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# McGILL UNIVERSITY

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## GEOLOGY AND MINERALOGY

No. 3

On the Amount of Internal Friction Developed in Rocks During Deformation and on the Relative Plasticity of Different Types of Rocks.

BY

FRANK D. ADAMS and J. AUSTEN BANCROFT.

On the Mathematical Theory of the Internal Friction and Limiting Strength of Rocks Under Conditions of Stress Existing in the Interior of the Earth.

BY

LOUIS VESSOT KING

[Reprinted from the Journal of Geology, Vol. XXV, No. 7, October-November, 1917.]

Montreal, 1918.

VOLUME XXV NUMBER 7

THE

# JOURNAL OF GEOLOGY

*OCTOBER-NOVEMBER 1917*

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## ON THE AMOUNT OF INTERNAL FRICTION DEVELOPED IN ROCKS DURING DEFORMATION AND ON THE RELATIVE PLASTICITY OF DIFFERENT TYPES OF ROCKS

FRANK D. ADAMS, D.Sc., F.R.S., AND J. AUSTEN BANCROFT, M.A., PH.D
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### INTRODUCTION

At the meeting of the Geological Society of America held in Albany in the year 1900, a brief résumé of the experimental work on the flow of marble carried out by Adams and Nicolson was presented to the Society, and in the discussion which followed the reading of this paper a number of interesting points were suggested by various speakers as worthy of experimental investigation. Among these was one put forward by Dr. G. K. Gilbert, which, in a letter to the authors, he subsequently formulated as follows:

> It has been thought that great pressure breaks down the structure called solidity and so reduces viscosity that very little differential stress is necessary to produce flow. It is thought that the strength of rocks is practically unaffected by pressure, in which case flow should begin only when differential stress equals the crushing strength of the material as conditioned by the temperature. It is certainly conceivable also that the strength of rocks is increased by pressure, so that the production of flow requires differential stress greater than the crushing stress as conditioned by the temperature. I hope your experimentation may be brought to throw light upon this point.

The sense in which certain terms are used in this quotation is not quite clear, but we understand the question put forward by Dr. Gilbert to be as follows:

A unit cube of any rock—granite for instance—is submitted to pressure in a testing machine on the earth's surface. It will give away or break down under a certain load—this is termed its crushing load.

If this cube of rock were imbedded deep within the earth's crust, great pressure would be exerted upon it from all sides. Such being the case, and omitting from consideration the influence of temperature, would the rock (1) be reduced to a condition which approaches fluidity and move at once if the pressure in one direction became slightly greater than that in another? Or (2) would the rock become deformed only when this additional pressure in one direction was equal to its crushing load at the surface? Or (3) would the rock show an increased resistance to deformation and require a much greater additional pressure in one direction to deform it than was required to crush it at the surface?

A few preliminary trials which served to open up the experimental investigation of this problem were undertaken some years ago by Dr. Adams in association with Dr. Ernest G. Coker, then Associate Professor of Civil Engineering at McGill University. Dr. Coker subsequently resigned his position at McGill University to accept the professorship of mechanical engineering and applied mathematics at the Finsbury Technical College in London, and for a time the work was discontinued. Dr. Bancroft, however, some years later coming to McGill University, the investigation was resumed. It has extended over a period of several years. The writers desire to acknowledge their indebtedness to the Carnegie Institute of Washington, the work having been carried out under a grant received from that body.

### ROCKS EXAMINED

The following rocks were examined:

White alabaster, Castelino, Italy.

White marble, Carrara, Italy.

Black Belgian marble ("Noir fin").

White dolomite, Cockeysville, Maryland, U.S.A.
Steatite ("Albarine"), Virginia, U.S.A.
Slate, New Rockland, Province of Quebec, Canada.
Sandstone, Cleveland, Ohio, U.S.A.
Granite, Baveno, Italy.
Olivine diabase, Sudbury, Province of Ontario, Canada.

For the purposes of comparison experiments were also conducted with metallic copper and metallic lead.

Detailed petrographical descriptions of these rocks, with the exception of the alabaster, dolomite, steatite, and slate, have been given in a former paper.[1] It is necessary here, therefore, to refer briefly to the character of these four rocks only.

*Alabaster, Castelino, Italy.*—Under the microscope the rock is seen to be composed of an aggregate of small grains of gypsum which are clear, colorless, and approximately equal in size. The individual grains display a tendency to elongation in one direction, thus giving the rock a very faint foliation. The columns of alabaster used in the experiments were cut from a single uniform block of this rock in such a manner that their longer axes were parallel to this indistinct foliation.

*Dolomite, Cockeysville, Maryland, U.S.A.*—This is a rather fine-grained, white, granular dolomite, very pure in character and uniform in composition, containing $CaCO_3$ and $MgCO_3$ in almost exactly their molecular proportions. It presents the appearance of a white marble and is extensively quarried as such. Thin sections of the rock, when examined under the microscope, show that it is composed of a mosaic of grains of the mineral dolomite, more or less irregular in shape and varying somewhat in size. Between crossed nicols, they present a uniform extinction or show only the faintest strain shadows. They are very seldom twinned.

*Steatite, Virginia, U.S.A.*—This steatite is placed on the market under the name of "albarine." The columns employed in the experiments were cut from a perfectly uniform slab of this rock

[1] "An Investigation into the Elastic Constants of Rocks More Especially with Reference to Their Cubic Compressibility," by F. D. Adams and E. G. Coker, The Carnegie Institute of Washington, 1906; see also *American Journal of Science*, XXII (August, 1906).

with dimensions of 10″×11″×1¼″. Under the microscope the rock is seen to possess a distinct foliation parallel to the broad surface of the slab. All of the columns were cut from this slab with their longer axes parallel to the foliation. In thin sections under the microscope the rock is seen to be composed chiefly of chlorite, talc and dolomite, numerous small crystals and grains of magnetite, and a few grains of pyrite are also present. The two minerals, chlorite and talc, make up by far the greater portion of the rock, the chlorite being somewhat more abundant than the talc. Both occur as plates and sheaflike aggregates, and both possess a very distinct cleavage parallel to which extinction takes place. The dolomite is present both in large rhombohedral individuals and as small irregular granules which possess a linear arrangement parallel to the foliation of the rock. None of the grains of dolomite show either twinning or strain shadows. Having been cut parallel to the foliation, it is not surprising that the columns of this rock employed in the experiments bulged assymetrically when deformed, and hence a larger number of experiments were made with the steatite than with the other rocks, in order that accurate average results might be secured.

*Slate, New Rockland, Quebec, Canada.*—This is a typical fine-grained slate, black in color, uniform in character, and possessing an excellent cleavage. By means of a diamond drill cores were taken perpendicular to the cleavage of the slate, and from these the columns of slate used in the experiments were prepared.

Under the microscope this slate is found to be composed essentially of minute flakes of two minerals, one of which is apparently kaolin and the other muscovite. In general, the kaolin is much more abundant than the muscovite, from which it can be distinguished in that it possesses a lower double refraction and is not quite so transparent. Within a few extremely narrow bands of the slate the muscovite preponderates. A few minute grains of quartz are interposed between the flakes of muscovite and kaolin. A considerable number of very small flakes of black, opaque, carbonaceous matter, abundant, minute, needle-like crystals of rutile, and a very few widely scattered grains of pyrrhotite are also present. The rutile crystals are brownish in color and occasionally display the geniculated twinning that is characteristic of this species.

The foliation of the slate explains the lack of symmetry in the expansion of columns of this rock during deformation.

The *Copper* used in these experiments was taken from a rod 1 inch in diameter, representing a good commercial grade of this metal. Prior to being turned into columns for the experiments, the pieces cut from the rod were annealed by being heated to bright redness in the coal fire of a forge, being then allowed to cool down gradually.

The *Lead* employed in the experiments was "assay lead" which, in order to free it from all air bubbles, was melted down and cast in a heated iron mold, which was then allowed to cool slowly.

### METHODS EMPLOYED

Several long round bars of nickel steel $2\frac{1}{8}$ inches in diameter, all of identical composition and from the same heat, and all having been submitted to identical treatment in their manufacture, were secured. For these the authors are indebted to the Bethlehem Steel Company, which placed them at their disposal for the purpose of the present investigation.

This steel, which is very uniform in character, possesses a high tensile strength, as well as a high elastic limit and has the following chemical composition:

| | |
|---|---|
| Carbon | .30 per cent |
| Manganese | .74 per cent |
| Silicon | .162 per cent |
| Phosphorus | .035 per cent |
| Sulphur | .038 per cent |
| Nickel | 4.740 per cent |

The bars were sawed into lengths of about $3\frac{1}{4}$ inches. These were then bored and turned into tubes, the longitudinal sections of which, with the final dimensions, are shown in the upper half of Fig. 1. Two sets of these tubes were prepared, differing only in the thickness of the wall of the central portion of the tube. In the first set this has a thickness of 0.33 centimeter, while in the second

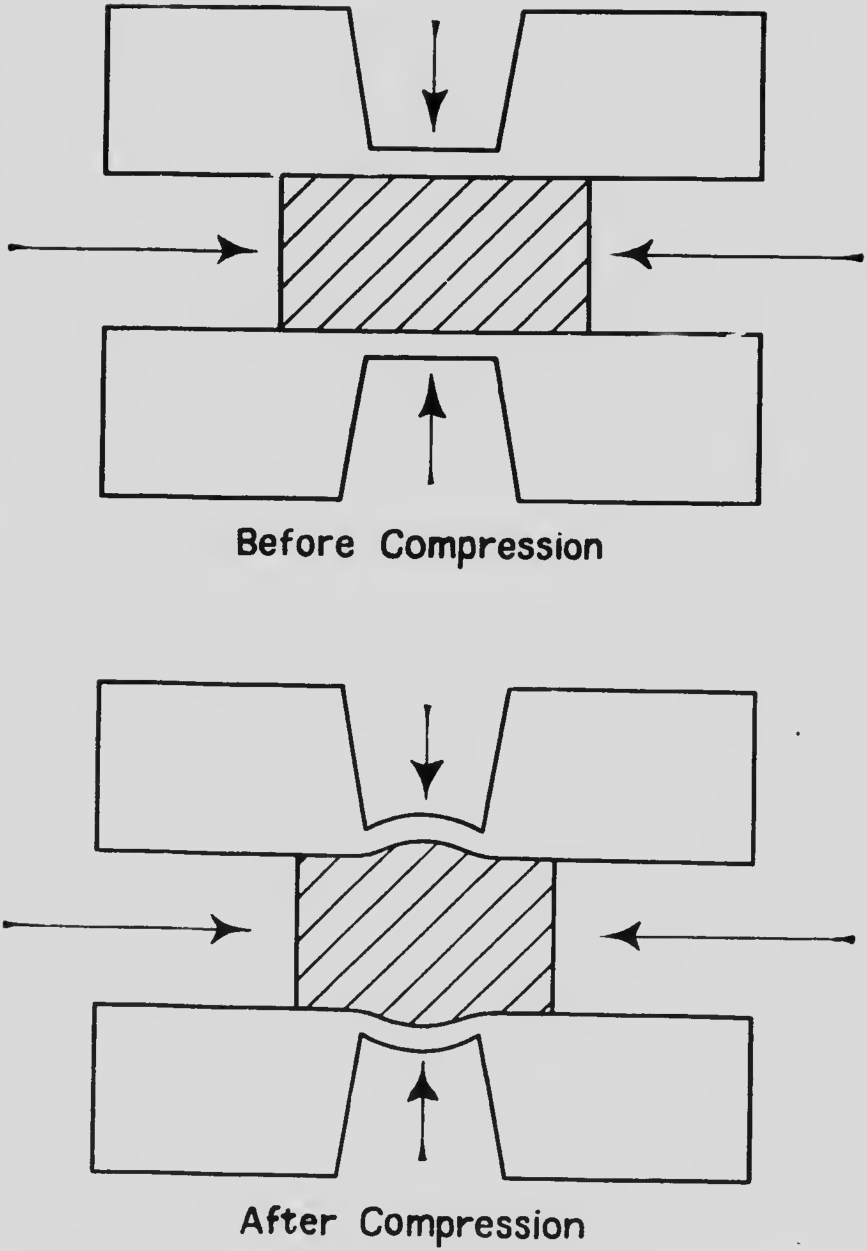


FIG. 1.—Longitudinal section through steel cylinder with wall 0.33 cm. thick, and inclosing one of the rock columns (natural scale).

set the thickness is 0.25 centimeter. The interior diameter of the tube in both sets is of such a size that it will just receive a column of rock 2 centimeters in diameter. The inner surface of the tube in every case was not only perfectly smooth, but highly polished. The angle of the bevel, by which the thickness of the wall is reduced at the middle of the tube, was adopted after a long series of preliminary experiments, which proved it to be that which was demanded by the conditions to be secured. Pistons fitting accurately into either end of these tubes were then made of chromium tungsten steel, suitably tempered by being heated, quenched in oil, and then ground to the exact dimensions required.

Large blocks of each of the rocks having been secured, rough columns of them were bored out by means of a hollow-bit diamond drill, care being taken in the case of each rock to have all the columns bored out of the rock in the same direction, that is, parallel to one another, so that any possible variations due to rift, grain, or incipient foliation were avoided. These rough columns were then reduced to the exact size required, by being ground down in a lathe by means of revolving carborundum wheels of different degrees of fineness, and were finally highly polished. When completed the columns were of such a size that they would just pass into the steel tubes at the ordinary temperature, the tube inclosing the column with an absolutely perfect mechanical fit. The column was in each case 4 centimeters long and 2 centimeters in diameter. While the column was thus fitted accurately into the tube, it could, by the exertion of a certain amount of pressure, be moved up and down within the tube. The column of rock, when inserted into the tube, was so placed that its center was exactly in the center of the thinner portion of the tube, as shown in the diagram, the extremities of the column being in this way supported by the walls of the thicker portion of the tube at either end.

The pressure to which the rock was submitted was obtained by a Wicksteed testing machine set up in the Testing Laboratory of the Macdonald Engineering Building of McGill University. This machine has a capacity of 100 tons and, when loaded to its capacity, is sensitive to a load of 4 pounds. Unfortunately, being graduated to read only in tons and pounds, it was necessary to obtain the data of the research in these units. In presenting the final results, however, the data for the conversion of these into a unit more generally employed in physical investigations are given.

The extensometer employed for the purpose of measuring the expansion of the tube under pressure was a simplified form of the type designed by Professor Coker and described in the *Proceedings of the Royal Society of Edinburgh*, XXV (1904–5). It was affixed to the opposite points of the steel tube on the plane of maximum deformation and showed the expansion, multiplied by two, by means of a fine line moving over a graduated scale, which was read by a telescope placed at a distance of several feet.

In a number of experiments two extensometers were employed, which were applied to the tube in the plane of maximum deformation, but in directions at right angles to one another. In this way it was ascertained that the bulge which the steel tube displayed under pressure was nearly symmetrical. but in order that any error which might arise from a single measurement might be eliminated, in almost all cases the two extensometers employed were affixed to the tube at right angles to one another, and the mean of the two readings was secured. By means of this form of extensometer and by reading with a telescope, it was possible to measure an increase on the diameter of the tube amounting to only 0.0005 inch. The steel tube inclosing the rock column, with the extensometers in position, the whole set up in the press ready for the application of pressure, is shown in Fig. 2.

The method adopted for measuring the internal friction developed in the rock by deformation was as follows:

A column of rock, Carrara marble, first was taken, having the dimensions already referred to. This was inclosed in a tube of nickel steel, as above described; the tube had a wall thickness of 0.25 centimeter at its thinner portion. As will be seen from Fig. 1, the middle portion of the marble column is inclosed by the thinner portion of the tube, while the ends of the column are held by the thicker portion of the tube wall. In this way the rock is prevented from flowing up between the tube and the pistons and thus from escaping from the tube. With a tube of this shape and these dimensions, the movement of the rock under pressure is confined to the middle portion of the column, which is surrounded by the thinner portion of the tube. The pistons being inserted and the whole properly set up in the testing machine, the pressure was

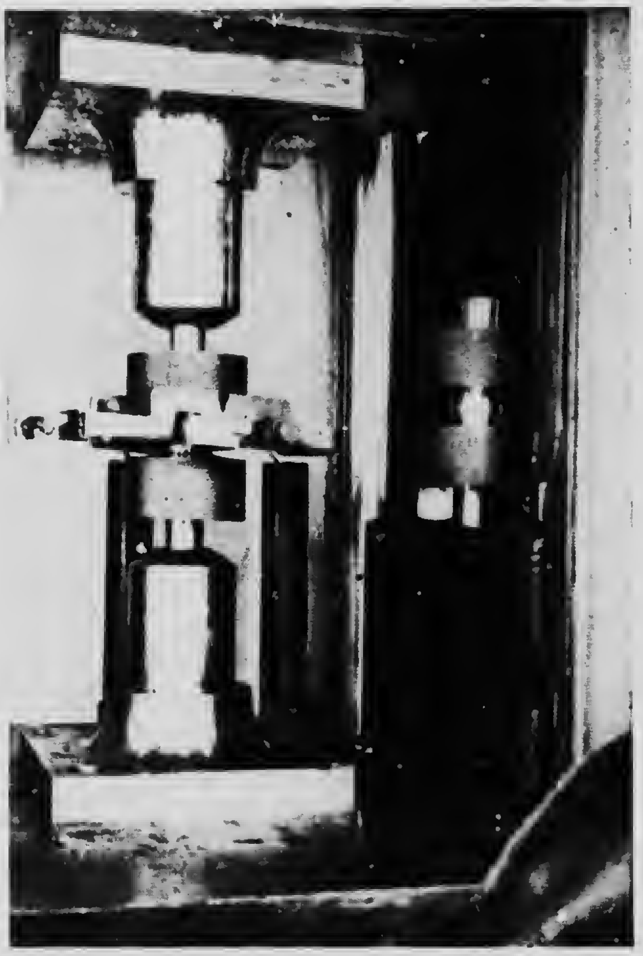

Fig. 2.—Steel cylinder, inclosing a rock column and with the two extensometers in position, set up in the Wicksteed press. To the right a bulged cylinder is shown as it appears at the close of an experiment.

applied in successive increments of 1,000 pounds. The extensometer showed no yielding of the inclosed rock until a load of about 12,000 pounds had been reached, when a very slight distension of the tube was indicated. Up to this point, the marble, being an elastic body, was undergoing cubic compression, the pressure exerted by the machine and the resistance exerted by the steel collar being equal. The slight distension of the steel tube at a load of 12,000 pounds is due to the elastic deformation of the marble. After each additional increase of 1,000 pounds to the load, extensometer readings were taken every 30 seconds until four successive readings were identical, that is to say, until no movement that could be registered on the scale took place during a period of 2 minutes. The pressure was then increased by another 1,000 pounds and a similar series of readings were taken. This was continued until the bulging steel tube showed signs of rupture or was actually ruptured by the movement of the inclosed rock. The time which elapsed between the first application of pressure and the final rupture of the tube, that is to say, the duration of the experiment, differed somewhat in the different experiments, but may be said to be about four hours.

During the time which elapses from the point when the elastic limit of the rock is exceeded to that at which the tube fails, the inclosed rock is undergoing deformation with extreme slowness and by internal movements of one kind or another, which give rise to what may be termed a plastic flow.

At the commencement of the experiment the column of marble had the form and dimensions represented in the upper half of Fig. 1. When at the conclusion of the experiment the test piece was placed in a lathe and the steel collar was turned off, the specimen of marble was set free. It was still intact, unbroken, and, when tested in compression, was found to be very nearly as strong as a piece of the original marble of the same shape and size. It now had the form represented in the lower half of Fig. 1.

A photograph of a column of rock, before and after deformation, the rock, however, in this particular case being steatite, is shown in Fig. 3.

The pressure which was applied to the marble column effected two results. It overcame the pressure (or resistance) exerted upon the sides of the column by the inclosing tube of steel, and it overcame the internal friction developed within the rock during its change of shape. If it were possible, therefore, to ascertain the amount of the pressure (or lateral resistance) exerted by the inclosing tube, it would be possible by subtracting this from the total load employed to determine the load which was required to overcome the internal friction of the rock under the conditions of the experiment.

In order to determine the amount of pressure required to effect the progressive deformation of the tube, i.e., the amount of pressure exerted by the tube on the inclosed rock during the successive stages of deformation, a series of steel tubes, identical in every respect with those employed in the experiment just described, were taken and were deformed in a precisely similar manner, except that these tubes were filled with soft tallow, instead of being occupied by a column of marble. This material was selected as being one which moves with the development of an amount of internal friction which is so small that it was negligible in the present case. In carrying out the experiment with tallow, we found it necessary to slightly alter the shape of the steel pistons, the ends inserted in the steel tube being turned so as to present a somewhat concave face, as shown in Fig. 4, the outer margins having a thin feather edge. When pressure is brought to bear upon these pistons, this thin edge expands slightly, thus pressing against the walls of the tube and preventing the tallow from escaping between the piston and the wall. It was found that in this way the deformation of the tube could be readily effected.

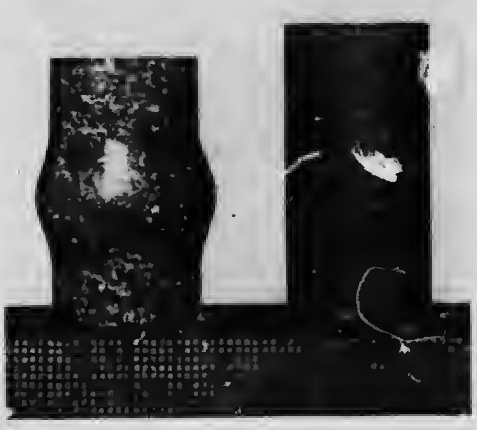

FIG. 3.—Photograph of columns of steatite before and after deformation. The smaller divisions of the scale below are millimeters.

The objection might be put forward that, while undoubtedly the tallow possesses at ordinary atmospheric pressure an internal friction which is quite negligible, this material under the pressure to which it must be subjected in order to deform the steel tube might develop an amount of internal friction and a rigidity which would be by no means negligible.

In order to ascertain whether such was the case, companion experiments were made, using the same pistons, but employing

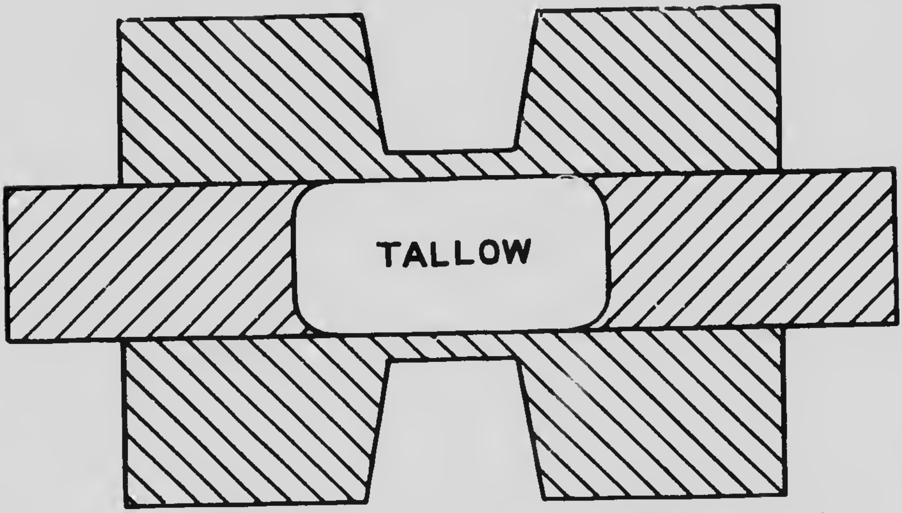


FIG. 4.—Longitudinal section through steel cylinder, showing the type of piston used when deforming the steel with tallow.

water in one case and oil in another, instead of tallow. It was found that the deformation of the tube could be effected by either of these materials, although, when water was employed, it was necessary to raise the pressure rapidly at first to cause the feather edges of the pistons to expand and make the joint tight, thus preventing the water from escaping. This series of comparative experiments was carried out with loads up to 19,000 pounds, at which pressure the tubes failed, and it was found that under these pressures the three substances mentioned—water, oil and soft tallow—showed no difference in viscosity which could be detected. The tallow, of course, undoubtedly possesses a somewhat greater internal friction than the water, but at the range of pressure to which it was submitted in the present investigation this difference is not noticeable and may therefore be neglected. The tallow, however, being more convenient for purposes of experiment, was employed in a further series of comparative experiments.

There was one other possible source of error, namely, the friction between the walls of the tube and the thin feather edge of the hollow-faced piston used in the experiments with the tallow. In the experiments with a column of rock a flat-faced piston was of course employed, and this source of friction was thus eliminated. In order to ascertain the amount of this friction in the case of the tallow, another steel tube was constructed, identical in all respects with those used in this investigation. One end of it, however, was closed so that it would be necessary to employ only a single piston, and through the closed end a small copper tube was inserted, which led to a powerful pump provided with an accurate pressure gage. The whole apparatus having been filled with water supplied by the pump, the steel tube with its cup-shaped piston was placed in a 75-ton Emery testing machine, and the piston slowly forced into the fluid, the pressure required to do this being noted at every stage on the testing machine and also on the gage fitted to the pump. In this way the pressure necessary to force the piston forward was measured at each additional increment of load applied to the piston by the Emery machine. As a result of a series of trials, it was ascertained that the friction on the feather edges of the piston amounted on an average to only 290 pounds, so that, in view of the very heavy pressure employed in this investigation, the error thus introduced is so small that it may be neglected.

It having been ascertained that soft tallow was a material which for the purposes of this investigation might be considered to move without the development of internal friction, a series of experiments were made with steel tubes identical in character and dimensions with those employed to inclose the marble, but soft tallow was substituted for marble. The two series of experiments were carried out in exactly the same manner in every detail, except that in the tubes filled with tallow the load was raised by increments of 500 pounds, instead of 1000 pounds, and the readings were taken every 15 seconds instead of every 30 seconds till they remained constant for at least 5 consequent readings. This change was necessitated in order to standardize the conditions in the two series of experiments, since, when the tube was filled with tallow, the whole load was applied to overcome the resistance of the tube, while, when the place of the tallow was taken by marble, a portion of the load was applied to overcome the internal friction of the rock, and the movement was slower. By modifying the procedure, as above mentioned, in the case of the tubes filled with tallow an identical deformation was secured in both cases.

When columns of rock are inclosed in the steel tubes and deformation is carried out in the manner described, the impending rupture of the steel tube, which marks the conclusion of the experiment, is indicated by the appearance of a series of sharply marked vertical lines on the bulged wall of steel which inclosed the deformed rock. If the experiment is continued, the tube splits along one of these vertical lines, and the inclosed rock becomes visible, and, if the pressure is still maintained, the resistance along the line of rupture being removed, the rock along this line crumbles and is forced out of the fissure in the form of a powder.

In the case of the experiments in which tallow was employed in place of a column of rock, the completion of the test is marked by the development of a vertical fissure in the thin portion of the steel tube in the usual manner. So soon as this appears, however, and usually before the load can be taken off the testing machine, a fragment of the thin steel wall, bounded on one side by the fissure in question and at the top and bottom by the thicker portion of the steel tube, opens out like a door on its hinges and is instantly torn off and with a loud report is shot across the room with great violence. It therefore was necessary in the case of these experiments that the observer should always be protected from these projectiles, the importance of this protection being emphasized in the case of one of the experiments by the fact that the piece of steel struck and split in two the piece of hard wood, a quarter of an inch thick, which protected the observer's head.

In order to make quite sure that the form and outline of the bulge assumed by the tube in the case of the experiments with the different rocks was the same, a special series of experiments to decide this question was made, employing copper, lead, marble, Belgian black, and granite. In each instance the experiment was carried to the point where the bulge or expansion of the diameter amounted to 0.030. The cylinder was then removed, and by using an electric arc light in a dark room a sharp shadow of the outline of the bulged cylinder was cast upon sensitive paper, removed at such a distance that the photograph enlarged the outline of the cylinder approximately 18 times. The cylinder was then placed in the Wicksteed machine, and the bulge increased to 0.110, and a similar photograph taken. By a comparison of the photographs it was found that the outline of the deformed wall was essentially identical in all cases.

As has been mentioned, from two to five experiments were made in the case of each rock when inclosed in a 0.25-centimeter tube and the same number with each rock inclosed in a tube having a wall thickness of 0.33 centimeter. The mean of the closely concordant results was then worked out in each case, and the figures obtained are presented in Tables I and II. These represent the data yielded by the experimental work.

The necessary data having been thus secured, a curve was plotted presenting these graphically in the case of each experiment. In these curves the exact amount of the load required to produce any required bulge or distension of the tube is shown from the point when the first movement can be detected until the final rupture of the tube takes place. The curves for the several experiments with Carrara marble inclosed in the steel tubes with a 0.25-centimeter wall are shown in Fig. 5 (p. 620). A curve representing the mean of the results obtained in the several experiments is also given. In Fig. 6 (p. 621) this curve of the mean of the results obtained from the marble inclosed in a 0.25-centimeter tube is reproduced, and below it is the mean of the curves obtained from tallow when inclosed in a 0.25-centimeter steel tube.

Since the tallow, as has been shown, offers itself no measurable resistance to deformation under the conditions of the experiment, the curve in the tallow experiments shows merely the resistance offered to deformation by the steel tube itself.

## TABLE I

AMOUNT OF DEFORMATION, IN INCHES, OF THE SEVERAL ROCKS WHEN INCLOSED IN THE STEEL CYLINDERS HAVING A WALL THICKNESS OF 0.25 CENTIMETER

| LOAD IN POUNDS* | TALLOW | LEAD | COPPER | ROCKS EMPLOYED | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Steatite, Virginia, U.S.A. | Alabaster, Castelino, Italy | Sandstone, Cleveland, Ohio, U.S.A. | White Marble, Carrara, Italy | Dolomite, Cockeysville, Maryland, U.S.A. | Black Belgian Marble ("Noir fin") | Slate, New Rockland, Quebec, Canada | Olivine Diabase, Sudbury, Ontario, Canada | Granite, Baveno, Italy |
| 2,000 | 0.0002 | | | | | | | | | | | |
| 3,000 | .0005 | | | | | | | | | | | |
| 4,000 | .0008 | | | | 0.0003 | | | | | | | |
| 5,000 | .0013 | 0.0004 | | | .0003 | | | | | | | |
| 6,000 | .0018 | .0010 | | | .0003 | | | | | | | |
| 7,000 | .0025 | .0015 | | 0.0003 | .0003 | | | | | | | |
| 8,000 | .0032 | .0020 | 0.0003 | .0004 | .0003 | | | | | | | |
| 9,000 | .0062 | .0038 | .0005 | .0005 | .0003 | | | | | | | |
| 10,000 | .0130 | .0075 | .0010 | .0006 | .0003 | | 0.0003 | | | | | |
| 11,000 | .0216 | .0139 | .0018 | .0006 | .0008 | | .0005 | | | | | |
| 12,000 | .0323 | .0220 | .0023 | .0008 | .0009 | | .0006 | | | | | |
| 13,000 | .0483 | .0330 | .0038 | .0009 | .0009 | | .0006 | | | | | |
| 14,000 | .0748 | .0550 | .0050 | .0009 | .0010 | | .0008 | | | | | |
| 15,000 | 0.1097 | .0843 | .0073 | .0013 | .0013 | | .0009 | | | | | |
| 16,000 | | 0.1133 | .0098 | .0013 | .0013 | 0.0003 | .0009 | | | | | |
| 17,000 | | | .0120 | .0016 | .0014 | .0005 | .0016 | | | | | |
| 18,000 | | | .0148 | .0021 | .0016 | .0005 | .0018 | | | | | |
| 19,000 | | | .0175 | .0026 | .0021 | .0005 | .0018 | | | | | |
| 20,000 | | | .0203 | .0043 | .0028 | .0008 | .0021 | | | | | |
| 21,000 | | | 0233 | .0068 | .0031 | .0008 | .0021 | | | | | |
| 22,000 | | | .0260 | .0103 | .0036 | .0010 | .0023 | | | | | |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 21,000 | | | 0233 | .0068 | .0031 | .0008 | .0021 | | | | | |
| 22,000 | | | .0260 | .0103 | .0036 | .0010 | .0023 | | | | | |
| 23,000 | | | .0295 | .0145 | .0045 | .0010 | .0028 | | | | | |
| 24,000 | | | .0328 | .0199 | .0063 | .0015 | .0030 | | | | | |
| 25,000 | | | .0365 | .0258 | .0080 | .0023 | .0035 | | | | | |
| 26,000 | | | .0403 | .0323 | .0098 | .0030 | .0043 | | | | 0.0003 | |
| 27,000 | | | .0445 | .0386 | .0126 | .0040 | .0051 | | | | .0003 | |
| 28,000 | | | .0488 | .0462 | .0151 | .0065 | .0068 | | | | .0004 | |
| 29,000 | | | .0535 | .0532 | .0184 | .0083 | .0078 | 0.0005 | | | .0004 | |
| 30,000 | | | .0583 | .0608 | .0218 | .0125 | .0085 | .0009 | 0.0005 | | .0004 | |
| 31,000 | | | .0640 | .0694 | .0258 | .0155 | .0110 | .0011 | .0010 | | .0006 | |
| 32,000 | | | .0708 | .0779 | .0318 | .0198 | .0126 | .0011 | .0010 | | .0006 | 0.0004 |
| 33,000 | | | .0768 | .0872 | .0353 | .0238 | .0139 | .0011 | .0010 | | .0009 | .0004 |
| 34,000 | | | .0840 | .0972 | .0419 | .0283 | .0169 | .0013 | .0010 | | .0009 | .0004 |
| 35,000 | | | .0923 | .1072 | .0473 | .0330 | .0181 | .0016 | .0010 | | .0009 | .0004 |
| 36,000 | | | .0995 | .1184 | .0556 | .0368 | .0208 | .0031 | .0013 | | .0009 | .0004 |
| 37,000 | | | 0.1093 | .1300 | .0868† | .0423 | .0234 | .0036 | .0013 | | .0014 | .0004 |
| 38,000 | | | | 0.1512 | .0893 | .0483 | .0268 | .0040 | .0018 | | .0014 | .0004 |
| 39,000 | | | | | .0935 | .0535 | .0296 | .0040 | .0020 | | .0014 | .0004 |
| 40,000 | | | | | .0990 | .0593 | .0326 | .0045 | .0020 | 0.0004 | .0014 | .0004 |
| 41,000 | | | | | .1048† | .0650 | .0354 | .0048 | .0020 | .0008 | .0014 | .0004 |
| 42,000 | | | | | .1123 | .0738 | .0386 | .0053 | .0023 | .0008 | .0018 | .0004 |
| 43,000 | | | | | .1231 | .0800 | .0416 | .0060 | .0023 | .0009 | .0018 | .0008 |
| 44,000 | | | | | .1324 | .0880 | .0464 | .0075 | .0025 | .0009 | .0018 | .0008 |
| 45,000 | | | | | .1458 | .0958 | .0511 | .0091 | .0030 | .0009 | .0020 | .0010 |
| 46,000 | | | | | 0.1559 | .1050 | .0560 | .0108 | .0033 | .0010 | .0020 | .0010 |
| 47,000 | | | | | | .1145 | .0613 | .0129 | .0038 | .0011 | .0025 | .0010 |
| 48,000 | | | | | | .1235 | .0658 | .0153 | .0040 | .0011 | .0025 | .0010 |
| 49,000 | | | | | | .1323 | .0716 | .0181 | .0053 | .0013 | .0028 | .0010 |
| 50,000 | | | | | | .1398 | .0768 | .0224 | .0063 | .0013 | .0031 | .0010 |
| 51,000 | | | | | | .1493 | .0819 | .0255 | .0085 | .0015 | .0031 | .0011 |
| 52,000 | | | | | | 0.1580 | .0879 | .0301 | .0108 | .0018 | .0035 | .0013 |
| 53,000 | | | | | | | .0936 | .0351 | .0135 | .0020 | .0040 | .0014 |
| 54,000 | | | | | | | .0989 | .0406 | .0165 | .0021 | .0043 | .0015 |
| 55,000 | | | | | | | 0.1063 | 0.0471 | 0.0210 | 0.0022 | 0.0043 | 0.0016 |

* Each 1,000 pounds of load as given in Column I = 2.052 7 pounds per square inch = 139.64 atmospheres.

† Between the daggers in the column pertaining to Alabaster the results are irregular because in one of the experiments there was an impact delivered to the specimen, as explained in the discussion of Fig. 12.

TABLE I—*Continued*

| Load in Pounds* | Tallow | Lead | Copper | Rocks Employed | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Steatite, Virginia, U.S.A. | Alabaster, Castelino, Italy | Sandstone, Cleveland, Ohio, U.S.A. | White Marble, Carrara, Italy | Dolomite, Cockeysville, Maryland, U.S.A. | Black Belgian Marble, ("Noir fin") | Slate, New Rockland, Quebec, Canada | Olivine Diabase, Sudbury, Ontario, Canada | Granite, Baveno, Italy |
| 56,000 | ...... | ...... | ...... | ...... | ...... | ...... | 0.1109 | 0.0546 | 0.0258 | 0.0026 | 0.0048 | 0.0016 |
| 57,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1155 | .0604 | .0313 | .0085 | .0051 | .0018 |
| 58,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1219 | .0671 | .0360 | .0091 | .0056 | .0021 |
| 59,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1282 | .0734 | .0413 | .0103 | .0070 | .0023 |
| 60,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1349 | .0818 | .0485 | .0108 | .0078 | .0023 |
| 61,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1405 | .0895 | .0543 | .0119 | .0084 | .0024 |
| 62,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1459 | .0968 | .0613 | .0150 | .0089 | .0026 |
| 63,000 | ...... | ...... | ...... | ...... | ...... | ...... | 0.1505 | .1041 | .0665 | .0335 | .0098 | .0026 |
| 64,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | .1110 | .0723 | .0447 | .0113 | .0029 |
| 65,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | .1178 | .0838 | .0540 | .0125 | .0031 |
| 66,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | .1265 | .0898 | .0680 | .0136 | .0033 |
| 67,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | .1344 | .0980 | .0801 | .0159 | .0036 |
| 68,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | .1410 | .1073 | .0905 | .0178 | .0036 |
| 69,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | .1494 | .1143 | .0981 | .0206 | .0041 |
| 70,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | .1569 | .1235 | .1116 | .0247 | .0045 |
| 71,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | 0.1051 | .1315 | .1223 | .0289 | .0049 |
| 72,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | .1395 | .1316 | .0330 | .0050 |
| 73,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | .1540 | .1432 | .0396 | .0055 |
| 74,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | 0.1648 | 0.1521 | .0475 | .0060 |
| 75,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | .0554 | .0069 |
| 76,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | .0650 | .0076 |
| 77,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | .0744 | .0089 |
| 78,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | .0839 | .0121 |
| 79,000 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | ...... | .0939 | .0290 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 80,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1038 | .0434 |
| 81,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1131 | .0554 |
| 82,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1230 | .0636 |
| 83,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1321 | .0708 |
| 84,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1405 | .0793 |
| 85,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | 0.1505 | .0868 |
| 86,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .0961 |
| 87,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1000 |
| 88,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1143 |
| 89,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1234 |
| 90,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1308 |
| 91,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1418 |
| 92,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | 0.1498 |
| Number of experiments performed | 3 | 2 | 2 | 4 | 2 | 2 | 4 | 2 | 2 | 2 | 2 | 2 |
| Average duration of each experiment | 0 hr. 52 min. | 1 hr. 10 min. | 1 hr. 32 min. | 2 hr. 11 min. | 2 hr. 22 min. | 4 hr. 0 min. | 4 hr. 0 min. | 4 hr. 32 min. | 4 hr. 14 min. | 3 hr. 49 min. | 4 hr. 56 min. | 4 hr. 11 min. |

* Each 1,000 pounds of load as given in Column I = 2,052.7 pounds per square inch = 139.64 atmospheres.

## TABLE II

Amount of Deformation, in Inches, of the Several Rocks When Inclosed in the Steel Cylinders Having a Wall Thickness of 0.33 Centimeter

| Load in Pounds* | Tallow | Lead | Copper | Rocks Employed | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Steatite, Virginia, U.S.A. | Alabaster, Castelino, Italy | Sandstone. Cleveland, Ohio, U.S.A. | White Marble, Carrara, Italy | Dolomite, Cockeysville, Maryland, U.S.A. | Black Belgian Marble, ("Noir fin") | Slate, New Rockland, Quebec, Canada | Olivine Diabase. Sudbury, Ontario, Canada | Granite, Baveno, Italy |
| 4,000 | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ |
| 5,000 | 0.0004 | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ |
| 6,000 | .0005 | 0.0005 | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ |
| 7,000 | .0008 | .0008 | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ |
| 8,000 | .0010 | .0013 | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ |
| 9,000 | .0012 | .0013 | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ |
| 10,000 | .0018 | .0019 | 0.0003 | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ |
| 11,000 | .0028 | .0025 | .0006 | 0.0004 | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ |
| 12,000 | .0055 | .0040 | .0010 | .0005 | ........ | ........ | ........ | ........ | ........ | ........ | ........ | ........ |
| 13,000 | .0095 | .0080 | .0013 | .0007 | 0.0004 | ........ | ........ | ........ | ........ | ........ | ........ | ........ |
| 14,000 | .0153 | .0116 | .0017 | .0009 | .0006 | ........ | 0.0005 | ........ | ........ | ........ | ........ | ........ |
| 15,000 | .0212 | .0173 | .0018 | .0010 | .0006 | ........ | .0008 | ........ | ........ | ........ | ........ | ........ |
| 16,000 | .0280 | .0240 | .0023 | .0013 | .0006 | 0.0003 | .0010 | ........ | ........ | ........ | ........ | ........ |
| 17,000 | .0367 | .0313 | .0026 | .0013 | .0008 | .0003 | .0010 | ........ | ........ | ........ | ........ | ........ |
| 18,000 | .0475 | .0419 | .0033 | .0017 | .0008 | .0005 | .0010 | ........ | ........ | ........ | ........ | ........ |
| 19,000 | .0645 | .0560 | .0040 | .0021 | .0009 | .0005 | .0010 | ........ | ........ | ........ | ........ | ........ |
| 20,000 | .0848 | 0.0805 | .0050 | .0025 | .0009 | .0008 | .0013 | ........ | ........ | ........ | ........ | ........ |
| 21,000 | 0.1152 | ........ | .0065 | .0028 | .0013 | .0008 | .0013 | ........ | ........ | ........ | ........ | ........ |
| 22,000 | ........ | ........ | .0078 | .0029 | .0014 | .0008 | .0015 | 0.0003 | ........ | ........ | ........ | ........ |
| 23,000 | ........ | ........ | .0098 | .0034 | .0015 | .0008 | .0018 | .0003 | ........ | ........ | ........ | ........ |
| 24,000 | ........ | ........ | .0117 | .0038 | .0019 | .0008 | .0020 | .0003 | ........ | ........ | ........ | ........ |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 25,000 | ... | ... | .0138 | .0050 | .0026 | .0008 | .0020 | .0003 | ... | ... | ... | ... |
| 26,000 | ... | ... | .0155 | .0071 | .0029 | .0010 | .0020 | .0003 | ... | ... | ... | ... |
| 27,000 | ... | ... | .0187 | .0093 | .0031 | .0010 | .0023 | .0003 | ... | ... | ... | ... |
| 28,000 | ... | ... | .0212 | .0116 | .0038 | .0010 | .0023 | .0004 | 0.0003 | ... | ... | ... |
| 29,000 | ... | ... | .0238 | .0146 | .0043 | .0010 | .0028 | .0006 | .0003 | ... | ... | ... |
| 30,000 | ... | ... | .0273 | .0173 | .0051 | .0011 | .0030 | .0008 | .0003 | ... | ... | ... |
| 31,000 | ... | ... | .0305 | .0212 | .0061 | .0013 | .0040 | .0009 | .0003 | ... | ... | ... |
| 32,000 | ... | ... | .0345 | .0237 | .0075 | .0013 | .0040 | .0010 | .0007 | ... | ... | ... |
| 33,000 | ... | ... | .0387 | .0265 | .0089 | .0018 | .0050 | .0011 | .0007 | ... | ... | ... |
| 34,000 | ... | ... | .0426 | .0306 | .0114 | .0043 | .0063 | .0011 | .0007 | ... | ... | ... |
| 35,000 | ... | ... | .0470 | .0333 | .0126 | .0058 | .0078 | .0013 | .0007 | ... | 0.0003 | ... |
| 36,000 | ... | ... | .0518 | .0379 | .0163 | .0084 | .0085 | .0014 | .0007 | ... | .0003 | 0.0005 |
| 37,000 | ... | ... | .0570 | .0415 | .0186 | .0106 | .0093 | .0014 | .0008 | ... | .0005 | .0005 |
| 38,000 | ... | ... | .0630 | .0456 | .0209 | .0141 | .0108 | .0015 | .0009 | ... | .0009 | .0005 |
| 39,000 | ... | ... | .0688 | .0481 | .0238 | .0164 | .0128 | .0018 | .0009 | ... | .0009 | .0005 |
| 40,000 | ... | ... | .0760 | .0537 | .0269 | .0188 | .0140 | .0019 | .0009 | ... | .0009 | .0005 |
| 41,000 | ... | ... | .0838 | .0601 | .0309 | .0210 | .0150 | .0021 | .0010 | ... | .0009 | .0005 |
| 42,000 | ... | ... | .0915 | .0658 | .0348 | .0243 | .0170 | .0021 | .0012 | 0.0007 | .0011 | .0005 |
| 43,000 | ... | ... | 0.0987 | .0714 | .0370 | .0273 | .0185 | .0024 | .0013 | .0007 | .0011 | .0005 |
| 44,000 | ... | ... | ... | .0789 | .0451 | .0298 | .0210 | .0026 | .0013 | .0009 | .0011 | .0010 |
| 45,000 | ... | ... | ... | .0856 | .0494 | .0325 | .0230 | .0028 | .0013 | .0009 | .0011 | .0010 |
| 46,000 | ... | ... | ... | .0939 | .0558 | .0349 | .0258 | .0031 | .0013 | .0009 | .0014 | .0010 |
| 47,000 | ... | ... | ... | .1023 | .0615 | .0389 | .0270 | .0040 | .0015 | .0009 | .0014 | .0010 |
| 48,000 | ... | ... | ... | .1117 | .0680 | .0427 | .0300 | .0044 | .0016 | .0009 | .0014 | .0010 |
| 49,000 | ... | ... | ... | .1217 | .0742 | .0452 | .0320 | .0054 | .0017 | .0011 | .0014 | .0010 |
| 50,000 | ... | ... | ... | .1333 | .0820 | .0493 | .0358 | .0063 | .0022 | .0011 | .0014 | .0010 |
| 51,000 | ... | ... | ... | .1428 | .0922 | .0535 | .0373 | .0068 | .0025 | .0013 | .0017 | .0010 |
| 52,000 | ... | ... | ... | 0.1525 | .0980 | .0570 | .0408 | .0086 | .0030 | .0013 | .0017 | .0010 |
| 53,000 | ... | ... | ... | ... | .1074 | .0610 | .0458 | .0096 | .0042 | .0013 | .0017 | .0010 |
| 54,000 | ... | ... | ... | ... | .1175 | .0656 | .0485 | .0114 | .0053 | .0013 | .0021 | .0010 |
| 55,000 | ... | ... | ... | ... | .1265 | .0695 | .0530 | .0131 | .0073 | .0015 | .0021 | .0015 |
| 56,000 | ... | ... | ... | ... | .1356 | .0740 | .0558 | .0155 | .0084 | .0016 | .0023 | .0015 |
| 57,000 | ... | ... | ... | ... | .1456 | .0783 | .0600 | .0179 | .0110 | .0018 | .0025 | .0015 |
| 58,000 | ... | ... | ... | ... | 0.1569 | 0.0830 | 0.0630 | 0.0204 | 0.0138 | 0.0019 | 0.0025 | 0.0015 |

* Each 1,000 pounds of load as given in Column I = 2,052 7 pounds per square inch = 139.64 atmospheres.

TABLE II—*Continued*

| Load in Pounds* | Tallow | Lead | Copper | Rocks Employed | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Steatite, Virginia, U.S.A. | Alabaster, Castelino, Italy | Sandstone, Cleveland, Ohio, U.S.A. | White Marble, Carrara, Italy | Dolomite, Cockeys-ville, Maryland, U.S.A. | Black Belgian Marble, ("Noir fin") | Slate, New Rockland, Quebec, Canada | Olivine Diabase, Sudbury, Ontario, Canada | Granite, Baveno, Italy |
| 59,000 | ...... | ...... | ...... | ...... | ...... | 0.0887 | 0.0658 | 0.0235 | 0.0175 | 0.0019 | 0.0028 | 0.0020 |
| 60,000 | ...... | ...... | ...... | ...... | ...... | .0952 | .0690 | .0256 | .0204 | .0020 | .0030 | .0021 |
| 61,000 | ...... | ...... | ...... | ...... | ...... | .1012 | .0728 | .0300 | .0237 | .0023 | .0030 | .0023 |
| 62,000 | ...... | ...... | ...... | ...... | ...... | .1060 | .0765 | .0336 | .0274 | .0025 | .0034 | .0024 |
| 63,000 | ...... | ...... | ...... | ...... | ...... | .1115 | .0825 | .0370 | .0312 | .0029 | .0038 | .0024 |
| 64,000 | ...... | ...... | ...... | ...... | ...... | .1168 | .0865 | .0410 | .0339 | .0030 | .0040 | .0026 |
| 65,000 | ...... | ...... | ...... | ...... | ...... | .1223 | .0900 | .0450 | .0380 | .0030 | .0043 | .0027 |
| 66,000 | ...... | ...... | ...... | ...... | ...... | .1278 | .0938 | .0496 | .0424 | .0035 | .0048 | .0028 |
| 67,000 | ...... | ...... | ...... | ...... | ...... | .1330 | .1013 | .0544 | .0466 | .0035 | .0050 | .0031 |
| 68,000 | ...... | ...... | ...... | ...... | ...... | .1378 | .1055 | .0588† | .0506 | .0039 | .0052 | .0031 |
| 69,000 | ...... | ...... | ...... | ...... | ...... | .1445 | .1103 | .0646† | .0550 | .0040 | .0056 | .0034 |
| 70,000 | ...... | ...... | ...... | ...... | ...... | .1498 | .1155 | .0681 | .0585 | .0041 | .0056 | .0035 |
| 71,000 | ...... | ...... | ...... | ...... | ...... | 0.1553 | .1205 | .0719 | .0649 | .0046 | .0066 | .0036 |
| 72,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1253 | .0761 | .0697 | .0050 | .0073 | .0036 |
| 73,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1270 | .0810 | .0760 | .0050 | .0083 | .0037 |
| 74,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1353 | .0858 | .0806 | .0056 | .0090 | .0039 |
| 75,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1400 | .0909 | .0844 | .0060 | .0096 | .0041 |
| 76,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1441 | .0964 | .0900 | .0135 | .0105 | .0043 |
| 77,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1488 | .1025 | .0960 | .0226 | .0115 | .0048 |
| 78,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1540 | .1080 | .1017 | .0273 | .0129 | .0052 |
| 79,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1583 | .1131 | .1059 | .0300 | .0139 | .0055 |
| 80,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1635 | .1174 | .1139 | .0356 | .0149 | .0058 |
| 81,000 | ...... | ...... | ...... | ...... | ...... | ...... | .1685 | .1229 | .1194 | .0525 | .0176 | .0065 |
| 82,000 | ...... | ...... | ...... | ...... | ...... | ...... | 0.1738 | .1290 | .1247 | .0609 | .0199 | .0072 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 83,000 | ... | ... | ... | ... | ... | ... | ... | .1365 | .1298 | .0675 | .0223 | .0079 |
| 84,000 | ... | ... | ... | ... | ... | ... | ... | .1428 | .1373 | .0748 | .0255 | .0087 |
| 85,000 | ... | ... | ... | ... | ... | ... | ... | .1476 | .1438 | .0814 | .0288 | .0098 |
| 86,000 | ... | ... | ... | ... | ... | ... | ... | 0.1543 | 0.1495 | .0894 | .0351 | .0117 |
| 87,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | .0970 | .0420 | .0136 |
| 88,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1053 | .0484 | .0159 |
| 89,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1120 | .0575 | .0183 |
| 90,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1194 | .0609 | .0213 |
| 91,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1263 | .0678 | .0249 |
| 92,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1320 | .0739 | 0308 |
| 93,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | 0.1407 | .0808 | .0341 |
| 94,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .0864 | .0402 |
| 95,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .0928 | .0444 |
| 96,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .0987 | .0493 |
| 97,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1067 | .0548 |
| 98,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1133 | 0607 |
| 99,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1198 | .0651 |
| 100,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1277 | .0718 |
| 101,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | 0.1349 | .0793 |
| 102,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .0859 |
| 103,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .0896 |
| 104,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .0978 |
| 105,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1048 |
| 106,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1106 |
| 107,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | .1170 |
| 108,000 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | 0.1231 |
| Number of experiments performed | 2 | [illegible] | 3 | 5 | 2 | 2 | 3 | 3 | 3 | 2 | 2 | 4 |
| Average duration of each experiment | 1 hr. 4 min. | 1 hr. 20 min. | 2 hr. 3 min. | 3 hr. 1 min. | 3 hr. 32 min. | 5 hr. 36 min. | 6 hr. 0 min. | 5 hr. 26 min. | 4 hr. 39 min. | 4 hr. 23 min. | 5 hr. 14 min. | 5 hr. 9 min. |

*Each 1,000 pounds of load as given in Column I = 2,052.7 pounds per square inch = 139.64 atmospheres.

† Between the daggers in the column pertaining to Dolomite the results are irregular because in one of the experiments the load was removed and when later restored a slight impact was delivered to the deforming rock column.

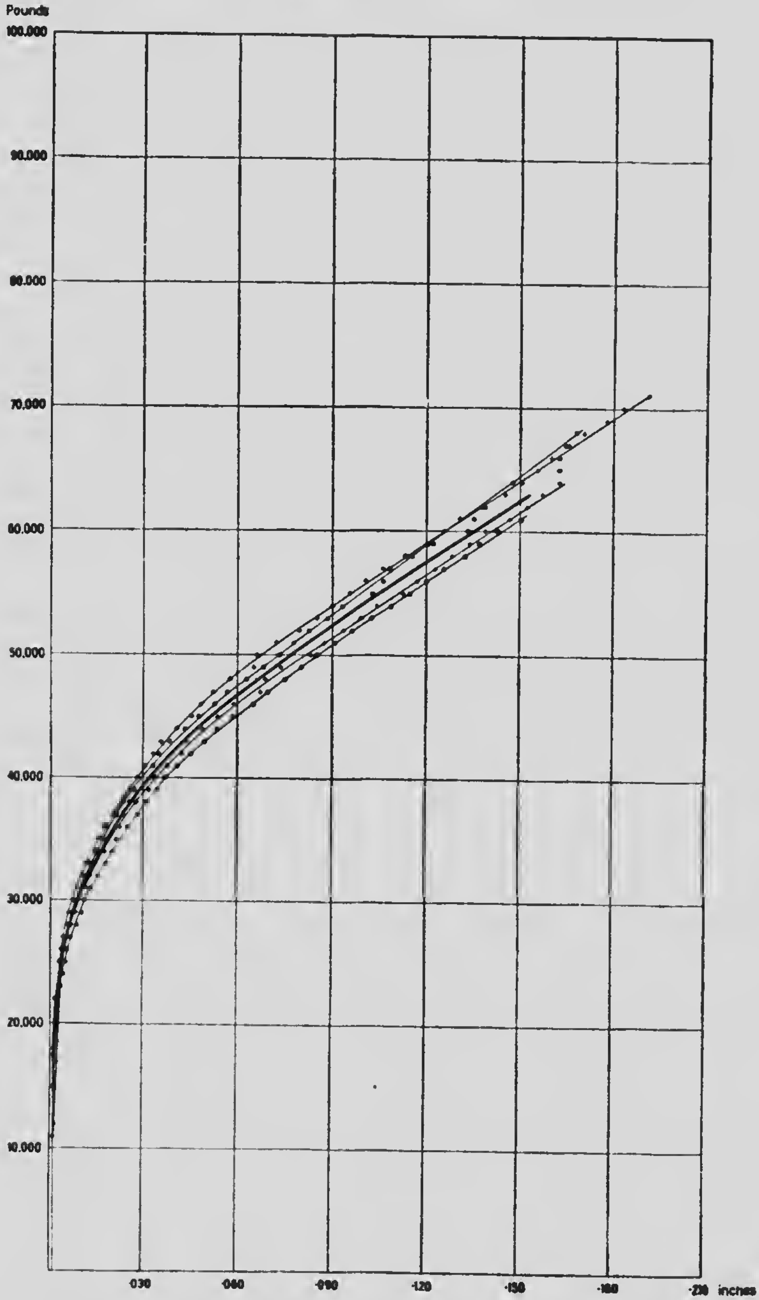


FIG. 5.—Curves showing graphically the results obtained in four experiments on the deformation of Carrara marble when it is inclosed in a steel cylinder with wall 0.25 cm. thick—also the mean of these curves (in heavy line).

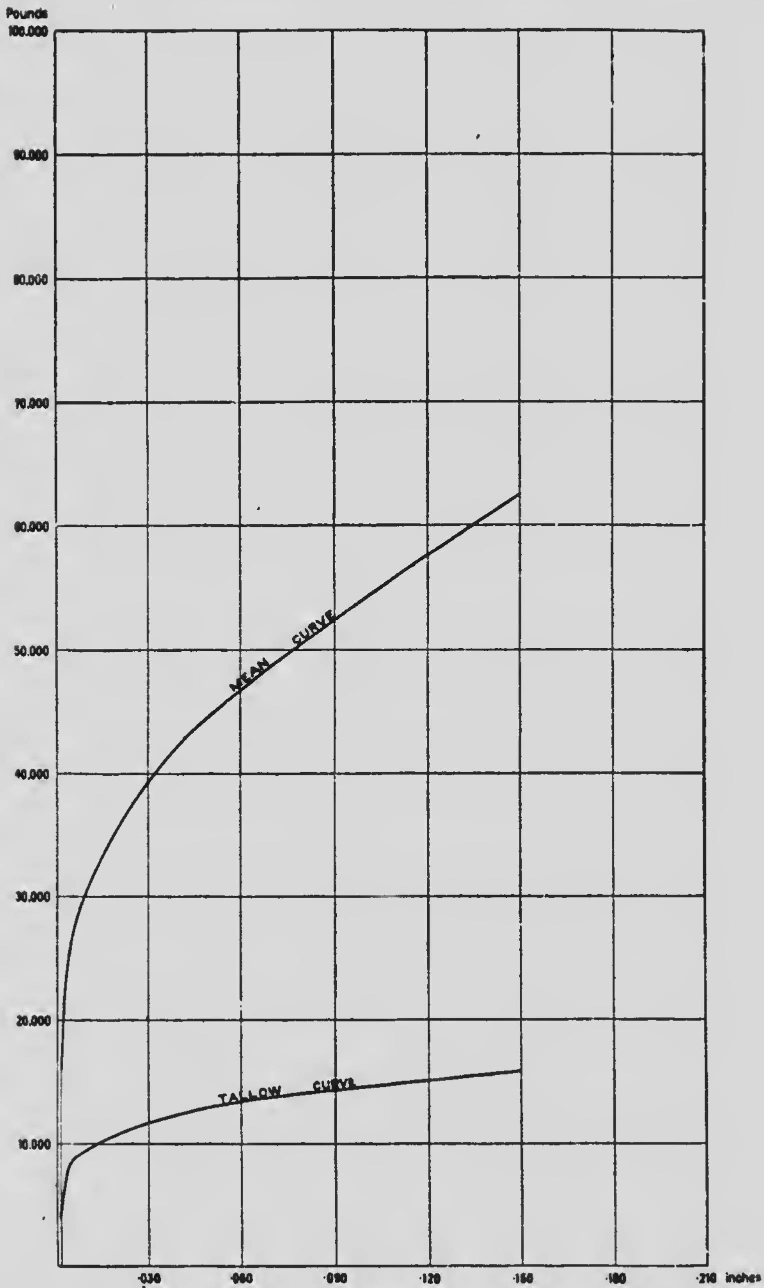


FIG. 6.—The mean of the curves obtained in the deformation of Carrara marble (see Fig. 5) when it is inclosed in a steel cylinder with wall 0.25 cm. thick—with the [illegible] obtained when [illegible] steel tube of identical dimensions is deformed when filled with [illegible]w.

Such being the case, with the information thus secured it is possible to separate the two components of the load, namely, that necessary to overcome the resistance offered by the tube and that required to effect the deformation of the marble. If at a series of points the load required to produce a certain distension or bulge in the steel tube when filled with the tallow is subtracted from the load required to produce the same bulge in the case of the tube containing the marble, values are obtained which represent that portion of the load which is expended in affecting the deformation of the marble. This may be termed the *true curve*, and that obtained for a standard column of Carrara marble deformed in a standard steel tube having a wall thickness of 0.25 centimeter is shown in Fig. 7. In the same manner the *true curve* for each of the other rocks may be plotted from the data presented in Tables I and II. It will be seen that, in the case of Carrara marble, this curve starting from a distension of 0.001, which may be considered to be due to elastic deformation, and which is produced by a load of 12,000 pounds, shows a rapid deflection to a point representing a distension of 0.052 which is produced by a load of 33,000 pounds, after which it develops into what is practically a straight line until the tube ruptures.

This shows that after the elastic limit of the marble has been passed, at about 12,000 pounds, and the marble commences to deform, the load which is required to start this movement and produce a unit of diametral expansion is relatively great. As the movement progresses the additional increment of load required to produce a unit of diametral expansion grows progressively less till a bulge of 0.052 is reached, after which there is a definite and constant ratio between the increase of load and the expansion which it produces. This ratio is 0.0065 for each increase of 1,000 pounds in load.

It will be noted that in the case of the slate, just after the rock began to deform, the curve shows a sudden break or sag which is repeated at a second point before the regular movement, indicated by the nearly straight line, is developed. This is due to the fact, above mentioned, that the slate, being a foliated and not a granular rock, is not isotropic in its response to pressure. It consists of little

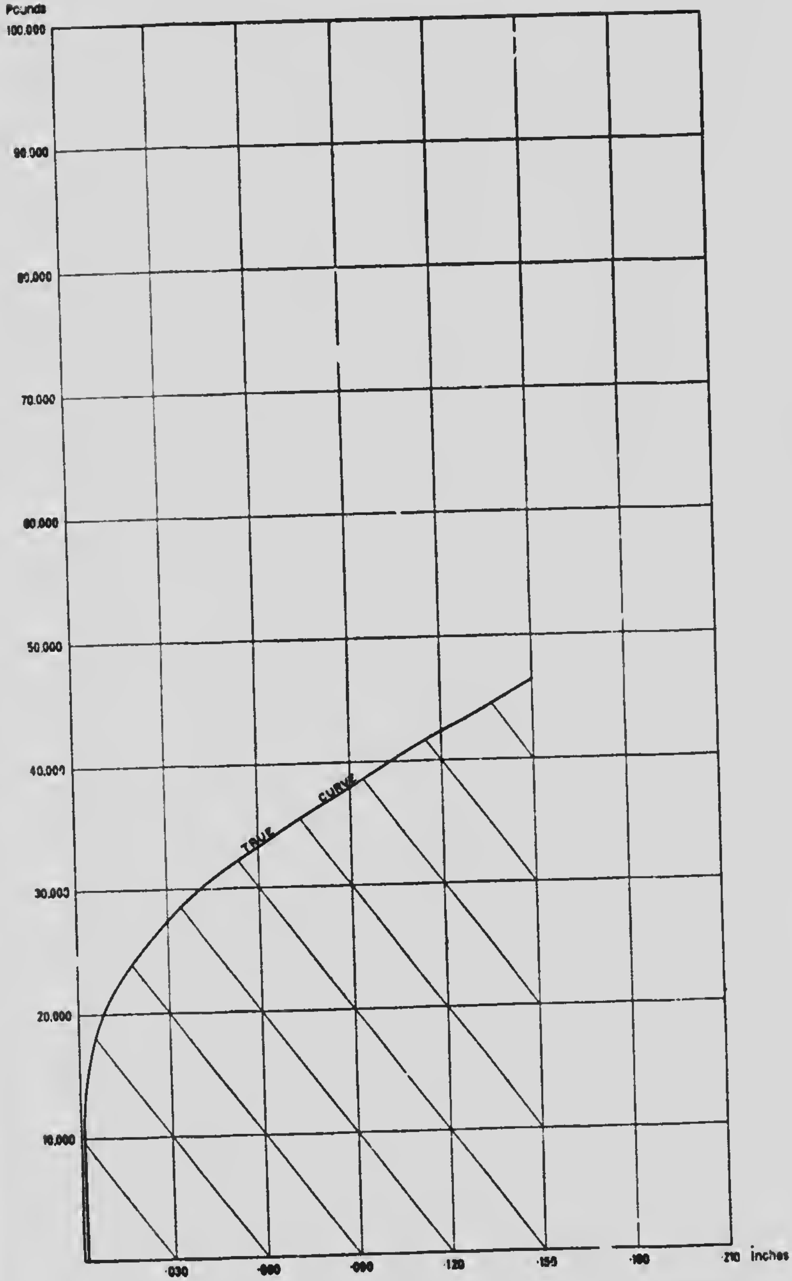


FIG. 7.—True curve obtained by the deformation of a standard column of Carrara marble in steel cylinder with wall 0.25 cm. thick. The area designated by oblique lines represents the work done in effecting the deformation of the marble to a bulge of 0.150 inch.

plates of kaolin and muscovite lying parallel to one another and at right angles to the direction in which the pressure is exerted. The breaking down of the foliated structure of the rock is indicated on the curves by the irregularities to which reference has been made.

It will also be seen that in the case of granite, when the lateral resistance is relatively low (e.g., when the rock is inclosed in the steel tube having a 0.25-centimeter wall), there is at the same point a sag, though much less marked, due to the fact that the lateral resistance offered by the tube is not quite sufficient to develop a uniform movement in this the strongest of all the rocks employed in the investigation.

Attention must be drawn to the manner in which deformation goes forward in a column of rock when deformed under the conditions of the experiment. As may be seen, if the tube and the inclosed rock are sawed in two vertically, the column of rock begins to move or flow at the middle, the motion taking place first along the well-known shearing cones, having an angle of approximately 45° (usually somewhat greater), seen when a column or cube of the rock is crushed between the faces of a testing machine in the ordinary determinations of the strength of rock for building purposes. Thus, as the movement progresses, there develops within the column two obtuse cones, having as their bases the faces of the advancing pistons and consisting of portions of the rock which show no evidences whatsoever of deformation, but which are, under the conditions of the experiment, subjected only to cubic compression. As the experiment progresses, these cones (see *A* and *B* in Fig. 8) advance into the deforming rock, additional amounts of the rock shearing off the surfaces of the cones and thus coming to participate in the movements which are going forward. Owing to the fact, therefore, that the quantity of flowing rock is continually increasing in an unknown ratio, it is impossible from the data mentioned above to determine whether the definite increase in the ratio of load to deformation is due to an increase of internal friction developed with increase of pressure, or to the increased amount of material which is being moved.

The answer to this question is obtained from another series of experiments which exactly duplicated those with the columns of

Carrara marble, described above, except that the lateral resistance to movement was increased by increasing the thickness of the walls of the steel tube inclosing the marble from a thickness of 0 25 centimeter to 0.33 centimeter. In these the amount of material moved is identical with that in the series of experiments just described, while the internal friction is increased by the increased thickness of the steel tube.

A series of additional experiments were also made to determine the resistance offered by such tubes when filled with soft tallow.

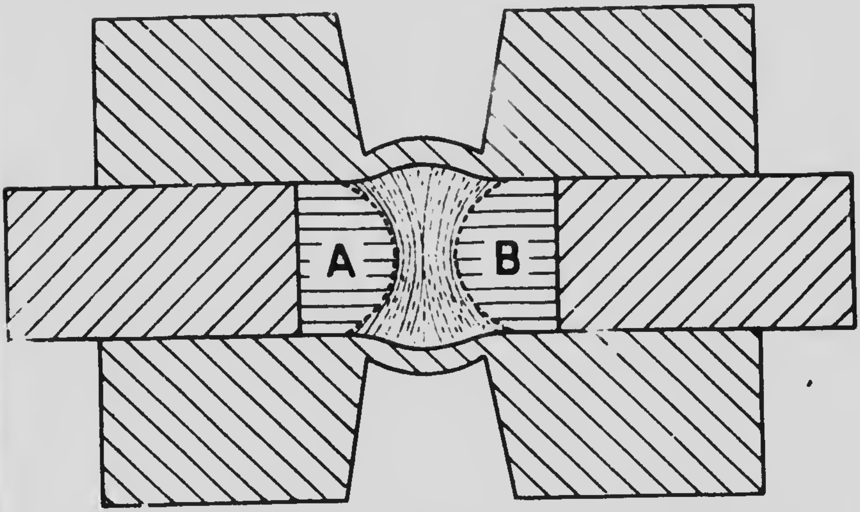


Fig. 8.—Longitudinal section through steel cylinder with pistons inserted and inclosing a deformed column of rock—showing the obtuse shearing cones which advance into the deforming rock.

In this way another series of curves were obtained for each material and another "true curve" for the deformation of a standard column of Carrara marble under conditions identical with those of the former experiments, except that the resistance to deformation offered by the steel tube was much greater. The "true curve" for the deformation of the marble in a steel tube having walls 0.33 centimeter thick is shown in Fig. 10.

An inspection of this curve will show that while, as before, starting from the limit of elastic expansion the rising load at first induces a relatively small amount of movement in the rock, the

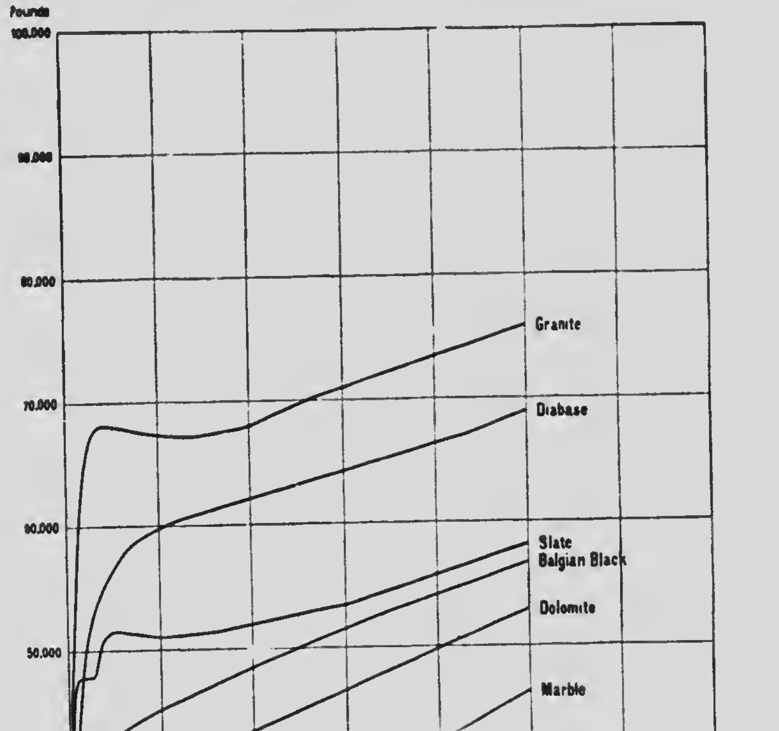


FIG. 9.—True curves obtained by the deformation of the several rocks when inclosed in the steel cylinders with wall 0.25 cm. thick.

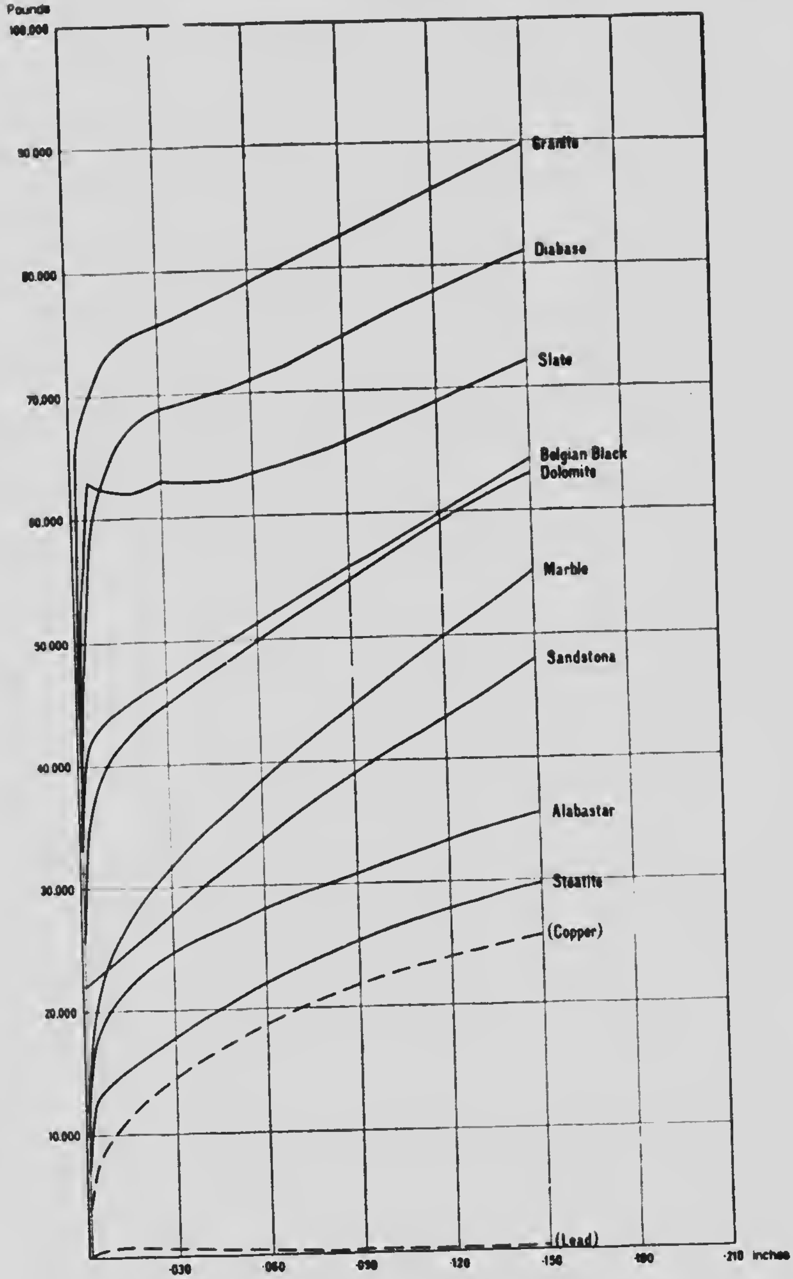


FIG. 10.—True curves obtained by the deformation of the several rocks when inclosed in the steel cylinders with wall 0.33 cm. thick.

ratio of the amount of this movement to increment of loads increases rather rapidly, and, after deformation amounting to about 0.06 has been brought about—which requires a load of 38,750 pounds—the ratio of increase of load to amount of deformation of the column becomes constant, as when the marble is deformed in the tubes with thinner walls. It will be seen, however, that for the experiments in the thicker-walled tube this ratio of increase is much less than when the wall was thinner, i.e., 0.25 centimeter being 0.0051 diametrical increase for each increase of 1,000 pounds in the load, instead of .0065, as in the first series of experiments.

This demonstrates that the moving rock possesses internal friction and that with the increase of the lateral resistance the amount or coefficient of friction rapidly increases, and at a constant ratio.

The investigation was then extended to the other rocks of the series enumerated on pp. 598 and 599. The conditions and method of conducting the experiments were in every case identical with those just described with Carrara marble. Two sets of standard steel tubes, having wall thicknesses of 0.25 centimeter and 0.33 centimeter, respectively, were employed, and the true curves were plotted representing the mean of a series of experiments in each case (see Figs. 9 and 10).

### "WORK DONE" IN THE DEFORMATION OF ROCKS

If $Px$ be the load to which the specimen is subjected and $Py$ be the resistance to movement offered by the inclosing walls of the steel cylinder, the data were first examined to ascertain whether the formula

$$Px - Py = \text{a constant}$$

represents the movement, and it was found that this was not the case. They were then studied to see whether each rock possessed a constant factor $K$, which might be termed its modulus of plasticity, as in the formula

$$Px - KPy = \text{a constant}$$

It was found that, if the data are calculated so as to take into consideration the bulge of the cylinder and are plotted to show vertical stress as compared with lateral stress, this formula represents the facts and that each of the softer rocks possesses a definite modulus of plasticity, this being also true in the harder rocks in the earlier stages of the deformation at least.

This interesting fact is discussed at length in the accompanying paper by Dr. King, where a mathematical treatment of some of the new data developed in the present investigation is also presented, illuminating certain parts at least of that hitherto unsubdued and almost unoccupied domain—the mathematics of the flow of solids.

In the present paper, without entering into a mathematical treatment of the subject, the following deductions from the experimental data may be indicated.

If a vertical line be drawn cutting off the "true curve" obtained in the case of any rock when the deformation of the tube amounting to 0.15 has been reached, and if the area inclosed by this line, the "true curve" itself, and the base line of the diagram be measured, this area represents the "work done" to effect the deformation of the rock. This area showing the "work done" in deforming a standard column of Carrara marble in a 0.25-centimeter steel tube in Fig. 7 is shaded. In Fig. 9 the "true curves" obtained in this deformation of all the rocks of the series, in steel tubes having a wall thickness of 0.25 centimeter, are shown, and in Fig. 10 the complete series of "true curves" obtained when the wall thickness of the tube is increased to 0.33 centimeter is set forth. In both figures the curves are cut off at the ordinate 0.15, and the area representing the "work done" in the case of each rock is clearly shown and may be compared.

Table III sets forth these comparative values in square inches. This table shows quite clearly that with the increased resistance, offered by the thicker-walled steel tube, the amount of work required to effect an equal deformation increased in the case of every rock. It also sets forth the comparative value of these increases and also the relative amount of work done to deform the different rocks of the series.

The table thus shows that the "work done" in deforming a column of marble of the size employed and under the conditions of the experiment, when inclosed in the thinner-walled tube, is to the "work done" when an identical column is deformed, when inclosed in the thicker-walled tube, as 51,708 is to 60,415. Or, again, that the "work done" in deforming a marble column, whether the resistance be small or great, is almost exactly one-half of that required to effect an equal amount of deformation in a column of granite under the same conditions. That is to say, almost exactly twice as much work is required to deform granite as is required to effect an equal deformation in the case of marble and nearly four times as much as is required to produce an equal deformation in the case of steatite.

TABLE III

RELATIVE AMOUNT OF "WORK DONE" IN EFFECTING AN EQUAL DEFORMATION IN UNIT COLUMNS OF DIFFERENT ROCKS

| | Under Resistance of | |
|---|---|---|
| | 0.25 cm. Steel Tube | 0.33 cm. Steel Tube |
| Steatite | 26,054 | 34,123 |
| Alabaster | 35,569 | 42,946 |
| Sandstone | 41,262 | 53,446 |
| Marble | 51,708 | 60,415 |
| Dolomite | 66,362 | 77,092 |
| Belgian Black | 73,754 | 79,362 |
| Slate | 79,069 | 97,154 |
| Diabase | 92,985 | 107,431 |
| Granite | 104,169 | 119,877 |

TABLE IV

RELATIVE AMOUNT OF "WORK DONE" IN EFFECTING AN EQUAL DEFORMATION IN UNIT COLUMNS OF DIFFERENT ROCKS CALCULATED ON THE BASIS OF MARBLE AS UNITY

| | Under Resistance of | |
|---|---|---|
| | 0.25 cm. Steel Tube | 0.33 cm. Steel Tube |
| Steatite | 0.50 | 0.56 |
| Alabaster | 0.69 | 0.71 |
| Sandstone | 0.80 | 0.88 |
| Marble | 1.00 | 1.00 |
| Dolomite | 1.28 | 1.28 |
| Belgian Black | 1.43 | 1.31 |
| Slate | 1.53 | 1.61 |
| Diabase | 1.80 | 1.78 |
| Granite | 2.01 | 1.98 |

If the "work done" to deform marble be taken as unity, these figures may be set forth as in Table IV.

In these tables there is expressed in actual values the phenomena which are displayed in such a striking manner in the great exposures of the Grenville series and in other terranes which have undergone deformation at great depths below the surface of the earth where the same force has acted on a complex of rocks of diverse character. In these occurrences some of these rocks are torn to fragments, which are then carried far apart in a flowing matrix formed of some other and more plastic member of the complex. This is seen in a striking manner where dykes of diabase or belts of granite cut through a limestone, and the whole complex is then deformed under conditions of deep-seated differential pressure. The diabase dyke or belt of granite is torn apart into angular fragments, which are floated along in sinuous curves in the plastic flowing limestone, like logs or drifting timber on the surface of a flowing river (see Fig. 11).

### EFFECT OF A CHANGE IN THE RAPIDITY OF THE APPLICATION OF PRESSURE

In Fig. 12 there are two curves: one showing the deformation of alabaster, the other, the deformation of marble. These also illustrate the effects of a change in the rate at which the pressure is applied.

In the former case, after a load of 36,000 pounds had been gradually applied in successive increments and no movement had taken place under the load for 2 minutes, the next increment of load was by mistake applied suddenly, thereby submitting the rock to an impact instead of to a slow increase of pressure. This, as will be seen, produced at once a movement of 0.045 inch. Following this, however, four increments of load, each of 1,000 pounds, had to be applied before the movement was resumed, and two additional increments, each of 1,000 pounds, had to be applied before the movement could be re-established in its regular course, after which the flow continued in the line followed by the normal curve.

In the second case—that of the marble—the normal course of the experiment was interrupted four times by postponing the time of reading the deformation produced by a new increment of load much longer than usual, namely, from 9 to 75 minutes. These were when the load on the column of rock was 40,000, 55,000, 60,000, and 65,000 pounds, respectively. It will be noted that the same effect, though on a smaller scale, was produced as that just described as the result of impact. An abnormal increase of load

FIG. 11.—Photograph of a specimen of Trenton limestone which has been cut by a narrow dyke of camptonite. The whole has then been distorted by pressure exerted by the intrusion of the igneous mass constituting Mount Royal. The harder camptonite has been broken into fragments which have been carried apart in the flowing mass of more plastic limeston. (Canadian Northern Railway Tunnel through Mount Royal, Montreal, Canada).

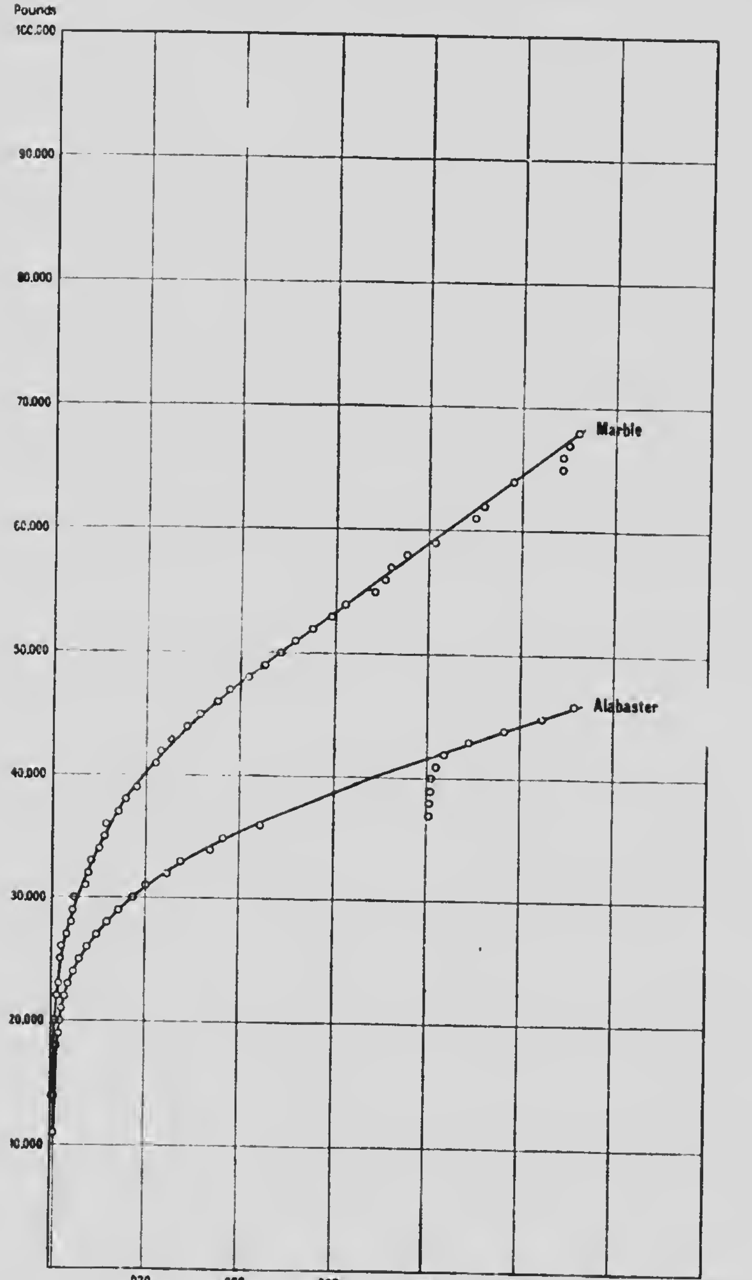


FIG. 12.—Curves showing the effect of change in rate of application of pressure

was required to bring about a re-establishment of the movement, which, however, eventually resumed its former course.

## BEARING OF THE RESULTS ON CERTAIN PROBLEMS PRESENTED BY THE EARTH'S CRUST

The experimental results afford a reply to the question propounded by Dr. Gilbert and set forth in the opening paragraph of this paper. They also have a direct bearing on the problems presented by the origin of "decken" and by the theory of isostasy.

When movement producing deformation is once started in the rock under the influence of tangential thrust, resulting in the breaking down of its texture, the rock, if deeply buried in the earth's crust, does not on that account offer a decreased resistance to further movement.

Some experiments by Karman[1] on the deformation of marble under differential pressure have yielded data with reference to the amount of this pressure which must be exerted in the case of marble in order to induce plastic flow in the rock. The data obtained represent maximum results, because in the experiments the pressure was applied rapidly as compared with that which would be developed in any earth movements, and, also, the factor of heat was not taken into account. It must be noted, however, that heat and a very slow application of the deforming force would produce movements under lower pressures than those made use of in the experimental work. Karman found that, if a column of marble were submitted to a supporting or containing pressure, such as that exerted by the steel tube in our experiments, amounting to 685 atmospheres—which would be equivalent to that exerted by the overlying strata at a depth of 2.53 miles below the surface[2]—it would flow uniformly and continuously under a load of 2,870 atmospheres applied to the ends of the column. If the containing pressure fell below the value mentioned, that is, if the rock occupied a position in the earth's crust nearer the surface, it would speedily crumble and break to pieces, presenting in this way a failure similar

[1] "Festigkeits Versuche unter allseitigem Druck," *Zeit. des Ver. deut. Ingenieure*, October 21, 1911.

[2] F. D. Adams, "Depth of the Zone of Flow in the Earth's Crust," *Journal of Geology*, February, 1912.

to that which is obtained in testing building stones in the laboratory. On the other hand, if the containing or supporting pressure is increased, the load required to produce deformation rapidly increases also, and the experiments seem to indicate that with a containing pressure of about 10,000 atmospheres, which would be equivalent to a depth of about 22 miles below the surface, it would be impossible to make the marble flow, except under a pressure which would be simply colossal.

Since with the increase of resistance to tangential thrust, that is, with increasing depth below the surface of the earth, the amount of such thrust required to produce movements in the earth's crust increases rapidly, it is evident that the great movements of adjustment by rock flow or transference of material in the earth's crust from one point to another—other than the transference of rock in a molten condition—must take place comparatively near the surface. That is, beneath the zone of fracture where adjustment takes place by faults and overthrusts—in the zone of flow—movements so far as they are determined by pressure are effected with an ease which increases rapidly in proportion to their nearness to the surface.

It would seem, therefore, that it is in the upper part of the zone of flow only that the great "decken," as, for instance, those which are developed in the Alps, are produced. This explains the fact that in the mountain range in question it is the upper "decken" which have moved more rapidly and have extended farther than the lower "decken," where the rock is under the increased load and is consequently much less plastic.

Since with the increase of depth there is a rapid increase in rigidity of the rocks of the earth's crust, it is not difficult to understand how it is that, while great movements may take place near the surface of the earth in the upper part of the zone of flow, the globe itself is "more rigid than steel or glass."

The experimental work also affords at least a first approximation to the determination of the dimensions of the forces which are required in order to effect deformation in the earth's crust in the case at least of the chief types of rocks which make up the crust in question.

In these measurements it must again be noted that the factor of pressure alone was considered, no account being taken of the element of heat in the crust, which would undoubtedly tend to increase the ease of movement.

In the experiments it has been shown, as mentioned, that the resistance to deformation exerted by the wall of the steel tube gradually increases as the experiment progresses. If, however, the value of the resistance is taken at a point where the regular column shows a diametral increase of 0.05 inch (or 6.35 per cent), i.e., when the deformation is well under way and after which it becomes proportional to the increased tangential pressure, this resistance, in the case of the experiment with the steel wall 0.25 centimeter thick, would be equivalent to 26,685 pounds to the square inch, or 1,815 atmospheres, that is, to a depth of 4.2 miles below the surface.

In the case of our experiment with a steel wall 0.33 centimeter thick it would be equivalent to 37,359 pounds per square inch, or 2,542 atmospheres, that is, to a depth of 5.8 miles below the surface.

Thus at these respective depths the additional tangential thrust required to induce a pronounced movement in the case of marble and granite, respectively, would be as shown in Table V.

TABLE V

| | At Depth of 4 2 Miles | | At Depth of 5 8 Miles | |
|---|---|---|---|---|
| | Pounds per Square Inch | Atmospheres | Pounds per Square Inch | Atmospheres |
| Marble........... | 66,400 | 4,517 | 74,500 | 5,068 |
| Granite.......... | 138,500 | 9,422 | 159,600 | 10,857 |

CONCLUSIONS

1. All the rocks employed in the present investigation can be deformed under differential pressure at ordinary temperatures.

2. In order to effect an equal deformation, it is necessary to employ differential pressures having different values in the case of the several rocks.

3. The ease with which these rocks are deformed has as one of its functions the hardness of the rock (or of the minerals composing it).

4. In the case of the softer rocks—alabaster, steatite, marble, etc.—the deformation is produced by movements due to a slipping within the constituent crystals of the rock on their gliding planes, often accompanied by twinning, the movement in this case being similar to that seen in metals when they are deformed. In the harder rocks the deformation is accompanied by granulation, the texture developed being similar to that found in mylonite.

5. Each of the softer rocks at least has a well-defined modulus of plasticity.

6. The "work done" when a rock is deformed by a tangential thrust, within the earth's crust, increases rapidly with the weight of the superincumbent strata, i.e., with its depth below the surface.

7. The relative ease with which the several rocks will flow under differential pressure is shown in Tables III and IV, which give mathematical expression of the "work done" in deforming standard columns of each rock.

8. A uniform thrust exerted on a prism of the earth's crust may deform and fold the upper portion of the mass, while it will be quite insufficient to produce any movement in the lower part of the same mass.

9. The thrust required to develop deformation, taking no cognizance of the influence of heat or the time effect which might result if the pressure were applied with extreme slowness, in the case of marble, and of granite, is shown by the values given in Table V.

10. To revert to the question propounded by Dr. Gilbert, in order to develop flow in any rock within the earth's crust the rock must be submitted to a differential stress which is greater than that which is required merely to break down its texture and very much greater than that which is sufficient to crush it to pieces under the ordinary conditions which obtain at the surface of the earth.

# ON THE MATHEMATICAL THEORY OF THE INTERNAL FRICTION AND LIMITING STRENGTH OF ROCKS UNDER CONDITIONS OF STRESS EXISTING IN THE INTERIOR OF THE EARTH

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## INTRODUCTION

That solid bodies could be permanently deformed and made to flow without rupture under sufficiently grea stress has long been known. The extensive experiments of Tresca on the flow of metals[1] (1864–72) directed the attention of several mathematicians of the time to the subject. Tresca announced as a result of his experiments the simple law that a stressed solid would commence to flow as soon as the maximum shearing stress exceeded a limiting value $K$ characteristic of the solid. This hypothesis was incorporated into the elastic solid theory by Saint-Venant[2] and others. The hope was expressed by these writers that by effecting the solution of simple problems in "plasticodynamics," corresponding to the experimental arrangements employed, it might be possible, not only to verify the theoretical results, but also to determine a specific constant $K$ characteristic of the various metals and related in an intimate manner to other physical constants. It was found possible, however, to solve only a very limited number of extremely simple problems: (1) circular cylinder under uniform pressure over the plane ends or subject to uniform lateral pressure; (2) cylindrical shell constrained to remain of constant length and subject to uniform internal and external pressure; (3) circular cylinder twisted beyond the elastic limit; (4) bar of rectangular section bent by a suitable distribution of forces to take the form of a circular arc.

[1] H. Tresca, *Par. Mém. Sav. Étr.*, XX (1872), 75 ff. and 281 ff. A summary of Tresca's experiments is given by L. S. Ware, *Journal of the Franklin Institute*, LXXIII (1877), 418 f.

[2] Saint-Venant, *Comptes Rendus*, LXVII (1868), 131 ff., 203 ff., 278 ff.; LXVIII (1869), 221 ff., 290 ff.

None of these simple problems corresponded, however, to any detailed observations available. The position with regard to the final mathematical interpretation of Tresca's observations was summed up by Saint-Venant in a communication to the French Academy[1] It was stated that, before much progress could be made in formulating a mathematical theory of plastic flow, it would be necessary to plan experiments more easily capable of mathematical specifications; in particular he recommended that means be taken to trace out in the *interior* of the solid the extent of the plastic deformations. The difficulty of doing this without at the same time interfering with the continuity of the solid under test has apparently not been overcome up to the present, so that data on plastic deformation available for mathematical treatment are still very meager.

It is interesting to notice, however, that we have available at the present day a method of exploring the internal structure of solids which seems to fulfil the need expressed by Saint-Venant. By the use of extremely powerful X-rays it has been found possible to detect internal cavities in steel castings not visible on the surface. The subject has recently been extensively studied by Davey,[2] who states that it is possible to detect an air-inclusion 0.021 inch thick in $1\frac{1}{4}$ inches of steel and an air-inclusion 0.007 inch thick in $\frac{5}{8}$ inch of steel. More recently Pilon,[3] making use of the Coolidge tube, has successfully penetrated 5.5 centimeters of steel. This method appears to the writer to offer the means of studying in successive stages the plastic deformation of specimens of various materials under conditions of intense stress. In these circumstances it would be necessary only to drill extremely fine holes in the specimen in various directions and to study the deformation of these as the solid is made to flow.

Tresca's hypothesis that flow in a solid commences and continues as long as the shearing stress exceeds a definite limit has been found

[1] Saint-Venant, "De la suite qu'il serait nécessaire de donner aux recherches expérimentales de plasticodynamique," *Comptes Rendus*, LXXXI (juillet, 1875), 115–21.

[2] W. P. Davey, *Trans. Am. Electrochem. Soc.*, XXVIII (1915), 407–18.

[3] H. Pilon, *Rev. de Mét.*, XII (Nov., 1915), 1017–23.

by later tests to be only approximately true. It is found that to produce continuous flow in a plastic solid it is necessary continuously to increase the distorting stress. A simple illustration of this fact is to be noticed in the manner in which a short circular cylinder crushed in a testing machine ultimately breaks down. According to Tresca's theory the surfaces of shear should be cones of semi-vertical angle of 45°, while experiments indicate that the angle is more often in the neighborhood of 55° for a material like cast iron.[1] These results have led to a modification of Tresca's hypothesis as already mentioned. The effect of this so-called "resistance to flow" does not appear to have been studied with a view to formulating the laws according to which solids may be made to flow continuously.

In the field of experimental ballistics the use of the permanent deformation of short copper cylinders to measure the enormously high pressures involved in testing explosives by means of the so-called "crusher-gauge" invented by Noble about 1875,[2] has led to the detailed study of the relation of applied stress and deformation produced in these special circumstances.[3] The results of these observations have recently been studied in detail by Brillouin.[4] The behavior of copper shows the existence of internal friction analagous to that observed by Adams and Bancroft in the case of various rock specimens.

In the experiments carried out by the latter investigators the use of nickel-steel jackets of standard thickness to incase the rock specimens subjected to flow is analagous to the use of short cylinders of annealed copper in the crusher-gauges just referred to. In order to obtain the lateral pressure on the specimen corresponding to a given deformation of the nickel-steel jacket, a calibration-curve is obtained by filling the cylinders with tallow. The hydrostatic pressures required to give a series of deformations give the required

[1] A. Morley, *Strength of Materials* (Longmans, Green, & Co., 1908), p. 55.

[2] See *Encyclopaedia Britannica*, 11th ed., article on "Ballistics," for a brief description of the crusher-gauge.

[3] Vieille, *Mémorial des poudres et salpêtres* (Gauthier-Villars, Paris), V, 12–61.

[4] M. Brillouin, "Les grandes déformations du cuivre par écrasement et par traction," *Ann. de Chimie et de Physique*, 9e série, II (1914), 489–96.

calibration-curve, just as the copper cylinders of the crusher-gauge are calibrated under known end pressures in a testing machine.

### MATHEMATICAL DISCUSSION OF THE OBSERVATIONS OF ADAMS AND BANCROFT DURING THE ELASTIC STAGE

Although the experiments which form the subject of the present discussion were all carried out when both rock and nickel-steel had been deformed beyond the elastic limit, it is not without interest, especially in view of further experiments on the subject, to follow out the distribution of stresses in the rock specimen and in the nickel-steel throughout the elastic stage. The necessary theory from which the formulas given below are derived has been given by the writer in a previous paper.[1] As in that discussion, it is sufficient for the present purpose to consider the ideal problem of plane stress, that is, one in which the end pressures and lateral pressures are such that the displacements at the outer surfaces, both of the rock specimen and of the nickel-steel jackets, are everywhere symmetrical with respect to the axis and everywhere constant for a given load. In reality the nickel-steel jacket shows a bulge over the center of the specimen. As long as this is not too great the analysis will give an approximate representation of the state of stress in the central portion of the specimen and nickel-steel jacket at which the measurements of displacement were taken by means of a sensitive extensometer. The justification for this mode of treatment has already been noticed in the writer's paper previously referred to in its application to a similar problem.

We denote by $\widehat{rr}$ the stress component along the radius $r$; by $\widehat{\theta\theta}$, the component at right angles to $r$; and by $\widehat{zz}$, that along the axis. According to Lamé's notation, $\mu$ is the modulus of rigidity of the rock specimen and $\lambda$ one of the moduli of elasticity such that $\kappa=(\lambda+\frac{2}{3}\mu)$ is the modulus of compression. Poisson's ratio is denoted by $\sigma=\frac{1}{2}\lambda/(\lambda+\mu)$. We denote by accented symbols the corresponding elastic constants for nickel-steel. In the problem under discussion we denote by $b$ the radius of the rock specimen and

[1] L. V. King, "On the Limiting Strength of Rocks under Conditions of Stress Existing in the Earth's Interior," *Journal of Geology*, XX (February–March, 1912), 121–26.

the interior radius of the nickel-steel jacket, and by $c$ the exterior radius of the nickel-steel jacket. If $P$ is the pressure per unit area applied to the end of the test specimen, we have $\widehat{zz} = -P$. The principal shearing stresses are one-half the algebraic difference of the principal stresses and are at once obtained by writing $a=0$ in the equations (13) of the writer's paper mentioned above. We then obtain

$$\left.\begin{aligned} &\text{(i)} \quad \tfrac{1}{2}\,|\,\widehat{rr}-\widehat{zz}\,| = \tfrac{1}{2}\,\frac{\sigma}{1-\sigma}\,P\left\{\frac{1-2\sigma}{\sigma}+\frac{\beta}{1+\beta}\right\} \\ &\text{(ii)} \quad \tfrac{1}{2}\,|\,\widehat{rr}-\widehat{\theta\theta}\,| = 0 \\ &\text{(iii)} \quad \tfrac{1}{2}\,|\,\widehat{\theta\theta}-\widehat{zz}\,| = \tfrac{1}{2}\,\frac{\sigma}{1-\sigma}\,P\left\{\frac{1-2\sigma}{\sigma}+\frac{\beta}{1+\beta}\right\} \end{aligned}\right\} \qquad (1)$$

where

$$\beta = \frac{1+\sigma}{1-\sigma}\cdot\frac{\mu}{\mu'}\left\{1+\frac{1-\sigma'}{1+\sigma'}\,\frac{b^2}{c^2}\right\}\cdot\frac{1}{1-b^2/c^2} \qquad (2)$$

The radial displacement $U$ at the outer surface of the rock specimen is given by

$$\frac{U}{b} = \frac{P}{2\mu}\cdot\frac{\sigma}{1+\sigma}\,\frac{\beta}{1+\beta} \qquad (3)$$

Each of the principal shearing stresses (i), (ii), (iii), is associated with a family of surfaces along which the material will crack or flow. These are illustrated in Fig. 1, reproduced from the writer's paper already mentioned. It is important to notice in the present connection that the principal shearing stresses in the interior of the rock, as given by (i) and (iii), are independent of the radius $r$ and remain equal throughout the elastic régime. It thus follows from Tresca's theory that the rock, when stressed under these ideal conditions, will commence to break down or flow *simultaneously* throughout its entire volume. The surfaces of shear which will be associated with the elastic breakdown may either be the system of cones (i) of semivertical angle 45° or the system of helicoidal surfaces (iii) of 45° pitch giving rise to the well-known Luder's lines on the curved surface of the specimen. The particular surfaces of shear which will be observed in any particular test will depend on accidental circumstances, as either system is equally likely to occur.

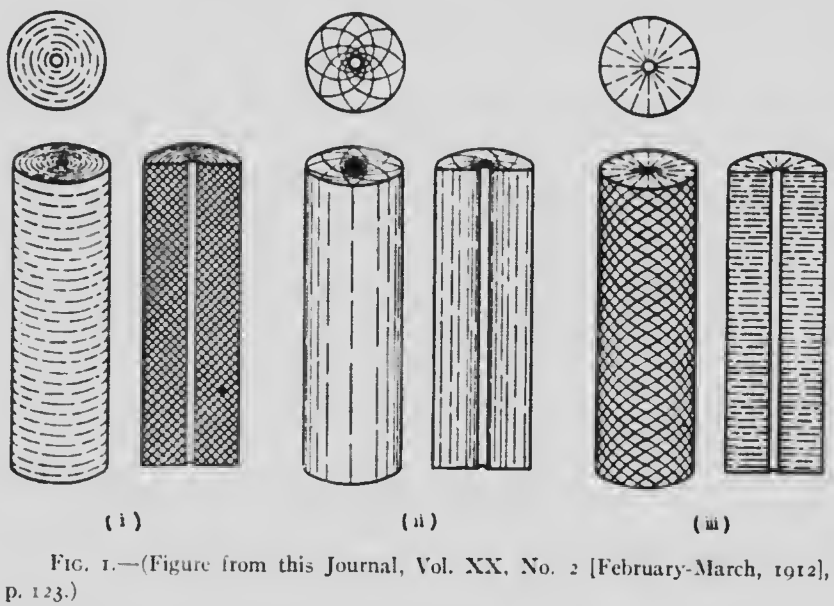
We easily derive expressions for the principal shearing stresses in the nickel-steel jacket. At points distant $r'$ from the axis these are

$$\left.\begin{aligned}
&\text{(i)}' \quad \tfrac{1}{2}\left|\widehat{rr}'-\widehat{zz}'\right| = \tfrac{1}{2}P\cdot\frac{\sigma}{1-\sigma}\cdot\frac{1}{1+\beta}\,\frac{c^2/r'^2-1}{c^2/b^2-1}\\
&\text{(ii)}' \quad \tfrac{1}{2}\left|\widehat{rr}'-\widehat{\theta\theta}'\right| = P\,\frac{\sigma}{1-\sigma}\,\frac{1}{1+\beta}\,\frac{c^2/r'^2}{c^2/b^2-1}\\
&\text{(iii)}' \quad \tfrac{1}{2}\left|\widehat{\theta\theta}'-\widehat{zz}'\right| = \tfrac{1}{2}P\,\frac{\sigma}{1-\sigma}\,\frac{1}{1+\beta}\,\frac{c^2/r'^2+1}{c^2/b^2-1}
\end{aligned}\right\} \quad (4)$$

The radial displacement $U'$ at the outer surface of the nickel-steel jacket is given by

$$\frac{U'}{c} = \frac{P}{\mu'}\cdot\frac{b^2}{c^2}\cdot\frac{1}{1-b^2/c^2}\cdot\frac{1}{1+\sigma'} \qquad (5)$$

By writing $\mu=0$ and therefore $\beta=0$, $\sigma=\frac{1}{2}$, the foregoing give the familiar results for stresses in a cylinder subject to internal hydro-



(i) (ii) (iii)

FIG. 1.—(Figure from this Journal, Vol. XX, No. 2 [February-March, 1912], p. 123.)

static pressure. The three principal shearing stresses given above all take their maximum value (independent of sign) at the interior surface $r=b$, and of these maxima (ii)′ is the greatest. The maximum shearing stress is therefore

$$\tfrac{1}{2}\,|\,\widehat{rr}'-\widehat{\theta\theta}'\,|_{max.}=P\frac{\sigma}{1-\sigma}\cdot\frac{1}{1+\beta}\cdot\frac{1}{1-b^2/c^2} \qquad (6)$$

It follows from this discussion that elastic breakdown of the nickel-steel jacket commences at the interior surface and, as deformation continues, extends gradually to the outer surface. The surfaces of shear in this case are the system of cylindrical surfaces whose traces on a plane perpendicular to the axis of the cylinder are equiangular spirals intersecting orthogonally and cutting all radii at angles of 45°. An examination of the nickel-steel jackets shows, in fact, that the surfaces of shear approximated roughly to this system. The polished outer surface of stressed specimens showed indications of fine longitudinal ribs, while in such as were actually ruptured it was noticed that the surface of rupture conformed to that predicted from theory. As the rupture occurred when the nickel-steel was stressed very much beyond the elastic limit, the actual surfaces of shear are determined by very complex conditions involving the effect of internal friction, with which we shall deal in a later section.

*Numerical results.*—A rough verification of the preceding results may be made by calculating the relation between the load and the increase of diameter of the nickel-steel jacket according to equation (5). For nickel-steel we take $\sigma'=0.327$ and $\mu'=10.8\times10^6$ pounds per sq. in., values employed in the writer's paper just referred to. In one set of experiments (referred to as 0.25-centimeter wall) $b=1.00$ cm., $c=1.25$ cm., giving from (2)

$$\beta=3.68\times\frac{1+\sigma}{1-\sigma}\cdot\frac{\mu}{\mu'}.$$

When the jacket is filled with tallow we may take $\sigma=\frac{1}{2}$, $\mu=0$, $\beta=0$, so that equation (5) gives

$$U'/c=1.34\times(P/\mu'),$$

or in terms of the total load, $W=\pi b^2P$, we obtain

$$2U'\ (\text{inches})=2.52\times10^{-7}\times W\ (\text{pounds}) \qquad (7)$$

When a specimen of Carrara marble is inserted we have[1] $\sigma = 0.2744$, $\mu = 3.154 \times 10^6$ pounds per sq. in., whence $\beta = 1.889$ and

$$U'/c = 0.176 \times (P/\mu'), \text{ or } 2U' \text{ (inches)} = 3.31 \times 10^{-8} \times W \text{ (pounds)} \quad (8)$$

In another set of experiments (referred to as 0.33-centimeter wall), $b = 1.00$ cm., $c = 1.33$ cm., giving

$$\beta = 2.96 \times \frac{1+\sigma}{1-\sigma} \frac{\mu}{\mu'}.$$

In the case of tallow filling we find as before,

$$U'/c = 0.980 \times (P/\mu') \text{ or } 2U' \text{ (inches)} = 1.96 \times 10^{-7} \times W \text{ (pounds)} \quad (9)$$

and in the case of the Carrara marble specimen

$$U'/c = 0.147 \times (P/\mu') \text{ or } 2U' \text{ (inches)} = 2.94 \times 10^{-8} \times W \text{ (pounds)} \quad (10)$$

In Fig. 2 are compared the observed and theoretical stress-strain diagrams corresponding to the cases calculated out in equations (8) to (10). In the case of tallow filling, the initial slope of the observed curves agrees approximately with the calculated slope. In the case of the marble filling, the agreement is within the limits of error involved in measuring these extremely small strains.

## MATHEMATICAL DISCUSSION OF THE OBSERVATIONS OF ADAMS AND BANCROFT DURING THE PLASTIC STAGE

1. *Navier's theory of internal friction.*—Let $\widehat{xx}$, $\widehat{yy}$, and $\widehat{zz}$ be the principal stresses in the solid at a point $P$ measured toward the origin (Fig. 3). Let $S$ be the shearing stress in a plane whose direction cosines with respect to the direction of the three principal stresses are $(l, m, n)$. Let $N$ be the stress normal to this plane. We then have

$$\left.\begin{aligned} S^2 + N^2 &= l^2\widehat{xx}^2 + m^2\widehat{yy}^2 + n^2\widehat{zz}^2 \\ \text{and} \qquad N &= l^2 xx + m^2\widehat{yy} + n^2\widehat{zz} \end{aligned}\right\} \quad (11)$$

Generalizing somewhat on Navier's hypothesis of elastic breakdown, we may state that the material will not break down as long as

$$S < K \quad (12)$$

[1] Adams and Coker, "Elastic Constants of Rocks," *Publication No. 46 of the Carnegie Institution of Washington*, 1906, p. 69.

where $K$ is a function, not only of the stress $N$ normal to the plan at which slide occurs, but also of the previous history of the

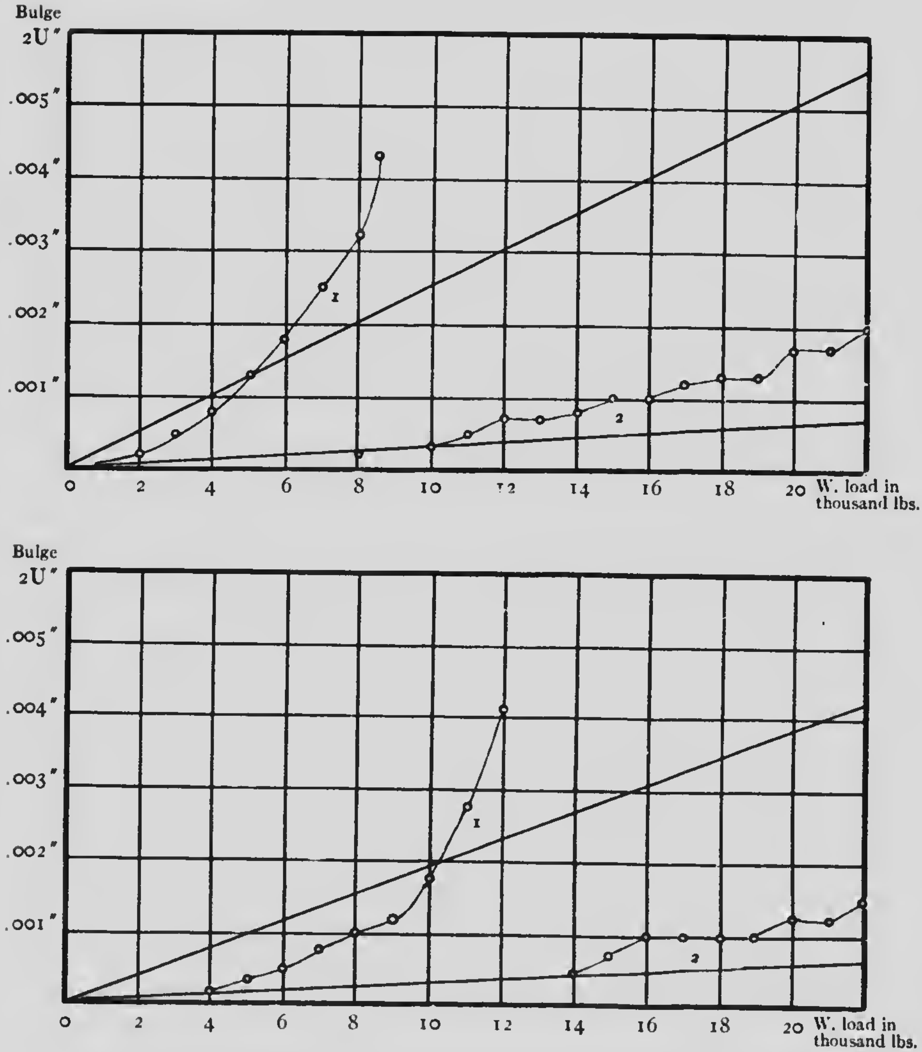



FIG. 2.—Theoretical and observed stress-strain diagrams. Curves 1, tallow filling. Curves 2, Carrara marble filling.

specimen. According to Tresca's hypothesis, $K$ was regarded as a constant, depending only on the nature of the specimen. An extension of this hypothesis due to Navier (the so-called internal-friction theory) replaces (12) by the condition

$$S<K+\mu N,$$

$\mu$ being a new constant somewhat analogous to the coefficient of friction of mechanics. In order to discover the relation between the principal stresses at the elastic limit, it is necessary to find the direction $(l, m, n)$ which makes $(S-\mu N)$ a maximum and equate the result to $K$. Suppose the principal stresses to be all of the same sign, two of them equal, $\widehat{yy}=\widehat{xx}$, and $\widehat{zz}>\widehat{xx}$ (corresponding to the state of affairs in the cylindrical rock specimens under test). We then have, writing $l=\sin\theta\cos\phi$, $m=\sin\theta\sin\phi$, $n=\cos\theta$,

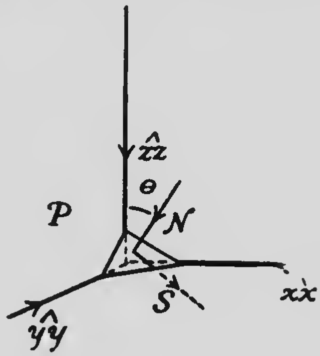



FIG. 3

$$\left.\begin{aligned} S^2+N^2=\widehat{xx}^2\sin^2\theta+\widehat{zz}^2\cos^2\theta, \quad N=xx\sin^2\theta+\widehat{zz}\cos^2\theta \\ S=(\widehat{zz}-\widehat{xx})\sin\theta\cos\theta \end{aligned}\right\} \tag{13}$$

$$S-\mu N=(\widehat{zz}-\widehat{xx})\sin\theta\cos\theta-\mu(\widehat{xx}\sin\theta+\widehat{zz}\cos^2\theta) \tag{14}$$

This expression reaches a maximum when

$$\cot 2\theta=-\mu, \tag{15}$$

i[illegible] circumstances

$$(S-\mu N)_{max.}=\tfrac{1}{2}(\widehat{zz}\cot\theta-\widehat{xx}\tan\theta), \tag{16}$$

and the relation between the principal stresses at breakdown is given by

$$\widehat{zz}=2K\tan\theta+\widehat{xx}\tan^2\theta \tag{17}$$

where $\theta$ is given in terms of $\mu$ (the coefficient of friction) by (15). This result indicates that the material in question will break down along a family of cones of semivertical angle $\alpha=\frac{1}{2}\pi-\theta$.

2. *Discussion of observations.*—In the experiments of Adams and Bancroft the cylindrical rock specimens were subjected to end loads transmitted by the steel pistons. As a result of the intense pressure

developed, the rock cylinders were caused to bulge out laterally over the central portion, where the thickness of the nickel-steel jacket was reduced to 0.25 centimeter and 0.33 centimeter, respectively, in the two sets of experiments. The rock was thus subjected to a continuous succession of breakdowns, so that it was possible from these observations to determine the relation between the end and lateral pressures required to keep the rock in movement.

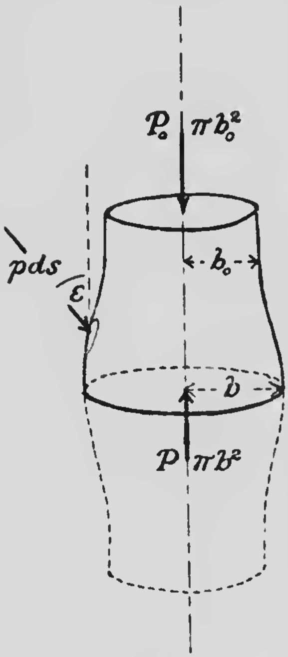



FIG. 4

Considering the central portion of the rock cylinder throughout which the flow takes place, we may reasonably assume, when the bulge is small, that the average pressure-intensity $P_0$ along the direction of the axis is given by

$$P_0 = W_0/(\pi b_0^2),$$

$W_0$ being the load on the steel piston and $b_0$ its radius. As the bulge becomes sensible, it is necessary to make a correction to allow for the increasing area over which the pressure is distributed. Referring to Fig. 4, we denote by $P$ the average pressure-intensity across a plane at right angles to the axis at the position of maximum bulge where the radius of the cross section is $b$. We denote by $p$ the resultant traction per unit area exerted by the nickel-steel jacket on the rock specimen in a direction making an angle $\epsilon$ with the axis. Then, considering the equilibrium of one-half of the rock specimen, we may write

$$\pi P_0 b_0^2 + \int p \cos \epsilon \, dS = \pi b^2 P, \tag{18}$$

the integral representing the total component of the tractions between the rock specimen and the nickel-steel jacket in a direction parallel to the axis of the cylinder. When an exactly similar jacket is filled with tallow and deformed by the application of a load on the steel pistons in the same way, we may consider the pressure in the interior to be hydrostatic. If $p_0$ be the hydrostatic pressure required to bulge the nickel-steel jacket to the same radius $b$, we have instead of (18) the equation

$$\pi p_0 b_0^2 + \int p_0 \cos \epsilon_0 dS = \pi b^2 p_0, \tag{19}$$

where $\epsilon_0$ now denotes the direction which the normal to the deformed surface makes with the axis of the cylinder. It was carefully ascertained in the experiments of Adams and Bancroft that the *shape* of the bulged nickel-steel jacket was the same when occupied by the softer rocks and such an easily flowing metal as lead, in which conditions of pressure approach very nearly to hydrostatic conditions under the very intense loads employed. As the deformation of the nickel-steel jacket is due to the distribution of surface tractions $p$, it is reasonable to assume that they are distributed in approximately the same way. This is equivalent to asserting that the tangential component of the surface traction between rock and nickel-steel is negligible compared to the normal component, a statement which seems to be reasonable in view of the fact that both rock and nickel-steel are highly polished over the surface of contact. We may thus write $\int p \cos \epsilon \, dS = \int p_0 \cos \epsilon_0 \, dS$ in (18) and (19) and arrive at the relation

$$P = p_0(1 - b_0^2/b^2) + P_0 b_0^2/b^2, \tag{20}$$

giving the average pressure-intensity at the center of the specimen to be identified with $\widehat{zz}$ of equation (17). The corresponding lateral pressure is given by $p_0$, which is identified with $\widehat{xx}$ of (17).

We are now in a position to test the theory of internal friction expressed by (17) from the observations of Adams and Bancroft. It is only necessary to plot against each other the end pressures $\widehat{zz}$ and the lateral pressures $\widehat{xx}$ as determined above. Such specimens as give straight lines may be said to possess a definite *modulus of plasticity*, $K$, and *coefficient of internal friction*, $\mu$. Curves obtained in this way are shown in the Appendix, where they are described in detail for the various specimens tested. The results show that for some kinds of rock the curves approximate closely to straight lines between certain limits of pressure. In the interpretation of these curves it must be kept in mind that the material is not broken down from an initially unstrained state[1] at each stage of the process. The constants of plasticity and internal friction, as determined by the present investigation, refer to rock which is being made to flow continuously. This state of affairs, however, approaches more nearly to that occurring in nature during slow geological deformations than to conditions existing when the rock is broken down from an initially unstrained state.

Under ideal conditions the curves for the observations taken with the nickel-steel jackets of the two wall thicknesses should be identical. Actually, however, they differ to some extent, indicating that the effect of stresses set up by the deformation of the nickel-steel has not been entirely eliminated. The two sets of observations are, however, sufficiently close to give approximate estimates of the relation between the principal stresses which must exist before the rock can be made to flow under conditions existing in the earth's crust. It will be noticed from the curves of Plate I that for the harder rocks, such as diabase and granite, the curves along which breakdown takes place show the existence of a very large coefficient of internal friction. Since the hydrostatic pressure is given by $\frac{1}{3}(2\widehat{xx}+\widehat{zz})$, this is equivalent to the statement that the *stiffness* or limiting shearing stress required to break down the rock increases with the hydrostatic pressure to which the rock is submitted. In other words, we come to the important conclusion that *the stress-difference required to break down rock material under conditions of pressure existing in the earth's crust increases with the depth.* In the application of this result to geophysical problems, the foregoing conclusion may have to be somewhat modified to take into account the rise of temperature with depth. It is highly desirable that further experiments be carried out with a view to ascertaining the influence of this factor.

## NOTE ON APPLICATIONS TO GEOPHYSICAL PROBLEMS

Up to the present the only quantitative data available for use in geodynamical problems have been obtained by crushing cubes of various rocks in a testing machine according to the ordinary rules

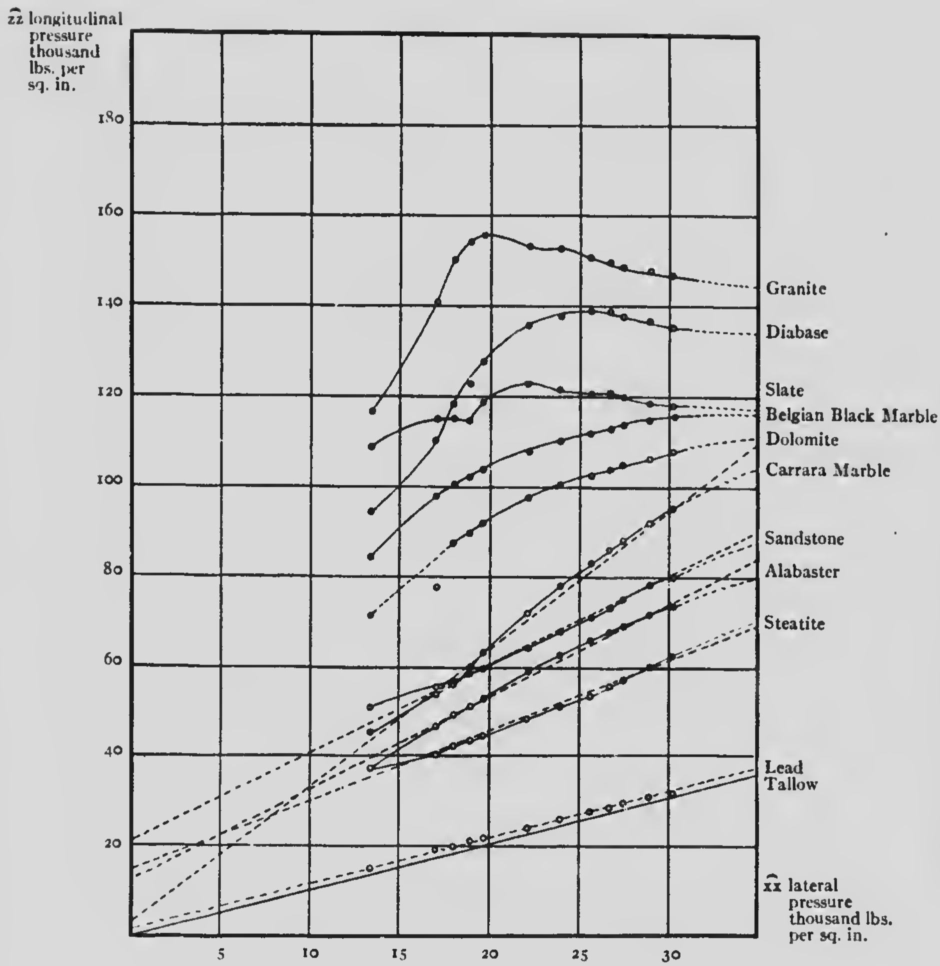
[1] Compare Karman's observations on marble and sandstone, *Zeit. des Vereins deutscher Ingenieure*, October 21, 1911.

of engineering practice. The unsatisfactory nature of such data as applied to conditions of stress deep down in the earth's crust has already been pointed out by the writer.[1] The results now available from the observations of Adams and Bancroft supply much needed data for the purposes of geophysics. Quoting from a classical paper by Sir George Darwin,[2] "With regard to the earth we require to know what is the limiting stress-difference under which a material takes permanent set or begins to flow rather than the stress-difference under which it breaks; for if the materials of the earth were to begin to flow, the continents would sink down, and the sea bottoms rise up." In the paper quoted Darwin estimates roughly the stress-difference in the interior of the earth due to a distribution of continental masses corresponding roughly to the actual distribution. For instance, it is estimated that the stress-difference under the continents of Africa and America is at a maximum at more than 1,100 miles from the earth's surface and amounts to about 4 tons per square inch. Darwin's conclusion that "marble would break under this stress, but that *strong* granite would stand" must be modified considerably in the light of the results of Adams and Bancroft, as the limiting strength of the rock material under the enormous pressure at the depth referred to would probably be increased many times. For the purposes of such calculations the curves of Plate I may be employed as they stand. If, for instance, it is desired to investigate the stability of mountain ranges or of continental elevations, the principal stresses at great depths must be derived from the theory of elasticity, making use of elastic constants derived from the interpretation of seismological records. If the principal stresses at any point be plotted as $\widehat{zz}$ and $\widehat{xx}$ on such a diagram as that of Plate I, a particular rock material will flow if the point falls between the axis $\widehat{zz}$ and the curve characteristic of the particular rock formation under consideration. The material will be on the point of flowing if the point falls on the curve itself, while the rock will stand the stress if the point falls between the

[1] L. V. King, *op. cit.*, p. 120.

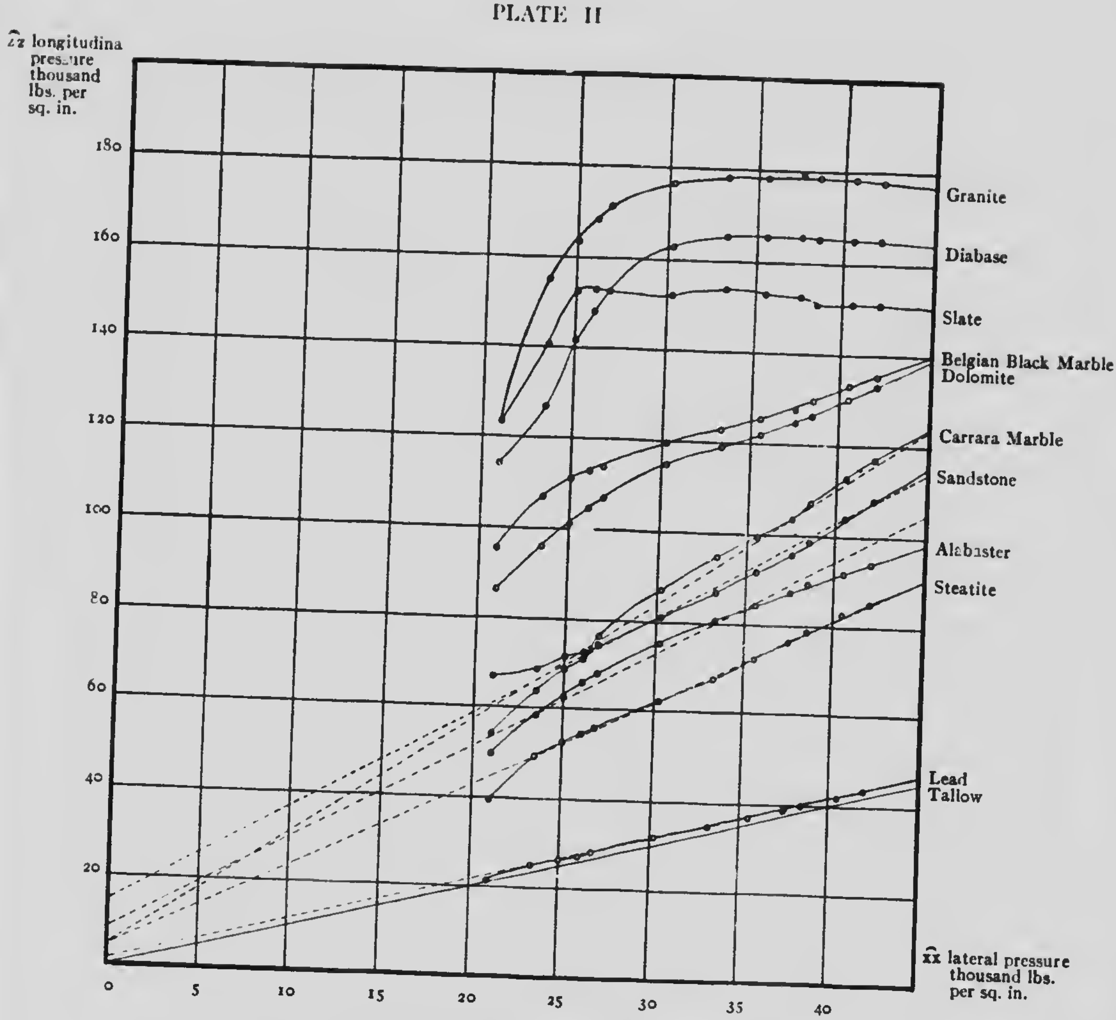
[2] Sir G. Darwin, "On the Stresses Caused in the Interior of the Earth by the Weight of Continents and Mountains," *Phil. Trans.*, CLXXIII (1882), 187–230; *Scientific Papers*, II (1908), 495.

PLATE I




Relative Plasticities of Various Rock Specimens
Nickel-steel jackets of 0.25 cm. wall

PLATE II




RELATIVE PLASTICITIES OF VARIOUS ROCK SPECIMENS
Nickel-steel jackets of 0.33 cm. wall

curve and the axis $\widehat{xx}$. Thus for complete stability the entire series of points representing stress-differences beneath a continental elevation must fall in this latter region. It is thus evident that the existing theories of isostasy should, in considering the equilibrium of stresses called into existence by continental elevations and mountain ranges, take account of a "*compensation* of *plasticity*"—i.e., of the increased stiffness or resistance to deformation—of the underlying rock when submitted to greater hydrostatic pressure. With the reservation already made as to the possible influence of temperature, we have a considerable basis of evidence in favor of the conclusion that at any time in the past history of the earth continental elevations might have attained much greater altitudes above sea-level than any at present existing, without giving rise to stress-differences in the earth's interior sufficiently great to have caused rupture or breakdown, owing to the much increased "resistance to flow" set up in the rock material by the great pressure of the overlying crust. We should conclude also that, in the event of flow occurring, the region of flow would be confined to a region of the earth's crust comparatively near the earth's surface. The increasing limiting stress, with pressure characteristic of rock material made to flow as in Adams' and Bancroft's experiments, leads one to the conclusion that great movements of the earth's crust have for the most part always proceeded by extremely slow and continuous adjustments to pressure conditions, and not, as supposed by some geologists,[1] by a series of cataclysmal collapses of the type which would occur if the material of the earth's crust possessed in all circumstances a unique and definite limiting strength analogous to that obtained by crushing a specimen, unsupported laterally, in a testing machine. The further consideration of these problems must, however, be left over for further discussion. Enough has been said to make it evident that the results of Adams and Bancroft have provided much needed data in the light of which many of the existing theories of geodynamics may require considerable modification.

[1] G. A. J. Cole, Presidential address delivered before the geological section of the British Association, Manchester meeting, 1915, *B. A. Report*, pp. 403–20.

## APPENDIX

In order to study the experimental data on the flow of rocks in the light of a theory of internal friction, the data reproduced in Tables I and II were obtained from the original large-scale curves obtained by Adams and Bancroft connecting the end load on the steel pistons with the bulge of the nickel-steel jacket. Each of the curves represented the mean of two, three, or more complete sets of observations. The first row of numbers for each specimen is the total load $W_0$ (in thousand pounds) on the steel pistons required to bulge the nickel-steel jacket by the amounts entered under the various columns. The second row gives the pressure-intensity $P_0 = W_0/\pi b_0^2$ in thousand pounds per square inch exerted on the end of the specimen of radius $b_0$. The third line gives the average pressure-intensity $P = \widehat{zz}$ in thousand pounds per square inch at the central portion of the specimen in the direction of the axis, correcting for the effect of the bulge from formula (20). It will be noticed that the average longitudinal pressure at the center is somewhat less than that over the ends by amounts which increase considerably with the harder rocks. The final results given in Tables I and II are shown graphically in Plates I and II, respectively. Against the lateral pressures (given by the experiment on tallow) are plotted the longitudinal pressures required to bulge the nickel-steel jacket to the same extent. For such of the rocks as give curves approximating to straight lines we may say that a definite *modulus of plasticity* and *coefficient of internal friction* exist. Rough estimates of these constants as determined for the soft rocks from large-scale curves are given in Table III, in which the first entry corresponds to the 0.25-centimeter wall nickel-steel jacket and the second to the 0.33-centimeter wall. It will be noticed that the two sets of results are in poor agreement for $K$, and are only in rough agreement for $\mu$, the difficulty arising from attempting to fit a straight line to a series of points which are only approximately colinear.

In the case of the harder rocks no definite coefficient of friction can be said to exist. In the case of dolomite and Belgian black marble it is noticed that the coefficient of friction tends to diminish with increasing longitudinal and lateral stress. Slate gives a very irregular curve due to the development of cracks while the material is stressed. The sudden bend in the curves for diabase and granite is attributed to the a[illegible] breakdown of the rock material. From the curves of Plates I and II it will be noticed that this occurs in the neighborhood of $\widehat{zz} = 150{,}000$, $\widehat{xx} = 25{,}000$, corresponding to a stress-difference of 125,000 pounds per square inch. In a general way this result confirms the conclusion already arrived at from a discussion of the experiments of Adams on the pressure required to close up small cylindrical cavities in specimens of Westerly granite. In the writer's paper already mentioned (p. 641, n. 1) it was pointed out that the stress-difference required to break down the rock material in the neighborhood of small cavities amounted to as much as

TABLE I

| 0.25-Centimeter Wall | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Bulge $2b_0 = 0.7874''$ | $2b - 2b_0$ | 0.002″ | 0.004″ | 0.006″ | 0.008″ | 0.010″ | 0.020″ | 0.030″ | 0.040″ | 0.050″ | 0.060″ | 0.080″ | 0.100″ |
| | $2b$ | .7894″ | .7914″ | .7934″ | .7954″ | .7974″ | .8074″ | .8174″ | .8274″ | .8374″ | .8474″ | .8674″ | .8874″ |
| | $1 - b_0^2/b^2$ | 0.004 | 0.010 | 0.014 | 0.020 | 0.023 | 0.048 | 0.071 | 0.095 | 0.115 | 0.136 | 0.176 | 0.213 |
| Tallow | $W_0$ | 6.5 | 8.2 | 8.8 | 9.2 | 9.6 | 10.8 | 11.7 | 12.5 | 13.0 | 13.4 | 14.1 | 14.7 |
| | $P_0 = \bar{x}x$ | 13.4 | 16.9 | 18.1 | 19.0 | 19.7 | 22.2 | 24.0 | 25.7 | 26.7 | 27.5 | 29.0 | 30.3 |
| | $\bar{z}z = x\bar{x}$ | 13.4 | [illegible] | 18.1 | 19.[illegible] | 19.7 | 22.2 | 24.0 | 25.7 | 26.7 | 27.5 | 29.0 | 30.3 |
| Steatite | $W_0$ | 18.0 | 19.7 | 20.5 | 21.3 | 21.8 | 24.0 | 25.6 | 27.1 | 28.5 | 29.8 | 32.2 | 34.3 |
| | $P_0$ | 36.9 | 40.4 | 42.1 | 43.7 | 44.7 | 49.3 | 52.6 | 55.6 | 58.4 | 61.1 | 66.1 | 70.4 |
| | $\bar{z}z$ | 36.8 | 40.2 | 41.8 | 43.2 | 44.1 | 48.0 | 50.5 | 52.8 | 54.8 | 56.5 | 59.6 | 61.9 |
| Alabaster | $W_0$ | 18.0 | 22.6 | 24.0 | 25.0 | 26.0 | 29.5 | 31.9 | 33.8 | 35.3 | 36.6 | 39.0 | 41.0 |
| | $P_0$ | 37.0 | 46.4 | 49.3 | 51.3 | 53.4 | 60.6 | 65.4 | 69.3 | 72.4 | 75.1 | 80.1 | 84.1 |
| | $\bar{z}z$ | 36.9 | 46.1 | 48.9 | 50.7 | 52.6 | 58.7 | 62.4 | 65.2 | 67.2 | 68.6 | 71.1 | 72.[illegible] |
| Sandstone | $W_0$ | 24.5 | 27.0 | 27.7 | 28.7 | 29.5 | 32.2 | 34.5 | 36.5 | 38.4 | 40.0 | 43.0 | 45.3 |
| | $P_0$ | 50.3 | 55.4 | 56.9 | 58.8 | 60.5 | 66.0 | 70.7 | 74.8 | 78.7 | 82.1 | 88.2 | 92.9 |
| | $\bar{z}z$ | 50.2 | 55.0 | 56.4 | 58.0 | 5[illegible].5 | 64.0 | 67.3 | 70.[illegible] | [illegible] | 74.6 | 77.8 | 79.6 |
| [illegible]rble | $W_0$ | 22.0 | 26.2 | 27.5 | 30.0 | 31.0 | 36.2 | 40.0 | [illegible] | [illegible] | [illegible] | [illegible] | 54.7 |
| | $P_0$ | 45.1 | 53.8 | 56.4 | 61.6 | 63.6 | 74.2 | 82.0 | [illegible] | [illegible] | [illegible] | [illegible] | 112.[illegible] |
| | $\bar{z}z$ | 45.0 | 53.5 | 55.9 | 60.8 | 62.6 | 71.7 | 77.8 | 82.5 | [illegible] | [illegible] | [illegible] | 94.[illegible] |
| Dolomite | $W_0$ | 35.0 | 38.0 | 43.0 | 4[illegible].2 | 45.5 | 49.2 | 51.7 | 53.7 | [illegible] | [illegible] | 59.7 | 62.[illegible] |
| | $P_0$ | 71.8 | 78.0 | 88.2 | [illegible].8 | 93.3 | 101.0 | 106.0 | 110.[illegible] | [illegible] | 116.[illegible] | [illegible] | 123.2 |
| | $\bar{z}z$ | 71.6 | 77.4 | 87.3 | 89.3 | 91.6 | 97.2 | 100.1 | [illegible] | [illegible] | [illegible] | [illegible] | 107.4 |
| Belgian black marble | $W_0$ | 41.0 | 48.0 | 49.5 | 50.6 | 51.6 | 54.6 | 5[illegible].8 | 58.7 | [illegible] | 61.9 | 64.8 | 67.2 |
| | $P_0$ | 84.1 | 98.4 | 101.5 | 103.7 | 105.8 | 112.0 | 116.[illegible] | [illegible].4 | [illegible].9 | 127.0 | 132.9 | 137.9 |
| | $\bar{z}z$ | 84.9 | 97.6 | 100.3 | 101.9 | 103.7 | 107.6 | 100.[illegible] | 11[illegible].5 | 112.7 | 113.4 | 11 46 | 115.0 |
| Slate | $W_0$ | 53.0 | (56.[illegible]) | ([illegible]) | (56.9) | 58.8 | 62.[illegible] | 62.8 | 63.6 | 64.5 | 65.4 | 67.0 | 69.0 |
| | $P_0$ | (108.7) | (115.7) | (116.[illegible]) | (116[illegible] | 120.6 | 128.0 | [illegible].8 | 13[illegible] | 13[illegible].3 | 134.1 | 137.4 | 141.5 |
| | $\bar{z}z$ | (108.3) | (11[illegible] | (114.9) | (11[illegible] | 118.2 | 122.8 | 121.1 | 120.3 | 120.2 | 119.5 | 118.2 | 117.8 |
| [illegible] | $W_0$ | 46.0 | [illegible] | 58.5 | [illegible].7 | 63.5 | 68.9 | 71.5 | 73.3 | 74.5 | 75.5 | 77.7 | 79.7 |
| | $P_0$ | 94.3 | [illegible].8 | 120.0 | 124.5 | 130.2 | 141.2 | 146.6 | 150.3 | 152.8 | 154.8 | 159.3 | 163.4 |
| | $\bar{z}z$ | 94.0 | [illegible] | 118.6 | 122.3 | 127.5 | 135.4 | 137.7 | 13[illegible].5 | 138.3 | 137.4 | 136[illegible] | 135.0 |
| [illegible]nite | $W_0$ | 57.0 | 69.0 | 74.0 | 76.5 | 77.4 | 79.0 | 79.2 | 79.7 | 80.5 | 81.6 | 84.1 | 86.0 |
| | $P_0$ | 116.9 | 142.0 | 151.[illegible] | 156.0 | [illegible].7 | 159.[illegible] | 162.4 | 163.4 | 165.1 | 167.3 | 172.5 | 177.6 |
| | $\bar{z}z$ | 116.6 | 140.6 | 140.[illegible] | 154.[illegible] | [illegible] | 1[illegible].0 | 152.4 | 150.4 | 149.2 | 148.2 | 147.2 | 146.3 |
| [illegible] | $W_0$ | 7.0 | 9.1 | 9.6 | 10.1 | [illegible] | 11.[illegible] | 12.0 | 13.3 | 13.8 | 14.3 | 14.9 | 15.4 |
| | $P_0$ | 14.4 | 18.7 | 19.7 | 20.8 | [illegible] | 24.0 | 25.9 | 27.3 | 28.3 | 29.4 | 30.6 | 31.9 |
| | $\bar{z}z$ | 14.4 | 18.7 | 19.7 | 20.7 | 21.4 | 23.9 | 25.7 | 27.2 | 28.1 | 29.2 | 30.3 | 31.4 |

TABLE II

TABLE II

| [illegible]ENTI- [illegible] WALL | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $2b_0 = 0.7874''$ | | [illegible] | 0 [illegible] | 0 006″ .7934″ 0 014 | 0 008″ .7954″ 0 020 | 0 010″ .79[illegible] | 0 020″ .8074″ 0 048 | 0 030″ .8174″ 0 071 | 0 040″ .827 0 09[illegible] | 0 050″ .8[illegible]″ [illegible] 115 | 0 060″ .8474″ 0 136 | 0 080″ .8674″ 0 176 | 0 100″ .8874″ 0 213 |
| Tallow | $W_0$ | | 11 4 | 12 1 | [illegible] | [illegible] 0 | 14 [illegible] | 1[illegible] | 17 3 | 18 2 | 18 7 | 19 8 | 20 5 |
| | $P_0 = 1$ | | 23 4 | 24 9 | [illegible] | 26.7 | | | 35 5 | 37 [illegible] | 38 4 | 40 6 | 42 1 |
| | ϖ = 11 | 20 9 | 23 1 | 21 9 | 25 9 | 26 7 | | 33 3 | 35 5 | 37 [illegible] | 38 4 | 40 6 | 42 1 |
| Steatite | $W_0$ | [illegible] | | 25 5 | | 27 5 | 30 9 | | [illegible] 7 | [illegible] 0 | 41 0 | 44 2 | [illegible] |
| | $P_0$ | | | 52.3 | | 56 | 63.3 | | | 80 0 | 84 0 | 90 7 | |
| | ϖ | | [illegible] 3 | 51.9 | [illegible] | [illegible] | 61 7 | | 71 | [illegible] 1 | 77 7 | 81 8 | 84 [illegible] |
| Alabaster | | 24 0 | 28 5 | 30 75 | 32 3 | [illegible] | | | [illegible] | [illegible] | 47 0 | 49 7 | 52 2 |
| | | 49 2 | 58 1 | | 66 2 | 68 7 | | [illegible] | | [illegible] 7 | 96 4 | 101 9 | 107 0 |
| | | 49 1 | 58 0 | | 65 4 | 67 6 | | | [illegible] | 86 3 | 88 4 | 91 0 | [illegible] |
| Sandstone | | 32 5 | | | | 30 [illegible] | [illegible] 5 | 44 1 | 47 1 | 5[illegible] | 5[illegible] | [illegible] | 61 |
| | | 66 6 | 68 7 | | | 75 2 | [illegible] 0 | 90 4 | 96 6 | [illegible] | [illegible] | [illegible] | [illegible] |
| | ϖ | 66 4 | 68 2 | [illegible] | | | [illegible] 4 | 86 3 | 90 8 | [illegible] | [illegible] 0 | 105 7 | [illegible] 4 |
| Marble | $W_0$ | 26 0 | 31 0 | 33 7 | [illegible] | 37 | | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 66 7 |
| | $P_0$ | 53 3 | 63.6 | 69 1 | 71 8 | 76 | | | [illegible] 6 | 111 7 | 11[illegible] | [illegible] | 136 7 |
| | ϖ | 53 2 | 63.2 | 68 | 70 9 | 7[illegible] | [illegible] | | 98 9 | 103 1 | 10[illegible] 5 | 112 4 | 1[illegible] 5 |
| Dolomite | $W_0$ | 42 0 | 47 0 | [illegible] | 51 5 | 53 0 | 57 8 | 61 0 | | | [illegible] | 72 | [illegible] 5 |
| | $P_0$ | 86.1 | 96 4 | 101 [illegible] | 105.6 | 108 7 | 118 5 | 125 | | | 140 [illegible] | 139 [illegible] | 150 9 |
| | ϖ | 85 9 | 95 6 | 100 3 | 104 0 | 106 7 | 114 [illegible] | 1[illegible] | | | [illegible] | [illegible] 1 | 132 4 |
| Belgian black marble | $W_0$ | 46 5 | 52 5 | 54 5 | 55 7 | 56 5 | 6[illegible] | | | | 70 0 | 71 1 | [illegible] 8 |
| | $P_0$ | 95 3 | 107 6 | 111.8 | 114 2 | 115 9 | 123 [illegible] | | [illegible] | | 143 6 | 152 0 | 1[illegible] |
| | ϖ | 95 0 | 106 7 | 110 5 | 112 2 | 113 7 | 118 8 | | [illegible] | [illegible] 3 | 129 1 | 13[illegible] | 134 5 |
| Slate | $W_0$ | 60.0 | 69 0 | 75.0 | (75 3) | (75 5) | 76 7 | | | [illegible] | 81 8 | 84 7 | 87 4 |
| | $P_0$ | 123 1 | 141 7 | 154.0 | (155 0) | (155.4) | 157 9 | | | [illegible] | 168 4 | 174 4 | 179 6 |
| | ϖ | 122 6 | 140 4 | 152.1 | (152.4) | (152 1) | 151 7 | [illegible] | | [illegible] | [illegible] 0 5 | 150 7 | 150 3 |
| Diabase | $W_0$ | 75.0 | 62 0 | 69 0 | 73 0 | 75 5 | 82 2 | 85 0 | 80 7 | [illegible] | 89 | [illegible] 0 | [illegible] 2 |
| | $P_0$ | 114 0 | 127 4 | 141 8 | 150 0 | 155 1 | 168 9 | 175 0 | 178 2 | 181 [illegible] | 184 | 1[illegible] 1 | 197 7 |
| | ϖ | 113 6 | 126.3 | 140 1 | 147 5 | 152.0 | 162 2 | 16[illegible] | [illegible] 3 7 | 164 9 | 164 | 164 6 | 164 6 |
| Granite | $W_0$ | (60.0) | 76.0 | 80 2 | 83 2 | 85 1 | 8[illegible] 5 | | | 96 0 | [illegible] 8 | 101 2 | 104 3 |
| | $P_0$ | (123.1) | 156 2 | 165 2 | 171 0 | 174 8 | 183 9 | | 193 2 | 197 3 | 200 9 | 207 6 | 213 8 |
| | ϖ | (122.6) | 154 8 | 163 1 | 168 1 | 171 2 | 176 4 | [illegible] 9 | 178 2 | 178 9 | 178 6 | 178 1 | 177 2 |
| Lead | $W_0$ | 10 0 | 12 0 | 12 5 | 13 0 | 13 7 | 15 4 | 16 8 | 17 8 | 18 [illegible] | [illegible] | 20 [illegible] | 2[illegible] |
| | $P_0$ | 20.5 | 24 6 | 25 7 | 26 7 | 28 1 | 31 6 | 34 5 | 36 5 | [illegible] 4 | 39 0 | 41 | [illegible] |
| | ϖ | 20.5 | 24.5 | 25 6 | 26 7 | 28 0 | 31.5 | 34 4 | 36 4 | 38.3 | 39 4 | 41 0 | 42 7 |

160,000–200,000 pounds per square inch. The conclusion, there limited to small cavities, is extended by the present experiments of Adams and Bancroft to continuous rock stressed under conditions approaching those existing in

TABLE III

| Specimen | $K$ | $\mu$ |
|---|---|---|
| | (Pounds per square inch) | |
| Steatite......... | 5,500–1,800 | 0.24–0.32 |
| Alabaster........ | 4,200–3,100 | 0.37–0.38 |
| Sandstone....... | 7,500–3,100 | 0.34–0.40 |
| Marble.......... | 850–1,500 | 0.58–0.52 |
| Lead............ | 850– 500 | 0.00 |

the earth's interior, in which circumstances a limiting stress-difference several times greater than that obtained by the usual crushing test must be employed.