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ON THE VARIATION OF THE COEFFICIENT OF DISCHARGE FOR SMALL ORIFICES.

By T. P. Strickland, B.E., M.Sc., S. Can. Soc. C.E.

(To be read before General Section, January 7, 1909.)

Introductory.—Early in 1898, during the preparation of apparatus for other experiments, the author made, at the suggestion of Professor Bovey, some determinations of the coefficient of discharge of two sharp-edged circular orifices. In working up the results for publication, it has been found that the variation of the coefficient of discharge, with the head, can be represented by a simple formula, which is also applicable to the results obtained by other observers. A study of all the results available throws some light on the manner in which the coefficient varies with the size and shape of the orifice.

It is, of course, unnecessary to enter here into any discussion of the ordinary theory of the discharge of fluids through sharp-edged orifices; suffice it to say that if Q be the quantity of water discharged in time T through an orifice of area A under a head h, the coefficient of discharge may be defined as the ratio of the actual discharge Q to the theoretical discharge $AT\sqrt{2gh}$; and that the coefficient may obviously be determined by measuring the quantities Q, T and h for an orifice of known area.

Description of Apparatus.—The phenomena of jets issuing from sharp-edged orifices are displayed in a very perfect form in the hydraulic laboratory at McGill University, and are doubtless familiar to most members of this Society; they have, moreover, been very fully described in a paper* read before the Royal Society of Canada. The apparatus there used has also been described in detail in a paper† read before this Society in May, 1898, by Professor Bovey and J. T. Farmer, and also in a paper‡ read before the Royal Society of Canada by J. T. Farmer. As the apparatus used, and the methods adopted by the author in his experiments did not differ materially from those described in the papers referred to, it will not be necessary here to enter into any lengthy descriptions which would only cover the same ground.

In J. T. Farmer's experiments, which were on smaller orifices, the jets were discharged through a bifurcated tube into a calibrated vessel; in the author's experiments, however, owing to the larger quantity of water to be measured, it was necessary to depart from this arrangement. The jets were accordingly discharged through a tube—which prevented loss by splashing—into the flume running along one side of the hydraulic laboratory below the floor level. At the end of this flume is a weir, beyond which is a flap-door with bevelled edges, along the centre line of which runs a piece of india-rubber cord, so that when the door is closed and pressed home, a perfectly water-tight joint is formed. This door is opened and shut by a lever with a spring clasp actuating a system of links acting as a toggle-joint, so that the door can be rapidly and firmly shut and locked in position, each movement of the lever being recorded on the chronograph.

When this door is open the water runs to waste; while, when it is closed, the water flows over it into one or more of a set of five 1000-gallon cast-iron tanks set firmly in concrete below the floor level. Each of these tanks is connected through a valve to a header so that they can be used separately or in any combination. To each tank is connected a vertical four-inch brass pipe forming a float chamber; the float is attached to a brass rod with a pointer at the upper end moving over a brass scale, and attached to a counterweight by a fine cord passing over a frictionless pulley. The scale of each tank is marked at each 100 gallons up to 1000 gallons, and then at each 10 gallons up to 1080 gallons.

As indicated above, the duration of each experiment was recorded on a chronograph connected to a standard and accurately-regulated clock in the adjacent testing laboratory, a mark being made (by a fountain pen) on the record every second. The time

* "The Phenomena of Jets springing from non-circular Orifices." Strickland and Farmer. Trans. Roy. Soc., Canada, 1898 99.

† "Hydraulic Laboratory, McGill University." H. T. Bovey and J. T. Farmer. Trans. Can. Soc. C.E., 1898.

‡ "The Determination of the Coefficient of Discharge for Sharp-edged Orifices." J. T Farmer. Trans. Roy. Soc., Canada, 1896-97.

of opening and closing the flap-door was recorded by a glass stylus, the point of which followed closely in the track of the fountain pen, and derived sufficient ink from the moist track to make its mark when the lever was moved. The mark indicating the movements of the door was on the opposite side of the line to that indicating the seconds, so that there could be no possible confusion between the two marks even when occurring at the same point of the record. A stop-watch was also used, as a check against large errors, and also to indicate during any experiment the time which had elapsed since the start; so that the observer would, after one run at a given head, know when the tank was nearly full. The duration of every run at a given head could thus be made approximately the same, so that the total quantities discharged would be approximately equal, and any large errors in reading the scale of the tank detected at once.

Experimental Work.—The water in the flume had, of course, to be kept up to the crest of the weir; before an experiment, therefore, the orifice was opened and the water run to waste for ten or fifteen minutes, to allow the water to attain a steady head above the crest of the weir, so that at the opening and closing of the flap-door the flow at every point would be steady. During this time the inlet valve of the tank was adjusted, the chronograph record placed in position, and the gauge of the measuring tank read. The temperature of the water in the flume did not differ by more than a few degrees from that of the water issuing from the orifice, or as finally measured in the tank, so that no appreciable error was introduced, especially as the quantity of water measured was so large and the temperature very nearly that of the maximum density of water, the experiments being made during the winter months.

During the course of each experiment the head was kept under constant observation, and the temperature of the water issuing from the orifice taken at frequent intervals. The temperature was also taken in the measuring tanks when the quantity was read, but no correction has been made for the difference between this reading and that of the orifice, as it never amounted to more than a few degrees Fahrenheit, and the resulting error was well within the limits of error of the other measurements. This introduces the subject of the probable accuracy of the determinations.

Q.—The total quantity discharged during one experiment was never less than 1000 gallons, and in some cases was 2000 gallons, these latter being in the experiments on the two-inch orifice at high heads. On the scale of the tank 0.87 inch corresponded to

10 gallons; the position of the pointer was referred to the nearest 10-gallon mark by means of a steel scale graduated in hundredths of an inch, so that the combined error of the initial and final readings would probably not exceed 1 in 5000 or .0002 as far as the relative values of the coefficient in these experiments are concerned. The absolute values would depend upon the accuracy of the calibration of the tank and on the measures used, so that in comparing the results of the author's experiments with others made with different apparatus, the error might be somewhat greater. The tanks had been calibrated by filling with weighed quantities of water, and the calibrations had been checked from time to time and found to be accurate and permanent. For converting gallons into cubic feet the multiplier used was .16037, as in calibrating the tank, the gallon was defined as 10 lbs. of water at 62° F.

T.—On the chronographic record the distance corresponding to one second was 0.40 inch, and the error in reading would probably not exceed 1/50th second. As the duration of an experiment varied from 500 seconds upwards, the greatest error should not exceed one in 25,000 or .00004. It should be remarked that the clock was compared daily with the time ball at the McGill Observatory.

A.—The diameters of the orifices were very carefully measured on a comparator in the geodetic laboratory, measurements being made on four different diameters. These measurements on the two-inch orifice were:

2.00460. 2.00460. 2.00462. 2.00466.

The diameter of this orifice has been taken at 2.00462; to take the error at .00005 is therefore probably over-estimating it, but this allows for the uncertainty of a few degrees in the temperature of the orifice plate, that of the laboratory in which they were kept for a day before being measured being 60° F. The error in area may, therefore, be taken as .0001, which is equivalent to 1 in 40,000 for the two-inch orifice, and 1 in 10,000 for the one-inch orifice, the actual diameter of which was found to be 1.00020 inch.

H.—It is somewhat difficult to estimate the probable error in the value of the head; there are really two sources of error, the first being in setting the adjustable indicator by means of the gauge-glass, and the second being due to slight variations in the head during an experiment; this variation, however, was never great, as the indicator was kept constantly under observation, and the means for regulating the inflow were so perfect that after some adjustment the head would sometimes maintain itself within 1/100 inch over an hour. The error from each source would not exceed 1/200 inch, and if both were in the same direction would total

1/100 inch, so that the maximum probable error would be 1 in 2000 at the lowest head, producing an error in the coefficient of 1 in 2400, or say .0004.

In the case of the one-inch orifice at the lowest head the probable error may therefore be summed up as follows:

Q	.0002
A	.0001
T	.00004
H	.0004
Total	.00074

being the greatest probable error in the worst case; taking the value of the coefficient as .6, the error in the figures of the coefficient should therefore not exceed .00045 in any one experiment.

Two experiments were always made at each head, and if these did not show good agreement, or if there were any reason to doubt one, a third was made; the greatest difference obtained between the values of the coefficient at the same head was .0009, and this was a single exception. The mean of the two observations has been taken as the probable value in each case, and it seems fair to assume that the error in any case does not exceed .0003.

As the diameter of the orifices were measured at a temperature of 60° F., and as the water during the experiments had an average temperature of about 40° F., the coefficients as given below are subject to a small correction on account of the diameters during the experiments being smaller than as measured. As the temperature of the orifice plate would be slightly higher than that of the water, the temperature of the room being about 60° F., the correction is uncertain, but in any case would not exceed two in the fourth place of decimals, and has, therefore, not been applied; this error should, of course, be taken into account in comparing the values given below with others of similar accuracy.

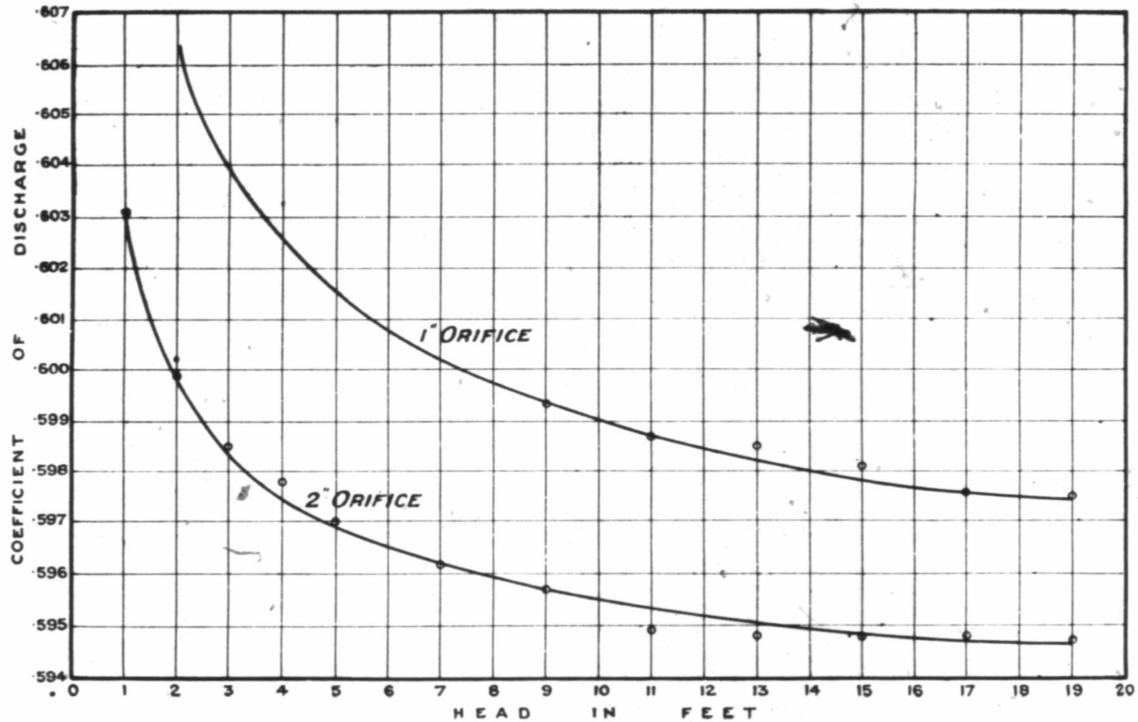
No attempt was made to determine the variation of the coefficient with temperature; such variation would, no doubt, be very slight (as it would have an effect only upon the coefficient of velocity and none upon the coefficient of contraction), and a considerable range of temperature would be necessary. Such a range could not at that time be obtained. At a later date, however, in connection with other work, arrangements were made for admitting steam to the tank, so that experiments could be made with hot water; and, as this is always desirable, and as the brass scale, when mounted, as in this case on the tank, would be affected by the increase in temperature, the author suggests the desirability of placing the gauge-glass and scale on a wall at some distance from the tank; care being taken to so support it that the relative positions of the zero line and the centre of the orifice would not be affected by the rise in temperature. As has been shown above, the probable error in the measurement of the head (at low heads) is greater than the sum of the probable errors in the other measurements, and it is therefore suggested that at low heads a hook gauge should be used, which could be arranged for use in small auxiliary vessels attached to the tank at suitable heights, and each provided with a valve to admit the water, when readings were being taken at the corresponding head.

With this refinement, and the use of a hook gauge in the measuring tank, greater accuracy would no doubt be attained, but at the same time it should be remembered that the condition of the orifice plate as regards cleanliness has its effect upon the coefficient, and it is therefore doubtful whether the coefficient can be determined to greater accuracy than one in the fourth decimal place. For practical purposes, of course, such accuracy is not required, and it would therefore only be useful as a help in obtaining a more accurate knowledge of the laws governing the variation of the coefficients than is obtainable from experiments hitherto published.

The head available in the supply pipe was 280 feet, and the range of head could therefore have been greatly extended had the tank been arranged so that it could be closed in at the top, and built strong enough to withstand the full pressure. This would necessitate the provision of a mercury gauge.

The value taken for g is 32.176, as determined for Montreal, in 1893, by Commandant Desforges.

The values of the coefficient of discharge obtained on these experiments are given in the following Table I and plotted in Fig. I.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COEFFICIENT OF DISCHARGE
1" ORIFICE
2" ORIFICE
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
HEAD IN FEET

Fig. I.

TABLE I.

VALUES OF THE COEFFICIENT OF DISCHARGE.

Head in feet	Orifice 1 in. diameter: By Experiment	Orifice 1 in. diameter: By Formula $Cd = .5930 + \frac{.019}{\sqrt{H}}$	Orifice 2 in. diameter: By Experiment	Orifice 2 in. diameter: By Formula $Cd = .5920 + \frac{.011}{\sqrt{H}}$
1	..	.6120	.6031	.6030
2	..	.6064	.5999	.5998
3	..	.6040	.5985	.5983
4	..	.6025	.5978	.5975
5	.6016	.6015	.5970	.5969
7	.5999	.6002	.5962	.5962
9	.5993	.5993	.5957	.5957
11	.5987	.5987	5949	.5953
13	.5985	.5983	.5948	.5951
15	.5981	.5979	.5948	.5949
17	.5976	.5976	.5948	.5947
19	.5975	.5974	.5947	.5946

Of the experimental values, judging by the agreement of the two observations, the best are: for the one-inch orifice, those at heads of 17 and 19 feet, and for the two-inch orifice, those at 1, 3, and 7 feet.

The values of the coefficients for the two-inch orifice at heads of 15 feet and above are a little uncertain, both on account of the disturbance caused by the large quantity of water (2000 gallons in 12 minutes) flowing into the tank, and of the difficulty of leading this quantity into the flume without losing any by leakage or splashing.

Inspection of the figures and curves will show that the coefficient increases as the head decreases, and also as the size of the orifice decreases; and again that the values tend to approach a constant value as the head is increased.

That the coefficient varies in the manner just stated is, of course, well known, but as far as the author is aware no general expression has yet been published giving Cd as a function of the head and the area of the orifice. The variation in Cd is so small as to be of little practical importance, and this also makes it necessary to obtain very accurate experimental values before an empirical equation can be deduced. Again it appears at present to be impossible to obtain a theoretical law which would give some indication of the correct form of an empirical equation.

The most complete mathematical treatment of the problem, with which the author is acquainted, is that given by Boussinesq in an article* in the *Journal de Physique* for 1892, in which are given approximate formulae for the coefficients of discharge for certain circular and rectangular orifices, which show how the rate of discharge varies from the centre towards the perimeter.

According to Boussinesq, it is impossible to determine the discharge exactly from considerations entirely theoretical, but by starting from a basework of four equations, of which two are theoretical, one empirical, and one partly theoretical and partly empirical, he was able to derive an approximate formula for the coefficient.

His theoretical equations are based, one on Torricelli's theorem giving the velocity at the free surface of the jet, the other on the consideration that the stream-lines at the perimeter must be tangential to the plane of the orifice.

His empirical equation is based on some experiments made by Bazin, which showed that the velocity of the central stream-line in a certain circular orifice was .632, and in a certain narrow rectangular orifice without end contractions .690, of the velocity at the free surface. His fourth equation is the well-known hydrodynamical equation,† which shows that the coefficient of contraction can never be less than .5.

From these four equations Boussinesq derives formulae for the discharge per unit area at the different points of the orifices in question as follow:

for the circular orifice

$$dq = V\left(.632 + 12.2329\,\frac{r^{10}}{R^{10}}\right)\left(1 - \frac{r^2}{R^2}\right)$$

and for the rectangular orifice

$$dq = V\left(.690 + 7.9272\,\frac{b^8}{B^8}\right)\left(1 - \frac{b^2}{B^2}\right)$$

in which V in each case represents the velocity of the steam-lines on the free surface as given by Torricelli's theorem, while for the circular orifice r represents the distance from the centre and R the radius, and for the rectangular orifice, b represents the distance from the horizontal axis and B is one-half the depth of the orifice.

The values of dq obtained from the above equations are given

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* "Ecoulement en Mince Paroi." Journal de Physique, Tome 1, 3me Ser., 1892.

† Basset's "Hydrodynamics," p. 29. Lamb's "Motion of Fluids," p. 26.

in Table II. and plotted in Fig. II. The curves show how erroneous is the assumption that the velocity and discharge depend only upon the head, on which are based calculations* for the total discharge by integrating in horizontal layers, as it is clear that the velocity of any stream-line depends not only upon the head but upon its position in the orifice.

TABLE II.

VALUES OF BOUSSINESQ'S FUNCTION.

$\frac{r}{R} = \frac{b}{B}$	f for circle	f for rectangle
.10	.6257	.683
.20	.6067	.662
.30	.5752	.628
.40	.5309	.584
.50	.4830	.541
.57826	..	.525 (min.)
.60	.4519	.527
.6075	.4516 (min.)	..
.70	.4792	.584
.80	.7005	.722
.85	.816	.786
.872	..	.800 (max.)
.90	..	.777
.90205	.93065 (max.)	..
.95	.777	.589
.96	.688	..
.98	.425	.297
1.00	0	0

It will have been noticed that the formulae just given do not take into account any variation of the coefficients with the head or area, except in so far as the experimental values of the velocity of the central stream-line may vary with the head and area: this variation it would be useful to determine by experiment. In fact, throughout the article it is apparently assumed that the coefficient of contraction is synonymous with the coefficient of discharge; which, by the formulae, would have the values .6073 and .620, as against the experimental values .598 and .626 obtained by Bazin for the orifices in question, which were 20 centimetres in diameter,

* Merriman's "Hydraulics," pp. 42, 43. Bovey's "Hydraulics," pp. 50-54

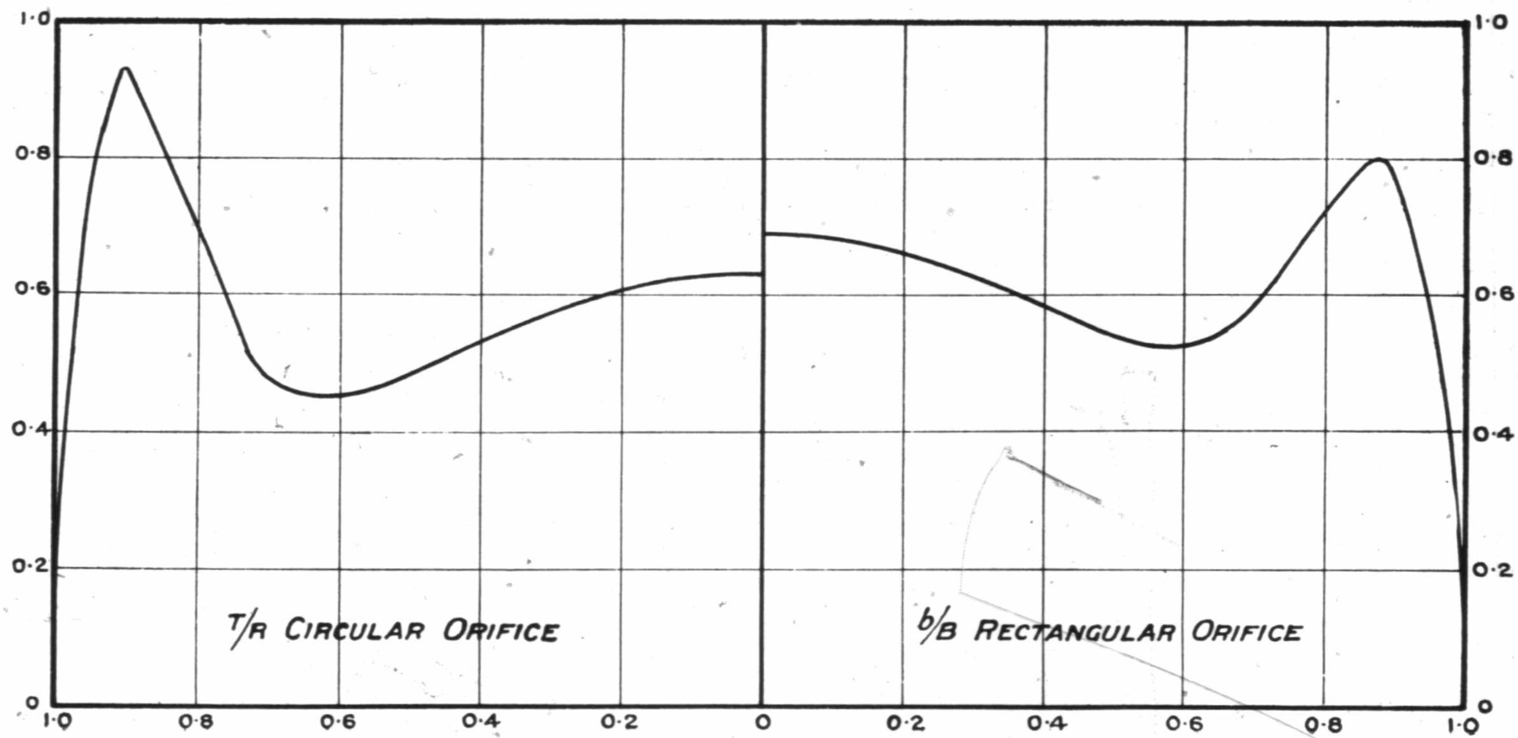

Fig. II.

and 80 centimetres long by 20 centimetres deep respectively, under a head of about one metre. The discussion, therefore, although invaluable for a proper understanding of the subject of the flow through orifices, is of very little guidance when dealing with the question of the variation of the coefficients concerned.

A formula showing the variation of the coefficient of discharge should preferably be based on formulae giving the variations of its two factors, the coefficients of contraction and of velocity. As far as the author is aware, however, these two coefficients have not yet been determined with accuracy over any range of head and area; owing, no doubt, to the experimental difficulties; and, therefore, until such experiments are available, all that can be done is to form some idea of the causes of the variations, and to base an empirical law on the data obtained.

It might be supposed, and Boussinesq's article seems to show, that the coefficient of contraction is a geometrical constant. It appears likely, however, that this coefficient would be affected to some extent by capillarity, the effect of which would probably be a function of the ratio of the perimeter of the orifice to its area, and therefore vary inversely as a function of the linear dimensions of the orifice, for symmetrical orifices. This theory receives some support from the fact that the value of the coefficient of discharge is altered if the orifice plate be at all greasy; and, as will be seen later, is also supported by the formulae derived by the author from the results of experiments.

Experiments on jets of mercury, which has a very much greater surface tension than water, would throw some light on the effect of surface tension on the coefficient of contraction.

The reduction in velocity expressed in the coefficient of velocity is generally ascribed to viscosity, which must have some effect, though probably a small one; but it has been suggested* that it is mainly due to the reaction of the outer stream-lines in being deflected between the orifice and the *vena contracta*; this reaction being similar to that which occurs during the passage of a stream over a curved vane.

Whatever be the causes, a series of careful experiments is certainly needed, to determine the variation of the coefficients of contraction and of velocity. Till these are available, it is impossible to derive a rational formula for the coefficient of discharge.

When the head on an orifice is lowered to a level near the top of the orifice a free surface is formed, and the conditions then become those for a weir. It is probable, therefore, that the law

* Strickland and Farmer. "The phenomena of jets springing from non-circular orifices," Trans. Roy. Soc. Canada, 1898-99.

governing the variation of the coefficient of discharge will cease to hold at very low heads; on the other hand, it is possible that a similar law may be found to answer for weirs, and that by making proper allowances for the alteration in head and area the one formula may fit both weirs and orifices, at all events in the case of rectangular ones.

Guided by the foregoing considerations, and by the curves, after some trial the author arrived at a fairly simple formula expressing the variation of the coefficient of discharge with the head. This formula is

$$Cd = m + \frac{n}{\sqrt{h}}$$

in which m and n are constants for the same orifice. It is found that an expression of the above form will express the variation of the coefficient of discharge for orifices of all sizes and shapes for which results are available, the values of m and n being suitably varied.

For the two orifices measured by the author the values of m and n, which appear to give the best results, are:

For the 1-inch orifice... .. ..$m = .5930$ $n = .019$
For the 2-inch orifice... .. ..$m = .5920$ $n = .011$

The values of Cd corresponding to these values of m and n are given alongside the experimental values in Table I, and it will be seen that the values given by the formulae agree very closely with those obtained experimentally. As in most of the other experimental values by which the formula has been tested the agreement is as good, there seems to be no reasonable doubt as to the correctness of the *form* of the expression given above.

The values of the constants m and n cannot, however, be stated with such certainty, as they may be varied slightly in opposite directions, and still give values of Cd agreeing with the experimental values within the limits of error. They would also be subject to alteration if required to express results over a greater range of head, m becoming smaller and n greater.

Although experiments on two orifices cannot be used to derive a formula for the variation of Cd with the area, the values given above indicated that m is approximately constant for circular orifices, though possibly larger for the smaller orifices, while it was noticed that the value of n for the two-inch orifice is about one-half of its value for the one-inch orifice, or inversely as the diameters. It therefore seemed possible that, with further data, the formula might be extended to cover the variation of Cd with the diameter.

The author therefore regrets that he did not take an opportunity to carry out, as he intended, experiments on other circular orifices which were available, as in that case the variation of *Cd* with the head and area could have been accurately expressed. In the absence of such data it was therefore necessary to resort to the results given by other observers.

The most complete table of the values of *Cd* for circular orifices, to which the author had access, is that given on page 79 of Merriman's Treatise on Hydraulics (4th edition); and an inspection of that table showed that the value of *Cd* for the highest heads is, for every orifice except the smallest, given as .592, pointing to a value of m agreeing very well with that given above. A portion of this table is reproduced in Table III, and alongside the experimental values are given values calculated from the formula $Cd = m + \frac{n}{\sqrt{h}}$ with suitable values of m and n.

TABLE III.

VALUES OF THE COEFFICIENT OF DISCHARGE FOR CIRCULAR ORIFICES.

Head in feet	0.48 inch Diameter		0.84 inch Diameter		1.20 inch Diameter	
	By Experiment	By Formula $Cd = .594 + \frac{.028}{\sqrt{h}}$	By Experiment	By Formula $Cd = .593 + \frac{.020}{\sqrt{h}}$	By Experiment	By Formula $Cd = .592 + \frac{.016}{\sqrt{h}}$
0.4	.637	.637	.624	.624	.618	.617
0.6	.630	.630	.618	.619	.613	.613
0.8	.626	.625	.615	.615	.610	.610
1.0	.623	.622	.612	.613	.608	.608
1.5	.618	.617	.608	.609	.605	.605
2.0	.614	.614	.607	.607	.604	.603
2.5	.612	.612	.605	.606	.603	.602
3.0	.611	.610	.604	.604	.603	.601
4	.609	.608	.603	.603	.602	.600
6	.607	.606	.602	.601	600	.599
8	.605	.604	.601	.600	.600	.598
10	.603	.603	.599	.599	.598	.597
20	.599	.600	.597	.597	.596	.596

With the results given as above to the third place of decimals only, there is not sufficient variation in the values of the coefficient for the larger orifices given by Merriman, to deduce any accurate values of m and n, while the values given for the smallest orifice cannot all be fitted to a curve of the required form, although they are not inconsistent with the general formula given hereinafter.

It will be noticed that the values given by Merriman for heads of 50 and 100 feet are not included, as the formulae are only offered as representing the values up to heads of 20 feet. To include higher heads the values of m must be somewhat decreased, with a corresponding increase in the values of n, and the results would not then be comparable with those obtained at McGill University.

It remained to be seen whether an expression could be formulated covering the variation of Cd with the area of the orifice; tabulating the values of m and n for the different diameters, it will be seen that, while m decreases as the diameter increases, the variation is so slight that it cannot be accurately expressed, and an average value of .5925 may therefore be taken. As regards n, the following table shows that the value of m may be expressed by the equation $n = \frac{k}{\sqrt[3]{d^2}}$, the average value of k being about .018.

TABLE IV.

d inches	n observed	$\frac{.018}{\sqrt[3]{d^2}}$
0.24	—	.0465
0.48	.028	.0294
0.84	.020	.0202
1.00	.019	.0180
1.20	.016	.0160
2.00	.011	.0113
2.40	—	.0100

As the probable error in the values of n as derived from the experiments is not less than .0005, the agreement may be considered good.

The general formula for the coefficient of discharge for sharp-edged circular orifices may therefore be written

$$Cd = m + \frac{k}{\sqrt{h}\ \sqrt[3]{d^2}}$$

where h is expressed in feet and d in inches
and m has an average value of .5925, increasing slightly as the diameter decreases
and k has an approximate value of .018.

In view of the nature of the data from which this formula is derived, it is desirable that a series of accurate experiments should be carried out by one observer with one set of apparatus under uniform conditions, when it is probable that the values of m and k may be somewhat modified. The formula is offered as a first approximation only, for orifices of diameters up to 3 inches, and for heads up to 20 feet. It has the merit of being fairly simple in form, as, given the values of m and k, the values of Cd may be obtained very quickly by the use of a slide rule.

It will now be useful to examine the table of values of Cd for square orifices, given on page 81 of Merriman's treatise. Taking as before the orifices of medium size, the approximate values of m, n, and k are as follows:

l inches	m	n	$k = n\sqrt[3]{l^2}$
.48	.598	.029	.0178
.84	.598	.020	.0178
1.20	.598	.015	.0170

As the values of the coefficient are given to three places of decimals only, the values of m and n (and consequently k) are necessarily approximate; but it may be stated that m is approximately constant, but has a higher value than for circular orifices, while the figures in the last column show that, as before, $n = \dfrac{k}{\sqrt[3]{l^2}}$ where the average value of k may be taken as .0175, or practically the same as for circular orifice. This agreement is in favor of the theory that the value of the coefficient depends upon the ratio of the perimeter to the area of an orifice. This is supported also by the value of n for the only other equilateral orifice for which the values of Cd are available, this orifice being triangular in form.

A series of experiments is needed on a set of equilateral triangular orifices of different areas, in order to determine the values of m and n for such orifices. The dependence of n upon the ratio of perimeter to area could then be verified, and some idea be obtained of the variation of m with the number of sides of the regular polygon. Experiments on other regular polygonal orifices would also be useful in throwing light on these points.

Fortunately, however, in practice, the variation of m and n with the shape of the orifice is not of great importance in the case of regular polygons, as circular and square orifices are the only ones generally used. It is therefore better for practical purposes to define n in terms of the diameter or length of side respectively, and to consider m as a separate constant for each case. Generally it may be stated that both m and n (and consequently Cd) are greater, the greater the departure from the circle, or the greater the ratio of perimeter to area, and this holds good also for rectangular orifices, which are frequently used in practice. Sufficient accurate data are not, however, available to determine a general law for rectangular orifices, though it is certain that the law $Cd = m + \frac{n}{\sqrt{h}}$ holds equally well for such orifices; and what evidence there is, is in favor of assuming that n varies as $\left(\frac{\text{perimeter}}{\text{area}}\right)^{\frac{2}{3}}$. It is, of course, easy to see that the value of Cd would be greater for elongated rectangles than for squares, as the end contractions produce a relatively smaller effect. As rectangular orifices are much used, it is desirable that a series of careful experiments should be made on such orifices, preferably on sets of orifices of the same perimeters but of different areas, or of the same areas with different perimeters. As indicated before, however, the first step towards the determination of a general expression for the values of Cd should be to ascertain the variation of the coefficients of contraction, and of velocity.

Although a vast number of experiments have been made on the discharge of jets from various orifices, it will have been seen that, like those described in this paper, they are of a disconnected nature; and it is therefore desirable that further experiments be carried out on a connected scheme. The author has indicated points on which further experiments are particularly required; and has described his own experiments and offers the formulae derived therefrom, not on account of any merit they may possess, but rather as a guide to future workers in the same field.

In conclusion, the author wishes to express his thanks to the authorities at McGill University for the use of the apparatus in their Hydraulic Laboratory, and to Professor Bovey for generous advice and assistance in carrying out the experiments.