much, very much, more important that he should *train his mind*, by striving to conceive and to express in full language the elementary reasoning by which these results are obtained. Therefore, instead of "saving time" by curtailing his solutions, he should rather seek to express them in full and accurate language.

EXERCISE XXI.

1. If 7 yards of cloth cost 497 cents, find the cost of 16 yards.

2. If 9 feet of hose cost $121\frac{1}{2}$ cents, how many feet of hose can be bought for $4242\frac{1}{2}$ cents?

3. If 25 boxes of berries sell for one dollar, how many cents should 56 boxes sell for?

4. If $\frac{2}{3}$ of my hay sells for $132 at $11 a ton, how much should the remainder sell for at $12 a ton?

5. If 10 lbs. of water occupy $277\frac{1}{4}$ cubic inches, how many lbs. of water will fill a vessel whose capacity is 1728 cubic inches?

6. If a man can walk 1800 yards in 25 minutes, how long will he require to walk 126 yards?

7. *A* can do a certain work in 7 days, how long will it take him to do $\frac{7}{11}$ of it?

8. A farmer received $124 for $\frac{4}{9}$ of his wheat, what should he receive for $\frac{7}{31}$ of it at the same price per bushel?

9. A man alone can do a certain work in 15 days, and his son alone can do it in 25 days; how long will it take both to do it working together?

10. A can do a work in 24 days, and B in 20 days. A works at it for 5 days alone, how long will it take B to finish it?

11. A can do as much work in 2 days as B can do in 3 days. If A and B working together do a certain job in 9 days, how long will it take each to do it alone?

12. A can reap a field in 12 days, B in 15 days, and C in 20 days. In what time can all do it working together?

13. A can run 25 yards in 10 seconds, B can run 33 yards in 12 seconds. Which can run the faster; and if at the start the faster is 25 yards behind the slower, how long will he be in catching up?

14. A and B together do a work in $3\frac{1}{2}$ days which A alone could do in 9 days. How long will it take B alone to do it?

15. I sold my berries at the rate of 11 boxes for 50 cents, my neighbor sold his at the rate of 6 boxes for 25 cents. What fractional rate will derive the price of a box of my berries from the price of a box of his?

16. Find the cost of $32\frac{5}{8}$ yards of cloth at $72\frac{1}{2}$ cents a yard.

17. Make out a bill of the following items:

$28\frac{1}{2}$ yards of flannel at 68 cents a yard;
35 yards of print at 15 cents a yard;
$3\frac{1}{2}$ doz. pairs of stockings at \$2·10 a doz.;
7 pairs of gloves at 90 cents a pair;
$12\frac{1}{2}$ yards of linen at \$1·12 a yard;
4 pairs of curtains at \$4·20 a pair.

18. A farmer sold the following articles to a merchant to whom he owed \$54·45: 1680 lbs. of hay at \$15 per 2000 lbs.; 3·75 cords of wood at \$4·80 per cord; 4 barrels of apples at \$2·75 per barrel; 3·50 cwt. of flour at \$2·50 per cwt.; 30·625 lbs. of butter at 16 cents per lb. How does the account now stand?

19. A farmer sells to a merchant 3015 lbs. of hay at $16 per 2000 lbs., and takes in payment 6lbs. of tea at 80 cents per lb.; $22\frac{1}{2}$ lbs. of coffee at 26 cents per lb.; 33 lbs. of sugar at 12 lbs. for a dollar; $32\frac{4}{5}$ lbs. of raisins at $18\frac{3}{4}$ cents per lb.; $14\frac{13}{16}$ lbs. of bacon at 16 cents per lb., and the balance in cash How much cash does the farmer receive?

20. If 5 men or 7 women can do a piece of work in 37 days, how long will a piece of work twice as great occupy 7 men and women?

21. *A* and *B* can do a work alone in 15 and 18 days, respectively; they work together at it for 3 days, when *B* leaves, and after 3 days *A* is joined by *C*; these two then finish it in 4 days. In what time would *C* do the work by himself?

22. A cistern can be filled in 18 hours by a pipe *A*, and can be emptied in 12 hours by a pipe *B*. If the cistern be $\frac{3}{4}$ full and both pipes are open, how long will it take to empty the cistern?

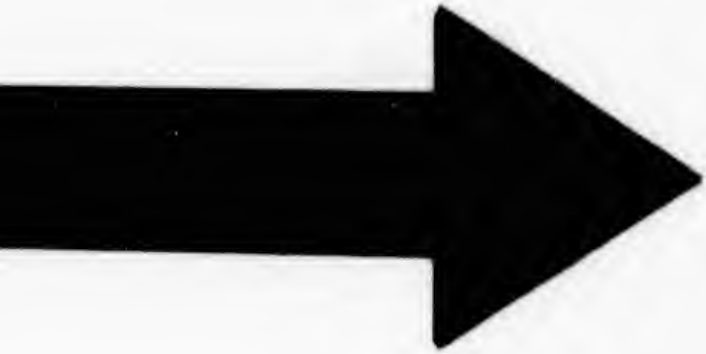
The following three examples are more difficult:

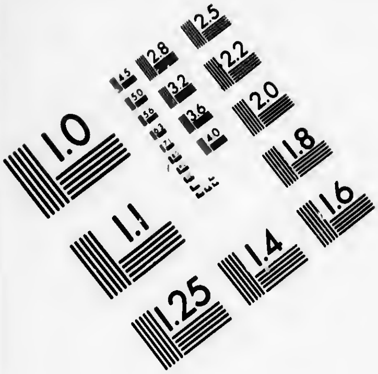
23. *A* can do a work in $13\frac{1}{3}$ hours, and *B* in $16\frac{2}{3}$ hours. They commence the work together, but after 4 hours, on account of an accident, *A's* efficiency is reduced by $\frac{1}{4}$ of itself. How long is the work in doing?

24. Two equal casks, *A* and *B*, are full of water. *A* can be emptied by a pipe in 4 hours, and *B* by a pipe in 5 hours. If both pipes be opened together, and closed when one cask contains twice as much water as the other, how long time will the pipes be running?

25. Tom and his father saw wood for a living. He finds out that he can split wood just as fast as his father saws it, but that his father can split wood four times as fast as he saws it. Now they saw and split a cord of wood for $1·20. Tom wishes to know how much of this money he should have. Tell him.

[redacted]





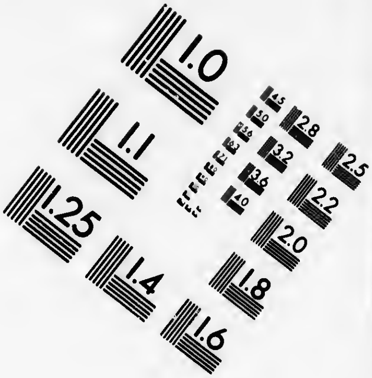
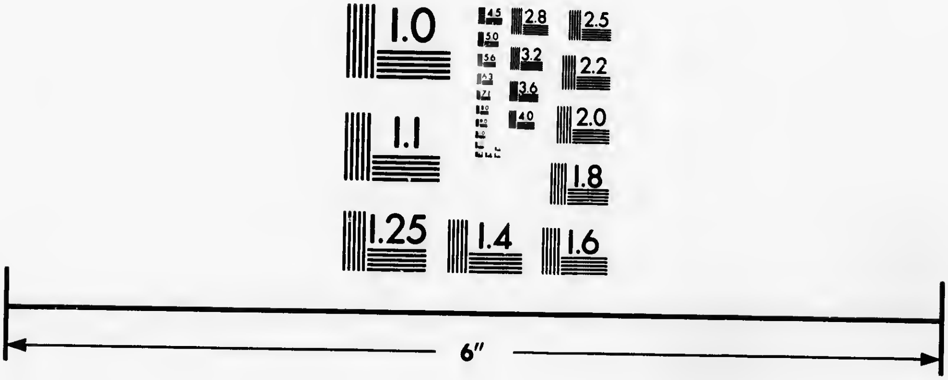
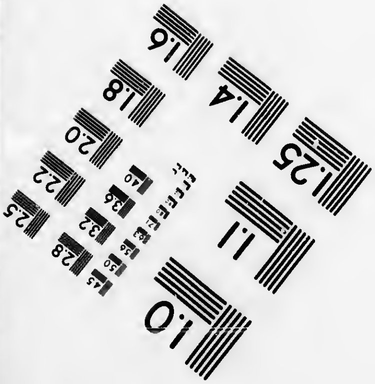
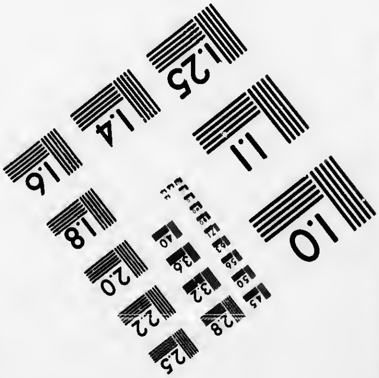


IMAGE EVALUATION
TEST TARGET (MT-3)







Photographic
Sciences
Corporation

23 WEST MAIN STREET
WEBSTER, N.Y. 14580
(716) 872-4503

2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.8
1.6
1.1
1.0

CHAPTER XI.

COMPOUND QUANTITIES—REDUCTION.

81. Sometimes a quantity is measured by units not derived from one another in any regular way. Thus to measure a long distance, we may choose a mile as the unit. The mile is then repeated until a part of the distance is left less than a mile, which the mile will not measure. To measure this part, the mile is cut into 8 equal parts (not 10, as in Article 41), each of which is called a furlong. The furlong is repeated until a part is left less than a furlong, which the furlong will not measure. Again, to measure this part, the furlong is cut into 40 equal parts, each of which is called a rod. The rod is then repeated until a part is left unmeasured less than a rod. To measure this the rod is cut into 11 equal parts, two of which put together make up a unit called a yard. The yard is repeated until a part is left unmeasured less than a yard. Finally this part is measured by a foot, and an inch, as units.

82. Compound Quantities.—When a quantity has been measured by, and written in terms of, units of different names irregularly derived from one another, the quantity is said to be a *Compound Quantity*. Thus, if the distance between two stations be 13 miles, 5 furlongs, 23 rods, 5 yards, we call the distance a compound quantity. To show the manner by which the distance is made up of these units, we write:

The distance = a mile × 13 + a furlong × 5 + a rod × 23 + a yard × 5.

83. Table of Rates.—Of the units which are used in English-speaking countries to measure such quantities as length, surface, volume, weight, mass, value, time and angle, the following tables give the names and manner of derivation. The student is expected to know these in the succeeding chapters of this book.

(1) *To measure Length or Distance.*

A league = a mile × 3, therefore a mile = a league × $\frac{1}{3}$.
A mile = a furlong × 8, therefore a furlong = a mile × $\frac{1}{8}$.
A furlong = a rod × 40, therefore a rod = a furlong × $\frac{1}{40}$.
A rod = a yard × $5\frac{1}{2}$, therefore a yard = a rod × $\frac{2}{11}$.
A yard = a foot × 3, therefore a foot = a yard × $\frac{1}{3}$.
A foot = an inch × 12, therefore an inch = a foot × $\frac{1}{12}$.
A chain = a rod × 4.

(2) *To measure Surface or Area.*

A square mile = an acre × 640, therefore, etc.
An acre = a square rod × 160.
A square rod = a square yard × $30\frac{1}{4}$.
A square yard = a square foot × 9.
A square foot = a square inch × 144.
An acre = a square chain × 10.

(3) *To measure Volume or Capacity.*

A cubic yard = a cubic foot × 27.
A cubic foot = a cubic inch × 1728.
A bushel = a cubic inch × 2218.
A bushel = a peck × 4.
A peck = a quart × 8.
A gallon = a cubic inch × 277·274.
A gallon = a quart × 4.
A quart = a pint × 2.
A barrel = a gallon × $31\frac{1}{2}$.

A hogshead = a gallon × 63.
A cord of wood or stone = a cubic foot × 128.

In the United States, however,

A gallon = a cubic inch × 231.

(4) *To measure the Weight or Mass* of ordinary commodities the Avoirdupois table of units is used.

A ton = a hundredweight (cwt.) × 20.
= a pound × 2000.
A long ton = a pound × 2240.
A cwt. = a quarter × 25.
A pound = an ounce × 16.

(5) *To measure the Weight of Gold, Silver, and Precious Stones* the Troy table of units is used.

A Troy pound = a Troy ounce × 12.
A Troy ounce = a pennyweight (dwt.) × 20.
A dwt. = a grain × 24.

Therefore a Troy pound = a grain × 5760.
But an Avoirdupois pound = a grain × 7000.

(6) *To measure Value.*

In Canada and the United States,

A cent = a mill × 10.
A dollar (\$) = a cent × 100.

In England,

A pound sterling (£) = a shilling (*s.*) × 20.
A shilling = a penny (*d.*) × 12.
A penny = a farthing × 4.

But a farthing is written, a penny × $\frac{1}{4}$, or $\frac{1}{4}d$.
A guinea = a shilling × 21.
Further, a pound sterling

$$= \text{a dollar} \times \frac{73}{15},$$

$$= \$4{\cdot}86\tfrac{2}{3}.$$

(7) *To measure Time.*

A year = a day × 365.
A leap year = a day × 366.
A day = an hour × 24.
An hour = a minute × 60.
A minute = a second × 60.

The length of the months is given in the following rhyme:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February alone,
Which has but twenty-eight days clear,
And twenty-nine in each leap year."

(8) *To measure Angle, Latitude, and Longitude.*

A quadrant = a right angle = a degree × 90.
A sextant = a degree × 60.
A circle = a degree × 360.
A degree (1°) = a minute (1′) × 60.
A minute = a second (1″) × 60.

(9) *Farmers' Produce.*—The weight of a bushel of wheat, peas, beans, clover seed, potatoes, turnips, carrots, parsnips, beets, or onions is 60 pounds; Indian corn, or rye, 56 pounds; flax seed, 50 pounds; barley, buckwheat, or timothy seed, 48 pounds; hemp seed, 44 pounds: oats, 34 pounds; blue grass seed, 14 pounds; dried apples, 22 pounds; coal, 66 to 70 pounds.

A barrel of pork or beef weighs 200 pounds, and a barrel of flour, 196 pounds.

(10) *Miscellaneous Units.*

A gallon of water = a pound of water × 10.
A cubic foot of water = a pound of water × $62\frac{1}{2}$ (nearly).

A dozen = 12, a score = 20, a gross = 144.
A quire of paper = 24 sheets, a ream = 20 quires.

84. To Reduce a Compound Quantity to a Simple Quantity; that is, when a quantity is expressed by naming more than one unit, to express it by only one unit.

Example 1.—To reduce £40 16*s.* 4*d.* to pence.

Here the quantity

$$\begin{aligned}
&= \text{a £} \times 40 + \text{a shilling} \times 16 + \text{a penny} \times 4,\\
&= \text{a shilling} \times (20 \times 40 + 16) + \text{a penny} \times 4,\\
&= \text{a shilling} \times 816 + \text{a penny} \times 4,\\
&= \text{a penny} \times (12 \times 816 + 4),\\
&= \text{a penny} \times 9796.
\end{aligned}$$

Example 2.—To reduce 3 furlongs, 17 rods, $2\frac{5}{14}$ feet to miles.

Here the quantity

$$\begin{aligned}
&= \text{a furlong} \times 3 + \text{a rod} \times 17 + \text{a foot} \times 2\frac{5}{14},\\
&= \text{a furlong} \times 3 + \text{a rod} \times 17 + \text{a yard} \times \frac{1}{3} \times \frac{33}{14},\\
&= \text{a furlong} \times 3 + \text{a rod} \times \left(17 + \frac{2}{11} \times \frac{11}{14}\right),\\
&= \text{a furlong} \times 3 + \text{a rod} \times 17\frac{1}{7},\\
&= \text{a furlong} \left(3 + \frac{1}{40} \times \frac{120}{7}\right),\\
&= \text{a furlong} \times 3\frac{3}{7},\\
&= \text{a mile} \times \frac{1}{8} \times \frac{24}{7},\\
&= \text{a mile} \times \frac{3}{7}.
\end{aligned}$$

85. To Reduce a Simple Quantity to a Compound Quantity.

Example 1.—To reduce $9\frac{120}{121}$ acres to a compound quantity as far as inches.

Here $9\frac{120}{121}$ acres $=$ an acre $\times \left(9+\frac{120}{121}\right)$,

$=$ an ac. $\times 9 +$ a sq. rod $\times 160 \times \frac{120}{121}$,

$=$ an ac. $\times 9 +$ a sq. rod $\times \left(158\frac{82}{121}\right)$,

$=$ an ac. $\times 9 +$ a sq. rod $\times 158 +$ a sq. yd. $\times \frac{121}{4} \times \frac{82}{121}$,

$=$ an ac. $\times 9 +$ a sq. rod $\times 158 +$ a sq. yd. $\times 20\frac{1}{2}$,

$=$ an ac. $\times 9 +$ a sq. rod $\times 158 +$ a sq. yd. $\times 20 +$ a sq. ft. $\times 4\frac{1}{2}$,

$=$ 9 ac., 158 sq. rods, 20 sq. yds., 4 sq. ft, 72 sq. in.

Example 2.—To reduce 52132 ounces to tons, cwts., etc.

52132 ounces $=$ an ounce $\times$ 52132;

$=$ a pound $\times \frac{52132}{16}$,

$=$ a pound $\times 3258 +$ an ounce $\times 4$,

$=$ a quarter $\times \frac{3258}{25} +$ an ounce $\times 4$,

$=$ a quarter $\times 130 +$ a pound $\times 8 +$ an ounce $\times 4$,

$=$ a cwt. $\times \frac{130}{4} +$ a pound $\times 8 +$ an ounce $\times 4$,

$=$ a cwt. $\times 32 +$ a quarter $\times 2 +$ a pound $\times 8 +$ an oz. $\times 4$,

$=$ 1 ton, 12 cwt., 2 qrs., 8 lbs., 4 oz.

86. To Derive a Compound Quantity from another by means of any Rate.

Example 1.—To simplify (£14 13*s.* 4*d.*) $\times \frac{13}{15}$.

The derived quantity

$=$ a £ $\times 14 \times \frac{13}{15} +$ a shilling $\times 13 \times \frac{13}{15} +$ a penny $\times 4 \times \frac{13}{15}$,

$$=\text{a }£\times 12\frac{2}{15}+\text{a shilling}\times\frac{169}{15}+\text{a penny}\times\frac{52}{15},$$

$$=\text{a }£\times 12+\text{a shilling}\times\left(20\times\frac{2}{15}+\frac{169}{15}\right)+\text{a penny}\times\frac{52}{15},$$

$$=\text{a }£\times 12+\text{a shilling}\times 13\frac{14}{15}+\text{a penny}\times\frac{52}{15},$$

$$=\text{a }£\times 12+\text{a shilling}\times 13+\text{a penny}\times\left(12\times\frac{14}{15}+\frac{52}{15}\right),$$

$$=\text{a }£\times 12+\text{a shilling}\times 13+\text{a penny}\times 14\frac{2}{3},$$

$$=\text{a }£\times 12+\text{a shilling}\times 14+\text{a penny}\times 2\frac{2}{3},$$

$$=£12\ 14s.\ 2\tfrac{2}{3}d.$$

Example 2.—To simplify (3 acres, 14 square rods, 16 square yards) × 2·35.

The derived quantity

$$=\text{an ac.}\times 3\times 2{\cdot}35+\text{a sq. rod}\times 14\times 2{\cdot}35+\text{a sq. yd.}\times 16\times 2{\cdot}35,$$

$$=\text{an ac.}\times 7{\cdot}05+\text{a sq. rod}\times 32{\cdot}9+\text{a sq. yd.}\times 37{\cdot}6,$$

$$=\text{an ac.}\times 7+\text{a sq. rod}\times(160\times{\cdot}05+32{\cdot}9)+\text{a sq. yd.}\times 37{\cdot}6,$$

$$=\text{an ac.}\times 7+\text{a sq. rod}\times 40{\cdot}9+\text{a sq. yd.}\times 37{\cdot}6,$$

$$=\text{an ac.}\times 7+\text{a sq. rod}\times 40+\text{a sq yd.}\times\left(\frac{121}{4}\times{\cdot}9+37{\cdot}6\right),$$

$$=\text{an ac.}\times 7+\text{a sq. rod}\times 40+\text{a sq. yd.}\times 63{\cdot}925,$$

$$=\text{an ac.}\times 7+\text{a sq. rod}\times 42+\text{a sq. yd.}\times 3{\cdot}425,$$

since 2 sq. rods = a sq. yd. × 60·5.

87. Examples solved.

Ex. 1.—To reduce \$52·25 to £ *s. d.*

Here the money

$$=\text{a dollar}\times 52{\cdot}25,$$

$$= \text{a £} \times \frac{15}{73} \times 52{\cdot}25, \quad = \text{a £} \times \frac{15}{73} \times \frac{209}{4}, \quad = \text{a £} \times 10\frac{215}{292},$$

$$= \text{a £} \times 10 + \text{a shilling} \times \frac{20 \times 215}{292},$$

$$= \text{a £} \times 10 + \text{a shilling} \times 14\frac{53}{73},$$

$$= \text{a £} \times 10 + \text{a shilling} \times 14 + \text{a penny} \times \frac{12 \times 53}{73},$$

$$= \text{£}10\ 14s.\ 8\tfrac{52}{73}d.$$

In the following example we give in full the mechanical labor necessary to complete the reasoning. In the preceding examples this has been suppressed for want of space; but the student should do his multiplying, adding, and so on, always neatly, and preserve it for inspection or correction.

Ex. 2.—To find the cost of 13 ac., 23 sq. rods, 24 sq. yds. of land at $120 per acre.

Solution.—Here the land purchased

$$= \text{an ac.} \times 13 + \text{a sq. rod} \times 23 + \text{a sq. yd.} \times 24,$$

$$= \text{an ac.} \times 13 + \text{a sq. rod} \times \left(23 + \frac{4}{121} \times 24\right),$$

$$= \text{an ac.} \times 13 + \text{a sq. rod} \times \frac{2879}{121},$$

$$= \text{an ac.} \times \left(13 + \frac{1}{160} \times \frac{2879}{121}\right),$$

$$= \text{an ac.} \times \frac{254559}{160 \times 121}.$$

Therefore the cost of the land $= \text{a dollar} \times \dfrac{120 \times 254559}{160 \times 121}$,

$$= \text{a dollar} \times \frac{254559 \times 3}{484} = \text{a dollar} \times 1577{\cdot}84\frac{61}{121},$$

$$= \$1577{\cdot}85 \text{ nearly}.$$

$$\begin{array}{r} 24 \\ \times 4 \\ \hline 96 \end{array}$$

$$\begin{array}{r} 121 \\ \times 23 \\ \hline 363 \\ 242 \\ \hline 2783 \\ +96 \\ \hline 2879 \end{array}$$

$$\begin{array}{r} 121 \\ \times 160 \\ \hline 7260 \\ 121 \\ \hline 19360 \\ \times 13 \\ \hline 58080 \\ 19360 \\ \hline 251680 \\ +2879 \\ \hline 254559 \\ \times \quad 3 \\ \hline 763677 \end{array}$$

$$\begin{array}{l} \begin{array}{r} 121 \\ \times 4 \\ \hline 484 \end{array}\ )\ 76367\dot{7}\ (\ 1577{\cdot}84 \\ \begin{array}{r} 484 \\ \hline 2796 \\ 2420 \\ \hline 3767 \\ 3388 \\ \hline 3797 \\ 3388 \\ \hline 4090 \\ 3872 \\ \hline 2180 \\ 1936 \\ \hline 244 \end{array} \end{array}$$

EXERCISE XXII.

1. Reduce £174 10*s.* to pence.

2. Reduce £432 15*s.* 10*d.* to shillings.

3. Reduce £12 17*s.* 9*d.* to pounds sterling.

4. Reduce 5 ac., 137 sq. rods, 13 sq. yds., 6 sq. ft., 15 sq. ins. to sq. ins.

5. Reduce 7 ac., 15 sq. rods, 5 sq. yds., 3 sq. ft. to sq. ins.

6. Reduce 15 sq. rods, 5 sq. yds., 3 sq. ft. to acres.

7. Reduce 74237 sq. yds. to a compound quantity.

8. Reduce 562934 sq. ins. to sq. rods.

9. Reduce 3 qrs., 14 lbs., 8 oz. to cwt.

10. Reduce 3 bush., 3 pecks, 3 qts., 1 pt. to bushels.

11. Reduce 4930 cubic inches to gallons.

12. How many seconds elapse from 2.30 p.m. on Monday to 8.40 a.m. on Tuesday?

13. How many minutes from 9 a.m. on May 24th to 12 m. on June 1st.

14. Reduce 2 days, 3 hours, 5 minutes to weeks.

15. What rate will derive 130 sq. rods, $6\frac{1}{20}$ sq. yds. from an acre?

16. Reduce 11 cwt., 3 qrs., $12\frac{1}{2}$ lbs. to tons.

17. Explain the meaning of, and simplify, $\dfrac{\pounds 19\ 16s.\ 7\frac{3}{4}d.}{\pounds 20\ 16s.\ 8\frac{3}{4}d.}$

18. Reduce $3\frac{4}{15}$ acres to a compound quantity as far as square inches.

19. Find the result of (3 days, 14 hours, 25 min.) $\times \frac{1}{6}$.

20. Express in acres the sum of $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{15}{16}$ of an acre, $\frac{2}{3}$ of $\frac{13}{21}$ of $\frac{63}{65}$ of 100 sq. rods, and $\frac{16}{17}$ of $2\frac{1}{8}$ times 605 sq. yds.

21. Find the aggregate of $\frac{3}{7}$ of £13, $\frac{1}{3}$ of $\dfrac{1}{2\frac{4}{5}}$ of $\frac{3}{5}$ of £2 12*s.*, and $\frac{5}{7}$ of 9*d.*

22. Express the ratio of 13*s.* $4\frac{1}{2}d.$ to 19*s.* 6*d.* as a decimal as far as the order $+5$.

23. Find the value of ·8596 lbs. at £10 18*s.* $7\frac{1}{2}d.$ per lb.

24. Find the cost of ·0625 of 112 lbs. of sugar at ·0703125 of 17*s.* $9\frac{1}{3}d.$ per lb.

25. Change £143 15*s.* $8\frac{3}{4}d.$ to dollars.

26. Change $432·15 to £ *s.* *d.*

27. Simplify (£3729 18*s.* 6*d.*) $\times \frac{25}{64}$.

28. Simplify (25 ac., 95 sq. rods) $\times 29\frac{3}{5}$.

29. Simplify a mile $\times \frac{3}{16}$ + a furlong $\times \frac{2}{3}$ + a yard $\times \frac{3}{8}$.

30. Express $\frac{3}{7}$ of $2\frac{4}{5}$ of 5 ac., 120 sq. rods as a fraction of $\frac{3}{5}$ of 18 ac., 80 sq. rods.

31. What is the weight of a bushel of water?

CHAPTER XII.

RENT—INTEREST AND DISCOUNT.

88. Rent.—When one man has the use of another man's property, that which he pays for the use of the property is called *Rent.* Thus, if I have the use of *A's* farm for a year, the money I pay him for the use of his farm for this time is called *a year's rent.* Just how much this rent is, and when I pay it, are previously agreed upon between *A* and myself.

89. In the same way, if I have the use of *B's* money for a certain time, that which I pay him for the use of his money may be called *Rent.* Thus, if I have the use of $435 of *B's* money for 6 months, the money I pay him for the use of this $435 may be called *6 months' rent.* Just how much this rent is, and when I pay it, are previously agreed upon between *B* and myself. In such a case, we say that "I *hired* or *borrowed* the money from *B*," and that "*B loaned* or *rented* the money to me." The money itself is called a *Loan.*

It is usual for me to pay the rent when the money hired is paid back. Then the rent and the sum hired together is called the *Amount* at the end of six months.

It is also the custom among business men that the rent of money shall be found by means of a *number* used as a rate. But they use the rate in two ways:

90. Interest and Discount Distinguished.

(1) *Interest.*—When the rent of money is derived, by the rate

agreed upon, from the sum hired at the beginning of the interval of time, the rent is called *Interest*, and the rate agreed upon is called the *Rate of Interest*. The sum hired is called the *Principal*.

(2) *Discount*.—But when the rent of money is derived, by the rate agreed upon, from the amount paid back at the end of the interval of time, the rent is called *Discount*, and the rate agreed upon is called the *Rate of Discount*. The amount to be returned at the end of the time is called the *Debt*.

The person who hired the money, that is, the person who has to pay a debt at a certain time, is called the *Debtor*; and the person who loaned him the money, that is, the person to whom the debt is to be paid, is called the *Creditor*.

91. Rate of Rent (Interest and Discount).—The rate of rent is usually given which will derive a *year's* rent, and is called the annual rate. The annual rate agreed upon by the debtor and creditor is spoken of, or given, in different ways; thus, the annual rate may be 5 per cent., 5 per centum, 5 %, $\frac{5}{100}$, or ·05, which all mean the same, namely: that a year's rent consists of 5 parts, made by dividing the principal, or debt, into 100 equal parts. In this case, then,

$$\text{a year's interest} = \text{the principal} \times \frac{5}{100},$$

$$= \text{the principal} \times \frac{1}{20},$$

$$\text{or} = \text{the principal} \times {\cdot}05.$$

So, also, if the annual rate of discount is $6\frac{2}{3}$ per cent., which we may write $\frac{20}{3} \times \frac{1}{100}$ or $\frac{1}{15}$,

$$\text{a year's discount} = \text{the debt} \times \frac{1}{15}.$$

Again, if the annual rate of rent is $6\frac{1}{4}$ per cent., the rate for 73 days will be $\frac{25}{4} \times \frac{1}{100} \times \frac{73}{365}$, or $\frac{1}{80}$; so that the interest for 73 days = the principal $\times \frac{1}{80}$, and the discount for 73 days = the debt $\times \frac{1}{80}$.

92. Examples solved.—(1) A man hired \$425·25 on April 4th at the rate of $5\frac{1}{2}$ per cent. interest, and paid his debt on July 27th; what rent (interest) did he pay?

Solution.—The time the money was in use is found thus:

In April are 26 days (the 4th April *is not* counted),
in May are 31 days,
in June are 30 days,
and in July are 27 days (the 27th July *is* counted).

Therefore the time is 114 days.

The rate of interest for 114 days $= \frac{11}{200} \times \frac{114}{365} = \frac{11 \times 57}{36500}$.

The principal also = a \$ × 425·25.

Therefore the rent = a \$ $\times \frac{425{\cdot}25 \times 11 \times 57}{36500}$.

= a \$ × 7·30 = \$7.30,

when the following operations are performed:

57	425·25	365) 2666·3175 (7·304
11	627	2555
627	297675	1113
	85050	1095
	255150	1810
	266631·75	1460
		350

If the last figure 4 of the quotient had been 5, 6, 7, 8 or 9, it is the custom to write 7·31 for the quotient. But, in this case, 7·30 is nearer than 7·31 to the exact quotient, $7{\cdot}30\frac{181}{365}$.

(2) A man paid \$300 on September 9th to cancel a debt contracted on May 28th at 6 per cent. discount. Find the rent (discount) and the sum hired.

Solution.—As before: the time the money was in use = 104 days.

The rate of discount for 104 days $= \dfrac{6}{100} \times \dfrac{104}{365}$.

The discount or rent $= \text{a } \$ \times \dfrac{300 \times 6 \times 104}{36500}$.

$= \text{a } \$ \times 5{\cdot}128 = \$5{\cdot}13,$

The money hired $= \$294{\cdot}87$.

(3) If the amount of a sum hired for 4 months at $4\frac{1}{2}$ per cent. interest was \$530, find the rent.

Solution.—The rate of interest for 4 months $= \dfrac{9}{200} \times \dfrac{4}{12} = \dfrac{3}{200}$.

Therefore the interest = the principal $\times \dfrac{3}{200}$.

But the principal = the principal × 1. Article 8.

Therefore the amount = the principal $\times \dfrac{203}{200}$. Article 16.

Therefore the principal = the amount $\times \dfrac{200}{203}$. Article 65.

Therefore the interest = the amount $\times \dfrac{200}{203} \times \dfrac{3}{200}$,

$= \text{a } \$ \times 530 \times \dfrac{3}{203} = \$7{\cdot}83$.

(4) *A* received from *B* \$520 for 6 months, agreeing to pay rent for it at 5 per cent. discount. Find the rent he paid at the end of 6 months.

Solution.—The rate of discount $= \dfrac{5}{100} \times \dfrac{1}{2} = \dfrac{1}{40}$.

Therefore the discount = the debt $\times \dfrac{1}{40}$.

But the debt = the debt × 1. Article 8.

Therefore the money *A* used = the debt $\times \dfrac{39}{40} = \$520$. Article 19.

Therefore the debt = a $ $\times\ 520 \times \dfrac{40}{39}$.

Therefore the rent (discount) = a $ $\times\ 520 \times \dfrac{40}{39} \times \dfrac{1}{40}$,

= a $ $\times\ 13\frac{1}{3}$ = $\$13\frac{1}{3}$.

(5) For how long time will the interest of $500 at 6 per cent. be $\$23\frac{1}{3}$?

Solution.—The rate of interest for the required time is the measure of $\$23\frac{1}{3}$ when $500 is the unit; that is,

the rate for the required time $= \dfrac{\text{a } \$ \times 23\frac{1}{3}}{\text{a } \$ \times 500} = \dfrac{23\frac{1}{3}}{500}$ (Article 33) $= \dfrac{7}{150}$.

But this rate $= \dfrac{6}{100} \times$ the measure of the time.

Therefore the measure of the time $= \dfrac{7}{150} \div \dfrac{6}{100} = \dfrac{7}{9}$.

Therefore the required time = a year $\times \frac{7}{9}$.

EXERCISE XXIII.

1. Calculate the rent in the following cases:
   (*a*) When $500 is hired for 2 years at 6 % interest.
   (*b*) When $325 is hired for 6 months at 4 % interest.
   (*c*) When $225·45 is hired for 9 months at $5\frac{1}{3}$ % interest.
   (*d*) When $1234·56 is hired for $1\frac{1}{2}$ years at $3\frac{3}{4}$ % interest.
   (*e*) When $235·21 is hired for 315 days at 6 % interest.
   (*f*) When $1111·11 is hired for 111 days at 11 % interest.

2. Calculate the rent in the cases when

(*a*) \$450 pays a debt contracted a year ago at 6 % discount.

(*b*) \$240 pays a debt contracted 3 months ago at $7\frac{1}{2}$ % discount.

(*c*) \$1000 pays a debt contracted 87 days ago at $6\frac{1}{2}$ % discount.

(*d*) \$135·29 pays a debt contracted 9 months ago at 5 % discount.

(*e*) \$463·13 pays a debt contracted 146 days ago at $3\frac{1}{3}$ % discount.

3. Find the interest paid for \$500 hired on January 15th and returned on August 3rd, at $5\frac{1}{2}$ %.

4. If the rent for a sum of money for a year be \$49·20, for what time will the rent be \$16·40?

5. If the interest of \$290 be \$14·50 for a year, what is the rate of interest?

6. If the interest of \$640 for a certain time is \$16, of what sum is \$42 the interest for the same time?

7. If the annual rate of rent be $7\frac{1}{2}$ %, for what time is the rate $2\frac{1}{2}$ %?

8. What rate will derive the principal from the amount, when the principal is hired for 73 days at $13\frac{1}{3}$ % interest?

9. What rate will derive the amount at the end of 8 months from the 8 months' interest, the rate of interest being 8 %?

10. What rate will derive the amount from the principal which is hired for 170 days at 6 % interest?

11. What principal loaned for $1\frac{1}{2}$ years at $4\frac{1}{2}$ % will amount to \$854?

12. If the interest = the amount × $\frac{5}{32}$, what rate will derive the interest from the principal?

13. A man hired \$50 on the first day of each month of a

certain year at 10 % interest, how much did he owe at the end of that year?

14. Find the interest of $125·62 from April 29th to September 16th at 5 %.

15. $600 was hired on May 9th at 6 % interest, and the debt was paid with $625. Find the day on which the debt was paid.

16. A debt was paid on May 8th with $620. The rent, which was at the rate of 5 % discount, was $20. Find the day on which the debt was contracted.

17. *A* received from *B* $400, for which he is to pay rent at 6 % discount. At the end of 6 months *A* paid his debt; find the rent.

18. *A* hired two equal sums of money, each for 6 months. For the one he paid 5 % interest, and for the other $6\frac{1}{2}$ % interest. The total interest was $46. Find the sums hired.

19. A man hired two equal sums of money, each for 6 months. For the one he paid rent at 6 % interest and for the other he paid rent at 6 % discount. If the total rent paid was $40·60, find the sums he hired.

20. A person borrows $500 on April 10th, and on June 22nd pays his debt with $510·20. At what rate per cent. per annum was he charged interest?

21. Find the rent of £243 6*s.* 8*d.* for 97 days at $6\frac{1}{4}$ %.

22. A sum amounts to $359·60 at the end of a year at 5 % interest. What was the amount at the end of 6 months?

23. If the amount of $400 at the end of a year be $430 at a certain rate of interest, what would be the amount at the end of 9 months at the same rate?

24. A sum at 8 % interest amounts in 9 months to $530; in how many months will it amount to $540?

25. A sum was borrowed for 8 months at $9\frac{1}{3}$ per cent. interest; what is the equivalent rate of discount?

93. Promissory Notes.—When one man, John Brown, hires a sum of money (say $500) from another man, Henry Smith, and agrees to pay him Rent for the money at 6 per cent. interest, it is the custom for John Brown to give to Henry Smith a paper, which reads as follows:

> *$500.* TORONTO, *May 9th, 1897.*
>
> *Six months after date, I promise to pay Henry Smith, or order, the sum of Five Hundred Dollars ($500), with interest at 6 per cent. per annum, value received.*
>
> *(Signed)* JOHN BROWN.

And when John Brown pays his debt, Henry Smith returns the paper to John Brown. This paper is called a promissory note. When John Brown pays the debt, he is said to have "redeemed the note," or to have "paid the note."

94. Three Days of Grace.—It would seem that John Brown here promises to pay the debt "6 months after" May 9th, that is, on November 9th; but, by an Act of Parliament of the Dominion of Canada, this time of payment is extended 3 days, so that John Brown is not required to pay the debt till November 12th. This later date is called the day of maturity of the note. It is plain that John Brown has the use of Henry Smith's money from May 9th to November 12th, that is, for 187 days.

$$\text{The rent, then, he pays} = a\ \$ \times 500 \times \frac{6}{100} \times \frac{187}{365} = \$15{\cdot}37.$$

95. Discounting Notes.—If, however, John Brown had asked for a loan of $500 from a Bank or Loan Company, with the understanding that he was to pay rent at 6 per cent. discount (or what is the same thing, to pay *in advance*, rent at 6 per cent. interest), he would then give a paper, which reads as follows:

$500. TORONTO, *May 9th, 1897.*

Six months after date I promise to pay the Bank of Commerce, or order, the sum of Five Hundred Dollars ($500), value received.

(Signed) JOHN BROWN.

Thus, as before, John Brown promises to pay $500 on November 12th. But of this a part is rent, which is derived from the debt, $500, by the rate, 6 per cent., thus:

$$\text{the rent} = \text{a } \$ \times \frac{500 \times 6 \times 187}{36500} = \$15{\cdot}37.$$

Therefore, John Brown would not have the use of $500, but of ($500 – $15·37) $484·63. Now, when the Bank pays $484·63 to John Brown for his note, as above, the Bank is said to "discount John Brown's note." The $484·63 is called the *Proceeds* of the note, and the $15·37 is called the *Discount* of the note.

In this case, then, John Brown uses $484·63 for 187 days, and pays $15·37 rent (discount); while in the case of Article 93, he uses $500 for 187 days, and pays $15·37 rent (interest).

96. Selling Notes.—By virtue of the words "or order," a note, such as that in Article 93, may be sold by Henry Smith to a third party, George Taylor, for money. Henry Smith will then have the use of George Taylor's money from the day he sold the note till the day John Brown pays it. For the use of this money Henry Smith usually agrees to pay rent at a certain rate of discount, so that the rent is derived from the amount of the note on November 12th by the rate agreed upon. On giving the note to George Taylor, Henry Smith signs his name across the back of it; and, by thus *endorsing* the note, will have to pay it in case John Brown fails to do so. Here, also, George Taylor is said to discount John Brown's note, and the money he pays for the note is called the proceeds of the note.

97. Demand Notes.—Sometimes, however, a man may hire money from another by giving a note, as follows:

> *$1000.* LONDON, *June 6th, 1897.*
>
> *On demand, I promise to pay Uriah Heep, or order, the sum of One Thousand Dollars ($1000), with interest at 7 per cent. per annum, value received.*
>
> WILKINS MICAWBER.

Such a loan may be paid back part at a time. For instance, on October 16th, $450 may be returned to Uriah Heep; but it is agreed that the rent for the $1000 up to October 16th is first paid out of the $450, that the balance of the $450 pays part of the loan, and that Wilkins Micawber pays rent for the use of the remaining principal. If, however, the part payment is not enough to pay the rent due at the time of payment, no rent is to be paid for the use of the unpaid rent; that is, the rent after the part payment is made, is derived from the same principal as it was before the part payment was made.

98. Examples solved.

(1) A three months' note for $350 was given on May 29th, and was sold on June 26th at 7 per cent. discount. Find the proceeds.

Solution.—The note matures on September 1st. The seller then has the use of the proceeds from June 26th to September 1st, that is, for 77 days.

The rate of discount for this time $= \frac{7}{100} \times \frac{77}{365}$.

Therefore the rent he is to pay September 1st,

$$= \text{a } \$ \times 350 \times \frac{7}{100} \times \frac{77}{365} = \$5{\cdot}17.$$

The proceeds, then, $= \text{a } \$ \times (350 - 5{\cdot}17) = \$344{\cdot}83$.

The student is to perform the operations as follows:

$$\begin{array}{r} 350 \\ 7 \\ \hline 2450 \\ 77 \\ \hline 17150 \\ 17150 \\ \hline 188650 \end{array} \qquad \begin{array}{l} 365\ )\ 1886{\cdot}50\ (\ 5{\cdot}168 \\ 1825 \\ \overline{615} \\ 365 \\ \overline{2500} \\ 2190 \\ \overline{3100} \end{array}$$

(2) For what sum shall I make a note for 4 months on May 9th, so that, if I immediately sell it at 6 per cent. discount, I may receive $400?

Solution.—The note matures on September 12th. I have the use of $400, then, from May 9th to September 12th, that is, for 126 days.

$$\begin{aligned} \text{The rent I pay, then, } &= \text{the debt} \times \frac{6}{100} \times \frac{126}{365}, \\ &= \text{the debt} \times \frac{756}{36500}, \\ \text{But the debt } &= \text{the debt} \times 1. \\ \text{Therefore the proceeds } &= \text{the debt} \times \left(1 - \frac{756}{36500}\right), \\ &= \text{the debt} \times \frac{35744}{36500}. \end{aligned}$$

$$\text{That is, the debt} \times \frac{35744}{36500} = \$400.$$

$$\begin{aligned} \text{Consequently, the debt } &= \text{a } \$ \times \frac{400 \times 36500}{35744}, \\ &= \$408{\cdot}46, \end{aligned}$$

which, of course, is the sum the note is made out for.

(3) A demand note for $1200 was made on January 28th, drawing interest at 7 per cent. It was partly paid as follows:

On April 11th, \$425; on October 12th, \$23·68. What remains due on December 31st?

Solution.—

The interest up to April 11th $= \$1200 \times \frac{7}{100} \times \frac{73}{365} = \$16{\cdot}80$.

When this interest is paid, the balance of the payment reduces the principal to \$791·80, for which rent is paid after April 11th. The interest, then, up to October 12th

$$= \$791{\cdot}80 \times \frac{7}{100} \times \frac{184}{365} = \$27{\cdot}94.$$

But the payment, \$23·68, leaves \$4·26 of this interest unpaid. With the same principal as before, then,

the interest up to December 31st $= \$791{\cdot}80 \times \frac{7}{100} \times \frac{80}{365} = \$12{\cdot}15$.

Hence, on December 31st there is due

$$\$4{\cdot}26 + \$12{\cdot}15 + \$791{\cdot}80 \text{ or } \$808{\cdot}21.$$

EXERCISE XXIV.

1. Find the interest of the following note:

\$850. LONDON, *September 16th, 1897.*

Eight months after date I promise to pay R. Johnson, or order, the sum of Eight Hundred and Fifty Dollars (\$850), with interest at $6\frac{1}{2}$ *per cent., value received.*

PETER RYAN.

2. What rent is paid when the following note is sold on July 8th, at $7\frac{1}{2}$ per cent. discount?

\$4352·58. STRATFORD, *May 9th, 1897.*

Nine months after date we promise to pay to the order of Charles Smith & Co., Four Thousand Three Hundred and Fifty-two $\frac{58}{100}$ *Dollars (\$4352·58), value received.*

CASH AND PENNY.

3. I wish to hire $500 by selling my note on June 1st, for three months, at 7 per cent. discount. Find what discount I shall pay, and what I shall make out the note for.

4. A demand note for $500 was given on January 6th, 1896, drawing interest at 7 per cent. On May 16th, $200 was paid on the note. How much paid the note on October 12th, 1896?

5. A note was discounted 73 days before it matured, at 7½ per cent. discount, and produced $394. Find its face value.

6. A note for $160 is to run for a year at 8 per cent. interest. Find the debt at the end of that time. Find also the rent the seller of the note would pay if he sold it after 6 months, at 6 per cent. discount.

7. I hired $1200 at 6 per cent. interest, agreeing to pay back $100 at the end of each month, together with the rent then due. How much rent did I pay in all?

8. The following note was sold by John Jones the day it was dated, at 6 per cent. discount:

$1400. GUELPH, *November 16th, 1896.*

Seventy days after date I promise to pay John Jones, or order, the sum of Fourteen Hundred Dollars ($1400), with interest at 6 per cent., value received.

GEORGE PUTNAM.

Find the proceeds. What rent did John Jones pay? What rent did George Putnam pay?

9. Find the face of a note drawn for 60 days that will realize $840, when discounted on the day it was made, at 6½ per cent.

10. The discount of a note for $380, maturing October 12th, was $9.10 at 6 per cent. On what day was the note dated?

11. What must be the face of a note made on January 19th, 1896, for 11 months, so that, when discounted at 7 per cent. the day it was made it may yield $486·45?

12. Find the interest of a note for $125369·45, maturing in 126 days after date, at $3\frac{1}{2}$ per cent. Find also the amount of the note at maturity.

13. *$4000.* TORONTO, *June 1st, 1897.*

On demand I promise to pay Richard Little, or order, Four Thousand Dollars ($4000), with interest at 7 per cent., value received.

STEPHEN THOMPSON.

This note was endorsed as follows :

September 15th, 1897, Received $400·50. *R. L.*
December 15th, 1897, Received $50·00. *R. L.*
March 1st, 1898, Received $500·00. *R. L.*

What is the note worth January 1st, 1899?

99. Compound Interest.—Again, *A* may have the use of *B's* money for a longer time than a year. It is usual then for *A* to agree to pay the rent (interest) at the end of every year, or at the end of every six months. If he fails to do so, the rent, when due, is put with the principal, and then *A* pays rent for the whole amount. In such a case, the rent *A* pays when he discharges his debt is called *Compound Interest.* If the rent is put with the principal at the end of every six months (say), the rent (or interest) is said to be convertible into principal half-yearly, or compounded half-yearly.

100. How Compound Interest is found.—Thus, if *A* has the use of *B's* money at 6 per cent. interest, convertible yearly;

the 1st year's rent = the principal × ·06. Article 91.
But the principal = the principal × 1.
This rent being unpaid, is put with the principal, so that
the 2nd year's principal = the 1st year's principal × 1·06;
that is, the 2nd year's principal is derived from the 1st year's principal by the rate 1·06.

Therefore, the 3rd year's principal is derived from the 2nd year's principal by the same rate, $1{\cdot}06$; that is,
the 3rd year's principal
= the 2nd year's principal × 1.06.
= the 1st year's principal × $1{\cdot}06 \times 1{\cdot}06$.

Similarly, the 4th year's principal
= the 1st year's principal × $1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06$,
and so on.

It has been agreed to denote the product $1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06$ thus, $(1{\cdot}06)^3$.

Hence we say that the amount of *A's* debt at the end of 4 years
= the principal × $(1{\cdot}06)^4$.

Further, this includes the principal and the 4 years' rent; therefore the 4 years' rent = the principal × $\{(1{\cdot}06)^4 - 1\}$.

We shall call $(1{\cdot}06)^4$ "the rate of the amount for 4 years," and $(1{\cdot}06)^4 - 1$ "the rate of interest for 4 years."

Thus, if the rate of interest for 1 year is $4\frac{3}{8}$ per cent.,
the rate of amount for 5 years is $(1{\cdot}04375)^5$,
and the rate of interest for 5 years is $(1{\cdot}04375)^5 - 1$.

101. Since *A's* debt at the end of 4 years
= the principal × $(1{\cdot}06)^4$;
therefore the principal

$$= A\text{'s debt at the end of 4 years} \times \frac{1}{(1{\cdot}06)^4}. \qquad \text{Article 65.}$$

102. Since the 4 years' rent = the principal × $\{(1{\cdot}06)^4 - 1\}$;
therefore the principal = the 4 years' rent × $\dfrac{1}{(1{\cdot}06)^4 - 1}$.

103. Examples solved.

(1) At 6 per cent. what is the rate of amount for 100 days?

Solution.—The interest for 100 days

$$= \text{the principal} \times \frac{5}{100} \times \frac{100}{365}.$$

$$= \text{the principal} \times \frac{1}{73}.$$

Therefore the amount at the end of 100 days

$$= \text{the principal} \times \frac{74}{73},$$

$$= \text{the principal} \times 1{\cdot}0150685, \text{ by dividing.}$$

Hence the required rate is 1·0150685.

(2) To find the compound interest of \$532 for 3 years, at 4 per cent.

Solution.—The rate of amount for 3 years is $(1{\cdot}04)^3$; so that the interest for 3 years $= \text{a } \$ \times 532 \times \{(1{\cdot}04)^3 - 1\} = \$66{\cdot}43$.
The student will multiply as indicated, thus:

$$\begin{array}{r} 1{\cdot}04 \\ 1{\cdot}04 \\ \hline 416 \\ 104 \\ \hline 1{\cdot}0816 \\ 1{\cdot}04 \\ \hline 43264 \\ 10816 \\ \hline 1{\cdot}124864 \end{array} \qquad\qquad \begin{array}{r} {\cdot}124864 \\ 532 \\ \hline 249728 \\ 374592 \\ 624320 \\ \hline 66{\cdot}427648 \end{array}$$

(3) What principal will amount, at the end of 2 years 6 months, at 6 per cent., to \$500?

Solution.—The rate of amount for 1 year = 1·06,
the rate of amount for 6 months = 1·03.
Hence the debt at the end of 2 years 6 months

$$= \text{the principal} \times (1{\cdot}06)^2 \times 1{\cdot}03;$$

that is, the principal $\times (1{\cdot}06)^2 \times 1{\cdot}03 = \$500$,

therefore the principal $= \text{a } \$ \times \dfrac{500}{(1{\cdot}06)^2 \times 1{\cdot}03} = \$432.04$,

as seen from the following operations:

```
    1·06
    1·06
    ----
     636
   106
  ------
  1·1236
    1·03
  ------
   33708
  11236
 --------
 1·157308
```

```
1·157308 ) 500·0000 ( 432·037
           4629232
           -------
            3707680
            3471924
            -------
             2357560
             2314616
             -------
              4294400
              3471924
              -------
               822476
```

EXERCISE XXV.

1. At 7 per cent., what rate will derive the debt at the end of 2 years from the sum hired?

2. What rate will derive the compound interest from the principal borrowed for 2 years at 5 per cent.?

3. What rate will derive the amount at the end of 3 years from a principal hired at 8 per cent. per annum?

4. What rate will derive the rent from a principal which is hired for 2 years at 8 per cent. per annum, convertible half-yearly?

5. Find the rate of amount for 3 years 4 mos. at 6 per cent.

6. Find the rate of interest for 2 years 4 mos. at 6 per cent.

7. A man hired $400 for 2 years at 6½ per cent. Find his debt.

8. Calculate the compound interest in the following cases:
   (*a*) When $300 is hired for 3 years at 10 per cent.
   (*b*) When $435 is hired for 2½ years at 8 per cent.
   (*c*) When $180 is hired for 4 years at 5 per cent.
   (*d*) When $1250 is hired for 3½ years at 4 per cent.
   (*e*) When $1234.56 is hired for 2 years 146 days at 5 per cent.

9. Calculate the rent when

(*a*) \$360 is hired for $1\frac{1}{2}$ years at 6 per cent., convertible half-yearly.

(*b*) \$600 is hired for 1 year at 8 per cent., convertible quarterly.

(*c*) \$825 is hired for 2 years at 5 per cent., convertible half-yearly.

(*d*) \$4000 is hired for 9 months at 6 per cent., convertible quarterly.

10. Find what principal will in 2 years at 4 per cent. amount to \$600?

11. If the compound interest of a sum of money borrowed for 2 years at 7 per cent. is \$28·98, find the sum of money.

12. If the compound interest of a sum of money for 3 years at 8 per cent. exceeds the simple interest of the same sum at the same rate by \$20, find the sum.

13. Find the interest when \$1234·60 is borrowed on June 18th, 1898, at 7 per cent. per annum, and paid on Dec. 31st, 1901.

104. Equation of Payments.—As the result of certain business transactions between *A* and *B*, it may happen that *A* owes *B* the following debts:

\$200 to be paid on May 9th.
\$300 to be paid on July 18th.
\$600 to be paid on August 3rd.

If these debts are not paid when due, *A* must pay *B* rent for the money he is using. There is a day, however, on which *A* may discharge his whole obligation by paying to *B* \$1100, the sum of these debts. This day is called "the equated time" of these debts; and the process of finding it is called "the equation of these payments."

105. To find the Equated Time.—If *A* waits till, say, August 15th before paying anything to *B*, he will then owe, evidently,

$200 + the rent of $200 for 98 days,
$300 + the rent of $300 for 28 days,
and $600 + the rent of $600 for 12 days;

that is, he will then owe

$200 + the rent of (200 × 98) $19600 for 1 day,
$300 + the rent of (300 × 28) $8400 for 1 day,
and $600 + the rent of (600 × 12) $7200 for 1 day,

making a total of

$1100 + the rent of $35200 for 1 day;

or of $1100 + the rent of $1100 for $\left(\frac{35200}{1100} = \right)$ 32 days.

Consequently, *A* could discharge his obligation by paying $1100 to *B* 32 days *before* August 15th, that is, on July 14th.

106. Averaging Accounts.—If, however, *A* pays *B* in part payment,

$150 on June 28th, and $300 on July 30th;

then the statement of these debts and payments is called the *Account* between *A* and *B*. As before, there is a day on which *A* may discharge the rest of his obligation to *B* by paying him $650, the balance of the debts. This day is called the "equated time of the account," and the process of finding it is called "averaging the account."

To find this time.—If these part payments were withheld by *A* till August 15th then they would be worth

$150 + the rent of (150 × 49) $7350 for 1 day,
and $300 + the rent of (300 × 16) $4800 for 1 day,

or in total, $450 + the rent of $12150 for 1 day.

But on August 15th his debt was

$1100 + the rent of $35200 for 1 day. See Art. 105.

Therefore he still owes on August 15th,

$650 + the rent of $23050 for 1 day;

that is, $650 + the rent of $650 for $\left(\frac{23050}{650}=\right)$ 35 days.

Hence, he should pay the $650 to *B* 35 days before August 15th, that is, on July 11th.

If *A* did not pay the $650 on July 11th, as he evidently did not, he must pay rent for the $650 from July 11th onwards at the rate agreed upon.

EXERCISE XXVI.

1. I owe James White $75 due in 4 months, $50 due in 2 months, $90 due in 5 months, and $65 due in 7 months. Find the equated time.

2. Equate the following debts: $123 due January 9th, $423 due March 18th, and $218 due May 28th.

3. *A* owes *B* the following debts: $350 due in 5 months, $425 due in 8 months, $560 due in 9 months, and $720 due in 12 months. Find the equated time.

4. Find the equated time of the following account: Peter Breehaw, *Dr.* to John Fitch, $135 due October 1st, $596 due November 29th, $384 due December 16h; *Cr.* by $400 paid December 3rd, and $300 paid December 27th.

5. *Dr.* ROBERT GUNN & Co. *Cr.*

May 18th.	To	$1230·00	July 11th.	By	$ 950·00
Aug. 20th.	To	2300·00	Oct. 18th.	By	1800·00
Sept. 30th.	To	1250·00	Dec. 5th.	By	1200·00
Nov. 8th.	To	2140·00			

Find the equated time of the above account; and find how much will pay it off on December 31st, at 7 per cent.

6. A man owed two notes, one for $625, maturing on September 17th, and the other for $750, maturing on December 29th. For these two notes he gives a note for $1375. On what day does this note mature?

CHAPTER XIII.

BUYING AND SELLING—GAIN AND LOSS.

107. When A buys an article from B, he gives B money for it; so that the article B owned before A now owns, and the money A had before B now has. A speaks of the money he paid for the article as its *Cost Price;* while B speaks of the money he got for it as its *Selling Price.* This exchange between A and B is called a *Business Transaction.* The person who makes or produces the article at first is called a *Manufacturer* or *Producer;* the person who uses the article is called the *Consumer;* while the person who buys the article from the producer and sells it to the consumer is called a *Merchant.*

108. Gain and Loss.—Now the merchant usually gets more money for the article from the consumer than he pays for it to the producer. The difference is what the merchant *gains* by buying and selling the article. The merchant, however, may get less. The difference, then, is what the merchant *loses.*

109. The Rate of Gain or Loss.—Again, the merchant's gain or loss is thought of as being derived from the cost of the article by means of a rate. So that the *Rate* of gain, or loss, is that number, or rate, by which his gain or loss is derived from the cost price of the article. Thus, if a merchant says he gained 25 per cent. by buying and selling an article, he means

$$\text{that his gain} = \text{the cost of the article} \times \frac{1}{4};$$

or if he loses 15 per cent.,

$$\text{that his loss} = \text{the cost of the article} \times \frac{3}{20}.$$

110. Capital.—Since the purchase of an article must take place some time before the sale of it, a merchant must have money to buy with before he can *gain* money by buying and selling. This money which he uses to commence and carry on his business is called his *Capital*, which is said to be invested in the business. And, again, the whole gain or profit which a merchant makes during a year, or six months, is regarded as derived from the capital invested by means of a rate. Thus, if, during the year 1896 a merchant made a profit of 8 per cent., we mean that

$$\text{the year's gain or profit} = \text{his capital} \times \frac{8}{100}.$$

111. Trade Discount.—When a merchant receives an article which he intends to sell, he usually marks upon it the price for which he intends to sell it. Sometimes, however, he does not mark the price on the article, but sets it down in a list, which he can conveniently consult. It may happen that, before he sells the article, he may see fit to reduce its marked or list price, in which case the amount he takes off the price is called *Trade Discount.* Moreover, as before, this trade discount is derived from the marked or list price by means of a rate. Thus, if a merchant advertises a discount of 10 per cent., he means that

$$\text{the reduction in his price} = \text{the list price} \times \frac{1}{10},$$

so that $\text{the selling price} = \text{the list price} \times \frac{9}{10}.$

112. Examples solved.

(1) A merchant marked an article $16\frac{2}{3}$ per cent. above its cost, and selling at this price, gained \$1.60; find his cost and selling prices.

Solution.—Since $16\frac{2}{3}$ per cent. $= \frac{1}{6}$,

the advance above cost $=$ the cost price $\times \frac{1}{6}$.

This also is the gain, so that the cost price $\times \frac{1}{6} = \text{a } \$ \times 1{\cdot}60$,

therefore the cost price $= \text{a } \$ \times 1{\cdot}60 \times 6$,

$$= \$9{\cdot}60.$$

But the gain $= \$1{\cdot}60$,

therefore the selling price $= \$11{\cdot}20$.

(2) A merchant marked his goods 20 per cent. above their cost, but sold them at a discount of 10 per cent. Find his rate of gain.

Solution.—The advance above the cost price $=$ the cost $\times \frac{1}{5}$,

therefore the marked price $=$ the cost $\times \frac{6}{5}$.

Again, the discount $=$ the marked price $\times \frac{1}{10}$,

therefore the selling price $=$ the marked price $\times \frac{9}{10}$,

$$= \text{the cost} \times \frac{6}{5} \times \frac{9}{10},$$

$$= \text{the cost} \times \frac{27}{25},$$

therefore the gain $=$ the cost $\times \frac{2}{25}$.

So that the rate of gain $= \frac{2}{25} = 8$ per cent.

(3) During the 1st year of his business a merchant increased his capital by $12\frac{1}{2}$ per cent., and the 2nd year he increased it by 15 per cent.; he was then worth \$4140. What was his original capital?

Solution.—The 1st year's gain $=$ the original capital $\times \frac{1}{8}$.

Therefore the 2nd year's capital $=$ the original capital $\times \frac{9}{8}$.

Again, the 2nd year's gain = the 2nd year's capital $\times \frac{3}{20}$.

Therefore the final capital = the 2nd year's capital $\times \frac{23}{20}$;

= the original capital $\times \frac{9}{8} \times \frac{23}{20}$.

that is, the original capital $\times \frac{9}{8} \times \frac{23}{20} = \$4140$.

Therefore the original capital $= \$4140 \times \frac{8 \times 20}{9 \times 23}$,

$= \$3200$.

(4) At what rate does a grocer reduce his price by giving $\frac{1}{2}$ an ounce with each pound, for good measure?

Solution.—The business transaction is:

The grocer gives $16\frac{1}{2}$ oz. of sugar (say), and gets the list price of 16 oz.

Therefore the selling price of $16\frac{1}{2}$ oz. of sugar
= the list price of 16 oz.;
that is, the selling price of an oz. of sugar $\times 16\frac{1}{2}$
= the list price of an oz. $\times 16$.
Therefore the selling price of an oz.

= the list price of an oz. $\times 16 \times \frac{2}{33}$.

Therefore his reduction = the list price $\times \frac{1}{33}$,

and his rate of reduction is $\frac{1}{33}$ or $3\frac{1}{33}$ per cent.

EXERCISE XXVII.

1. A merchant paid \$3250 for a certain line of goods, which he sold at a gain of 7 per cent. Find his selling price.

2. A man gained \$12 by selling an article at $12\frac{1}{2}$ per cent. above its cost. Find its cost.

3. A merchant paid \$4.50 for an article which he sold for \$6. Find his rate per cent. of gain.

4. The selling price of an article was \$25 when the rate of gain was 25 per cent. Find its cost price.

5. A merchant paid \$1432·25 for a certain line of goods, which he marked at 20 per cent. above cost. He disposed of them at a discount of 5 per cent.; find his gain.

6. A merchant marked an article 30 per cent. above its cost, and sold it without reduction for \$22·10. Find its cost.

7. A merchant began business by investing \$4525. He gained the first year at the rate of 6 per cent., and the second year at the rate of 8 per cent. What was he then worth?

8. A merchant's capital at first was \$7500, at the end of the year it was \$7000. Find his rate of loss.

9. The first year a merchant increased his capital by the rate of $12\frac{1}{2}$ per cent., the second year by the rate of 10 per cent.; his profit for the two years was \$1520. Find his original capital.

10. *A* bought some oranges at the rate of 7 for 12 cents, and sold them at the rate of 2 for 5 cents. Find his rate of gain.

11. The manufacturer of a certain article made a profit of 20 per cent., the merchant made a profit of 25 per cent. If the merchant's selling price was 12 cents more than the manufacturer's outlay, find what the consumer paid for the article.

12. I bought 325 barrels of apples at \$1·40 per barrel; I prepaid the freight on them to Montreal at 6 per cent. of their cost. I sold them there at a profit of $12\frac{1}{2}$ per cent. of the whole outlay. Find my profit.

13. I sold two houses and lots for \$1600 each, gaining on the one at the rate of $12\frac{1}{2}$ per cent., and losing on the other at the rate of $12\frac{1}{2}$ per cent. Find my gain or loss on these transactions.

14. If \$1·40 is gained by selling at 20 per cent. above cost, find what selling price would make the rate of gain 25 per cent.

sold for $6.

the rate of

of goods,
disposed of

ove its cost,
ost.
He gained
nd year at

end of the

by the rate
r cent.; his
l capital.
cents, and
of gain.
profit of 20
nt. If the
e manufac-
ticle.
rel; I pre-
their cost.
hole outlay.

ning on the
ther at the
ansactions.
above cost,
5 per cent.

---

15. I marked my goods $22\frac{2}{9}$ per cent. above cost, but in selling gave a discount of $9\frac{1}{11}$ per cent., gaining, however, $125. Find what the goods cost.

16. If 4 articles are sold for the cost of 3, find the rate of gain?

17. A grocer sold butter with a pound weight $\frac{1}{2}$ an ounce light; at what rate did he thereby increase his price?

18. A merchant measured off 37 inches of cloth for a yard; by what rate did he thereby decrease his list price?

19. A merchant decreases the list price by three successive rates of discount, each equal to 10 per cent.; what single rate of discount is equivalent to these three rates?

20. Half of my goods I sold at a profit of 15 per cent., and the other half at a profit of 29 per cent. Find the rate of profit for the whole.

21. An egg buyer purchased 300 dozen eggs at 7 cents a dozen, 400 dozen at $7\frac{1}{2}$ cents, 516 dozen at 8 cents, and 600 dozen at $7\frac{1}{4}$ cents. He sold them all at $7\frac{7}{8}$ cents a dozen. Find his gain and also his rate of gain.

22. I bought 4 barrels of molasses at 32 cents a gallon. In selling 2 gallons were wasted. I made a profit, however, of $13\frac{1}{3}$ per cent. Find my selling price per gallon.

23. A dry goods merchant marked his cloth $42\frac{6}{7}$ per cent. in advance of its cost. He sold it at a discount of 10 per cent., and gained 15 cents a yard. Find the cost per yard.

24. During 6 months of business a merchant's gain was at the rate of 8 per cent. Of this gain the merchant withdrew $400 for private use. The next 6 months his gain was at the rate of 10 per cent. At the end of the year he was worth $11440. Find his original capital.

25. A merchant bought $225\frac{3}{4}$ yards of cloth at $\$1.17\frac{1}{2}$ a yard. His profit on selling was at the rate of $23\frac{1}{3}$ per cent. Find his total selling price.

The following six examples are more difficult.

26. A merchant sold 25 per cent. of a certain stock, gaining at the rate of $22\frac{1}{2}$ per cent., 10 per cent. of it gaining at the rate of $18\frac{3}{4}$ per cent., 30 per cent. of it gaining at the rate of 15 per cent., and the rest losing at the rate of 50 per cent. His gain from these transactions was $242. Find the cost of the stock.

27. The 1st year a merchant gained at the rate of $12\frac{1}{2}$ per cent., the 2nd year at the rate of $11\frac{1}{9}$ per cent., the 3rd year at the rate of 10 per cent., the 4th year at the rate of $9\frac{1}{11}$ per cent., and the 5th year at the rate of $8\frac{1}{3}$ per cent. What rate will derive the 5 years' gain from the original capital?

28. A dishonest merchant pretended to buy and sell at the same price, but cheated to the extent of $\frac{1}{2}$ an ounce every time he bought or sold a pound. What rate will derive his dishonest gain from his outlay?

29. A grain buyer bought 5000 bushels of wheat at 64 cents a bushel. When wheat had risen to 66 cents he sold it all. He then invested all this money again in wheat at 65 cents a bushel, and sold it all at 67. Again, he invested all his money in wheat at 66 cents, and sold out at 68 cents. Find his total gain from these transactions.

30. A miller had ground together 34 bushels of oats at 37 cents a bushel, 25 bushels of corn at 58 cents, 25 bushels of peas at 43 cents, and 48 bushels of barley at 45 cents. The cost in wages for grinding was 5 cents a bushel. He sold the feed to farmers at $1·50 per cwt.; find his profit.

31. An apple woman bought half a lot of apples at 7 for 2 cents, and the other half at 10 for 3 cents; she sold them all at 17 for 5 cents. Find her rate of gain or loss.

113. Partnership.—It often happens that one man has not enough capital to successfully commence and carry on a business. Then two or more men put together their several capitals, and conduct the business, thus forming a *Partnership.* It is fair that, if their capitals are in the business during the same time, each should gain or lose at the same rate; so that whatever rate will derive the whole gain from the whole capital will also derive each partner's gain from his capital. Thus, if *A*, *B* and *C* form a partnership for a year, contributing, respectively, $700, $900 and $800, and gain $1200,

The whole capital being $2400, and the whole gain $1200, the rate of gain $= \frac{\$1200}{\$2400} = \frac{1200}{2400} = \frac{1}{2}$.

Therefore *A*'s share of the gain $= \$700 \times \frac{1}{2} = \$350$.

B's share of the gain $= \$900 \times \frac{1}{2} = \$450$.

and *C*'s share of the gain $= \$800 \times \frac{1}{2} = \$400$.

114. Example solved.—*A*, *B* and *C* form a partnership, *A* contributing $400, which was in business for 8 months; *B* $500 for 6 months, and *C* $400 for 12 months. The whole gain was $900. How much of this belongs to each?

Solution.—*A* should get what $400 earns in 8 months; that is, *A* should get what $3200 earns in 1 month; also, *B* should get what $3000 earns in 1 month, and *C* should get what $4800 earns in 1 month. Therefore, all should get what $11000 earns in 1 month.

The whole gain is $900.

Hence, the rate of gain for a month $= \frac{\$900}{\$11000} = \frac{9}{110}$.

Therefore *A*'s share should $= \$3200 \times \frac{9}{110} = \$261\frac{9}{11}$.
and so for the others.

EXERCISE XXVIII.

1. *A*, *B* and *C* contribute, respectively, $500, $600 and $700 to carry on a business for a year. Divide the whole profit, $240, among them.

2. *A*, *B* and *C* put into a business for a year the capitals, $650, $910 and $1300. *A's* share of the gain was $143; find the shares of *B* and *C*.

3. *A* and *B* form a year's partnership, *A* contributing $4900, and *B* $9100. After 6 months *A* contributed $5000 more. The whole gain was $3300. How much of this belongs to *A*?

4. *A* and *B* start a business, *A* putting in $5000 and *B* $4000. After 6 months they admit another partner, *C*, with a capital of $3000. At the end of the year the books show a loss of $660. The business is then wound up. How much capital does each now possess?

5. *A* put in a business $480 and *B* $560. Their gains were $240 and $420. If *A's* capital was in 9 months, how long was *B's* capital employed?

6. *A* and *B* contribute, respectively, $2342.35 and $3571.69, and gain $6235.24. How much was *A's* profit?

7. *A*, *B* and *C* rent a pasture field for $100. *A* had 25 head of cattle in it for 4 months, *B* 35 head for 6 months, and *C* 20 head for 5 months. How much rent should each pay?

8. *A* commenced business on July 4th, investing $429.10. On September 25th he admitted a partner, *B*, with a capital of $324.10. On December 31st, the profit was found to be $185.25. Find the share of each.

9. *A*, *B* and *C* contribute equal sums of money to carry on a business. At the end of 3 months *A* withdraws $\frac{1}{2}$ of his capital, at the end of 6 months *B* withdraws $\frac{1}{3}$ of his capital, and at the end of 9 months *C* withdraws $\frac{1}{4}$ of his capital. Divide $3950, the year's profit, equitably among them.

115. Commission. A merchant, however, may have no capital with which to conduct a business for himself. In such a case another person, called a *Principal*, furnishes the money with which to buy goods, and receives the money for which the goods are sold. Such a merchant is called a *Commission Merchant*, or *Agent*, and the money he gets for his work in buying and selling is called his *Commission*. Of the money he receives from his Principal to buy with, it is the custom for him to take a part for his commission and to pay the rest for the goods; and of the money he receives for goods sold, to take a part for his commission and to send the rest to his Principal. Further, as in other cases, this commission is derived from the money the Agent pays for goods bought, or from the money the Agent receives for goods sold, by means of a rate previously agreed upon. Thus, if this rate be 3 per cent., the commission for buying goods

$$= \text{the money the agent paid for the goods} \times \frac{3}{100},$$

and the commission for selling goods

$$= \text{the money the agent received for the goods} \times \frac{3}{100}.$$

116. Examples solved.

(1) An agent received \$430 from his principal to invest, on a commission at the rate of 2 per cent. Find his commission.

Solution.—

The agent's commission $= \text{the cost of the goods} \times \frac{1}{50}$,

therefore the money he received, which includes

the commission and the cost $= \text{the cost of the goods} \times \frac{51}{50}$;

that is, the cost of the goods $\times \frac{51}{50} = \text{a } \$ \times 430$.

Therefore the cost of the goods $= \text{a } \$ \times 430 \times \frac{50}{51}$,

$$\text{Therefore the commission} = \text{a } \$ \times 430 \times \frac{50}{51} \times \frac{1}{50},$$

$$= \text{a } \$ \times 430 \times \frac{1}{51} = \$8{\cdot}43.$$

(2) An agent's rates are 2 per cent. for buying and 3 per cent. for selling. Upon advice from his principal he sold 300 barrels of apples at \$1.75 per barrel, and with the proceeds, less his charges, bought wheat at 65 cents per bushel. Find the quantity of wheat purchased.

Solution.—The apples sold for a \$ × 1·75 × 300; that is, for a \$ × 525.

$$\text{The agent's charge for selling} = \text{a } \$ \times 525 \times {\cdot}03,$$

$$= \text{a } \$ \times 15{\cdot}75.$$

Therefore the net proceeds of the sale = a \$ × 509·25.

$$\text{Again, his charge for buying the wheat} = \text{its cost} \times \frac{1}{50}.$$

This commission, together with the cost of the wheat,

$$= \text{the cost of the wheat} \times \frac{51}{50};$$

$$\text{so that the cost of the wheat} \times \frac{51}{50} = \text{a } \$ \times 509{\cdot}25.$$

$$\text{Therefore the cost of the wheat} = \text{a } \$ \times 509{\cdot}25 \times \frac{50}{51}.$$

$$\text{But the cost of a bushel} = \text{a } \$ \times \frac{65}{100}.$$

Therefore the cost of the wheat (Article 33)

$$= \text{the cost of a bushel} \times \left\{509{\cdot}25 \times \frac{50}{51} \div \frac{65}{100}\right\}.$$

Therefore the amount of wheat (Article 77)

$$= \text{a bushel of wheat} \times \left\{509{\cdot}25 \times \frac{50}{51} \div \frac{65}{100}\right\}.$$

$$= \text{a bushel of wheat} \times \frac{509{\cdot}25 \times 50 \times 100}{51 \times 65},$$

= a bushel of wheat × $768\frac{22}{221}$,

= 768 bushels and 6 lbs. nearly.

EXERCISE XXIX.

1. Calculate the commission in the following cases:

(*a*) For buying 4369 bushels of wheat at $63\frac{1}{2}$ cents per bushel, at $2\frac{1}{2}$ per cent.

(*b*) For selling 537 barrels of flour at \$2·$12\frac{1}{2}$ per cwt., at $3\frac{1}{8}$ per cent.

(*c*) For buying 2849 pounds of pork at \$5·85 per cwt., at $1\frac{7}{8}$ per cent.

(*d*) For selling 18 tons 620 pounds of hay at \$12·64 per ton, at $3\frac{1}{4}$ per cent.

2. I received from my principal \$4030 to invest in tea at $22\frac{1}{2}$ cents per pound, on a commission of $\frac{3}{4}$ per cent. Find the quantity of tea bought.

3. I sold on commission 25 tons of coal at \$6·80 per ton; and with the proceeds, less my charges at 3 per cent., bought wood at \$5·10 per cord. Find my total commission, and the quantity of wood bought.

4. An agent sold 28 head of fat cattle, whose average weight was 1262 pounds, at \$2·90 per cwt., on commission at the rate of $1\frac{1}{2}$ per cent. What sum did he return to his principal?

5. If an agent's commission for buying tea at 20 cents per pound was \$25·30, his rate being $1\frac{1}{4}$ per cent., find what the tea cost the principal, and how much tea he bought.

6. I sent my agent \$2900, with instructions to purchase cotton for me on commission, at the rate of $1\frac{1}{3}$ per cent. Find his commission.

7. My charge for buying or selling is at the rate of 5 per cent. I sold some cloth, and with the proceeds, less my charges, amounting to $300, I bought cotton. How much did I sell the cloth for?

117. Stocks and Shares.—Again, the capital required to carry on a business may be furnished by a great number of men, who thus form a *Company*. These men elect a few of themselves, whose work it is to direct the general affairs of the business, and who are called the *Directors*, or the *Directorate*. They also elect a man to manage the business in detail, who is called the *Manager*. The names of all the men who have contributed capital are entered in a book, with the sums they have contributed. These men are called *Stockholders*. At the time any one (say *A. B.*) of these stockholders paid in his money he was given a paper, which reads in effect as follows:

MONTREAL, *June 8th, 1897*.

This is to certify that Arthur Backus has standing in his name $5000 stock in the Dominion Express Company.

(Signed) P. Q., *Manager*.

This is a Stock Certificate.

118. How Stock is Bought and Sold.—After a while, *A. B.* may wish to sell his stock to *C. D.* for money. To do this *A. B.* and *C. D.* employ a Broker, whose business is to buy and sell stocks, and who charges each man the same amount of money, which is called brokerage, for effecting the transaction. The manner of selling is as follows: *A. B.* gives the Broker his stock certificate, who, in turn, gives it to the Manager of the business. The Manager then removes *A. B.'s* name from the books, and, in its place, puts *C. D.'s* name; he destroys *A. B.'s* certificate, and issues a new one in *C. D.'s* name, which he gives to the Broker. The Broker then hands this certificate to *C. D.*, who pays him the price agreed upon; and finally, he takes out his own charges, and gives the balance of the money to *A. B.*

119. Shares.—The stock which a man has standing in his name is usually divided into *Shares* of \$100 each, and this \$100 is called the *par value* of a share.

(*a*) Now, the price agreed upon between *A. B.* and *C. D.* is derived from the par value of the stock by means of a rate. Thus, if *A. B.* sold his stock to *C. D.* at 87, we mean that

$$\text{the price of the stock} = \text{its par value} \times \frac{87}{100}.$$

(*b*) Also the money (brokerage) the broker charges *A. B.* or *C. D.* is derived from the par value of the stock by means of a rate. Thus, if the broker's rate is $\frac{1}{4}$ per cent.,
the brokerage the buyer pays

$$= \text{the par value of the stock} \times \frac{1}{400},$$

and also, the brokerage the seller pays

$$= \text{the par value of the stock} \times \frac{1}{400},$$

(*c*) Further, the profits of the business for a year or 6 months are distributed among the stockholders in such a way that each man's share of the profits is derived from the par value of his stock by means of a rate. The whole distributed profits is called the *Dividend* for a year or half-year; but a stockholder would speak of his share of it as income, rent or interest. Thus, if a company declares for distribution a dividend at the rate of 6 per cent., a stockholder understands that

$$\text{his income from his stock} = \text{the par value of his stock} \times \frac{6}{100}.$$

(*d*) If a man purchased stock at the rate of 90 (per cent.), which declared an annual dividend at the rate of $7\frac{1}{2}$ per cent.,

$$\text{his interest} = \text{his investment} \times \frac{7\frac{1}{2}}{90},$$

which makes his rate of interest $8\frac{1}{3}$ per cent.

120. Therefore, if we say, "I bought \$5000 six per cent. stock at 83, brokerage $\frac{1}{8}$," we mean that

I bought 50 shares of stock ;
I paid \$83 for each share ;
I paid \$$\frac{1}{8}$, in addition, to the broker ;
my income from each share is \$6 ;

and my rate of interest $= \dfrac{6}{83\frac{1}{8}}$.

121. Examples solved.

(1) I bought \$5000 stock in the 6 per cents. at 83, the brokerage being $\frac{1}{8}$ per cent. Find my money invested, my annual income, the brokerage and the rate of interest.

1st Solution.—

My stock = one share of stock × 50.

Therefore my investment

$= \text{the price of a share} \times 50,$ Article 77.

$= \text{a } \$ \times 83\frac{1}{8} \times 50 = \$4156{\cdot}25.$

Therefore my annual income

$= \text{the income from a share} \times 50,$ Article 77.

$= \text{a } \$ \times 6 \times 50 = \$300.$

Therefore the brokerage

$= \text{the brokerage for a share} \times 50,$ Article 77.

$= \text{a } \$ \times \frac{1}{8} \times 50 = \$6{\cdot}25.$

Therefore the rate of interest $= \dfrac{6}{83\frac{1}{8}} = 6\dfrac{132}{133}$ per cent.

2nd Solution.—

My investment $= \$5000 \times \dfrac{83\frac{1}{8}}{100},$ Article 119 (*a*).

$= \$4156{\cdot}25,$ on reducing.

My income $= \$5000 \times \dfrac{6}{100} = \$300.$ Article 119 (*c*).

The brokerage $= \$5000 \times \dfrac{1}{800} = \$6{\cdot}25.$ Article 119 (*b*).

The rate of interest $= \dfrac{6}{83\frac{1}{8}} = 6\dfrac{132}{133}$ per cent.

(2) I invested \$25000 in Consols at $82\frac{3}{8}$, paying 3 per cent., brokerage at $\frac{1}{8}$. Find my income.

Solution.—The total cost of a share

$$= \text{a } \$ \times (82\tfrac{3}{8} + \tfrac{1}{8}) = \text{a } \$ \times 82\tfrac{1}{2}.$$

The income from a share $= \text{a } \$ \times 3.$

Therefore my rate of interest $= \dfrac{3}{82\frac{1}{2}}.$ Article 119 (*d*).

Therefore my income $= \$25000 \times \dfrac{3}{82\frac{1}{2}}.$ Article 119 (*c*).

$$= \$909{\cdot}09, \text{ on reducing.}$$

(3) I transferred \$8000 stock in the 6 per cents. at 113, to 5 per cent. stock at 102, employing a broker, whose rate is $\frac{1}{2}$ per cent. Find the change in my income.

Solution.—My income from the 6 per cent. stock

$$= \$8000 \times \frac{6}{100} = \$480.$$

I sold this 6 per cent. stock, and with the money I bought the 5 per cent. stock.

The money I received for the 6 per cent. stock

$$= \$(113 - \tfrac{1}{2}) \times 80 = \$9000.$$

With this money I bought $\dfrac{9000}{102\frac{1}{2}}$ shares of the 5 per cent. stock,

from which my income $= \$5 \times \dfrac{9000}{102\frac{1}{2}} = \$439\dfrac{1}{41}.$

Therefore my income is decreased by $\$40\frac{40}{41}$.

EXERCISE XXX.

1. A man purchased $4500 Bank Stock at 165, which pays 8 per cent. dividends. Find its cost and his income from it.

2. A man invested $4020 in Railway Stock at $80\frac{2}{5}$. How much stock did he buy?

3. I sold $5600 Telegraph Stock at 93, paying the broker $\frac{1}{4}$ per cent. Find the money I received.

4. A man invested $4325 in stock at 90, paying $4\frac{1}{2}$ per cent. dividends. Find his income.

5. My income of $1200 is obtained from an investment in 8 per cent. stock when it was at 160. Find my capital and the stock I purchased.

6. If $550 stock in the 6 per cents. sell for $558·25, what is the price of a share?

7. Which is the better investment, buying 9 per cent. stock at 125 or 6 per cent. stock at 75?

8. I sold $6400 stock in the 6 per cents. at $83\frac{1}{3}$, and purchased 7 per cent. stock at 128. Find the decrease of my income.

9. Find the alteration in my income occasioned by transferring $3200 stock in the 3 per cents. at $86\frac{3}{8}$ to 4 per cent. stock at $114\frac{7}{8}$, the brokerage being $\frac{1}{8}$.

10. By investing in 6 per cent. stock I make 5 per cent. interest. Find the price of the stock.

11. I have $19200 to invest. I can buy 3 per cent. Consols at $93\frac{3}{8}$, or 4 per cent. Bonds at $113\frac{7}{8}$, the brokerage being $\frac{1}{4}$ per cent. Which investment gives the better income, and by how much better is it?

12. A man sold out $12500 stock in the 5 per cents. at 90. He invested 40 per cent. of the proceeds in 6 per cent. stock at $112\frac{1}{2}$, and the rest in 4 per cent. stock at 75. Find the change in his income.

which pays 8
from it.
t $80\frac{2}{5}$. How

the broker $\frac{1}{4}$

g $4\frac{1}{2}$ per cent.

vestment in 8
ipital and the

58·25, what is

per cent. stock

and purchased
income.
d by transfer-
per cent. stock

e 5 per cent.

r cent. Consols
ge being $\frac{1}{4}$ per
, and by how

r cents. at 90.
cent. stock at
nd the change

---

13. The whole capital stock of a company is \$50000. The profits for a year were \$7528·30, How much of this can they reserve after declaring a dividend of $12\frac{1}{2}$ per cent.?

14. The capital stock of a company is \$60000. The whole dividend for a year was \$4500. How much of this should a stockholder receive who holds \$5300 stock?

15. A man increased his income \$10 by transferring his 5 per cent. stock at 90 to 6 per cent. stock at 96. Find the capital he has invested.

16. A person sells \$1200 stock in the 5 per cents. at 96, and invests the money he gets in 8 per cent. stock, without changing his income. Find the price of the 8 per cent. stock.

122. Duties.—When a merchant buys goods in a foreign country, he receives from the producer a paper, on which is written an accurate description of these goods and their prices. This paper is called an Invoice of the Goods. On the arrival of the goods in the place where the merchant conducts his business, they are taken possession of by an officer of the government, until the merchant pays him a sum of money which is called the *Duty* on the goods. Generally the amount of this duty is derived from the invoice price of the goods by means of a rate fixed by the government. This duty, when found in this way, is called an *ad valorem* duty. Thus, if the duty is at the rate of $22\frac{1}{2}$ per cent. for a certain article, we mean that

$$\text{the duty paid} = \text{the invoice price of the article} \times \frac{9}{40}.$$

Sometimes, however, the duty is derived in a different way, as, for instance, "$2\frac{1}{4}$ cents for a pound weight of the goods." It is then called *specific* duty.

123. Example solved.—A merchant in Toronto imported from Manchester 1260 yards of cloth invoiced at 8*s* 3*d*. a yard. Find the duty at $18\frac{1}{2}$ per cent.

Solution.—The price of a yard $= a\ s. \times 8\frac{1}{4} = a\ £ \times \frac{33}{80}$,

$$= a\ \$ \times \frac{73}{15} \times \frac{33}{80}. \quad \text{Article 83 (6).}$$

Therefore the price of the cloth $= a\ \$ \times \frac{73}{15} \times \frac{33}{80} \times 1260$.

Therefore the duty paid $= a\ \$ \times \frac{73}{15} \times \frac{33}{80} \times 1260 \times \frac{18\frac{1}{2}}{100}$.

$= \$467 \cdot 95$, on reducing.

EXERCISE XXXI.

1. Calculate the duty on the following invoice of goods at $22\frac{1}{2}$ per cent.:

5 pcs. Linen, 120 yds. each, at $35\frac{1}{2}$ cents per yd.
40 pcs. Cotton, 110 yds. each, at $4\frac{3}{8}$ cents per yd.
1600 Handkerchiefs, at $7\frac{7}{8}$ cents each.
25 pcs. Lace, 75 yds. each, at $9\frac{1}{5}$ cents per yd.

2. An importer at Toronto received the following invoice of goods, shipped from Liverpool: 250 yds. Tweed, at 2*s.* 3*d.* per yd.; 400 yds. Worsted, at 3*s.* $2\frac{1}{2}$*d.* per yd.; 120 yds. Blue Serge, at 2*s.* 9*d.* per yd.; 300 doz. Buttons, at 1*s.* 3*d.* per doz. Find the duty at $18\frac{3}{4}$ per cent.

3. A merchant imported some goods at an invoice cost of \$5231·25. He paid duty at $27\frac{1}{2}$ per cent. He sold them at a gain of $16\frac{2}{3}$ per cent. Find what they cost the consumer.

4. A merchant imported a certain article, on which he paid a duty of $22\frac{1}{2}$ per cent. He marked upon it a price $37\frac{1}{2}$ per cent. above the total cost of it, and finally sold it at a discount of 10 per cent., gaining, however, \$45·57. Find its invoice cost.

5. If the duty at $33\frac{1}{3}$ per cent. on half a bill of goods, and at 20 per cent. on the other half, amounts to \$22·40, find the invoice price of the goods.

124. Insurance.—A company may undertake the business of insuring property against loss by fire. The business transaction they are a party to is as follows:

The company, in return for a certain sum of money paid to them by the owner of property, gives the owner a paper on which is written their promise to pay the owner a sum of money in case the property is destroyed by fire within a certain time. This paper is called a *Policy of Insurance*. The sum the company promises to pay is called the *Face of the Policy*, the *Amount of Insurance*, or the *Risk*. So long as the policy remains in force the company is said to carry the risk. The sum of money the owners pay the company is called the *Premium of Insurance*, and it is derived from the face of the policy by means of a rate. Thus, if a company's rate for a certain class of buildings is $\frac{4}{5}$ per cent., we mean that

$$\text{the premium a man pays for insuring} = \text{the risk} \times \frac{4}{500}.$$

125. Examples solved.

(1) A man had his house insured for $\frac{3}{4}$ of its value, paying a premium of \$12·60, which was at the rate of $\frac{4}{5}$ per cent. Find the value of his house.

Solution.—Here the risk = the value of the house $\times \dfrac{3}{4}$.

$$\text{The premium he pays} = \text{this value} \times \frac{3}{4} \times \frac{4}{500} = \text{a } \$ \times 12{\cdot}60.$$

$$\text{Therefore the value of the house} = \text{a } \$ \times 12{\cdot}60 \times \frac{500}{3} = \$2100.$$

(2) For what must I have my house, which I value at \$1500, insured at $1\frac{1}{4}$ per cent., so that, in case it is burned, I may recover my premium and $\frac{3}{4}$ of its value?

Solution.—The premium I pay = the risk $\times \dfrac{1}{80}$, and the condition of the question is that the risk = the value $\times \frac{3}{4}$ + the premium;

that is, the risk = $\$1500 \times \frac{3}{4}$ + the risk $\times \frac{1}{80}$.

Therefore the risk $\times \frac{79}{80} = \$1500 \times \frac{3}{4}$.

Therefore the risk = a $\$ \times \frac{1500 \times 3 \times 80}{4 \times 79} = \$1139.24$;

which is what I must have my house insured for.

EXERCISE XXXII.

1. Calculate the premium to be paid in the following cases:
   (*a*) At $\frac{5}{8}$ per cent., for a risk of $2600.
   (*b*) At $\frac{7}{8}$ per cent., for a risk of $11100.
   (*c*) At $1\frac{1}{8}$ per cent., for a risk of $6250.

2. My property was insured for $1800 at a premium of $\frac{7}{8}$ per cent. It was destroyed. Find what I saved by insuring.

3. A company took a risk of $16000 on a building at $\frac{2}{3}$ per cent. The building was destroyed. Find the loss to the company in taking the risk.

4. I insured my house for $\frac{2}{3}$ of its value, paying a premium of $16, which was at the rate of $1\frac{1}{4}$ per cent. Find the value of my house.

5. Find what I must insure my house for, which is worth $5070, at $\frac{5}{8}$ per cent., so that I may recover, in case of loss, both the value of the house and the premium.

6. My store is worth $\frac{2}{3}$ as much as my stock. The store is insured for $\frac{2}{3}$ of its value, at $\frac{4}{5}$ per cent.; and the stock for $\frac{9}{10}$ of its value, at $\frac{3}{4}$ per cent. The total premium I paid was $37.10. Find the value of my store and stock.

7. I insured my store so as to cover $\frac{2}{3}$ of its value, and the

premium at $\frac{4}{5}$. What rate will derive the face of the policy from the value?

8. A company took a risk of \$50000 upon a vessel at $1\frac{1}{2}$ per cent., but afterwards placed $\frac{2}{5}$ of this risk with another company, paying them a premium of $1\frac{1}{4}$ per cent. Find the net sum each company would lose in case the vessel were lost.

9. A company took a risk upon some property worth \$1500 for $\frac{7}{8}$ of its value, charging a premium of \$14.40. Find their rate of premium.

10. A cattle dealer purchased 120 head of fat cattle at an average cost of \$32.25. He shipped them to Liverpool at an additional cost of \$10.30 per head. He had them insured for their original cost, at $1\frac{1}{2}$ per cent. He sold them in Liverpool at £11 6*s.* 3*d.* per head, and had the money changed into Canadian money at the rate of \$4.85 for a £. Find his gain.

126. Taxes.—The Council of a city, a town, a county, or a township having to keep the roads, streets and bridges in repair, and to perform various other works required by the people, collect the money they need for these purposes from the people. This money is called *Taxes*. The manner of collecting is as follows:

The Council appoints a man called an *Assessor*, whose work is to determine the value of each man's property and to set it down in a book, and also to set down in the book that part of a man's income which is not exempted by law. This amount of money is then called the man's *Assessment*, or the assessed value of his property or income. The Council also fixes a *Rate of Taxation* by which the taxes a man must pay is derived from his assessment. Thus, if the rate of taxation is fixed at 21 mills to the dollar, the tax a property owner would pay

$$= \text{the assessed value of his property} \times \frac{21}{1000}.$$

127. Example solved.—A man's net income was \$1484·80 after he paid an income tax of 19 mills to the dollar. If \$700 of his income was not subject to assessment, find his total income.

Solution.—The tax he paid $=$ his assessed income $\times \frac{19}{1000}$.

Therefore his net income $= \$700 +$ his assessed income $\times \frac{981}{1000}$.

Therefore his assessed income $\times \frac{981}{1000} =$ a \$ $\times$ 784·80.

Therefore his assessed income $=$ a \$ $\times 784{\cdot}80 \times \frac{1000}{981} = \$800$.

Therefore his total income $= \$1500$.

EXERCISE XXXIII.

1. The whole assessed value of a town is \$960000, and the money needed during a certain year is \$25600. Find to the nearest mill on the dollar the rate of taxation. How much more money will be collected than is needed at this rate?

2. A man's taxes were \$48·10 when the rate of taxation was $18\frac{1}{2}$ mills on the dollar. Find his assessment.

3. In a certain town \$3093·75 was raised from a tax at 15 mills to the dollar. What was the assessed value of the property in the town?

4. The assessed value of the property of a town is \$1493250. What sum of money can be raised by a rate of $20\frac{1}{2}$ mills to the dollar, after paying the collector of the taxes 2 per cent. of them?

5. The assessed value of a town is \$2503400. During the year the Council estimates the expenditure as follows: interest on the debt, \$9575; the Board of Works, \$10930; the Public Schools, \$8570; the High School, \$6825; miscellaneous expenditure, \$3295. If it costs 2 per cent. of the taxes for collecting,

find the least rate, in exact mills on the dollar, which will be necessary to raise the money required.

6. If a man's income up to \$700 is exempt from taxation, and his net income, after paying a tax at the rate of $1\frac{1}{3}$ per cent., is \$1240·10, find his tax.

7. A school section, whose property is assessed for \$180000, built a school-house costing \$3600, which they pay for in four equal annual payments. If the rent of money is not taken into account, find what is the annual rate of taxation.

8. *A* owes *B* \$400 due in 1 year, \$300 due in 2 years, and \$200 due in 3 years. What sum paid now will discharge these debts, if money is worth 5 per cent. compound interest?

9. A four months' note for \$2450 is dated Kingston, June 1st, 1898, and bears interest at 6 per cent. It was sold on August 15th at 8 per cent. discount. Find the proceeds.

10. How much money must be invested in buying stock at $97\frac{1}{2}$ which pays 6 per cent. dividends, so as to produce an income of \$600 per annum?

11. Distinguish interest and discount, showing clearly how each is found. In what respect have these words the same meaning?

12. Describe the formation of a Joint Stock Company, explaining the terms manager, stockholders, stock, stock certificate and dividend. How is stock bought and sold?

13. State definitely in what cases numbers are used as rates in business affairs.

14. What is a business transaction? Is it the same as an agreement?

15. What is capital? Why is it necessary for a merchant to have capital?

CHAPTER XIV.

SQUARE AND CUBE ROOTS.

I. SQUARE ROOT.

128. When a single rate is the product of two equal rates, the single rate is called the *square* of either of the equal rates, and each of the equal rates is called the *square root* of their product.

Thus, since $64 = 8 \times 8$, 64 is the square of 8, and may be written 8^2; while 8 is the square root of 64, and may be written $\sqrt{64}$.

Again, since $\frac{9}{25} = \frac{3}{5} \times \frac{3}{5}$, $\frac{9}{25}$ is the square of $\frac{3}{5}$, and may be written $\left(\frac{3}{5}\right)^2$; while $\frac{3}{5}$ is the square root of $\frac{9}{25}$, and may be written $\sqrt{\frac{9}{25}}$. So also, $\cdot 0081 = (\cdot 09)^2$, and $\cdot 09 = \sqrt{\cdot 0081}$.

129. Rule of Order in Square Root.—Since the order of the product of two simple numbers is the sum of the orders of the simple numbers, therefore the order of the square of a simple number is twice the order of the simple number; that is, the order of the square of a simple number is even. See Article 48.

Thus, since the order of 6 in ·006 is -3, therefore the order of 36 in $(\cdot 006)^2$ is -6; so that $(\cdot 006)^2 = \cdot 000036$.

Again, since the order of 4 in 400 is $+2$, therefore the order of 16 in $(400)^2$ is $+4$; so that $(400)^2 = 160000$.

Now, from the Multiplication Table, the student knows that 1, 4, 9, 16, 25, 36, 49, 64 and 81 are the squares of 1, 2, 3, 4, 5, 6, 7, 8 and 9, respectively. Hence, if any one of these square numbers be written in an even order, the order of its square root is half this order.

Thus, since the order of 49 in ·0049 is -4, therefore the order of 7 in $\sqrt{\cdot 0049}$ is -2; so that $\sqrt{\cdot 0049} = \cdot 07$.

Again, since the order of 81 in 81000000 is $+6$, therefore the order of 9 in $\sqrt{81000000}$ is $+3$; so that $\sqrt{81000000} = 9000$.

130. To Find the Square Root of any Number.—In order to find the square root of a compound number other than those in Article 129, we shall consider the following manner of squaring a compound number.

Since $47 = 4$ in the order $+1$ and 7 in the order $0 = 40 + 7$; we may multiply thus:

$$\begin{array}{r} 40+7 \\ 40+7 \\ \hline 40\times 7+7\times 7 \\ 40\times 40+40\times 7 \\ \hline \end{array}$$

$$\begin{aligned} \text{Then } 47^2 &= 40\times 40+40\times 7\times 2+7\times 7, \text{ on adding} \ldots . & (a) \\ &= 1600+(40\times 2+7)\times 7 & (b) \\ &= 1600+560+49, \text{ from } (a) & (c) \\ &= 2209. \end{aligned}$$

If now, in the number 2209 we mark the digits whose orders are even, thus, $2\overset{|}{2}0\overset{|}{9}$, we find that 22 is the first part of the number whose order is even, and that this order is $+2$. We also see that 22 is more than the square number 16 and less than the square number 25. Therefore the first part of the square root of 2209 is 4 in order $+1$, that is, 40.

Further, if $(40)^2$, that is, 1600, be subtracted from 2209, the remainder is 609; but, from line (a) above, we see that this remainder is also $40\times 7\times 2+7\times 7$, the major part of which is

$40 \times 7 \times 2$, as we can see in line (*c*). Therefore, when we divide $60\overset{|}{9}$ by 40×2, that is, by 4×2 in order $+1$, we obtain 7 in order 0, which is probably the next figure of the square root. This 4×2 is called the Trial Divisor, which, we see, is double the part of the square root already found.

Finally, from line (*b*) we see that the complete divisor is $40 \times 2 + 7$, that is, 87. When this is multiplied by 7, the product is 609, thus verifying that 7 is the next figure of the root.

In practice, the operations described above are conveniently arranged thus:

$$\begin{array}{r|l} & 2\overset{|}{2}0\overset{|}{9}\ (\ 47 \\ & 16 \\ 87 & \ \ 609 \\ & \ \ 609 \\ \cline{2-2} \end{array}$$

131. If, however, the compound number be not a complete square, we shall proceed in the same way to find its square root. Thus, to find the square root of ·05742.

We mark the digits whose orders are even, thus, $\cdot 0\overset{|}{5}7\overset{|}{4}2\overset{|}{0}$. The first part of the number whose order is even is 5, in the order -2. Therefore, the 1st figure of the square root is 2, in the order -1, that is ·2. The first trial divisor by which the 2nd figure of the root is obtained is 2×2, in the order -1, and so on. The operations are as follows:

$$\begin{array}{r|r} & \cdot 0\overset{|}{5}7\overset{|}{4}2\overset{|}{0}\ (\ \cdot 2396 \\ & 4 \\ \cline{2-2} 43 & 174 \\ & 129 \\ \cline{2-2} 469 & 4520 \\ & 4221 \\ \cline{2-2} 4786 & 29900 \\ & 28716 \\ \cline{2-2} & 1184 \end{array}$$

Here 46 is the 2nd trial divisor, by which 9 is obtained; and 478 is the 3rd trial divisor, by which 6 is obtained.

It would seem that, when the 1st trial divisor, 4, is divided into 17, the quotient would be 4. But, on trial, the product of the complete divisor 44 and 4, namely, 176, is seen to be greater than 174. It is rarely necessary to find a square root to more than 6 figures, except when a very large quantity is to be derived from a very small one by using this square root as a rate.

EXERCISE XXXIV.

1. Write down the squares of the following numbers, giving the reasons for the orders of the digits in the same:

(*a*) 40.
(*b*) 700.
(*c*) 8000.
(*d*) 30000.
(*e*) 120000.
(*f*) ·4.
(*g*) ·007.
(*h*) ·08.
(*i*) ·00003.
(*j*) ·001.
(*k*) 200.
(*l*) ·002,
(*m*) ·1.
(*n*) ·9.
(*o*) ·0000012.

2. Write down the square roots of the following numbers, giving the reasons for the orders of the digits in the roots:

(*a*) 64.
(*b*) 6400.
(*c*) 810000.
(*d*) 900.
(*e*) 121000000.
(*f*) ·09.
(*g*) ·0064.
(*h*) 000004.
(*i*) ·0001.
(*j*) ·000121.
(*k*) 3600.
(*l*) ·36.
(*m*) ·000025.
(*n*) ·04.
(*o*) ·009.

3. Extract the square roots of the following numbers:

(*a*) 576.
(*b*) 3125.
(*c*) 15625.
(*d*) 815409.
(*e*) 687241.
(*f*) 1522756.
(*g*) 72900.
(*h*) ·09732.
(*i*) ·00004.
(*j*) ·1.
(*k*) 231·29.
(*l*) ·2.
(*m*) 2.
(*n*) ·00001.
(*o*) ·025.

4. Find the square roots of:

(*a*) $\frac{4}{9}$. (*b*) $\frac{25}{64}$. (*c*) $\frac{81}{121}$. (*d*) $\frac{64}{49}$. (*e*) $5\frac{1}{16}$.

5. Reduce the following fractional rates to decimal rates, and thence find their square roots to 5 figures:

(a) $\frac{1}{25}$. (e) $19\frac{3}{4}$. (i) $\frac{2496}{38247}$.

(b) $\frac{8}{13}$. (f) $\frac{21}{19}$. (j) $\frac{\cdot 173}{2\cdot 5}$.

(c) $\frac{1}{2}$. (g) $\frac{100}{99}$. (k) $\frac{\cdot 0072}{\cdot 032}$.

(d) $\frac{1}{125}$. (h) $\frac{2}{3}$. (l) $\frac{1}{\cdot 0625}$.

6. An article was marked to sell for $160; but, on the price being reduced by two equal successive rates, it sold for $122.50. Find these rates.

7. In marking goods for sale, a merchant increased the cost price by two equal rates, and so gained at the rate of 44 per cent. Find these rates.

8. A book contains 123201 words. It has as many pages as there are words on each page. Find how many pages in the book.

II. CUBE ROOT.

132. When a single rate is the product of three equal rates, the single rate is called the *cube* of any one of the equal rates: and any one of the equal rates is called the *cube root* of their product.

Thus, since $\frac{64}{125} = \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$, $\frac{64}{125}$ is the cube of $\frac{4}{5}$, and may be written $\left(\frac{4}{5}\right)^3$; while $\frac{4}{5}$ is the cube root of $\frac{64}{125}$, and may be written $\sqrt[3]{\frac{64}{125}}$. So, also, for decimal rates.

133. Rule of Order in Cube Root.—Since the order of the product of three simple numbers is the sum of the orders of the simple numbers, therefore, the order of the cube of a simple number is three times the order of the simple number; that is, the order of the cube of a simple number is a multiple of 3.

Thus, since the order of 4 in 400 is $+2$, therefore the order of 64 in $(400)^3$ is $+6$; so that $(400)^3 = 64000000$.

Again, since the order of 2 in $\cdot 2$ is -1, therefore the order of 8 in $(\cdot 2)^3$ is -3; so that $(\cdot 2)^3 = \cdot 008$.

Now, since 1, 8, 27, 64, 125, 216, 343, 512 and 729 are the cubes of 1, 2, 3, 4, 5, 6, 7, 8 and 9, respectively, if any one of these cube numbers be written in an order which is a multiple of 3, its cube root will be written in an order which is one-third of that order.

Thus, since the order of 125 in $\cdot 000125$ is -6, therefore the order of 5 in $\sqrt[3]{\cdot 000125}$ is -2; so that $\sqrt[3]{\cdot 000125} = \cdot 05$.

Again, since the order of 8 in 8000 is $+3$, therefore the order of 2 in $\sqrt[3]{8000}$ is $+1$; so that $\sqrt[3]{8000} = 20$.

134. To Find the Cube Root of any Number.—In order to find the cube root of a compound number other than those in Article 133, we shall consider the following manner of cubing a compound number.

$$\begin{aligned}
\text{Since}\quad (57)^2 &= (50+7) \times (50+7) \\
&= (50)^2 \quad + 50 \times 7 \times 2 \quad + 7^2. \qquad \text{Article 130 } (a). \\
\text{Therefore } (57)^3 &= \{(50)^2 \quad + 50 \times 7 \times 2 \quad + 7^2\} \times (50+7) \\
&= (50)^3 \quad + (50)^2 \times 7 \times 2 \quad + 50 \times 7^2, \text{ on multiplying by } 50; \\
&\qquad\quad + (50)^2 \times 7 \quad + 50 \times 7^2 \times 2 \quad + 7^3, \text{ on mult. by } 7; \\
&= (50)^3 \quad + (50)^2 \times 7 \times 3 \quad + 50 \times 7^2 \times 3 \quad + 7^3, \text{ on adding}; \quad (a) \\
&= 125000 \quad + \{(50)^2 \times 3 \quad + 50 \times 7 \times 3 \quad + 7^2\} \times 7, \ldots\ldots \quad (b) \\
&= 125000 \quad + (5^2 \times 300 \quad + 5 \times 7 \times 30 \quad + 7^2) \times 7. \qquad (c)
\end{aligned}$$

But $(57)^3 = 185193$ by ordinary multiplication.

If, now, in 185193 we mark the digits whose orders are multiples of 3, thus, $18\overset{|}{5}19\overset{|}{3}$, we find that 185 is the first part of the number whose order is a multiple of 3, and that this order is $+3$. We also see that 185 is more than the cube number 125 and less than the cube number 216. Therefore, the first part of the cube root of the number is 5 in the order $+1$; that is, 50.

Further, if $(50)^3$, that is, 125000, be subtracted from $18\overset{|}{5}19\overset{|}{3}$, the remainder is $6019\overset{|}{3}$. But, from line (*a*) above, we see that this remainder is also $(50)^2 \times 7 \times 3 + 50 \times 7^2 \times 3 + 7^3$, the major part of which is $(50)^2 \times 7 \times 3$ or $5^2 \times 7 \times 300$. Therefore, when $6019\overset{|}{3}$ is divided by $5^2 \times 300$ the quotient is 7, which is *probably* the next figure of the cube root. This $5^2 \times 300$ is called the *trial divisor*, which, we see, is always formed by multiplying the square of the part of the root already found by 300.

From line (*c*) we see that the complete divisor is $5^2 \times 300 + 5 \times 7 \times 30 + 7^2$, that is, 8599. When this is multiplied by 7 the product is 60193, thus verifying that 7 is the next figure of the root.

In practice, the operations described above are conveniently performed as follows:

$$\begin{array}{r|l}
 & 18\overset{|}{5}19\overset{|}{3} \quad (\ 57 \\
5^3 = & 125 \\
\hline
5^2 \times 300 \quad = 7500 & 60193 \\
5 \times 7 \times 30 = 1050 & \\
7^2 \quad = \quad 49 & 60193 \\
\hline
8599 &
\end{array}$$

135.—If, however, the number be not a complete cube, we proceed in the same way to find its cube root. Thus, to find the cube root of ·0756.

We first mark those digits in the number whose orders are multiples of 3, thus, $\cdot 07\overset{|}{5}60\overset{|}{0}$.

Therefore, the first part of the number whose order is a multiple of 3 is 75 in the order -3.

Therefore, the first digit in the cube root is 4 in the order -1, that is, ·4.

The first trial divisor by which the second digit of the root is found is $4^2 \times 300$, or 4800, and so on. The process is shown below:

			·075600 (·4228
		$4^3 =$	64
$4^2 \times 300$	=	4800	11600
$4 \times 2 \times 30$	=	240	
2^2	=	4	10088
1st complete divisor is		5044	1512000
$(42)^2 \times 300$	=	529200	
$42 \times 2 \times 30$	=	2520	
2^2	=	4	1063448
2nd complete divisor is		531724	448552000
$(422)^2 \times 300$	=	53425200	
$422 \times 8 \times 30$	=	101280	
8^2	=	64	428212352
3rd complete divisor is		53526544	20339648

136. To Obtain more Figures in the Root.—Since the 1st and 2nd complete divisors have their first digits the same, and 2nd and 3rd complete divisors have their first two digits the same, we may expect, then, that the 3rd and 4th complete divisors will have their first *three* digits the same. Therefore, using these three digits, 535, of the 3rd trial divisor as a divisor, and the last remainder as a dividend, we may expect to obtain at least two more digits in the cube root, thus:

$$\begin{array}{r} 535\ )\ 20339\ (\ 38 \\ \underline{1605} \\ 4289 \\ \underline{4280} \\ 9 \end{array}$$

Therefore the cube root of ·0756 is ·422838 nearly.

EXERCISE XXXV.

1. Write down the cubes of the following numbers, giving the reasons for the orders in which you write them:

(*a*) 20.
(*b*) 500.
(*c*) 7000.
(*d*) 90000.
(*e*) ·3.
(*f*) ·01.
(*g*) ·004.
(*h*) ·0005.
(*i*) ·1.
(*j*) ·008.
(*k*) 800.
(*l*) 70.
(*m*) 10000.
(*n*) ·0001.
(*o*) ·5.
(*p*) ·2.

2. Write down the cube roots of the following numbers, giving the reasons for the orders in which you write them:

(*a*) 8000000.
(*b*) 125000.
(*c*) 27000.
(*d*) 512000000.
(*e*) ·064.
(*f*) ·216.
(*g*) ·000008.
(*h*) ·000027.
(*i*) 729000.
(*j*) ·000729.
(*k*) ·125.
(*l*) 125.

3. Write down the cube roots of the following fractional rates:

(*a*) $\frac{64}{125}$.
(*b*) $\frac{729}{512}$.
(*c*) $\frac{125}{27}$.
(*d*) $\frac{8000}{729}$.
(*e*) $\frac{512}{343000}$.
(*f*) $\frac{512}{125000}$.
(*g*) $15\frac{5}{8}$.
(*h*) $18\frac{26}{27}$.
(*i*) ·000000729.

4. Find the cube roots of the following numbers:

(*a*) 91125.
(*b*) 32768.
(*c*) 14706125.
(*d*) 322828856.
(*e*) ·039304.
(*f*) ·001092727.
(*g*) 74·088.
(*h*) ·000004096.

5. Find to four figures the cube roots of:

(*a*) 90.
(*b*) ·08.
(*c*) ·8.
(*d*) ·0125.
(*e*) 7290.
(*f*) 12500.
(*g*) 64.
(*h*) 6·4.
(*i*) ·64.

6. Find to six figures the cube roots of:

(*a*) $\frac{5}{7}$.
(*b*) 3·1415926.
(*c*) 123·456.
(*d*) 2.
(*e*) 3.
(*f*) 4.
(*g*) 10.
(*h*) 100.
(*i*) ·1.

137. Examples in Square and Cube Roots.

(1) At what rate will \$500 amount at the end of 2 years to \$600, interest convertible yearly?

Solution.—The amount at the end of 2 years (Article 100)

$$= \$500 \times (\text{the rate of amount for a year})^2.$$

So that a \$ × 500 × (the rate of amount for a year)^2 = a \$ × 600.

Therefore (the rate of amount for a year)$^2 = \dfrac{600}{500} = 1{\cdot}2$.

Therefore the rate of amount for a year $= \sqrt{1{\cdot}2} = 1{\cdot}095445$.

Therefore the rate of interest $= {\cdot}095445 = 9{\cdot}5445$ per cent.

2. In 1861 the population of a town was 125000 individuals, in 1891 it was 216000 individuals. If the rate of increase of the population for each decade was the same, find what the population will be in 1901 at the same rate.

Solution.—The increase during the 1st decade (1861-1871)

$$= \text{a person} \times 125000 \times \text{that rate.}$$

Therefore the population in 1871

$$= \text{a person} \times 125000 \times (1 + \text{the rate}).$$

Similarly, the population in 1881

$$= \text{a person} \times 125000 \times (1 + \text{the rate})^2,$$

and the population in 1891

$$= \text{a person} \times 125000 \times (1 + \text{the rate})^3;$$

that is, a person × 125000 × (1 + the rate)^3 = a person × 216000.

Therefore (1 + the rate of increase)$^3 = \dfrac{216}{125}$.

Therefore (1 + the rate of increase) $= \sqrt[3]{\dfrac{216}{125}} = \dfrac{6}{5}$.

Therefore the population in 1901 will

$$= 216000 \times \frac{6}{5} \text{ individuals} = 259200 \text{ individuals.}$$

EXERCISE XXXVI.

1. At what rate compound interest will the interest for the use of \$800 for 2 years be \$90·42?

2. When interest is convertible yearly, at what annual rate of interest will \$7000 amount to \$12096 at the end of 3 years?

3. The population of *A* in 1871 was 136900, and its population in 1891 was 148225. If the rate of increase for each decade was the same, find the population in 1881.

4. The manufacturer and the merchant each made the same rate of profit; and an article which cost the manufacturer \$152 was sold by the merchant for \$201·02. Find what it cost the merchant.

5. What single rate of discount will give the same selling price as two successive rates of discounts, each 10 per cent.?

6. A merchant marked his goods at \$10, but reduced this price by two successive equal rates, and sold for \$6·40. Find these equal rates of reduction.

7. What annual rate of interest is equivalent to a quarterly rate of 2 per cent., convertible quarterly?

8. What half-yearly rate, convertible half-yearly, is equivalent to a yearly rate of 8 per cent.?

9. Show that $348 \times 347 \times 346 \times 345 + 1$ is a perfect square.

10. The population of a city in 1871 was 12,800, in 1891 it was 16,200, the rate of increase for the two decades being the same. At the same rate of increase what will be the population in 1901?

11. What rate of trade discount used three times in succession is equivalent to the single rate of 30 per cent.?

12. Find in rods the side of a square field whose surface is $22\frac{1}{2}$ acres.

CHAPTER XV.

MENSURATION—LENGTH, SURFACE AND VOLUME.

138. In Chapter VII. we showed how length is measured. If, now, we measure the width of a rectangular plot of ground by means of the standard unit, a yard, and find that we can say

$$\frac{\text{the width of the plot}}{\text{a yard}} = 2{\cdot}4\ ;$$

then we call 2·4 "the measure of the width of the plot."

Further, we may call 2·4 the rate which tells how we derive the width of the plot from a yard; so that we may say,

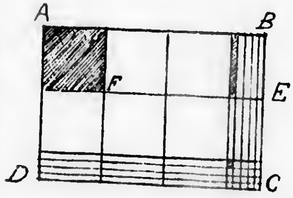
$$\text{the width of the plot} = \text{a yard} \times 2{\cdot}4,$$

which is read, "the width of the plot is derived from a yard by the rate 2·4."

Similarly, after measuring, we may say,

$$\text{the length of the plot} = \text{a yard} \times 3{\cdot}5.$$

139. To Measure the Surface of the Rectangle, whose sides were measured in Article 138.



To do this, we cut the surface, as in the diagram, so that each of the large squares, as *AF*, is the unit, a square yard; each of the narrow pieces at the left and at the bottom is a sub-unit of the 1st order; and each of the small squares at the lower right-hand corner is a sub-unit of the 2nd order. On counting these, we find that the surface consists of 6 units, 22 sub-units of the 1st order, and 20 sub-units of the 2nd order.

When this is expressed in another way, we may say,
the surface of the plot = a square yard × $(\dot{6} + \dot{2}2 + \dot{0}20)$,
= a square yard × 8·4.

This is a tedious process; and we shall show in the next Article that, if the width and the length be measured, as in Article 138, we may find what the measure of the surface is without actually measuring it, as we did above.

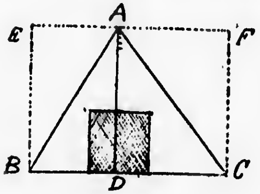
140. The Rule for the Surface of a Rectangle.

—On observing the diagram of the preceding article, we see that
the length AB = a yard × 3·5;
therefore the surface AE = a square yard × 3·5. Article 77.

Again, since the width BC = a yard × 2·4,
therefore the surface AC = the surface AE × 2·4. Article 77.
Therefore the surface of the rectangle
= a square yard × 3·5 × 2·4,
= a square yard × 8·4. Article 22.

So that the *Rate* by which the surface of a rectangle is derived from the unit of surface is the product of the measures of the length and the width.

141. The Rule for the Surface of a Triangle.



Let us draw about the triangle ABC the rectangle $EBCF$, as in the diagram. Also, let us draw the height (or altitude) of the triangle AD, which is equal to a side of the rectangle EB or FC. Suppose that, after measuring AD and BC, we may say
the height of the triangle = an inch × 2·4,
and the base of the triangle = an inch × 3·3.
where 2·4 and 3·3 are the measures of the sides of the rectangle.

Then the surface of the rectangle

$= \text{a square inch} \times 3{\cdot}3 \times 2{\cdot}4.$ Article 140.

But the surface of the triangle

$$= \text{the surface of the rectangle} \times \frac{1}{2}.$$

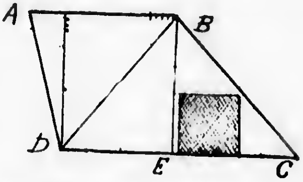
Therefore the surface of the triangle

$$= \text{a square inch} \times 3{\cdot}3 \times \frac{2{\cdot}4}{2} = \text{a square inch} \times 3{\cdot}96.$$

Hence, the *Rate* by which the surface of a triangle is derived from the unit of surface is the product of the measure of the base and the measure of half the height.

142. The Rule for the Surface of a Trapezium.

The figure *ABCD* is a trapezium when the side *AB* is parallel to the side *DC*; that is, when the distance between *AB* and *CD* is the same, wherever it is measured.



Now, when *BD* is drawn, it is seen that the trapezium consists of two triangles, *ABD* and *BDC*, whose heights are each equal to *BE*.

Now, by Article 141, when a foot is the unit of length, the surface of the triangle *ABD*

$$= \text{a square foot} \times \frac{\text{the measure of } BE}{2} \times \text{the measure of } AB.$$

Also, the surface of the triangle *BDC*

$$= \text{a square foot} \times \frac{\text{the measure of } BE}{2} \times \text{the measure of } DC.$$

Therefore, when these triangles are put together, the surface of the trapezium

$$= \text{a square foot} \times \frac{\text{the measure of its width}}{2} \times \text{the sum of the}$$

measures of the parallel sides. This is the rule.

143. Examples solved.

(1) The sides of a rectangle are 25·4 feet and 20·21 feet. Find its surface.

Solution.—Since its length = a foot × 25·4,
and its width = a foot × 20·21 ;
therefore its surface = a square foot × 25·4 × 20·21,
= a square foot × 513·334.

(2) The surface of a rectangle is $\frac{4}{3}$ square feet, and its length is $\frac{8}{5}$ feet. Find its width.

Solution.—Since its surface = a square foot × $\frac{4}{3}$,
and its length = a foot × $\frac{8}{5}$,
therefore its width = a foot × $\left(\frac{4}{3} \div \frac{8}{5}\right)$ = a foot × $\frac{5}{6}$.

(3) It is required to find the worth, at $120 an acre, of a farm in the shape of a trapezium. It fronts 110 rods on the road, while the line fence at the back, which is 88 rods long, is 170 rods from the front.

Solution.—The surface of the farm

= a square rod × $\frac{170}{2}$ × (110 + 88), Article 142.

= an acre × $\frac{170}{2} \times \frac{110+88}{160}$ = an acre × $\frac{17}{2} \times \frac{198}{16}$.

Therefore the worth of the farm = \$120 × $\frac{17}{2} \times \frac{198}{16}$, Article 77.
= \$12622·50.

(4) Required the cost of painting the walls, ceiling and floor of a room, 24 feet long, 18 feet wide and 11 feet high, at 12½ cents per square yard, no allowance being made for windows or doors.

Solution.—The surface of the floor and ceiling
= a square foot × 24 × 18 × 2 = a square foot × 864.

The surface of the side walls

$$= \text{a square foot} \times 24 \times 11 \times 2 = \text{a square foot} \times 528,$$

and the surface of the end walls

$$= \text{a square foot} \times 18 \times 11 \times 2 = \text{a square foot} \times 396.$$

Therefore the whole surface to be painted

$$= \text{a square foot} \times (864 + 528 + 396),$$

$$= \text{a square foot} \times 1788.$$

Therefore the cost of the work

$$= \text{a } \$ \times \cdot 12\frac{1}{2} \times \frac{1788}{9} = \text{a } \$ \times \frac{1}{8} \times \frac{1788}{9} = \$21\frac{5}{6}.$$

(5) The length and width of a field are as 5 is to 3, and its surface is 5 acres. Find its length within an inch.

Solution.—Here the measure of the width

$$= \text{the measure of the length} \times \frac{3}{5}.$$

Therefore, if we choose a rod as the unit of length, the surface of the field = a square rod × the measure of the length

$$\times \text{ the measure of the length} \times \frac{3}{5},$$

$$= \text{a square rod} \times (\text{the measure of the length})^2 \times \frac{3}{5}.$$

But 5 acres = a square rod × 800.

$$\text{Therefore } (\text{the measure of the length})^2 \times \frac{3}{5} = 800.$$

$$\text{Therefore } (\text{the measure of the length})^2 = 800 \times \frac{5}{3}.$$

$$\text{Therefore the measure of the length} = \sqrt{\frac{4000}{3}} = 36\cdot 5148.$$

So that the length = a rod × 36·5148 where the sub-units of the 5th and higher orders are neglected.

$$\text{But an inch} = \text{a rod} \times \frac{2}{33} \times \frac{1}{12} = \text{a rod} \times \cdot 00505.$$

Therefore less than an inch has been neglected.

EXERCISE XXXVII.

1. A board is 9 inches wide and $12\frac{1}{2}$ feet long. Find its surface in square yards.

2. A floor is $15\frac{1}{2}$ feet long and 14 feet wide; how many square yards of carpet will just cover it?

3. A farm is 80 rods in frontage and 110 rods deep; how many acres does it contain?

4. Show by a diagram that a sq. rod consists of $30\frac{1}{4}$ sq. yards.

5. How much surface is there on the outside of a block of wood 8 inches long, 7 inches wide, and 6 inches high.

6. How many square feet of tin will it take to line on the inside an open box whose length, width and depth, internally measured, are 26 inches, 32 inches and 30 inches, respectively?

7. How much will it cost to paint the walls and ceiling of a room 18 feet long, 14 feet wide, and 10 feet high, at $7\frac{1}{2}$ cents per square yard?

8. If it cost $20 to sod a plot of ground 30 feet wide, at 15 cents per square yard, find the length of the plot.

9. A flower-bed is in the form of a 4-pointed star, made by putting triangles 6 feet high on the sides of a 4-foot square. Find the surface of the bed.

10. Find in feet the side of a square field whose surface is $\frac{1}{10}$ of an acre.

11. The width of a field = its length × $\frac{2}{3}$, and its area = a square yard × 2400. Find its length and its width.

12. A room is 15 feet long, 13 feet wide, and 9 feet high. How much will it cost to have its walls and ceiling papered with paper 27 inches wide, at 15 cents per yard of paper, no allowance being made for doors or windows?

13. How far does a man walk in a day who ploughs $1\frac{1}{2}$ acres, cutting a furrow 9 inches wide?

14. The length is to the width of a field as 4 is to 3, and it contains $7\frac{1}{2}$ acres. How much did it cost to fence it, at $22\frac{1}{2}$ cents a rod?

15. If $\frac{1}{4}$ of an inch on a map represents a mile in the country, how much surface in the country will a square inch on the map represent?

16. Required the cost of a field whose length is 1 furlong, 16 rods, $2\frac{1}{2}$ yards, and whose width is 35 rods, 4 yards, at $62.50 an acre?

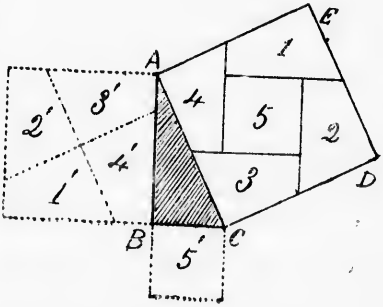
17. If a square mile of territory is represented on a map by $\frac{9}{16}$ of a square inch, what will be the length and width of a map to represent a township 14 miles long and 10 miles wide?

18. If the measure of a square field, which contains 2·5 acres, is $2\frac{1}{4}$, how many rods does the unit of length consist of?

19. Reduce the surface of a township, whose length is 11 miles, 5 furlongs, 25 rods, 5 yards, and width 8 miles, 6 furlongs, 15 rods, 2 yards, to a compound quantity from square miles to square yards.

144. The 47th Proposition of Euclid.

Upon a stiff piece of white paper draw a triangle *ABC*, so that it has a square corner, *B*: that is, so that the angle at *B* is a right angle. On *AC*, the longest side of the triangle, draw the square *ACDE*. Find the middle points of the sides of this square, and from these points draw, as in the diagram, lines parallel to *AB* and *CB*, the other sides of the triangle; thus marking off



---

g. Find its

many square

deep; how

$\frac{1}{4}$ sq. yards.

a block of

line on the
, internally
pectively?

ceiling of a
at $7\frac{1}{2}$ cents

wide, at 15

ar, made by
foot square.

urface is $\frac{1}{10}$

ts area = a

feet high.
ng papered
f paper, no

s $1\frac{1}{2}$ acres,

this square into five parts, which are numbered 1, 2, 3, 4 and 5. Next, with a sharp knife, cut out these five pieces.

Now, it will be found that, without turning these pieces, they can be slid into the positions marked 1′, 2′, 3′, 4′ and 5′, respectively; thus making squares drawn upon *AB* and *BC*, the other sides of the triangle.

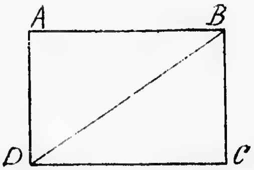
Hence, if one corner of a triangle be a square corner, the surface of the square drawn upon the longest side is equal to the surface of the two squares drawn upon the two other sides of the triangle. This is Euclid's Proposition.

145. To Find the Diagonal of a Rectangle, when the sides have been measured.

Suppose that the sides of the rectangle *ABCD* have been measured, and it is found that

AB = an inch × 91,

and AD = an inch × 60.



Therefore, the square drawn upon AB

= a square inch × 91 × 91,

= a square inch × 8281.

and the square drawn upon AD

= a square inch × 3600.

Now, since the triangle *ABD* has a square corner at *A*, these two squares can be made into the square drawn upon *BD*. Article 144.

So that the square drawn upon *BD*

= a square inch × 11881.

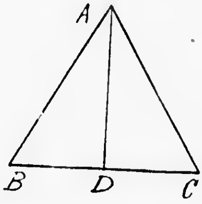
Therefore *BD*, the diagonal of the rectangle,

= an inch × $\sqrt{11881}$.

= an inch × 109.

Thus, we have found the length of *BD* without actually measuring it.

146. The Surface of an Equilateral Triangle, whose side is 24 inches.



Draw AD the height of the triangle.

Then AB, AC or BC = an inch × 24,

and BD = an inch × 12.

Further, since the triangle ABD has a square corner at D, therefore the square upon AD can be made by taking the square upon BD from the square upon AB. Article 144.

Now, the square upon AB = a square inch $\times (24)^2$,

and the square upon BD = a square inch $\times (12)^2$,

therefore the square upon AD = a square inch $\times \{(24)^2 - (12)^2\}$,

= a square inch × 432.

Therefore AD = an inch $\times \sqrt{432}$.

Hence the surface of the triangle

$$= \text{a square inch} \times \frac{24 \times \sqrt{432}}{2}, \qquad \text{Article 139.}$$

$$= \text{a square inch} \times 249{\cdot}415,$$

when the operations are performed as follows:

```
       | 432·00 ( 20·7846          20·7846
       | 4                              12
       |------                    --------
  407  |  3200                    249·4152
       |  2849
       |-------
 4148  |  35100
       |  33184
       |--------
41564  |  191600
       |  166256
       |--------
       |   25344
```

EXERCISE XXXVIII.

1. A field is 40 rods long and 30 rods wide. How far is it between its opposite corners?

2. How much farther will a man walk in going half-way around a square mile than in going from corner to corner across the field?

3. The longest side of a right-angled triangle is 85 inches, and one of the other sides is 75 inches. Find the length of the 3rd side.

4. The side of a square = an inch × ·00351. Find to 6 figures its diagonal.

5. What is the length of the longest string which can be stretched in a room 24 feet long, 18 feet wide, and 16 feet high?

6. A boy walked 13 rods north, 16 rods east, 31 rods north, 43 rods east, 59 rods north, and finally 11 rods east. How far in a straight line is he from his starting point?

7. If a boy in taking a walk were to go 144 rods north, 210 rods west, 115 rods south, and then 323 rods east, what is the shortest distance he can now walk to reach home

8. Find the surface of a triangle, each side of which is 15 inches.

9. A boy, flying a kite with a string 60 yards long, found out that he was 48 yards from a pine tree when the kite caught fast in the tree top. How high was the tree?

10. A rectangular field contains $2\frac{1}{2}$ acres, and its sides are as 4 is to 9. How far is it from corner to corner across the field?

147. The Circumference of a Circle.—One of the hard problems which puzzled the ancient people of the world, was how to find, without measuring, the Rate by which the length of the circumference of a circle is derived from its diameter; and only within the last 300 years has it been solved. We shall, therefore, simply give the result, which the student may verify by measuring.

The length of the circumference of a circle

$$= \text{its diameter} \times 3{\cdot}1416.$$

This rate is not absolutely exact, but it is near enough to give the first five figures correct in nearly every result obtained by using it.

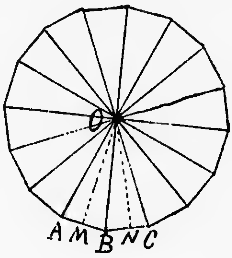
Again, since $\frac{22}{7} = 3{\cdot}142857$, $\frac{22}{7}$ is sometimes used as the rate, but it is a little too large, and will give only three figures correct in the result. This rate is sometimes called π (pronounced *pi*), the Greek letter *p*.

So that the length of the circumference of a circle

$$= \text{its diameter} \times \pi.$$

148. The Rule for the Surface of a Circle.

Draw enough thin triangles, such as OAB, to fill up the space about the point O, so that their heights are equal, thus forming a figure of many sides.



Now, the surface of the triangle OAB = a square inch × the measure of half the height × the measure of the base AB; and the surface of the triangle OBC, next it,

= a square inch × the measure of half the height × the measure of the base BC.

Therefore the surface of both triangles

= a square inch × the measure of half the height
× the sum of the measures of the bases.

And so, when all the triangles are put together,

the surface of the many-sided figure

= a square inch × the measure of half the height
× the sum of the measures of all the bases.

If, now, the triangles had been made very thin and their number very many, so that they still fill up the space about O, then, in the most extreme case, the many-sided figure is a circle, the bases of the triangle form its circumference, the height of the triangle is the radius of the circle, and the statement above is

The surface of the circle

$$= \text{a square inch} \times \text{the measure of half its radius} \times \text{the measure of its circumference.}$$

But the measure of the circumference

$$= \text{the measure of the radius} \times 2 \times \pi.$$

Therefore the surface of a circle

$$= \text{a square inch} \times \frac{\text{the measure of the radius}}{2} \times \text{the measure of the radius} \times 2 \times \pi,$$

$$= \text{a square inch} \times (\text{the measure of the radius})^2 \times \pi.$$

149. Examples solved.

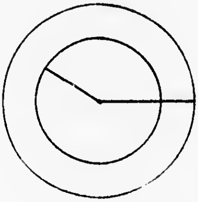
(1) Find the surface of a circle whose radius is 24 inches.

Solution.—The radius = an inch × 24.

Therefore the surface $= \text{a square inch} \times (24)^2 \times 3{\cdot}1416,$

$= \text{a square inch} \times 1809{\cdot}56,$

when the multiplication is performed.



(2) The circumferences of two circles which have the same centre are 120 feet and 80 feet, respectively. Find the surface of the ring outside the smaller circle.

Solution.—Since the circumference of the large circle

$$= \text{the radius} \times 2 \times \pi,$$

therefore the radius $= \text{the circumference} \times \dfrac{1}{2 \times \pi},$

$$= \text{a foot} \times 120 \times \frac{1}{2 \times \pi} = \text{a foot} \times \frac{60}{\pi}.$$

Therefore the surface of the large circle

$$= \text{a square foot} \times \left(\frac{60}{\pi}\right)^2 \times \pi = \text{a square foot} \times \frac{3600}{\pi}.$$

Similarly, the radius of the small circle $= \text{a foot} \times \frac{40}{\pi}$.

Therefore the surface of the small circle

$$= \text{a square foot} \times \left(\frac{40}{\pi}\right)^2 \times \pi = \text{a square foot} \times \frac{1600}{\pi}.$$

Therefore the surface of the ring, which is made by taking out the small circle,

$$= \text{a square foot} \times \frac{2000}{\pi},$$

$$= \text{a square foot} \times 636{\cdot}618,$$

when the student has performed the operations, as follows:

```
3·1416 ) 200000 ( 636·618
         188496
         ------
          115040
           94248
          ------
           207920
           188496
           ------
            194240
            188496
            ------
              57440
              31416
              -----
              26024
```

(3) The surface of a circular field is 10 acres. Find the cost of fencing it at 60 cents a rod.

Solution.—The surface of the field

$= \text{a square rod} \times 1600$,

and also $= \text{a square rod} \times (\text{the measure of the radius})^2 \times \pi$.

Therefore $(\text{the measure of the radius})^2 \times \pi = 1600$.

Therefore (the measure of the radius)$^2 = \dfrac{1600}{\pi}$.

Therefore the measure of the radius $= \sqrt{\dfrac{1600}{\pi}} = \dfrac{40}{\sqrt{\pi}}$.

Therefore the circumference $=$ a rod $\times \dfrac{40}{\sqrt{\pi}} \times 2 \times \pi$,

$=$ a rod $\times 80 \times \sqrt{\pi}$, since $\sqrt{\pi} \times \sqrt{\pi} = \pi$.

The cost of fencing, then,

$=$ a \$ $\times \cdot 60 \times 80 \times \sqrt{\pi}$.

$=$ a \$ $\times 48 \times \sqrt{\pi}$.

$=$ \$85·08, as seen below:

$$
\begin{array}{r|l}
 & 3{\cdot}1416\ (\ 1{\cdot}7724 \\
 & 1 \\
27 & 214 \\
 & 189 \\
\hline
347 & \ \ 2516 \\
 & \ \ 2429 \\
\hline
3542 & \ \ \ \ 8700 \\
 & \ \ \ \ 7084 \\
\hline
 & \ \ \ \ 1616
\end{array}
\qquad\qquad
\begin{array}{r}
1{\cdot}7724 \\
48 \\
\hline
141792 \\
70896 \\
\hline
85{\cdot}0752
\end{array}
$$

EXERCISE XXXIX.

1. Find the surfaces of the circles whose diameters are:

(*a*) 100 feet. (*c*) 12·34 inches. (*e*) $\frac{3}{8}$ of an inch.

(*b*) ·032 yards. (*d*) ·016 miles. (*f*) $4\frac{3}{5}$ feet.

2. The circumference of a circle is 1000 yards. Find its diameter.

3. Find the diameter of a circle whose surface is 4 acres. (Give result in rods.)

4. Find the surface of the largest circle which can be made out of a square whose side is 10 inches.

5. The surface of a circle is 25 square inches. Find its circumference.

6. The diameters of two circles having the same centre are 73 feet and 53 feet, respectively. Find the surface of the ring between their circumferences.

7. A circular pond is ½ a mile in circumference. How much will it cost to gravel a roadway 4 rods wide around it, at 75 cents per square rod?

8. A plot of ground 12 yards long and 10 yards wide is to be laid out, as follows: A path 4 feet wide is to run across it, both ways; in each of the four plots thus formed is to be a circular flower bed, 8 feet in diameter; around the outside, except at the paths, are to be flower beds 3 feet wide, and the rest is to be sodded. If the sodding costs 35 cents per square yard, the paths 25 cents per square yard, and the flower beds 20 cents per square yard, make out the bill.

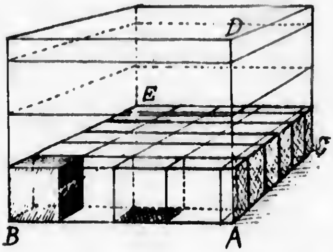
9. Find, in rods, the difference between the circumference of a circular field, and the perimeter of a square field, each of which contains 2½ acres.

10. A designer has a floor 15¾ feet long and 11¼ feet wide, on which he wishes to draw equal circles as large as possible, so that they touch without overlapping. Tell him how large to draw his circles, how many there will be, and how much space will not be covered by circles.

11. A cow is tethered by a rope 40 feet long to one corner of a square enclosure whose side is 20 feet, and into which the cow cannot go. Find in square yards the surface over which the cow can graze.

12. The outer circumference of a race-course is 170 rods, and the width of the track is 4 rods. Find the inner circumference.

150. The Rule for the Volume of a Block.



In the figure, suppose that the length, AC, of the block, $= \text{a foot} \times 5{\cdot}6$; the width, AB, of the block, $= \text{a foot} \times 4{\cdot}3$; and the height, AD, of the block, $= \text{a foot} \times 3{\cdot}4$.

Now, whatever rate will derive the bottom surface of the block from a square foot, the same rate will derive the volume of the layer AE from the cubic foot at B. . . . . Article 77.

But the bottom surface of the block

$= \text{a square foot} \times 5{\cdot}6 \times 4{\cdot}3$. Article 139.

Therefore the volume of the layer $AE = \text{a cubic foot} \times 5{\cdot}6 \times 4{\cdot}3$.

Again, in the same way,

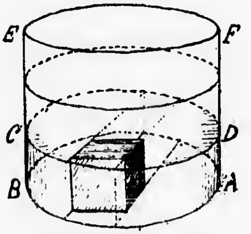
since $AD = \text{a foot} \times 3{\cdot}4$,

therefore the volume of the block = the layer $AE \times 3{\cdot}4$, Art. 77.

$= \text{a cubic foot} \times 5{\cdot}6 \times 4{\cdot}3 \times 3{\cdot}4 = \text{a cubic foot} \times 81{\cdot}872$.

Hence, the rate by which the volume of a block is derived from the unit of volume is the product of the measures of the length, width and height.

151. The Rule for the Volume of a Cylinder.



Let the radius of the bottom surface of the cylinder

$= \text{a foot} \times 2{\cdot}9$.

Then the surface of the bottom

$= \text{a square foot} \times (2{\cdot}9)^2 \times 3{\cdot}1416$.

Therefore the volume of the layer $ABCD$

$= \text{a cubic foot} \times (2{\cdot}9)^2 \times 3{\cdot}1416$.

And since the height of the cylinder AF

$= \text{a foot} \times 3$,

therefore the volume of the cylinder

$$= \text{a cubic foot} \times (2{\cdot}9)^2 \times 3{\cdot}1416 \times 3 = \text{a cubic foot} \times 79{\cdot}2626,$$

since only the first 6 figures are correct.

Hence the rate by which the volume of the cylinder is derived from the unit of volume, is the product of the measures of the height and the surface of its end.

152. The Rule for the Surface of a Cylinder.—It is left here for the student to prove that the Rate by which the surface of a cylinder is derived from the square unit, is the product of the measures of the height and the distance around it.

153. Examples solved.

(1) How many gallons will a cistern hold which is 7 feet deep and 6 feet across?

Solution.—$\dfrac{\text{The surface of the bottom}}{\text{a square foot}} = 3 \times 3 \times \pi.$ Article 33.

Therefore the volume of the cistern

$$= \text{a cubic foot} \times 9 \times \pi \times 7,$$

$$= \text{a cubic inch} \times 63 \times \pi \times 1728,$$

$$= \text{a gallon} \times \frac{63 \times \pi \times 1728}{277{\cdot}274},$$

$$= \text{a gallon} \times 1233{\cdot}4, \text{ as seen below:}$$

```
     3·1416
         63
     ------
      94248
    188496
   --------
   197·9208
       1728
   --------
   15833664
   3958416
 13854456
 1979208
-----------
 342007·1424
```

```
   ✓ ✓✓✓
 277·274 ) 342007 ( 1233·4
           277274
           ------
            64723
            55455
            -----
             9268
             8318
             ----
              950
              832
              ---
              118
              111
              ---
```

Only the first 6 figures of the product are used for the dividend, and in dividing we check off the figures of the divisor, instead of bringing down the figures of the dividend. But in multiplying the divisor by the figure in the quotient, we "carry" from the figure checked off.

(2) How many feet of lumber will it take to make a closed box 4 feet long, 3 feet 6 inches wide, and 2 feet 8 inches high, out of boards 1 inch thick?

Solution.—If the box were a solid block of wood, its volume would = a cubic inch × 48 × 42 × 32, Article 150.

and the volume of the inside of the box

$$= \text{a cubic inch} \times 46 \times 40 \times 30.$$

Therefore the volume of the boards

$$= \text{a cubic inch} \times (48 \times 42 \times 32 - 46 \times 40 \times 30).$$

But a foot of lumber = a cubic inch × 144.

Therefore the lumber in the box

$$= \text{a foot of lumber} \times \frac{48 \times 42 \times 32 - 46 \times 40 \times 30}{144},$$

$$= \text{a foot of lumber} \times \left(14 \times 32 - \frac{46 \times 5 \times 5}{3}\right),$$

$$= \text{a foot of lumber} \times 64\frac{2}{3}.$$

EXERCISE XL.

1. How many bushels will a bin hold which is 8 feet long, 7 feet wide, and 6 feet deep?

2. How many bushels of corn will a waggon box hold whose length is 14 feet, width 3 feet 4 inches, and depth 14 inches?

3. How many gallons of water will a cistern 6 feet deep and 5 feet in diameter hold?

4. How many feet of lumber can be cut from a log 16 feet long and 26 inches in diameter, if in sawing $\frac{1}{3}$ of it is used up in sawdust, slabs and edgings?

5. How many cords of stone in a pile 50 feet by 30 feet by 6 feet?

6. Required the cost, at 75 cents per square foot, of polishing a marble column 12 feet high and 10 inches in diameter.

7. A farmer's roller is 9 feet long and 3 feet in diameter. How far will the farmer drive to roll 10 acres, not reckoning the turnings?

8. The water which fell during a shower upon the roof of a house 36 feet long and 28 feet wide, filled a cistern 6 feet deep and $4\frac{1}{2}$ feet in diameter. Required, in inches, the depth of the rainfall and the number of tons of water which fell on one acre.

9. Find exactly the number of feet of lumber required to make a box 3 feet long, $2\frac{2}{3}$ feet wide, and $2\frac{1}{6}$ feet high, without a cover, out of boards $1\frac{1}{2}$ inches thick.

10. Find the cost of digging a ditch across a township 12 miles wide, at 15 cents per cubic yard. The average width of the ditch at the top is 15 feet and at the bottom 7 feet, and the depth is 5 feet.

11. Find the depth of a bin 6 feet square, which will hold 100 bushels.

12. Find the edge of a cube whose volume is 10648 cubic inches.

13. A cistern 6 feet in diameter is filled with water; then from it a tank, 8 feet long and $2\frac{1}{2}$ feet in diameter, is filled. It is found that the water in the cistern is now 4 feet 2 inches deep.. Required the depth of the cistern.

14. A circular vat 12 feet in diameter and 4 feet deep is filled by a wind-pump, which makes a stroke every 5 seconds, and which discharges a volume of water equal to that of a cylinder 4 inches in diameter and 5 inches long at every stroke. How long is it in filling?

154. The Rule for the Volume of a Pyramid.

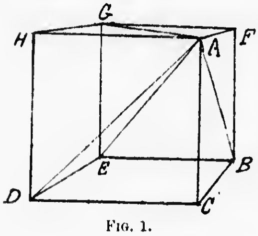


FIG. 1.

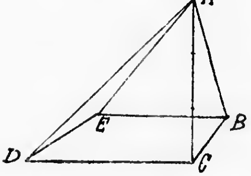


FIG. 2.

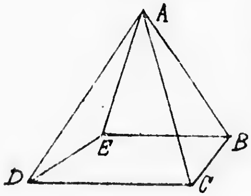


FIG. 3.

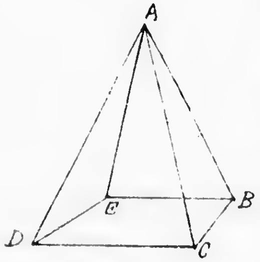


FIG. 4.

If one corner, A, of a cube be joined to all the other corners, as in the diagram, it will be seen, on observation, that the cube consists of three equal pyramids, one of which is shown in Fig. 2.

Now, the volume of the cube

= a cubic inch × the measure of the base × the measure of its height.

Therefore the volume of the pyramid

= a cubic inch × the measure of the base

$$\times \frac{\text{the measure of its height}}{3}.$$

Next, suppose the pyramid to be made up of very thin layers piled upon the base $BCDE$, and then carefully tilted till the vertex A is over the centre of the base, as in Fig. 3. Then the volume is the same as before.

Finally, suppose each layer to swell or shrink in thickness, each by the same rate, so that the pyramid becomes as it is in Fig. 4; then, whatever rate will derive the height in Fig. 4, from the height in Fig. 3, the same rate will derive the volume in Fig. 4 from the volume in Fig. 3.

Now, the volume in Fig. 3

= a cubic inch × the measure of the base

$$\times \frac{\text{the measure of the height in Fig. 3}}{3}$$

Therefore the volume in Fig. 4

= a cubic inch × the measure of the base

$$\times \frac{\text{the measure of the height in Fig. 4}}{3}$$

amid.
ube be joined
as in the dia
observation,
f three equal
is shown in

e cube
the measure
of its height.
the pyramid
re of the base
height

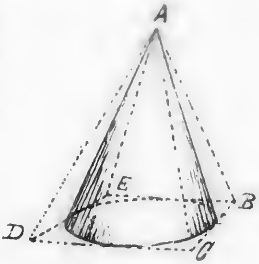
ramid to be
rs piled upon
hen carefully
is over the
Fig. 3. Then
s before.
ayer to swell
each by the
pyramid be-
; then, what-
he height in
n Fig. 3, the
he volume in
n Fig. 3.
g. 3
re of the base
ght in Fig. 3

n Fig. 4
re of the base
ght in Fig. 4

Hence, the rate by which the volume of a pyramid is derived from the cubic unit is the product of the measure of the base surface and the measure of one-third the height.

155. The Rule for the Volume of a Cone. If now, upon the base of the pyramid, in Figure 4, Article 154, we describe the largest circle possible, and join its circumference at all points to the vertex *A*, we shall thus form a cone, as in the diagram.

It will be seen, also, that whatever rate will derive the surface of the circle from the surface of the square *BCDE*, the same rate will derive the volume of the cone from the volume of the pyramid.



Now, the surface of the circle

$$= \text{the surface of the square} \times \frac{\pi}{4}.$$

Therefore the volume of the cone

$$= \text{the volume of the pyramid} \times \frac{\pi}{4}.$$

$$= \text{a cubic inch} \times \frac{\text{the measure of the height}}{3}$$

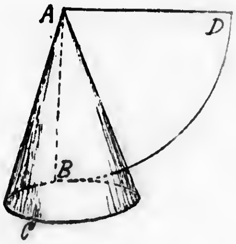
$$\times \text{ the measure of the square base} \times \frac{\pi}{4}, \quad \text{Art. 154.}$$

$$= \text{a cubic inch} \times \frac{\text{the measure of the height}}{3}$$

$$\times \text{ the measure of the circular base.}$$

So then, the rule for the volume of a cone is the same as that for the volume of a pyramid.

156. The Rule for the Curved Surface of a Cone.—



If a paper be carefully fitted about a cone so that the edges are together along the line *AB*, and the paper then unrolled, we shall find it to be of the shape *ABD*, which is a part (sector) of a circle. It is left here for the student to show, by putting together thin triangles to fill up the space between *AB* and *AD*, as in Article 148, that

The surface of the sector of a circle
= a square inch × the measure of half the radius
× the measure of the arc *BD*.

But the arc *BD* = the circumference of the base of the cone,
and the radius *AB* = the slant height of the cone.

Therefore the curved surface of a cone
= a square inch × the measure of half the slant height
× the measure around the base.

157. Example solved.—A pile of wheat on the barn floor is the shape of a cone; it is 12 feet in diameter, and 3 feet 3 inches high. Required the number of bushels in it.

Solution.—The pile of the wheat

$$= \text{a cubic inch} \times 72 \times 72 \times 3{\cdot}1416 \times \frac{39}{3}, \qquad \text{Article 155.}$$

and the pile of a bushel = a cubic inch × 2218.

$$\text{Therefore the pile of wheat} = \text{a bushel} \times \frac{72 \times 72 \times 3{\cdot}1416 \times 39}{2218 \times 3},$$

$$= \text{a bushel} \times \frac{72 \times 72 \times 3{\cdot}1416 \times 13}{2218},$$

$$= \text{a bushel} \times 95{\cdot}454.$$

EXERCISE XLI.

1. A pyramid has a square base whose side is 25 inches; if the volume is 1000 cubic inches, find its height.

2. How many quarts will a vessel in the form of an inverted cone hold, if it is 2 feet deep and 1 foot across the top?

3. How much tin will it take to make the vessel in question 2, with a circular cover, not allowing for seams?

4. How many cubic feet in a pile of sand shaped like a cone, 50 feet across the base and 20 feet high?

5. Find the volume of the figure formed by setting square pyramids, 6 inches high, on the sides of a cube whose edge is 7 inches.

6. How many square yards of canvas will be required to make a conical tent, 20 feet in diameter and 16 feet in slant height?

7. Omit the word slant, and read question 6.

8. A piece of lead, 15 inches by 12 inches by 2 inches, is moulded into a cone, whose base is 6 inches in diameter. Find the height of the cone.

9. How many bushels of peas in a conical pile 10 feet in diameter and 2 feet high?

10. How many cords of stone in a pyramidal pile which is 12 feet high, and covers a square piece of ground whose side is 18 feet?

158. The Rules for the Sphere and Triangle.—The following rules we give here, without proof.

(*a*) The surface of a sphere is *four* times the surface of a circle of the same diameter.

(*b*) The volume of a sphere

$$= \text{the volume of a cube into which the sphere fits} \times \frac{\pi}{6}.$$

(*c*) Let the measures of the three sides of a triangle be called a, b and c; and let $s = \frac{1}{2} \times (a + b + c)$.

Then the surface of the triangle

$$= \text{the unit surface} \times \sqrt{s \times (s - a) \times (s - b) \times (s - c)}.$$

Example.—Find the surface of a triangle whose sides are 3·5 inches, 4·3 inches and 5·6 inches.

Solution.—Here $s = \frac{1}{2} \times (3{\cdot}5 + 4{\cdot}3 + 5{\cdot}6) = 6{\cdot}7$.

Therefore $s - a = 3{\cdot}2,\ s - b = 2{\cdot}4,\ s - c = 1{\cdot}1$.

Therefore the surface of the triangle

$$= \text{a square inch} \times \sqrt{6{\cdot}7 \times 3{\cdot}2 \times 2{\cdot}4 \times 1{\cdot}1},$$
$$= \text{a square inch} \times \sqrt{56{\cdot}6016},$$
$$= \text{a square inch} \times 7{\cdot}5234.$$

EXERCISE XLII.

1. A triangular field has its sides 65 rods, 70 rods and 75 rods. Find its surface in acres.

2. How far is the longest side of the field in No. 1 from the opposite corner?

3. Find the weight of a cannon ball 6 inches in diameter, if iron weighs $7\frac{1}{2}$ times as heavy as water.

4. Find the surface of a triangle whose sides are ·05 inches, ·12 inches and ·13 inches.

5. Find the surface of an equilateral triangle whose side is 30 inches.

6. A sphere just fits into a hollow closed cylinder whose diameter and height are each 16 inches. How much space is occupied by air in the cylinder?

7. Find the diameter of a sphere whose volume is 1000 cubic inches.

8. If the diameter of one sphere A = the diameter of another sphere $B \times 3$, find what rate will derive

(*a*) The surface of A from the surface of B;

(*b*) The volume of A from the volume of B.

9. If the volume of the sphere $A =$ the volume of the sphere $B \times 64$, find what rate will derive

(*a*) The diameter of A from the diameter of B:

(*b*) The surface of A from the surface of B.

10. If the surface of the sphere $A =$ the surface of the sphere $B \times 2$, find what rate will derive

(*a*) The diameter of A from the diameter of B;

(*b*) The volume of A from the volume of B.

11. The sides of a right-angled triangle are 80 inches and 150 inches. Find the distance of the right angle from the hypothenuse.

12. A ship sails east 30 miles, and then north 10 miles. In the meantime another ship, starting from the same port, sails west 25 miles, then south 12 miles, and then east 15 miles. How far are the ships now apart?

13. Find the surface of a regular hexagon whose side is 6 inches.

14. By heating a block of metal its length, width and thickness are each increased by the rate ·00028. By what rate is the volume increased?

15. Find the surface of a triangle whose sides are 245 feet, 246 feet and 3 feet.

16. Three equal circles, whose diameters are 30 inches, touch one another. Find the triangular space between them.

17. A cylinder is 21 inches in diameter and 24 inches long. Find the length of a thread which passes spirally once around it, one end being on the circumference of the base, and the other on that of the top.

CHAPTER XVI.

THE METRIC UNITS.

159. The irregular system of measuring Length, Surface, Volume, and Weight or Mass, which is in use in English-speaking countries, and which we described in Chapter XI., causes much unnecessary labor in addition, subtraction, multiplication and division. To avoid this unnecessary labor, the French and other peoples of Europe now use the regular system of measuring these quantities, which we described in Chapters I. and VII. The only difference is in the language used to describe the system.

160. **The Metre.**—In English-speaking countries the standard unit of length is the yard, and the student is supposed to have a nearly correct idea of its length. The French unit of length is the *Metre*, which is derived from our yard by the rate 1·093633; that is,

a metre = a yard × 1·093633,
or a metre = an inch × 39·37079.

Now, the unit is their metre,
a multiple unit of the 1st order is their *Deka*metre,
a multiple unit of the 2nd order is their *Hecto*metre,
a multiple unit of the 3rd order is their *Kilo*metre,
a sub-unit of the 1st order is their *deci*metre,
a sub-unit of the 2nd order is their *centi*metre,
a sub-unit of the 3rd order is their *milli*metre.

Hence if, in Articles 41 and 42, we had chosen the metre as the unit of length, then the whole line or distance would consist of 7 metres, 5 decimetres, 8 centimetres, and 4 millimetres, or, as we agreed there to write it,

the length of the line = a metre × $\dot{7}584$.

It is also evident that

the length of the line = a decimetre × $7\dot{5}84$,

or = a Kilometre × $\dot{0}007584$.

161. **The Are**.—The standard unit of surface in France is the surface of a square whose side is a Dekametre. This unit is called the *Are*. As in the case of the metre, the multiple units of the 1st, 2nd and 3rd orders are called the Dekare, the Hectare and the Kilare; while the sub-units of the 1st, 2nd and 3rd orders are called the deciare, the centiare and the milliare. It is, however, as convenient and more expressive to call the units of surface the square metre, the square Dekametre, the square centimetre, and so on.

We may say, then, that

the surface of a field = an are × $3\dot{4}76$,

or = a centiare × $347\dot{6}$,

or = a square Dekametre × $3\dot{4}76$,

or = a square metre × $347\dot{6}$, and so on.

162. **The Litre**.—The standard unit of volume in France is the volume of a cube whose edge is a decimetre. This unit is called the *Litre*. As in the others, the multiple units of the 1st, 2nd and 3rd orders are called the Dekalitre, the Hectolitre and the Kilolitre; while the sub-units of the 1st, 2nd and 3rd orders are called the decilitre, the centilitre and the millilitre. But it is as convenient and more expressive to say a cubic metre, a cubic decimetre, a cubic millimetre, and so on.

163. **The Gram**.—The standard unit of weight (or mass) is a weight which is as heavy as a cubic centimetre of water measured just as it begins to expand in freezing. This unit is called the *Gram.* As before, the multiple units of the 1st, 2nd and 3rd orders are called the Dekagram, the Hectogram and the Kilogram; while the sub-units of the 1st, 2nd and 3rd orders are called the decigram, the centigram and the milligram.

164. Examples solved.

(1) Find in square metres (centiares) the surface of a rectangle whose sides are 43·2 decimetres and 25·5 decimetres.

Solution.—The length = a dm. × 43·2 = a metre × 4·32,
and the width = a metre × 2·55.
Therefore the surface = a square metre × 4·32 × 2·55,
= a square metre × 11·016.

(2) Express an acre in ares.

Solution.—An acre = a square rod × 160,

$$= \text{a square inch} \times 9 \times \frac{121}{4} \times 144 \times 160,$$

and a Dekametre = an inch × 393·7.
Therefore a square Dekametre, or an are,
= a square inch × $(393{\cdot}7)^2$.

$$\text{Therefore an acre} = \text{an are} \times \frac{9 \times 121 \times 144 \times 40}{393{\cdot}7 \times 393{\cdot}7} = \text{an are} \times 40{\cdot}4687.$$

(3) Express a litre in quarts.

Solution.—A decimetre = an inch × 3·937.
Therefore a cubic decimetre, or a litre,
= a cubic inch × $(3{\cdot}937)^3$.

$$\text{But} \quad \text{a quart} = \text{a cubic inch} \times \frac{277{\cdot}274}{4}.$$

$$\text{Therefore a litre} = \text{a quart} \times (3{\cdot}937)^3 \div \frac{277{\cdot}274}{4} = \text{a quart} \times {\cdot}8803,$$

when the multiplying and dividing is done.

EXERCISE XLIII.

1. Find, in square metres, the area of a triangle whose base is 2·53 metres and height 20·5 metres.

2. The distance between two towns is 325 Kilometres. How many miles is it?

3. How many litres will a box hold whose dimensions are ·056 Dm., 3·2 dm., and 2·5 m. measured internally.

4. Find, in ares, the area of a circle whose diam. is 4000 metres.

5. How many cubic centimetres of copper are there [illegible] wire a Kilometre long and ·3 centimetres in diameter?

6. The circular shaft of a mine is 5 metres in diameter and 11 Hectometres deep. How many cubic metres of earth and rock have been excavated to make it?

7. If the diameter of the earth at the equator is 8000 miles, how many Kilometres is it in circumference at the equator?

8. How many Kilograms of water are there in a cylindrical vessel 10 dm. in diameter and 8 dm. deep, if the vessel be full?

9. Find the diagonal of a rectangle whose sides are 2·35 metres and 3·16 metres.

10. Find the diameter of a circle whose circumference is 3521·764 millimetres.

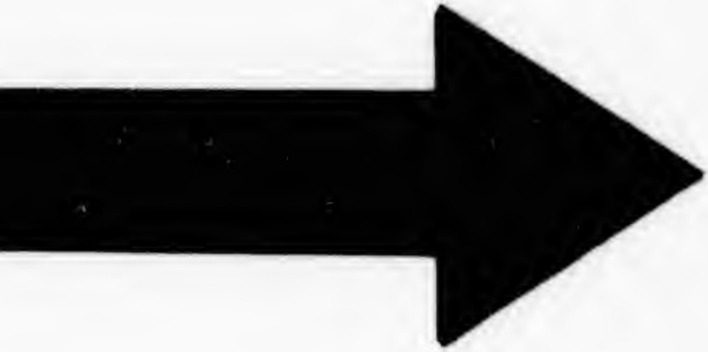
165. The Number of Figures in a Rate.—Suppose we are told that the length of a line AB = a metre × $\dot{3}769524$; then we regard the number $\dot{3}769524$ as abbreviated instructions, which tell us how to use a metre to make up the length of AB. In full, these instructions are:

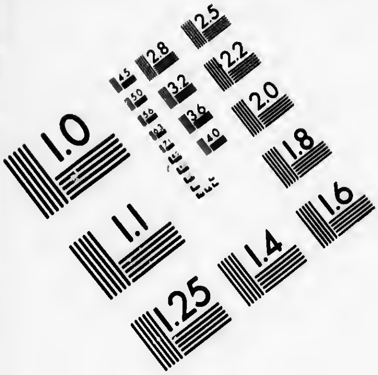
(*a*) Place a metre down 3 times in the same straight line without missing or overlapping.

(*b*) Cut the metre into 10 equal parts, and place one of the parts down 7 times, as before, in this same straight line.

(*c*) Cut one of the parts placed down in (*b*) into 10 equal parts, and place one of them down 6 times, as before.

(*d*) Cut one of the parts placed down in (*c*) into 10 equal parts, and place one of them down 9 times, as before.





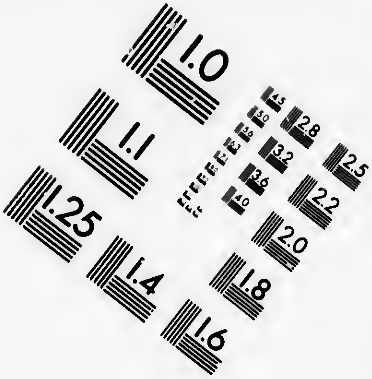
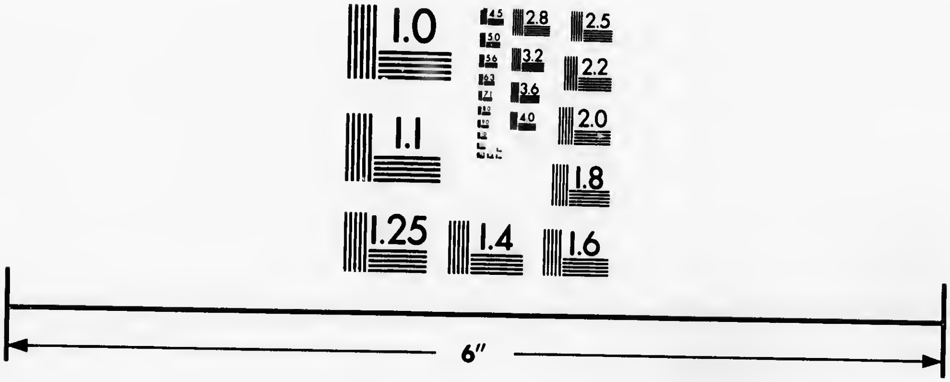
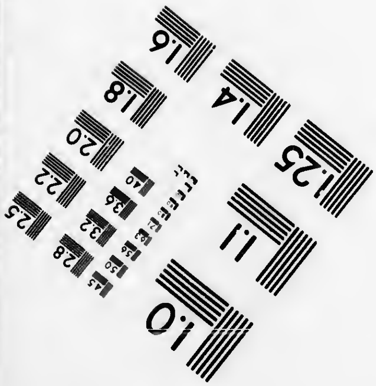
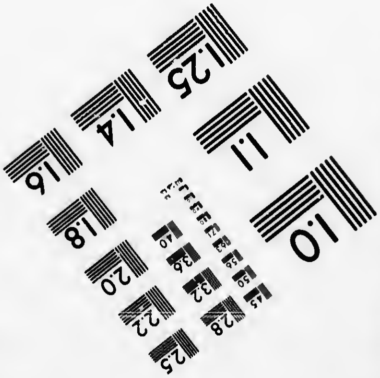
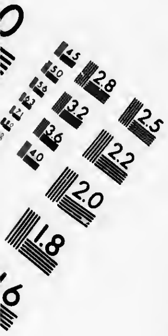


IMAGE EVALUATION
TEST TARGET (MT-3)








4.5
5.0
5.6
6.3
2.8
3.2
2.5
3.6
4.0
2.2
2.0
1.8
1.6


1.1
1.0

(*e*) Cut one of the parts placed down in (*d*) into 10 equal parts, and place one of them down 5 times, as before; and so on.

If these instructions be carried out, it will be found that one of the parts placed down in (*e*) is about as long as the thickness of ordinary paper. Hence, the instructions given by the last two figures, 2 and 4, may, or must, be disregarded. In ordinary cases, then, a rate with 5 figures will give instructions for deriving a quantity from the unit sufficiently accurate.

166. Unnecessary Labor in Multiplication.—If, then, we have to find a single rate equivalent to $\dot{3}769524 \times 4\dot{0}21836$, it is clear that the 6 figures at the left in the product are all that are needed, and that the labor in multiplying to get the figures after the 6th is unnecessary. Thus (see Article 48):

Since the order of 3 in the 1st number is 0, and the order of 4 in the 2nd number is + 1, therefore the order of 3 × 4 is + 1, so that 3 × 4 will be written $1\dot{2}0$.

Again, the order of 4 in the first number is − 6, and the order of 6 in the 2nd number is − 5, so that the order of 4 × 6, or 24, is in the order − 11. Therefore the whole product of the two numbers extends from the order + 2 to the order − 11, and consists, consequently of 14 figures. Of these, the 6 at the left are needed and the other 8 are not.

167. To Find the First Six Figures of $\dot{3}769524 \times 4\dot{0}21836$. The complete product is found by multiplying every figure in the 1st number by every figure in the 2nd, setting the products down in their proper orders, and adding these products. Now, since the product of the first figures of these numbers is 12 in the order + 1, that is, 120, therefore the first six figures of the whole product extends as far as $1\dot{2}0000$, that is, to the order − 3. Hence, we may omit all those products which are in the orders − 5, − 6, − 7, etc.; but we must multiply those figures together which give a product in the order − 4, for this product may have a figure in the order − 3.

Again, since the order of 4 in the 2nd number is + 1, and the order of 2 in the 1st number is − 5, therefore 2 × 4, or 8 in the product, is in the order − 4. In the same way, we may show that 5 × 0, 9 × 2, 6 × 1, 7 × 8 and 3 × 3 are in the order − 4. Let us then write:

4 of the 2nd number under 2 of the 1st,
0 of the 2nd number under 5 of the 1st,
2 of the 2nd number under 9 of the 1st,

and so on.

$$\begin{array}{r|l} \dot{3}769 & 524 \\ 63812 & \dot{0}4 \\ \hline \end{array}$$

Let us also draw a vertical line between the orders − 3 and − 4 of the upper number. This is shown at the left. Then the product of each figure and the one above it is of the order − 4, and all the products which fall to the right of the vertical line may be omitted. Now the manner of multiplying

by 4 is: $2 \times 4 = 8$ in the order − 4 = 1 in the order − 3 (nearly),
$5 \times 4 + 1 = 21$ in the order − 3,
$9 \times 4 + 2 = 38$ in the order − 2, and so on;
by 2 is: $9 \times 2 = 18$ in the order − 4 = 2 in the order − 3,
$6 \times 2 + 2 = 14$ in the order − 3,
$7 \times 2 + 1 = 15$ in the order − 2, and so on;
and similarly by 1, 8 and 3.

This work is then seen as below:

$$\begin{array}{r|l} \dot{3}769 & 524 \\ 63812 & \dot{0}4 \\ \hline 150781 & \\ 754 & \\ 38 & \\ 30 & \\ 1 & \\ \hline 15\dot{1}604 & \end{array}$$

If the last figure of a product which is in the order − 4 is 5, 6, 7, 8 or 9, 1 is added to the figure of this product which is in the order − 3; but if the last figure be 0, 1, 2, 3 or 4, 1 is not added.

It will be observed, also, that the multiplier is written backwards, and that the units figure in it is always next the vertical line and to the right of it.

168. Examples solved.

(1) To obtain the first five figures in $\dot{0}0320479 \times \dot{0}318296$.

Solution.—Since the order of 3 in the 1st number is -2, and of 3 in the 2nd number is -1, therefore the order of the *first* figure 9 of the product is -3; that is, the first figure is $\dot{0}009$. Therefore the first 5 figures will extend as far as $\dot{0}0090000$; that is, to the order -7. We shall, therefore, arrange the numbers so that the product of each figure and the one above it is in the order -8. This is shown in the two upper lines at the left.

The operations are also shown. But, since, in adding, $1+9$ is 10, it happens that we have found the first 6 figures.

$$\begin{array}{r|l} \dot{0}0320479 & 0 \\ 692813 & \dot{0} \\ \hline 96144 & \\ 3205 & \\ 2563 & \\ 64 & \\ 29 & \\ 2 & \\ \hline \dot{0}0102007 & \end{array}$$

(2) To find the compound interest of \$495·28 for 2 years at $6\frac{1}{3}$ per cent.

Solution.—The rate of amount for one year
$= 1·0633333\ldots$

Therefore the amount of the sum at the end of 2 years $=$ a \$ $\times\, 495·28 \times (1·06333\ldots$
$=$ a \$ $\times\, 560·00$,
where the multiplying is shown at the left.
The principal also $=$ a \$ $\times\, 495·28$.
Therefore the interest $=$ a \$ $\times\, 64·72 =$ \$64·72.

$$\begin{array}{r|l} 495·28 & 0 \\ \ldots 333\ 60· & 1 \\ \hline 495\ 28 & \\ 29\ 71 & \\ 1\ 49 & \\ 15 & \\ 1 & \\ \hline 526·64 & 0 \\ \ldots 333\ 60· & 1 \\ \hline 526\ 64 & \\ 31\ 60 & \\ 1\ 58 & \\ 16 & \\ 2 & \\ \hline 560·00 & \\ 495·28 & \\ \hline 64·72 & \end{array}$$

(3) Find the surface of a circle whose diameter is a metre $\times\, 3·29345$.

Solution.—The radius
$=$ a metre $\times\, 1·646725$.
Then the surface (Article 148)
$=$ a square metre $\times\, (1·646725)^2 \times 3·1416$.
We shall neglect in the surface all sub-units

beyond the 5th order, by finding the product of these three numbers to the order – 5, thus :

```
1·64672|5              2·71170|
5 27646|1̇                 6141|3̇
-------|-              -------|-
1 64673|               8 13510|
  98803|                 27117|
   6587|                 10847|
    988|                   271|
    115|                   163|
      3|                      |
      1|                      |
-------|-              -------|-
2·71170|               8·51908|
```

The surface then = a square metre × 8·51908.

EXERCISE XLIV.

1. Tell without multiplying how many figures there will be in the following products :

   (*a*) $38{\cdot}7642 \times 29{\cdot}76346$.

   (*b*) $\cdot 021486 \times \cdot 0053729$.

   (*c*) 1823×219574.

2. To what order must the products in the following cases be found in order to obtain the first 6 figures of them :

   (*a*) $3\dot{2}76415 \times 24\dot{5}721$.

   (*b*) $50\dot{3}7694 \times \dot{0}05218349$.

   (*c*) $\dot{0}485132 \times \dot{0}000745863$.

   (*d*) 284793×85218614.

   (*e*) $4{\cdot}2135 \times 62{\cdot}3147$.

3. Obtain the first 6 figures in each of the products in question 2.

4. Multiply $64\dot{3}582 \times \dot{2}57639$ to the order – 1.

5. Multiply $12\dot{3}45678$ by $\dot{1}2345678$ to the order – 4.

6. Multiply $\dot{0}0006543219$ by $54\dot{3}7692$ to the order – 3.

7. Find the product of $783\dot{2}85961$ and $\dot{0}000008356923$ to the order – 6.

8. Find the product of $\dot{0}0009037402$ and $1234\dot{5}2147$ to the order – 4.

9. Obtain the first 5 figures of $30\dot{0}79421 \times \dot{0}084321$.

10. Obtain the first 6 figures of $\dot{0}015700834 \times \dot{0}217006894$.

11. Explain fully how to obtain the first 6 figures of $\dot{0}3125679 \times 123\dot{4}60072$.

12. Find to 5 figures the surface of a rectangle whose length = a metre × 300·4279 and whose width = a metre × ·008214687.

13. Obtain 5 figures of 603·5721 × 2·34786.

14. Obtain (1·045555. . .)^6 to the order – 6.

15. The base of a cone is 3·4721 feet in diameter and 5·8234 feet in height. Find in cubic feet its volume to the order – 4.

169. To Obtain Five Figures in the Quotient when ·03734216 is divided by 59·216438.

Since the order of 5 in the divisor is + 1,
and the order of 37 in the dividend is – 3;
therefore the order of the first figure of the quotient is – 4, and the figure itself is 6.
Hence, the quotient to have five figures in it extends as far as $\dot{0}00060000$, that is, to the order – 8.

Again, the order of the last figure of the quotient is – 8,
and the order of the first figure of the divisor is + 1,
therefore the order of the last figure of the dividend which we need is – 8 + 1, that is, – 7.

Hence, if we draw a vertical line between the orders – 7 and – 8 in the dividend, we may omit all the work which is set down at the right of the line.

Further, it will be seen, then, that we need not use the figures of the divisor which are beyond the order – 3, except the one in order – 4, which we carry from in multiplying. The arrangement then is:

$$59{\cdot}216\overset{\checkmark}{4}\ )\ {\cdot}0373422\ (\ {\cdot}0006$$

In dividing, each step shortens the divisor by one figure at the right, which is checked off. The work then stands;

$$\begin{array}{lr}
59{\cdot}\overset{\checkmark}{2}\overset{\checkmark}{1}\overset{\checkmark}{6}\overset{\checkmark}{4}\ )\ & {\cdot}0373422\ (\ {\cdot}00063060 \\
 & \underline{355298} \\
 & 18124 \\
 & \underline{17765} \\
 & 359 \\
 & \underline{355} \\
 & 4
\end{array}$$

In short, then, to obtain 5 figures of the quotient only 5 figures of the divisor are needed, and a corresponding number of figures of the dividend; but, in multiplying by the figure in the quotient, we carry from the 6th figure of the divisor.

170. Example solved.—Divide 305·72438 by 4732·1935 to 5 figures.

Solution.—Since the order of 30 in the dividend is + 1, and of 4 in the divisor is + 3, therefore the order of the first figure of the quotient is – 2. We need to use 5 figures of the divisor and 6 of the dividend, the others are checked off, and the work then is as follows:

$$\begin{array}{lr}
47\overset{\checkmark}{3}\overset{\checkmark}{2}{\cdot}\overset{\checkmark}{1}\overset{\checkmark}{9}\overset{\checkmark}{3}\overset{\checkmark}{5}\ )\ & 305{\cdot}724\overset{\checkmark}{3}\overset{\checkmark}{8}\ (\ {\cdot}064601 \\
 & \underline{283931} \\
 & 21793 \\
 & \underline{18928} \\
 & 2865 \\
 & \underline{2839} \\
 & 26 \\
 & \underline{47}
\end{array}$$

Here 26 is more than half of 47, then we put 1 for the last figure of the quotient.

EXERCISE XLV.

1. Divide to 5 figures :
   - (*a*) 1234·5678 by 34·21596.
   - (*b*) ·00325718 by 8·5769231.
   - (*c*) ·000415238 by 5·3164197.
   - (*d*) 25 by 3·1415926.
   - (*e*) 1 by 3·1416.

2. Divide 300·215 by 12345·6789 to 6 figures.

3. Obtain $\frac{23{\cdot}56421 \times 51{\cdot}315214}{9{\cdot}35284 \times 2{\cdot}9653721}$ to 5 figures.

4. Divide 304·56 by 1·0422222 . . . . to 5 figures.

5. Obtain 6 figures of $\frac{1000}{(1{\cdot}0333\,.\,.\,.)^2 - 1}$.

7. Obtain 8 figures of $3 \div 2{\cdot}78287828$.

171. Miscellaneous Exercise.—We give here, in conclusion, an exercise which, we believe, will be a guide to the teacher in teaching, and a help to the student in reviewing the subject; but it must not be regarded as furnishing a complete list of questions.

EXERCISE XLVI.

1. Describe how to count toothpicks to obtain the number 2357.

2. If a carpet tack be the unit, how do you make a multiple unit of the 3rd order?

3. How is a multiple unit of the 5th order made from multiple units of the 3rd order? Express the manner in one sentence.

4. Tell in detail the information given by the number 1435 when we are told that the matches in a box = a match × 1435. What does the "×" denote here?

5. Describe the operation denoted by the line in

$$\frac{\text{a pile of pebbles}}{\text{one pebble}}.$$

6. Read in full English the sentences:

(*a*) $\frac{\text{A pile of pebbles}}{\text{one pebble}} = 325.$

(*b*) A pile of pebbles = one pebble × 325.

7. Why is a number called a rate, and why is it called the measure of a quantity?

8. Explain how you put together a match × 329 and a match × 476 so as to obtain a match × 805.

9. Explain how you take a grain of wheat × 493 from a grain of wheat × 631, to find that the quantity left is a grain of wheat × 138.

10. How do you subtract 1493 from 5182?

11. Of what use is Subtraction and Addition? How does Addition save labor?

12. Make a diagram having 49 spaces arranged in 7 rows and 7 columns. In these spaces write numbers of 5 figures each, so that the numbers are all different, the figures in each number are different, and the figures 1, 2, 3, and 0 do not occur. Find the sums of the rows and columns; then add the sums of the rows and the sums of the columns. The two totals should agree.

13. Repeat No. 12, with modifications, till you can make the totals agree the first time adding five times out of six.

---

14. What is the order of a digit in a number? Write a scheme which shows the orders of all the digits of a number.

15. In the number 197$\dot{6}$04314 what are the orders of 43, of 7, of 4314, of 19, of 760, and of 604?

16. What is the use of the point above the 6 in "4327$\dot{6}$14 dollars?"

17. What is the "Rule of Order" in Multiplication, and how is it proved?

18. Multiply 3421 by 8634, using 8 first, 6 next, 3 next, and 4 next.

19. Multiply 3421 by 8634, going in reverse order in both numbers.

20. Multiply 123456789 by 987654321, and then 987654321 by 123456789, and see if your products agree.

21. Repeat No. 20, with modifications, till you make the products agree the first time multiplying five times out of six.

22. Why does 7 × 9 = 63?

23. When is "×" the sign of Derivation, and when is it the sign of Multiplication?

24. Of what use is Multiplication? How does it save labor?

25. Tell without multiplying how many figures are in the product 43976 × 8571487?

26. In what order will you set the single product 9 × 5 in question No. 25?

27. The length of a furrow in a field is 438 yards, and the width of a field = the width of a furrow × 416. Find the whole length of the furrows.

---

28. Describe how to perform the operation denoted by the line in "$\frac{\text{the length of the desk}}{\text{the length of a pencil}}$."

29. Read the sentence "$\frac{448 \text{ yards}}{8 \text{ yards}} = 56$."

30. When does the line denote the operation of measuring and when does it denote the operation of dividing?

31. Read "$\frac{36 \text{ feet}}{14 \text{ feet}} = \frac{36}{14}$."

32. What is meant by saying "divide 1496 by 8"?

33. What is the purpose of Division in Arithmetic?

34. Show that the number 85134 is made up of the products of 7 found in the Multiplication Table written in different orders.

35. Multiply 7328 by 6, 7, 8 and 9, and divide the whole product by 6, 7, 8 and 9, using the figures in succession. The final quotient should be 7328.

36. Repeat No. 35, with modifications, until you can multiply with accuracy and rapidity.

37. What is the measure of 832796 cubic feet when 329 cubic feet is the unit?

38. If a line AB = a line $CD \times 4375$, and a line PQ = the line $CD \times 175$, express AB in terms of PQ.

39. Multiply 68379 by 85769, and divide the product by 22793.

---

40. Write a scheme showing the orders of all the figures in 304274176098.

41. Write 160 in the orders $+2$, -5, $+1$, -1, -3, $+4$, $+3$, -4 and 0.

42. Tell without multiplying the number of figures in 380472×2976417.

43. Multiply 37·642 by ·024.

44. Simplify $\text{a dollar} \times \dfrac{24{\cdot}31 \times 6{\cdot}27}{{\cdot}0363}$.

45. Simplify $\$275 \times \dfrac{145}{365} \times \dfrac{23}{8}$.

46. Divide the sum of ·075 and ·0075 by the difference between 7·5 and ·75.

47. Divide 1 by ·0375 for 5 figures.

48. Simplify (*a*) $\frac{\cdot 1735}{\cdot 0905}$, (*b*) $\frac{37\cdot 8}{200}$, (*c*) $\frac{\cdot 005}{3\cdot 2}$, giving the reasoning for the position of the units point in each case.

49. Simplify $\frac{20}{\cdot 05}+\frac{\cdot 2}{50}+\frac{60}{\cdot 12}+\frac{\cdot 603}{\cdot 09}$.

50. Define a fraction and a decimal, and prove the rules for
(*a*) Reducing a fraction,
(*b*) Adding two fractions,
(*c*) Subtracting one fraction from another,
(*d*) Multiplying two fractions,
and (*e*) Dividing one fraction by another.

51. Reduce $\frac{14652}{15048}$ to its simplest form.

52. Change the following mixed numbers into fractions:

$$139\frac{9}{151},\ 496\frac{138}{571} \text{ and } 2134\frac{71}{314}.$$

53. Find the G. C. D. of 132288 and 107328.

54. Find the L. C. M. of 11, 14, 28, 22, 7, 56, 42 and 81, and explain your method.

55. Divide $\frac{1}{2}-\frac{2}{3}$ of $\frac{5}{8}+\frac{7}{78}$ by $\frac{3}{8}\div\left(\frac{7}{9}\text{ of }\frac{3}{14}-\frac{1}{8}\right)$.

56. Simplify $\frac{60\times 60\times 60}{277\cdot 274}$.

57. Reduce to one vulgar fraction $\frac{7}{8}+\frac{8}{9}+\frac{9}{10}+\frac{10}{11}+\frac{11}{12}$.

58. Reduce to a decimal of 5 figures $13\frac{5}{19}\times 5\frac{6}{13}-4\frac{4}{13}\times 6\frac{8}{19}$.

59. What is the greatest number that will divide 11067 and 35602, leaving as remainders 17 and 21 respectively?

60. Make out the following bill of goods: 23 yds. cotton at 11 cents, 13 yds. gingham at 23 cents, 25 yds. flannel at 37 cents, $18\frac{1}{3}$ yds. tweed at \$1·50, $12\frac{1}{2}$ yds. serge at \$1·75, $6\frac{1}{2}$ yds. broadcloth at \$4·50.

61. Find the amount of the following bill: $12\frac{1}{2}$ yds. cassimere at $\$2\frac{3}{4}$ per yd., $18\frac{1}{3}$ yds. silk at \$1·17, $23\frac{3}{4}$ yds. flannel at $37\frac{1}{2}$ cents, 112 yds. print at $9\frac{1}{3}$ cents, 55 yds. shirting at $17\frac{1}{2}$ cents, $37\frac{1}{2}$ yds. tweed at \$1·12.

62. A boy can do a piece of work in $4\frac{2}{3}$ days, and a man can do the same in $\frac{3}{7}$ of the time. How long will both require to do it, working together?

63. Divide \$1300 between *A* and *B*, so that their shares are as $1\frac{1}{2}$ and $1\frac{3}{4}$.

64. Divide \$5609 among *A*, *B* and *C*, so that $22\frac{1}{2}$ per cent. of *A's* share, $18\frac{3}{4}$ per cent. of *B's* share, and $16\frac{2}{3}$ per cent. of *C's* share may be equal.

65. *A* has \$18·93 more than *B*, and together they have \$59·77. Find *A's* money.

66. How much water must be added to 92 gallons of brandy, worth \$4·60 per gallon, to make a mixture worth \$3·60 per gallon?

67. A farmer marketed his wheat in 3 loads, as follows: 38 bushels 17 lbs. at $68\frac{1}{2}$ cents, 37 bushels 29lbs. at $69\frac{1}{2}$ cents, 41 bushels 51 lbs. at 69 cents. Find the total money he received.

68. Find the cost of 11750 feet of lumber, at \$17·25 per thousand.

69. A man's salary is \$1150 per year for 5 years. In these years he saved $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$ and $\frac{1}{9}$ of his salary, respectively. Not counting interest, what are his total savings during these 5 years?

70. Change £194 18*s.* 3*d.* to Canadian money, when £1 = \$4·86$\frac{2}{3}$.

71. How often is 6 yards 2 feet, contained in 25 furlongs?

72. A man has 5 tons 6 cwt. of flour; after selling 25 barrels of it, how many sacks, each holding 150 lbs., can be filled with the remainder?

73. A man has 703 acres 142 square rods 14$\frac{1}{4}$ square yards of land. He sold 19 acres 70 square rods 2$\frac{1}{4}$ square yards. He then divided the remainder among his sons, giving each 45 acres 100 square rods 25 square yards. How many sons had he?

---

74. A person hired \$500 on April 10th, and on June 22nd he paid his debt with \$510·20. At what rate was he charged interest?

75. For what time will the interest of \$30441 be \$2210·10, if, at the same rate, the interest of \$24944·10 for 1 year 15 days is \$2596·92? Also, what is the rate of interest?

76. Calculate the interest of \$9348·56 from January 9th, 1896, to September 18th, 1896, at 7$\frac{3}{4}$ per cent.

77. On March 23rd a bank gave me \$845 for a note of \$860, charging discount at 8 per cent. When was the note due?

78. On January 1st, 1897, a person borrowed \$2417·50 at 6$\frac{3}{4}$ per cent. simple interest, promising to pay his debt as soon as it amounted to \$2582·50. On what day did the loan expire?

79. Find the proceeds of a note for \$1389·25, drawn on May 8th, 1897, for 4 months, and discounted on July 21st, at 8 per cent.

80. \$3420$\frac{00}{100}$. Ottawa, September 9th, 1897.

Nine months after date I promise to pay *A. B.*, or order, the sum of Three Thousand Four Hundred and Twenty Dollars (\$3420), with interest at 6 per cent. per annum, value received.

M. N.

The above note was sold on December 18th, 1897, at 7 per cent. discount. Find what was paid for it.

81. For what sum must a 70 day note be made out for on May 19, so that when it is immediately sold it may yield $160, discount being at the rate of 8 per cent.?

82. Toronto, December 1st, 1896.—On demand I promise to pay *A. B.* $1500, with interest at the rate of 8 per cent., value received.

P. Q.

This note was endorsed as follows:

January 23rd, 1897. Received $400. *P. Q.*.
August 20th, 1897. Received $500. *P. Q.*.

What was due on the note December 1st, 1897?

---

83. Find the accrued interest on a loan of $600 at the end of 4 years at 6 per cent., convertible yearly.

84. Find the difference between the simple and the compound interests of $\$9902\frac{2}{3}$ for $2\frac{1}{2}$ years, at $3\frac{1}{2}$ per cent. per annum.

85. Explain clearly the distinction between discount, interest and compound interest.

86. A man has the choice of loaning his capital, $10000, for 3 years, at $7\frac{1}{2}$ per cent. per annum compound interest, or at 8 per cent. simple interest. Which is the better investment?

87. Find the accrued compound interest upon $4530 borrowed January 16th, 1893, at 6 per cent., when the debt is paid July 31st, 1897.

88. A money lender has at interest $1500 at 8 per cent., $1200 at $7\frac{1}{2}$ per cent., and $1000 at 6 per cent. Find his average rate of interest.

89. A man hired $1200 on May 1st, and paid it back July 25th, with rent at 7 per cent. (*a*) Calculate the rent at 7 per cent. interest; (*b*) calculate the rent at 7 per cent. discount.

90. If the difference between the simple and the compound interests of a sum of money hired for 3 years, at 6 per cent., is $38·556, what is the sum of money?

91. A man put in the bank $10 on the 1st day of each month for 3 years. What should be to his credit at the end of the 3 years, if the bank pays 3 per cent. interest, convertible every 6 months?

92. Find the equated time of the following debts: $500 due January 15th, $600 due February 26th, $800 due March 13th, and $900 due July 10th.

93. By selling an article for $10·80 I gained 20 per cent. How much should I sell it for to gain $16\frac{2}{3}$ per cent.?

94. I marked my goods to sell at an advance of 30 per cent. of their cost. I sold them, however, at a discount of 10 per cent., and gained $3·74. Find the cost of the goods.

95. A merchant buys his goods at two successive rates of discount of 20 per cent. and 10 per cent. off the retail price. He gains by so doing $1·96. Find the cost price.

96. A barrel of coal oil, containing 36 gallons, was bought at $12\frac{1}{2}$ cents a gallon. In selling, 2 gallons were spilled. The retail price was $16\frac{1}{2}$ cents per gallon. Find the rate of gain.

97. 277·274 cubic inches of water weighs 10 lbs. How much will a cubic foot of ice weigh, if, in freezing, water increases in bulk by 10 per cent.?

98. Half my goods I sold at a gain of 25 per cent., a third at a gain of 20 per cent., and the rest at a gain of 15 per cent. Find my average rate of gain.

99. I bought a certain lot of goods, half of them I marked 30 per cent. above cost, and the other half 20 per cent. above cost.

In selling, I gave a discount of 10 per cent. all round, and gained \$133·20. Find what the lot cost me.

100. I sold two houses, each for \$1600, gaining on the one at the rate of $12\frac{1}{2}$ per cent., and losing on the other at the rate of $11\frac{1}{9}$ per cent. Find my total gain or loss.

101. If interest is at the rate of 8 per cent., what relation must the 3 months' credit price of an article bear to its cash price that the prices may be equivalent?

102. Read "discount" instead of "interest" in No. 101.

103. Brown purchased $\frac{7}{25}$ of a timber limit for \$4064·55, and Smith purchased $\frac{9}{35}$ of the same property at a rate 5 per cent. higher. What did Smith's part cost him, and how much of the property remains unsold?

104. *A*, *B* and *C* invested, respectively, \$2300, \$2900 and \$3700 in a partnership. At the end of the year their combined capital was \$9880·78. Find the gain of each partner.

105. *A*, *B* and *C* contribute, respectively, \$4295·25, \$5612·34 and \$8156·41 to conduct a business. At the end of a year *A* received \$593·21 as his share of the gain. Find the shares the others received and the total capital.

106. *A* and *B* form a partnership for half a year, contributing \$4000 and \$5000, respectively. *A* withdraws \$90 and *B* \$80 at the end of each month. At the end of the time their capital was \$10150. Find each partner's share of it.

107. The stock of an insurance company sells at $137\frac{1}{4}$, and pays yearly dividends at 10 per cent. If the brokerage is at $\frac{1}{4}$ per cent., what rate of interest will a purchaser receive for the money he invests?

108. A person invested one portion of $1000 in 3½ per cent. stock at 80, and the rest of it in 5 per cent. stock at 112. His whole income from it was $44·06¼. Find each investment.

109. A retired farmer invests 40 per cent. of his money in 3½ per cent. stock at 90, and the remainder in 4 per cent. stock at 95. His income is now $1340 per year. Find his capital.

110. I sold $2000 stock in the 6 per cents. at 90, and purchased 5 per cent. stock at 75. Find the change in my income.

111. A man sold his 5 per cents. at 78, and invested the proceeds in 6 per cent. stock at 104. The change in his income was $385. Find how much 5 per cent. stock he had.

112. A man invests $4875 in the 3 per cents. at 97½; he afterwards sells out at 99, and reinvests the money in R. R. shares at 110, paying 4 per cent. By how much has he increased his income?

---

113. I sold on commission, at 2½ per cent., a lot of goods for $3592·20, and sent my principal his share of the money at a cost of ¼ per cent. of the money he received. Find what he received.

114. An agent sold 1350 lbs. tea at 32½ cents per lb., on a commission of 3 per cent., and invested the net proceeds in sugar on a commission of 2 per cent., at 2⅞ cents per lb. Find the quantity of sugar bought.

115. A dealer shipped 400 bushels of wheat which cost $1·40 per bushel, 800 bushels at $1·62½, and 300 bushels at $1·20, to his agent, who sold the first lot at an advance of 20 per cent., the second at an advance of 15 per cent., and the third at 4⅙ per cent. less than cost. The agent's commission was at 3 per cent., and other charges were $83·44. Find the dealer's rate of gain.

116. My house is worth $5000, I insure it for 3 years for $3800 at 85 cents per $100. Calculate my annual premium.

$3\frac{1}{2}$ per cent.
; 112. His
tment.

noney in $3\frac{1}{2}$
nt. stock at
ipital.

0, and pur-
my income.

ed the pro-
income was

at $97\frac{1}{2}$; he
ey in R. R.
ne increased

of goods for
ey at a cost
he received.

per lb., on a
proceeds in
r lb. Find

cost \$1·40
at \$1·20, to
0 per cent.,
third at $4\frac{1}{6}$
as at 3 per
ealer's rate

3 years for
remium.

---

117. Describe the process of insuring property.

118. For what must I insure my barns, worth \$2500, at $1\frac{1}{4}$ per cent., so that in case of loss I may recover \$2000 and the premium?

---

119. With a ruler and pole measure the length, width and height of the school-room, and calculate how many pupils the room should hold, so that each may have 200 cubic feet of air to breathe.

120. How many square yards of ground are there in the school grounds, not reckoning that covered by the side-walk and the buildings?

121. How much lumber, not counting the fence posts, has it taken to fence the school-yard?

122. How much wood is there in the pile at the school-house, and what is its cost at $\$3{\cdot}62\frac{1}{2}$ per cord?

123. How far is it between the opposite corners of the school-room floor?

124. How many gallons does a milk can hold which is 24 inches in diameter and 36 inches deep?

125. *ABCD* is an irregular field of 4 sides, so that *AB* is 51 yards, *BC* is 40 yards, *CD* is 75 yards, and *DA* is 68 yards, and so that the corners at *A* and *C* are square corners. Find its surface.

126. A bin is 6 feet square and 5 feet deep. Standing upright in each corner is a 2×3 inch scantling, to which the boards are nailed. How many bushels will it contain?

127. An even bushel of coal weighs 66 lbs. How high up in a coal bin 15 feet by 12 feet will 20 tons fill it?

128. How many barrels of water will a tank hold whose average diameter is 12 feet and height 3 feet, inside measurement?

129. Calculate the quantity of wheat (in bushels) in a conical pile 16 feet in diameter and 4 feet 6 inches high.

130. A circle is 4321 inches in circumference. Find its diameter to 5 figures.

131. The surface of a circle is 314·16 square inches. Find its circumference.

132. A globe is 2 feet in diameter. Find its volume and also its surface.

133. Find the side of a cube whose volume is 1906·624 cubic inches.

134. Find within the hundredth part of an inch the edge of a cube whose volume is a bushel.

135. If it cost $11·20 for paper for the walls of a room 25 feet 3 inches long, 19 feet 9 inches wide, and 12 feet high, when the paper is 27 inches wide, find the cost of the paper per yard, (no allowance for doors or windows).

136. What is the cost of polishing a cylindrical marble pillar 2 feet 6 inches in diameter and 12 feet long, at $1·25 per square foot?

137. A square field, containing 16 acres 401 square yards, has a walk 4 yards wide around it inside the fence. Find the area of the walk in yards.

138. How many bricks, 9 inches long, $4\frac{1}{2}$ inches wide and $2\frac{1}{2}$ inches thick, will be required to build a wall 45 feet long, 17 feet high and 4 feet thick, supposing the mortar to increase the volume of each brick $6\frac{1}{4}$ per cent.

139. Find the side of the largest square stick of timber that can be sawed from a 30-inch log.

140. A rectangular piece of ground, whose sides are as 2 is to 3, containing 15 acres, is fenced at a cost of 45 cents per rod. Find whole cost.

141. A hollow cylinder 4 feet long has its outside and inside diameters 3 feet and 2 feet 6 inches. Find its whole surface and its volume.

142. If iron is $7\frac{1}{2}$ times as heavy as water, find the weight of an iron cannon ball 1 foot in diameter.

143. A circular race-course, 4 rods wide, is to be laid out in a square field containing $22\frac{1}{2}$ acres, so as to be as long as possible. Find its length measured along its middle.

144. Use the method of the last articles to find 5 figures of $(3{\cdot}1415926)^2 \times 1{\cdot}92043$.

145. Simplify to 5 figures $\sqrt{2} \times \sqrt[3]{3}$.

146. Simplify to 6 figures $(1{\cdot}0345)^8$.

---

147. What is a number?

148. What is number?

149. What is a quantity?

150. What is quantity?

151. Has a number magnitude, so that we may truthfully say that one number is greater than another?

152. Has a quantity magnitude?

153. What is Arithmetic?

ANSWERS.

EXERCISE I. (PAGE 14.)

1. Let the student actually perform these operations.

3. (*a*) "A load of wheat is got (derived) from a bushel of wheat in the way that is told by the number 25."

4. (*a*) The number of beans in the handful when counted by one bean is (say) 534.

EXERCISE II. (PAGE 16.)

1. (*a*) Five hundred and thirty-two, etc.

(*b*) One hundred and eighty-two thousand, three hundred and fourteen, etc.

(*d*) One billion, two hundred and thirty-four million, five hundred and sixty-seven thousand, eight hundred and ninety, etc.

2, (*a*) 268. (*c*) 936268. (*e*) 259234513.
(*b*) 936. (*d*) 300004002.

EXERCISE III. (PAGE 19.)

1. (*a*) 65, 120, 714, 5151. (*c*) 36121.
(*b*) 3369. (*d*) 9225544.

2. An apple × 2022. 3. 911456. 4. 80661.

EXERCISE IV. (PAGE 23.)

1. 20, 40, 30, 80, 75, 50, 25, 36, 79, 81, 64, 58, 15, 1, 11, 12 and 89.

2. (*a*) 4598.
(*b*) 45.
(*c*) 73737.
(*d*) 87655.
(*e*) 111110101.
(*f*) 702900.
(*g*) 9582690.
(*h*) 99786.

3. (*a*) A foot × 402.
(*b*) A dollar × 1075.
(*c*) A book × 24.
(*d*) A grain of sand × 108922.

4. An apple × 298,

5. 7 times.

6. 6 times, 746398.

7. A cent × 1273.

EXERCISE V. (Page 24.)

1. 1 = I.
2 = II.
3 = III.
4 = IV.
5 = V.
6 = VI.
7 = VII.
8 = VIII.
9 = IX.
10 = X.
11 = XI.
12 = XII.
13 = XIII.
14 = XIV.
15 = XV.
16 = XVI.
17 = XVII.
18 = XVIII.
19 = XIX.
20 = XX.
21 = XXI.
25 = XXV.
29 = XXIX.
40 = XL.
41 = XLI.
45 = XLV.
49 = XLIX.
79 = LXXIX.
83 = LXXXIII.
90 = XC.
95 = XCV.
99 = XCIX.

2. (*a*) CCCXXIX.
(*b*) CXLVIII.
(*c*) DCXCIII.
(*d*) MCDXXXVII.
(*e*) MMCIX.
(*f*) MMMXVI.
(*g*) MIX.
(*h*) CMXCIX.
(*i*) DCCCLXXXVIII.
(*j*) DCCLXXVII.
(*k*) CCCLVIII.
(*l*) CDXXI.

3. (*a*) 1666. (*b*) 94. (*c*) 980. (*d*) 119. (*e*) 144.

EXERCISE VI. (Page 30.)

1, (*a*) In 4200 the order of 4 is +3, and of 2 is +2.
(*b*) In 5321761 the order of 5 is +6, of 7 is +2, etc.
(*c*) 2 is in the order +8, etc.

2. +5, +4, +3, +1.

3. (*a*) 600.
(*b*) 50000.
(*c*) 70.
(*d*) 9000000.
(*e*) 100000.
(*f*) 4.
(*g*) 13.
(*h*) 28000.
(*i*) 14900000.
(*j*) 2000000.
(*k*) 12000.
(*l*) 560.

4. (*a*) +3. (*c*) +2. (*e*) +3. (*g*) +9.
(*b*) +5. (*d*) +4. (*f*) 0. (*h*) +7.

5. (*a*) 32000. (*c*) 140000. (*e*) 1000000. (*g*) 20000000.
(*b*) 90000. (*d*) 3000000. (*f*) 72000000. (*h*) 160000.

6. In 4 (*a*). Since the order of 2 in 200 is +2 and of 3 in 30 is +1, therefore the order of 6 in the product is (+1+2 or)+3, so that 200 × 30 = 6000, and so on.

7. 36, 4862, 30963, 1604.

8. 1728, 2064, 1284, 78572, 208704.

9. 6172835, 4938268, 3703701. 10. 845136.

11. 22511909, 19295922. 12. 2295702. 13. 5142848.

14. (*a*) 11776. (*c*) 16184. (*e*) 1106665. (*g*) 2068614.
(*b*) 25914. (*d*) 109620. (*f*) 2475775. (*h*) 2625205.

15. +6, see Article 28. 8 figures. One more than the order of the left-hand figure of the product.

16. (*a*) 39375612. (*c*) 844150524. (*e*) 8742416652.
(*b*) 97495568. (*d*) 117796978.

17. 63360. 18. 490. 19. 1118.

20. 988. 21. 3888. 22. 92715875.

23. (*a*) 18995. (*b*) 333164. (*c*) 1397.

24. (*a*) 3125. (*c*) 823543. (*e*) 387420489.
(*b*) 46656. (*d*) 16777216. (*f*) 4753771243.

25. 19900. 26. 2345. 27. 106322.

28. 5247957. 29. 66795 cents.

30. 220244960 matches.

EXERCISE VII. (PAGE 38.)

1. (*a*) 7000. (*c*) 30. (*e*) 600. (*g*) 5000000.
(*b*) 90. (*d*) 60. (*f*) 5000. See Art 35.

2. See Article 36. 87. 3. 48869.

4. (*a*) 1069608, 713072, 534804, 267102.
   (*b*) 1257. (*c*) 81803. (*d*) and (*e*) are the same.

5. 1760. 6. 4200 sheaves, 14 loads.

7. 4 matches. 8. 99.

9. (*a*) $649\frac{1}{5}$. (*d*) $9222\frac{2}{9}$. (*f*) $785\frac{3}{8}$
   (*b*) $9512\frac{7}{8}$. (*e*) $1071\frac{1}{2}$. (*g*) 701.
   (*c*) $1428\frac{4}{7}$.

10. (*a*) 204. (*c*) $1040\frac{1}{8}$. (*e*) $1669\frac{2}{5}$.
    (*b*) 743. (*d*) $3053\frac{3}{8}$. (*f*) 8000.

11. See Article 33. 12. $11896\frac{1}{4}$, $1189\frac{25}{40}$, $118\frac{385}{400}$, $11\frac{3585}{4000}$.

13. $100634\frac{31}{60}$, $1006\frac{2071}{6000}$, $7547\frac{471}{800}$, $67\frac{8071}{9000}$, $6038\frac{71}{1000}$.

EXERCISE VIII. (PAGE 41.)

1. (*a*) 6. (*c*) 144. (*e*) 4345.
   (*b*) 48. (*d*) 4632. (*f*) 1309.

2. (*a*) $222\frac{110}{629}$. (*c*) $85\frac{64586}{84162}$. (*e*) $13\frac{32730}{61723}$.
   (*b*) $151\frac{1308}{5389}$. (*d*) $11031\frac{550}{725}$. (*f*) $8\frac{77997}{96301}$.

3. (*a*) $\frac{1234567}{4321} = 285\frac{3082}{4321}$. (*b*) ....

4. (*a*) 3. (*b*) 3. (*c*) 4.

5. 390. 6. 1178. 7. 729. 8. 102.

9. (*a*) 582978. (*b*) 400205. (*c*) 700403.

10. (*a*) $126\frac{1011}{4307}$. (*b*) $7\frac{4685}{13617}$. (*c*) $8487\frac{57266}{67298}$.
    (*d*) $32\frac{635660}{998001}$.

11 See Article 33. 12. 54 times, 2311 inches.

13. $\frac{52763}{732} = 72\frac{59}{732}$.

EXERCISE IX. (PAGE 45.)

1. See Article 43.

2. A pound × $68\dot{8}113$. 3. A \$ × $8\dot{9}75$.

4. $704\dot{3}46978$. 5. $81397\dot{2}65256$.

6. (*a*) A pound × $7\dot{7}879$. (*d*) A \$ × $144\dot{4}8$.
(*b*) A minute × $9\dot{2}47$. (*e*) A cubic foot × $31\dot{6}68$.
(*c*) A yard × $9265\dot{7}56$.

7. (*a*) $\dot{0}5$. (*d*) $\dot{0}09$. (*g*) $\dot{2}4995$.
(*b*) $\dot{1}7$. (*e*) $\dot{0}47$. (*h*) $\dot{0}0011$.
(*c*) $\dot{2}2$. (*f*) $\dot{0}0001$. (*i*) $1898\dot{5}794$.
8. A gallon × $2\dot{3}8$. (*j*) $1111\dot{1}01$.

EXERCISE X. (PAGE 48.)

1. In $100\dot{0}304$, 1 is in the order + 3, 3 is in the order − 1, and 4 is in the order − 3, and so for the others.

2. $300\dot{0}$, $\dot{0}003$, $\dot{0}00003$, $3\dot{0}$, $\dot{3}$, $\dot{0}3$ and $30\dot{0}$.

3. $720\dot{0}$, $\dot{0}72$, $72\dot{0}$, $\dot{7}2$, $\dot{0}00072$, $72000\dot{0}$ and $7\dot{2}$.

4. $300\dot{0}$. $300000\dot{0}$, $\dot{0}30$, $3\dot{0}$, $\dot{0}00000030$, $30\dot{0}$, $\dot{3}0$ and $\dot{0}03$.

5. $293000\dot{0}$, $\dot{0}0293$, $2930\dot{0}$, $\dot{2}93$, $293\dot{0}$, $2\dot{9}3$ and $29\dot{3}$.

6. $\dot{0}0008200$, $8200\dot{0}$, $\dot{0}000082$, $8200000\dot{0}$, $\dot{8}2$, $82\dot{0}$, $820\dot{0}$ and $82000\dot{0}$.

7. (*a*) − 1. (*e*) − 2. (*i*) + 12. (*m*) + 614.
(*b*) − 2. (*f*) + 5. (*j*) 0. (*n*) − 1684.
(*c*) + 2. (*g*) − 4. (*k*) 0.
(*d*) − 9. (*h*) + 2. (*l*) − 7.

8. (*a*) − 5. (*e*) + 2. (*i*) + 2. (*m*) − 1310.
(*b*) − 14. (*f*) − 1. (*j*) + 2. (*n*) − 722.
(*c*) + 8. (*g*) + 10. (*k*) + 16.
(*d*) − 7. (*h*) − 12. (*l*) + 1.

9. (*a*) + 5. (*e*) − 2. (*i*) − 2. (*m*) + 1310.
(*b*) + 14. (*f*) + 1. (*j*) − 2. (*n*) + 722.
(*c*) − 8. (*g*) − 10. (*k*) − 16.
(*d*) + 7. (*h*) + 12. (*l*) − 1.

EXERCISE XI. (PAGE 49.)

1. (*a*) $\dot{1}2$. (*c*) $\dot{1}2$. (*e*) $\dot{0}56$. (*g*) $\dot{0}01$.
(*b*) [illegible] (*d*) $16\dot{0}$. (*f*) $1\dot{6}$. (*h*) $\dot{0}0004$.

2. (*a*) $3\dot{0}12$.
   (*b*) $\dot{0}71706$.
   (*c*) $38\dot{7}099$.
   (*d*) $41\dot{1}2832$.
   (*e*) $5\dot{0}94432$.
   (*f*) $2\dot{8}63656$.
   (*g*) $1811\dot{1}3$.
   (*h*) $\dot{0}31328064$.
   (*i*) $8160\dot{1}739$.
   (*j*) $446\dot{8}43002983$.

3. 198.

4. A cubic inch × $873\dot{1}131$.

5. $8\dot{0}8962$ feet.

6. $1\dot{0}3977$.

EXERCISE XII. (Page 52.)

1. (*a*) $\dot{0}4$.
   (*b*) $\dot{0}08$.
   (*c*) 6000.
   (*d*) 6000.
   (*e*) 7000.
   (*f*) $\dot{0}008$.
   (*g*) 40000.
   (*h*) $\dot{0}00004$.
   (*i*) $\dot{0}04$.
   (*j*) 4000.
   (*k*) 20.
   (*l*) $\dot{0}005$.

2. (*a*) $\dot{0}3125$.
   (*b*) $\dot{0}21875$.
   (*c*) $\dot{0}1125$.
   (*d*) $\dot{0}72$.
   (*e*) $\dot{0}640625$.
   (*f*) $\dot{0}32$.
   (*g*) $\dot{0}015625$.
   (*h*) $\dot{2}416015625$.

3. (*a*) + 1. (*b*) − 2. (*c*) − 2. (*d*) + 2.

4. (*a*) $\dot{0}0863070$.
   (*b*) $25\dot{7}567$.
   (*c*) $37\dot{6}485$.
   (*d*) 640.
   (*e*) $10\dot{0}239$.
   (*f*) $\dot{0}0190032$.

5. (*a*) $\dot{0}5$.
   (*b*) $\dot{0}33333$,
   (*c*) $\dot{0}2$.
   (*d*) $\dot{0}14285$.
   (*e*) $\dot{0}11111$.
   (*f*) $\dot{0}090909$.
   (*g*) $\dot{0}076923$.
   (*h*) $\dot{0}058823$.
   (*i*) $\dot{0}052631$.
   (*j*) $\dot{0}0085470$.
   (*k*) $\dot{0}00090009$.
   (*l*) $\dot{0}0010010$.

6. $\dot{0}914401$.

7. $\dot{0}231716$.

8. $2\dot{6}4$.

9. $\dot{2}49726$.

10. $\dot{1}4765625$.

11. $1\dot{0}497$.

12. $2\dot{6}6970$.

13. $\dot{0}052013$.

14. $\dot{3}9434784$.

15. (*a*) $6\dot{5}44297$. (*b*) $\dot{6}76848$. (*c*) $\dot{7}34417$.

EXERCISE XIII. (Page 56.)

1. $9 = 3 \times 3$, $16 = 2 \times 2 \times 2 \times 2$, $24 = 2 \times 2 \times 2 \times 3$, and so on. Test your results by multiplying again.

2. $46 = 2 \times 23$, $76 = 2 \times 2 \times 19$, and so on.

3. (*a*) $2^9 \times 5$. (*d*) $2^5 \times 3^3 \times 5^2$. (*f*) 2^{10}.
(*b*) $2^6 \times 3^2 \times 5$. (*e*) $2^2 \times 3^3$. (*g*) $7 \times 11 \times 13$.
(*c*) 5^6.

4. Let the student prove his factors.

5. (*a*) 4×2, 4×3. (*b*) 12×3, 12×2.
(*c*) 10×3, 10×4. (*d*) 8×3, 8×4, and so on.
The greatest divisors are:
(*e*) 17. (*j*) 6. (*n*) 11. (*r*) 4.
(*f*) 21. (*k*) 7. (*o*) 50. (*s*) 600.
(*g*) 8. (*l*) 38. (*p*) 13. (*t*) 15.
(*h*) 7. (*m*) 6. (*q*) 15. (*u*) 625.
(*i*) 3.

6. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

7. 101, 103, 107, 109, 113, 127, 131, 137, 139, 141, 149, 151, 157, 163, 167, 181, 191, 193, 197, 199.

8. 1 foot, 2 feet, 3 feet, 5 feet, 6 feet, 7 feet, 10 feet, 14 feet, 15 feet, 21 feet, 35 feet, 30 feet, 42 feet, 70 feet, 105 feet.

9. A ℔. × 1, a ℔. × 2, a ℔. × 3, a ℔. × 4, a ℔. × 6, a ℔. × 11, a ℔. × 12, a ℔. × 33, a ℔. × 44, a ℔. × 66.

10. 7. 11. No. 12. 5 yards. 13. 825.

14. Cut them into equal parts.

EXERCISE XIV. (PAGE 59.)

1. (*a*) 12. (*c*) 8. (*e*) 111. (*g*) 40701.
(*b*) 2. (*d*) 27. (*f*) 375. (*h*) 12345679.

2. An inch × 121. 3. 14 feet, 880. 4. 3085.

5. 13 rods square, 35.

EXERCISE XV. (PAGE 62.)

1. (*a*) 200. (*b*) 360. (*c*) 3901. (*d*) 84.

× 11 × 13.

× 2.
, and so on.

) 4.
) 600.
) 15.
) 625.

, 47, 53, 59,

41, 149, 151,

feet, 14 feet,
5 feet.
, a lb. × 11, a

13. 825.

40701.
12345679.
. 3085.

84.

(*e*) 272. (*h*) 585. (*k*) 734877. (*n*) 219282.
(*f*) 162. (*i*) 1500. (*l*) 5688384. (*o*) 198257998536.
(*g*) 1089. (*j*) 13000. (*m*) 76724655

2. (*a*) 5040. (*c*) 8640. (*e*) 4800.
(*b*) 27720. (*d*) 10080.

3. (*a*) 137343465. (*b*) 21584640. (*c*) 105815565.

4. A foot × 1512. 5. 48 feet, 3 and 4 turns.

6. 792 feet, 132 lots. 7. 713.

8. 68590142. 9. 9828 seconds; *A* 182 rounds, *B* 156 rounds, *C* 117 rounds, *D* 108 rounds.

10. Use ruler and compasses. 11. 8 times the longer line.

12. 685377 inches. 13. An inch.

14. 84 feet, 62 times. 15. 384 inches, 3053 plots.

15. 27720 lbs.

EXERCISE XVI. (PAGE 66.)

1. See Article 65. 2. 75, ·3125, ·1125, ·136, ·0153125.

3. A lb. × 1·4125. 4. $\frac{7}{9}$. 5. See Article 65.

6. ·390625. 8. *A*'s farm × 1·6.

EXERCISE XVII. (PAGE 68.)

1. (*a*) $\frac{3}{4}$. (*e*) $\frac{5}{6}$. (*i*) $\frac{2}{3}$.
(*b*) $\frac{2}{3}$. (*f*) $\frac{7}{5}$. (*j*) $\frac{6}{7}$.
(*c*) $\frac{7}{12}$. (*g*) $\frac{1}{4}$. (*k*) $\frac{39}{34}$.
(*d*) $\frac{3}{4}$. (*h*) 4. (*l*) $\frac{3}{5}$.

2. (*a*) $\frac{5}{6}$. (*b*) $\frac{221}{370}$. (*c*) $\frac{5}{32}$.

3. (*a*) $\frac{31}{7}$. (*e*) $\frac{33573}{8}$. (*i*) $\frac{1594475}{193214}$.
(*b*) $\frac{46}{9}$, (*f*) $\frac{5750}{7}$. (*j*) $\frac{193574}{517}$.
(*c*) $\frac{15634}{11}$; (*g*) $\frac{9710}{8214}$. (*k*) $\frac{642892}{99}$.
(*d*) $\frac{24601}{30}$. (*h*) $\frac{7431}{2196}$. (*l*) $\frac{30129812}{3571}$.

4\. (*a*) $60\frac{3}{7}$. (*d*) $16\frac{124}{1351}$. (*g*) $176\frac{3464}{12345}$.
(*b*) $4\frac{4}{23}$. (*e*) $19\frac{163}{200}$. (*h*) $1\frac{1}{7963}$.
(*c*) $30\frac{19}{36}$. (*f*) $631\frac{243}{843}$. (*i*) $3\frac{3}{33332}$.

5\. See Article 67. 6. 5.2069.

EXERCISE XVIII. (Page 70.)

1\. (*a*) $\frac{5}{6}$. (*c*) $\frac{22}{15}$. (*e*) $\frac{31}{36}$. (*g*) $\frac{163}{60}$. (*i*) $\frac{3}{35}$.
(*b*) $\frac{13}{12}$. (*d*) $\frac{7}{6}$. (*f*) $\frac{77}{60}$. (*h*) $\frac{173}{144}$. (*j*) $\frac{395}{576}$.

2\. (*a*) $2\frac{3}{4}$. (*d*) $\frac{1}{36}$. (*f*) $\frac{23}{91}$. (*h*) $45\frac{5}{6}$. (*j*) $14\frac{80}{99}$.
(*b*) $26\frac{1}{12}$. (*e*) $\frac{47}{480}$. (*g*) $201\frac{20}{63}$. (*i*) $150\frac{7}{12}$. (*k*) $7\frac{251}{288}$.
(*c*) 736.

3\. A metre × $3\frac{41}{72}$. 4. $\frac{21}{1600}$. 5. A yard × $\frac{7}{11}$.

6\. The property × $\frac{7}{24}$. 7. $\frac{4}{15}$ of the work. 8. $\frac{89}{280}$.

9\. $2\frac{5}{6}$. 10. $\frac{165807}{110620}$.

EXERCISE XIX. (Page 75.)

1\. (*a*) $\frac{1}{2}$. (*d*) $\frac{12}{25}$. (*g*) 45. (*j*) 22. (*m*) $162\frac{3}{5}$.
(*b*) $\frac{3}{7}$. (*e*) $\frac{9}{26}$. (*h*) $12\frac{7}{24}$. (*k*) $12\frac{4}{7}$. (*n*) $14\frac{58}{143}$.
(*c*) $\frac{10}{27}$. (*f*) $15\frac{5}{9}$. (*i*) $\frac{4}{15}$. (*l*) $156\frac{29}{56}$. (*o*) $4\frac{4}{1001}$.

2\. (*a*) $\frac{2}{13}$. (*c*) $3\frac{1}{3}$. (*e*) $41\frac{19}{20}$.
(*b*) 40. (*d*) $1\frac{1}{11}$. (*f*) 0.

4\. His money × $\frac{9}{20}$. 5. The goods × $\frac{5}{18}$.

6\. The whole work × $\frac{1}{35}$. 7. 315 cents.

8\. 100 acres. 9. $\frac{15}{104}$.

10\. $\frac{6}{5}$. 11. *A's* farm = *C's* farm × $\frac{8}{9}$.

12\. $6\frac{258}{1109}$. 13. *A*, \$28 ; *B*, \$16.

14\. The younger \$4268·75; the elder \$5976·25.

15\. $140\frac{5}{8}$ lbs., $109\frac{3}{8}$ lbs.

16\. *A*, \$380 ; *B*, \$684 ; *C*, \$399.

17\. *A*, \$700 ; *B*, \$1120 ; *C*, \$1600.

18\. Father, \$35 ; mother, \$25 ; a son, \$15 ; a daughter, \$10.

19\. *A*, \$240 ; *B*, \$187·50 ; *C*, \$150 ; *D*, \$200.

EXERCISE XX. (PAGE 77.)

1. $\frac{9688}{835425}$.
2. $1\frac{113}{240}$.
3. $4\frac{23}{64}$.
4. 1.
5. A cwt. × $\frac{14}{195}$.
6. $\frac{314513}{362880}$.
7. A pound × $\frac{11}{20}$.
8. A £ × $2\frac{13}{60}$.
9. 9.
10. $\frac{912247}{3757000}$.
11. $60000.
12. ··2347776.
13. 2·44.
14. $\frac{9}{13}$.
15. 119.
16. 149668.
18. $110\frac{23}{216}$.
21. 31622777.
22. 1·2499999.
23. *A*, $2500 ; *B*, $2300 ; *C*, $2050 ; *D*, $1750 ; *E*, $1400.
24. $433\frac{1}{3}$ lbs , 65 lbs., $43\frac{1}{3}$ lbs.
25. $94 85.
26. *A*, 144 cents ; *B*, 351 cents ; *C*, 156 cents.

EXERCISE XXI. (PAGE 83.)

1. $11·36.
2. $314\frac{7}{27}$ feet.
3. $2·24.
4. $216.
5. $62\frac{362}{1109}$.
6. $1\frac{3}{4}$ minutes.
7. $1\frac{10}{11}$ days.
8. $63.
9. $4\frac{5}{11}$ days.
10. $15\frac{5}{6}$ days.
11. *A*, 15 days ; *B*, $22\frac{1}{2}$ days.
12. 5 days.
13. *B*, 100 seconds.
14. $5\frac{8}{11}$ days.
15. $\frac{12}{11}$.
16. $23·47$\frac{3}{16}$.
17. $69·08.
18. The merchant owes 80 cts.
19. $2·20.
20. $17\frac{1}{2}$ days.
21. 24 days.
22. 27 hours.
23. $7\frac{89}{93}$ hours.
24. $3\frac{1}{3}$ hours.
25. 40 cents.

EXERCISE XXII. (PAGE 94.)

1. 41880*d*.
2. $8655\frac{5}{6}$*s*.
3. £$12\frac{71}{80}$.
4. 36751875 square inches.
5. 44425044 square inches.
6. $\frac{5509}{58080}$ acres.
7. 2454 sq. rods, $3\frac{1}{2}$ sq. yards.
8. $14\frac{7039}{19602}$ square rods.
9. $\frac{179}{200}$ cwt.
10. $3\frac{55}{64}$ bushels.
11. 17·780 gallons.
12. 65400.
13. 11700.
14. $\frac{613}{2016}$ weeks.
15. $\frac{651}{800}$.
16. $\frac{19}{32}$ tons.
17. ·951807.
18. An acre × 3 + a square rod × 49 + a square yard × 6 + a square foot × 8 + a square inch × $119\frac{1}{13}$.

19. An hour × 14 + a minute × 24 + a second × 10.
20. An acre × $\frac{7}{8}$.   21. £5 15*s*. 8$\frac{1}{4}$*d*.   22. ·68589.
23. £9 7*s*. 11·1606*d*.   24. 8*s*. 9*d*.   25. $699·76.
26. A £ × 88 + a *s*. × 15 + a *d*. × 11$\frac{37}{73}$.
27. A £ × 1457 + a *d*. × $\frac{15}{32}$.
28. An acre × 757 + a square rod × 92.
29. A furlong × 2 + a rod × 6 + a yard × 4 + an inch × 1$\frac{1}{2}$.
30. $\frac{23}{39}$ of ($\frac{3}{5}$ of 18 acres, 80 square rods).   31. 80 lbs.

EXERCISE XXIII. (Page 100.)

1. (*a*) $60.   (*c*) $9·02.   (*e*) $12·18.
(*b*) $6·50.   (*d*) $69·44.   (*f*) $37·17.

2. (*a*) $27.   (*c*) $15·49.   (*e*) $6·18.
(*b*) $4·50.   (*d*) $5·07.

3. $15·07.   4. 4 months   5. 5 per cent.
6. $1680.   7. 4 months.   8. $\frac{75}{77}$.
9. 19$\frac{3}{4}$.   10. $\frac{1876}{1825}$.   11. $800.
12. $\frac{5}{27}$.   13. $632·50.   14. $2·41.
15. January 18th next year.   16. September 14th previous year.
17. $12·37.   18. $400.   19. $666·36.
20. 10$\frac{1}{5}$ per cent.   21. £4. 0*s*. 10*d*.   22. $351·04
23. $422·50.   24. 12 months.   25. 9$\frac{201}{211}$ per cent.

EXERCISE XXIV. (Page 107.)

1. $37·09.   2. $195·87.   3. $9·28, $509·28.
4. $321·49.   5. $400.   6. $172·80, $5·18.
7. $39.   8. $1399·80, $17, $16·80.
9. $849·53.   10. May 20th.   11. $520·49.
12. $1514·74.   13. $3439·39.

EXERCISE XXV. (Page 112.)

1. 1·1449.   2. ·1025.   3. 1·259712.
4. ·16985856.   5. 1·21483632.   6. ·146072.

7. \$453·69.

8. (*a*) \$99·30. (*b*) \$92·68. (*c*) \$38·79. (*d*) \$184·20. (*e*) \$153·76.

9. (*a*) \$33·38. (*b*) \$49·46. (*c*) \$85·65. (*d*) \$182·71.

10. \$551·73.

11. \$200.

12. \$1014·61.

13. \$334·69.

EXERCISE XXVI. (PAGE 115.)

1. In $4\frac{37}{56}$ months.
2. March 27th.
3. In $9\frac{67}{111}$ months.
4. November 1st.
5. July 21st, \$3062·84.
6. November 12th.

EXERCISE XXVII. (PAGE 119.)

1. \$3477·50.
2. \$96.
3. 25 per cent.
4. \$20.
5. $\$200·51\frac{1}{2}$.
6. \$17.
7. \$5180·22.
8. $6\frac{2}{3}$ per cent.
9. \$6400.
10. $46\frac{5}{8}$ per cent.
11. 36 cents.
12. $\$64·26\frac{1}{4}$.
13. Loss \$50·79.
14. \$8·75.
15. \$375.
16. $33\frac{1}{3}$ per cent.
17. $3\frac{7}{31}$ per cent.
18. $2\frac{26}{37}$ per cent.
19. 27·1 per cent.
20. 22 per cent.
21. \$7 23.
22. 36·85 cents.
23. $52\frac{1}{2}$ cents.
24. \$9259·26.
25. \$326·32.
26. \$1600.
27. $62\frac{1}{2}$ per cent.
28. $6\frac{14}{31}$ per cent.
29. \$356·15.
30. \$29·37.
31. $\frac{300}{897}$ per cent. loss.

EXERCISE XXVIII. (PAGE 124.)

1. *A*, $\$66\frac{2}{3}$; *B*, \$80 ; *C*, $\$93\frac{1}{3}$.
2. *B*, \$200·20 ; *C*, \$286.
3. \$1480.
4. *A*, $\$4685\frac{5}{7}$; *B*, $\$3748\frac{4}{7}$; *C*, $2905\frac{5}{7}$.
5. $13\frac{1}{2}$ months.
6. \$2469·57.
7. *A*, \$24·39 ; *B*, \$51·22 ; *C*, \$24·39.
8. *A*, \$131·66 ; *B*, \$55·59.
9. *A*, \$1030·44 : *B*, \$1373·91 ; *C*, \$1545·65.

EXERCISE XXIX. (PAGE 127.)

1. (*a*) \$69·36. (*b*) \$69·89. (*c*) \$3·12. (*d*) \$7·52.
2. $17777\frac{7}{9}$ lbs. of tea. 3. 31 cords, 50 cubic feet of wood.
4. \$1009·37. 5. \$2055·30, 10120 lbs. of tea.
6. \$38·16. 7. \$3150.

EXERCISE XXX. (PAGE 132.)

1. \$7425, \$360. 2. \$5000 stock or 50 shares.
3. \$5194. 4. \$216·25.
5. \$24000, \$15000 stock. 6. $101\frac{1}{2}$.
7. The latter by $\frac{4}{5}$ per cent. 8. \$$92\frac{1}{3}$.
9. No change. 10. 120.
11. The latter by \$57·72. 12. \$25 decrease.
13. \$1278·30. 14. \$397·50.
15. \$1440. 16. $153\frac{3}{5}$.

EXERCISE XXXI. (PAGE 134.)

1. \$160·93. 2. \$116·38. 3. \$7781·48.
4. \$156·63. 5. \$84.

EXERCISE XXXII. (PAGE 136.)

1. (*a*) \$16·25. (*b*) \$97·12$\frac{1}{2}$. (*c*) \$70·31$\frac{1}{4}$.
2. \$1784·25. 3. \$15893·33. 4. \$1920.
5. \$5101·89. 6. \$3600. 7. $\frac{125}{186}$.
8. \$29500, \$19750. 9. $\frac{64}{175}$ per cent. 10. \$1419·82$\frac{1}{2}$.

EXERCISE XXXIII. (PAGE 138.)

1. 27 mills, \$320. 2. \$2600. 3. \$206250.
4. \$29999·39. 5. 16 mills. 6. \$9·90.
7. 5 mills. 8. \$825·83. 9. \$2474·22.
10. \$9750.

EXERCISE XXXIV. (Page 143.)

1. (*a*) 1600.
(*b*) 490000.
(*c*) 64000000.
(*d*) 90000000000.
(*e*) 14400000000.
(*f*) ·16.
(*g*) ·000049.
(*h*) ·0064.
(*i*) ·0000000009.
(*j*) ·000001.
(*k*) 40000.
(*l*) ·000001.
(*m*) ·01.
(*n*) ·81.
(*o*) ·00000000000144.

2. (*a*) 8.
(*b*) 80.
(*c*) 900.
(*d*) 30.
(*e*) 11000.
(*f*) ·3.
(*g*) ·08.
(*h*) ·002.
(*i*) ·01.
(*j*) ·011.
(*k*) 60.
(*l*) ·6.
(*m*) ·005.
(*n*) ·2.
(*o*) ·0948683.

3. (*a*) 24.
(*b*) 55·9017.
(*c*) 125.
(*d*) 903.
(*e*) 829.
(*f*) 1234.
(*g*) 270.
(*h*) ·311961.
(*i*) ·00632455.
(*j*) ·316228.
(*k*) 15·2082.
(*l*) ·447213.
(*m*) 1·414213.
(*n*) ·00316228.
(*o*) ·158114.

4. (*a*) $\frac{2}{3}$. (*b*) $\frac{5}{8}$. (*c*) $\frac{9}{11}$. (*d*) $\frac{8}{7}$. (*e*) $2\frac{1}{4}$.

5. (*a*) ·4.
(*b*) ·78446.
(*c*) ·70710.
(*d*) ·089442.
(*e*) 4·4441.
(*f*) 1·0513.
(*g*) 1·00503.
(*h*) ·81649.
(*i*) ·25546.
(*j*) ·26306.
(*k*) ·47434.
(*l*) ·40000.

6. $12\frac{1}{2}$ per cent. 7. 20 per cent. 8. 351 pages.

EXERCISE XXXV. (Page 148.)

1. (*a*) 8000.
(*b*) 125000000.
(*c*) 343000000000.
(*d*) 729000000000000.
(*e*) ·027.
(*f*) ·000001.
(*g*) ·000000064.
(*h*) ·000000000125.
(*i*) ·001.
(*j*) ·000000512.
(*k*) 512000000.
(*l*) 343000.
(*m*) 1000000000000.
(*n*) ·000000000001
(*o*) ·125.
(*p*) ·008.

2. (*a*) 200. (*d*) 800. (*g*) ·02. (*j*) ·09.
(*b*) 50. (*e*) ·4. (*h*) ·03. (*k*) ·5.
(*c*) 30. (*f*) ·6. (*i*) 90. (*l*) 5.

3. (*a*) $\frac{4}{5}$. (*d*) $\frac{20}{9}$. (*g*) $2\frac{1}{2}$.
(*b*) $\frac{9}{8}$. (*e*) $\frac{4}{35}$. (*h*) $2\frac{2}{3}$.
(*c*) $\frac{5}{3}$. (*f*) $\frac{4}{25}$. (*i*) ·009.

4. (*a*) 45. (*d*) 686. (*g*) 4·2.
(*b*) 32. (*e*) ·34. (*h*) ·016.
(*c*) 245. (*f*) ·103.

5. (*a*) 4·481. (*d*) ·23207. (*g*) 4.
(*b*) ·4308. (*e*) 19·389. (*h*) 1·856.
(*c*) ·9283. (*f*) 23·207. (*i*) ·8617.

6. (*a*) ·893903. (*d*) 1·25992. (*g*) 2·15443.
(*b*) 1·46459. (*e*) 1·44225. (*h*) 4·64158.
(*c*) 4·97932. (*f*) 1·58740. (*i*) ·464158.

EXERCISE XXXVI. (Page 150.)

1. $5\frac{1}{2}$ per cent. 2. 20 per cent. 3. 142450.
4. \$174·80. 5. 19 per cent. 6. 20 per cent.
7. 8·243216 per cent. 8. 3·923 per cent. 9. $(120061)^2$.
10. 18225. 11. 11·2 per cent. 12. 60 rods.

EXERCISE XXXVII. (Page 156.)

1. $1\frac{1}{24}$ sq. yds. 2. $24\frac{1}{9}$ sq. yds. 3. 55 acres.
5. 292 sq. inches. 6. $36\frac{1}{3}$ sq. feet. 7. \$7·43$\frac{1}{3}$.
8. 40 feet. 9. 64 sq. feet. 10. 66 feet.
11. 60 yards long by 40 yards wide. 12. \$15·53$\frac{1}{3}$.
13. $16\frac{1}{2}$ miles. 14. \$31·50. 15. 16 sq. miles.
16. \$787·88.
17. $10\frac{1}{2}$ inches by $7\frac{1}{2}$ inches. 18. $13\frac{1}{3}$ rods.
19. A sq. mile × 102 + an acre × 633 + a sq. rod × 16
+ a sq. yard × $7\frac{1}{4}$.

EXERCISE XXXVIII. (PAGE 160.)

1. 50 rods. 2. 87·452 rods. 3. 40 inches.
4. An inch × ·00496389.
5. 34 feet. 6. 124·535 rods. 7. 116·66 rods.
8. A sq. inch × 97·428. 9. 108 feet.
10. 32·829 rods.

EXERCISE XXXIX. (PAGE 164.)

1. (*a*) 7854 sq. feet. (*d*) ·00020106 sq. miles.
(*b*) ·00080425 sq. yards. (*e*) ·110447 sq. inches.
(*c*) 119·597 sq. inches. (*f*) 16·619 sq. feet.
2. 318·309 yards. 3. 28·546 rods. 4. 78·54 sq. ins.
5. 17·7245 inches. 6. 1979·2 sq. feet. 7. $517·70.
8. $30·69. 9. 9·102 rods.
10. $2\frac{1}{4}$ feet diameter, 35 circles, 38·02 sq. feet.
11. 488·69 sq. yards. 12. 144·867 rods.

EXERCISE XL. (PAGE 168.)

1. 261·77. 2. 42·416. 3. 734·2.
4. 472. 5. 70·3125. 6. $23·56.
7. $9\frac{1}{6}$ miles. 8. 1·136 inches, 128·499 tons.
9. $45\frac{17}{96}$. 10. $19360. 11. 3 ft. 6·785 ins.
12. 22 inches. 13. 5 ft. $6\frac{2}{3}$ ins. 14. 17·28 hours.

EXERCISE XLI. (PAGE 173.)

1. $4\frac{4}{5}$ inches. 2. 13·0525. 3. 579·4.
4. 13090. 5. 931 cubic ins. 6. 55·85.
7. 65·86. 8. 38·19 inches. 9. 40·79.
10. $10\frac{1}{8}$.

EXERCISE XLII. (PAGE 174.)

1. $13\frac{1}{8}$ acres. 2. 56 rods. 3. 33·09 lbs.
4. ·003 sq. inches. 5. 389·71 sq. ins. 6. 1072·3 cub. ins.

7. 12·407 inches. 8. (*a*) 9, (*b*) 27. 9. (*a*) 4, (*b*) 16.
10. (*a*) $\sqrt{2}$, (*b*) $2 \times \sqrt{2}$.
11. $70\frac{10}{17}$ inches. 12. 58 miles.
13. 374·12 square inches. 14. ·00084.
15. 347·18 square feet. 16. 36·14 square inches.
17. 70·22 inches.

EXERCISE XLIII. (Page 178.)

1. 25·9325 sq. metres. 2. 201·945 miles. 3. 4·48 litres.
4. 125664 ares. 5. 7068·6 cm. 6. 21598·5 cub. m.
7. 40447 Km. 8. 628·32 Kg. 9. 3·938 metres.
10. 1121 mm.

EXERCISE XLIV. (Page 183.)

1. (*a*) 12. (*b*) 9. (*c*) 8.
2. (*a*) − 2. (*b*) − 4. (*c*) − 9. (*d*) + 8. (*e*) − 3.
3. (*a*) 8050·82. (*c*) ·000361840. (*e*) 262·563.
(*b*) 16·2883. (*d*) 24269400000000.
4. 1658·0. 5. 152·4156. 6. $\dot{0}355$.
7. ·065457. 8. 11·1568. 9. 25·362.
10. ·00340718. 11. 385·895.
12. A square mile × 2·4679. 13. 1417·0.
14. 1·306447. 15. 18·3793 cubic feet.

EXERCISE XLV. (Page 186.)

1. (*a*) 36·082. (*c*) ·000078105. (*e*) ·31831.
(*b*) ·00037976. (*d*) 7·9578.
2. ·0243178. 3. 43·600. 4. 292·23.
5. 14753·9. 6. 1·0780206.

EXERCISE XLVI. (PAGE 186.)

3\. See Article 26. 15. $-3,\ +1,\ -5,\ +2,\ -1,\ -2$.

25\. 12. 26. $+7$. 27. 182208 yards.

37\. 2531·29 + 38. $PQ \times 25$. 39. 257307.

42\. 13. 43. 1·580964. 44. $4199.

45\. $314·08. 46. ·01222.... 47. ·31799.

48\. (*a*) 947, (*b*) ·189, (*c*) ·0015625. 49. 906·704.

51\. $\frac{37}{38}$. 52. $\frac{20998}{151}$, $\frac{283351}{571}$, $\frac{670147}{811}$.

53\. 2496. 54. 49896. 5. $\frac{1}{52}$.

56\. 779·01. 57. $\frac{17779}{39960}$. 58. 44·777.

59\. 221. 60. \$93·39$\frac{1}{2}$. 61. $126 81.

62\. 1$\frac{2}{5}$ days. 63. $600, $700.

64\. *A*, $1580 ; *B*, $1896 ; *C*, $2133. 65. $39·35.

66\. 25$\frac{5}{9}$ gallons. 67. $81·15. 68. $202·69.

69\. \$857·48. 70. \$948·57$\frac{5}{12}$. 71. 825 times.

72\. 38. 73. 15. 74. 10$\frac{1}{5}$ per cent.

75\. 265 days,10 per cent. 76. $502·20.

77\. June 10th. 78. Jan. 6th, 1898. 79. $1373·42.

80\. $3454·49. 81. $162·60. 82. $681·56.

83\. $157·49. 84 $24·47.

86\. The former by $2·30. 87. $1373·28.

88\. 7$\frac{11}{37}$ per cent. 89. (*a*) \$19·56, (*b*) \$19·89.

90\. $3500. 91. $376·73. 92. April 7th.

93\. $10·50. 94. $22. 95. $5·60.

96\. 24$\frac{2}{3}$ per cent. 97. 52·482 lbs. 98. 21$\frac{2}{3}$ per cent.

99\. \$1065·60. 100. Loss \$22$\frac{2}{9}$.

101\. Credit price = cash price $\times \frac{51}{50}$.

102\. Credit price = cash price $\times \frac{50}{49}$.

103\. \$4076·16, $\frac{431}{910}$ of the property.

104\. *A*, $253·46 ; *B*, $319·38 ; *C*, $407·74.

105\. *B*, $755·11 ; *C*, $1126·47, $20558·79.

106\. *A*. $4507·93 ; *B*, $5642·07. 107. 7$\frac{3}{11}$ per cent.

108\. $650, $350. 109. $32828. 110. $0.

111. $77000 stock.
112. $30.
113. $3493·66.
114. 14513 lbs.
115. 6 per cent.
116. $10·77.
118. $2025·32.
124. 58·74 gals.
125. 3234 sq. yds.
126. 139·58 bush.
127. 4 ft. 3.86 ins.
128. 67·13 bbls.
129. 235 bush.
130. 1375·4 ins.
131. 62·832 ins.
132. 4·1888 cub. feet, 12·5664 sq. feet.
133. 12·4 ins.
134. 13·04 ins.
135. 7 cents.
136. $117·81.
137. 4400 sq. yds.
138. 49152 bricks.
139. 21·21 ins.
140. $90.
141. 73·434 sq. feet, 8·6394 cub. feet.
142. 244·73 lbs.
143. ½ a mile 15 rods 15 feet.
144. 18·954.
145. 2·0396.
146. 1·31173.
147. See Articles.
151. No.
152. Yes.

93·66.
·77.
sq. yds.
3 bbls.
32 ins.
ins.
·81.
1 ins.
. feet.

173.