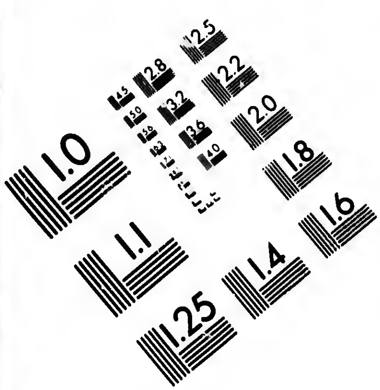


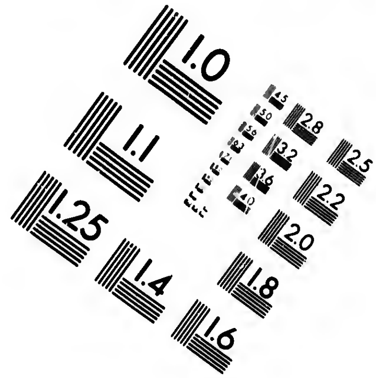
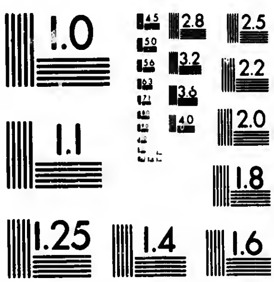
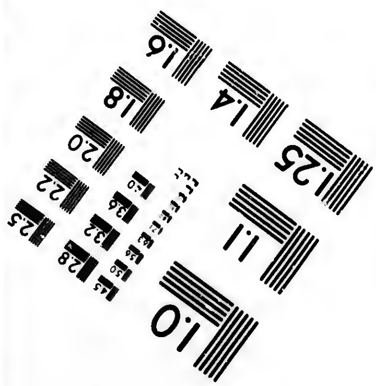
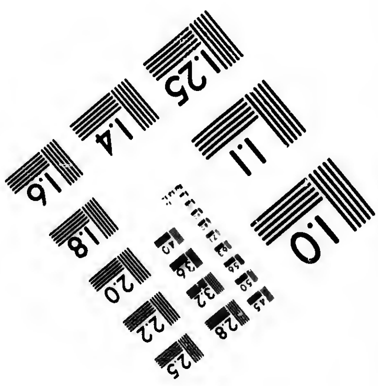


IMAGE EVALUATION TEST TARGET (MT-3)







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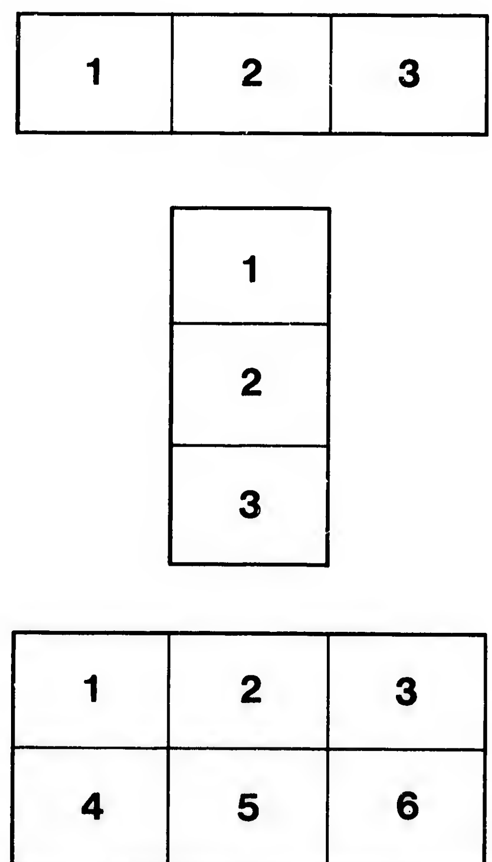
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CUTHBERT'S

PRIMARY

NUMBER-WORK; AND COMPANION

TO THE COMMON-SENSE

ARITHMETICAL CALCULATOR.

BY

W. N. CUTHBERT,

TORONTO.

"Seeing is Believing."

TORONTO:

THE COPP, CLARK COMPANY, LIMITED.

1896.

Entered according to Act of the Parliament of Canada, in the year one thousand eight hundred and ninety-six, by THE COPP, CLARK COMPANY, LIMITED, Toronto, Ontario, in the Office of the Minister of Agriculture.

PREFACE.

This book is published as a Companion to the Common-Sense Arithmetical Calculator, an invention in the form of a Calculator of new design (a full description of which is given—with illustrations—in the introductory part of the book).

The merits of the Calculator are manifold : (1) As an adjunct in the *primary department* of a Public School it forms an invaluable aid in *teaching in a rational way the first lessons* in Arithmetic ; (2) It makes good use of Number-Forms *("Picture-Numbers")* in representing Numbers, that simple, but admirable means of developing in the minds of children an appreciation of NUMBER ; (3) By giving a *clear perception* of Numbers, it develops the observing and reasoning faculties, cultivates the memory, and sharpens the intellect ; (4) It presents Number on a sound *psychological basis*, and thus lays the foundation of a good Mathematical Education by giving the child a proper conception of Number ; the *arbitrary signs*, by which numbers are known, *are made intelligent* to children, *by a comparison* with *real objects.*

Believing that the Calculator will prove of practical value in the school-room, and a boon to teachers, and especially so to those engaged in rural schools, I have been induced to place both the Calculator and the accompanying book of explanation in the hands of the publishers.

I am indebted to Dr. McLellan, Principal of the Ontario School of Pedagogy, and to Wm. Scott, Esq., M.A., Vice-Principal of the Toronto Normal School, for many suggestions during the progress of the book.

The book is designed for Teachers using the Calculator, and is not intended as a book to be placed in the hands of the pupils.

W. NELSON CUTHBERT.

TORONTO, April 13th, 1896.

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INTRODUCTION.

This little book, as stated in the preface, is intended as a Companion to the Common-Sense Arithmetical Calculator, an invention consisting of two series of eleven rectangular number-tablets, each (on which hemispheres or half-balls are glued) placed one below the other at the top of the framework. Immediately beneath these number-tablets a board is reversibly mounted so as to revolve vertically, thus permitting of either side being turned, at pleasure, towards the class. To the revolving board are affixed several appliances and devices to be used as an aid in the *psychological* teaching of Number. The whole is enclosed in a portable frame.

The devices shown in Figures *A* and *B* may be briefly described as follows :—

(1) The Fundamental Number-Forms (Fig. A[1]) from ONE to TEN, as represented by dots in suitable *symmetrical grouping*, each group being adjacent to the arbitrary sign or symbol by which it is known, thus giving the child the idea of the number which the sign represents ; (2) A Small Calculator (Fig. A[2]) consisting of ten balls (movable) on wires for teaching in *Synthetic-Analytic Number-Form* (by objects other than dots) the numbers up to TEN ; (3) An Automatic Numeral-Frame (Fig. B[3]) wherein all the *Number-Forms* from ZERO to TEN are arranged on rectangular number-tablets (by hemispheres or half-balls arranged in symmetrical grouping and fixed on these tablets) in two horizontal rows, the lower one of which is movable between two grooved pieces of wood so as to admit (by moving the said row—the lower series of eleven number-tablets—of Number-Forms successively in the same direction)

[1] See top of Calculator ("Picture-Numbers"). [2] See top of Calculator (Ten Balls).
[3] See top of Calculator (Tablets).

of the formation in number-form of ALL THE COMBINATIONS which can possibly be made in Addition with *any two* of the said number-forms, thus forming in number-form, *all the addition combinations* on Numbers from ONE to TWENTY; (4) A correspondent Automatic Mechanical Schedule (Fig. B[1]) wherein all the number-symbols from Zero to Ten are arranged in two horizontal rows, the lower one of which is movable in a rabbeted groove so as to admit (by moving the said row of Number-Symbols successively in the same direction) of the formation, in symbol-form, of the said combinations; (5) A Larger Calculator (Fig. B[2]) consisting of twenty balls (movable) on wires, for illustrating, in symmetrical number-form, the numbers to twenty, together with their *combinations* and *separations (additions* and *subtractions*). All the combinations possible on the Numeral-Frame and Schedule may be performed upon this larger calculator. Both calculators, having movable balls, have been placed lowest on the calculator for the convenience and benefit of the children who should be required to go to the calculator and form for themselves any of the said numbers and the combinations thereof; (6) A movable circular disc (Fig. B[3]) of flexible material, cut in such a manner that it may be readily transformed into a rectangle, for the purpose of teaching, by ocular demonstration, the area of the circle. The circle of course does not belong to primary work; but the disc has been added to the appliances for the proper explaining of more advanced work involving mensuration.

In entering upon school-life, all children have some idea of Numbers; some, however, have the sounds (words) that stand for numbers without any real conception of the numbers themselves. It is, therefore, necessary to teach, thoroughly, the numbers by means of OBJECTS of some kind, as clear ideas of Numbers should be given. The Common-Sense Calculator will be found to furnish a means by which this may be successfully accomplished.

---

[1] See bottom of Calculator (Small Tablets). [2] See bottom of Calculator (Twenty Balls). [3] See bottom of Calculator (Circle).

CUTHBERT'S

COMMON-SENSE ARITHMETICAL CALCULATOR

FIG. A.

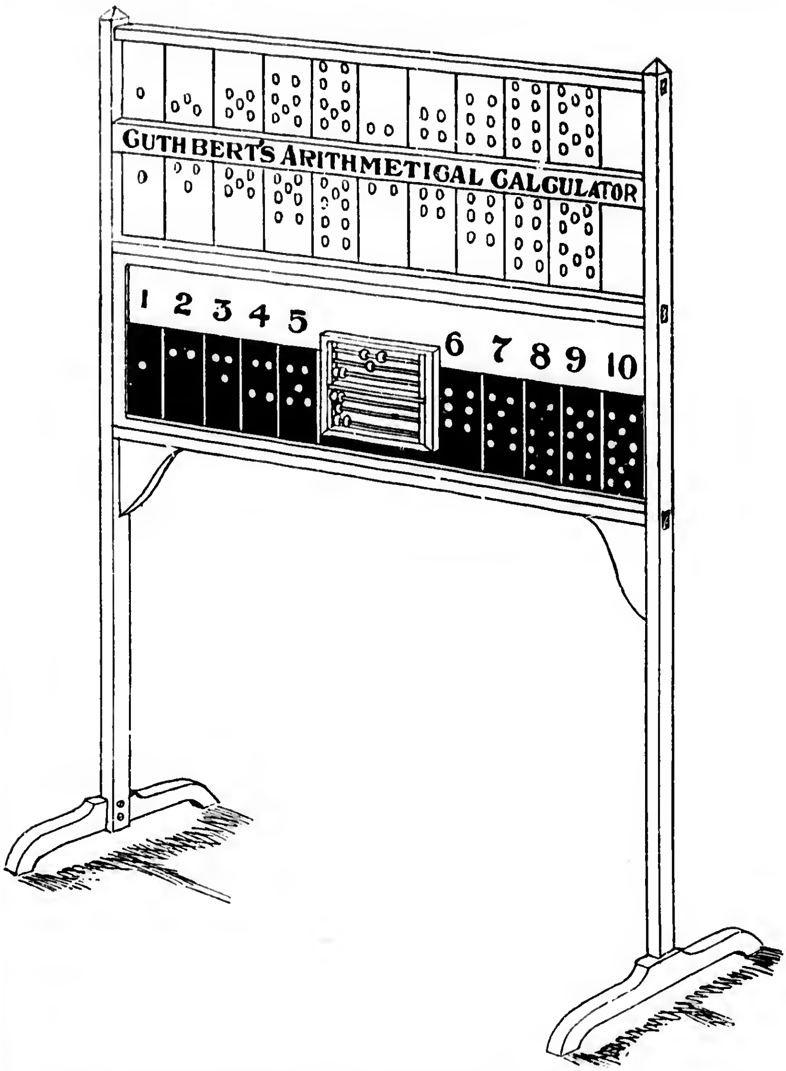
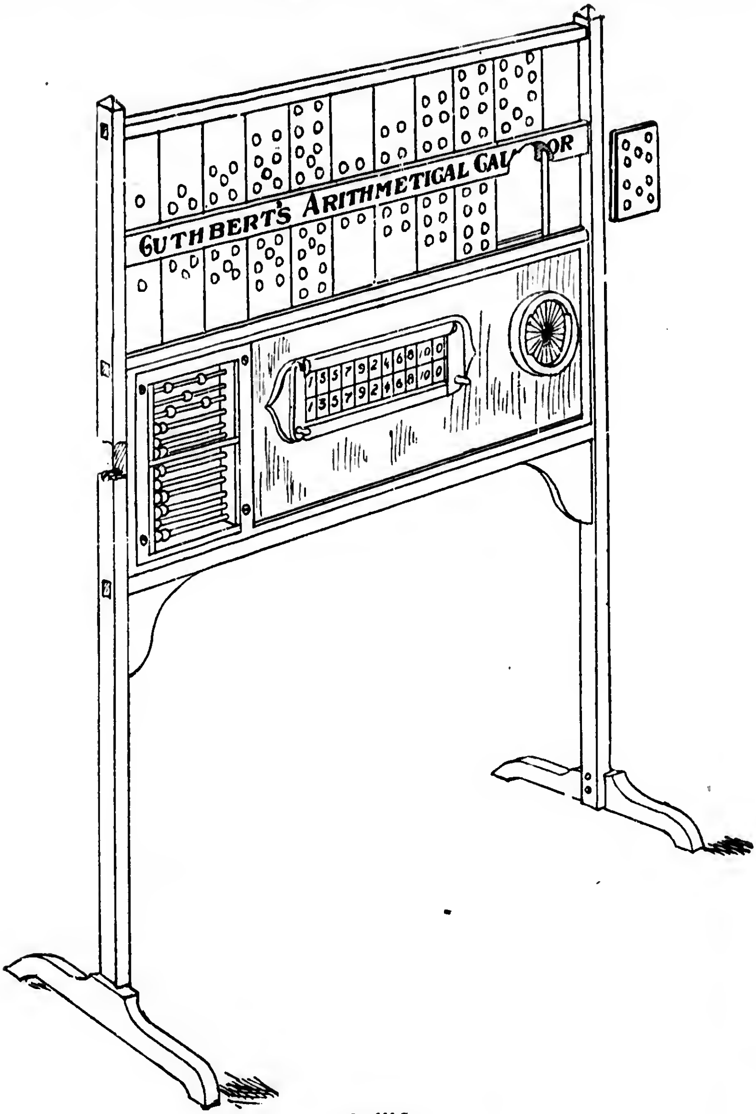


Fig. *A* represents one face of the Common-Sense Arithmetical Calculator, and Fig. *B* represents the reverse face.

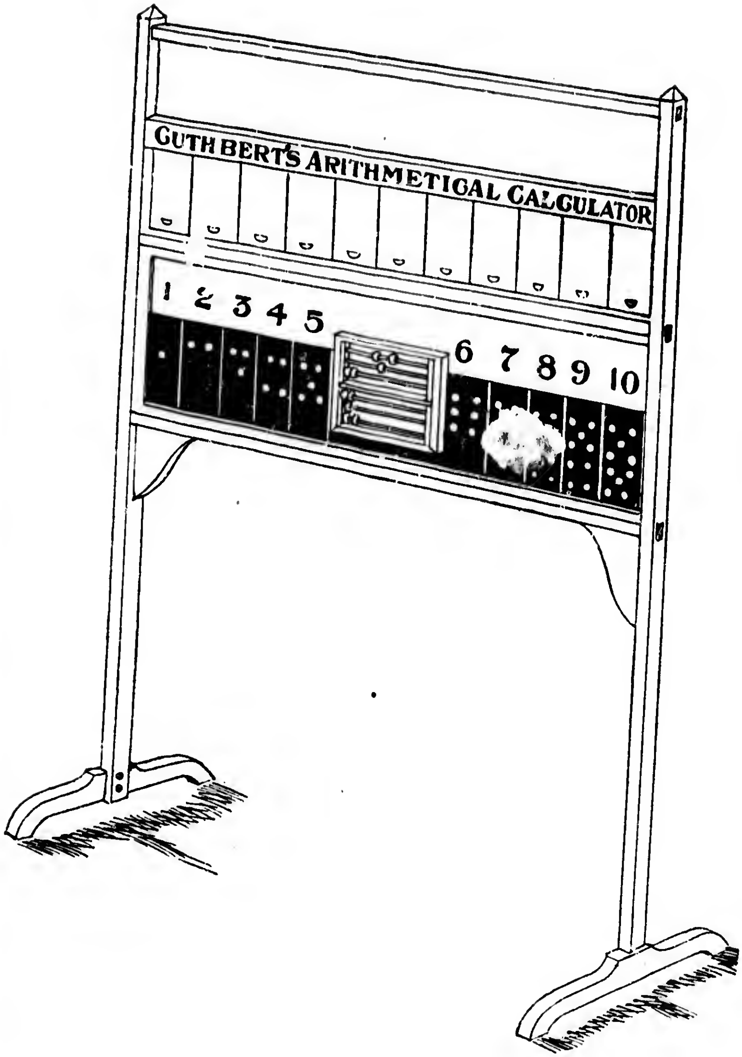
CUTHBERT'S

COMMON-SENSE ARITHMETICAL CALCULATOR

Fig. B.



BACK VIEW OF CALCULATOR.



COMPANION TO

The Common-Sense Arithmetical Calculator

The different devices and appliances found on the Calculator (as shown in Figures *A* and *B*) will be referred to under the following names :—

Fig. *A*.
1. The Fundamental Number-Forms.
2. The Smaller Calculator or Ball-Frame.
3. The Automatic Numeral-Frame.

Fig. *B*.
3. The Automatic Numeral-Frame.
4. The Automatic Mechanical Schedule.
5. The Larger Calculator or Ball-Frame.
6. The Circle.

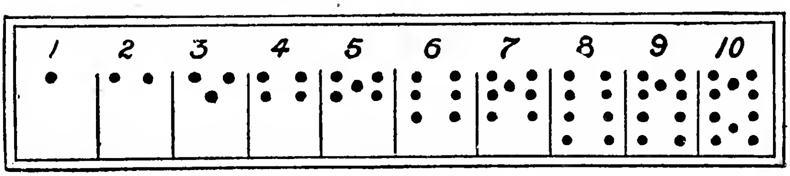
EXPLANATION OF THE SEVERAL DEVICES

FOUND UPON THE

Common-Sense Arithmetical Calculator,

AND THE

METHOD PURSUED IN USING THEM.



The above cut represents the *Fundamental Number-Forms* from ONE to TEN arranged in symmetrical grouping by dots painted on a slated back-ground, so as to admit of partition by a chalk-line which may be erased at pleasure, and also the *Arabic Number-Symbols* which represent these Number-Forms. In order *to be clear* and to be "*taken in at sight*," these intuitions must be symmetrically arranged. They must be repeatedly brought before the mind until the instant the "Picture" or Number-Form comes up in the mind, so does the word (and symbol) and *vice versâ*. These Fundamental Number-Forms and the *sixty-six combinations* on Numbers from One to Twenty, as formed by the Automatic Numeral-Frame, are *all-important*, as they lay the FOUNDATION for the subsequent study of Arithmetic. The Fundamental Numbers, and the Combinations on Numbers to Twenty may justly be called the FOUNDATION OF MATHEMATICS.

Each of the Fundamental Numbers is here represented (in Number-Form) as a whole, and the pupils get a symmetrical picture of the Number. The pupils see also the *relation* of the different *pairs of addends* (combinations or related parts) which make up the Number. Pupils should see the Number (in question) in its *entirety*, and should be required to give, *from observation*, all the possible combinations of pairs of addends (related parts) which make up the number. ***(The idea of measuring with the simple, undefined unit comes in just here.)*** If they cannot give you *all* the possible combinations from observation, then a chalk-line drawn through the Number-Form will assist the pupils in *seeing* the required combinations *in pairs or related parts.*

For instance, take the Number-Form Three (∵), and by drawing, with the chalk, a line through it, between the two (●●) dots and the one (●) dot, pupils may be led to see clearly that "two and one make three" (2+1=3), thus :—

Now, draw a chalk line immediately below the Number-Form, thus :—

Ask the pupils how many dots are below the line.

They will be led to see, by this means, that "three and nothing (zero) make three" (3+0=3).

These are *all the pairs of addends* contained in THREE, or which compose or make up Three.

$$3 = 3 + 0$$
$$= 2 + 1$$

Have the pupils make an oral statement of what they *observe* as each pair of addends *is revealed to the mind through the eye.*

Pupils may be led to see these combinations more clearly perhaps by the teacher taking in his hand a piece of card-board, and placing it edgewise between any two pairs of addends (or entirely below the number to represent the relation of the number to zero, that is, the number plus zero), and moving it so as to hide, alternately, one addend of the pair of addends (or related parts) forming the number, and then the other addend of the said pair. *E.g.*:—

By placing the edge of the card-board between the two (●●) dots and the one (●) dot (as indicated by the line drawn between the two and the one in the example given), and turning it so as to hide the two (●●) and the one (●), alternately, the pupils may be led to see clearly that "*two and one make three*" (*2*+*1*=3). *One is one-third of three ; two is two-thirds of three*, etc.

Require from the pupils an oral statement of what they observe as you turn the card-board as described.

Place the edge of the card-board immediately below the Number-Form, three, thus :—

By placing the edge of the card-board (as indicated by the line in the example) and turning it so as first to reveal the Number-Form, and second to hide it, the pupils will be led to see that three and nothing (zero) are three (3+0=3). Require the pupils, as before, to make an *oral statement* of *what they observe* when you turn the card-board, as indicated in the example given.

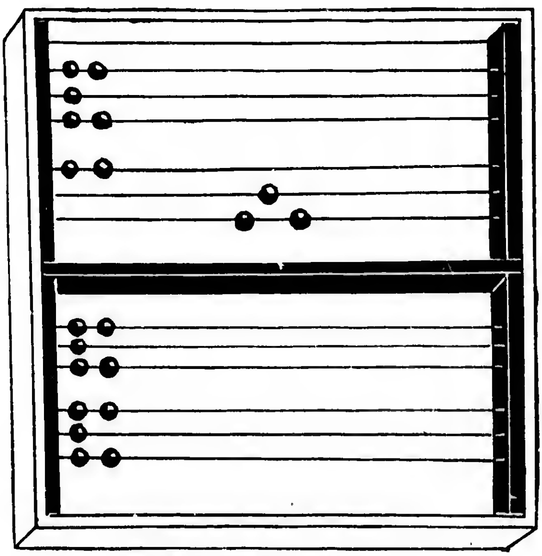
These are all the pairs of addends (related parts) which make up the number three :—

3 = *1* + *2*
0 + *3*

This exercise on Three may be varied (and variety of exercise always pleases children) in the following way :—

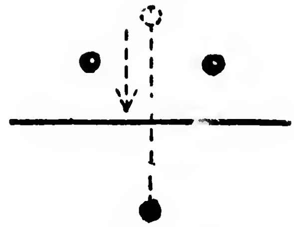
Draw a heavy chalk line (about five feet in length) upon the school-room floor, in front of the teacher's table ; now, have three pupils come out from the class and stand on one side of the said line, so as to represent the Number-Form, three, thus :—

Have some pupil in the class go to the Larger Calculator and move out upon the wires the lowest three balls of the upper ten on calculator,—these balls when moved out will be found to be immediately above the cross-bar of the calculator which equally divides the space (back-ground) behind the calculator, and which corresponds, here, to the chalk line on the floor,—so that the three balls occupy positions on the calculator similar to the positions occupied by the pupils upon the floor, thus :—

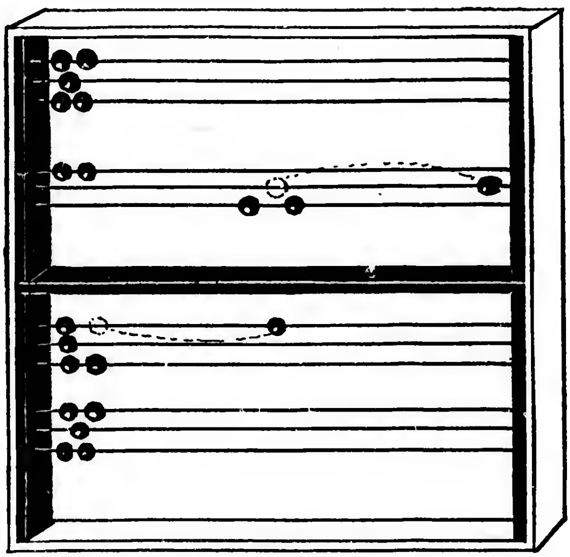


THE LARGER CALCULATOR.

Now, of the three pupils, have the one standing farthest away from the chalk line, cross over the line and take up a position upon the other side of the line, thus :—



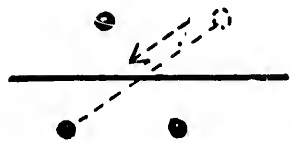
Have the pupil at the calculator make the *corresponding* change there, by moving, to one side, the upper ball in the group of three balls (∴) and then moving out one of the two balls, on the top wire in the lower ten, so that it shall occupy a position immediately below the said cross-bar, thus :—



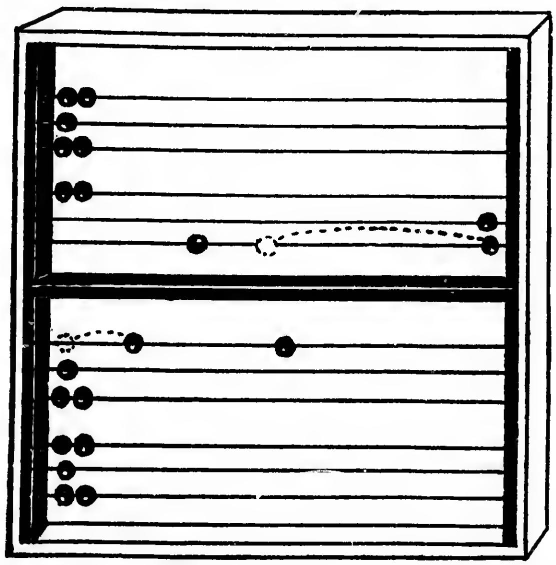
Have the pupils in the class *express orally what they have observed* :—

"TWO AND ONE ARE THREE."
(2 + 1 = *3*)

Have another pupil cross over the chalk line, as before, and take up a position upon the other side of the line, thus :—



Have the pupil at the calculator again make the *corresponding* change there, by moving, to one side as before, one of the remaining two of the upper balls, and then moving into position the other of the two balls on the top wire in the lower ten, thus :—

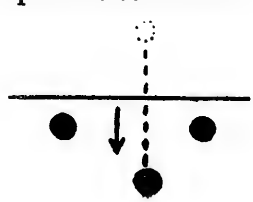


Now, have the pupils in the class *express orally what they have observed* :—

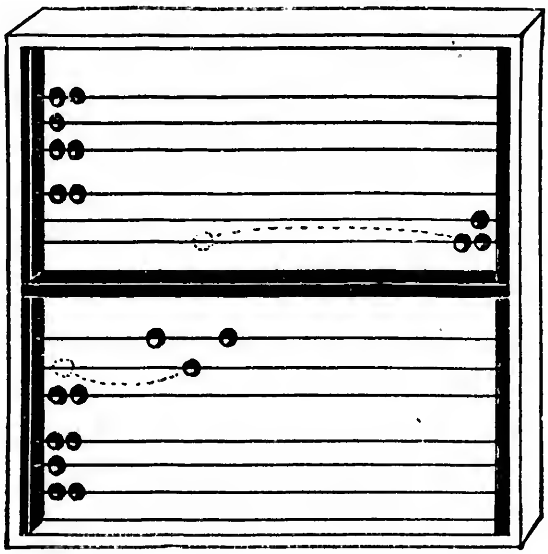
"ONE AND TWO ARE THREE."

(1 + 2 = *3*)

Have the remaining pupil cross over the chalk line, as before, and take up a position upon the other side of the line, thus :—



Have the pupil at the calculator once more make the *corresponding* change there, by moving, to one side as before, the remaining ball (of the upper ten) above the cross-bar, and then moving into position the remaining one of the two balls on the top wire of the lower ten, thus :—



Now ask the pupils, in the class, how many pupils are standing on either side of the chalk line ; or how many balls they see out on the wires above the cross-bar, and how many balls they see out upon the wires below the said bar.

Have them *express orally*, as before, *what they have observed* :—

"NOTHING (ZERO) AND THREE ARE THREE."

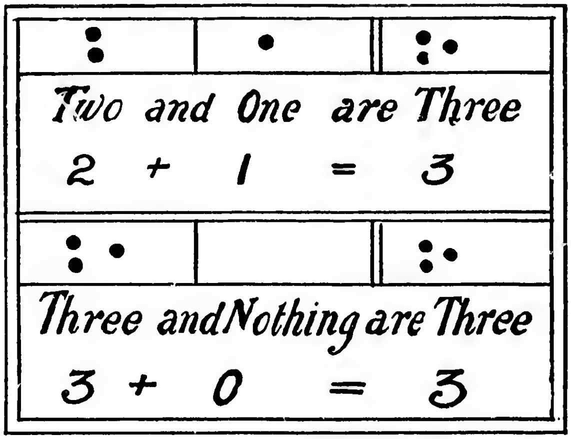
$$(0 + 3 = 3)$$

Thus they will have gone, again, over the possible combinations that make up Three.

Pupils will observe, too, that THREE is composed of *three simple primary units* or related parts, and they will thus have *measured* three in every sense ; they have conceived a whole of related parts, and have related these parts in the whole.

Now it will be well to have the pupils reproduce in their Exercise Books what they have learned. Some such scheme as the following, drawn upon the black-board by the teacher, will be found convenient, the pupils copying the same from the black-board, and "filling into place," what they have seen, as follows:—

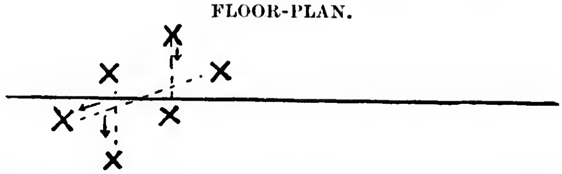
SCHEME AS FILLED UP BY THE PUPILS.



The teacher draws, on the black-board, the skeleton scheme and the pupils copy the skeleton and "fill it up" as above indicated; or better, have the pupils get the "Primary Number-Work" Exercise Book which accompanies this book.

Before beginning the exercise on the number Three, by placing the three pupils on the floor, it might be well for the teacher to indicate (by lightly-drawn chalk lines and crosses), on the floor, the way in which the pupils must move in order to give the *two combinations* of THREE. This will save the teacher *placing the pupils* at each move:—

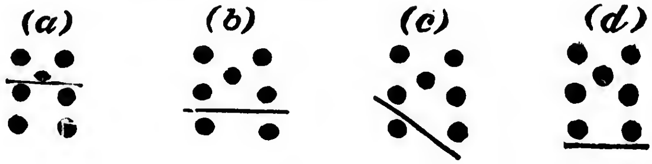
FLOOR-PLAN.



☞ The ×'s represent the stations the pupils assume, and the *arrows* and *dotted lines* the several courses taken in making the changes.

We must have the conscious "recognition of THREE (∴) as ONE WHOLE (unity), and of the *three* things as individuals and of the *unity* as made up of the three things, by *comparing*, *relating*, *measuring*, *counting*, etc."

All the FUNDAMENTAL NUMBERS should be dealt with in a similar way. Take the number Seven, for instance, and by pursuing the method as outlined, the *relation* of the *several combinations (pairs of addends or related parts)* which make up that number may be clearly brought before the minds of the pupils :—



(*a*) 3 + 4 = 7
(*b*) 2 + 5 = 7
(*c*) 1 + 6 = 7
(*d*) 0 + 7 = 7

These are *all the pairs of addends* which make up the number SEVEN. The strokes seen represent chalk lines which may be erased when no longer needed. Use Number-Form SEVEN, Fundamental Numbers on Calculator (Fig. A).

The same ideas may be brought clearly before the pupils' minds by using the Smaller Calculator (Fig. A) with the ten balls. It is intended as a means of *illustrating* all the Fundamental Numbers.

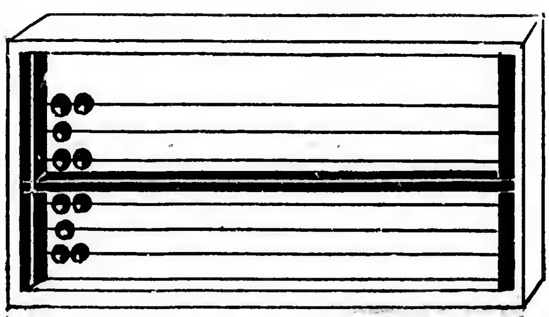
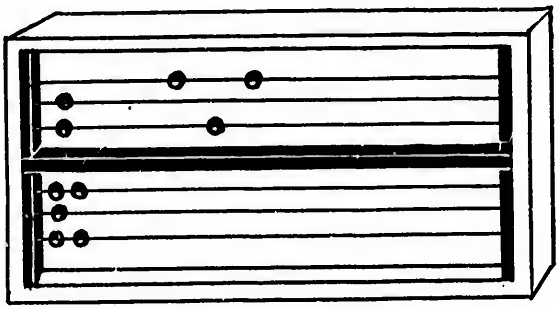


FIG. A.

The above cut represents the Smaller Calculator, which consists of six wires on which move ten balls of various colours, the whole being fixed in a ball-frame, as seen in Fig. A.

The significance of the numbers from One to Ten, and the SYMBOLS (FIGURES) used to represent them may be taught, *objectively*, by means of the balls above referred to. "Objects (and measured things) aid the mind in its work of constructing numerical ideas."

For the sake of variety the numbers three and seven, already referred to, may be more clearly presented to the minds of the pupils by using the Smaller Ball-Frame, as follows :—



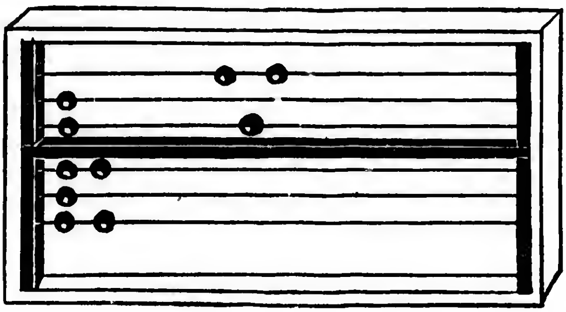
The above is the Smaller Ball-Frame with the number three represented upon it, by moving out three balls upon the wires as indicated.

ANALYTIC-SYNTHETIC PROCESS.

(Analysis and Synthesis of the Number Three.)

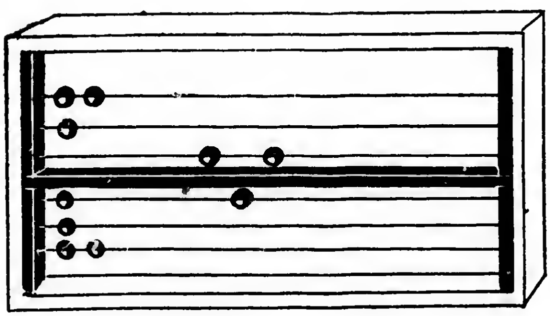
ADDITIONS.

(*a*)



3 + 0 = 3
0 + 3 = 3

(*b*)



2 + 1 = 3
1 + 2 = 3

Move out upon the wires *three balls* as indicated in (*a*) above; then place the edge of the card-board, held in the hand, immediately below the three balls as indicated (the cross-bar between the wires indicating the position of the card-board), and turning it so as to reveal the *three balls*, ask the pupils how many balls they see. They answer :—

"THREE BALLS."

Then, tu .ing the card-board the opposite way, so as to hide the three balls, ask the pupils how many balls they see now. They answer :—

"NO BALLS."

Now, have the pupils make the *oral statement*—as you turn the card-board to hide, alternately, the *nothing* (*no ball*) and the *three* (*three balls*) :—"Three balls and no balls are three balls," (3+0=3); and *vice versâ:* "No balls and three balls are three balls," (0+3=3).

Again, move out upon the wires, three balls, as indicated in (*b*) above ; and, placing the card-board between the *two* balls and the *one* ball as indicated (the line drawn between the wires indicating the position of the card-board), and turning it so as *to hide* the *one* ball and *reveal* the *two* balls, ask the pupils, as before, how many balls they see. They answer :—

"TWO BALLS."

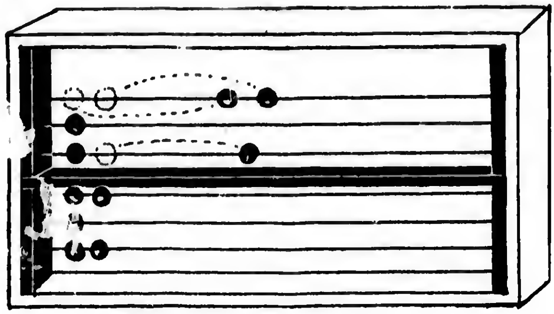
Then, turning the card-board the opposite way so as to hide the two balls, and reveal the one ball, ask the pupils how many balls they see now. They answer :—

"ONE BALL."

Now, have the pupils make an oral statement, as before, of what they saw as you turned the card-board, revealing and hiding the parts of the number, alternately :—"Two balls and one ball are three balls" (2+1=3), and *vice versâ* :—"One ball and two balls are three balls" (1+2=3).

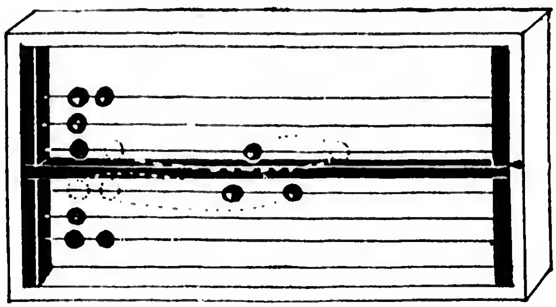
This may be illustrated also by moving the balls upon the wires, as follows :—

(*a*)



$0+3=3$
$3+0=3$

(*b*)



$1+2=3$
$2+1=3$

Having no balls out upon the wires, ask the pupils how many balls are out upon the wires. They answer :—

"No Balls."

Then, move out, together, three balls, as indicated in (*a*) above; now, ask the pupils how many balls are out upon the wires. They answer :—

"Three Balls."

Now, have them make *the oral statement*, as you move the balls :

"No Balls and Three Balls are Three Balls."

Leave the balls as they stand, and ask the pupils how many you must *add to the three balls* to make three balls. They answer :—

"No Balls."

From this have them make the statement :

"Three Balls and No Balls are Three Balls."

Again, move out upon the wires one ball as indicated in (*b*), and ask the pupils how many balls you moved out upon the wires. They answer :

"One Ball."

Then, move out two more balls on another wire as indicated, and ask the pupils how many you moved out the second time. They answer :

"Two Balls."

The three balls will then occupy such positions as to indicate *the two parts of three*, that is, one and two (• and • •). Now, move the one ball to the left, from the position into which it had previously been moved, so as to occupy the position in which they are in (*b*). Have the pupils make the statement :—

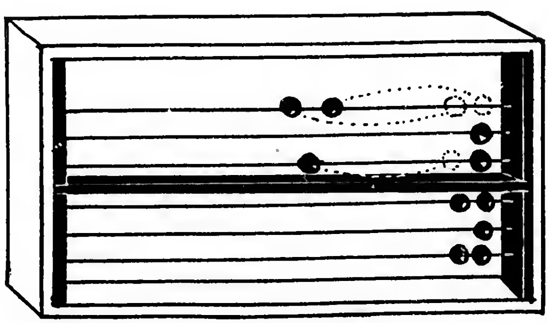
"One Ball and Two Balls are Three Balls."

Now, move the balls back to the left side of the ball-frame, as indicated by the rings in (*b*), so as to occupy their original position, and then reverse the operation, by moving out the two balls first, etc. The statement will then be :—

"Two Balls and One Ball are Three Balls."

SUBTRACTION.

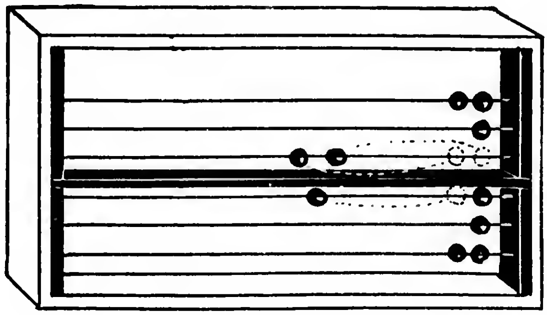
(*a*)



$3 - 3 = 0$

$3 - 0 = 3$

(*b*)



3 − 2 = 1
3 − 1 = 2

Move out upon the wires three balls, as in (*a*), so as to form the number THREE; then ask the pupils how many balls are out on the wires. They answer :—

"THREE BALLS."

Now, move the balls back to their former positions as indicated by the rings in (*a*), and the pupils will see that :—"Three balls from three balls leaves no balls," (3 − 3 = 0); also, by moving the balls back into their former positions, so as to form the number three again [see (*a*)], and leaving them as they stand upon the wires, it may be shown that :—"No balls from three balls leaves three balls," (3 − 0 = 3).

Again, move out upon the wires three balls, so as to form the number three, as in (*b*); now, ask the pupils how many balls are out upon the wires. They answer :—

"THREE BALLS."

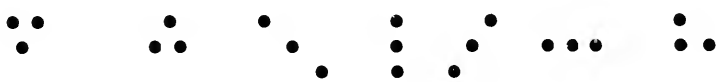
Now, move two of the balls back to the places indicated by the rings [see (*b*)], and the pupils will see that: "Two balls from three balls leaves one ball," (3 − 2 = 1). Now, move the balls back, so as to form the number three as before; then, move away to the left the lower ball forming the number three, so as to occupy the place indicated by the ring, and the pupils will see that :—"One from three leaves two," (3 − 1 = 2).

These *subtractions* may be performed, also, by using the card-board *to partition off the parts* for subtracting.

The ADDITIONS and SUBTRACTIONS should be taught together, as the two operations go hand in hand; because they are *inverse* operations.

This ANALYSIS and SYNTHESIS, or "*partition* and *re-combination*" of numbers is of *vital importance in teaching* them. *Objective teaching of number* APPEALS TO THE CHILD'S UNDERSTANDING. "He learns," says Dr. McLellan, "to think relations by seeing them," and "seeing is believing."

After reasonable drill on one form, other forms of a number should be given, as for example, the different forms of three:—



Etc.

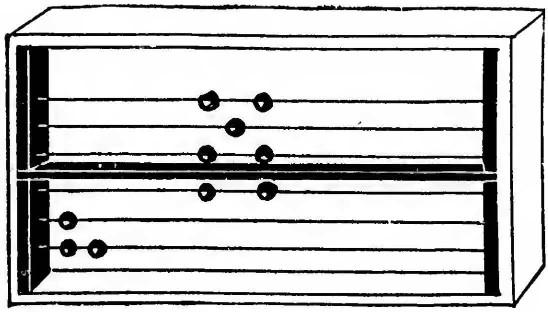
These may be shown either upon the ball frame or upon the black-board, using dots instead of balls. Three should be taught in the form  lastly, and before proceeding to  (4), so that pupils may see clearly the relation of three to four. *Each number should be thoroughly understood before going to the next.*

The Analysis and Synthesis of the Number Three may be put in the following form:—

THREE.

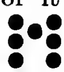
Additions.	*Subtractions.*
3 + 0 = 3	3 − 3 = 0
2 + 1 = 3	3 − 2 = 1
1 + 2 = 3	3 − 1 = 2
0 + 3 = 3	3 − 0 = 3

Analysis and Synthesis of the Number Seven.

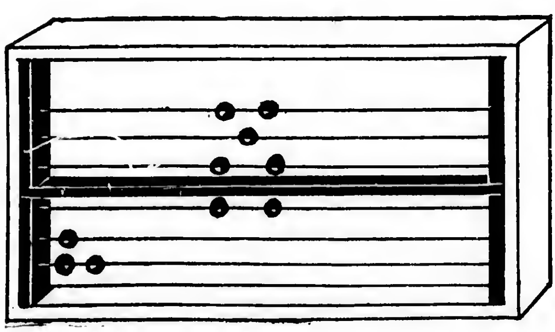


SMALLER BALL-FRAME.

The above represents the number SEVEN AS A WHOLE.

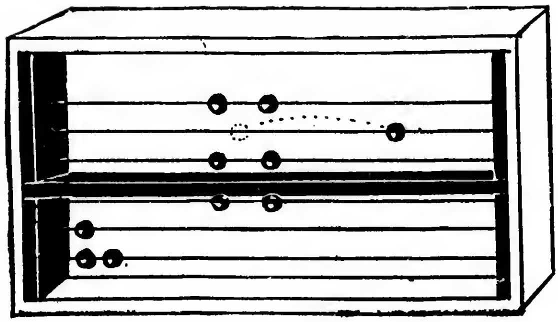
By means of a piece of card-board to partition off or separate the related pairs of addends forming the number SEVEN, the analysis and synthesis, "partition" and "re-combination," (or "*parting*" and "*wholing*"), of it may be taught, either from the fundamental number-form  as seen in Fig. (A), p. vii., or from the ball-frame, as seen in the above cut; but, the better way to teach it is by moving the balls on the wires as illustrated by the following diagrams :

(*a*)



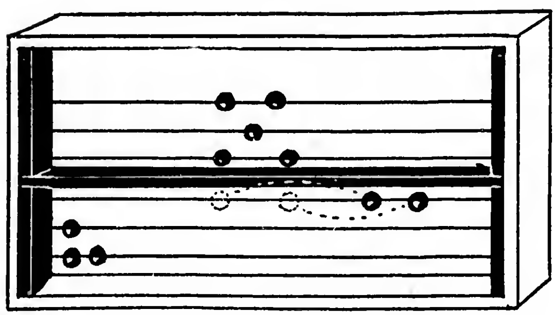
$7 + 0 = 7$

(*b*)



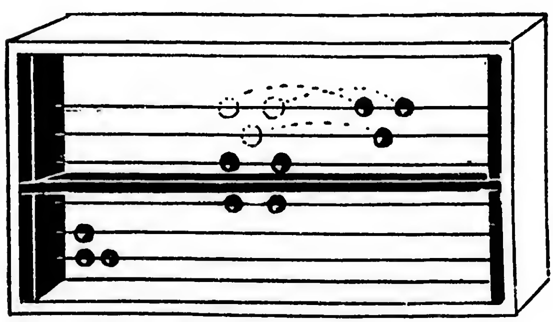
6 + 1 = 7

(*c*)



5 + 2 = 7

(*d*)



4 + 3 = 7

These are *all the pairs of addends* which make up the number SEVEN ; these have their *related forms*, however, that is, $7+0=0+7$, $6+1=1+6$, $5+2=2+5$, and $4+3=3+4$, in all eight combinations :—

$$(a) \begin{Bmatrix} 7+0=7 \\ 0+7=7 \end{Bmatrix}; \ (b) \begin{Bmatrix} 6+1=7 \\ 1+6=7 \end{Bmatrix}; \ (c) \begin{Bmatrix} 5+2=7 \\ 2+5=7 \end{Bmatrix}; \ (d) \begin{Bmatrix} 4+3=7 \\ 3+4=7 \end{Bmatrix}.$$

The subtractions may be taught by the same diagrams, so that, by means of the balls as seen in (a), the following four things may be taught :—$7+0=7$; $0+7=7$; $7-7=0$, and $7-0=7$; so also in (b):—$6+1=7$; $1+6=7$; $7-6=1$, and $7-1=6$; therefore we have the following :—

SEVEN.

Additions.	*Subtractions.*
$7+0=7$	$7-7=0$
$6+1=7$	$7-6=1$
$5+2=7$	$7-5=2$
$4+3=7$	$7-4=3$
$3+4=7$	$7-3=4$
$2+5=7$	$7-2=5$
$1+6=7$	$7-1=6$
$0+7=7$	$7-0=7$

In teaching the Subtractions, put out upon the wires the requisite number of balls to represent the number under consideration, and, by sliding away one or more, as the case may require, the subtractions may be taught; however, instead of sliding away the balls, a piece of card-board may be held in the hand (as described) and a part of the number (that is one of the pair of related addends) may be covered up by placing the card-board between the related parts of the number, *e.g.*,  (seven), so as to hide, first the ● ● (2), and then the  (5); in this way the pupils will *readily* see that  and ● ● make seven; have them tell you orally what they see as you change the position of the card-board to form the combinations, as "*Two and five are seven*" ; "*Five and two are seven,*" etc.; or —"*Three and four are seven,* and "*Four and three are seven,*" etc.

Thus it will be seen that, in the Addition and Subtraction stages, there are FOUR THINGS *to learn about every pair of addends which form a number; e.g.*,

FOUR and THREE forming *Seven.*

(*a*) **4 + 3** = *7*
(*b*) **3 + 4** = *7*
(*c*) **7 - 4** = *3*
(*d*) **7 - 3** = *4*

Two Additions.	(*a*) FOUR and THREE are *seven.* (*b*) THREE and FOUR are *seven.* (*c*) FOUR from SEVEN leaves *three.* (*d*) THREE from SEVEN leaves *four.*	Two Subtractions.

Addition and Subtraction are inverse operations; and hence should be taught together. "They have their origin in the operation of counting with an unmeasured unit, with the idea of aggregation—of more or less."

So also it will be seen that, in the Multiplication and Division stages, there are FOUR THINGS *to learn about every pair of factors which compose a number*; *e.g.*,

FOUR and THREE composing *Twelve.*

(*a*) **4 × 3** = *12*
(*b*) **3 × 4** = *12*
(*c*) **12 ÷ 4** = *3*
(*d*) **12 ÷ 3** = *4*

Two Multiplications.	(*a*) FOUR *times* THREE are *twelve.* (*b*) THREE *times* FOUR are *twelve.* (*c*) FOUR into TWELVE *three times.* (*d*) THREE into TWELVE *four times.*	Two Divisions.

Multiplication and Division, being inverse operations, should go hand in hand together, and should follow the other two rules.

This *analysis and synthesis* of a number *gives a clear perception* of that number.

The Fundamental Numbers should be all taken up in logical order and analyzed, as outlined, care being taken *to have each one thoroughly understood before the next is taken up.*

Pupils should have practice in counting backward and forward as soon as they have learned the Fundamental Numbers ; afterwards, when they have learned the symbols (figures) which represent numbers, and the numbers from One to Twenty, they should write out these numbers, and count backward and forward from One to Twenty.

The Numbers from One to Twenty may be divided into three divisions, as follows :—

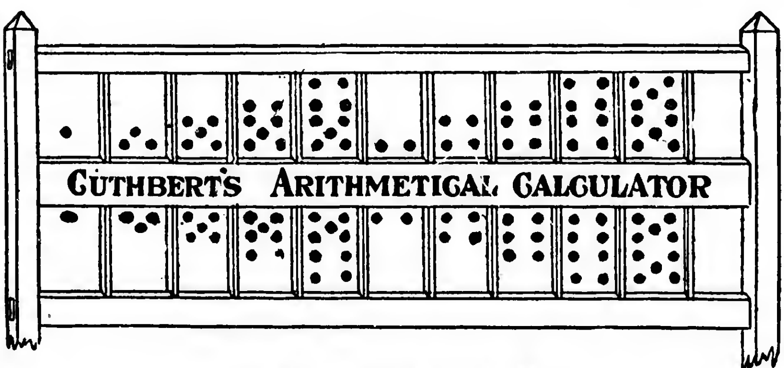
(1) Numbers from One to Five.—Numbers the sum of a pair of which does not exceed ten.

(2) Numbers from Six to Ten.—Numbers the sum of a pair of which exceeds ten.

(3) Numbers from Eleven to Twenty.—Numbers formed by Ten (as a unit) and some Fundamental Number.

The Numbers from One to Five and from Six to Ten may be taken up, by means of the Fundamental Number-Forms and the Smaller Ball-Frame ; then the Numbers from Eleven to Twenty may be taken up, by means of the Automatic Numeral-Frame (Fig. B), the Automatic Mechanical Schedule (Fig. B), and the Larger Calculator or Ball-Frame (Fig. B). For Figures, see p. viii.

FIG. B.



THE AUTOMATIC NUMERAL-FRAME.

The above represents the Automatic Numeral-Frame wherein are arranged (by hemispheres or half-balls in symmetrical grouping) all the *Number-Forms* from ZERO to TEN, in two horizontal rows. Those in the lower row are fixed upon rectangular tablets movable between two grooved pieces of wood, between which, as these tablets are moved forward successively in the same direction, they may be made to assume different relative positions with regard to the *Number-Forms* in the upper row which is stationary, being in one piece and spaced off into correspondent divisions with the lower row of movable tablets. The pressing forward of the lower series of tablets admits of the formation of *all the combinations* in Addition which can possibly be made by *any two* of the said *number-forms.*

These Number-Forms are arranged in two series of eleven each.

Each of the tablets in the lower series (except one which represents Zero, there being no hemispheres or half-balls fixed upon its surface) contains a number of hemispheres varying from One to Ten, so that the series contains all the *Number-Forms* from *Zero* to *Ten.* The two series are alike (in design) and correspond with the Automatic Mechanical Schedule.

Both of these work alike, the Schedule showing the combination in *Symbol-Form*, and the Numeral-Frame in *Number-Form.*

The tablets in the lower series of the Numeral-Frame can be removed from, or inserted into, the lower series; by the groove, in the bars above and below, being made deeper so that the tablets can be lifted clear of the bottom groove.

By means of the Numeral-Frame there can be formed ELEVEN SETS of the SERIES of COMBINATIONS which it is possible *to form in Simple Addition* by *any two of the Number-Forms* from ZERO to TEN.

The following Figures will better illustrate the workings of the Numeral-Frame :—

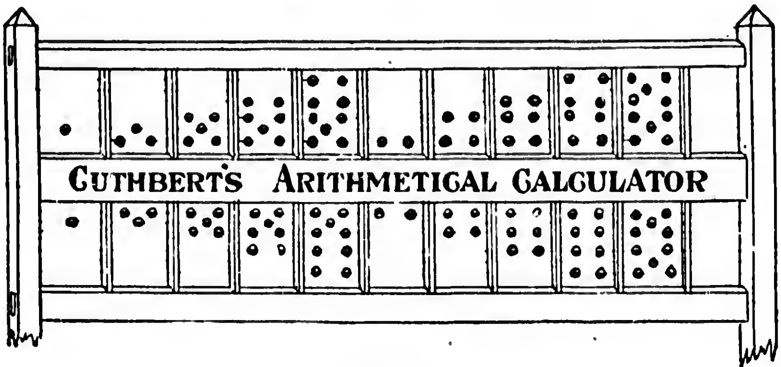


Fig. (*a*).

Fig. (*a*) represents the Automatic Numeral-Frame with the Number-Forms inserted therein, in the order in which they are used to form the *First Set* of the Series of Combinations possible to form with the Numeral-Frame, by moving the lower row of tablets. The order in which the Number-Forms in the Numeral-Frame are inserted therein is as follows: ONE, THREE, FIVE, SEVEN, NINE, TWO, FOUR, SIX, EIGHT, TEN, ZERO. The Number-Forms, as thus arranged, form the First Set of the Series.

The upper row remains stationary; but the lower row is movable between two grooved pieces of wood, between which, as the frames are pressed forward successively, in the same direction and in the order in which they are seen above, the Number-Forms may be made to assume different relative positions with regard to the Number-Forms in the upper row. All the different Combinations in Addition, which can possibly be made by any two of the Number-Forms, may thus be represented.

In order to move the lower row of Number-Forms, the tablet on the extreme left-hand side *is removed*, by pushing it upward (by the thumb-catch at back of tablet,—see back view of Calculator, page ix.) into the upper groove, so as to free it at the bottom, when it may be removed from the Numeral-Frame.

The remainder of the lower row is then *shifted to the left*, and the tablet removed is then *inserted*, at the other end of the said row, so as to fill the vacancy caused by the moving forward of the said row, thus:—

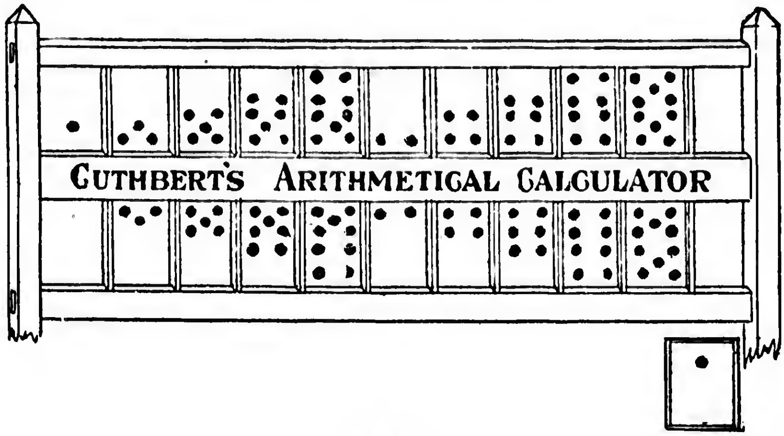


FIG. (*b*).

Fig. (*b*) represents the Numeral-Frame with the tablet, representing the Number-Form "One" removed and ready to be inserted at the other end of the row.

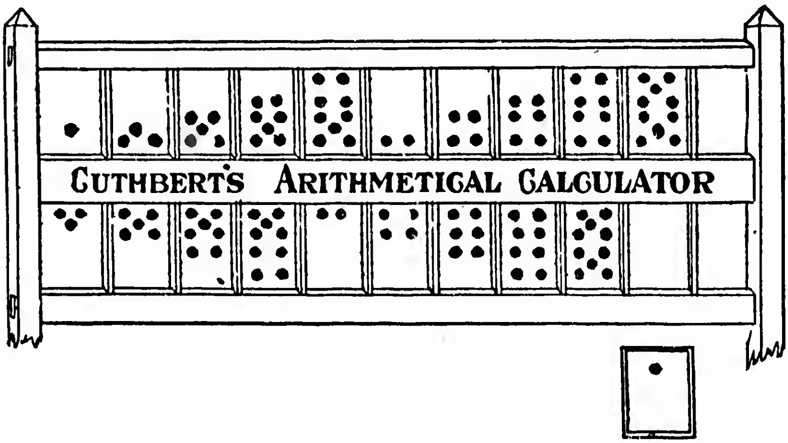


FIG. (*c*).

Fig. (*c*) represents the Numeral-Frame with the remainder of the lower series of tablets moved one place to the left, and the tablet representing the Number-Form "One," ready for insertion at the other end of the series of tablets.

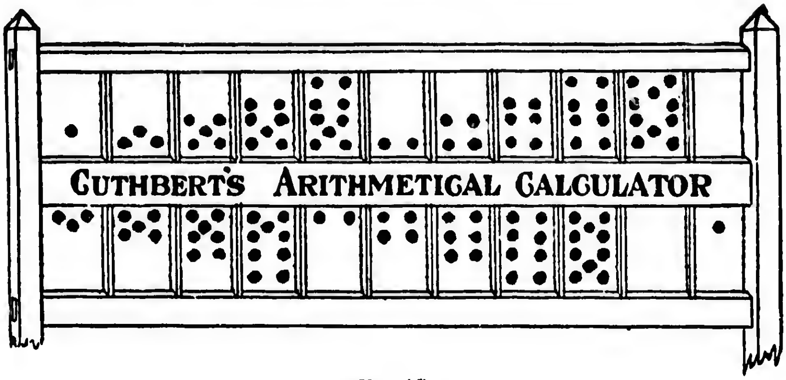


FIG. (*d*).

Fig. (*d*) represents the Numeral-Frame with the remainder of the lower series of tablets moved one place to the left, and the tablet representing the Number-Form "One" inserted at the other end of the series of tablets, thus forming the *Second Set* of the Series of Combinations.

By removing the tablet containing the Number-Form "Three," as it stands in Fig. (*d*), shifting the remainder of the row and inserting it at the other end of the series, the *Third Set* of the Series of Combinations may be formed, as follows :—

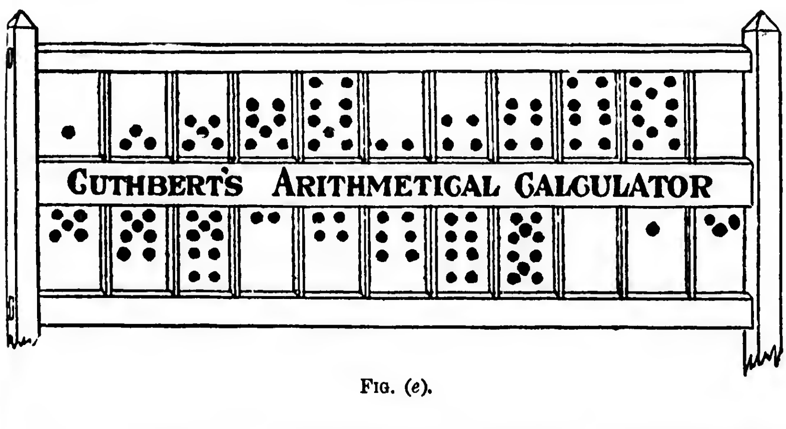


FIG. (*e*).

Fig. (*e*) represents the *Third Set* of the Series of Combinations, formed by the removal and re-insertion of the tablet representing the Number-Form Three.

The remaining Eight Sets completing the Series are formed in a similar way, by removing the tablet to the extreme left in the lower row, shifting the remainder of the said row as before, and inserting the removed tablet at the other end of the row, in the place vacated by the shifting of the remainder of the lower row.

The following Figures will illustrate the remaining Eight Sets of the Series.

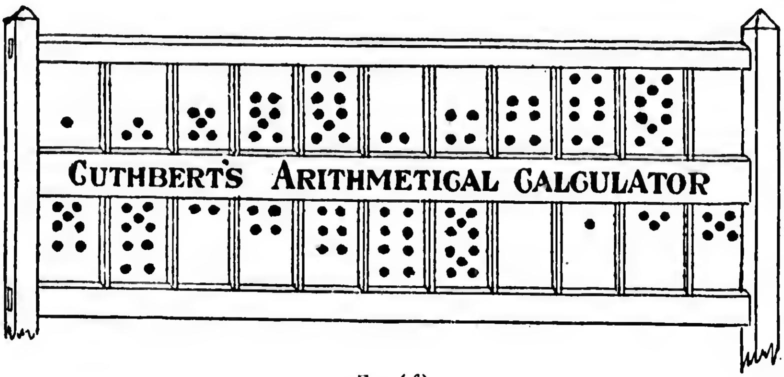


FIG. (*f*).

Fig. (*f*) represents the *Fourth Set* of the Series.

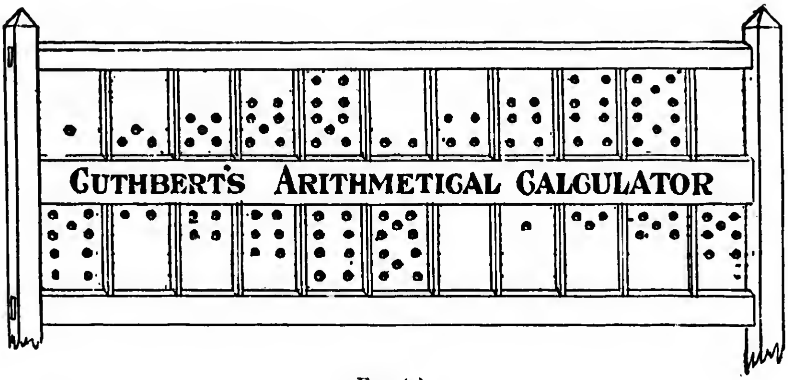


FIG. (*g*).

Fig. (*g*) represents the *Fifth Set* of the Series.

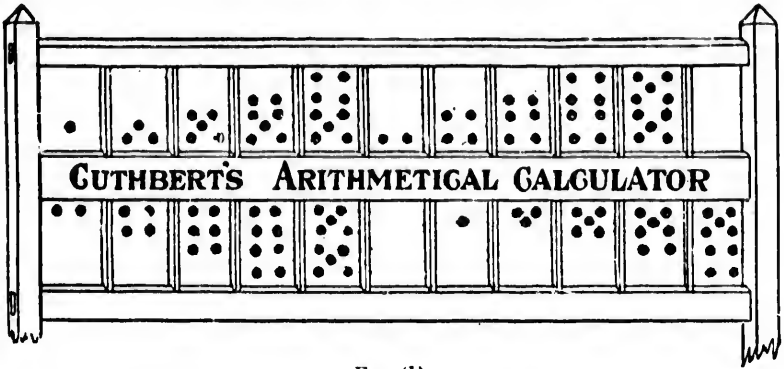


FIG. (*h*).

Fig. (*h*) represents the *Sixth Set* of the Series.

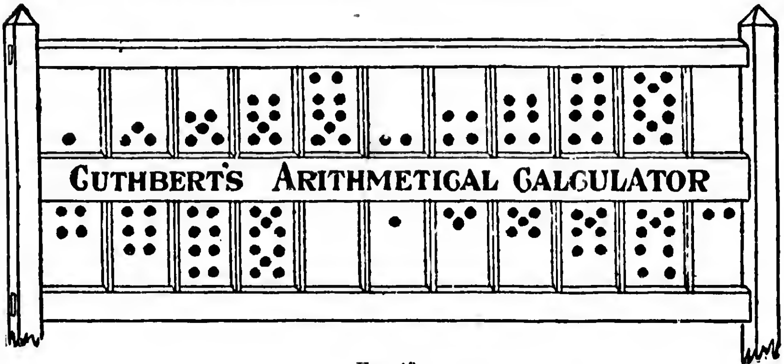


FIG. (*i*).

Fig. (*i*) represents the *Seventh Set* of the Series.

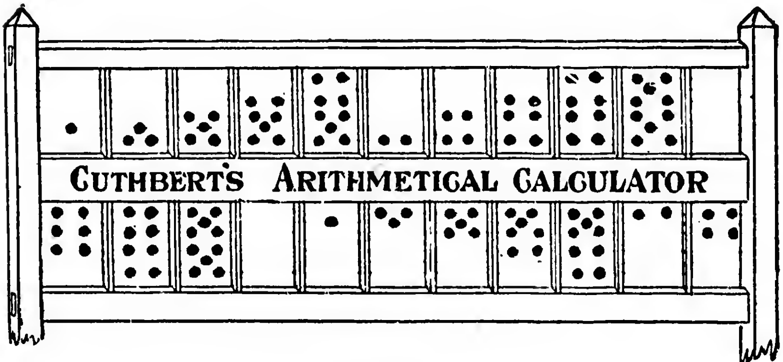


FIG. (*j*).

Fig. (*j*) represents the *Eighth Set* of the Series.

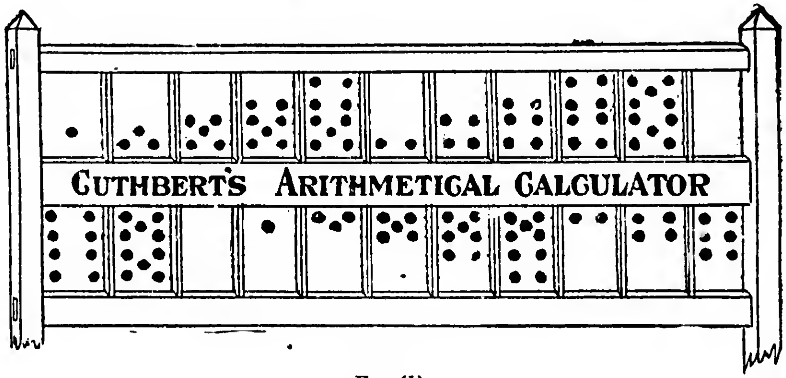


FIG. (*k*).

Fig. (*k*) represents the *Ninth Set* of the Series.

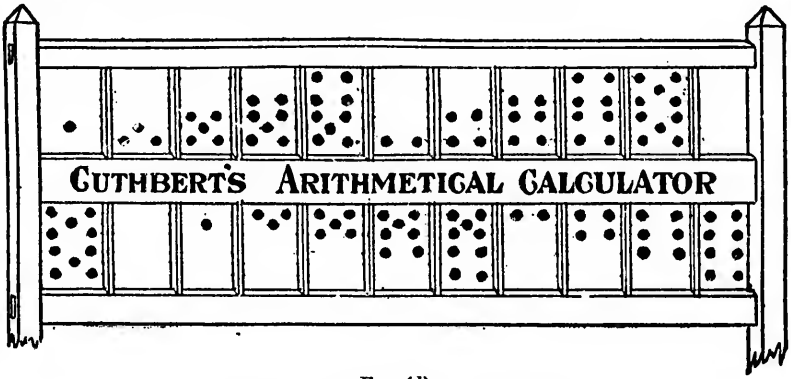


FIG. (*l*).

Fig. (*l*) represents the *Tenth Set* of the Series.

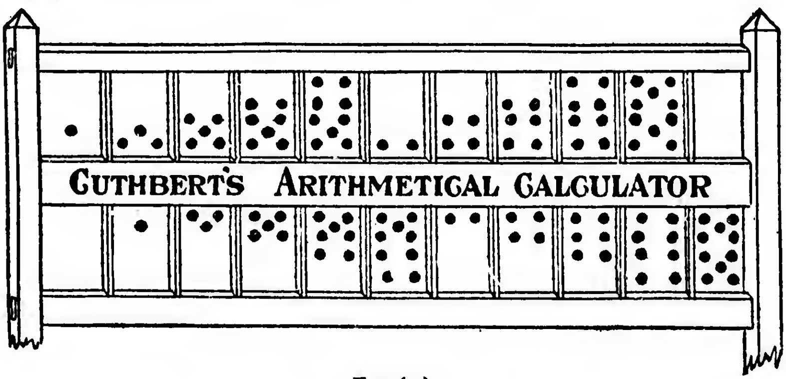


FIG. (*m*).

Fig. (*m*) represents the *Eleventh Set* of the Series.

This completes the Series, making in all Eleven changes, and ONE HUNDRED AND TWENTY-ONE COMBINATIONS.

Of these Combinations, One Hundred and Twenty are significant, and One (0+0) is insignificant, inasmuch as it represents nothing or zero.

Of the Significant Combinations, taking into account the Combinations with Zero, there are, in all, SIXTY-FIVE Combinations, and no more. The other *Fifty-Five* Combinations, shown on the Numeral-Frame, are simply *related forms.* (See the Table of Combinations on Numbers from One to Twenty, page 38).

The next Device upon the Calculator is The Automatic Mechanical Schedule whereon are painted all the NUMBER-SYMBOLS, from ZERO to TEN, in two horizontal rows. Those in the lower row are painted upon blocks, movable in a rabbeted groove, within which, as they are pressed forward, successively in the same direction, they may be made to assume different relative positions with regard to the Number-Symbols in the upper row. ALL THE DIFFERENT COMBINATIONS in Addition, which can possibly be made by *any two* of the said Number-Symbols, may thus be represented.

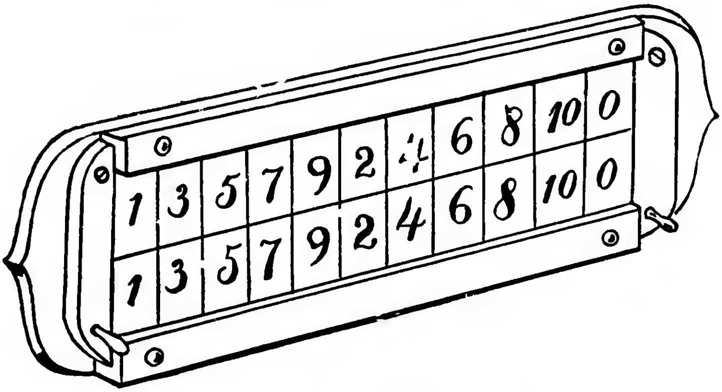
The said Schedule is provided at the ends with stops, each of which is made to open outward on a pivot, so as to admit of the removal or reception of the movable blocks in the lower row with the view of forming New Combinations of Number-Symbols, in Addition, as described above.

The Schedule *is correspondent to* the Numeral-Frame ; the *former* makes the *Combination in Symbol-Form* and the *latter* in *Number-Form.*

The Schedule may be better explained by the following Figures:—

The Automatic Mechanical Schedule.

SET 1 OF THE SERIES OF COMBINATIONS.



THE AUTOMATIC MECHANICAL SCHEDULE.

The above cut represents the Automatic Mechanical Schedule with the Number-Symbols painted upon it (as described in the foregoing paragraph) in two rows, the upper row being stationary and the lower one movable in a groove. The Symbols as thus arranged form the *First Set* of the *Series of Eleven Combinations* which it is possible to form, with the said Schedule, by moving the lower row of blocks. The lower row, moved *eleven times*, gives the ELEVEN SETS of the SERIES OF COMBINATIONS, thus forming ONE HUNDRED AND TWENTY-ONE *simple additions*, **the greatest number** possible with *any two* of the said *symbols*.

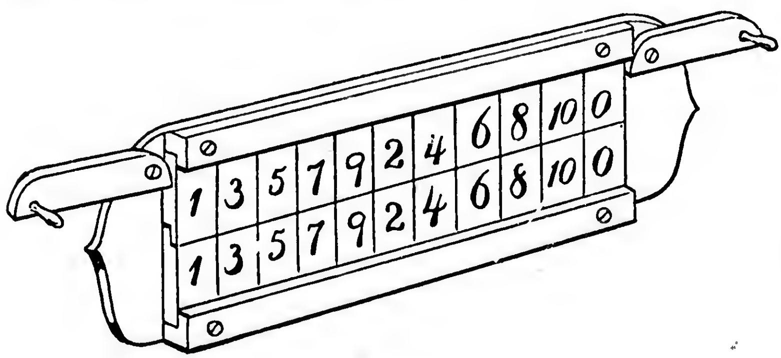


FIG. (*a*).

Fig. (*a*) represents the Schedule with the stops pushed back, so as to admit of the removal, from the lower horizontal row, of block marked 1, preparatory to the pushing of the remainder of the blocks, in the said row, *one place to the left.*

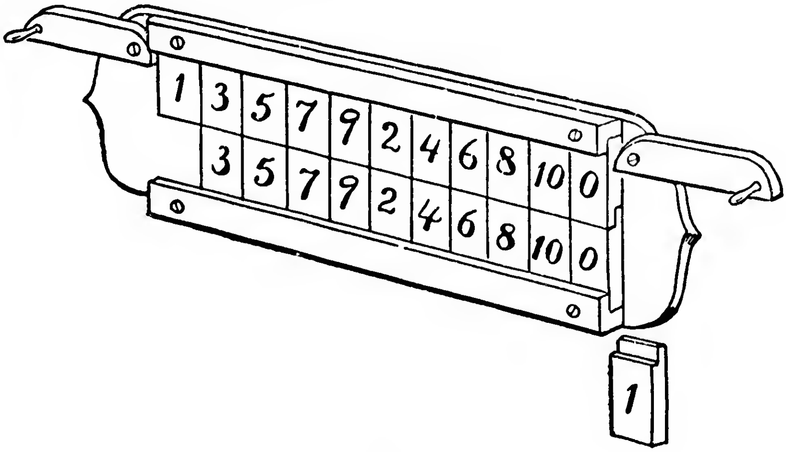


FIG. (*b*).

Fig. (*b*) represents the Schedule with block marked 1 removed and the remainder of the lower row of blocks ready for shifting *one place to the left*, block marked 1 being ready in position for insertion at the other end of the said lower row.

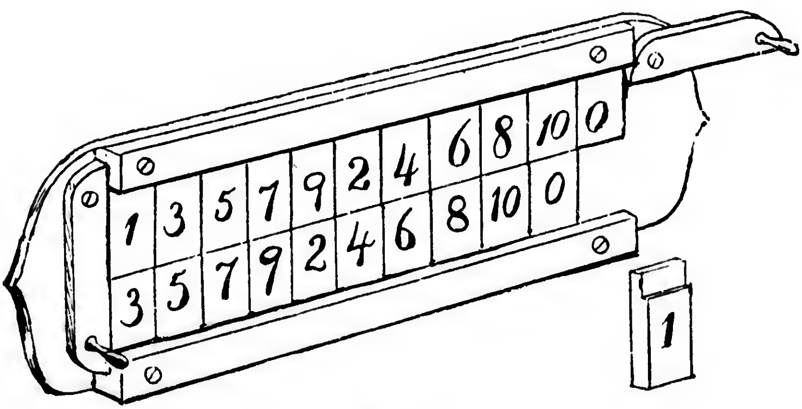


FIG. (*c*).

Fig. (*c*) represents the Schedule with the remainder of the lower row of blocks moved *one place to the left;* the stop on the left of the Schedule closed and block marked 1 ready for insertion in the place vacated by the movement forward of block marked zero (0).

SET 2 OF THE SERIES OF COMBINATIONS.

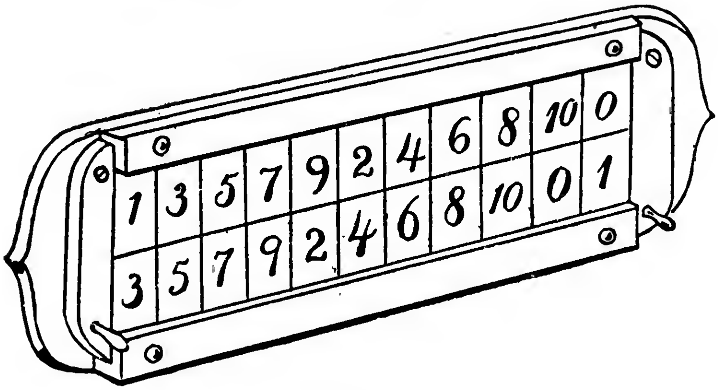


FIG. (*d*).

Figure (*d*) represents the Schedule with the lower row moved one place to the left; block marked 1 inserted in the place previously occupied by block marked 0, and the stop in the right of the Schedule closed, so that a New Set of the Series of the Eleven Combinations for Simple Additions is formed.

The next Set of the Series of Combinations is formed in a similar way, pushing back the stops, removing block marked 3, closing the stop on the left of the Schedule, and moving the remainder of the lower row of blocks one place to the left, as before; then inserting block marked 3 in the place vacated by block marked 1, and then closing the stop on the right of the Schedule. This will give another Set of the Series of the Eleven Combinations of the Number-Symbols for Simple Additions.

The remaining *Nine Sets* of the *Series of Combinations*, as formed by the Schedule to complete the Eleven Sets of the Series, may be represented by the following Figures, each of which represents a New Set, after the shifting of the lower row of blocks one place to the left, as already explained.

The Remaining Nine Sets of the Schedule Combinations.

Set 3 of the Series of Combinations.

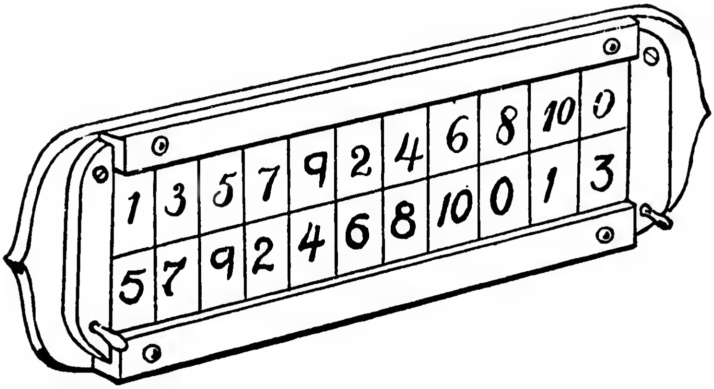


Fig. (*e*).

Set 4 of the Series of Combinations.

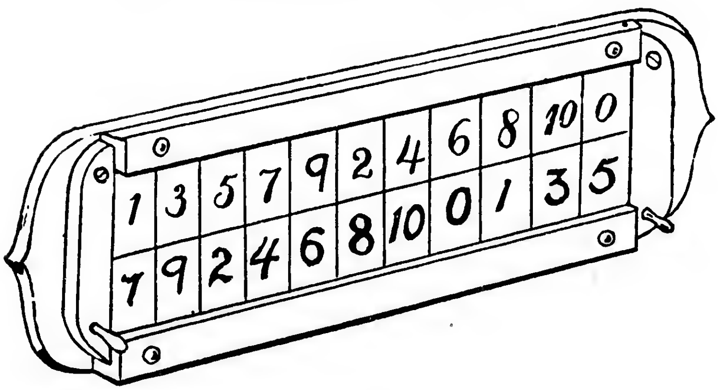


Fig. (*f*).

SET 5 OF THE SERIES OF COMBINATIONS.

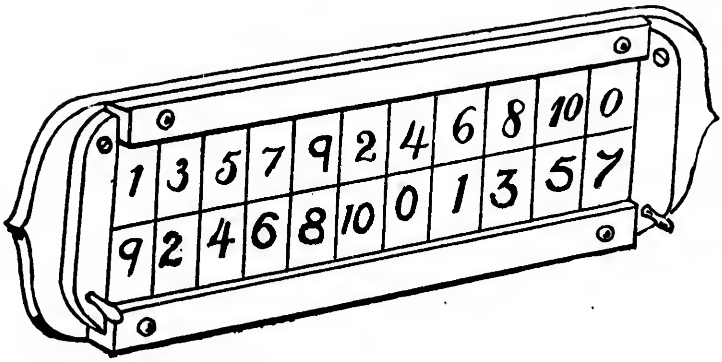


FIG. (*g*).

SET 6 OF THE SERIES OF COMBINATIONS.

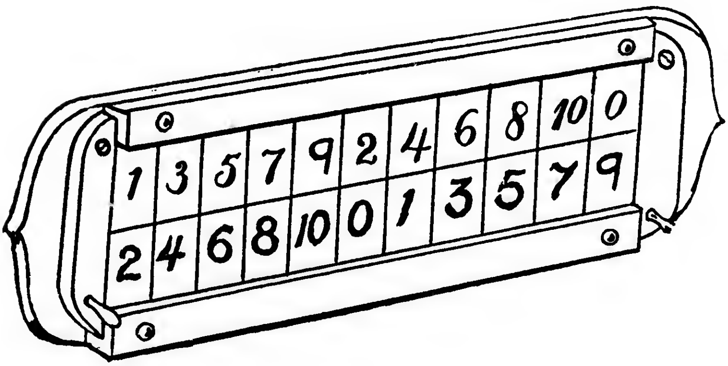


FIG. (*h*).

SET 7 OF THE SERIES OF COMBINATIONS.

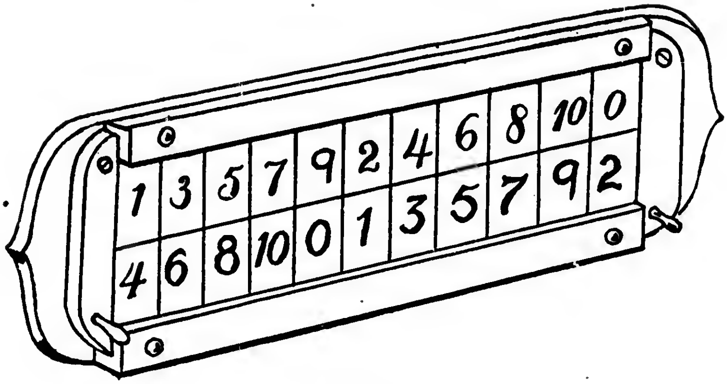


FIG. (*i*).

SET 8 OF THE SERIES OF COMBINATIONS.

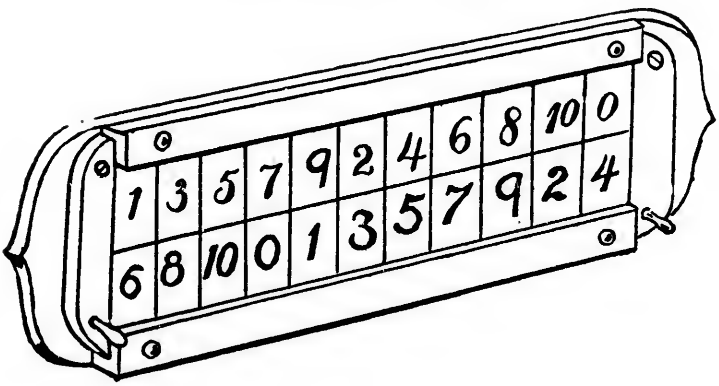


FIG. (*j*).

SET 9 OF THE SERIES OF COMBINATIONS.

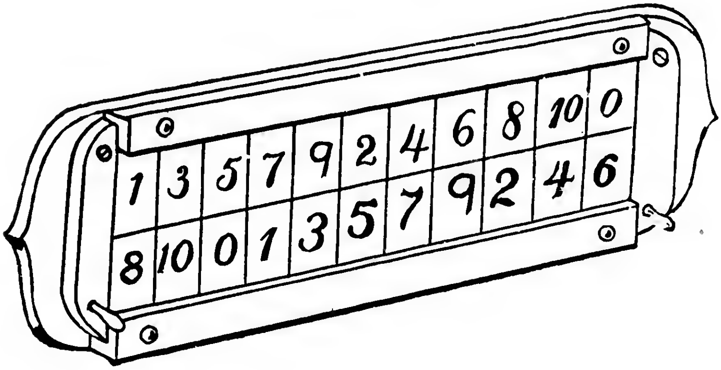


FIG. (k).

SET 10 OF THE SERIES OF COMBINATIONS.

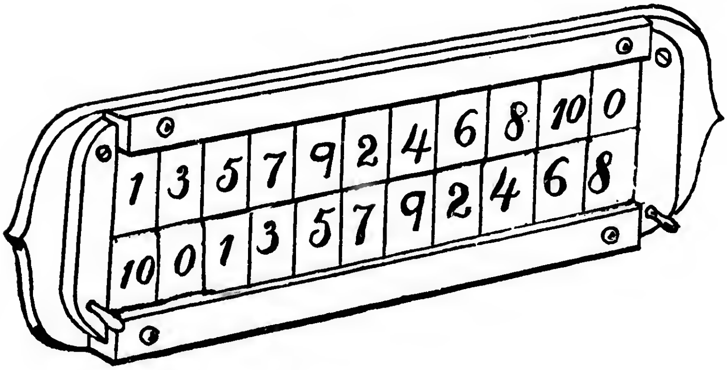


FIG (l).

SET 11 OF THE SERIES OF COMBINATIONS.

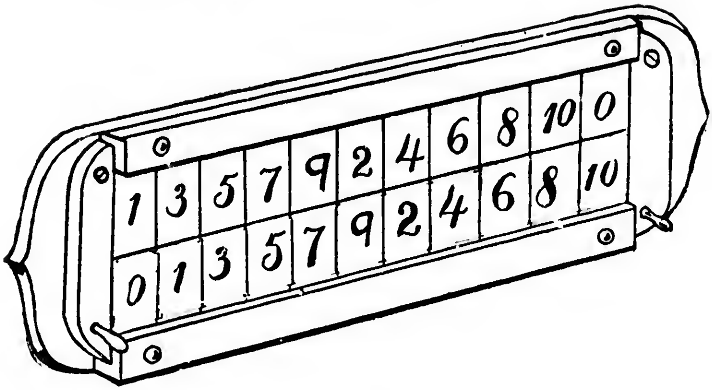


FIG. (*m*).

Sums of Schedule Combinations.

TABLE OF SUMS.

SETS.	1	2	3	4	5	6	7	8	9	10	11
	2	4	6	8	10	3	5	7	9	11	1
	6	8	10	12	5	7	9	11	13	3	4
	10	12	14	7	9	11	13	15	5	6	8
	14	16	9	11	13	15	17	7	8	10	12
SUMS	18	11	13	15	17	19	9	10	12	14	6
	4	6	8	10	12	2	3	5	7	9	11
	8	10	12	14	4	5	7	9	11	13	6
	12	14	16	6	7	9	11	13	15	8	10
	16	18	8	9	11	13	15	17	10	12	14
	20	10	11	13	15	17	19	12	14	16	18
	0	1	3	5	7	9	2	4	6	8	10

The above Table shows the Sums of the ONE HUNDRED AND TWENTY-ONE (121) COMBINATIONS of Two Number-Symbols each, or it may be said of Two Digits each.

The following is *another form* of Table showing all the Combinations possible in Addition with any two of the Number-Symbols from Zero to Ten.

Table of Combinations on Numbers from One to Twenty.

1	2	3	4	5	6	7	8	9	10
1+0	2+0	3+0	4+0	5+0	6+0	7+0	8+0	9+0	10+0
	1+1	2+1	3+1	4+1	5+1	6+1	7+1	8+1	9+1
0+1			2+2	3+2	4+2	5+2	6+2	7+2	8+2
11	0+2	1+2			3+3	4+3	5+3	6+3	7+3
	12	0+3	1+3	2+3			4+4	5+4	6+4
		13	0+4	1+4	2+4	3+4			5+5
10+1			14	0+5	1+5	2+5	3+5	4+5	
9+2	10+2			15	0+6	1+6	2+6	3+6	4+6
8+3	9+3	10+3			16	0+7	1+7	2+7	3+7
7+4	8+4	9+4	10+4			17	0+8	1+8	2+8
6+5	7+5	8+5	9+5	10+5			18	0+9	1+9
	6+6	7+6	8+6	9+6	10+6			19	0+10
5+6			7+7	8+7	9+5	10+7			20
4+7	5+7	6+7			8+8	9+8	10+8		
3+8	4+8	5+8	6+8	7+8			9+9	10+9	
2+9	3+9	4+9	5+9	6+9	7+9	8+9			
1+10	2+10	3+10	4+10	5+10	6+10	7+10	8+10	9+10	10+10

Or, understanding that 10^9 means that the number Ten is formed as a sum, nine times, by nine different combinations of two number-symbols (from 0 to 10) each, we may have the following Summary of the said One Hundred and Twenty-One Combinations of Simple Additions :—

0^1 1^2 2^3 3^4 4^5 5^6 6^7 7^8 8^9 9^{10}
10^{11}
11^{10} 12^9 13^8 14^7 15^6 16^5 17^4 18^3 19^2 20^1

The foregoing Table shows :—

65 Combinations with numbers 0 to 20. These are seen in the upper portion to the spacing.

The 55 related forms with numbers 0 to 20. These are seen below the spacing.

Then the Calculator makes 0+0, which is an insignificant Combination (inasmuch as it stands for nothing), and it is not shown on the Table of Combinations.

The 65 Combinations (*primary*) may be arranged as follows into two classes :—

(i) Combinations to Ten.

${0 \atop 1}$; ${0 \atop 2}$, ${1 \atop 1}$; ${0 \atop 3}$, ${1 \atop 2}$; ${0 \atop 4}$, ${1 \atop 3}$, ${2 \atop 2}$; ${0 \atop 5}$, ${1 \atop 4}$, ${2 \atop 3}$; ${0 \atop 6}$, ${1 \atop 5}$, ${2 \atop 4}$, ${3 \atop 3}$; ${0 \atop 7}$, ${1 \atop 6}$, ${2 \atop 5}$, ${3 \atop 4}$;

${0 \atop 8}$, ${1 \atop 7}$, ${2 \atop 6}$, ${3 \atop 5}$, ${4 \atop 4}$; ${0 \atop 9}$, ${1 \atop 8}$, ${2 \atop 7}$, ${3 \atop 6}$, ${4 \atop 5}$; ${0 \atop 10}$, ${1 \atop 9}$, ${2 \atop 8}$, ${3 \atop 7}$, ${4 \atop 6}$, ${5 \atop 5}$.

35 Combinations, *0 to 10.*

(ii) Combinations of Ten with the Intermediate Units to 20.

${1 \atop 10}$, ${2 \atop 9}$, ${3 \atop 8}$, ${4 \atop 7}$, ${5 \atop 6}$; ${2 \atop 10}$, ${3 \atop 9}$, ${4 \atop 8}$, ${5 \atop 7}$, ${6 \atop 6}$; ${3 \atop 10}$, ${4 \atop 9}$, ${5 \atop 8}$, ${6 \atop 7}$; ${4 \atop 10}$, ${5 \atop 9}$, ${6 \atop 8}$, ${7 \atop 7}$;

${5 \atop 10}$, ${6 \atop 9}$, ${7 \atop 8}$; ${6 \atop 10}$, ${7 \atop 9}$, ${8 \atop 8}$; ${7 \atop 10}$, ${8 \atop 9}$; ${8 \atop 10}$, ${9 \atop 9}$; ${9 \atop 10}$; ${10 \atop 10}$.

30 Combinations, *11 to 20.*

(35 + 30 = 65 Combinations.)

The upper part of the foregoing Table gives the Combinations from One to Ten, and the lower part, the Combinations from Eleven to Twenty, both Numbers inclusive in each case. Leaving out of account the Combination "zero plus zero" (which is insignificant, inasmuch as it represents the symbol zero, and is not shown in Table, though it is formed by the Schedule and the Numeral-Frame), we have, in all, *Sixty-Five Combinations.* The other

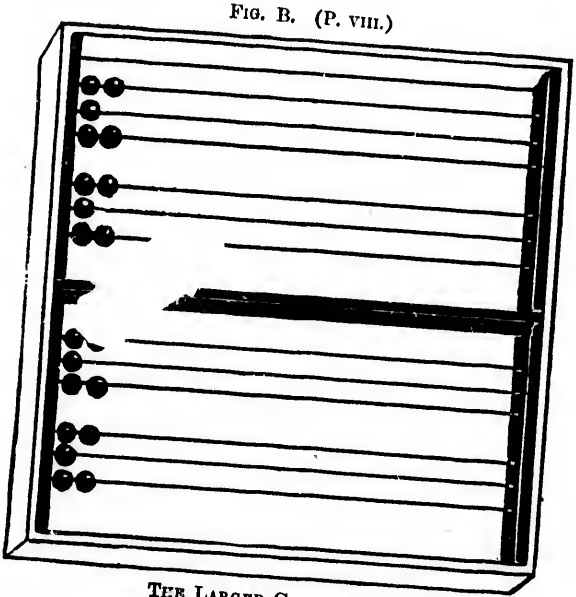
FIFTY-FIVE COMBINATIONS are *related forms*, that is, other ways of expressing the same Combinations. The Number SEVEN, for example, is formed *eight times*, by the Numeral-Frame and Schedule respectively:—

$$\text{SEVEN} = \left\{ \begin{array}{c} \left\{\begin{array}{c}7+0\\6+1\\5+2\\4+3\end{array}\right\} \\ \text{and} \\ \left\{\begin{array}{c}3+4\\2+5\\1+6\\0+7\end{array}\right\}\end{array}\right\} \text{ as per Numeral-Frame and Schedule;}$$

but of these *eight forms* there are only *four different combinations*, as the lower four are simply other ways of making the upper four. A wider spacing in the Table separates the fundamental from the related forms.

A child must be familiar with the whole One Hundred and Twenty Significant Combinations as shown in the aforesaid Table, as he does not know a number until he knows all the Combinations which make up that number.

FIG. B. (P. VIII.)



THE LARGER CALCULATOR.

The above cut represents the Larger Ball-Frame and consists of twelve wires, arranged in groups of three each, on which move TWENTY BALLS, the whole being fixed in a frame and attached to the Calculator, for the purpose of *illustrating*, *objectively*, the ONE HUNDRED AND TWENTY-ONE COMBINATIONS formed upon the Numeral-Frame and Schedule, already described.

For example, in teaching the Fourth Simple Combination, in Addition, as shown in the First Set of the Series of Combinations, move out upon the wires, "seven and seven" balls to indicate, objectively, the said Combination, thus :—

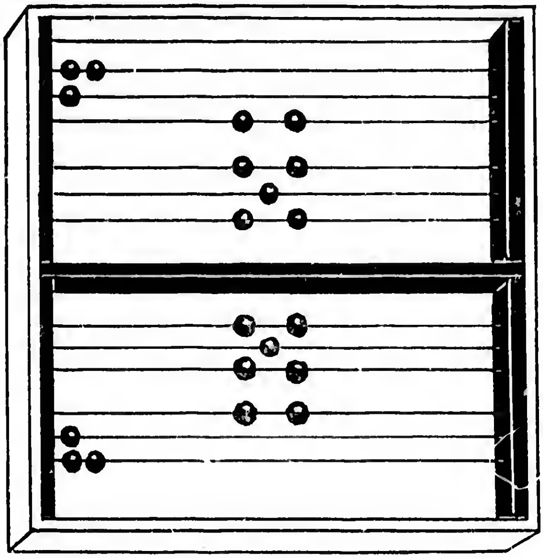


FIG. (*a*).

Fig. (*a*) represents the fourth combination in the First Set of the Series of Combinations—

"SEVEN and SEVEN are *Fourteen*"

$7 + 7 = 14$.

By means of the Larger Ball-Frame all the different Combinations, which make up the number Fourteen, may be illustrated :—

Fourteen.

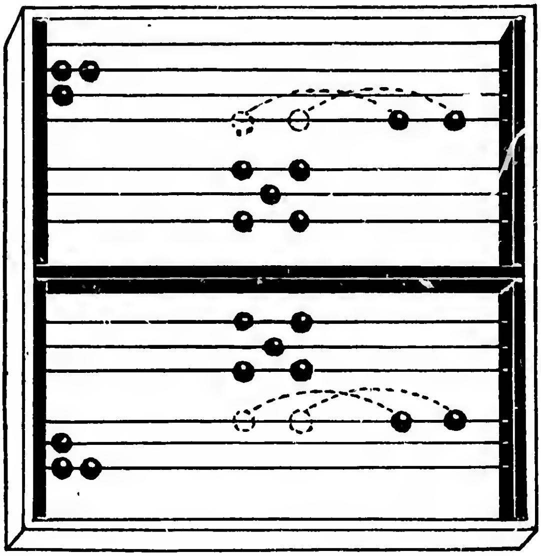


Fig. (b).

10 + 4 = 14 4 + 10 = 14

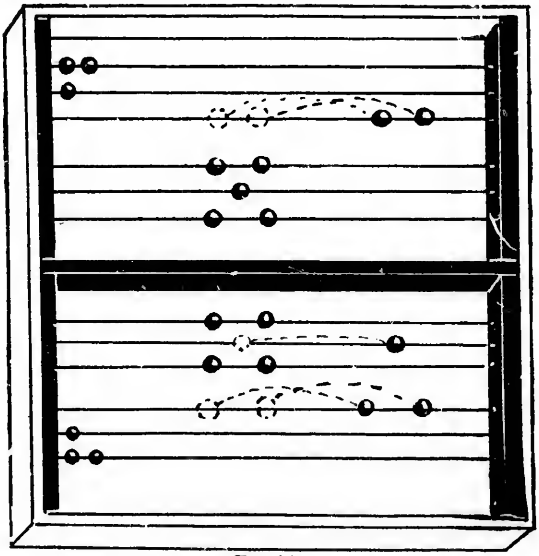


Fig (c).

9 + 5 = 14 5 + 9 = 14

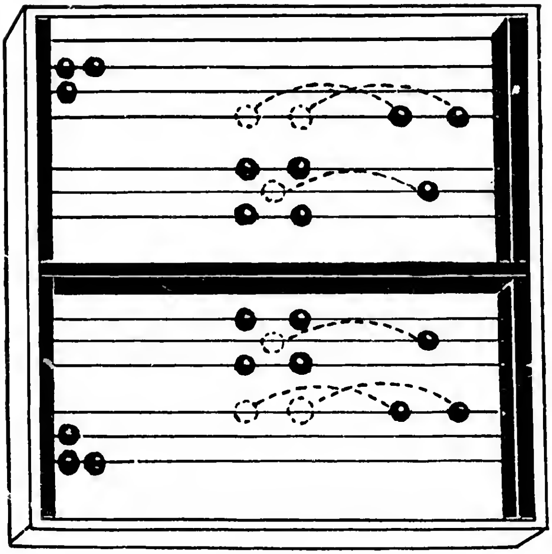


FIG. (*d*).

8 + 6 = 14 6 + 8 = 14

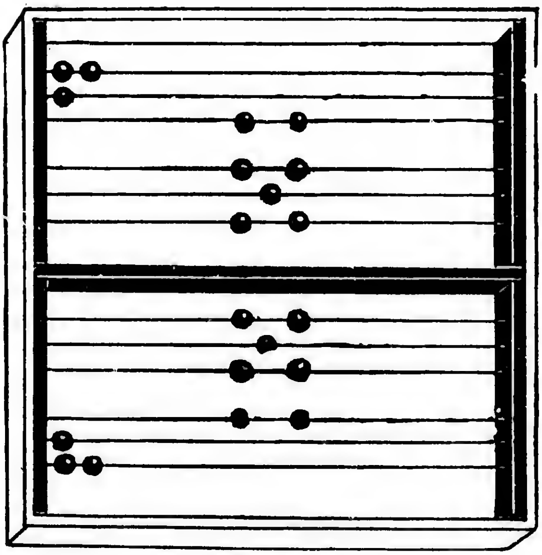


FIG. (*e*).

7 + 7 = 14 7 + 7 = 14

ADDITION COMBINATIONS.

10 + 4 = 14
9 + 5 = 14
8 + 6 = 14
7 + 7 = 14
6 + 8 = 14
5 + 9 = 14
4 + 10 = 14

SUBTRACTIONS.

14 − 10 = 4
14 − 9 = 5
14 − 8 = 6
14 − 7 = 7
14 − 6 = 8
14 − 5 = 9
14 − 4 = 10

Four Things About Each Combination.

10 + 4 = 14
4 + 10 = 14
14 10 = 4
14 − 4 = 10

The Number Thirteen.

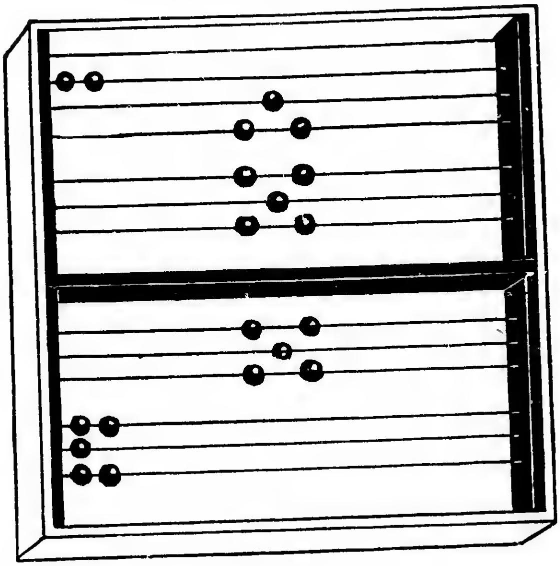


Fig. (*f*).

Fig. (*f*) represents the Number-Form Thirteen.

The four following Figures will illustrate the possible Combinations to form Thirteen on the Ball-Frame :—

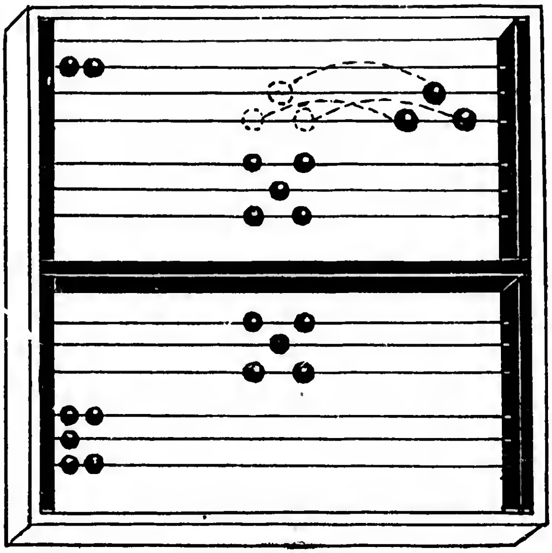


FIG. (*g*).

$$10 + 3 = 13$$
$$3 + 10 = 13$$
$$13 - 10 = 3$$
$$13 - 3 = 10$$

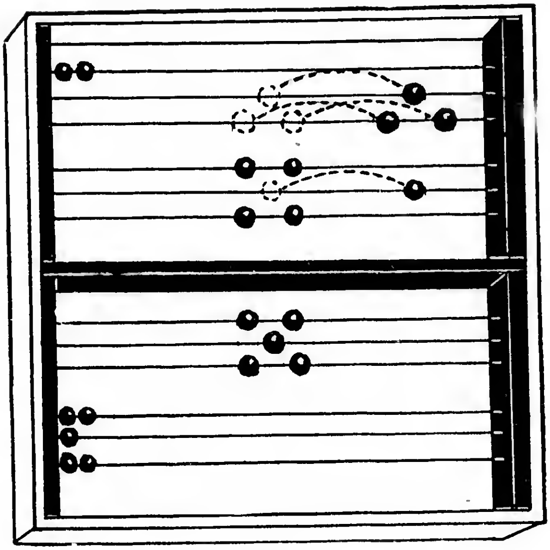


FIG. (*h*).

$$9 + 4 = 13$$
$$4 + 9 = 13$$
$$13 - 9 = 4$$
$$13 - 4 = 9$$

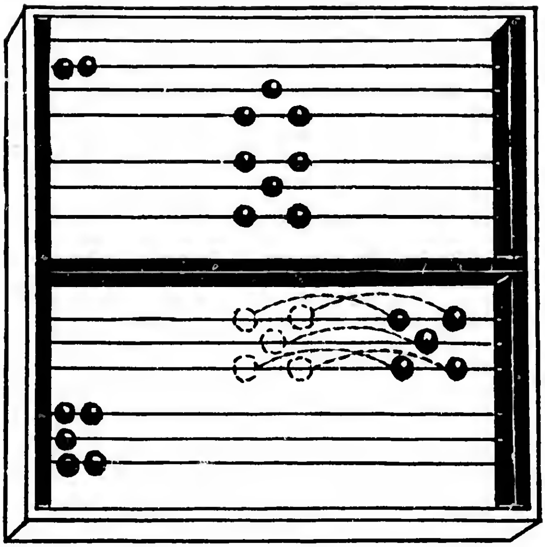


Fig. (*i*).

$$8 + 5 = 13$$
$$5 + 8 = 13$$
$$13 - 8 = 5$$
$$13 - 5 = 8$$

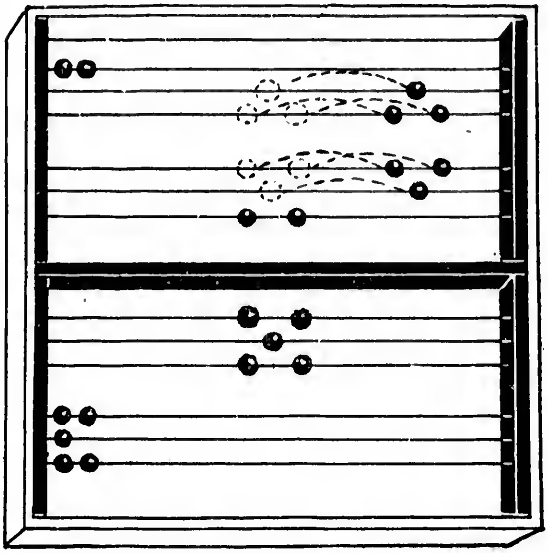


Fig. (j).

7 + 6 = 13
6 + 7 = 13
13 − 7 = 6
13 − 6 = 7

ADDITIONS.

10 + 3 = 13
9 + 4 = 13
8 + 5 = 13
7 + 6 = 13
6 + 7 = 13
5 + 8 = 13
4 + 9 = 13
3 + 10 = 13

SUBTRACTIONS.

13 − 10 = 3
13 − 9 = 4
13 − 8 = 5
13 − 7 = 6
13 − 6 = 7
13 − 5 = 8
13 − 4 = 9
13 − 3 = 10

Thirteen will be recognized, too, as being *thirteen ones*, or as being composed of *thirteen primary units.*

The plan followed in the FUNDAMENTAL NUMBERS is to be followed in proceeding from *Eleven* to *Twenty.*

The Larger Ball-Frame (on which the balls are movable) may be used to form all the Combinations on Numbers from One to Twenty; but all these Combinations are formed by the Numeral-Frame (in which the half balls are fixed) and Automatic Schedule, as the lower row or series, in each, is pressed forward to form New Combinations.

In teaching any of the Numbers, use the Number-Form first; and when the pupils understand thoroughly *the Number as presented objectively*, then teach the *corresponding Arabic character* (figure); but *keep the two ideas separate at first.*

Dr. J. A. McLellan says :—"Giving the symbol as soon as the idea is mastered is justified by common sense, as well as by psychology. There is variety and therefore interest; dealing with the objects too long becomes monotonous; symbols open up a new field. There comes also a feeling of *power*, of advance, etc. There is *economy* of time and power for both teacher and pupil. It affords means of *self-instruction.* In short, the justification is on the same ground for the child as for the race. The human mind always economizes by means of some condensed symbol as soon as the idea is familiar. It is worse than useless to be always going back to beginnings; this would render progress extremely slow."—See Dr. McLellan's remarks on Arithmetic in his "Applied Psychology." See, also, "Psychology of Number," by McLellan and Dewey.

Following the Table of Combinations on Numbers from One to Twenty, the pupils should be required to *write out* in their Exercise Books *all the different Combinations* for each; *e.g.*, the number THIRTEEN—whose *combinations are* TEN and THREE (10+3); NINE and FOUR (9+4); EIGHT and FIVE (8+5); SEVEN and SIX (7+6)—should be written:—

These Combinations may be formed either on the Large Ball-Frame, the Numeral-Frame (at the top of the Calculator), or various forms may be used on the black-board. In writing out, in Number-Form and in Symbol-Form, the different Sets as they are formed by the Automatic Numeral-Frame and Schedule, all these Combinations will be taken; but not in the same order in which they are seen in the Table of Combinations on Numbers from One to Twenty. However, they should be written out both as they appear in the Eleven Sets and in the Table of Combinations.

After the Fundamental Numbers have all been thoroughly taught, and their *correspondent* Arabic Symbols, the next step is to teach the combinations with the number TEN AS A UNIT.

10 + 1 = 11
10 + 2 = 12
10 + 3 = 13
10 + 4 = 14
10 + 5 = 15
10 + 6 = 16
10 + 7 = 17
10 + 8 = 18
10 + 9 = 19
10 + 10 = 20

This may be successfully accomplished upon the Larger Calculator or Ball-Frame, which is divided by a cross-bar of wood into two tens, the upper ten and the lower ten. Now the upper ten, or the ten balls on the upper half of the said Calculator, may be brought out upon the wires and by being brought into close contact (as close as the frame and wires will admit and so as to fall within the limits of the slides and cross-bar) in the centre of the upper half of the enclosed space, the ten balls may be considered as the Unit Ten (a One Ten or a Ten Unit). The intermediate units may then be combined with the said Unit Ten, and thus the Combinations to Twenty may be taught. But before considering Ten as a Unit (Ten Unit or a One Ten) in this way, on the Calculator, let us develop Ten as a Unit (Multiplex Unit) by means of crayons of chalk.

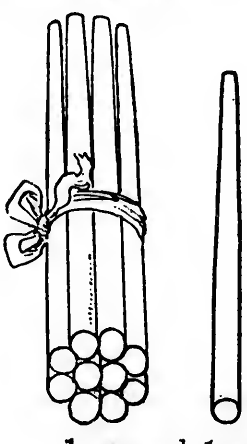
"*Variety is the spice of life.*"

Take a box of crayons (a box of whole ones if possible, as broken crayons will not be so handy to tie into bundles) and tie (or fasten with rubber bands) the crayons into bundles of ten each, until you have tied up ten bundles; now, with these and ten single crayons besides, you can teach, *understandingly*, the Numbers to One Hundred.

In order to develop Ten as a Unit and how to represent it in symbol-form, it is necessary to first teach the number Eleven (11).

Now take *one* of your *bundles of ten* crayons and lay it out on the table before your class; then place one *single crayon* on the right of it, as seen from where the pupils stand.

You have before the class ONE TEN and *one unit*, thus:—



1 TEN and *1 unit*
(10 + 1)

[a TEN-UNIT (DERIVED UNIT) and a *Primary* unit].

Represent this on the black-board by symbols (figures), thus:

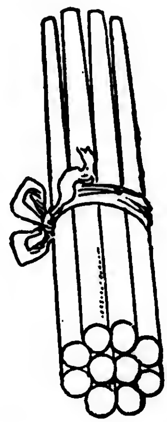
ten	unit
1	1

ELEVEN.

(A *multiplex* and a *simple* unit.)

Show the pupils plainly *which* of the ones (1's) stands for the TEN and *which* for the *unit*; *i.e.*, show them which of the 1's stands for the ten-bundle, or bundle of ten crayons (the ten-unit) and which of the 1's stands for the single crayon (or unit).

Now remove the single crayon, and you have left ONE TEN and *no units*, thus :—



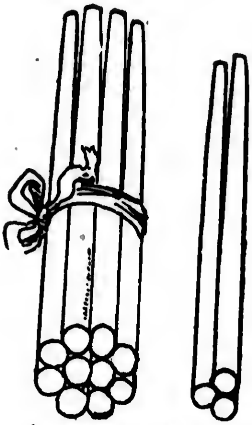
1 TEN and no (0) *units*.
(10 + 0)

Show the pupils that when you have 1 ten and no (0) units, you represent it on the black-board, thus :—

ten	units
1	0

TEN.

Place *three* crayons beside the *ten-bundle* (*ten-unit*), and you have ONE TEN and *three units*, thus :—



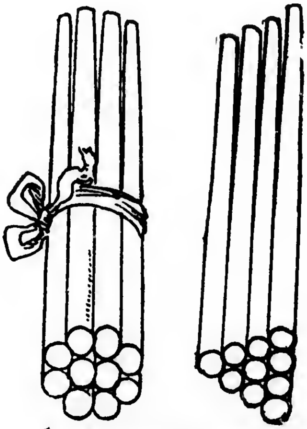
1 TEN and *3 units.*
(10 + 3)

Represent this on the black-board, as before, in symbol-form, thus :—

ten	units
1	3

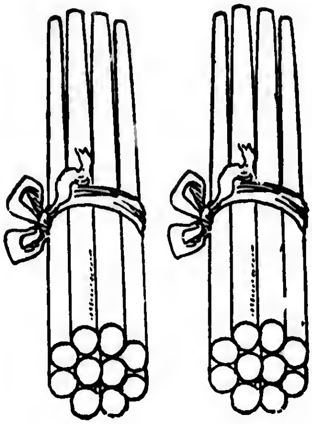
THIRTEEN
(13=10+3).

Teach the *intermediate units* with TEN AS A UNIT, in this way, taking them up in logical order (after ten is developed as a unit) and when you arrive at 1 ten and 10 units, thus :—



1 TEN and *10 units,*
(10 + 10)

you should explain to the pupils that the *ten units* are not represented (on the black-board) in *symbol-form* by writing 10 in the units' place, but by tying the ten units (10 units) into another bundle of ten, and then representing it as ANOTHER TEN-UNIT in the *tens' place*, thus :—



2 TENS and *no* (0) *units*

(20 + 0).

2 tens is simply a shorter way of writing 1 ten and 1 ten.

$= 10 + 10 + 0$

$= 20 + 0$

$= 20$

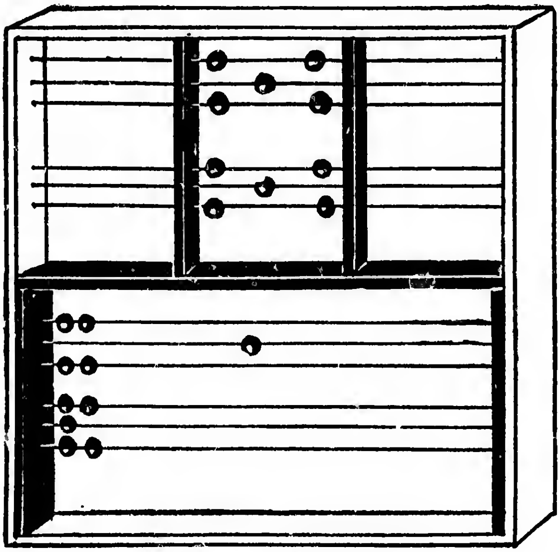
TWENTY

(A. S. twentig)

= twa tens

= two tens

or two ten-units.



THE LARGER CALCULATOR.

By moving up the "slides" at sides of upper half of Calculator, so as to draw together the ten movable balls at top, a ten-unit may be made.

As it stands we have: TEN + ONE = ELEVEN. Any of the *intermediate units* may now be added until TWENTY is reached; when, by moving up the lower slides TWO TEN-UNITS may be shown, and TWENTY revealed as *two tens*.

Similarly with the Smaller Calculator with slides, TEN may be shown to be TWO FIVES. Thus on the two calculators may be shown TEN-UNITS and FIVE-UNITS.

Multiplex Unity as Units.

Teach in this way all the other TENS *as units*, that is 3 tens (30); 4 tens (40); 5 tens (50); 6 tens (60); 7 tens (70); 8 tens (80); 9 tens (90), with the intermediate units between the tens.

Now, from 9 TENS and *9 units* you *can easily develop* ONE HUNDRED *as a* UNIT (A HUNDRED-UNIT).

Lay out on the table before the class 9 bundles of ten crayons each, and beside them place 9 single crayons; now add another single crayon to the nine crayons, and you will have 9 tens and 10 units.

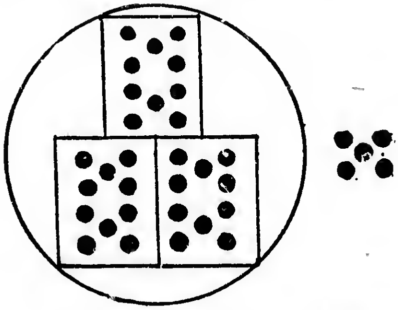
But these 10 units are equal to another bundle of ten crayons—a ten-unit—which may be tied into a ten-bundle; now add this ten-bundle to the 9 tens and it makes 10 ten-bundles and leaves no units. But ten tens, or ten ten-bundles may be tied together, as were the ten single crayons, so as to form a Unit. You have now a unit of the next higher order—One Hundred (1 Hundred) which is not written in the tens' place, but in the hundreds' place, when it is represented in symbol-form; so that out of your 9 tens and 9 units plus 1 unit, you have now 1 HUNDRED, *no tens*, and *no units:*

H.	T.	U.
1	0	0

Carry on this operation until the pupils know *how to express any number* in the Hundreds, the Tens and the Units.

Bundles of sticks of equal size and length, fastened together into bundles of tens, hundreds, etc., with rubber bands, will be found to answer even better than the chalk; but the chalk is always to hand, whereas the sticks require to be prepared, specially.

Have pupils practise making "PICTURES" of such numbers as:—56; 127; 347; 604; 592; 35, etc.; *e.g.*, 35—



3 TENS and *5 units*

=30+5

=35, etc.

Pupils taught in this way will not say that Thirty-Five is Three and Five, or that One Hundred and Twenty-Seven is One, Two, and Seven.

Ten may be taught as a unit by using (as mentioned before) the Larger Calculator or Ball-Frame.

The Combinations with *Ten as a unit* are all formed to Twenty, as each Number-Form, in the lower series of the Automatic Numeral-Frame, comes under the Number-Form Ten in the upper series as the said lower series of blocks is pressed forward. By removing the blocks in the lower series, and then inserting the said blocks at the right-hand side, so that they may each be moved *immediately* and *successively* under the NUMBER-FORM TEN, the whole *Ten Combinations* may be formed in logical (natural) order—10 + 1, 10 + 2, etc.; but the blocks must be inserted in logical order to accomplish this—1, 2, 3, 4, etc., instead of 1, 3, 5, 7, etc. (the order in which they are—*for a purpose*—arranged upon the Calculator).

Next, the teaching of the intermediate numbers between the tens may be taken up. As in going from eleven to twenty, so proceed from twenty to thirty; thirty to forty; forty to fifty, etc., *keeping clear the idea* that we have always a certain number of TENS *plus a certain number of* UNITS; *e.g.*, 99 = 9 TENS *plus 9 units.*

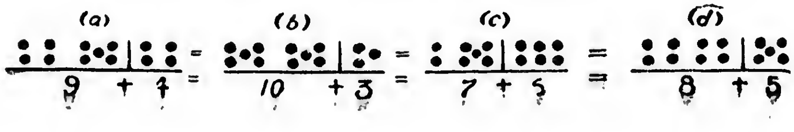
A thorough understanding of the Fundamental Numbers will make the teaching of the tens and the intermediate units an easy task to accomplish.

Counting backward and forward past ten may be left off until after the Combinations from One to Twenty are mastered, as the pupils who are able to count readily to twenty, will resort to this means in finding out the Combinations, and instead of making these Combinations a test of Sight and Sound (Seeing and Hearing) they will be finding them out by counting. Permit no counting in giving the Combinations.

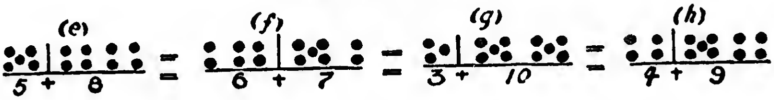
To *know* the Number Thirteen, a pupil must know it:

(1) BY SIGHT (in *Number-Form* and in *Symbol-Form*), thus:—

FUNDAMENTAL FORMS.



Related Forms.



See Numeral-Frame and Schedule Sets ; also, the Table of Sums of Schedule Sets. The foregoing Combinations of Thirteen will be seen by referring to the aforesaid Sets, as follows :—

	Set.	Combination.
(*a*)	No. 3	No. 5
(*b*)	" 4	" 10
(*c*)	" 5	" 4
(*d*)	" 6	" 9
(*e*)	" 7	" 3
(*f*)	" 8	" 8
(*g*)	" 9	" 2
(*h*)	" 10	" 7

(2) By Sound or Sense (in *Word-Form, either spoken or written*), thus :—

(*a*)	Nine	and	Four	are	Thirteen.
(*b*)	Ten	"	Three	"	"
(*c*)	Seven	"	Six	"	"
(*d*)	Eight	"	Five	"	"
(*e*)	Five	"	Eight	"	"
(*f*)	Six	"	Seven	"	"
(*g*)	Three	"	Ten	"	"
(*h*)	Four	"	Nine	"	"

Dr. McLellan says :—" It seems plain that, if the child is led by clear intuitions to think the relations as presented in these Number-Forms, the 'mental experiences' will blend into a lasting conception of the number."

The Numeral-Frame gives *one definite Number-Form for each number;* but these may be varied if desirable, as in the case of the number Three in the example given, when going over the Fundamental Numbers. A *variety of forms* may be made upon the Larger Ball-Frame, on which the balls are *movable* for that purpose.

Pupils *must know the pairs of addends or related parts* which form any number before they know that number. They must know all the combinations, (1) by additions of pairs of addends (re-combinations or "*wholing*"), (2) subtraction or resolution of a number into its several combinations (partition or "*parting*").

This involves the FOUR THINGS to be taught in connection with the additions and subtractions of every pair of addends (in combination) which form any number, *e.g.:*

Five as made up of the addends three and two.

THE FOUR IDEAS.

(1) THREE *and* TWO *are* FIVE.

$3 + 2 = 5$

(2) TWO *and* THREE *are* FIVE.

$2 + 3 = 5$

(3) TWO *from* FIVE *leaves* THREE.

$5 - 2 = 3$

(4) THREE *from* FIVE *leaves* TWO.

$5 - 3 = 2$

These Combinations will all be found on the Numeral-Frame, as the lower row or series of tablets containing the Number-Forms are pressed forward to form new Combinations. In connection with all the Combinations made by the Numeral-Frame *these four ideas* should be taught.

The Schedule must be worked so as to correspond to the Numeral-Frame; the former making in Symbol-Form what the latter makes in Number-Form.

By the Numeral-Frame and Schedule the pupils must know:—

(1) The *Combinations* in NUMBER-FORM.

(2) The *Combinations* in SYMBOL-FORM in *connection* therewith; and

(3) The *Symbol-Combinations* IN ABSTRACTION.

When the pupils know the Combinations in abstraction, they are ready to take up work prescribed in Cuthbert's "Desk-Work in the Simple Rules," Nos. 1 to 4, in Addition and Subtraction.

When the pupils have proceeded a little farther, the multiplications and divisions must be taught. (We have here the "*multiplex*" *unit*).

The four ideas in connection with these combinations should be taught; *e.g.*,

(1) SIX TIMES SEVEN *are* FORTY-TWO.

$$6 \times 7 = 42$$

(2) SEVEN TIMES SIX *are* FORTY-TWO.

$$7 \times 6 = 42$$

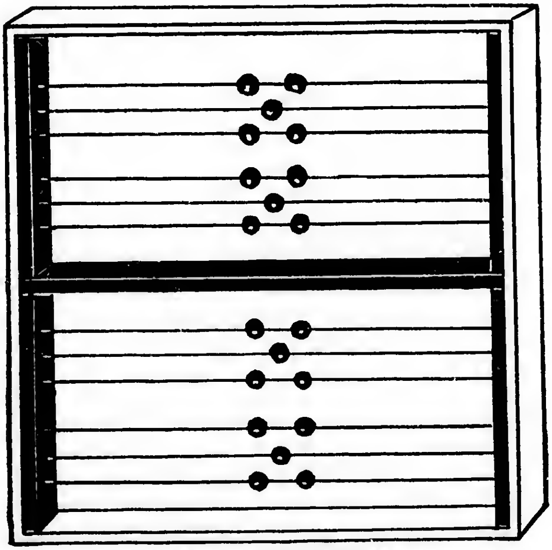
(3) SEVEN *into* FORTY-TWO *goes* SIX *times*.

$$42 \div 7 = 6$$

(4) SIX *into* FORTY-TWO *goes* SEVEN *times*.

$$42 \div 6 = 7$$

These may be taken up to a limited extent on the Larger Ball-Frame; *e.g.*,



THE LARGER BALL-FRAME.

The Number Twenty on the Larger Ball-Frame.

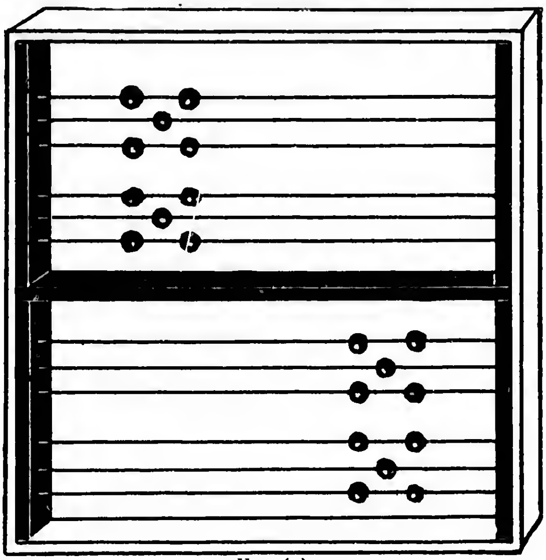


FIG. (*a*).

$2 \times 10 = 20 \qquad 20 \div 10 = 2$

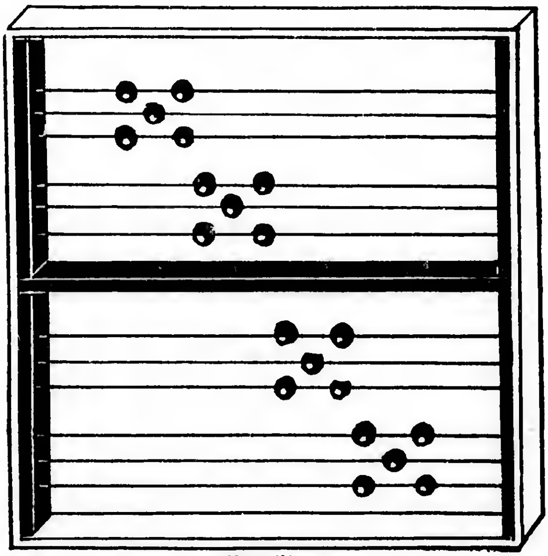


FIG. (*b*).

$4 \times 5 = 20 \qquad 20 \div 5 = 4$

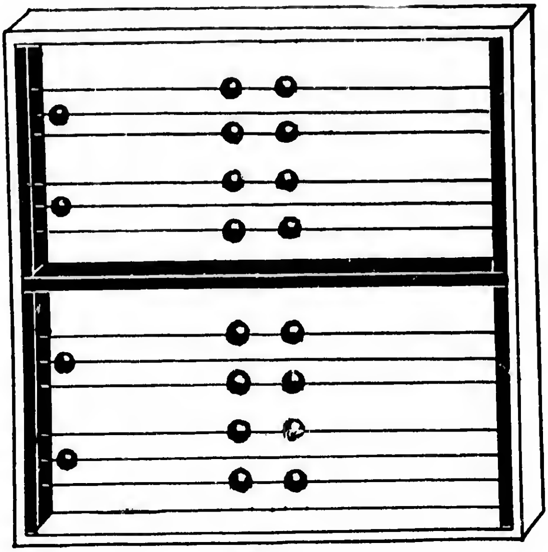


FIG. (c).

$5 \times 4 = 20 \qquad 20 \div 4 = 5$

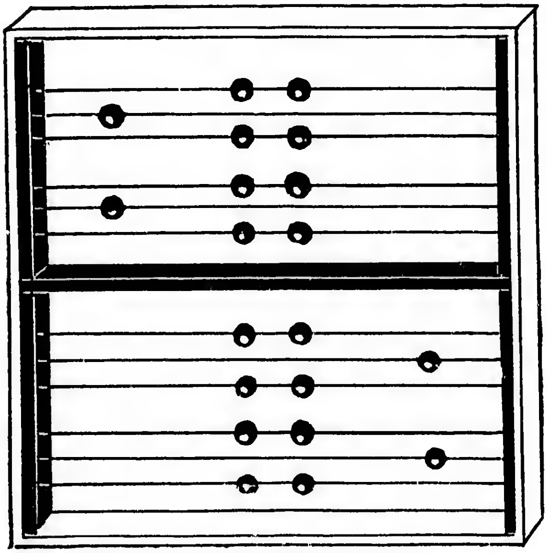


FIG. (d).

$10 \times 2 = 20 \qquad 20 \div 2 = 10$

MULTIPLICATIONS.

$2 \times 10 = 20$

$4 \times 5 = 20$

$5 \times 4 = 20$

$10 \times 2 = 20$

DIVISIONS.

$20 \div 2 = 10$

$20 \div 4 = 5$

$20 \div 5 = 4$

$20 \div 10 = 2$

The Number Sixteen.

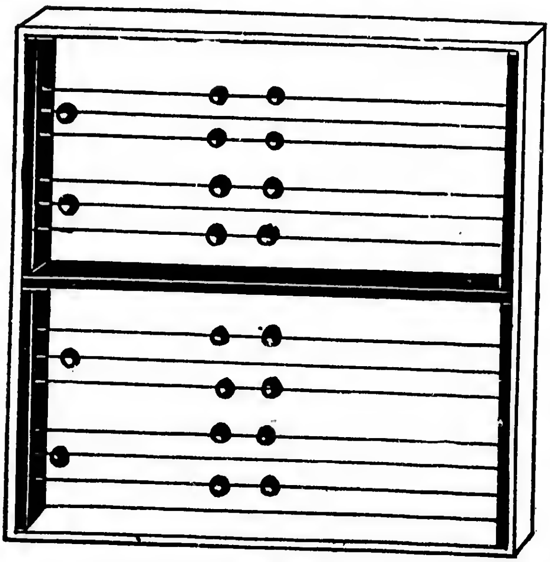


FIG. (*a*).

$2 \times 8 = 16$

$16 \div 8 = 2$

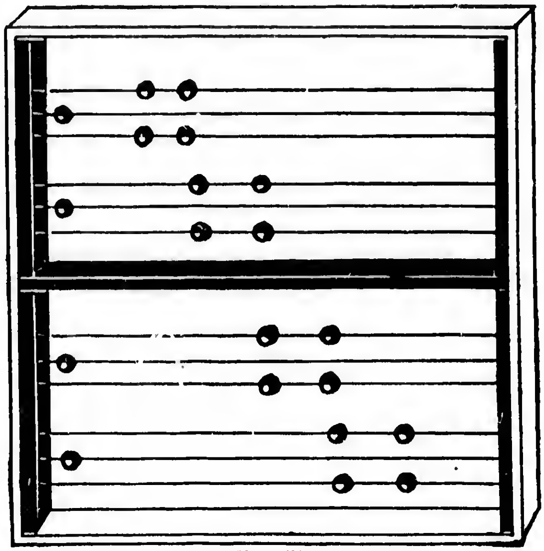


FIG. (b).

4 × 4 = 16　　　16 ÷ 4 = 4

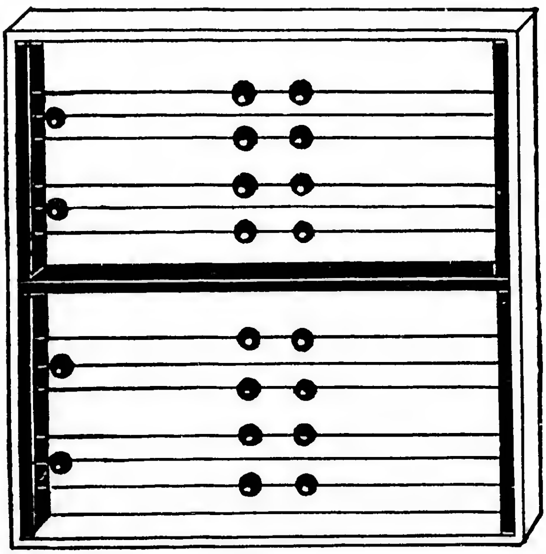


FIG. (c).

4 × 4 = 16　　　16 ÷ 4 = 4

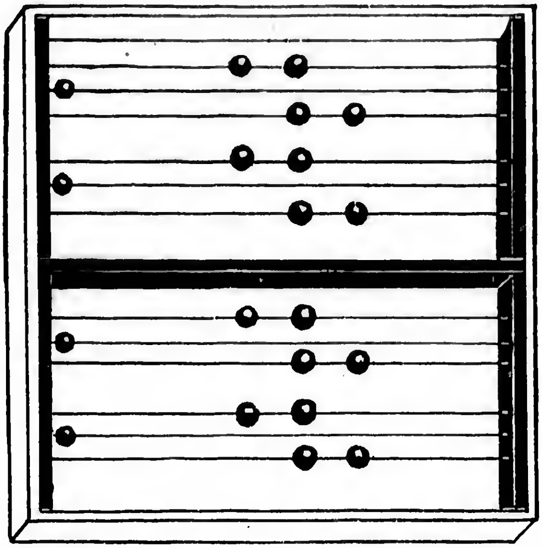


Fig. (*d*).

8 × 2 = 16

16 ÷ 2 = 8

MULTIPLICATIONS.

2 × 8 = 16

4 × 4 = 16

4 × 4 = 16

8 × 2 = 16

DIVISIONS.

16 ÷ 2 = 8

16 ÷ 4 = 4

16 ÷ 4 = 4

16 ÷ 8 = 2

Two Eights *are* Sixteen
(2 × 8 = 16).

Eight Twos *are* Sixteen
(8 × 2 = 16).

Four Fours *are* Sixteen
(4 × 4 = 16).

The FOURS in SIXTEEN *are* FOUR
(16 ÷ 4 = 4).

The TWOS in SIXTEEN *are* EIGHT.
(16 ÷ 2 = 8).

The EIGHTS in SIXTEEN *are* TWO
(16 ÷ 8 = 2).

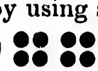
EIGHT is the HALF of SIXTEEN, etc.

As soon as the pupils have mastered (*a*) the additions, (*b*) the subtractions, (*c*) the multiplications, and (*d*) the divisions (exact), in connection with any number, then practical problems (in Mental Arithmetic) should be given to suit the requirements of the case in hand.

For example, in teaching the combination  and  (6 and 7), some such question as the following may be given :—I have six balls in one frame and seven balls in another frame ; how many balls have I in both frames? And from this :—Tom has six apples in one pocket and seven in another pocket ; how many apples has he in both pockets?

Again, taking the same combination ( and ) and removing the frame, with number-form seven, from the lower series, we may illustrate such a problem as the following :

I have thirteen balls in two frames ; if I take away seven balls, how many have I left? Or, we may say : Tom has 13 apples, and gives away seven of them ; how many has he left?

Similar problems may be constructed in Multiplication and Division and illustrated upon the Numeral-Frame by using such Combinations as  and  (10 and 10)  and  (8 and 8), etc. See examples given on Larger Ball-Frame.

In teaching the Combinations in Addition from the Numeral-Frame, it is well to have the pupils (1) *repeat orally the different Combinations* in each Set of the Series, as the eye scans the said Frame from end to end of the Series; (2) write out in their Exercise Books, in Number-Form, the different Combinations in each series; (3) write out on their Exercise Books the *Corresponding Combina-*

tions in Symbol-Form, from the Schedule, which should be *arranged to correspond with* the Numeral-Frame; the last mentioned exercise may be conveniently done by the pupils in their Primary Number-Work Exercise Books, as per the following scheme :—

SCHEME.

The foregoing is the scheme which the pupils have in their Exercise Books, ready to be "filled in" from the Numeral-Frame.

The following represents the scheme "filled in" by the pupils, from SET SEVEN of the SERIES of COMBINATIONS formed by the Numeral-Frame :—

SCHEME.

1	3	5	7	9	2	4	6	8	10	0
4	6	8	10	0	1	3	5	7	9	2
5	9	13	17	9	3	7	11	15	19	2

It is well to reverse the Board upon the Calculator so that the Schedule Combinations are not seen by the class while the Scheme is being "filled in" from the Numeral-Frame; the pupils, of course, must make the ADDITIONS (that is, the *sums* of the *pairs of addends*) mentally, themselves.

They should also be required to put their work down as follows :—

$1 + 4 = 5$
$3 + 6 = 9$
$5 + 8 = 13$
$7 + 10 = 17$

9 + 0 = 9
2 + 1 = 3
4 + 3 = 7
6 + 5 = 11
8 + 7 = 15
10 + 9 = 19
0 + 2 = 2

The teacher may correct Exercise Books either *by examining them individually* (which should often be done), or by *turning or reversing the board on Calculator, and referring the pupils to the Schedule Combinations as they are formed upon the Schedule, which should correspond with the Numeral-Frame.* For the sum totals, the teacher may refer to the Table of Sums of Schedule Combinations, as already laid down. For the foregoing, see Set Seven.

The pupils should be required to write the foregoing Combinations in words, after writing them in Number-Form (and before writing them in Symbol-Form), as they are supposed to be able, by the time they have reached this stage, to write the whole of the Fundamental Numbers:—

(1) In NUMBER-FORM,
(2) In WORD-FORM,
(3) In SYMBOL-FORM;

e.g.:—

FIVE and TWO are SEVEN
5 + 2 = 7

As already stated, the Numbers from One to Twenty should be taught in THREE STAGES, viz.:—

(1) From ONE TO FIVE inclusive.

(2) From SIX TO TEN inclusive.
These may be taught, either by means of the Fundamental Number-Forms, painted upon the Board, or by means of the Smaller Ball-Frame, as already described.

(3) From ELEVEN TO TWENTY inclusive.

These may be taught, either by means of the Larger Ball-Frame or by means of the Numeral-Frame, using the Number-Form Ten, *in the Lower Series*, to make the several *Combinations* with the Number-Forms *in the Upper Series*, as already described.

The last mentioned stage seems to be the most difficult for the pupils to grasp, and it requires both time and patience.

Pupils should be required to tell all they know about each Number. All about the NUMBER EIGHT, for example :—

(1) *Additions.*	(2) *Subtractions.*
$8 + 0 = 8$	$8 - 8 = 0$
$7 + 1 = 8$	$8 - 7 = 1$
$6 + 2 = 8$	$8 - 6 = 2$
$5 + 3 = 8$	$8 - 5 = 3$
$4 + 4 = 8$	$8 - 4 = 4$
$3 + 5 = 8$	$8 - 3 = 5$
$2 + 6 = 8$	$8 - 2 = 6$
$1 + 7 = 8$	$8 - 1 = 7$
$0 + 8 = 8$	$8 - 0 = 8$

They will readily give the *additions* :—

Eight and Nothing are Eight,
Seven " One " "
Six " Two " " etc. ;

but the *subtractions* will require *questioning*, unless the Combinations are learned by taking the Additions and Subtractions together, which is the better plan ; *e.g.*,

7 and 1 are 8 $(7 + 1 = 8)$
1 " 7 " 8 $(1 + 7 = 8)$
1 from 8 leaves 7 $(8 - 1 = 7)$
7 " 8 " 1 $(8 - 7 = 1)$

FOUR THINGS ABOUT EACH PAIR OF ADDENDS.

(*a*) The *addition* of *pairs of addends* ("wholing").

(*b*) The *subtractions* by *resolution* ("parting") *into pairs of addends.*

Addition and Subtraction should be taught together; so Multiplication and Division, as already mentioned.

Pupils should be required, when they reach Multiplication and Division, to tell you all they know about each combination, as in Addition and Subtraction; *e.g.*, the Combination three times four are twelve ($3 \times 4 = 12$).

THREE *times* FOUR *are* TWELVE ($3 \times 4 = 12$).
FOUR *times* THREE *are* TWELVE ($4 \times 3 = 12$).
THREE *into* TWELVE *goes* FOUR *times* ($12 \div 3 = 4$).
FOUR *into* TWELVE *goes* THREE *times* ($12 \div 4 = 3$).

FOUR THINGS ABOUT EACH PAIR OF FACTORS.

(*a*) The *multiplications* of *pairs of factors.*

(*b*) The *divisions* by *resolution into pairs of factors.*

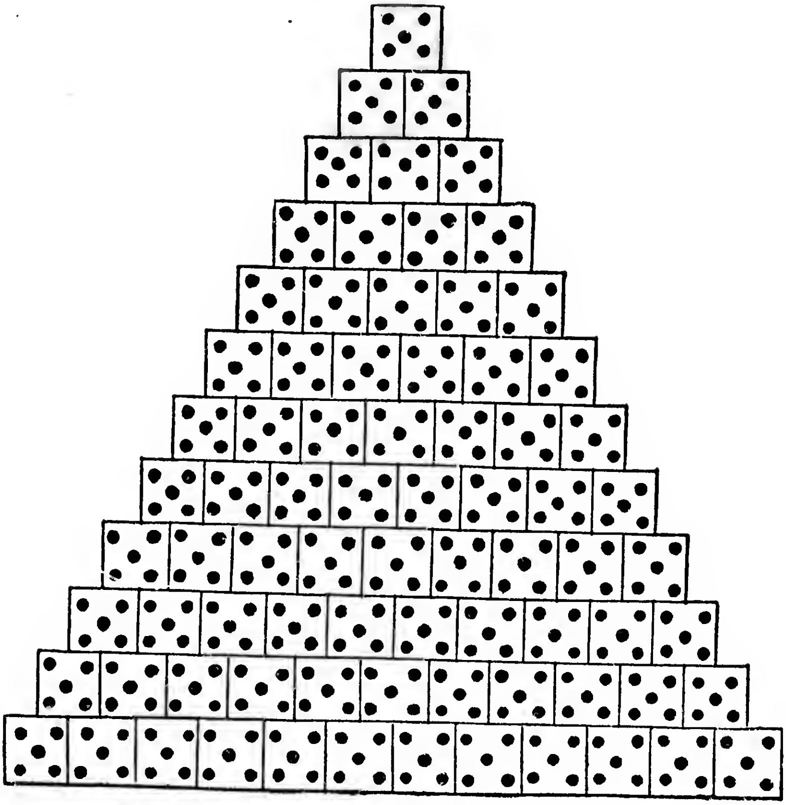
When pupils are familiar with the *multiplications* and *divisions*, have them give *all the pairs of factors* which make up a given number; *e.g.*, the number 36:

$$3 \times 12 = 36$$
$$4 \times 9 = 36$$
$$6 \times 6 = 36$$
$$9 \times 4 = 36$$
$$12 \times 3 = 36$$

also

$$2 \times 18 = 36$$
$$18 \times 2 = 36$$

By means of the Number-Forms, pupils may be led to a better understanding of the Multiplication Tables; *e.g.*, Five Times:



= One	times	Five	is	Five.
= Two	"	"		Ten.
= Three	"	"		Fifteen.
= Four	"	"		Twenty.
= Five	"	"		Twenty-Five.
= Six	"	"		Thirty.
= Seven	"	"		Thirty-Five.
= Eight	"	"		Forty.
= Nine	"	"		Forty-Five.
= Ten	"	"		Fifty.
= Eleven	"	"		Fifty-Five.
= Twelve	"	"		Sixty.

For other forms of writing the Tables see Exercise Books 5 and 6 in "Desk-Work in the Simple Rules."

The study of Number may be taken up in the order in which I have taken up the explanation of the different Devices found upon the Calculator :—(1) The Fundamental Numbers ; (2) Combinations with Ten (as a unit) plus each of the Fundamental Numbers ; (3) The Numeral-Frame and Schedule Combinations in conjunction.

After pupils have become familiar with the Schedule Combinations in the Arabic or Symbol-Form, they may be given desk-work in the *mechanical operations* from the exercises in ***"Desk-work in the Simple Rules" Nos. 1, 2, 3 and 4.***

Before these Exercises are taken up for drill and practice, the additions and subtractions of the Fundamental Numbers should be thoroughly understood.

"Making these partitions and re-combinations, and expressing the process in words and figures, affords good self-instruction work."

Dr. McLellan says :—"From the beginning, arithmetic should supply useful examples for desk-work."

By means of "Desk-Work in the Simple Rules," all the Junior Division of the school may be employed with suitable desk-work.

In the lower classes of our Public Schools we must aim at RAPIDITY and ACCURACY in the mechanical operations.

"ALL AT WORK ; AND ALWAYS AT WORK."

Plenty of drill in the mechanical operations begets rapidity and accuracy ; therefore the work prescribed in "Desk-Work in the Simple Rules" will be found helpful in connection with the work laid down in this book.

(i) Pupils should be required *to recite orally* (1) from the Numeral-Frame ; and (2) from the Schedule in connection, ALL THE COMBINATIONS "AT SIGHT."

(ii) Pupils should be able to write out (1) in Number-Form ; (2) in Word-Form ; and (3) in Symbol-Form, all these Combinations.

Little problems in mental arithmetic should accompany the *mechanical* work.

"In mental work, rapidity, correct language, and logical order of thought and statement must be constantly aimed at."

In giving practical problems in mental work, suit the problem to the rule in hand :—

Addition—*e.g.*, John has 3 marbles in one pocket and 4 in another ; how many marbles has he in both pockets?

Subtraction—*e.g.*, Tom had 9 marbles and lost 4 of them ; how many has he left?

Afterwards, when the pupils have become acquainted with both, use a combination of the two ; *e.g.*,

Jim got 4 candy-sticks from his father and 5 from his mother ; he gave 6 away to his sister ; how many had he left?

Similarly in the other two rules.

In teaching the Fundamental Numbers as they are represented upon the Calculator, perhaps drawing the chalk through the number, so as to make the partitions, would be preferable to hiding each addend (of the pair of addends forming the number) with a piece of card-board, as already described. Both methods may be employed to advantage.

Rapid mental work must be given to assist the mechanical work ; *e.g.*,

$$2+5\times 7+1\div 5\times 6+3\div 7\times 9+9, \text{ etc., equals what?}$$

In introducing Addition and Subtraction, perform the work by means of the balls, having the pupils to assist you.

Exercises in Nos. 1 to 11 in "Desk-Work in the Simple Rules," are intended to supply an ample amount of "desk-work" for the pupils, in the mechanical operations ; but, in the earlier stages, the pupils should have practice in making and varying the number-forms ; writing out all the Combinations of each number, and in the Numeral-Frame and Schedule Combinations.

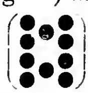
Teach thoroughly the *Fundamental Numbers*, as on these *the con-*

struction of Mathematics depends. Following the Table of Combinations on Numbers from One to Twenty, have the pupils write out the Combinations of some of the numbers each day, until all the numbers with their Combinations are understood.

Pupils should be required to learn these Numbers and their Combinations so thoroughly that they *can readily repeat orally, from memory,* all the different Combinations that make each of the Numbers from One to Twenty.

Teach Numbers higher than Ten on the basis of ten as a unit.

Children may be led to see the meaning of Twenty (Two Tens, 10+10), Thirteen (Three and Ten, 3+10), etc.

All the Combinations with Ten, in going from Ten to Twenty, may be nicely shown by removing all the tablets from the lower series in the Numeral-Frame (Fig. B) and then inserting the block with the Number-Form "TEN"  upon it, and moving it along, so as to come under each number, in the upper series in regular order ; *e.g.*,

1+10, 2+10, 3+10, 4+10, 5+10, 6+10, 7+10, 8+10, 9+10, 10+10.

The "ten-tablet" should be moved along so as to do this in regular or natural order, the numbers in the upper series being placed there out of their regular or natural order :—1, 3, 5, 7, etc.

When a pupil knows a number, he knows it in all its combinations in addition or subtraction.

When he has learned by tuition, that 8 + 5 are 13, he thinks of the eight and the five as wholes ("complex units"), and of the number thirteen as a whole ("Unity") ; he will also know that each of these numbers is composed of a certain number (as the case may be) of *single or primary units* (ONES).

A clear idea of unity is necessary to Number-Teaching.

"The idea of a unit can begin only from analysis of a whole ; it is completed only by relating the part to the whole, so that it is finally conceived at once in its isolation and its unity to the whole."

A unit is *any measuring part* by which the whole quantity is numerically defined. *It is a unit only when it is employed to measure some greater like magnitude.*

Similar objects, symmetrically arranged, present the best example for the conscious recognition of number as being a unity made up of related parts.—"Units constituting the defined unity."

The *objective idea* has been kept in view in constructing the COMMON-SENSE ARITHMETICAL CALCULATOR, and it is hoped that it may prove useful as an aid to the busy teacher in placing "NUMBER" in an *attractive* and *intelligible* form before his pupils.

"Objects are not number. No numerical concept or idea can enter into consciousness till the mind orders the objects—that is, compares and relates them in a certain way."

In a DOZEN apples (12), the *unit* may be a HALF DOZEN (6), which *measures* 12 *twice;* or it may be *a* THIRD OF A DOZEN (4), which *measures* 12 *three* times; or it may be A QUARTER OF A DOZEN (3), which *measures* 12 *four* times. Again, twelve, the UNITY in the foregoing, may be considered the UNIT in *measuring* eggs, oranges, lemons, etc.

Ten balls may be marked off upon the Calculator into *unit-groups* of 5's or 2's, which are *derived measuring parts* with regard to the *unity ten.* Ten may be counted out as composed of ten *primary measuring parts* or units (ones); or it may be regarded as a *unit* to measure 20, 30, 40, 50, 60, 100, 500, 1000, etc. ***We have the primary units in the derived unit, and the derived units (units of the same scale) in the measured unity or quantity.***

Twelve should be *measured out* by 2's, 3's, 4's, 6's (*derived units*); and in 1's (*primary units*).

The twenty balls upon the Larger Calculator may be taken as a unit to measure 40, 60, 80, 100, etc. Five times over it is 100. How many times, then, does twenty as a unit measure 100?

The twenty balls may be regarded as a unity, and analyzed, as on pages 61, 62, 63, 64.

For more minute and extended explanation of the unit, unity, stages of measurement, etc., see "Psychology of Number."

Each of the Combinations, after having been taught, *intuitively*, should be mastered "*at sight*" in the Arabic characters (figures). The teacher may accomplish this by pointing, promiscuously, to the different Combinations, which are contained in each Set of the Series of Schedule Combinations.

The "Hundred" Tables.

Teach pupils to write out the numbers 1 to 100; 100 to 200; 200 to 300; 300 to 400; 400 to 500; 500 to 600; 600 to 700; 700 to 800; 800 to 900; 900 to 1000.

See PRIMARY NUMBER-WORK *Exercise Book.*

Several days may be spent in teaching, for the first time, each of the Sets in the Series of Numeral-Frame and Schedule Combinations; consequently, the Numeral-Frame tablets and the Schedule blocks will not require to be shifted oftener than once a week, except for "review." Teach *slowly* and *thoroughly.*

Pupils should be taught to count from 1 to 100, taking the numbers in regular order; but this need not be undertaken until the pupils have learned the numbers and their combinations to Twenty.

The Larger Ball-Frame may be used for this purpose; it contains TWENTY balls, and five times over it will bring the pupils to *100*, etc.

Pupils should be taught to *count by* **2's, 3's, 4's, 5's, 6's, 7's,** etc.

In conjunction with the Schedule Combinations (Combinations in Symbol-Form) the addition of numbers "*by terminations*" should be taught and practised, as this is an *important feature* in teaching Addition. This may be done when going over the Schedule Combinations, say, the second time.

A thorough understanding of the Combinations on the Fundamental Numbers will greatly assist in learning the higher Combinations of Tens and the Intermediate Units; *e.g.*,

1 + 1 = 2 leads to 11 + 1 = 12; 21 + 1 = 22; 31 + 1 = 32, etc.

3 + 3 = 6 leads to 13 + 3 = 16; 23 + 3 = 26; 33 + 3 = 36, etc.

However, in teaching Addition in this way,— by "terminations" —select at first only those *pairs of addends whose sum is less than ten* (numbers from 1 to 5); afterwards take those *pairs of addends whose sum is either ten or more than ten* (numbers from 6 to 10); *e.g.*,

$5 + 5 = 10$ leads to $15 + 5 = 20$; $25 + 5 = 30$; $35 + 5 = 40$, etc.

$7 + 7 = 14$ leads to $17 + 7 = 24$; $27 + 7 = 34$; $37 + 7 = 44$, etc.

$9 + 9 = 18$ leads to $19 + 9 = 28$; $29 + 9 = 38$; $39 + 9 = 48$, etc.; because $9 + 9 = 18$, and 18 terminates in 8; therefore all Combinations with $9 + 9$ will also terminate in 8, etc.

When using these "terminations," have the pupils count by 2's, 3's, 5's, etc., as mentioned before; also have them name all the numbers between 1 and 100, which TERMINATE in a given number, say 7; these numbers will then be :—

7, 17, 27, 37, 47, 57, 67, 77, 87, 97.

It is well to illustrate, by some such means as the following, the fact, say, that there is no other number between 18 and 28 *that terminates* in 8; *e.g.*,

18
19
20
21
22
23
24
25
26
27
28

Thus it will be seen that *28* is the next number after 18 that *terminates in 8*. Pupils will by the foregoing see this for themselves.

Give plenty of oral drill on the "terminations" in connection with the Numeral-Frame and Schedule Combinations; also in repeating numbers, between 1 to 100, that terminate in a given number, say in 7, as in example given.

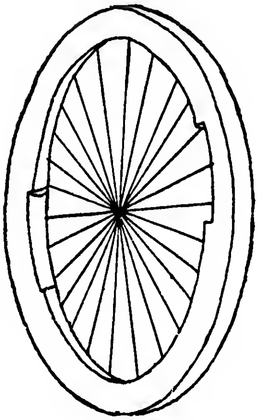
For "terminations," see No. 1 "Desk-Work in Simple Rules."

The Numeral-Frame at the top of the Calculator is a silent teacher of Number, as a pupil will never look at it without getting an idea on number, just as he never sees a good Map on the walls of a school-room without learning some fact in Geography, or having some fact engraven on the memory.

As *abstraction must ultimately take the place of observation (intuition)* in the teaching of Number, the numbers onward from Twenty may be taught *in the abstract*, which will be comparatively easy to the pupil who has thoroughly mastered the Numbers up to Twenty.

The next device upon the Calculator is seen in the following figure :—

FIG. B.



THE CIRCLE

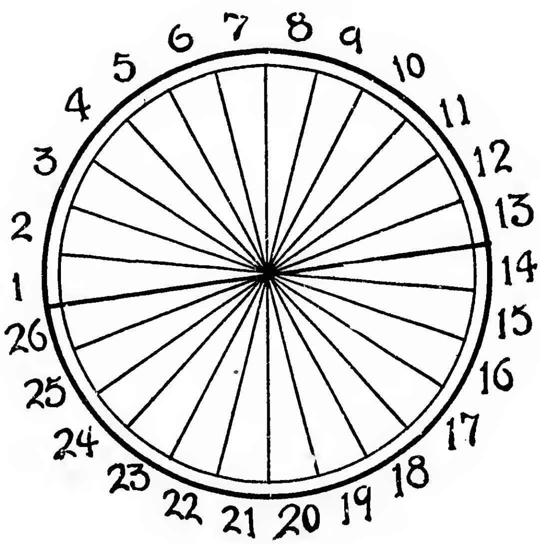


FIG. (*a*).

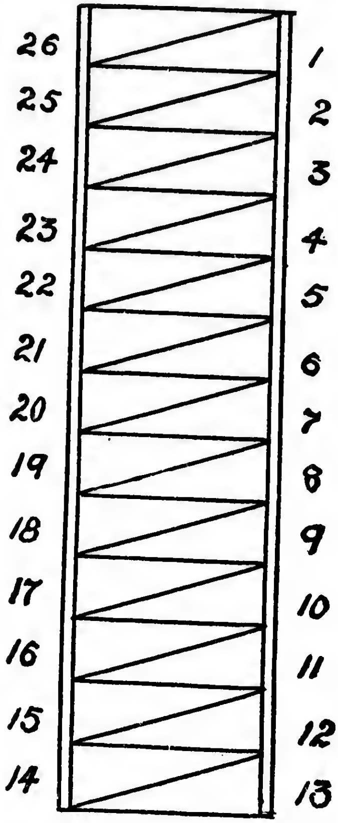


Fig. (*b*).

Figure (*a*) represents the Leather Circle as found in the rabbeted groove on said Calculator. (See Fig. B, page viii.)

Figure (*b*) represents the said Circle taken apart and put together again in the form of a rectangle.

The leather composing the Circle may be pinned upon the blackboard in either of the foregoing forms by means of a few pins, so as to hold it in position while demonstrating it to the pupils.

☞ For explanation of the Circle, see introduction to Fourth Class work, *Cuthbert's Exercises in Arithmetic, Part II.*

N.B.—While the device for illustrating the Circle is out of place in teaching Primary Arithmetic, yet I have added it to the Calculator, in the hope that it may prove useful in explaining, to more advanced pupils, the area of the Circle.

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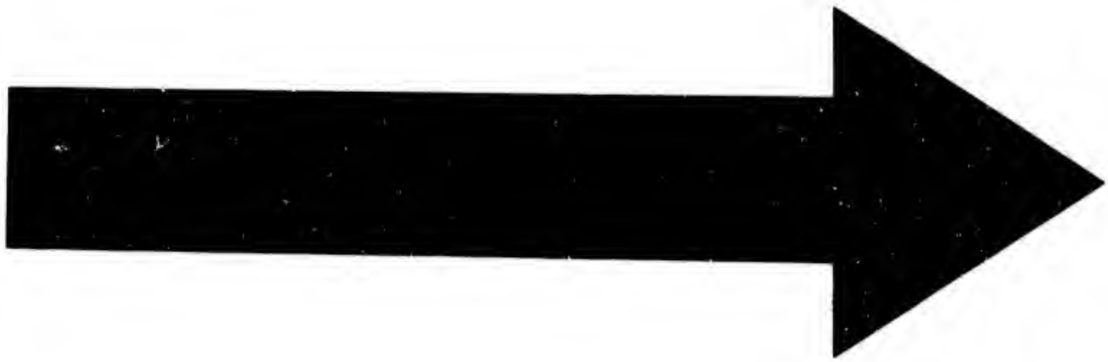

FICHE 2 NOT REQUIRED