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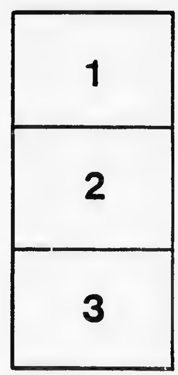

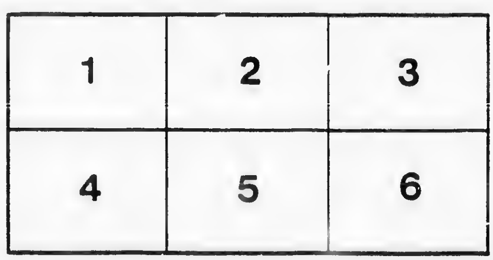

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TREATISE

ON

THEORETICAL AND PRACTICAL

Land Surveying,

ADAPTED PARTICULARLY TO THE PURPOSES OF WOOD-LAND SURVEYS;

TO WHICH IS ADDED

AN INVESTIGATION AND DEMONSTRATION OF THE RULES GIVEN IN THE WORK,

WITH ALL THE NECESSARY TABLES.

BY

ALEXANDER MONRO,

LAND SURVEYOR.

PICTOU, NOVA-SCOTIA:

Printed by Geldert & Patterson, Eastern Chronicle Office,

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DEDICATION.

TO THE

HON. AMOS EDWIN BOTSFORD,

MEMBER OF HER MAJESTY'S LEGISLATIVE COUNCIL,

MEMBER OF THE BOARD OF EDUCATION,

LIEUTENANT-COLONEL,

&c. &c.

SIR,—

Of the merits of the following Treatise it is not for me to judge. That point must he left to the decision of the impartial reader. I may, however, be permitted to express my regret that it is not more worthy of your patronage. At the same time it affords me great pleasure to dedicate it to one, to whom mind the urgent necessity which exists for a work of this kind must have frequently suggested itself, whilst superintending the affairs of the public, and particularly the proceedings in our Courts of Law,—to one whose familiar acquaintance with the subject discussed, affords some guarantee that the work itself is not altogether unworthy of notice, and favourable acceptance,—to one from whom its numerous imperfections are sure to meet with almost parental indulgence,—and to one whose patronage will shield it from the obloquy or neglect to which, on account of the obscurity, and want of personal or relative influence on the part of the author, it might otherwise be exposed.

Other considerations have likewise induced me to solicit the honour of dedicating to you this, my first attempt at authorship. I was exceedingly desirous to avail myself of this opportunity publicly to express my grateful sense of the courtesy, kindness, and attention, which you have so generously extended towards the humble author of the following work, ever since he had first the happiness of being introduced to your notice. Be assured, Honoured Sir, I am not ungrateful. Your name will ever be associated in my recollection with the most lively emotions of esteem and respect.

Besides, the countenance and encouragement which you have uniformly extended to the industrious and enterprising youth, and the interest which you have ever manifested in the cause of Education, and in every movement in which the prosperity of British North America is involved, justly entitle you to this public expression of grateful approval.

Hoping that the work itself may not altogether disappoint your expectation,—hoping even that it may meet with some degree of approbation,—and praying that you may be long spared to enjoy the confidence and respectful esteem of those who may be honoured with your acquaintance,—and to witness with delight the rapid progress of intellectual improvement, and the growing prosperity of your native country.

I remain, Sir,

With much esteem and respect,

Your Most Obedient,

And Very Humble Servant,

ALEXANDER MONRO.

Bay De Verte, N. B., October, 1844.

TO THE LAND SURVEYORS OF BRITISH NORTH AMERICA.

GENTLEMEN,—

It is with much diffidence that I submit the following Treatise to the examination of a body of men so well qualified to judge of its merits. To your experience, however, I can confidently appeal for the great importance of, and crying necessity for, a work of this nature. You very well know how imperfectly adapted the British works upon this subject are to the purposes of Colonial Land Surveying,—particularly to the running, tracing, and re-tracing of lines, and the Location of Lands,—two departments of the Profession indespensably necessary to be understood by the Colonial Land Surveyor. This defect I have endeavoured to supply. In an Appendix I have given the investigation of the Problems in Trigonometry, and in the Mensuration and Location of Lands, by the perusal of which the reasons of the rules given in the body of the work may be easily understood. I have added also in the end of the work all the Tables necessary to be used in Land Surveying, thus rendering it complete for all practical purposes.

I have the Honour to be,

Gentlemen,

Your Obedient,

And Very Humble Servant,

ALEXANDER MONRO.

Bay De Verte, N. B., October, 1844.

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PREFACE.

THE TERM GEOMETRY, in its literal acceptation, denotes *the measurement of the Earth, or of Land.* In its ordinary acceptation, it is the name of that science which treats of the extension, magnitude, figure, divisibility, and distances of bodies. It is applied to the measurement of the magnitude, distances, and motions, of the heavenly bodies, as well as of bodies on or near the earth's surface.

Respecting the origin of this science various opinions have been entertained. The ancients agree in giving to the Egyptians the credit of being the earliest cultivators of geometry. The annual overflowings of the Nile disturbing their landmarks rendered it necessary frequently to adjust their boundaries by measurement. Hence the supposed origin of Land Surveying.

Achilles Tatius, a Mathematician of Greece, informs us that the Egyptians were the first who measured the heavens and the earth. Moses is even said to have acquired a knowledge of the science, when he resided at the Egyptian Court.

Herodotus ascribes its origin to Sesostris, who is supposed to be the Shishak mentioned in scripture, and who made war upon Rehoboam the son of Solomon. This King is reported to have intersected Egypt by Canals, and to have divided the land among his subjects, giving to each an allotment, for which an annual tribute was to be paid.

Aristotle has attributed its origin to the Egyptian priests, who living secluded from the world had abundant leisure for study.

The antiquity of this science affords some evidence of its utility and importance. Necessity, the prolific parent of invention, compelled mankind to study it. It is nearly, if not altogether, coeval with, because indispensable to, civilization. From its principles, rules for measuring distances, heights, surfaces, and solids, are derived. Its principles are the basis of Land and Marine Surveying. They direct all the calculations in Navigation and Astronomy. They guide the labours of the Optician and Mechanist. Civil and Military engineering are only particular applications of its elements. It enables the Seaman to traverse the pathless ocean, and the Miner to prosecute his subterranean excavations. Painting and Drawing, and the construction of Maps and Charts, are indebted to it for nearly all their accuracy and justness of proportions. In a word, wherever measurement, computation, or construction is required, its aid is indispensable.

The subject of the present treatise is *Land Surveying.* This constitutes one of the most important and useful branches of the Mathematics. The Surveyor, in the discharge of his professional duties, directs his attention

1. To the tracing and measurement of lines;

2. To the position of these lines, in reference to the angles which they form with other lines;

3. To the Calculation of the area of the field or tract surveyed; and

4. To the delineation of its boundaries upon a plan or Map. He should be able also, when required, to determine the position of all the remarkable objects within or near the scene of his operations,—to measure their distances and heights,—to ascertain the variation of the compass,—the latitude and longitude of particular places and their bearings,—to delineate not only the boundaries of an ordinary survey, but also of coasts and harbours,—or to give a correct representation of the inequalities of the earth's surface.

Among the ancients, Archimedes and Tantaglia made considerable progress in evolving the principles by which areas are determined. The former bestowed much attention upon the areas of curves particularly, and ascertained that the area of a circle is equal to the area of half the rectangle contained by its circumference and radius. The latter is said to have demonstrated the rule for obtaining the area of a triangle when its three sides are given.

Of those who have cultivated this department of Mathematical science in modern times, the most celebrated are Hutton, Nesbit, Gale, Milne, Crocker, and the late Robert Gibson. Notwithstanding the attention bestowed upon this subject by these and several other eminent individuals, still it cannot be denied that Land Surveying, as a science, has been too much neglected. If this remark has any degree of truth when applied to old and extensively cultivated countries, it is still more just in its application to the British North American Colonies. Wood-Land Surveying differs in many respects from surveying in an open and clear country. For this reason none of the treatises upon Land Surveying with which I have met, are so suited as they might be to the circumstances and wants of the inhabitants of these Provinces. The necessity for some work on the subject adapted to our condition, appears to me to be great and pressing. From the frequency with which disputes about boundaries are introduced into our Courts of Law for adjudication, it is evident that some acquaintance with Land Surveying ought to form a part of the necessary education of every member of the legal profession. Scarcely less necessary is an acquaintance with this science to qualify any individual to be a juror or an arbitrator. In a word, every owner of land ought to have some knowledge of the subject. A very little acquaintance with Land Surveying would have saved many an individual from ruinous litigation, in which not only his property has been squandered, but his peace of mind disturbed, and strife and contention spread through the community. Impressed with a deep sense of the necessity and importance of such a work, I have been induced to undertake the following treatise, which I now offer to the acceptance of an indulgent public. How far I have succeeded, it is not for me to determine. The decision is left to the judgment of the discriminating reader. I have only to remark in conclusion, that I make no pretensions to purity or great accuracy of composition. If I have succeeded in rendering my meaning intelligible, I have arrived at the summit of my ambition on this point.

New Brunswick, October, 1844.

CONTENTS.

DECIMAL FRACTIONS.

THE TERM FRACTION, literally denotes something *broken off*.

To form a distinct conception of the nature of fractions, employed in calculations, let the Student suppose any object or quantity *broken*, or divided, into several equal parts. Any number of these parts, considered in their relation to the whole object or quantity, constitutes a *fraction*.

A fraction is expressed by two numbers, placed the one above the other, with a line between them, thus: $\frac{3}{5}$.

The figure below the line (5), called the *denominator*, expresses the number of equal parts into which any object or quantity is supposed to be divided; and the figure above the line (3), called the *numerator*, specifies the number of these parts which the fraction represents.

DECIMAL FRACTIONS are such as have for their denominators, 10, or some multiple of 10, that is, 10 multiplied into itself a certain number of times, as 100, 1,000, 10,000, &c. Expressed in the common form, they appear thus: $\frac{3}{10}$ $\frac{45}{100}$ $\frac{261}{1000}$ &c. In the decimal form, the denominator, being easily ascertained, is omitted; and its place is supplied by a dot or decimal point, (.) prefixed to the numerator, thus: .3, .45, .261, &c.

To ascertain the denominator of a decimal fraction, it is only necessary to write down as many cyphers as there are figures in the fraction; and then to place the figure 1 before them.

Cyphers on the right hand of decimal fractions, do not affect their value; but every cypher on the left hand diminishes their value tenfold.

The value of figures in decimals as in whole numbers is determined by their position. The following table, in which the figures on the left hand of the decimal point are whole numbers, and those on the right are decimals, will illustrate the influence of position in determining their value:—

Integers.								Decimals.					
8	1	3	4	2	4	6	.	4	1	2	3	5	7
Millions,	Hundreds of Thousands,	Tens of Thousands,	Thousands,	Hundreds,	Tens,	Units,		Tenths,	Hundredths,	Thousandths,	Ten Thousandths,	Hund. Thousandths,	Millionths,

The notation and numeration of decimals will be obvious from the following examples:—

4.7	signifies	four, and seven tenth parts.
.47	"	four tenth parts, and seven hundredth parts, or 47 hundredth parts.
.047	"	four hundredth parts, and seven thousandth parts, or 47 thousandth parts.
4.07	"	four, and seven hundredth parts.
4.007	"	four and seven thousandth parts.

ADDITION.

RULE.

Place the figures directly underneath those of the same value, whether they be mixed numbers, or pure decimals, paying particular attention to the separating points. These should always appear in a direct line, one under another. Then add as in whole numbers.

EXAMPLES.

1. Add 2.81, 5.50, 1.6, 4.334, 6.2431, together.

```
   2.81
   5.50
   1.6
   4.334
   6.2431
   ------
Ans. 20.4871.
```

2. Add 4.28, 3.2187, .0024, 342.501, .223, and 1.2324101 together.

```
     4.28
     3.2187
      .0024
   342.501
      .223
     1.2324101
   -----------
Ans. 351.4575101.
```

SUBTRACTION.

RULE.

Place the figures as directed in Addition, then deduct as in whole numbers.

EXAMPLES.

1. From 28.4 take 24.35.

```
   28.4
   24.35
   -----
Ans. 4.05.
```

2. From 70.38 take .829.

```
       70.38
        .829
       ------
Ans.   69.551.
```

MULTIPLICATION.

RULE.

Place the factors and multiply, as in whole numbers. Then from the product, point off as many places for decimals from the right hand as there are decimals in both Factors together. If there are not as many figures in the product for decimals as the rule requires, supply the defect by prefixing cyphers on the left hand.

EXAMPLES.

1. Multiply 3.141592 by 52.7438.

```
              3.141592
               52.7438
              --------
              25132736
              9424776
            12566368
           21991144
           6283184
         15707960
         -------------
Ans.   165.6995001296.
```

2. Multiply .15 by .3.

```
        .15
        .3
       ----
Ans.   .045
```

DIVISION.

RULE.

Divide as in whole numbers, annexing cyphers to the dividend when necessary, observing that the divisor and quotient must together contain as many decimal figures as are contained in the dividend. If at the conclusion of the work the divisor and quotient do not contain as many decimal figures as are contained in the dividend, the deficiency must be supplied by prefixing cyphers to the quotient.

EXAMPLES.

1. Divide 66.993548 by 27.4.

```
27.4)66.993548(2.44502 Ans.
     548
     ----
     1219
     1096
     ----
      1233
      1096
      ----
       1375
       1370
       ----
         548
         548
         ---
```

2. Divide .45695 by 12.5.

```
12.5).4569500(.036556 Ans.
      375
      ---
      819
      750
      ---
       695
       625
       ---
        700
        625
        ---
         750
         750
         ---
```

REDUCTION.

To reduce a Vulgar Fraction to a Decimal of the same value.

RULE.

Annex cyphers to the numerator, and divide by the denominator. The quotient arising thence will be the decimal required.

EXAMPLES.

1. Reduce $\frac{3}{4}$ to a Decimal Fraction.

$$\begin{array}{r} 4)3.00 \\ \hline .75 \text{ Ans.} \end{array}$$

2. Reduce $\frac{5}{64}$ to a Decimal Fraction.

$$\begin{array}{l} 64)5.00(.078125. \text{ Ans.} \\ \quad\ 4\ 48 \\ \hline \quad\ \ \ 520 \\ \quad\ \ \ 512 \\ \hline \quad\ \ \ \ 80, \text{ \&c.} \end{array}$$

Every quantity may be considered as a fraction of a larger quantity of the same kind; as an inch is the $\frac{1}{12}$ of a foot, a pole or perch is $\frac{1}{40}$ of a rood, or $\frac{1}{160}$ of an acre, &c., and may be reduced to a decimal fraction by the preceding rule, observing that the given quantity is the numerator of the fraction, and the number of that denomination contained in the higher denomination is its denominator.

EXAMPLES.

1. Reduce 9 inches to the decimal of a foot.

In this example 9 is the numerator, and 12, the number of inches in a foot, is the denominator; thus: $\frac{9}{12}$. The operation is as follows:—

$$\begin{array}{r} 12)9.00 \\ \hline .75 \text{ Ans.} \end{array}$$

2. Reduce 20 perches to the decimal of an acre.

In this example 20 is the numerator, and 160, the number of perches in an acre, is the denominator. Then

160)20.0(.125 *Ans.*
160
400
320
800
800

When the given quantity is of different denominations, reduce them to the lowest denomination for a numerator. The number of the same denomination contained in the integer will be the denominator. Then proceed as above.

EXAMPLES.

1. Reduce 1 rood 14 perches to the decimal of an acre.

r.	*p.*
1	14
40	
54 perches, Numerator.	

Then 160)54.0(.3375 *Ans.*
480
600
480
1200
1120
800
800

2. Reduce 21 min. 54 sec. to the decimal of a degree.

21′ 54″	1°	3600)1314.0(.365 *Ans.*
60	60	10800
1314 Numerator.	60	23400
	60	21600
	3600 Denominator.	18000
		18000

To determine the value of a Decimal.

RULE.

Multiply the decimal by the number of parts of the next inferior denomination contained in the integer, pointing off in the product as many places for decimals, to the right hand, as the given decimal consists of. The figure or figures on the left hand of the decimal point will be an integer number, while the figures on the right hand will be decimals. Then multiply these decimals by the number of parts contained in the next inferior denomination, and point off as before. Proceed thus until it is brought to the lowest denomination.

EXAMPLES.

1. What is the value of .6 of an acre?

$$\begin{array}{r} .6 \\ 4 \\ \hline 2.4 \\ 40 \\ \hline 16.0 \end{array}$$

Ans. 2 r. 16 p.

2. What is the value of .175 of a Pound?

$$\begin{array}{r} .175 \\ 20 \\ \hline 3.500 \\ 12 \\ \hline 6.000 \end{array}$$

Ans. 3 s. 6 d.

3. What is the value of .42 of a degree?

$$\begin{array}{r} .42 \\ 60 \\ \hline 25.20 \\ 60 \\ \hline 12.00 \end{array}$$

Ans. 25′ 12″

THE EXTRACTION OF THE SQUARE ROOT.

The Square Root of any number is the quantity or number, which, when *squared* or multiplied by itself, will yield the given number as its product. Thus, 4 is the square root of 16, as 4 *squared* or multiplied by itself will yield 16 as its product. 16 is also the square of 4.

To Extract the Square Root.

RULE.

Point the given number into periods of two figures each, beginning at the units place; then find the greatest number the square of which shall be equal to or les than the first period, or the quantity before the first point towards the left hand. Place that number in the quotient. Write the square of that number under the first period, and subtract. To the remainder bring down the second period, and call the whole quantity *the resolvend.* On the left hand of the resolvend write the double of the figure placed in the quotient, after the manner of a divisor. Enquire how often this divisor is contained in the resolvend, omitting the figure in the units place of the resolvend. Write that number in the quotient and also on the right hand of the divisor. Multiply this divisor by the figure last placed in the quotient, and subtract the product from the resolvend. To the remainder bring down the third period for a new resolvend. To the last divisor add the figure last placed in the quotient and write the sum on the left hand of the resolvend. Then proceed as before until all the periods are brought down. The quotient will be the square root required.

Note.—When there is a remainder at the termination of the process after the last period has been brought down, the operation may be continued at pleasure by annexing periods of cyphers for the formation of new resolvends: remembering always that all the figures placed in the quotient after the annexation of the first period of cyphers, are decimals.

EXAMPLES.

1. What is the square root of the square number 2025?

```
   20.25 (45 Ans.
   16
   ---
35)425
   425
   ---
```

2. What is the square root of 22071204?

```
      22.07.12.04 (4698 Ans.
      16
      ---
  86) 607
   6  516
  ---  ----
 929) 9112
   9  8361
 ---  -----
9388) 75104
      75104
      -----
```

3. What is the square root of 180000000?

```
        1.80.00.00.00 (13416 Ans.
        1
        --
   23)  80
    3   69
  ---   ---
  264) 1100
    4  1056
  ---  ----
 2681)  4400
    1   2681
 ----   -----
26826)171900
      160956
      ------
       10944
```

N. B.—If to the square of this answer we add the remainder 10944, we will obtain the given number, whose root was required.

GEOMETRY.

DEFINITIONS.

1. Geometry is that science which treats of the properties and relations of magnitudes.

2. A Point is that which has position, but not magnitude.

3. A Line is that which has length, without breadth or thickness.

N. B.—The extremities of a line are points.

4. A Straight or Right Line is the shortest line which can be drawn betweeen two points.

5. Every line which is neither straight nor composed of straight lines is a Curve Line; as A B. (*Fig.* 1.)

6. A Superficies or Surface is that which has length and breadth, without thickness; as A B C D. (*Fig.* 2.)

7. Convexity, when applied in reference to a curve line, denotes its exterior or outward part; as A B C; and Concavity, its interior or inner part; as D E F. (*Fig.* 3.)

8. An Angle is the inclination of two straight lines toward each other, which meet in a point; as A B C. (*Fig.* 4.) The point in which the straight lines meet is called the *angular point*.

Note.—When an angle is expressed by three letters, the letter denoting the angular point is always placed in the middle, between the other two; as A B C. An angle, however, is frequently expressed by one letter, which in the figure is always placed at the angular point; as B. (*Fig.* 5.)

9. When a straight line, standing on another straight line, makes the adjacent angles equal to each other, each of the angles is called a *right angle;* and the straight line which stands on the other straight line is called a *perpendicular* to it. (*Fig.* 6.)

10. A MIXED ANGLE is an angle formed by one straight line and one curved line. (*Fig.* 7.)

11. An OBTUSE ANGLE is that which is greater than a right angle. (*Fig.* 8.)

12. An ACUTE ANGLE is that which is less than a right angle. (*Fig.* 9.)

13. A FIGURE is that which is enclosed by one or more boundaries. Two straight lines cannot enclose a space. The space contained within the boundary, or boundaries, is called the *area* of the Figure.

14. A CIRCLE is a plane figure contained by one line, which is called the *circumference,* and is such that all straight lines drawn from a certain point within the figure to the circumference are equal to one another. This point is called the *centre* of the circle. Thus A B C D E, is the circumference, and F, the centre; and the lines F A, F C, F D, and F E, are all equal to each other. (*Fig.* 10.)

15. A DIAMETER of a circle is a straight line drawn through the centre, and terminated both ways by the circumference; as A D.

16. A RADIUS or SEMIDIAMETER is a straight line drawn from the centre, and terminated by the circumference; as F A.

17. An ARC or ARCH of a circle is any part of its circumference; as C D. The *chord* of an arc is the straight line which joins its extremities; as C *a* D.

18. A SEMICIRCLE is the figure contained by a diameter, and the part of the circumference cut off by it. (*Fig.* 11.)

19. The circumference of every circle is supposed to be divided into 360 equal parts, called DEGREES; each degree is supposed to be subdivided into 60 equal parts, called MINUTES; each minute into 60 equal parts, called SECONDS; and

so on. If therefore a circle, or a part of a circle, be described from the vertex of any angle as its centre, the number of degrees, minutes, &c., contained in the arc of that circle, intercepted between the lines forming the angle, is the measure of that angle. Thus in the figure at definition 14, the number of degrees, minutes, &c., contained in the arc C D, is the measure of the angle C F D.

20. PARALLEL OR COLLATERAL LINES are lines equi-distant from each other in all their parts, or lines which, being in the same plane and produced ever so far both ways, will never meet.

21. RECTILINEAL FIGURES are those which are bounded by straight lines.

22. MIXTILINEAL FIGURES are those which are bounded partly by straight and partly by curved lines.

23. TRILATERAL FIGURES OF TRIANGLES are those which are contained by three straight lines.

24. QUADRILATERAL FIGURES are those which are bounded by four straight lines.

25. MULTILATERAL FIGURES OF POLYGONS are those which are bounded by more that four straight lines. A figure of five sides is sometimes called a *Pentagon;* of six sides, a *Hexagon*, &c. If their sides are all equal they are called *Regular Polygons*,—if unequal they are denominated *Irregular Polygons.*

26. Of three-sided figures, an EQUILATERAL TRIANGLE is that which has all its sides equal. (*Fig.* 12.)

27. An ISOSCELES TRIANGLE is that which has only two sides equal. (*Fig.* 13.)

28. A SCALENE TRIANGLE is that which has three unequal sides. (*Fig.* 14.)

29. A RIGHT-ANGLED TRIANGLE is that which has a right angle. (*Fig.* 15.)

Note.—In a right-angled triangle, the side opposite the right angle is called the *Hypotenuse;* as C B. The side A B is called the *base;* and C A, the perpendicular.

30. An OBTUSE-ANGLED TRIANGLE is that which has an obtuse angle. (*Fig.* 16.)

31. An Acute-angled Triangle is that which has three acute angles. (*Fig.* 17.)

32. Of four-sided figures, a square is that which has all its sides equal, and all its angles right angles. (*Fig.* 18.)

33. A Rectangle or Oblong, is that which has all its angles right angles, but whose sides are not all equal. (*Fig.* 19.)

34. A Rhombus is that which has all its sides equal, but its angles are not right angles. (*Fig.* 20.)

35. A Rhomboid is that which has its opposite sides equal to one another, but all its sides are not equal, nor its angles right angles. (*Fig.* 21.)

Note 1.—A Rhombus is sometimes denominated an inclined square; and a Rhomboid, an inclined rectangle.

2.—Quadrilateral figures, whose opposite sides are parallel, are called Parallelograms.

36. All other four-sided figures are called Trapeziums. (*Fig.* 22.)

37. A straight line, joining the opposite points or angles of a quadrilateral figure, is called a diagonal.

Note.—When an angle of a rectilineal figure is less than two right angles, it is called a *salient* angle; and when it is greater than two right angles, it is said to be *re-entrant.*

38. Any side of a rectilineal figure may be called the base.

39. The angular point, opposite to the base of a triangle, is called the *vertex*, and the angle at that point is called the vertical angle.

40. The Altitude of any triangle or parallelogram, is a perpendicular drawn from the opposite angle or side upon the base.

POSTULATES.

1. Let it be granted that a right line may be drawn from any one point to any other point;

2. That a terminated straight line may be produced or continued in a straight line at pleasure; and

3. That a circle may be described from any centre, and with any radius.

AXIOMS.

1. Things which are equal to the same thing, are equal to one another.

2. If equals be added to equals the wholes are equal.

3. If equals be taken from equals, the remainders are equal.

4. If equals be added to unequals, the wholes are unequal.

5. If equals be taken from unequals, the remainders are unequal.

6. Things which are halves of the same thing, are equal to one another.

7. Things which are doubles of the same thing, are equal to one another.

8. Magnitudes which exactly fill the same space are equal to one another.

9. The whole is greater than its part.

10. All right angles are equal to one another.

11. Two straight lines cannot be drawn through the same point parallel to the same straight line, without coinciding with one another.

12. It is possible for a figure to exist equal in every respect to another given figure.

Explanation of Terms.

1. A *Theorem* is a truth which becomes evident by a process of reasoning called demonstration.

2. A *Problem* is a proposal of something to be done; as the construction of a figure, or the solution of a question.

3. A *Lemma* is a truth stated and demonstrated in order to assist in some following demonstration.

4. A *Proposition* is a general term, employed to denote either a Theorem, a Problem, or a Lemma.

5. An *Hypothesis* is a supposition, or a principle assumed without proof either in the enunciation of a proposition, or in the course of a demonstration.

6. A *Corollary* is a consequence which follows from one or more propositions.

7. A *Scholium* is a remark made upon some preceding

proposition or propositions, for the purpose of illustrating their connexion, their restriction, their extension, or the manner of their application.

8. An *Investigation* is a process employed for the discovery of unknown truths.

9. The *Construction* of a figure is an operation in which lines are drawn and points determined, according to certain specified conditions.

10. The *Data*, or *Premises* of a proposition, are the magnitudes, quantities, relations, and conditions stated or given, from which new relations, &c., are to be deduced, or from which a figure is to be constructed.

CONTRACTIONS EMPLOYED IN THE FOLLOWING PART OF THIS WORK.

Prob.	signifies	Problem.
Geo.	"	Geometry, Geometrical,
Trig.	"	Trigonometry, Trigonometrical,
Log.	"	Logarithm, Logarithmic.
Euc.	"	Euclid.
Theo.	"	Theorem.
Hyp.	"	Hypotenuse.
Per.	"	Perpendicular.
Men.	"	Mensuration.
Div.	"	Division.
Loc.	"	Location.
Tan.	"	Tangent.
Sec.	"	Secant.
Nat.	"	Natural.
Scho.	"	Scholium.
Rad.	"	Radius.
App.	"	Appendix.
Amp.	"	Amplitude.
Diff. Lat.	"	Difference of Latitude.
Dep.	"	Departure.
Ex.	"	Example.
Ch.	"	Chain.

L.	signifies	Link.
∠ or ∠s.	"	Angle or Angles.
△	"	Triangle.
60°, 30′, 20″	"	Sixty degrees, Thirty minutes, Twenty seconds.
S 20° W.	"	South, Twenty degrees West.

Explanation of Signs.

= is the sign of equality. It shows that the quantities between which it is placed are equal, or of equal value. Thus $A = B$ signifies that the quantity or magnitude represented by A, is equal to the quantity or magnitude represented by B.

+ (read *plus*) is the sign of addition. It indicates that the quantities between which it is written are to be added together, and the whole expression denotes the sum of the quantities between which it is placed; thus $A + B + C$ expresses the sum of the three quantities represented by these letters.

— (read *minus*) is the sign of subtraction. It denotes that the latter of the two quantities between which it is written is to be subtracted from the former; and the whole expression denotes the excess of the former above the latter; thus $A - B$, expresses the excess of the quantity represented by the letter A above the quantity represented by the letter B.

× is the sign of multiplication. It denotes that the quantities between which it is placed are to be multiplied into each other; and the whole expression denotes the product which results from their multiplication into each other, thus: $A \times B \times C$ denotes the product which results from multiplying the quantity represented by A into the quantity denoted by the letter B, and that product multiplied again by the quantity represented by C.

÷ is the sign of division. It denotes that the former of the two quantities between which it is placed is to be divided by the latter, and the whole expression denotes the quotient which will result from the division of the former quantity by the latter, thus A $\div$ B indicates that the quantity represented by A, is to be divided by the quantity represented by B. Division is also frequently expressed in the form of a fraction, by writing the quantity to be divided above the quantity by which it is to be divided, with a line between them; thus $\frac{A}{B}$ expresses the quotient of A divided by B.

: :: : is the sign of Proportion. Thus, A: B:: C: D denotes that A bears the same proportion to B, which C bears to D, and is read thus, as A is to B so is C to D.

A^2 denotes the square described on a line A. If the line is expressed by two letters, A B, then the square described upon it is denoted by the sign A B^2.

The principal signs employed in Algebra are the same with those explained above. It may be observed, however, that Algebraists generally employ the small letters of the alphabet in their calculations. Instead of $\times$ the sign of multiplication, a dot (.) is frequently employed in Algebra, or the letters are written together without any sign between them. Thus $a \times b$, $a.b$, and ab, all express the product of a multiplied by b, $\frac{a+b-c}{ax}$ expresses the quotient which results from the division of the excess of the sum of a added to b above c, by a multiplied into x. It is read thus, a plus b minus c, divided by a multiplied by x.

$x = a + b - c$ shews that the quantity represented by x is equal to the excess of the sum of a added to b above c. It is read x equal to a plus b minus c.

$\surd$ is called the *radical* sign, and denotes that some root of the quantity before which it is placed is to be extracted. Thus, $\surd a$ denotes the square root of a. $\sqrt[3]{a}$ denotes the cube root of a. Instead of the radical sign, a fraction is sometimes employed; thus, $a^{\frac{1}{2}}$ denotes the square root of a, $a^{\frac{2}{3}}$ denotes the cube root of the square of a.

Positive or *affirmative* quantities are those which are to be added, or which have the sign + before them.

Negative quantities are those which have the sign — before them.

A *co-efficient* is a letter or number prefixed to any quantity into which it is to be multiplied. In the expressions ax, $3x$, a and 3 are the co-efficients of x. When a quantity appears without any co-efficient unity or 1 is understood as being its co-efficient.

A *Vinculum* is a line —— drawn over several quantities, for the purpose of collecting them into one. Thus $\overline{a+b} \times c$ denotes that the compound quantity $a+b$ is to be multiplied by c. So in like manner $\sqrt{\overline{ab+c^2}}$ denotes the square root of the compound quantity $ab+c^2$. Instead of the vinculum, parentheses are frequently employed, thus $(a+b) \times c$, or $(a+b)\,c$.

A quantity without any sign prefixed to it is a positive quantity, the sign + being understood as placed before it.

GEOMETRICAL PROBLEMS.

PROBLEM I.

To bisect a right line A B. (*Fig.* 23.)

From A and B, as centres, with any distance A*b*, B*a* greater than half the line A B, describe two arcs, *c a d* and *c b d*, cutting each other in *c* and *d*. Through the points of intersection, *c* and *d*, draw the line, *c e d*, cutting A B in *e*. Then A*e* = *e*B.

PROBLEM II.

To raise a perpendicular from a given point C, *in a given right line* A C. (*Fig.* 24.)

CASE I.

When the point C *is at the end of the line.*

From any point *a*, out of the line with the radius *a*C, draw the arc *c* C *b*, cutting A C in *b*; from the point of intersection *b*, through the central point *a*, draw the straight line *b a c*, cutting the arc *c* C *b* in *c*; join *c* C and it will be the perpendicular required.

CASE II.

When the point is near the middle of the line.

Take the points *a* and *b*, (*Fig.* 25,) at equal distances from C, and from them as centres, with any radius greater than aC, describe arcs cutting each other in n; then draw a straight line from C, through n and it will be the perpendicular required.

PROBLEM III.

From a given point D, (*Fig.* 26,) *to let fall a perpendicular on a given right line* A B.

CASE I.

When the given point is nearly opposite to the middle of the line.

On D, as a centre with a radius sufficiently great describe an arc intersecting the line A B in m and n; then on m and n, as centres, with a radius greater than half of m n, describe arcs cutting each other in C: then draw a straight line through the points C and D, intersecting A B in e: the line e D is the perpendicular required.

CASE II.

When the given point is nearly opposite the end of the line.

Draw a straight line from D, (*Fig.* 27,) to any point m in the line A B; bisect the line D m; from the point of bisection n, with a radius n m, or n D describe the arc D C m intersecting A B in C; then join D C and the line of junction will be the perpendicular required.

PROBLEM IV.

At a given point A, (*Fig.* 28,) *in a given line* A B, *to make an angle equal to a given angle* E.

From the point E, with any radius describe an arc meeting the lines containing the angle E in a and b; with the same radius on the point A as a centre describe the arc c d: apply the distance a b on the former arc to the arc c d, from d to c; then through the points Ac draw the line A D, which will form an angle with the line A B, equal to the angle at E.

PROBLEM V.

To draw a straight line through a given point parallel to a given straight line.

Let A (*Fig.* 29,) be the given point, and B C the given straight line.

From the point A draw a straight line meeting the line B C in any point D; then at the point A form an angle D A E equal to the angle A D C, and produce the straight line E A to F, the straight line E F will be parallel to the straight line B C. (*Euc.* i. 27).

PROBLEM VI.

To describe an equilateral triangle upon a given straight line.

Let A B (*Fig.* 30,) be the given straight line.

From the points A and B with the radius A B describe arcs cutting each other in the point C. Then draw the lines A C and C B, and the figure A B C will be the triangle required.

PROBLEM VII.

To construct a triangle, the sides of which may be equal to three given straight lines A, B, *and* C. (*Fig.* 31.)

Lay off a straight line D E equal to one of the given straight lines A; on D as a centre with a radius equal to another of the given lines B, describe an arc; from E as a centre with a distance equal to the remaining given line C, describe another arc, cutting the former in F; then join D F and E F, and D E F will be the triangle required.

Scholium.—It is necessary that the sum of any two of the given lines be greater than the third line.

PROBLEM VIII.

To describe a square upon a given straight line A B. (*Fig.* 32.)

From the point A draw A D perpendicular and equal to

A B: then from the points A and D with the distance A B or A D, describe arcs intersecting each other in E. Draw the lines D E and B E, and the figure A B E D will be the square required.

PROBLEM IX.

To find a third line proportional to two given straight lines A *and* B. (*Fig.* 33.)

From any point C, draw two straight lines, the one C D equal to A, the other C E equal to B forming any angle: join D E: produce C D and C E; lay off D F equal to B. or C E, then draw F K parallel to D E, meeting C E produced in K, the line E K will be a third proportional.*

PROBLEM X.

To find a fourth proportional to three given right lines A, B, *and* C. (*Fig.* 34).

Draw two straight lines D E, and D F forming any angle E D F; upon the line D E lay off D G equal to A, and G E equal to B; and upon the line D F lay off D H equal to C: join G H: from the point E, draw E F parallel to G H, meeting the line D F in F; then H F will be a fourth proportional to the lines A, B, and C.†

PROBLEM XI.

To find a mean proportional between two given straight lines A *and* B. (*Fig.* 35).

Draw any right line C D, on it lay off C P = A and P D =

* *Demonstration.* Because D E is parallel to F K, a side of the triangle C F K, according to Euc. vi. 2, C D : D F : : C E: E K; but according to the construction C D = A, and D F = C E = B; therefore A : B : : B : E K.

† *Dem.* Because G H is parallel to E F, one of the sides of the triangle D E F, according to Euc. vi. 2, D G: G E: : D H: H F; but, according to the construction of the Figure, D G = A, G E = B, and D H = C; therefore A: B: : C : H F.

B. Bisect C D in o, and with o C or o D as radius, describe the semicircle C F D. Again from the point P, draw P F perpendicular to C D: P F will be a mean proportional between A and B: *i. e.*, C P (A) : P F :: P F : P D (B). (*Euc.* vi. 13.)

PROBLEM XII.

At a given point D, (*Fig.* 36,) *to make an angle equal to a given rectilineal angle* A B C.

From the points B and D, as centres, describe two arcs, $a\ b$, and $m\ n$; make $m\ n = a\ b$; then through the points D, n, draw the straight line D E, and through the points D, m, draw the straight line D F: the angle E D F will be equal to the angle A B C.

PROBLEM XIII.

To make an angle of any proposed number of degrees.

Draw any straight line A B, (*Fig.* 37,) take the first 60 degrees from the scale of chords,* and with this distance as a radius, describe the arc $m\ n$. From the same scale, take the chord of the proposed number of degrees and apply it to the arc from n to m; then from the point A draw the line A C through the point m, the angle C A B will be the angle required.

N. B. If the proposed angle exceed 90 degrees, lay off first one half, and then the other: *e. g.* if the proposed number of degree be 130, from the point n, towards m, lay off first 65°, to o, then from o, towards m lay off 65° more, and it will give the measure of an angle of 130°.

* A line of chords, adapted to 90°, or the fourth part of a circle, is commonly put upon the plain scale, which will be found in almost every portable case of Mathematical Instruments.

PROBLEM XIV.

To find the number of degrees contained in any given angle C A B. (*Fig.* 37).

From the angular point A, with the chord of 60°, describe the arc *m n*, intersecting the lines A C and A B, *in m* and *n*. Then take the distance *m n*, and apply it to the same line of chords and it will show the number of degrees contained in the angle C A B.

N. B. If the distance *m n* exceed 90°, the number of degrees which it contains must be ascertained by two measurements.

PROBLEM XV.

To bisect a given angle A C B, (*Fig.* 38,) *i. e., to divide it into two equal parts.*

From the angular point C, with any distance describe the arc A B, and from the points A, B, with any distance describe arcs cutting each other in *n*; then through the points C *n*, draw the straight line C *n*, and it will bisect the angle A C B, as was required.

PROBLEM XVI.

To inscribe, in a given circle, a regular polygon of any proposed number (5,) *of sides.*

Divide 360 (the number of degrees in a circle,) by the number of sides, (5,) and at the centre O (*Fig.* 39,) of the circle make an angle A O B, the number of degrees in the measure of which shall be equal to the quotient, (72;) join the points A B, and apply the chord A B to the circumference, the number of times that there are to be sides to the polygon, (5,) and they will form the figure required.

PROBLEM XVII.

To describe a parallelogram whose area and perimeter shall be respectively equal to the area and perimeter of a given triangle A B C. (*Fig.* 40.)

Produce A B to D, making B D = B C. Bisect A D in E, and draw B F parallel to A C. With the radius A E, and centre A, describe a circle intersecting B F in G, then join A G. Bisect A C in H, and draw H F parallel to A G. The parallelogram A G F H will be equal to the triangle A B C, both in area and perimeter.

PROBLEM XVIII.

To describe a circle about a triangle A B C. (*Fig.* 41.)

Bisect the line* A C by the perpendicular D E; bisect also the line C B by the perpendicular F G, intersecting the perpendicular D E in H. On H, the point of intersection, as a centre with any of the distances H A, H B, or H C, as a radius describe the circle A B C, passing through the points A, B, and C, and it will be the circle required.

PROBLEM XIX.

To construct a triangle that shall be equal to a given trapezium A B C D. (*Fig.* 42).

Draw the diagonal D B, and make C E parallel to it, meeting the side A B produced in E. Join the points D, E, and A D E will be the triangle required.

PROBLEM XX.

To describe a triangle that shall be equal to a given rectilineal figure A B C D E A. (*Fig.* 43.)

Produce the side A B both ways. Join D B, and from C draw C G parallel to D B. Join also D A, and through E draw E F parallel to D A. Then join D G, and D F, and the triangle F D G will be equal to the figure A B C D E A.

PROBLEM XXI.

To draw a square equal to a given Rectangular Parallelogram A B C D. (*Fig.* 44.)

Produce the line D A, and on the part thus produced, lay

*When the term *line* is employed, a *straight* line is always intended, unless the contrary is expressed.

off the distance A B from A; bisect the whole line in G, and on G, as a centre with the radius G D describe a semicircle D F. Produce B A to F; A F is the side of a square whose area is equal to the Rectangle A C.

PROBLEM XXII.

To describe three equal circles which shall touch, without intersecting each other.

Take any straight line A B, (*Fig.* 45,) and bisect it in D: On A and B as centres, with the distance A D, or D B, describe circles, and they will touch each other in the point D. On the line A B, draw an equilateral triangle A B C. On the angular point C, as a centre with the same distance A D or D B, describe another circle, and it will touch the other circles in the points E and F, and will also be equal to them, as was required.

PROBLEM XXIII.

To describe an ellipse, the transverse diameter or major axis A B, and the conjugate diameter or minor axis C D (Fig. 46,) being given.

On the transverse axis A B, describe two circles of such diameters that while they intersect each other in the points F and G, they will also pass through the points A and B, the extremities of the transverse axis: through the points of intersection F and G, draw the straight line O P which will bisect the transverse axis A B in E, and also be perpendicular to it. On O P lay off the conjugate diameter C D bisected in E by the transverse axis A B: then find two points in the right line O P such that if one foot of the compasses be placed in them successively, the other will describe arcs passing through the points C and D, the extremities of the conjugate axis C D, and also touch the circles G B F and G A F in the points *r*, *s*, *t*, *u*. Draw the arcs *u* C *r*, and *t* D *s*, and a figure will be formed sufficiently near an ellipse for the most of practical purposes. Where greater nicety or accuracy is required the following method may be adopted:—

Another method to describe an Ellipse.

Draw the transverse and conjugate axis A B, C D, (*Fig.* 47,) bisecting each other perpendicularly, then with half the longest diameter as a radius and centre C, describe arcs cutting A B in F, G; the points F, G, will be the foci of the ellipse. Then take two pins and fasten a thread upon them in such a way that when the thread is stretched the distances between the pins shall be equal to the length of the transverse axis A B. Fasten the pins in the foci F, G, then by moving a pin or pencil round within the thread and keeping the thread always stretched by it, a curve will be traced out forming the ellipse required.

PROBLEM XXIV.

To project lines of Chords, Signs, Tangents, Secants, &c., to any Radius.

On the line A B, (*Fig.* 48,) describe the semicircle A D B. Upon the centre C, erect the perpendicular C D, continued at pleasure to F; through B draw B E parallel to C F, and consequently perpendicular to A B; and draw the right line D B. Divide the quadrant D B into 9 equal parts, and with one foot of the compasses in B, and the distances B 10, B 20, B 30, &c., on the curve line B D transfer them to the right line D B, and it will be a LINE OF CHORDS.

From the points 10, 20, 30, &c., on the arc B D draw a line parallel to D C, and it will divide the radius C B into a LINE OF SINES, reckoning from C to B, or of VERSED SINES, if reckoned from B to C.

From the centre C through the several divisions of the quadrants D B, viz: 10, 20, 30, &c., draw right lines, until they meet the line B E, and it will be a LINE OF TANGENTS.

Transfer the distances between the centre C and the divisions on the line of Tangents to the line D F, and it will give a LINE OF SECANTS, which must be numbered from D to F.

In the figure the divisions are only given to every tenth degree; but by subdividing each of the 9 divisions, we may have a line of chords, sines, tangents, &c., graduated to degrees; and if we transfer them to lines on a rule, we shall have scales of chords, sines, &c., ready for use.

Concerning Scales of Equal Parts.

Scales of equal parts are nothing more than measures in miniature, employed for laying down upon paper, &c., any known measure as chains, yards, feet, &c.; each part on the scale answering to one chain, yard, foot, &c., and the plan will be larger or smaller, as the scale contains a smaller or a greater number of parts in an inch. If, for example, an inch be taken for a common measure and divided into any specified number of equal parts, and if these parts be continued onward in the same straight line, and if the first or the last division be subdvided into ten equal parts, and if the first or primary divisions be numbered 1, 2, 3, 4, &c., as far as the first line has been continued it will be a scale of equal parts. The numbers 1, 2, 3, 4, &c, may represent 10, 20, 30, 40, &c, and then the subdivisions will stand for 1, 2, 3, 4, &c.; or they may represent 100, 200, 300, 400, &c., and then the subdivisions will stand for 10, 20, 30, 40, &c.; or they may represent 1, 2, 3, 4, &c., and then the subdivisions will stand for the decimal numbers .1, .2, .3, .4, &c.

Of scales of equal parts there is great variety. In almost every case of Mathematical Instruments are to be found scales of equal parts, and a diagonal scale which is a scale of equal parts in a particular form, The one side is divided into lines commencing with 55, 45, 40, 35, 30, &c., expressing the number of equal parts into which an inch is divided on that scale or line. The last part of the upper line is usually changed into a line of chords, which commences at the letter C. When the given number consists of three places of figures the figure in the last or units place may be determined accurately by the diagonal scale on the opposite side.

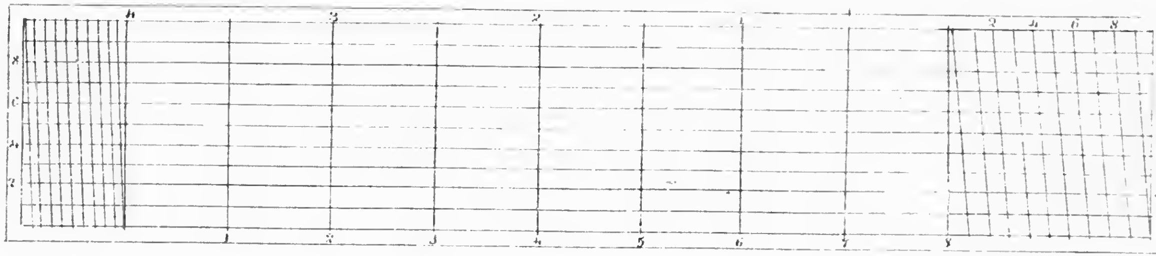
The most useful scales for a Surveyor are those of 12 or 14 inches, one of whose sides is flat and the other convex, and having on the convex side th divisions and subdivisions marked on the edges, and continued to the end of the scale. On the centre will be found the numbers 20 and 40, 25 and 50, &c., distinctly marked. (*See Plate.*)

By means of the scale of equal parts it is easy to measure any line laid down upon a plan if we only know the scale by which the plan has been drawn, and also to lay down any distance upon any given scale.

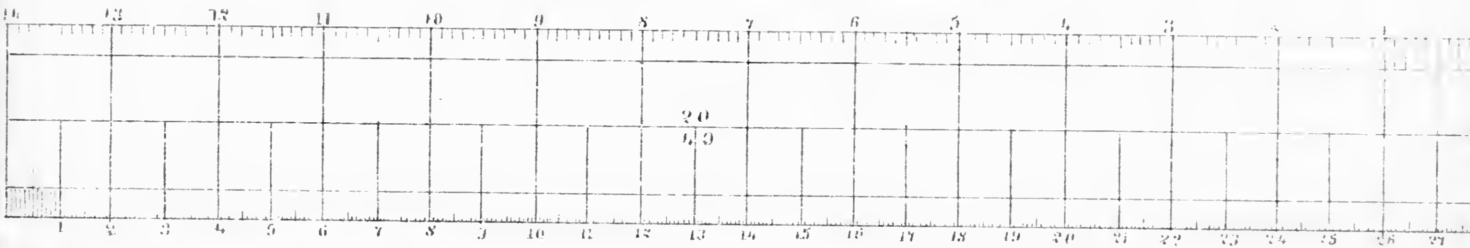
f the scale.
40, 25 and

to measure
w the scale
y down any

DIAGONAL SCALE



SCALE OF EQUAL PARTS



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LOGARITHMS.

About the end of the sixteenth century and the beginning of the seventeenth, several Mathematicians began to consider by what means they might simplify the arithmetical operations of multiplication, division, and the extraction of roots, which formed no inconsiderable obstacles to the improvement of those branches of knowledge, in the prosecution of which tedious calculations were indispensable. For abridging these calculations several ingenious expedients were suggested. Of these by far the most complete, was the system of numbers called *Logarithms*, invented by John Napier, Baron of Merchiston, in Scotland, and afterwards improved and extended by Mr. Briggs, and others; forming, doubtless, one of the happiest and most useful contrivances of modern times.

Let two series of numbers be formed, the one in geometrical progression, whose first term is unity or one, and the common ratio, 2; and the other in arithmetical progression, whose first term is 0, and the common difference 1, thus:—

Geo. Prog.	*Arith. Prog.*
1	0
2	1
4	2
8	3
16	4
32	5
64	6
128	7
256, &c.,	8, &c.

The terms in the arithmetical series will be the logarithms of the corresponding terms in the geometrical series; that is, 0 is the logarithm of 1, and 1 is the logarithm of 2, and 2 is the logarithm of 4, and 3 is the logarithm of 8, &c.

It will be evident, from a moment's inspection, that the sum of the logarithms of any two numbers in the foregoing series, is equal to the product of the numbers themselves. For example, the product of 4 multiplied by 32 is 128. Now the number in the arithmetical series corresponding to the term 4 in the geometrical series, or in other words, the logarithm of 4 is 2. In like manner the logarithm of 32 is 5. Again 2 added to 5 is equal to 7, and the term in the geometrical series corresponding to 7 in the arithmetical series is 128, or the product of 4 multiplied by 32. In like manner it is evident that the difference of the logarithms of any two numbers ÷ the logarithm of the quotient arising from the division of the one number by the other.

From these statements it appears that multiplication of natural numbers can be effected by the addition of their logarithms, and that division of natural numbers may be effected by the subtraction of their logarithms.

In the logarithmic tables usualy employed the series in geometrical progression is

1, 10, 100, 1,000, 10,000, &c.,

and the corresponding series in arithmetical progression is

0, 1, 2, 3, 4, &c.,

that is, the logarithm of 1 is 0, the logarithm of 10 is 1, the logarithm of 100 is 2, and so on.

The logarithms of the terms of the progression 1. 10, 100, 1,000, &c., being thus determined; in order to find the logarithms of the numbers between 1 and 10, and between 10 and 100, &c., we must conceive a great number of geometrical means to be interposed between each two adjoining terms of the preceding geometrical series, and as many arithmetical means between the corresponding terms of the arithmetical series. Then as the terms of the arithmetical series 0, 1, 2, 3, &c., are the logarithms of the corresponding terms of the geometrical series 1, 10, 100, 1,000, &c., the interposed terms of the former will also be the logarithms of the corresponding interposed terms of the latter.

The integral part of any logarithm, usually called its *index* or *characteristic*, is always less by 1, than the number of integers of which the natural number consists. In the logarithm of a decimal it is the integral part also, or the index, which determines the distance of the first significant figure from the decimal point. Thus, the logarithm of the

Nat. Num. 2651.	is	3.423410
265.1		2.423410
26.51		1.423410
2.651		0.423410
.2651		—1.423410 or 9.423410
.02651		—2.423410 or 8.423410
.002651		—3.423410 or 7.423410, &c.

N. B. The Negative sign (—) is frequently written over the index, instead of before it; thus, $\overline{1}.423410$.

Explanation of the Table of Logarithms of Numbers.

I. *To find the logarithm of any whole number, under* 100.

On the first page of the logarithmic Table, in the column marked N, or No. look for the given number; immediately to the right of it and in the same line with it, in the column marked Log. you will find the logarithm sought, with its proper index prefixed. Thus, the logarithm of 63 is 1.799-341; and the log. of 74 is 1.869232.

II. *To find the logarithm of any whole number between* 100 *and* 1,000.

Find the given number in the left hand column marked N, or No. and immediately opposite to it in the column marked 0 at the top and bottom you will find the decimal part of the logarithm, to which prefix the proper index and you have the logarithm required. Thus, the log. of 364 is 2.561101, and the log. of 333 is 2.522444.

III. *To find the logarithm of any number consisting of four figures.*

Find the first three figures as before, and opposite to it in the column marked at the top and bottom with the fourth figure, you will find the decimal part of the logarithm, to which the proper index must be prefixed. Thus, the Log. of .7854 is $\bar{1}.895091$, and the log. of .0795 is $\bar{2}.900367$.

IV. To find the logarithm of any number consisting of five or more places of figures.

Find the Log. of the first four figures as above directed; then take the number opposite to it in the right hand column, marked D. or Diff., (called the tabular difference, that is the difference between two adjacent logarithms). Multiply this number by the remaining figures of the natural number. From the product point off from the right hand as many figures as are contained in the multiplier, the remaining figures of the product being added to the Log. of the first four figures with the proper index prefixed will be the Log. required.

Ex. Required the Log. of 36548.

The Log. of 36540 is 4.562769, and the tabular difference 119: this tab. diff. multiplied by 8, the fifth figure, gives 952, from which strike off the last figure, and the remainder 95 added to 4.562769, will give 4.562864, the logarithm required.

N. B. If the figure, or the first of the figures cut off be 5, or any figure above five, it is usual to add 1 to the unit figure of the remainder.

V. To find the Log. of a Vulgar Fraction, or of a mixed number.

Reduce the Vulgar Fraction to a Decimal, and then proceed as directed above.

VI. To find the natural number corresponding to a given logarithm

Look for the given Log. in the Logarithmic Tables from 1000 to 10,000, without respect to the index. If you find the exact logarithm given, mark the number corresponding to it in the column on the left hand of the page marked N, or No. Then if the index of the given Log. be less than 3, cut off from the right hand of the number found as many figures as the index is less than 3, the figures so cut off will be decimals and the remainder a Nat. No. or Nat. Nos. Thus, if the Nat. No. corresponding to the logarithm 2.326-950 be required: the log. 326950 being found in the tables, opposite to it in the left column is 212, and at the top and bottom of the column in which it is found is 3, which, placed after 212, gives 2123: and since the index 2 is less by 1 than 3, one figure is to be cut off from the right hand as a decimal which will give 212.3 as the Nat. No. corresponding to the given log. 2.326950. If however the index exceed 3, annex as many cyphers to the number found as the index exceeds 4, and you have the Nat. No. required.

But if a Logarithm exactly corresponding to the given Log. cannot be found in the table, take the Log. next to the given one and less than it: then take the difference between that and the given Log., to which annex cyphers and divide it by the tabular difference found opposite to the Log. which you have taken from the table. Annex the quotient to the Nat. Nos. corresponding to the Log. taken from the table, and place the decimal point wherever the index points out, and you have the Nat. No. required. For example:—To find the Nat. No. corresponding to the Log. 4.478309; the Log. nearest to it, and less than it, is 478278, answering to the Nat. No. 3008. The difference between 478278 and the given Log. 478309 is 31. By annexing cyphers to this number and dividing by 145, the tab. diff. found opposite to the Log. 478278 you have for a quotient 213 +, which annexed to figures 3008 makes 30082132 +. But the index 4 shows that there can only be five places of whole numbers. The decimal point being therefore placed after the fifth figure, gives 30082.13 + as the Nat. No. corresponding to the given Log. 4.478309.

If the number required is to consist altogether of decimal figures, the same method must be used to obtain it as directed above; only observe, that 9 cyphers, less the index are to be prefixed to the No. found. Thus, to find the decimal

No. corresponding to the Log. 7.819083; look in the Table for the Log. 819083, and you will find the corresponding Nat. No. to be 6593 Now 9 cyphers less the index 7, leave two cyphers to be prefixed to the No. found, giving .006593 as the decimal number answering to the Log. 7.819083.

VII. *To find the Arithmetical complement of a Logarithm.*

The arithmetical complement of a Logarithm is the logarithm of the reciprocal of the corresponding natural number, or it is the number it wants of 10.000000 or 20.000000. To find it, begin at the left hand and subtract every figure from 9 except the last significant figure, which is to be subtracted from 10. If the index exceed 9 it is to be subtracted from 19, or if it be negative, it is to be added to 9, and the rest subtracted as before. In taking the sum of the Logarithms, observe that for every arithmetical complement employed, 10 must be subtracted from the sum of the indices, in order to obtain the proper index of the result. The arithmetical complement is frequently used in proportions, and in trigonometrical calculations, to change subtractions into additions.

MULTIPLICATION BY LOGARITHMS.

RULE.

Add the Logarithms of the numbers to be multiplied and their sum will be the product in Logarithms. If there be negative and affirmative indices, their difference, with the proper sign prefixed will be the index of the Log. of the product. If, in any consequence of either of the factors or of both of them being decimals, the index of the sum exceed 10, reject the 10, and the remainder will be the index of the Logarithm of the product.

EXAMPLES.

1. Required the product of 23.14 multiplied by 5.062.

The Log. of	23.14	is	1.364363
The Log. of	5.062	is	0.704322
Product,	117.134.		2.068685 Log. of the Product.

2. What is the continued product of 3.902, 597.16, and .031473?

The Log. of	3.902	is	0.591287	or	0.591287
The Log. of	597.16	is	2.776091	or	2.776091
The Log. of	.031473	is	$\bar{2}$.497938	or	8.497938
Product,	73.3357		1.865316		1.865316, rejecting 10 from the index.

DIVISION BY LOGARITHMS.

RULE.

From the Logarithm of the Dividend subtract the Logarithm of the Divisor, and the natural number answering to the remainder will be the Quotient required.

If the Log. of the divisor exceed the Log. of the dividend, proceed as before until you come to the index. If the decimal part of the Log. of the divisor exceed the decimal part of the Log. of the dividend add 1 to the index of the Log. of the divisor. Change the sign of the index of the Log. of the divisor, add the index of the Log. of the dividend to it, and with its proper sign prefixed it will be the index of the Log. of the quotient; or, when the index of the Log. of the divisor exceeds the index of the Log. of the dividend, borrow 10, and the remainder will be the index of the Log. of the quotient.

EXAMPLES.

1. Divide 4768.2 by 36.954.

The Log. of the dividend	4768.2	is	3.678354
The Log. of the divisor	36.954	is	1.567661
Quotient,	129.0307		2.110693, Log. of Quo.

2. Divide 4.6257 by .17608.

Log. of	4.6257	is	0.665177		0.665177
Log. of	.17608	is	$\bar{1}$.245710	or	9.245710
Quo.	26.2704		1.419467		1.419467, Log. of Quotient.

Here, in the first case, the index of the Log. of the divisor is changed from —1, or $\bar{1}$, to +1, or 1, and the index of the Log. of the dividend, 0, being added to it gives 1 as the index of the result. In the second case, 10 is borrowed for the index of the dividend, and the index of the divisor being subtracted from it leaves the same result.

3. Divide .19876 by .0012345.

Log. of	.19876 is	$\overline{1}.298329$	or 9.298329
Log. of	.0012345 is	$\overline{3}.091491$	or 7.091491
Quo.	161.0044	2.206838	2.206838, Log. of Quo.

Here again, the index of the Log. of the divisor is changed from -3, to $+3$, and this added to -1, the index of the dividend, gives $+2$, or 2, as the index of the Log. of the Quotient.

RULE OF THREE, OR PROPORTION BY LOGARITHMS.

RULE.

From the sum of the logarithms of the second and third terms, subtract the logarithm of the first term; the remainder will be the logarithm of the fourth term: or, add the arithmetical complement of the first term to the logarithms of the second and third terms, and the sum, after subtracting 10 from the index will be the logarithm of the fourth term.

In any Compound Proportion the term sought may be found by subtracting the sum of the logarithms of all those terms which, when multiplied into each other, are to form the divisor, from the sum of the logarithms of all the terms which, when multiplied into each other, form the dividend, the remainder is the logarithm of the term required.

Instead of subtracting one logarithm from another, you may add the arithmetical complement of the subtrahend to the logarithm of the minuend, and reject 10 from the index of the sum.

EXAMPLES.

1. If 72.34 lb. cost £2.519, what will 357.486 lb. cost?

As	72.34	Log.	1.859379	Arith.	Comp.	8.140621
Is to	2.519	Log.	0.401228	or	Log.	0.401228
So is	357.486	Log.	2.553259	or	Log.	2.553259
To	12.4483		1.095108			1.095108

2. Find a third proportional to 12.796 and 3.24718.

As	12.796	Arith. Comp.	8.892926
Is to	3.24718	Log.	0.511506
So is	3.24718	Log.	0.511506
To	.824021		$\bar{1}$.915938, third propor.

3. Find a fourth proportional to the three numbers, 36, 48, and 66.

Multiply	48	Log.	1.681241
by	66	Log.	1.819544
Divide the Pro.	3168		3.500785
by	36	Log.	1.556303
Quotient,	88		1.944482, fourth proportional.

INVOLUTION BY LOGARITHMS.

RULE.

Multiply the logarithm of the given number by the index of the power, and the product will be the logarithm of the power sought.

Note 1.—In multiplying a logarithm whose index is negative by an affirmative number, the product is negative, but the number carried from the decimal part of the logarithm to the product of the index is affirmative. Wherefore the difference between the number carried from the decimal part of the logarithm to the product of the index, and that product, with the sign of the greater number prefixed, is the true index of the Log. of the root sought.

Note 2.—When affirmative indices are employed in the logarithms of decimal fractions, prefix as many cyphers less one, as the index of the product wants of being the product of 10 multiplied by the index of the power.

EXAMPLES.

1. Required the square or second power of 25.791.

25.791	Log.	1.411468
25.791	Index.	2
665.175		2.822936

2. Required the cube or third power of 30.7146.

30.7146	Log.	1.487345
30.7146	Index	3
Cube, 28975.7		4.462035, Log. of Cube or 3rd power.

3. Required the cube or third power of .008.

.008		Log.	$\bar{3}.903090$
.008		Index	3
.000000512			$\bar{7}.709270$

Here, the index of the Log. multiplied by the index of the power gives — 9, but as the number 2 is to be carried from the decimal part of the Log. this reduces it to — 7, as above.

4. Required the fifth power of .2.

.2		Log.	9.301030
.2		Index	5
.00032			46.505150.

In this example the affirmative Log. for the decimal fraction is used. The excess of the product of 10 multiplied by 5, the index of the power, above 46 the index of the Log. is 4. This number, less one, that is 3 is the number of cyphers which must be fixed to the natural quantity corresponding to the Log. .301030.

EVOLUTION, OR THE EXTRACTION OF ROOTS BY LOGARITHMS.

RULE.

Divide the Logarithm of the given number by the index of the power, and the quotient is the root required.

Note 1.—When the index of the logarithm is negative, and the divisor is not exactly contained in it, increase the index by the smallest number that will make it exactly divisible. Carry this borrowed number as so many tens to the left hand figure of the decimal part of the Logarithm. Then proceed with the division as usual.

Note 2.—When affirmative indices are used for the logarithms of decimal fractions, prefix to the index of the Log. a figure less by 1 than the index of the power; then divide the whole by the index of the power.

EXAMPLES.

1. Required the square root of 365.

Index of the power 2)2.562293 Log. of 365.

The root required 1.281146 Log. of 19.105 *Ans.*

2. Required the Cube Root of 12345.

Index of the power 3)4.091491 Log. of 12345

1.363830 Log. of 23.1116 the root required.

3. Required the Cube Root of .000000512.

Index of the power 3)$\bar{7}$.709270 Log. of .000000512

$\bar{3}$.903090 Log. of .008 the root required.

Here the index of the Log. is not exactly divisible by the index of the Power. Two, the smallest figure which will render it exactly divisible, is added to it, making it 9. This two is then carried forward as so many tens to the decimal part of the Log. Say 3 is into 27, &c.

4. Required the fifth root of .00032.

Index of the power 5)46.505150 Log. of .00032

9.301030 Log. of .2 the root required.

Here the affirmative index to the Log. of .00032 is 6, to which a figure less by 1 than the index of the power, that is 5 — 1 = 4 is to be prefixed, making 46 as above.

Explanation of the Tables of Logarithmic Sines, Tangents, &c.

From the manner in which Lines of Chords, Sines, Tangents, &c., are projected, (*See Prob. 23 of Geometry*,) it is evident that if the Radius consist of any number of equal parts, the Sine, Tangent, Secant, &c., of every arc described on that Radius, bearing a determinate proportion to it, must also consist of a determinate proportional number of these equal parts. The computation of the number of these parts in the Sines, Tangents, &c., contained in every arc of the Quadrant, form Tables of Sines, Tangents, &c. In this form they are called Natural Sines, Tangents, Secants, &c., and the Logarithms of these numbers give us Tables of Logarithmic Sines, Tangents, Secants, &c.

To find the Logarithmic Sine, Tangent, &c., of any number of Degrees and Minutes,

If the number of degrees given be less than 45, look for them at the top of the page, then look for the number of given minutes, in the left hand column; opposite to which, in the column marked Sine, Tangent, &c., you will find the Logarithmic Sine, Tangent, &c., of the arc proposed.

If the number of degrees exceed 45, and less than 90, look for the given number of degrees at the bottom of the page, and for the minutes in the right hand column; opposite to which, in the column marked at the foot, Sine, Tangent, &c., you have the Logarithmic Sine, Tangent, &c., of the arc of the specified number of degrees.

If the number of degrees exceed 90, take out the Logarithmic Sine, Tangent, &c., of its supplement, that is of an arc consisting of the number of degrees contained in the remainder, which will result from the subtraction of the given number of degrees and minutes from 180°.

EXAMPLES.

Arcs.	Sine.	Co-Sine.	Tangents.	Co-Tang.	Secant.
18° 15′	9.495772	9.977586	9.518185	10.481815	10.022414
64° 56′	9.957040	9.627030	10.330009	9.669991	10.372970

The Natural Sine for any number of degrees and minutes will be found most readily from a Table of Natural Sines, the arrangement and uses of which must be sufficiently obvious from the explanation of the Table of Logarithmic Sines already given. When the Natural Sine and Co-sine are known, the Natural Tangent, Secant, &c., are easily calculated.

EXAMPLES.

1. Required the Nat. Sine of an arc of 23° 20′. *Ans.* .396080.

2. Required the Nat. Co-sine of an arc of 87° 15′. *Ans.* .047978.

TRIGONOMETRY.

Plane Trigonometry treats of the relations subsisting between the sides and angles of plane triangles. The principal parts of a triangle are, the three sides and the three angles. The main object of Plane Trigonometry is to give rules by which, when some of these parts are known, the others may be determined.

Triangles are either right-angled or oblique-angled. Plane Trigonometry is therefore very naturally divided into two parts. The first treats of right-angled triangles; and the second, of oblique-angled triangles.

Part I.

OF RIGHT-ANGLED PLANE TRIANGLES.

DEFINITIONS.

1. In a right-angled triangle the *Hypotenuse* is the side opposite to the right angle.
2. The *Base* is the side opposite to the vertical angle.
3. The *Perpendicular* is the side which forms a right angle with the base.

Note.—The base and perpendicular are sometimes called *legs.*

Corollary.—In a right angled triangle, if one of the acute angles is given the other angle is given also.

Demonstration.—The three angles of any plane triangle are together equal to two right angles, or to 180°. A right angle is 90°. The two acute angles therefore must together be equal to 90° also. Now, when one of the acute angles is known, the other is at once ascertained by subtracting the known angle from 90° and the remainder is the measure of the other angle, being the complement of the given angle.

In a right-angled triangle, any two of the principal parts (*i. e.* of the three sides or of the three angles) being given, and one of these given parts being a side, the other parts may be ascertained by rules easily deduced from the following theorems:—

CASE I.

When a leg and the angle opposite to it, or when two sides are given, to find the other part:

Theorem.—As any side is to the sine of its opposite angle, so is any other side to the sine of its opposite angle: and *vice versa*, as the sine of any angle is to its opposite side, so is the sine of any other angle to its opposite side.

CASE II.

When the legs, that is the sides about the right angle are given, to find the angles and the hypotenuse:

Theorem.—As one of the given sides is to the other given side, so is radius to the tangent of the acute angle at the end of the side at which the proportion commenced.

PROBLEM I.

Given the angles and hypotenuse of a right-angled plane triangle to find the base and perpendicular:

EXAMPLE.

In the triangle A B C, (*Fig.* 49,) right-angled at B, given the angle at C, 55° 30′, and the hypotenuse A C 121 yards; required the sides A B and B C.

According to the preceding corollary ∠ C 55° 30′ — ∠ B 90° = ∠ A = 34° 30′.

To find the side A B.

As radius, (*i. e.* the sine of ∠ B,)	= 10.00000
Is to the side A C 121	= 2.08278
So is the sine ∠ C 55° 30′	= 9.91599
To the perpendicular A B 99.72 yds.	= 1.99877

Here we add the logarithms of the second and third term of the proportion, and subtract the logarithm of the first term from that sum. The remainder is the logarithm of the fourth term, or the answer.

To find the side C B.

As radius	10.00000
Is to hyp. A C 121	2.08278
So is sine ∠ A 34° 30′	9.75312
To the base C B 68.54 yds.	1.83590.

PROBLEM II.

Given the angles and one side, to find the hypotenuse and the other side.

EXAMPLE.

In the right angled triangle A B C (*Fig.* 50,) right angled at B, given the angle at A 35° 30′, and the side A B 294 chains, required the base C B and the hypotenuse A C.

90° — ∠ A 35° 30′ = ∠ C 54° 30′.

To find the hypotenuse A C.		*To find the base* C B.	
As sine ∠ C 54° 30′	9.91068	As sine ∠ C 54° 30′	9.91068
Is to per. A B 294 ch.	2.46834	Is to per. A B 294 ch.	2.46834
So is radius	10.00000	So is sine ∠ A 35° 30′	9.76395
To hyp. A C 361.1 ch.	2.55766	To C B 209.7 ch.	2.32161

PROBLEM III.

Given the hypotenuse and one side, to find the angles and the other side.

EXAMPLE.

In the right angled triangle A B C, (*Fig.* 51,) right angled at B, given the hypotenuse A C 3 chains and 50 links, and the perpendicular A B 2 chains and 45 links; required the angles A and C, and the base B C.

To find the angle C.		*To find the side* B C.	
As hyp. A C 3.50	0.54407	As radius	10.00000
Is to radius	10.00000	Is to hyp. A C 3.50	0.54407
So is side A B 2.45	0.38917	So is sine ∠ A 45° 35′	9.85386
To sine ∠ C 44° 25′	9.84510	To base B C 2.499	0.39793

The hypotenuse may be found independently of the angles; for, according to Euc. i. 47. we have $A C = \sqrt{A B^2 + B C^2}$ $= \sqrt{A B \left(A B + \frac{B C^2}{A B}\right)}$. This latter form of expression A C is by far the most convenient for logarithmic calculation.

From the same property of a right angled triangle, viz: that the square of the hypotenuse is equal to the sum of the squares of the other two sides, any one of the two sides about the right angle may be found independently of the angles, if the hypotenuse and the other side are given or ascertained. For since $A C^2 = A B^2 + B C^2$, it follows that $B C^2 = A C^2 - A B^2 = (A C + A B).(A C - A B)$; and therefore $B C = \sqrt{(A C + A B).(A C - A B)}$, from which expression B C is easily determined; or, Let H denote the hypotenuse; B, the base; and P, the perpendicular: then, $H^2 = B^2 + P^2$; and $H^2 - B^2 = P^2$; and $H^2 - P^2 = B^2$.

Part II.

OF OBLIQUE-ANGLED PLANE TRIANGLES.

In an oblique-angled plane triangle, a side and any other two of the principal parts being given, the other principal parts may be ascertained.

In every plane triangle the sum of the three interior angles is equal to two right angles, or to 180°.

Corollary 1.—Two angles of any plane triangle being given, the third is also given; for it is the supplement of the other two, and may be found by subtracting their sum from 180°.

Corollary 2.—One angle of any plane triangle being given, the sum of the other two is also given, and may be found by subtracting the given angle from 180°.

The principles or rules by which unknown parts of oblique-angled triangles may be determined from those which are known, are evolved in the following theorems.

CASE I.

When a side and two angles, or when two sides and the angle opposite to one of them, are given:

Theorem.—The sides of a plane triangle are to one another as the sines of their opposite angles, and *vice versa.*

CASE II.

When two sides and the included angle are given:

Theorem.—The sum of any two sides of a plane triangle is to the difference between them, as the tangent of half the sum of the opposite angles is to the tangent of half their difference.

Scholium.—Having ascertained half the sum and half the difference of the unknown angles, the angles are easily determined; for half the difference being added to half the sum gives the greater angle; and half the difference subtracted from half the sum gives the less.

CASE III.

When the three sides are given:

Theorem.—As the base of any plane triangle is to the sum of the other two sides, so is the difference between these two sides to the difference between the segments into which the base is divided by a perpendicular let fall upon it from the opposite angle.

Scholium.—Having obtained by the preceding theorem, half the difference of the segments, the segments themselves are easily found; for half the difference added to half the sum gives the greater segment, and half the difference subtracted from half the sum gives the less.*

PROBLEM I.

Given the angles and one side of an oblique-angled triangle to find the other sides.

EXAMPLE.

In the oblique-angled triangle A B C, (*Fig.* 52,) given the angle at B 46° 22′, the angle at C 54° 15′, and consequently the angle at A 79° 23′; and the side B C 1 *ch.* 35 *l*: required the sides A B and A C.

* For the demonstration of these theorems, see the Appendix.

To find the side A B.		*To find the side* A C.	
As Sine $\angle$ A 79° 23′	9.99250	As Sine $\angle$ A 79° 23′	9.99250
Is to side B C 1.35 *l.*	2.13033	Is to side B C 1.35 *l.*	2.13033
So is Sine $\angle$ C 54° 15′	9.90932	So is Sine $\angle$ B 46° 22′	9.85960
To side A B 111.1 *l.*	2.04715	To side A C 99.4 *l.*	1.99743

PROBLEM II.

Given two sides and an angle opposite to one of them, to find the other angles and the remaining side.

EXAMPLE.

In the oblique-angled triangle A B C, (*Fig.* 53,) obtuse at B, given the side A C 3 *ch.* 18 *l.* the side B C 1 *ch.* 95 *l.* and the angle at A 32° 40′; required the angles at B and C, and the side A B.

To find the angle at B.		*To find the side* A B.	
As the side B C 195 *l.*	2.29003	As Sine $\angle$ A 32° 40′	9.73219
Is to Sine $\angle$ A 32° 40′	9.73219	Is to side B C 195 *l.*	2.29003
So is side A C 318 *l.*	2.50242	So is Sine $\angle$ C 29°	9.68557
To Sine of $\angle$ B 61° 40′	9.94458	To side A B 175.1 l.	2.24341

But by the data, B is an obtuse angle. It is therefore the supplement of an angle of 61° 40′, or 180° — 61° 40′ = 118° 20′ = $\angle$ B.

PROBLEM III.

Given two sides and the included angle to find the other angles and the remaining side.

EXAMPLE.

In the triangle A B C, (*Fig.* 54,) given the side A C 919.95 *l.* the side A B 500 *l.* and the included angle at A 36° 52′; required the angles at B and C, and the side B C.

To find the angles at B *and* C.

As A C + A B = 1419.95	3.15227
Is to A C — A B = 419.95	2.62319
So is Tan. ½ $\angle$s B + C = 71° 34′	10.47716
To Tan. ½ $\angle$s B — C 41° 35′	9.94808

Then 71° 34′ + 41° 35′ = 113° 9′ = $\angle$ B; and 71° 34′ — 41° 35′ = 29° 59′ = $\angle$ C.

To find the side B C.

As Sine ∠ C 29° 59′	9.69875
Is to side A B 500 *l.*	2.69897
So is Sine ∠ A 36° 52′	9.77811
To side B C 600.26	2.77833

PROBLEM IV.

Given the three sides of an oblique-angled plane triangle to find the angles.

EXAMPLE.

In the triangle A B C, (*Fig.* 55,) given A B 5 *ch.* 62 *l.*, A C 8 *ch.*, and B C 3 *ch.* 20 *l.*; required the angles.

To find the segments into which the base A C *is divided by a perpendicular* D B *let fall upon it from the opposite angle* B.

As the base A C 800 *l.*	2.90309
Is to A B + B C = 562 + 320 = 882	2.94546
So is A B — B C = 562 — 320 = 242	2.42619
To A D — D C 266.8	2.46756

Now $\frac{1}{2}$ (A D + D C) = 400, and $\frac{1}{2}$ (A D — D C) = 133.4; therefore 400 + 133.4 = 533.4 = A D greater segment, and 400 — 133.4 = 266.6 = D C less segment.

The segments of the base may also be obtained by the following rule:

From the sum of the squares of the two greatest sides subtract the square of the least side, and divide the remainder by twice the greatest side; the quotient will be the greatest segment.

Thus:

$$A C^2 = 640000$$
$$B C^2 = 102400$$
$$A C^2 + B C^2 = 742400$$
$$A B^2 = 315844$$
$$\frac{(A C^2 + B C^2) - A B^2 = 426556}{2 A C \quad 1600} = 266.6 = D C$$

Then

$$A C = 800$$
$$D C = 266.6$$
$$A D = 533.4$$

To find the angle at C.

As side B C 320	2.50515
Is to Rad.	10.00000
So is side D C 266.6	2.42586
To Sine ∠ C B D 56° 26′	9.92071

Then 90° — 56° 26′ = 33° 34′ = ∠ at C.

To find the angle at A.

As side A B 562	2.74973
Is to Rad.	10.00000
So is side A D 533.4	2.72705
To Sine ∠ A B D 71° 39′	9.97732

Then 90° — 71° 39′ = 18° 21′ = ∠ A.

Lastly, ∠ C B D 56° 26′ + ∠ A B D 71° 39′ = 128° 5′ = ∠ B.

MENSURATION OF HEIGHTS AND DISTANCES.

Any of the instruments employed in surveying may be used to determine lines and angles which are inaccessible. For determining vertical angles, the Quadrant is the least expensive. It is the fourth part of a circle divided into degrees, &c., and furnished with a plummet suspended from the centre, and with sights fastened upon one of its radii.

PROBLEM I.

At the distance of 3 *ch.* 10 *l.* (*Fig.* 56,) from a wall, and on a level with its foundation, the angle of elevation is observed to be 15° 40′; required the height of the wall.

As Sine ∠ C 74° 20′	9.98355
Is to the base A B 310 *l.*	2.49136
So is Sine ∠ A 15° 40′	9.43142
To height of wall B C 86.94 *l.*	1.93923

PROBLEM. II. (*Fig.* 57.)

Standing on the top of a tower 136½ feet in height, I observed a tree at a distance on the plane, a straight line to which from the top of the tower makes with the wall an angle of 67° 20′; required the distance of the tree from the bottom of the tower.

As Sine ∠ A 22° 40′	9.58587
Is to the height of the wall 136.5 *ft*,	2.13513
So is Sine ∠ C 67° 20′	9.96509
To the distance A B 326.8 *ft*.	2.51435

PROBLEM III.

Wishing to know the breadth of a river, (*Fig*. 58,) I measured for a base a straight line, 250 links in length, close to the bank. From each end of this base line I found the angles subtended by it and a tree at the brink of the river on the opposite side to be respectively 53° and 79° 12′; required the breadth of the river.

Let a perpendicular fall on the base from the opposite angle at A, the length of that perpendicular is the breadth of the river. First find the length of the side A B. Now we have ∠ B 53°, and ∠ C 79° 12′, consequently ∠ A is 47° 48′ and the side B C 250 links. Then,

To find the length of the side A B:

As Sine ∠ A 47° 48′	9.86970
Is to side B C 250 *l*.	2.39794
So is Sine ∠ C 79° 12′	9.99223
To side A B 331.5	2.52047

Now we have the right-angled triangle B A D, in which are known the side A B 331.5 links and the angle at B 53°.

Then	As Rad.	10.00000
	Is to side A B 331.5 *l*.	2.52048
	So is Sine ∠ B 53°	9.90234
	To side A D 264.7	2.42282

The perpendicular breadth of the river accordingly is 264.7 *l*.

PROBLEM IV. (*Fig*. 59.)

Wishing to know the height of, and my distance from an object apparently on a level with the place on which I stood, on the opposite side of a river; and being unable to measure backward on the same plane on account of the immediate rise of the bank, I placed a mark at the place on which I stood. I then measured a distance of 264 links up the ascending ground in a straight direction from the object. At this station it was evident that I was above the level of the object. Looking through the sights of the quadrant first to the mark at my first station, I found the angle of depression (A E D) 42°. Looking in the same way to the bottom of the object, I found the angle of depression (D B A) to be 27°. Directing the instrument in like manner to the top of the object, the angle of depression (D C F) was found to be 19°; required the height of the object and the distance between it and the mark placed at the first station.

Let fall the perpendicular D A on the straight line A B, the angle at A will be a right angle. Find first the length of the sides A D and A E.

In the triangle A E D, right-angled at A, we have the hypotenuse D E 264 links, and the ∠ E 42°; and consequently the ∠ E D A 48°.

Therefore As Rad.	10.00000
Is to hyp. 264 *l.*	2.42160
So is Sine ∠ E 42°	9.82551
To side A D 176.7	2.24711.

And As Rad.	10.00000
Is to hyp. 264 *l.*	2.42160
So is Sine ∠ A D E 48°	9.87107
To side A E 196.2	2.29267.

Find next the length of the line A B. Now we have in the right-angled triangle A D B the side A D 176.7 *l.*, and the ∠ B 27°, and consequently the ∠ A D B 63°.

Therefore As Sine ∠ B 27°	9.65704
Is to side A D 176.7 *l.*	2.24711
So is Sine ∠ A D B 63°	9.94988
To side A B 346.7 *l.*	2.53995

We found before A E 196.2 *l.*, and now find A B 346.7 *l.*, therefore B E must be 150.5 *l.*, which is the distance between the first station and the bottom of the object.

We have still to find the height of the object, or the length of the side C B. Draw C F parallel to A B, and because D A and C B are perpendicular to A B, they must be parallel to each other; and because F C is parallel to A B, F A and C B must be equal. Having already found the length of A D to be 176.7 *l.* if we can ascertain the length of D F, the length of F A, and consequently of C B will be easily determined. Now in the triangle F D C right-angled at F, we have the base F C = A B, which we have found to be 346.7 *l.*, and the angle at C 19°, and consequently the angle F D C 71°, to find the side D F.

As Sine ∠ C D F 71°	9.97567
Is to side F C or A B 346.7 *l.*	2.53995
So is Sine ∠ C 19°	9.51264
To side D F 119.4 *l.*	2.07692

It was before ascertained that A D is 176.7 *l.*, and now that D F is 119.4 *l* in length, A F must therefore be 57.3 *l.* But A F and C B are equal: C B is therefore 57.3 *l.*, which is the height of the object.

The methods of ascertaining the heights and distances of inaccessible objects are numerous. If, however, the student fully comprehends the preceding examples, he will have no difficulty in applying the principles of Trigonometry for the solution of any problem that may occur.

Wherever sufficient level space can be obtained, the following rule is applicable and expeditious:

Let the observer retire from the object until the angle of elevation is 45°, the distance between the place of observation and the object is equal to the height of the object. But if the object be inaccessible, let the observer find the point at which the angle of elevation is 26° 34′. Having marked the spot let him advance towards the object until he finds the angle of elevation is 45°; or let him retire from it until the angle is 18° 26′, in either case the distance between the first and second places of observation will be equal to the height of the object.

LAND SURVEYING.

INSTRUMENTS EMPLOYED.

I. The Chain.

The Chain is a measure consisting of a certain number of links of strong iron wire, very generally employed in surveying for the purpose of measuring lines or distances. It is in length four poles, or sixty-six feet, or one hundred links. A link is therefore 7.92 inches long. Hence it is easy to reduce any number of links to feet, and *vice versa*.

To assist in counting the number of links, when any distance does not amount to an exact number of chains, a small piece of brass, generally marked, is attached to the end of every tenth link, dividing the chain into ten equal parts.

Instead of the chain, a half chain, or, as it is often called, a two-pole chain is very frequently employed. For measuring lines on cleared level land, such as marshes, intervales, &c., the whole chain or four-pole chain is the more convenient. In British North America, however, for which this treatise is principally designed, as a great part of the business of a Surveyor consists in running lines, and making surveys in the forest, and on every variety of surface, the half chain is generally to be preferred. The reason is obvious. It is much easier to keep a two-pole chain in a straight and horizontal position, than one of twice its length.

By frequent use the rings which connect the links of the chain are apt to open, and thus the length of the chain is increased. Before proceeding to measure any line, the Surveyor should therefore carefully examine and *measure* his chain. To this point too much attention cannot be paid.

Two chain-men or chain-bearers are generally employed to carry the chain. Upon their carefulness and strict compliance with the directions of the Surveyor, the correctness of the survey in a great measure depends. Over their conduct therefore it is of great importance that he exercise a careful supervision.

Before commencing the measurement of a stationary distance, an object easily seen is to be placed at one extremity of the line to be measured. The hindmost chain-man then takes up his position at the other extremity, holding the end of the chain exactly at the end of the line. The other chain-bearer being previously furnished with, and now carrying ten pins, sharpened at one end, about ten inches in length, if the surface to be measured is smooth, and about eighteen inches in length if the surface be uneven, or if it be in the woods, and holding the other end of the chain, proceeds towards the object placed at the farthest extremity of the line. It is now the duty of the hindmost chain-bearer to direct the course of the foremost. Having advanced until the chain is stretched, the latter is directed, if necessary, by the former to move to the right hand or to the left until he is in a direct line with the object toward which they are advancing. When he covers that object from the sight of the hindmost chain-bearer, the latter knows that he is in the proper course, and with a motion of the hand, or otherwise, directs his companion to stick one of his pins exactly at the end of the chain. Both chain-bearers then advance simultaneously toward the object at the end of the line, until the hindmost arrives at the place where the pin was deposited. He then directs the foremost chain-bearer as formerly, and pulling up the pin carries it carefully with him. Thus they proceed until either the whole line is measured or until all the pins carried by the foremost chain-man are exhausted. In the former case, if the line do not contain an exact number of chains, the distance between the last pin and the object at the end of the line is measured in links. The number of pins held by the hindmost chain-bearer expresses the number of chains or half chains measured. This, with the odd links (if any) added to it, is the length of the line. In the latter case, at the end of the first ten chains, the hindmost chain-man returns all the pins to the foremost,—

a note of the transfer is taken, which is sometimes called *keeping tally*,—and the chain-bearers proceed as before, until the whole line is measured. Then the number of transfers, or tallies, each counting ten chains,—the number of pins held by the hindmost chain-bearer, counting each a chain, and the number of odd links (if any) shew the length of the line.

It must be very evident that, in a survey, much depends upon the hindmost chain-bearer. Inaccuracies very frequently occur in consequence of bad chaining. If, therefore, a Surveyor cannot procure, for a hindmost chain-bearer, an individual upon whom he can rely, he ought to act in that capacity himself. This is another point on which he cannot be too careful.

The surveyor will require likewise to caution the chain-bearers against losing any of their pins, and also to teach them that inclined planes, such as the sides of hills are to be measured horizontally, and not along the inclined plane. This subject however, we will discuss more particularly when we come to treat of the running of lines.

II. The Circumferentor.

This instrument is employed by surveyors for taking angles. It consists of several parts:

1. A brass box, about five or six inches in diameter. Within this box are; 1*st*, a brass graduated circle, the upper surface of which is divided into 360 degrees, and numbered 10, 20, 30, &c., to 360. The bottom of the box is divided into four parts or quadrants, each of which is subdivided into 90 degrees numbered from the meridian, each way to the east and west points. And 2*ndly*, a steel pin in the centre, called a centre pin or pivot, finely pointed, upon which is nicely balanced a needle, touched by a loadstone, which, when at rest, and when the box containing it is in a horizontal position, always points in a North and South direction nearly. To the bottom of this box is also attached a slide, by means of which the needle may be raised from the centre pin, or pivot, to prevent the fine point of the lat-

ter from being blunted when the instrument is carried from one place to another. The box is also covered with a glass lid to preserve the needle from being disturbed by wind or injured by rain at the time of using. When the instrument is to be carried from one place to another a brass lid or cover is placed over the glass to protect it from accident.

2. A brass index or ruler, about 11 or 12 inches in length, to the ends of which, and perpendicular to it, sights are attached, and to which the box above described is fastened by screws. In each of these sights are two apertures, or slits, the one above the other, and the one much wider than the other. In one of the sights the wide slit is uppermost; in the other, it is below. Through the widest of these apertures is placed longitudinally, a horse hair or fine silk thread, to assist in taking an observation with greater exactness.

3. Two levels at right angles to each other are attached to the bottom of the instrument, by the aid of which it may be readily levelled.

4. A ball and socket on which the instrument may be placed, and by which, with the assistance of a screw, it may easily be adjusted horizontally in any direction. The whole is supported, when used, by a common surveyor's staff.

Note.—The levelling of instruments is generally done by the *Spirit Level* or *Magnetic Needle.*

By the former, as the Needle is subject to what is called the dip, in consequence of which, there will be a difference between an instrument levelled by the Spirit Level and one levelled by the Needle; hence, the latter way will be preferable, which is done by placing the instrument so that the Needle will play freely and parallel to the bottom of the Compass box.

III. The Theodolite.

The Theodolite is a complex, but most convenient and valuable instrument. On account of its expense few Surveyors in British North America are able or disposed to purchase it. In consequence of its complexity it is difficult to give a description of it which would be at all intelligible, without an inspection and examination of the instrument itself. The following remarks, however, may serve to give some idea of its intricacy and importance. Its principal parts are,

1. The Horizontal Limb.—This consists of two circular plates, the upper or vernier plate, and the graduated limb. The former moves freely above the latter, without actual contact, and both have a horizontal motion about a vertical axis, consisting of two parts, the one external, fixed to the graduated limb, the other internal, fastened to the vernier plate.

2. The Verniers.—These are short scales on the upper plate, and on opposite sides of it, or 180° asunder. They are minutely graduated, and so placed as to subdivide the divisions of the lower plate into minutes. By the assistance of microscopes frequently placed over the verniers, the half or even the fourth of a minute may be estimated.

3. The Parallel Plates, which serve for levelling the instrument, and are held together by a ball and socket.

4. Spirit Levels.—Two of these are placed at right angles to each other, with adjusting screws on the plane of the vernier plate, to determine when the horizontal limb is truly level.

5. A Compass or Circumferentor.—This is placed upon the vernier plate, and is very useful in pointing out the meridional line, or the situation of the land.

6. The Vertical Arc and Telescope. The arc is placed on a horizontal axis, the ends of which are supported by two frames. One side of it by means of a vernier, may be read off to single minutes; the other side shews the difference between the hypotenuse and the base of a right angled triangle, or the number of links to be deducted from each chain length to reduce hypotenusal to horizontal lines.—The level, which is under and parallel to the telescope, is fastened to it at one end by a joint, and at the other end by a screw, by which that end may be elevated or depressed. There is also a screw at the jointed end for lateral adjustment. By their assistance, the level may be placed parallel to the axis or *line of collimation* of the telescope.

By the Theodolite angles whether vertical or horizontal may be measured with great accuracy. It will give the angles of a field, and the bearing of any stationary distance line from the meridian, in the same manner in which these may be obtained by the Circumferentor, and Quartered Compass. Before this instrument can be applied to practical purposes with accuracy, its parts must be adjusted to each other by means of the screws and levels. The first adjustment is that of the line of collimation. The second places the level attached to the telescope parallel to the rectified line of collimation. The third makes the axis of the horizontal limb truly vertical by means of the telescope level, which is most to be depended upon. Then the levels on the vernier plate are adjusted by their screws, so that their air bubbles may remain stationary in the middle of their tubes, while it makes a complete revolution on its axis. When these adjustments are perfect, the vernier of the vertical arc must be so set that its index will point to 0, or zero on the arc, or else its deviation from zero must be marked and applied as an index error.

Several other instruments may be employed in measuring surface, such as the *Semicircle, Plane Table, and Cross Staff,* which do not require an extended notice.

The Semicircle may be employed for taking angles. It will be observed, however, that in using this instrument only one end of the index rests upon it. The number of degrees however, marked by the other end may be obtained from the end resting upon the semicircle, by substituting 181 for 1, 182 for 2, &c., proceeding onwards to 360.

The Plane Table may also be employed for taking angles, and in many respects serves the purposes of a Theodolite or Semicircle. It is indeed a very valuable instrument, but in consequence of its clumsiness it is not at all adapted to the survey of wilderness land.

The Cross Staff is a very simple instrument, used principally for laying off perpendiculars, or offsets, on or from a straight line. In measuring fields where no obstructions lie in the way, it is very useful.

The Protractor is an instrument for laying down and measuring, with accuracy and despatch, angles upon paper, by which the use of the line of chords is superseded. It is principally employed in delineating or drawing a plan from Field Notes. It is variously formed and constructed of different materials. It usually contains three concentric circles, at such distance from each other that figures may be contained between them. The outward circle is numbered from the right to the left hand, with 10, 20, 30, &c., to 180°; the middle circle is numbered in the same direction from 180° to 360°; and the inner, from the upper edge both ways, with 10, 20, 30, &c., to 90°.

Instruments useful to a Land Surveyor.

A Case of good Pocket or Mathematical Instruments.
A Set of Feather-edged Plotting Scales.
Two or three Parallel Rulers.
A Cross Staff.
A Circumferentor.
A Sextant.
A Theodolite.
A Surveying Compass.
Measuring Chains.
A Spirit Level with a Telescope.
A Protractor.
A Quadrant.
A Copying-Glass.
An Azimuth Compass.

In selecting Mathematical Instruments particular attention should be paid to their co-aptation and adjustments. Reject those whose principal parts are immovable, for they cannot, at least without difficulty and expense, be rectified or adjusted. Whatever pains may have been bestowed upon their original construction, it need not be expected that they will continue as correct as when they came from the hand of the Mathematical Instrument Maker.

Use of the Chain.

PROBLEM I.

To reduce half chains or two-pole chains and links to whole or four-pole chains and links.

RULE I.

If the given number of half or two-pole chains be even divide it by 2, and the quotient with the given number of links annexed will be the number of chains and links required.

EXAMPLE.

In 18 half chains and 40 links how many chains and links?

2)18 40

ch. 9 40 *l.* *Ans.*

RULE II.

But if the given number of half chains be odd, divide as before, and add the remainder which is equal to 50 links to the given number of links, and it will give the number of chains and links required.

EXAMPLE.

In 15 half chains and 20 links, how many chains and links?

2)15 20

ch. 7 70 *l.* *Ans.*

PROBLEM II.

To reduce chains and links to half, or two-pole chains and links.

RULE.

Reduce the whole to links, and divide by 50; the quotient will be the number of half chains, and the remainder will be links.

EXAMPLE.

In 8 *ch.* 82 *l.* how many half chains and links?

ch.	*l.*
8	82
100	

50)882 *l.*

half chs. 17 32 *l.* *Ans.*

PROBLEM III.

To reduce chains and links to perches and decimal parts of a perch.

RULE.

Write down the chains as whole numbers, and the links as decimals; then multiply by 4, and the product will be the number of perches and decimal parts of a perch.

Note.—The reason of this rule is obvious. As there are 100 links in a chain, a link is the hundredth part of a chain. By writing them with the decimal point before them, they become decimal parts of a chain, observing only that if the number of links do not exceed 9, a cypher must be written before it, in order that it may express hundredth parts.

EXAMPLE.

In 10 *ch.* 64 *l.* how many perches?

ch.
10.64
4

42.56 *per.* *Ans.*

PROBLEM IV.

To reduce half or two-pole chains and links to perches and decimal parts of a perch.

RULE.

Reduce the given number of half chains and links to chains and links by Prob. I; then reduce them to perches and decimal parts of a perch by the preceding rule.

EXAMPLE.

In 11 half chains and 21 links how many perches?

$$\begin{array}{l} 2)11\ \ 21 \\ \hline \textit{chs.}\ 5\ \ 71\ \textit{l.} \end{array} \qquad \begin{array}{r} \textit{ch.} \\ 5.71 \\ 4 \\ \hline 22.84\ \textit{per. Ans.} \end{array}$$

PROBLEM V.

To reduce perches and decimals of a perch to chains and links.

RULE.

Divide by 4 so as to have at least two places of decimals, the whole numbers in the quotient will be chains, the first two places of decimals will be links, and the remainder will be decimals of a link.

EXAMPLE.

In 22.32 perches how many chains and links?

$$\begin{array}{l} 4)22.32 \\ \hline \textit{chs.}\ 5\ \ 58\ \textit{l. Ans.} \end{array}$$

PROBLEM VI.

To reduce perches and decimals of a perch to half or two-pole chains and links.

RULE.

Divide the whole numbers by 2, the quotient will be the number of half chains. To the remainder annex the decimals and divide by 4, the quotient will be the number of links.

EXAMPLE.

In 27.52 perches how many half chains and links?

$$\begin{array}{l} 2)27. \\ \hline \textit{half chs.}\ 13.152 \end{array}$$

$$\frac{152}{4} = 38\ \textit{l.} \quad \textit{Ans.}\ 13\ \textit{half chs.}\ 38\ \textit{l.}$$

PROBLEM VII.

To reduce chains and links to feet and decimal parts of a foot.

RULE.

Write down the chains as whole numbers, and the links as decimals; then multiply by the number of feet in a chain, and the product will be the number of feet and decimal parts of a foot required.

EXAMPLE.

In 7 *chs.* and 21 *l.* how many feet and decimal parts of a foot?

$$\begin{array}{r} 7.21 \\ 66 = \textit{feet in a chain.} \\ \hline 508.86\ \textit{ft. Ans.} \end{array}$$

PROBLEM VIII.

To reduce feet and inches to chains and links.

RULE.

Reduce the inches to the decimal of a foot, and annex it to the given number of feet, then divide by 66. Continue the division by annexing cyphers to the dividend until you have two places of decimals in the quotient. Then the whole numbers in the quotient will be the chains, and the decimals will be the links required.

EXAMPLE.

In 210 *ft.* 6 *in.* how many chains and links?

$$\frac{6.0}{12} = .5\ \textit{ft.}, \text{ then } \frac{210.50}{66} = 3.18 \text{ or } 3\ \textit{chs.}\ 18\ \textit{l.}\quad \textit{Ans.}$$

PROBLEM IX.

To take a survey by the chain only.

EXAMPLE.

Let A B C D E A (*Fig.* 60,) be a field. It is required to survey it and to lay off the angles with the chain only.

RULE.

Commence at any angular point A, and on the straight line A E measure from the point A one chain towards E, and set up a stake or other mark exactly at the end of the first chain as at *f*. In the same manner measure from the point A in a direct line towards B, one chain, at the end of which set up another stake as at *g*. Then measure the distance between *f* and *g*, and enter the same in your field book. Proceed in the measurement of the line A B, and enter the result in your field book. Measure the angle at B as you measured the angle at A. Proceed in the same way to measure all the lines which enclose the field, and also the angles, observing to take the *external* angle at D. From the data which your field notes will afford, according to the principles of Practical Geometry, already laid down in a preceding part of this work, with the aid of a scale of equal parts, a figure may be constructed in which the angles will be laid off, and which will be a correct plan or map of the field. Or: divide the field into three triangles A B D, C B D, and A E D. Then measure the sides of the triangles, in succession, and you have data by which to construct figures and lay off the angles as before.

PROBLEM X.

To find the distance of an inaccessible object by the chain only.

Let A, (*Fig.* 61,) be the position of an inaccessible object, it is required to determine its distance from the point B by the chain only.

Place any conspicuous object for a mark at B, and from it measure backwards in a straight line with A B, any convenient distance C. From B, at right angles and equal to B C, lay off B E. Complete the square B E D C. Standing at D, cause a pole to be set up at F, the point in which a straight line drawn from D to A would intersect B E. Measure the distance between E and F, the distance between F and B, will then also be known. Then, as the triangles D E F and A B F are similar: As E F is to E D, so is F B to B A.

Note.—The above proportion holds good in any parallelogram.

Another Method.

Let the distance between A and B (*Fig.* 62,) represent the width of a river, it is required to ascertain that distance by the chain only.

Make A D perpendicular to A B. Bisect A D in C. Draw D E perpendicular to A D. Measure along the line D E until you arrive at a point E, in a direct line with C B. The distance between D and E will be equal to the distance between A and B, the width of the river.

Because A C and C D are equal, and the angles at A and D right angles, it is evident that the triangles A B C and D E C are not only similar but equal, and therefore D E is equal to A B.

Note.—It is not necessary that the station A should be exactly on the brink of the river. It may be taken at any convenient distance from it. Having determined the distance between the station and the inaccessible object at the opposite edge of the river, measure carefully the distance between the station and the river's brink. This distance being subtracted from the whole distance, the remainder will be the breadth of the river.

Use of the Circumferentor.

To take field notes by the Circumferentor.

1. *By fore-sights.*

Place the instrument at any angle A, (*Fig.* 63,) as your first station; cause a staff or pole to be erected perpendicularly at the next angle B: having levelled the Circumferentor, turn the flower-de-luce, or north part of the box, to your eye. Looking through the small aperture, or slit in the sight, turn the index until you can see the staff at B, through the large slit in the opposite sight, and until the thread or hair which is in it divides or cuts that object; the degrees pointed out by the south end of the needle* will shew the number of degrees by which the stationary line is distant from a north course, counting quite round with the sun.

Having entered the number of degrees, or the bearing of the line A B in your field book, measure the line, and insert its length in chains and links in the field book likewise, under the title "distance."

Being arrived now at your second station B, cause a pole to be erected at the next angle C. Place the instrument in a horizontal position over the spot on which the object at B stood. Direct the sights to the object at C, with the north of the box to your eye. When the instrument is in such a position, that, looking through the sights the thread in the large slit of the opposite sight exactly cuts the pole at C, count your degrees to the south end of the needle, and Register them in your field book as before. Measure the line B C, and make the necessary entry in your field book. Proceed thus from angle to angle, until you arrive at the place of beginning, noting as you proceed the names of the owners of the contiguous lands, and the names of the roads, rivers, &c., which bound the lot, or intersect the boundaries which you are running.

2. By back and fore-sights.

Proceeding as directed above, before you leave the station at A, set up a stake or pole in the spot over which the circumferentor stood. Having arrived at the second station B, and levelled your instrument, with the south part of the box to your eye, direct the sights to the object at A. When the thread in the opposite sight cuts the object at A, count the degrees to the south end of the needle. If both

* Some needles are pointed at the south end, and have a small ring or cross at or near the north end; while others are pointed at both ends. The latter kind is to be preferred, as it enables the Surveyor to count the surplus numbers with greater accuracy. The insertion of an agate into the cap of the needle, that it may rest on the pivot or centre pin, is a great improvement, as it causes the needle to move or play with greater freedom.

observations have been correct, the number of degrees will be the same with the number reckoned at the first station, the direction of the index being the same. Then direct the sights to the next station, and proceed as formerly. At this station leave also an object, and when you arrive at the next station C, proceed in every respect as at the station B. In this manner proceed until you return to the first station.

N B. It was remarked before that the brass ring in the box of the Circumferentor is divided into 360°. It is numbered from the North to the West 10, 20, 30, &c., to 90°; from the North to the East, from the South to the West, and from the South to the East in the same manner.

The North, South, East, and West, called the *cardinal points*, are engraved on the bottom of the box. When the needle points directly to one of these, it is written in the field book simply N., S., E., or W. But if the needle points to any intermediate part, for example, 10 from the North towards the East, it is to be entered thus: N. 10° E.

To find the number of degrees contained in any angle formed by the two adjacent lines that bound a field.

Place the instrument on the angular point, and direct the sights along the lines or legs of the triangle. Note down their respective bearings. The number of degrees marked upon the brass ring between the points cut by the end of the needle will be the measure of the angle required. Thus, in the angle A B C, (*Fig.* 64,) having placed the circumferentor on B, and having directed the sights to A, the bearing is found to be N. 30° W. Then turning the instrument about on its stand and directing the sights to C, the bearing is S. 55° W. The number of degrees on the brass ring between N. 30° W. and S. 55° W. is 95°, which is the measure of the angle A B C.

To measure an angle of altitude by the circumferentor.

Let the glass lid be taken off, and the needle removed. Then turn the instrument on one side with the stem of the ball in the notch of the socket, so that the circle may be perpendicular to the plane of the horizon. Having suspended a plummet from the centre pin, direct the sights to the top of the object, and the complement of the number of degrees between the thread of the plummet and that part of the instrument which is next your eye will be the angle of altitude.

Use of the Theodolite.

To take the angles of a field by the Theodolite. (*Fig.* 65.)

Set the instrument on some angular point of the field as at A, then lay one end of the index to 360°, when the other end will cut 180°; turn the whole about, until the part marked "360" is from you. Direct the sights from A to E, and screw the instrument fast. Direct them then from A to B, and the degree cut by the end of the index opposite to you will be the quantity of the angle E A B; which note in your field book, with the distance A B in chains and links. Proceed to the next station at B, and place the Theodolite on the angular point, and unscrew it. Then lay the moveable index so that it shall coincide with 360 and 180, with 180 next you as before, causing the thread or hair in the sight to cut the object at A. Screw the instrument fast and direct the sights to the object at C. The degree cut by the end of the index opposite to you will be the quantity of the angle A B C. Enter it in your field book, with the distance B C. Proceed thus from station to station, until you return to the place of beginning, or first station.

Lemma.

All the angles of any polygon are together equal to twice as many right angles as the figure has sides, less four.

Let the polygon be laid off into triangles by lines drawn from any assigned point F within the figure, as by the lines F A, F B, F C, &c. Now since the three angles of every triangle are together equal to two right angles, (Euc. i. 32,) it is evident that the angles in all the triangles contained in this figure must be equal to twice as many right angles as the figure has sides. But, according to Euc. i. 13, all the angles about the point F are together equal to four right angles. Therefore the remaining angles are equal to twice as many right angles as the figure has sides, less four. If, then, the angles of a field be correctly taken in any survey, their sum will be equal to twice as many right angles, less four, as there have been stations taken in making the survey.

Use of the Protractor.

To protract or draw a plan from a field book.

It is required to protract, or draw a plan from the following notes in a field book:

Commencing at the point F, (*Fig.* 66,) running thence N. 8 chains, thence N. 15° E. 8½ chains, thence S. 80° E. 7½ chains, thence S. 15° E. 7 chains and 90 links, thence S. 20° W. 10 chains, thence N. 75° W. 8 chains and 45 links, to the first station at F, or place of beginning.

Consider what part of your paper will suit best for the first station, as at F. From that point draw a meridional, or a North and South line. Now from the field book it appears that the first course is due North, it is only necessary that, on that line from F towards A, the given distance 8 chains be laid off, from a scale of equal parts. Lay now your protractor on the meridional line, so that the centre may be exactly on the point A. As the next course is N. 15° E., from the meridional line prick off towards the East or right hand 15°, as pointed out by the chamfered edge of the protractor. Through this point from the point A, draw a blank line A B, forming, of course, with the meridional line, an angle of 15°. On that blank line lay off the given distance 8½ chains to B. Next, through the point B draw another meridian parallel to the meridional line F A. Lay the centre of the protractor on the point B, and prick off the next course S. 80° E. Through that mark or dot from B, draw the blank line B C, on which lay off the given distance 7½ chains. Proceed in the same way until you have completed the figure.

Though it is the best way, in all protractions when the courses are given, to draw meridians parallel to each other through every angle of the field, as above; yet where the angle at each station is given, the centre of the protractor may be placed on the extremity of the distances when laid off, and the number of degrees contained in the given angle pricked off for the direction of the next line.

The method of using the Protractor is obvious, from the nature and use of the Circumferentor and Theodolite.

Of the Field Book.

The accuracy of every survey, and the ease with which plans may be drawn depend, to a great extent, on the manner in which the field book is kept. The adoption of a convenient and perspicuous method of keeping field notes is therefore a matter of great importance. The subjoined form is simple, concise, and plain.

Note.—In keeping the field book it is customary and useful, where the distance line crosses a brook, lake, &c., to enter the same at the proper place by a line drawn between the respective entries, representing the course of such stream, &c., as between 35 and 40 in the 4th or N. W. course, and between 40 and 45 in the last course, in the adjoining field book, in which they are represented by straight lines, in consequence of the Printer not having lines of the proper direction.

FORM OF A FIELD BOOK.

	3	38 *ch.*
	35	
	30	
	25	
	20	
	15	
	10	
	5	
Thence	N 50° E	
	40	40 *ch.*
Boggy Land.	35	Cross Lake
	30	
	25	
	20	
	15	
	10	
	5	
Thence	N 45° W	
	30	30 *ch.*
	25	
	20	
	15	
	10	
	5	
Thence	S 85° W	
	5	25 *ch.*
	20	
	15	
	10	
	5	
Thence	South	
Estate of Mr. A. W.	4	24 *ch.*
	20	
	15	
	10	
	5	
Thence	West	

of D.
Parish of C., and County
tate of A B., Esq., in the
easterly angle of the Es-
Commenced at the most

to the place of beginning.

Road to A. W.'s Estate.	70	70 *ch.* and 7 *l.*
	65	
	60	
	55	
	50	
	45	
	40	Narrow Lake
	35	
	30	
	25	
	20	
	15	
	10	
	5	
Thence	S 3° 54′ E	
	4	34 *ch.*
	30	
	25	
	20	
	15	
	10	
	5	
Thence	S 81° E	
	35	35 *ch.*
	30	
	25	
	20	
	15	
	10	
	5	
Thence	N 60° E	
	3	38 *ch.*
	35	
	30	
	25	
	20	
	15	
	10	
	5	
Thence	N 25° W	

(CONTINUED.)

REMARKS ON THE PRECEDING FORM OF A FIELD BOOK.

The preceding notes are supposed to be entered in the field book while surveying the Estate of A. B., Esq., (*See Fig.* 67,) the courses being taken from the meridian by a circumferentor, and the stationary distances measured by a half or two-pole chain.

Determine first at what part of the Estate it will be most convenient to commence the survey. Having concluded to begin at the angle No. 1., which is the most Easterly corner in the Estate, you insert in the field book the place of beginning, thus: "Commenced at the most Easterly corner of," &c. In keeping a field book it is found most convenient to begin at the bottom of the page, and to write upwards. Set the instrument on the angular point, and take the course to the second station at the angle No. 2, which is seen to be due West. Write in your field book "Thence West," or "Thence W." Proceed to measure the distance between ∠ 1 and ∠ 2, and every time that the ten pins carried by the foremost chain-bearer are all transferred to the hindmost chain-man, insert 5 in your field notes, writing upwards; for 10 half or two-pole chains, only make 5 chains. When you have arrived at the station write down the odd chains and links, if any, and place the sum of the whole in the right hand column, thus: "24 *ch.*" Observe whose land lies to the left of the land which you are surveying, and enter a note of it in your field book, in the left hand column, thus: "Estate of Mr. A W." Observe also if any particular objects appear in the immediate vicinity and adjoining the stationary line, such as stream, fence, road, &c., and insert it also in the left hand column of your field book, thus: "Road." Proceed in this manner from station to station, until you return to the place of beginning, always carefully noting to whom the adjoining land belongs, the roads, waters, &c., which bound, the Field or Estate which you are surveying, and also the streams, lakes, &c., which you cross in running the lines, and at what part of the line you cross them; as all these particulars must be expressed and accurately laid down upon the plan which is afterwards to be drawn.

VARIATION OF THE COMPASS.

The natural magnet or loadstone was for a long time supposed to be the only body which possessed magnetic properties. It is an ore of iron, whose specific gravity is about five times that of water. Its colour is iron-black, and its lustre metallic. It is found in almost every part of the world, and occurs in beds, often of vast thickness, and of great extent. Its attractive power over small pieces of iron has been known from the remotest antiquity. It is said to be distinctly referred to by Homer, Pythagoras, Aristotle, Pliny, &c. It has been asserted that its directive power or polarity was known to the Chinese in the earliest ages, and that the needle was employed to guide travellers by land a thousand years before the commencement of the Christian Era.

To whatever amount of importance these statements may be entitled, it may nevertheless be confidently asserted, that nearly all our knowledge of the magnetic virtue, and nearly all its applications to practical purposes, are discoveries and inventions of comparatively modern date. Notwithstanding, however, of the numerous and valuable additions, which have recently been made to our acquaintance with the laws by which magnetic influences are regulated, many interesting and important points still remain to be determined. To the determination of some of these points the attention of scientific men has recently been directed. Magnetic observatories have been erected, and powerful and delicate instruments have been constructed for the advancement of this branch of Science. Members of the British Government were requested some years ago, to establish magnetic observatories not only in Britain but also in these Colonies. About the same time Baron Humbolt addressed an interesting letter to the late Duke of Sussex, President of the Royal Society, soliciting that learned body to extend in the colonies of Great Britain, the line of simultaneous observations, and to establish permanent magnetic stations either in the tropical regions on each side of the magnetic Equator, or in the high Latitudes of the Southern Hemisphere, and in Canada. I mention these facts for the purpose of attracting the attention of the North American Colonist to this interesting and important subject.

The property of magnets or magnetized bodies upon which nearly all their value depends, is their *polarity* or *directive power*. By these terms, is intended to be expressed the tendency of such bodies when suspended, or made to float on water or mercury by being placed on a thin piece of wood or cork, to assume a position in which the one end will be directed to the North and the other to the South, nearly. In consequence of their possessing this property, by their assistance, we can at any time ascertain the direction of the meridian at any given place, and the bearings of other objects in relation to that line. The magnetic instrument commonly employed for this purpose is the *Compass*. It is a matter of regret that the name of the inventor of this curious and invaluable instrument is unknown. It deserves to be written in letters of gold and to be handed down with honour to the latest ages.

Though magnetic instruments are useful in determining several curious and important points, their principal value consists in their application to the arts of Navigation and Land Surveying. By the aid of the compass the mariner guides his vessel through the trackless ocean, and establishes an intercourse with the most distant nations. By the assistance of the compass the surveyor divides large tracts of country covered with dense forests, assigns to the future occupant his portion, and determines the bounds of his habitation, or assigns to the respective claimants their just proportion of an improved and valuable estate.

It must not however be supposed that the compass, however useful, affords to the surveyor infallible direction in his operations. The magnetic effects are liable to various and varying influences and disturbances, which the surveyor must be able to detect and estimate. The source of these influences is still involved in obscurity, but the effects are well known and fully established. Of these one of the most important and therefore first deserving attention is:

The variation of the Needle.—It has already been observed that the magnet does not point North and South exactly: or, in other words, that the magnetic meridian and the terrestrial meridian seldom coincide. The deviation of the magnetic needle from a true North and South line, or from a true meridian, is called its *variation.* This deviation is continually varying. Accordingly, the variation is different in different places, and in the same place at different times. In some places there is little or no perceptible variation. In other places the variation is great. At some times it appears to be stationary at a particular place, at other times in the same place it increases or diminishes with great rapidity. At London, in the year 1576, the variation was 10° 15′ Easterly. In 1580, it had arrived at its maximum, or greatest Easterly deviation, being 11° 17′ E. The Easterly variation then began to decrease. From 1657 to 1662, no variation was perceptible. In 1666, it was 34′ Westerly. The Westerly variation continued to increase until 1815, when it arrived at its maximum or greatest Westerly deviation, being then 24° 27′ 18″ W. It has since been decreasing. In 1831 it was 24° W.

At Paris, in 1541, the north end or pole of the magnetic needle pointed 7° to the East of North. In 1580 it had attained its maximum of Easterly variation, which was 11° 30′. The Easterly variation declined gradually from that time until about 1666, when it became imperceptible. In 1667 the variation was 15′ W. The Westerly variation increased gradually until 1814, when it had arrived at 22° 54′. It has since been decreasing. In 1829 it was 22° 12½′ W.

According to the report of Dr. Gesner, Provincial Geologist, made to His Excellency Sir William MacLean George Colebrooke, R.H., Lieutenant-Governor of New-Brunswick, the variation in that Province ranges from 17° to 21° W.

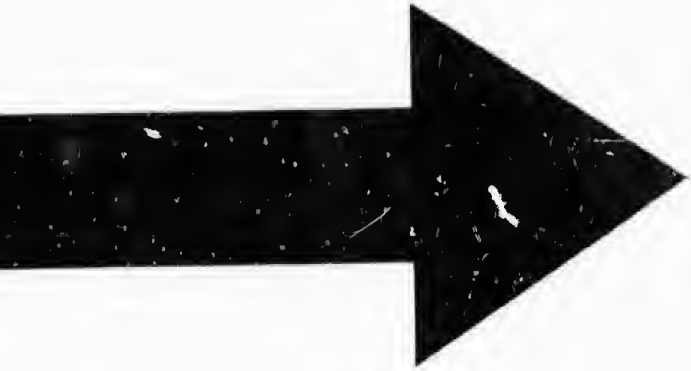
And on the 6th of Sept., 1843, I found the variation at Bay de Verte to be 18° 10′ W., nearly.

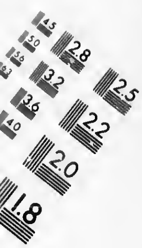
Besides these progressive changes in the variation of the compass, the needle is also subject to more minute yet appreciable oscillations, at different seasons of the year, and at different times of the day and night. The observations which have been made to determine the precise periods of the year in which this annual variation attains its maximum and minimum do not exactly correspond. Probably they are different in different parts of the world. In general, however, it may be stated to be least in winter, amounting to about 7 minutes, and greatest in summer, when it is about 13 minutes. The same discrepancy exists in the numerous observations made to determine the maximum and minimum of the daily variation. In London the general mean daily variation is 60′ 44″. In Geneva it is not quite 16′. In Nicolajef it is only 10′ 53.4″. In Kasan it amounts to 1° 10′ 36.5″.

The following diurnal variations have been observed in Paris. During the night it was nearly stationary. At sunrise its North extremity moved to the Westward, as if avoiding the solar influence. Towards noon, or more generally from noon to 3 o'clock, it attained its maximum Westerly deviation. Then it returned Eastward, till 9, 10, or 11 o'clock, when it had regained its original position, where it remained until morning. In April, May, June, July, August, and September, the daily variation was from 13′ to 15′. During the other six months it was from 8′ to 10′. On some days it amounted to 25′, while on others it did not exceed 5′ or 6′.

Hence it will follow that a line run by the compass, partly about mid-day and partly late in the afternoon or evening, will not be a straight line.

Since then the magnetic needle does not always point due North and South,—since this variation differs widely in different places, and in the same place at different times,—and since this variation is not governed by any rule hitherto dis-





covered, by which, independent of observation, it may exactly be ascertained,—it becomes a matter of great importance to the Surveyor, as well as to the Mariner, to understand, and to be able to employ the expedients necessary to determine the amount of this deviation, at any particular place, and at any specified time. In order to understand these methods, the student should make himself familiar with the meaning of the following geographical and astronomical terms:—

The Equator is a great circle, equally distant in every part of its circumference from the poles, and dividing the globe into two equal parts or hemispheres. It is called also *The Equinoctial Line.* Its distance from the pole is 90°. If the plane of the Terrestrial Equator be produced to the heavens, it will describe *The Celestial Equator.*

The Latitude of any place is its distance from the Equator reckoned in degrees and minutes. If the place is North of the Equator it is said to be in North Latitude. If it be South of the Equator it is in South Latitude. The Latitude of any place cannot exceed 90° degrees.

The Complement of the Latitude of any place is the difference between the Latitude of that place and 90°.

The Horizon is a great circle of the sphere cutting the Equator at right angles, and dividing the world into two parts.

The Rational Horizon is a great circle, cutting the Equator at right angles, and dividing the world into two equal parts, the plane of which passes through the centre of the earth.

The Sensible Horizon is a less circle, likewise at right angles to the Equator, and dividing the globe into two unequal parts. It is the circle which bounds our vision, and of which our eye is the centre.

The Declination of the sun or star is its distance from the Celestial Equator, reckoned in degrees, minutes, &c. It is North or South, according as the sun or star is to the North or South of the Equator. Declination cannot exceed 90°.

The Complement of the declination of the sun or of a star is the distance between it and the pole, and may be ascertained by subtracting the declination from 90°.

The Azimuth of the sun or of a star is an arc of the horizon, which measures the distance of the sun or a star at its rising or sitting, from the North or South cardinal points. Azimuth distances accordingly are measured on the horizon from the North or South points.

Amplitude is the complement of the Azimuth, or its defect from a right angle; or it is the distance of the sun or star at its rising or setting, from the East or West points, in degrees, minutes, &c., measured on the horizontal circle.

The Magnetic Amplitude is the amplitude indicated by the circumferentor, or quartered compass; or it is the number of degrees, &c., which the sun or star rises or sets to the North or South of the magnetic East or West points.

To find the variation of the Compass by Amplitudes.

The Latitude of the place, the Sun's Declination, and the magnetic amplitude being given, find first the true amplitude by the following rule:

As the Co-Sine of the Latitude
Is to the Sine of the Sun's Declination,*
So is radius
To the Sine of the true Amplitude.

Then, if both the magnetic and the true amplitudes be either North or South, their difference is the variation; but if the one is North and the other South, their sum is the variation.

To know whether the variation is E. or W.

Let the observer's face be turned towards the sun, and if the true amplitude is on the right-hand side of the magnetic or observed amplitude, the variation is E., but if to the left, it is W.

* The sun's declination must be taken from an *Ephemeris*. It is always contained in the *Nautical Almanack* and *Astronomical Ephemeris*, published annually by order of the LORDS COMMISSIONERS of the ADMIRALTY. It will be found also in some of the Almanacks published in the Colonies.

EXAMPLES.

1. On the 10th of August, 1842, in Lat. 46° N., the bearing of the sun at its rising was observed to be N. 85° E. by the compass; required the true amplitude and the variation of the needle.

To find the true Amplitude.

As the Co-sine of the Lat. 46°	9.84177
Is to the Sine of the Sun's Dec. 15° 35′	9.42917
So is Radius	10.00000
To the Sine of the true Amplitude 22° 45′	9.58740

The bearing of the sun by the compass being N. 85° E., the magnetic amplitude is 90° — 85° = 5° N. Therefore,

True Amplitude E. 22° 45′ N., for the Dec. is N.
Mag. Amplitude E. 5° 0′ N.

Variation 17° 45′;

which is West, because the observation being taken in the morning, and the observer's face being turned towards the sun, the true Amplitude is on the left of the magnetic, or observed Amplitude.

2. Suppose the bearing of the sun at setting to be S. 80° W., and consequently the magnetic Amplitude W. 10° S., while the true Amplitude is found to be W. 25° S. What is the variation?

True Amp. W. 25° S.
Mag. Amp. W. 10° S.

Variation 15° W.,

because the true Amplitude is to the left of the Magnetic.

3. At the time of rising, suppose the sun's bearing to be S. 75° E. by the compass, and consequently the Mag. Amp. E. 15° S., while the true Amp. is found to be E. 8° 15′ N.; required the variation.

True Amp. E. 8° 15′ N.
Mag. Amp. E. 15° 0′ S.

Variation 23° 15′ W.,

because the true Amplitude is to the left of the Magnetic.

4. Suppose the sun's bearing at setting to be S. 83° W., when the true Amp. is found to be W. 7° 30′ N.; required the variation.

True Amp. W. 7° 30′ N.
Mag. Amp. W. 7° 0′ S.

Variation 14° 30′ E.,

because the true Amplitude is to the right of the Magnetic.

To find the variation by Concentric Circles.

On a smooth board of a convenient size, (about a foot square,) draw several concentric circles. In the centre, place a pin, about 4 or 5 inches long, perpendicular to the plane, or surface of the board. Then let the board be carefully levelled. Observe in the forenoon the particular spot on any of the circles, upon which the shadow of the head of the pin rests, and mark it carefully. Observe also in the afternoon the particular spot on the same circle, on which the shadow of the same part of the pin rests; and there also make a mark. A right line joining these spots or marks will be an *East and West line.* Bisect this line, and from the centre, through the point of bisection, draw another right line, and it will be a *North and South line.* Remove the pin from the centre, and in its place substitute another, having a fine point on its upper end. On this place a magnetic needle, (the needle of a compass suits very well,) and when it has settled make a mark under the North end of the needle. From this mark draw a straight line to the centre. Then the angle formed by that line and the North and South line will be the variation.

By this method, with care and a little experience, the variation may be determined with considerable accuracy.

To find the variation by the NORTH STAR *and* ALIOTH.

The constellation *Ursa Major*, or the *Great Bear*, well known by the name of *Charles' Wain*, and in some places called the *Plough*, is situated in the Northern part of the heavens. Four remarkable stars in the body of the bear are so situated as to form a quadrilateral figure, nearly resembling a rectangle, or square, which is considered as forming the body of the wain. The two hindmost of these are called *pointers*, because a right line drawn from them towards the North will pass near the pole star in the tail of the *Little Bear*. Following this square are three other stars of about the same size, forming the tail of the Great Bear, or the handle of the plough, visible every clear night. The one next the body of the bear is *Alioth*. Now it so happens that the pole star and Alioth come to the meridian at the same time. This star is accordingly often employed by Navigators to determine the latitude. By its assistance also we may ascertain the variation of the compass with considerable accuracy, by the following method:—

About midnight, in the beginning of October, or about 8 o'clock, P. M., towards the end of November, the pole star comes to the meridian, and at the same time Alioth is at the meridian below the pole, and consequently both stars are due North. Alioth is also exactly perpendicular to the pole star, or directly below it. While Alioth is still to the Westward, let the compass be duly levelled and prepared to take its bearing. Then suspend a plummet by a white thread, in some convenient place, so that without moving the body much the plummet and compass may both be attended to. Bring the thread to cut the North star. Watch carefully until Alioth comes to the thread, and is cut by it. Then take the bearing of A th by the compass, and its divergence from the North will be the variation.

Note.—By causing the plummet to fall into a vessel of water, the wind (if there should be any at the time) will be prevented from moving it. In this way the object may be gained with greater ease and accuracy.

Besides these progressive, annual, and diurnal variations, the magnetic needle is liable to disturbances from local and from temporary causes. Of these the most important are, the existence of masses of ferruginous matter in the vicinity, the Aurora Borealis, and thunder storms.

It is well known that the magnet is powerfully affected by iron in most of its combinations. It is well known also that iron is a substance very extensively diffused, being found in one or another of its numerous forms, throughout the whole suite of rock formations. Few mineral bodies or stones, are without an admixture of it. It can be detected in clays, and sands, and even in parts of vegetable and animal substances, and in the atmosphere. Other metals, likewise, in certain conditions affect the needle. We might therefore reasonably expect that the disturbances of the magnetic influence would be a phenomenon of frequent occurrence. Doubtless numerous errors arising from this cause escape detection from their minuteness, and the inadvertence of surveyors. In hilly and broken ground, and when a survey is made over or near a bed of iron ore, or other metallic substance, the disturbance is very considerable, amounting often to several degrees. The best method of detecting departures from the right course in this case, and indeed in all cases, is the use of back-sights.

As brass frequently becomes magnetic by hammering, and is often mixed with iron, and other metallic substances, which affect the magnet, Surveyors should exercise a prudent caution in the purchase of their Instruments. I would recommend the young Surveyor especially, before parting with his money in the purchase of any magnetic instrument, to try it by the following test. Remove the needle from the instrument, and place it upon a finely pointed pin, fastened in a board. When the needle, thus balanced, has ceased to vibrate, then bring the instrument gradually near to it. If the presence of the instrument does not affect the needle, the former is in this respect faultless. But if it evidently appear that the presence of the instrument affects the needle, the instrument should be condemned.

The Aurora Borealis also exercises an irregular action on the magnetic needle, affecting it with sudden and extraordinary movements, to which Baron Humbolt has given the name of *magnetic hurricanes.* During these apparently capricious movements, the needle traverses with a shivering motion, and frequently oscillates several degrees on each side of its mean position. When the Aurora only rises a few degrees above the horizon, the disturbance is small and often inappreciable. But when the Aurora rises to the zenith, the disturbance is generally very considerable. M. Arago, who has studied the influence of the Northern Lights upon the needle with particular care, states that the part of the heavens at which all the beams or radiations of an Aurora meet and unite, is precisely the point to which a magnetic needle directs itself when suspended by its centre of gravity.

The needle may be affected by an Aurora which is invisible at the place of disturbance. M. Arago assures us that Auroras visible only in America, at St. Petersburg, and in Siberia, produced very perceptible derangements of the needle at Paris. It seems however, that some kinds of Aurora, though exceedingly brilliant and rapid in their movements, scarcely affect the magnetic needle; producing only at most, a slight tremulous motion.

The needle of the compass may be affected also by the electric fluid, previous to or during a thunder-storm. The effect of an electric shock is peculiar. Sometimes it communicates the magnetic virtue to unmagnetised iron or steel. Sometimes it entirely destroys the magnetic virtue of a magnetised body. And sometimes it reverses the poles of a magnet. It may therefore be worth while to examine magnetic instruments after violent thunder-storms.

There is still another affection of the magnet, called *the dip of the needle*, to which we must shortly advert. This expression denotes the angle which a well balanced needle forms with the horizon, after it has been magnetised, and when it is allowed to move freely in the plane of the magnetic meridian. This angle, like the angle of variation, has different values in different places; being, generally speaking, very small at the Equator, and increasing towards the poles. At the magnetic pole, which Commander Ross found to be situated in North Latitude 70° 5′ 17″, and West Longitude 96° 45′ 48″, the dip was 89° 59′, or within one minute of being perpendicular. There is much reason to suppose that every place has its own magnetic axis, with its own pole, and equator.

Like the variation, the dip of the needle also undergoes a continual change, increasing in some places and diminishing in others. At London, in 1720, the observed dip was 74° 42′, and the computed dip was 76° 27′, while in 1830 the observed dip was 69° 38′, and in 1833 the computed dip was 69° 21′.

Whenever the needle of a compass is perceived to vary from the horizontal position while resting freely on the pivot, it becomes unfit for service until it is corrected. For this purpose, make the instrument perfectly level by means of the spirit-level, which will be indicated by the air bubbles remaining in the centre. Then supply the end of the needle with an additional quantity of magnetism, until it resumes its horizontal position. Then both ends of the needle will be equally distant from the bottom of the instrument. Then too, if the compass be a good one, both ends will point to the same degree on the graduated brass circle within which it revolves. This operation should be repeated every three or four years.

THE RUNNING OF LINES.

In the running of lines four things are to be observed, viz: —*Course*, *Distance*, *Difference of Latitude*, *and Departure*.

1. THE COURSE is the angle which the line run forms with the meridian of the place from which you started.

2. THE DISTANCE is the length of the line run, reckoned in chains and links, rods, &c.

3. THE DIFFERENCE OF LATITUDE is the distance of the one end of a line from the other end, North or South, and is reckoned on a meridian.

4. THE DEPARTURE is the distance between one end of a line, and the meridian passing through its other end. It is East or West, and is measured on a parallel of Latitude.

PROBLEM I.

The Course and Distance of any line being given, to find the Difference of Latitude and the Departure.

The Distance, the Difference of Latitude, and the Departure, from a right-angled triangle. Therefore,

To find the Difference of Latitude.

As Radius
Is to the Distance,
So is the Co-Sine of the Course
To the Difference of Latitude. And,

To find the Departure.

As Radius
Is to the Distance,
So is the Sine of the Course
To the Departure.

EXAMPLE.

Let the course or bearing of the line S A (*Fig.* 68,) be N. 25° W.,* the Distance, or length of the line 71 *ch.* 20 *l.*; required the Difference of Latitude or *Northing*, and the Departure or *Westing*.

To find S. N. the Difference of Latitude.

As Radius 90°	10.00000
Is to Dist. S A 7120 *l.*	3.85248
So is the Co-Sine of the Course 25°	9.95728
To the Diff. Lat. S N 6452 *l.*	3.80976

To find A N, the Departure.

As Radius 90°	10.00000
Is to the Dist. S A 7120 *l.*	3.85248
So is the Sine of the Course 25°	9.62595
To the Departure A N 3009 *l.*	3.47843.

Thus, the Difference of Latitude, or the Northing, is ascertained to be 6452 *l.*, or 64 *ch.* and 52 *l.*; and the Departure, or Westing, to be 3009 *l.*, or 30 *ch.* and 9 *l.*

By the same method, the Difference of Latitude and the

* The Course of the line S. A. as laid down on diagram 68, is wrong.

Departure may be found after running any line, when the Course and Distance are known.

Note.—When the course is due North or South, the Difference of Latitude is equal to the Distance, and the Departure is 0; and when the Course is due East or West, the Departure and Distance are equal, and the Difference of Latitude is 0.

PROBLEM II.

The Difference of Latitude and the Departure being given, to find the Course and Distance.

To find the Course.

As the Diff. Lat.
Is to the Departure,
So is Radius
To the Tangent of the Course.

To find the Distance.

And As the Sine of the Course
Is to the Departure
So is Radius
To the Distance.

EXAMPLE.

Suppose I run in the North East quarter until my Diff. of Lat. is 6 *ch.* 85 *l.*, (*Fig.* 69,) and my Dep. 4 *ch.* 76 *l.*, wh[illegible] was the Course and Dist.

To find the Course.

As the Diff. Lat. 685 *l.*	2.83569
Is to the Dep. 476 *l.*	2.67661
So is Radius 90°	10.00000
To the Tan. of the Course 34° 44′	9.84092

To find the Distance.

As the Sine of the Course 34° 44′	9.75569
Is to the Dep. 476 *l.*	2.67661
So is Radius 90°	10.00000
To the Dist. 833.5 *l.*	2.92092

Hence the course is N. 34° 44′ E., and the stationary distance, or length of the line is 8 *ch.* 33.5 *l.*

PROBLEM III.

To find the direct Course and Distance made good in Traverse Running.

Make a Table divided into six colums. In the first column set down the several courses, and opposite to them set down in the second column their corresponding distances. The third and fourth columns are to contain the Difference of Latitude, the third being marked N., and the fourth, S. The fifth and sixth columns are to contain the Departure, the fifth being marked E., and the sixth, W.

Having entered the Course and Distance in their respective columns, find by the preceding problems, the Difference of Latitude and the Departure, for each Course and Distance, and insert them in the Table, in their proper columns opposite to the Course and Distance; observing that the Diff. of Lat. must be placed in the column marked N. when the course is Northerly, and in the column marked S. when it is Southerly; and that the Departure must be inserted in the column marked E. if the Course is Easterly, and in the column marked W. if it is Westerly. Add up the columns of Northing and Southing, and of Easting and Westing. The Difference between the sums of the N. and of the S. columns will be the whole Diff. of Lat. made good, and will be of the same name with the greater: and the difference between the sums in the E. and W. columns will be the whole Departure made good, of the same name with the larger sum. With this Diff. of Lat. and Departure find a corresponding Course and Distance. The defective columns of the Table will show the direction of the home Course.

EXAMPLE.

A surveyor on an exploration, run the following Courses and Distances, viz:—S. 22½° W. 54 *ch.*, (*Fig.* 70,) thence S. 78¾° W. 39 *ch.*, thence N. 33¾° W. 40 *ch.*, thence N. 56¼° E. 69 *ch.*,* thence N. 22½° W. 60 *ch.*; required the direct Course and Distance to the place of beginning.

* The distance on the 4th stationary line of the diagram belonging to *Fig*. 70, is wrong.

TRAVERSE TABLE.

Courses.	Dist. in chs.	Diff. of Lat. N.	Diff. of Lat. S.	Departure. E.	Departure. W.
S. 22½° W.	54		49.89		20.66
S. 78¾° W.	39		7.60		38.25
N. 33¾° W.	40	33.25			22.22
N. 56¼° E.	69	38.33		57.36	
N. 22½° W.	60	55.43			22.96
		127.01	57.49	57.36	104.09
		57.49			57.36
Diff. of Lat.		69.52 N		Dep.	46.73 W

To find the Course.

As the Diff. of Lat. 6952 *l.*	3.84204
Is to the Dep. 4673 *l.*	3.66959
So is Radius 90°	10.00000
To Tan. of Course 33° 55′	9.82755

To find the Distance.

As Sine of Course 33° 55′	9.74662
Is to the Dep. 4673 *l.*	3.66959
So is Radius 90°	10.00000
To the Dist. 8361 *l.*	3.91297

As the deficiency in the Table is in the S. and E. columns, the home course is S. 33° 55′ E., and the distance to the place of beginning 83 *ch.* 61 *l.*

PROBLEM IV.

To lay out a straight road from A *to* D, (*Fig* 71,) *instead of the crooked line* A B C D.

Set your compass at A, and take the course and distance to B, which, insert in a Traverse Table as below, according to the preceding problem. In the same manner take the courses and distances from B to C, and from C to D. Then find the course and distance from D to A, as directed in the foregoing example, and run it accordingly.

Courses.	Dist.	Diff. of Lat. N.	Diff. of Lat. S.	Departure. E.	Departure. W.
S. 80° E.	7.50		1.30	7.39	
S. 15° E.	7.90		7.63	2.05	
S. 20° W.	10.00		9.40		3.42
			18.33	9.44	3.42
				3.42	
		Diff. of Lat. S.	18.33	6.02	Dep. E.

The course from D to A is N. 18° 6′ W., and the distance 19 *ch.* 39 *l.*

By traverse running, a Surveyor being furnished with a correct outline of the boundaries of any County or Province, may lay off the courses and distances of roads through the forest, or explore a body of Wilderness Land. Traverse running may also be applied to the determination of inaccessible distances.

PROBLEM V.

To find the Difference of variation on an old line.*

If the marks made at the time of the first survey remain visible, trace the original line with your compass. Then the Difference between the Course now given by the compass, and the Course laid down upon the plan or map is the Difference of variation, since the time at which the former survey was made.

But if only the extremities of the old line, or if only two points in the line not visible from each other, can be satisfactorily determined, set your compass to the course laid down on the plan, or mentioned in the grant or deed, and run out the distance. This course and distance will generally bring you out in sight of the end of the original line. From the end of the old line, and at a right angle thereto, measure the exact distance to the line which you have just run. Then,

* *Difference of Variation* denotes the difference between the variation of the compass at the time when the line was first run, and the time when it becomes necessary to run it again, or retrace it.

As the length of the old line
Is to the Distance,
So is 57.3
To the degrees, minutes, &c., in the Diff. of Variation.

If the course bring you out to the left of the old line, the difference of variation is Westerly; if to the right, it is Easterly, and must be allowed accordingly.

Note.—In changing the course by the compass to suit the difference of variation, remember to add when the course of the line to be run is in the N. E. or S. W. quarter, and subtract when it is in the N. W. or S. E. quarter, if the difference of variation is Westerly. But if the difference of variation is Easterly, subtract when the course to be run is N. E. or S. W., and add when it is N. W. or S. E.

EXAMPLES.

1. The original course of the old line A B, (*Fig.* 72,) was N. 55° W., and length, 100 *ch.* Running that course and distance I came out 6 *ch.* to the left of B. Required the difference of variation, and the home course.

ch. *ch.*
As 100 : 6 : : 57.3° : 3.438° = 3° 26′ + the Diff. of Var. W. Then N. 45° W. — 3° 26′ W. = 41° 34′. Therefore, S. 41° 34′ E., is the home course.

The difference of variation may also be found by the 2nd case of right angled Trigonometry.

Thus, As the distance A B 100 *ch.*	2.00000
Is to the Distance B C 6 *ch.*	0.77815
So is Radius 90°	10.00000
To Tan. of ∠ B A C 3° 26′	8.77815

giving 3° 26′ for the difference of variation as before.

2. Running the old line A B, (*Fig.* 73,) by its original course N. 10° E. 50 *ch.* I came out at C, 4 *ch.* to the left; required the correct present course from B to A.

ch. *ch.*
As 50 : 4 : : 57.3° : 4° 34′ = Diff. of Var. W. Then N. 10° E. + 4° 34′ W. = N. 14° 34′ E. The course from B to A, therefore is S. 14° 34′ W.

3. Being employed to find the present course of an old line, originally run N. 20° E, (*Fig.* 74.,) one mile. By run-

ning that course and distance I came out 8 *chs.* to the right; required the correct home course.

ch. *ch.*
As 80 : 8 : : 57.3° : 5° 43′ E. Diff. of Var. Then N. 20 E. — 5° 43′ E = 14° 17′. The home course therefore is S. 14° 17′ W.

4. Being employed to trace an old line A B, (*Fig.* 75,) formerly run N. 15° W. 60 *ch.*, by running that course and distance I came out $5\frac{1}{2}$ *chs.* to the right; required the present course from A to B.

ch. *ch.*
As 60 : 5.5 : : 57.3° : $5\frac{1}{4}$° Diff. of Var. Then 15° + 5° 15′ = 20° 15′. Hence the present course of A B is N. 20° 15′ W.

PROBLEM VI.

To find the difference of variation on coeval or contemporary lines.

Ascertain by the compass the courses of a number of old lines, the more the better, then the difference between their mean and the original course is the difference of variation sought, nearly.

EXAMPLE.

In tracing the several lines of an old grant, in which the courses were laid down as running N. 20° E., I found them to be as follows, viz: N. 21° 30′ E., N. 21° 15′ E., N. 21° 40′ E., N. 21° 50′ E., N. 21° 45′ E., N. 21° 30′ E., and N 21° 15′ E.; required the difference of variation.

21° 30′ + 21° 15′ + 21° 40′ + 21° 50′ + 21° 45′ + 21° 30′ + 21° 15′ = 150° 45′

Then 150° 45′ ÷ 7 = 21° 32′ mean course. Then 21° 32′ — 20° = 1° 32′ = diff. of var. E.

Directions for blazing, and for running lines when the course is obstructed by trees, houses, hills, ravines, &c.

1. Corner trees, or bounds, are generally blazed on four sides; and the initials of the Surveyor's name, the initials

of the owner's name, and the year on which the survey was made, should be impressed on them with a marking iron.

2. Trees standing on the line are generally blazed and marked with three notches, made by striking the axe upwards.

3. Lines should be well cleared out and bushed; and all trees standing within four feet of the line, on each side, should be blazed on two places in the direction of the line. Large trees that obstruct the course, ought to be blazed and hacked exactly on the part of the tree which is cut by the course. They should likewise be blazed and hacked in the same way on the opposite side.

Suppose you are employed to run the line A B, (*Fig.* 76,) commencing at, or bounded by the spruce tree at A. Having marked the tree as may be necessary, place your compass between A and C, as close to A as may be convenient, and set it to the given course. Let the Bushman go forward and cut away all the bushes that obstruct the sight, until they arrive at the tree at C, standing exactly on the line. This tree is therefore to be blazed and hacked in the centre. It is to be blazed and hacked in the centre likewise on the opposite side. Remove the compass and place it between C and D, as near to C as may be convenient, and set it to the original course. Be careful that your compass is level, and that the back-sights cut the tree at C, on the centre. Direct the bushmen again until they arrive at the tree at D, which does not stand exactly on the line. Cause it to be blazed and hacked, quartering on the left side. Again remove your compass, and place it a little beyond D, and set it to the course. Let the bushmen proceed as before to the tree at F, which stands on the left side of the line. Cause it to be blazed and hacked on the right side. Proceed in the same manner until you arrive at the end of the distance at B. A little practice will render the whole familiar.

Again suppose you are called to run the line A B, (*Fig.* 77.,) the course of which is obstructed at *c* by a church and its enclosure. Proceed as before until you arrive at the point *c*, within a little distance of the church. Then strike off to the right or left, as may be most convenient, exactly at a right angle with your course. Run in this direction until you are fairly clear of the obstructions, as at *d*. Then resume your original course, until you can, without any obstruction, return to the first line, as at *e*. Then strike off towards the original line, at a right angle with your last course, and run the same distance towards the first line as you formerly run from it, making *ef* equal to *cd*. Add the distance *de* to the distance A*c*. Place your compass at *f*, set it to the course and proceed as before.

4. Allowance must also be made in running lines for inequalities of surface. A line run over steep hills, or across streams with high banks, will evidently be longer than it would be were the line throughout its whole extent perfectly level or horizontal. In a survey it is generally this level or base line, and not the surface line, that is required. Now as this base line cannot be determined by direct measurement, other expedients must be adopted for its determination. For this purpose different methods may be employed. Were it always practicable to measure the ascents and descents in direct lines, it would be easy to determine the base line by Trigonometry. Let the direct distance from A to B, (*Fig.* 78,) up a hill, be 88 *ch.*, and from B to C, down its opposite side, be 54 *ch.*, and the angle C A B 29° 49′, and the angle A C B 54° 8′, then the horizontal distance from A to C will be found to be 108 *ch.* But it is seldom possible to measure acclivities or declivities in a straight surface line by the chain. In certain cases the principles laid down in the *mensuration of heights and distances* might be applied to ascertain its direct length. It is, however, seldom convenient to employ this method. The usual way is, in ascending a hill to direct the hindmost chain-man to hold the chain directly over the pin left by the leader, at such an height that when the chain is stretched it will be horizontal; and in descending to direct the leader to hold his end so high that the chain, when straight, will be level, and then stick his pin perpindicularly below the end of it. To determine when the end of the chain is directly over the pin in ascending a

hill, and the exact spot in which the pin should be placed in descending, it is customary to recommend to let a stone fall from the end of the chain. The use of a line and plummet is an improvement upon this method. In ordinary cases the eye can determine when the chain is horizontal. When greater accuracy is required, the Quadrant may be employed.

In running lines over hilly ground, a two-pole or half chain is preferrable to a whole chain. It is much easier to stretch the former, until it becomes nearly straight, than the latter. In some cases it may be proper to employ even a shorter measure.

5. The manner of setting the boundaries of lands upon a road or line may be understood from the following example of a field book.

Example of a Field Book.

Each lot is 16 chains in breadth.

		and 1840.
	160	160 *chs.* to a juniper tree marked H. W.
	155	
	150	on four sides.
	145	144 *chs.* to a maple squared and hacked
	140	
	135	
	130	128 *chs.* to a pine squared.
	125	
	120	
	115	112 *chs.* to a birch squared.
	110	
	105	
	100	96 *chs.* to a spruce squared.
	95	
	90	
	85	
	80	80 *chs.* to a pine do.
	75	
	70	
	65	64 *chs.* to an ash marked do.
	60	
	55	
	50	48 *chs.* and bounded by a spruce stake
	45	
	40	
	35	32 *chs.* to a hemlock marked S. B.
	30	
	25	
	20	16 *chs.* to a fir marked T. D.
	15	
	10	
	5	
Thence N.	20°	W.

Commenced at a spruce tree marked H. W. and 1840.

Additional Remarks on the Running of Lines.

The great secret, the grand security of success in surveying, is to run the correct course, to run the lines exactly straight from bound to bound, and to measure with accuracy the distance between them. To obtain these results, every thing depends upon the quality of the surveyor's instruments, and on his skill and dexterity in using them.

As lands are frequently bounded by curved, as well as by straight lines, it is the duty of every surveyor to make himself thoroughly acquainted with the properties of curves.

It is customary to bound lands on rivers, roads, &c. When lands are to be bounded by rivers, great care should be taken to place the bounds at a proper distance from the edge of the stream or shore. Banks are liable to be undermined, and if the bound be too near the brink it may fall in, and leave its exact position in uncertainty. From this source disputes frequently arise, leading to lawsuits, and resulting in the loss of the property, peace, and character of the parties, and the reputation of the surveyor.

Difficulties likewise frequently arise from bounding lands upon roads. The road is liable to be changed for the purpose either of straightening or levelling it, and their boundaries are in danger of being removed. Surveyors therefore should, as far as possible, measure from known and well established boundaries, and run in straight lines. It is particularly desirable that the place of beginning be distinctly marked, and not liable to be removed. Lines then would be easily retraced either by the surveyor by whom they were run, or by his successors.

In the tracing and retracing of lines, great care should also be taken to follow closely the original line, especially if it be straight. No trees should be blazed except those which were blazed before. When individuals unacquainted with the properties of lines, mark trees which stand perhaps two rods to the right or left of the original line, interested parties may be deceived, or an opportunity afforded to the litigious to embroil his peaceful neighbour in the anxieties and losses of a lawsuit. The surveyor too comes in for a full share of the blame. No unqualified or unauthorized person has any right to take such liberties. By such improper conduct peaceful settlements are frequently thrown into confusion, and evils of incalculable magnitude have been produced.

MENSURATION OF LANDS.

The ***area*** or ***contents*** of any figure is the measure of the surface contained within its lines or boundaries.

In Land Measure.

16½ feet, or 25 links make	1 Rod, Pole, or Perch.
4 Rods, or 66 *ft.*, or 100 *l.*, or 22 *yds.*	1 Chain.
40 square poles or perches	1 Rood.
4 Roods. or 10 sq. chs., or 160 sq. poles	1 Acre.
10,000 Square Links	1 Square Chain.
625 Square Links	1 Square Perch.
25,000 Square Links	1 Square Rood.
100,000 Square Links	1 Square Acre.
640 Square Acres	1 Square Mile.
80 Chains in length	1 Mile.

Note.—In surveying land it will be found most convenient to take all the measures in four-pole chains and links.

PROBLEM I.

To find the area of a Parallelogram, whether it be a Square, a Rectangle, a Rhombus, or a Rhomboid.

RULE I.

Multiply the length by the perpendicular breadth, and the product will be the area.

RULE II.

As radius
Is to the sine of any angle of the parallelogram,
So is the product of the sides containing the angle
To the area of the parallelogram.

RULE. III.

Multiply the product of any two adjacent sides of the parallelogram by the natural sine of the included angle.

EXAMPLES.

1. A square tract of land, B A C D, (*Fig.* 79,) fronts on the road B A, which runs N. 80° W., and the length of its side 44 *ch.* 72 *l.*; required the area, the courses of the other sides, and a plan of the same.

A $B^2 = 44.72 \times 44.72 = 1999.8784$ *ch.* $\div 10 = 199.98784$ *ac.* $= 199$ *ac.* 3 *r.* 38 *p.* = area.

The course of A C is N. 10° E.—of C D, S. 80° E.—and of D B, S. 10° W.

Draw the road B A. Take any point on that road as at B, and lay off 44 *ch.* 72 *l.* towards A, according to the given scale. At A raise the perpendicular A C, and according to the same scale lay off 44 *ch.* 72 *l.* Draw C D equal and parallel to C A. Then join D B, and the plan is drawn.

2. Required the area of the Rectangle A G F E, (*Fig.* 80,) whose length A G is 80 *ch.*, and breadth A E 30 *ch.*

A G $\times$ A E $= 80 \times 30 = 2400$ *ch.* $= 240$ *ac.* = area.

3. Required the area of a Rhombus A B C D, (*Fig.* 81,) whose front A B runs E. 15 *ch.*, and the side B C, N. 10° E., and the perpendicular D K is 14 *ch.* 77 *l.*

$$\begin{array}{r} 1477\ l. = \text{D K.} \\ 1500\ l. = \text{A B} = \text{B C.} \\ \hline 22.15500 \\ 4 \\ \hline .62000 \\ 40 \\ \hline 24.80000 \end{array}$$

Ans. 22 *ac.* 0 *r.* 24 *p.*

4. Requ[ired] the area of a Rhomboid A B E F, (*Fig.* 82,)

whose front A B is 15 *ch.*, the side B E, 50 *ch.* 86 *l.*, and the perpendicular, 50 *ch.* 8 *l.*

5008 *l.* × 1500 *l.* = 7512000 *l.* = 75 *ac.* 0 *r.* 19 *p.* = area.

5. Required the area of the above Rhomboid the angle B A F being 80°.

By Rule 2:	As Radius 90°	10.00000
	Is to Sine ∠ B A F 80°	9.99335
	So is B A × A F 762.9 *ch.*	2.88246
	751.3 *ch.*	2.87581

751.3 *ch.* = 75 *ac.* 0 *r.* 21 *p.* = area. Or, by Rule 3:

B A × A F × Nat. Sine ∠ 80° = 762.9 *ch.* × .98481 = 751.311549 *ch.* = 75 *ac.* 0 *r.* 20.98 *p.*, or 75 *ac.* 0 *r.* 21 *p.*, nearly.

PROBLEM II.

To find the area of a Triangle.

RULE I.

Multiply one of its sides by a perpendicular let fall upon it from the opposite angle, and half the product will be the area.

RULE II.

Multiply the product of any two of its sides by the natural sine of their included angle and half the product will be the area.

EXAMPLES.

1. In the triangle A B C, (*Fig.* 83,) A B is 15 *ch.*, and the perpendicular C D let fall upon it from the opposite angle at C is 14 *ch.* 77 *l.*; required the area.

A B = 1500 *l.*
C D = 1477 *l.*

2)2215500

Area = 11.07700 *l.*
4

.30800
40

12.32000 [illegible] 11 *ac.* 0 *r.* 12 *p.*

2. In the triangle A B C (*Fig.* 84,) the side A B runs S. 80° E. 8 *ch.*, and A C runs N. 20° E. 26 *ch.* 65 *l.*; required the area.

S. 80° E.	180°
N. 20° E.	100°
100 Sum,	80° = ∠ B A C.

Then by Rule 2, A C × A B × Nat. Sine ∠ 80° = 26.65 *ch.* × 8 *ch.* × .98481 = 209.9614920 *ch.* ÷ 2 = 104.9807460 *ch.* = 10.4980746 *ac.* = 10 *ac* 2 *r.* nearly, = area.

The area may also be determined by Logarithms, according to the following rule:

As Radius
Is to the sine of any angle of a triangle
So is the product of the sides containing the angle
To twice the area of the triangle.

Then, As Radius 90°	10.00000
Is to the Sine of ∠ B A C 80°	9.99335
So is A B × A C = 26.65 × 8 = 213.2	2.32878
To twice the area 210. *ch.*	2.32213

Then 210 ÷ 2 = 105 *ch.* = 10 *ac.* 2 *r.* = area.

PROBLEM III.

The three sides of a triangle being given to find the area.

RULE.

From half the sum of the three sides subtract each side separately. Then multiply the half of the sum of the sides by the three remainders successively, and the square root of the product will be the area.

EXAMPLE.

Required the area of a triangle whose three sides are respectively 20, 30, and 40 *ch.* (*Fig.* 85.)

ch.		
20		45 — 20 = 25 1st rem.
30		45 — 30 = 15 2nd rem.
40		45 — 40 = 5 3rd rem.
2)90	sum,	
45	half-sum,	
25	1st rem.	
1125		
15	2nd rem.	
16875		
5	3rd rem.	

$\sqrt{84375} = 290.47$ *ch.* $= 29.047$ *ac.* $= 29$ *ac.* 0 *r.* 7 *p.* = area.

By Logarithms.

Half-sum	45	Log.	1.65321
1st rem.	25		1.39794
2nd rem.	15		1.17609
3rd rem.	5		0.69897
			2)4.92621 sum.
290.4 *ch.*			2.463105 square root.

Now 290.4 *ch.* = 29.04 *ac.* = 29 *ac.* 0 *r.* 6 *p.* = area.

PROBLEM IV.

To find the area of a Trapezoid.

RULE.

Add together the two parallel sides, and multiply their sum by the perpendicular breadth, or by the distance between them. Then half the product will be the area.

EXAMPLE.

Required the area of a Trapezoid A B C D, (*Fig.* 86,) whose parallel sides are 12 *ch.* 41 *l.* and 8 *ch.* 22 *l.*, and whose perpendicular distance is 5 *ch.* 15 *l.*

A B = 12.41
C D = 8.22

Sum of parallel sides 20.63
And rectangular breadth = 5.15

2)106.2445 = product.

Half prod. = 53.12225 *ch.* = 5.312225 *ac.* = 5 *ac.* 1 *r.* 10 *p.* nearly.

PROBLEM V.

To find the area of a Regular Polygon.

RULE.

Multiply the perimeter of the polygon or the sum of its sides, by the perpendicular let fall from the centre upon one of the sides, and half the product will be the area.

EXAMPLE.

The sides of the regular pentagon A B C D E A, (*Fig.* 87,) measure each 25 *ch.*, and the perpendicular O P measures 17 *ch.* 2 *l.*; required the area.

25 *ch.* × 5 = 125 *ch.* = perimeter. Then 125 *ch.* × 17.02 *ch.* = 2127.50 *ch.* ÷ 2 = 1063.75 *ch.* = 106.375 *ac.* = 106 *ac.* 1 *r.* 20 *p.* = area.

PROBLEM VI.

To find the area of a Trapezium.

RULE.

Draw a diagonal dividing the trapezium into two triangles; then find the areas of these triangles, and their sum will be the area required.

Note.—If two perpendiculars be let fall on the diagonal from the opposite angles, and their sum multiplied by the diagonal, half the product will be the area.

EXAMPLE.

In the trapezium A B C D, (*Fig.* 88.) the diagonal A C is 13 *ch.* 50 *l.*, the perpendicular D*a* 6 *ch.* 50 *l.*, and B*b* 5 *ch.* 70 *l.*; required the area.

$5.70 + 6.50 = 12.20 \times 13.50 = 164.7000 \div 2 = 82.3500$ *ch.* $= 8.235$ *ac.* $= 8$ *ac.* 0 *r.* 37 *p.* $=$ area.

PROBLEM VII.

To find the area of any Rectilineal Figure.

RULE.

Divide the figure into triangles, find their areas separately, then the sum of their areas will be the area required.

EXAMPLE.

Required the number of acres contained in the farm, the field notes of which are contained in the following Feld Book. (*Fig.* 89.)

No. of Triangle.	Bases in links.	Perpend. in links.	Double Areas in links.
1	5210	1700	8857000
2	6400	2500	16000000
3	6100	2900	17690000
4	6100	2500	15250000
5	4500	1450	6525000
	Sum of double areas		64333000

Area in links = 321.61000
4
2.44000
40

Ans. 321 *ac.* 2 *r.* 17 *p.* 17.60000

In running the base line, the perpendicular may be taken by the cross staff. Set up the cross staff at that point in the base at which, while one of the lines of the cross ranges with the base, the other points exactly to the opposite angle. Measure the distance from that point to the opposite angle, and enter in the Feld Book in the column marked *perpendiculars*. Finish the running of the base line, and insert its entire length in the column marked *bases*. Multiply the base by the perpendicular, and insert the product in the column marked *double areas*. Proceed in this manner until all the triangles of which the figure is composed are measured and computed. Divide the sum of the double areas by 2, and you have the area of the whole figure or farm.

In order to *plot* the survey it will be necessary to note the distance of the point at which the perpendicular was taken from the end of the base line.

N. B.—A cross staff with sights is to be preferred to one which has only small points at the extremities of the cross lines.

PROBLEM VIII.

To find the area of a Mixtilineal Figure, by means of equidistant ordinates.

RULE.

Measure the perpendicular breadths of the figure in several places, equidistant from each other, then divide the sum of these perpendicular breadths by their number, the quotient multiplied by the whole length of the line will give a near approximation to the area of the figure.

EXAMPLE.

The length of the base of a field, curvilinear on one side, is 7 *ch.* 20 *l.*, (*Fig.* 90.); and the lengths of seven equidistant ordinates erected upon it are respectively 200 *l.*, 225 *l.*, 230 *l.*, 248 *l.*, 260 *l.*, 280 *l.*, and 300 *l.*; required the area of the field.

	links.	Then, 249 mean breadth.
A *i* =	200	720
D *h* =	225	
E *g* =	230	1.79280 area in links.
F *f* =	248	4
G *e* =	260	
H *d* =	280	3.17120
B C =	300	40
	7)1743	6.84800
	249 mean breadth.	*Ans.* 1 *ac.* 3 *r.* 6 *p.* nearly.

If greater accuracy be required, take half the sum of the two extreme breadths for one of the ordinates, and add it to the others as before; then divide the sum by the number of parts in the base, instead of by the number of ordinates, and this mean breadth multiplied by the length of the base will give the area. It is still, however, only an approximation, but sufficiently near the truth in ordinary circumstances. It may be observed, farther, that the greater the number of ordinates employed, the nearer the result will be to the exact area.

When the curved boundary meets the base, as is often the case in surveying, the area is found by dividing the sum of all the ordinates by the number of parts in the base, and then multiplying the quotient by the length.

If it is particularly inconvenient or impracticable to erect ordinates at equal distances, perpendiculars may be raised at unequal distances, and the parts into which the figure is then divided may be computed as so many trapezoids, and the sum of their areas taken as the area of the whole.

PROBLEM IX.

To find the diameter of a circle whose circumference is given, or the circumference when the diameter is given.

RULE I.

As 7 is to 22, so is the diameter to the circumference.
As 22 is to 7, so is the circumference to the diameter.

RULE II.

As 1 is to 3.1416, so is the diameter to the circumference.
As 3.1416 is to 1, so is the circumference to the diameter.

EXAMPLES.

1. If the diameter of a circle be 1 *ch.* 12.68 *l.*, what is the length of the circumference.

As 1 : 3.1416 : : 112.68 *l.* : 354 *l.*, or 3 *ch.* 54 *l.*, *Ans.*

2. If the circumference of a circle is 3 *ch.* 54 *l.*, what is the diameter.

As 3.1416 : 1 : : 354 *l.* : 112.68 *l.*, or 1 *ch.* 12.68 *l.* *Ans.*

By this problem the number of degrees contained in the radius of a circle may be determined. For since the radius is half the diameter, and the circumference contains 360°, it follows that half the quotient of 360° divided by 3.1416 will give the number of degrees contained in the radius.

PROBLEM X.

To find the area of a circle whose diameter is given.

RULE.

Multiply the square of the diameter by .7854, and the product will be the area.

EXAMPLE.

Required the area of a circle whose diameter is 10 *ch.*

$10^2 \times .7854 = 78.5400$ *ch.* $= 7.85400$ *ac.* $= 7$ *ac.* 3 *r.* 16 *p.* $=$ area.

PROBLEM XI.

To find the area of a circle whose circumference is given.

RULE.

Multiply the square of the circumference by .07958, and the product will be the area.

EXAMPLE.

Required the area of a circle whose circumference is 31 *ch.* 40 *l.*

$3140^2 \times .07958 = 784626.96800$ *l.* $= 7$ *ac.* 3 *r.* 15 *p.* $=$ area.

PROBLEM XII.

To find the circumference of an Ellipse, the transverse and conjugate diameters being given.

RULE.

Multiply the square root of half the sum of the squares of the two diameters by 3.1416, and the product will be the circumference, nearly.

EXAMPLE.

Required the circumference of an Ellipse, whose transverse diameter is 9 *ch.*, and conjugate 6 *ch.*

$$\sqrt{\left\{\frac{9^2 + 6^2}{2}\right\}} \times 3.1416 = \sqrt{\left\{\frac{81 + 36}{2}\right\}} \times 3.1416 =$$

$\sqrt{58.5} \times 3.1416 = 7.648 \times 3.1416 = 23.9269 =$ the circumference.

PROBLEM XIII.

To determine the area of an Ellipse, the transverse and conjugate diameters being given.

RULE.

Multiply the product of the transverse and conjugate diameters by .7854, and the result will give the area.

EXAMPLE.

Required the area of an Ellipse whose transverse diameter is 9 *ch.*, and conjugate 6 *ch.*

$9 \times 6 \times .7854 = 42.4116$ *ch.* = 4 *ac.* 0 *r.* 38 *p. Ans.*

Note.—In practice, a surveyor is seldom required to measure a circle or ellipse.

PROBLEM XIV.

To find the area of a farm by drawing a plan of it from a scale of equal parts.

RULE.

On good paper, draw a correct plan by a scale of equal parts, (say a scale of 2 *ch.* to an inch,—the larger the scale employed the more accurate the work will be). Draw large lines dividing the figure into triangles, on these bases let perpendiculars fall from the opposite angles. Measure the length of these bases and perpendiculars by the same scale by which the plan was drawn, and enter them into their respective columns in the Calculation Table. Multiply the several bases by their corresponding perpendiculars, and insert their products in the column of double areas. Then half the sum of this column will be the area of the field, nearly.

EXAMPLE.

Required the area of the farm, the field notes of which are contained in the following Field Book. (*Fig.* 91.)

Field Book.

No. of Station.			Course.	Distance.
1	at	A B	N. 18° 34′ E.	4892
2	,,	B C	N. 20° E.	5000
3	,,	C D	East	4000
4	,,	D E	S. 20° E.	5100
5	,,	E F	N. 80° W.	4500
6	,,	F G	S. 15° E.	5500
7	,,	G A	West	6000

Calculation Table.

No of Trian.		Base.	Perp.	Double Area.
1	C E D	7503	2506	18802518
2	C E F	7503	2300	17256900
3	C B F	5000	2600	13000000
4	B G F	6404	2603	16669612
5	A G B	6000	4609	27654000

2)93383030

466.91515
4

3.66060
40

26.42400

Ans. 466 *ac.* 3 *r.* 26 *p.*

Remarks.—In the preceding Field Book the first column contains the No. of the stations, and consequently the number of the sides of the farm; the second, the bearings of the lines from the meridian; the third, the stationary distances in links. The second and third columns contain all the data necessary to protract the plan of the farm, A B C D E F G A.

The first column of the *Calculation Table* contains the Nos. of the triangles into which the figure is divided; the second, the base of each triangle; the third, the perpendicular; and the fourth, the product of the base and perpendicular, and is called *the column of double areas*, half the sum of which gives the area, nearly.

This method, however, will only give an approximation to the true area. It is not possible by it to determine the lengths of base and perpendicular lines within several links, especially if these lines are of considerable length.

PROBLEM XV.

To find the area of any Rectilinear Figure (the courses and distances round the same being given,) without the assistance of a plan, by Rectangular Surveying; i. e. by calculation from Tables of Northing, Southing, Easting, and Westing.

RULE.

Prepare a Table with ten columns. In the first, headed "No. of Stat.," write the number of the station, 0, 1, 2, 3, &c. In the second, headed "Bearing," write the course. In the third, marked "Dist. in *ch. l.*," insert the distance in chains and links. In the fourth and fifth columns, headed "Diff. Lat.," insert the Difference of Latitude, according to the directions contained in the III. and IV. Prob. of Transverse running of lines.

Fill up likewise, according to the same directions, the sixth and seventh columns, headed "Half Dep.," observing only that instead of the whole, the half of the Departure is to be entered. Place the sign + before the Eastings, and the sign — before the Westings. If the field notes have been correctly taken, and the entries of Diff. of Lat. and of Half. Dep. been accurately made, the sum of the North and South columns will be equal. The sum of the East and West columns will also be equal.

In the eighth column headed "Mer. Dist.," and opposite to 0 in the first column, insert the whole Departure, or double the sum of the Half-Eastings, or Half-Westings, contained in the sixth and seventh columns. Observe what sum has been entered in the first line of either of the columns headed "Half Dep.," opposite to the figure 1 in the first column. Observe also whether the entry has been made in the E. or W. column. If the entry has been made in the E. column, add the sum to the whole Departure. If the entry has been made in the W. column, subtract the sum from the whole Departure. Insert the sum or difference in the eighth column marked "Mer. Dist." Observe particularly, that the sum now inserted is the *meridional distance* *of the middle of the first line.* To this sum add or subtract according as it is East or West the same sum or Half Departure, and you have the meridional distance at the end of the first line. Observe again the sum that is entered in the column of Half Dep., opposite to the figure 2. in the first column of the Table; and according as it has the sign + or — prefixed, add it to or subtract it from the meridional distance at the end of the first line. This will give the meridional distance of the *middle* of the second line. Proceed in this manner until the column is completely filled. If the operation has been correctly performed, the last sum will be equal to the sum at the head of the column.

Next, multiply the several meridional distances at the *middle* of each line into the Northing or Southing, which will be found opposite to that meridional distance in one of the columns marked Diff. Lat. in the Table. When the sum has been taken from the column marked "N." the product is to be inserted in the ninth column, marked "North Areas;" but if it have been taken from the column marked "S." the product must be entered in the tenth column, headed "South Areas." Then the difference between the sum of the products contained in the column of North Areas and the sum of the products in the column of South Areas will be the area of the Figure.

EXAMPLES.

1. Required the area of a farm whose field notes are as follows, viz:—N. 20° E. 50 *ch.*, East 40 *ch.*, S. 20° E. 51 *ch.*, N. 80° W. 45 *ch.*, S. 15° E. 55 *ch.*, West 60 *ch.*, and N. 18° 34′ E. 48 *ch.* 92 *l.*

Having prepared your Table, and entered your stations, courses and distances in their respective columns, by the directions and principles laid down for the running of lines, or from the annexed Tables, find the Diff. Lat., and the Half Dep., and insert them in their proper place in the Table. Thus the first course is N. 20° E., and distance 50 *ch.*, the Northing is 46 *ch.* 98 *l.*, and the Half Easting 8 *ch.* 55 *l.* Insert the former in the column marked N.; and the latter n the column marked E., and place the sign + before it. Proceed in this manner until the columns of Diff. Lat. and Half Dep. are filled up.

Add up the columns of Diff. Lat. and of Half Dep. The sum of the Northings is 101.05. The sum of the Southings is also 101.05. The Eastings and Westings are likewise equal. These agreements shew that the survey has been correctly taken.

Proceed next to fill up the column headed "Mer. Dist." In this column, in the same line with 0 in the first column, write 104 *ch.* 32 *l.*, the whole departure or double the sum of the Half Eastings or Half Westings. Under the heading "Half Dep.," and in the column marked E., you will find the sum + 8.55 has been entered. Then 104.32 + 8.55 = 112.87. This sum is the meridional distance at the *middle* of the first line. Insert it in the column of Mer. Dist., opposite to 1 in the first column. Then to this sum add 8.55, and you have 121.42, the meridional distance at the end of the first line, or at the beginning of the second line. Place this sum in the column of Mer. Dist., perpendicularly below the Mer. Dist. at the middle of the first line. Again to the sum last entered add 20.00, which you will find in the E. Column, of Half Dep., and it will give you the meridional distance at the *middle* of the second line. Thus 121.42 + 20.00 = 141.42. Insert this sum in the column of Mer. Dist., immediately below the last entry, and directly opposite to 2 in the first column. Proceed in this manner until the column is filled up. The last sum must be equal to the first, or the sum at the head of this column.

Next, multiply the meridional distance at the middle of the first line, which in this example is 112.87, by the difference of latitude which will be found under "Diff. Lat." in the column marked N., and which in this Example is 46.98; and insert the product 5302.6326 in the column of North Areas, because the Diff. Lat. is N. The second meridional distance in the middle of the line in this column of Mer. Dist. is 141.42, but as the course is due East there is no difference of Latitude, and consequently no product to be inserted in either column of areas. The third meridional distance in the middle of the line is 170.14, and the difference of Latitude is 74.92 in the column marked S. The product of these numbers is 8153.1088, and is to be placed in the column of South Areas, because the Diff. Lat. is S. Proceed in this manner until these columns are completed.

N. B.—The meridional distance for the *middle* of the line will always be found in the column of Mer. Dist., opposite to, and in the same line with, the No. of the Station, the Bearing, Distance, Diff. Lat. and Half. Dep.

The sum of the products contained in the column of North Areas is 10989.0006, and the sum of the products in the column of South Areas 15679.5046. The difference between them 4690.5040 *ch.*, or 469 *ac.* 0 *r.* 8 *p.* is the area of the Farm.

EXAMPLE I.

CALCULATION TABLE.*

No. of Stat.	Bearing.	Dist. I.	N.	S.	E.	W.	Mer. Dist.	North	South
		ch. lin.	Diff.	Lat.	Half.	Dep.		Areas.	Areas.
0							104.32		
1	N. 20° E.	50.00	46.98		+ 8.55		112.87	5302.6826	
							121.42		
2	East	40.00			+20.00		141.42		
							161.42		
3	S. 20° E.	51.00		47.92	+ 8.72		170.14		8153.1088
							178.86		
4	N. 80° W.	45.00	7.81			— 22.16	156.70	1223.8270	
							134.54		
5	S. 15° E.	55.00		53.15	+ 7.12		141.66		7526.3958
							148.78		
6	West	60.00				— 30.00	181.78		
							88.78		
7	N. 18° 34′ E.	48.92	46.26		+ 7.77		96.55	4462.5410	
							104.32		
			101.05	101.05	52.16	52.16		10989.0006	15679.5046
						2			
						104.32			

* From the above Table a plan or map may be constructed in the following manner. Draw the line N S, (*Fig.* 92.) for a North and South line. On N S assume any point as at A. From A to 1, lay the Diff. Lat. of Stat. 7, which is 46.26. Again from 1 to 2 lay off the Diff. Lat. 46.98. Again from 2 to 3 lay off 47.92. And from 3

EXAMPLE II.

CALCULATION TABLE.

* From the above Table a plan or map may be constructed in the following manner. Draw the line N S, (*Fig.* 92.) for a North and South line. On N S assume any point as at A. From A to 1, lay the Diff. Lat. of Stat. 7, which is 46.26. Again from 1 to 2 lay off the Diff. Lat. 46.98. Again from 2 to 3 lay off 47.92. And from 3

EXAMPLE II.

CALCULATION TABLE.

No. of Stat.	Bearing.	Dist. in ch. lin.	N. Diff.	S. Lat.	E. Half.	W. Dep.	Mer. Dist.	North Areas.	South Areas.
0							11.64		
1	South	8.00		8.00			11.64 11.64		93.1200
2	S. 15° W.	8.50		8.20		— 1.10	10.54 9.44		86.5334
3	N. 80° W.	7.50	1.30			— 3.69	5.75 2.06	7.4750	
4	N. 15° W.	7.90	7.63			— 1.03	1.03 0.00	7.8589	
5	N. 20° E.	10.00	9.40		+ 1.71		1.71 3.42	16.0740	
6	S. 75° E.	8.40		2.13	+ 4.11		7.53		15.9636
			18.33	18.33	5.82	5.82	11.64	31.4079	195.6178
						2			31.4079
						11.64			16.42091
									4
									1.68364
									40
									27.34560

Ans. 16 *ac.* 1 *r.* 27 *p.*

to 4 lay off 781. Now since we commenced with station No. 7, and since the 6th course is West, A will be the ter-

EXAMPLE III.

CALCULATION TABLE.

No. of Stat.	Bearing.	Dist. in	N.	S.	E.	W.	Mer. Dist.	North	South
0		ch. lin.	Diff. Lat.		Half. Dep.		61.54	Areas.	Areas.
1	N. 45° W.	35.50	25.07			— 12.55	48.99	1228.1793	
2	N. 10° E.	29.50	28.05		+ 2.56		39.00	1093.9500	
3	N. 60° E.	30.00	15.00		+ 12.99		54.55	818.2500	
4	S. 68° 10′ E.	32.64		12.18	+ 15.22		82.76		1008.0168
5	South.	30.00		30.00			97.98		2939.4000
6	S. 20° W.	25.00		23.50		— 4.27	93.71		2202.1850
7	S. 85° W.	28.00		2.44		= 13.95	75.49		184.1956

The difference between the sum contained in the column of North Areas and the sum contained in the columns of South Areas, is 3193.4181 *ch.*, or 319 *ac.* 1 *r.* and 14 *p.*

minating point. Then as the plan is to lie on the East side of the line N S, lay off the departures perpendicularly to the North and South line, according to the following calculation of Departures:

$$7.77 \times 2 = 15.54 = 1\ \text{B}$$
$$8.55 \times 2 = 17.10 + 15.54 = 32.64 = 2\ \text{C}$$
$$20.00 \times 2 = 40.00 + 32.64 = 72.64 = 2\ \text{D}$$
$$8.72 \times 2 = 17.44 + 72.64 = 90.08 = 3\ \text{E}$$
$$22.16 \times 2 = 44.32 - 90.08 = 45.76 = 4\ \text{F}$$
$$7.12 \times 2 = 14.24 + 45.76 = 60.00 = \text{A G}$$

Then join A B, B C, C D, D E, E F, F G, and G A, and it will be a plan or map of the farm.

In this way a map or plan may be drawn without paying any attention to the courses; for, by setting off the Northings and Southings on the meridian or North and South line, and the whole departures upon perpendiculars

Remarks.—In the last Table the meridian distances at the beginning of each line are omitted. When the distances both at the beginning and at the middle of each line are inserted, mistakes are very apt to be made from multiplyign the Northings and Southings into a wrong number.

In taking your field notes, you turn always to the right, so that the land to be surveyed lies always on your right hand, the sum of the column of South Areas will be greater than the sum of the column of North Areas, and *vice versa.*

In balancing the columns of the Calculation Table, if the error do not exceed 4 links for each stationary distance, the columns may be made to balance, and the area determined from them. If, however, on balancing the columns, or after drawing a plan from the notes, there is reason to believe that an error has been committed, the safest course to be pursued is to repair again to the ground and take the field notes anew.

DIVISION OF LAND.

PROBLEM I.

To divide a parallelogram in any proportion by a line running parallel to a given side.

RULE.

Since parallelograms of the same altitude are to one another as their bases, (Euc. vi. 1,) find first the area or quantity of land contained in the whole figure. Then, as the area of the whole parallelogram is to its base, so is the area or quantity of land to be laid off, to its base.

EXAMPLE.

In the parallelogram A B C D, (*Fig.* 94,) the base A B is 20 *ch.*, and the side A D is 16 *ch.* It is required to de-

$20.00 \times 2 = 40.00 + 32.64 = 72.64 = 2\ D$
$8.72 \times 2 = 17.44 + 72.64 = 90.08 = 3\ E$
$22.16 \times 2 = 44.32 - 90.08 = 45.76 = 4\ F$
$7.12 \times 2 = 14.24 + 45.76 = 60.00 = A\ G$

Then join A B, B C, C D, D E, E F, F G, and G A, and it will be a plan or map of the farm.

In this way a map or plan may be drawn without paying any attention to the courses; for, by setting off the Northings and Southings on the meridian or North and South line, and the whole departures upon perpendiculars raised from these points, to the East or West, according as the map is to be drawn on the East or West side of the first meridian, and then drawing right lines from the extremities of these perpendiculars, the plan will be complete.

The above map might also be constructed by drawing meridians through every station, and then by laying off the Diff. Lat., and the Dep. of each course and distance on and from these respective meridians.

In Rectangular Surveying the plan is intended to lie entirely on one side of the first meridian.

termine the point in the base A B, from which a right line must commence, which, running parallel to B C until it strikes the opposite side D C, shall form a rectangle E F B C, containing 10 *ac.* A D $\times$ A B = 16 $\times$ 20 = 320 *ch.* = 32 *ac.* = whole area. Then, as 32 *ac.* : 20 *ch.* : : 10 *ac.* : 6 *ch.* 25 *l.* = B F, the base of a rectangle E F B C, containing 10 *ac.* Then from the point B lay off 6 *ch.* 25 *l.* towards A, and you will have the point required.

The preceding rule applies to all parallelograms, whether Squares, Rectangles, Rhombuses, or Rhomboids,

PROBLEM II.

From a given point in the boundaries of a Square or Rectangle, to run a line which shall cut off a given quantity of land in a given direction.

EXAMPLE.

In the Rectangle A B C D, (*Fig.* 95,) containing 28 *ac.* the line A B runs E. 14 *ch.*, and the line A D runs N. 20 *ch.* It is required to lay off 15 *ac.* to the east by a line commencing at E, 8 *ch.* from the ∠ at B. The course and distance of this division line are also required.

RULE.

From the given point E run the line E F parallel to A D or B C. Then determine the area of the rectangle A E F D; thus, A D $\times$ A E = 20 $\times$ 6 = 120 *ch.* = 12 *ac.* = area of A E F D. Subtract this area from the area of the whole figure; thus, 28 — 12 = 16 *ac.* = area of the remaining rectangle E F C B. Find the difference between this area and the area to be laid off; thus, 16 — 15 = 1 *ac.* Now it will be seen at once that this area is in the form of a right angled triangle. Of this triangle there are known or determined the area which is 1 *ac.*, and the base line E F, which is 20 *ch.* From these data, determine the length of the perpendicular F S, by the following rule:

Divide the area by half the base, and the quotient will be the length of the perpendicular; thus, 1 *ac.* or 10 *ch.* $\div \frac{\text{E F}}{2}$

or 10 *ch.*, = 1 *ch.*, which is the length of the perpendicular F S. As the area contained in the rectangle E B C F, cut off by the line E F, exceeds the quantity of land (15 *ac.*) required to be laid off, it is evident that the 1 *ac.* must be taken from that area; or, that the perpendicular F S must be laid off towards C, or the right. If, however, instead of the quantity to be laid off being less than the area or quantity of land contained in the remaining rectangle E B C F, it had exceeded that quantity, the difference would have to be taken from the area contained in the rectangle A D F E; or in other words, the perpendicular F S would require to be laid off towards the West, or towards the left hand.

The course is ascertained by Prob. V. *Running of Lines*; thus, as E F or 20 : F S or 1 : : 57.3° : $\angle$ E F S = 2° 52′, nearly.

Since the triangle E F S is right angled at F, $\overline{EF}^2$ or 400 + $\overline{FS}^2$ or 1 = $\overline{SE}^2$ or 401 *ch.*, the square root of which is 20 *ch.* 3 *l.* = E S. The course of E S therefore is N. 20° 52′ E., and the distance 20 *ch.* 3 *l.*

PROBLEM III.

To divide a Trapezoid into two equal parts by a line running perpendicularly to the base or front.

EXAMPLE.

In the Trapezoid A B C D, (*Fig.* 96,) containing 12 *ac.* 3 *r.* 8 *p.*, the parallel sides run due East 12 and 20 *ch.* respectively, the side A D runs N. 10° E. 8 *ch.* 12 *l.*, and the side B C N. 39° 34′ W. 10 *ch.* 37 *l.* It is required to divide the Trapezoid into two equal parts by a line running due North from the base or front A B.

From the angle at D let a perpendicular D S fall upon the base A B. By Rule I., Right-angled Trigonometry, we find that A S is 1 *ch.* 39 *l.*, and that D S is 8 *ch.* in length. Then by Prob. II., Mensuration, we find the area of the triangle A D S to be 55600 *l.* Now the area of the whole Trapezoid is 12 *ac.* 3 *r.* 8 *p.* or 1280000 *l.* Half the area of the whole Trapezoid is therefore 640000 *l.* But we have already the area of the triangle A D S = 55600, which, subtracted from 640000 leaves 584400 *l.* as the area remaining to be laid off, which from an inspection of the figure, it is evident must be laid off in the shape or figure of a rectangle. In this rectangle we have the area 584400 *l.*, and one of the sides, D S, 8 *ch.* Hence, $\frac{584400}{8.00} = 7$ *ch.* 30 *l.* Then from the point S run off 7 *ch.* 30 *l.* due East to E.; and from E run E F due North and the the trapezoid is divided by it as was required.

PROBLEM IV.

To divide a parallelogram into two equal parts by a line running from a given point.

EXAMPLE.

Let A B C D, (*Fig.* 97,) be a parallelogram. It is required to run a line from the point F which shall divide the parallelogram into two equal parts.

Draw the diagonal D B, and bisect it. Draw a line from the point F through the point of bisection of the diagonal, and continue it till it strikes the opposite side D C in the point E; the line F E will divide the parallelogram A B C D into two equal parts, as was required. (Euc. i. 34.)

PROBLEM V.

To divide a Trapezium.

EXAMPLE.

In the trapezium A B C D, (*Fig.* 98,) the two sides B C and A D are parallel. It is required to cut off one third of the whole area by a line running from the point A.

Produce the line B C to E, so that C E may be equal to A D. From B lay off B G equal one third of B E, and join A G; the triangle A G B is the third part of the trapezium A B C D. (Euc. vi. 1.)

PROBLEM VI.

To divide a Trapezium into two equal parts, by a line drawn from one of its angles.

EXAMPLE.

Let A B C D, (*Fig.* 99,) be the given trapezium, and A the angle from which the dividing line is to be drawn. Draw the diagonals A C and B D. Bisect D B in E. Through E draw G E F parallel to A C. Join A F and it will divide the trapezium A B C D into two equal parts.

PROBLEM VII.

To divide a Triangle into any proposed number of equal parts, by lines running from a given angle.

EXAMPLE.

It is required to divide the triangle A B C, (*Fig.* 100,) into three equal parts, by lines running from the angle at C.

According to Euc. i. 38, triangles upon equal bases, and between the same parallels, are equal to each other, therefore divide the line opposite the angle at C into three equal parts, and from the points of division D, E, draw the lines D C and E C. The areas of the triangles A D C, D E C, and E B C, will be equal to each other.

PROBLEM VIII.

To divide a Triangle by lines running parallel to a given side.

EXAMPLES.

1. In the triangle A B C, (*Fig.* 101,) the side A B measures 40 *ch.*, the side A C 58 *ch.*, and the side C B 56 *ch.* It is required to divide the triangle A B C into two equal parts, by a line running parallel to A B.

According to Euc. vi. 19, similar triangles are to each other in the duplicate ratio of their homologous sides: therefore as the area of the whole triangle is to the square of its side, so is the area to be cut off to the square of its side

The square root of this sum will be the side of the triangle to be cut off, thus: the area of the triangle A B C is 106 *ac.*, and consequently the area to be cut off is 53 *ac.* Then, as A B C or 106 *ac.* : $\overline{A\ C}^2$ or 3364 *ch.* : : C D E or 53 *ac.*, : $\overline{C\ D}^2$ or 1682 *ch.*; and $\sqrt{1682} = 41 =$ C D. Then, draw D E from the point D, parallel to A B, and the triangle is divided into two equal parts, as was required.

2. Divide the above triangle into three equal parts, by lines running parallel to the base A B, (*Fig.* 102.)

The whole area being 106 *ac.*, one third of it is 35.3333 + *ac.*; therefore as A B C or 106 *ac.* : $\overline{A\ C}^2$ or 3364 *ch.* : : C E D or 35.3333 + *ac.* : $\overline{C\ E}^2$ or 1121.33 +, and $\sqrt{1121.33+} = 33$ *ch.* 48 *l.* = C E.

Again as 106 : 3364 : : C G H or 70.6666 + : $\overline{C\ G}^2$ or 2242.65; then $\sqrt{2242.65} = 47$ *ch.* 35 *l.* = C G.

Then through the points E and G draw E D and G H parallel to A B, and the triangle is divided into three equal parts, as was required.

By the same principle any specified amount of area may be laid off in a trapezium by a line running parallel to one of its sides, for by producing some two of its sides until they meet, a triangle will be formed.

EXAMPLE.

In the trapezium A B C D, (*Fig.* 103,) the side A D runs S. 17° E. 15 *ch.*, A B runs E. 20 *ch.*, and B C runs S. 20° W. 17 *ch.* It is required to lay off 6 *ac.* towards the North by a line running parallel to A B.

Produce the sides A D and B C until they meet in E. Then $180° - (\angle E A B + \angle E B A) = 37° = \angle A E B$.

By Oblique Angled Trigonometry.

To find the side A E.

As Sine ∠ A E D 37°	9.77946
Is to A B 20	1.30103
So is Sine ∠ A B E 70°	9.97299
To A E 31.23	1.49456

To find the side B E.

As Sine ∠ A E B 37°	9.77946
Is to A B 20	1.30103
So is Sine ∠ B A E 73°	9.98060
To B E 31.78	1.50217

Next, by Mensuration, Prob. II. Rule 2, the area of the triangle A B E is found to be 29.86334 *ac.*, from which take the area to be cut off 6 *ac.*, and the remainder is 23.86334 *ac.* Then as 29.86334 : $\overline{\text{B E}}^2$ or 1009.9684 : : 23.86334 : $\overline{\text{E F}}^2$ or 807.0507, and $\sqrt{807.0507} = 28$ *ch.* 40 *l.* Lay off B F = 28 *ch.* 40 *l.*, and from F run F G due West, and A B F G will contain 6 *ac.*

By the same rule we find that the triangle E C D contains 7 *ac.* 2 *r.* and 35 *p.*, and that the trapezium G F C D contains 16 *ac.* and 23 *p.*

PROBLEM IX.

To divide land by Calculation.

EXAMPLE.

It is required to divide the farm A B C D E F A, (*Fig.* 104,) into two equal parts by a line running from the point A.

Calculate first the area of the whole farm. Then draw a line, or suppose a line to be drawn from the given point to some other known station as at D, which will divide the farm in the required proportion, as nearly as you can judge. Then fill up the columns of a Calculation Table with the courses and distances from A round to D. The difference between the sum of the North and of the South columns will show the Difference of Latitude, and the difference between the sum of the East and of the West columns will show the Departure of D A. From these data find the area of the part cut off A B C D A. Find the difference between this area and the area of the half of the whole farm. That difference will be a triangle. If the area of the piece cut off exceed the half area of the whole figure, that triangle lies within the figure cut off, but if it be less than half the area of the whole figure, the triangle lies on the opposite side of the line A D. Suppose that the quantity cut off exceed the half of the whole area. Find the area of the triangle A D C. Then as the area of the triangle A D C is to the square of the side D C so is the area of the triangle A D H to the square of the side D H. Then having the course and distances of A D and D H, by a Traverse Table find the course and distance of A H, the true dividing line. Set your compass at A, and run the course and distance A H, and the farm is divided as required.

Suppose the farm to contain 16 *ac.* 1 *r.* 27 *p.*, and the field notes of the survey to be as follows:

Commenced at station A, and run thence S. 80° E. 7 *ch.* 50 *l.*, thence S. 15° E. 7 *ch.* 90 *l.*, thence S. 20° W. 10 *ch.*, thence N. 75° W. 8 *ch.* 45 *l.*, thence N. 8 *ch.*, thence N. 15° E. 8.50 *l.*, to the place of beginning. Let the plan be laid by a scale of 10 *ch.* to an inch.

CALCULATION TABLE FOR A B C D A.

CALCULATION TABLE FOR A B C D A.

No. of Stat.	Bearing.	Dist. in ch. lin.	N. Diff. Lat.	S. Diff. Lat.	E. Half Dep.	W. Half Dep.	Mer. Dist.	North Areas.	South Areas.
0							9.44		
1	S. 80° E.	7.50		1.30	+3.69		13.13 16.82		17.0690
2	S. 15° E.	7.90		7.63	+1.03		17.85 18.88		136.1955
3	S. 20° W.	10.00		9.40		— 1.71	17.17 15.46		161.3980
4	*N. 18° 11′ W.	19.29	18.33			— 3.01	12.45	228.2085	
			18.33	18.33	4.72	4.72	9.44		314.6625

	228.2085
Area of A B C D A	86.4540
Half area of whole farm	82.1066
Area of A D H	4.3474

Then to find the area of the triangle A D C.

D C N. 20° 00′ E.
D A N. 18° 11′ E.
∠ C D A 38° 11′

Nat. Sine of ∠ 38° 11′ = .61818
D A = 19.29
119.256922
D C = 10.00
2)119.246922000
Area of triangle A D C = 59.623461000

Then as 59.6234 : 10.00 : : 4.3474 : 72 *l.* = D H.

The course and distance AH, the true division line, is found by traverse running to be S. 19° 34′ E. 18*ch.* 72 *l.*

* This course and distance is ascertained by traverse running.

N. B.—Lands may be divided either by the method of *Calculation* or of *Construction.*

By *Calculation* we can ascertain the true courses and distances of Division Lines. *Construction* gives only a moderate approximation to the truth.

For this reason the method by *Construction* ought never to be employed in the division of very valuable land.

PROBLEM X.

To divide a Triangular Lot of Land in certain proportions.

EXAMPLE.

Being employed to divide a ten acre lot of Marsh between three claimants,—A, B, and C; A claiming 4 acres, and B and C, 3 acres each. The lot is triangular. The base A B (Fig. 105,) measuring 20 *ch.*, and a perpendicular let fall thereon from the opposite angle measuring 8 *ch.* 50 *l.* The lot, therefore, it is evident, will not hold out its measurement. Now supposing the division lines to run from the angle opposite to the base, how much land, and what proportion of the base should each claimant receive.

C D × A B = 20.00 × 8.50 = 170.0000 ÷ 2 = 85.0000 = 8 *ac.* 2 *r.* = area of the lot.

As 10 : $8\frac{1}{2}$: : 4 : 3 *ac.* 1 *r.* 24 *p.* = A's share.

As 10 : $8\frac{1}{2}$: : 3 : 2 *ac.* 2 *r.* 8 *p.* = B's or C's share.

As 8.5 *ac.* : 20 *ch.* : : 3 *ac.* 1 *r.* 24 *p.* : 8 *ch.* = A's share of base.

As 8.5 *ac.* : 20 *ch.* : : 2 *ac.* 2 *r.* 8 *p.* : 6 *ch.* = B's and C's share of base.

PROBLEM XI.

To divide by Calculation a lot of land of a certain amount of value among different claimants, in proportion to the amount of their claims and the estimated value of the land.

EXAMPLE.

A Testator leaves by will a lot of land containing 500 *ac.*, the value of which he estimates at £1470, to be divided among his servants A B C D E F in the following proportions, according to the value of the land, viz: to A he bequeaths a portion worth £40, to B £20, to C £10, to D £100, to E £400, and to F £1,000. Now the value of the land most convenient for A, B, and C, is estimated at 7s. per acre, while the land most convenient for D is worth 10s. per acre, for E 15s. per acre, and for F 12s. per acre. It is required to determine the quantity of land which falls to the share of each.

RULE.

Divide the sum bequeathed to each by the value of the land per acre which is to be allotted to him, add the quotients together, divide the whole given quantity of land by that sum; this quotient will be a common multiplier, by which multiply each particular quotient and the product will give the amount of land which falls to the share of each individual, or: Say as the sum of all the quotients is to the whole quantity of land, so is each particular quotient to its proportional share.

$$\text{Thus:}\quad \left.\begin{array}{lrrl} A & 7) & 40(& 5.71428 \\ B & 7) & 20(& 2.85714 \\ C & 7) & 10(& 1.42857 \\ D & 10) & 100(& 10. \\ E & 15) & 400(& 26.66666 \\ F & 12) & 1000(& 83.33333 \end{array}\right\}\text{Quotients.}$$

Sum of Quotients, 129.99998 or 130

Now, $500 \div 130 = 3.846153$, the common multiplier.

Then,

$$\begin{array}{rcl} 5.71428 \times 3.846153 = & 21.9778\ ac. & = \text{A's share.} \\ 2.85714 \times 3.846153 = & 10.9918\ ac. & = \text{B's share.} \\ 1.42857 \times 3.846153 = & 5.4941\ ac. & = \text{C's share.} \\ 10. \quad \times 3.846153 = & 38.4615\ ac. & = \text{D's share.} \\ 26.66666 \times 3.846153 = & 102.5641\ ac. & = \text{E's share.} \\ 83.33333 \times 3.846153 = & 320.5127\ ac. & = \text{F's share.} \end{array}$$

Sum of the whole shares, $= 500.0020\ ac.$

On the same principle if any single share should contain land of different values.

Again, if there be different quantities of land as well as different values, find what each quantity is worth at its valuation, and add their sums together: then, as the sum of the quantities is to this sum, so is one acre to its mean value.

LOCATION OF LANDS.

This section will treat of the method of laying off any given quantity of land, in any specified form, from the least possible data.

As the quantity of land is generally given in acres, roods, and perches, it is necessary to reduce them to square links, which may be performed by the following

RULES:

1. To the acres annex five cyphers on the right hand, and the whole will be links.
2. Having annexed five cyphers to the right of the number of roods, divide the sum by 4, the quotient will be the links.
3. To the right hand of the perches annex four cyphers, and divide by 16, the quotient will be the links.
4. Add these sums together, and you have the square links contained in the given quantity.

PROBLEM I.

To lay out a given quantity of land, in the form of a Square.

RULE.

Extract the square root of the area, and you have the side of the square required.

EXAMPLE.

It is required to lay out 200 acres of land in the form of a squa[illegible] [illegible]n the East side of a road running N. 10° E., (*Fig.* [illegible]) What is the length of the side of the square? Lay down also a plan from a scale of 25 chains to an inch.

200 *ac.* = 2000000 *l.*, the square root of which is 4472 + *l.* = 44 *ch.* 72 *l.* = the side of the square; and since the course of A C is N. 10° E., the course of A B must be S. 80° E.

PROBLEM II.

To lay out land in the form of a Rectangle, the length of one side being given.

RULE.

Divide the area by the given side, and the quotient will express the length of the other side.

EXAMPLE.

Being employed to lay out 80 *ac.* 2 *r.* 20 *p.* on the West side of, and fronting on, a road running N. 3° E., (*Fig.* 107,) the lot to measure 12 chains in front, along said road; required the course and distances, with a plan of the same.

80 *ac.* 2 *r.* 20 *p.* = 8062500 *l.* ÷ 12 *ch.* or 1200 *l.* = 6719 *l.* or 67 *ch.* 19 *l.* = length of side required.

The course of A B is S. 87° E., and distance 67 *ch.* 19 *l.*, and B C runs N. 3° E. 12 *ch.*

PROBLEM III.

To lay out land in the form of a Rhombus, one of the angles being given.

RULE.

Divide the area by the Nat. Sine of the given angle, and the square root of the quotient will be the side required.

EXAMPLE.

Being employed to lay out 100*ac.* of land, in the form of a rhombus, on the East side of a road running due North, (*Fig* 108,) the course of the line from said road to be S. 80° E.; required the length of the side, and also a map of the lot laid down by a scale of 25 chains to an inch.

The Nat. Sine of 80°, when radius is 1, is .98481, and 100 *ac.* = 10000000 *l.* Then 10000000 ÷ .98481 = 10154241 .92, the square root of which is 3187 *l.*, or 31 *ch.* 87 *l.*, the length of the side.

PROBLEM IV.

To lay off land in the form of a Rhomboid, a side and an angle being given.

RULE.

Divide the area by the product of the given side multiplied into the Nat. Sine of the given angle, the quotient will be the other side.

EXAMPLE.

It is required to lay off 75 *ac.* 23 *p.* in the form of a rhomboid, on the North side of, and fronting on, a river, (*Fig.* 109,) which runs due East, the front to measure 15 chains along said river, and the line from the river to the rear to run N. 10° E. What must be the length of the side line? Draw also a plan of the lot.

The Nat. Sine, as in the preceding Example, is .98481 × 1500 *l.*, the width of the front = 1477.21500. Then 75 *ac.* 0 *r.* 23 *p.* = 7514375 ÷ 1477.21500 ≡ 5086 *l.* = 50 *ch.* 86 *l.*, the length of the side line A D.

PROBLEM V.

To lay off land in a rectangular form, so that the length may be a given multiple of the breadth.

RULE.

Divide the area by the given multiple and the square root of the quotient will be the width, and the width multiplied by the given multiple will be the length.

EXAMPLE.

Being employed to lay off 78 *ac.* 2 *r.* 36 *p.* on the East side of a line running N. 4° E. (*Fig.* 110,) in the form of a rectangle, whose length shall be three times that of its breadth; required the courses, distances, and a plan of the lot.

78 *ac.* 2 *r.* 36 *p.* = 7872500 *l.* ÷ 3 = 2624166, the square root of which is 1620* very nearly. The breadth therefore 16 *ch.* 20 *l.*, and the length = 16.20 × 3 = 48.60 or 48 *ch.* 60 *l.*

A D runs N. 4° E. 16 *ch.* 20 *l.*
D C runs S. 86° E. 48 *ch.* 60 *l.*

* By assuming 1620 *l.* as the width instead of 1619.9 + the true width, the above lot contains about 4 *r.* more than the given quantity. Unless where land is exceedingly valuable, a surveyor would probably take 1620 *l.* as the width, and lay off accordingly.

PROBLEM VI.

To lay out land in a rectangular form, so that the length may be to the breadth in a certain proportion.

RULE.

Multiply the area by the less and divide the product by the greater number of the proportion, and the square root of the quotient will be the width: And the width multiplied by the greater and divided by the less number of the proportion will be the length.

EXAMPLE.

If it be required to lay on 109 *ac.* 17 *p.* in a rectangular form, (*Fig.* 111,) so that the breadth may be to the length as 5 is to 8, what must the length and the breadth be respectively? Draw a map of the lot by a scale of 25 *ch.* to an inch.

109 *ac.* 17 *p.* $= 10910625$ *l.* $\times 5 = 54553125 \div 8 = 6819140$, the square root of which is $2611 =$ breadth of the farm. Then $2611 \times 8 = 20888 \div 5 = 4179$. The length A B therefore is 41 *ch.* 79 *l.*, and the breadth A D 26 *ch.* 11 *l.*

PROBLEM VII.

To lay out land in the form of a Rectangle, so that the length may exceed the breadth, by a certain given quantity.

RULE.

Add the square of one fourth of the given difference to the area, and from the square root of the sum subtract half the given difference for the less side. To the remainder add the whole difference for the greater side.

EXAMPLE.

It is required to lay out 200 *ac.* in a rectangular form (*Fig.* 112,) so that the length may exceed the breadth by 10 *ch.*

$\frac{10^2}{4} = 250000 + 2000000$ area, $= 20250000$, the square root of which is $4500 - \frac{1000}{2}$ or half the given difference =

4000 *l.* = the less side, and 4000 + 1000 the whole difference = 5000 *l.*, the greater side. The side A D therefore must be 40 *ch.*, and the side A B 50 *ch.*

PROBLEM VIII.

To lay out land in the form of a Rhomboid, so that the length may be a given multiple of the breadth.

RULE.

Divide the area by the product of the given multiple and the natural sine of the given angle, and the quotient will be the breadth; and the breadth multiplied by the given multiple will be the length.

EXAMPLE.

Required the sides of a Rhomboid containing 10 *ac.*, (*Fig.* 113,) whose acute angle is 80°, and whose length is three times greater than the breadth.

Nat. Sine 80° to Rad. 1 is .98481 × 3 = 2.95443, and 10 *ac.* = 10.00000 *l.* ÷ 2.95443 = 338474, the square root of which is 582 *l.* or 5 *ch.* 82 *l.* = width, and 5 *ch.* 82 *l.* × 3 = 17 *ch.* 46 *l.* = length.

PROBLEM IX.

To lay out land in the form of a Rhomboid, so that the length may be to the breadth in any given proportion.

RULE.

Multiply the area by the less number in the given proportion, and divide the product by the product of the Nat. Sine of the given angle multiplied by the greater number of the proportion: the square root of the quotient will be the breadth; and the breadth multiplied by the greater and divided by the less number in the given proportion gives the length.

EXAMPLE.

Being employed to lay out 10 *ac.* in the form of a Rhomboid whose length shall be to its breadth as 2 to 5, and whose included angle shall be 80°; required the length and breadth. (*See the preceding Figure.*)

Nat. Sine 80° is .98481 × 5 = 4.92405, and 10 *ac.* = 1000000 *l.* × 2 = 2000000 ÷ 4.92405 = 406169, the square root of which is 638 *l.* or 6 *ch.* 38 *l.*, = the breadth, and $\frac{638 \times 5}{2}$ = 1595 *l.* or 15 *ch.* 95 *l.*, = the length.

PROBLEM X.

To lay out land in the form of a Trapezoid, whose central length shall be any given multiple of the width.*

RULE.

Divide the area by the given multiple, and the square root of the quotient will be the width; which multiplied by the given multiple will give the central length.

EXAMPLE.

It is desired to lay off 200 *ac.* on the East side of a road running N. 30° W., (*Fig.* 114,) in the form of a trapezoid, whose parallel sides shall run due East, and whose central length shall be double its breadth; required the courses and distances, and a plan of the lot.

$\sqrt{200\ ac. \text{ or } 2000000\ l. \div 2}$ = 3162 *l.* or 31 *ch.* 62 *l.* = the breadth, and 31 *ch.* 62 *l.* × 2 = 63 *ch.* 12 *l.* = central length.

Then to find the length of the parallel sides, since in the triangle A D N right angled at N, the side N D = B C is known to be 31 *ch.* 62 *l.*, the angle at D is also known to be 30°, and consequently the angle at A must be 60°; by right angled Trigonometry, say as Sine ∠ A : D N : : Sine ∠ D : A N, from which we find A N to be 18 *ch.* 26 *l.* Again, since the central length E F = $\frac{A\,B + D\,C}{2}$ and A N = A B — D C; if to the central length E F you add half the difference or half A N, the sum will be the longest side A B; and if from the central length E F you subtract half

* A Trapezoid is a rectilineal quadrilateral figure, only two of whose opposite sides are parallel.

the difference or half A N, the remainder will be the length of the shortest side D C.

Now A N = 1826 *l.* which ÷ 2 = 913 + 6324 or E F = 7237 *l.* or 72 *ch.* 37 *l.* = A B, the longest side; and 913 — 6324 = 5411 or 54 *ch.* 11 *l.* = D C, the shortest side. Wherefore commencing at A, A B runs East 72 *ch.* 37 *l.*, B C runs South 31 *ch.*, 62 *l.*, C D runs West 54 *ch.* 11 *l.*, and D A runs N. 30° W., and the distance is ascertained by Trigonometry to be 36 *ch.* 51 *l.*

PROBLEM XI.

To lay out land in the form of a Trapezium, having one of its sides given.

RULE.

Divide the given area into two parts, either equal or unequal, and then find the perpendicular that will lay out one of these parts in a right-angled triangle upon the given side as a base. This perpendicular will be a diagonal of the trapezium, and a base upon which the remaining triangle must be constructed. Then find the perpendicular, which, falling upon the opposite side of this base, will lay out the other part.

N. B.—These perpendiculars are found by dividing double the area of the triangle by the given or known sides.

EXAMPLE.

It is required to locate 8 *ac.* in the form of a trapezium A D B C, (*Fig.* 115,) one of whose sides B C shall measure 8 *ch.*

Let the area be supposed to be divided into triangles, one of which contains 5 *ac.* and the other 3.

$\dfrac{5\ ac.\ \text{or}\ 500000\ l. \times 2}{8\ ch.\ \text{or}\ 800\ l.} = 1250\ l.$ = perpendicular. Then from the point B in the given side B C, and perpendicular to it draw B A = 1250 *l.*, and join A C; the triangle A B C contains 5 *ac.*

Next, $\dfrac{3\ ac.\ \text{or}\ 300000\ l. \times 2}{1250} = 480\ l.$ = perpendicular of remaining triangle. From A, and perpendicular to A B, draw A D 480 *l.*; join D B, and the triangle A D B will contain 3 *ac.*

The trapezium A C B D will also contain 8 *ac.*, and the side B C is 10 *ch.*, as was required.

PROBLEM XII.

To lay out land in the form of a Triangle, of which one side and the angle at one of its extremities are given.

RULE.

Divide double the area by the product of the given side multiplied into the Nat. Sine of the given angle, and the quotient will be the other side, including the given angle.

EXAMPLE.

From the Northern extremity of the line N C, (*Fig.* 116,) which runs due North 25 *ch.*, it is required to run another line C O, S. 34° 41′ E., so that the triangle N O C may contain 80 *ac.*

Nat. Sine of 34° 41′ is .56904 × 2500 or N C = 1422, and 80 *ac.* × 2 = 16000000 *l.* ÷ 1422 = 11245 *l.* or 112 *ch.* 45 *l.*, = side C O.

PROBLEM XIII.

To lay off any quantity of land in a triangular form, between two lines forming an angle, one of the sides of the triangle being given.

RULE.

Divide double the area to be laid off by the length of the given side, and the quotient will be the length of a perpendicular let fall from the opposite angle upon some part of the base or line given. From the extremity of the given line raise a perpendicular of the ascertained length. From the end of that perpendicular run another line parallel to the given line until it intersects the other line. Then a line drawn from the point of intersection to the point from which the perpendicular was raised will complete the triangle containing the required number of acres.

EXAMPLE.

In the corner or angle formed by the road A B (*Fig.* 117,) and A C, I am required to lay off 4 *ac.* fronting on the road A B 10 *ch.*; required the termination of the line A C.

Double of the area = 400000 × 2 = 800000 ÷ 1000 *l.*, (the length of A B) = 800 *l.* or 8 *ch.* = length of perpendicular. Then from B and perpendicular to the line A B run a line B S, 8 *ch.* in length. From the end of this perpendicular or from the point S run a line S C parallel to B A, until it strikes or intersects the side line A C aforesaid in C. Join C B. The triangle A B C contains 4 *ac.*, as was required.

N. B.—In this case the quantity of the included angle does not affect the accuracy of the rule. It may be an angle of 80° as C A B, or of 75° as C′ A B, or of 50° as C″ A B.

PROBLEM XIV.

To lay off land in the form of an Isosceles Triangle, the angle contained between the equal sides and the area being given.

RULE.

Divide double the area by the Nat. Sine of the given angle, and the square root of the quotient will be the length of one of the equal sides.

EXAMPLE.

It is required to lay out 38 *ac.* 2 *r.* 18 *p.* in the form of an Isosceles Triangle A B C, (*Fig.* 118,) the course of A B is S. 25° W., and the course of A C is S. 26° E.; What must be the length of the sides and the course of B C.

38 *ac.* 2 *r.* 18 *p.* = 3861250 *l.* × 2 = 772250, the double area, ÷ .77715, the Nat. Sine of ∠ A, (25° + 26° = 51°) = 9936940, the square root of which is 3151 = A C or A B. Then 180° — ∠ A or 51° = (∠ B + ∠ C) or 129°. Now since the ∠ B and the ∠ C are equal each of them is 64° 30′. Wherefore, by Trigonometry:

As Sine of $\angle$ C 64° 30′ : B A 3151 : : Sine $\angle$ A 51° : B C 2713.

The course of A B is S. 25° W., and the $\angle$ B is 64° 30′; the course of B C is N. 89° 30′ E.

Hence the sides A B and A C are each 31 *ch.* 51 *l.*, and the side B C runs N. 89° 30′ E. 27 *ch.* 13 *l.*

PROBLEM XV.

To locate land in the form of a Circle.

RULE.

Divide the area by .7854, and the square root of the quotient will be the diameter.

EXAMPLE.

Required the diameter of a circle containing one acre. (*Fig.* 119.)

$\sqrt{1\ ac. = 100000\ l. \div .7854} = \sqrt{127323.65} = 356.8\ l.$ or 3 *ch.* 56.8 *l.*

PROBLEM XVI.

To lay out land in the form of an Ellipse.

CASE 1.

When the Transverse Diameter exceeds the Conjugate by a given quantity.

RULE.

Divide the area by .7854, to the quotient add the square of half the difference between the diameters, from the square root of the sum subtract one half of the difference between the diameters, and the remainder will give the Conjugate. The difference added to the Conjugate will give the Transverse.

EXAMPLE.

Required the Transverse and Conjugate diameters of an Ellipse containing one acre, whose Transverse diameter shall exceed the Conjugate by one chain. (*Fig.* 120.)

$1\ ac. = 100000\ l. \div .7854 = 127323 + \surd\ \frac{100}{2}$ or $2500 =$ 129823, the square root of which is $360 - \frac{100}{2}$ or $50 =$ $310 =$ Conjugate, and $310 + 100 = 410 =$ Transverse. The Transverse diameter therefore is 4 *ch.* 10 *l.*, and the Conjugate 3 *ch.* 10 *l.*

CASE II.

When the Transverse and Conjugate Diameters are to each other in a certain ratio.

RULE.

Multiply the area by the greater number in the proportion, and divide that product by the product of the less number multiplied into .7854, and the square root of the quotient will be the Transverse diameter; then multiply the Transverse by the less number in the proportion and divide by the greater, and it will give the Conjugate. (*See the preceding Figure.*)

EXAMPLE.

It is required to lay out one acre in the form of an Ellipse whose Transverse diameter shall be to its Conjugate in the ratio of 5 to 3.

$\sqrt{\frac{100000 \times 5}{.7854 \times 3}} = 400 =$ the Transverse, and $\frac{400 \times 3}{5}$ $= 276 =$ the Conjugate.

APPENDIX.

It is presumed that in the preceding treatise nothing occurs requiring a formal demonstration until the student arrives at

RECTANGULAR TRIGONOMETRY.

THEOREM.

The sine, versed-sine, tangent, and secant, of an arc which is the measure of any given angle, is to the sine, versed-sine, tangent, and secant, of any other arc which is the measure of the same angle, as the radius of the first arc is to the radius of the second.

Let B D (*Fig.* 121,) and F H be two arcs which measure the same angle B A C; and let A B be the radius of the arc B D, and A H the radius of the arc F H. Let D C be the sine, B C the tangent, and A C the secant, of the arc B D; and let F I be the sine, H E the tangent, and A E the secant, of the arc F H; since B C, D C, E H, and F I, are parallel, according to Euc. vi. 4, tang. B C : tang. H E : : rad. A B : rad. A H; and sine D C : sine F I : : rad. A D : rad A F; and the sec. A C : sec. A E : : rad. A B : rad. A H. Hence the truth of the Theorem is obvious.

From this Theorem it is evident, that, as the *Trigonometrical Tables* exhibit in numbers, the sines, tangents, secants, &c., of certain angles to a given radius, they exhibit also the ratio of the sines, tangents, secants, &c., of the same angles to any radius whatever.

Upon this principle the solutions of the different cases of right-angled plane triangles depend; and from this Theorem the Rules for Rectangular Trigonometry are deduced.

OBLIQUE-ANGLED TRIGONOMETRY.

THEOREM I.

The sides of a plane triangle are to each other as the sines of the angles opposite to them.

Let A B C (*Fig.* 122,) be a triangle, and C D a perpendicular let fall from the vertical angle at C upon the opposite side A B; because the $\triangle$ C A D is right-angled at D, C A : C D = R : Sine $\angle$ A. For the same reason C B : C D = R : Sine $\angle$ B; and inversely, C D : C B = Sine B : R; therefore, by indirect equality, C A : C B = Sine $\angle$ B : Sine $\angle$ A. In the same way it may be demonstrated that C A : A B = Sine $\angle$ B : Sine $\angle$ C.

THEOREM II.

If to half the sum of two quantities be added half their difference, that sum will be the greater quantity; and if from half the sum of two quantities be subtracted half the difference, the remainder will be the less quantity.

Let the two quantities be represented by A E and E B, (*Fig.* 123,) A E being the greater, and E B the less. Then it is evident that A D is the sum, and C E the difference of the two quantities, and A D or D B their half sum, and C D or D E their half difference. Now if to A D we add D E we have A E, the greater quantity; and if from D B we take D E we have E B, the less quantity.

THEOREM III.

The sum of the two sides of a triangle is to their difference as the tangent of half the sum of the angles at the base is to the tangent of half the difference.

Let A B C (*Fig.* 124,) be any triangle. From A as a centre with the radius A B describe the semicircle D B E. Produce C A to D. Join D B and B E, and draw E F parallel to B C. Then because the angle D A B is the exterior angle of the triangle A B C, it is equal to the sum of the two interior and opposite angles A B C and A C B. But the angle D E B is equal to half the angle D A B; therefore the angle D E B is equal to half the sum of the angles A B C and A C B. Now since A B is equal to A E, the angle A B E is equal to the angle A E B. But the angle A E B is equal to the two angles E B C and B C E; wherefore, also, the angle A B E is equal to the sum of the angles E B C and B C E. To each of these add the angle E B C; then the whole angle A B C is equal to twice the angle E B C together with the angle B C E. Whence it is plain that the angle E B C or the alternate angle B E F is equal to half the difference of the angles A B C and B C A. Now the angle D B E is a right angle. (Euc. iii. 31.) Therefore to the same radius E B, D B will be the tangent of the angle D E B, and F B will be the tangent of B E F; so that B D : B F : : tan. D E B : : tan. B E F : : tan. $\frac{1}{2}$ (A B C + A C B) : tan. $\frac{1}{2}$ (A B C — A C B). Also, A D and A E are each equal to A B, it is evident that D C is the sum of the sides A B and A C, and that C E is their difference. But because E F is parallel to B C, D C : C E : : D B : B F; that is, the sum of the two sides of the triangle A B C is to their difference, as the tangent of half the sum of the angles opposite to these sides is to the tangent of half their difference.

THEOREM IV.

In any right lined plane triangle the base is to the sum of the other sides as the difference of these sides is to the the difference of the segments of the base, made by a perpendicular let fall upon it from the angle opposite to it.

In the oblique-angled triangle A B D, (*Fig.* 125,) produce B D until B G is equal to A B, the shortest side. On B as a centre, with the distance B G or B H describe a circle A H G, cutting B D and A D in the points H and F. Then D G is evidently equal to the sum of the sides D B and B A, and H D their difference. And since A E is equal to E F, D F is the difference between D E and E A, the segments into which the base is divided by the perpendicular let fall upon it from the opposite angle. Now (by Euc. iii. 36,) the rectangle contained by D G and D H is equal to the rectangle contained by D A and D F; therefore, A D : G D : : H D : F D, *i. e.* the base is to the sum of the other sides as the difference of these sides is to the difference of the segments of the base.

RULE II.

This is merely another application of the same principle.

RULE. III.

Let $a =$ D A, $b =$ D B, $c =$ A B, and $x =$ D E, the greater segment; then $a - x =$ E A, the less segment.

$$\text{Then, } a : b + c :: b - c : 2x - a$$
$$\text{and } 2ax - a^2 = b^2 - c^2$$
$$\text{Hence, } 2ax = a^2 + b^2 - c^2$$
$$\text{and } x = \frac{a^2 + b^2 - c^2}{2a}$$

Hence the Rule.

MENSURATION OF SUPERFICIES.

PROBLEM I.

RULE I.

The measuring unit of a superficies may be one inch, one foot, one yard, one chain, or any determinate figure and magnitude. Let A B C D (*Fig.* 126,) be a rectangle, and M the unit of measure. When M is contained a certain number of times in A B and B C, it is only necessary to multiply together the figures which express the number of times the linear unit M is contained in A B and B C. The parallelogram A B E F is equal to the rectangle A B C D. (Euc. i. 35). Hence the reason of the rule is obvious.

RULE II.

Let A B C D (*Fig.* 126,) be a parallelogram, and C E its perpendicular altitude. Then in the right-angled triangle C B E, Rad. : Sine B : : C B : C E; then by multiplying the last two terms of this proportion by B A, as Rad. : Sine B : : C B $\times$ B A : C E $\times$ B A: but A B $\times$ C E = area of the parallelogram, and hence the rule.

RULE III.

The demonstration of this rule is evidently comprised in the preceding.

PROBLEM II.

RULE I.

The truth of this rule will be evident by comparing Euc. i. 41, with Rule I. Prob. II.

RULE II.

This rule follows evidently and directly from Rule II. Prob. I.

PROBLEM III.

Let the sides opposite to the angles A, B, and C, (*Fig.* 122,) be represented by a, b, and c, respectively.

Then $b^2 = a^2 + c^2 - 2\,c \times DB$, and $DB = \frac{a^2+c^2-b^2}{2\,c}$.

Hence $DC = \sqrt{a^2 - \left(\frac{a^2+c^2-b^2}{2\,c}\right)^2} = \sqrt{a - \frac{a^2+c^2-b^2}{2\,c}} \times \sqrt{a + \frac{a^2+c^2-b^2}{2\,c}} = \sqrt{\frac{b^2-(a-c)^2}{2\,c}} \times \sqrt{\frac{(a+c)^2-b^2}{2\,c}}$
$= \tfrac{1}{2}c\sqrt{(b+a-c)\times(b-a+c)\times(a+c+b)\times(a+c-b)}$
$= \frac{2}{c}\sqrt{\frac{a+b+c}{2}} \times \sqrt{\frac{a+b+c}{2} - a} \times \sqrt{\frac{a+b+c}{2} - b} \times \sqrt{\frac{a+b+c}{2} - c}$. Let S = the sum of the sides of the

triangle; then $\frac{DC\times c}{2}=\sqrt{\frac{S}{2}\times\left\{\frac{S}{2}-a\right\}\times\left\{\frac{S}{2}-b\right\}\times\frac{S}{2}-c}$.
But $\frac{AB\times CD}{2}=$ area of the triangle. Therefore the area $=\sqrt{\frac{S}{2}\times\left\{\frac{S}{2}-a\right\}\times\left\{\frac{S}{2}-b\right\}\times\frac{S}{2}-c}$. This formula expressed in words, is the rule.

PROBLEM IV.

The area of the triangle A B D, $=\frac{AB\times DE}{2}$ (*Fig.* 128,) and the area of the $\triangle$ B D C $=\frac{DC}{2}\times BF=\frac{DC}{2}\times DE$. Then $AB\times DC\times\frac{DE}{2}=$ the sum, from which formula the rule is sufficiently obvious.

PROBLEM V.

In the Pentagon A B C D E, (*Fig.* 129,) let a perpendicular fall from the centre R. Then $AB\times\frac{RP}{2}=$ area of $\triangle$ A B R. Now the area of the polygon is plainly equal to the areas of as many $\triangle$s, each equal to the triangle A B R, as the polygon contains sides. Hence the reason of the Rule is manifest.

PROBLEMS VI & VII.

These two problems are simple applications of the rule for determining the area of triangles.

PROBLEM VIII.

The area $At=\frac{AD\times mt}{2}+Am$, (*Fig.* 130;) the area $ms=\frac{mt\times ns}{2}+mn$, &c. The area of the whole figure

$\frac{AD + mt + ns + or + pg + BC}{6} \times AB$. Now, $\frac{AD + BC}{2}$ is an arithmetical mean between the two ends. Hence the Rule.

PROBLEMS IX—XIII.

These Problems are of so little importance in practical land surveying that I think it unnecessary to give the investigations.

PROBLEM XIV.

This Problem is merely the application of the rule for ascertaining the area of a triangle.

PROBLEM XV.

THEOREM I.

If a survey has been accurately made the sum of the Northings will equal the sum of the Southings, and the sum of the Eastings will equal the sum of the Westings.

Let $a\ b\ c\ d\ e$, (*Fig.* 131,) represent a field. Let a be the first station, b the second, &c., and let N S be a meridian line. Then $a\,n$, $b\,h$, and $c\,p$, will be meridians, and $n\,b$, $h\,c$, and $p\,d$, will be departures, or East and West lines.

Then it is evident that the Northings $d\,f + e\,g$, will be exactly equal to the Southings, $a\,n + b\,h + c\,p$; *i. e.*, the sum of the Northings is equal to the sum of the Southings.

Also, the Departures, $c\,h + a\,g = b\,n + p\,d + f\,e$. For $a\,g + k\,n = p\,d + f\,e$, and $b\,n = h\,c - n\,k$; if $h\,c - n\,k$ be added to the first part of the preceding equation, and $b\,n$ to the latter, then $c\,h + a\,g = n\,b + p\,d + f\,e$, *i. e.*, the sum of the Eastings is equal to the sum of the Westings.

THEOREM II.

In any trapezium, as A B C D, (*Fig.* 132,) having two sides perpendicular to a given side, the product of half the sum of the parallel sides multiplied by the base on which they stand will be the area of the figure.

Let Ds and $m\,n$ be drawn parallel to A B, and E F parallel to B C or A D; Cs is the difference between the sides A D and B C, and C n = $n\,s$ = m D = their half difference, and the perpendicular E F will bisect Ds and $m\,n$. Now as the angles D F m and n F C are equal, Euc. i. 22, and the side F n = the side F m, and n C = m D, the triangles are equal and similar. Now if A B be multiplied by $\frac{BC + AD}{2}$ = E F, the product will be the area of the Trapezoid.

THEOREM III.

In rectangular surveying (in which the work is always on one side of the first meridian,) if the departure of any stationary distance is East, and the work lies on the East side of the North and South line, the farther that that course is run the greater will be the departure or the distance from the first meridian. But the farther that a stationary distance having West departure extends, the nearer will the first meridian be approached.

Draw N S (*Fig.* 133,) for a first meridian. Let a, b, c, and d, be stations on the East side of N S. Let also the perpendiculars $l\,a$, $b\,i$, $g\,c$, and $e\,d$, be raised upon the N. and S. line. Draw $a\,r$, $b\,t$, and $c\,p$, parallel to N S. Now the departure of the first stationary line $a\,b$, is $b\,r$, and lies to the West side of its meridian $a\,r$. As the point b in the line $a\,b$, is nearer the line N S than the point a, for $a\,l - r\,b = b\,i$; therefore $b\,i$ is less than $a\,l$, and consequently the point b is nearer the first meridian N S, than the point a is.

In the next stationary distance $b\,c$, the departure lies to the Eastward of its meridian $b\,t$; but the point c, or any point in this distance line $b\,c$, is more remote from the first meridian N S, than the point b is, for $b\,i + c\,t = c\,g$.

THEOREM IV.

If the meridian distance in the middle of every stationary line be multiplied into the particular Northings or Southings of that line, and the difference between the sum of the North and the sum of the South products be taken, that difference will be the area of the survey.

Let A F B C D E (*Fig.* 134,) be the plan of a survey whose area is required. Draw N S on the west side of the plan for a first meridian. Let perpendiculars fall upon N S from the beginning and middle of every stationary line, and let meridians be drawn through each station. Then A c, F m, and B i, will be the Northings; and C l, D p, and E x, will be the Southings.

The meridian distances to the middle of the stationary lines whose courses are Northward, are $o\,a$, $d\,n$, and $g\,k$.

Now by Theor. II, $ZH \times a\,o =$ area Z A F H Z; and $Hf \times d\,n =$ area H F B f H; and $f\,b \times g\,k =$ area f B C b f, and the sum of these areas will be the area of Z A F B C b Z, or the whole North area. Again the meridian distances to the middle of the stationary lines whose courses are Southward, are G R, $e\,q$, and $u\,v$.

By Theor. II, $b\,y \times$ G R = area of the trapezoid y D C $b\,y$, and $y\,h \times e\,q =$ area h E D $y\,h$, and $h\,Z \times u\,v =$ area Z A E h Z. The sum of these areas will be the area of Z A E D C b Z, or the whole of the South area. Then if the area Z A F B C b Z be taken from the area Z A E D C b Z, the remainder will be the area of the figure A F B C D E A, or the difference between the sum of the North areas and the sum of the South areas will be the area of the survey. Hence the foundation of the rule is evident.

DIVISION OF LAND.

The principles on which the Rules for the Division of Land are based are so evident from Euclid's Elements, that a formal demonstration of them is considered unnecessary.

LOCATION.

In perusing this part of the Appendix, the Student is referred to the figures contained in that part of the preceding work, which treats of the Location of Lands.

He will take notice, likewise, that in the following investigations the letter a, is employed to represent the area or contents of a field, farm, &c.

PROBLEMS I & II.

The area of a rectangle is equal to the product of one side $\times$ by the other. It is evident then that the area $\div$ by one of the sides will give the other side. The area of a square is the square of one of its sides. The square root of the area therefore, must be equal to the length of one of its sides. For a square, the formula stands thus: $\sqrt{a} =$ side of the square. For a rectangle it is thus expressed: $\frac{a}{b} =$ the side required, in which expression b represents the given side.

PROBLEM III.

Let S = nat. sine of the given angle, and $x =$ A B or B D. Then $S x^2 = a$, and $x^2 = \frac{a}{S}$; therefore $x = \sqrt{\frac{a}{S}}$, which affords the Rule.

PROBLEM IV.

Let S = nat. sine of the given angle, $b =$ the side given, and $x =$ the side required. Then $S \times b \times x$ or $S b x = a$; therefore $x = \frac{a}{bS}$, which is the rule.

PROBLEM V.

Let $m =$ the given multiple, and $x =$ the breadth. Then $m x =$ the length, and $m x \times x$ or $m x^2 = a$; hence $x^2 = \frac{a}{m}$. Therefore, $x = \sqrt{\frac{a}{m}}$, which gives the rule.

PROBLEM VI.

Let $x =$ width, then as $5 : 8 :: x : \frac{8x}{5}$, wherefore $\frac{8x}{5}$

$=$ length. Then $\frac{8\,x}{5} \times x = a$, or $\frac{8\,x^2}{5} = a$; hence $8\,x^2 = 5\,a$ and $x^2 = \frac{5\,a}{8}$. Therefore, $x = \sqrt{\frac{5a}{8}}$, which yields the Rule.

PROBLEM VII.

Let $x =$ breadth, and $b =$ the difference between the length and breadth; then $x + b =$ the length; hence $(x+b) \times x = a$ and $x^2 + b\,x = a$. Complete the square $x^2 + b\,x + \frac{b^2}{4} = a + \frac{b^2}{4}$. Then, $x + \frac{b}{2} = \sqrt{a + \frac{b^2}{4}}$. Therefore $x = \sqrt{a + \frac{b^2}{4}} - \frac{b}{2}$ or $x = \frac{b}{2} \pm \sqrt{a + \frac{b^2}{4}}$, which is the Rule.

PROBLEM VIII.

Let $x =$ breadth, S$=$ nat. sine of given $\angle$, and $m =$ the given multiple. Then $m\,x =$ the length. Then $x + m\,x \times S = a$, and $m\,x^2\,S = a$, and $x^2 = \frac{a}{m\,S}$. Therefore $x = \sqrt{\frac{a}{mS}}$, which is the Rule.

PROBLEM IX.

Let $S =$ nat. sine of $\angle$ B A D, and $x =$ width; then as $2 : 5 :: x : \frac{5x}{2} =$ C B, and $S\,x =$ D E, the perpendicular breadth; hence $\frac{5x}{2} + S\,x = a$, or $\frac{5Sx^2}{2} = a$, and $5\,S\,x^2 = 2\,a$, and $x^2 = \frac{2a}{5S}$, therefore $x = \sqrt{\frac{2a}{5S}} =$ A D, which affords the Rule.

PROBLEM X.

Let $x =$ width, then $2\,x =$ central length. Then $2\,x \times x$ or $2\,x^2 = a$ and $x = \sqrt{\frac{a}{2}}$, which is the Rule.

PROBLEM XI.

The reason of this rule appears from the rule given at Prob. II, because a triangle is just half a rectangle of the same base and altitude.

PROBLEM XII.

Let b = given side, S = the sine of the given $\angle$, and x = the side required; then, $\frac{b\,x\,S}{2} = a$; hence we have $b\,s\,x = 2\,a$ and $x = \frac{2a}{bs}$, from which the reason of the rule is obvious.

PROBLEM XIII.

This Problem is merely a particular application of the rule given at Prob. XI.

PROBLEM XIV.

Let S = the nat. sine of the given $\angle$, and x = one of the equal sides; then $s\,x^2 = 2\,a$, and $x^2 = \frac{2a}{s}$ and $x = \sqrt{\frac{2a}{s}}$, which is the Rule.

PROBLEM XV.

According to Euc. xii. 2, circles are to each other as the squares of their diameters. Now the area of a circle whose diameter is 1, according to the calculation of the celebrated Van Ceulen, is .785398 +, but for practical purposes .7854 is sufficiently near the truth. When therefore x = diameter, $x^2 \times .7854 = a$. Then $x^2 = \frac{a}{.7854}$, and $x = \sqrt{\frac{a}{.7854}}$, which gives the Rule.

PROBLEM XVI.

Let b = .7854, and x = the Conjugate diameter, then

$x + 100\ l. =$ the Transverse diameter; then $(x + 100) \times x \times b = a$, and $b\,x^2 + 100\,b\,x = a$; and $x^2 + 100\,x = \frac{a}{b}$. Complete the Square, and $x^2 + 100\,x + 2500 = \frac{a}{b} + 2500$. Then $x + 50 = \sqrt{\frac{a}{b} + 2500}$ and $x = \sqrt{\frac{a}{b} + 2500} - 50$, which affords the Rule.

PROBLEM XVII.

Let $x =$ the Transverse diameter, and $b = .7854$. Then as $5 : 3 :: x : \frac{3x}{5} =$ the Conjugate; hence $\frac{3\,b\,x^2}{5} = a$, and $3\,b\,x^2 = 5\,a$ and $x^2 = \frac{5\,a}{3\,b}$. Therefore $x = \sqrt{\frac{5\,a}{3\,b}}$, which formula expressed in words is the Rule.

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A COLLECTION OF

PROMISCUOUS PROBLEMS,

FOR THE FARTHER ILLUSTRATION OF THE PRECEDING RULES.

PROBLEM I.

In the triangle A B D, (*Fig.* 135,) are given A B = 5, B D = 3, and A D = 4, and the line P D *in position*, viz: A P = 1. Required the construction of the figure, and the length of the lines, A O, and O B, drawn from the angles A and B to a windmill at O, on such a point in the line P D, that the angle A O B shall contain 120°.

CONSTRUCTION.

Make the base, A B, the chord of 120°, and find the centre C of the corresponding circle by making the angles B A C and A B C each 30°. Because in every quadrilateral, as A O B F, inscribed in a circle, the sum of the opposite angles is equal to 180°, (Euc. iii. 22;) and as the angle A O B must contain 120° by the terms of the Problem, its supplement A F B must contain 60°. And since the angle at the centre of a circle is double the angle at the circumference, (Euc. iii. 20,) the ∠ A C B must be 120°. And as the triangle A B C is an isosceles, each of the angles C A B and A B C must contain 30°. The point in which the circumference of the circle cuts the line P D, given in position, will be the situation of the windmill, and the angle A O B will contain 120°, as was required.

Proportion by which to find the segments A H *and* H B.

As A B (5) : A B + D B (7) : : A D — D B (1) : A H — H B (1.4). Now A B = 5, and 5 + 1.4 = 6.4, the half of which is 3.2 = A H; and 5 — 1.4 = 3.6, the half of which is 1.8 = H B.

To find the perpendicular D H.

As the triangle A D B is right-angled at D, the perpendicular D H is a mean proportional between the segments A H and H B; therefore $\sqrt{3.2 \times 1.8} = \sqrt{5.76} = 2.4 =$ D H, and A H — P A = 2.2 = P H.

Proportion to find the ∠ D P H.

As the side P H = 2.2	0.34242
Is to H D = 2.4	0.38021
So is rad. 90°	10.00000
To the tan. ∠ D P H = 47° 29′	10.03779

Hence 180° — 47° 29′ = 132° 31′ = ∠ D P A.

To find C B.

As the sine ∠ B C A 120°	9.93753
Is to A B = 5	0.69897
So is sine ∠ C A B 30°	9.69897
To the side C B = 2.88	0.46041

To find the ∠ *s* B C P *and* C P B.

As B P + B C = 6.88	0.83759
Is to B P — B C = 1.12	0.04922
So is tan. ½ sum = 75°	10.57195
To tan. ½ diff. = 30° 17′	9.78358

Then 75° + 31° 17′ = 106° 17′ = ∠ B C P.
And 75° — 31° 17′ = 43° 43′ = ∠ C P B.

To find C P.

As the sine ∠ B C P = 106° 17′	9.98222
Is to side B P = 4	0.60206
So is sine ∠ C B P = 30°	9.69897
To the side C P = 208	0.31881

To find ∠ *s* P O C *and* P C O.

As side C O = 2.88	0.46041
Is to sine ∠ C P O 91° 12′	9.99991
So is C P = 2.08	0.31881
To sine ∠ P O C 46° 11′	9.85831

Then 180° — 46° 11′ — 91° 12′ = 42° 37′ = ∠ P C O.

To find the dist. P O.

As sine ∠ C P O 91° 12′	9.99991
Is to the side C O = 2.88	0.46041
So is sine ∠ P C O 42° 37′	9.83065
To the side P O = 1.95	0.29115

To find the ∠ *s* P O B *and* P B O.

As P B + P O = 5.95	0.77452
Is to P B — P O = 2.05	0.31175
So is tan. ½ sum ∠ s 66° 15′	10.35654
To tan. ½ diff. 38° 4′	9.89377

Then 66° 15′ + 38° 4′ = 104° 9′ = ∠ P O B.
And 66° 15′ — 38° 4′ = 27° 11′ = ∠ P B O.

To find the side O B.

As sine ∠ P O B 104° 19′	9.98630
Is to side P B = 4	0.60206
So is sine ∠ B P O 47° 29′	9.86752
To side O B = 3.04	0.48328

To find the side A O.

As sine ∠ A O B 120°	9.93753
Is to side A B = 5	0.69897
So is sine ∠ P B O 27° 11′	9.65976
To side A O = 2.63	0.42120

Note.—This Problem might have been solved more expeditiously by Algebra.

PROBLEM II.

Near the middle of a certain farm or tract of land A B C, (*Fig.* 136,) whose form is that of an equilateral triangle, and whose side A C runs due North, is a spring of water at O, so situated that the perpendiculars O T, O D, and O S. let fall from it upon the three sides of the triangle, are respectively 18, 20, and 24 chains, viz: O T = 18 *ch.*, O D = 20 *ch.*, and O S = 24 *ch.* The owner of the farm has bequeathed it to his three daughters, O, P, and Q; and to O is bequeathed the triangle A O C, to P the triangle A O B, and to Q the triangle B O C. Required the area of the whole farm, the area of the portion bequeathed to each of the daughters respectively, and the courses and distances of the division lines.

It can be proved that the perpendicular C R is exactly equal to the sum of the perpendiculars O D, O T, and O S, in whatever part of the triangle the point O is situated. The Problem is then solved by *Trigonometry* and *Mensuration of Superficies*, as follows:

To find the side A C, *and consequently the other sides.*

As the triangle A B C is equilateral. each of its ∠ s = 60°; therefore ·

As the sine of the angle at A 60°	9.93753
Is to C R = O T + O D + O S = 62	1.79239
So is rad. 90°	10.00000
To the side A C = 71.59	1.85486

Then 71.59 *ch.* (A B) $\times$ 62 *ch.* (C R) = 4437.58 *ch.* $\div$ 2 = 2218.79 *ch.* = 221 *ac.* 3 *r.* 20 *p.* = the area of the triangle A B C.

To find the share of each Daughter.

$\frac{71.59\ ch.\ (A\ C) \times 24\ ch.\ (O\ S)}{2}$ = 859.08 *ch.* = 85 *ac.* 3 *r.* 25 *p.* = O's portion.

Again $\frac{71.59\ ch.\ (A\ B) \times 20\ ch.\ (O\ D)}{2}$ = 715.90 *ch.* = 71 *ac.* 2 *r.* 14 *p.* = P's portion.

And $\frac{71.59\ ch.\ (B\ C) \times 18\ ch.\ (O\ S)}{2}$ = 464.31 *ch.* = 46 *ac.* 1 *r.* 28 *p.* = Q's portion.

To find the Courses and Distances of the Division Lines.

Join S D, D T, and T S; then in the $\triangle$ D O S we have given S O, O D, and $\angle$ S O D, to find the side S D and the $\angle$ S D O; then the $\angle$ A D O = 90° — $\angle$ S D O = $\angle$ A D S. Having ascertained this side and these angles, then in the triangle A S D, all the $\angle$s and the side S D will be given to find A D and A S. Then D B and S C may be found by Subtraction, and B O and O C by the Square Root.

Next, the $\angle$ S O D = 120° subtracted from 180° = 60°, the half of which is 30° = half the sum of the opposite angles.

Also, D O + O S = 20 + 24 = 44 = sum of the sides.

And O S — D O = 24 — 20 = 4 = difference of sides.

To find the $\angle$*s* S D O *and* D S O.

As D O + O S = 44	1.64345
Is to O S — D O = 4	0.60206
So is ½ sum of $\angle$s 30°	9.76144
To tan. ½ diff. 3°	8.72005

To find the side S D.

As sine $\angle$ D S O 27°	9.65705
Is to side D O = 20	1.30103
So is sine $\angle$ S O D 120°	9.93753
To the side S D = 38.15	1.58151

Then $30° - 3° = 27° = \angle$ D S O, and $30° + 3° = 33° = \angle$ S D O, and $90° - 27°$ ($\angle$ D S O) $= 63° = \angle$ A S D, and $90° - 33° = 57° =$ A D S.

To find A D.

As sine $\angle$ D A S 60°	9.93753
Is to side D S = 38.15	1.58151
So is sine $\angle$ A S D 63°	9.94988
To side A D = 39.29	1.59376

To find A S.

As sine $\angle$ D A S 60°	9.93753
Is to side D S = 38.15	1.58151
So is sine $\angle$ A D S 57°	9.92359
To side A S = 36.95	1.56757

To find the $\angle$ D A O.

As side A D = 39.29	1.59376
Is to side D O = 20	1.30103
So is rad. 90°	10.00000
To the tan. $\angle$ D A O 27°	9.70727

Now the $\angle$ C A B $= 60°$, and $\angle$ D A O $= 27°$; their difference is 33°, and the course of A O is N. 33° E.

To find the $\angle$ D B O.

As the side D B = 32.30	1.50920
Is to the side D O = 20	1.30103
So is rad. 90°	10.00000
To the tan. $\angle$ D B O 31° 43′	9.79183

Since $\angle$ C A B $= 60°$ and $\angle$ D B O $= 31° 43'$, their sum is $91° 43' - 90° = 1° 43'$ and $90° - 1° 43' = 88° 17'$. Hence the course of the line B O is S. 88° 17′ E.

To find the length of A O.

$\sqrt{(AD^2 + DO^2)} = \sqrt{1943.7041} = 44.08 =$ A O. The length of A O therefore is 44 *ch.* 8 *l.*

To find the length of B O.

$\sqrt{(DE^2 + DO^2)} = \sqrt{1443.2900} = 37.99 =$ B O. The length of B O therefore is 37 *ch.* 99 *l.*

To find the $\angle$ O C S.

As the side C S = 34.64	1.53958
Is to the side S O = 24	1.38021
So is rad. 90°	10.00000
To tan. S C O 34° 43′	9.84063

To find the length of C O.

$\sqrt{(SC^2+SO^2)}=\sqrt{1776.0296}=42.14=CO$. Therefore the course of C O is S. 34° 43′ E., and the distance is 42 *ch.* 14 *l.*

PROBLEM III.

The Figure (137) is intended to represent that part of a lake which is included within the boundaries of a farm, the area of which part I am employed to determine. Commencing at that point C at which the rear line intersects the lake, I run S. 20 *ch.*, thence W. 15 *ch.*, thence N. 15 *ch.*, striking the rear line at D. Setting my compass at D, I find the course of D C, the rear line, to be E. I am satisfied therefore, that the survey has been accurately made, and that the distance between C and D is 15 *ch.* I then measure the offsets and enter them into my field book as follows:

No. of $\triangle$s.	Base.	Perp.	Double Areas.
1	800 *l.*	500 *l.*	400000
2	300	200	60000
5	500	300	150000
6	600	200	120000
			2)730000
			365000

No. of Trapezoid.	$\frac{1}{2}$ Sum of Par. Sides	Base.	Area.
3	450 *l.*	150 *l.*	67500
4	600	200	120000
7	225	300	67500
8	225	400	90000
			345000

The area of the whole Rectangle A B C D is

B C $\times$ D C $=$ 1500 *l.* $\times$ 2000 *l.* $=$	30.00000 *l.*
And the area of all the offsets is 365000 + 345000 =	7.10000
	22.9000
	4
	3.60000
	40
Area of the Lake 22 *ac.* 3 *r.* 24 *p.*	24.00000

PROBLEM IV.

The multilineal Plot (*Fig.* 138,) represents the boundaries of an ungranted or reserved lot of land, of which the bases and perpendiculars are given in the following field book. Required the contents. It is required also to lay off 200 acres from the point B, towards A.

No. of $\triangle$.	Base.	Perp.	Double Areas
1	1800	750	1350000
2	1800	850	1530000
3	1270	350	444500
4	2100	600	1260000
5	1800	350	630000
6	1400	250	350000
7	3100	540	1674000
8	2500	1100	2750000
9	2850	1280	3648000
10	2850	340	969000
11	3050	700	2135000
12	4740	1740	8247600
13	4200	2860	12012000
14	2200	1200	2640000
15	2260	1600	3616000
16	3400	1510	5134000
17	3700	1200	4440000
18	10800	4200	45360000
19	10800	4400	47520000
			2)145710100
			72855050

TRAPEZOIDS.

$$\begin{aligned} r\,g &= 1600 \\ o\,f &= 1000 \\ n\,e &= 1500 \\ md &= 1060 \\ c\,c &= 2050 \\ S\,b &= 1900 \\ t\,a &= 2500 \\ \hline & \ 11610 \end{aligned}$$

Then the sum of these offsets $= 11610 \div 7$, the number of them $= 1658$ the mean offset; and $1658 \times t\,r = 6400 = 10611200 =$ the area of the trapezoids, *r f*, *o e*, *n d*, *m c*, *e b*, and *s a*, which added to the area of the triangles, (Nos. 1—19,) gives $83466250 = 834$ *ac.* 2 *r.* 26 *p.* = area of the whole lot.

To lay off 200 acres from the point B.

Let a conjectural line be drawn or run from B to M. The area of B M A B will be found to be 26288200, from which subtract the quantity to be laid off, 20000000, the remainder is 6288200, which $\times$ by $2 = 12576400 =$ the double area of the $\triangle$ B A M, which divided by the conjectural line B M, or 9150, gives 1374 for a perpendicular let fall from M on the line A B. Then run B A and the operation is completed.

PROBLEM V.

The following method of ascertaining the contents of a field, whose boundaries are curvilinear or irregular, is sometimes successfully adopted by skilful and experienced surveyors.

Let the mixtilineal figure *a c d e f g h* C A, (*Fig.* 139,) represent the boundaries of a field or tract of land, the contents of which are required. Run the lines A B and B C, so that the parts *a*, *c*, *e* and *g*, included between these lines, shall be, as nearly as can be estimated, equal to the parts *b*, *d*, and *f*, lying beyond them; then find the area of the triangle A B C.

In the same manner a curvilinear figure may be reduced to the form of a parallelogram, or any other rectilinear figure, and its area ascertained by the ordinary rules for determining the area of such figures.

By this method a surveyor, of good judgment and extensive experience, will come very near the true contents of a field. As, however, much will depend upon the formation of a just estimate of the quantity of land contained in detached pieces, this method should only be adopted by skilful practitioners, and where lands are not very valuable.

PROBLEM VI.

The triangular lot of land, A B C, (*Fig*. 140,) lies on the side of the road A B. It is required to divide it by a line running from the opposite angle C to the road A B, so that the areas of the parts may be as 9 to 7, the side A B being $9\frac{1}{2}$ chains. What extent of front must be assigned to each part?

As the two $\triangle$ s into which the field is to be divided have the same altitude, it is evident that they must be to each other as their bases.

Therefore as 16 : 9 : : 950 *l.* : 534.375 or 5 *ch.* 54 + *l.* = A D.
And as 16 : 7 : : 950 *l.* : 415.625 or 4 *ch.* 15 + *l.* = D B.

PROBLEM VII.

Divide the straight line A B, (*Figs*. 141 & 142,) into three parts, in the proportion of 3, 5, 7.

From the points A and B, draw the parallel lines A C and B D, on opposite sides of the line A B. From a scale of equal parts lay off 3 from A towards E, and 7 from B towards F. From the same scale lay off also 5 from E towards C, and from F towards D. Then draw the lines E D and C F, intersecting A B in H and R.

Then A H : H R : : 3 : 5, and H R : R B : : 5 : 7.

In this way A B may be divided similarly to any given divided line.

Note.—This method of dividing lines might be advantageously employed to find the point D in the preceding Prob., and the point E in that which follows.

PROBLEM VIII.

The side A B of a triangular field containing 6 acres, is 466, and A C 420. It is required to lay off 2 *ac.* by a straight line running from the point D, which is 230 distant from A.

By Prob. VI. or VII., find the point E in the base A B. If E falls between A and D, the point F will be in A C; but if not, the point F will be in B C. Now if we join D C, and draw E F parallel to it, it is evident that the $\triangle$s A C D and A F E would be parallel.

Therefore, As A D : A E : : A C : A F.

To find A E.

By Prob. VI., As 6 : 2 : : A B (466) : A E (155).

To find A F.

As A D (230) : A E (155) : : A C (420) : A F (283).

Lay off 283 from A to F, in the direction of C, and run the line F D, and the work is completed.

If E had fallen between D and B, the point F would have fallen in the line C B.

PROBLEM IX.

A Gentleman after having taken the dimensions of a square field forgot all the distances, and only recollected that having occasion to measure the diagonal he found it to exceed the side by 10 *ch.* Required a rule by which to find the side.

Let x = one of the sides of the square A B C D, (*Fig.* 143,) and d = the difference between the side and the diagonal or 10 *ch.* Then $x + d$ = the hypotenuse B D. But according to Euc. i. 47, $BC^2 + CD^2 = BD^2$ or $2\,BC^2 = BD^2$. Hence we have $2x^2 = x^2 + 2dx + d^2$ or $x^2 - 2dx = d^2$. Complete the Square and $x^2 - 2dx + d^2 = d^2 + d^2$ or $x^2 + 2dx + d^2 = 2d^2$. Then $x - d = \sqrt{2d^2}$, and $x = \sqrt{2d^2} + d$ or x [illegible]4.14 = side of the square. Hence results the following

RULE:

To the square root of twice the square of the difference, add the difference, and the sum will be the side of the square.

PROBLEM X.

Required a rule by which to lay off a given quantity of land in the form of a triangle, two sides of which shall be equal, the included angle being given.

Let a represent the area, S the nat. sine of the given $\angle$, and x one of the equal sides. (*Fig.* 144.) Then $S\,x^2 = 2\,a$ and $x^2 = \frac{2\,a}{S}$, therefore $x = \sqrt{\frac{2\,a}{S}}$. Whence results the rule. Divide double the area by the nat. sine of the included angle, and the square root of the quotient will be the length of one of the equal sides.

PROBLEM XI.

To re-establish marks upon an old line, without running it.

RULE.

Run a line at random in the direction of the old line, upon which set up stakes equi-distant from each other. When you arrive opposite to the end of the original line, the marks upon which you wish to restore, remove your instrument thereto, and let a perpendicular fall upon the random line. Then as the length of that line is to the perpendicular let fall upon it from the extremity of the old line, so is the distance to the first stake to the distance which it must be moved to the old line. Then if this distance be multiplied by 2, 3, 4, &c., as circumstances may require, the products will be the several distances which the stakes must be moved, in order to stand on the old line.

Let C B, (*Fig.* 145,) be an old line, the marks upon which it is required to re-establish, while at the same time it is inconvenient to run the same. Run the random line C A 20 *ch.* The length of the perpendicular A B, or the distance of A from the end of the old line is found to be 5 *ch*. On this line, A C, set up stakes at a, c, and e, each 5 *ch*. distant from each other.

Then, As C A (20) : A B (5) : : C e (5) : $e\,f$ (1.25.)
As C A (20) : A B (5) : : C c (10) : $c\,d$ (2.50.)
As C A (20) : A B (5) : : C a (15) : $a\,b$ (3.75.)

Or thus: Having found by the first proposition that $e\,f = 1.25$, then $1.25 \times 2 = 2.50 = c\,d$, and $1.25 \times 3 = 3.75 = a\,b$.

PROBLEM XII.

A surveyor having run 30 *ch*. in a direction between South and East, finds that the sum of his difference of latitude and departure is 40 *ch*. Required the course, and the area included within the line run, the difference of latitude, and the departure.

INVESTIGATION.

Let a = the distance B C, (*Fig.* 146,) and b = diff. of lat. + dep., or A B + A C; also let x = the dep. or A C, then B — x = diff. of lat.

Now, $B\,C^2 = B\,A^2 + A\,C^2$ (Euc. i. 47,) *i. e.* $a^2 = b^2 + 2\,b\,x + 2\,x^2$ or $2\,x^2 - 2\,b\,x = a^2 - b^2$. Therefore, $x^2 - b\,x = \frac{a^2 - b^2}{2}$, then $x^2 - b\,x + \frac{b^2}{4} = \frac{a^2 - b^2}{2} + \frac{b^2}{4} = \frac{2\,a^2 - b^2}{4}$, and $x - \frac{b}{2} = \sqrt{\left\{\frac{2\,a^2 - b^2}{4}\right\}}$ or $x = \sqrt{\left\{\frac{2\,a^2 - b^2}{4}\right\}} + \frac{b}{2}$ $= 27.07 =$ A C the dep. Then $40 - 27.07 = 12.93 =$ A B the diff. lat.

Hence we have the following rule to find the remaining sides of a right-angled triangle, when the hypotenuse and the sum of the other two sides are given, viz:

From twice the square of the hypotenuse, or distance run, subtract the square of the sum of the other sides, viz. the diff. lat. and dep.; divide the remainder by 4, and to the square root of the quotient add half the sum of the two sides, and that sum will be the departure.

To find the Course.

As the diff. of lat. 12.92	1.11126
Is to the dep. 27.07	1.43248
So is rad. 90°	10.00000
To the tan. of course 64° 30′	10.32122

To find the Area.

$\frac{27.07 \times 12.93}{2} = 17\frac{1}{2}$ *ac.* nearly. The course, therefore, is S. 64° 30′ E., and the area 17½ *ac.* nearly.

PROBLEM XIII.

To locate land in the form of a Right-angled Triangle, the Area and the Hypotenuse being given.

Let a = area, b = the hypotenuse A C, (*Fig.* 147,) and x = base A B, then $\frac{2a}{x}$ = perpendicular C B. Then, by Euc. i. 47, $x^2 + \frac{4a^2}{x^2} = b^2$; then $x^4 + 4a^2 = b^2 x^2$, and $x^4 - b^2 x^2 = -4a^2$. Next, $x^4 - b^2 x^2 + \frac{b^4}{4} = \frac{b^4}{4} - 4a^2$ Then, $x^2 - b^2 = \sqrt{\left\{\frac{b^4}{4} - 4a^2\right\}}$ and $x^2 = b^2 + \sqrt{\left\{\frac{b^4}{4} - 4a^2\right\}}$ Wherefore $x = \sqrt{\left\{ b^2 \pm \sqrt{\left(\frac{b^4}{4} - 4a^2\right)}\right\}}$ which expression affords us the following

RULE:

From the biquadrate of the hypotenuse divided by 4, subtract four times the square of the area, and extract the square root of the remainder. To the root add, or from it subtract (according as you wish to ascertain the longest or shortest side of the triangle) the square of the hypotenuse divided by 2, the square root of the sum and of the difference will give the length of the sides respectively.

PROBLEM XIV.

Required the length and breadth of a rectangular meadow, whose perimeter is 60 chains, and area 20 acres.

Let a = area, b = perimeter, and x = length, then $\frac{a}{x}$ = breadth, and $2x + \frac{2a}{x} = b$. Then $2\,x^2 + 2\,a = b\,x$. This equation reduced gives $x = \frac{b}{4} + \sqrt{\left\{\frac{b^2}{16} - a.\right\}}$ Hence $x = 20\,ch.$ = length, and $\frac{a}{x} = 10\,ch.$ = breadth. This formula, expressed in words, affords the following

RULE:

Divide the square of the perimeter by 16, and from the quotient subtract the area. To the square root of the remainder add one fourth of the perimeter, and the sum will be the length, and the area divided by the length will be the width.

PROBLEM XV.

An easy method of locating land in the form of a Rhomboid.

Let the figure A B C D, (*Fig.* 148,) represent a Rhomboid whose area and front A D are given.

By Trigonometry find the base and perpendicular A e and e D, of the right-angled triangle A E D = C b B. Next, find the areas of these triangles; then by Prob. II., Loc., lay out the remainder in a rectangle as A b C e, and D e + e C = D C, a side of the Rhomboid.

PROBLEM XVI.

The owner of a square field A B C D, (*Fig.* 149,) containing 10 *ac.*, wishes to lay off a walk half way around which shall take up one acre. Required the width of the walk.

Let a = one side of the square, and x = width of the walk; then $2\,a\,x - x^2 = b$, the area of the walk. Now this equation reduced gives $x = a \pm \sqrt{(a^2 - b)} = 52\ l.$, the width of the walk. Hence the

RULE.

From the area of the square subtract the area of the walk, subtract the square root of the remainder from the side of the square, and this last remainder will be the width of the walk.

Note.—Perhaps it may not be altogether superfluous to remark that the square root of any quantity may be either positive or negative, *i. e.*, it may have either the sign + or — before it, because $-x \times -x = x^2$, as well as $x \times x$. Quadratic equations therefore admit of two solutions. Hence the reason of the use of the ambiguous expression $\pm$ placed before the unknown quantity. The learner must not, however, on that account, suppose that every Problem leading to a Quadratic equation may have two answers. The Problem may be of such a nature as to render one of the results wholly inadmissable, and which of them is to be rejected will be easily determined by the conditions of the Problem itself.

PROBLEM XVII.

It is required to locate 2 *ac.* in the form of the figure C A B n c d, (*Fig.* 150,) so that the width C d shall be equal to the width B n, the side C A being 6 *ch.*, and A B 4 *ch.* What must be the width of C d or B n?

Let a = area, b = C A or 6 *ch.*, c = A B or 4 *ch.*, and x = width, or C d = B n. Then $b + c - x$ = length, and $(b + c - x) \times x = b\,x + c\,x - x^2$ = a substitute t for $b + c$, then $t\,x - x^2 = a$, and this equation reduced gives the value of x; thus, $x = \frac{t}{2} \pm \sqrt{\left\{\frac{t^2}{4} - a\right\}} = 277\ l.$ = C d or B n, and from this expression we obtain the following

RULE:

Add the distances A B and A C. Divide the square of their sum by 4. From the quotient subtract the area. Subtract the square root of the remainder from half the sum of the two sides A B and A C: the remainder is the width.

PROBLEM XVIII.

To find the scale by which any plan has been drawn when the contents are given, but the scale omitted.

RULE.

Find the area by any scale, and then institute the following proportion:

As the content found
Is to the square of the scale by which you found it,
So is the given area
To the square of the scale by which the plan was drawn.
The square root of this number will be the scale required.

PROBLEM XIX.

To find the true area of a survey, though it has been taken by a chain which is either too long or too short.

RULE.

Let the area be completed, as if the chain had been of the proper length. Then form the following proportion:

As the square of the true chain
Is to the content found by the chain employed,
So is the square of the chain by which the survey was made
To the true area.

PROBLEM XX.

To find the area of the inaccessible field A B C D E A. (*Fig.* 151.)

RULE.

Run a line as R S at some convenient distance from the field. At the extremities of this line take the bearings of the respective angles, as R A, R B, R C, &c., and S A, S B, S C, &c. Then by protracting these several courses and joining their points of intersection, as A B, A C, &c., you have the sides of the field.

Its sides may also be found by Trigonometry, thus: In the △ R S A, you have all the ∠ s and the side R S to find the other sides. Again in the △ R S B, you have all the ∠ s and the side R S to find the other sides. Having ascertained the sides R A and R B, and knowing the ∠ A R B, the side A B is easily determined. In the same way all the remaining sides of the field may be ascertained.

PROBLEM XXI.

Another method of finding the area of right-lined figures by Calculation.

Find the Difference of Latitude and the whole Departure, and fill up the Table as in Prob. XV. of Mensuration. Having filled up the column of Meridian Distances, add the first M. D. to the second, for the first or upper number in the Column of *Sums;* add the second M. D. to the third for the second number in the column of *Sums*, &c.; and add the lowest M. D. to the uppermost for the last number in the column.

Then multiply each number in the column of sums by its respective Northing or Southing. Insert the product in the column of North or South areas, and half the diference between these columns will be the area of the field.

The following example will serve to illustrate the Rule.

CALCULATION TABLE.

Course.	Dist.	North.	South.	East.	West.	Mer. Dist.	Sums.	North areas.	South areas.
N. 10° E.	2900	2856		504		0	504	1439424	
N. 60° E.	3000	1500		2598		504	3606	5409000	
S. 66° E.	3300		1342	3015		3102	9219		12371898
South	5500		5500			6117	12234		67287000
S. 10° W.	2000		1969		347	6117	11887		23405500
S. 80° W.	3000		521		2954	5770	8566		44733000
N. 58° W.	3300	1749			2798	2316	2334	4957666	
N. 19° 19′ W.	3257	3227			18	18	18	58086	
		9332	9332	6117	6117	24444	48888	11864176	107587707

107587707
11864176
2)[illegible]
478.61765
4
1.47060
40
18.82400

Ans. 478 *ac.* 1 *r.* 18 *p.*

Note.—In this method the whole Departure must be taken.

Remark.—The Pennsylvanian method of Calculation is very correct, but I cannot perceive any advantage which it possesses over the methods herein laid down and illustrated. There are several other methods by which the contents of land may be calculated, but as the methods already laid down are so correct,—so easily understood,—and so universally applicable to the purposes of Practical Land Surveying, especially when the Surveyor must depend upon calculations from courses and distances, that I

IMAGE EVALUATION
TEST TARGET (MT-3)

6"


2.8
2.5
3.2
2.2
2.0
1.0

have not thought it advisable to swell the size or increase the price of this Treatise by their insertion. A Surveyor will find his advantage in selecting some particular system, in making himself thoroughly and familiarly acquainted with it, and adhering to it with very few deviations.

Before concluding this branch of the subject, it may be serviceable to lay down a few RULES *for finding contained angles.*

1. If the first letters of the course are unlike, and the last likewise, subtract the less course from the greater;

Thus: S. 62° W.
N. 15° E.

47° = included ∠.

2. If the first letters are unlike and the last alike, add the bearings;

Thus: S. 30° W.
N. 40° W.

70° = included ∠.

3. If the first and last letters are both alike, add the less bearing to the supplement of the greater.

4. If the first letters are alike and the last unlike, subtract the sum of the bearings from 180°.

General Rule.—In order to find the quantity of an angle suppose yourself standing at the angular point. Then reverse one or both courses, (as necessity may require,) and the quantity of the angle will be easily ascertained.

LEVELLING.

Though it is not expected that every Land Surveyor will possess all the qualifications, and provide himself with all the instruments necessary for civil engineering, yet he will often find an acquaintance with the general principles of Levelling of very great service. He may be employed to survey a course for conducting supplies of water, to determine the most suitable site for the erection of a mill, or the most eligible route for the location of a road. If he make himself completely master of the subject he is in this respect qualified to superintend the construction of a rail-road or the cutting of a Canal. W. & S. Jones' Circumferentor, a cheap and simply constructed instrument, will enable any individual, otherwise qualified for the task, to perform all the ordinary operations in which a surveyor can expect to be employed.

With regard to the theory of Levelling, we may observe that a plumb line, hanging freely in the air, marks the direction of gravity or the true perpendicular. A line drawn at right angles with this perpendicular is a level line. There is an *apparent level* and a *true level.* The line drawn at right angles to the perpendicular or plumb, and touching the earth's surface, if extended each way in a straight line, is the apparent level. Such a line, however, will soon leave the surface of the earth, because the earth is globular, and the rising of the line will be in the proportion of the square of the distance to which it is produced. It will be raised 8 *in.* at the end of the first mile. At the end of the second mile it is raised 8 *in.* $\times$ by the square of the distance. The distance is 2 miles, the square of which is 4, which $\times$ 8 = 32 *in.*, which is the height to which it has risen at the end of the second mile. At the end of the third mile it is 3 times 3 or 9 $\times$ 8 = 72 *in.*, &c. The true level is a curve of the earth's surface, after the inequalities and irregularities of that surface have been removed, or a curve of the earth's surface equally distant in all its parts, from the centr. The apparent level is the tangent of that curve.

For the purpose of Levelling the surveyor should be furnished with two Observation Staves, about 10 feet long, divided into feet, inches, and tenths, and having a moveable vane or a slide attached to it, which may be made fast in any particular part. Before commencing operations he should also secure the assistance of two competent men to carry the staves, to place them in the proper stations, to keep them in an erect or exactly perpendicular position, and to elevate or depress the vane as occasion may require. He needs also two men to measure the stationary distances, and one or more to remove obstructions from the way.

Being thus furnished, the Surveyor directs the chain-bearers to proceed, and measure or run the stationary distance. In consequence of the curvature of the earth's surface the stationary distances should not exceed a few chains. (The curvature of the earth in 10 *ch.* amounts to an eighth or .125 of an inch.) He then plants his instrument in the centre of this distance, or midway between the two stations. Then having carefully adjusted and levelled his instrument, observing particularly that the air bubble is exactly in the centre, he directs the men at the stations to raise or depress the vane until it is cut by the hair of the instrument, and to note the height upon the staves. Suppose the height of the vane upon one staff is 6 feet from the surface, and upon the other 3; then it is evident that the rise between the places is exactly 3 feet.

In a continuous process of levelling, or what is termed compound levelling, it is not necessary to find by Subtraction the differences between every stationary distance. It is sufficient to enter each observation in its respective column, under its title of fore-sight or back-sight. Having made the necessary entries, the back-staff should be carried forward and placed at a convenient distance before the fore-staff, which now becomes the back-staff. Then place the instrument in the centre, and proceed as before. In this manner complete the survey.

Add up the column of back-sights and the column of fore-sights. If these sums are equal, and the survey has been correct, the first station and the last have the same elevation, or they are on the same level. If the sum of the back-sights exceed the sum of the foresights, the terminating point or last station is higher than the place of beginning or first station, and *vice versa*, as will appear by the following examples:

EXAMPLE I.

No. of Stat.	Course.	Dist. in lin.	Back-sights. Ft.	In.	Tenths	Fore-sights. Ft.	In.	Tenths
1	N. 10° E.	300	6	0	2	0	8	4
2	,,	300	7	2	0	1	2	3
3	,,	300	7	3	5	3	0	0
4	,,	300	6	8	0	2	0	7
5	N. 15° E.	300	5	4	3	6	4	0
6	,,	300	4	8	4	7	6	4
7	,,	300	2	0	0	8	0	0
8	,,	300	3	2	0	8	5	4
		2400	42	4	4	37	3	2
			37	3	2			
			5	1	2			

The terminating station is therefore 5 *ft.* 1 *in.* 2 *tenths* higher than the first.

EXAMPLE II.

No. of Stat.	Course.	Dist. in lin.	Back-sights. Ft.	In.	Tenths	Fore-sights. Ft.	In.	Tenths
1	S. 45° W.	300	4	0	1	6	0	2
2	,,	300	5	2	0	5	0	1
3	,,	300	4	3	2	4	5	0
4	,,	300	6	0	1	5	1	3
5	S. 50° W.	300	2	1	4	7	2	0
6	,,	300	4	2	0	5	0	2
		1800	25	8	8	32	8	8
						25	8	8
						7	0	0

Hence it appears that the terminating point is 7 feet lower than the first station.

The height of hills may be ascertained likewise by Trigonometrical Calculations, by taking the angles of elevation and depression, and measuring the slant sides.

MISCELLANEOUS.

RE-ESTABLISHMENT OF LOST BOUNDARIES,—RELEASE.

In attempts to settle disputes about old lines, it often happens that the parties who were present at the original survey have forgotten the circumstances or have removed from the country, or have themselves been removed by death, and consequently sufficient evidence cannot be procured to restore the original boundaries. When a new line has been agreed to, in order to prevent litigation afterwards, it is advisable that the parties formally release to each other all the land quite to the newly established line.

The following is a copy of a Legal Release for this purpose:—

KNOW ALL MEN, by these presents, that I, *A. B.*, of in the County of and Province of for and in consideration of the sum of good and lawful money of the said Province of to me in hand, well and truly paid by *C. D.*, of in the County of and Province of at and before the ensealing and delivery of these presents, the receipt whereof I do hereby acknowledge, and that I am fully satisfied and paid, HAVE remised, released, and quit claim, and by these presents DO remise, release, and quit claim, to the said *C. D.*, his heirs and assigns forever, all my right, title, interest, claim, and demand, of what nature or kind soever, whether in law or equity, of, in, and to the following piece or parcel of Land, situate, lying and being in and bounded as follows, namely: beginning at* containing by

* It might be useful in a release of this kind to insert here the name of the Surveyor, the date of the survey, and the course of the line now established.

estimation be the same, more or less. TO HAVE and TO HOLD the above described premises, to him the said *C. D.*, his heirs, and assigns, to his and their only proper benefit and behoof forever, together with all and singular, the buildings, privileges, and appurtenances thereto belonging, or in any wise appertaining, and to every part and parcel thereof.

IN WITNESS whereof I have hereunto my Hand and Seal subscribed and set, this day of in the year of our Lord One Thousand Eight Hundred and

Signed, Sealed, and Delivered in the presence of
E. F.
O. P.

A. B. L. S.

Facts Respecting Magnetism,—Variation of the Compass.

Iron, with its oxides, and alloys, is one of the substances most generally diffused through nature. It is not, however, the only substance possessing the property of becoming magnetical. The influence of Magnetism has been distinctly observed in Nickel, Cobalt, and Titanium. It may be detected in many clays, sands, stones, springs, and rivers, in rain and snow, and even in many animal and vegetable substances. According to M. Arago there is no substance which, under favourable circumstances, is incapable of exhibiting unequivocal evidences of magnetic virtue.

The opposite poles or ends of magnets attract each other, *i. e.*, the North pole of one magnet will attract the South pole of another magnet.

The electric fluid, or lightning, generally destroys the polarity of the needle.

Heat has a great influence on magnetism. A white heat entirely destroys the magnetic virtue. According to the experiment of Barlow on malleable iron, soft shear steel, and hard shear steel, the magnetic power is about 4 times as strong when the metal is at a red heat or at a blood red heat as when it is cold.

Some substances will not exhibit any symptoms of magnetic virtue until gently heated. Minerals which are not metallic are almost all acted upon by the magnet after they have been subjected to the action of fire.

The effects of heat upon the magnetic virtue is very variable. The principle upon which it operates is not understood. No rule can therefore be given, by which its effects may be determined.

Chemical action is said also to affect the magnetic needle.

The statements of experimenters upon magnetism are often very opposite and contradictory.

These considerations would naturally lead us to expect considerable disagreement between magnetic instruments, and a want of uniformity of action in the magnetic needle. This accordingly we find to be the case. These facts therefore afford sufficient evidence of the inaccuracy of a method sometimes employed by surveyors to ascertain the difference of variation. The following is the method to which I allude: Knowing the original course and date of an old line, and having ascertained the present course, they divide the difference between the two courses by the number of the years which have intervened between the dates of the respective surveys, and use the quotient as the mean annual difference of variation, and allow it accordingly on all lines near that place. The inaccuracy of this rule may be placed beyond dispute, by the consideration of the following facts:

The line of division between the Townships of Sackville and Westmorland, in the Province of New Brunswick, runs nearly four miles through a part of Sackville marsh, and therefore affords peculiar facilities for observation.

In 1762 its course was due North. In 1791 it was traced, and found to run by the compass N. 2° 40′ E., which gives nearly 5′ 30″ as the yearly difference of variation. According to a grant and plan made by George Sprowle, then Surveyor-General of New Brunswick, the course in 1813 was N. 2° 30′ E, which exhibits a retrogade change of variation of 10′ in twelve years. In 1843 I found it to be N. 6° 15′ E, or 3° 45′ in thirty years, giving an annual change of variation of 7′ 30″. From 1762, when the line was first run, until 1843, are eighty-one years, and the change of variation during that time amounts to 6° 15′, which gives 4′ 37″ for the mean annual change of variation. Again, by other lines situated very near this line, and run in 1823 the difference of variation in 1843 was found to be 1° 20′ or 4′ every year.

Whether we impute these discordant results to the inaccuracy of the surveys, to the disagreement of instruments, or to the irregularities of the magnetic influence, or to all these causes combined, the inaccuracy of the above rule is equally proved.

Surveyors while using instruments governed by the magnet should take care that they have no substance about their persons by which its actions may be affected. A delicate needle may be affected by a knife in the pocket, or buttons composed of magnetic brass upon their clothes. Of this fact any person may satisfy himself by placing a compass on some solid object, and after the needle has settled, causing some person having any ferruginous substance about him to approach within two feet of the instrument. The movement of the needle from its true position will indicate the magnetic disturbance which his presence occasions. To this cause may be referred many of the inaccuracies which are so perplexing in old surveys.

Surveyors would do well to devote some of their leisure hours to the study of the geological structure of the earth. Some acquaintance with this important and interesting science is so intimately connected with practical land surveying, that it may legitimately be said to come within the range of his professional qualifications.

Suggestion about Meridian Lines.

I would before dismissing this subject take the liberty of suggesting the propriety and expediency of establishing true meridian lines in every County, or at convenient distances from each other, by which every Surveyor might compare his instrument as often as should be deemed necessary. These lines should be determined with great accuracy by astronomical observation, and established by permanent marks not liable to an alteration of position. The courses contained in all documents conveying a title to landed property, and in all documental records should be taken from this true meridian, instead of from the magnetic. By this means many inaccuracies would be avoided, much assistance would be afforded in after times to Surveyors in tracing lines which will then be old, and become eventually the source of much valuable information upon the science of magnetism,—a science of great importance in the affairs of mankind, and a science as yet but imperfectly understood.

Concluding Remarks.

My object in the preceding treatise has been to select and include within as narrow limits as possible the most simple and most extensively applicable methods of Land Surveying, and to adapt the statement and illustration of them to the most ordinary capacity.

The Surveyor must not expect to find in any treatise examples of every case with which he may meet in the course of his practice. When we consider the numerous unaccountable irregularities to which the magnetic virtue is subject, the numerous and unappreciable causes by which it may be disturbed, the differences between instruments governed by the needle, and the loose manner in which the original surveys were frequently executed, he need not be astonished if after his utmost care and diligence, he find it almost impossible to restore lost boundaries with any degree of certainty.

Most of the old grants in these Provinces contain more land than they express. Sometimes, however, they contain less. In either case the recorded description differs from their true dimensions and contents. Still the law requires the Surveyor, in re-surveys, to follow as nearly as possible the original description.

Many errors have arisen from following courses as they are literally expressed in the grant or recorded document, without making the necessary allowance for change of variation, &c. When this course has been pursued in subsequent transfers of land, expensive litigation has sometimes been the result.

Until meridian lines are established by competent authority as suggested in a former part of this work, in drawing deeds which convey lands granted or conveyed many years ago, a new survey should be made, and the courses and distance inserted accordingly. True it is that surveys are expensive, but lawsuits are often ruinous.

As in most cases the oldest line must be allowed, it often becomes a matter of much importance to ascertain the comparative ages of different lines. Among the numerous difficulties which the young Surveyor has to encounter, this is about the greatest. When several lines run from the same point forming with each other angles of considerable magnitude, and when angry disputants are equally positive about the date of their favourite line, to determine which is the oldest is often an exceedingly difficult task. In a few instances the appearance of blazes on the trees may afford some assistance. An appeal to the oldest inhabitants of the place who were present at the original survey, or from other circumstances acquainted with the lines, is generally the most safe and satisfactory course to pursue. In this case, however, it is frequently difficult, and often utterly impossible to reconcile the conflicting statements given in evidence.

Notwithstanding the heavy responsibility which devolves on the Surveyor, frequently he must depend, to a great extent, upon his own judgment and discretion. In these circumstances an intimate and extensive knowledge of Mathematical principles, and a general acquaintance with collateral sciences, will guide the intelligent Surveyor through the labyrinth, by suggesting to him numerous expedients which would never occur to the mind of a mere routine practitioner.

ERRATA.

Page 76, third line from the bottom—for "Sir William MacLean George Colebrooke, R. H." &c., read Sir William MacBean George Colebrooke, K. H. &c.

Page 116, last line of the Calculation Table, in the column Half Departures—for "= 13.95" read — 13.95.

Page 133, at the head—for "Division of Land" read Location of Land.

A

TRAVERSE TABLE,

CONTAINING THE

DIFFERENCE OF LATITUDE AND HALF DEPARTURE,

TO EVERY QUARTER DEGREE OF THE COMPASS; THE DISTANCE SUPPOSED TO BE ONE CHAIN, OR FOUR RODS.

To find the Diff. of Lat. and Dep. by the following Table.

In the first column under D. M. find the Degrees and Minutes contained in the bearing; in a line with which, under N. S. and E. W., you have the Diff. of Lat. and half the Dep. for that bearing. Multiply this Diff. of Lat. and Dep. by the length of the Stationary Distance; and the product will be the Diff. of Lat. and half Dep. for that line.

Thus, let the bearing be N. 10° 15′ W., Distance 20 *ch.* 30 *l.* Required the Diff. of Lat. and Departure.

.9840 N. S.	.0889 E. W.
20.30	20.30
295200	26670
196800	17780
1.9975200 Northing.	1.804670 half Westing.
	2
	3.609340 Westing.

N. B.—In the following Table, if the Dep. be multiplied by two, the Dep. and Diff. of Lat. will be the natural sines and co-sines of the corresponding Courses, respectively; the Radius being supposed to be one.

D.	M.	N. S.	E.W.	D.	M.	N. S.	E.W.	D.	M.	N. S.	E.W.
0.	0	1.0000	.0000	11.	0	.9810	.0954	22.	0	.9272	.1873
	15	1.0000	.0022		15	.9808	.0975		15	.9255	.1893
	30	1.0000	.0044		30	.9793	.0997		30	.9239	.1913
	45	.9999	.0065		45	.9790	.1081		45	.9222	.1933
1.	0	.9998	.0087	12.	0	.9871	.1039	23.	0	.9205	.1953
	15	.9997	.0109		15	.9772	.1061		15	.9188	.1973
	30	.9996	.0131		30	.9763	.1082		30	.9171	.1993
	45	.9996	.0153		45	.9753	.1103		45	.9153	.2013
2.	0	.9994	.0175	13.	0	.9744	.1124	24.	0	.9153	.2033
	15	.9992	.0196		15	.9734	.1146		15	.9117	.2053
	30	.9990	.0218		30	.9724	.1167		30	.9099	.2073
	45	.9988	.0240		45	.9713	.1188		45	.9081	.2093
3.	0	.9986	.0262	14.	0	.9703	.1209	25.	0	.9063	.2113
	15	.9984	.0283		15	.9692	.1231		15	.9044	.2133
	30	.9981	.0305		30	.9681	.1252		30	.9020	.2152
	45	.9978	.0327		45	.9670	.1273		45	.9007	.2172
4.	0	.9975	.0349	15.	0	.9659	.1294	26.	0	.8988	.2192
	15	.9972	.0370		15	.9647	.1315		15	.8968	.2210
	30	.9969	.0392		30	.9636	.1336		30	.8949	.2230
	45	.9969	.0414		45	.9624	.1357		45	.8929	.2250
5.	0	.9962	.0436	16.	0	.9613	.1378	27.	0	.8910	.2270
	15	.9958	.0457		15	.9600	.1399		15	.8890	.2289
	30	.9954	.0479		30	.9588	.1420		30	.8870	.2308
	45	.9949	.0501		45	.9576	.1441		45	.8850	.2328
6.	0	.9945	.0522	17.	0	.9563	.1462	28.	0	.8829	.2347
	15	.9940	.0544		15	.9550	.1482		15	.8309	.2366
	30	.9936	.0566		30	.9537	.1503		30	.8788	.2385
	45	.9931	.0587		45	.9524	.1524		45	.8767	.2405
7.	0	.9925	.0609	18.	0	.9510	.1545	29.	0	.8746	.2424
	15	.9920	.0631		15	.9497	.1566		15	.8725	.2443
	30	.9914	.0652		30	.9483	.1586		30	.8703	.2462
	45	.9908	.0674		45	.9469	.1607		45	.8684	.2481
8.	0	.9903	.0696	19.	0	.9451	.1627	30.	0	.8660	.2500
	15	.9898	.0717		15	.9445	.1648		15	.8638	.2519
	30	.9890	.0739		30	.9426	.1669		30	.8616	.2537
	45	.9883	.0760		45	.9412	.1689		45	.8594	.2556
9.	0	.9877	.0782	20.	0	.9397	.1710	31.	0	.8571	.2575
	15	.9870	.0804		15	.9382	.1730		15	.8549	.2594
	30	.9863	.0825		30	.9366	.1751		30	.8527	.2612
	45	.9855	.0847		45	.9351	.1771		45	.8503	.2631
10.	0	.9848	.0868	21.	0	.9336	.1792	32.	0	.8480	.2649
	15	.9840	.0889		15	.9320	.1812		15	.8457	.2668
	30	.9832	.0911		30	.9304	.1832		30	.8434	.2686
	45	.9824	.0933		45	.9288	.1852		45	.8410	.2705

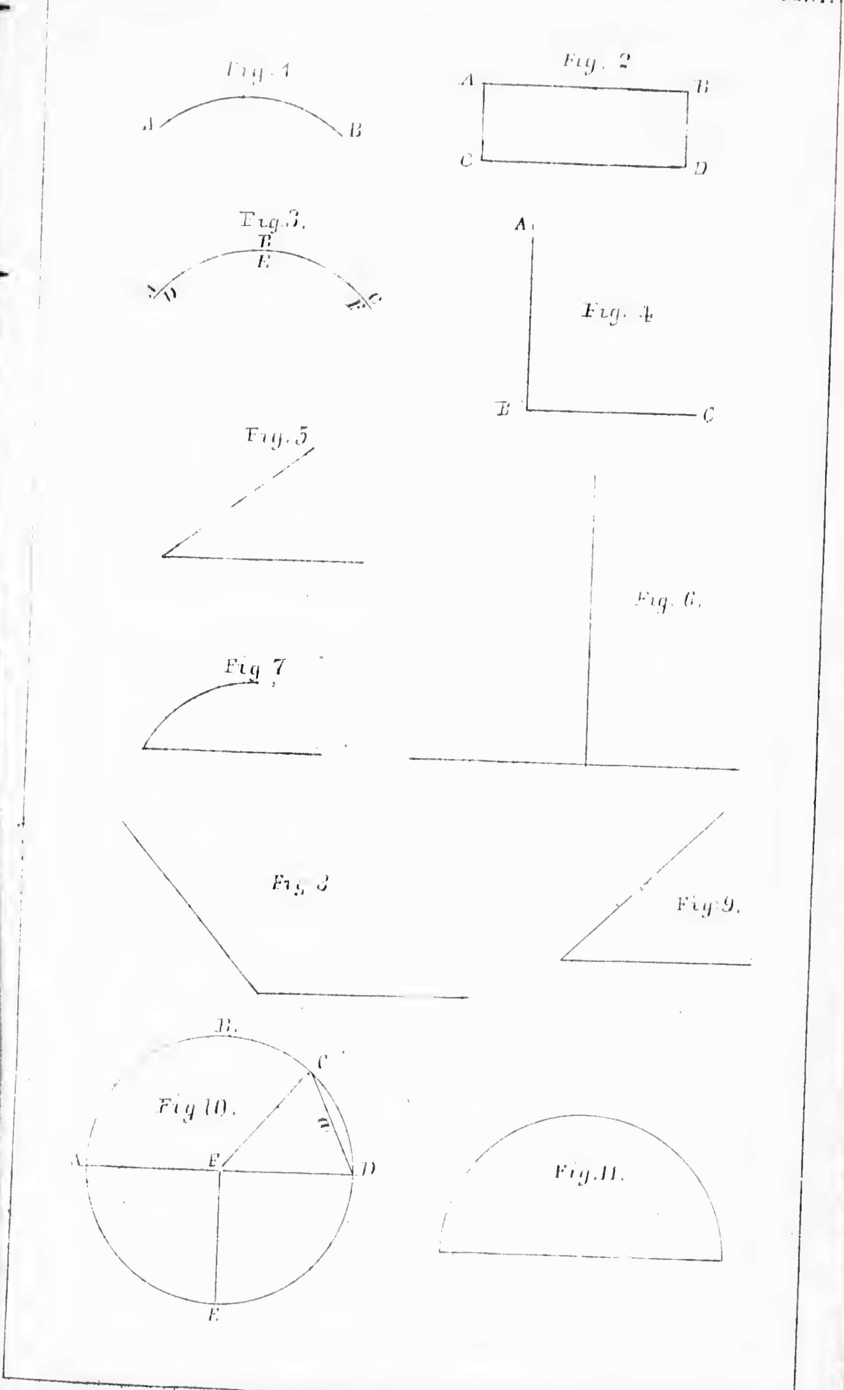
DIFFERENCE OF LATITUDE AND DEPARTURE.

D.	M.	N. S.	E.W.	D.	M.	N. S.	E.W.	D.	M.	N. S.	E.W.
33.	0	.8387	.2723	44.	0	.7192	.3473	55.	0	.5736	.4095
	15	.8363	.2741		15	.7163	.3489		15	.5700	.4108
	30	.8339	.2759		30	.7132	.3504		30	.5664	.4120
	45	.8315	.2718		45	.7102	.3520		45	.5628	.4133
34.	0	.8290	.2796	45.	0	.7071	.3535	56.	0	.5592	.4155
	15	.8266	.2814		15	.7041	.3551		15	.5555	.4157
	30	.8241	.2832		30	.7009	.3566		30	.5519	.4169
	45	.8216	.2850		45	.6978	.3581		45	.5483	.4181
35.	0	.8191	.2667	46.	0	.6946	.3596	57.	0	.5446	.4193
	15	.8166	.2885		15	.6915	.3612		15	.5408	.4205
	30	.8141	.2903		30	.6883	.3627		30	.5373	.4216
	45	.8116	.2921		45	.6852	.3642		45	.5336	.4228
36.	0	.8090	.2939	47.	0	.6820	.3656	58.	0	.5299	.4240
	15	.8064	.2956		15	.6788	.3671		15	.5269	.4251
	30	.8058	.2974		30	.6756	.3686		30	.5225	.4263
	45	.8012	.2991		45	.6724	.3701		45	.5188	.4274
37.	0	.7986	.3009	48.	0	.6691	.3715	59.	0	.5150	.4286
	15	.7960	.3026		15	.6659	.3730		15	.5113	.4297
	30	.7933	.3044		30	.6626	.3745		30	.5075	.4308
	45	.7907	.3061		45	.6593	.3759		45	.5033	.4319
38.	0	.7880	.3078	49.	0	.6560	.3773	60.	0	.5000	.4330
	15	.7853	.3095		15	.6527	.3788		15	.4962	.4341
	30	.7826	.3112		30	.6494	.3802		30	.4924	.4352
	45	.7799	.3129		35	.6461	.3816		45	.4886	.4362
39.	0	.7771	.3146	50.	0	.6428	.3830	61.	0	.4848	.4373
	15	.7744	.3163		15	.6394	.3844		15	.4810	.4383
	30	.7716	.3180		30	.6361	.3858		30	.4771	.4394
	45	.7688	.3197		45	.6327	.3872		45	.4733	.4405
40.	0	.7660	.3214	51.	0	.6293	.3885	62.	0	.4695	.4415
	15	.7632	.3230		15	.6259	.3899		15	.4656	.4425
	30	.7604	.3247		30	.6225	.3913		30	.4617	.4435
	45	.7575	.3263		45	.6191	.3926		45	.4579	.4445
41.	0	.7547	.3280	52.	0	.6157	.3940	63.	0	.4540	.4455
	15	.7518	.3296		15	.6122	.3953		15	.4501	.4465
	30	.7489	.3313		30	.6037	.3966		30	.4462	.4475
	45	.7460	.3329		45	.6052	.3980		45	.4423	.4485
42.	0	.7431	.3345	53.	0	.6018	.3993	64.	0	.4383	.4494
	15	.7402	.3361		15	.5983	.4006		15	.4344	.4503
	30	.7373	.3378		30	.5948	.4019		30	.4305	.4513
	45	.7343	.3394		45	.5912	.4032		45	.4265	.4522
43.	0	.7313	.3410	54.	0	.5878	.4045	65.	0	.4226	.4532
	15	.7282	.3426		15	.5842	.4058		15	.4186	.4541
	30	.7254	.3441		30	.5807	.4070		30	.4140	.4550
	45	.7224	.3457		45	.5771	.4083		45	.4177	.4559

D.	M.	N. S.	E. W.	D.	M.	N. S.	E. W.	D.	M.	N. S.	E. W.
66.	0	.4067	.4567	74.	0	.2756	.4806	82.	0	.1392	.4951
	15	.4027	.4576		15	.2714	.4812		15	.1348	.4954
	30	.3987	.4585		30	.2762	.4818		30	.1305	.4957
	45	.3947	.4594		45	.2630	.4823		45	.1262	.4960
67.	0	.3907	.4602	75.	0	.2588	.4829	83.	0	.1218	.4962
	15	.3867	.4611		15	.2546	.4835		15	.1175	.4965
	30	.3827	.4619		30	.2504	.4840		30	.1132	.4968
	45	.3736	.4627		45	.2461	.4846		45	.1088	.4970
68.	0	.3746	.4636	76.	0	.2419	.4851	84.	0	.1045	.4972
	15	.3705	.4644		15	.2377	.4856		15	.1002	.4974
	30	.3665	.4652		30	.2344	.4862		30	.0958	.4977
	45	.3624	.4660		45	.2292	.4867		45	.0915	.4979
69.	0	.3584	.4668	77.	0	.2249	.4872	85.	0	.0871	.4981
	15	.3543	.4676		15	.2207	.4876		15	.0826	.4983
	30	.3502	.4683		30	.2164	.4881		30	.0784	.4984
	45	.3461	.4691		45	.2122	.4886		45	.0741	.4986
70.	0	.3420	.4698	78.	0	.2079	.4890	86.	0	.0697	.4988
	15	.3379	.4706		15	.2036	.4895		15	.0654	.4989
	30	.3338	.4713		30	.1993	.4899		30	.0610	.4990
	45	.2297	.4720		45	.1951	.4904		45	.0566	.4992
71.	0	.3255	.4727	79.	0	.1908	.4908	87.	0	.0523	.4993
	15	.3214	.4734		15	.1865	.4912		15	.0480	.4994
	30	.3173	.4741		30	.1822	.4916		30	.0436	.4995
	45	.3131	.4748		45	.1779	.4920		45	.0392	.4596
72.	0	.3090	.4755	80.	0	.1736	.4924	88.	0	.0349	.4997
	15	.3048	.4762		15	.1693	.4927		15	.0305	.4997
	30	.3007	.4768		30	.1650	.4931		30	.0262	.4998
	45	.2965	.4775		45	.1607	.4935		45	.0218	.4998
73.	0	.2924	.4781	81.	0	.1564	.4938	89.	0	.0174	.4999
	15	.2882	.4788		15	.1521	.4941		15	.0131	.4999
	30	.2840	.4794		30	.1478	.4945		30	.0087	.5000
	45	.2798	.4800		45	.1435	.4948		45	.0043	.5000
								90.	0	.0000	.5000

S.	E.W.
92	.4951
48	.4954
05	.4957
62	.4960
18	.4962
75	.4965
32	.4968
88	.4970
45	.4972
02	.4974
58	.4977
15	.4979
71	.4981
26	.4983
84	.4984
41	.4986
97	.4988
54	.4989
10	.4990
66	.4992
23	.4993
80	.4994
36	.4995
92	.4596
349	.4997
305	4997
262	.4998
218	.4998
174	.4999
131	.4999
087	.5000
043	.5000
0000	.5000

Pl. 1.



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Thayer & Co's Lith Boston

Fig. 12.

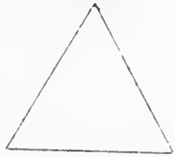
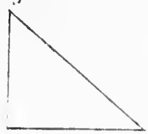


Fig. 13.

Fig. 14.

Fig. 15.



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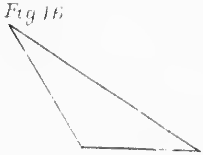

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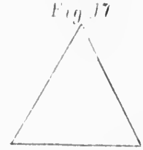

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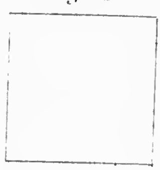

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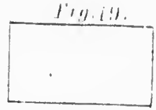

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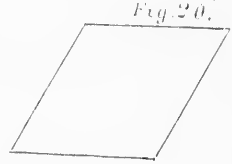

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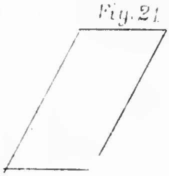

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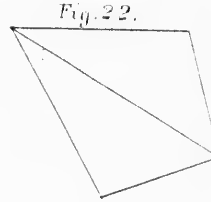

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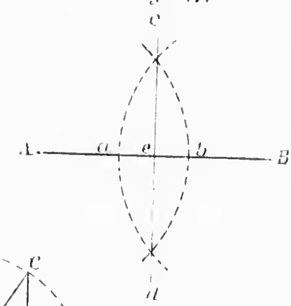

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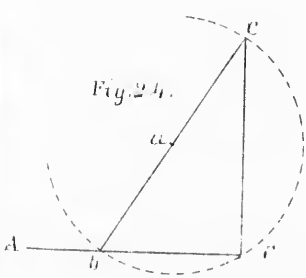

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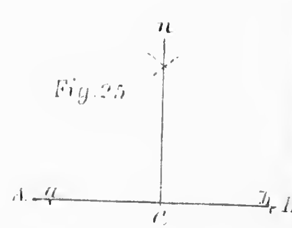

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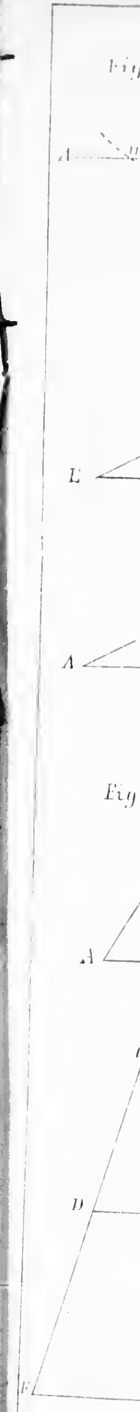
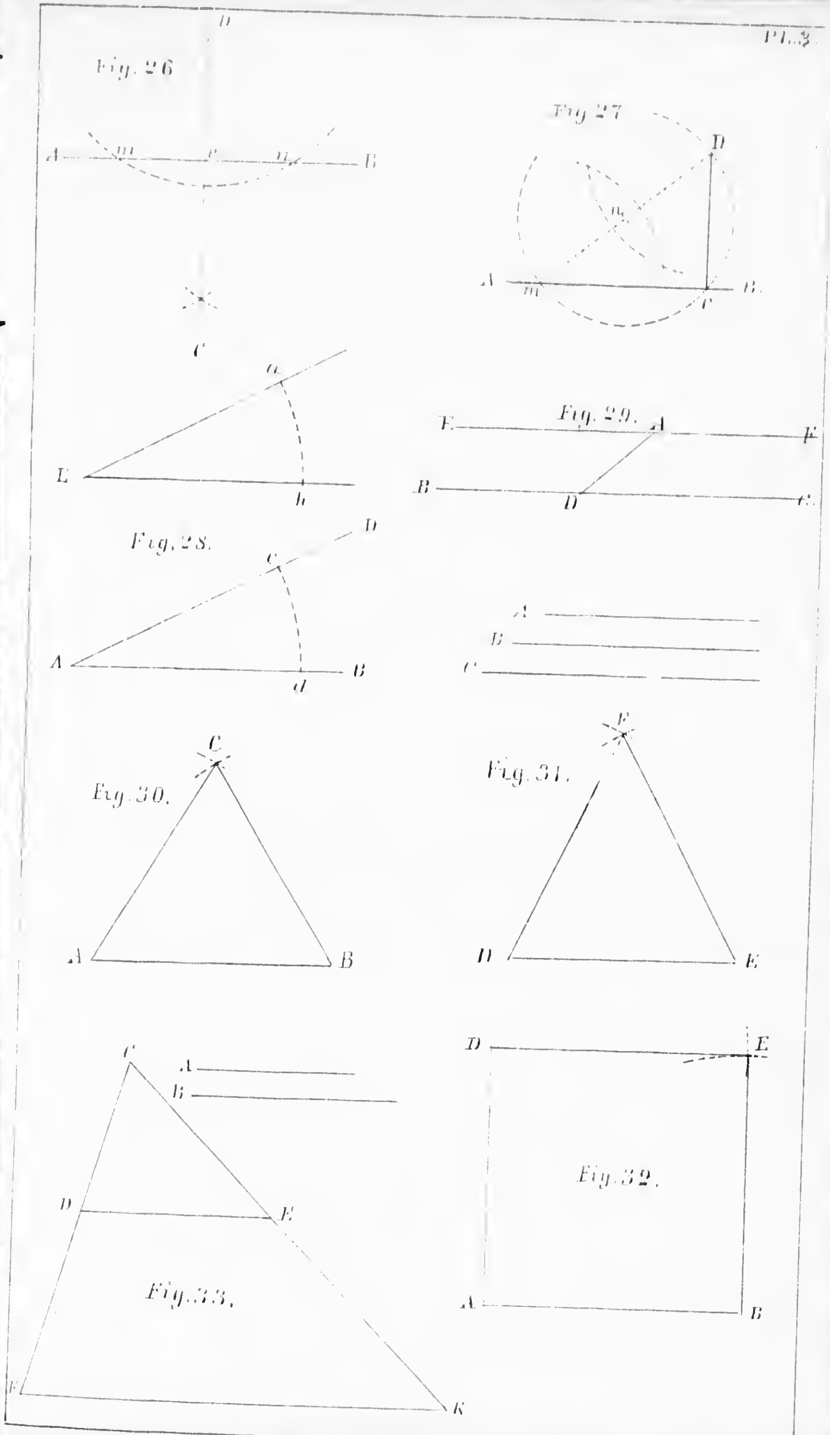
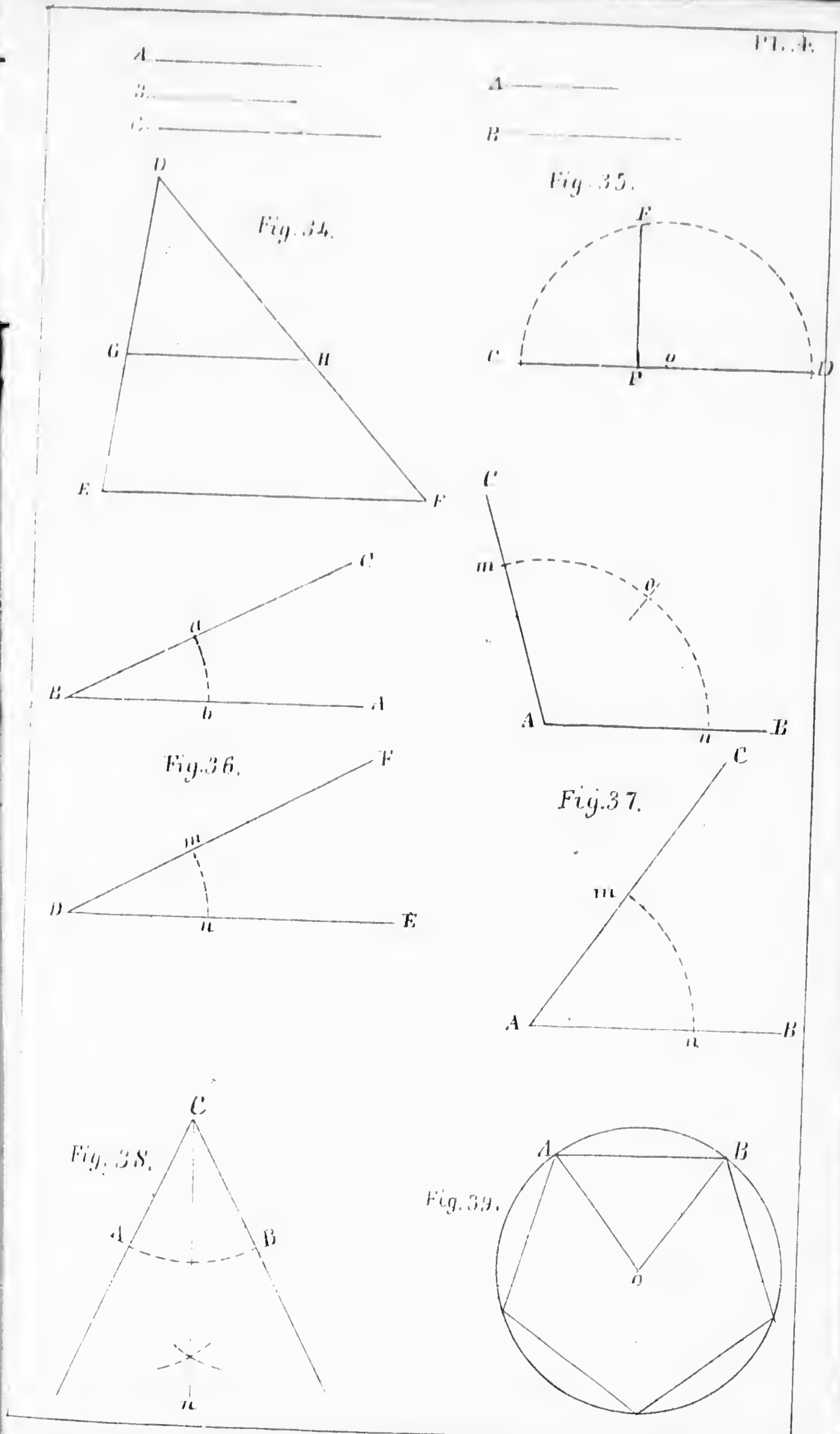


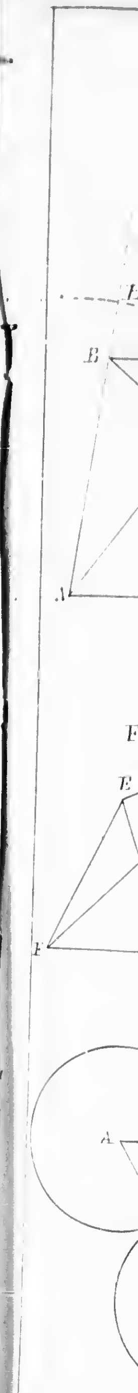


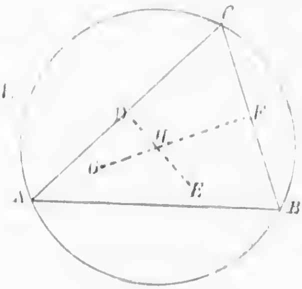
Lith. by Alexr Munroe

Fig. 3



Pub. by Alex.r Monroe





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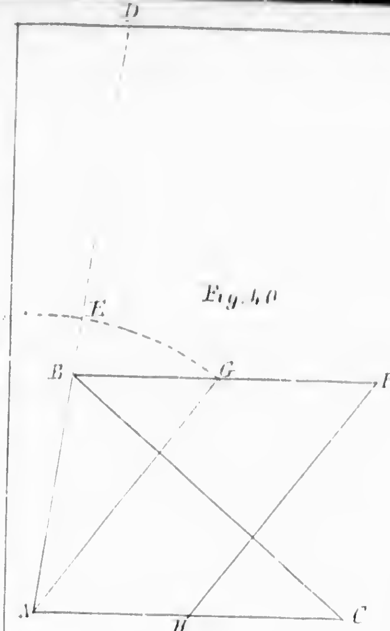
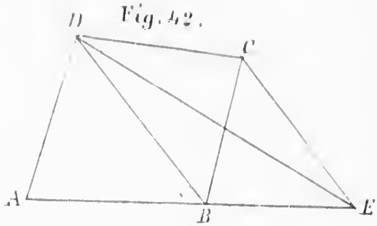


Fig. 40



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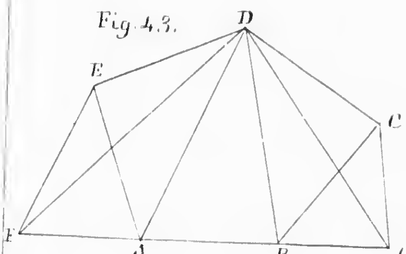

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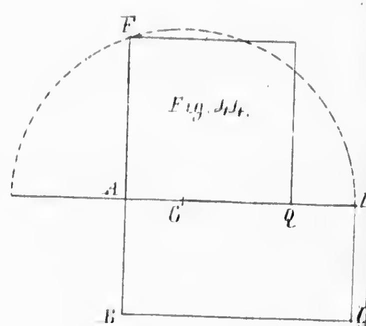

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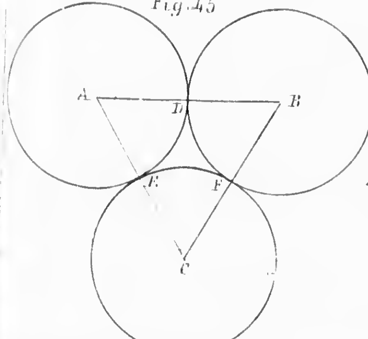

Fig. 45

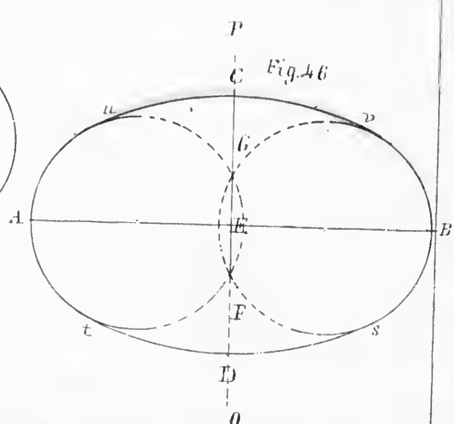

Fig. 46

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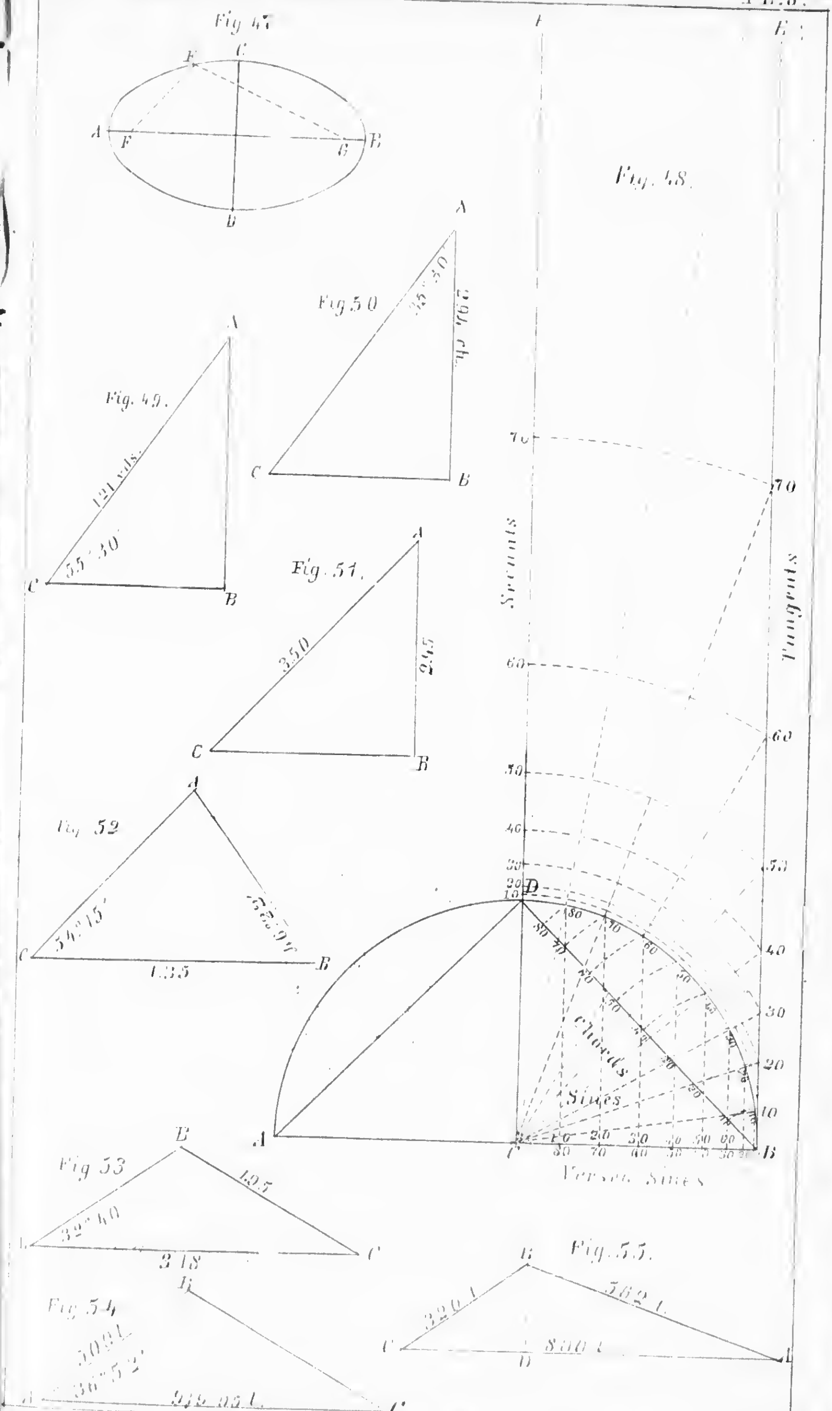


Fig. 47.
A
B
C
D
E
F
G
Fig. 48.
Secants
Tangents
Chords
Sines
Versed Sines
Fig. 49.
121 yds.
55° 30'
Fig. 50.
35° 30'
296 ch.
Fig. 51.
350
245
Fig. 52.
54° 15'
135
Fig. 53.
32° 40'
105
318
Fig. 54.
Fig. 55.
320 l.
562 l.
800 l.

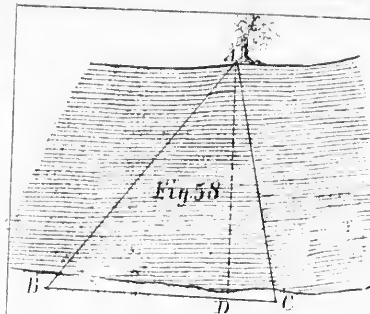


Fig. 58

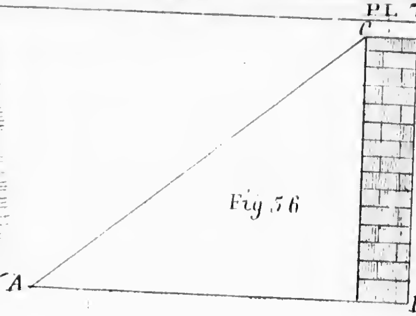


Fig. 56

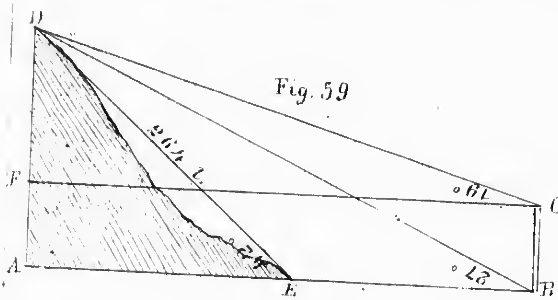
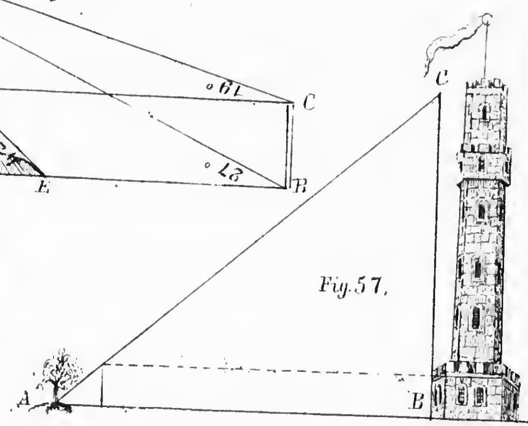


Fig. 59



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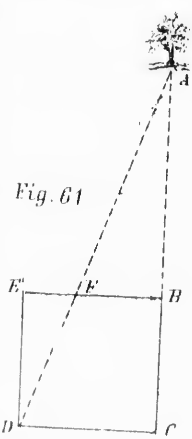
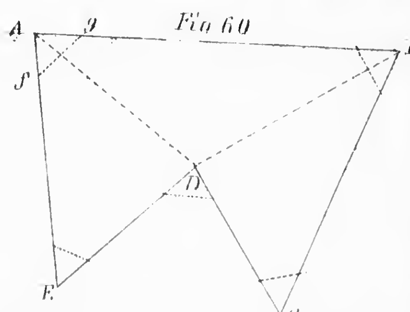


Fig. 61



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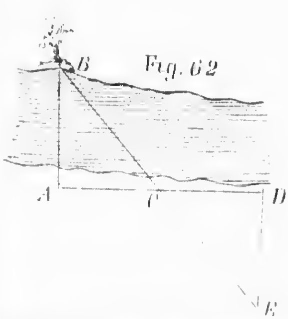
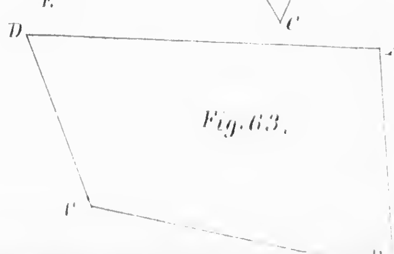


Fig. 62



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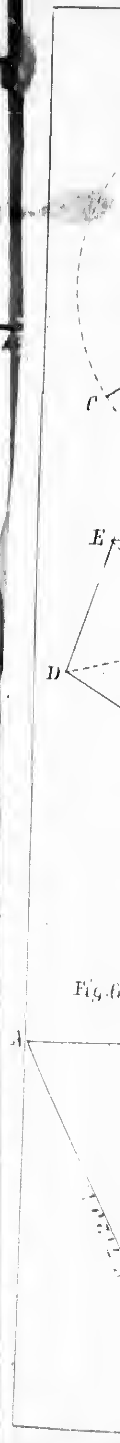

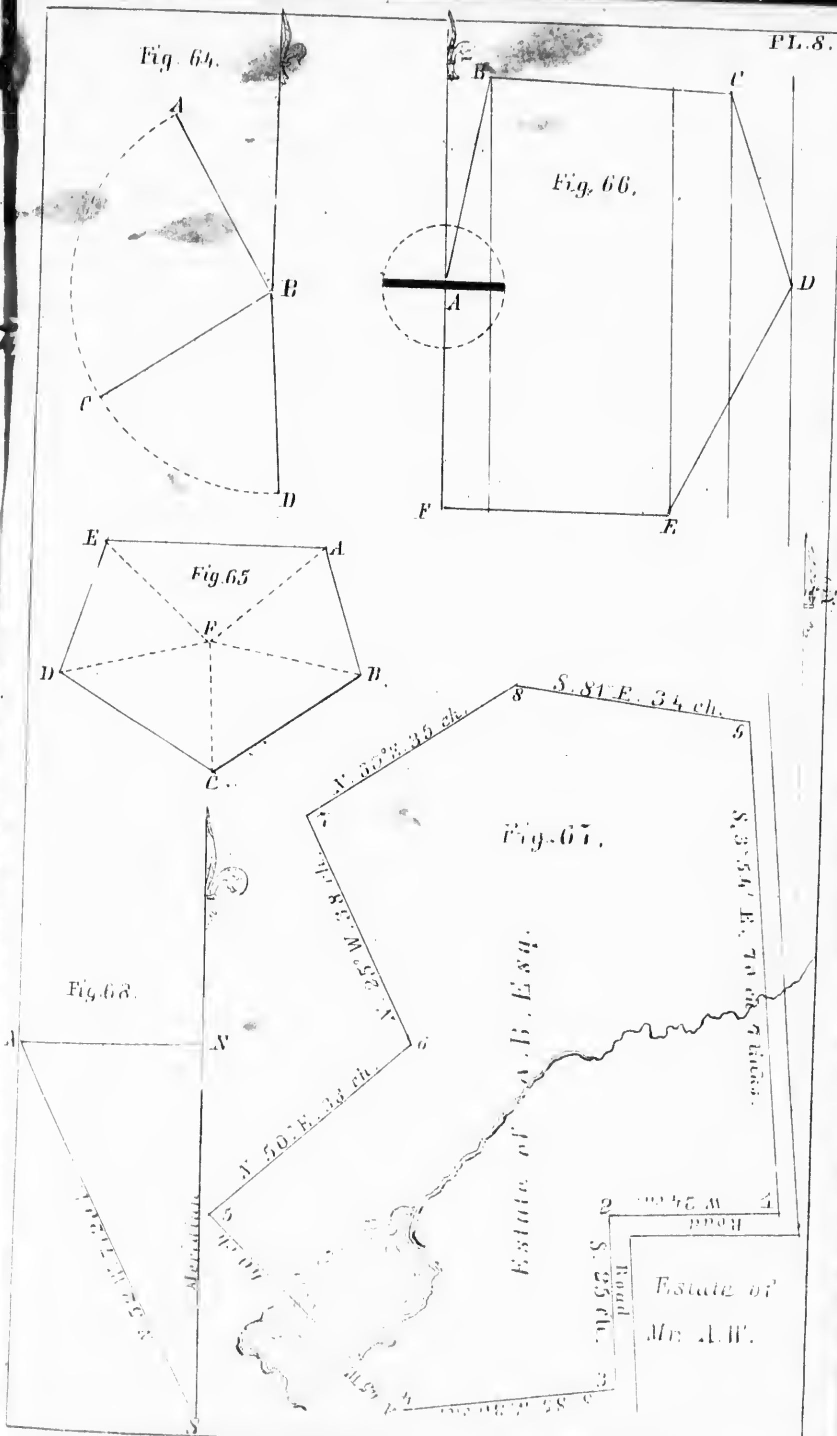

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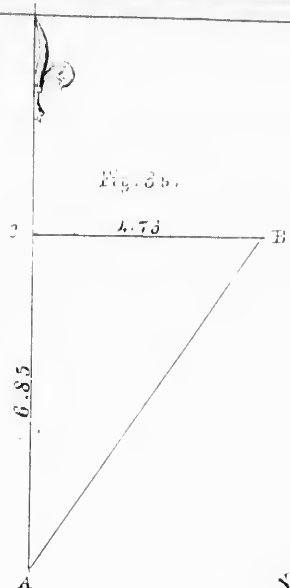

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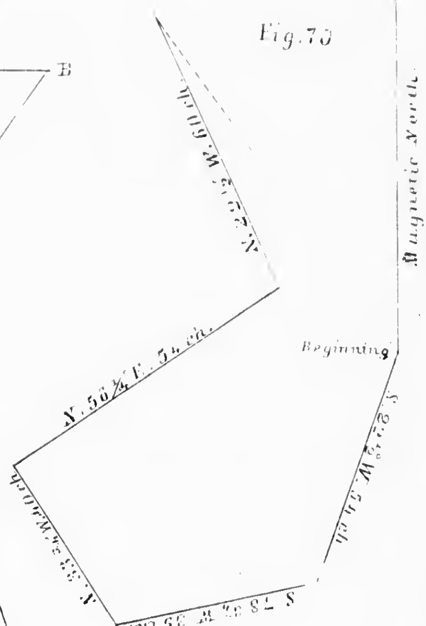
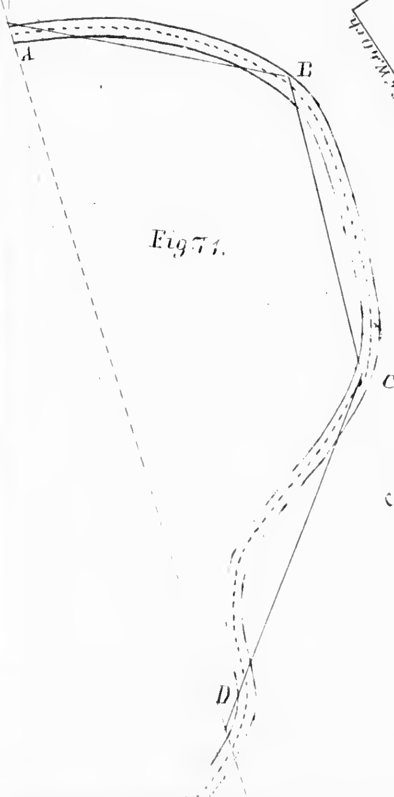


Fig. 70



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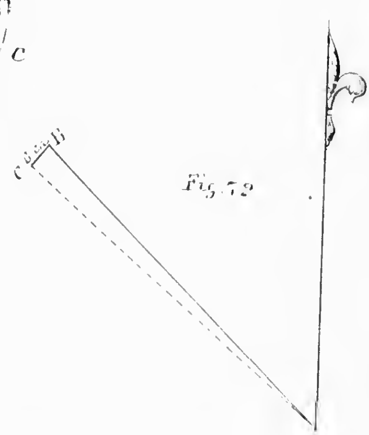


Fig. 72

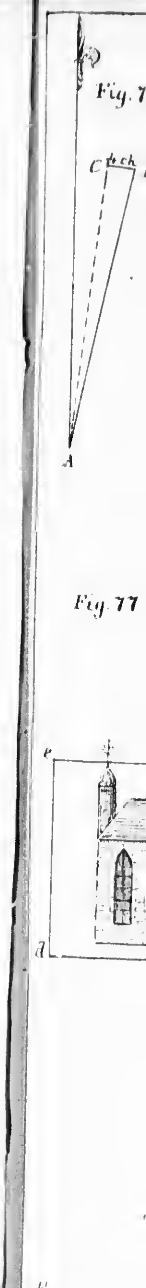
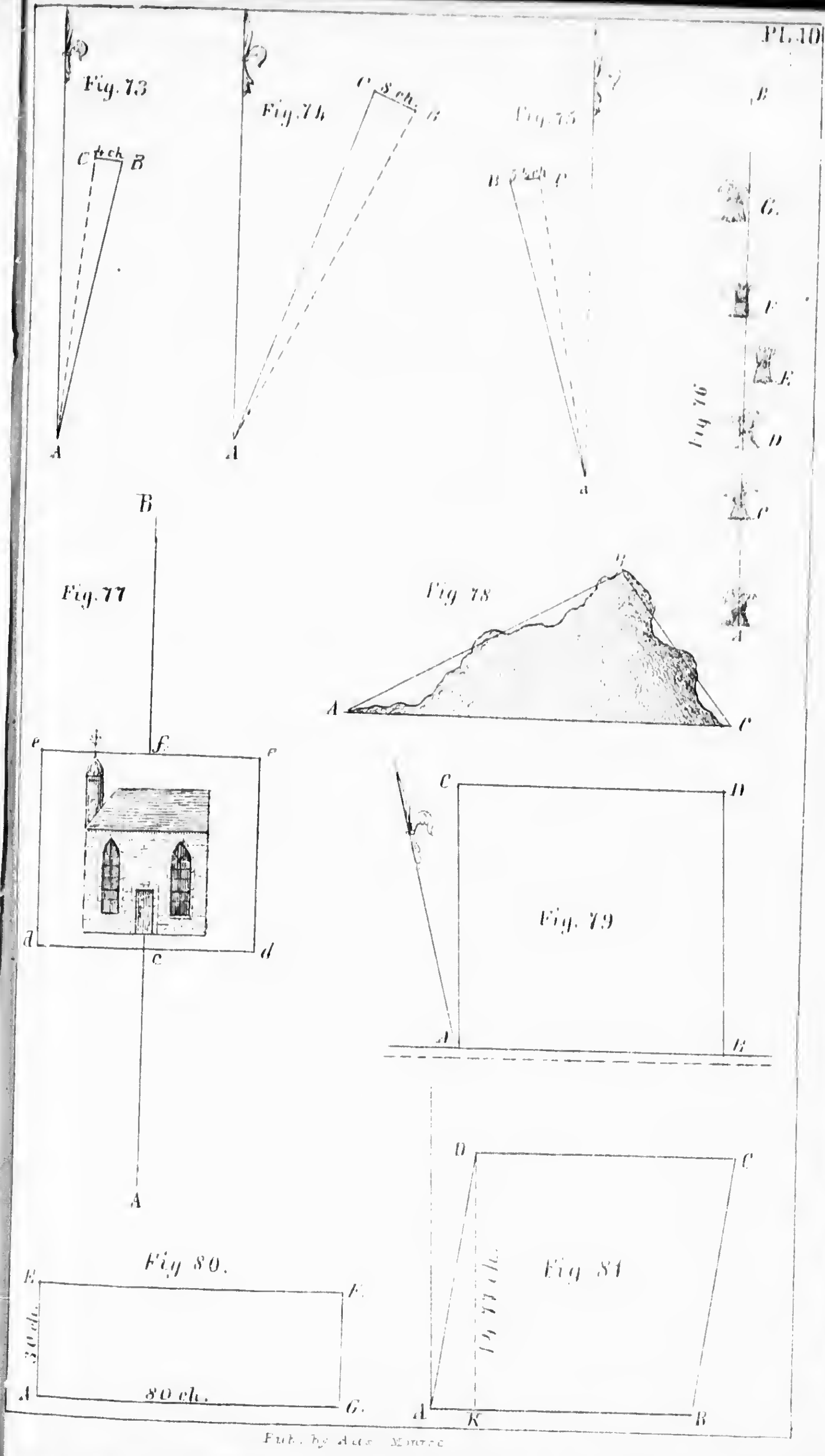


Fig. 7
C
A
Fig. 77
e
d
E
A



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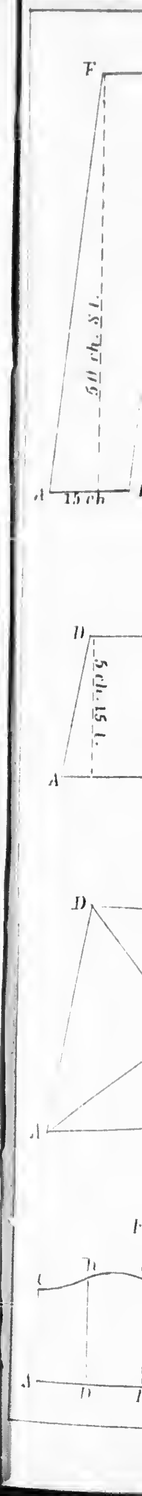
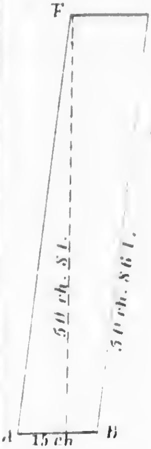

F
15 ch.
D
5 ch. 15 l.
A
D
A



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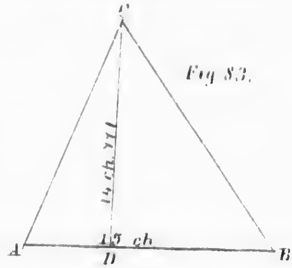

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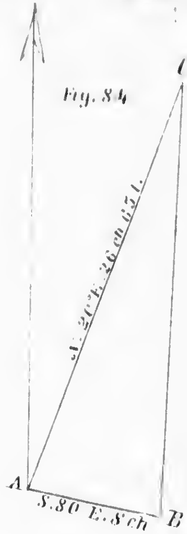


Fig. 84

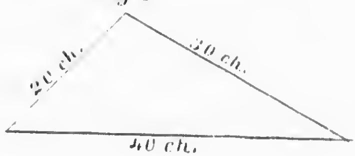


Fig. 85

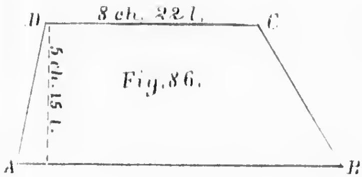


Fig. 86.

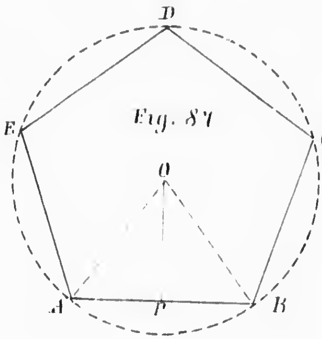


Fig. 87

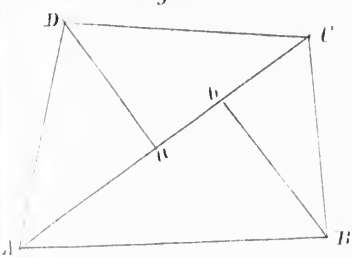


Fig. 88.

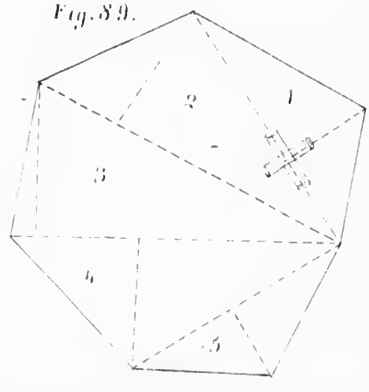


Fig. 89.

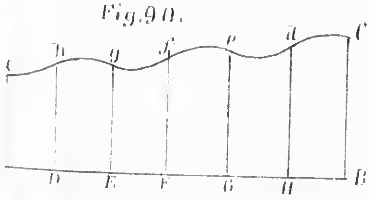


Fig. 90.


B
A
N
4
1
3
A

Pl. 12.

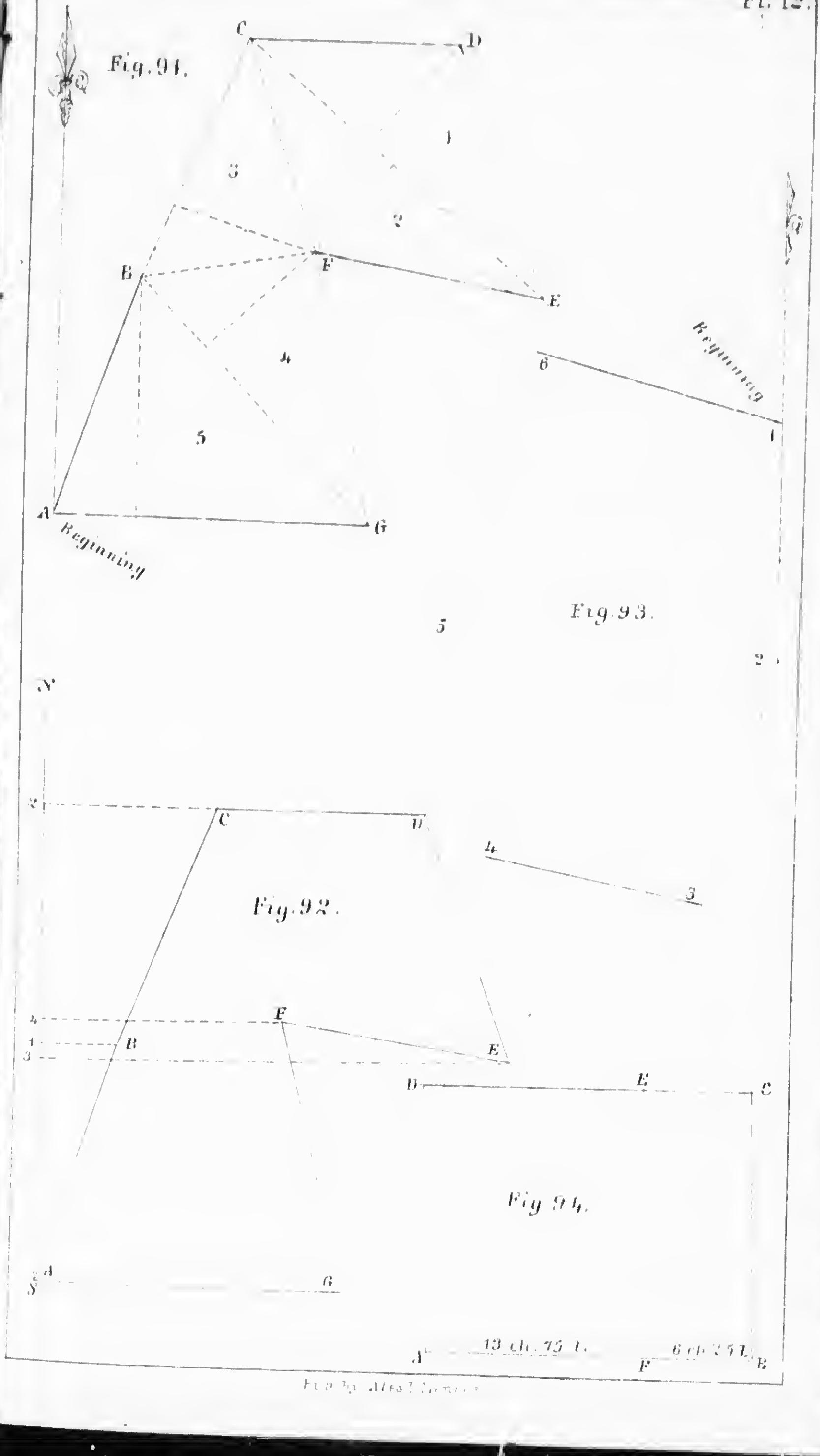


Fig. 91.
Beginning
Fig. 92.
Fig. 93.
Beginning
Fig. 94.
13 ch. 75 l.
6 ch. 25 l.

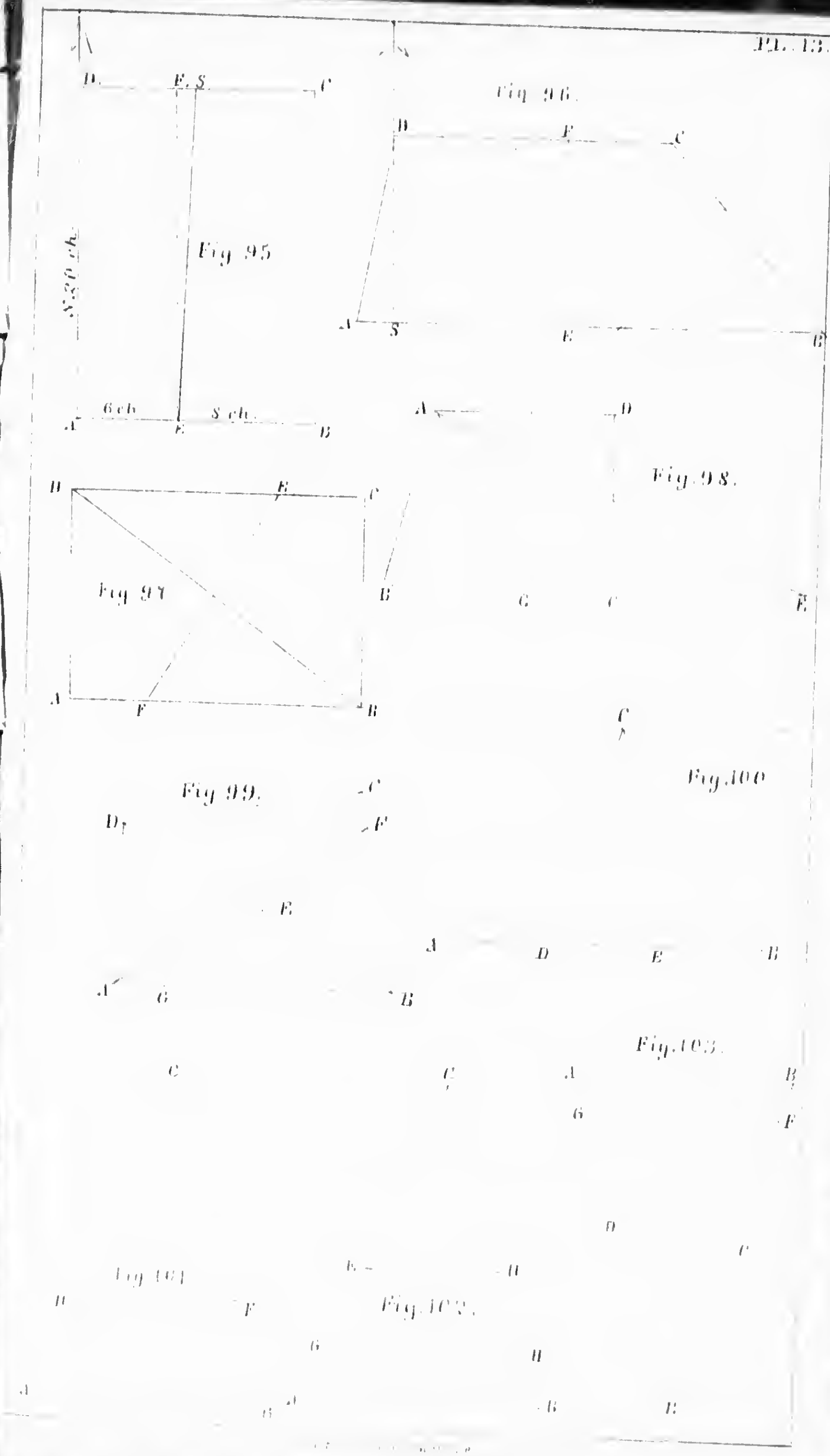
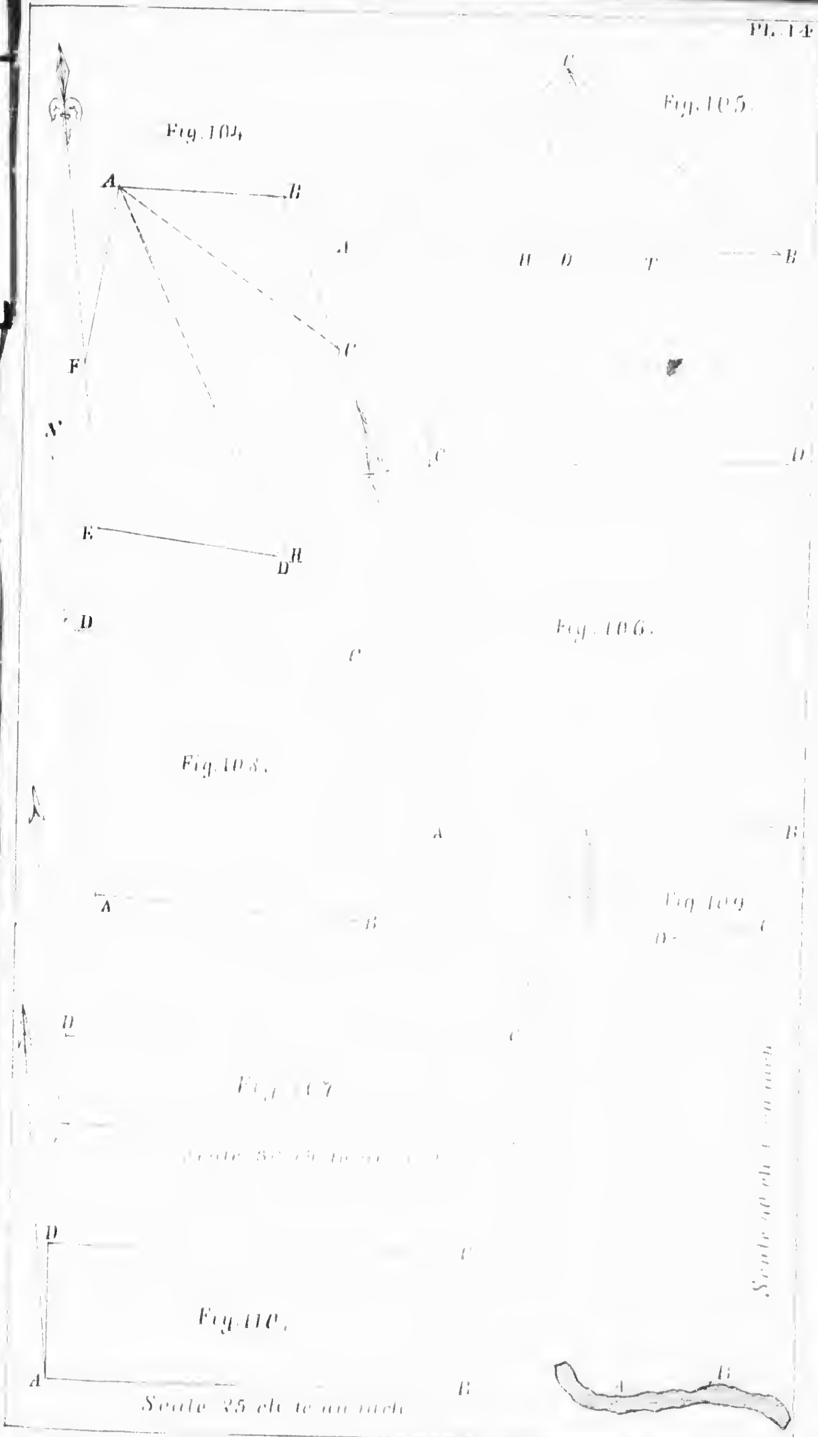


Fig. 95
6 ch.
8 ch.
Fig. 96.
Fig. 97
Fig. 98.
Fig. 99.
Fig. 100
Fig. 101
Fig. 102.
Fig. 103.


Pl. 14
Fig. 104.
A
B
F
N
E
D
H
Fig. 105.
Fig. 106.
Fig. 108.
Fig. 109.
Fig. 110.
Scale 25 ch. to an inch

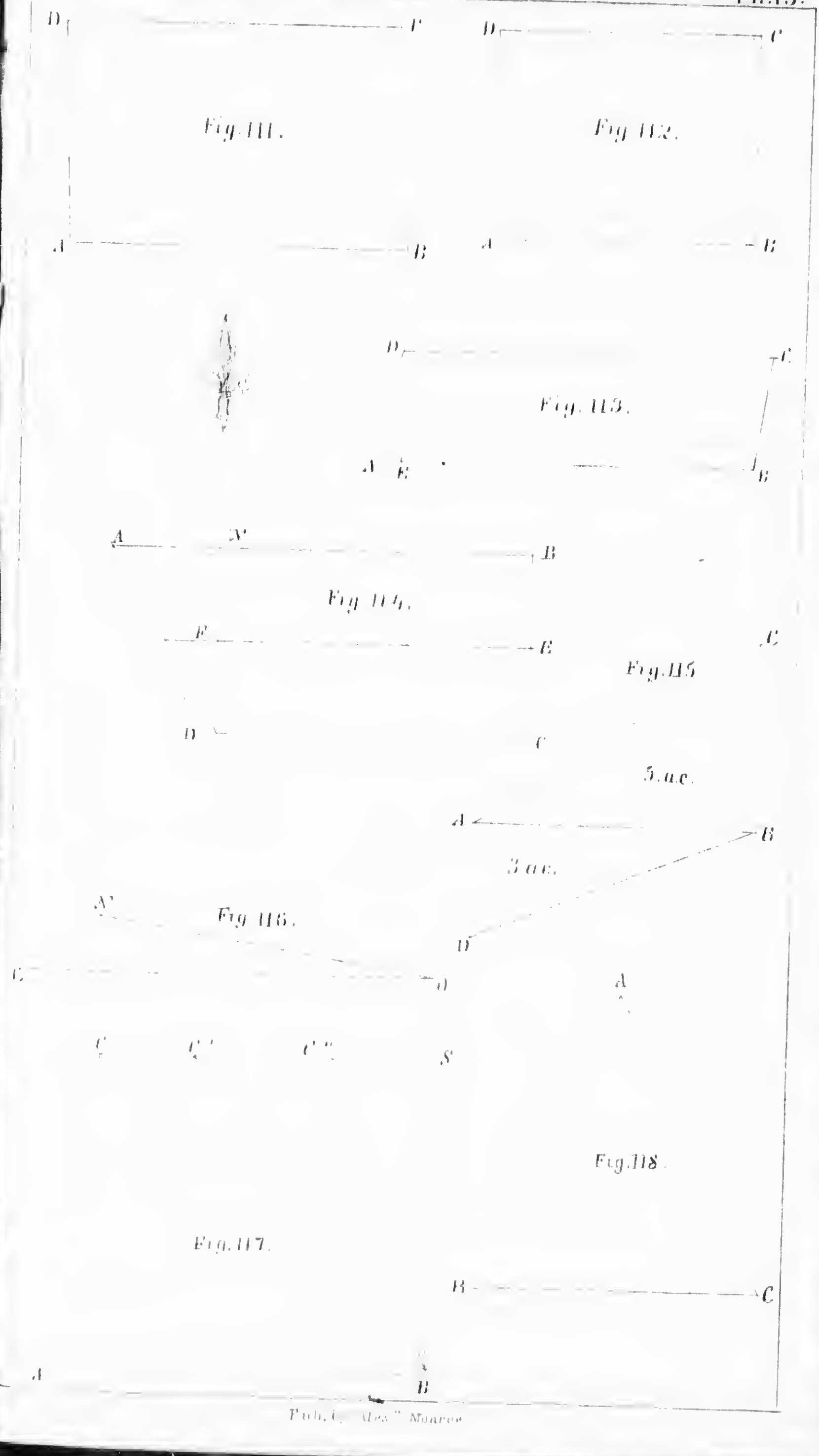
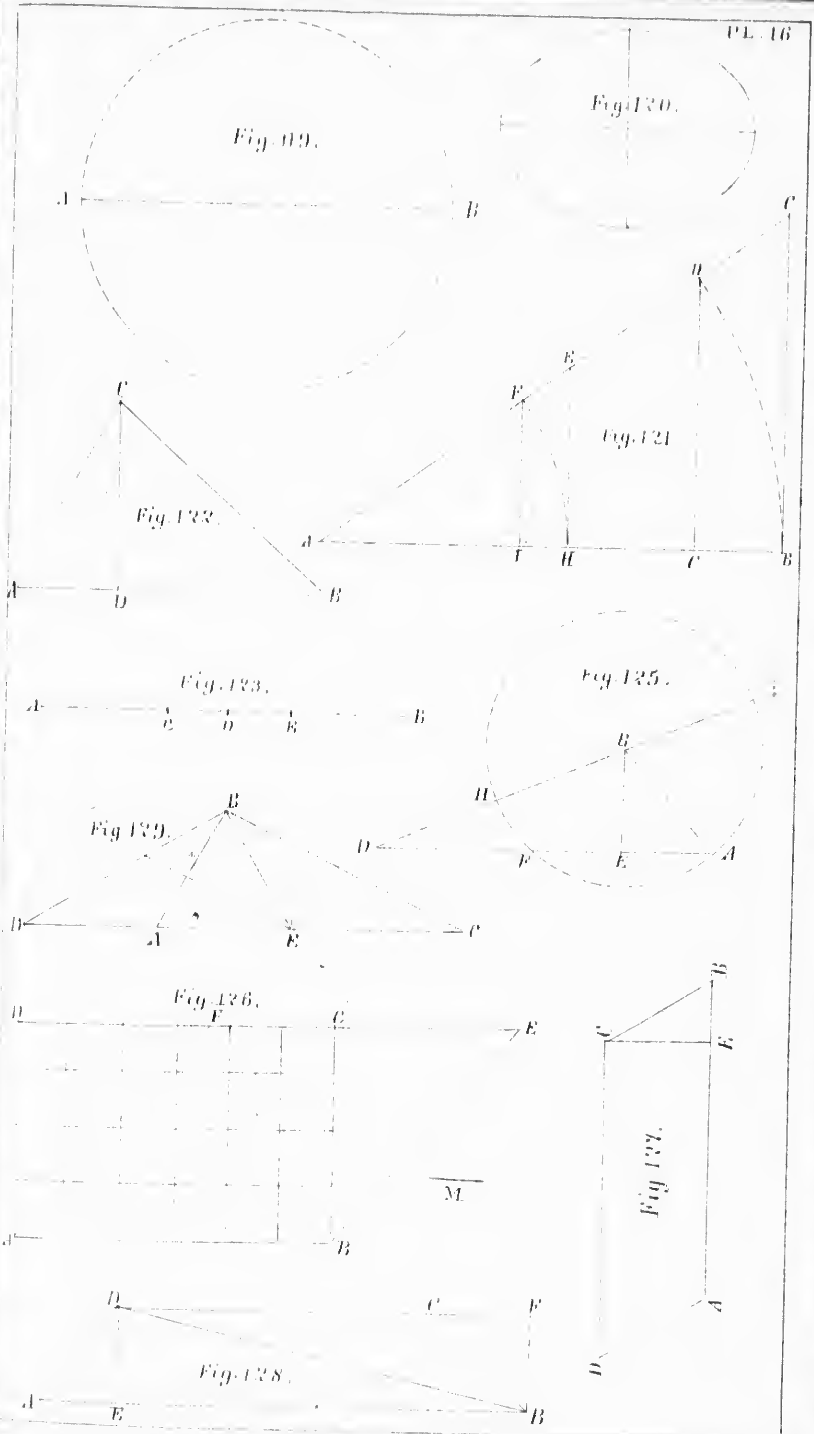
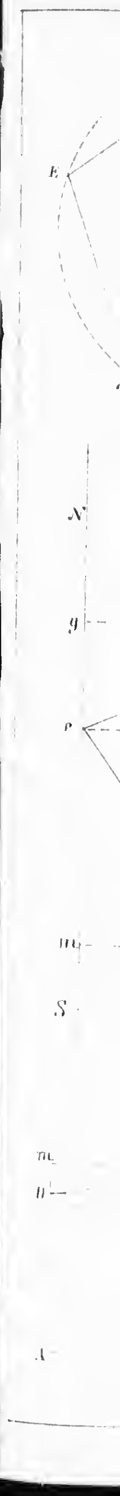


Fig. 111.
Fig. 112.
Fig. 113.
Fig. 114.
Fig. 115.
5 a.c.
3 a.c.
Fig. 116.
Fig. 117.
Fig. 118.

PL. 16



Etched by Alex.r Monroe


E
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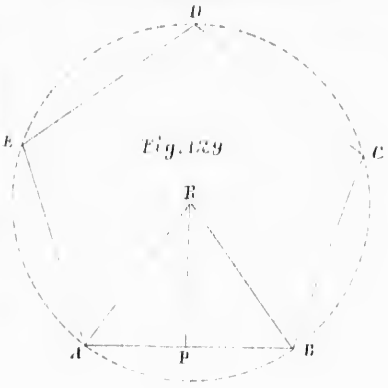


Fig. 129

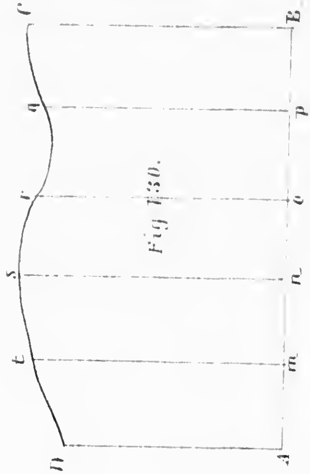


Fig. 130.

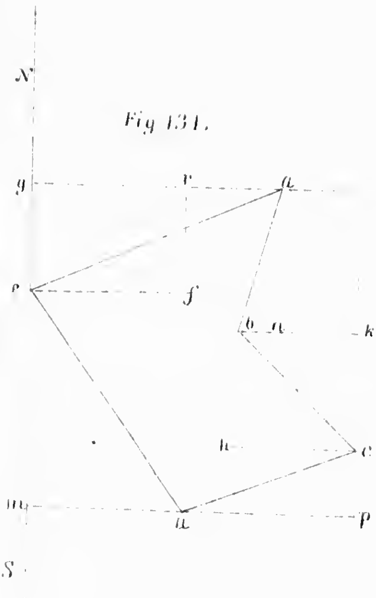


Fig. 131.

Fig. 132.

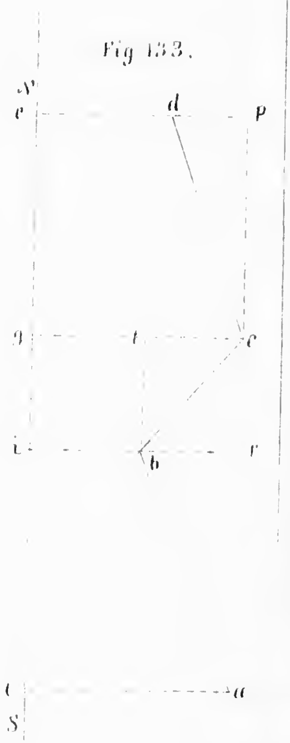
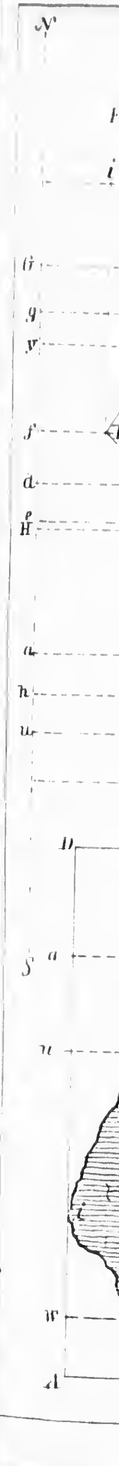


Fig. 133.



N
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Fig. 134.

Fig. 135.

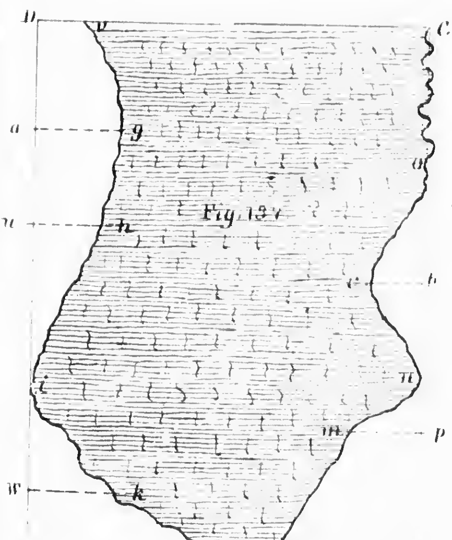

Fig. 136.

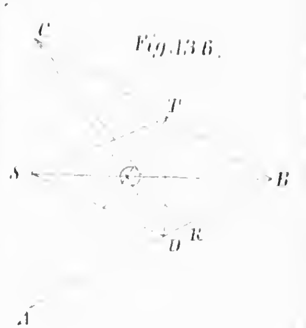

Fig. 138.

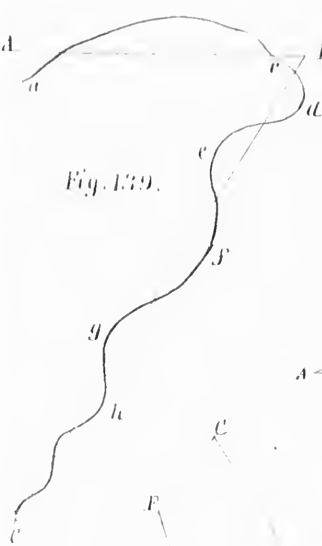

Fig. 139.

Fig. 140.

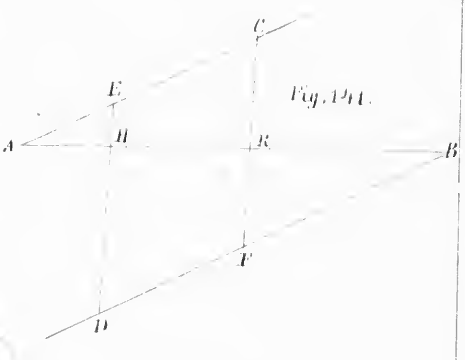

Fig. 141.

Fig. 142.

Pub. by Alexr Monroe

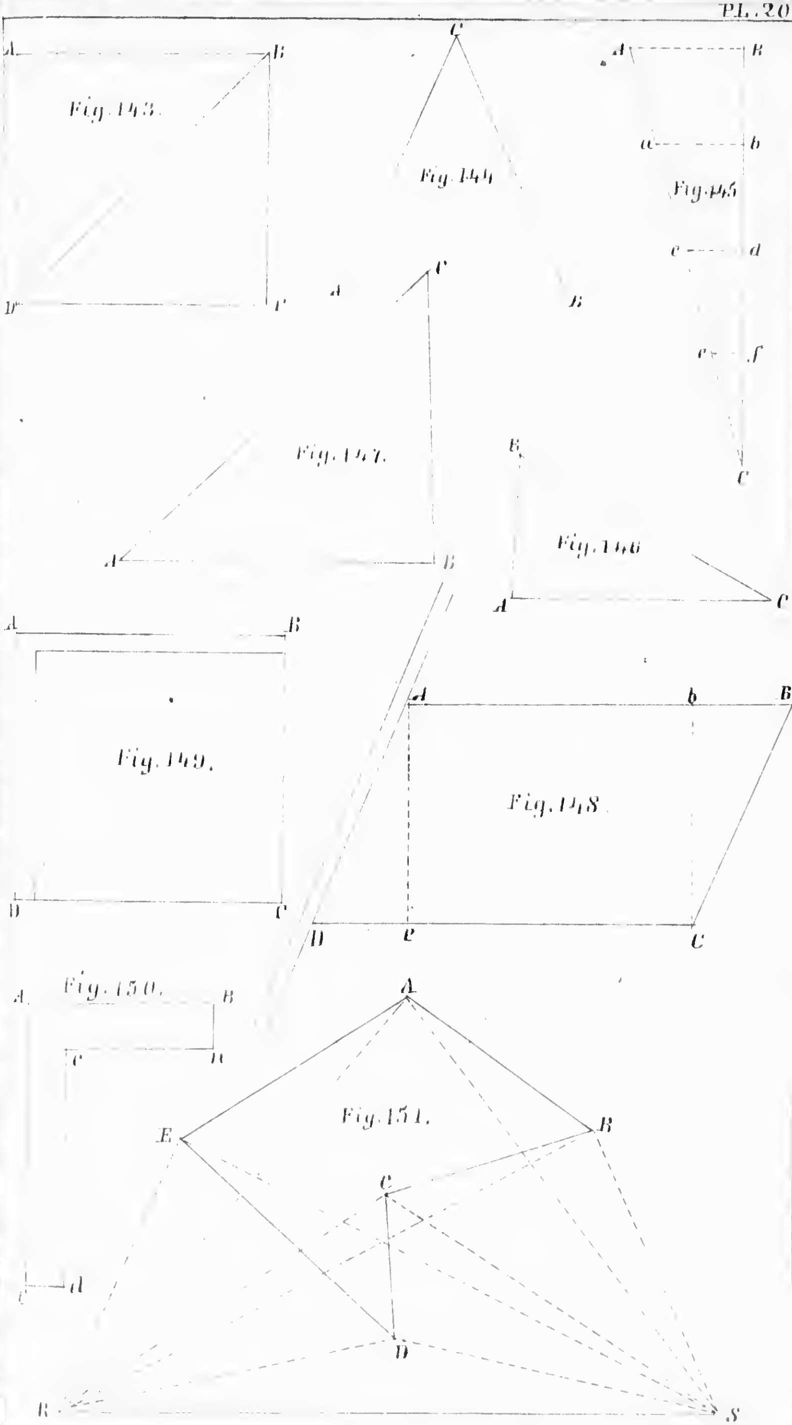
Fig. 143.
Fig. 144.
Fig. 145.
Fig. 146.
Fig. 147.
Fig. 148.
Fig. 149.
Fig. 150.
Fig. 151.

N.	
1	0.
2	0.
3	0.
4	0.
5	0.
6	0.
7	0.
8	0.
9	0.
10	1.
11	1.
12	1.
13	1.
14	1.
15	1.
16	1.
17	1.
18	1.
19	1.
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22	1.
23	1.
24	1.
25	1.

N. B

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A TABLE

OF

LOGARITHMS OF NUMBERS

FROM 1 o 10,000.

N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	26	1.414973	51	1.707570	76	1.880814
2	0.301030	27	1.431364	52	1.716003	77	1.886491
3	0.477121	28	1.447158	53	1.724276	78	1.892095
4	0.602060	29	1.462398	54	1.732394	79	1.897627
5	0.698970	30	1.477121	55	1.740363	80	1.903090
6	0.778151	31	1.491362	56	1.748188	81	1.908485
7	0.845098	32	1.505150	57	1.755875	82	1.913814
8	0.903090	33	1.518514	58	1.763428	83	1.919078
9	0.954243	34	1.531479	59	1.770852	84	1.924279
10	1.000000	35	1.544068	60	1.778151	85	1.929419
11	1.041393	36	1.556303	61	1.785330	86	1.934498
12	1.079181	37	1.568202	62	1.792392	87	1.939519
13	1.113943	38	1.579784	63	1.799341	88	1.944483
14	1.146128	39	1.591065	64	1.806180	89	1.949390
15	1.176091	40	1.602060	65	1.812913	90	1.954243
16	1.204120	41	1.612784	66	1.819544	91	1.959041
17	1.230449	42	1.623249	67	1.826075	92	1.963788
18	1.255273	43	1.633468	68	1.832509	93	1.968483
19	1.278754	44	1.643453	69	1.838849	94	1.973128
20	1.301030	45	1.653213	70	1.845098	95	1.977724
21	1.322219	46	1.662758	71	1.851258	96	1.982271
22	1.342423	47	1.672098	72	1.857333	97	1.986772
23	1.361728	48	1.681241	73	1.863323	98	1.991226
24	1.380211	49	1.690196	74	1.869232	99	1.995635
25	1.397940	50	1.698970	75	1.875061	100	2.000000

N. B. In the following table, in the last nine columns of each page, where the first or leading figures change from 9's to 0's, points or dots are introduced instead of the 0's through the rest of the line, to catch the eye, and to indicate that from thence the annexed first two figures of the Logarithm in the second column stand in the next lower line.

N.	0	1	2	3	4	5	6	7	8	9	D.
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	8600	9026	9451	9876	.300	.724	1147	1570	1993	2415	424
103	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616	419
104	7033	7451	7868	8284	8700	9116	9532	9947	.361	.775	416
105	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
107	9384	9789	.195	.600	1004	1408	1812	2216	2619	3021	404
108	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	7426	7825	8223	8620	9017	9414	9811	.207	.602	.998	396
110	041393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	389
112	9218	9606	9993	.380	.766	1153	1538	1924	2309	2694	386
113	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524	382
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	.320	379
115	060698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	372
117	8186	8557	8928	9298	9668	..38	.407	.776	1145	1514	369
118	071882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	079181	9543	9904	.266	.626	.987	1347	1707	2067	2426	360
121	082785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
123	9905	.258	.611	.963	1315	1667	2018	2370	2721	3071	351
124	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
125	6910	7257	7604	7951	8298	8644	8990	9335	9681	..26	346
126	100371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	340
128	7210	7549	7888	8227	8565	8903	9241	9579	9916	.253	338
129	110590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	113943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
131	7271	7603	7934	8265	8595	8926	9256	9586	9915	.245	330
132	120574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
134	7105	7429	7753	8076	8399	8722	9045	9368	9690	..12	323
135	130334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	315
138	9879	.194	.508	.822	1136	1450	1763	2076	2389	2702	314
139	143015	3327	3639	3951	4263	4571	4885	5196	5507	5818	311
140	146128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
141	9219	9527	9835	.142	.449	.756	1063	1370	1676	1982	307
142	152288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
144	8362	8664	8965	9266	9567	9868	.168	.469	.769	1068	301
145	161368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
146	4353	4650	4917	5244	5541	5838	6134	6430	6726	7022	297
147	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
148	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	176091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
151	8977	9264	9552	9839	.126	.413	.699	.985	1272	1558	287
152	181844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
154	7521	7803	8084	8366	8647	8928	9209	9490	9771	..51	281
155	190332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	5899	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	8657	8932	9206	9481	9755	..29	.303	.577	.850	1124	274
159	201397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
160	204120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
161	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
162	9515	9783	..51	.319	.586	.853	1121	1388	1654	1921	267
163	212188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
165	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
166	220108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
169	7887	8144	8400	8657	8913	9170	9426	9682	9938	.193	256
170	230449	0704	0960	1215	1470	1724	1979	2234	2488	2742	254
171	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
173	8046	8297	8548	8799	9049	9299	9550	9800	..50	.300	250
174	240549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
175	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
176	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	7973	8219	8464	8709	8954	9198	9443	9687	9932	.176	245
178	250420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	255273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	260071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
183	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
185	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
186	9513	9746	9980	.213	.446	.679	.912	1144	1377	1609	233
187	271842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
189	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	278754	8982	9211	9439	9667	9895	.123	.351	.578	.806	228
191	281033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	290035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
196	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
197	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
198	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
199	8853	9071	9289	9507	9725	9943	.161	.378	.595	.813	218
200	301030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
201	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
202	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
203	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
204	9630	9843	..56	.268	.481	.693	.906	1118	1330	1542	212
205	311754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
206	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
207	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
209	320146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	322219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
211	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
212	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
213	8380	8583	8787	8991	9194	9398	9601	9805	...8	.211	203
214	330414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202
215	2438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
216	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
217	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
218	8456	8656	8855	9054	9253	9451	9650	9849	..47	.246	199
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226	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
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229	9835	..25	.215	.404	.593	.783	.972	1161	1350	1539	189
230	361728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
231	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
234	9216	9401	9587	9772	9958	.143	.328	.513	.698	.883	185
235	371068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
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237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
239	8398	8580	8761	8943	9124	9306	9487	9668	9849	..30	181
240	380211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
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242	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
243	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
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245	9166	9343	9520	9698	9875	..51	.228	.405	.582	.759	177
246	390935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
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248	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
249	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	397940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
251	9674	9847	..20	.192	.365	.538	.711	.883	1056	1228	173
252	401401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
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259	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	414973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
261	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
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266	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
267	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
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273	6163	6322	6481	6640	6798	6957	7116	7275	7433	7592	159
274	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
275	9333	9491	9648	9806	9964	.122	.279	.437	.594	.752	158
276	440909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
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278	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
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284	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
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291	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
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293	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
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295	9822	9969	.116	.263	.410	.557	.704	.851	.998	1145	147
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307	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
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312	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
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319	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
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357	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
358	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
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368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	568202	8319	8436	8554	8671	8788	8905	9023	91[illegible]	9257	117
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374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
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376	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
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410	612784	2890	2996	3102	3207	3313	3419	3525	3630	3736	106
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442	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
443	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
444	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
445	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
446	9335	9432	9530	9627	9724	9821	9919	..16	.113	.210	97
447	650308	0405	0502	0599	0696	0793	0890	0987	1084	1181	97
448	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
449	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	653213	3309	3405	3502	3598	3695	3791	3888	3984	4080	96
451	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
452	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
453	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
454	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
455	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
456	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
457	9916	..11	.106	.201	.296	.391	.486	.581	.676	.771	95
458	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
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461	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
462	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
463	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
464	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
465	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
466	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
467	9317	9410	9503	9596	9689	9782	9875	9967	..60	.153	93
468	670246	0339	0431	0524	0617	0710	0802	0895	0988	1080	93
469	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	672098	2190	2283	2375	2467	2560	2652	2744	2836	2929	92
471	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
472	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
473	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
474	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
475	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
476	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
477	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
478	9428	9519	9610	9700	9791	9882	9973	..63	.154	.245	91
479	680336	0426	0517	0607	0698	0789	0879	0970	1060	1151	91
480	681241	1332	1422	1513	1603	1693	1784	1874	1964	2055	90
481	2145	2235	2326	[illegible]	2506	2596	2686	2777	2867	2957	90
482	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
483	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
484	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
485	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
486	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
487	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
488	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
489	9309	9398	9486	9575	9664	9753	9841	9930	..19	.107	89
490	690196	0285	0373	0462	0550	0639	0728	0816	0905	0993	89
491	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
492	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
493	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
494	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
495	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
496	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
497	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
498	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
499	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	698970	9057	9144	9231	9317	9404	9491	9578	9664	9751	87
501	9838	9924	..11	..98	.184	.271	.358	.444	.531	.617	87
502	700704	0790	0877	0963	1050	1136	1222	1309	1395	1482	86
503	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
504	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
505	3291	3377	3463	3549	3635	3721	3807	3895	3979	4065	86
506	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
507	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
508	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
509	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
510	707570	7655	7740	7826	7911	7996	8081	8166	8251	8336	85
511	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
512	9270	9355	9440	9524	9609	9694	9779	9863	9948	..33	85
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514	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
515	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
516	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
517	3491	3575	3650	3742	3826	3910	3994	4078	4162	4246	84
518	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
519	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84

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522	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
523	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
524	9331	9414	9497	9580	9663	9745	9828	9911	9994	..77	83
525	720159	0242	0325	0407	0490	0573	0655	0738	0821	0903	83
526	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
528	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
529	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	724276	4358	4440	4522	4604	4685	4767	4849	4931	5013	82
531	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
532	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
533	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
534	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
535	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
536	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
537	9974	..55	.136	.217	.298	.378	.459	.540	.621	.702	81
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539	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	732394	2474	2555	2635	2715	2796	2876	2956	3037	3117	80
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542	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
543	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
544	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
545	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
547	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
549	9572	9651	9731	9810	9889	9968	..47	.126	.205	.284	79
550	740363	0442	0521	0600	0678	0757	0836	0915	0994	1073	79
551	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2646	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
555	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
556	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
559	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	748188	8266	8343	8421	8498	8576	8653	8731	8808	8885	77
561	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
562	9736	9814	9891	9968	..45	.123	.200	.277	.354	.431	77
563	750508	0586	0663	0740	0817	0894	0971	1048	1125	1202	77
564	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
565	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
566	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
567	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
568	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
569	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	755875	5951	6027	6103	6180	6256	6332	6408	6484	6560	76
571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
572	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
573	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
574	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
575	9668	9743	9819	9894	9970	..45	.121	.196	.272	.347	75
576	760422	0498	0573	0649	0724	0799	0875	0950	1025	1101	75
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578	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
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582	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
583	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
584	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
585	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
586	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
587	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
588	9377	9451	9525	9599	9673	9746	9820	9894	9968	..42	74
589	770115	0189	0263	0336	0410	0484	0557	0631	0705	0778	74
590	770852	0926	0999	1073	1146	1220	1293	1367	1440	1514	74
591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
597	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
598	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
599	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
600	778151	8224	8296	8368	8441	8513	8585	8658	8730	8802	72
601	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
602	9596	9669	9741	9813	9885	9957	..29	.101	.173	.245	72
603	780317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
604	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
605	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
606	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
607	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
608	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
609	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	785330	5401	5472	5543	5615	5686	5757	5828	5899	5970	71
611	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
612	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
613	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
615	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
616	9581	9651	9722	9792	9863	9933	...4	..74	.144	.215	70
617	790285	0356	0426	0496	0567	0637	0707	0778	0848	0918	70
618	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
619	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
620	792392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
621	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
622	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
623	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
624	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
625	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
626	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
628	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
629	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
630	799341	9409	9478	9547	9616	9685	9754	9823	9892	9961	69
631	800029	0098	0167	0236	0305	0373	0442	0511	0580	0648	69
632	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
633	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	69
635	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
639	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
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640	806180	6248	6316	6384	6451	6519	6587	6655	6723	6790	68
641	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
642	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
643	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
644	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
645	9560	9627	9694	9762	9829	9896	9964	..31	..98	.165	67
646	810233	0300	0367	0434	0501	0569	0636	0703	0770	0837	67
647	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
648	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
649	2245	[illegible]	2379	2445	2512	2579	2646	2713	2780	2847	67
650	812913	2980	3047	3114	3181	3247	3314	3381	3448	3514	67
651	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
652	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
653	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
654	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
655	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
656	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
657	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
658	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
659	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	819544	9610	9676	9741	9807	9873	9939	...4	..70	.136	66
661	820201	0267	0333	0399	0464	0530	0595	0661	0727	0792	66
662	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
663	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
664	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
665	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
666	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
667	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	826075	6140	6204	6269	6334	6399	6464	6528	6593	6658	65
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672	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
673	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
674	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
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679	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
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682	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
683	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
684	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
685	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
686	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
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688	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
689	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	838849	8912	8975	9038	9101	9164	9227	9289	9352	9415	63
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694	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
695	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
696	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
697	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
698	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
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706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
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744	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
745	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
746	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
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749	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
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752	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
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755	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
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763	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
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767	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
768	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
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776	9862	9918	9974	..30	..86	.141	.197	.253	.309	.365	56
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779	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	892095	2150	2206	2262	2317	2373	2429	2484	2540	2595	56
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789	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	897627	7682	7737	7792	7847	7902	7957	8012	8067	8122	55
791	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
792	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
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798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
799	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
800	903090	3144	3199	3253	3307	3361	3416	3470	3524	3578	54
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809	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
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839	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
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841	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
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843	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
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849	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	929419	9470	9521	9572	9623	9674	9725	9776	9827	9879	51
851	9930	9981	..32	..83	.134	.185	.236	.287	.338	.389	51
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855	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
856	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
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882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
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886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
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891	9878	9926	9975	..24	..73	.121	.170	.219	.267	.316	49
892	950365	0414	0462	0511	0560	0608	0657	0706	0754	0803	49
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
895	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
898	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
899	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	954243	4291	4339	4387	4435	4484	4532	4580	4628	4677	48
901	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
902	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
904	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
905	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
908	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
909	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	959041	9089	9137	9185	9232	9280	9328	9375	9423	9471	48
911	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
912	9995	..42	..90	.138	.185	.233	.280	.328	.376	.423	48
913	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
914	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	47
915	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
916	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
917	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
918	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
919	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	963788	3835	3882	3929	3977	4024	4071	4118	4165	4212	47
921	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
922	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
924	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
925	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
926	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
929	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	968483	8530	8576	8623	8670	8716	8763	8810	8856	8903	47
931	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
932	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
933	9882	9928	9975	..21	..68	.114	.161	.207	.254	.300	47
934	970347	0393	0440	0486	0533	0579	0626	0672	0719	0765	46
935	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
940	973128	3174	3220	3266	3313	3359	3405	3451	3497	3543	46
941	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	977724	7769	7815	7861	7906	7952	7998	8043	8089	8135	46
951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
955	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412	45
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	982271	2316	2362	2407	2452	2497	2543	2588	2633	2678	45
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	986772	6817	6861	6906	6951	6996	7040	7085	7130	7175	45
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
975	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
977	9895	9939	9983	..28	..72	.117	.161	.206	.250	.294	44
978	990339	0383	0428	0472	0516	0561	0605	0650	0694	0738	44
979	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	991226	1270	1315	1359	1403	1448	1492	1536	1580	1625	44
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
985	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030	44
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
995	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
N.	0	1	2	3	4	5	6	7	8	9	D.

A TABLE

OF

LOGARITHMIC

SINES AND TANGENTS

FOR EVERY

DEGREE AND MINUTE

OF THE QUADRANT.

N. B The minutes in the left-hand column of each page, increasing downwards, belong to the degrees at the top; and those increasing upwards, in the right-hand column, belong to the degrees below.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	0.000000		10.000000		0.000000		Infinite.	60
1	6.463726	501717	000000	00	6.463726	501717	13.536274	59
2	764756	293485	000000	00	764756	293483	235244	58
3	940847	208231	000000	00	940847	208231	059153	57
4	7.065786	161517	000000	00	7.065786	161517	12.934214	56
5	162696	131968	000000	00	162696	131969	837304	55
6	241877	111575	9.999999	01	241878	111578	758122	54
7	308824	96653	999999	01	308825	99653	691175	53
8	366816	85254	999999	01	366817	85254	633183	52
9	417968	76263	999999	01	417970	76263	582030	51
10	463725	68988	999998	01	463727	68988	536273	50
11	7.505118	62981	9.999998	01	7.505120	62981	12.494880	49
12	542906	57936	999997	01	542909	57933	457091	48
13	577668	53641	999997	01	577672	53642	422328	47
14	609853	49938	999996	01	609857	49939	390143	46
15	639816	46714	999996	01	639820	46715	360180	45
16	667845	43881	999995	01	667849	43882	332151	44
17	694173	41372	999995	01	694179	41373	305821	43
18	718997	39135	999994	01	719003	39136	280997	42
19	742477	37127	999993	01	742484	37128	257516	41
20	764754	35315	999993	01	764761	35136	235239	40
21	7.785943	33672	9.999992	01	7.785951	33673	12.214049	39
22	806146	32175	999991	01	806155	32176	193845	38
23	825451	30805	999990	01	825460	30806	174540	37
24	843934	29547	999989	02	843944	29549	156056	36
25	861662	28388	999988	02	861674	28390	138326	35
26	878695	27317	999988	02	878708	27318	121292	34
27	895085	26323	999987	02	895099	26325	104901	33
28	910879	25399	999986	02	910894	25401	089106	32
29	926119	24538	999985	02	926134	24540	073866	31
30	940842	23733	999983	02	940858	23735	059142	30
31	7.955082	22980	9.999982	02	7.955100	22981	12.044900	29
32	968870	22273	999981	02	968889	22275	031111	28
33	982233	21608	999980	02	982253	21610	017747	27
34	995198	20981	999979	02	995219	20983	004781	26
35	8.007787	20390	999977	02	8.007809	20392	11.992191	25
36	020021	19831	999976	02	020045	19833	979955	24
37	031919	19302	999975	02	031945	19305	968055	23
38	043501	18801	999973	02	043527	18803	956473	22
39	054781	18325	999972	02	054809	18327	945191	21
40	065776	17872	999971	02	065806	17874	934194	20
41	8.076500	17441	9.999969	02	8.076531	17444	11.923469	19
42	086965	17031	999968	02	086997	17034	913003	18
43	097183	16639	999966	02	097217	16642	902783	17
44	107167	16265	999964	03	107202	16268	892797	16
45	116926	15908	999963	03	116963	15910	883037	15
46	126471	15566	999961	03	126510	15568	873490	14
47	135810	15238	999959	03	135851	15241	864149	13
48	144953	14924	999958	03	144996	14927	855004	12
49	153907	14622	999956	03	153952	14627	846048	11
50	162681	14333	999954	03	162727	14336	837273	10
51	8.171280	14054	9.999952	03	8.171328	14057	11 828672	9
52	179713	13786	999950	03	179763	13790	820237	8
53	187985	13529	999948	03	188036	13532	811964	7
54	196102	13280	999946	03	196156	13284	803844	6
55	204070	13041	999944	03	204126	13044	795874	5
56	211895	12810	999942	04	211953	12814	788047	4
57	219581	12587	999940	04	219641	12590	780359	3
58	227134	12372	999938	04	227195	12376	772805	2
59	234557	12164	999936	04	234621	12168	765379	1
60	241855	11963	999934	04	241921	11967	758079	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D	Cotang.	
0	8.241855	11963	9.999934	04	8.241921	11967	11.758079	60
1	249033	11768	999932	04	249102	11772	750898	59
2	256094	11580	999929	04	256165	11584	743835	58
3	263042	11398	999927	04	263115	11402	736885	57
4	269881	11221	999925	04	269956	11225	730044	56
5	276614	11050	999922	04	276691	11054	723309	55
6	283243	10883	999920	04	283323	10887	716677	54
7	289773	10721	999918	04	289856	10726	710144	53
8	296207	10565	999915	04	296292	10570	703708	52
9	302546	10413	999913	04	302634	10418	697366	51
10	308794	10266	999910	04	308884	10270	691116	50
11	8.314954	10122	9.999907	04	8.315046	10126	11.684954	49
12	321027	9982	999905	04	321122	9987	678878	48
13	327016	9847	999902	04	327114	9851	672886	47
14	332924	9714	999899	05	333025	9719	666975	46
15	338753	9586	999897	05	338856	9590	661144	45
16	344504	9460	999894	05	344610	9465	655390	44
17	350181	9338	999891	05	350289	9343	649711	43
18	355783	9219	999888	05	355895	9224	644105	42
19	361315	9103	999885	05	361430	9108	638570	41
20	366777	8990	999882	05	366895	8995	633105	40
21	8.372171	8880	9.999879	05	8.372292	8885	11.627708	39
22	377499	8772	999876	05	377622	8777	622378	38
23	382762	8667	999873	05	382889	8672	617111	37
24	387962	8564	999870	05	388092	8570	611908	36
25	393101	8464	999867	05	393234	8470	606766	35
26	398179	8366	999864	05	398315	8371	601685	34
27	403199	8271	999861	05	403338	8276	596662	33
28	408161	8177	999858	05	408304	8182	591696	32
29	413068	8086	999854	05	413213	8091	586787	31
30	417919	7996	999851	06	418068	8002	581932	30
31	8.422717	7909	9.999848	06	8.422869	7914	11.577131	29
32	427462	7823	999844	06	427618	7830	572382	28
33	432156	7740	999841	06	432315	7745	567685	27
34	436800	7657	999838	06	436962	7663	563038	26
35	441394	7577	999834	06	441560	7583	558440	25
36	445941	7499	999831	06	446110	7505	553890	24
37	450440	7422	999827	06	450613	7428	549387	23
38	454893	7346	999823	06	455070	7352	544930	22
39	459301	7273	999820	06	459481	7279	540519	21
40	463665	7200	999816	06	463849	7206	536151	20
41	8.467985	7129	9.999812	06	8.468172	7135	11.531828	19
42	472263	7060	999809	06	472454	7066	527546	18
43	476498	6991	999805	06	476693	6998	523307	17
44	480693	6924	999801	06	480892	6931	519108	16
45	484848	6859	999797	07	485050	6865	514950	15
46	488963	6794	999793	07	489170	6801	510830	14
47	493040	6731	999790	07	493250	6738	506750	13
48	497078	6669	999786	07	497293	6676	502707	12
49	501080	6608	999782	07	501298	6615	498702	11
50	505045	6548	999778	07	505267	6555	494733	10
51	8.508974	6489	9.999774	07	8.509200	6496	11.490800	9
52	512867	6431	999769	07	513098	6439	486902	8
53	516726	6375	999765	07	516961	6382	483039	7
54	520551	6319	999761	07	520790	6326	479210	6
55	524343	6264	999757	07	524586	6272	475414	5
56	528102	6211	999753	07	528349	6218	471651	4
57	531828	6158	999748	07	532080	6165	467920	3
58	535523	6106	999744	07	535779	6113	464221	2
59	539186	6055	999740	07	539447	6062	460553	1
60	542819	6004	999735	07	543084	6012	456916	0
	Cosine		Sine		Cotang.		Tang.	M.

Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8.542819	6004	9.999735	07	8.543084	6012	11.456916	60
1	546422	5955	999731	07	546691	5962	453309	59
2	549995	5906	999726	07	550268	5914	449732	58
3	553539	5858	999722	08	553817	5866	446183	57
4	557054	5811	999717	08	557336	5819	442664	56
5	560540	5765	999713	08	560828	5773	439172	55
6	563999	5719	999708	08	564291	5727	435709	54
7	567431	5674	999704	08	567727	5682	432273	53
8	570836	5630	999699	08	571137	5638	428863	52
9	574214	5587	999694	08	574520	5595	425480	51
10	577566	5544	999689	08	577877	5552	422123	50
11	8.580892	5502	9.999685	08	8.581208	5510	11.418792	49
12	584193	5460	999680	08	584514	5468	415486	48
13	587469	5419	999675	08	587795	5427	412205	47
14	590721	5379	999670	08	591051	5387	408949	46
15	593948	5339	999665	08	594283	5347	405717	45
16	597152	5300	999660	08	597492	5308	402508	44
17	600332	5261	999655	08	600677	5270	399323	43
18	603489	5223	999650	08	603839	5232	396161	42
19	606623	5186	999645	09	606978	5194	393022	41
20	609734	5149	999640	09	610094	5158	389906	40
21	8.612823	5112	9.999635	09	8.613189	5121	11.386811	39
22	615891	5076	999629	09	616262	5085	383738	38
23	618937	5041	999624	09	619313	5050	380687	37
24	621962	5006	999619	09	622343	5015	377657	36
25	624965	4972	999614	09	625352	4981	374648	35
26	627948	4938	999608	09	628340	4947	371660	34
27	630911	4904	999603	09	631308	4913	368692	33
28	633854	4871	999597	09	634256	4880	365744	32
29	636776	4839	999592	09	637184	4848	362816	31
30	639680	4806	999586	09	640093	4816	359907	30
31	8.642563	4775	9.999581	09	8.642982	4784	11.357018	29
32	645428	4743	999575	09	645853	4753	354147	28
33	648274	4712	999570	09	648704	4722	351296	27
34	651102	4682	999564	09	651537	4691	348463	26
35	653911	4652	999558	10	654352	4661	345648	25
36	656702	4622	999553	10	657149	4631	342851	24
37	659475	4592	999547	10	659928	4602	340072	23
38	662230	4563	999541	10	662689	4573	337311	22
39	664968	4535	999535	10	665433	4544	334567	21
40	667689	4506	999529	10	668160	4526	331840	20
41	8.670393	4479	9.999524	10	8.670870	4488	11.329130	19
42	673080	4451	999518	10	673563	4461	326437	18
43	675751	4424	999512	10	676239	4434	323761	17
44	678405	4397	999506	10	678900	4417	321100	16
45	681043	4370	999500	10	681544	4380	318456	15
46	683665	4344	999493	10	684172	4354	315828	14
47	686272	4318	999487	10	686784	4328	313216	13
48	688863	4292	999481	10	689381	4303	310619	12
49	691438	4267	999475	10	691963	4277	308037	11
50	693998	4242	999469	10	694529	4252	305471	10
51	8.696543	4217	9.999463	11	8.697081	4228	11.302919	9
52	699073	4192	999456	11	699617	4203	300383	8
53	701589	4168	999450	11	702139	4179	297861	7
54	704090	4144	999443	11	704646	4155	295354	6
55	706577	4121	999437	11	707140	4132	292860	5
56	709049	4097	999431	11	709618	4108	290382	4
57	711507	4074	999424	11	712083	4085	287917	3
58	713952	4051	999418	11	714534	4062	285465	2
59	716383	4029	999411	11	716972	4040	283028	1
60	718800	4006	999404	11	719396	4017	280604	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8.718800	4006	9.999404	11	8.719396	4017	11.280604	60
1	721204	3984	999398	11	721806	3995	278194	59
2	723595	3962	999391	11	724204	3974	275796	58
3	725972	3941	999384	11	726588	3952	273412	57
4	728337	3919	999378	11	728959	3930	271041	56
5	730688	3898	999371	11	731317	3909	268683	55
6	733027	3877	999364	12	733663	3889	266337	54
7	735354	3857	999357	12	735996	3868	264004	53
8	737667	3836	999350	12	738317	3848	261683	52
9	739969	3816	999343	12	740626	3827	259374	51
10	742259	3796	999336	12	742922	3807	257078	50
11	8.744536	3776	9.999329	12	8.745207	3787	11.254793	49
12	746802	3756	999322	12	747479	3768	252521	48
13	749055	3737	999315	12	749740	3749	250260	47
14	751297	3717	999308	12	751989	3729	248011	46
15	753528	3698	999301	12	754227	3710	245773	45
16	755747	3679	999294	12	756453	3692	243547	44
17	757955	3661	999286	12	758668	3673	241332	43
18	760151	3642	999279	12	760872	3655	239128	42
19	762337	3624	999272	12	763065	3636	236935	41
20	764511	3606	999265	12	765246	3618	234754	40
21	8.766675	3588	9.999257	12	8.767417	3600	11.232583	39
22	768828	3570	999250	13	769578	3583	230422	38
23	770970	3553	999242	13	771727	3565	228273	37
24	773101	3535	999235	13	773866	3548	226134	36
25	775223	3518	999227	13	775995	3531	224005	35
26	777333	3501	999220	13	778114	3514	221886	34
27	779434	3484	999212	13	780222	3497	219778	33
28	781524	3467	999205	13	782320	3480	217680	32
29	783605	3451	999197	13	784408	3464	215592	31
30	785675	3431	999189	13	786486	3447	213514	30
31	8.787736	3418	9.999181	13	8.788554	3431	11.211446	29
32	789787	3402	999174	13	790613	3414	209387	28
33	791828	3386	999166	13	792662	3399	207338	27
34	793859	3370	999158	13	794701	3383	205299	26
35	795881	3354	999150	13	796731	3368	203269	25
36	797894	3339	999142	13	798752	3352	201248	24
37	799897	3323	999134	13	800763	3337	199237	23
38	801892	3308	999126	13	802765	3322	197235	22
39	803876	3293	999118	13	804758	3307	195242	21
40	805852	3278	999110	13	806742	3292	193258	20
41	8.807819	3263	9.999102	13	8.808717	3278	11.191283	19
42	809777	3249	999094	14	810683	3262	189317	18
43	811726	3234	999086	14	812641	3248	187359	17
44	813667	3219	999077	14	814589	3233	185411	16
45	815599	3205	999069	14	816529	3219	183471	15
46	817522	3191	999061	14	818461	3205	181539	14
47	819436	3177	999053	14	820384	3191	179616	13
48	821343	3163	999044	14	822298	3177	177702	12
49	823240	3149	999036	14	824205	3163	175795	11
50	825130	3135	999027	14	826103	3150	173897	10
51	8.827011	3122	9.999019	14	8.827992	3136	11.172008	9
52	828884	3108	999010	14	829874	3123	170126	8
53	830749	3095	999002	14	831748	3110	168252	7
54	832607	3082	998993	14	833613	3096	166387	6
55	834456	3069	998984	14	835471	3083	164529	5
56	836297	3056	998976	14	837321	3070	162679	4
57	838130	3043	998967	15	839163	3057	160837	3
58	839956	3030	998958	15	840998	3045	159002	2
59	841774	3017	998950	15	842825	3032	157175	1
60	843585	3000	998941	15	844644	3019	155356	0
	Cosine		Sine		Cotang.		Tang.	M.

86 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D	Cotang.	
0	8.843585	3005	9.998941	15	8.844644	3019	11.155356	60
1	845387	2992	998932	15	846455	3007	153545	59
2	847183	2980	998923	15	848260	2995	151740	58
3	848971	2967	998914	15	850057	2982	149943	57
4	850751	2955	998905	15	851846	2970	148154	56
5	852525	2943	998896	15	853628	2958	146372	55
6	854291	2931	998887	15	855403	2946	144597	54
7	856049	2919	998878	15	857171	2935	142829	53
8	857801	2907	998869	15	858932	2923	141068	52
9	859546	2896	998860	15	860686	2911	139314	51
10	861283	2884	998851	15	862433	2900	137567	50
11	8.863014	2873	9.998841	15	8.864173	2888	11.135827	49
12	864738	2861	998832	15	865906	2877	134094	48
13	866455	2850	998823	16	867632	2866	132368	47
14	868165	2839	998813	16	869351	2854	130649	46
15	869868	2828	998804	16	871064	2843	128936	45
16	871565	2817	998795	16	872770	2832	127230	44
17	873255	2806	998785	16	874469	2821	125531	43
18	874938	2795	998776	16	876162	2811	123838	42
19	876615	2786	998766	16	877849	2800	122151	41
20	878285	2773	998757	16	879529	2789	120471	40
21	8.879949	2763	9.998747	16	8.881202	2779	11.118798	39
22	881607	2752	998738	16	882869	2768	117131	38
23	883258	2742	998728	16	884530	2758	115470	37
24	884903	2731	998718	16	886185	2747	113815	36
25	886542	2721	998708	16	887833	2737	112167	35
26	888174	2711	998699	16	889476	2727	110524	34
27	889801	2700	998689	16	891112	2717	108888	33
28	891421	2690	998679	16	892742	2707	107258	32
29	893035	2680	998669	17	894366	2697	105634	31
30	894643	2670	998659	17	895984	2687	104016	30
31	8.896246	2660	9.998649	17	8.897596	2677	11.102404	29
32	897842	2651	998639	17	899203	2667	100797	28
33	899432	2641	998629	17	900803	2658	099197	27
34	901017	2631	998619	17	902398	2648	097602	26
35	902596	2622	998609	17	903987	2638	096013	25
36	904169	2612	998599	17	905570	2629	094430	24
37	905736	2603	998589	17	907147	2620	092853	23
38	907297	2593	998578	17	908719	2610	091281	22
39	908853	2584	998568	17	910285	2601	089715	21
40	910404	2575	998558	17	911846	2592	088154	20
41	8.911949	2566	9.998548	17	8.913401	2583	11.086599	19
42	913488	2556	998537	17	914951	2574	085049	18
43	915022	2547	998527	17	916495	2565	083505	17
44	916550	2538	998516	18	918034	2556	081966	16
45	918073	2529	998506	18	919568	2547	080432	15
46	919591	2520	998495	18	921096	2538	078904	14
47	921103	2512	998485	18	922619	2530	077381	13
48	922610	2503	998474	18	924136	2521	075864	12
49	924112	2494	998464	18	925649	2512	074351	11
50	925609	2486	998453	18	927156	2503	072844	10
51	8.927100	2477	9.998442	18	8.928658	2495	11.0[illegible]1342	9
52	928587	2469	998431	18	930155	2486	069845	8
53	930068	2460	998421	18	931647	2478	068353	7
54	931544	2452	998410	18	933134	2470	066866	6
55	933015	2443	998399	18	934616	2461	065384	5
56	934481	2435	998388	18	936093	2453	063907	4
57	935942	2427	998377	18	937565	2445	062435	3
58	937398	2419	998366	18	939032	2437	060968	2
59	938850	2411	998355	18	940494	2430	059506	1
60	940296	2403	998344	18	941952	2421	058048	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8.940296	2403	9.998344	19	8.941952	2421	11.058048	60
1	941738	2394	998333	19	943404	2413	056596	59
2	943174	2387	998322	19	944852	2405	055148	58
3	944606	2379	998311	19	946295	2397	053705	57
4	946034	2371	998300	19	947734	2390	052266	56
5	947456	2363	998289	19	949168	2382	050832	55
6	948874	2355	998277	19	950597	2374	049403	54
7	950287	2348	998266	19	952021	2366	047979	53
8	951696	2340	998255	19	953441	2360	046559	52
9	953100	2332	998243	19	954856	2351	045144	51
10	954499	2325	998232	19	956267	2344	043733	50
11	8.955894	2317	9.998220	19	8.957674	2337	11.042326	49
12	957284	2310	998209	19	959075	2329	040925	48
13	958670	2302	998197	19	960473	2323	039527	47
14	960052	2295	998186	19	961866	2314	038134	46
15	961429	2288	998174	19	963255	2307	036745	45
16	962891	2280	998163	19	964639	2300	035361	44
17	964170	2273	998151	19	966019	2293	033981	43
18	965534	2266	998139	20	967394	2286	032606	42
19	966893	2259	998128	20	968766	2279	031234	41
20	968249	2252	998116	20	970133	2271	029867	40
21	8.969600	2244	9.998104	20	8.971496	2265	11.028504	39
22	970947	2238	998092	20	972855	2257	027145	38
23	972289	2231	998080	20	974209	2251	025791	37
24	973628	2224	998068	20	975560	2244	024440	36
25	974962	2217	998056	20	976906	2237	023094	35
26	976293	2210	998044	20	978248	2230	021752	34
27	977619	2203	998032	20	979586	2223	020414	33
28	978941	2197	998020	20	980921	2217	019079	32
29	980259	2190	998008	20	982251	2210	017749	31
30	981573	2183	997996	20	983577	2204	016423	30
31	8.982883	2177	9.997984	20	8.984899	2197	11.015101	29
32	984189	2170	997972	20	986217	2191	013783	28
33	985491	2163	997959	20	987532	2184	012468	27
34	986789	2157	997947	20	988842	2178	011158	26
35	988083	2150	997935	21	990149	2171	009851	25
36	989374	2144	997922	21	991451	2165	008549	24
37	990660	2138	997910	21	992750	2158	007250	23
38	991943	2131	997897	21	994045	2152	005955	22
39	993222	2125	997885	21	995337	2146	004663	21
40	994497	2119	997872	21	996624	2140	003376	20
41	8.995768	2112	9.997860	21	8.997908	2134	11.002092	19
42	997036	2106	997847	21	999188	2127	000812	18
43	998299	2100	997835	21	9.000465	2121	10.999535	17
44	999560	2094	997822	21	001738	2115	998262	16
45	9.000816	2087	997809	21	003007	2109	996993	15
46	002069	2082	997797	21	004272	2103	995728	14
47	003318	2076	997784	21	005534	2097	994466	13
48	004563	2070	997771	21	006792	2091	993208	12
49	005805	2064	997758	21	008047	2085	991953	11
50	007044	2058	997745	21	009298	2080	990702	10
51	9.008278	2052	9.997732	21	9.010546	2074	10.989454	9
52	009510	2046	997719	21	011790	2068	988210	8
53	010737	2040	997706	21	013031	2062	986969	7
54	011962	2034	997693	22	014268	2056	985732	6
55	013182	2029	997680	22	015502	2051	984498	5
56	014400	2023	997667	22	016732	2045	983268	4
57	015613	2017	997654	22	017959	2040	982041	3
58	016824	2012	997641	22	019183	2033	980817	2
59	018031	2006	997628	22	020403	2028	979597	1
60	019235	2000	997614	22	021620	2023	978380	0
	Cosine		Sine		Cotang.		Tang.	M

M	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	M
0	9.019235	2000	9.997614	22	9.021620	2023	10.978380	60
1	020435	1995	997601	22	022834	2017	977166	59
2	021632	1989	997588	22	024044	2011	975956	58
3	022825	1984	997574	22	025251	2006	974749	57
4	024016	1978	997561	22	026455	2000	973545	56
5	025203	1973	997547	22	027655	1995	972345	55
6	026386	1967	997534	23	028852	1990	971148	54
7	027567	1962	997520	23	030046	1985	969954	53
8	028744	1957	997507	23	031237	1979	968763	52
9	029918	1951	997493	23	032425	1974	967575	51
10	031089	1947	997480	23	033609	1969	966391	50
11	9.032257	1941	9.997466	23	9.034791	1964	10.965209	49
12	033421	1936	997452	23	035969	1958	964031	48
13	034582	1930	997439	23	037144	1953	962856	47
14	035741	1925	997425	23	038316	1948	961684	46
15	036896	1920	997411	23	039485	1943	960515	45
16	038048	1915	997397	23	040651	1938	959349	44
17	039197	1910	997383	23	041813	1933	958187	43
18	040342	1905	997369	23	042973	1928	957027	42
19	041485	1899	997355	23	044130	1923	955870	41
20	042625	1894	997341	23	045284	1918	954716	40
21	9.043762	1889	9.997327	24	9.046434	1913	10.953566	39
22	044895	1884	997313	24	047582	1908	952418	38
23	046026	1879	997299	24	048727	1903	951273	37
24	047154	1875	997285	24	049869	1898	950131	36
25	048279	1870	997271	24	051008	1893	948992	35
26	049400	1865	997257	24	052144	1889	947856	34
27	050519	1860	997242	24	053277	1884	946723	33
28	051635	1855	997228	24	054407	1879	945593	32
29	052749	1850	997214	24	055535	1874	944465	31
30	053859	1845	997199	24	056659	1870	943341	30
31	054966	1841	9.997185	24	9.057781	1865	10.942219	29
32	056071	1836	997170	24	058900	1869	941100	28
33	057172	1831	997156	24	060016	1855	939984	27
34	058271	1827	997141	24	061130	1851	938870	26
35	059367	1822	997127	24	062240	1846	937760	25
36	060460	1817	997112	24	063348	1842	936652	24
37	061551	1813	997098	24	064453	1837	935547	23
38	062639	1808	997083	25	065556	1833	934444	22
39	063724	1804	997068	25	066655	1828	933345	21
40	064806	1799	997053	25	067752	1824	932248	20
41	9.065885	1794	9.997039	25	9.068846	1819	10.931154	19
42	066962	1790	997024	25	069938	1815	930062	18
43	068036	1786	997009	25	071027	1810	928973	17
44	069107	1781	996994	25	072113	1806	927887	16
45	070176	1777	996979	25	073197	1802	926803	15
46	071242	1772	996964	25	074278	1797	925722	14
47	072306	1768	996949	25	075356	1793	924644	13
48	073366	1763	996934	25	076432	1789	923568	12
49	074424	1759	996919	25	077505	1784	922495	11
50	075480	1755	996904	25	078576	1780	921424	10
51	9.076533	1750	9.996889	25	9.079644	1776	10.920356	9
52	077583	1746	996874	25	080710	1772	919290	8
53	078631	1742	996858	25	081773	1767	918227	7
54	079676	1738	996843	25	082833	1763	917167	6
55	080719	1733	996828	25	083891	1759	916109	5
56	081759	1729	996812	26	084947	1755	915053	4
57	082797	1725	996797	26	086000	1751	914000	3
58	083832	1721	996782	26	087050	1747	912950	2
59	084864	1717	996766	26	088098	1743	911902	1
60	085894	1713	996751	26	089144	1738	910856	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.085894	1713	9.996751	26	9.089144	1738	10.910856	60
1	086922	1709	996735	26	090187	734	909813	59
2	087947	1704	996720	26	091228	1730	908772	58
3	088970	1700	996704	26	092266	1727	907734	57
4	089990	1696	996688	26	093302	1722	906698	56
5	091008	1692	996673	26	094336	1719	905664	55
6	092024	1688	996657	26	095367	1715	904633	54
7	093037	1684	996641	26	096395	1711	903605	53
8	094047	1680	996625	26	097422	1707	902578	52
9	095056	1676	996610	26	098446	1703	901554	51
10	096062	1673	996594	26	099468	1699	900532	50
11	9.097065	1668	9.996578	27	9.100487	1695	10.899513	49
12	098066	1665	996562	27	101504	1691	898496	48
13	099065	1661	996546	27	102519	1687	897481	47
14	100062	1657	996530	27	103532	1684	896468	46
15	101056	1653	996514	27	104542	1680	895458	45
16	102048	1649	996498	27	105550	1676	894450	44
17	103037	1645	996482	27	106556	1672	893444	43
18	104025	1641	996465	27	107559	1669	892441	42
19	105010	1638	996449	27	108560	1665	891440	41
20	105992	1634	996433	27	109559	1661	890441	40
21	9.106973	1630	9.996417	27	9.110556	1658	10.889444	39
22	107951	1627	996400	27	111551	1654	888449	38
23	108927	1623	996384	27	112543	1650	887457	37
24	109901	1619	996368	27	113533	1646	886467	36
25	110873	1616	996351	27	114521	1643	885479	35
26	111842	1612	996335	27	115507	1639	884493	34
27	112809	1608	996318	27	116491	1636	883509	33
28	113774	1605	996302	28	117472	1632	882528	32
29	114737	1601	996285	28	118452	1629	881548	31
30	115698	1597	996269	28	119429	1625	880571	30
31	9.116656	1594	9.996252	28	9.120404	1622	10.879596	29
32	117613	1590	996235	28	121377	1613	878623	28
33	118567	1587	996219	28	122348	1615	877652	27
34	119519	1583	996202	28	123317	1611	876683	26
35	120469	1580	996185	28	124284	1607	875716	25
36	121417	1576	996168	28	125249	1604	874751	24
37	122362	1573	996151	28	126211	1601	873789	23
38	123306	1569	996134	28	127172	1597	872828	22
39	124248	1566	996117	28	128130	1594	871870	21
40	125187	1562	996100	28	129087	1591	870913	20
41	9.126125	1559	9.996083	29	9.130041	1587	10.869959	19
42	127060	1556	996066	29	130994	1584	869006	18
43	127993	1552	996049	29	131944	1581	868056	17
44	128925	1549	996032	29	132893	1577	867107	16
45	129854	1545	996015	29	133839	1574	866161	15
46	130781	1542	995998	29	134784	1571	865216	14
47	131706	1539	995980	29	135726	1567	864274	13
48	132630	1535	995963	29	136667	1564	863333	12
49	133551	1532	995946	29	137605	1561	862395	11
50	134470	1529	995928	29	138542	1558	861458	10
51	9.135387	1525	9.995911	29	9.139476	1555	10.860524	9
52	136303	1522	995894	29	140409	1551	859591	8
53	137216	1519	995876	29	141340	1548	858660	7
54	138128	1516	995859	29	142269	1545	857731	6
55	139037	1512	995841	29	143196	1542	856804	5
56	139944	1509	995823	29	144121	1539	855879	4
57	140850	1506	995806	29	145044	1535	854956	3
58	141754	1503	995788	29	145966	1532	854034	2
59	142655	1500	995771	29	146885	1529	853115	1
60	143555	1496	995753	29	147803	1526	852197	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.143555	1496	9.995753	30	9.147803	1526	10.852197	60
1	144453	1493	995735	30	148718	1523	851282	59
2	145349	1490	995717	30	149632	1520	850368	58
3	146243	1487	995699	30	150544	1517	849456	57
4	147136	1484	995681	30	151454	1514	848546	56
5	148026	1481	995664	30	152363	1511	847637	55
6	148915	1478	995646	30	153269	1508	846731	54
7	149802	1475	995628	30	154174	1505	845826	53
8	150686	1472	995610	30	155077	1502	844923	52
9	151569	1469	995591	30	155978	1499	844022	21
10	152451	1466	995573	30	156877	1496	843123	50
11	9.153330	1463	9.995555	30	9.157775	1493	10.842225	49
12	154208	1460	995537	30	158671	1490	841329	48
13	155083	1457	995519	30	159565	1487	840435	47
14	155957	1454	995501	31	160457	1484	839543	46
15	156830	1451	995482	31	161347	1481	838653	45
16	157700	1448	995464	31	162236	1479	837764	44
17	158569	1445	995446	31	163123	1476	836877	43
18	159435	1442	995427	31	164008	1473	835992	42
19	160301	1439	995409	31	164892	1470	835108	41
20	161164	1436	995390	31	165774	1467	834226	40
21	9.162025	1433	9.995372	31	9.166654	1464	10.833346	39
22	162885	1430	995353	31	167532	1461	832468	38
23	163743	1427	995334	31	168409	1458	831591	37
24	164600	1424	995316	31	169284	1455	830716	36
25	165454	1422	995297	31	170157	1453	829843	35
26	166307	1419	995278	31	171029	1450	828971	34
27	167159	1416	995260	31	171899	1447	828101	33
28	168008	1413	995241	32	172767	1444	827233	32
29	168856	1410	995222	32	173634	1442	826366	31
30	169702	1407	995203	32	174499	1439	825501	30
31	9.170547	1405	9.995184	32	9.175362	1436	10.824638	29
32	171389	1402	995165	32	176224	1433	823776	28
33	172230	1399	995146	32	177084	1431	822916	27
34	173070	1396	995127	32	177942	1428	822058	26
35	173908	1394	995108	32	178799	1425	821201	25
36	174744	1391	995089	32	179655	1423	820345	24
37	175578	1388	995070	32	180508	1420	819492	23
38	176411	1386	995051	32	181360	1417	818640	22
39	177242	1383	995032	32	182211	1415	817789	21
40	178072	1380	995013	32	183059	1412	816941	20
41	9.178900	1377	9.994993	32	9.183907	1409	10.816093	19
42	179726	1374	994974	32	184752	1407	815248	18
43	180551	1372	994955	32	185597	1404	814403	17
44	181374	1369	994935	32	186439	1402	813561	16
45	182196	1366	994916	33	187280	1399	812720	15
46	183016	1364	994896	33	188120	1396	811880	14
47	183834	1361	994877	33	188958	1393	811042	13
48	184651	1359	994857	33	189794	1391	810206	12
49	185466	1356	994838	33	190629	1389	809371	11
50	186280	1353	994818	33	191462	1386	808538	10
51	9.187092	1351	9.994798	33	9.192294	1384	10.807706	9
52	187903	1348	994779	33	193124	1381	806876	8
53	188712	1346	994759	33	193953	1379	806047	7
54	189519	1343	994739	33	194780	1376	805220	6
55	190325	1341	994719	33	195606	1374	804394	5
56	191130	1338	994700	33	196430	1371	803570	4
57	191933	1336	994680	33	197253	1369	802747	3
58	192734	1333	994660	33	198074	1366	801926	2
59	193534	1330	994640	33	198894	1364	801106	1
60	194332	1328	994620	33	199713	1361	800287	0
	Cosine		Sine		Cotang.		Tang.	M.

81 Degrees.

M.	Sine	D.	Cosine	D	Tang.	D.	Cotang.	
0	9.194332	1328	9.994620	33	9.199713	1361	10.800287	60
1	195129	1326	994600	33	200529	1359	799471	59
2	195925	1323	994580	33	201345	1356	798655	58
3	196719	1321	994560	34	202159	1354	797841	57
4	197511	1318	994540	34	202971	1352	797029	56
5	198302	1316	994519	34	203782	1349	796218	55
6	199091	1313	994499	34	204592	1347	795408	54
7	199879	1311	994479	34	205400	1345	794600	53
8	200666	1308	994459	34	206207	1342	793793	52
9	201451	1306	994438	34	207013	1340	792987	51
10	202234	1304	994418	34	207817	1338	792183	50
11	9.203017	1301	9.994397	34	9.208619	1335	10.791381	49
12	203797	1299	994377	34	209420	1333	790580	48
13	204577	1296	994357	34	210220	1331	789780	47
14	205354	1294	994336	34	211018	1328	788982	46
15	206131	1292	994316	34	211815	1326	788185	45
16	206906	1289	994295	34	212611	1324	787389	44
17	207679	1287	994274	35	213405	1321	786595	43
18	208452	1285	994254	35	214198	1319	785802	42
19	209222	1282	994233	35	214989	1317	785011	41
20	209992	1280	994212	35	215780	1315	784220	40
21	9.210760	1278	9.994191	35	9.216568	1312	10.783432	39
22	211526	1275	994171	35	217356	1310	782644	38
23	212291	1273	994150	35	218142	1308	781858	37
24	213055	1271	994129	35	218926	1305	781074	36
25	213818	1268	994108	35	219710	1303	780290	35
26	214579	1266	994087	35	220492	1301	779508	34
27	215338	1264	994066	35	221272	1299	778728	33
28	216097	1261	994045	35	222052	1297	777948	32
29	216854	1259	994024	35	222830	1294	777170	31
30	217609	1257	994003	35	223606	1292	776394	30
31	9.218363	1255	9.993981	35	9.224382	1290	10.775618	29
32	219116	1253	993960	35	225156	1288	774844	28
33	219868	1250	993939	35	225929	1286	774071	27
34	220618	1248	993918	35	226700	1284	773300	26
35	221367	1246	993896	36	227471	1281	772529	25
36	222115	1244	993875	36	228239	1279	771761	24
37	222861	1242	993854	36	229007	1277	770993	23
38	223606	1239	993832	36	229773	1275	770227	22
39	224349	1237	993811	36	230539	1273	769461	21
40	225092	1235	993789	36	231302	1271	768698	20
41	9.225833	1233	9.993768	36	9.232065	1269	10.767935	19
42	226573	1231	993746	36	232826	1267	767174	18
43	227311	1228	993725	36	233586	1265	766414	17
44	228048	1226	993703	36	234345	1262	765655	16
45	228784	1224	993681	36	235103	1260	764897	15
46	229518	1222	993660	36	235859	1258	764141	14
47	230252	1220	993638	36	236614	1256	763386	13
48	230984	1218	993616	36	237368	1254	762632	12
49	231714	1216	993594	37	238120	1252	761880	11
50	232444	1214	993572	37	238872	1250	761128	10
51	9.233172	1212	9.993550	37	9.239622	1248	10.760378	9
52	233899	1209	993528	37	240371	1246	759629	8
53	234625	1207	993506	37	241118	1244	758882	7
54	235349	1205	993484	37	241865	1242	758135	6
55	236073	1203	993462	37	242610	1240	757390	5
56	236795	1201	993440	37	243354	1238	756646	4
57	237515	1199	993418	37	244097	1236	755903	3
58	238235	1197	993396	37	244839	1234	755161	2
59	238953	1195	993374	37	245579	1232	754421	1
60	239670	1193	993351	37	246319	1230	753681	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.239670	1193	9.993351	37	9.246319	1230	10.753681	60
1	240386	1191	993329	37	247057	1228	752943	59
2	241101	1189	993307	37	247794	1226	752206	58
3	241814	1187	993285	37	248530	1224	751470	57
4	242526	1185	993262	37	249264	1222	750736	56
5	243237	1183	993240	37	249998	1220	750002	55
6	243947	1181	993217	38	250730	1218	749270	54
7	244656	1179	993195	38	251461	1217	748539	53
8	245363	1177	993172	38	252191	1215	747809	52
9	246069	1175	993149	38	252920	1213	747080	51
10	246775	1173	993127	38	253648	1211	746352	50
11	9.247478	1171	9.993104	38	9.254374	1209	10.745626	49
12	248181	1169	993081	38	255100	1207	744900	48
13	248883	1167	993059	38	255824	1205	744176	47
14	249583	1165	993036	38	256547	1203	743453	46
15	250282	1163	993013	38	257269	1201	742731	45
16	250980	1161	992990	38	257990	1200	742010	44
17	251677	1159	992967	38	258710	1198	741290	43
18	252373	1158	992944	38	259429	1196	740571	42
19	253067	1156	992921	38	260146	1194	739854	41
20	253761	1154	992898	38	260863	1192	739137	40
21	9.254453	1152	9.992875	38	9.261578	1190	10.738422	39
22	255144	1150	992852	38	262292	1189	737708	38
23	255834	1148	992829	39	263005	1187	736995	37
24	256523	1146	992806	39	263717	1185	736283	36
25	257211	1144	992783	39	264428	1183	735572	35
26	257898	1142	992759	39	265138	1181	734862	34
27	258583	1141	992736	39	265847	1179	734153	33
28	259268	1139	992713	39	266555	1178	733445	32
29	259951	1137	992690	39	267261	1176	732739	31
30	260633	1135	992666	39	267967	1174	732033	30
31	9.261314	1133	9.992643	39	9.268671	1172	10.731329	29
32	261994	1131	992619	39	269375	1170	730625	28
33	262673	1130	992596	39	270077	1169	729923	27
34	263351	1128	992572	39	270779	1167	729221	26
35	264027	1126	992549	39	271479	1165	728521	25
36	264703	1124	992525	39	272178	1164	727822	24
37	265377	1122	992501	39	272876	1162	727124	23
38	266051	1120	992478	40	273573	1160	726427	22
39	266723	1119	992454	40	274269	1158	725731	21
40	267395	1117	992430	40	274964	1157	725036	20
41	9.268065	1115	9.992406	40	9.275658	1155	10.724342	19
42	268734	1113	992382	40	276351	1153	723649	18
43	269402	1111	992359	40	277043	1151	722957	17
44	270069	1110	992335	40	277734	1150	722266	16
45	270735	1108	992311	40	278424	1148	721576	15
46	271400	1106	992287	40	279113	1147	720887	14
47	272064	1105	992263	40	279801	1145	720199	13
48	272726	1103	992239	40	280488	1143	719512	12
49	273388	1101	992214	40	281174	1141	718826	11
50	274049	1099	992190	40	281858	1140	718142	10
51	9.274708	1098	9.992166	40	9.282542	1138	10.717458	9
52	275367	1096	992142	40	283225	1136	716775	8
53	276024	1094	992117	41	283907	1135	716093	7
54	276681	1092	992093	41	284588	1133	715412	6
55	277337	1091	992069	41	285268	1131	714732	5
56	277991	1089	992044	41	285947	1130	714053	4
57	278644	1087	992020	41	286624	1128	713376	3
58	279297	1086	991996	41	287301	1126	712699	2
59	279948	1084	991971	41	287977	1125	712023	1
60	280599	1082	991947	41	288652	1123	711348	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.280599	1082	9.991947	41	9.288652	1123	10.711348	60
1	281248	1081	991922	41	289326	1122	710674	59
2	281897	1079	991897	41	289999	1120	710001	58
3	282544	1077	991873	41	290671	1118	709329	57
4	283190	1076	991848	41	291342	1117	708658	56
5	283836	1074	991823	41	292013	1115	707987	55
6	284480	1072	991799	41	292682	1114	707318	54
7	285124	1071	991774	42	293350	1112	706650	53
8	285766	1069	991749	42	294017	1111	705983	52
9	286408	1067	991724	42	294684	1109	705316	51
10	287048	1066	991699	42	295349	1107	704651	50
11	9.287687	1064	9.991674	42	9.296013	1106	10.703987	49
12	288326	1063	991649	42	296677	1104	703323	48
13	288964	1061	991624	42	297339	1103	702661	47
14	289600	1059	991599	42	298001	1101	701999	46
15	290236	1058	991574	42	298662	1100	701338	45
16	290870	1056	991549	42	299322	1098	700678	44
17	291504	1054	991524	42	299980	1096	700020	43
18	292137	1053	991498	42	300638	1095	699362	42
19	292768	1051	991473	42	301295	1093	698705	41
20	293399	1050	991448	42	301951	1092	698049	40
21	9.294029	1048	9.991422	42	9.302607	1090	10.697393	39
22	294658	1046	991397	42	303261	1089	696739	38
23	295286	1045	991372	43	303914	1087	696086	37
24	295913	1043	991346	43	304567	1086	695433	36
25	296539	1042	991321	43	305218	1084	694782	35
26	297164	1040	991295	43	305869	1083	694131	34
27	297788	1039	991270	43	306519	1081	693481	33
28	298412	1037	991244	43	307168	1080	692832	32
29	299034	1036	991218	43	307815	1078	692185	31
30	299655	1034	991193	43	308463	1077	691537	30
31	9.300276	1032	9.991167	43	9.309109	1075	10.690891	29
32	300895	1031	991141	43	309754	1074	690246	28
33	301514	1029	991115	43	310398	1073	689602	27
34	302132	1028	991090	43	311042	1071	688958	26
35	302748	1026	991064	43	811042	1070	688315	25
36	303364	1025	991038	43	311685	1068	687673	24
37	303979	1023	991012	43	312327	1067	687033	23
38	304593	1022	990986	43	312967	1065	686392	22
39	305207	1020	990960	43	313608	1064	685753	21
40	305819	1019	990934	44	314247	1062	685115	20
41	9.306430	1017	9.990908	44	9.315523	1061	10.684477	19
42	307041	1016	990882	44	316159	1060	683841	18
43	307650	1014	990855	44	316795	1058	683205	17
44	308259	1013	990829	44	317430	1057	682570	16
45	308867	1011	990803	44	318064	1055	681936	15
46	309474	1010	990777	44	318697	1054	681303	14
47	310080	1008	990750	44	319329	1053	680671	13
48	310685	1007	990724	44	319961	1051	680039	12
49	311289	1005	990697	44	320592	1050	679408	11
50	311893	1004	990671	44	321222	1048	678778	10
51	9.312495	1003	9.990644	44	9.321851	1047	10.678149	9
52	313097	1001	990618	44	322479	1045	677521	8
53	313698	1000	990591	44	323106	1044	676894	7
54	314297	998	990565	44	323733	1043	676267	6
55	314897	997	990538	44	324358	1041	675642	5
56	315495	996	990511	45	324983	1040	675017	4
57	316092	994	990485	45	325607	1039	674393	3
58	316689	993	990458	45	326231	1037	673769	2
59	317284	991	990431	45	326853	1036	673147	1
60	317879	990	990404	45	327475	1035	672525	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.317879	990	9.990404	45	9.327474	1035	10 672526	60
1	318473	988	990378	45	328095	1033	671905	59
2	319066	987	990351	45	328715	1032	671285	58
3	319658	986	990324	45	329334	1030	670666	57
4	320249	984	990297	45	329953	1029	670047	56
5	320840	983	990270	45	330570	1028	669430	55
6	321430	982	990243	45	331187	1026	668813	54
7	322019	980	990215	45	331803	1025	668197	53
8	322607	979	990188	45	332418	1024	667582	52
9	323194	977	990161	45	333033	1023	666967	51
10	323780	976	990134	45	333646	1021	666354	50
11	9.324366	975	9.990107	46	9.334259	1020	10.665741	49
12	324950	973	990079	46	334871	1019	665129	48
13	325534	972	990052	46	335482	1017	664518	47
14	326117	970	990025	46	336093	1016	663907	46
15	326700	969	989997	46	336702	1015	663298	45
16	327281	968	989970	46	337311	1013	662689	44
17	327862	966	989942	46	337919	1012	662081	43
18	328442	965	989915	46	338527	1011	661473	42
19	329021	964	989887	46	339133	1010	660867	41
20	329599	962	989860	46	339739	1008	660261	40
21	9.330176	961	9.989832	46	9.340344	1007	10.659656	39
22	330753	960	989804	46	340948	1006	659052	38
23	331329	958	989777	46	341552	1004	658448	37
24	331903	957	989749	47	342155	1003	657845	36
25	332478	956	989721	47	342757	1002	657243	35
26	333051	954	989693	47	343358	1000	656642	34
27	333624	953	989665	47	343958	999	656042	33
28	334195	952	989637	47	344558	998	655442	32
29	334766	950	989609	47	345157	997	654843	31
30	335337	949	989582	47	345755	996	654245	30
31	9.335906	948	9.9895[illegible]3	47	9.346353	994	10.653647	29
32	336475	946	[illegible]	47	346949	993	653051	28
33	337043	945	[illegible]	47	347545	992	652455	27
34	337610	944	[illegible]	47	348141	991	651859	26
35	338176	943	[illegible]	47	348735	990	651265	25
36	338742	941	989413	47	349329	988	650671	24
37	339306	940	989384	47	349922	987	650078	23
38	339871	939	989356	47	350514	986	649486	22
39	340434	937	989328	47	351106	985	648894	21
40	340996	936	989300	47	351697	983	648303	20
41	9.341558	935	9.989271	47	9.352287	982	10.647713	19
42	342119	934	989243	47	352876	981	647124	18
43	342679	932	989214	47	353465	980	646535	17
44	343239	931	989186	47	354053	979	645947	16
45	343797	930	989157	47	354640	977	645360	15
46	344355	929	989128	48	355227	976	644773	14
47	344912	927	989100	48	355813	975	644187	13
48	345469	926	989071	48	356398	974	643602	12
49	346024	925	989042	48	356982	973	643018	11
50	346579	924	989014	48	357566	971	642434	10
51	9.347134	922	9.988985	48	9.358149	970	10.641851	9
52	347687	921	988956	48	358731	969	641269	8
53	348240	920	988927	48	359312	968	640687	7
54	348792	919	988898	48	359893	967	640107	6
55	349343	917	988869	48	360474	966	639526	5
56	349893	916	988840	48	361053	965	638947	4
57	350443	915	988811	49	361632	963	638368	3
58	350992	914	988782	49	362210	962	637790	2
59	351540	913	988753	49	362787	961	637213	1
60	352088	911	988724	49	363364	960	636636	0
	Cosine		Sine		Cotang.		Tang	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.352088	911	9.988724	49	9.363364	960	10.636636	60
1	352635	910	988695	49	363940	959	636060	59
2	353181	909	988666	49	364515	958	635485	58
3	353726	908	988636	49	365090	957	634910	57
4	354271	907	988607	49	365664	955	634336	56
5	354815	905	988578	49	366237	954	633763	55
6	355358	904	988548	49	366810	953	633190	54
7	355901	903	988519	49	367382	952	632618	53
8	356443	902	988489	49	367953	951	632047	52
9	356984	901	988460	49	368524	950	631476	51
10	357524	899	988430	49	369094	949	630906	50
11	9.358064	898	9.988401	49	9.369663	948	10.630337	49
12	358603	897	988371	49	370232	946	629768	48
13	359141	896	988342	49	370799	945	629201	47
14	359678	895	988312	50	371367	944	628633	46
15	360215	893	988282	50	371933	943	628067	45
16	360752	892	988252	50	372499	942	627501	44
17	361287	891	988223	50	373064	941	626936	43
18	361822	890	988193	50	373629	940	626371	42
19	362356	889	988163	50	374193	939	625807	41
20	362889	888	988133	50	374756	938	625244	40
21	9.363422	887	9.988103	50	9.375319	937	10.624681	39
22	363954	885	988073	50	375881	935	624119	38
23	364485	884	988043	50	376442	934	623558	37
24	365016	883	988013	50	377003	933	622997	36
25	365546	882	987983	50	377563	932	622437	35
26	366075	881	987953	50	378122	931	621878	34
27	366604	880	987922	50	378681	930	621319	33
28	367131	879	987892	50	379239	929	620761	32
29	367659	877	987862	50	379797	928	620203	31
30	368185	876	987832	51	380354	927	619646	30
31	9.368711	875	9.987801	51	9.380910	926	10.619090	29
32	369236	874	987771	51	381466	925	618534	28
33	369761	873	987740	51	382020	924	617980	27
34	370285	872	987710	51	382575	923	617425	26
35	370808	871	987679	51	383129	922	616871	25
36	371330	870	987649	51	383682	921	616318	24
37	371852	869	987618	51	384234	920	615766	23
38	372373	867	987588	51	384786	919	615214	22
39	372894	866	987557	51	385337	918	614663	21
40	373414	865	987526	51	385888	917	614112	20
41	9.373933	864	9.987496	51	9.386438	915	10.613562	19
42	374452	863	987465	51	386987	914	613013	18
43	374970	862	987434	51	387536	913	612464	17
44	375487	861	987403	52	388084	912	611916	16
45	376003	860	987372	52	388631	911	611369	15
46	376519	859	987341	52	389178	910	610822	14
47	377035	858	997310	52	389724	909	610276	13
48	377549	857	987279	52	390270	908	609730	12
49	378063	856	987248	52	390815	907	609185	11
50	378577	854	987217	52	391360	906	608640	10
51	9.379089	853	9.987186	52	9.391903	905	10.608097	9
52	379601	852	987155	52	392447	904	607553	8
53	380113	851	987124	52	392989	903	607011	7
54	380624	850	987092	52	393531	902	606469	6
55	381134	849	987061	52	394073	901	605927	5
56	381643	848	987030	52	394614	900	605386	4
57	382152	847	986998	52	395154	899	604846	3
58	382661	846	986967	52	395694	898	604306	2
59	383168	845	986936	52	396233	897	603767	1
60	383675	844	986904	52	396771	896	603229	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.383675	844	9.986904	52	9.396771	896	10.603229	60
1	384182	843	986873	53	397309	896	602691	59
2	384687	842	986841	53	397846	895	602154	58
3	385192	841	986809	53	398383	894	601617	57
4	385697	840	986778	53	398919	893	601081	56
5	386201	839	986746	53	399455	892	600545	55
6	386704	838	986714	53	399990	891	600010	54
7	387207	837	986683	53	400524	890	599476	53
8	387709	836	986651	53	401058	889	598942	52
9	388210	835	986619	53	401591	888	598409	51
10	388711	834	986587	53	402124	887	597876	50
11	9.389211	833	9.986555	53	9.402656	886	10.597344	49
12	389711	832	986523	53	403187	885	596813	48
13	390210	831	986491	53	403718	884	596282	47
14	390708	830	986459	53	404249	883	595751	46
15	391206	828	986427	53	404778	882	595222	45
16	391703	827	986395	53	405308	881	594692	44
17	392199	826	986363	54	405836	880	594164	43
18	392695	825	986331	54	406364	879	593636	42
19	393191	824	986299	54	406892	878	593108	41
20	393685	823	986266	54	407419	877	592581	40
21	9.394179	822	9.986234	54	9.407945	876	10.592055	39
22	394673	821	986202	54	408471	875	591529	38
23	395166	820	986169	54	408997	874	591003	37
24	395658	819	986137	54	409521	874	590479	36
25	396150	818	986104	54	410045	873	589955	35
26	396641	817	986072	54	410569	872	589431	34
27	397132	817	986039	54	411092	871	588908	33
28	397621	816	986007	54	411615	870	588385	32
29	398111	815	985974	54	412137	869	587863	31
30	398600	814	985942	54	412658	868	587342	30
31	9.399088	813	9.985909	55	9.413179	867	10.586821	29
32	399575	812	985876	55	413699	866	586301	28
33	400062	811	985843	55	414219	865	585781	27
34	400549	810	985811	55	414738	864	585262	26
35	401035	809	985778	55	415257	864	584743	25
36	401520	808	985745	55	415775	863	584225	24
37	402005	807	985712	55	416293	862	583707	23
38	402489	806	985679	55	416810	861	583190	22
39	402972	805	985646	55	417326	860	582674	21
40	403455	804	985613	55	417842	859	582158	20
41	9.403938	803	9.985580	55	9.418358	858	10.581642	19
42	404420	802	985547	55	418873	857	581127	18
43	404901	801	985514	55	419387	856	580613	17
44	405382	800	985480	55	419901	855	580099	16
45	405862	799	985447	55	420415	855	579585	15
46	406341	798	985414	56	420927	854	579073	14
47	406820	797	985380	56	421440	853	578560	13
48	407299	796	985347	56	421952	852	578048	12
49	407777	795	985314	56	422463	851	577537	11
50	408254	794	985280	56	422974	850	577026	10
51	9.408731	794	9.985247	56	9.423484	849	10.576516	9
52	409207	793	985213	56	423993	848	576007	8
53	409682	792	985180	56	424503	848	575497	7
54	410157	791	985146	56	425011	847	574989	6
55	410632	790	985113	56	425519	846	574481	5
56	411106	789	985079	56	426027	845	573973	4
57	411579	788	985045	56	426534	844	573466	3
58	412052	787	985011	56	427041	843	572959	2
59	412524	786	984978	56	427547	843	572453	1
60	412996	785	984944	56	428052	842	571948	0
	Cosine		Sine		Cotang.		Tang	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.412996	785	9.984944	57	9.428052	842	10.571948	60
1	413467	784	984910	57	428557	841	571443	59
2	413938	783	984876	57	429062	840	570938	58
3	414408	783	984842	57	429566	839	570434	57
4	414878	782	984808	57	430070	838	569930	56
5	415347	781	984774	57	430573	838	569427	55
6	415815	780	984740	57	431075	837	568925	54
7	416283	779	984706	57	431577	836	568423	53
8	416751	778	984672	57	432079	835	567921	52
9	417217	777	984637	57	432580	834	567420	51
10	417684	776	984603	57	433080	833	566920	50
11	9.418150	775	9.984569	57	9.433580	832	10.566420	49
12	418615	774	984535	57	434080	832	565920	48
13	419079	773	984500	57	434579	831	565421	47
14	419544	773	984466	57	435078	830	564922	46
15	420007	772	984432	58	435576	829	564424	45
16	420470	771	984397	58	436073	828	563927	44
17	420933	770	984363	58	436570	828	563430	43
18	421395	769	984328	58	437067	827	562933	42
19	421857	768	984294	58	437563	826	562437	41
20	422318	767	984259	58	438059	825	561941	40
21	9.422778	767	9.984224	58	9.438554	824	10.561446	39
22	423238	766	984190	58	439048	823	560952	38
23	423697	765	984155	58	439543	823	560457	37
24	424156	764	984120	58	440036	822	559964	36
25	424615	763	984085	58	440529	821	559471	35
26	425073	762	984050	58	441022	820	558978	34
27	425530	761	984015	58	441514	819	558486	33
28	425987	760	983981	58	442006	819	557994	32
29	426443	760	983946	58	442497	818	557503	31
30	426899	759	983911	58	442988	817	557012	30
31	9.427354	758	9.983875	58	9.443479	816	10.556521	29
32	427809	757	983840	59	443968	816	556032	28
33	428263	756	983805	59	444458	815	555542	27
34	428717	755	983770	59	444947	814	555053	26
35	429170	754	983735	59	445435	813	554565	25
36	429623	753	983700	59	445923	812	554077	24
37	430075	752	983664	59	446411	812	553589	23
38	430527	752	983629	59	446898	811	553102	22
39	430978	751	983594	59	447384	810	552616	21
40	431429	750	983558	59	447870	809	552130	20
41	9.431879	749	9.983523	59	9.448356	809	10.551644	19
42	432329	749	983487	59	448841	808	551159	18
43	432778	748	983452	59	449326	807	550674	17
44	433226	747	983416	59	449810	806	550190	16
45	433675	746	983381	59	450294	806	549706	15
46	434122	745	983345	59	450777	805	549223	14
47	434569	744	983309	59	451260	804	548740	13
48	435016	744	983273	60	451743	803	548257	12
49	435462	743	983238	60	452225	802	547775	11
50	435908	742	983202	60	452706	802	547294	10
51	9.436353	741	9.983166	60	9.453187	801	10.546813	9
52	436798	740	983130	60	453668	800	546332	8
53	437242	740	983094	60	454148	799	545852	7
54	437686	739	983058	60	454628	799	545372	6
55	438129	738	983022	60	455107	798	544893	5
56	438572	737	982986	60	455586	797	544414	4
57	439014	736	982950	60	456064	796	543936	3
58	439456	736	982914	60	456542	796	543458	2
59	439897	735	982878	60	457019	795	542981	1
60	440338	734	982842	60	457496	794	542504	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.440338	734	9.982842	60	9.457496	794	10.542504	60
1	440778	733	982805	60	457973	793	542027	59
2	441218	732	982769	61	458449	793	541551	58
3	441658	731	982733	61	458925	792	541075	57
4	442096	731	982696	61	459400	791	540600	56
5	442535	730	982660	61	459875	790	540125	55
6	442973	729	982624	61	460349	790	539651	54
7	443410	728	982587	61	460823	789	539177	53
8	443847	727	982551	61	461297	788	538703	52
9	444284	727	982514	61	461770	788	538230	51
10	444720	726	982477	61	462242	787	537758	50
11	9.445155	725	9.982441	61	9.462714	786	10.537286	49
12	445590	724	982404	61	463186	785	536814	48
13	446025	723	982367	61	463658	785	536342	47
14	446459	723	982331	61	464129	784	535871	46
15	446893	722	982294	61	464599	783	535401	45
16	447326	721	982257	61	465069	783	534931	44
17	447759	720	982220	62	465539	782	534461	43
18	448191	720	982183	62	466008	781	533992	42
19	448623	719	982146	62	466476	780	533524	41
20	449054	718	982109	62	466945	780	533055	40
21	9.449485	717	9.982072	62	9.467413	779	10.532587	39
22	449915	716	982035	62	467880	778	532120	38
23	450345	716	981998	62	468347	778	531653	37
24	450775	715	981961	62	468814	777	531186	36
25	451204	714	981924	62	469280	776	530720	35
26	451632	713	981886	62	469746	775	530254	34
27	452060	713	981849	62	470211	775	529789	33
28	452488	712	981812	62	470676	774	529324	32
29	452915	711	981774	62	471141	773	528859	31
30	453342	710	981737	62	471605	773	528395	30
31	9.453768	710	9.981699	63	9.472068	772	10.527932	29
32	454194	709	981662	63	472532	771	527468	28
33	454619	708	981625	63	472995	771	527005	27
34	455044	707	981587	63	473457	770	526543	26
35	455469	707	981549	63	473919	769	526081	25
36	455893	706	981512	63	474381	769	525619	24
37	456316	705	981474	63	474842	768	525158	23
38	456739	704	981436	63	475303	767	524697	22
39	457162	704	981399	63	475763	767	524237	21
40	457584	703	981361	63	476223	766	523777	20
41	9.458006	702	9.981323	63	9.476683	765	10.523317	19
42	458427	701	981285	63	477142	765	522858	18
43	458848	701	981247	63	477601	764	522399	17
44	459268	700	981209	63	478059	763	521941	16
45	459688	699	981171	63	478517	763	521483	15
46	460108	698	981133	64	478975	762	521025	14
47	460527	698	981095	64	479432	761	520568	13
48	460946	697	981057	64	479889	761	520111	12
49	461364	696	981019	64	480345	760	519655	11
50	461782	695	980981	64	480801	759	519199	10
51	9.462199	695	9.980942	64	9.481257	759	10.518743	9
52	462616	694	980904	64	481712	758	518288	8
53	463032	693	980866	64	482167	757	517833	7
54	463448	693	980827	64	482621	757	517379	6
55	463864	692	980789	64	483075	756	516925	5
56	464279	691	980750	64	483529	755	516471	4
57	464694	690	980712	64	483982	755	516018	3
58	465108	690	980673	64	484435	754	515565	2
59	465522	689	980635	64	484887	753	515113	1
60	465935	688	980596	64	485339	753	514661	0
	Cosine		Sine		Cotang.		Tang.	M.

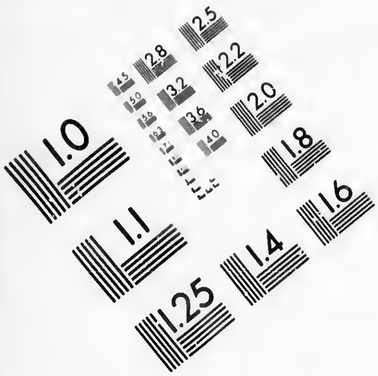
73 Degrees.

M.	Sine	D	Cosine	D.	Tang.	D.	Cotang.	
0	9.465935	688	9.980596	64	9.485339	755	10.514661	60
1	466348	688	980558	64	485791	752	514209	59
2	466761	687	980519	65	486242	751	513758	58
3	467173	686	980480	65	486693	751	513307	57
4	467585	685	980442	65	487143	750	512857	56
5	467996	685	980403	65	487593	749	512407	55
6	468407	684	980364	65	488043	749	511957	54
7	468817	683	980325	65	488492	748	511508	53
8	469227	683	980286	65	488941	747	511059	52
9	469637	682	980247	65	489390	747	510610	51
10	470046	681	980208	65	489838	746	510162	50
11	9.470455	680	9.980169	65	9.490286	746	10.509714	49
12	470863	680	980130	65	490733	745	509267	48
13	471271	679	980091	65	491180	744	508820	47
14	471679	678	980052	65	491627	744	508373	46
15	472086	678	980012	65	492073	743	507927	45
16	472492	677	979973	65	492519	743	507481	44
17	472898	676	979934	66	492965	742	507035	43
18	473304	676	979895	66	493410	741	506590	42
19	473710	675	979855	66	493854	740	506146	41
20	474115	674	979816	66	494299	740	505701	40
21	9.474519	674	9.979776	66	9.494743	740	10.505257	39
22	474923	673	979737	66	495186	739	504814	38
23	475327	672	979697	66	495630	738	504370	37
24	475730	672	979658	66	496073	737	503927	36
25	476133	671	979618	66	496515	737	503485	35
26	476536	670	979579	66	496957	736	503043	34
27	476938	669	979539	66	497399	736	502601	33
28	477340	669	979499	66	497841	735	502159	32
29	477741	668	979459	66	498282	734	501718	31
30	478142	667	979420	66	498722	734	501278	30
31	9.478542	667	9.979380	66	9.499163	733	10.500837	29
32	478942	666	979340	66	499603	733	500397	28
33	479342	665	979300	67	500042	732	499958	27
34	479741	665	979260	67	500481	731	499519	26
35	480140	664	979220	67	500920	731	499080	25
36	480539	663	979180	67	501359	730	498641	24
37	480937	663	979140	67	501797	730	498203	23
38	481334	662	979100	67	502235	729	497765	22
39	481731	661	979059	67	502672	728	497328	21
40	482128	661	979019	67	503109	728	496891	20
41	9.482525	660	9.978979	67	9.503546	727	10.496454	19
42	482921	659	978939	67	503982	727	496018	18
43	483316	659	978898	67	504418	726	495582	17
44	483712	658	978858	67	504854	725	495146	16
45	484107	657	978817	67	505289	725	494711	15
46	484501	657	978777	67	505724	724	494276	14
47	484895	656	978736	67	506159	724	493841	13
48	485289	655	978696	68	506593	723	493407	12
49	485682	655	978655	68	507027	722	492973	11
50	486075	654	978615	68	507460	722	492540	10
51	9.486467	653	9.978574	68	9.507893	721	10.492107	9
52	486860	653	978533	68	508326	721	491674	8
53	487251	652	978493	68	508759	720	491241	7
54	487643	651	978452	68	509191	719	490809	6
55	488034	651	978411	68	509622	719	490378	5
56	488424	650	978370	68	510054	718	489946	4
57	488814	650	978329	68	510485	718	489515	3
58	489204	649	978288	68	510916	717	489084	2
59	489593	648	978247	68	511346	716	488654	1
60	489982	648	978206	68	511776	716	488224	0
	Cosine		Sine		Cotang.		Tang.	M.

72 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.489982	648	9.978206	68	9.511776	716	10.488224	60
1	490371	648	978165	68	512206	716	487794	59
2	490759	647	978124	68	512635	715	487365	58
3	491147	646	978083	69	513064	714	486936	57
4	491535	646	978042	69	513493	714	486507	56
5	491922	645	978001	69	513921	713	486079	55
6	492308	644	977959	69	514349	713	485651	54
7	492695	644	977918	69	514777	712	485223	53
8	493081	643	977877	69	515204	712	484796	52
9	493466	642	977835	69	515631	711	484369	51
10	493851	642	977794	69	516057	710	483943	50
11	9.494236	641	9.977752	69	9.516484	710	10.483516	49
12	494621	641	977711	69	516910	709	483090	48
13	495005	640	977669	69	517335	709	482665	47
14	495388	639	977628	69	517761	708	482239	46
15	495772	639	977586	69	518185	708	481815	45
16	496154	638	977544	70	518610	707	481390	44
17	496537	637	977503	70	519034	706	480966	43
18	496919	637	977461	70	519458	706	480542	42
19	497301	636	977419	70	519882	705	480118	41
20	497682	636	977377	70	520305	705	479695	40
21	9.498064	635	9.977335	70	9.520728	704	10.479272	39
22	498444	634	977293	70	521151	703	478849	38
23	498825	634	977251	70	521573	703	478427	37
24	499204	633	977209	70	521995	703	478005	36
25	499584	632	977167	70	522417	702	477583	35
26	499963	632	977125	70	522838	702	477162	34
27	500342	631	977083	70	523259	701	476741	33
28	500721	631	977041	70	523680	701	476320	32
29	501099	630	976999	70	524100	700	475900	31
30	501476	629	976957	70	524520	699	475480	30
31	9.501854	629	9.976914	70	9.524939	699	10.475061	29
32	502231	628	976872	71	525359	698	474641	28
33	502607	628	976830	71	525778	698	474222	27
34	502984	627	976787	71	526197	697	473803	26
35	503360	626	976745	71	526615	697	473385	25
36	503735	626	976702	71	527033	696	472967	24
37	504110	625	976660	71	527451	696	472549	23
38	504485	625	976617	71	527868	695	472132	22
39	504860	624	976574	71	528285	695	471715	21
40	505234	623	976532	71	528702	694	471298	20
41	9.505608	623	9.976489	71	9.529119	693	0.470881	19
42	505981	622	976446	71	529535	693	470465	18
43	506354	622	976404	71	529950	693	470050	17
44	506727	621	976361	71	530366	692	469634	16
45	507099	620	976318	71	530781	691	469219	15
46	507471	620	976275	71	531196	691	468804	14
47	507843	619	976232	72	531611	690	468389	13
48	508214	619	976189	72	532025	690	467975	12
49	508585	618	976146	72	532439	689	467561	11
50	508956	618	976103	72	532853	689	467147	10
51	9.509326	617	9.976060	72	9.533266	688	10.466734	9
52	509696	616	976017	72	533679	688	466321	8
53	510065	616	975974	72	534092	687	465908	7
54	510434	615	975930	72	534504	687	465496	6
55	510803	615	975887	72	534916	686	465084	5
56	511172	614	975844	72	535328	686	464672	4
57	511540	613	975800	72	535739	685	464261	3
58	511907	613	975757	72	536150	685	463850	2
59	512275	612	975714	72	536561	684	463439	1
60	512642	612	975670	72	536972	684	463028	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.512642	612	9.975670	73	9.536972	684	10.463028	60
1	513009	611	975627	73	537382	683	462618	59
2	513375	611	975583	73	537792	683	462208	58
3	513741	610	975539	73	538202	682	461798	57
4	514107	609	975496	73	538611	682	461389	56
5	514472	609	975452	73	539020	[illegible]	460980	55
6	514837	608	975408	73	539429	[illegible]	460571	54
7	515202	608	975365	73	539837	680	460163	53
8	515566	607	975321	73	540245	680	459755	52
9	515930	607	975277	73	540653	679	459347	51
10	516294	606	975233	73	541061	679	458939	50
11	9.516657	605	9.975189	73	9.541468	678	10.458532	49
12	517020	605	975145	73	541875	678	458125	48
13	517382	604	975101	73	542281	677	457719	47
14	517745	604	975057	73	542688	677	457312	46
15	518107	603	975013	73	543094	676	456906	45
16	518468	603	974969	74	543499	676	456501	44
17	518829	602	974925	74	543905	675	456095	43
18	519190	601	974880	74	544310	675	455690	42
19	519551	601	974836	74	544715	674	455285	41
20	519911	600	974792	74	545119	674	454881	40
21	9.520271	600	9.974748	74	9.545524	673	10.454476	39
22	520631	599	974703	74	545928	673	454072	38
23	520990	599	974659	74	546331	672	453669	37
24	521349	598	974614	74	546735	672	453265	36
25	521707	598	974570	74	547138	671	452862	35
26	522066	597	974525	74	547540	671	452460	34
27	522424	596	974481	74	547943	670	452057	33
28	522781	596	974436	74	548345	670	451655	32
29	523138	595	974391	74	548747	669	451253	31
30	523495	595	974347	75	549149	669	450851	30
31	9.523852	594	9.974302	75	9.549550	668	10.450450	29
32	524208	594	974257	75	549951	668	450049	28
33	524564	593	974212	75	550352	667	449648	27
34	524920	593	974167	75	550752	667	449248	26
35	525275	592	974122	75	551152	666	448848	25
36	525630	591	974077	75	551552	666	448448	24
37	525984	591	974032	75	551952	665	448048	23
38	526339	590	973987	75	552351	665	447649	22
39	526693	590	973942	75	552750	665	447250	21
40	527046	589	973897	75	553149	664	446851	20
41	9.527400	589	9.973852	75	9.553548	664	10.446452	19
42	527753	588	973807	75	553946	663	446054	18
43	528105	588	973761	75	554344	663	445656	17
44	528458	587	973716	76	554741	662	445259	16
45	528810	587	973671	76	555139	662	444861	15
46	529161	586	973625	76	555536	661	444464	14
47	529513	586	973580	76	555933	661	444067	13
48	529864	585	973535	76	556329	660	443671	12
49	530215	585	973489	76	556725	660	443275	11
50	530565	584	973444	76	557121	659	442879	10
51	9.530915	584	9.973398	76	9.557517	659	10.442483	9
52	531265	583	973352	76	557913	659	442087	8
53	531614	582	973307	76	558308	658	441692	7
54	531963	582	973261	76	558702	658	441298	6
55	532312	581	973215	76	559097	657	440903	5
56	532661	581	973169	76	559491	657	440509	4
57	533009	580	973124	76	559885	656	440115	3
58	533357	580	973078	76	560279	656	439721	2
59	533704	579	973032	77	560673	655	439327	1
60	534052	578	972986	77	561066	655	438934	0
	Cosine		Sine		Cotang.		Tang.	M.



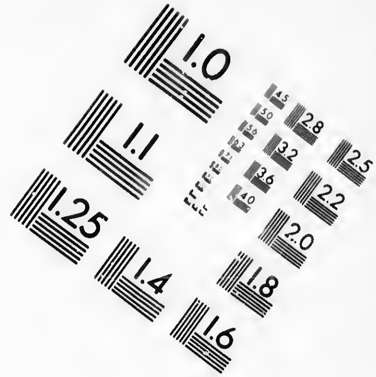
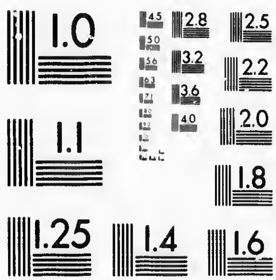
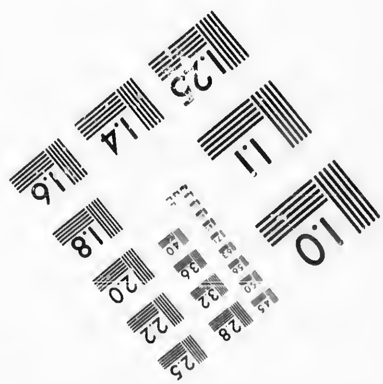


IMAGE EVALUATION
TEST TARGET (MT-3)



6″




2.8
2.5
3.2
2.2
2.0


1.0

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.534052	578	9.972986	77	9.561066	655	10.438934	60
1	534399	577	972940	77	561459	654	438541	59
2	534745	577	972894	77	561851	654	438149	58
3	535092	577	972848	77	562244	653	437756	57
4	535438	576	972802	77	562636	653	437364	56
5	535783	576	972755	77	563028	653	436972	55
6	536129	575	972709	77	563419	652	436581	54
7	536474	574	972663	77	563811	652	436189	53
8	536818	574	972617	77	564202	651	435798	52
9	537163	573	972570	77	564592	651	435408	51
10	537507	573	972524	77	564983	650	435017	50
11	9.537851	572	9.972478	77	9.565373	650	10.434627	49
12	538194	572	972431	78	565763	649	434237	48
13	538538	571	972385	78	566153	649	433847	47
14	538880	571	972338	78	566542	649	433458	46
15	539223	570	972291	78	566932	648	433068	45
16	539565	570	972245	78	567320	648	432680	44
17	539907	569	972198	78	567709	647	432291	43
18	540249	569	972151	78	568098	647	431902	42
19	540590	568	972105	79	568486	646	431514	41
20	540931	568	972058	78	568873	646	431127	40
21	9.541272	567	9.972011	78	9.569261	645	10.430739	39
22	541613	567	971964	78	569648	645	430352	38
23	541953	566	971917	78	570035	645	429965	37
24	542293	566	971870	78	570422	644	429578	36
25	542632	565	971823	78	570809	644	429191	35
26	542971	565	971776	78	571195	643	428805	34
27	543310	564	971729	79	571581	643	428419	33
28	543649	564	971682	79	571967	642	428033	32
29	543987	563	971635	79	572352	642	427648	31
30	544325	563	971588	79	572738	642	427262	30
31	9.544663	562	9.971540	79	9.573123	641	10.426877	29
32	545000	562	971493	79	573507	641	426493	28
33	545338	561	971446	79	573892	640	426108	27
34	545674	561	971398	79	574276	640	425724	26
35	546011	560	971351	79	574660	639	425340	25
36	546347	560	971303	79	575044	639	424956	24
37	546683	559	971256	79	575427	639	424573	23
38	547019	559	971208	79	575810	638	424190	22
39	547354	558	971161	79	576193	638	423807	21
40	547689	558	971113	79	576576	637	423424	20
41	9.548024	557	9.971066	80	9.576958	637	10.423041	19
42	548359	557	971018	80	577341	636	422659	18
43	548693	556	970970	80	577723	636	422277	17
44	549027	556	970922	80	578104	636	421896	16
45	549360	555	970874	80	578486	635	421514	15
46	549693	555	970827	80	578867	635	421133	14
47	550026	554	970779	80	579248	634	420752	13
48	550359	554	970731	80	579629	634	420371	12
49	550692	553	970683	80	580009	634	419991	11
50	551024	553	970635	80	580389	633	419611	10
51	9.551356	552	9.970586	80	9.580769	633	10.419231	9
52	551687	552	970538	80	581149	632	418851	8
53	552018	552	970490	80	581528	632	418472	7
54	552349	551	970442	80	581907	632	418093	6
55	552680	551	970394	80	582286	631	417714	5
56	553010	550	970345	81	582665	631	417335	4
57	553341	550	970297	81	583043	630	416957	3
58	553670	549	970249	81	583422	630	416578	2
59	554000	549	970200	81	583800	629	416200	1
60	554329	548	970152	81	584177	629	415823	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.554329	548	9.970152	81	9.584177	629	10.415823	60
1	554658	548	970103	81	584555	629	415445	59
2	554987	547	970055	81	584932	628	415068	58
3	555315	547	970006	81	585309	628	414691	57
4	555643	546	969957	81	585686	627	414314	56
5	555971	546	969909	81	586062	627	413938	55
6	556299	545	969860	81	586439	627	413561	54
7	556626	545	969811	81	586815	626	413185	53
8	556953	544	969762	81	587190	626	412810	52
9	557280	544	969714	81	587566	625	412434	51
10	557606	543	969665	81	587941	625	412059	50
11	9.557932	543	9.969616	82	9.588316	625	10.411684	49
12	558258	543	969567	82	588691	624	411309	48
13	558583	542	969518	82	589066	624	410934	47
14	558909	542	969469	82	589440	623	410560	46
15	559234	541	969420	82	589814	623	410186	45
16	559558	541	969370	82	590188	623	409812	44
17	559883	540	969321	82	590562	622	409438	43
18	560207	540	969272	82	590935	622	409065	42
19	560531	539	969223	82	591308	622	408692	41
20	560855	539	969173	82	591681	621	408319	40
21	9.561178	538	9.969124	82	9.592054	621	10.407946	39
22	561501	538	969075	82	592426	620	407574	38
23	561824	537	969025	82	592798	620	407202	37
24	562146	537	968976	82	593170	619	406829	36
25	562468	536	968926	83	593542	619	406458	35
26	562790	536	968877	83	593914	618	406086	34
27	563112	536	968827	83	594285	618	405715	33
28	563433	535	968777	83	594656	618	405344	32
29	563755	535	968728	83	595027	617	404973	31
30	564075	534	968678	83	595398	617	404602	30
31	9.564396	534	9.968628	83	9.595768	617	10.404232	29
32	564716	533	968578	83	596138	616	403862	28
33	565036	533	968528	83	596508	616	403492	27
34	565356	532	968479	83	596878	616	403122	26
35	565676	532	968429	83	597247	615	402753	25
36	565995	531	968379	83	597616	615	402384	24
37	566314	531	968329	83	597985	615	402015	23
38	566632	531	968278	83	598354	614	401646	22
39	566951	530	968228	84	598722	614	401278	21
40	567269	530	968178	84	599091	613	400909	20
41	9.567587	529	9.968128	84	9.599459	613	10.400541	19
42	567904	529	968078	84	599827	613	400173	18
43	568222	528	968027	84	600194	612	399806	17
44	568539	528	967977	84	600562	612	399438	16
45	568856	528	967927	84	600929	611	399071	15
46	569172	527	967876	84	601296	611	398704	14
47	569488	527	967826	84	601662	611	398338	13
48	569804	526	967775	84	602029	610	397971	12
49	570120	526	967725	84	602395	610	397605	11
50	570435	525	967674	84	602761	610	397239	10
51	9.570751	525	9.967624	84	9.603127	609	10.396873	9
52	571066	524	967573	84	603493	609	396507	8
53	571380	524	967522	85	603858	609	396142	7
54	571695	523	967471	85	604223	608	395777	6
55	572009	523	967421	85	604588	608	395412	5
56	572323	523	967370	85	604953	607	395047	4
57	572636	522	967319	85	605317	607	394683	3
58	572950	522	967268	85	605682	607	394318	2
59	573263	521	967217	85	606046	606	393954	1
60	573575	521	967166	85	606410	606	393590	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.573575	521	9.967166	85	9.606410	606	10.393590	60
1	573888	520	967115	85	606773	606	393227	59
2	574200	520	967064	85	607137	605	392863	58
3	574512	519	967013	85	607500	605	392500	57
4	574824	519	966961	85	607863	604	392137	56
5	575136	519	966910	85	608225	604	391775	55
6	575447	518	966859	85	608588	604	391412	54
7	575758	518	966808	85	608950	603	391050	53
8	576069	517	966756	86	609312	603	390688	52
9	576379	517	966705	86	609674	603	390326	51
10	576689	516	966653	86	610036	602	389964	50
11	9.576999	516	9.966602	86	9.610397	602	10.389603	49
12	577309	516	966550	86	610759	602	389241	48
13	577618	515	966499	86	611120	601	388880	47
14	577927	515	966447	86	611480	601	388520	46
15	578236	514	966395	86	611841	601	388159	45
16	578545	514	966344	86	612201	600	387799	44
17	578853	513	966292	86	612561	600	387439	43
18	579162	513	966240	86	612921	600	387079	42
19	579470	513	966188	86	613281	599	386719	41
20	579777	512	966136	86	613641	599	386359	40
21	9.580085	512	9 966085	87	9.614000	598	10.386000	39
22	580392	511	966033	87	614359	598	385641	38
23	580699	511	965981	87	614718	598	385282	37
24	581005	511	965928	87	615077	597	384923	36
25	581312	510	965876	87	615435	597	384565	35
26	581618	510	965824	87	615793	597	384207	34
27	581924	509	965772	87	616151	596	383849	33
28	582229	509	965720	87	616509	596	383491	32
29	582535	509	965668	87	616867	596	383133	31
30	582840	508	965615	87	617224	595	382776	30
31	9.583145	508	9.965563	87	9 617582	595	10.382418	29
32	583449	507	965511	87	617939	595	382061	28
33	583754	507	965458	87	618295	594	381705	27
34	584058	506	965406	87	618652	594	381348	26
35	584361	506	965353	88	619008	594	380992	25
36	584665	506	965301	88	619364	593	380636	24
37	584968	505	965248	88	619721	593	380279	23
38	585272	505	965195	88	620076	593	379924	22
39	585574	504	965143	88	620432	592	379568	21
40	585877	504	965090	88	620787	592	379213	20
41	9.586179	503	9.965037	88	9.621142	592	10.378858	19
42	586482	503	964984	88	621497	591	378503	18
43	586783	503	964931	88	621852	591	378148	17
44	587085	502	964879	88	622207	590	377793	16
45	587386	502	964826	88	622561	590	377439	15
46	587688	501	964773	88	622915	590	377085	14
47	587989	501	964719	88	623269	589	376731	13
48	588289	501	964666	89	623623	589	376377	12
49	588590	500	964613	89	623976	589	376024	11
50	588890	500	964560	89	624330	588	375670	10
51	9.589190	499	9.964507	89	9.624683	588	10.375317	9
52	589489	499	964454	89	625036	588	374964	8
53	589789	499	964400	89	625388	587	374612	7
54	590088	498	964347	89	625741	587	374259	6
55	590387	498	964294	89	626093	587	373907	5
56	590686	497	964240	89	626445	586	373555	4
57	590984	497	964187	89	626797	586	373203	3
58	591282	497	964133	89	627149	586	372851	2
59	591580	496	964080	89	627501	585	372499	1
60	591878	496	964026	89	627852	585	372148	0
	Cosine		Sine		Cotang.		Tang.	M.

67 Degrees

M.	Sine	D.	Cosine	D.	Tang	D.	Cotang.	
0	9.591878	496	9.964026	89	9.627852	585	10.372148	60
1	592176	495	963972	89	628203	585	371797	59
2	592473	495	963919	89	628554	585	371446	58
3	592770	495	963865	90	628905	584	371095	57
4	593067	494	963811	90	629255	584	370745	56
5	593363	494	963757	90	629606	583	370394	55
6	593659	493	963704	90	629956	583	370044	54
7	593955	493	963650	90	630306	583	369694	53
8	594251	493	963596	90	630656	583	369344	52
9	594547	492	963542	90	631005	582	368995	51
10	594842	492	963488	90	631355	582	368645	50
11	9.595137	491	9.963434	90	9.631704	582	10.368296	49
12	595432	491	963379	90	632053	581	367947	48
13	595727	491	963325	90	632401	581	367599	47
14	596021	490	963271	90	632750	581	367250	46
15	596315	490	963217	90	633098	580	366902	45
16	596609	489	963163	90	633447	580	366553	44
17	596903	489	963108	91	633795	580	366205	43
18	597196	489	963054	91	634143	579	365857	42
19	597490	488	962999	91	634490	579	365510	41
20	597783	488	962945	91	634838	579	365162	40
21	9.598075	487	9.962890	91	9.635185	578	10.364815	39
22	598368	487	962836	91	635532	578	364468	38
23	598660	487	962781	91	635879	578	364121	37
24	598952	486	962727	91	636226	577	363774	36
25	599244	486	962672	91	636572	577	363428	35
26	599536	485	962617	91	636919	577	363081	34
27	599827	485	962562	91	637265	577	362735	33
28	600118	485	962508	91	637611	576	362389	32
29	600409	484	962453	91	637956	576	362044	31
30	600700	484	962398	92	638302	576	361698	30
31	9.600990	484	9.962343	92	9.638647	575	10.361353	29
32	601280	483	962288	92	638992	575	361008	28
33	601570	483	962233	92	639337	575	360663	27
34	601860	482	962178	92	639682	574	360318	26
35	602150	482	962123	92	640027	574	359973	25
36	602439	482	962067	92	640371	574	359629	24
37	602728	481	962012	92	640716	573	359284	23
38	603017	481	961957	92	641060	573	358940	22
39	603305	481	961902	92	641404	573	358596	21
40	603594	480	961846	92	641747	572	358253	20
41	9.603882	480	9.961791	92	9.642091	572	10.357909	19
42	604170	479	961735	92	642434	572	357566	18
43	604457	479	961680	92	642777	572	357223	17
44	604745	479	961624	93	643120	571	356880	16
45	605032	478	961569	93	643463	571	356537	15
46	605319	478	961513	93	643806	571	356194	14
47	605606	478	961458	93	644148	570	355852	13
48	605892	477	961402	93	644490	570	355510	12
49	606179	477	961346	93	644832	570	355168	11
50	606465	476	961290	93	645174	569	354826	10
51	9 606751	476	9.961235	93	9.645516	569	10.354484	9
52	607036	476	961179	93	645857	569	354143	8
53	607322	475	961123	93	646199	569	353801	7
54	607607	475	961067	93	646540	568	353460	6
55	607892	474	961011	93	646881	568	353119	5
56	608177	474	960955	93	647222	568	352778	4
57	608461	474	960899	93	647562	567	352438	3
58	608745	473	960843	94	647903	567	352097	2
59	609029	473	960786	94	648243	567	351757	1
60	609313	473	960730	94	648583	566	351417	0
	Cosine		Sine		Cotang.		Tang.	M

66 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.609313	473	9.960730	94	9.648583	566	10.351417	60
1	609597	472	960674	94	648923	566	351077	59
2	609880	472	960618	94	649263	566	350737	58
3	610164	472	960561	94	649602	566	350398	57
4	610447	471	960505	94	649942	565	350058	56
5	610729	471	960448	94	650281	565	349719	55
6	611012	470	960392	94	650620	565	349380	54
7	611294	470	960335	94	650959	564	349041	53
8	611576	470	960279	94	651297	564	348703	52
9	611858	469	960222	94	651636	564	348364	51
10	612140	469	960165	94	651974	563	348026	50
11	9.612421	469	9.960109	95	9.652312	563	10.347688	49
12	612702	468	960052	95	652650	563	347350	48
13	612983	468	959995	95	652988	563	347012	47
14	613264	467	959938	95	653326	562	346674	46
15	613545	467	959882	95	653663	562	346337	45
16	613825	467	959825	95	654000	562	346000	44
17	614105	466	959768	95	654337	561	345663	43
18	614385	466	959711	95	654674	561	345326	42
19	614665	466	959654	95	655011	561	344989	41
20	614944	465	959596	95	655348	561	344652	40
21	9.615223	465	9.959539	95	9.655684	560	10.344316	39
22	615502	465	959482	95	656020	560	343980	38
23	615781	464	959425	95	656356	560	343644	37
24	616060	464	959368	95	656692	559	343308	36
25	616338	464	959310	96	657028	559	342972	35
26	616616	463	959253	96	657364	559	342636	34
27	616894	463	959195	96	657699	559	342301	33
28	617172	462	959138	96	658034	558	341966	32
29	617450	462	959081	96	658369	558	341631	31
30	617727	462	959023	96	658704	558	341296	30
31	9.618004	461	9.958965	96	9.659039	558	10.340961	29
32	618281	461	958908	96	659373	557	340627	28
33	618558	461	958850	96	659708	557	340292	27
34	618834	460	958792	96	660042	557	339958	26
35	619110	460	958734	96	660376	557	339624	25
36	619386	460	958677	96	660710	556	339290	24
37	619662	459	958619	96	661043	556	338957	23
38	619938	459	958561	96	661377	556	338623	22
39	620213	459	958503	97	661710	555	338290	21
40	620488	458	958445	97	662043	555	337957	20
41	9.620763	458	9.958387	97	9 662376	555	10.337624	19
42	621038	457	958329	97	662709	554	337291	18
43	621313	457	958271	97	663042	554	336958	17
44	621587	457	958213	97	663375	554	336625	16
45	621861	456	958154	97	663707	554	336293	15
46	622135	456	958096	97	664039	553	335961	14
47	622409	456	958038	97	664371	553	335629	13
48	622682	455	957979	97	664703	553	335297	12
49	622956	455	957921	97	665035	553	334965	11
50	623229	455	957863	97	665366	552	334634	10
51	9.623502	454	9.957804	97	9.665697	552	10.334303	9
52	623774	454	957746	98	666029	552	333971	8
53	624047	454	957687	98	666360	551	333640	7
54	624319	453	957628	98	666691	551	333309	6
55	624591	453	957570	98	667021	551	332979	5
56	624863	453	957511	98	667352	551	332648	4
57	625135	452	957452	98	667682	550	332318	3
58	625406	452	957393	98	668013	550	331987	2
59	625677	452	957335	98	668343	550	331657	1
60	625948	451	957276	98	668672	550	331328	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.625948	451	9.957276	98	9.668673	550	10.331327	60
1	626219	451	957217	98	669002	549	330998	59
2	626490	451	957158	98	669332	549	330668	58
3	626760	450	957099	98	669661	549	330339	57
4	627030	450	957040	98	669991	548	330009	56
5	627300	450	956981	98	670320	548	329680	55
6	627570	449	956921	99	670649	548	329351	54
7	627840	449	956862	99	670977	548	329023	53
8	628109	449	956803	99	671306	547	328694	52
9	628378	448	956744	99	671634	547	328366	51
10	628647	448	956684	99	671963	547	328037	50
11	9.628916	447	9.956625	99	9.672291	547	10.327709	49
12	629185	447	956566	99	672619	546	327381	48
13	629453	447	956506	99	672947	546	327053	47
14	629721	446	956447	99	673274	546	326726	46
15	629989	446	956387	99	673602	546	326398	45
16	630257	446	956327	99	673929	545	326071	44
17	630524	446	956268	99	674257	545	325743	43
18	630792	445	956208	100	674584	545	325416	42
19	631059	445	956148	100	674910	544	325090	41
20	631326	445	956089	100	675237	544	324763	40
21	9.631593	444	9.956029	100	9.675564	544	10.324436	39
22	631859	444	955969	100	675890	544	324110	38
23	632125	444	955909	100	676216	543	323784	37
24	632392	443	955849	100	676543	543	323457	36
25	632658	443	955789	100	676869	543	323131	35
26	632923	443	955729	100	677194	543	322806	34
27	633189	442	955669	100	677520	542	322480	33
28	633454	442	955609	100	677846	542	322154	32
29	633719	442	955548	100	678171	542	321829	31
30	633984	441	955488	100	678496	542	321504	30
31	9.634249	441	9.955428	101	9.678821	541	10.321179	29
32	634514	440	955368	101	679146	541	320854	28
33	634778	440	955307	101	679471	541	320529	27
34	635042	440	955247	101	679795	541	320205	26
35	635306	439	955186	101	680120	540	319880	25
36	635570	439	955126	101	680444	540	319556	24
37	635834	439	955065	101	680768	540	319232	23
38	636097	438	955005	101	681092	540	318908	22
39	636360	438	954944	101	681416	539	318584	21
40	636623	438	954883	101	681740	539	318260	20
41	9.636886	437	9 954823	101	9.682063	539	10.317937	19
42	637148	437	954762	101	682387	539	317613	18
43	637411	437	954701	101	682710	538	317290	17
44	637673	437	954640	101	683033	538	316967	16
45	637935	436	954579	101	683356	538	316644	15
46	638197	436	954518	102	683679	538	316321	14
47	638458	436	954457	102	684001	537	315999	13
48	638720	435	954396	102	684324	537	315676	12
49	638981	435	954335	102	684646	537	315354	11
50	639242	435	954274	102	684968	537	315032	10
51	9.639503	434	9.954213	102	9.685290	536	10.314710	9
52	639764	434	954152	102	685612	536	314388	8
53	640024	434	954090	102	685934	536	314066	7
54	640284	433	954029	102	686255	536	313745	6
55	640544	433	953968	102	686577	535	313423	5
56	640804	433	953906	102	686898	535	313102	4
57	641064	432	953845	102	687219	535	312781	3
58	641324	432	953783	102	687540	535	312460	2
59	641584	432	953722	103	687861	534	312139	1
60	641842	431	953660	103	688182	534	311818	0
	Cosine		Sine		Cotang.		Tang.	M.

64 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.641842	431	9.953660	103	9.688182	534	10.311818	60
1	642101	431	953599	103	688502	534	311498	59
2	642360	431	953537	103	688823	534	311177	58
3	642618	430	953475	103	689143	533	310857	57
4	642877	430	953413	103	689463	533	310537	56
5	643135	430	953352	103	689783	533	310217	55
6	643393	430	953290	103	690103	533	309897	54
7	643650	429	953228	103	690423	533	309577	53
8	643908	429	953166	103	690742	532	309258	52
9	644165	429	953104	103	691062	532	308938	51
10	644423	428	953042	103	691381	532	308619	50
11	9.644680	428	9.952980	104	9.691700	531	10.308300	49
12	644936	428	952918	104	692019	531	307981	48
13	645193	427	952855	104	692338	531	307662	47
14	645450	427	952793	104	692656	531	307344	46
15	645706	427	952731	104	692975	531	307025	45
16	645962	426	952669	104	693293	530	306707	44
17	646218	426	952606	104	693612	530	306388	43
18	646474	426	952544	104	693930	530	306070	42
19	646729	425	952481	104	694248	530	305752	41
20	646984	425	952419	104	694566	529	305434	40
21	9.647240	425	9.952356	104	9.694883	529	10.305117	39
22	647494	424	952294	104	695201	529	304799	38
23	647749	424	952231	104	695518	529	304482	37
24	648004	424	952168	105	695836	529	304164	36
25	648258	424	952106	105	696153	528	303847	35
26	648512	423	952043	105	696470	528	303530	34
27	648766	423	951980	105	696787	528	303213	33
28	649020	423	951917	105	697103	528	302897	32
29	649274	422	951854	105	697420	527	302580	31
30	649527	422	951791	105	697736	527	302264	30
31	9.649781	422	9.951728	105	9.698053	527	10.301947	29
32	650034	422	951665	105	698369	527	301631	28
33	650287	421	951602	105	698685	526	301315	27
34	650539	421	951539	105	699001	526	300999	26
35	650792	421	951476	105	699316	526	300684	25
36	651044	420	951412	105	699632	526	300368	24
37	651297	420	951349	106	699947	526	300053	23
38	651549	420	951286	106	700263	525	299737	22
39	651800	419	951222	106	700578	525	299422	21
40	652052	419	951159	106	700893	525	299107	20
41	9.652304	419	9.951096	106	9.701208	524	10.298792	19
42	652555	418	951032	106	701523	524	298477	18
43	652806	418	950968	106	701837	524	298163	17
44	653057	418	950905	106	702152	524	297848	16
45	653308	418	950841	106	702466	524	297534	15
46	653558	417	950778	106	702780	52[illegible]	297220	14
47	653808	417	950714	106	703095	523	296905	13
48	654059	417	950650	106	703409	523	296591	12
49	654309	416	950586	106	703723	523	296277	11
50	654558	416	950522	107	704036	522	295964	10
51	9.654808	416	9.950458	107	9.704350	522	10.295650	9
52	655058	416	950394	107	704663	522	295337	8
53	655307	415	950330	107	704977	522	295023	7
54	655556	415	950266	107	705290	522	294710	6
55	655805	415	950202	107	705603	521	294397	5
56	656054	414	950138	107	705916	521	294084	4
57	656302	414	950074	107	706228	521	293772	3
58	656551	414	950010	107	706541	521	293459	2
59	656799	413	949945	107	706854	5[illegible]	293146	1
60	657047	413	949881	107	707166	520	292834	0
	Cosine		Sine		Cotang.		Tang.	M.

M	Sine	D	Cosine	D.	Tang.	D.	Cotang.	
0	9.657047	413	9.949881	107	9.707166	520	10.292834	60
1	657295	413	949816	107	707478	520	292522	59
2	657542	412	949752	107	707790	520	292210	58
3	657790	412	949688	108	708102	520	291898	57
4	658037	412	949623	108	708414	519	291586	56
5	658284	412	949558	108	708726	519	291274	55
6	658531	411	949494	108	709037	519	290963	54
7	658778	411	949429	108	709349	519	290651	53
8	659025	411	949364	108	709660	519	290340	52
9	659271	410	949300	108	709971	518	290029	51
10	659517	410	949235	108	710282	518	289718	50
11	9.659763	410	9.949170	108	9.710593	518	10.289407	49
12	660009	409	949105	108	710904	518	289096	48
13	660255	409	949040	108	711215	518	288785	47
14	660501	409	948975	108	711525	517	288475	46
15	660746	409	948910	108	711836	517	288164	45
16	660991	408	948845	108	712146	517	287854	44
17	661236	408	948780	109	712456	517	287544	43
18	661481	408	948715	109	712766	516	287234	42
19	661726	407	948650	109	713076	516	286924	41
20	661970	407	948584	109	713386	516	286614	40
21	9.662214	407	9.948519	109	9.713696	516	10.286304	39
22	662459	407	948454	109	714005	516	285995	38
23	662703	406	948388	109	714314	515	285686	37
24	662946	406	948323	109	714624	515	285376	36
25	663190	406	948257	109	714933	515	285067	35
26	663433	405	948192	109	715242	515	284758	34
27	663677	405	948126	109	715551	514	284449	33
28	663920	405	948060	109	715860	514	284140	32
29	664163	405	947995	110	716168	514	283832	31
30	664406	404	947929	110	716477	514	283523	30
31	9.664648	404	9.947863	110	9.716785	514	10.283215	29
32	664891	404	947797	110	717093	513	282907	28
33	665133	403	947731	110	717401	513	282599	27
34	665375	403	947665	110	717709	513	282291	26
35	665617	403	947600	110	718017	513	281983	25
36	665859	402	947533	110	718325	513	281675	24
37	666100	402	947467	110	718633	512	281367	23
38	666342	402	947401	110	718940	512	281060	22
39	666583	402	947335	110	719248	512	280752	21
40	666824	401	947269	110	719555	512	280445	20
41	9.667065	401	9.947203	110	9.719862	512	10.280138	19
42	667305	401	947136	111	720169	511	279831	18
43	667546	401	947070	111	720476	511	279524	17
44	667786	400	947004	111	720783	511	279217	16
45	668027	400	946937	111	721089	511	278911	15
46	668267	400	946871	111	721396	511	278604	14
47	668506	399	946804	111	721702	510	278298	13
48	668746	399	946738	111	722009	510	277991	12
49	668986	399	946671	111	722315	510	277685	11
50	669225	399	946604	111	722621	510	277379	10
51	9.669464	398	9.946538	111	9.722927	510	10.277073	9
52	669703	398	946471	111	723232	509	276768	8
53	669942	398	946404	111	723538	509	276462	7
54	670181	397	946337	111	723844	509	276156	6
55	670419	397	946270	112	724149	509	275851	5
56	670658	397	946203	112	724454	509	275546	4
57	670896	397	946136	112	724759	508	275241	3
58	671134	396	946069	112	725065	508	274935	2
59	671372	396	946002	112	725369	508	274631	1
60	671609	396	945935	112	725674	508	274326	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.671609	396	9.945935	112	9.725674	508	10.274326	60
1	671847	395	945868	112	725979	508	274021	59
2	672084	395	945800	112	726284	507	273716	58
3	672321	395	945733	112	726588	507	273412	57
4	672558	395	945666	112	726892	507	273108	56
5	672795	394	945598	112	727197	507	272803	55
6	673032	394	945531	112	727501	507	272499	54
7	673268	394	945464	113	727805	506	272195	53
8	673505	394	945396	113	728109	506	271891	52
9	673741	393	945328	113	728412	506	271588	51
10	673977	393	945261	113	728716	506	271284	50
11	9.674213	393	9.945193	113	9.729020	506	10.270980	49
12	674448	392	945125	113	729323	505	270677	48
13	674684	392	945058	113	729626	505	270374	47
14	674919	392	944990	113	729929	505	270071	46
15	675155	392	944922	113	730233	505	269767	45
16	675390	391	944854	113	730535	505	269465	44
17	675624	391	944786	113	730838	504	269162	43
18	675859	391	944718	113	731141	504	268859	42
19	676094	391	944650	113	731444	504	268556	41
20	676328	390	944582	114	731746	504	268254	40
21	9.676562	390	9.944514	114	9.732048	504	10.267952	39
22	676796	390	944446	114	732351	503	267649	38
23	677030	390	944377	114	732653	503	267347	37
24	677264	389	944309	114	732955	503	267045	36
25	677498	389	944241	114	733257	503	266743	35
26	677731	389	944172	114	733558	503	266442	34
27	677964	388	944104	114	733860	502	266140	33
28	678197	388	944036	114	734162	502	265838	32
29	678430	388	943967	114	734463	502	265537	31
30	678663	388	943899	114	734764	502	265236	30
31	9 678895	387	9.943830	114	9.735066	502	10.264934	29
32	679128	387	943761	114	735367	502	264633	28
33	679360	387	943693	115	735668	501	264332	27
34	679592	387	943624	115	735969	501	264031	26
35	679824	386	943555	115	736269	501	263731	25
36	680056	386	943486	115	736570	501	263430	24
37	680288	386	943417	115	736871	501	263129	23
38	680519	385	943348	115	737171	500	262829	22
39	680750	385	943279	115	737471	500	262529	21
40	680982	385	943210	115	737771	500	262229	20
41	9.681213	385	9.943141	115	9.738071	500	10.261929	19
42	681443	384	943072	115	738371	500	261629	18
43	681674	384	943003	115	738671	499	261329	17
44	681905	384	942934	115	738971	499	261029	16
45	682135	384	942864	115	739271	499	260729	15
46	682365	383	942795	116	739570	499	260430	14
47	682595	383	942726	116	739870	499	260130	13
48	682825	383	942656	116	740169	499	259831	12
49	683055	383	942587	116	740468	498	259532	11
50	683284	382	942517	116	740767	498	259233	10
51	9.683514	382	9.942448	116	9.741066	498	10.258934	9
52	683743	382	942378	116	741365	498	258635	8
53	683972	382	942308	116	741664	498	258336	7
54	684201	381	942239	116	741962	497	258038	6
55	684430	381	942169	116	742261	497	257739	5
56	684658	381	942099	116	742559	497	257441	4
57	684887	380	942029	116	742858	497	257142	3
58	685115	380	941959	116	743156	497	256844	2
59	685343	380	941889	117	743454	497	256546	1
60	685571	380	941819	117	743752	496	256248	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.685571	380	9.941819	117	9.743752	496	10.256248	60
1	685799	379	941749	117	744050	496	255950	59
2	686027	379	941679	117	744348	496	255652	58
3	686254	379	941609	117	744645	496	255355	57
4	686482	379	941539	117	744943	496	255057	56
5	686709	378	941469	117	745240	496	254760	55
6	686936	378	941398	117	745538	495	254462	54
7	687163	378	941328	117	745835	495	254165	53
8	687389	378	941258	117	746132	495	253868	52
9	687616	377	941187	117	746429	495	253571	51
10	687843	377	941117	117	746726	495	253274	50
11	9.688069	377	9.941046	118	9.747023	494	10.252977	49
12	688295	377	940975	118	747319	494	252681	48
13	688521	376	940905	118	747616	494	252384	47
14	688747	376	940834	118	747913	494	252087	46
15	688972	376	940763	118	748209	494	251791	45
16	689198	376	940693	118	748505	493	251495	44
17	689423	375	940622	118	748801	493	251199	43
18	689648	375	940551	118	749097	493	250903	42
19	689873	375	940480	118	749393	493	250607	41
20	690098	375	940409	118	749689	493	250311	40
21	9.690323	374	9.940338	118	9.749985	493	10.250015	39
22	690548	374	940267	118	750281	492	249719	38
23	690772	374	940196	118	750576	492	249424	37
24	690996	374	940125	119	750872	492	249128	36
25	691220	373	940054	119	751167	492	248833	35
26	691444	373	939982	119	751462	492	248538	34
27	691668	373	939911	119	751757	[illegible]	[illegible]243	33
28	691892	373	939840	119	752052	[illegible]	[illegible]47948	32
29	692115	372	939768	119	752347	[illegible]	[illegible]7653	31
30	692339	372	939697	119	752642	[illegible]	[illegible]58	30
31	9.692562	372	9.939625	119	9.752937	[illegible]	[illegible]	29
32	692785	371	939554	119	753231	4[illegible]	[illegible]	28
33	693008	371	939482	119	753526	49[illegible]	[illegible]4	27
34	693231	371	939410	119	753820	490	[illegible]0	26
35	693453	371	939339	119	754115	490	24[illegible]85	25
36	693676	370	939267	120	754409	490	245591	24
37	693898	370	939195	120	754703	490	245297	23
38	694120	370	939123	120	754997	490	245003	22
39	694342	370	939052	120	755291	490	244709	21
40	694564	369	938980	120	755585	489	244415	20
41	9.694786	369	9.938908	120	9.755878	489	10.244122	19
42	695007	369	938836	120	756172	489	243828	18
43	695229	369	938763	120	756465	489	243535	17
44	695450	368	938691	120	756759	489	243241	16
45	695671	368	938619	120	757052	489	242948	15
46	695892	368	938547	120	757345	488	242655	14
47	696113	368	938475	120	757638	488	242362	13
48	696334	367	938402	121	757931	488	242069	12
49	696554	367	938330	121	758224	488	241776	11
50	696775	367	938258	121	758517	488	241483	10
51	9.696995	367	9.938185	121	9.758810	488	10.241190	9
52	697215	366	938113	121	759102	487	240898	8
53	697435	366	938040	121	759395	487	240605	7
54	697654	366	937967	121	759687	487	240313	6
55	697874	366	937895	121	759979	487	240021	5
56	698094	365	937822	121	760272	487	239728	4
57	698313	365	937749	121	760564	487	239436	3
58	698532	365	937676	121	760856	486	239144	2
59	698751	365	937604	121	761148	486	238852	1
60	698970	364	937531	121	761439	486	238561	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.698970	364	9.937531	121	9.761439	486	10.238561	60
1	699189	364	937458	122	761731	486	238269	59
2	699407	364	937385	122	762023	486	237977	58
3	699626	364	937312	122	762314	486	237686	57
4	699844	363	937238	122	762606	485	237394	56
5	700062	363	937165	122	762897	485	237103	55
6	700280	363	937092	122	763188	485	236812	54
7	700498	363	937019	122	763479	485	236521	53
8	700716	363	936946	122	763770	485	236230	52
9	700933	362	936872	122	764061	485	235939	51
10	701151	362	936799	122	764352	484	235648	50
11	9.701368	362	9.936725	122	9.764643	484	10.235357	49
12	701585	362	936652	123	764933	484	235067	48
13	701802	361	936578	123	765224	484	234776	47
14	702019	361	936505	123	765514	484	234486	46
15	702236	361	936431	123	765805	484	234195	45
16	702452	361	936357	123	766095	484	233905	44
17	702669	360	936284	123	766385	483	233615	43
18	702885	360	936210	123	766675	483	233325	42
19	703101	360	936136	123	766965	483	233035	41
20	703317	360	936062	123	767255	483	232745	40
21	9.703533	359	9.935988	123	9.767545	483	10.232455	39
22	703749	359	935914	123	767834	483	232166	38
23	703964	359	935840	123	768124	482	231876	37
24	704179	359	935766	124	768413	482	231587	36
25	704395	359	935692	124	768703	482	231297	35
26	704610	358	935618	124	768992	482	231008	34
27	704825	358	935543	124	769281	482	230719	33
28	705040	358	935469	124	769570	482	230430	32
29	705254	358	935395	124	769860	481	230140	31
30	705469	357	935320	124	770148	481	229852	30
31	9.705683	357	9.935246	124	9.770437	481	10.229563	29
32	705898	357	935171	124	770726	481	229274	28
33	706112	357	935097	124	771015	481	228985	27
34	706326	356	935022	124	771303	481	228697	26
35	706539	356	934948	124	771592	481	228408	25
36	706753	356	934873	124	771880	480	228120	24
37	706967	356	934798	125	772168	480	227832	23
38	707180	355	934723	125	772457	480	227543	22
39	707393	355	934649	125	772745	480	227255	21
40	707606	355	934574	125	773033	480	226967	20
41	9.707819	355	9.934499	125	9.773321	480	10.226679	19
42	708032	354	934424	125	773608	479	226392	18
43	708245	354	934349	125	773896	479	226104	17
44	708458	354	934274	125	774184	479	225816	16
45	708670	354	934199	125	774471	479	225529	15
46	708882	353	934123	125	774759	479	225241	14
47	709094	353	934048	125	775046	479	224954	13
48	709306	353	933973	125	775333	479	224667	12
49	709518	353	933898	126	775621	478	224379	11
50	709730	353	933822	126	775908	478	224092	10
51	9.709941	352	9.933747	126	9.776195	478	10.223805	9
52	710153	352	933671	126	776482	478	223518	8
53	710364	352	933596	126	776769	478	223231	7
54	710575	352	933520	126	777055	478	222945	6
55	710786	351	933445	126	777342	478	222658	5
56	710997	351	933369	126	777628	477	222372	4
57	711208	351	933293	126	777915	477	222085	3
58	711419	351	933217	126	778201	477	221799	2
59	711629	350	933141	126	778487	477	221512	1
60	711839	350	933066	126	778774	477	221226	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.711839	350	9.933066	126	9.778774	477	10.221226	60
1	712050	350	932990	127	779060	477	220940	59
2	712260	350	932914	127	779346	476	220654	58
3	712469	349	932838	127	779632	476	220368	57
4	712679	349	932762	127	779918	476	220082	56
5	712889	349	932685	127	780203	476	219797	55
6	713098	349	932609	127	780489	476	219511	54
7	713308	349	932533	127	780775	476	219225	53
8	713517	348	932457	127	781060	476	218940	52
9	713726	348	932380	127	781346	475	218654	51
10	713935	348	932304	127	781631	475	218369	50
11	9.714144	348	9.932228	127	9.781916	475	10.218084	49
12	714352	347	932151	127	782201	475	217799	48
13	714561	347	932075	128	782486	475	217514	47
14	714769	347	931998	128	782771	475	217229	46
15	714978	347	931921	128	783056	475	216944	45
16	715186	347	931845	128	783341	475	216659	44
17	715394	346	931768	128	783626	474	216374	43
18	715602	346	931691	128	783910	474	216090	42
19	715809	346	931614	128	784195	474	215805	41
20	716017	346	931537	128	784479	474	215521	40
21	9.716224	345	9.931460	128	9.784764	474	10.215236	39
22	716432	345	931383	128	785048	474	214952	38
23	716639	345	931306	128	785332	473	214668	37
24	716846	345	931229	129	785616	473	214384	36
25	717053	345	931152	129	785900	473	214100	35
26	717259	344	931075	129	786184	473	213816	34
27	717466	344	930998	129	786468	473	213532	33
28	717673	344	930921	129	786752	473	213248	32
29	717879	344	930843	129	787036	473	212964	31
30	718085	343	930766	129	787319	472	212681	30
31	9.718291	343	9.930688	129	9.787603	472	10.212397	29
32	718497	343	930611	129	787886	472	212114	28
33	718703	343	930533	129	788170	472	211830	27
34	718909	343	930456	129	788453	472	211547	26
35	719114	342	930378	129	788736	472	211264	25
36	719320	342	930300	130	789019	472	210981	24
37	719525	342	930223	130	789302	471	210698	23
38	719730	342	930145	130	789585	471	210415	22
39	719935	341	930067	130	789868	471	210132	21
40	720140	341	929989	130	790151	471	209849	20
41	9.720345	341	9.929911	130	9.790433	471	10.209567	19
42	720549	341	929833	130	790716	471	209284	18
43	720754	340	929755	130	790999	471	209001	17
44	720958	340	929677	130	791281	471	208719	16
45	721162	340	929599	130	791563	470	208437	15
46	721366	340	929521	130	791846	470	208154	14
47	721570	340	929442	130	792128	470	207872	13
48	721774	339	929364	131	792410	470	207590	12
49	721978	339	929286	131	792692	470	207308	11
50	722181	339	929207	131	792974	470	207026	10
51	9.722385	339	9.929129	131	9.793256	470	10.206744	9
52	722588	339	929050	131	793538	469	206462	8
53	722791	338	928972	131	793819	469	206181	7
54	722994	338	928893	131	794101	469	205899	6
55	723197	338	928815	131	794383	469	205617	5
56	723400	338	928736	131	794664	469	205336	4
57	723603	337	928657	131	794945	469	205055	3
58	723805	337	928578	131	795227	469	204773	2
59	724007	337	928499	131	795508	468	204492	1
60	724210	337	928420	131	795789	468	204211	0
	Cosine		Sine		Cotang.		Tang.	M.

58 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.724210	337	9.928420	132	9.795789	468	10.204211	60
1	724412	337	928342	132	796070	468	203930	59
2	724614	336	928263	132	796351	468	203649	58
3	724816	336	928183	132	796632	468	203368	57
4	725017	336	928104	132	796913	468	203087	56
5	725219	336	928025	132	797194	468	202806	55
6	725420	335	927946	132	797475	468	202525	54
7	725622	335	927867	132	797755	468	202245	53
8	725823	335	927787	132	798036	467	201964	52
9	726024	335	927708	132	798316	467	201684	51
10	726225	335	927629	132	798596	467	201404	50
11	9.726426	334	9.927549	132	9.798877	467	10.201123	49
12	726626	334	927470	133	799157	467	200843	48
13	726827	334	927390	133	799437	467	200563	47
14	727027	334	927310	133	799717	467	200283	46
15	727228	334	927231	133	799997	466	200003	45
16	727428	333	927151	133	800277	466	199723	44
17	727628	333	927071	133	800557	466	199443	43
18	727828	333	926991	133	800836	466	199164	42
19	728027	333	926911	133	801116	466	198884	41
20	728227	333	926831	133	801396	466	198604	40
21	9.728427	332	9.926751	133	9.801675	466	10.198325	39
22	728626	332	926671	133	801955	466	198045	38
23	728825	332	926591	133	802234	465	197766	37
24	729024	332	926511	134	802513	465	197487	36
25	729223	331	926431	134	802792	465	197208	35
26	729422	331	926351	134	803072	465	196928	34
27	729621	331	926270	134	803351	465	196649	33
28	729820	331	926190	134	803630	465	196370	32
29	730018	330	926110	134	803908	465	196092	31
30	730216	330	926029	134	804187	465	195813	30
31	9.730415	330	9.925949	134	9.804466	464	10.195534	29
32	730613	330	925868	134	804745	464	195255	28
33	730811	330	925788	134	805023	464	194977	27
34	731009	329	925707	134	805302	464	194698	26
35	731206	329	925626	134	805580	464	194420	25
36	731404	329	925545	135	805859	464	194141	24
37	731602	329	925465	135	806137	464	193863	23
38	731799	329	925384	135	806415	463	193585	22
39	731996	328	925303	135	806693	463	193307	21
40	732193	328	925222	135	806971	463	193029	20
41	9.732390	328	9.925141	135	9.807249	463	10.192751	19
42	732587	328	925060	135	807527	463	192473	18
43	732784	328	924979	135	807805	463	192195	17
44	732980	327	924897	135	808083	463	191917	16
45	733177	327	924816	135	808361	463	191639	15
46	733373	327	924735	136	808638	462	191362	14
47	733569	327	924654	136	808916	462	191084	13
48	733765	327	924572	136	809193	462	190807	12
49	733961	326	924491	136	809471	462	190529	11
50	734157	326	924409	136	809748	462	190252	10
51	9.734353	326	9.924328	136	9.810025	462	10.189975	9
52	734549	326	924246	136	810302	462	189698	8
53	734744	325	924164	136	810580	462	189420	7
54	734939	325	924083	136	810857	462	189143	6
55	735135	325	924001	136	811134	461	188866	5
56	735330	325	923919	136	811410	461	188590	4
57	735525	325	923837	136	811687	461	188313	3
58	735719	324	923755	137	811964	461	188036	2
59	735914	324	923673	137	812241	461	187759	1
60	736109	324	923591	137	812517	461	187483	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.736109	324	9.923591	137	9.812517	461	10.187482	60
1	736303	324	923509	137	812794	461	187206	59
2	736498	324	923427	137	813070	461	186930	58
3	736692	323	923345	137	813347	460	186653	57
4	736886	323	923263	137	813623	460	186377	56
5	737080	323	923181	137	813899	460	186101	55
6	737274	323	923098	137	814175	460	185825	54
7	737467	323	923016	137	814452	460	185548	53
8	737661	322	922933	137	814728	460	185272	52
9	737855	322	922851	137	815004	460	184996	51
10	738048	322	922768	138	815279	460	184721	50
11	9.738241	322	9.922686	138	9.815555	459	10.184445	49
12	738434	322	922603	138	815831	459	184169	48
13	738627	321	922520	138	816107	459	183893	47
14	738820	321	922438	138	816382	459	183618	46
15	739013	321	922355	138	816658	459	183342	45
16	739206	321	922272	138	816933	459	183067	44
17	739398	321	922189	138	817209	459	182791	43
18	739590	320	922106	138	817484	459	182516	42
19	739783	320	922023	138	817759	459	182241	41
20	739975	320	921940	138	818035	458	181965	40
21	9.740167	320	9.921857	139	9.818310	458	10.181690	39
22	740359	320	921774	139	818585	458	181415	38
23	740550	319	921691	139	818860	458	181140	37
24	740742	319	921607	139	819135	458	180865	36
25	740934	319	921524	139	819410	458	180590	35
26	741125	319	921441	139	819684	458	180316	34
27	741316	319	921357	139	819959	458	180041	33
28	741508	318	921274	139	820234	458	179766	32
29	741699	318	921190	139	820508	457	179492	31
30	741889	318	921107	139	820783	457	179217	30
31	9.742080	318	9.921023	139	9.821057	457	10.178943	29
32	742271	318	920939	140	821332	457	178668	28
33	742462	317	920856	140	821606	457	178394	27
34	742652	317	920772	140	821880	457	178120	26
35	742842	317	920688	140	822154	457	177846	25
36	743033	317	920604	140	822429	457	177571	24
37	743223	317	920520	140	822703	457	177297	23
38	743413	316	920436	140	822977	457	177023	22
39	743602	316	920352	140	823250	456	176750	21
40	743792	316	920268	140	823524	456	176476	20
41	9.743982	316	9.920184	140	9.823798	456	10.176202	19
42	744171	316	920099	140	824072	456	175928	18
43	744361	315	920015	140	824345	456	175655	17
44	744550	315	919931	141	824619	456	175381	16
45	744739	315	919846	141	824893	456	175107	15
46	744928	315	919762	141	825166	456	174834	14
47	745117	315	919677	141	825439	455	174561	13
48	745306	314	919593	141	825713	455	174287	12
49	745494	314	919508	141	825986	455	174014	11
50	745683	314	919424	141	826259	455	173741	10
51	9.745871	314	9.919339	141	9.826532	455	10.173468	9
52	746059	314	919254	141	826805	455	173195	8
53	746248	313	919169	141	827078	455	172922	7
54	746436	313	919085	141	827351	455	172649	6
55	746624	313	919000	141	827624	455	172376	5
56	746812	313	918915	142	827897	454	172103	4
57	746999	313	918830	142	828170	454	171830	3
58	747187	312	918745	142	828442	454	171558	2
59	747374	312	918659	142	828715	454	171285	1
60	747562	312	918574	142	828987	454	171013	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D	Cotang.	
0	9.747562	312	9.918574	142	9.828987	454	10.171013	60
1	747749	312	918489	142	829260	454	170740	59
2	747936	312	918404	142	829532	454	170468	58
3	748123	311	918318	142	829805	454	170195	57
4	748310	311	918233	142	830077	454	169923	56
5	748497	311	918147	142	830349	453	169651	55
6	748683	311	918062	142	830621	453	169379	54
7	748870	311	917976	143	830893	453	169107	53
8	749056	310	917891	143	831165	453	168835	52
9	749243	310	917805	143	831437	453	168563	51
10	749429	310	917719	143	831709	453	168291	50
11	9.749615	310	9.917634	143	9.831981	453	10.168019	49
12	749801	310	917548	143	832253	453	167747	48
13	749987	309	917462	143	832525	453	167475	47
14	750172	309	917376	143	832796	453	167204	46
15	750358	309	917290	143	833068	452	166932	45
16	750543	309	917204	143	833339	452	166661	44
17	750729	309	917118	144	833611	452	166389	43
18	750914	308	917032	144	833882	452	166118	42
19	751099	308	916946	144	834154	452	165846	41
20	751284	308	916859	144	834425	452	165575	40
21	9.751469	308	9.916773	144	9.834696	452	10.165304	39
22	751654	308	916687	144	834967	452	165033	38
23	751839	308	916600	144	835238	452	164762	37
24	752023	307	916514	144	835509	452	164491	36
25	752208	307	916427	144	835780	451	164220	35
26	752392	307	916341	144	836051	451	163949	34
27	752576	307	916254	144	836322	451	163678	33
28	752760	307	916167	145	836593	451	163407	32
29	752944	306	916081	145	836864	451	163136	31
30	753128	306	915994	145	837134	451	162866	30
31	9 753312	306	9.915907	145	9.837405	451	10.162595	29
32	753495	306	915820	145	837675	451	162325	28
33	753679	306	915733	145	837946	451	162054	27
34	753862	305	915646	145	838216	451	161784	26
35	754046	305	915559	145	838487	450	161513	25
36	754229	305	915472	145	838757	450	161243	24
37	754412	305	915385	145	839027	450	160973	23
38	754595	305	915297	145	839297	450	160703	22
39	754778	304	915210	145	839568	450	160432	21
40	754960	304	915123	146	839838	450	160162	20
41	9.755143	304	9.915035	146	9.840108	450	10.159892	19
42	755326	304	914948	146	840378	450	159622	18
43	755508	304	914860	146	840647	450	159353	17
44	755690	304	914773	146	840917	449	159083	16
45	755872	303	914685	146	841187	449	158813	15
46	756054	303	914598	146	841457	449	158543	14
47	756236	303	914510	146	841726	449	158274	13
48	756418	303	914422	146	841996	449	158004	12
49	756600	303	914334	146	842266	449	157734	11
50	756782	302	914246	147	842535	449	157465	10
51	9.756963	302	9.914158	147	9.842805	449	10.157195	9
52	757144	302	914070	147	843074	449	156926	8
53	757326	302	913982	147	843343	449	156657	7
54	757507	302	913894	147	843612	449	156388	6
55	757688	301	913806	147	843882	448	156118	5
56	757869	301	913718	147	844151	448	155849	4
57	758050	301	913630	147	844420	448	155580	3
58	758230	301	913541	147	844689	448	155311	2
59	758411	301	913453	147	844958	448	155042	1
60	758591	301	913365	147	845227	448	154773	0
	Cosine		Sine		Cotang.		Tang.	M

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.758591	301	9.913365	147	9.845227	448	10.154773	60
1	758772	300	913276	147	845496	448	154504	59
2	758952	300	913187	148	845764	448	154236	58
3	759132	300	913099	148	846033	448	153967	57
4	759312	300	913010	148	846302	448	153698	56
5	759492	300	912922	148	846570	447	153430	55
6	759672	299	912833	148	846839	447	153161	54
7	759852	299	912744	148	847107	447	152893	53
8	760031	299	912655	148	847376	447	152624	52
9	760211	299	912566	148	847644	447	152356	51
10	760390	299	912477	148	847913	447	152087	50
11	9.760569	298	9.912388	148	9.848181	447	10.151819	49
12	760748	298	912299	149	848449	447	151551	48
13	760927	298	912210	149	848717	447	151283	47
14	761106	298	912121	149	848986	447	151014	46
15	761285	298	912031	149	849254	447	150746	45
16	761464	298	911942	149	849522	447	150478	44
17	761642	297	911853	149	849790	446	150210	43
18	761821	297	911763	149	850058	446	149942	42
19	761999	297	911674	149	850325	446	149675	41
20	762177	297	911584	149	850593	446	149407	40
21	9.762356	297	9.911495	149	9.850861	446	10.149139	39
22	762534	296	911405	149	851129	446	148871	38
23	762712	296	911315	150	851396	446	148604	37
24	762889	296	911226	150	851664	446	148336	36
25	763067	296	911136	150	851931	446	148069	35
26	763245	296	911046	150	852199	446	147801	34
27	763422	296	910956	150	852466	446	147534	33
28	763600	295	910866	150	852733	445	147267	32
29	763777	295	910776	150	853001	445	146999	31
30	763954	295	910686	150	853268	445	146732	30
31	9.764131	295	9.910596	150	9.853535	445	10.146465	29
32	764308	295	910506	150	853802	445	146198	28
33	764485	294	910415	150	854069	445	145931	27
34	764662	294	910325	151	854336	445	145664	26
35	764838	294	910235	151	854603	445	145397	25
36	765015	294	910144	151	854870	445	145130	24
37	765191	294	910054	151	855137	445	144863	23
38	765367	294	909963	151	855404	445	144596	22
39	765544	293	909873	151	855671	444	144329	21
40	765720	293	909782	151	855938	444	144062	20
41	9.765896	293	9.909691	151	9.856204	444	10.143796	19
42	766072	293	909601	151	856471	444	143529	18
43	766247	293	909510	151	856737	444	143263	17
44	766423	293	909419	151	857004	444	142996	16
45	766598	292	909328	152	857270	444	142730	15
46	766774	292	909237	152	857537	444	142463	14
47	766949	292	909146	152	857803	444	142197	13
48	767124	292	909055	152	858069	444	141931	12
49	767300	292	908964	152	858336	444	141664	11
50	767475	291	908873	152	858602	443	141398	10
51	9.767649	291	9.908781	152	9.858868	443	10.141132	9
52	767824	291	908690	152	859134	443	140866	8
53	767999	291	908599	152	859400	443	140600	7
54	768173	291	908507	152	859666	443	140334	6
55	768348	290	908416	153	859932	443	140068	5
56	768522	290	908324	153	860198	443	139802	4
57	768697	290	908233	153	860464	443	139536	3
58	768871	290	908141	153	860730	443	139270	2
59	769045	290	908049	153	860995	443	139005	1
60	769219	290	907958	153	861261	443	138739	0
	Cosine		Sine		Cotang.		Tang.	M.

54 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.769219	290	9.907958	153	9.861261	443	10.138739	60
1	769393	289	907866	153	861527	443	138473	59
2	769566	289	907774	153	861792	442	138208	58
3	769740	289	907682	153	862058	442	137942	57
4	769913	289	907590	153	862323	442	137677	56
5	770087	289	907498	153	862589	442	137411	55
6	770260	288	907406	153	862854	442	137146	54
7	770433	288	907314	154	863119	442	136881	53
8	770606	288	907222	154	863385	442	136615	52
9	770779	288	907129	154	863650	442	136350	51
10	770952	288	907037	154	863915	442	136085	50
11	9.771125	288	9.906945	154	9.864180	442	10.135820	49
12	771298	287	906852	154	864445	442	135555	48
13	771470	287	906760	154	864710	442	135290	47
14	771643	287	906667	154	864975	441	135025	46
15	771815	287	906575	154	865240	441	134760	45
16	771987	287	906482	154	865505	441	134495	44
17	772159	287	906389	155	865770	441	134230	43
18	772331	286	906296	155	866035	441	133965	42
19	772503	286	906204	155	866300	441	133700	41
20	772675	286	906111	155	866564	441	133436	40
21	9.772847	286	9.906018	155	9.866829	441	10.133171	39
22	773018	286	905925	155	867094	441	132906	38
23	773190	286	905832	155	867358	441	132642	37
24	773361	285	905739	155	867623	441	132377	36
25	773533	285	905645	155	867887	441	132113	35
26	773704	285	905552	155	868152	440	131848	34
27	773875	285	905459	155	868416	440	131584	33
28	774046	285	905366	156	868680	440	131320	32
29	774217	285	905272	156	868945	440	131055	31
30	774388	284	905179	156	869209	440	130791	30
31	9.774558	284	9.905085	156	9.869473	440	10.130527	29
32	774729	284	904992	156	869737	440	130263	28
33	774899	284	904898	156	870001	440	129999	27
34	775070	284	904804	156	870265	440	129735	26
35	775240	284	904711	156	870529	440	129471	25
36	775410	283	904617	156	870793	440	129207	24
37	775580	283	904523	156	871057	440	128943	23
38	775750	283	904429	157	871321	440	128679	22
39	775920	283	904335	157	871585	440	128415	21
40	776090	283	904241	157	871849	439	128151	20
41	9.776259	283	9.904147	157	9.872112	439	10.127888	19
42	776429	282	904053	157	872376	439	127624	18
43	776598	282	903959	157	872640	439	127360	17
44	776768	282	903864	157	872903	439	127097	16
45	776937	282	903770	157	873167	439	126833	15
46	777106	282	903676	157	873430	439	126570	14
47	777275	281	903581	157	873694	439	126306	13
48	777444	281	903487	157	873957	439	126043	12
49	777613	281	903392	158	874220	439	125780	11
50	777781	281	903298	158	874484	439	125516	10
51	9.777950	281	9.903203	158	9.874747	439	10.125253	9
52	778119	281	903108	158	875010	439	124990	8
53	778287	280	903014	158	875273	438	124727	7
54	778455	280	902919	158	875536	438	124464	6
55	778624	280	902824	158	875800	438	124200	5
56	778792	280	902729	158	876063	438	123937	4
57	778960	280	902634	158	876326	438	123674	3
58	779128	280	902539	159	876589	438	123411	2
59	779295	279	902444	159	876851	438	123149	1
60	779463	279	902349	159	877114	438	122886	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.779463	279	9.902349	159	9.877114	438	10.122886	60
1	779631	279	902253	159	877377	438	122623	59
2	779798	279	902158	159	877640	438	122360	58
3	779966	279	902063	159	877903	438	122097	57
4	780133	279	901967	159	878165	438	121835	56
5	780300	278	901872	159	878428	438	121572	55
6	780467	278	901776	159	878691	438	121309	54
7	780634	278	901681	159	878953	437	121047	53
8	780801	278	901585	159	879216	437	120784	52
9	780968	278	901490	159	879478	437	120522	51
10	781134	278	901394	160	879741	437	120259	50
11	9.781301	277	9.901298	160	9.880003	437	10.119997	49
12	781468	277	901202	160	880265	437	119735	48
13	781634	277	901106	160	880528	437	119472	47
14	781800	277	901010	160	880790	437	119210	46
15	781966	277	900914	160	881052	437	118948	45
16	782132	277	900818	160	881314	437	118686	44
17	782298	276	900722	160	881576	437	118424	43
18	782464	276	900626	160	881839	437	118161	42
19	782630	276	900529	160	882101	437	117899	41
20	782796	276	900433	161	882363	436	117637	40
21	9.782961	276	9.900337	161	9.882625	436	10.117375	39
22	783127	276	900240	161	882887	436	117113	38
23	783292	275	900144	161	883148	436	116852	37
24	783458	275	900047	161	883410	436	116590	36
25	783623	275	899951	161	883672	436	116328	35
26	783788	275	899854	161	883934	436	116066	34
27	783953	275	899757	161	884196	436	115804	33
28	784118	275	899660	161	884457	436	115543	32
29	784282	274	899564	161	884719	436	115281	31
30	784447	274	899467	162	884980	436	115020	30
31	9.784612	274	9.899370	162	9.885242	436	10.114758	29
32	784776	274	899273	162	885503	436	114497	28
33	784941	274	899176	162	885765	436	114235	27
34	785105	274	899078	162	886026	436	113974	26
35	785269	273	898981	162	886288	436	113712	25
36	785433	273	898884	162	886549	435	113451	24
37	785597	273	898787	162	886810	435	113190	23
38	785761	273	898689	162	887072	435	112928	22
39	785925	273	898592	162	887333	435	112667	21
40	786089	273	898494	163	887594	435	112406	20
41	9.786252	272	9.898397	163	9.887855	435	10.112145	19
42	786416	272	898299	163	888116	435	111884	18
43	786579	272	898202	163	888377	435	111623	17
44	786742	272	898104	163	888639	435	111361	16
45	786906	272	898006	163	888900	435	111100	15
46	787069	272	897908	163	889160	435	110840	14
47	787232	271	897810	163	889421	435	110579	13
48	787395	271	897712	163	889682	435	110318	12
49	787557	271	897614	163	889943	435	110057	11
50	787720	271	897516	163	890204	434	109796	10
51	9.787883	271	9.897418	164	9.890465	434	10.109535	9
52	788045	271	897320	164	890725	434	109275	8
53	788208	271	897222	164	890986	434	109014	7
54	788370	270	897123	164	891247	434	108753	6
55	788532	270	897025	164	891507	434	108493	5
56	788694	270	896926	164	891768	434	108232	4
57	788856	270	896828	164	892028	434	107972	3
58	789018	270	896729	164	892289	434	107711	2
59	789180	270	896631	164	892549	434	107451	1
60	789342	269	896532	164	892810	434	107190	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.789342	269	9.896532	164	9.892810	434	10.107190	60
1	789504	269	896433	165	893070	434	106930	59
2	789665	269	896335	165	893331	434	106669	58
3	789827	269	896236	165	893591	434	106409	57
4	789988	269	896137	165	893851	434	106149	56
5	790149	269	896038	165	894111	434	105889	55
6	790310	268	895939	165	894371	434	105629	54
7	790471	268	895840	165	894632	433	105368	53
8	790632	268	895741	165	894892	433	105108	52
9	790793	268	895641	165	895152	433	104848	51
10	790954	268	895542	165	895412	433	104588	50
11	9.791115	268	9.895443	166	9.895672	433	10 104328	49
12	791275	267	895343	166	895932	433	104068	48
13	791436	267	895244	166	896192	433	103808	47
14	791596	267	895145	166	896452	433	103548	46
15	791757	267	895045	166	896712	433	103288	45
16	791917	267	894945	166	896971	433	103029	44
17	792077	267	894846	166	897231	433	102769	43
18	792237	266	894746	166	897491	433	102509	42
19	792397	266	894646	166	897751	433	102249	41
20	792557	266	894546	166	898010	433	101990	40
21	9.792716	266	9.894446	167	9.898270	433	10.101730	39
22	792876	266	894346	167	898530	433	101470	38
23	793035	266	894246	167	898789	433	101211	37
24	793195	265	894146	167	899049	432	100951	36
25	793354	265	894046	167	899308	432	100692	35
26	793514	265	893946	167	899568	432	100432	34
27	793673	265	893846	167	899827	432	100173	33
28	793832	265	893745	167	900086	432	099914	32
29	793991	265	893645	167	900346	432	099654	31
30	794150	264	893544	167	900605	432	099395	30
31	9.794308	264	9.893444	168	9.900864	432	10.099136	29
32	794467	264	893343	168	901124	432	098876	28
33	794626	264	893243	168	901383	432	098617	27
34	794784	264	893142	168	901642	432	098358	26
35	794942	264	893041	168	901901	432	098099	25
36	795101	264	892940	168	902160	432	097840	24
37	795259	263	892839	168	902419	432	097581	23
38	795417	263	892739	168	902679	432	097321	22
39	795575	263	892638	168	902938	432	097062	21
40	795733	263	892536	168	903197	431	096803	20
41	9.795891	263	9.892435	169	9.903455	431	10.096545	19
42	796049	263	892334	169	903714	431	096286	18
43	796206	263	892233	169	903973	431	096027	17
44	796364	262	892132	169	904232	431	095768	16
45	796521	262	892030	169	904491	431	095509	15
46	796679	262	891929	169	904750	431	095250	14
47	796836	262	891827	169	905008	431	094992	13
48	796993	262	891726	169	905267	431	094733	12
49	797150	261	891624	169	905526	431	094474	11
50	797307	261	891523	170	905784	431	094216	10
51	9.797464	261	9.891421	170	9.906043	431	10.093957	9
52	797621	261	891319	170	906302	431	093698	8
53	797777	261	891217	170	906560	431	093440	7
54	797934	261	891115	170	906819	431	093181	6
55	798091	261	891013	170	907077	431	092923	5
56	798247	261	890911	170	907336	431	092664	4
57	798403	260	890809	170	907594	431	092406	3
58	798560	260	890707	170	907852	431	092148	2
59	798716	260	890605	170	908111	430	091889	1
60	798872	260	890503	170	908369	430	091631	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine.	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.798872	260	9.890503	170	9.908369	430	10.091631	60
1	799028	260	890400	171	908628	430	091372	59
2	799184	260	890298	171	908886	430	091114	58
3	799339	259	890195	171	909144	430	090856	57
4	799495	259	890093	171	909402	430	090598	56
5	799651	259	889990	171	909660	430	090340	55
6	799806	259	889888	171	909918	430	090082	54
7	799962	259	889785	171	910177	430	089823	53
8	800117	259	889682	171	910435	430	089565	52
9	800272	258	889579	171	910693	430	089307	51
10	800427	258	889477	171	910951	430	089049	50
11	9.800582	258	9.889374	172	9.911209	430	10.088791	49
12	800737	258	889271	172	911467	430	088533	48
13	800892	258	889168	172	911724	430	088276	47
14	801047	258	889064	172	911982	430	088018	46
15	801201	258	888961	172	912240	430	087760	45
16	801356	257	888858	172	912498	430	087502	44
17	801511	257	888755	172	912756	430	087244	43
18	801665	257	888651	172	913014	429	086986	42
19	801819	257	888548	172	913271	429	086729	41
20	801973	257	888444	173	913529	429	086471	40
21	9.802128	257	9.888341	173	9.913787	429	10.086213	39
22	802282	256	888237	173	914044	429	085956	38
23	802436	256	888134	173	914302	429	085698	37
24	802589	256	888030	173	914560	429	085440	36
25	802743	256	887926	173	914817	429	085183	35
26	802897	256	887822	173	915075	429	084925	34
27	803050	256	887718	173	915332	429	084668	33
28	803204	256	887614	173	915590	429	084410	32
29	803357	255	887510	173	915847	429	084153	31
30	803511	255	887406	174	916104	429	083896	30
31	9.803664	255	9.887302	174	9.916362	429	10.083638	29
32	803817	255	887198	174	916619	429	083381	28
33	803970	255	887093	174	916877	429	083123	27
34	804123	255	886989	174	917134	429	082866	26
35	804276	254	886885	174	917391	429	082609	25
36	804428	254	886780	174	917648	429	082352	24
37	804581	254	886676	174	917905	429	082095	23
38	804734	254	886571	174	918163	428	081837	22
39	804886	254	886466	174	918420	428	081580	21
40	805039	254	886362	175	918677	428	081323	20
41	9.805191	254	9.886257	175	9.918934	428	10.081066	19
42	805343	253	886152	175	919191	428	080809	18
43	805495	253	886047	175	919448	428	080552	17
44	805647	253	885942	175	919705	428	080295	16
45	805799	253	885837	175	919962	428	080038	15
46	805951	253	885732	175	920219	428	079781	14
47	806103	253	885627	175	920476	428	079524	13
48	806254	253	885522	175	920733	428	079267	12
49	806406	252	885416	175	920990	428	079010	11
50	806557	252	885311	176	921247	428	078753	10
51	9.806709	252	9.885205	176	9.921503	428	10.078497	9
52	806860	252	885100	176	921760	428	078240	8
53	807011	252	884994	176	922017	428	077983	7
54	807163	252	884889	176	922274	428	077726	6
55	807314	252	884783	176	922530	428	077470	5
56	807465	251	884677	176	922787	428	077213	4
57	807615	251	884572	176	923044	428	076956	3
58	807766	251	884466	176	923300	428	076700	2
59	807917	251	884360	176	923557	427	076443	1
60	808067	251	884254	177	923813	427	076187	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D	Tang.	D.	Cotang.	
0	9.808067	251	9.884254	177	9.923813	427	10.076187	60
1	808218	251	884148	177	924070	427	075930	59
2	808368	251	884042	177	924327	427	075673	58
3	808519	250	883936	177	924583	427	075417	57
4	808669	250	883829	177	924840	427	075160	56
5	808819	250	883723	177	925096	427	074904	55
6	808969	250	883617	177	925352	427	074648	54
7	809119	250	883510	177	925609	427	074391	53
8	809269	250	883404	177	925865	427	074135	52
9	809419	249	883297	178	926122	427	073878	51
10	809569	249	883191	178	926378	427	073622	50
11	9.809718	249	9.883084	178	9.926634	427	10.073366	49
12	809868	249	882977	178	926890	427	073110	48
13	810017	249	882871	178	927147	427	072853	47
14	810167	249	882764	178	927403	427	072597	46
15	810316	248	882657	178	927659	427	072341	45
16	810465	248	882550	178	927915	427	072085	44
17	810614	248	882443	178	928171	427	071829	43
18	810763	248	882336	179	928427	427	071573	42
19	810912	248	882229	179	928683	427	071317	41
20	811061	248	882121	179	928940	427	071060	40
21	9.811210	248	9.882014	179	9.929196	427	10.070804	39
22	811358	247	881907	179	929452	427	070548	38
23	811507	247	881799	179	929708	427	070292	37
24	811655	247	881692	179	929964	426	070036	36
25	811804	247	881584	179	930220	426	069780	35
26	811952	247	881477	179	930475	426	069525	34
27	812100	247	881369	179	930731	426	069269	33
28	812248	247	881261	180	930987	426	069013	32
29	812396	246	881153	180	931243	426	068757	31
30	812544	246	881046	180	931499	426	068501	30
31	9.812692	246	9.880938	180	9.931755	426	10.068245	29
32	812840	246	880830	180	932010	426	067990	28
33	812988	246	880722	180	932266	426	067734	27
34	813135	246	880613	180	932522	426	067478	26
35	813283	246	880505	180	932778	426	067222	25
36	813430	245	880397	180	933033	426	066967	24
37	813578	245	880289	181	933289	426	066711	23
38	813725	245	880180	181	933545	426	066455	22
39	813872	245	880072	181	933800	426	066200	21
40	814019	245	879963	181	934056	426	065944	20
41	9.814166	245	9.879855	181	9.934311	426	10.065689	19
42	814313	245	879746	181	934567	426	065433	18
43	814460	244	879637	181	934823	426	065177	17
44	814607	244	879529	181	935078	426	064922	16
45	814753	244	879420	181	935333	426	064667	15
46	814900	244	879311	181	935589	426	064411	14
47	815046	244	879202	182	935844	426	064156	13
48	815193	244	879093	182	936100	426	063900	12
49	815339	244	878984	182	936355	426	063645	11
50	815485	243	878875	182	936610	426	063390	10
51	9.815631	243	9.878766	182	9.936866	425	10.063134	9
52	815778	243	878656	182	937121	425	062879	8
53	815924	243	878547	182	937376	425	062624	7
54	816069	243	878438	182	937632	425	062368	6
55	816215	243	878328	182	937887	425	062113	5
56	816361	243	878219	183	938142	425	061858	4
57	816507	242	878109	183	938398	425	061602	3
58	816652	242	877999	183	938653	425	061347	2
59	816798	242	877890	183	938908	425	061092	1
60	816943	242	877780	183	939163	425	060837	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.816943	242	9.877780	183	9.939163	425	10.060837	60
1	817088	242	877670	183	939418	425	060582	59
2	817233	242	877560	183	939673	425	060327	58
3	817379	242	877450	183	939928	425	060072	57
4	817524	241	877340	183	940183	425	059817	56
5	817668	241	877230	184	940438	425	059562	55
6	817813	241	877120	184	940694	425	059306	54
7	817958	241	877010	184	940949	425	059051	53
8	818103	241	876899	184	941204	425	058796	52
9	818247	241	876789	184	941458	425	058542	51
10	818392	241	876678	184	941714	425	058286	50
11	9.818536	240	9.876568	184	9.941968	425	10.058032	49
12	818681	240	876457	184	942223	425	057777	48
13	818825	240	876347	184	942478	425	057522	47
14	818969	240	876236	185	942733	425	057267	46
15	819113	240	876125	185	942988	425	057012	45
16	819257	240	876014	185	943243	425	056757	44
17	819401	240	875904	185	943498	425	056502	43
18	819545	239	875793	185	943752	425	056248	42
19	819689	239	875682	185	944007	425	055993	41
20	819832	239	875571	185	944262	425	055738	40
21	9.819976	239	9.875459	185	9.944517	425	10.055483	39
22	820120	239	875348	185	944771	424	055229	38
23	820263	239	875237	185	945026	424	054974	37
24	820406	239	875126	186	945281	424	054719	36
25	820550	238	875014	186	945535	424	054465	35
26	820693	238	874903	186	945[illegible]0	424	054210	34
27	820836	238	874791	186	946045	424	053955	33
28	820979	238	874680	186	946299	424	053701	32
29	821122	238	874568	186	946554	424	053446	31
30	821265	238	874456	186	946808	424	053192	30
31	9.821407	238	9.874344	186	9.947063	424	10.052937	29
32	821550	238	874232	187	947318	424	052682	28
33	821693	237	874121	187	947572	424	052428	27
34	821835	237	874009	187	947826	424	052174	26
35	821977	237	873896	187	948081	424	051919	25
36	822120	237	873784	187	948336	424	051664	24
37	822262	237	873672	187	948590	424	051410	23
38	822404	237	873560	187	948844	424	051156	22
39	822546	237	873448	187	949099	424	050901	21
40	822688	236	873335	187	949353	424	050647	20
41	9.822830	236	9.873223	187	9.949607	424	10.050393	19
42	822972	236	873110	188	949862	424	050138	18
43	823114	236	872998	188	950116	424	049884	17
44	823255	236	872885	188	950370	424	049630	16
45	823397	236	872772	188	950625	424	049375	15
46	823539	236	872659	188	950879	424	049121	14
47	823680	235	872547	188	951133	424	048867	13
48	823821	235	872434	188	951388	424	048612	12
49	823963	235	872321	188	951642	424	048358	11
50	824104	235	872208	188	951896	424	048104	10
51	9.824245	235	9.872095	189	9.952150	424	10.047850	9
52	824386	235	871981	189	952405	424	047595	8
53	824527	235	871868	189	952659	424	047341	7
54	824668	234	871755	189	952913	424	047087	6
55	824808	234	871641	189	953167	423	046833	5
56	824949	234	871528	189	953421	423	046579	4
57	825090	234	871414	189	953675	423	046325	3
58	825230	234	871301	189	953929	423	046071	2
59	825371	234	871187	189	954183	423	045817	1
60	825511	234	871073	190	954437	423	045563	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.825511	234	9.871073	190	9.954437	423	10.045563	60
1	825651	233	870960	190	954691	423	045309	59
2	825791	233	870846	190	954945	423	045055	58
3	825931	233	870732	190	955200	423	044800	57
4	826071	233	870618	190	955454	423	044546	56
5	826211	233	870504	190	955707	423	044293	55
6	826351	233	870390	190	955961	423	044039	54
7	826491	233	870276	190	956215	423	043785	53
8	826631	233	870161	190	956469	423	043531	52
9	826770	232	870047	191	956723	423	043277	51
10	826910	232	869933	191	956977	423	043023	50
11	9.827049	232	9.869818	191	9.957231	423	10.042769	49
12	827189	232	869704	191	957485	423	042515	48
13	827328	232	869589	191	957739	423	042261	47
14	827467	232	869474	191	957993	423	042007	46
15	827606	232	869360	191	958246	423	041754	45
16	827745	232	869245	191	958500	423	041500	44
17	827884	231	869130	191	958754	423	041246	43
18	828023	231	869015	192	959008	423	040992	42
19	828162	231	868900	192	959262	423	040738	41
20	828301	231	868785	192	959516	423	040484	40
21	9.828439	231	9.868670	192	9.959769	423	10.040231	39
22	828578	231	868555	192	960023	423	039977	38
23	828716	231	868440	192	960277	423	039723	37
24	828855	230	868324	192	960531	423	039469	36
25	828993	230	868209	192	960784	423	039216	35
26	829131	230	868093	192	961038	423	038962	34
27	829269	230	867978	193	961291	423	038709	33
28	829407	230	867862	193	961545	423	038455	32
29	829545	230	867747	193	961799	423	038201	31
30	829683	230	867631	193	962052	423	037948	30
31	9.829821	229	9.867515	193	9.962306	423	10.037694	29
32	829959	229	867399	193	962560	423	037440	28
33	830097	229	867283	193	962813	423	037187	27
34	830234	229	867167	193	963067	423	036933	26
35	830372	229	867051	193	963320	423	036680	25
36	830509	229	866935	194	963574	423	036426	24
37	830646	229	866819	194	963827	423	036173	23
38	830784	229	866703	194	964081	423	035919	22
39	830921	228	866586	194	964335	423	035665	21
40	831058	228	866470	194	964588	422	035412	20
41	9.831195	228	9.866353	194	9.964842	422	10.035158	19
42	831332	228	866237	194	965095	422	034905	18
43	831469	228	866120	194	965349	422	034651	17
44	831606	228	866004	195	965602	422	034398	16
45	831742	228	865887	195	965855	422	034145	15
46	831879	228	865770	195	966109	422	033891	14
47	832015	227	865653	195	966362	422	033638	13
48	832152	227	865536	195	966616	422	033384	12
49	832288	227	865419	195	966869	422	033131	11
50	832425	227	865302	195	967123	422	032877	10
51	9.832561	227	9.865185	195	9.967376	422	10.032624	9
52	832697	227	865068	195	967629	422	032371	8
53	832833	227	864950	195	967883	422	032117	7
54	832969	226	864833	196	968136	422	031864	6
55	833105	226	864716	196	968389	422	031611	5
56	833241	226	864598	196	968643	422	031357	4
57	833377	226	864481	196	968896	422	031104	3
58	833512	226	86436[illegible]	[illegible]	969149	422	030851	2
59	833648	226	864245	[illegible]	969403	422	030597	1
60	833783	226	864127	[illegible]	969656	422	030344	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.833783	226	9.864127	196	9.969656	422	10.030344	60
1	833919	225	864010	196	969909	422	030091	59
2	834054	225	863892	197	970162	422	029838	58
3	834189	225	863774	197	970416	422	029584	57
4	834325	225	863656	197	970669	422	029331	56
5	834460	225	863538	197	970922	422	029078	55
6	834595	225	863419	197	971175	422	028825	54
7	834730	225	863301	197	971429	422	028571	53
8	834865	225	863183	197	971682	422	028318	52
9	834999	224	863064	197	971935	422	028065	51
10	835134	224	862946	198	972188	422	027812	50
11	9.835269	224	9.862827	198	9.972441	422	10.027559	49
12	835403	224	862709	198	972694	422	027306	48
13	835538	224	862590	198	972948	422	027052	47
14	835672	224	862471	198	973201	422	026799	46
15	835807	224	862353	198	973454	422	026546	45
16	835941	224	862234	198	973707	422	026293	44
17	836075	223	862115	198	973960	422	026040	43
18	836209	223	861996	198	974213	422	025787	42
19	836343	223	861877	198	974466	422	025534	41
20	836477	223	861758	199	974719	422	025281	40
21	9.836611	223	9.861638	199	9.974973	422	10.025027	39
22	836745	223	861519	199	975226	422	024774	38
23	836878	223	861400	199	975479	422	024521	37
24	837012	222	861280	199	975732	422	024268	36
25	837146	222	861161	199	975985	422	024015	35
26	837279	222	861041	199	976238	422	023762	34
27	837412	222	860922	199	976491	422	023509	33
28	837546	222	860802	199	976744	422	023256	32
29	837679	222	860682	200	976997	422	023003	31
30	837812	222	860562	200	977250	422	022750	30
31	9.837945	222	9.860442	200	9.977503	422	10.022497	29
32	838078	221	860322	200	977756	422	022244	28
33	838211	221	860202	200	978009	422	021991	27
34	838344	221	860082	200	978262	422	021738	26
35	838477	221	859962	200	978515	422	021485	25
36	838610	221	859842	200	978768	422	021232	24
37	838742	221	859721	201	979021	422	020979	23
38	838875	221	859601	201	979274	422	020726	22
39	839007	221	859480	201	979527	422	020473	21
40	839140	220	859360	201	979780	422	020220	20
41	9.839272	220	9.859239	201	9.980033	422	10.019967	19
42	839404	220	859119	201	980286	422	019714	18
43	839536	220	858998	201	980538	422	019462	17
44	839668	220	858877	201	980791	421	019209	16
45	839800	220	858756	202	981044	421	018956	15
46	839932	220	858635	202	981297	421	018703	14
47	840064	219	858514	202	981550	421	018450	13
48	840196	219	858393	202	981803	421	018197	12
49	840328	219	858272	202	982056	421	017944	11
50	840459	219	858151	202	982309	421	017691	10
51	9.840591	219	9.858029	202	9.982562	421	10.017438	9
52	840722	219	857908	202	982814	421	017186	8
53	840854	219	857786	202	983067	421	016933	7
54	840985	219	857665	203	983320	421	016680	6
55	841116	218	857543	203	983573	421	016427	5
56	841247	218	857422	203	983826	421	016174	4
57	841378	218	857300	203	984079	421	015921	3
58	841509	218	857178	203	984331	421	015669	2
59	841640	218	857056	203	984584	421	015416	1
60	841771	218	856934	203	984837	421	015163	0
	Cosine		Sine		Cotang.		Tang.	M.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.841771	218	9.856934	203	9.984837	421	10.015163	60
1	841902	218	856812	203	985090	421	014910	59
2	842033	218	856690	204	985343	421	014657	58
3	842163	217	856568	204	985596	421	014404	57
4	842294	217	856446	204	985848	421	014152	56
5	842424	217	856323	204	986101	421	013899	55
6	842555	217	856201	204	986354	421	013646	54
7	842685	217	856078	204	986607	421	013393	53
8	842815	217	855956	204	986860	421	013140	52
9	842946	217	855833	204	987112	421	012888	51
10	843076	217	855711	205	987365	421	012635	50
11	9.843206	216	9.855588	205	9.987618	421	10.012382	49
12	843336	216	855465	205	987871	421	012129	48
13	843466	216	855342	205	988123	421	011877	47
14	843595	216	855219	205	988376	421	011624	46
15	843725	216	855096	205	988629	421	011371	45
16	843855	216	854973	205	988882	421	011118	44
17	843984	216	854850	205	989134	421	010866	43
18	844114	215	854727	206	989387	421	010613	42
19	844243	215	854603	206	989640	421	010360	41
20	844372	215	854480	206	989893	421	010107	40
21	9.844502	215	9.854356	206	9.990145	421	10.009855	39
22	844631	215	854233	206	990398	421	009602	38
23	844760	215	854109	206	990651	421	009349	37
24	844889	215	853986	206	990903	421	009097	36
25	845018	215	853862	206	991156	421	008844	35
26	845147	215	853738	206	991409	421	008591	34
27	845276	214	853614	207	991662	421	008338	33
28	845405	214	853490	207	991914	421	008086	32
29	845533	214	853366	207	992167	421	007833	31
30	845662	214	853242	207	992420	421	007580	30
31	9.845790	214	9.853118	207	9.992672	421	10 007328	29
32	845919	214	852994	207	992925	421	007075	28
33	846047	214	852869	207	993178	421	006822	27
34	846175	214	852745	207	993430	421	006570	26
35	846304	214	852620	207	993683	421	006317	25
36	846432	213	852496	208	993936	421	006064	24
37	846560	213	852371	208	994189	421	005811	23
38	846688	213	852247	208	994441	421	005559	22
39	846816	213	852122	208	994694	421	005306	21
40	846944	213	851997	208	994947	421	005053	20
41	9.847071	213	9.851872	208	9.[illegible]95199	421	10.004801	19
42	847199	213	8[illegible]747	208	995452	421	004548	18
43	847327	213	851622	208	995705	421	004295	17
44	847454	212	851497	209	995957	421	004043	16
45	847582	212	851372	209	996210	421	003790	15
46	847709	212	851246	209	996463	421	003537	14
47	847836	212	851121	209	996715	421	003285	13
48	847964	212	850996	209	996968	421	003032	12
49	848091	212	850870	209	997221	421	002779	11
50	848218	212	850745	209	997473	421	002527	10
51	9.848345	212	9.850619	209	9.997726	421	10.002274	9
52	848472	211	850493	210	997979	421	002021	8
53	848599	211	850368	210	998231	421	001769	7
54	848726	211	850242	210	998484	421	001516	6
55	848852	211	850116	210	998737	421	001263	5
56	848979	211	849990	210	998989	421	001011	4
57	849106	211	849864	210	999242	421	000758	3
58	849232	211	849738	210	999495	421	000505	2
59	849359	211	849611	210	999748	421	000253	1
60	849485	211	849485	210	10.000000	421	000000	0
	Cosine		Sine		Cotang.		Tang.	M.

A TABLE OF NATURAL SINES.

	0 Deg.		1 Deg.		2 Deg.		3 Deg.		4 Deg.		
M	Nat. Sine	N. Co-Sine	Nat. Sine	N. Co-Sine	Nat. Sine	N. Co-Sine	Nat. Sine	N. Co-Sine	Nat. Sine	N. Co-Sine	M
0	00000	Unit.	01745	99985	03490	99939	05234	99863	06976	99756	60
1	00029	00000	01774	99984	03519	99938	05263	99861	07005	99754	59
2	00058	00000	01803	99984	03548	99937	05292	99860	07034	99752	58
3	00087	00000	01832	99983	03577	99936	05321	99858	07063	99750	57
4	00116	00000	01862	99983	03606	99935	05350	99857	07092	99748	56
5	00145	00000	01891	99932	03635	99934	05379	99855	07121	99746	55
6	00175	00000	01920	99982	03664	99933	05408	99854	07150	99744	54
7	00204	00000	01949	99981	03693	99932	05437	99852	07179	99742	53
8	00233	00000	01978	99980	03723	99931	05466	99851	07208	99740	52
9	00262	00000	02007	99980	03752	99930	05495	99849	07237	99738	51
10	00291	00000	02036	99979	03781	99929	05524	99847	07266	99736	50
11	00320	99999	02065	99979	03810	99927	05553	99846	07295	99734	49
12	00349	99999	02094	99978	03839	99926	05582	99844	07324	99731	48
13	00378	99999	02123	99977	03868	99925	05611	99842	07353	99729	47
14	00407	99999	02152	99977	03897	99924	05640	99341	07382	99727	46
15	00436	99999	02181	99976	03926	99923	05669	99839	07411	99725	45
16	00465	99999	02211	99976	03955	99922	05698	99838	07440	99723	44
17	00495	99999	02240	99975	03984	99921	05727	99836	07469	99721	43
18	00524	99999	02269	99974	04013	99919	05756	99831	07498	99719	42
19	00553	99998	02298	99974	04042	99918	05785	99833	07527	99716	41
20	00582	99998	02327	99973	04071	99917	05814	99831	07556	99714	40
21	00611	99998	02356	99972	04100	99916	05844	99829	07585	99712	39
22	00640	99998	02385	99972	04129	99915	05873	99827	07614	99710	38
23	00669	99998	02414	99971	04159	99913	05902	99826	07643	99708	37
24	00698	99998	02443	99970	04188	99912	05931	99824	07672	99705	36
25	00727	99997	02472	99969	04217	99911	05960	99822	07701	99703	35
26	00756	99997	02501	99969	04246	99910	05989	99821	07730	99701	34
27	00785	99997	02530	99968	04275	99909	06018	99819	07759	99699	33
28	00814	99997	02560	99967	04304	99907	06047	99817	07788	99696	32
29	00844	99996	02589	99966	04333	99906	06076	99815	07817	99694	31
30	00873	99996	02618	99966	04362	99905	06105	99813	07846	99692	30
31	00902	99996	02647	99965	04391	99904	06134	99812	07875	99689	29
32	00931	99996	02676	99964	04420	99902	06163	99810	07904	99687	28
33	00960	99995	02705	99963	04449	99901	06192	99808	07933	99685	27
34	00989	99995	02734	99963	04478	99900	06221	99806	07962	99683	26
35	01018	99995	02763	99962	04507	99898	06250	99804	07991	99680	25
36	01047	99995	02792	99961	04536	99897	06279	99803	08020	99678	24
37	01076	99994	02821	99960	04565	99896	06308	99801	08049	99676	23
38	01105	99994	02850	99959	04594	99894	06337	99799	08078	99673	22
39	01134	99994	02879	99959	04623	99893	06366	99797	08107	99671	21
40	01164	99993	02908	99958	04653	99892	06395	99795	08136	99668	20
41	01193	99993	02938	99957	04682	99890	06424	99793	08165	99666	19
42	01222	99993	02967	99956	04711	99889	06453	99792	08194	99664	18
43	01251	99992	02996	99955	04740	99888	06482	99790	08223	99661	17
44	01280	99992	03025	99954	04769	99886	06511	99788	08252	99659	16
45	01309	99991	03054	99953	04798	99885	06540	99786	08281	99657	15
46	01338	99991	03083	99952	04827	99883	06569	99784	08310	99654	14
47	01367	99991	03112	99952	04856	99882	06598	99782	08339	99652	13
48	01396	99990	03141	99951	04885	99881	06627	99780	08368	99649	12
49	01425	99990	03170	99950	04914	99879	06656	99778	08397	99647	11
50	01454	99989	03199	99949	04943	99878	06685	99776	08426	99644	10
51	01483	99989	03228	99948	04972	99876	06714	99774	08455	99642	9
52	01513	99989	03257	99947	05001	99875	06743	99772	08484	99639	8
53	01542	99988	03286	99946	05030	99873	06773	99770	08513	99637	7
54	01571	99988	03316	99945	05059	99872	06802	99768	08542	99635	6
55	01600	99987	03345	99944	05088	99870	06831	99766	08571	99632	5
56	01629	99987	03374	99943	05117	99869	06860	99764	08600	99630	4
57	01658	99986	03403	99942	05146	99867	06889	99762	08629	99627	3
58	01687	99986	03432	99941	05175	99866	06918	99760	08658	99625	2
59	01716	99985	03461	99940	05205	99864	06947	99758	08687	99622	1
M	N. Co-Sine	Nat. Sine	N. Co-Sine	Nat. Sine	N. Co-Sine	Nat. Sine	N. Co-Sine	Nat. Sine	N. Co-Sine	Nat. Sine	M
	89 Deg.		88 Deg.		87 Deg.		86 Deg.		85 Deg.		

M	5 Deg. N. S.	5 Deg. N. CS.	6 Deg. N. S.	6 Deg. N. CS.	7 Deg. N. S.	7 Deg. N. CS.	8 Deg. N. S.	8 Deg. N. CS.	9 Deg. N. S.	9 Deg. N. CS.	M
0	08716	99619	10453	99452	12187	99255	13917	99027	15643	98769	60
1	08745	99617	10482	99449	12216	99251	13946	99023	15672	98764	59
2	08774	99614	10511	99446	12245	99248	13975	99019	15701	98760	58
3	08803	99612	10540	99443	12274	99244	14004	99015	15730	98755	57
4	08831	99609	10569	99440	12302	99240	14033	99011	15758	98751	56
5	08860	99607	10597	99437	12331	99237	14061	99006	15787	98746	55
6	08889	99604	10626	99434	12360	99233	14090	99002	15816	98741	54
7	08918	99602	10655	99431	12389	99230	14119	98998	15845	98737	53
8	08947	99599	10684	99428	12418	99226	14148	98994	15873	98732	52
9	08976	99596	10713	99424	12447	99222	14177	98990	15902	98728	51
10	09005	99594	10742	99421	12476	99219	14205	98986	15931	98723	50
11	09034	99591	10771	99418	12504	99215	14234	98982	15959	98718	49
12	09063	99588	10800	99415	12533	99211	14263	98978	15988	98714	48
13	09092	99586	10829	99412	12562	99208	14292	98973	16017	98709	47
14	09121	99583	10858	99409	12591	99204	14320	98969	16046	98704	46
15	09150	99580	10887	99406	12620	99200	14349	98965	16074	98700	45
16	09179	99578	10916	99402	12649	99197	14378	98961	16103	98695	44
17	09208	99575	10945	99399	12678	99193	14407	98957	16132	98690	43
18	09237	99572	10973	99396	12706	99189	14436	98953	16160	98686	42
19	09266	99570	11002	99393	12735	99186	14464	98948	16189	98681	41
20	09295	99567	11031	99390	12764	99182	14493	98944	16218	98676	40
21	09324	99564	11060	99386	12793	99178	14522	98940	16246	98671	39
22	09353	99562	11089	99383	12822	99175	14551	98936	16275	98667	38
23	09382	99559	11118	99380	12851	99171	14580	98931	16304	98662	37
24	09411	99556	11147	99377	12880	99167	14608	98927	16333	98657	36
25	09440	99553	11176	99374	12908	99163	14637	98923	16361	98652	35
26	09469	99551	11205	99370	12937	99160	14666	98919	16390	98648	34
27	09498	99548	11234	99367	12966	99156	14695	98914	16419	98643	33
28	09527	99545	11263	99364	12995	99152	14723	98910	16447	98638	32
29	09556	99542	11291	99360	13024	99148	14752	98906	16476	98633	31
30	09585	99540	11320	99357	13053	99144	14781	98902	16505	98629	30
31	09614	99537	11349	99354	13081	99141	14810	98897	16533	98624	29
32	09642	99534	11378	99351	13110	99137	14838	98893	16562	98619	28
33	09671	99531	11407	99347	13139	99133	14867	98889	16591	98614	27
34	09700	99528	11436	99344	13168	99129	14896	98884	16620	98609	26
35	09729	99526	11465	99341	13197	99125	14925	98880	16648	98604	25
36	09758	99523	11494	99337	13226	99122	14954	98876	16677	98600	24
37	09787	99520	11523	99334	13254	99118	14982	98871	16706	98595	23
38	09816	99517	11552	99331	13283	99114	15011	98867	16734	98590	22
39	09845	99514	11580	99327	13312	99110	15040	98863	16763	98585	21
40	09874	99511	11609	99324	13341	99106	15069	98858	16792	98580	20
41	09903	99508	11638	99320	13370	99102	15097	98854	16820	98575	19
42	09932	99506	11667	99317	13399	99098	15126	98849	16849	98570	18
43	09961	99503	11696	99314	13427	99094	15155	98845	16878	98565	17
44	09990	99500	11725	99310	13456	99091	15184	98841	16906	98561	16
45	10019	99497	11754	99307	13485	99087	15212	98836	16935	98556	15
46	10048	99494	11783	99303	13514	99083	15241	98832	16964	98551	14
47	10077	99491	11812	99300	13543	99079	15270	98827	16992	98546	13
48	10106	99488	11840	99297	13572	99075	15292	98823	17021	98541	12
49	10135	99485	11869	99293	13600	99071	15327	98818	17050	98536	11
50	10164	99482	11898	99290	13629	99067	15356	98814	17078	98531	10
51	10192	99479	11927	99286	13658	99063	15385	98809	17107	98526	9
52	10221	99476	11956	99283	13687	99059	15414	98805	17136	98521	8
53	10250	99473	11985	99279	13716	99055	15442	98800	17164	98516	7
54	10279	99470	12014	99276	13744	99051	15471	98796	17193	98511	6
55	10308	99467	12043	99272	13773	99047	15500	98791	17222	98506	5
56	10337	99464	12071	99269	13802	99043	15529	98787	17250	98501	4
57	10366	99461	12100	99265	13831	99039	15557	98782	17279	98496	3
58	10395	99458	12129	99262	13860	99035	15586	98778	17308	98491	2
59	10424	99455	12158	99258	13889	99031	15615	98773	17336	98486	1
M	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	M
	84 Deg.		83 Deg.		82 Deg.		81 Deg.		80 Deg.		

	10 Deg.		11 Deg.		12 Deg.		13 Deg.		14 Deg.		
M	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	M
0	17365	98481	19081	98163	20791	97815	22495	97437	24192	97030	60
1	17393	98476	19109	98157	20820	97809	22523	97430	24220	97023	59
2	17422	98471	19138	98152	20848	97803	22552	97424	24249	97015	58
3	17451	98466	19167	98146	20877	97797	22580	97417	24277	97008	57
4	17479	98461	19195	98140	20905	97791	22608	97411	24305	97001	56
5	17508	98455	19224	98135	20933	97784	22637	97404	24333	96994	55
6	17537	98450	19252	98129	20962	97778	22665	97398	24362	96987	54
7	17565	98445	19281	98124	20990	97772	22693	97391	24390	96980	53
8	17594	98440	19309	98118	21019	97766	22722	97384	24418	96973	52
9	17623	98435	19338	98112	21047	97760	22750	97378	24446	96966	51
10	17651	98430	19366	98107	21076	97754	22778	97371	24474	96959	50
11	17680	98425	19395	98101	21104	97748	22807	97365	24503	96952	49
12	17708	98420	19423	98096	21132	97742	22835	97358	24531	96945	48
13	17737	98414	19452	98090	21161	97735	22863	97351	24559	96937	47
14	17766	98409	19481	98084	21189	97729	22892	97345	24587	96930	46
15	17794	98404	19509	98079	21218	97723	22920	97338	24615	96923	45
16	17823	98399	19538	98073	21246	97717	22948	97331	24644	96916	44
17	17852	98394	19566	98067	21275	97711	22977	97325	24672	96909	43
18	17880	98389	19595	98061	21303	97705	23005	97318	24700	96902	42
19	17909	98383	19623	98056	21331	97698	23033	97311	24728	96894	41
20	17937	98378	19652	98050	21360	97692	23062	97304	24756	96887	40
21	17966	98373	19680	98044	21388	97686	23090	97298	24784	96880	39
22	17995	98368	19709	98039	21417	97680	23118	97291	24813	96873	38
23	18023	98362	19737	98033	21445	97673	23146	97284	24841	96866	37
24	18052	98357	19766	98027	21474	97667	23175	97278	24869	96858	36
25	18081	98352	19794	98021	21502	97661	23203	97271	24897	96851	35
26	18109	98347	19823	98016	21530	97655	23231	97264	24925	96844	34
27	18138	98341	19851	98010	21559	97648	23260	97257	24953	96837	33
28	18166	98336	19880	98004	21587	97642	23288	97251	24982	96829	32
29	18195	98331	19908	97998	21616	97636	23316	97244	25010	96822	31
30	18224	98325	19937	97992	21644	97630	23345	97237	25038	96815	30
31	18252	98320	19965	97987	21672	97623	23373	97230	25066	96807	29
32	18281	98315	19994	97981	21701	97617	23401	97223	25094	96800	28
33	18309	98310	20022	97975	21729	97611	23429	97217	25122	96793	27
34	18338	98304	20051	97969	21758	97604	23458	97210	25151	96786	26
35	18367	98299	20079	97963	21786	97598	23486	97203	25179	96778	25
36	18395	98294	20108	97958	21814	97592	23514	97196	25207	96771	24
37	18424	98288	20136	97952	21843	97585	23542	97189	25235	96764	23
38	18452	98283	20165	97946	21871	97579	23571	97182	25263	96756	22
39	18481	98277	20193	97940	21899	97573	23599	97176	25291	96749	21
40	18509	98272	20222	97934	21928	97566	23627	97169	25320	96742	20
41	18538	98267	20250	97928	21956	97560	23656	97162	25348	96734	19
42	18567	98261	20279	97922	21985	97553	23684	97155	25376	96727	18
43	18595	98256	20307	97916	22013	97547	23712	97148	25404	96719	17
44	18624	98250	20336	97910	22041	97541	23740	97141	25432	96712	16
45	18652	98245	20364	97905	22070	97534	23769	97134	25460	96705	15
46	18681	98240	20393	97899	22098	97528	23797	97127	25488	96697	14
47	18710	98234	20421	97893	22126	97521	23825	97120	25516	96690	13
48	18738	98229	20450	97887	22155	97515	23853	97113	25545	96682	12
49	18767	98223	20478	97881	22183	97508	23882	97106	25573	96675	11
50	18795	98218	20507	97875	22212	97502	23910	97100	25601	96667	10
51	18824	98212	20535	97869	22240	97496	23938	97093	25629	96660	9
52	18852	98207	20563	97863	22268	97489	23966	97086	25657	96653	8
53	18881	98201	20592	97857	22297	97483	23995	97079	25685	96645	7
54	18910	98196	20620	97851	22325	97476	24023	97072	25713	96638	6
55	18938	98190	20649	97845	22353	97470	24051	97065	25741	96630	5
56	18967	98185	20677	97839	22382	97463	24079	97058	25769	96623	4
57	18995	98179	20706	97833	22410	97457	24108	97051	25798	96615	3
58	19024	98174	20734	97827	22438	97450	24136	97044	25826	96608	2
59	19052	98168	20763	97821	22467	97444	24164	97037	25854	96600	1
M	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S	M
	79 Deg.		78 Deg.		77 Deg.		76 Deg.		75 Deg.		

M	15 Deg. N. S.	15 Deg. N. CS.	16 Deg. N. S.	16 Deg. N. CS.	17 Deg. N. S.	17 Deg. N. CS.	18 Deg. N. S.	18 Deg. N. CS.	19 Deg. N. S.	19 Deg. N. CS.	M
0	25882	96593	27564	96126	29237	95630	30902	95106	32557	94552	60
1	25910	96585	27592	96118	29265	95622	30929	95097	32584	94542	59
2	25938	96578	27620	96110	29293	95613	30957	95088	32612	94533	58
3	25966	96570	27648	96102	29321	95605	30985	95079	32639	94523	57
4	25994	96562	27676	96094	29348	95596	31012	95070	32667	94514	56
5	26022	96555	27704	96086	29376	95588	31040	95061	32694	94504	55
6	26050	96547	27731	96078	29404	95579	31068	95052	32722	94495	54
7	26079	96540	27759	96070	29432	95571	31095	95043	32749	94485	53
8	26107	96532	27787	96062	29460	95562	31123	95033	32777	94476	52
9	26135	96524	27815	96054	29487	95554	31151	95024	32804	94466	51
10	26163	96517	27843	96046	29515	95545	31178	95015	32832	94457	50
11	26191	96509	27871	96037	29543	95536	31206	95006	32859	94447	49
12	26219	96502	27899	96029	29571	95528	31233	94997	32887	94438	48
13	26247	96494	27927	96021	29599	95519	31261	94988	32914	94428	47
14	26275	96486	27955	96013	29626	95511	31289	94979	32942	94418	46
15	26303	96479	27983	96005	29654	95502	31316	94970	32969	94409	45
16	26331	96471	28011	95997	29682	95493	31344	94961	32997	94399	44
17	26359	96463	28039	95989	29710	95485	31372	94952	33024	94390	43
18	26387	96456	28067	95981	29737	95476	31399	94943	33051	94380	42
19	26415	96448	28095	95972	29765	95467	31427	94933	33079	94370	41
20	26443	96440	28123	95964	29793	95459	31454	94924	33106	94361	40
21	26471	96433	28150	95956	29821	95450	31482	94915	33134	94351	39
22	26500	96425	28178	95948	29849	95441	31510	94906	33161	94342	38
23	26528	96417	28206	95940	29876	95433	31537	94897	33189	94332	37
24	26556	96410	28234	95931	29904	95424	31565	94888	33216	94322	36
25	26584	96402	28262	95923	29932	95415	31593	94878	33244	94313	35
26	26612	96394	28290	95915	29960	95407	31620	94869	33271	94303	34
27	26640	96386	28318	95907	29987	95398	31648	94860	33298	94293	33
28	26668	96379	28346	95898	30015	95389	31675	94851	33326	94284	32
29	26696	96371	28374	95890	30043	95380	31703	94842	33353	94274	31
30	26724	96363	28402	95882	30071	95372	31730	94832	33381	94264	30
31	26752	96355	28429	95874	30098	95363	31758	94823	33408	94254	29
32	26780	96347	28457	95865	30126	95354	31786	94814	33436	94245	28
33	26808	96340	28485	95857	30154	95345	31813	94805	33463	94235	27
34	26836	96332	28513	95849	30182	95337	31841	94795	33490	94225	26
35	26864	96324	28541	95841	30209	95328	31868	94786	33518	94215	25
36	26892	96316	28569	95832	30237	95319	31896	94777	33545	94206	24
37	26920	96308	28597	95824	39265	95310	31923	94768	33573	94196	23
38	26948	96301	28625	95816	30292	95301	31951	94758	33600	94186	22
39	26976	96293	28652	95807	30320	95293	31979	94749	33627	94176	21
40	27004	96285	28680	95799	30348	95284	32006	94740	33655	94167	20
41	27032	96277	28708	95791	30376	95275	32034	94730	33682	94157	19
42	27060	96269	28736	95782	30403	95266	32061	94721	33710	94147	18
43	27088	96261	28764	95774	30431	95257	32089	94712	33737	94137	17
44	27116	96253	28792	95766	30459	95248	32116	94702	33764	94127	16
45	27144	96246	28820	95757	30486	95240	32144	94693	33792	94118	15
46	27172	96238	28847	95749	30514	95231	32171	94684	33819	94108	14
47	27200	96230	28875	95740	30542	95222	32199	94674	33846	94098	13
48	27228	96222	28903	95732	30570	95213	32227	94665	33874	94088	12
49	27256	96214	28931	95724	30597	95204	32254	94656	33901	94078	11
50	27284	96206	28959	95715	30625	95195	32282	94646	33929	94068	10
51	27312	96198	28987	95707	30653	95186	32309	94637	33956	94058	9
52	27340	96190	29015	95698	30680	95177	32337	94627	33983	94049	8
53	27368	96182	29042	95690	30708	95168	32364	94618	34011	94039	7
54	27396	96174	29070	95681	30736	95159	32392	94609	34038	94029	6
55	27424	96166	29098	95673	30763	95150	32419	94599	34065	94019	5
56	27452	96158	29126	95664	30791	95142	32447	94590	34093	94009	4
57	27480	96150	29154	95656	30819	95133	32474	94580	34120	93999	3
58	27508	96142	29182	95647	30846	95124	32502	94571	34147	93989	2
59	27536	96134	29209	95639	30874	95115	32529	94561	34175	93979	1
M	74 Deg. N. CS	74 Deg. N. S.	73 Deg. N. CS.	73 Deg. N. S.	72 Deg. N. CS.	72 Deg. N. S.	71 Deg. N. CS.	71 Deg. N. S.	70 Deg. N. CS.	70 Deg. N. S.	M

M	20 Deg.		21 Deg.		22 Deg.		23 Deg.		24 Deg.		M
	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	
0	34202	93969	35837	93358	37461	92718	39073	92050	40674	91355	60
1	34229	93959	35864	93348	37488	92707	39100	92039	40700	91343	59
2	34257	93949	35891	93337	37515	92697	39127	92028	40727	91331	58
3	34284	93939	35918	93327	37542	92686	39153	92016	40753	91319	57
4	34311	93929	35945	93316	37569	92675	39180	92005	40780	91307	56
5	34339	93919	35973	93306	37595	92664	39207	91994	40806	91295	55
6	34366	93909	36000	93295	37622	92653	39234	91982	40833	91283	54
7	34393	93899	36027	93285	37649	92642	39260	91971	40860	91272	53
8	34421	93889	36054	93274	37676	92631	39287	91959	40886	91260	52
9	34448	93879	36081	93264	37703	92620	39314	91948	40913	91248	51
10	34475	93869	36108	93253	37730	92609	39341	91936	40939	91236	50
11	34503	93859	36135	93243	37757	92598	39367	91925	40966	91224	49
12	34530	93849	36162	93232	37784	92587	39394	91914	40992	91212	48
13	34557	93839	36190	93222	37811	92576	39421	91902	41019	91200	47
14	34584	93829	36217	93211	37838	92565	39448	91891	41045	91188	46
15	34612	93819	36244	93201	37865	92554	39474	91879	41072	91176	45
16	34639	93809	36271	93190	37892	92543	39501	91868	41098	91164	44
17	34666	93799	36298	93180	37919	92532	39528	91856	41125	91152	43
18	34694	93789	36325	93169	37946	92521	39555	91845	41151	91140	42
19	34721	93779	36352	93159	37973	92510	39581	91833	41178	91128	41
20	34748	93769	36379	93148	37999	92499	39608	91822	41204	91116	40
21	34775	93759	36406	93137	38026	92488	39635	91810	41231	91104	39
22	34803	93748	36434	93127	38053	92477	39661	91799	41257	91092	38
23	34830	93738	36461	93116	38080	92466	39688	91787	41284	91080	37
24	34857	93728	36488	93106	38107	92455	39715	91775	41310	91068	36
25	34884	93718	36515	93095	38134	92444	39741	91764	41337	91056	35
26	34912	93708	36542	93084	38161	92432	39768	91752	41363	91044	34
27	34939	93698	36569	93074	38188	92421	39795	91741	41390	91032	33
28	34966	93688	36596	93063	38215	92410	39822	91729	41416	91020	32
29	34993	93677	36623	93052	38241	92399	39848	91718	41443	91008	31
30	35021	93667	36650	93042	38268	92388	39875	91706	41469	90996	30
31	35048	93657	36677	93031	38295	92377	39902	91694	41496	90984	29
32	35075	93647	36704	93020	38322	92366	39928	91683	41522	90972	28
33	35102	93637	36731	93010	38349	92355	39955	91671	41549	90960	27
34	35130	93626	36758	92999	38376	92343	39982	91660	41575	90948	26
35	35157	93616	36785	92988	38403	92332	40008	91648	41602	90936	25
36	35183	93606	36812	92978	38430	92321	40035	91636	41628	90924	24
37	35211	93596	36839	92967	38456	92310	40062	91625	41655	90911	23
38	35239	93585	36867	92956	38483	92299	40088	91613	41681	90899	22
39	35266	93575	36894	92945	38510	92287	40115	91601	41707	90887	21
40	35293	93565	36921	92935	38537	92276	40141	91590	41734	90875	20
41	35320	93555	36948	92924	38564	92265	40168	91578	41760	90863	19
42	35347	93544	36975	92913	38591	92254	40195	91566	41787	90851	18
43	35375	93534	37002	92902	38617	92243	40221	91555	41813	90839	17
44	35402	93524	37029	92892	38644	92231	40248	91543	41840	90826	16
45	35429	93514	37056	92881	38671	92220	40275	91531	41866	90814	15
46	35456	93503	37083	92870	38698	92209	40301	91519	41892	90802	14
47	35484	93493	37110	92859	38725	92198	40328	91508	41919	90790	13
48	35511	93483	37137	92849	38752	92186	40355	91496	41945	90778	12
49	35538	93472	37164	92838	38778	92175	40381	91484	41972	90766	11
50	35565	93462	37191	92827	38805	92164	40408	91472	41998	90753	10
51	35592	93452	37218	92816	38832	92152	40434	91461	42024	90741	9
52	35619	93441	37245	92805	38859	92141	40461	91449	42051	90729	8
53	35647	93431	37272	92794	38886	92130	40488	91437	42077	90717	7
54	35674	93420	37299	92784	38912	92119	40514	91425	42104	90704	6
55	35701	93410	37326	92773	38939	92107	40541	91414	42130	90692	5
56	35728	93400	37353	92762	38966	92096	40567	91402	42156	90680	4
57	35755	93389	37380	92751	38993	92085	40594	91390	42183	90668	3
58	35782	93379	37407	92740	39020	92073	40621	91378	42209	90655	2
59	35810	93368	37434	92729	39046	92062	40647	91366	42235	90643	1
M	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	M
	69 Deg.		68 Deg.		67 Deg.		66 Deg.		65 Deg.		

	25 Deg.		26 Deg.		27 Deg.		28 Deg.		29 Deg.		
M	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	M
0	42262	90631	43837	89879	45399	89101	46947	88295	48481	87462	60
1	42288	90618	43863	89867	45425	89087	46973	88281	48506	87448	59
2	42315	90606	43889	89854	45451	89074	46999	88267	48532	87434	58
3	42341	90594	43916	89841	45477	89061	47024	88254	48557	87420	57
4	42367	90582	43942	89828	45503	89048	47050	88240	48583	87406	56
5	42394	90569	43968	89816	45529	89035	47076	88226	48608	87391	55
6	42420	90557	43994	89803	45554	89021	47101	88213	48634	87377	54
7	42446	90545	44020	89790	45580	89008	47127	88199	48659	87363	53
8	42473	90532	44046	89777	45606	88995	47153	88185	48684	87349	52
9	42499	90520	44072	89764	45632	88981	47178	88172	48710	87335	51
10	42525	90507	44098	89752	45658	88968	47204	88158	48735	87321	50
11	42552	90495	44124	89739	45684	88955	47229	88144	48761	87306	49
12	42578	90483	44151	89726	45710	88942	47255	88130	48786	87292	48
13	42604	90470	44177	89713	45736	88928	47281	88117	48811	87278	47
14	42631	90458	44203	89700	45762	88915	47306	88103	48837	87264	46
15	42657	90446	44229	89687	45787	88902	47332	88089	48862	87250	45
16	42683	90433	44255	89674	45813	88888	47358	88075	48888	87235	44
17	42709	90421	44281	89662	45839	88875	47383	88062	48913	87221	43
18	42736	90408	44307	89649	45865	88862	47409	88048	48938	87207	42
19	42762	90396	44333	89636	45891	88848	47434	88034	48964	87193	41
20	42788	90383	44359	89623	45917	88835	47460	88020	48989	87178	40
21	42815	90371	44385	89610	45942	88822	47486	88006	49014	87164	39
22	42841	90358	44411	89597	45968	88808	47511	87993	49040	87150	38
23	42867	90346	44437	89584	45994	88795	47537	87979	49065	87136	37
24	42894	90334	44464	89571	46020	88782	47562	87965	49090	87121	36
25	42920	90321	44490	89558	46046	88768	47588	87951	49116	87107	35
26	42946	90309	44516	89545	46072	88755	47614	87937	49141	87093	34
27	42972	90296	44542	89532	46097	88741	47639	87923	49166	87079	33
28	42999	90284	44568	89519	46123	88728	47665	87909	49192	87064	32
29	43025	90271	44594	89506	46149	88715	47690	87896	49217	87050	31
30	43051	90259	44620	89493	46175	88701	47716	87882	49242	87036	30
31	43077	90246	44646	89480	46201	88688	47741	87868	49268	87021	29
32	43104	90233	44672	89467	46226	88674	47767	87854	49293	87007	28
33	43130	90221	44698	89454	46252	88661	47793	87840	49318	86993	27
34	43156	90208	44724	89441	46278	88647	47818	87826	49344	86978	26
35	43182	90196	44750	89428	46304	88634	47844	87812	49369	86964	25
36	43209	90183	44776	89415	46330	88620	47869	87798	49394	86949	24
37	43235	90171	44802	89402	46355	88607	47895	87784	49419	86935	23
38	43261	90158	44828	89389	46381	88593	47920	87770	49445	86921	22
39	43287	90146	44854	89376	46407	88580	47946	87756	49470	86906	21
40	43313	90133	44880	89363	46433	88566	47971	87743	49495	86892	20
41	43340	90120	44906	89350	46458	88553	47997	87729	49521	86878	19
42	43366	90108	44932	89337	46484	88539	48022	87715	49546	86863	18
43	43392	90095	44958	89324	46510	88526	48048	87701	49571	86849	17
44	43418	90082	44984	89311	46536	88512	48073	87687	49596	86834	16
45	43445	90070	45010	89298	46561	88499	48099	87673	49622	86820	15
46	43471	90057	45036	89285	46587	88485	48124	87659	49647	86805	14
47	43497	90045	45062	89272	46613	88472	48150	87645	49672	86791	13
48	43523	90032	45088	89259	46639	88458	48175	87631	49697	86777	12
49	43549	90019	45114	89245	46664	88445	48201	87617	49723	86762	11
50	43575	90007	45140	89232	46690	88431	48226	87603	49748	86748	10
51	43602	89994	45166	89219	46716	88417	48252	87589	49773	86733	9
52	43628	89981	45192	89206	46742	88404	48277	87575	49798	86719	8
53	43654	89968	45218	89193	46767	88390	48303	87561	49824	86704	7
54	43680	89956	45243	89180	46793	88377	48328	87546	49849	86690	6
55	43706	89943	45269	89167	46819	88363	48354	87532	49874	86675	5
56	43733	89930	45295	89153	46844	88349	48379	87518	49899	86661	4
57	43759	89918	45321	89140	46870	88336	48405	87504	49924	86646	3
58	43785	89905	45347	89127	46896	88322	48430	87490	49950	86632	2
59	43811	89892	45373	89114	46921	88308	48456	87476	49975	86617	1
M	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	M
	64 Deg.		63 Deg.		62 Deg.		61 Deg.		60 Deg.		

M	30 Deg.		31 Deg.		32 Deg.		33 Deg.		34 Deg.		M
	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	
0	50000	86603	51504	85717	52992	84805	54464	83867	55919	82904	60
1	50025	86588	51529	85702	53017	84789	54488	83851	55943	82887	59
2	50050	86573	51554	85687	53041	84774	54513	83835	55968	82871	58
3	50076	86559	51579	85672	53066	84759	54537	83819	55992	82855	57
4	50101	86544	51604	85657	53091	84743	54561	83804	56016	82839	56
5	50126	86530	51628	85642	53115	84728	54586	83788	56040	82822	55
6	50151	86515	51653	85627	53140	84712	54610	83772	56064	82806	54
7	50176	86501	51678	85612	53164	84697	54635	83756	56088	82790	53
8	50201	86486	51703	85597	53189	84681	54659	83740	56112	82773	52
9	50227	86471	51728	85582	53214	84666	54683	83724	56136	82757	51
10	50252	86457	51753	85567	53238	84650	54708	83708	56160	82741	50
11	50277	86442	51778	85551	53263	84635	54732	83692	56184	82724	49
12	50302	86427	51803	85536	53288	84619	54756	83676	56208	82708	48
13	50327	86413	51828	85521	53312	84604	54781	83660	56232	82692	47
14	50352	86398	51852	85506	53337	84588	54805	83645	56256	82675	46
15	50377	86384	51877	85491	53361	84573	54829	83629	56280	82659	45
16	50403	86369	51902	85476	53386	84557	54854	83613	56305	82643	44
17	50428	86354	51927	85461	53411	84542	54878	83597	56329	82626	43
18	50453	86340	51952	85446	53435	84526	54902	83581	56353	82610	42
19	50478	86325	51977	85431	53460	84511	54927	83565	56377	82593	41
20	50503	86310	52002	85416	53484	84495	54951	83549	56401	82577	40
21	50528	86295	52026	85401	53509	84480	54975	83533	56425	82561	39
22	50553	86281	52051	85385	53534	84464	54999	83517	56449	82544	38
23	50578	86266	52076	85370	53558	84448	55024	83501	56473	82528	37
24	50603	86251	52101	85355	53583	84433	55048	83485	56497	82511	36
25	50628	86237	52126	85340	53607	84417	55072	83469	56521	82495	35
26	50654	86222	52151	85325	53632	84402	55097	83453	56545	82478	34
27	50679	86207	52175	85310	53656	84386	55121	83437	56569	82462	33
28	50704	86192	52200	85294	53681	84370	55145	83421	56593	82446	32
29	50729	86178	52225	85279	53705	84355	55169	83405	56617	82429	31
30	50754	86163	52250	85264	53730	84339	55194	83389	56641	82413	30
31	50779	86148	52275	85249	53754	84324	55218	83373	56665	82396	29
32	50804	86133	52299	85234	53779	84308	55242	83356	56689	82380	28
33	50829	86119	52324	85218	53804	84292	55266	83340	56713	82363	27
34	50854	86104	52349	85203	53828	84277	55291	83324	56736	82347	26
35	50879	86089	52374	85188	53853	84261	55315	83308	56760	82330	25
36	50904	86074	52399	85173	53877	84245	55339	83292	56784	82314	24
37	50929	86059	52423	85157	53902	84230	55363	83276	56808	82297	23
38	50954	86045	52448	85142	53926	84214	55388	83260	56832	82281	22
39	50979	86030	52473	85127	53951	84198	55412	83244	56856	82264	21
40	51004	86015	52498	85112	53975	84182	55436	83228	56880	82248	20
41	51029	86000	52522	85096	54000	84167	55460	83212	56904	82231	19
42	51054	85985	52547	85081	54024	84151	55484	83195	56928	82214	18
43	51079	85970	52572	85066	54049	84135	55509	83179	56952	82198	17
44	51104	85956	52597	85051	54073	84120	55533	83163	56976	82181	16
45	51129	85941	52621	85035	54097	84104	55557	83147	57000	82165	15
46	51154	85926	52646	85020	54122	84088	55581	83131	57024	82148	14
47	51179	85911	52671	85005	54146	84072	55605	83115	57047	82132	13
48	51204	85896	52696	84989	54171	84057	55630	83098	57071	82115	12
49	51229	85881	52720	84974	54195	84041	55654	83082	57095	82098	11
50	51254	85866	52745	84959	54220	84025	55678	83066	57119	82082	10
51	51279	85851	52770	84943	54244	84009	55702	83050	57143	82065	9
52	51304	85836	52794	84928	54269	83994	55726	83034	57167	82048	8
53	51329	85821	52819	84913	54293	83978	55750	83017	57191	82032	7
54	51354	85806	52844	84897	54317	83962	55775	83001	57215	82015	6
55	51379	85792	52869	84882	54342	83946	55799	82985	57238	81999	5
56	51404	85777	52893	84866	54366	83930	55823	82969	57262	81982	4
57	51429	85762	52918	84851	54391	83915	55847	82953	57286	81965	3
58	51454	85747	52943	84836	54415	83899	55871	82936	57310	81949	2
59	51479	85732	52967	84820	54440	83883	55895	82920	57334	81932	1
M	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	M
	59 Deg.		58 Deg.		57 Deg.		56 Deg.		55 Deg.		

M	35 Deg.		36 Deg.		37 Deg.		38 Deg.		39 Deg.		M
M	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	M
0	57358	81915	58779	80902	60182	79864	61566	78801	62932	77715	60
1	57381	81899	58802	80885	60205	79846	61589	78783	62955	77696	59
2	57405	81882	58826	80867	60228	79829	61612	78765	62977	77678	58
3	57429	81865	58849	80850	60251	79811	61635	78747	63000	77660	57
4	57453	81848	58873	80833	60274	79793	61658	78729	63022	77641	56
5	57477	81832	58896	80816	60298	79776	61681	78711	63045	77623	55
6	57501	81815	58920	80799	60321	79758	61704	78694	63068	77605	54
7	57524	81798	58943	80782	60344	79741	61726	78676	63090	77586	53
8	57548	81782	58967	80765	60367	79723	61749	78658	63113	77568	52
9	57572	81765	58990	80748	60390	79706	61772	78640	63135	77550	51
10	57596	81748	59014	80730	60414	79688	61795	78622	63158	77531	50
11	57619	81731	59037	80713	60437	79671	61818	78604	63180	77513	49
12	57643	81714	59061	80696	60460	79653	61841	78586	63203	77494	48
13	57667	81698	59084	80679	60483	79635	61864	78568	63225	77476	47
14	57691	81681	59108	80662	60506	79618	61887	78550	63248	77458	46
15	57715	81664	59131	80644	60529	79600	61909	78532	63271	77439	45
16	57738	81647	59154	80627	60553	79583	61932	78514	63293	77421	44
17	57762	81631	59178	80610	60576	79565	61955	78496	63316	77402	43
18	57786	81614	59201	80593	60599	79547	61978	78478	63338	77384	42
19	57810	81597	59225	80576	60622	79530	62001	78460	63361	77366	41
20	57833	81580	59248	80558	60645	79512	62024	78442	63383	77347	40
21	57857	81563	59272	80541	60668	79494	62046	78424	63406	77329	39
22	57881	81546	59295	80524	60691	79477	62069	78405	63428	77310	38
23	57904	81530	59318	80507	60714	79459	62092	78387	63451	77292	37
24	57928	81513	59342	80489	60738	79441	62115	78369	63473	77273	36
25	57952	81496	59365	80472	60761	79424	62138	78351	63496	77255	35
26	57976	81479	59389	80455	60784	79406	62160	78333	63518	77236	34
27	57999	81462	59412	80438	60807	79388	62183	78315	63540	77218	33
28	58023	81445	59436	80420	60830	79371	62206	78297	63563	77199	32
29	58047	81428	59459	80403	60853	79353	62229	78279	63585	77181	31
30	58076	81412	59482	80386	60876	79235	62251	78261	63608	77162	30
31	58094	81395	59506	80368	60899	79318	62274	78243	63630	77144	29
32	58118	81378	59529	80351	60922	79300	62297	78225	63653	77125	28
33	58141	81361	59552	80334	60945	79282	62320	78206	63675	77107	27
34	58165	81344	59576	80316	60968	79264	62342	78188	63698	77088	26
35	58189	81327	59599	80299	60991	79247	62365	78170	63720	77070	25
36	58212	81310	59622	80282	61015	79229	62388	78152	63742	77051	24
37	58236	81293	59646	80264	61038	79211	62411	78134	63765	77033	23
38	58260	81276	59669	80247	61061	79193	62433	78116	63787	77014	22
39	58283	81259	59693	80230	61084	79176	62456	78098	63810	76996	21
40	58307	81242	59716	80212	61107	79158	62479	78079	63832	76977	20
41	58330	81225	59739	80195	61130	79140	62502	78061	63854	76959	19
42	58354	81208	59763	80178	61153	79122	62524	78043	63877	76940	18
43	58378	81191	59786	80160	61176	79105	62547	78025	63899	76921	17
44	58401	81174	59809	80143	61199	79087	62570	78007	63922	76903	16
45	58425	81157	59832	80125	61222	79069	62592	77988	63944	76884	15
46	58449	81140	59856	80108	61245	79051	62615	77970	63966	76866	14
47	58472	81123	59879	80091	61268	79033	62638	77952	63989	76847	13
48	58496	81106	59902	80073	61291	79015	62660	77934	64011	76828	12
49	58519	81089	59926	80056	61314	78998	62683	77916	64033	76810	11
50	58543	81072	59949	80038	61337	78980	62706	77897	64056	76791	10
51	58567	81055	59972	80021	61360	78962	62728	77879	64078	76772	9
52	58590	81038	59995	80003	61383	78944	62751	77861	64100	76754	8
53	58614	81021	60019	79986	61406	78926	62774	77843	64123	76735	7
54	58637	81004	60042	79968	61429	78908	62796	77824	64145	76717	6
55	58661	80987	60065	79951	61451	78891	62819	77806	64167	76698	5
56	58684	80970	60089	79934	61474	78873	62842	77788	64190	76679	4
57	58708	80953	60112	79916	61497	78855	62864	77769	64212	76661	3
58	58731	80936	60135	79899	61520	78837	62887	77751	64234	76642	2
59	58755	80919	60158	79881	61543	78819	62909	77733	64256	76623	1
M	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	M
	54 Deg.		53 Deg.		52 Deg.		51 Deg.		50 Deg.		

M	40 Deg. N. S.	40 Deg. N. CS.	41 Deg. N. S.	41 Deg. N. CS.	42 Deg. N. S.	42 Deg. N. CS.	43 Deg. N. S.	43 Deg. N. CS.	44 Deg. N. S.	44 Deg. N. CS.	M
0	64279	76604	65606	75471	66913	74314	68200	73135	69466	71934	60
1	64301	76586	65628	75452	66935	74295	68221	73116	69487	71914	59
2	64323	76567	65650	75433	66956	74276	68242	73096	69508	71894	58
3	64346	76548	65672	75414	66978	74256	68264	73076	69529	71873	57
4	64368	76530	65694	75395	66999	74237	68285	73056	69549	71853	56
5	64390	76511	65716	75375	67021	74217	68306	73036	69570	71833	55
6	64412	76492	65738	75356	67043	74198	68327	73016	69591	71813	54
7	64435	76473	65759	75337	67064	74178	68349	72996	69612	71792	53
8	64457	76455	65781	75318	67086	74159	68370	72976	69633	71772	52
9	64479	76436	65803	75299	67107	74139	68391	72957	69654	71752	51
10	64501	76417	65825	75280	67129	74120	68412	72937	69675	71732	50
11	64524	76398	65847	75261	67151	74100	68433	72917	69696	71711	49
12	64546	76380	65869	75241	67172	74080	68455	72897	69717	71691	48
13	64568	76361	65891	75222	67194	74061	68476	72877	69737	71671	47
14	64590	76342	65913	75203	67215	74041	68497	72857	69758	71650	46
15	64612	76323	65935	75184	67237	74022	68518	72837	69779	71630	45
16	64635	76304	65956	75165	67258	74002	68539	72817	69800	71610	44
17	64657	76286	65978	75146	67280	73983	68561	72797	69821	71590	43
18	64679	76267	66000	75126	67301	73963	68582	72777	69842	71569	42
19	64701	76248	66022	75107	67323	73944	68603	72757	69862	71549	41
20	64723	76229	66044	75088	67344	73924	68624	72737	69883	71529	40
21	64746	76210	66066	75069	67366	73904	68645	72717	69904	71508	39
22	64768	76192	66088	75050	67387	73885	68666	72697	69925	71488	38
23	64790	76173	66109	75030	67409	73865	68688	72677	69946	71468	37
24	64812	76154	66131	75011	67430	73846	68709	72657	69966	71447	36
25	64834	76135	66153	74992	67452	73826	68730	72637	69987	71427	35
26	64856	76116	66175	74973	67473	73806	68751	72617	70008	71407	34
27	64878	76097	66197	74953	67495	73787	68772	72597	70029	71386	33
28	64901	76078	66218	74934	67516	73767	68793	72577	70049	71366	32
29	64923	76059	66240	74915	67538	73747	68814	72557	70070	71345	31
30	64945	76041	66262	74896	67559	73728	68835	72537	70091	71325	30
31	64967	76022	66284	74876	67580	73708	68857	72517	70112	71305	29
32	64989	76003	66306	74857	67602	73688	68878	72497	70132	71284	28
33	65011	75984	66327	74838	67623	73669	68899	72477	70153	71264	27
34	65033	75965	66349	74818	67645	73649	68920	72457	70174	71243	26
35	65055	75946	66371	74799	67666	73629	68941	72437	70195	71223	25
36	65077	75927	66393	74780	67688	73610	68962	72417	70215	71203	24
37	65099	75908	66414	74760	67709	73590	68983	72397	70236	71182	23
38	65122	75889	66436	74741	67730	73570	69004	72377	70257	71162	22
39	65144	75870	66458	74722	67752	73551	69025	72357	70277	71141	21
40	65166	75851	66480	74703	67773	73531	69046	72337	70298	71121	20
41	65188	75832	66501	74683	67795	73511	69067	72317	70319	71100	19
42	65210	75813	66523	74664	67816	73491	69088	72297	70339	71080	18
43	65232	75794	66545	74644	67837	73472	69109	72277	70360	71059	17
44	65254	75775	66566	74625	67859	73452	69130	72257	70381	71039	16
45	65276	75756	66588	74606	67880	73432	69151	72236	70401	71019	15
46	65298	75738	66610	74586	67901	73412	69172	72216	70422	70998	14
47	65320	75719	66632	74567	67923	73393	69193	72196	70443	70978	13
48	65342	75699	66653	74548	67944	73373	69214	72176	70463	70957	12
49	65364	75680	66675	74528	67965	73353	69235	72156	70484	70937	11
50	65386	75661	66697	74509	67987	73333	69256	72136	70505	70916	10
51	65408	75642	66718	74489	68008	73314	69277	72116	70525	70896	9
52	65430	75623	66740	74470	68029	73294	69298	72095	70546	70875	8
53	65452	75604	66762	74451	68051	73274	69319	72075	70567	70855	7
54	65474	75585	66783	74431	68072	73254	69340	72055	70587	70834	6
55	65496	75566	66805	74412	68093	73234	69361	72035	70608	70813	5
56	65518	75547	66827	74392	68115	73215	69382	72015	70628	70793	4
57	65540	75528	66848	74373	68136	73195	69403	71995	70649	70772	3
58	65562	75509	66870	74353	68157	73175	69424	71974	70670	70752	2
59	65584	75490	66891	74334	68179	73155	69445	71954	70690	70731	1
60	65606	75471	66913	74314	68200	73135	69466	71934	70711	70711	0
M	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	M
	49 Deg.		48 Deg.		47 Deg.		46 Deg.		45 Deg.