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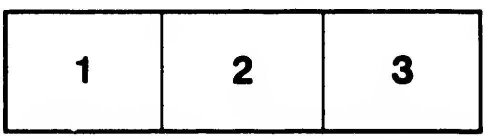
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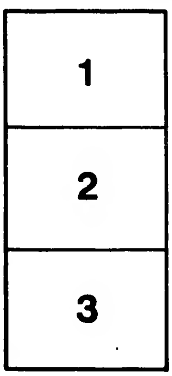
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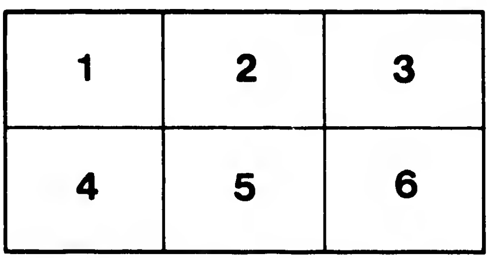
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A VERY INTERESTING SELECTION

OF

IMPORTANT MATHEMATICAL PROBLEMS

WITH SOLUTIONS

Designed as an Appendix or Supplement

TO

Arithmetic and Mensuration.

BY

A. DOYLE.

OTTAWA:
Ottawa Printing Co., (Limited), 3 and 5 Mosgrove Street.
1898.

PREFACE.

This valuable variety of useful exercises, is estined to inspire students with an ardent desire r more extended mathematical attainments than ose acquired from a limited study of abridged ementary school-books. With a view of pro- oting intellectual progress, I have given many eorems of great utility, with the greatest ssible variety of useful problems and their lutions, in a limited space. No two are alike, id from each, a rule or formula may be deduced r the working of similar questions. Those ho have acquired a knowledge of algebra and eometry, will find these exercises really attractive s a source of profitable recreation.

This little work, containing elaborate solutions f all its exercises[3] comprehends more proposi- ons than the first four books of Euclid. It must ndoubtedly secure a wide circulation and meritori- s success. The principal propositions have een contributed by the distinguished mathe- atical correspondents of the Canadian Almanac nd Journal of Education; selected and solved y their Mathematical Editor, the Author.

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The

IMPORTANT MATHEMATICAL PROBLEMS.

1. Find the area of a triangle whose sides are, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$. Ans. $\sqrt{3\frac{1}{2}}$.
2. The square inscribed in a circle : square in a simicircle :: 5 : 2.
3. The square inscribed in a semicircle : square in quadrant :: 8 : 5.
4. If an isosceles triangle inscribed in a circle have each of its sides double of the base, the squares described upon the radius of the circle and one of the sides of the triangle, shall be to each other as 4 : 15.
5. If r denote the radius of a circle, the side of the inscribed square will be $r\sqrt{2}$, and the side of the circumscribed square will be $2r$.
6. If a denote the side of a given square, rad. of inscribed circle shall be $\frac{1}{2}a$, and radius of the circumscribed circle will be $\frac{a}{2}\sqrt{2}$.

The rectangle under two sides of any triangle is equal to the rectangle under the perpendicular to the base and diameter of the circumscribing circle.

7. Circle.————. The rectangle under the hypothenuse and ⊥ of a right angled triangle is equal to the rectangle under the sides.

8. The perimeter multiplied by half the radius of the inscribed circle is = area of triangle, radius of inscribed circle = $2a \div P$, a = area and P = perimeter.

9. The continued product of the 3 sides of a triangle = 2 area × diameter of the circumscribing circle ∴ diam. = $\frac{a\,b\,c}{2 \text{ area}}$

10. The square on the diameter of a globe = 3 times side of the inscribed cube.

11. If r denote radius of a circle, side of the inscribed regular decagon = $\frac{1}{2} r (\sqrt{5} - 1)$.

12. If r denote the rad. of circle, the side of the inscribed regular pentagon will be $\frac{1}{4} r \sqrt{10 - 2\sqrt{5}}$

13. If a denote a side of a given regular pentagon, rad. of circumscribed circle will be = $\frac{1}{10} a \sqrt{50 + 10\sqrt{5}}$.

14. The sum of the sides of right angled triangle divided by 5 = ⊥ from the r ∠

15. The square on the side of an equilateral triangle inscribed in *a* circle $= 3r^2$

16. The ⊥ of an equilateral triangle is equal 3 times the radius of the inscrib. circle.

17. The side of a square inscribed in an equilateral triangle is = the excess of 4 times the ⊥ height of the triangle above its perimeter.

18. If the line bisecting the vertical angle of a triangle be divided into two parts which are to each other as the base to the sum of the sides, the point of division is the center of the inscribed circle.

19. Given the base, the area and line bisecting the base of a triangle, to determine the remaining parts.—Let AB = 16; bisecting line CE = 11, area = 82; then $82 \div 8 = 10\frac{1}{4} = \perp$ DC.

$\sqrt{11^2 - 10\frac{1}{4}^2} = 3.99218 = DE$; then

$8 - 3{\cdot}99218 = 4{\cdot}00782 = AD \therefore$

$\sqrt{AD^2 + DC^2}\ 11.006 = AC$.

In like manner BC may be found.

20. Given the area of right angled triangle 48 and difference of the base and ⊥, 4, to find the

sides. Let x denote the base, then $x+4=$ ⊥ $\frac{(x+4)\,x}{2}=48$ ∴ $x^2+4x=96$; hence $x=8$ and $x+4=12=$ the ⊥ ∴ $\sqrt{12^2+8^2}=\sqrt{208^2}$ $=14.4222=$ hypothenuse.

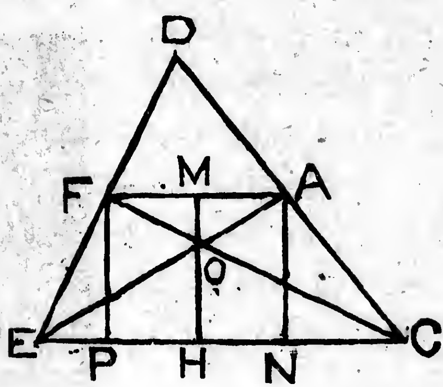
21. Find the length of a straight line bisecting a given triangle from a given point in one of its sides.

Let A C $=14$; BC $=13$ and A B $=15$; area $=84$ and A E $=2$. Let B F $=x$, then $15\times13:13\times::2:1$ ∴ $x=7\frac{1}{2}=$ B F, and C F $=5\frac{1}{2}$. A G $=8\frac{2}{5}$; G B $=6\frac{3}{5}$ ∴ $\sqrt{14^2-8\frac{2}{5}^2}=$ ⊥ C G $=11\frac{1}{5}$; $8\frac{2}{5}-7\frac{1}{2}=.9=$ DG; $\sqrt{.9^2+11.2^2}=$ C D $=11.236102$. Area of triangle EFB $=42$; $15-2=13=$ BE; ∴ $42\div6\frac{1}{2}=$ ⊥ F M $=6.46154$; G B $=6.6$ As $11.2:6.6::6.46154:$ B M $:3.80769$; $13-3.80769=9.19231=$ E M. Then

$\sqrt{9.19231^2+6.46154^2}=$ E F $=11,23615$ length of the bisector.

22. Given one side of a triangle and the lines drawn from the angles at it, bisecting the other two sides, to find the sides.

Let $CE = 16$; $AE = 12$ and $CF = 15$. $\frac{2}{3} CF = CO = 10$ $\therefore$ $OF = 5$; $\frac{2}{3} AE = EO = 8$, $\therefore$ $AO = 4$. On CE describe the triangle CEO and produce EO to $A = 12$, and CO to $F = 15$. Join AC, FE and AF; then AF is $\parallel CE$ and $= \frac{1}{2}$ of it, $= 8$. In triangle AOF, $8 : 9 :: 1 : 1\frac{1}{8}$ diff. of segts. AM and MF; $\therefore$ $4\frac{9}{16} = MF$, and $3\frac{7}{16} = MA$. $\sqrt{4^2 - 3\frac{7}{16}^2} = MO = 2.04538$. Again, $16 : 18 :: 2 : 2\frac{1}{4}$ = diff. of segts, CH and HE; then $CH = 9\frac{1}{8}$, and $HE = 6\frac{7}{8}$. $\sqrt{10^2 - 9\frac{1}{8}^2} = HO = 4.09076 + 2.04538 = 6.13614 = HM = AN$. $CH - AM = 9\frac{1}{8} - 3\frac{7}{16} = CN = 5\frac{11}{16}$. $\sqrt{HM^2 - CN^2}$ $\sqrt{6.13614^2 + 5.6875^2} = AC = 8.36653 \times 2$



$= 16.73316 =$ CD. MF $= 4\frac{9}{16}$; HE — MF $= 6\frac{7}{8} - 4\frac{9}{16} =$ PE $= 2\frac{5}{16}$. $\sqrt{PE^2 + HM^2} = \sqrt{2\frac{5}{16}^2 + 6.13614^2} = 6.55743 =$ E F; ∴ E F $\times 2 = 13.11486 =$ ED.

23\. A pole 16 feet high is broken by the wind in D; in falling DC touches the ground $2\frac{1}{2}$ feet from base of the pole; or the base of right angled triangle and sum of the hypotenuse and perpendicular are given, to find the sides.

Let x denote AD; AC = 16; AB $= 2\frac{1}{2} = a$; AC $= b$; DB $= b - x$. Then $x^2 + a^2 = b^2 - 2bx + x^2$; ∴ $2bx + a^2 = b^2$, and $x = \frac{b^2 - a^2}{2b}$; ∴ AD $= \frac{16^2 + 2\frac{1}{2}^2}{2 \times 16} = 7.8046875$; DB or DC $= 8.1953125$.

24\. If a line bisecting the vertical angle of a triangle, be divided into parts which are to each other as the base to the sum of the sides, the point of division is the center of the inscribed circle.

25. In any triangle, it is required to inscribe a rectangle whose sides shall bear a given ratio to each other.— Let ABC be the given triangle; AB=b; ⊥ CD=h, and side of rect < ⊥ to base=x, and adjacent side = nx, n

denoting the given ratio. Then b ; h :: nx : $h - x$; ∴ $x = \frac{b\,h}{b + n\,h}$ = G H, one side of the required rectangle.

When side of the inscribed square is required. As $b : h :: x : h - x$ ∴ $x = \frac{b\,h}{h + b}$.

26. A farmer has a triangular field, the distances from whose three angles to the middle of the opposite sides are: 110, 140, and 160 yards, respectively. Required the area of the field? Put AD $= b$ (110), E C = 160 (d); B C = x : AB = y; AC = z. Then $z^2 + y^2 = 2b^2 + \frac{1}{2} x^2$; $y^2 + x^2 = 2d^2 + \frac{1}{2} z^2$; $z^2 + x^2 = 2z^2 + \frac{1}{2} y^2$ The sum of these gives $2x^2 + 2y^2 + 2z^2 = 2b^2 + 2c^2 + 2d^2 + \frac{1}{2}$

$x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2$; and $x^2 + z^2 + y^2 = \frac{4}{3}b^2 + \frac{4}{3}c^2 + \frac{4}{3}d^2$; from this subt. the first

equation; then $x^2 = \frac{3}{4}d^2 - \frac{2}{3}b^2 + \frac{4}{3}c^2 - \frac{1}{2}x^2$, or $9x^2 = 8c^2 + 8d^2 - 4c^2$; $9z^2 = 8b^2 + 8c^2 - 4d^2$; whence $x = \frac{1}{3}\sqrt{8c^2 + 8d^2 - 4b^2} =$ C B $= 186.547 +$ $y = \frac{1}{3}\sqrt{8b^2 + 8d^2 - 4c^2}$; $z = \frac{1}{3}\sqrt{8b^2 + 8c^2 - 4d^2}$ $\therefore$ 186.547, 157.48, 129.615 = Sides.

27. A stone is weighed in a pair of scales which are known to be incorrect; when placed in one scale, it weighs $71\frac{3}{7}$ lbs; but being put in the other, it only weighs $37\frac{4}{5}$ lbs; required its true weight.

$\sqrt{71\frac{3}{7} \times 37\frac{4}{5}} = \sqrt{2700} = 51.9615$. (nean proportional.

28. The base of a triangle is 80, and sides including the vertical angle are 65 and 55 perches

respectively; required the length of a line drawn from a point within the triangle, 8.53 perches from the side AB, so as to cut off $\frac{5}{7}$ of the area. CB = 65 = f; BF = x; ratio 5 : 7; AB = 55 = b; $m+n = s = 12$; AC = 80 = d; BE = $9\frac{2}{7}$ = g; EP = 58.67 = p; AP = 8.53 : CG = 59.71; BH = 8.53; EH = 3.67. By similar triangle s, we have : $g + x : p :: x : \frac{p\,x}{g+x}$ = BI. Then B I × BF : BA × BC :: $m : m+n$. $\frac{p\,x^2}{g+x} : b\,f :: m : s$. Then bmf $= \frac{s\,p\,x^2}{g \times x}$, and $S\,p\,x^2$ = bmfx + bmfg ∴ x = 32.6172 = BF ∴ FI is easily found.

29. In a given triangle, the base AC = 100; AB = 70; BC = 90; what is the length of a line (1) drawn ∥ to the base. (2), ⊥ to the base (3), inclined to the base at a given ∠ 15°, so as to cut off $\frac{7}{11}$ of the area?—

(1). Let x denote the length of the line ∥ base AC: then (XIX.VI. cr) 100^2; x^2 :: 3059.41 : $\frac{7}{11}$ of 3059.41. From this we get x = 79.77 = QR.

29. (II.) Area of triangle B D C = 61.1882 × $\frac{66}{2}$ = 2019 2106. Let x = required ⊥, H L then, as B D^2 : x^2 :: B D C : $\frac{7}{11}$ × 3059.4 — 3744 : x^2 : : 2019 2106 : 1946.89$\frac{8}{11}$. From this proportion we obtain the value of x = ⊥ to base. A C = 60.08258 = H L.

(III.) The ∠ A = 60° 56′ 28″ : ∠ ACE = 15°, ∴ AE is found = 26.68, and E C = 90.11; triangle A E C = 1166.03, and C B E = 1893.38. From triangle A E C, cut off $\frac{4}{11}$ of area of whole by a line 11 C E; remainder will be $\frac{7}{11}$ of the whole.

$EC^2 : x^2 :: 1166.08 : \frac{4}{11}$ of 3059.41, $\therefore x =$ 88.0176; then $8119.812 : x^2 :: 1166.083 : 1112.152$; $\therefore x = 88.0176$, the line required.

(IIII.) Bisect the triangle by a line whose length is 49.32 perches.

$AC \times BC = 2\ KC \times HC$, or $2\,abx \times CH$, and $CH = \frac{ab}{2x}$. By similar triangles $b : d :: \frac{ab}{2x} : \frac{abd}{2bx} = CL$; $KL = KC - CL = X - \frac{abd}{2bx} = \frac{2bx^2 - abd}{2bx}$ $\therefore$

$$\frac{4b^2c^2x^2 - 4b^2x^2 - a^2b^2d^2 + 4ab^2dx^2}{4b^2x^2} = \frac{a^2b^2 - a^2b^2d^2}{4b^2x^2} : \therefore 4b^2x^4 - 4b^2c^2x^2 - 4adx^2 = -a^2b^2 \therefore x = 70.$$

30. There is a room in shape of a rhomboid whose adjacent sides are 12 and 7 yards respectively; the shortest diagonal is 11, find the length of the other. Sum the squares of the diagonals = sum of the squares of the sides $\therefore (12^2 + 7^2) \times 2 = 386$: then $(386 - 11^2) = 265$ then $\sqrt{265} = 16.28$.

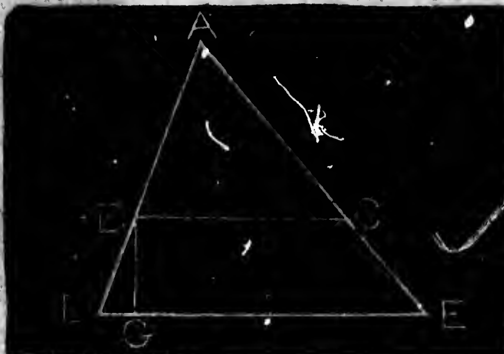
31. The base of an isosceles triangle is 30, and a segment of one of the equal sides made by a perpendicular from one of the base angles, on the opposite side is 10. Let

A D $= x$: $x + 10 =$ A B; $(x+10) \times 10 = \frac{30^2}{2}$ $\therefore$ $x = 35$, and A B or A C $= 45 - \sqrt{45^2 - 15^2} = 42.4264 = \perp \times 15 = 636.$-3961, the area.

32. The radius of a circle is 10, find the sides of an isosceles triangle inscribed in it having the base equal one half each of the other sides. Let x denote the side. The square on

the radius : that upon one of the sides : : 4 : 15 $\therefore$ $15r = 4x^2$. $r^2 : x^2 :: 4 : 15$, $\therefore$ $x = \frac{r}{2}\sqrt{15}$

33. The three sides of a triangle are 15, 14 and 13, how far beyond the base must the sides (14 and 13) be produced so as to form a trapizium containing an area equal $2\frac{1}{2}$ times that of the given triangle.



Area of triangle A B C $=84 \times 2\frac{1}{2}=210=$ trapezium B D E C $+ 84=294$ $BC^2 : DE^2 :: 84 : 294$: whence $DE = 28.0624$. $\therefore (15+28.0624) \div 2 = \frac{1}{2}$ sum of II sides, B C and D E $= 21.5312$. $210 \div 21.5312 = 9.7523 = \perp$ BG.

34. A stick of fimber is 4 inches wide at one end; and 8 inches at the other, and 12 feet long, where must it be cut, so that one half may be at each side of the cut? AD = 4 : BC = 8 : AE or PN = 144; AP = EN = BE = 2 inches.

When BA and CD are produced to meet in V, it is easily shewn that BA = AV, and PV

= PN ; ∴ NV = 288. Then $4 \times 288 = 1152$ = area of the triangle VBC, and $1152 \div 4 = 288$ = triangle AVD. NV = 288. $1152 - 288 = 864$; then $432 + 288 = 720$ = triangle VGH $1152 : 720 :: 288^2 : VL^2$; $8 : 5 :: 82944 : VL^2$ ∴ $VL^2 = 51840$, and VL = 227.684 + ; $227.684 - 144 = 83.684$ = PL ∴ LN = 60.316. $BC^2 : GH^2 :: 1152 : 720$: from this proportion we find the dividing line GH = 6.32455 inches.

35. Given the differences between the diagonal and side of a square to find the side. Rule.—Square the diff., double it, extract the $\sqrt{\ }$, and add the diff. to it. When sum of diagonal and side is given to find the side. Rule.—Square the sum, double it, extract the $\sqrt{\ }$ and subtract the sum fram the last result.

36. Given the sum of the diagonal and longer side of a r < d parrallogram, to describe it, when

the square of the diagonal is equal $(n+1)$ times the square of the shorter side. Let AB = sum. From B draw BC ⊥ to AB =, and make $BC^2 = n\ AB^2$. Join AC, and make CD = CB. From D erect ⊥ D E, meeting A B in

E ; join E C. Then $EB^2 + BC^2 = ED^2 + DC^2$. ∴ $EB^2 = ED^2$, and EB = ED. Then AE is the 11^m required. Let $AB^2 = 25$; then $BC^2 = 125$; n being denoted by 5. $\sqrt{25+125} = \sqrt{150}$ AC = 12.24744, and $\sqrt{125} = 11.18034$. $\sqrt{25} = 5$. 12.24744—11.1803 = 1.0671 = AD. Now in similar triangles ADE and ABC, we have 11.18034 : 5 : : E D : 1.0671, ∴ 5 E D = 11.18034 × 1.0671 = 11.930540814 ; then E D = 11.930540814 ÷ 5 = 2.386108^2 = 5.693511 ; $1.0671^2 \times 5 = 5.693512$.

37. The sides of a triangle are 26, 28, and 30, what must be the sides of a similar triangle

containing $3\frac{1}{4}$ times its area? Area of given triangle = 336 : 336 × $3\frac{1}{4}$ or as 1 : $3\frac{1}{4}$: : 26^2; x^2 . ·. x = 46.87216. In like manner, we find AB = 50.4777 and BC = 54.08329.

38. The perimeter of a *r* < *d* triangle, is 74.4, and ⊥ from the *r* < on hypothenuse is 14.88; find the sides.—Let P = Perimeter; CD = *a*; AC = x; BC = y.

Then AB = P—(x + y), and $AC^2 + BC^2 = AB$; whence $x^2 + y^2 = P^2 - 2P(x+y) + x^2 + 2xy + y^2$ transpose and divide by 2 and $P(x+y) - \frac{1}{2}P^2 = xy$ (I). By similar triangle s, AB : BC : : AC : CD . ·. AB × AC, or $aP - a(x+y) = xy$. (II). By Substitution $(a+P) \times (x+y) = aP + \frac{1}{2}P^2$; whence $x + y = \frac{P(a+\frac{1}{2}P)}{a+P}$ or $y = \frac{P(a+\frac{1}{2}P)}{a+P} - x$. Substitute these values for (x + y) and y in Eq. (II.); the result when simplified and reduced, gives $(a+P)x^2 - P(a+\frac{1}{2}P)x = -\frac{1}{2}aP^2$. From last Eq and value of y above is found, x or $AC = \frac{P(a+\frac{1}{2}P)}{2(a+P)} +$

$\frac{P.}{2(a+P)}\sqrt{(a-\frac{1}{2}P)^2-2a^2}$ and if the result of the two sides be taken from P, the result will given AB = P $p-(x+y)=\frac{p^2}{2(a+p)}$ which expressions are = the values of the 3 sides of the triangle, which may be found to be AB = 31 ; BC = 18.6 ; AC = 24.8.

39. Given the difference between the side and ⊥ of an equilateral triangle to find the side. Let the side AB = x ; AD = y ; and x − y = 1.071797; then, $\sqrt{x^2-\frac{x^2}{4}}=y$, or $x-\sqrt{\frac{3x^2}{4}}=1.071797$, ∴ $x-\frac{x}{2}\sqrt{3}=1.071797$ ∴ $2x-\sqrt{3}=2.143594$ $2x-1.73205$; $x=2.143594$, ∴ $.26795\ x=\frac{2.143594}{.26795}$ and x = 8.

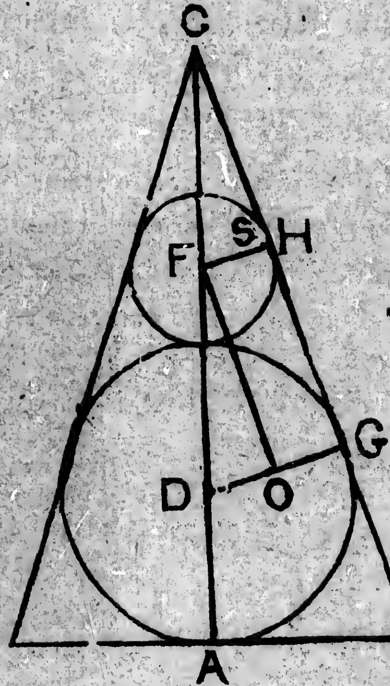
40. The inside dimensions of a school house are: the inside length is 3 feet more than 3 times the height ; the inside breath is 4 ft greater than twice the height ; and the inside surface of the walls is 508 less than the surface of the floor and ceiling together. Required the dimensions? Let y = height ; then 3 y + 3 = length ; 2 y + 4 = breadth. C y^2 × C y = area

of the sides; $4y^2 + 8y$ = area of ends; $12y^2 + 36y + 24$ = area of floor and ceiling. $12y^2 + 36y + 24 - 508 = 10y^2 + 14y$; and $y^2 + 11y = 242$. Solved gives $y = 11$, the height; $3x \times 11 + 3 = 36$, the length; $2 \times 11 + 4 = 26$, the breadth. W. D.

41. The wheel of a carriage is 5 feet in diameter, required the length of the successive cycloidal arcs generated by a nall in the circumference of the wheel, in a distance of 24 miles? By means of the calculus, we find the height of arc of a cycloid as follows: $y = \sqrt{2rx - x^2} + \text{vers}^{-1}x$, the equation of the curve; $\frac{dy}{dx} = \frac{r-x}{\sqrt{2rx-x^2}} + \frac{r}{\sqrt{2rx-x^2}} = \frac{\sqrt{2rx-x}}{x}$ $\therefore ds = dx \times \sqrt{1 + \frac{dy^2}{dx^2}} = \sqrt{2r} \cdot \frac{dx}{\sqrt{x}}$ $\therefore S = 2rS\, x^{-\frac{1}{2}}\, dx = 2\sqrt{2rx} + C$. When $x = O$, $S = O$, $\therefore C = O$, and when $x = 2r$, then the semicycloidal arc $= 2\sqrt{2r^2} = 4r$ and the whole length of the cycloid is $8r = 4$ times the diameter of the gener-

ating circle. $\therefore \dfrac{24 \times 4}{3.1416} = 30.55767$ miles travelled by the nail.

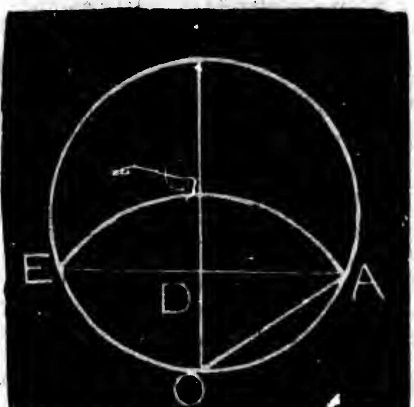
42. There are two circles inscribed in an isosceles triangle touching each other and the sides of the triangle; their diameters are 4 and 6 respectively. Find the sides.



From the centers D and F draw DG and FH perpendiculars to BC, and FO ∥ CB: draw CFDA. Put $DG = r$: $FH = s$: $DO = r - s = c$: $FD = r + s = b$. Then $FO = \sqrt{b^2 - c^2} = d$; $CB = a$. By similar triangles D

F O and B C A, we have $b : d :: a : \frac{a d}{b} = AC$; and $c : b :: r : \frac{r b}{c} = CD$, then $r + \frac{r b}{c} = AC = \frac{d a}{b}$. $\therefore a = \frac{b^2 r}{c d} + \frac{b r}{d}$.

43. Find the radius of a circle whose centre being taken in the circumference of another circle containing two acres, shall cut off one half its area? Radius of first circle = 20.1850118. Now, suppose A O = 11.7 (nearly); then 20.1850118 x = $(11.7)^2$ = 136.89 $\therefore$ x = 6.7817615 = height of the lower segment. Then 11.7 — 6.7817615 = 4.9182385 = height of upper segment.

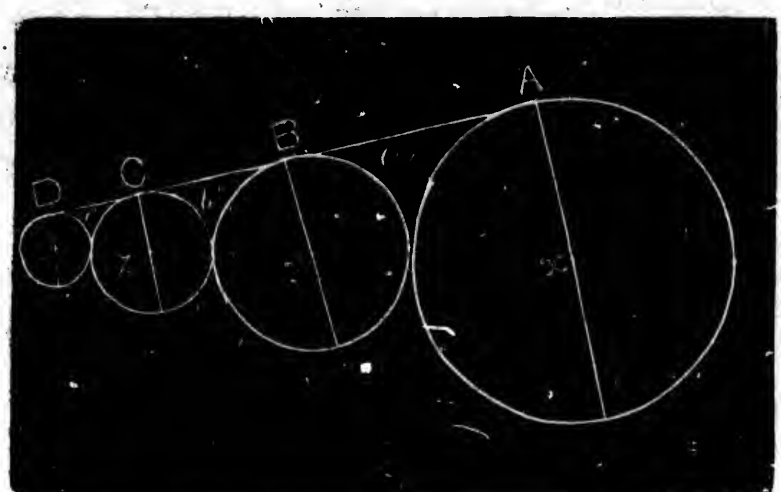


Then the area of upper segment A C E is found = 65.7318402, and area of lower segment A O E = 94.3903972 : sum of segments = 160.1222374, too much. Again, suppose 11.69, and proceed in like manner we have :

65.7299442268 = area of upper segment.
94.1699210470 = area of lower segment.

159.8998652738, too little; error = .1001347262 and .122237465. Using these errors by the rule of *trial and error*, the radius approximates to 11.6944789, and sum of segments = 160,000,000 + a decimal.

44. A line 16 inches long is a common tangent to 4 circles touching one another. It is divided



in the points of contact A B C D, such that $AB^2 = 3\,BC^2 = 5\,DC^2$. The difference between the diameters of the 1st and 4th circles is 7.311521963. Reqd. areas of the 4 circles.

$AB^2 = 3\,BC^2$, and $BC^2 = 5\,DC^2$ ∴ $3\,BC^2 = 15\,DC^2$, hence $AB^2 = 15\,DC^2$; $BC^2 = 5\,CD^2$, then $AB = \sqrt{15} \times DC$: $BC = \sqrt{5} \times DC$ ∴ $(\sqrt{15} + \sqrt{5} + 1) \times DC = 16$ and $DC = \frac{16}{\sqrt{15} + \sqrt{5} + 1} = c$. Then $AB = \sqrt{15} \times z = 8.71673739$. $BC = \sqrt{5} \times CD = 5.03261066992$ ∴ $CD = 2.2506$. Let x. y, z, and u represent the diameters respectively: then $xy = a^2 = 3b^2 = 3yz = 5uz$ ∴ $3y = 5u$, and $y = \frac{5u}{3}$; $x = 3z$ ∴ $u + d\ (7.3115219) = 3z$. ∴ $z = \frac{u+d}{3} \times u = c^2$ or $u^2 + du = 3c^2$. This Eq. gives $u = \frac{1}{2}\sqrt{d^2 + 12c^2} - \frac{d}{2} = 1.688478$-$037$; $z = \frac{c^2}{u} = 3.00001361$; $y = \frac{b^2}{z} = 8.442351418$: $x = \frac{a^2}{y} = 9$ then

$3.00001361^2 \times .7854 = \ldots\ 7.0686$ = area of 3rd circle.
$1.688478^2 \times .7854 = \ldots\ 2.2391$ = area of 4th circle.
$9^2 \times .7854 \ldots\ldots\ 63.6174$ = area of 1st circle.
$8.442351^2 \times .7854 = \ldots\ 55.978$ = area of 2nd circle.

45. An aged man two daughters had,
And they were very fair;
To each he gave a piece of land,
A circle and a square,
At twenty shillings an acre just
The land its value had;
The money that enclosed the whole
Just for the land was paid.
If every shilling be an inch,
As it is very near,
Required the acres in each piece
The circle and the square?

For the circle.—Let x denote the circumference in inches; then $x^2 \times$ ied by .0795775 = area in square inches, and 6272640 square inches in an acre $\therefore \frac{x^2 \times .0795775 \times 20}{6272640}$ = price of all. $= x = 3941214.54$ = circumference $\therefore 394214.54^2 \times .0795775 \div 6272640 = 197060 \cdot 7338$ = area of the circle.

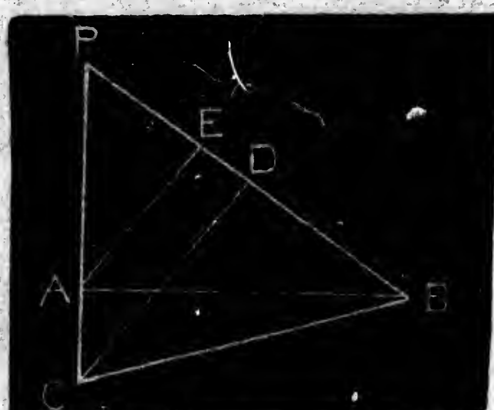
For the square.—$3941214.54 \div 20 = 197060.727$ = area as before for circle. Let x denote the side of square, then x^2 = area in inches. $\frac{x^2}{6272640} = 4x \therefore x = 1254528$ inches.

$\frac{1254528^2 \times 20}{6272640} = 4\ x \quad \therefore \quad x = 1254528.$

$1254528 \times 4 \div 20 = 250905.6$ = area of the square.

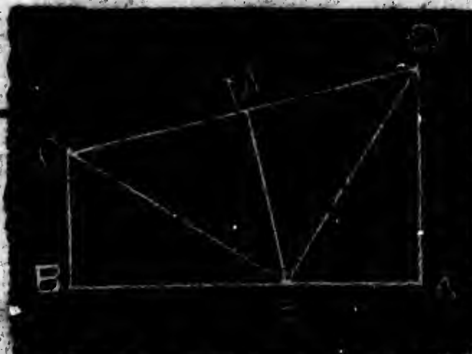
46. A tree standing in the water is just 15 feet above the surface. When the wind is blowing the tree is bent over and touches the surface 20 feet from where it stood. Find the length of the pole or tree.

Let the line A B represent the water ; C P the pole or tree. When P touches the water at B, C B must be equal C P $\therefore$ triangle B C P is isosceles — AB = 20 feet : AP = 15 $\therefore$ $\sqrt{20^2 + 15^2}$



= 25 = PB $\therefore$ PD or DB = $12\frac{1}{2}$; $\perp$ AE = 12, and PE = 9 ; then $9 : 12 :: 12\frac{1}{2} : 16\frac{2}{3}$ = CD. Then $\sqrt{CD^2 + PD^2} = \sqrt{12\frac{1}{2}^2 + 16\frac{2}{3}^2}$ = CP = $20\frac{5}{6}$, length CP, as required.

47. AD = 24; BC = 18; DE = EC; required ED, AB, AE, EB and DC. It is easily proved that triangles ADE and BCE are equiangular, and DE = EC ∴ they are = in every respect,



and AE = 18 : BE = 24; ∴ AB = 42; DE = 30; DC = 42.4264. The ⊥ from M, middle pt. of DC. finds pt. E.

48. The parallel sides of a trapezium are 20 and 26; the area is 996; find the ⊥ height.—Area ÷ by ½ sum of sides = ⊥ height.

49. To find the area a rhombus.—Multiply ½ of one diagonal by the other.

50. To inscribe a circle in a rhombus.—Intersection of diagonals is the center.

51. To find the ⊥ let fall from the *r* ∠ on the hypotenuse, when the sides are represented by 3, 4, 5, or *n* times these number, sum of 3 sides ÷ 5 = ⊥.

52. Find the diagonal of a cube, the length of whose side being 18 inches.

The square [illegible] ot of 3 [illegible] the square of side = the internal diagonal.

53. The length of a room is 18 feet; breath $13\frac{1}{3}$, and $10\frac{1}{2}$ feet high; required the distance from any angle of floor [illegible] the farthest corner of the ceiling? $\sqrt{18^2 + 13\frac{1}{3}} = 22.5$; [illegible] 8 [illegible] the required diagonal.

54. Find the size of the largest square stick that can be cut from a cylindrical piece of wood, $5\frac{1}{2}$ feet in circumference and $12\frac{1}{2}$ feet high. Inscribe a square in circle representing the base, and multiply its area by the height.

55. The sides of a trapezium inscribed in a circle are 40, 60, 80, and 67, to find the angles. Triangles A D O and B C O are similar ∴ AD :

BC : : OD : OC ; for a similar reason, AB : DC : : AO : OD : OB : OC ; hence we have the proportional lengths of AO, OB, OC, and OD that is AO = $1.79\frac{7}{17}$; OB = $.37\frac{37}{67}$; OC = 1, and OD = 2. The relative lengths taken two by two, give the rates AC ÷ BD, and AC × BD = AB × CD + AD × BC, calling *ac*, bd, the hypothetical values of the diagonals, we have bd : *ac* :: 7220 : AC^2 , or $2.89\frac{37}{67}$: $2.79\frac{7}{67}$ :: 7220 : AC^2 ∴ AC = 83.42 +, and 7220 ÷ 83.42 = 86.54. We have now the 3 sides of each triangle to find he ∠s ∴ ABC = 111° 29′ 42½″ ; DAB = 74° 50′ 17″ ; then ADC = 68″ 30′ 19½″, and BCD = 105° 9′ 43″.

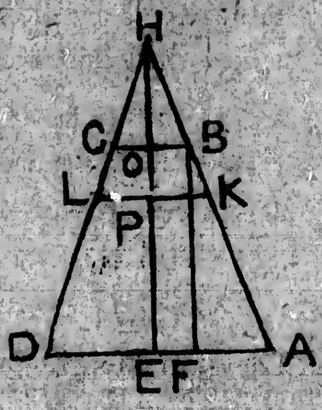
56. Given the diagonal of cube, 20.78461 feet ; find the length of the side or edge. The square of the diagonal of a cube is equal 3 times the square of the side.

57. A cone 28 inches high is bisected by a circle ‖ base ; how far from the vertex is this circle ? Let x = height of upper cone, cut off. Then $28^3 \cdot x^3 :: 2 : 1$ ∴ $2x^3 = 28^3$; $x^3 = 10976$ ∴ $x = 14\sqrt[3]{4} = 22.22$ in.

58. The slant-height of a cone is 12, and its solidity 521.1537408 : required the height ?

Let y denote the height, and $2x$ the diameter of the base; then $\sqrt{12^2 - x^2} = y$, and $4x^2 \times .7854 \times \frac{y}{3} = 521.1537408$. $3.1416x^2y = 1563.4612224 \therefore x^2 = \frac{1563 \cdot 4612224}{3.1416y} \therefore 144 - \frac{1563 \cdot 4612224}{3.1416y} = y^2$, and $y^3 - 144y = -497.664$, solved, $y = 9 \cdot 6$, the height.

59. A piece of square timber is 12 feet long; each side of the greater base is 11 inches and that of the less, 5 inches. What length must be cut off from the less end, so as to contain a solid foot? . . . . . AD = 11 ∴ AE = 5½; BC is 5; BO = 2½. Through B draw BF ∥ HE; then AF = 3, and FE = 2½; AF : FB :: AE : EH. or, 3 : 144 :: 5½ : 264 inches = EH; but EG = 144 ∴ OH = 120 inches. $5^2 \times 40 = 1000$ cubic inches; 1000 + 1728 = 2728 = solidity of pyramid KHL; $11^2 \times \frac{264}{3} = 10648$, solidity ADH. Let KL = x; then $11^3 : x^3$



:: 10648:2728; whence x^3 = 341 ∴ x = 6.9863, length of the dividing line As $2\frac{1}{2}$: 120 :: 3.4932 : 167.6736 = PH ; ∴ 167.6736 − 120 = 47,6736 = PO.

60. A bubble of air having a diameter of 4 inches, passes from the bottom of a lake to the top 120 fathoms. Required the diameter of the bubble on reaching the surface.

$\frac{120 \times 6 \times 12}{1728} \times 62\frac{1}{2} = 312\frac{1}{2}$ = pressure per square inch on the bubble. $4^2 \times 3.1416 = 50.2656$ = surface of the bubble. $312\frac{1}{2} \times 50.2656$ = 15708 = pressure on bubble at the bottom. $\frac{125}{1728} \times 50.2656 = 3.636$ = pressure of = size at the top. As 3.636 : 15708 :: 1 : 4320 ; $\sqrt{4320 \div 3.1416}$ = 11.13.

61. Two men purchase a circular race course, 1 mile in diameter, and divide it by a line // diameter ; required the length of the dividing fence, so that one may have ⅔ of the area, and the other ⅓ ?

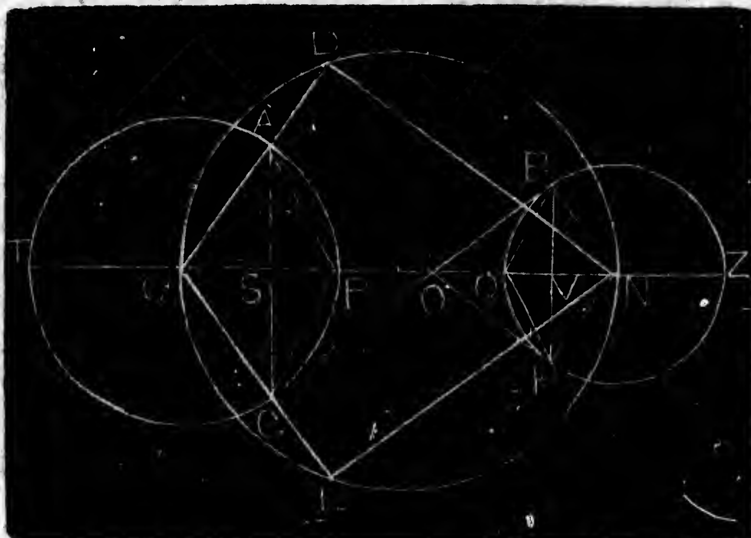
The area of a circle whose diameter is 1 mile, is .7854 mile, ⅓ of which is .2618 of a mile. The versed line answering to this area is .36753395 which taken from the diameter 1, leaves .63246605, remainder of diam. Then semichord is a mean proportional between the segments of the diam. ∴ we have : $2\sqrt{.63246605 \times .36753395} = .96426162$ mile $= 1697$ yards.

62. A can mow a field in 15 days by getting 7 days helph from B ; and B can mow it in 24 days by getting $2\frac{6}{7}$ days help from A. In what time could both working together mow it ?

B A A
As 1 $\frac{34}{83}$: 7 :: 1 : 5
A
A + B B ∴ 1 of A $= 1\frac{34}{85}$ of B As 1 : $2\frac{6}{7}$ :: 1 $\frac{34}{85}$: 4
A B
$15 + 7 = 24 + 2\frac{6}{7}$ $\frac{15}{20} : 1 :: 1 : \frac{1}{20} = 6\frac{140}{7}$ D As $\frac{3}{35}$ *w* : 1*d* :: 1 :
$11\frac{2}{3}$ = time req.*d*.
$2\frac{6}{7} + 7 = 7 + 2\frac{2}{7}$ $\frac{24}{28} : 1 :: 1 : \frac{1}{28} = \frac{5}{12} / 140 =$
$12\frac{1}{7} = 17$
$\frac{3}{35}$ *w*.

63. Find the length of a band to surround two wheels, the distance between whose centers being 14 feet, and diameters 10 and 6 feet respectively. C N = 14 ; C P = 5 ; Q N = 3. When tangents are on alternate sides of the circles, on the line joining centers, discribe a semicircle, and make C D = sum of the radii, = 8. Then $\sqrt{(14^2 - 8^2)} = 11.489125$, DN. The

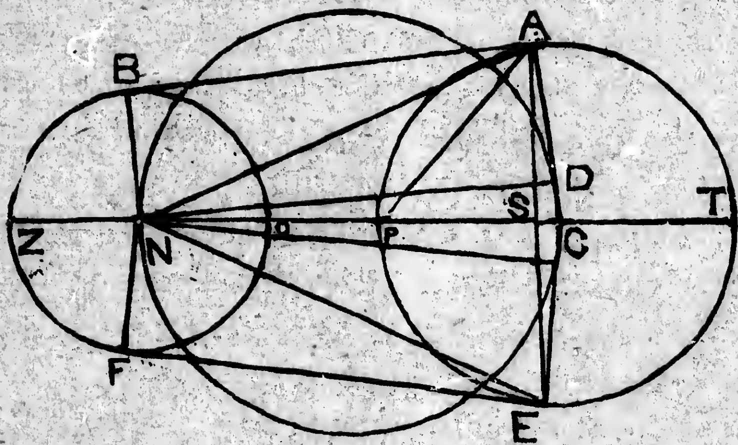
triangles C D N and O H N are similar; ∴ 8 : 11.489125 :: 3 : 4.308422 = H O, = B O;



∴ 11.489125 − 4.308422 = 7.180703. = A O, = G O. $\sqrt{(A\,O^2 + A\,C^2)}$ = C O = $\sqrt{(7.180703^2 + 5^2)}$ = 8.75. Then, in triangle C A O, 8.75 : 7.180703 + 5 :: 7.180703 − 5 : 3.035713 = diff. of segments = O S − C S; hence C S = 2.857144; and S O = 5.892856. $\sqrt{(5^2 - 2.857144^2)}$ = SA = SG = 4.10325. PS= $\sqrt{(2.142856^2 + 4.10325^2)}$ = 4.6291 = P A or P G = chord of ½ the arc A P G, and 2 A S = 8.206512 = chord of the whole arc A P G; hence the arc A P G = 9.608762. Circumference of the whole circle = 31.415926, −9.608762 = arc ATG, = 21.807164 + 2 × 7.180703 = 36.16857, length of band O A T G O. Again, AH−GO = OB or HO = 4.308422. NB or NH = 3; 14−8.75 = ON = 5.25. Then, in triangle OBN; the base : sum of sides &c &c ∴ 5.25 : 7.308422 :: 1.308422 : 1.821428 = diff. of the segts. ∴ OV = 3.535714; NV = 1.714286; QV = 1.285714. $\sqrt{(BN^2 - VN^2)}$ = B V = 2.461955. $(BV^2 + QV^2)^{\frac{1}{2}}$ = B Q = 2.77746 = chord of ½ arc B Q H. Then, chord of whole arc = 2×2.461955 = 4.92391. Hence, the arc B Q H may be found to be = 5.765225 .6 × 3.1415926 = 18.8495556. −5.76525 = 13.0843 = BZH, +2×4.308422 = 21.70114 = belt OBZH, +36.16857 = 57.86971, the entire length of the band.

64. When the tangents are not on alternate sides: CN = 14; CP = 5; NO = 3; PO = 6; CD = 2. Then, $\sqrt{(14^2 - 2^2)}$ = 13.855406 = D N = AB = EF. $\sqrt{(DN^2 + DA^2)}$ = AN = 14.177446. In triangle C A N, we have the base to sum of sides &c. &c. ∴ 14 : 19.177446 :: 9.177446 : 12.571427 = N S − C S, and the sum = 14, ∴ N S = 13.285713, and C S = .714287. Then

$\sqrt{}(5^2 - .714287^2) = SA = 4.94876, \times 2 = AE$
$= 9.897432$, chord of the whole arc APE. $\sqrt{}$



$(AS^2 + PS^2) = AP = 6.550353$ = chord of ½ arc APE. Hence, length of whole arc A P E = 14.168464. 3.1415926 × 10 -- 14.168464 = 17.246462 = arc ATE. PT being diameter; 10 : 6 :: 14.168464 : 8.5010784 = arc B Z F. A T E = 17.246462 ; B Z F = 8.501078 ; 2AB = 27.712812, ∴ 53.46135 = length of the whole band ATEFZBA.

65. A promissory note is offered for sale, on which is due $24,3225 interest for one year, at P per cent, simple interest. The owner of the note agrees to cancel the interest, and sell it for $23 less than the principal, which is at a

discount of the above per cent. Required the amount of note?

Let x = the amount; P = rate ℔ unit; n equal number of years. Then x P n = int. for n years at rate. Also 1+P n = amt. of $1 for 1 year. $1 + Pn : 1 :: xPn : \frac{xPn}{1+Pn}$ = discount.

$\frac{xPn}{1+Pn}$ = 23 ℔ question; x P n = 23+2 P n:

x P = 23 + 23 P .·. n – 1; but x P n = 24.3225; .·. 23+23 P=24.3225, and 23 P=1.3225 .·. P= .0575; but x P n=x×.0175×1=24.3225 .·. x= 24.3225÷.0575=amt. reqd = 423.

66. A boatman rows $6\frac{1}{2}$ miles down a river, and up again in 182 minutes, the stream having a uniform current of $2\frac{1}{4}$ miles an hour: find at what speed he can row in still water. Let x denote the rate in still water, and $x+2\frac{1}{2}$ = rate downwards; then $6\frac{1}{2}$—$(x - 2\frac{1}{4}) = 26 - (4x - 9)$ = time of ascent; but 182 minutes = $3\frac{1}{30}$ = 91 ÷ 30 = whole time; .·. (26 ÷ (4x + 9) + 26 ÷ (4x – 9) = 91 ÷ 30; x=$5\frac{1}{4}$, the answ.

67. The interest on a certain sum of money for 1 year, at simple interest, is 317.0465 and the discount on the same sum for the same time, at the same rate, is $297. Find the sum. Let

x denote the rate; then $317.0465 \times \frac{100}{x} =$ $31704.65 \div x$P. $100+x : x :: 31704.65 \div x : 297$. From this proportion, $x = 6.7496633$ $\therefore$ $31704.65 \div 6.7496633 = 4697$, the principle required.

68. Two railroad trains 109 and 111 feet long respeetively, are moving on parallel rails; when they move in opposite directions, they pass each other in $2\frac{1}{2}$ seconds; but when they move in the same direction, the faster train passes the other in 15 sec. Find the speed of the trains. Let x = ft. travelled by the faster per second; and y = ft. per second travelled by the slower train. Then, $\frac{109}{x+y} + \frac{111}{x+y}$ or $\frac{220}{x+y} = 2\frac{1}{2}$ = combined speed of both; and $2\frac{1}{2}x + 2\frac{1}{2}y = 220$ $\therefore$ $5x + 5y = 440$, and $x + y = 88$. Again, $\frac{109}{x-y} + \frac{111}{x-y}$ or $= \frac{220}{x-y} = 15$ $\therefore$ $15x - 15y = 220$, and $x - y = 14\frac{2}{3}$. We have now the sum and diff. $\therefore$ $x = 51\frac{1}{3}$, and $y = 36\frac{2}{3}$. $\therefore$ speed of faster per hour = 35 m. speed of the slower 25 miles.

69. A person changed a fifty dollar gold piece for 31 pieces of foreign coin; some of which were worth \$2.26 each; others \$3.01, and the rest 77 cents each; how many did he get of each sort?

Let $x+y+z=31$;

$$\begin{aligned} 226x+301y+77z &= 5000 \\ 77x+\ 77y+77z &= 2387 \\ \hline 149x+224y \qquad &= 2613 \end{aligned}$$

and $224y=2613-149x=$ a whole number; $\frac{x-y}{224}=$ a whole number $=P$. If $P=O$, $x=1$; $y=11$ and $z=19$. $\therefore$ the required numbers are 1, 11, and 19.

70. A cubic foot of gold weighs 11 cwt 10$\frac{6}{7}$ lbs, and a grain can be beaten out so thin as to form a leaf of 60.25 square inches; how many of these leaves will be required to form an inch in thickness?

144 : 175 :: 1110$\frac{6}{7}$: 1350 Troy $=$ 7776000 grains $\div$ 1728 $=$ 4500 grains in a cubic inch; but one grain covers 60.25 $\therefore$ 60.25$\times$4500 = 271125 leaves form an inch in thickness, and the thickness of a leaf is 1 $\div$ 271125.

71. A vessel [A] contains 20 gallons of wine; another [B] 8 gallons of water. How many gallons must be taken from each and poured

into the other, so that after repeating the process any number of times, the quantity of wine in each vessel may be the same as after the first operation?

WINE WATER
20, and 8; Let x denote the quantity
WATER
reqd. x + 20 − x, and 8 − x + x = first remainders. Then $\frac{19\,x}{20}$, and $\frac{380 - 19\,x}{20}$, and $\frac{54 - 7\,x}{8}$, $\frac{7x}{8}$: $\frac{8 - x}{20}$, and $\frac{x}{8}$ .... $\frac{x}{20}$; $\frac{20 - x}{26}$ ∴ $\frac{20 - x}{20} = \frac{380 - 19\,x}{20} + \frac{x}{8}$; or, $800 - 4\,x = 760 - 38\,x + 5\,x$ ∴ $7\,x = 40$, and $x = 5\frac{5}{7}$.

Otherwise.—Let x gallons to be taken from [A]; y = number to be taken from [B]; then $20 - x : y :: 20 : 8$; but $x = y$ ∴ $x = 5\frac{5}{7}$.

72. An Arab tent is composed of canvas a yard wide and $\frac{1}{86}$ inch thick; and when not pitched is wound on a pole 2 inches in diameter, forming 115 rounds of cloth; how many yards in the tent?

$$2.04 \times 3.1416 = 6.403864$$
$$4.56 \times 3.1416 = 14.325696$$
$$20.73456 \times 115$$

$= 2384.4744$ in. $= 66.2354$ yards.

Otherwise.—$115 \times \frac{1}{25}$ = 4.6 inches = thickness of cloth ; then $2 \times 4.6 + 2 = 11.2$ = diameter of pole and cloth ; ∴ $(11.2)^2 - 2^2 = 121.44 \times .7854 =$ 95.378976 inches, surface of end of cloth ; then $95.378976 \div 36 = 2.649416 \div \frac{1}{25} = 66.2314$.

73. A grain merchant having a quantity of barley, sold $\frac{1}{5}$ of it at a certain gain per cent ; ⅔ at twice that gain, and the remainder at 3 times the gain on the first lot. He gained upon the whole 30%. What was the gain on each lot ? Let P denote the required price ; r = rate per cent.

Then $\frac{P}{5} \times \frac{R}{100} = \frac{PR}{500}$ = profit on the first lot.

$\frac{2P}{3} \times \frac{2R}{100} = \frac{2PR}{300}$ = profit on the second.

$\frac{2P}{15} \times \frac{3R}{100} = \frac{2PR}{500}$ = profit on the third.

Their sum is $\frac{29PR}{1500}$ = profit on the whole ;

∴ $\frac{29PR}{1500} = \frac{30P}{100}$; $29r = 450$ ∴ $r =$ $15\frac{15}{29}$ = profit % on the first.

$31\frac{1}{29}$ = profit % on the second.

$46\frac{16}{29}$ = profit % on the third.

74. If A had travelled $\frac{2}{11}$ mile an hour faster, he would have finished his journey in $\frac{37}{49}$ of the time ; but if he had travelled $\frac{2}{11}$ mile an hour slower he would have been $1\frac{31}{35}$ hours longer

on the road. How many miles did he travel?

Let x = miles travelled; y = miles per hour. Then $\frac{x}{y}$ = time he couldfinish; y + $\frac{2}{11} : x :: 1 : \frac{11x}{11y+2} = \frac{37x}{39y}$, or, $429xy = 407xy + 74x$, or, $22xy = 74x \therefore 11y = 37$, and $y = 3\frac{4}{11}$ miles. Again, $y - \frac{2}{11} : x :: 1 : \frac{11x}{11y-2} = \frac{x}{y} + \frac{31}{35} \therefore 35x = 363y^2 - 66y = 3885 \therefore x = 111$ miles.

75. A broker has two kinds of money. It takes *m* pieces of the first to make a dollar, and *n* pieces of the second, to make the same sum. Some one offers him a dollar for *r* pieces; how many of each sort shall he take?

Let x = pieces of the first; and y = pieces of the second. Then $mx = ny = 1.00$. If $m = 1$, $x = 100$; $m = 2$; $x = 50$; $m = 4$, $x = 25$; $m = 5$, $x = 20$; $m = 10$, $x = 10$; $m = 100$, $x = 1$. Again, if $n = 100$, $y = 1$; $n = 20$, $y = 5$; $n = 10$, $y = 10$; $n = 5$, $y = 20$. If $r = 8$, then we have $m = 5$, and $n = 20$. $\therefore 4 \times 20 + 4 \times 5 = 100$; $\therefore$ he takes 4 twenty cent pieces, and 4 five cent pieces.

76. A, B, C, are 3 equal vessels ; the first contains water ; the second, wine and the third contains wine and water. If the contents of B and C be put together, it is found that the mixture is 11¼ times as strong as if the contents of A and C had been treated in like manner. Find the proportion of wine to water in C.

Let x denote de wine in C ; then C − x = water in C. Then $\frac{B+x}{C-x}$ = strength of Ist mixture. $\frac{x}{A+C-x}$ = strength of second mixture ; $\frac{1+x}{1-x} = \frac{11\frac{1}{4}}{2-x}$; ∴ $2+x-x^2 = 11\frac{1}{4}x - 11\frac{1}{4}x^2$, and $x^2 - x = -\frac{8}{41}$ ∴ x = .73425 = wine in C ; 1 − .73425 = .26575 = water in C.

77. The discount on P dollars due x years hence : \$ P :: *m* : *n*, find rate of int.

Let r = rate, and x = time ; then $100+rx : rx :: P : \frac{Prx}{100+rx}$ = discount on \$ P for x years ; $\frac{P\,r\,x}{100+r\,x} : P :: m : n$, or, $\frac{r\,x}{100+r\,x} : 1 :: m : n : r\,x : 100+r\,x :: m : n$; this proportion gives $x = \frac{100}{r} \times \left(\frac{m}{n-1}\right)$ or $r = \frac{100}{x}\left(\frac{m}{n-m.}\right)$

78. A steamboat started from Hamilton, and sailed down Lake Ontario. Owing to roughness of the lake, during the first four hours, she only sailed 23 miles ; of this, the third hour's sailing was double the first ; the fourth was 1¼ miles less than sum, and the second was ⅓ of the distance = 18 miles more than twice the third hour's sailing. Continuing this progression, how far was the boat from Hamilton, at the end of the tenth hour ?

Let x = 1st hour's sailing ; 2 x = 3rd ; $3x - 1\frac{1}{4}$ = 4th ; $\frac{4x+1\frac{1}{4}}{3}$ = 2nd. ∴ 22 x = 71½ ; x = 3¼. Then 3¼ + 4¾ + 6½ + 8½ + 10¾ + 13¼ + 16 + 19 + 22¼ + 25¾ = 130 miles.

79. A local superintendent divides $360 between four schools in G. P., and each receives as many dollars as there were pupils attending it on an average. N°. 1 receiving the smallest share. It was then found that the amount received by N° 3 exceeded ¼ of that received by N° 1, by $90. Required the sum apportioned to each?

Let x = first term, and r the rate; then $x + rx + r^2 x + r^3 x = 360$, and $r^2 x - \frac{x}{4} = 90$ $\therefore 4 r^2 x - x = 360$; and $(4 x^2 - 1) \times 360$, and $x = \frac{360}{4 r^2 - 1.}$ Then $\frac{360}{4 r^2 - 1} + \frac{360 r}{4 x^2 - 1} + \frac{360 r^2}{4 r^2 - 1} + \frac{360 r^3}{4 r^2 - 1} = 360$ or $\frac{r^3 + r^2 + r + 1}{4 r^2 - 1} = 1 \therefore r^3 + r^2 + r + 1 = 4 r^2 - 1$, and $r^3 - 3 r^2 + r = -2$, solved, $x = 24 \therefore r = 2$; hence the respective values or shares are: 24, 48, 96, and 192 = 360. W. D.

80. A, B, and C commence bunsiness with an aggregate capital of $1200; C's capital exceeds A's by $100. A sells out in 10 months, B in 7½ months, and C in 5⅘ months, and each receives in stock and profit $700. Find each person's stock.

Let x = A's capital ; $100+x$ = C's ; then $1100-2x$ = B's. $x \times 10+(1100-2x)\,7\frac{1}{2}+(100+x)\times 5\frac{5}{7}=\frac{5x+79250}{9}$; consequently, $\frac{5x+79250}{9} : 10x :: 900 : \frac{16200x}{x+15850}=x \therefore 162000x=x^2+15850x \therefore x=350$ = A's profit. Then $700-350=350$ = A's stock. $1100-700=400$ = B's stock ; $x+100=450$ = C's stock. J.C.

81. **A miller mixes flower which cost him $5 a barrel, with some which cost him only $3 a barrel, and sells the mixture at $5.40 per barrel, making 42.5 per cent. Required the proportions of the mixture?**

Let x = first ; y = second ; then $5x+3y$ = first cost ; $5\frac{2}{5}(x+y)$ = selling price ; or $5\frac{2}{5}x+5\frac{2}{5}y=(5x+3y)=\frac{2}{5}x+2\frac{2}{5}y$ = profit. As $5x+3y : \frac{2}{5}x+\frac{2}{5}y :: 100 : 42\frac{1}{2}$. Whence, $x=15$, $y=23$, and the ratio is $15:23$ or $23:15$.

82. **When two metals are mixed in equal volumes, they form a compound of specific gravity 9. When mixed in equal weights,**

they form a compound of specific gravity $8\frac{8}{9}$. Find the specific gravities of the metals.

Let W, w, and w′ denote the weights of the compound; w, w′ being equal. S, s, and s′ = specific gravities. $(S-s')s \div (s-s')\,S \times W = w$; and $\frac{(s-S)\times s'}{(s-s')\times S} \times W = w'$. Then $\frac{(8\frac{8}{9}-s)\times s'}{(s-s')} \times 10 = 5$. From this $40(s+s') = 9\,ss'$. Again $\frac{5000\,s}{s} + \frac{5000\,s'}{s'} = \frac{5000\,s + 5000\,s'}{9}$ $\therefore S + s' = 18$. By substraction, transposition &c., &c., we obtain $s = 10$ and $s' = 8$, the specific gravities reqd.

83. **Two men, A and B, take shares in a Petrolia oil well to the amount of $1850. They sell out at *par*; A, at the end of 2½ years; B at the end of 7½, and each receives in capital and profit $1400. How much did each embark?**

Let x = A's capital; $1850 - x$ = B's; $x \times 2\frac{1}{2} + (1850 - x) \times 7\frac{1}{2} = 13875 - 5x$ = sum of products. $13875 - 5x : 950 ::$

$2\frac{1}{2}x : \frac{475x}{2775-x} =$ A's profit; $13875-5x :$ $950 :: 13875-7\frac{1}{2}x : \frac{2636250-1425x}{2775-x} =$ B's profit. As $950 : 13875-5x :: 1400 -x : \frac{3885000-4175x+x^2}{190} =$ product of A's capital and time ∴ we have, $(3885000-4175x+x^2) \div 475 =$ A's capital $= x$. From this equation, we obtain $x = 1091.863755$; and $1850 - 1091.863755$ B's capital.

84. A refiner had a tank full of alcohol, containing 177147 gallons, from which he drew a certain vessel full, and filled up the tank with water. He repeated this process 11 times, when he found only 2048 gls of alcohol left in the tank. Find the capacity of the vessel? The quantity left after the 11th draw is found to be $\frac{(177147-x)^{11}}{177147^{10}} = 2048$; ∴ $177147 - x = 177147^{\frac{10}{11}} \times 2$: and $177147 - x = 118098$ ∴ $x = 59049 = \frac{1}{3}$ tank.

85. There are two quantities, *m* and *n* whose arithmetial mean is x; the geometrical

mean is y; and the harmonic mean is z; if $x-y=3\frac{3}{5}$, and $x-z=4\frac{76}{125}$, find m and n. $\frac{m+n}{2}=x$; $\sqrt{mn}=y$; $\frac{2\,mn}{m+n} = z$.

Put $x=m$; $y=n$. Then $\frac{m+n}{2}-\sqrt{mn} = 3\frac{3}{5}$; and $\frac{m+n}{2}-\frac{2\,mn}{m+n}=4\frac{76}{125}$. Then $x-y=3\frac{3}{5}$, and $x-z=4\frac{76}{125}$. $x+y=2\sqrt{xy}+7\frac{1}{5}$, and $x+y=\frac{4\,x\,y}{m+n}+9\frac{27}{125}$. ∴ we have

$$\frac{4\,x\,y}{2\sqrt{x\,y}+7\frac{1}{5}} + 9\frac{27}{125} = 2\sqrt{xy} + 7\frac{1}{5}.$$

$$\frac{4\,x\,y}{2\sqrt{xy}+7\frac{1}{5}}+2\frac{2}{125}=2\sqrt{xy}:$$ clear of fractions, $$\frac{500\,x\,y}{2\sqrt{xy}+7\frac{1}{5}}+252=250\sqrt{x\,y}$$ $\therefore 500\,x\,y+504\sqrt{x\,y}+1814\frac{2}{5}=500\,xy+1800\sqrt{xy}\,1296\sqrt{xy}=1814\frac{2}{5}\ \therefore 5\sqrt{xy}=7$, and $\sqrt{xy}=\frac{7}{5}$, and $x\,y=\frac{49}{25}$; $x+y=2\frac{4}{5}+7\frac{1}{5}=10$. Now, we have the sum, and product given, hence $x=9\frac{4}{5}$, $m=9\frac{4}{5}$; $y=\frac{1}{5}\ \therefore\ n=\frac{1}{5}$.

86. The difference between the quotient and divisor is 81239, and sum of the divisor and

dividend is 3007249. It is required to find each.

Let x = divisor; then 81239 + x = quotient; but (81239 + x) × x = dividend, and 3007249 − x = dividend; ∴ $x^2 + 81239x = 3006249 - x$; whence $x^2 + 81240\,x = 3007249$. Then x = 37.

87. The time, rate, principal, and gain at compound interest are all equal. Required the time?

Let x denote each; $pR^t = s$; per question, $p = x$; $r = \frac{x}{100}$; $R = 1 + \frac{x}{100}$; $t = x$;

$$s = 2x \;\therefore\; x \times \left(1 \times \frac{x}{100}\right)^x = 2x;$$ divide

by x; $$\left(1 + \frac{x}{100}\right)^x = 2;$$ by the nature

logs, we have $$x \times \left(1 + \frac{x}{100}\right) \times M =$$

$$.3010300.x \times : \frac{x}{100} - \frac{x^2}{20000} + \frac{x^3}{3000000} \&c =$$

$$\frac{3010300}{M} \frac{x^2}{100} - \frac{x^3}{20000} + \frac{x^4}{3000000} - \&c., =$$

69317; by reversion, x = 8.49824, the answ.

88. In what time could $25 amount to the same, if placed at 6 per cent simple, and 3 per cent compound interest.

By a few trials, the time is found to be between 43 and 44 years; then by the general rule of "Trial and Error," the answer is 43⅓.

89. A merchant bought a cask of spirits for £48, and sold a quantity exceeding three fourths of the whole by 2 gallons, at a profit of 25%. He afterwards sold the remainder at such a price as to clear 60°/. on the whole transaction; and had he sold the whole quantity at the latter price, he would have gained 175 per cent. Find contents of cask.

NOTE.—Henri Mondeau. the Shepherd of Touraine, a wonderful French youth of extraordinary powers of mental calculation. having visited Jersey in order to exibit these powers, and when asked the foregoing question, answered it almost instantaneously as folllows:

He sells the first portion at a profit of 25 per cent, and the last at 175 per cent, and gained 60 per cent on the whole. The first profit is less than the mean profit by 35 per cent, and the second is greater by 115 per cent : he has ∴ sold 115 parts of the first against 35 of the second, that is, the first portion sold was $\frac{115}{150}$ of what the whole cost; and the last $\frac{35}{150}$; but the first portion was $\frac{3}{4}$ of cask and 2 gls. more; and the difference between $\frac{115}{150}$ or $\frac{23}{30}$ and $\frac{3}{4}$, is $\frac{1}{60}$ ∴ 2 gls. = $\frac{1}{60}$ of cask ∴ 120 gls = whole.

90. Which is the best interest, at 6 per cent compounded annually on $1000 for 20 years; 5% compounded every instant?

Let $x = \infty$ all the instants in 20 years; $a = 1000$; then $A = a\left(1+\frac{r}{x}\right)^{x} = a$ $(1+x)\frac{r}{x} + \frac{x(x-1)}{1 \cdot 2} \times \frac{r^2}{x^2} + \frac{x(x-1)\times(x-2)}{1 \cdot 2 \cdot 3} \times \frac{r^3}{x}$ &c., hence, since $x =$ infinity, $x(x-1) \div x^2 = 1$, and the series is. $(1+1+\frac{1}{2}+\frac{1}{1.2}+\frac{1}{1.2.3.4}$ &c.$)a = 1000 \times 2.7828 = 1000x$, Naperion base of logs $= 2718.28$, at 6% it is 3207.18; diff. $= 448.90$ in favor of 6%. J. Ireland.

91. A cylindrical piece of wood is 12 feet long, and $3\frac{1}{2}$ ft. in diameter. Find the solidity

of the greatest parallelopipedon that can be cut out of it. Find the area of a square whose diagonal is 3½, and multiply this area by the given length.

92. A farmer uses a roller, 4 feet 8 inches wide, and 2 ft. 8 in. in diameter. How many revolutions does it make over 7A 3R 25P? 8809.32, the answer.

93. A cow is tithered with a rope so as to graze over 1A 35 P of pasture; but the grass being insufficient to feed her, what additional length of rope will allow her the use of another acre? Answer 16 yds 1 foot.

94. Required the dimensions of an upright cylindrical vessel, capable of containing 16 gallons, when the depth is equal diameter of the base? $.7854\ x^2 \times x = 277.264 \times 16$. This equation gives $x = 17.809$ = height, or diam. of the base.

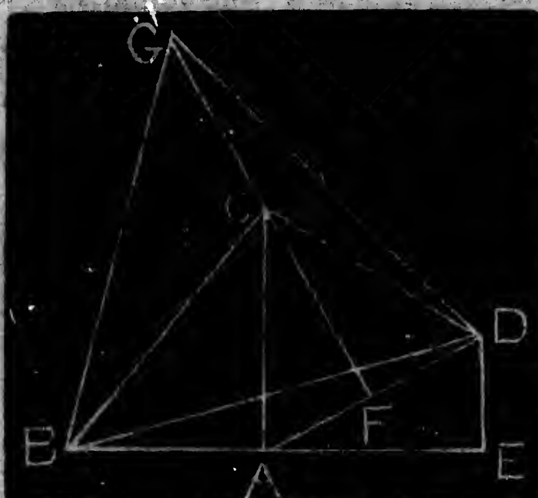
95. If into a cylindrical vessel whose inner diameter is 3 inches, we put as many wires of $\frac{1}{14}$ inch diameter, as possible; how much water can be afterwards poured in, allowing the height of the vessel to be 12 feet?

Put radius of the cylindrical vessel = *a*, and small radius = $\frac{1}{28}$; then the number of small circles that can touch the original O[ce] and one another, in one round, is found as follows: Let the required number of circles = x; then sin. $(360 \div 2x) = s$, and $1\frac{1}{2} s \div (s+1) = \frac{1}{28}$; $\therefore$ x will be found = 128. There are 21 terms in the progression, or the rad. must be divided into 21 = parts, each = diam. of the small circles. Then by the foregoing formula, we have 21 concentric circles to be described whose diameters are: $1\frac{1}{2}$, $1\frac{6}{14}$, $1\frac{5}{14}$, $1\frac{4}{14}$, $1\frac{3}{14}$, &c., &c., and the number of small circles of $\frac{1}{14}$ inch in diameter that can be described between each pair of circumferences, in succession are: 128, 122, 116, 110, 106, 98, 92, 86, 80, 74, 68, 62, 56, 50, 44, 38, 32, 26, 20, 14, 8, 2; their sum = 1420, total n° of wires. Area of end of each $= [\frac{1}{14}]^2 \times .7854 = .004007 \times 1420 = 5.7$ square inches covered by the wires. $3^2 \times .7854 = 7.0689 - 5.7 = 1.3686$, area of empty spaces at the bottom $1.3686 \times 12 \times 12 = 197.0784$ cubic inches of water the vessel can hold between the wires.

96. There is a point in an equilatorial triangle,

from which 3 lines are drawn to the angles, measure 2½, 2, and 1½ respectively, construct the triangle and find length of side.

The 3 given lines are 1½, 2, and 2½ respectively equal half 3, 4, and 5 ∴ they form a *r* < *d* triangle. Now draw a line A B = 1½; from A erect ⊥. A C = 2; join B C = 2. On A C describe an equilateral triangle ACD. From D let fall the ⊥ DE on B A produced; and from



C, draw C F at *r* <s to A D, bisecting A D in F. Join E F; join also D B, and on D B describe an equilateral triangle D B G. Then DBG is the triangle required.

It is easily proved by the principles of equilateral triangles, that triangle D C G and D A B are equiangular ∴ C G = A B = 1½; C D = 2, and C B = 2½; ∴ C is the point within the triangle G D B, from which the lines C B, C D, C G, drawn to the <s at B D G are = $2\frac{1}{2}$, 2, $1\frac{1}{2}$. Triangle D E F is easily proved to be equilateral ∴ D F = 1; A F = 1 ∴ $\sqrt{2^2 - 1^2} = \sqrt{3} = 1.73205$ = C F. $\sqrt{(CF+CG)^2 + 1} = \sqrt{11.4461472025}$ = 3.383214, side of required triangle.—D. E. Scott.

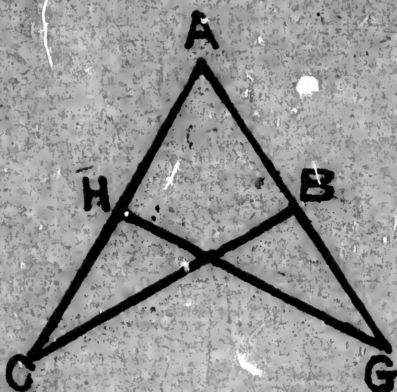
97. PRIZE PROBLEM. — If a rifleman man can plant 11 per cent of his bullets within a circle of 1 foot in diameter, at the distance of 100 yards; find the diameter of that circular target which he might make an even venture to hit the first shot.

Formula.—$a = r\sqrt{\frac{\text{Log. } 2}{\text{Log. } m}} = r\sqrt{\frac{\text{Log. } 2.}{\text{Log.}(1-H)}}$.
Let $r = \frac{1}{2}$ foot: $H = \frac{11}{100}$; $m = \frac{89}{100}$, whose log. is 0.050610, and log. 2 = 0.30103, then $a = \frac{1}{2}\sqrt{\frac{30103}{5061}} = 1.2195+$ feet, which doubled, gives 2 feet 5.2656 inch, diameter required.—A. D.

98. ANOTHER PRIZE PROBLEM.—A sledge moves down an inclined snow-plane, 140 feet long, and 30^0 inclination; and when arrived at the bottom, proceeds along a horizontal one

until friction brings it to rest. If the coefficient of friction between the snow and sledge be taken equal .05, what space will the sledge describe along the horizontal plane, neglecting resistance of the air?

A B = 70 ; B C = 121.2435 ; A C = 140. Bisect A C in H, and draw the ⊥ H G. Then by similar triangles H G = B C ; but H G is the cos. *a* (30°), or cos. *a* × coeff. of friction = friction ∴ 121.2435 × .05 = 6.062175 units of work destroyed. Or, the Normal reaction multiplied by the coefficient of friction is the friction. ∴ .866025 × .05 = 04330125 × 140 = 6.062175, units of work destroyed ∴ 70 w − 6.062175 = 63.937825 = units of work accumulated in the sledge at C ∴ 63.937825 ÷ .05 = 1278.7565, the answer.



Formula $W\left(h - \text{base} \times \text{coef. frict.} \div \frac{W}{20}\right)$ $\left(70 - 6.062175 \div \frac{W}{20}\right) = 1278.7565$. The following is another solution :

The pressure $x : w :: \text{base} : 140 \therefore 140x = 121.2435565w$, and $x = \frac{121.2435565}{140}$ = pressure. $\therefore \frac{121.2435565}{140} \times \frac{1}{20} = \frac{121.2435565}{2800}$ = work of friction. $\frac{70w}{140} - \frac{121.2435565}{2800}$ = work of gravity over friction. Then, $V = \left(\frac{64\frac{1}{3} \times 1278.7564435\, w \times 140}{2800}\right)^{1/2}$; $\therefore$ the units of work corresponding to this velocity $= 63.937822175\, w \div \frac{w}{20} = 1278.756435435$.

A. D

99. The frustum of a right cone is 24 inches in diameter at one end, and 16 inches at the other end, and 60 inches in height. At what height will half the solidity of the whole be found? Answer: The segments of the line representing the height are: height of lower frustum = 24.1712847, and height of upper frustum = 35.828716. Limited space prevents an entire solution.

100. In a railroad excavation there is a rock to be cut, ¼ mile long, 40 feet wide at the top, and 25 feet deep. A contractor agrees to

complete the work for $59888\frac{8}{9}$, or $.175 per cubic yard. How wide must he leave the cut at the bottom? Answer 16 feet.

Let x denote breadth at the bottom. Then

$$\frac{40+x}{2} \times 25 = \frac{1000+22x}{2}$$ [redacted]

$$\therefore \frac{1000+25x}{2} \times 1320 =$$ [redacted]

$$\frac{1320000 + 33000x}{2 \times 27} =$$ [redacted]

cubic yards of excavation $= 1\frac{3}{4} \times \frac{660000+16500x}{27}$ $= 59888\frac{8}{9}$ $\therefore$ $\frac{660000+16500x}{27} = 34222\frac{2}{9}$; from this $x=16$.

101. The height of a cone is 41.088 ft., and diameter of the base 8.56 feet? Find the length of a straight line || to base, and diameter of a circle that will bisect the cone.

Let $41.088 = h$, and x = the bisector.; then $8.56^2 \times .7854 \times \frac{41.088}{3} = 788.192274$ = solidity of the cone, and its ½ = 394.096137. Then $8.56^3 : x^3 :: 2 : 1$; hence $x = 6.78575^2 \times 7854 = 36.17550475$ = bisecting circle $\therefore$ 394.096137 $\div 36.17550475 \times \frac{h'}{3}$; whence $h' = 32.68$.

102. Any part may be cut off, if instead of the ratio 2 : 1, we use the ratio of the whole solidity : the part to be cut off.

103. Find the side of the largest cube that can be cut from a sphere 10 feet in diameter? The solidity of the greatest cylinder that can be inscribed in a given sphere, is the revolution of the greatest inscribed cube, and the side of the latter, is $\sqrt{\frac{10^2}{3}} = 5{\cdot}7735$.

104. Three men stand on a plane, in the same straight line; the first is 6 ft 2; second 6 ft; and the third is 5 ft 9 inches high. The distance between 1st and 2nd is 10 ft. Find the distance between the 2nd and 3rd, when

the tops of their heads are in the same straight line.

A C=74 inches; E N=72; and B D=69. 74 − 69=5=L C; 72 − 69=3=3=S N or L P ∴ 2 : 10 :: 3 : 15=S D or E B.

MAXIMA AND MINIMA.

105. Find the greatest rectangle that can be inscribed in a given triangle. The greatest inscribed rectangle is when its height $= \frac{1}{2} h$ of given triangle.

106. To find the shortest line that can be drawn through a given point between two other lines forming a right angle.

Let T B and B V be the two indefinite straight lines, B the right angle, and P the given point. From P draw P M $\perp$ base T B, and draw P A $\parallel$ T B; then P M is given $= b$, and P A $= a =$ M B. Denote T M by x; then x is found $= \sqrt[3]{a\ b^2}$; the two points T and B being then given, we determine the direction and lenght of T V.

107. To find the greatest trapezium that can be inscribed in a given semicircle. When the radius $= a$, and height of trapezium $= x$; then $x =$

$$= \frac{a}{2}\sqrt{3} \ . \ h = x$$

108. Of all the cylinders that can be inscribed in a right cone, determine that which has the greatest solidity.—Let height S C of the cone $=a$; rad. A C of the base $=b$; then let S D, height of upper section of the cone $=x$, $\therefore$ the lower section D C $=a-x$; then the solution gives $a-x$, height of the required cylinder $=a-\frac{2a}{3}=\frac{a}{3}$ $\therefore$ height of greatest insc, cy'r. = ⅓ height of cone.

By this result, we can find the side of greatest inscribed cube or globe.

109. A B represents the front of a street lot; required the minimum length of the three remaining lines bounding the lot which contains 80 perches. Let x = A B; A C = $\frac{80}{x}\times$ $2+2x$ = sum of the 4 sides = $\frac{160}{x}+2x=\frac{160+2x^2}{x}$ = min. solved gives $x=4\sqrt{5}$

110. To find the greatest rectangle that can be inscribed in a given semicircle. The greatest rectangle that can be inscribed in a semi-circle is, when its height is equal the radius divided by the square root of 2.

111. The equal sides of an isosceles triangle are opened from the vertex; what must be the length of a line joining their extremities, so that the quadrilateral thus formed, may be the greatest possible?

Answer, $4x = \sqrt{a^2 + 8b^2} - a.$

112. If the given triangle is equilateral, substituting a for b, the result is $4x = \sqrt{a^2 + 8a^2} - a$; or $3a - a = 2a \therefore x = \frac{a}{2}$, and the maximum trapezoid equals 3 times area of the original triangle.

113. Two cords, one of 6, the other of 8 feet in length are attached to a weight 100 lbs, and fastened at their extremities to hooks in the ceiling, 10 feet apart. Required the strian on each cord.

The sum of the sides represents the whole strain on both sides; and the strains on the sides are in the inverse ratio of the length of the sides. Then, as 10 : 14 :: 100 : 140, the total strain; ∴ 14 : 140 :: 8 : 80 = strain on 6 ft cord. As 14 : 140 :: 6 : 60 = strain on 8 ft cord.

114. A bar of wrought iron 150 ft long, and $\frac{1}{5}$ inch square in section, lengthens .289 inch under a certain strain; what must be the additional strain necessary to produce rupture?

L = 150; strain 2240 lbs gives .289; l : L :: $\frac{2240}{\frac{1}{25}}$: 29.000000, or modulus of elasticity. ∴ 290 l = 84, and l = .289. The strain sufficient to produce rupture is $\frac{1}{25}$ × 6720 tenacity = 2688 − 2240 = 448 lbs the additional strain.

115. An iron wedge whose angle is 14°, is driven into a mass of oak by a force of 125 lbs. What force is necessary to extract it?

$$W_{,} = W \times \frac{\sin.\ (31°\ 50' - 7°)}{(31°\ 50' + 7°)} \quad \text{or,} \quad W \times$$

$$\frac{\sin.\ 24°\ 50'}{\sin.\ 38°\ 50'} = 125 \times \frac{419980}{627057}.\ W_{,} = .66976 \times 125$$

= 83.72 lbs, the force required.

116. A beam of oak 1 foot square, has its end firmly embedded in Masonry, from which it projects 9 feet; to what height could a wall of brickwork, 2 feet thick, and resting on the beam, be carried without producing rupture? A cubic foot of brickwork is equal to 112 lbs.

Let a = natural length of the beam ; b its depth, and c its breath. $W = \frac{s}{3} \times \frac{c\ b^2}{a^2}$; s being the modulus of elasticity. Then, $w = \frac{499^2}{3} \times \frac{12 \times 144}{108^2} = 246\frac{42}{89}$ = pressure for every inch of length of beam. Then $246\frac{42}{89} \times 108 = 26618\frac{86}{89}$ lbs, sufficient to produce rupture. $9 \times 2 \times 1 \times 112 = 2016$ ∴ $26618\frac{86}{89} \div 2016 = 13.203$ feet high.

117. The rafters of a house are each 18 feet long and tied by a wrought iron rod 30 feet long, and section ¼ square inch. What weight must be suspended from the vertical angle so as to break the rod? $W \times 30 \div 4$ A D = horizontal pressure ; A D = 9.95 ∴ $\frac{W \times 30}{39.8} = \frac{67200}{4}$ ∴ $W \times 30 = 39.8 \times 16800$ ∴ $W = 39.8 \times 560 = 22288$ lbs.

118. A bar of wrought iron suspended vertically breaks by its own weight; what is its length? The tenacity of wrought iron = 67200 lbs. Let x = length of the bar and n the area of its section; 67200 n = breaking weight of the bar. Specific gravity of wrought iron = 7.788 $\therefore \frac{7788}{144 \times 16} \times$ nx = weight of bar; $\therefore \frac{7.788}{2304} \times n\,x = 67200\,n$ $\therefore 7.788\,x = 67200 \times 2304$; $x = 19880$ feet, length of bar.

119. If the traction power of 97 lbs is required to draw the forewheel of a carriage over an obstacle 6 inches high, what power will be required to draw the hind wheel over it; the diameters of the wheels being 3½ and 4½ feet respectively?

As the required power varies inversely as the radii of the wheels, we have 4½ : 3½ :: 97 lbs : 75.4 = the required power.

120. To what depth may an empty glass vessel, capable of bearing a pressure of 216 lbs to the square inch, be sunk in water before it breaks?

.03616 lbs avoir du poids = weight of one cubic inch of water, at a temperature of 60°;

then $216 \div .03616 = 5973.45$ inches $\therefore 5973.45 \div 12 = 497.7875$ = the required depth.

121. If team of horses draw 3500 lbs, and the center pin is removed one inch from the center, how much will each horse draw? If the center pin is moved one inch from the center to to the right or left, the horse drawing on the short end will pull about $\frac{1}{20}$ more than the other. $3500 \times \frac{1}{20} = 175$ lbs, difference. $175 + 1750 = 1925$; $1750 - 175 = 1575$; hence the diff. of draft = 350 lbs.

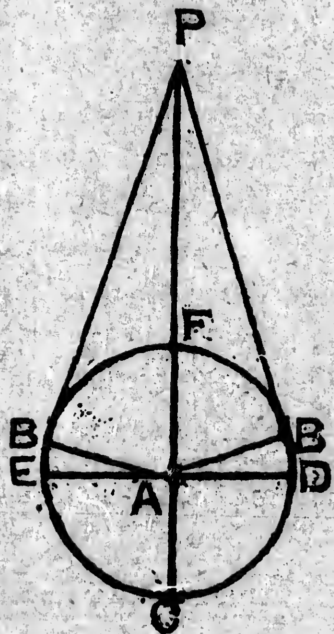
122. A owes B $455, payable in 14 years, viz, at the end of every two years $65. But he agrees to pay him in 7 years by *equal payments* each year, which B agrees to, and at the rate of 6% compound interest. What must be the annual payment? First, find the present worth of the seven payments, which were at first to be made, which is found to be $293.2583. Then find what annuity to continue 7 yrs at the given rate $293.2583 will purchase, which you will find to be 52.5\frac{1}{3}$, the answer required.

123. If a body weighs 60 lbs at a distance of 3000 miles above the earth's surface, what

will it weigh at 3000 miles below the surface? Find the weight at the surface. Bodies weigh directly as the mass, and inversely as the square of distance above the earth; but they weigh directly as the mass, below the surface. Then the body is 7000 miles from the center, what will it weigh at the surface? less $\therefore$ $7000^2 : 4000^2 :: 60$ lbs : $19\frac{29}{49}$. If a body weighs $19\frac{29}{49}$ lbs at 4000 miles from the center, what will it weigh at 1000 miles from the center? less.

As $4000 : 1000 :: 19\frac{29}{49} : 4\frac{44}{49}$ lbs, the answer.

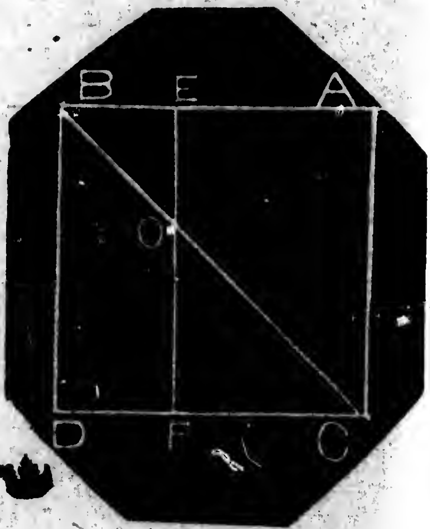
124. A belt is 16 inches long, and drawn round a circle whose diameter is 4 feet, until it reaches a point (P) in the diameter produced. Find the distance FP, BP. Also, if the circumference is equal to the length of the belt, B P must be equal to the arc B F, and B P + B′ P



= the arc B F B′. Let belt PBDCEB′ = 16; $3.1416 \times 4 = 12.5664 \div 2 =$ arc DCE $= 6.2832$. Then $2x + 6.2832 =$ BD CB′; $2y + 2x = 9.7168$, and $y + x = 4.8584$; which is an indeterminate equation.

If $x = 1$; $= y = 3.8584$; and $\sqrt{3.8584^2 + 2^2}$ $= 4.34590 =$ A P, ∴ F P $= 2.34594$. F P $+ 4 =$ C P $= 6.34594$ ∴ $\sqrt{6.34594 \times 2.34594} = y =$ 3.8584; $2y = 7.7168$; B D C E B′ $= 2 + 6.2832$ $= 8.2832 + 7.7168 = 16$.

125. Two men engage to excavate 100 square yards, divided by its diagonal into equal parts of sand and rock. They work from opposite sides, each, at sand and rock, until they meet in a line parallel to the sides at which they commenced. Each must receive equal parts of the whole cost which is $100; the sand at 75c., and rock at $1.25 per yard. How much rock and sand must each escavate and where will they



meet? Let $BE = x$; $AE = 10 - x$; then $EB = EO$, and $AE = CF = FO$. Side of square $= 10$; A B C is sand and B C D is rock. Then

$$\frac{10+x}{2} \times (10-x) \times \tfrac{3}{4} + \frac{10-x}{2} \times (10-x) \times \frac{5}{4}$$
$$= \frac{300-3x^2}{8} + \frac{500-100x+5x^2}{8}; \therefore \frac{300-3x^2}{8}$$
$$+ \frac{500-100x+5x^2}{8} = \frac{3x^2}{8} + \frac{100x-5x^2}{8};$$

hence we whave $x^2 - 50x = -200$: solved gives $x = 4.38448$; $10 - x = 5.61552$.

126. If E and D be the points of trisection of the sides A B, A C of a triangle (nearer to A), and F the point ot intersection of C D and B E; prove that the triangle B F C is half the triangle A B C, and the quadrilateral A D F E is equal to either ot the triangles C F E or B D F. On A B, side of triangle A B C, make $AD = \tfrac{1}{3} AB$, and make $AE = \tfrac{1}{3} AC$. Join B E, C D, E D and A F. The triangle B C F will be $\tfrac{1}{2}$ of triangle A B C, and the triangles E F C, D F B, and the quadrilateral A D F E will be equal to one another. For in similar triangles A D E and A B C, $ED = \tfrac{1}{3} BC$, and in similar triangles DEF and BCF, side $EF = \tfrac{1}{3} FB$; but

triangle C E B, by construction, = $\frac{2}{3}$ triangle A B C or $\frac{4}{6}$ A B C; but the base E F = $\frac{1}{4}$ B E $\therefore$ triangle E F C = $\frac{1}{6}$ A B C and B C F = $\frac{3}{6}$ or $\frac{1}{2}$ A B C. Triangles B C E and B C D on same base are equal; take away common triangle B C F, and EFC and DFB will be left equal. It can be readily seen that the two triangles forming quadrilateral = E F C or D F B.—D. Scott, C.E.

127. The girth of a heifer is $6\frac{1}{2}$ feet, and length from the shoulder blade to the tail bone 5.25; $6\frac{1}{2}^2 = 42.25$, and $5 \times 5.25 = 26.25$; multiplying this together, and dividing 1.5, gives 739.375 lbs the approximate weight of the heifer when dressed. A shorter method is, to multiply the square of the girth (back of the fore shoulder) by the length; then multi-

ply that result by 7 and divide the product by 2.

128. To find how many bricks in a wall or building, multiply the length, height and thickness in feet by 20. A brick 8 × 4 × 2 = 64 inches.

129. To find the content of barrels or casks. Square one half the sum of the bung and head diameters in inches, and multiply by the height in inches; then multiply by 8, and cut off the right hand figure; this gives the cubic inches which divided by 277¼ gives the number of gallons, and divided by 2150.4 gives the number of bushels.

130. Required the contents of a barrel whose middle or bung diameter is 22 inches, and diameter 18 inches, and 30 inches high? 22 + 18 ÷ 2 = 20, the average diam. 20 × 20 × 30 × 8 = 9600 ÷ 277¼ = 34¾ gls.

131. How many gallons in a round tank, 6 feet in diameter and 6 feett high? 6 × 6 × 8 = 288. 288 × 6 = 1728 gls or 1440 Canadian gls.

132. A cistern is 5 feet in diameter and 8 feet deep, how many barrels will it hold? 5 × 5 × 8 = 200 ÷ 5 = 40 barrels.

133. From the top of a mountain, h miles high the visible horizon appeared depressed a degrees; it is required to show that, if d be the distance of the boundary of the visible horizon, and D the diameter of the earth, $d = h \cot. \frac{1}{2} a$; $D + h = \cot^{2} \frac{1}{2} a$. Let A B be the height of the mountain, BC the diameter of the earth; $\angle$ DAF the dip, and AF the distance of the horizon. Join CF and FB, and produce them to meet a line through A at $r \angle s$ to ABC in D and E. Now the $\angle$ D is common to the $r \angle d$ triangles D F E and D E C; ∴ $\angle$ C = $\angle$ E; but $\angle$ A F E = $\angle$ C; ∴ A F E = E; hence A F E and A E F are each = $\frac{1}{2}$ D A F = $\frac{1}{2} a$, and A F = A E. Again $\angle$ D is the complement of $\angle$ E, and A F D the comp. of A F E; hence $\angle$ A F D = D and A F =

A D. The $r <$ *d* triangle A B E gives A E = A B cot. E; hence $d = h$ cot. ½ *a*. Again $r <$ *d* triangle C A D gives A C = A D cot C; hence $D + h = d$ cot. ½ $a = h$ cot² ½ *a*.

134. EXAMPLE.—From the top of a mountain 3 miles in height, the visible horizon appeared depressed 2° 13′ 27″; Find the diameter of the earth and the distance of the boundary of the visible horizon.

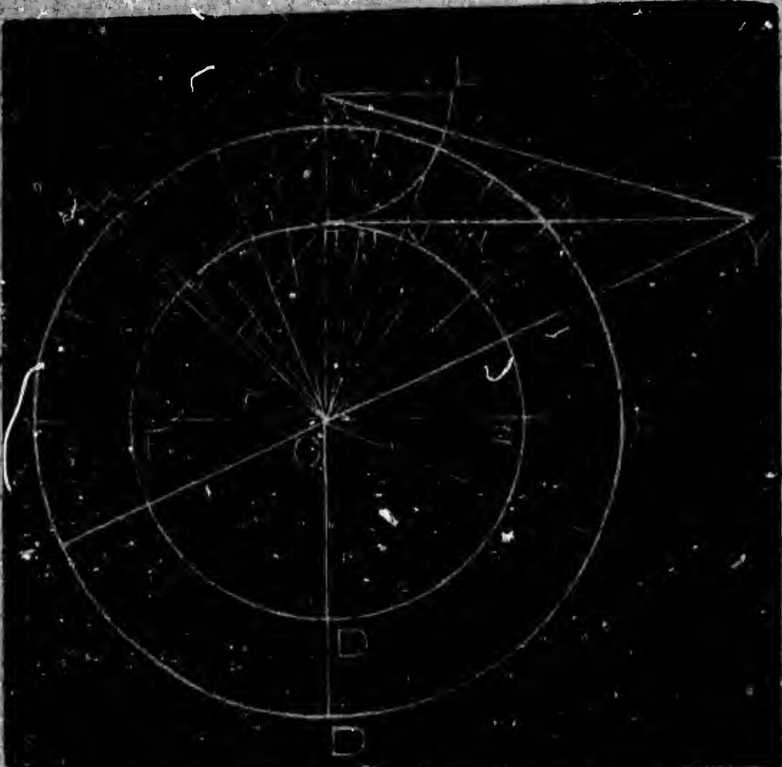
Log. h = 0.477121 }
Cot. ½ a = 11.711941 } sum = 2.189062 ; d =

= 154.54 ; $D + h$ = 7961.3.901003.

h = 3

∴ D = 7958 miles the diameter of the earth.

135. To construct a horizontal sundial.—In any straight line A B take a point O, and from O erect a ⊥ O T; take O as center withe the chord of 60° as radius and describe a circle F T S D. F S will represent the 6 o'clock line, and O T the meredian or 12 o'clock. From T on the quadrant arc, lay off T H = the latitude and join O H; from T draw the ⊥ T P which will be the sine of the latitude. Make T G = T P, and from G as center with



G T as radius describe the quadrant G T L and divide it into 6 = parts. Through each of these parts draw the lines G U, G V, &c., meeting a tangent from T, indefinitely produced in the points U, V, W, X, and Y. Through these points respectively, draw the lines O U, O V, O W, O X, and O Y, which will be the hour lines from 12 to 6, p.m. Make the arcs on the quadrant FT = respectively to these, and we have the hour lines for the forenoon. 7 (a.m.) produced gives the hour line for 7 (p.m.) and 5 (a.m.) gives 5 (p.m.) &c. To find half hour lines, divide the quadrant G T L into 12 = parts and proceed similarly. The quarter hours will be sufficiently approximate by bisecting the half hours arcs.

The angle of the gnomon or style, standing ⊥ ly on the meridian line O T, must exactly touch F S at O, and the meridian line must coincide with the breath of the gnomon. The ∠ of gnomon must be equal to latitude of the place for which the dial is constructed. To describe a south inclining vertical dial, we use the complement of the latitude; then the construction is the same as the foregoing. To describe a dial for the equator. Divide a circle into 24 equal parts and place a ⊥ style in the center. This placed on an inclined plane, having an angle equal the latitude of a given place, will show correct solar time. A dial engraved for a given lat., will show correct time by placing it on a wedge having an ∠ = the diff. of the two latitudes.

136. The three sides of triangle are 18, 12, and 10. It is required to bisect it by the shortest

line possible. Describe an isosceles triangle A F G = ½ triangle A C B, having a common < C A B, by the VI. 15, and the base F G is the line required.

Bisect A B in E; then A E C = ½ ACB. $\sqrt{12\times 9}$ = 10.392304 = A G or A F. Area of triangle A B C = 56.56854, hence A F G = 28.28427. Denote ½ F G by x, then we have the following equation; $\sqrt{a^2 - x^2} \times x = s = 28.28427$: this equation gives $x = 2.828427 \times 2 = 5.656854$ = F G, the required minimum line.

137. A, B, and C in partnership gain $1800. If we take C's time from the sum of A's and B's, 7 times the remainder will be equal to 11 times the sum of A's and C's dimished by B's C's stock is to the sum of A's and B's stocks; as A's time is to 6 times B's time; the sum of all their times divided by the sum of B's and C's minus A's, equals 19; and 3 times the difference between the stocks of A and B, is equal to twice C's stock. Required each person's gain, by simple proportion.

7 (A's + B's − C's) time = 11 (A's + C's − B's); hence 7 : 11 :: A's + C's − B's ; A's + B's − C's; then 18 : 4 :: 2 A's : 2 B's − 2 C's ; ∴ 36 B's − 36 C's = 8 A's or 9 B's − 9 C's = 2 A's also, 9 B's + 9 C's = 10 A's, hence 18 C's = 8 A ∴ C's time = $\frac{4}{9}$ A's time, and 18 B's = 12 A's ∴ B's = 9 × 12 ÷ 18 = 6 ; then A's = 9, B's = 6, and C's = 4. Again, C's stock ; A's + B's stocks :: 9 : 36 ; from this proportion, we find C's = (A's + B's) ÷ 4 = (3 A's − 3 B's) ÷ 2 ; this gives B's stock = $\frac{5}{7}$ A's.

Let A's stock = 1 × 9 = 9	15 : 1800 :: 9 : 1080 = A's gain.
" B's " = $\frac{5}{7}$ × 6 = $4\frac{2}{7}$	15 : 1800 :: $4\frac{2}{7}$: $514\frac{2}{7}$ = B's gain.
" C's " = $\frac{3}{7}$ × 4 = $1\frac{5}{7}$	15 : 1800 :: $1\frac{5}{7}$: $205\frac{5}{7}$ = C's g'n
sum of products of S & T. . . . . . . = 15	$1800 = sum of gains.

138. A and B are candidates at an election when 680 persons vote, and A is defeated. The same electors vote the following year, when A and B are again candidates, and A is successful, having carried his election by $1\frac{4}{9}$ times as many votes as he before lost by, and his majority ; B's the year before :: 9 : 5 ;

how many electors changed their minds during the year?

Let 110 denote B's gain and A's loss the first year; then $110 \times 1\frac{4}{5} = 198$ = A's gain the second year. Then 198 : 110 :: 9 : 5, and 198 − 110 = 88 who have changed there minds at the second election. This question admits 36 solutions clear of fractions, giving as many sets of answers; and the majorities change 36 times from 10 to 360 included.

139. A owes B $1000, and agrees to pay him in ten equal annual instalments, at a rate per cent, simple interest, equal to the TRUE equated time for all the payments; how much must B receive annually?

Let x = the rate = time; then x^2 = int. on each payment, and $100 + x^2$ = each annual payment. The most correct method of finding the equated time is, when the interest of the sums payable *before* the equated time, from the times when they are due till that time, should be equal to the discount of the sums payable *after* the equated time for the intervals between that time and the times at which they are due. Then, when x is the equated time, the times for interest are = $x - 1$, $x - 2$, $x - 3$, $x - 4$, and $x - 5$ years. The times for discount are: $6 - x$, $7 - x$, $8 - x$, $9 - x$, and $10 - x$ years. $x \times (100 + x^2) \times (x - 1) = x^4 - x^3 + 100x^2 - 100x$, $x \times (100 + x^2) \times (x - 2) = x^4 - 2x^3 + 100x^2 - 200x$; $x \times (100 + x^2) \times (x - 3) = x^4 - 3x^3 + 100x^2 - 300x$; $x \times (100 + x^2) \times (x - 4) = x^4 - 4x^3 + 100x^2 - 400x$; $x \times (100 + x^2) \times (x - 5) = x^4 - 5x^3 + 100x^2 - 500x$; The sum of these products = $\frac{(5x^4 - 15x^3 + 500x^2 - 1500x)}{100}$ = Interest.

DISCOUNT.

As $100 + 6x - x^2 : 6x - x^2 :: 100 + x^2 : \frac{6x^3 + 600x - x^4 - 100x^2}{100 + 6x - x^2}$

As $100 + 7x - x^2 : 7x - x^2 :: 100 + x^2 : \frac{7x^3 + 700x - x^4 - 100x^2}{100 + 7x - x^2}$

As $100 + 8x - x^2 : 8x - x^2 :: 100 + x^2 ; \frac{8x^3 + 800x - x^4 - 100x^2}{100 + 8x - x^2}$

As $100 + 9x - x^2 : 9x - x^2 :: 100 + x^2 ; \frac{9x^3 + 900x - x^4 - 100x^2}{100 + 9x - x^2}$

As $100 + 10x - x^2 : 10x - x^2 :: 100 + x^2 ; \frac{10x^3 + 1000x - x^4 - 100x^2}{100 + 10x - x^2}$

We now have—

$$\frac{6x^2 + 600 - x^3 - 100x}{100 + 6x - x^2} + \frac{7x^2 + 700 - x^3 - 100x}{100 + 7x - x^2} +$$

$$\frac{8x^2 + 800 - x^3 - 100x}{100 + 8x - x^2} + \frac{9x^2 + 900 - x^3 - 100x}{100 + 9x - x^2} +$$

$$\frac{10x^2 + 1000 - x^3 - 100x}{100 + 10x - x^2} = \frac{5x^3 - 15x^2 + 500x - 1500}{100}$$

This equation solved, gives $x = 5.29484$ = rate = time, and $x^2 = 28.0353306 2565$, the annual payment therefore, $= 128.0353$

140. The sum of the squares of two numbers minus their sum is 14; and their product added to their sum = 14; find the numbers.

$x^2 + y^2 - (x + y) = 14$, and $XY + (x + y) = 14$. Denote $x + y$ by u; then $x^2 + y^2 = u + 14$; and $XY = 14 - u$. Then $2XY = 28 - 2u$; $X^2 + 2XY + y^2 = u + 14 + 28 - 2u = 42 - u$, $\therefore x + y = \sqrt{42 - u}$, or $u = \sqrt{42 - u}$; $u^2 = 42 - u$, $\therefore u^2 + u = 42$; this equation gives $u = 6 = x + y$. $\therefore x^2 + y^2 = 20$, and $2xy = 28 - 12 = 16$; $\therefore x^2 - XY + y^2 = 20 - 16 = 4$ $\therefore x - y = 2$; and $x + y = 6$, hence $x = 4$ and $y = 2$.

141. Solve $x^2 + \sqrt{x} = 18$, by a simple equation.

142. Find the compound interest of \$80 for one month, at 6 per cent per annum; for 2 months, 3 months, etc.

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