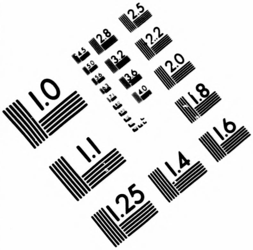

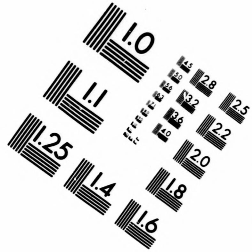

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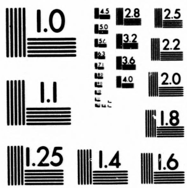


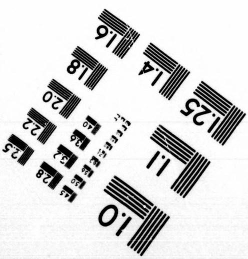

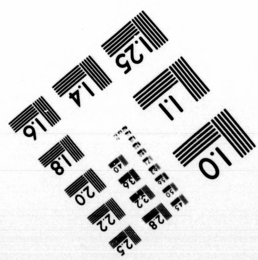

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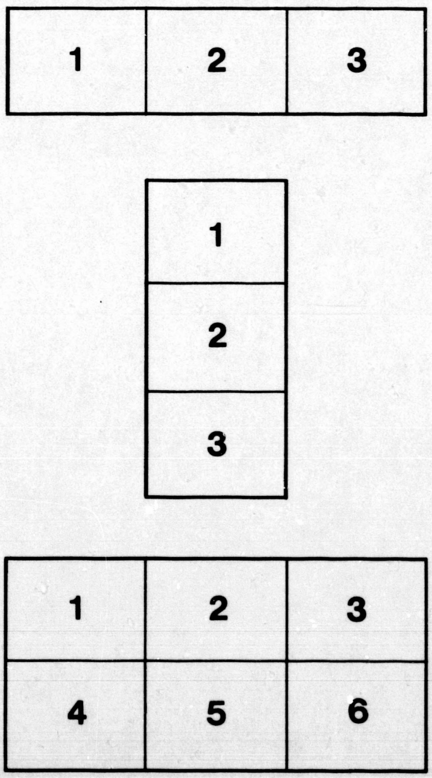
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BAILLAIRGÉ

THE

# STEREOMETRICON.

NEW SYSTEM OF MEASURING

ALL BODIES

BY ONE AND THE SAME RULE.

GENERAL APPLICATION OF THE PRISMOIDAL FORMULA.

NOMENCLATURE AND GENERAL FEATURES OF EACH OF THE

200 MODELS ON THE BOARD.

THE AREAS OF SPHERICAL TRIANGLES AND POLYGONS TO ANY RADIUS OR DIAMETER.

TABLES

*of the Areas of Circles, Segments, Zones — see index, table of Specific gravities.*

QUEBEC
PRINTED BY C. DARVEAU

1884

# BAILLAIRGE'S STER

**New system of determining the solid contents of a body of a**

*(Extract from the "Quebec Daily Mercury"*

Mr. Baillairgé's lecture on Wednesday evening last before the Literary and Historical Society of Quebec, proved once more how very interesting, even in a popular sense, an otherwise dry and abstruse subject, may become, when ably handled.

The lecturer showed the relationship of geometry to all the industries of life. He traced its origin from remote antiquity, its gradual developement up to the present time. He showed how it is the basis of all our public works, and how we are indebted to it for all the constructive arts ; its relationship to mechanics, hydraulics, optics, and all the physical sciences. The fairer portion of mankind, said Mr. B., have the keenest, most appreciative perception of its advantages and beauties, as evidenced in the ever-varying combinations so cunningly devised in their designs for needle tracery, laces and embroidery. He showed its relationship to chemistry in crystallization and polarization ; to botany and zoology in the laws of morphology ; to theology, and so on. In treating of the circle and other conic sections, he drew quite a poetical comparison between the engineer who traces out his curves among the woods and waters of the earth, and the astronomer who sweeps out his mighty circuits amidst the starry forests of the heavens. The parabola was fully illustrated in its application to the throwing of projectiles of war, also as evidenced in jets of water, the speaking trumpet, the mirror and the reflector, which, in lighthouses, gathers the rays of light, as it were, into a bundle, and sends them forth together on their errand of humanity. In treating of the ellipse, this almost magic curve which is traced out in the heavens by every planet that revolves about the sun, by every satellite about its primary, he alluded to that most beautiful of all ovals—the face of lovely woman. He showed how the re-appearance of a comet may now be predicted even to the very day it heaves in sights, and though it has been absent for a century, and how in former ages, when these phenomena were unpredicted, they burst upon the world in unexpected moments, carrying terror everywhere and giving rise to the utmost anxiety and consternation, as if the end of all things were at hand ; in a word, Mr. Baillairgé went over the whole field of geometry and mensuration, both plane and spherical ; a difficult feat within the limits of a single lecture ; and kept the audience, so to say, entranced with interest for two whole hours, which the president, Dr. Anderson, remarked : were to him as but one ; and no doubt it must have been so to others, since Mr. Wilkie, in seconding the vote of thanks proposed by Capt. Ashe, alluded to the pleasure with which he had listened to the lecture as if, he said, it were like poetry to him, instead of the unpromising matter foreshadowed in the title. Mr. Baillairgé next explained in detail his stereometrical tableau, which we hope to see soon introduced into all the schools of this Dominion. He showed how conducive it will be in shortening the time heretofore devoted to the study of solids and even to that of plan
superficies, spherical trigonometry, geometric
perspective, drawing the developement of surf
and shadows, and the like. Mr. Wilkie, so
tunity had been afforded him of proving the
corroborated Mr. B.'s statement in relation to
saving in time, where many abstruse proble
nerally required hours or days to solve, can no
be, as Mr. Baillairgé asserts, so generally app
as has been certified by so many persons in
over their own signatures,) with the help of
mula and tableau, be performed in as many
say nothing of the use the models are in in
glance a knowledge of their nomenclature or
an acquaintanceship with their varied shapes
He showed how, to the architect and enginee
and mechanic, the models are suggestive of th
relative proportions of buildings, roofs, dom
quays, cisterns and reservoirs, cauldrons, vats
and other vessels of capacity, earthworks
comprising railroad and other cuttings and e
the shaft of the Greek and Roman column
waney timber, saw-logs, the camping tent, t
splayed opening of a door or window, nich o
a wall, the vault or arched ceiling of a church
billiard or the cannon ball, or, on a larger sca
earth, sun and planets. Mr. Baillairgé, we
received an order for a tableau from the Mini
cation of New-Brunswick, with the view of i
into all the schools of that Province ; and Mr
writing to Mr. Baillairgé, from France, on
January last, to advise him of the granting
patent for that country, says that Messrs. Hu
the Président and secretary of the society fo
lization of education in France, have intima
tention, at their next general meeting of havin
of distinction conferred on him for the bene
invention and discovery are likely to confer
Mr. Giard, in writing to Mr. Baillairgé, on th
Hon. Mr. Chauveau, Minister of Public Instr
" Il se fera un devoir d'en recommander l'a
" toutes les maisons d'éducation et dans toute
From the Seminary and Laval University.
writes : " Plus on étudie, plus on approfondit
" du cubage des corps, plus on est enchant
" one marvels) de sa simplicité, de sa clarté
" sa grande généralité." Rév. Mr. McQu
" shall be delighted to see the old and tedio
" superseded by a formula so simple and so e
ton, or Yale College, United States : " consi
" bleau a most useful arrangement for sho
" riety and extent of the applications of the fo
College l'Assomption " will adopt Mr. Bai
" tem as part of their course of instruction."
has written to the author that " the rule is

# STEREOMETRICON!

**ontents of a body of any shape, by one and the same rule.**

*bec Daily Mercury" of 30th March,* 1872.)

olids and even to that of plane and convex rical trigonometry, geometrical projection, ving the developement of surfaces, shades, d the like. Mr. Wilkie, so far as oppor- afforded him of proving the calculations, B.'s statement in relation to the immense where many abstruse problems which ge- hours or days to solve, can now (if the rule irgé asserts, so generally applicable, and, fied by so many persons in testimonials guatures,) with the help of the new for- ı, be performed in as many minutes; to he use the models are in imparting at a lge of their nomenclature or names, and hip with their varied shapes and figures. to the architect and engineer, the builder e models are suggestive of the forms and ons of buildings, roofs, domes, piers and nd reservoirs, cauldrons, vats, casks, tubs s of capacity, earthworks of all kinds, ad and other cuttings and embankments, Greek and Roman column, square and w-lors, the camping tent, the square or of a door or window, nich or loophole in or arched ceiling of a church or hall, the nnon ball, or, on a larger scale, the moon, anets. Mr. Baillairgé, we may add, has for a tableau from the Minister of Edu- runswick, with the view of introducing it ols of that Province; and Mr. Vannier, in Baillairgé, from France, on the 10th of advise him of the granting of his letters ountry, says that Messrs. Humbert & Noé, d secretary of the society for the genera- ion in France, have intimated their in- next general meeting of having some mark ferred on him for the benefit which his covery are likely to confer on education. iting to Mr. Baillairgé, on the part of the eau, Minister of Public Instruction, says: evoir d'en recommander l'adoption dans ons d'éducation et dans toutes les écoles." ary and Laval University. Mr. Maingui n étudie, plus on approfondit cette formule corps, plus on est enchanté (the more e sa simplicité, de sa clarté et surtout de néralité." Rév. Mr. McQuarries, B. A. ted to see the old and tedious processes a formula so simple and so exact." New- ege, United States: "considers the ta- seful arrangement for showing the va- t of the applications of the formula." The otion "will adopt Mr. Baillairgé's sys- their course of instruction." Mr. Wilkie e author that "the rule is precise and "simple, and will greatly shorten the processes of calcu- lation. "The tableau," says this competent judge, "com- "prising as it does a great variety of elementary models, "will serve admirably to educate the eye, and must great- "ly facilitate the study of solid mensuration." "Again," says Mr. Wilkie, "the Government would confer a boon "on schools of the middle and higher class by affording "access to so suggestive a collection." There are others who, irrespective of considerations as to the comparative accuracy of the formula, or of its advantages, as applied to mere mensuration, are awake to the fact that the models are so much more suggestive to the pupil and the teacher than their me e representation on a blackboard or on paper, and who, in their written opinion, have alluded especially to this feature of the proposed system. M. Joly President of the Quebec Branch of the Montreal School of Arts and Design, in a letter on the subjects to Mr. Wearer, the Pre- sident of the Board, and after having himself witnessed its advantages on more than one occasion, says, in his expressive style, "the difference is enormous." Professor Tousaint, of the Normal School, Dufresne, of the Mont- magny Academy, Boivin, of St. Hyacinthe, and many others, are of the same opinion; among them MM. R. S. M. Bouchette, O'Farrell, Fletcher, St. Aubin, Steckel, Juneau, Venner, Gallagher, Lafrance, and the late Brother Anthony, &c., &c. Neither will it be forgotten that the professors of the Laval University, after reading the enun- ciation of Mr. B.'s formula, as given in his treatise of 1866, expressed themselves thus: "Un doute involontaire s'em- pare d'abord de l'esprit, lorsqu'on lit le No. 1521; "mais "un examen attentif des paragraphes suivants, dissipe "bientôt ce doute et l'on reste étonné à la vue d'une for- "mule, si claire, si aisée à retenir et dont l'application est "si générale." Mr Fletcher, of the Crown Lands De- partment, says: "I have compared, in the case of seve- "ral solids, the results obtained by your mode of compu- "tation with those resulting from the ordinary and more "lengthy processes, and congratulate you sincerely on "your enunciation of a formula so brief and simple in its "character, and so precise and satisfactory in its results." Mr. Baillairgé also took occasion during his lecture to allude, in other relations, to his treatise on geometry and mensuration, in which he showed he has introduced many important modifications in the usual mode of treating the subject of plane and spherical geometry and trigonometry. In conclusion, we must add that the Council of Public Instruction, at its last meeting, appointed a Committee, composed of the Lord Bishop of Quebec, and of Bishops Langevin and Labrecque, to report to the Council at its next general meeting in June, and who, it may be taken for granted, after the many flattering testimonials in re- lation to the util[illegible] and many advantages of the stereo- metrical tableau for purposes of education, cannot but recommend and direct its adoption in all the schools of the Dominion.

# BAILLAIRGE'S STER

HONORARY MEMBER OF THE SOCIETY FOR THE GENERALIZAT

**New system of measuring all bodies, segments, frustums and ung**

*Thirteen Medals of honor and Seventeen Diplomas from France, Italy, Belgiu*
*United-States of America, and*

This is a Case 5 feet long, 3 feet wide and 5 inches deep, with a hinged exhibiting and affording free access to some 200 well-finished Hardwood Mo form, each of which being merely attached to the board, by means of wire, pe Student or Professor.

The use of the Tableau and accompanying Treatise, reduces the whole science and art of Mensuration from the study of a year to that of a day or two, and so simplifies the study and teaching of Solid Geometry, the Nomenclature of Geometrical and other forms, the developement of surfaces, geometrical projection and perspective, plane and curved areas, Spherical Geometry and Trigonometry, and the mensuration of surfaces and solids, that the several branches hereinbefore mentioned may now be taught even in the most elementary schools, and in convents, where such study could not even have been dreamed of heretofore.

Each tableau is accompanied by a Treatise explanatory of the mode of measurement by the " Prismoïdal Eormula, and an explanation of the solid, its nature, shape, opposite bases, and middle section, its lateral surface developed, etc.

—

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# STEREOMETRICON.

OR THE GENERALIZATION OF EDUCATION IN FRANCE, ETC., ETC.

**nts, frustums and ungulas of these bodies, by one and the same rule.**

*France, Italy, Belgium, Russia, Canada, Japan, etc.* (*Patented in Canada, in the ates of America, and in Europe.*)

deep, with a hinged Glass Cover, under Lockand Key, so as to exclude dust while ished Hardwood Models of every conceivable Elementary, Geometrical or other y means of wire, peg or nail, can be removed and replaced at pleasure, by the

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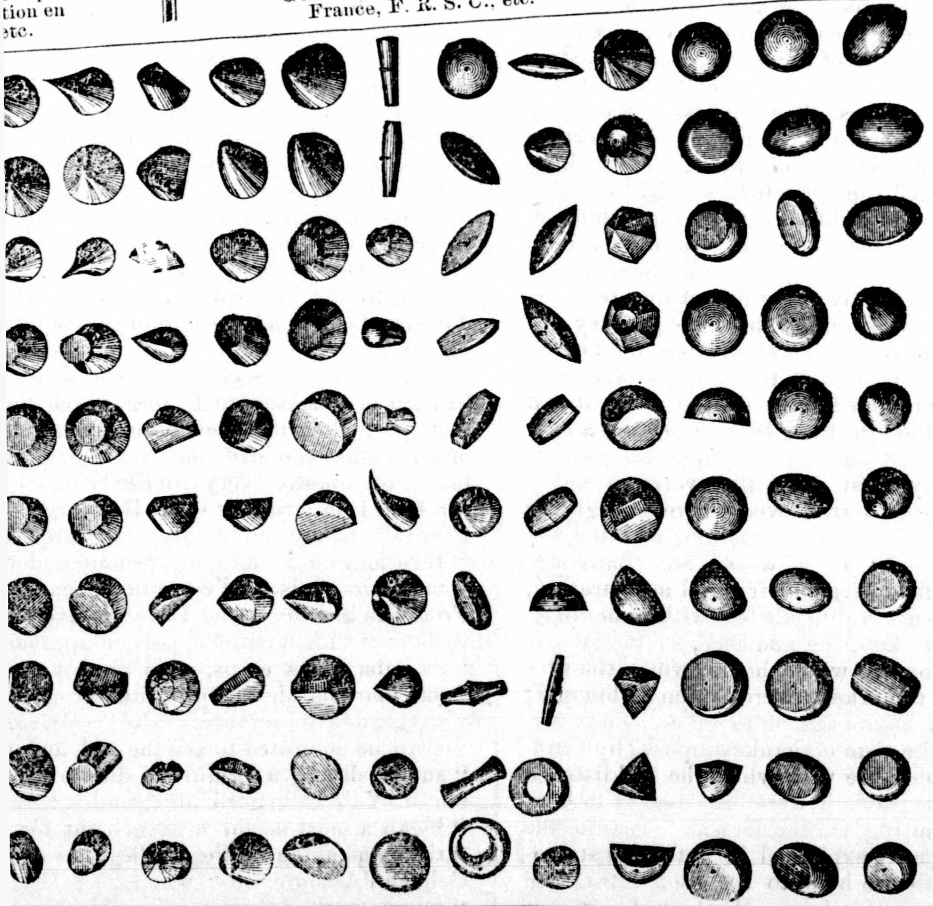
Patented in CANADA, in the UNITED STATES and in EUROPE.

Honorary Member of the Society for the Generalization of Education in France, F. R. S. C., etc.

To find the solid content of any body.

RULE: To the sum of the parallel end areas, add four times the middle area and multiply the whole by one sixth part of the height or length of the body.



Approved by the Council of Public Instruction of the Province of Quebec, and already adopted and ordered by many Establishments in Canada and elsewhere. For information and testimonials apply (free of charge) to

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CANADA.

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**Etc., Etc., Etc.**

ents and Apprentices, Customs and Excise Officers, Professors of Geometry and and other Educational Establishments, Schools of Art and Design, Mechanics, nd others in Canada and elsewhere.

# BAILLAIRG

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**New system of measuring all bo**

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This is a Case 5 feet long, 3 feet wide a while exhibiting and affording free access to some 2 ther form, each of which being merely attached to t the Student or Professor.

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—

*Agents wanted for the sale of the Stereometricon in Canada, the United States, &c.*

Pour trouver le volume d'un corps quelconque.

—

RÈGLE : A la somme des surfaces des extrémités parallèles, ajouter quatre fois la surface au centre, et multiplier le tout par la sixième partie de la hauteur ou longueur du solide.

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For the use of Architects, Engineers, Surv and Mathematics, Universities, Colleges, Seminarie anics, Measurers, Gaugers, Ship-builders, Contractors

# THE

# STEREOMETRICON.

ORIGINATOR : C. BAILLAIRGÉ, M. S.

MEMBER OF THE SOCIETY FOR THE GENERALIZATION OF EDUCATION IN FRANCE, AND OF SEVERAL LEARNED AND SCIENTIFIC SOCIETIES ; CHEVALIER OF THE ORDER OF ST. SAUVEUR DE MONTE-REALE, ITALY ; ETC., ETC. FELLOW OF THE ROYAL SOCIETY, CANADA.

MEASUREMENT OF ALL SOLIDS BY ONE AND THE SAME RULE.

UNIVERSAL APPLICATION OF THE PRISMOIDAL FORMULA.

THIRTEEN MEDALS AND SEVENTEEN DIPLOMAS AND LETTERS AWARDED THE AUTHOR FROM RUSSIA, FRANCE, ITALY, BELGIUM, JAPAN, ETC.

PROMOTER : THOMAS WHITTY,

PROFESSOR AT ST. DENIS ACADEMY, MONTREAL.

---

Comprises 200 Solids representative of all conceivable elementary forms, as of the Component parts of Compound bodies.

---

Name and description of each solid. What it is representative or suggestive of, or that of which it forms a component part.

---

Nature and name of opposite bases and of middle section, as of lateral faces and remainder of bounding Area, including every species of Plane, Spherical, Spheroidal, and Conoidal figures.

---

Division I, classes I to X : plane faced Solids and Solids of single curvature.
Division II, classes XI to XX : Solids of double curvature.

QUEBEC
PRINTED BY C. DARVEAU

1884

# INDEX



# TABLES

[illegible]

[illegible]

V. [illegible]

VI. [illegible]

VII. [illegible]

VIII. [illegible]

IX. [illegible]

X. [illegible]

# THE STEREOMETRICON

ORIGINATOR: **C. BAILLAIRGÉ, M .S.**

Member of the Society for the Generalisation of Education in France and of several learned and scientific Societies: Chevalier of the Order of St. Sauveur de Monte. Reale, Italy; Fellow of the Royal Society of Canada, etc., etc.. etc.

Measurement of all solids by one and the same rule.
Universal application of the prismoïdal formula.

Thirteen Medals and seventeen Diplomas and letters awarded the author, from France, Russia, Italy, Belgium, Japan, etc.

PROMOTER: THOMAS WHITTY, professor at St. Denis Academy, Montreal, etc.

**RULE:** *To the sum of the opposite and parallel end areas, add four times the area of a section midway between and parallel to the opposite bases; multiply the whole by $\frac{1}{6}$ part of the length or height or diameter of the solid, perpendicular to the bases; the result will be the solidity or volume, the capacity or contents of the body, figure or vessel under consideration.*

For application of the rule and examples of all kinds fully worked out, see "Key to Stereometricon."

For areas of all kinds, plane, and of single and double curvature, see also "Key to Stereometricon," with tables of areas of circles to eighths, tenths and twelfths of an inch, or of any other unit of measure, tables of segments and zones of a circle, etc., etc., at end of "Key."

The tableau comprises 200 models, disposed in 10 horizontal and 20 vertical rows, series, families or classes. The solids may be indifferently placed, and numbered from the right or left and from below upwards or the contrary.

The solids are representative of all conceivable elementary forms and figures, as of the component parts of all compound bodies.

---

# DIVISION I.

Plane faced solids and solids of single curvature, or of which the surfaces are capable of being developed in a plane.

## CLASS I.

## Prisms.

**Note.—The author uses the term "trapezium" and not "trapezoid," as the termination "oid" conveys the idea of a solid as paraboloid, hyperboloid, conoid, prismoid, etc. For the same reason he uses the French "trapeziform" instead of trapezoidal.**

| | |
|---|---|
| Name of solid, object of which it is representative or suggestive, or of which it forms a component part. | Nature and name of opposite bases and middle section, lateral faces and remainder of bounding surface. |
| Reference to "Key to Stereometricon," for computation of contents and of factors necessary thereto. | Reference to page or paragraph of "Key" for calculation of areas and of factors necessary thereto. |

---

| | |
|---|---|
| **1—The cube or hexaedron — one of the five platonic bodies** | Each of its three pairs of opposite and parallel faces or of its six faces or bases and middle sections, **perfect and equal squares.** For developed surface. See "Key to Ster.," page 131. |
| Representative of any other rectangular prism, of a building or block of buildings or of one of the component parts thereof; a brick or | Representative of the floor, ceiling, |

cut stone, a pedestal, a die or dado; a pier or quay; a box, chest, package of merchandise or parcel; a cistern, bin, vat or other vessel of capacity; a pile of bricks, stones, lumber, books, etc., etc., etc.

"Key to Ster.," p. 61, par. (78).

wall or partitions of a rectangular room or apartment, or of the bases and sides of the various objects mentioned under the name of the solid.

See "Key to Ster.," page 60.

---

**2—A right isosceles triangular prism**

On end, a triangular block or building; on its base, a ridge roof; on one of its sides, the roof of a penthouse or lean-to. "Key to Ster. p. 61.

Its opposite and parallel bases and middle section, **equal right-angled isosceles triangles.** Its sides or lateral faces **rectangles.** For areas, see "Key to Ster.," pages 19, 22 and 60. Sides suggestive of those of objects alluded to.

---

**3—A right regular pentagonal prism.**

On end, the base or component part of the shaft of a pentagonal pier or column; on one of its sides, a baker's, butcher's or other van; an ambulance, etc. "Key," page 61.

Its opposite and parallel bases and middle section, **regular and equal pentagons**; sides or lateral faces, **rectangles.**

Areas suggestive of those of objects mentioned in adjoining column. "Key," pages 35 and 19.

---

**4—A right regular octagonal prism.**

Base or shaft of a column, a pier or post, a bead, baluster, hand-rail, etc. "Key to Ster.," page 61.

Its parallel and opposite bases and section, **regular and equal octagons**; its sides or lateral faces, **rectangles.** "Key," pages 36, 19.

---

**5—Oblique hexagonal prism**

An inclined post or strut or the section of a stair-rail, a baluster on a rake, etc. Mitred section of a rail or bead. "Key to Ster,," page 64.

Its parallel bases and section, **symmetrical and equal hexagons**; its sides, **parallelograms.** "Key," pp. 26, 19 and 63. compute **half of sym. hex. as a trapezium.**

**6—Oblique rectangular prism.**

On end, an inclined strut or post, etc; on its parallelogram base, the pier of a skew bridge, portion of a mitred fillet, etc.

See "Key to Ster.," page 64.

Two of its three pairs of opposite and paralled faces or bases and sections, **equal rectangles**; the other bases and section, **equal parallelograms.** "Key," page 63.

---

**7—Oblique prism or parallelopipedon.**

Section of mitred fillet on an inclined or oblique surface, etc.

Each of its three pairs of parallel faces or bases and sections, **equal parallelograms.**

---

**8—A righ rectangular trapeziform prism, or a prism of which the base or section is a rectangular trapezium,**

On end, a pier or block of that shape; on its larger parallel face or base, the partially flat roof of a pent-house or lean-to; the base of a rectangular stack of chimneys on a sloped roof or gable, a corbel, etc. See "Key to Ster.," page 61.

Its opposite and parallel bases and section: on end, **equal rectangular trapeziums**; its lateral faces, **rectangles**; on either of its parallel sides or faces: its bases, **rectangles**; its lateral faces, **rectangles and trapeziums** See "Key to Ster.," pages 60 and 29.

May be treated indifferently as a prism or prismoid.

---

**9—A right trapeziform prism.**

On end, the splayed opening of a door or window or loop-hole in a wall; on broader base, a partially flat roof; on its lesser parallel base, a bin or through or other vessel of capacity, section of a ditch excavation or of a railroad embankment on level ground, a scow or pontoon.

On end, its bases and section, **trapeziums**, and sides, **rectangles**; on either of its parallel faces, its bases and section, **rectangles**; its sides, **rectangles** and **trapeziums.**

N. B. – Its solid contents, like those of Nos. 2 and 8, may be computed either as prisms or prismoids.

---

**10—A right or oblique polygonal compound prism, decom-**

Rule for solid contents: multiply one-third the sum of the three vert-

**posable into right or oblique triangular prisms or frusta of prisms**

An excavation or filling, etc.

A spoil bank or a borrowing pit.

Each frustum or component part may be treated as a prismoid, one of its sides being the base.

ical edges or depths of each of the component triangular prisms, or frusta of triangular prisms by the area of a section perpendicular to sides or horizontal, and add the results. Page 67, rule II, "Key."

---

CLASS II.

## Prisms, Frusta and Ungulae of Prisms.

**11—A right regular triangular prism.**

On end, a triangular building, pier or block; on one of its sides, the gable of a wall, the roof of a gabled house, etc.

"Key to Ster.," page 61.

Its parallel bases and section, equal **equilateral triangles** ; its faces, **rectangles.** Compute **as** prismoid with rectangular bases, the upper base then being **an arris** or **line.**

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**12—Lateral wedge or ungula of a right hexagonal prism, by a plane through edge of base,**

Portion of a mitred bead or hand-rail, end of stair baluster under hand-rail, ridge roof of an octagonal tower against a wall; base of a chimney stack on a sloped roof or gable.

One of its parallel bases a **regular hexagon** ; its middle base a **half hexagon or trapezium** ; its upper base **a line** ; its lateral faces **a line, a rectangle, triangles** and **trapeziums** ; its sloped face **a symmetrical hexagon or 2 trapeziums, base to base.**

**13—Lateral ungula of a right hexagonal prism, by a plane through opposite angles of the solid.**

Base of a chimney stack, vase or ornament on a sloped roof or gable, etc.

N. B.—This solid and the last, are not prismoids according to the definition thereof, page 163, par. (206), "Key to Ster.;" but the upper half, folded over and applied to the lower half, evidently completes the prism, and hence the solidity is exactly obtained by the prismoidal formula, as it is of a like frustum of a cylinder or of an ungula thereof by a plane through edge of base.

One of its opposite and parallel bases, a **regular hexagon** ; the other, a **point** ; its middle section a **half hexagon or two rectangular trapeziums base to base** ; its lateral faces, **trapeziums** and **triangles** ; its plane of section, a **symmetrical hexagon**, which, for area, regard as two equal trapeziums base to base, compute and add.

See "Key to Ster.," page 29.

Or the symmetrical hexagon may be decomposed into a rectangle and two equal triangles, for computation of area.

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**14—Central wedge or ungula of a right hexagonal prism; a prismoid.**

A wedge, the ridge roof of a tower, the base of a chimney stack, vase or ornament between two gables.

One of its parallel bases, a **hexagon** ; the other, a **line** ; its middle section, a **symmetrical hexagon or two trapeziums, base to base** ; its lateral faces, **triangles** and **trapeziums.**

See "Key to Ster.," page 29.

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**15—An oblique trapeziform prism.**

The partially flat roof to a dormer window, the roof of a building abutting against another roof, the splayed opening of a basement window, mitred portion of a batten or moulding, section of a ditch excavation, or of an embankment on a slope.

Treated as a prismoid: its opposite and parallel bases, **unequal rectangles** ; its lateral faces, **trapeziums.**

The factors of its middle section arithmetical means between those of its opposite and parallel bases.

**16—An oblique triangular prism.**

The roof of a dormer window or of a wing to a house with a sloped roof, a mitred moulding or fillet, etc.

Treated as a prismoid: one of its opposite and parallel bases, a **rectangle**; the other, a **line**; its lateral faces, **equal triangles** and **parallelograms.**

---

**17—Frustum of a right triangular prism.**

Ridge roof of a building against a wall, a mitred moulding, etc.

As a prismoid: one of its parallel bases, a **rectangle**; its opposite base, a **line**; its middle section, a **rectangle.**

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**18—Irregular frustum of an oblique triangular prism.**

Ridge roof of a building of irregular plan abutting on the unequally sloped roof of another building, etc.

Considered as a prismoid: one base, a **trapezium**, the other, a **line**; its middle section, a **trapezium**; its ends, **non-parallel triangles**; its sides, **trapeziums.**

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**19—A right prism on a mixtilinear base.**

On end, the unsplayed opening of a door or window in a wall, etc.

Note, for area of segment of circle or ellipse, "Key," pages 33, 44, 51, 53, 57, tables II, III, IV, VIII.

Parallel bases and section **mixtilinear figures**, decomposable into a rectangle and the segment or half of a circle or ellispis; the lateral face, a **continuous rectangle.**

Note.—The segment of a circle or ellipse may be equal to, less or greater than a semi-circle.

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**20—Regular frustrum of an oblique triangular prism.**

A ridge roof, mitred fillet, etc.

As a prismoid: one base, a **rectangle**; the other, a **line**; the middle section, a **rectangle.**

CLASS III.

## Frusta of Prisms, Prismoids, Wedges.

**21—The dodecahedron, or twelve-sided solid, one of the five platonic bodies.**

Assemblage of twelve equal pyramids with pentagonal bases, their apices or summits meeting in the centre of the solid or of the circumscribed sphere.

The capital or intermediate section of a pentagonal shaft or column, a finial or other ornament.

The six pairs of parallel bases or twelve component faces of the solid, **equal and regular pentagons**; **the** middle section a **regular decagon,** the side of which is equal to half the diagonal of the pentagon, tor area of which see "Key to Ster.," page 36, rule II; or compute one of the component pyramids and multiply by twelve. For developed surface, see "Key to Ster," page 132.

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**22—A rectangular wedge, the head or heel broader than the blade or edge.**

The frustum of a triangular prism, or may be treated as a prismoid, using either of its three pairs of parallel bases.

An inclined plane, a low pent roof, an ordinary wedge, etc.

On end: its opposite and parallel bases, a **rectangle** and a **line**; its middle base or section, a **rectangle. On** one of either of its other two pairs of parallel bases; one base, a **trapezium**, the other, a **line**; the middle section a **trapezium**; side faces, a **rectangle** and **triangles.**

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**23—A rectangular wedge or inclined plane the head or heel of equal breadth** with **the edge or blade.**

A right triangular prism, body of a dormer window or base of a chimney stack on a low or steep roof, etc.

Each of its three pairs of parallel bases, a **rectangle** and a **line**; its middle sections, **rectangles,** respectively equal to half the corresponding bases. May also be treated as a triangular prism, with bases and section **equal triangles.**

**24—An isosceles wedge, the edge or blade broader than the heel.**

May also be considered, the frustum of a triangular prism or prismoid with three pairs of parallel bases.

As a prismoid: one of its pairs of parallel bases, a **rectangle** and a **line**; middle section, a **rectangle**; each other pair of parallel bases, a **trapezium** and a **line**; middle section, a **trapezium**.

---

**25—Frustum of a right rectangular trapeziform prism, or a prismoid.**

A roof, partially flat, abutting against a vertical wall at one end and in rear, against a sloped roof at the other, etc.

As a prismoid: its opposite and parallel bases, **rectangles**; the longer side of the one corresponding to the shorter side of the other; its middle section, a **rectangle**; all its lateral faces, **trapeziums**.

---

**26—Irregular frustum of an oblique trapeziform prism.**

A roof between two others not parallel, irregular section of a ditch or embankment.

As a prismoid: its opposite and parallel bases and middle section, **trapeziums**; its lateral faces, **trapeziums**.

Factors of middle section arithmetic means between those of the bases.

---

**27—Frustum of a right isosceles trapeziform prism, a prismoid.**

On its larger base, a roof, section of an embankment, etc.; on its lesser base, a bin or vessel of capacity; the capital of a pilaster, a corbel; on end, a splayed opening in a wall.

As a prismoid: its opposite and parallel bases and middle section, **rectangles**; lateral faces, **trapeziums**.

In all such solids, the half way factors need never be measured, as they are always means between the parallel bases of the trapezium faces.

---

**28—Frustum of an isosceles triangular prism, a prismoid.**

Ridge roof with ends unequally sloped, mitred moulding, etc.

As a prismoid: one of its opposite and parallel bases, a **rectangle**; the other, a **line**; its middle section, a **rectangle**. "Key," page 19.

**29—Frustum of a trapeziform prism, a prismoid.**

A flat roof, etc.; on its lesser parallel base, a bin or reservoir, a vehicle of capacity, a scow, a pontoon; on end or its parallel faces vertical, the splayed opening of a window.

As a prismoid: its opposite parallel bases and middle section **rectangles**; its lateral faces, **trapeziums.** Factors of intermediate section or middle base, arithmetic means between those of the end bases.

"Key to Ster.," page 29.

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**30—A prismoid on a mixtilinear base.**

The roof of a building, circular at one end or coved celling of a room; on its lesser base, a bathing tub, etc.; vertically, the splayed opening of a circular headed window in a wall.

Its opposite and parallel bases and middle section, **mixtilinear figures**; the one a **rectangle** and a **semi-circle**; the other two, **rectangles and semi-ellipses**; its arched end developed, a sort of trapezium with curved bases; its area equal to half sum of bases by mean breadth or height.

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CLASS IV.

# Prismoids, etc.

**31—The icosahedron, or twenty-sided solid; one of the five platonic bodies.**

An assemblage of twenty equal pyramids on triangular bases, their apices or summits meeting in a common point, the centre of the solid or of the circumscribed or inscribed sphere.

The ten pairs of parallel bases or twenty component faces of the solid are **equal equilateral triangles.** Its middle section, a **regular dodecagon.** Its middle section parallel to two opposite apices or to the bases of any two opposite pentagonal pyramids of the solid, a **regular decagon,** whose side is

A finial or other ornament, etc. More expeditious to treat it for solidity by computing one of its component pyramids, and multiplying the result by twenty.

equal to half that of one of the edges of the solid. For developed surface, see "Key to Ster," p. 133.

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**32—A prismoid, both its bases, lines. Irregular triangular pyramid.**

Dormer or gablet abutting on a sloped roof. Component section of No. 79. "Key" p. 165, par. (212).

Its opposite bases — considering the solid as a prismoid resting on one of its parallel edges—**lines**; its middle section a **rectangle.** See "Key to Ster.," page 164, par, (208).

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**33—A prismold on a trapeziform base.**

A cutting or embankment, etc.

One of its paralled bases, a **trapezium**; the other, a **line**; its middle section, a **trapezium.**

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**34—A railroad prismold on a side slope.**

Section of a railroad cutting or embankment on ground, sloping laterally or in one direction only.

Its end sections or bases and middle parallel section **equal quadrilaterals,** for area of which see "Key to Ster.," page 30.

This prismoid is a prism on an irregular base, and may be so treated.

---

**35—A railroad prismold on a grade and side slope, or on ground sloping both laterally and longitudinally.**

Its narrow base upwards, an embankment; the same downwards, a cutting or excavation.

Its opposite and parallel end bases and middle section, **quadrilaterals,** the factors of the middle section being all arithmetic means between those of the corresponding end areas.

---

**36—A square or rectangular prismoidal stick of timber.**

A squared log, a tapering post,

Its end bases and middle section **squares or rectangles**.

Timber is usually measured by

the shaft of a chimney or high tower, a reducer between rectangular conduits of unequal size, etc.

Note.—25 per cent. of the whole or true content is 33⅓ per cent., or one-third of the erroneous result.

multiplying its middle section into its length. This gives an erroneous result; the more tapering the timber is, the more so. If it tapered to a point the error would be 25 per cent., or one-quarter of the whole in defect.

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**37—A prismoidal stick of waney timber**

A log of waney timber; on end, the shaft of a chimney, a high tower, a tapering post.

Its opposite bases and middle section, **symmetrical octagons,** for area of which see "Key," p. 176, par. (272), or squares or rectangles with chamfered corners or angles.

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**38—A concavo-convex prismoid or curved wedge.**

A corbel, spandrel, finial, etc.; a brake, a cam, etc. "Key to Ster.," par. (141).

Its opposite bases, a **rectangle** and a line; its middle section, a **rectangle**; its developed faces, **trapeziums**; sides, **mixtilinear triangle.**

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**39—A recto-concave prismoid, or frustun of a curved wedge.**

A corbel, spandrel, buttress, etc. May be decomposed, as also No. 38, into two sections for more exact computation of solid contents.

Its opposite and parallel bases and middle section, **rectangles**; its developed faces **trapeziums**; its lateral faces **mixtilinear trapeziums**

For areas see "Key," page 57.

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**40—Frustum of a rectangular trapeziform prism, a prismoid**

A flat roof in a rectangular corner; on its lesser base, an angular corbel, a sink, cistern, bin, etc.

As a prismoid, its opposite and parallel bases and middle section, **rectangles**; its lateral faces, **trapeziums.**

"Key," page 104, par. (141).

CLASS V.

# Prismoids, etc.

**41—The octahedron or eight-sided figure; one of the five platonic bodies.**

Assemblage of eight equal pyramids on triangular bases, their apices meeting in a common point, the centre of the solid; or two quadrangular pyramids, base to base.

Its four pairs of parallel bases or eight component faces, **equilateral triangles**; its middle section, a **regular hexagon**; its middle section through opposite apices and perpendicular to intervening arris or edge a **lozenge**; through four apices, a **square**. For developed surface see "Key to Ster," page 132.

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**42—A prismoid, one of its bases a square, the other an octagon**

Base or capital of a column, roof of a square tower, a tower, pier, vessel of capacity, component section of a steeple, etc.

Its opposite and parallel bases, a **square and an octagon**; the middle section, a **symmetrical octagon**; its lateral faces, **triangles** and **trapeziums.** For area of symmetrical octagon, see "Key," par. (272).

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**43—A prismoid, its opposite bases, a square and a circle.**

Base or capital of a column, roof of a square tower, a tower, pier, vessel of capacity, a lighthouse, a section of steeple or belfry, a reducer between a square and circular conduit.

One of its opposite and parallel bases a **square**; the other, a **circle**; the middle section, a **mixtilinear figure or a square with rounded corners.**

Its lateral surface capable of development into a plane **trapeziform figure,** one base circular, the other polygonal.

---

**44—A prismoid, its bases unequal squares set diagonally.**

Representative of the same objects as solids, Nos. 42 and 43.

Its opposite bases **unequal squares** set diagonally to each other; the middle section, a **symmetrical octagon**; its lateral faces, **triangles.**

**45—A prismoid its bases a hexagon and a rectangle.**

Representative of nearly the same objects as the three last solids.

One of its bases, a **hexagon**; other a **rectangle**; its middle section a **symmetrical octagon**; its lateral faces, **rectangles** and **triangles**.

---

**46—The lateral frustum of a rectangular prolate spindle.**

Roof of a square tower, component part of a steeple, etc.

Its parallel bases and section, **squares**; its lateral surface, **mixtilinear figures** capable of development into plane surfaces. For area of these see "Key," page 57.

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**47—A prismoid, its bases, an ellipsis and a square.**

A reducer between an elliptic and square conduit, a roof, etc.

Its middle section, a **mixtilinear figure** or **approximate oval.** Its lateral surface developed, a **curved trapezium**, one base curved, the other **polygonal.** See "Key to Ster.," page 166.

---

**48—A prismoid, its bases a symmetrical hexagon and a line.**

Ridge roof, coping or finial to a post, panel ornament, etc.

Its middle base, a **symmetrical octagon**; its lateral surface, **triangles.** For symmetrical hexagon, area equal to double that of half the figure, which is a trapezium.

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**49—A prismoid, its bases, a symmetrical hexagon and a lozenge**

Flat roof, ornament, etc.; on its lesser base, a fancy basket, a disk, etc.

Its middle section or base, a **symmetrical decagon**; its lateral faces, **triangles**. Area of hexagon, double that of component trapezium.

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**50—A groined ceiling or the half of a rectangular oblate spindle.**

A roof, panel ornament, etc. For more exact computation of contents, decompose into two parts.

Its base and middle section, **squares**; its opposite base, a **point**; its lateral faces, **mixtilinear figures.**

For areas of mixtilinear figures see "Key to Ster.," page 57.

CLASS VI.

# Pyramids and Frusta of Pyramids.

**51—The tetrahedon, or four-sided figure ; one of the five platonic bodies. A regular triangular pyramid.**

Apex roof of a triangular building, finial or other ornament, the component element of the icosahedron and octahedron.

Its base and middle section, **equilateral triangles**, the lesser equal in area to one-quarter the greater, its upper or opposite base, a **point** ; its faces, **triangles.** For development of surface see " Key to Ster.," page 131. For area of bases and faces, see page 36, rule II.

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**52—A regular square or rectangular pyramid.**

The spire of a steeple, a pinnacle, roof of square tower, a bin, a vessel of capacity, a finial or other ornament, etc.

One of its parallel bases, a **square** ; the other, a **point** ; its middle section, a **square,** of which the area is one quarter that of the base. Lateral faces, **isosceles triangles.**

---

**53—A pyramid, two of its faces perpendicular to base. The ungula of a rectangular prism on either of its bases.**

An apex roof, section of cutting or embankment, component portion of other solids, a roof saddle.

Its base and middle section, **triangles** ; apex, a **point.** Factors of middle section half those of the base.

Affords a demonstration of the theorem that in right-angled spherical triangles the sines of the sides are as the sines of the angles.

---

**54—Frustum of a right triangular pyramid.**

Roof, base or capital of a post or column, base of a table-lamp or vase, a vessel of capacity, component section of other solids.

Its parallel bases and middle section **similar triangles** ; lateral faces, **trapeziums.** Factors of section arithmetic means between those of bases.

**55—Frustum of an oblique triangular pyramid.**

Flat roof of triangular building abutting against a sloped or battered wall; portion of a ditch excavation, component portion of other solids.

Its bases and middle parallel section, **similar triangles**; lateral faces, **trapeziums**; factors of section, arithmetic means between those of the bases. For areas see "Key to Ster.," pages 19, 22 and 29.

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**56—Frustum of a right rectangular pyramid.**

Flat roof to tower; reducer between conduits of varied size, component portion of an obelisk, capital or base of a post or column, a bin, vat or other vessel of capacity, the body of a lantern, etc., etc.

Its opposite bases and middle section, **squares or rectangles** whose factors or sides are each equal to half the sum of the corresponding sides of the bases, or arithmetic means between them. For areas see "Key to Ster.," pages 19 and 29.

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**57—A regular octangular or octagonal pyramid.**

Roof of a tower, spire of a steeple, finial or other ornament, a funnel, strainer or filter, etc.

Its base and middle section, **similar octagons**; lesser area one-quarter of the greater; its upper base or opposite one, an apex or a **point**; lateral faces, **isosceles triangles.**

---

**58—The frustum of a regular octagonal pyramid.**

On its broader base, a roof, tower, pier, quay, component part of a steeple, etc.; base of a column, lamp or vase, etc.; on its lesser base, a vat, bin, vase, or other vessel of capacity; the body of a lantern, etc., etc.

Its opposite and parallel bases and middle section, **regular octagons**; factors of section means to those of the bases; its lateral faces, **trapeziums.** For expeditious mode of arriving at area of octagon, see "Key to Ster.," page 176 or page 26, rule II. Developed surface a regular polygonal sector or trapezium.

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**59—Irregular and oblique pyramid on a quadrilateral base.**

Apex roof of an irregularly shaped building against a battered wall or roof, a roof saddle, etc.

Its base, a **quadrilateral or irregular trapezium**; its summit or apex, a **point**. Middle section similar to base and equal in area to one-quarter that of base.

---

**60—Frustum of a pyramid with non parallel bases.**

Decomposable into the frustum of a pyramid with parallel bases, and an irregular pyramid, by a plane parallel to the base and passing through the nearest corner or point of the upper, or non parallel base.

When decomposed for computation of solid contents: bases and section of frustum, **similar triangles**; bases and section of component pyramid or upper portion, **similar quadrilaterals.** This pyramid has its base in one of the lateral faces of the solid.

CLASS VII.

## Cylinder, Frusta and Ungulae.

**61—A right cylinder or infinitary prism.**

A tower or circular apartment; a bin, vat, tub, bucket, pail, vase, drinking vessel, cauldron or other vessel of capacity; a road or other roller: the cylinder of a steam or other engine; a gasometer, the barrel of a pump, etc., etc., etc.

Its parallel bases and middle section, **equal circles**; its lateral surface developed in a plane, a **rectangle**; its height, that of the cylinder; its length, the circumference of the solid.

For areas of circles calculated to eighths, tenths and twelfths of unity, see tables II., III., IV. at end of "Key to Ster."

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**62—Frustum of lateral ungula or wedge of a right cylinder.**

May represent a cylindrical win-

Its base, a **circle**; its opposite base, a **semi-circle or other segment**; its middle section, a **seg-**

dow or opening in a sloped roof abutting to a vertical wall or surface, the liquid in a closed cylindrical vessel held obliquely, base to chimney or vase partly on a horizontal, partly on a gabled wall.

**ment greater than a semi-circle**; its plane of section the segment of an ellipsis; its cylindrical surface decomposable by lines parallel to bases into **trapeziums**. For areas of segments, see table VIII., "Key," pages 53, 38, 44.

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**63—A rectangular circular ring;**

The difference between two concentric cylinders, or a solid annulus.

Horizontal section of a tower wall, cross section of a brick, iron or other conduit, section of a boiler, vat, tub, or other vessel of capacity, etc., etc.

Its bases and parallel section, **concentric annuli**; its interior and exterior surfaces **continuous rectangles.** The area of annulus equal to the difference of the inner and outer circles, or to the breadth of annulus into half the sum of its circumferences. See "Key," p. 39.

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**64—Central ungula or wedge of a right cylinder.**

Ridge roof of a tower, a wedge, loop hole in a wall component portion of compound solid, a finial or other ornament, a strainer, etc.

Its base, a **circle**; its opposite base, a **line**; its middle section, the **zone or a circle**; its sloped faces, each a **semi-ellipsis**. Its cylindrical surface decomposable into **trapeziums** by arcs parallel to base. See tables II., III. IV., IX., of "Key to Ster.," also pages 38, 46, 53.

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**65—Frustum of central wedge or ungula of cylinder No. 64**

Flat roof of tower or other building, base or capital of rectangular pillar, vessel of capacity, component portion of compound solid, base of chimney stack or vase between two gables.

Its greater base a **circle**; its lesser base, the **central zone of a circle**; its intermediate base, the **zone of a circle**; its lateral faces, **equal segment of equal ellipses.** Its cylindrical surface decomposable into **trapeziums** parallel to bases. See "Key to Ster.," page 51.

**66—Lateral ungula of right cylinder or recto-cylindrical wedge.**

Lunette or arched headway of a door or window, etc., in a sloped roof, component of a compound solid, the liquid in an inclined cylindrical vessel, base of a salient chimney shaft over a roof, etc., etc.

Its base, a **semi-circle**; its intermediate base or middle section parallel to base also a **segment**; its opposite base, a **point**; its plane of section or sloped face, a **semi-ellipsis.** Its curved surface developed an **approximate parabola, trapeziums.** etc. See "Key," pages 38, 44, 51, tables II., III., IV., VIII.

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**67—Frustum of lateral wedge or ungula of a right cylinder.**

Lunette to arched opening in a sloped roof or ceiling abutting on a vertical wall or surface; liquid in an inclined closed cylindrical vessel; base of engaged column against a battered wall, etc.

Its parallel bases and middle section, **segments of a circle, less than, more than, and equal to half**; sloped face, the **excentric zone of an ellipsis**; cylindrical surface, **trapezium** parallel to base. For areas of segment, see "Key," page 44, rule I., rule II., table VIII.; for zone of ellipsis, see p. 53, art. (62).

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**68—Irregular ungula or wedge of right cylinder.**

Lunette to a partially circular opening in an inclined ceiling, etc. Component portion of a compound solid. For areas, see "Key to Ster.," pages 44, 46, 53, articles (61) and (62), tables VIII. and IX.

Ist base, the **segment of a circle greater than half**; its opposite base, a **line**; its middle section, an **eccentric zone of a circle**; one of its side faces, the **segment of an ellipsis**; the other plane face, an **eccentric zone of an ellipsis.**

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**69—Concavo-convex prismoid or cylindro-cylindrical solid or concave frustum of a wedge or ungula of right cylinder.**

Deposit of sediment in a cylindrical sewer, section of additional

One of its bases, the **lune of a circle greater than a semi-circle**; the other the **lune of a circle less than a semi-circle**; the middle section, a **lune equal** or thereabouts **to a semi-circle.** Its side surfaces, convex and concave excavation or filling, or difference between two lunettes.

**70—Frustum of an oblique cylinder.**

May be decomposed into **an oblique cylinder and the ungula of one** by a plane parallel to base, and passing through nearest point of other base.

**approximate trapeziums.** For areas of lunes, see "Key," page 47.

When decomposed, its bases and section **ellipses**; the base of ungula, an **ellipsis** equal to each of those of the inclined cylinder; its middle section **half an ellipsis.** For ungulæ, see Nos. 72, 73, 75.

CLASS VIII.

## Oblique Cylinder, Frusta, Ungulae, Cylindroids, etc.

**71—Oblique cylinder or infinitary prism**

Mitred section of conduit, hand rail, moulding; inclined column, post, strut or brace, etc.; inclined cylindrical opening in a wall, etc.

Its parallel bases and section **equal ellipses**; its lateral surface capable of development into a **plane mixtilineal figure.** See "Key to Ster.," fig. n. page 57. For area of ellipsis, see page 51 of same.

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**72—Obtuse frustum or ungula of oblique cylinder.**

Oblique lunette inclined upwards or arched headway to a circular or elliptical opening in a sloped roof or ceiling. Component mitred portion of hand-rail, bead molding, etc.

One of its opposite bases, an **ellipsis of sligh eccentricity**; its opposite base, a **point**; its middle section, a **semi-ellipsis** equal to half of base; its plane of section or lateral face, an **ellipsis of greater eccentricity**; its lateral cylindrical face developed, a figure like m page 57 of "Key."

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**73—Acute frustum or ungula of oblique cylinder.**

Representative of same as No.

Same as No. 72. For developed cylindrical surface, see fig. h. page 57 of "Key to Stereometricon."

72, but inclined downwards.

For area of ellipsis, "Key to Ser." pages 51 and 53.

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**74 Concave ungula or frustum of oblique cylinder.**

Representative of same as No. 73, but in arch roof or ceiling instead of sloped roof.

Same as No. 73, with curved instead of plane section. Its cylindrical surface developed similar to fig. h, page 57 of "Key;" its curved or concave section developed an oval or fig. like a, p. 57, "Key."

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**75—Frustum, ungula or wedge of right cylinder.**

Base of chimney shaft on sloped roof, or same as No. 72 not inclined.

Same as No. 72. For developed cylindrical surface, see fig. g; for ellipsis, fig. b. p. 57, "Key."

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**76—A cylindroid; its bases, a circle and an elipsis; infinitary prismoid,**

Base or capital of elliptic column, reducer or connecting link between a circular and an elliptic conduit; a tub, vat or other vessel of capacity; a hat with elliptic or oval head and a circular crown, etc.

Its middle section, an **ellipsis** of which the conjugate or lesser diameter or axis is an arithmetic mean between those of the opposite bases. For area of circle, see table II, III, IV, and of ellipses, p. 51, "Key." Lateral surface developed, a **plane trapeziform fig**.; its greater base, convex; lesser, concave; its area, equal to periphery of middle section into mean height.

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**77—Cylindroid or infinitary prismold; its bases, an elipsis and a circle.**

Same as No. 76, or frustum of a conic metallic vessel, which has become flattened or battered at one end.

Its lateral surface developes into **a plane trapeziform figure,** with greater periphery convex; and lesser concave. Area equal to periphery of middle section into mean height.

**78—Cylindroid; its bases ellipses at right angles to each other.**

Capital or base of elliptic column, connecting link between conduits; metallic envelope or tube flattened at ends in opposite directions.

Factors of middle section, arithmetic means between those of the bases. Lateral surface developed, a **plane trapeziform figure** of area equal to periphery of middle section into mean height, page 51 of "Key."

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**79—Cylindroid or prismoid; its bases an ellipsis and a line.**

Ridge roof to elliptical building or tower; a hut, camping tent, a strainer of filter; a finial or other ornament.

Middle section, a **mixtilineal figure** with factors, arithmetic means between those of bases. For area of middle section, page 57 of "Key." Lateral surface developed, a **plane trapeziform figure**; its base, convex; its opposite base, angular. Area equal circumference of middle section mean height.

---

**80—A compound solid; a cylinder and a cone.**

A tower or other building, a hut, tent, or camp with conical roof; a hay rick, canister, finial; reversed: a cauldron, cistern, tub, filter, etc.

For cylinder, see No. 61, class VII; for cone, see No. 81, class IX. The developed surface of a right cone is the **sector of a cercle.** For area, see "Key," page 42.

CLASS IX.

## Right and inclined Cone, Frusta, Ungulae, etc.

**81—A right cone or infinitary pyramid.**

Roof of tower, spire, finial or other ornament, pile of shot or shells, cornet, filter or strainer, funnel, etc.

Its base, **a circle**; its opposite base, **a point**; its middle section, **a circle** equal in area to one quarter that of the base. Its lateral surface developed, the sector of a circle. For area of circle, see tables II, III, IV, "Key to Ster."

**82—Frustum of a right cone, considered as a prismoid.**

A tower, quay, pier, base or capital of a column, flat roof of tower, component portion of a spire, a salting tub, etc., reversed: a butter firkin, a tub or vat in a brewery or distillery, etc., a drinking goblet, bucket, pail, dish, basket, lamp shade; a vessel of capacity, the plug of a stop cock, etc., etc.

Its opposite and parallel bases and middle section, **circles**; its lateral surface developed, the **sector of a circular ring,** or a **curved trapezium**. The diameter of middle section an arithmetic mean between those of the opposite bases. For area of bases and section see "Key to Ster.," page 38, for lateral surface, page 43. Tables of areas of circles to eighths, tenths & twelfths, II, III, IV.

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**83—Inclined or oblique cone.**

Loop hole in a wall, the liquid or fluid substance in a conical vessel inclined to the horizon; a finial or ornament adapted to a raking cornice or pediment, etc.

Its base and middle section, **similar ellipses**—the latter equal in area to one quarter the former; the upper base, an apex or **point**; lateral surface developed an **irregular sector**, which, for computation of area, divide into triangles.

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**84—Frustum of inclined cone.**

Unequally splayed circular opening in a wall; a coal scuttle: reducer or connecting link between two conduits of different diameters laid eccentrically etc.

Its opposite and parallel bases and middle section, **similar ellipses**; its lateral surface developed **portion of an eccentric annulus,** art. 39, page 33, of "Key to Ster.," Diameters of middle section, arithmetic means between those of bases.

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**85—Flat or low cone.**

Roof to tower or circular construction; cover of a box, basket, cauldron, etc, ; finial or other ornament; a chinese hat, a pile of shot or shells, a sun shade; reversed: a

Its base, a **circle**; opposite base or apex, a **point**; its middle section, a **circle** equal in area to one quarter that of base; its lateral face developed in a plane, the **sector of a circle.**

spinning top, bottom of cauldron or reservoir, a funnel, stainer or filter, etc.

For area of circle, see tables II, III, IV, of "Key to Ster.;" for sector, see page 42 of same.

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**86 – Frustum of a low or surbased cone.**

Flat roof to a pavillion, tower, etc.; a hat, the cover of a vessel of capacity; an unfinished or truncated pile of shot or shells; a lamp shade; a finial or other ornament; the bottom, base, top or other component section of a compound solid, as of No. 100; reversed: a dish, pan, saucer, cauldron, cistern,

Its opposite bases and paralled middle section or intermediate base, **circles**; diameter of middle section, an arithmetic mean between those of the opposite bases; the lateral area developed in a plane, the **sector of a circular annulus.**

For areas of circles, see tables II., III., IV. of "Key to Ster.," sector, page 43 of same.

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**87—Parabolic conic ungula by a plane parallel to side of cone.**

Lunette to a circular headed opening in a wall and sloped ceiling; liquid in a closed conic vessel inclined to the horizon.

N.B.–For ratio of chord of middle section or segment to that of base, see "Key to Ster.," page 143, where it is shown that the squares of the chords are proportional to the abscissae.

The base, the **segment of a circle**; the opposite base, a **point**; the middle section, the **segment of a circle**; the plane of section a **parabola.** For areas of segment, see "Key to Ster.," page 44 and table VIII.; for area of parabola, page 54 of same. The lateral surface developed an **approximate sector of a circle.** The height or versed sine of middle section segment is half that of base.

---

**88 Frustum of parabolic conic ngula by a plane parallel to base of cone.**

Splayed opening or embrasure to a segment-shaped window or loop hole in a wall; lunette to opening

The parallel bases and middle section, **segments of a circle**; the lateral plane face or figure, the **zone of a parabola**, for area of which see "Key to Ster.," page 55, art, (66); the developed conical in sloped ceiling terminating in a vertical surface ; liquid in a closed vessel in the shape of the frustum of a cone, No. 82, when inclined from the vertical.

For chord of middle segment, measure solid or compute by page 143 of " Ster."

surface, an **approximate sector of a circular annulus** or, more correctly, a **trapezium with curved concentric or parallel bases**, for area of which see note page 29, " Key to Ster.," For area of segment, table VIII, and page 44 of same.

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**89—Frustum of a right elongated cone.**

Shaft of Crecian column, tapered post, high tower or chimney shaft, funnel, pipe reducer, speaking trumpet or horn, plug of a stopcock or tap, deep drinking goblet, or other vessel of capacity large or small, shaft of a gun, component portion of many compound solids, etc.

Like No. 82, its opposite and parallel bases and middle section **circles** ; diameter of middle section equal to the half sum of those of the bases ; the developed lateral surface, the **sector of a concentric annulus.**

For areas of circles to eighths, tenths and twelfths, see tables II., III., IV., of " Key to Ster. ;" for that of sector, page 43 of same.

---

**90—A compound solid,** composed of or decomposable into **the frustum of a right cone and the segment or half of a sphere or spheroid.**

May represent a piece of ordnance, a deep conical vessel with hemi-spherical, hemi-spheroidal or segmental bottom or top to it.

For hemi-sphere, hemi-spheroid, or segments thereof, greater or less than half, see classes 18, 19, 20.

For diameter of middle section in segment of spheroid, see " Key to Ster.," pages 139 and 140, where

For nature and areas of bases and middle section of the component frustum or a cone and of its lateral surface, see Nos. 82 and 89.

For areas of bases and middle section of hemisphere or hemispheroid or of the segment of either, greater or less than a hemisphere, see tables II., III., IV. in " Key to Ster."

For diameter of middle section in hemisphere or in segment thereof, see " Baillairgé Geometry," par. 539 or "Key to Ster.," par. 154, where $oa = \sqrt{Co \cdot oD}$, and $oD =$

$AB : CD :: \sqrt{Ao \cdot oB} : o\ M$ and $CD : AB :: \sqrt{Co \cdot oD} : o\ M.$, or, the rectangle under the required radius and either axis of the spheroid is equal to that under the square root of the rectangle or product of the abscissas of the first axis and the other axis.

diam. $AB$ minus versed sine $oC$; or, the square of the half cord equals the rectangle under the versed sine and remainder of the diameter; or, may be obtained directly by measuring the solid.

## CLASS X.

## Conic Frusta and Ungulae, etc,

**91—Conic wedge or central ungula of a cone by planes drawn from opposite edges of the base to meet in the axis of the cone.**

Ridge roof to a tower, splayed opening or embrasure to a long narrow vertical loop hole in a wall; component section of compound solid of a cone and cylinder or of cones having their bases or apices in opposite directions.

The base, a **circle**; the parallel upper base, an arris or **line**; the middle section parallel to bases, the **zone of a circle**; the lateral plane faces **equal segments of equal ellipses, each greater than half**; the curved or conical faces developed, equal **curvilinear triangles**

For areas, see pages 38, 46, 53 and 57, and tables II, III, IV., of "Ster." For area of zone, see table IX, of same.

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**92—Frustum of a conic wedge or of the central ungula of a cone by a plane parallel to base**; or, may be considered the frustum of a right cone, laterally and equally truncated on opposite sides.

Arched and splayed embrasure in a wall, component portion of a compound solid.

The base, a **circle**; the opposite and parallel base, a **zone of a circle**; the middle section, a **zone**; the lateral plane faces, equal **segments of equal ellipses** the developped conical surfaces resolvable into **trapeziform figures.**

For area of trapezium, page 29, "Key to Ster."

**93—Lateral elliptic ungula of a cone, by a plane passing through edge of base.**

Splayed embrasure to elliptic opening in wall and through sloped roof or ceiling; etc.

Its base, a **circle** ; its upper or opposite base, a **point** ; its middle section parallel to base, the **segment of a circle** ; its plane face an **ellipsis** ; its conical surface developed a **concavo - convex figure** like h, page 97 of " Key to Ster."

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**94—Lateral elliptic conic ungula, by a plane passing within the base.**

The liquid in an inclined conical vessel, lunette head of opening in sloped roof or ceiling ; base of structure rising from an inclined surface, roof, pediment, etc.

For area of parabola see key to Ster., page 54 ; for area of hyperbola, page 55, or figure e, page 57 ; for ellipsis, page 51 and 53.

The base, a **segment of a circle** ; the upper base, a **point** ; the middle section, a **segment of a circle** ; the plane lateral face, the **segment of an ellipsis** ; the developed conical surface as in No. 87 or 94. If the cutting plane be parallel to side of cone the face will be a **parabola** ; if at an angle greater than side of cone to base, a **hyperbola** ; if less, an **ellipsis.**

---

**95—Central ungula of cone or conic wedge, by planes through opposite edges of upper or lesser base and meeting in the axis of the cone.**

An embrasure, etc., etc.

The plane lateral faces, segments of ellipses if cutting planes more inclined to base than side of cone ; if less, hyperbolas ; if equally, parabolas.

Bases and sections same as No. 91 ; developed conical surface, a **concavo-convex triangle** computible as per page 57 of "Key."

The lateral plane faces, **equal segments of equal ellipses, equal parabolas or equal hyperbolas,** as case may be.—See No. 94.

**96 Frustum of conic wedge, No 85, by a plane parallel to the base.**

An embrasure; a reducer or connecting link between a rectangular and circular conduit, etc.

Its base, a **circle**; other base and middle section, **zones of circles**, for areas of which see "Key to Stereometricon, table IX.

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**97—Concave ungula of a cone or a conical recto-concave wedge.**

Lunette of circular headed opening in wall, reaching through vaulted, groined or arched ceiling; cone scribed to cylindrical surface, or to a shaft of elliptical section.

The base, the **segment of a circle**; the other base, a **point** or curved arris; its intermediate base or section, or its bases or sections if divided for computation of cubical contents, **segments of circles.** Its sides like No. 94.

---

**98—Portion of frustum of right cone, by a plane through both bases.**

Splayed segment headed opening in wall, liquid in closed tub lying on its side; base or capital of half column against sloped wall; component section of base or capital of clustered, gothic or other column.

Its parallel end bases and middle section, **segments of circles**; its conical surface developed a figure of trapezium form, having parallel or concentric arcs of circles for its bases; its plane face, the **zone of an ellipsis or of a parabola or hyperbola** according to inclination of cutting plane.

---

**99.—Lateral conic ungula or wedge, by a plane through edge of lesser base of frustum**

Embrasure, liquid in inclined conical vessel, section of conical elbow or mitre, base of chimney stack to sloped roof. May be treated also as lying on its lateral plane face.

Its base, a **circle**; opposite base, a **point**; intermediate section a **segment of a circle**; its plane face an **ellipsis**, its conical surface developed a concavo-convex figure like g or h, page 97 of Ster. but with concave base. Treat on circular base as easier of computation.

---

**100—A compound solid** composed of, decomposable or resolvable into **two conic frusta and a low or flat cone**.

May represent a covered dish, a basket or hamper, a vase, a finial or other ornament, an urn, a cauldron on a stand, etc., etc.

All its areas to be used in computation of solid contents or capacity are **circles**, and can be measured to eighths, tenths or twelfths of an inch or other unity, and the areas found by mere inspection in tables II., III. and IV. at end of Baillargé's "Key to Ster."

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## DIVISION 2.

Solids of double curvature, or of which the surfaces are not capable of development in a plane.

### CLASS XI.

## Concave Cones, Frusta and Ungulae.

**101—Right concave cone or spindle.**

Camping tent; roof of tower, pavillon, hut, etc.; spire, funnel, strainer, trumpet; finial or other ornament.

May be decomposed into two or more frusta by planes parallel to base, to admit of more accurate determination of solid contents.

Its base and parallel sections, **circles**; its upper or opposite base, an apex or **point**. Its lateral surface not capable of development in a plane or into a sector of a circle as is the case with a regular right cone, but may be readily and very approximately computed by division into **continuous trapeziums** by lines parallel to circumference of base. See "Key to Ster.," page 96.

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**102—Frustum of a right concave cone between parallel planes.**

Its bases and parallel sections, **circles**. Intermediate diameters not, as in No. 82, arithmetical means

Illustrative of most of the objects mentioned in No. 82, which see.

For more accurate computation of contents, divide into two sections or more, according to greater or lesser curvature of the solid, and treat each section as a separate prismoid and add the results.

between those of the opposite or end bases, but must be measured or computed. Lateral area may be conceived as made up of a series of super or juxta-posed **continuous trapeziums.**

---

**103—Inclined concave cone.**

Finial, or ornament on a raking cornice; liquid in an inclined vessel, etc., as for No. 101, may be decomposed by imaginary planes parallel to base into two or more sections or slices, so that slant side of each may be sensibly a straight line. See p. 103, par. 139 " Key."

Its base and section, **approximate ellipses of slight excentricity** or **ovoid figures**; its other base, a **point.**

In developing the lateral surface into a series of **continuous trapeziums**, the lines are not as in the right cone parallel to base or to circumferences of parallel sections but are drawn equidistant from the apex, thus leaving at the base a figure like h, page. 57 of "Key."

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**104—Frustum of oblique concave cone between parallel planes.**

Representative of same as No. 84.

Its bases and sections parallel thereto, **approximate ellipses** or **ovoid figures.** See remarks to No. 102

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**105 - Flat or low concave cone.**

Representative of many of the objects mentioned in No. 85.

Its bases, a **circle** and a **point**; section, a **circle**; lateral area reducible to **continuous trapeziums**, par. 126, " Key to Ster."

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**106—Frustum of flat or low cone.**

Representative of objects under head of No. 86.

Its bases and section, **circles,** for areas of which see tables II., III. and IV. of " Key to Ster.," to eighths, tenths and twelfths of inch or other unity.

**107—Ungula of concave cone by a plane through outer edge of base.**

See No. 92, as to what it represents, etc.

See No. 92. Lateral surface reducible to **trapeziums** and **triangles.**

Base and sections, ovoid figures; areas, page 57 of Key.

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**108—Ungula of concave cone by a plane cutting the base.**

See No. 93 as to what it represents, etc.

Bases and section. **segments of circles**; upper base, a **point.** Lateral surface as No. 107.

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**109—Ungula of hollow cone by a plane through edge of lesser base of frustum.**

See No. 99, base of chimney stack to a sloped roof.

Base, a **circle**; opposite base, a **point**; middle section, the **segment of a circle**; lateral area, **trapeziums** and **triangles.**

---

**110—Frustum of** (No. 109) **ungula by a plane parallel to base.**

See Nos. 98, 116, 126.

Base or capital of a column, or base of chimney shaft, etc., on or outside of sloped roof or gable.

Its base, a **circle**; other base, a **segment of a circle**; its middle section parallel to bases, also a **segment.** For areas of segments of circles, see "Key to Ster.," table VIII., or rules, page 44 of same.

CLASS XII.

## Paraboloid or Parabolic Conoid, Frusta and Ungulae, etc.

**111—Right paraboloid or parabolic conoid.**

Dome, hut, hive, roof, finial or other ornament, shade, globe, cover, hood, cowl, etc.; reversed: a filter,

Its base and middle section, **circles**; its opposite base or apex, a **point**; its lateral surface resolvable into a small circle at apex, and **continuous trapeziums.** The cauldron, or other vessel of capacity, the bowl of a cup or drinking goblet, etc., etc.

squares of its intermediate diameters, proportional to abscissae. See "Key to Ster.," page 96.

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**112—Frustum of right paraboloid, between parallel planes.**

Represents mostly the same objects as the frustum of a cone, No. 82.

See page 142 "Key to Ster."

End and middle bases, **circles**; squares of diameters proportional to abscissae. For areas of circles, see "Key to Ster.," tables II., III., and IV.

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**113—Oblique paraboloid.**

"Key to Ster.," page 142.

Liquid in a parabolic vessel inclined to the horizon, metal in an inclined crucible, finial or ornament on an inclined or raking molding or pediment, etc.

Its base and middle section, **similar ellipses**; its opposite base or other end, an apex or **point.** For areas of ellipses see "Key to Ster.," page 51; for lateral area see No. 103.

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**114—Frustum of oblique paraboloid between parallel planes.**

Represents same as frustum of inclined cone No. 84, "Key to Ster.," page 142.

Its bases and middle section, **similar ellipses**; for areas of which see "Key to Ster., page 51. For lateral area, see No. 103 or reduce to **trapeziums** by lines from base to base.

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**115—Parabolic wedge or central ungula of paraboloid.**

See No. 91.

Lateral or paraboloidal surface capable of approximate development. See No. 91.

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**116—Portion of a paraboloidal frustum, by a plane through its greater base and edge of other or opposite base.**

Its lesser base, a **circle**; opposite base, the **segment of a circle**; middle section, also a **segment.** Its lateral plane face, the **segment**

See No. 98 as to what it represents. Also, base of chimney stack, partly on a horizontal and partly on an inclined base, or sloped roof, etc.

**of an ellipsis.** This face would be a parabola if angle of face equalled that of side; if greater, a hyperbola.

---

**117—Lateral ungula of paraboloid**

Very similar to No. 92, as to what it represents.

Its base, a **circle**; opposite base, a **point**; middle section, the **segment of a circle.** Its plane face an **ellipsis.**

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**118—Lateral ungula of paraboloid; elliptic, parabolic or hyperbolic, according as plane of section cuts the base at an angle less than, equal to, or greater than that of the side and base.**

Its base, the **segment of circle;** its middle section, a **segment;** its upper or opposite base, a **point;** its plane face, the **segment of an ellipsis, parabola or hyperbola,** according to angle of plane of section.

---

**119—Obtuse eliptic ungula of a paraboloid, by a plane through edge of lesser base of frustum.**

Base of chimney stack, etc., to sloped roof; base of vase, statue, etc., on a pediment; a lunette, scoop, etc.

Its base, a **circle**; middle section, a **segment**; other base, a **point**; its plane face, **an ellipsis.** For areas of segments of circles, table VIII of "Key to Ster." For area of ellipsis, page 51 of same.

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**120—Frustum af a paraboloid between non-parallele bases.**

"Key to Ster.," page 145.

Lunette through a vertical wall and inclined ceiling, etc. For computation of solid contents decom-

Its factor areas, **circles** and a **segment**; its plane face, **an ellipsis.** For areas of segments of circles, table VIII of "Key." Area of circle, tables II, III and IV, of same; ellipsis, page 51 of same;

pose into a frustum with parallel bases, and an ungula by a plane parallel to base, through nearest point of upper base.

lateral area, page 95; solidity, page 145 of same.

CLASS XIII.

## Hperboloid or Hyperbolic Conoid, Frusta and Ungulae, eto.

**121—Right hyperboloid or hyperbolic conoid.**

Page 146, "Key to Ster." Representative of same as No. 111.

For intermediate diameter or that of middle section, see "Key to Ster.," page 147, 3rd line, or by direct measurement.

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**122—Frustum of right hyperboloid.**

Representative of same, nearly as Nos. 112 and 82.

Except for diameter of middle section, same as No. 112, or the diameter may be measured directly.

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**123—Oblique hyperboloid.**

See "Key to Ster.," p. 146. Representative of same, as No. 113.

Same as No. 113, except for diameter of middle section for which see "Key to Ster.," page 147, line 3, or the diameter may be measured.

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**124—Frustum of oblique hyperboloid.**

Representative of same, nearly as Nos. 84 and 114.

Same as No. 114, except for diameter of middle section for which see "Key to Ster.," page 147, line 3, or may be had by measurement.

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**125--Hyperboloid wedge or central ungula.**

Similar solid to No. 95 of a cone and representative of same objects.

Except for diameter of middle section, same as No. 91 or 95. For area of **zone**, see "Key to Ster.," page 46 or table IX of same.

**126—Ungula of hyperboloid by a plane through edge of base.**

For solid content, treat as prismoid or by par. 185 of "Key."

Solid similar to No. 93 of cone, or to No. 117 of paraboloid.

Its base, a **circle**; middle section, the **segment of a circle**; other base, a **point**. Plane lateral face, an **ellipsis**, its lateral surface of double curvature, as all such figures are, not capable of development, but reducible as required.

---

**127—Frustum of hyperboloid wedge.**

Similar to No. 116 of paraboloid. Base of chimney stack, etc., resting partly on a sloped roof.

Bases same as in No. 116. Lateral area developes into **trapeziums** by lines parallel to bases. For areas of circles, segments, zones, see tables of "Key to Ster."

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**128—Ungula of hyperboloid by a plane through base.**

Similar to No. 118 of paraboloid.

Bases and section same as No. 118 of paraboloid. See table VIII, of "Key to Ster.," for areas of **segments.**

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**129—Frustum of hyperboloid wedge, or of central ungula of hyperboloid.**

Similar to No. 92 of cone.

Same as No. 92. For area of circles to eighths, tenths & twelfths, see tables II, III, and IV of "Key to Ster." For area of zone, see table IX, of same. Lateral surface decomposable into trapeziums.

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**130—A compound solid: two equal frusta of cone or conoid, base to base.**

Illustrative of a keg or cask, barrel, hogshead, etc., of any size or shape.

Treat one-half of solid as Nos. 92, 112, 122, and double the result.

See "Key to Ster.," fig. on page 155, for mode of measuring halfway diameter, when the half solid is not the frustum of a cone, but that of a conoid or of an ellipsoid or spheroid. When of a cone middle diameter equal to arithmetic mean of end diameters.

## CLASS XIV.

# Sundry Solids.

**131—Three axed spheroid.**

See "Key to Ster.," page XXXIX. May for measurement be supposed to lie or stand on either of its sides or apices.

Representative of a pebble, a bean, spindle, torpedoe, a shell fish, a flattened ellipsoid, etc., etc.

All its sections, **ellipses**; all its parallel sections, **similar ellipses.** For areas of ellipses, "Ster.," page 51. Lateral area, see general formula, page 95, "Key to Ster." Or, as with the spheroid, suppose the surface divided as a melon is or orange into ungulae, terminating in apices or poles of the fig.

---

**132—An ovoid or solid of the shape of an egg.**

Divide into two or three sections and treat separately as conoid, segment of sphere or spheroid, and frustum of conoid.

All parallel areas perpendicular to longer or fixed axis, **circles,** which find ready calculated for all sized diameters to eighths, tenths and twelfths of an inch, or other unity of measure, tables II., III., and IV., of Key to Ster. For lateral area, see page 96 of same.

---

**133—Circular disc with rounded edge.**

Treat as a **compound solid,** to wit: **a flat or low cylinder, and a ring semi-circular or segmental in section.** Add the results.

For cylinder, see No. 61. For ring compute area of section thereof as **semi-circle** or **segment,** and multiply into circumference. For area, mean circumference of ring into circumference of section.

---

**134—Twisted prism.**

Portion of a circular stair rail, a twisted pillar or column, spiral ornament, etc.

Its bases and sections similar and equal figures. The lateral surface of each face can be developed in a plane, a **trapezium** or **rectangle.**

**135—A compound solid.**

**Two frusta of cones, their lesser basses joined.**

A windlass, spool, handle, shaft, axle-tree, etc.

Treat half the solid as the frustum of a cone, and double the result, either for solid content or area of figure.

---

**136—A compound solid.**

**Two frusta of hollow cones joined by their lesser bases.**

A windlass, spool, handle, shaft, axle-tree, etc.

Treat one half the solid as frustum of cone No. 102, and double the result.

Lateral area resolvable into continuous trapeziums.

---

**137—Compound solid.**

**Two frusta of concave cones joined by their greater bases**

A windlass, shaft, axle-tree, etc.

Treat half the solid, and double the result. For areas of circles, see tables II., III. and IV. of Ster.

---

**138—Compound solid.**

**The segment or half of an elongated or prolate spindle, No. 151, and the segment or half of an oblate spindle, No. 141, or the segment of a sphere or spheroid,** classes XVII, and XIX., a buoy, etc.

Sections perpendicular to axis, **circles**; Area resolvable into **continuous trapeziums**, a **circle** and the **sector of a circle**. The circle at apex of segment of sphere or spheroid ; the sector at apex of spindle. See page 55 of "Key to Ster."

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**139—Compound solid like the last with hollow cone instead of spindle.**

A finial or other ornament, a cul-de-lampe or pendant.

Sections perpendicular to axis, **circles.** Lateral surface, **continuous trapeziums**, a **circle**, and the **sector of a circle** at apex of cone.

---

**140 — Compound solid : the frustum of a sphere or sphe-**

Bases and sections, **circles.** Lateral surface resolvable into

**roid and a hollow cone.**

A Moorish dome, a minaret, chimney of a coal oil lamp, a decanter, a vase, a pitcher.

**continuous trapeziums.** See general formula, page 95 of "Key to Ster."

CLASS XV.

## Oblate or Flattened Spindle, Frusta, Segments, Sundry.

**141—Oblate spindle, as two equal segments of sphere or spheroid base to base.**

A quoit, etc.

Treat one half as segment of sphere or spheroid, and double the result. See classes 17 and 19.

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**142—Semi-oblate spindle by a plane parallel to fixed axis.**

Floating caisson to entrance of dock, etc.

Treat its two halves together as one segment of sphere or spheroid. See classes 17 and 19.

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**143—Middle frustum of oblate spindle.**

Fixed caisson or coffer-dam. Treat as prismoid.

The bases and middle section each a **double segment of a circle or ellipsis, or two segments thereof, base to base.** Table VIII., "Key to Ster."

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**144—Lateral frustum of oblate spindle, between planes parallel to fixed axis.**

A flat-bottomed boat or other sailing vessel or a caisson, etc.

The bases and section half-way between them, **double segments of circles or ellipses**, for areas of which see table VIII., "Key to Ster.," and page 53 of same.

**145—Lateral frustum of oblate spindle truncated at one end.**

A flat-bottomed boat or other sailing vessel.

Bases and middle section, **double segments, base to base, of circles or ellipses truncated at one end.** For areas, see page 57 "Key to Ster."

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**146—Lateral frustum of oblate spindle truncated at both ends.**

A flat-bottomed boat or pontoon, a scow, lighter, etc.

Bases, **double segments of circles or ellipses truncated at both ends.** Divide into trapeziums and compute areas by page 57 "Key to Ster."

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**147—Quarter of an oblate spheroid, No. 181.**

The arched ceiling, roof or vault of the apsis of a church or half-groined ceiling of a circular apartment. On its lesser base, the head of a shallow niche in a wall, etc.

Its base and middle section, **semi-circles**, if treated on its broader base ; if on its lesser face, its base and middle section, **semi-ellipses**. On whatever base it stands, treat as if on broader base, it being easier to compute circles than ellipses.

---

**148—A compound body, a cone, and the segment of a sphere or spheroid.**

A buoy, covered filter, etc.

Treat separately as cone No. 81, and as segment of sphere, No. 173, or of spheroid No. 182.

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**149—Elliptic ring, or may be called an eccentric ring.**

Treat as circular or cylindrical ring, taking for bases, its least, its greater, and its mean sections ; and for length the mean of the inner and outer circumferences.

Compute half of solid as the lateral frustum of a half-prolate spindle or the frustum of an elongated cone. The solid may be conceived to be formed of the middle frustum of an elongated spindle bent till its ends meet.

**150—Compound solid: a cylinder and the segment of a spere or speroid.**

A mortar, a tower with domed roof, a hall or room with groined ceiling, a hut, hive, hood.

For area of sphere or spheroid, see page 95 "Key to Ster.," or page 105, 110, 124, Ex. 3. Areas of circles tables II., III. and IV. of same. Half-way diameter in segment of circle or sphere a mean proportional between abscissae of diameter.

CLASS XVI.

## Prolate or Elongated Spindle, Frusta, Segments, etc.

**151—Prolate spindle.**

A shuttle, a torpedoe, a cigar, a sheath, case, etc.

Its sections perpendicular to axis, **circles.** Decompose its lateral area into **continuous trapeziums** and a **sector.**

---

**152—Semi-prolate spindle by a plane through its greater or fixed axis.**

A boat or sailing vessel, a canoe, etc.

For solidity, compute planes perpendicular to fixed axis, as **segments of circles, semi-circles,** while the sections parallel thereto are not so readily computed.

---

**153—Semi-prolate spindle by a plane perpendicular to fixed axis.**

A hut, roof, filter or vessel of capacity, a minaret or finial.

For greater accuracy, divide into a frustum and segment, compute and add cubical contents. Areas of bases, tables II., III. and IV. of "Key to Ster."

---

**154—Middle frustum of prolate spindle between planes perpendicular to fixed axis.**

A cask or keg, puncheon, hogshead, etc.; see page 155 "Key."

See page 149 of "Key to Ster.," and for lateral surface, page 95 of same. See page 155 of same. Bases and sections, **circles,** tables II., III. and IV. of Key to Ster."

**155—Semi-middle frustum of prolate spindle.**

The liquid in a cask lying on its side, a boat with truncated ends. Compute as No. 154 and take half.

Bases and middle section, **semi-circles**, see page 160 of "Key to Ster." Lateral surface decomposable into **trapeziums.**

---

**156—Lateral frustum of prolate spindle by planes parallel to fixed or longer axis.**

A flat-bottomed boat or other sailing vessel.

Treat as prismoid, the greater base, a **double segment of a circle**. The other base and section, **oval figures** for areas of which see page 57 of "Key to Ster."

---

**157—Eccentric frustum of a prolate spindle by planes perpendicular to fixed or larger axis of solid.**

The shaft of a Roman column. Compute each frustum from centre and add the results.

Its bases and sections, **circles,** for areas of which to eighths, tenths and twelfths of inch or other unit of measure, see tables II., III. and IV., "Key to Ster."

Its lateral surface decomposable into **continuous trapeziums**, or nearly equal to length of side into mean circumference.

---

**158—Middle frustum of elongated spindle by planes perpendicular to fixed or longer axis.**

The shaft of a windlass, a drum or pulley, a cigar, torpedoe, etc.

Its bases and sections, **circles,** for areas of which see "Key to Ster.," page 38, or tables II., III. and IV. of same.

Lateral area equal nearly length of curved side into mean of circumferences.

---

**159—A curved half-spindle or cone.**

A horn, powder flask, tusk or tooth of an elephant, etc., a supporting bracket from face of wall.

Base and sections **circles** or **ellipses of slight eccentricity**. Lateral area decomposable into **continuous trapeziums** and sector at apex.

**160—Frustum of a prolate spindle between non parallel bases.**

Decompose into a **frustum with parallel bases and an ungula** by a plane through nearest point of one of the bases.

Base and sections parallel thereto, **circles**, base of ungula a **circle** ; middle base of ungula, a **semi-circle**; apex of ungula or opposite base, a **point** ; lateral surface, **continuous trapeziums**, and a fig. like h, page 57 " Key to Ster."

CLASS XVII.

## Sphere, Segments, Frusta and Ungulae, etc.

**161—The sphere.**

A billiard or other playing ball, the ball of a vane or steeple, spherical shot and shell, school spheres, lamp globe or well, component part of compound solid, etc. Solid content may be had by computing one of the component ungulae and multiplying into number thereof.

The opposite bases, **points** ; the middle section, a **circle.** The area of surface admits of approximate development into a series of equal figures in the shape of the longitudinal section of a prolate spindle, or of **double segments of a circle, base to base.**

Surface equal to four great circles or to four times that of a great circle.

---

**162.—A hemisphere.**

A dome, arched celling, globe, shade, cover, hut,hive, etc. ; reversed : a bowl, cauldron, copper, vase, etc.

Contents more easily computable as half of those of a whole sphere, where there is no intermediate diameter to calculate or measure.

Its base, a **circle** ; opposite base, a **point** ; its middle section, a **circle,** the half diameter of which equals the square root of the rectangle under the versed and suversed sines or portions of the diameter of the sphere. The lateral area equal to two great circles of the sphere.

**163.—Segment of a sphere less than a hemisphere.**

Representative of same objects as No. 162, cover or bottom of a boiler. Solid contents also equal to one of the component ungulae into the number thereof.

Base and section, **circles**; other base, a **point**; radius of middle section for area thereof, equal to root of rectangle of parts into which it divides the diameter of the sphere of which the segment forms part. For lateral area see "Key to Ster.," page 110, or General Formula, page 95.

---

**164.—Segment of sphere, greater than a hemisphere.**

Representative of same as No. 162, and of a Moorish or Turkish or horse-shoe dome.

Its base and section **circles**; other base a **point**; radius of middle section the root of rectangle of parts into which it divides diameter of sphere. Lateral area, see "Key to Ster.," pages 117 and 123.

---

**165. — Middle frustum of a sphere.**

Base, capital or middle section of a column or post, a puncheon, hogshead, crusher, roller, lamp shade, etc., etc.

Bases, equal **circles**; middle sections, a **circle**; see tables of areas of circles to eighths, tenths, and twelfths of an inch or other unity of measure, II., III., and IV. of "Key to Ster."

---

**166. — Lateral frustum of sphere.**

Base or capital of column, coved ceiling, cauldron, dish, soup plate, saucer, etc. Radii of bases and sections proportional to square roots of rectangles of portions into which such radii or ordinates divide the diameter of which the solid forms a part.

Bases and section, **circles; lateral** area resolvable into **continuous trapeziums**; or lateral area may be had very nearly at one operation, if the frustum be low or flat and that its lateral curvature be not considerable.

**167.—Sherical wedge or central ungula of a sphere by planes from opposite edges of base of hemisphere to meet in apex.**

Component portion of a compound solid.

Its base, a **circle** ; opposite base, a ridge, or axis, or **line** ; middle section, the **zone of a circle** ; its plane faces, **circles** ; and lateral area resolvable into **trapeziums and triangles**

---

**168.—Frustum of a spherical wedge or central ungula between parallel planes.**

Component portion of compound solid.

Base, a **circle** ; other base and middle section, **zones of circles.** For areas of zones, see table IX., " Key to Ster. "

---

**169—Spherical pyramid, obtuse-angled and triangular.**

Illustrative of the tri-obtuse-angular spherical triangle, and of the fact that the sum of the angles of a spherical triangle, may reach to six right angles, when each of the component angles increases to 180°.

Base, a **spherical triangle having three obtuse angles ;** apex or opposite base, a **point ;** middle section, a **similar tri-obtuse angular spherical triangle,** and whose area is equal to one-quarter that of base, its factors being halves of those of base, and $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

---

**170.—Frustum of sphere between non-parallel bases.**

Elbow or connecting link between two portions of a rail or bead ; base of a vase or other ornament on a raking cornice.

Decompose into frustum and ungula of a sphere by a plane parallel to one of the bases and passing through nearest point of other base, or more readily and exactly, compute whole sphere, and deduct segment.

CLASS XVIII.

# Spherical Ungulae, Sectors, Pyramids and Frusta.

**171 —Quarter-sphere or rectangular ungula of a sphere.**

Domed roof to a semi-circular plan, vault of the apsis of a church, head of a niche, " Key to Ster., " page 117.

Compute as a whole sphere, and divide by 4, or treat as an ungula. See opposite par.

On its base : one base, a **semi-circle**; opposite base, a **point**; middle section, the **segment of a circle**. On end : each of its opposite bases, **points**; its middle section, the **sector of a circle.** Only one area to compute, and easier and quicker than a segment.

---

**172.— Acute-angled spherical ungula.**

Component portion of the ball of a vane or steeple; natural section of an orange, or of a ribbed melon, section of a buoy, cauldron, etc., etc., elbow of two semi-cylindrical mouldings, etc., at an obtuse angle.

Its opposite bases, **points**; its middle section, the **sector of a circle**; the spherical surface, the component of a hollow metallic or other sphere or spherical vessel, or of the covering for a racket or other playing ball, etc.

For spherical area see " Key to Ster.," page 117.

---

**173.—Obtuse-angled ungula of a sphere.**

Head of niche reaching into a sloped ceiling; elbow of two half-beads at an acute angle, etc.

Opposite bases **points**; middle sections, the **sector of a circle**; its plane faces, **semi-circles**. Spherical area, page 117 " Key to Ster."

**174—Spherical sector or cone,** or, to avoid computing spherical areas, may be treated as a **compound body, a cone and the segment of a sphere.**

A buoy, a finial or ornament, a top, etc., a covered filter. For areas of circles see tables II, III and IV, of "Key to Ster."

Its base, a **spherical segment**; the other base, a **point**; middle section, a **spherical segment** concentric to the base and equal in area one quarter of base; its height equal to radius of sphere, its lateral face developed, the **sector of a circle.** See "Key to Ster.," page 110.

---

**175—Frustum of a spherical sector between parallel spherical bases.**

Portion of a shell or bomb or hollow sphere. To avoid computing spherical areas, treat as frustum of cone, adding greater and deducting lesser segment.

Its bases and middle section parallel thereto, **concentric and similar segments of spheres of corresponding radii.** Its height, the length of slant side. Solidity also equal to difference between whole and partial spherical sectors.

---

**176—Hexagonal spherical pyramid.**

Its base illustrative of a spherical polygon, page 127 of "Key."

Component portion of a solid sphere or ball; keystone of a vault, finial or other ornament; decomposable for computation into six equal triangular spherical pyramids, "Key to Ster.," page 129. See rule for spherical areas at end of this pamphlet.

Its base, a regular **six-sided spherical polygon**; its middle section a figure similar to the last, and equal in area to one-quarter thereof; its opposite base, a **point,** the centre of the sphere of which it forms part. For area of base, see "Key to Ster.," page 127. For area of component spherical triangle of base, see page 123 of same. Its plane faces **equal sectors of a circle.**

---

**177—Frustum of hexagonal spherical pyramid between parallel bases.**

Keystone of vault. Component

Its bases and middle section, **similar spherical polygons**; factor of middle section, as in cone, an arithmetic mean between those of portion of hollow sphere. Surfaces illustrative of similar spherical polygons. Height of solid equal slant height of side.

the bases. Its lateral faces, **equal frusta of equal sectors of a circle,** or **concavo - convex trapeziums.** See rule at end of this work.

---

**178— Half-quarter or one-eighth of sphere or tri-rectangular spherical pyramid.**

Termination or stop to chamfer on angle of wall or pillar.

Compute whole sphere and divide by eight.

Its base illustrative of the **tri-rectangular spherical triangle,** page 123 of "Key."

May compute for solid contents as the half of an ungula where only one area is required, that of a sector of a circle. See rule at end of this work.

---

**179—Acute equilateral triangular spherical pyramid.**

Its base illustrative of the equilateral spherical triangle.

Base and middle section **similar equilateral spherical triangles,** for areas of which, see "Key to Ster.," page 123, and rule at end of this work.

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**180—Frustum of triangular spherical pyramid.**

Illustrative in its bases of similar spherical triangles. Keystone of a vault to a triangular plan.

Bases and middle section, **similar spherical triangles** whose areas are as the squares of the corresponding radii ; or factors of middle section, arithmetic means between those of the opposite bases.

CLASS XIX.

## Oblate Spheroid, Frusta and Segments.

**181—Oblate spheroid.**

Representative, in a less exaggerated ratio of its diameters or axes, of the Earth and planets which are

Treated perpendicularly to its fixed axis, its opposite bases are considered **points**, as in the sphere, a plane touching the solid only in flattened at the poles or extremities of fixed axis and protuberant at the equator. An orange, lamp-shade, or globe, or bowl.

a point; its middle section, a **circle**. If considered parallel to its fixed axis, its middle section, an **ellipsis**. For spheroidal surface or area, see N. 161.

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**182—Semi-oblate spheroid by a plane perpendicular to its fixed or lesser axis.**

Elliptical celling, dome, cauldron, basin, dish, vase, shade, globe, etc.

Base, a **circle**; opposite base, a **point**; middle section, a **circle**; for diameter of which, if not from direct measurement, see "Key to Ster.," page 139, line 10 and page 140, line 20.

---

**183—Semi-oblate spheroid by a plane parallel to its fixed or lesser axis.**

Dome or ceiling to an elliptic plan; glass globe or shade, dish cover, hut, a trough, cauldron, etc.

Equal in area and solid contents to No. 182 and of easier and quicker computation, if considered such, the factors being **circles** instead of **ellipses**. As it stands, its base and middle section, **similar ellipses.**

---

**184—Segment of oblate spheroid, greater than half by a plane perpendicular to fixed axis,**

Turkish, Moorish or horse-shoe, dome or ceiling; a cauldron or copper, etc.

Its base and middle section, **circles**; opposite base, **point.** Spheroidal surface **continuous trapeziums** and a **circle** at apex. For areas of circles, see tables II., III. and IV. of "Key to Ster." For factors of middle section, see No. 182.

---

**185—Middle frustum or solid zone of an oblate spheroid between planes perpendicular to fixed or shorter axis.**

Representative of same as No. 165.

Opposite bases and middle section, **circles**; for areas of circles to eighths, tenths and twelfths of an inch or other unity, see tables II., III. and IV. of "Key to Ster." Spheroidal area, see page 95 of same.

186—**Middle frustum or solid zone of oblate spheroid by planes parallel to fixed or lesser axis of solid.**

Its bases and middle section **similar ellipses,** for areas of which see page 51 of "Key to Ster." Spheroidal area, page 95 of same.

---

187—**Segment of oblate spheroid less than half, by a plane parallel to its fixed or lesser axis.**

Representative of same as as No. 183.

Its base, an **ellipsis**; opposite base, a **point**; middle section, an **ellipsis** similar to base. For factors of middle section, see No. 182.

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188—**Lateral frustum of oblate spheroid by planes parallel to fixed or shorter axis.**

Coved ceiling of elliptic plan; reversed: a boat, a scow, a vessel of capacity, etc.

Its opposite parallel bases and middle section, **ellipses**, for areas of which see "Key to Ster." p. 51.

Its spheroidal surface decomposable into **continuous trapeziums** of variable height.

---

189—**Half or segment of oblate spheroid by a plane inclined to axis of solid**

Liquid or fluid in a semi-spheroidal vessel inclined from the vertical. Finial on a pediment or sloped surface.

Its base and middle section, **similar ellipses**; its opposite base, a **point**; its spheroidal surface **trapeziums**, with **ellipsis** at apex and a **curvilinear triangle** at base of shape similar to fig. h. page 57 of "Key to Ster.," or lateral area may be divided and computed as triangles.

---

190—**Frustum of oblate spheroid between non-parallel bases.**

Decompose into a **frustum with parallel bases, and an ungula** by a plane parallel to one base and drawn through nearest point of

Bases and middle section of component frustum with parallel bases, **ellipses**; base of ungula, an **ellipsis**; middle section of ungula the **segment of an ellipsis**; its other base, a **point.**

For factors of middle sections,

other base, or compute whole spheroid and deduct segments.

see "Key to Ster.," page 139, line 10 and page 140, line 20, where $AB : CD :: \sqrt{Ao.oB} : oM$ and $CD : AB :: \sqrt{Co.oD} : oM$.

CLASS XX.

## Prolate Spheroid, Frusta and Segments.

**191—Prolate spheroid**

Representative of a lemon, melon, cucumber, etc.; a case, sheath, etc.

The work of computation expedited by treating circles instead of ellipses; that is, areas perpendicular instead of parallel to fixed axis.

Its middle section perpendicular to fixed or longer axis, a **circle**; its opposite end bases, **points.** Spheroidal surface, **continuous trapezoids**, or a series of **double segments base to base** as the component ribs a of melon. May treat as plane segment with length of cord equal to semi-elliptical section.

---

**192—Semi-prolate spheroid by a plane parallel to fixed axis.**

Vaulted ceiling to elliptic plan; reversed: a boat or other sailing vessel, a cauldron or vessel of capacity, etc., etc.

For solid contents and spheroidal surface, treat perpendicular to fixed axis, where factors are **circles** or **semi-circles** instead of **ellipses.** For areas of circles, see tables II., III. and IV. of "Key to Ster."

---

**193—Semi-prolate spheroid by a plane perpendicular to fixed axis.**

A hive, hut, roof or dome to circular tower or apartment; reversed: a copper or boiler.

Base, a **circle**; other base, a **point**; middle section, a **circle.** For radius of middle section, see formula given in No. 190, or at page 139, line 10, page 140, line 20 of "Key to Ster." Spheroidal area, see No. 191.

**194—Segment of prolate spheroid greater than half, by a plane perpendicular to fixed axis.**

A hut, hive, dome, a cauldron or copper, etc.

Base and middle section, **circles**; its other base, an apex or **point.** Its spheroidal surface resolvable into **continuous trapeziums** and a **circle** at apex.

---

**195—Middle frustum or solid zone of prolate spheroid by parallel planes perpendicular to fixed axis.**

A cask, keg, barrel, puncheon, hogshead, etc., "Key." page 138.

End bases, equal **circles**; middle section, a **circle.** Unlike the middle frustum of a spindle, the solid contents of this solid are obtained exactly by treating the whole figure at once.

---

**196—Middle frustum or solid zone of prolate spheroid by parallel planes oblique to axis.**

A boss on raking strut, etc.

Opposite bases and middle section, **similar ellipses**. Spheroidal surface, **trapeziums** of which take mean height.

---

**197—Lateral frustum or solid zone of prolate spheroid by planes perpendicular to fixed axis.**

Coved ceiling, base of column, etc.; reversed: capital of column, dish, basin, bowl, tub, hamper or basket, stew pan, cauldron or other vessel of capacity, etc., etc.

Bases and section, **circles,** for areas of which see tables II., III. and IV. "Key to Ster." For diameter of middle section, measure solid or compute by formula of page 139, line 10; page 140. line 20, where it is shown that the rectangle under the required radius, and either axis of the spheroid, is equal to that under the square root of the rectangle or product of the abscissæ of the first axis and the other axis.

---

**198—Lateral frustum or solid zone of prolate spheroid by planes parallel to each other, and to longer or fixed axis.**

Coved ceiling of elliptical plan, etc.; reversed: a flat-bottomed boat, a scow, a dish, basket, etc., etc.

Its parallel bases and middle section, **similar ellipses**; for areas of which see "Key to Ster." page 51. Its lateral area resolvable into **continuous trapeziums** of varying height if parallel to bases, but of uniform height, if lines be drawn from extremities of fixed axis.

---

**199—Segment of prolate spheroid by a plane inclined to axis.**

Liquid in spheroidal vessel inclined from the vertical, a scoop, scuttle, etc.

Its base and middle section, **similar ellipses**; its other base, a **point**; its spheroidal surface resolvable by circles drawn from extremity of fixed axis, into a **circle, trapeziums and a triangle.**

---

**200—Frustum of prolate spheroid between non-parallel planes.**

The one, perpendicular to fixed axis, the other oblique or inclined thereto.

Decompose into frustum with parallel bases, and an ungula. Compute separately, and add; or compute whole segment due to frustum and deduct lesser segment.

# THE AREAS OF
# SPHERICAL TRIANGLES & POLYGONS
## TO ANY RADIUS OR DIAMETER.

**Read before the mathematical, physical and chemical section of the Royal Society of Canada, May 22nd 1883.**

Last year I laid before this section of the Royal Society my proposal to substitute in schools the prismoidal formula for all other known formulae pertaining to the cubing of solid forms.

I then showed that on this sole condition, the computation of solidities, even the most difficult by ordinary rules, as of the segments, frusta and ungulae of Conoids and Spheroids, was susceptible of generalisation and of being taught in the most elementary institutions.

I then submitted that the advantage of the proposed system consisted in this; that while he who had gone through a course of mathematics would, in three months thereafter or out of college, have completely forgotten or have inextricably mixed up in his mind the numerous and ever varying formulae for arriving at the contents of solids; the simple artisan, on the contrary, who at an elementary, school would have been taught the universal formula, and who from the fact of having to learn but one, could not forget it nor mix it up in his mind with any others, could apply it always and everywhere during a life time without the aid even of any book excepting may be, to save time, a table of the areas of circles or of other figures lengthy of computation.

What I then did for the measurement of solid forms, I now propose to do for the mensuration of areas of spherical triangles and polygons on a sphere of any radius; I mean a simple and expeditious mode of getting at the doubly curved area of any portion of the terrestrial spheroid as of every sphere great or small: interior or exterior surface of a dome for example or of one of its component parts, as well of the bottom or roof of a gasometer, boiler, or of one of the constituent sections thereof, descending even to the surface of the ball of a spire, a shell, a cannon or a billard ball.

## TO THIS END :

| | |
|---|---|
| The area of a sphere to diameter I. being | =3.141,592,653,589,793+ |
| Dividing by 2, we get that of the hemisphere | =1,570,796,326,794,896,5 |
| This divided by 4=area of tri-rectgl'r sph. triangle | =0,392,699,081,698,724,1 |
| ÷90=area of 1° or of bi-rect. sph. tri. with sp. ex=1° | =0,004,363,323,129,985,8 |
| ÷60= " of 1' or of " " " " 1' | =0,000,072,722,052,166,43 |
| ÷60= " of 1" or of " " " " 1" | =0,000,001,212,034,202,77 |
| ÷10= " of 0.1" or of " " " " 0.1" | =0,000,000,121,203,420,277 |
| ÷10= " of 0.01" or of " " " " 0.01" | =0,000,000,012,120,342,027,7 |
| ÷10= " of 0.001" or of " " " " 0.001" | =0,000,000,001,212,034,202,77 |

Find the spherical excess, that is, the excess of the sum of the three spherical angles over two right angles, or from the sum of the three spherical angles deduct 180°. Multiply the remainder, that is, the spherical excess, by the tabular number herein above given: the degrees by the number set opposite to 1°, the minutes by that corresponding to 1' and so on of the seconds and fractions of a second; add these areas and multiply their sum by the square of the diameter of the sphere of the surface of which the given triangle forms part; the result is the area required.

## EXAMPLE.

Let the spherical excess of a triangle described on the surface of a sphere of which the diameter is an inch, a foot, or a mile, etc., be 3°—4'—2.235". What is the area ?

| | | | | | |
|---|---|---|---|---|---|
| Area of 1° | = 0.004,363,323,129,985,8 | × | 3 | = | 0.013,089,969,389,955 |
| " 1' | = 0.000,072,722,052,166,43 | × | 4 | = | 0.000,290,888,208,664 |
| " 1" | = 0.000,001,212,034,202 | × | 2 | = | 0.000,002,424,068,404 |
| " 0.1" | = 0.000,000,121,203,420 | × | 2 | = | 0.000,000,242,406,840 |
| " 0.01" | = 0.000,000,012,120,342 | × | 3 | = | 0.000,000,036,361,026 |
| " 0.001" | = 0.000,000,001,212,034 | × | 5 | = | 0.000,000,006,060,170 |
| | Area required | | | | 0.013,383,566,495,059 |

The answer is of course in square units or fractions of a square unit of the same name with the diameter. That is, if the diameter is an inch, the area is the fraction of a square inch; if a mile, the franction of a square mile, and so on.

Remark.— If the decimals of seconds are neglected, then of course the operation is simplified by the omission of the three last lines for tenths, hundredths and thousandths of a second or of so many of them as may be omitted.

If the seconds are omitted, as would be the case in dealing with any other triangle but one on the earth's surface, on account of its size; there will in such case remain only the two upper lines for degrees and minutes, which will prove of ample accuracy when dealing with any triangular space, compartment, or component section of a sphere of the size of a dome, vaulted ceiling, gasometer, or large copper or boiler, etc ; and in dealing with such spheres as a billiard or other playing ball, a cannon ball or shell, the ball of a vane or steeple, or any boiler, copper, etc., of ordinary size, it will generally suffice to compute for degrees only. Whence the following

RULE TO DEGREES ONLY.

Multiply the spherical excess in degrees by 0.004,363 and the result by the square of the diameter for the required area. For greater accuracy use—0.004,363,323.

RULE TO DEGREES AND MINUTES.

Proceed as by last rule for degrees. Multiply the spherical excess in minutes by 0.000,073, or for greater accuracy by 0.000,072,722. Add the results, and multiply their sum by the square of the diameter for the required area.

### EXAMPLE I.

Sum of angles 140° + 92° + 68° = 300; 300 — 180 = 120° spherical excess. Diameter = 30. Answer area of 1° 0.004,363

| | |
|---|---|
| Multiply by spherical excess | 120° |
| We get | 0.523,560 |
| This multiplied by square of diameter 30 = | 900 |
| Required area = | 471.194,000 |

A result correct to units. If now greater accuracy be required, it is be obtained by taking in more decimals; thus, say area $1° = 0.004,363,323$

$$\begin{array}{r} 0.004,363,323 \\ 120 \\ \hline 0.523,598,760 \\ 900 \\ \hline 471.238,884,000 \end{array}$$

EXAMPLE II.

The three angles each 120° their sum 360°, from which deducting 180° we get spherical excess = 180°. Diameter 20, of which the square = 400.

Answer..............................Area to $1° = 0.004,363.323$

$$\begin{array}{r} 180 \\ \hline 0.785,398,140 \\ 400 \\ \hline 314.159,256,000 \end{array}$$

EXAMPLE III.

The sum of the three angles of a triangle traced on the surface of the Terrestrial sphere exceeds by (1″) one second, 180°; what is the area of the triangle, supposing the Earth to be a perfect sphere with a diameter = 7,912 English miles, or, which is the same thing, that the diameter of the Terrestrial spheroid or of its osculatory circle at the given point on its surface be 7,912 miles.

Answer. Area of 1″ to diameter 1. = 0.000,001,212,034,202
Square of diameter 62,598,744

$$\begin{array}{r} 0.000,001,212,034,202 \\ 62,598,744 \\ \hline 75.871,818,730,242,288 \end{array}$$

Remark.—This unit 75.87 etc., as applied to the Terrestrial sphere, becomes a tabular number, which may be used for computing the area of any triangle on the earth's surface, as it evidently suffices to multiply the area 75.87 etc., corresponding to one second (1″) by the number of seconds in the spherical excess, to arrive at the result; and the result may be had true to the tenth, thousandth, or millionth of a second, or of any other fraction thereof by successively adding the same figures

75.87 etc., with the decimal point shifted to the left, one place for every place of decimals in the given fraction of such second: the tenth of a second giving 7.587 etc., square miles, the $0.01'' = .7587$ of a square mile, the $0.001'' = .07587$ etc., of a square mile, and so on; while, by shifting the decimal point to the right, we get successively $10'' = 758.7$ square miles, $100'' = 7587.$ etc., square miles, or $1' = 75.87 \times 60$ (number of seconds in a minute), $1° = 75.87 \times 60 \times 60$ (number of seconds in a degree).

RULE.

To compute the area of any spherical polygon.

Divide the polygon into triangles, compute each triangle separately by the foregoing rules for triangles and add the results.

OR,

From the sum of all the interior angles of the polygon subtract as many times two right angles as there are sides less two. This will give the spherical excess. This into the tabular area for degrees, minutes, seconds and fractions of a second, as the case may be, and the sum of such areas into the square of the diameter of the sphere on which the polygon is traced, will give the correct area of the proposed figure.

It may be remarked here that the area of a spherical lune or the convex surface of a spherical ungula is equal to the tabular number into twice the spherical excess, since it is evident that every such lune is equivalent to two bi-rectangular spherical triangles of which the angle at the apex, that is the inclination of the planes forming the ungula, is the spherical excess.

Remark.—The area found for any given spherical excess, on a sphere of given diameter, may be reduced to that, for the same spherical excess, on a sphere of any other diameter; these areas being as the squares of the respective diameters.

The area found for any given spherical excess on the earth's surface, where the diameter of the osculatory circle is supposed to be 7912 miles, may be reduced to that for the same spherical excess where the osculatory circle is of different radius; these areas being as the squares of the respective radii or diameters.

# ON THE APPLICATION OF THE

# PRISMOIDAL FORMULA

## TO THE MEASUREMENT OF ALL SOLIDS

By CHS. BAILLAIRGÉ, M. A.,

**Member of the Society for the Generalization of Education in France, and of several learned and scientific Societies, Chevalier of the Order of St. Sauveur de Monte-Reale, Italy, &c. Recepient of 13 medals of honor and 17 diplomas and letters from Russia, France, Italy, Belgium, Japan, &c. Member of the Royal Society of Canada.**

**Read before the mathematical section of the Society on Saturday the 28th of May. 1882. ***

" Cette formule $V = \frac{H}{6}(B + B' + 4 M)$ (Says " the late Revd, N. " Maingui of the Laval University) que Mr. Baillargé travaille à " vulgariser, a l'immense avantage de pouvoir remplacer toutes les " autres formules de stéréométrie,"

The prismoidal formula reads thus: " *To the sum of the opposite and parallel end areas of a prismoid, add four times the middle area and multiply the whole into one sixth the length or height of the solid.*"

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*** See this formula at article " Stéréométrie of " Le grand dictionnaire universel du XIXème siècle par P. Larousse."**

The following letter from the Minister of Education, Russia, may be considered interesting in its bearings on the subject matter of this communication

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MINISTERE DE L'INSTRUCTION PUBLIQUE.

Saint-Petersburg, le $\frac{20}{14}$ février 1877.

No. 1823.

A M. BAILLAIRGÉ,

Architecte à Québec,

Monsieur,

Le comité scientifique du ministère de l'Instruction Publique, (de Russie,) reconnaissant l'incontestable utilité de votre "Tableau Stéréométrique" pour l'enseignement de la géométrie en général de même que pour son application pratique à d'autres sciences, éprouve un plaisir tout particulier à joindre aux suffrages des savants de l'Europe et de l'Amérique sa complète approbation, en vous informant que le susdit tableau, avec toutes ses applications, sera recommandé aux écoles primaires et moyennes, pour en compléter les cabinets et les collections mathématiques, et inscrit dans les catalogues des ouvrages approuvés par le ministère de l'Instruction Publique.

Agréez, monsieur, l'assurance de ma haute considération.

Le chef du département au ministère de l'Instruction Publique,

E. DE BRADKER.

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*The following extract from the Quebec Mercury, July* 10, 1878 *further corroborates its importance.*

" It will be remembered that in February, 1877, Mr. Baillairgé received an official letter from the Minister of Public Instruction, of St. Petersburg, Russia, informing him that his new system of mensuration had been adopted in all the primary and medium schools of that vast empire. After a lapse of eighteen months, the system having been found to work well, Mr. Baillairgé has received an additional testimonial from the same source informing him that the system is to be applied in all the polytechnic shools of the Russian Empire."

Should the Royal Society of Canada prove instrumental in the introduction of the new system throughout the remainder of the civilized world. It will have shown that its creation by the Marquis of Lorne, the Govr. Gen. of Canada, has been in no way premature.

The definition of a prismoid as generally given is understood to apply to a solid having parallel end areas bounded by parallel sides.

This parallelism of the sides or edges of the opposite bases or end areas does not imply, not does it exclude any proportionality between such sides or edges.

Therefore is the frustum of a pyramid a prismoid, as also that of a cone which is nothing but an infinitary pyramid, or one having for its base a polygon of an infinite number of sides.

Now let two of the parallel edges of either base of the frustum approach each other until they meet or merge in a single line or arris, when we have the wedge which is therefore to all intents and purposes a prismoid.

Further let this edge or arris become shorter and shorter until it reduces to a point and then have we the pyramid which is again a prismoid, as is the cone.

It need hardly be said that the prism and cylinder are prismoids, whose opposite edges are equal as well as parallel in the same way as for the frusta of the pyramid and cone the opposite edges are proportional while parallel.

Now, nine tenths or more of all the vessels of capacity, the world over, and either on a large or reduced scale, have the shape of the frustum of a cone or pyramid ; the latter as evidenced in bins, troughs and cisterns of all sizes, in vehicles of capacity ; the former, in the brewers vat, the salting tub, the butter firkin, the commom wooden pail, the drinking goblet, the pan or pie dish, the wash tub — of whatever shape its base — the milk pan and what not else ; again the lamp shade, the shaft of a gun or mortar, the buoy, quai, pier, reservoir, tower, hay-rick, hamper, basket and the like.

These are forms which in every-day life the otherwise untutored hand and eye are called upon to estimate. Why then not teach a mode of doing it which every one can learn, and not only learn but what is of greater import, retain in mind or memory when mastered.

Why continue the old routine when, as here evidenced, it is so much more simple and concise, so much quicker to apply the prismoidal formula to all these forms, than resort to one more difficult of apprehension and which to carry or work out requires tenfold the time the other does.

Legendre's formula requires a geometric mean between the areas of the opposite bases of the solid under consideration. This mean is far less easily conceivable than the arithmetic one ; and to arrive at it the end areas are to be multiplied into each other, and the square root extracted of their product ; a long and tedions operation, one known only to the few, most difficult to retain, forgotten as soon as learnt and therefore useless.

With the formula proposed on the contrary, the operation is one which the merest child can master, the mere mechanic or the artisan remember all his life and readily apply ; for he has been taught at school to compute areas, that of the circle as well as others, a figure which he readily sees is resolvable into triangles by lines drawn from the centre to equidistant points, or not, in the circumference, and the area thence equal to the circumference—sum of the bases of the component triangles —into half the radius, or height of the successive sectors which make up the figure.

Now, of almost all the solide herein above alluded to, the opposite bases and middle section are circles and the operation can be further expedited by taking the areas ready made, to inches and even lines or less, from tables prepared for the purpose.

The labour then reduces to the mere arithmetic of adding the areas so found, that is the end areas and four times the middle area, and of multiplying the sum thereof into one sixth the altitude, or depth ; that is, to the simplest form of arithmetic taught in the most elementary schools, to wit : addition and multiplication, with division added when the cubical contents in feet, inches or other unit of capacity, are to be reduced, as of inches into gallons and the like.

I would have but one formula applicable to all bodies, and it will of course be asked : why, for instance in the case of the cylinder, the whole cone or pyramid, substitute the more complex for the simpler form of computation. My reason for doing so has its untold importance to thousands of the human race. Memory is not a gift to every one. I have none of it myself or hardly any, and its absence only entails a little reasoning as I am now to show.

I have seen students, only three months out of college doubtful as to which of the ordinary formulae to apply, to this pyramid or cone, the conoid, the spheroid. In one—the first—the volume is due to the base and one third the height ; in the second, the base and one half the height ; in the other, the base and two thirds the height. Any mistake is fatal to the result.

But with the one and only one, the unique and universal formula which I propose to substitute for every other, no error can obtain. Take hold of the pyramid or cone : set down its upper or one end area or that of its apex, equal nought (0) or zero, its other end area, whatever that may be. Its middle area, you see at once is one quarter that of its base ; for the middle or half way diameter is half that of the base, and the areas of similar figures as the squares of their homologous or like dimensions. Now, ere you have put this down on paper ; ere you have had time to do so, the reasoning process is going on within your mind and in far less time than it takes me to relate it — that four times the middle area plus the area of the base is equal to twice the base, and that twice the base into one sixth the altitude is precisely the same thing as once the base, that is, the base into one third the altitude, and so come you back to the old or ordinary rule, the simpler of the two in this case, and without the necessity of having this formula stored in your mind as a separate process.

And so with the cylinder where you see at once that the area of each base and of the middle section being all equal quantities, the sum of these bases and of four times the middle section is the same thing as six time the base, and again that six times the base into one sixth the altitude is the old rule of the base into the altitude, without the necessity of remembering it as a separate and additional formula.

But the great advantage of this one universal rule, its beauty so to say is further evidenced and more strikingly in the computation of the more difficult solids, that is of those which are more difficult under the old or ordinary rules.

In the sphere, spheroid and conoids, the one area, that at the apex or crown is always nought or nothing, as a plane there touches them in one and only one point. The formula applled to the sphere and spheroid therefore reduces to four times the middle area into one sixth the altitude or diameter or axis perpendicular to the plane of section.

Now, let it be required to measure the liquid in a conoidal or spheroidal vessel inclined to the horison or out of the vertical. This by ordinary rules, becomes an operation of much time, trouble and anxiety, as the size of the whole body or solid of which the portion or figure under consideration forms a part, has to be made known, its factors entering into the formula for the content required; whereas by the prismoidal formula, no concern need be had as to the dimensions of the entire body of which the figure submitted to computation is a segment.

That the rule applies to all such cases, is and has been abundantly proven by myself (see my treatise of 1866) as applied to any segment of a sphere or spheroid, to any ungula of such solids contained between planes passing in any direction through the centre, to any frustum of these bodies, — lateral or central — contained between parallel planes inclined in any way to the axes; to any parabolic or hyperbolic conoid, right or inclined, as well to any parallel frustum of either.

This proof has been substantiated by MM. Steckel of the Dept. of Dominion Public Works, Deville a member of this society, and the late Revd. M. Maingui, professor of Mathematics at the Laval University, as well by the Revd. M. Billion, of the Seminary of St. Sulpice—Montreal; by His Grace, bishop Langevin of Rimouski, and by many other mathematicians fully adequate to the task.

M. Maingui says (page IX of his pamphlet and as already quoted from the french version): "This formula $V = \frac{H}{6}(B + B + 4\,M)$ is that "which Mr. Baillairgé is endeavouring to introduce; *it has the im-" mense advantage of replacing all other stereometrical formulae."*

This is the only formula which will allow of teaching stereometry in all schools however elementary, and as has just been shown, the application of it is the more simple, so to say, the more complex the body is, since in the conoid and segment of spheroid, one of the factors at least is zero, while two of them are zeros in the sphere and spheroid as in their ungulae.

Thus while the student at college or from a University after having devoted much time to the acquisition of a hundred rules for the cubing of as many solids, has hopelessly forgotten them in after life, the comparatively illiterate artisan, tradesman, merchant, &c. who has never frequented ought but a village school, will, having but one rule wherewith to charge his memory, remember it all his life and be ever ready to apply it ?

In the case of spindles and the masurement of their middle frusta — the representatives of casks of all varieties and sizes, — the prismoidal formula does not bring out the true content to within the tenth or twentieth and up to the half or thereabout of one per cent; notwithstanding which, it is the only practical formula which can bring out anything like a reliable result. The true formulae for casks never can, nor will they ever be applied; they are too lengtly, too abstruse, and the wine merchant will tell you that the nearest the guage rod can come to within the truth, the guage rod founded on these formulae, is to within from one to three and even four per cent. This stands to reason, as when operating on the half cask—which is always done with all figures having symmetrical and equal halves—the half way diameter between the head and bung, the very element by which the cask varies its capacity, enters as a factor into the occupation, while the guaging rod can take no note of it.

It remains but to say that in the case of hoofs and ungulae of cones and cylinders, of conoids and of spheroids, when the bounding planes do not pass through the centre, the prismoidal formula is still the best to be employed in practice, and again brings out the volume to within one half or so of one per cent. The true rules applicable to these ungulae can never be remembered, nor are or will they ever be applied in practice. Rather than that, the fudging or so called rule of thumb system, some averaging of the dimensions is sure to be resorted to and a result arrived at, where two or three to five per cent of error is considered near enough, while the proposed application of the prismoidal formula would reduce the error to almost nothing.

Compound bodies must of course be treated separately or in parts. Thus, a gun or mortar, as made up of a cylinder or the frustum of a cone and the segment or half of a sphere or spheroid ; a morish or turkish dome, as the frustum of a spheroid surmounted by a hollow cone ; a roofed tower, as a cone and cylinder, a cone and frustum of a cone or two conic frusta as the case may be and so of other compound forms.

Again when frusta between non parallel bases are to be treated, the solid is to be divided by a plane parallel to one of its bases and passing through the nearest edge or point of its opposite base, into a frustum proper and an angula, subject to the percentage of error already noticed in the volume of the angula ; while, by cubing the whole conoid on segment of a spheroid of which the frustum forms a part, and then the segment which is wanting to make up the whole, the true content can be arrived it.

There are a class of solid forms where it would appear at first sight that a departure from the prismoidal formula becomes necessary ; not so however as will presently be seen. I allude to the cubing of the fragment of a shell for instance, or of the material forming the vaulting of a dome as contained between its intrades and extrados. This is simply arrived at, when the inner and outer faces are parallel or when the dome or arch is of uniform thickness by applying the spherical, spheroidal or cylindrical surfaces of the opposite bases, and the equally curved surface of the middle section; while, when the faces are not parrallel or the thickness of varying dimensions, as well when the faces are everywhere aquidistant, the volume may be had by cubing the outer and inner component pyramids and taking the difference between them.

And in the making out of such spherical areas as may enter as factors into any computation, a most concise and easy rule will be found at page 35 of my "stereometricon" published in 1880; when any such area can in a few minutes be made up the mere multiplication and addition of the elemental quantities given in the text, and any portion of the earths surface thus arrived at when the radius of the osculatory circle for the given latitude is known.

With irregular forms, the figure can be sliced up and treated by the formula, and those forms when small and still more complex, such as carving, statuary, bronzes and the like, can be measured with minute accuracy by the indirect process of the quantity of fluid of any kind displaced, as of water when non obsorbent or of sand or sawdust etc., when the contrary.

Again may the specific gravities of bodies be applied, or their weights to making out their, volumes by simple rule of three, or the reverse process of weighing them by ratio when their volumes are ascertained.

Finally the quantities and respective weights of the separate substances which enter into amalgams or alloys are obtainable as taught by a comparison of their weights in air and water, that is of the amalgam itself and of its unalloyed constituents.

The whole field of solid meusuration is thus gone over in these few pages, instead of the volume required to contain the many separate and varied formulae which the old process of computation gives rise to and renders indispensable. The whole I say is gone over in as many minutes as the oll process requires hours or even days.

# TABLES

OF

I. Squares and Square Roots of numbers from 1 to 1600.

II. Circumferences and areas of circles of diameter $\frac{1}{64}$ to 150 advancing by $\frac{1}{8}$.

III. Circumferences and areas of circles of diameter $\frac{1}{10}$ to 100 advancing by $\frac{1}{10}$.

IV. Circumferences and areas of circles of diameter 1 to 50 feet, advancing by 1 inch.

V. Sides of Squares equal in area to a circle of a diameter 1 to 100 advancing by a $\frac{1}{4}$.

VI. Lengths of circular arcs, to diameter 1 divided into 1000 equal parts.

VII. Lengths of semi-elliptic arcs to transverse diameter 1 divided into 1000 equal parts.

VIII. Areas of the segments of a circle to diameter 1 divided into 1000 equal parts.

IX. Areas of the zones of a circle to a diameter 1 divided into 1000 equal parts.

X. Specific gravities or weights of bodies of all kinds solid, fluid, liquid and gazeous.

| No. | Square. | Sqre. root. | No. | Square. | Sqre. root. | No. | Square. | Sqre. root. |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0000000 | 61 | 3721 | 7.8102497 | 121 | 14641 | 11.0000000 |
| 2 | 4 | 1.4142136 | 62 | 3844 | 7.8740079 | 122 | 14834 | 11.0453610 |
| 3 | 9 | 1.7320508 | 63 | 3969 | 7.9372539 | 123 | 15129 | 11.0905365 |
| 4 | 16 | 2.0000000 | 64 | 4096 | 8.0000000 | 124 | 15376 | 11.1355287 |
| 5 | 25 | 2.2360680 | 65 | 4225 | 8.0622577 | 125 | 15625 | 11.1803399 |
| 6 | 36 | 2.4494897 | 66 | 4356 | 8.1240384 | 126 | 15876 | 11.2249722 |
| 7 | 49 | 2.6457513 | 67 | 4488 | 8.1853528 | 127 | 16129 | 11.2694277 |
| 8 | 64 | 2.8284271 | 68 | 4624 | 8.2462113 | 128 | 16384 | 11.3137085 |
| 9 | 81 | 3.0000000 | 69 | 4761 | 8.3066239 | 129 | 16641 | 11.3578167 |
| 10 | 100 | 3.1622777 | 70 | 4900 | 8.3666003 | 130 | 16900 | 11.4017543 |
| 11 | 121 | 3.3166248 | 71 | 5041 | 8.4261498 | 131 | 17161 | 11.4455231 |
| 12 | 144 | 3.4641016 | 72 | 5184 | 8.4852814 | 132 | 17424 | 11.4891253 |
| 13 | 169 | 3.6055513 | 73 | 5329 | 8.5440037 | 133 | 17689 | 11.5325626 |
| 14 | 196 | 3.7416574 | 74 | 5476 | 8.6023253 | 134 | 17956 | 11.5758369 |
| 15 | 225 | 3.8229833 | 75 | 5625 | 8.6602540 | 135 | 18225 | 11.6189500 |
| 16 | 256 | 4.0000000 | 76 | 5776 | 8.7177979 | 136 | 18496 | 11.6619038 |
| 17 | 289 | 4.1231056 | 77 | 5929 | 8.7749644 | 137 | 18769 | 11.7046999 |
| 18 | 324 | 4.2426407 | 78 | 6084 | 8.8317609 | 138 | 19044 | 11.7473401 |
| 19 | 361 | 4.3585989 | 79 | 6241 | 8.8881944 | 139 | 19321 | 11.7898261 |
| 20 | 400 | 4.4721360 | 80 | 6400 | 8.9442719 | 140 | 19600 | 11.8321596 |
| 21 | 441 | 4.5825757 | 81 | 6561 | 9.0000000 | 141 | 19881 | 11.8743421 |
| 22 | 484 | 4.6904158 | 82 | 6724 | 9.0553851 | 142 | 20164 | 11.9163753 |
| 23 | 529 | 4.7958315 | 83 | 6889 | 9.1104336 | 143 | 20349 | 11.9582607 |
| 24 | 576 | 4.8989795 | 84 | 7056 | 9.1651514 | 144 | 20736 | 12.0000000 |
| 25 | 625 | 5.0000000 | 85 | 7225 | 9.2195445 | 145 | 21025 | 12.0415946 |
| 26 | 676 | 5.0990195 | 86 | 7396 | 9.2736185 | 146 | 21316 | 12.0830460 |
| 27 | 729 | 5.1961524 | 87 | 7569 | 9.3273791 | 147 | 21609 | 12.1243557 |
| 28 | 784 | 5.2915026 | 88 | 7744 | 9.3808315 | 148 | 21904 | 12.1655251 |
| 29 | 841 | 5.3851648 | 89 | 7921 | 9.4339811 | 149 | 22201 | 12.2065556 |
| 30 | 900 | 5.4772256 | 90 | 8100 | 9.4868330 | 150 | 22500 | 12.2474487 |
| 31 | 961 | 5.5677644 | 91 | 8281 | 9.5393920 | 151 | 22801 | 12.2882057 |
| 32 | 1024 | 5.6568542 | 92 | 8464 | 9.5916634 | 152 | 23104 | 12.3288280 |
| 33 | 1089 | 5.7445626 | 93 | 8649 | 9.6436508 | 153 | 23409 | 12.3693169 |
| 34 | 1156 | 5.8309519 | 94 | 8836 | 9.6953597 | 154 | 23716 | 12.4096736 |
| 35 | 1225 | 5.9160798 | 95 | 9025 | 9.7467943 | 155 | 24025 | 12.4498996 |
| 36 | 1296 | 6.0000000 | 96 | 9216 | 9.7979590 | 156 | 24336 | 12.4899960 |
| 37 | 1369 | 6.0827625 | 97 | 9409 | 9.8488578 | 157 | 24649 | 12.5299641 |
| 38 | 1444 | 6.1644140 | 98 | 9604 | 9.8994949 | 158 | 24964 | 12.5698051 |
| 39 | 1521 | 6.2449980 | 99 | 9801 | 9.9498744 | 159 | 25281 | 12.6095202 |
| 40 | 1600 | 6.3245553 | 100 | 10000 | 10.0000000 | 160 | 25600 | 12.6491106 |
| 41 | 1681 | 6.4031242 | 101 | 10201 | 10.0498756 | 161 | 25921 | 12.6885775 |
| 42 | 1764 | 6.4807407 | 102 | 10404 | 10.0995049 | 162 | 26244 | 12.7279221 |
| 43 | 1849 | 6.5574385 | 103 | 10609 | 10.1488916 | 163 | 26569 | 12.7671453 |
| 44 | 1936 | 6.6332496 | 104 | 10816 | 10.1980390 | 164 | 26896 | 12.8062485 |
| 45 | 2025 | 6.7082039 | 105 | 11025 | 10.2469508 | 165 | 27225 | 12.8452326 |
| 46 | 2116 | 5.7823300 | 106 | 11236 | 10.2956301 | 166 | 27556 | 12.8840987 |
| 47 | 2209 | 6.8556546 | 107 | 11449 | 10.3440804 | 167 | 27889 | 12.9228480 |
| 48 | 2304 | 6.9282032 | 108 | 11664 | 10.3923048 | 168 | 28224 | 12.9614814 |
| 49 | 2401 | 7.0000000 | 109 | 11881 | 10.4403065 | 169 | 28561 | 13.0000000 |
| 50 | 2500 | 7.0710678 | 110 | 12100 | 10.4880885 | 170 | 28900 | 13.0384048 |
| 51 | 2601 | 7.1414284 | 111 | 12321 | 10.5356538 | 171 | 29241 | 13.0766968 |
| 52 | 2704 | 7.2111026 | 112 | 12544 | 10.5830052 | 172 | 29584 | 13.1148770 |
| 53 | 2809 | 7.2801099 | 113 | 12769 | 10.6301458 | 173 | 29929 | 13.1529464 |
| 54 | 2916 | 7.3484692 | 114 | 12996 | 10.6770783 | 174 | 30276 | 13.1909060 |
| 55 | 3025 | 7.4161985 | 115 | 13225 | 10.7238053 | 175 | 30625 | 13.2287566 |
| 56 | 3136 | 7.4833148 | 116 | 13456 | 10.7703296 | 176 | 30976 | 13.2664992 |
| 57 | 3249 | 7.5498344 | 117 | 13689 | 10.8166538 | 177 | 31329 | 13.3041347 |
| 58 | 3364 | 7.6157731 | 118 | 13924 | 10.8627805 | 178 | 31684 | 13.3416641 |
| 59 | 3481 | 7.6811457 | 119 | 14161 | 10.9087121 | 179 | 32041 | 13.3790882 |
| 60 | 3600 | 7.7459667 | 120 | 14400 | 10.9544512 | 180 | 32400 | 13.4164079 |

| No. | Square. | Sqre. root. | No. | Square. | Sqre. root. | No. | Square. | Sqre. root. |
|---|---|---|---|---|---|---|---|---|
| 181 | 32761 | 13.4536240 | 241 | 58081 | 15.5241747 | 301 | 90601 | 17.3493516 |
| 182 | 33124 | 13.4907376 | 242 | 58564 | 15.5563492 | 302 | 91204 | 17.3781472 |
| 183 | 33489 | 13.5277493 | 243 | 59049 | 15.5884573 | 303 | 91809 | 17.4068952 |
| 184 | 33856 | 13.5646600 | 244 | 59536 | 15.6204994 | 304 | 92416 | 17.4355958 |
| 185 | 34225 | 13.6014705 | 245 | 60025 | 15.6524758 | 305 | 93025 | 17.4642492 |
| 186 | 34596 | 13.6381817 | 246 | 60516 | 15.6843871 | 306 | 93636 | 17.4928557 |
| 187 | 34969 | 13.6747943 | 247 | 61009 | 15.7162336 | 307 | 94249 | 17.5214155 |
| 188 | 35344 | 13.7113092 | 248 | 61504 | 15.7480157 | 308 | 94864 | 17.5499288 |
| 189 | 35721 | 13.7477271 | 249 | 62001 | 15.7797338 | 309 | 95481 | 17.5783958 |
| 190 | 36100 | 13.7840488 | 250 | 62500 | 15.8113883 | 310 | 96100 | 17.6068169 |
| 191 | 36481 | 13.8202750 | 251 | 63001 | 15.8429795 | 311 | 96721 | 17.6351921 |
| 192 | 36864 | 13.8564065 | 252 | 63504 | 15.8745079 | 312 | 97344 | 17.6635217 |
| 193 | 37249 | 13.8924400 | 253 | 64009 | 15.9059737 | 313 | 97969 | 17.6918060 |
| 194 | 37636 | 13.9283883 | 254 | 64516 | 15.9373775 | 314 | 98596 | 17.7200451 |
| 195 | 38025 | 13.9642400 | 255 | 65025 | 15.9687194 | 315 | 99225 | 17.7482393 |
| 196 | 38416 | 14.0000000 | 256 | 65536 | 16.0000000 | 316 | 99856 | 17.7763888 |
| 197 | 38809 | 14.0356688 | 257 | 66049 | 16.0312195 | 317 | 100489 | 17.8044938 |
| 198 | 39204 | 14.0712473 | 258 | 66564 | 16.0623784 | 318 | 101124 | 17.8325545 |
| 199 | 39601 | 14.1067360 | 259 | 67081 | 16.0934769 | 319 | 101761 | 17.8605711 |
| 200 | 40000 | 14.1421356 | 260 | 67600 | 16.1245155 | 320 | 102400 | 17.8885438 |
| 201 | 40401 | 14.1774469 | 261 | 68121 | 16.1554944 | 321 | 103041 | 17.9164729 |
| 202 | 40804 | 14.2126704 | 262 | 68644 | 16.1864141 | 322 | 103684 | 17.9443584 |
| 203 | 41209 | 14.2478068 | 263 | 69169 | 16.2172747 | 323 | 104329 | 17.9722008 |
| 204 | 41616 | 14.2828569 | 264 | 69696 | 16.2480768 | 324 | 104976 | 18.0000000 |
| 205 | 42025 | 14.3178211 | 265 | 70225 | 16.2788206 | 325 | 105625 | 18.0277564 |
| 206 | 42436 | 14.3527001 | 266 | 70756 | 16.3095064 | 326 | 106276 | 18.0554701 |
| 207 | 42849 | 14.3874946 | 267 | 71289 | 16.3401346 | 327 | 106929 | 18.0831413 |
| 208 | 43264 | 14.4222051 | 268 | 71824 | 16.3707055 | 328 | 107584 | 18.1107703 |
| 209 | 43681 | 14.4568323 | 269 | 72361 | 16.4012195 | 329 | 108241 | 18.1383571 |
| 210 | 44100 | 14.4913767 | 270 | 72900 | 16.4316767 | 330 | 108900 | 18.1659021 |
| 211 | 44521 | 14.5258390 | 271 | 73441 | 16.4620776 | 331 | 109561 | 18.1934054 |
| 212 | 44944 | 14.5602198 | 272 | 73984 | 16.4924225 | 332 | 110224 | 18.2208672 |
| 213 | 45369 | 14.5945195 | 273 | 74529 | 16.5227116 | 333 | 110889 | 18.2482876 |
| 214 | 45796 | 14.6287388 | 274 | 75076 | 16.5529454 | 334 | 111556 | 18.2756669 |
| 215 | 46225 | 14.6628783 | 275 | 75625 | 16.5831240 | 335 | 112225 | 18.3030052 |
| 216 | 46656 | 14.6969385 | 276 | 76176 | 16.6132477 | 336 | 112896 | 18.3303028 |
| 217 | 47089 | 14.7309199 | 277 | 76729 | 16.6433170 | 337 | 113569 | 18.3575598 |
| 218 | 47524 | 14.7648231 | 278 | 77284 | 16.6733320 | 338 | 114244 | 18.3847763 |
| 219 | 47961 | 14.7986486 | 279 | 77841 | 16.7032931 | 339 | 114921 | 18.4119526 |
| 220 | 48400 | 14.8323970 | 280 | 78400 | 16.7332005 | 340 | 115600 | 18.4390889 |
| 221 | 48841 | 14.8660687 | 281 | 78961 | 16.7630546 | 341 | 116281 | 18.4661853 |
| 222 | 49284 | 14.8996644 | 282 | 79524 | 16.7928556 | 342 | 116964 | 18.4932420 |
| 223 | 49729 | 14.9331845 | 283 | 80089 | 16.8226038 | 343 | 117649 | 18.5202592 |
| 224 | 50176 | 14.9666295 | 284 | 80656 | 16.8522995 | 344 | 118336 | 18.5472370 |
| 225 | 50625 | 15.0000000 | 285 | 81225 | 16.8819430 | 345 | 119025 | 18.5741756 |
| 226 | 51076 | 15.0332964 | 286 | 81796 | 16.9115345 | 346 | 119716 | 18.6010752 |
| 227 | 51529 | 15.0665192 | 287 | 82369 | 16.9410743 | 347 | 120409 | 18.6279360 |
| 228 | 51984 | 15.0996689 | 288 | 82944 | 16.9705627 | 348 | 121104 | 18.6547581 |
| 229 | 52441 | 15.1327460 | 289 | 83521 | 17.0000000 | 349 | 121801 | 18.6815417 |
| 230 | 52900 | 15.1657509 | 290 | 84100 | 17.0293864 | 350 | 122500 | 18.7082869 |
| 231 | 53361 | 15.1986842 | 291 | 84681 | 17.0587221 | 351 | 123201 | 18.7349940 |
| 232 | 53824 | 15.2315462 | 292 | 85264 | 17.0880075 | 352 | 123904 | 18.7616630 |
| 233 | 54289 | 15.2643375 | 293 | 85849 | 17.1172428 | 353 | 124609 | 18.7882942 |
| 234 | 54756 | 15.2970585 | 294 | 86436 | 17.1464282 | 354 | 125316 | 18.8148877 |
| 235 | 55225 | 15.3297097 | 295 | 87025 | 17.1755640 | 355 | 126025 | 18.8414437 |
| 236 | 55696 | 15.3622915 | 296 | 87616 | 17.2046505 | 356 | 126736 | 18.8679623 |
| 237 | 56169 | 15.3948043 | 297 | 88209 | 17.2336879 | 357 | 127449 | 18.8944436 |
| 238 | 56644 | 15.4272486 | 298 | 88804 | 17.2626765 | 358 | 128164 | 18.9208879 |
| 239 | 57121 | 15.4596248 | 299 | 89401 | 17.2916165 | 359 | 128881 | 18.9472953 |
| 240 | 57600 | 15.4919334 | 300 | 90000 | 17.3205081 | 360 | 129600 | 18.9736660 |

| No. | Square. | Sqre. root. | No. | Square. | Sqre. root. | No. | Square. | Sqre. root. |
|---|---|---|---|---|---|---|---|---|
| 361 | 130321 | 19.0000000 | 421 | 177241 | 20.5182845 | 481 | 231361 | 21.9318122 |
| 362 | 131044 | 19.0262976 | 422 | 178084 | 20.5426386 | 482 | 232324 | 21.9544984 |
| 363 | 131769 | 19.0525589 | 423 | 178928 | 20.5669638 | 483 | 233289 | 21.9772610 |
| 364 | 132496 | 19.0787840 | 424 | 179776 | 20.5912603 | 484 | 234256 | 22.0000000 |
| 365 | 133225 | 19.1049732 | 425 | 180625 | 20.6155281 | 485 | 235225 | 22.0227155 |
| 366 | 133966 | 19.1311265 | 426 | 181476 | 20.6397674 | 486 | 236196 | 22.0454077 |
| 367 | 134689 | 19.1572441 | 427 | 182329 | 20.6639783 | 487 | 237169 | 22.0680765 |
| 368 | 135424 | 19.1833261 | 428 | 183184 | 20.6881609 | 488 | 238144 | 22.0907220 |
| 369 | 136161 | 19.2093727 | 429 | 184041 | 20.7123152 | 489 | 239121 | 22.1133444 |
| 370 | 136900 | 19.2353841 | 430 | 184900 | 20.7364414 | 490 | 240100 | 22.1359436 |
| 371 | 137641 | 19.2613603 | 431 | 185761 | 20.7605395 | 491 | 241081 | 22.1585198 |
| 372 | 138384 | 19.2873015 | 432 | 186624 | 20.7846097 | 492 | 242064 | 22.1810730 |
| 373 | 139129 | 19.3132079 | 433 | 187489 | 20.8086520 | 493 | 243049 | 22.2036033 |
| 374 | 139876 | 19.3390796 | 424 | 188356 | 20.8326667 | 494 | 244036 | 22.2261108 |
| 375 | 140625 | 19.3649167 | 435 | 189225 | 20.8566536 | 495 | 245025 | 22.2485955 |
| 376 | 141376 | 19.3907194 | 436 | 190096 | 20.8845130 | 496 | 246016 | 22.2710575 |
| 377 | 142129 | 19.4164878 | 437 | 190969 | 20.9045450 | 497 | 247009 | 22.2934968 |
| 378 | 142884 | 19.4422221 | 438 | 191844 | 20.9284495 | 498 | 248004 | 22.3159136 |
| 379 | 143641 | 19.4679223 | 439 | 192721 | 20.9523268 | 499 | 249001 | 22.3383079 |
| 380 | 144400 | 19.4835887 | 440 | 193600 | 20.9765770 | 500 | 250000 | 22.3606798 |
| 381 | 145161 | 19.5192213 | 441 | 194481 | 21.0000000 | 501 | 251001 | 22.3830293 |
| 382 | 145924 | 19.5448203 | 442 | 195364 | 21.0237960 | 502 | 252004 | 22.4053565 |
| 383 | 146689 | 19.5703858 | 443 | 196249 | 21.0475652 | 503 | 253009 | 22.4276615 |
| 384 | 147456 | 19.5959179 | 444 | 197136 | 21.0713075 | 504 | 254016 | 22.4499443 |
| 385 | 148225 | 19.6214169 | 445 | 198025 | 21.0950231 | 505 | 252025 | 22.4722051 |
| 386 | 148996 | 19.6468827 | 446 | 198916 | 21.1187121 | 506 | 256036 | 22.4944438 |
| 387 | 149769 | 19.6723156 | 447 | 199809 | 21.1423745 | 507 | 257049 | 22.5166605 |
| 388 | 150544 | 19.6978156 | 448 | 200704 | 21.1660105 | 508 | 258064 | 22.5388553 |
| 389 | 151321 | 19.7230829 | 449 | 201601 | 21.1896201 | 509 | 259041 | 22.5610283 |
| 390 | 152100 | 19.7484177 | 450 | 202500 | 21.2132034 | 510 | 260100 | 22.5831796 |
| 391 | 152881 | 19.7737199 | 451 | 203401 | 21.2367606 | 511 | 261121 | 22.6053091 |
| 392 | 153664 | 19.7989899 | 452 | 204304 | 21.2602916 | 512 | 262144 | 22.6274170 |
| 393 | 154449 | 19.8242276 | 453 | 205209 | 21.2837967 | 513 | 263169 | 22.6495033 |
| 394 | 155236 | 19.8494332 | 454 | 206116 | 21.3072758 | 514 | 264196 | 22.6715681 |
| 395 | 156025 | 19.8746069 | 455 | 207025 | 21.3307290 | 515 | 265225 | 22.6936114 |
| 396 | 156816 | 19.8997487 | 456 | 207936 | 21.3501565 | 516 | 266256 | 22.7156334 |
| 397 | 157609 | 19.9248588 | 457 | 208849 | 21.3775583 | 517 | 267289 | 22.7376340 |
| 398 | 158404 | 19.9499373 | 458 | 209764 | 21.4009346 | 518 | 268324 | 22.7596134 |
| 399 | 159201 | 19.9749844 | 459 | 210681 | 21.4242853 | 519 | 269361 | 22.7815715 |
| 400 | 160000 | 20.0000000 | 460 | 211600 | 21.4476106 | 520 | 270400 | 22.8035085 |
| 401 | 160801 | 20.0249844 | 461 | 212521 | 21.4709106 | 521 | 271411 | 22.8254244 |
| 402 | 161604 | 20.0499377 | 462 | 213444 | 21.4941853 | 522 | 272484 | 22.8473193 |
| 403 | 162409 | 20.0748599 | 463 | 214369 | 21.5174348 | 523 | 273529 | 22.8691933 |
| 404 | 163216 | 20.0997512 | 464 | 215296 | 21.5406592 | 524 | 274576 | 22.8910463 |
| 405 | 164025 | 20.1246118 | 465 | 216225 | 21.5638587 | 525 | 275625 | 22.9128775 |
| 406 | 164836 | 20.1494417 | 466 | 217156 | 21.6870331 | 526 | 276676 | 22.9346899 |
| 407 | 165649 | 20.1742410 | 467 | 218089 | 21.6101828 | 527 | 277729 | 22.9564806 |
| 408 | 166464 | 20.1990099 | 468 | 219024 | 21.6333077 | 528 | 278784 | 22.9782506 |
| 409 | 167281 | 20.2237484 | 469 | 219961 | 21.6564078 | 529 | 279841 | 23.0000000 |
| 410 | 168100 | 20.2484567 | 470 | 220900 | 21.6794834 | 530 | 280900 | 23.0217289 |
| 411 | 168921 | 20.2731349 | 471 | 221841 | 21.7025344 | 531 | 280961 | 23.0434372 |
| 412 | 169744 | 20.2977831 | 472 | 222784 | 21.7255610 | 532 | 283024 | 23.0651252 |
| 413 | 170569 | 20.3224014 | 473 | 223729 | 21.7485632 | 533 | 284089 | 23.0867928 |
| 414 | 171396 | 20.3469899 | 474 | 224676 | 21.7715411 | 534 | 285156 | 23.1084400 |
| 415 | 172225 | 20.3715488 | 475 | 225625 | 21.7944947 | 535 | 286225 | 23.1300670 |
| 416 | 173056 | 20.3960781 | 476 | 226576 | 21.8174242 | 536 | 287296 | 23.1516738 |
| 417 | 173889 | 20.4205779 | 477 | 227529 | 21.8403297 | 537 | 288369 | 23.1732605 |
| 418 | 174724 | 20.4450483 | 478 | 228484 | 21.8632111 | 538 | 289444 | 23.1948270 |
| 419 | 175561 | 20.4694895 | 479 | 229441 | 21.8860686 | 539 | 290521 | 23.2163735 |
| 420 | 176400 | 20.4939015 | 480 | 230400 | 21.9089023 | 540 | 291600 | 23.2379001 |

| No. | Square. | Sqre. root. | No. | Square. | Sqre. root. | No. | Square. | Sqre. root. |
|---|---|---|---|---|---|---|---|---|
| 541 | 292681 | 23.2594067 | 601 | 361201 | 24.5153013 | 661 | 436921 | 25.7099203 |
| 542 | 293764 | 23.2808935 | 602 | 362404 | 24.5356883 | 662 | 438244 | 25.7293607 |
| 543 | 294849 | 23.3023604 | 603 | 363609 | 24.5560583 | 663 | 439569 | 25.7487864 |
| 544 | 295936 | 23.3238076 | 604 | 364816 | 24.5764115 | 664 | 440896 | 25.7681975 |
| 545 | 297025 | 23.3452351 | 605 | 366025 | 24.5967478 | 665 | 442225 | 25.7875939 |
| 546 | 298116 | 23.3666429 | 606 | 367236 | 24.6170673 | 666 | 443555 | 25.8069758 |
| 547 | 299209 | 23.3880311 | 607 | 368449 | 24.6373700 | 667 | 444839 | 25.8263431 |
| 548 | 300304 | 23.4093998 | 608 | 369664 | 24.6576560 | 668 | 446224 | 25.8456960 |
| 549 | 301401 | 23.4307490 | 609 | 370881 | 24.6779254 | 669 | 447561 | 25.8650343 |
| 550 | 302500 | 23.4520788 | 610 | 372100 | 24.6981781 | 670 | 448900 | 25.8843582 |
| 551 | 303601 | 23.4733892 | 611 | 373321 | 24.7184142 | 671 | 450241 | 25.9036677 |
| 552 | 304704 | 23.4946802 | 612 | 374544 | 24.7386338 | 672 | 451584 | 25.9229628 |
| 553 | 305809 | 23.5159520 | 613 | 375769 | 24.7588368 | 673 | 452929 | 25.9422435 |
| 554 | 306916 | 23.5372046 | 614 | 376996 | 24.7790234 | 674 | 454276 | 25.9615100 |
| 555 | 308025 | 23.5584380 | 615 | 378225 | 24.7991935 | 675 | 455625 | 25.9807621 |
| 556 | 309136 | 23.5796522 | 616 | 379456 | 24.8193473 | 676 | 456976 | 26.0000000 |
| 557 | 310249 | 23.6008474 | 617 | 380689 | 24.8394847 | 677 | 458329 | 26.0192237 |
| 558 | 311364 | 23.6220236 | 618 | 381924 | 24.8596058 | 678 | 459684 | 26.0384331 |
| 559 | 312481 | 23.6431808 | 619 | 383161 | 24.8797106 | 679 | 461041 | 26.0576284 |
| 560 | 313600 | 23.6643191 | 620 | 384400 | 24.8997992 | 680 | 462400 | 26.0968096 |
| 561 | 314721 | 23.6854386 | 621 | 385641 | 24.9198716 | 681 | 463761 | 26.0759767 |
| 562 | 315844 | 23.7065392 | 622 | 386884 | 24.9399278 | 682 | 465124 | 26.1151297 |
| 563 | 316969 | 23.7276210 | 623 | 388129 | 24.9699679 | 683 | 466489 | 26.1342687 |
| 564 | 318096 | 23.7486842 | 624 | 389376 | 24.9799920 | 684 | 467856 | 26.1533837 |
| 565 | 319225 | 23.7697286 | 625 | 390625 | 25.0000000 | 685 | 469225 | 26.1725047 |
| 566 | 320356 | 23.7807545 | 626 | 381876 | 25.0199920 | 686 | 470596 | 26.1916017 |
| 567 | 321489 | 23.8117618 | 627 | 393129 | 25.0399681 | 687 | 471969 | 26.2106848 |
| 568 | 322624 | 23.8327506 | 628 | 394384 | 25.0599282 | 688 | 473344 | 26.2297541 |
| 569 | 323764 | 23.8537209 | 629 | 395641 | 25.0798724 | 689 | 474721 | 26.2488095 |
| 570 | 324900 | 23.8746728 | 630 | 396900 | 25.0998008 | 690 | 476100 | 26.2678511 |
| 571 | 326041 | 23.8956063 | 631 | 398161 | 25.1197134 | 691 | 477481 | 26.2868789 |
| 572 | 327184 | 23.9165215 | 632 | 399424 | 25.1396102 | 692 | 478864 | 26.3058929 |
| 573 | 328329 | 23.9374184 | 633 | 400689 | 25.1594913 | 693 | 480249 | 26.3248932 |
| 574 | 329476 | 23.9582971 | 634 | 401956 | 25.1793566 | 694 | 481636 | 26.3438797 |
| 575 | 330625 | 23.9791576 | 635 | 403225 | 25.1992063 | 695 | 483025 | 26.3628527 |
| 576 | 331776 | 24.0000000 | 636 | 404496 | 25.2190404 | 696 | 484416 | 26.3818119 |
| 577 | 332929 | 24.0208243 | 637 | 405769 | 25.2388589 | 697 | 485809 | 26.4007576 |
| 578 | 334084 | 24.0416306 | 638 | 407044 | 25.2586619 | 698 | 487204 | 26.4196896 |
| 579 | 335241 | 24.0624188 | 639 | 408321 | 25.2784493 | 699 | 488601 | 26.4386081 |
| 580 | 336400 | 24.0831891 | 640 | 409600 | 25.2982213 | 700 | 490000 | 26.4575131 |
| 581 | 337561 | 24.1039416 | 641 | 410881 | 25.3179778 | 701 | 491401 | 26.4764046 |
| 582 | 338724 | 24.1246762 | 642 | 412164 | 25.3377189 | 702 | 492804 | 26.4952826 |
| 583 | 339889 | 24.1453929 | 643 | 413449 | 25.3574447 | 703 | 494209 | 26.5141472 |
| 584 | 341056 | 24.1660919 | 644 | 414736 | 25.3771551 | 704 | 495616 | 26.5329983 |
| 585 | 342225 | 24.1867732 | 645 | 416025 | 25.3968502 | 705 | 497025 | 26.5518361 |
| 596 | 343396 | 24.2074369 | 646 | 417316 | 25.4165301 | 706 | 498436 | 26.5706605 |
| 587 | 344569 | 24.2280829 | 647 | 418609 | 25.4361947 | 707 | 499849 | 26.5894716 |
| 588 | 345744 | 24.2487113 | 648 | 419904 | 25.4558441 | 708 | 501264 | 26.6082694 |
| 589 | 346921 | 24.2693222 | 649 | 421201 | 25.4754784 | 709 | 502681 | 26.6270539 |
| 590 | 348100 | 24.2899156 | 650 | 422500 | 25.4950976 | 710 | 504100 | 26.6458252 |
| 591 | 349281 | 24.3104916 | 651 | 423801 | 25.5147016 | 711 | 505521 | 26.6645833 |
| 592 | 350464 | 24.3310501 | 652 | 425104 | 25.5342907 | 712 | 506944 | 26.6833281 |
| 593 | 351649 | 24.3515913 | 653 | 426409 | 25.5538647 | 713 | 508369 | 26.7020598 |
| 594 | 352836 | 24.3721152 | 654 | 427716 | 25.5734237 | 714 | 509796 | 26.7207784 |
| 595 | 354025 | 24.3926218 | 655 | 429025 | 25.5929678 | 715 | 511225 | 26.7394839 |
| 596 | 355216 | 24.4131112 | 656 | 430336 | 25.6524969 | 716 | 512656 | 26.7581763 |
| 597 | 356409 | 24.4335834 | 657 | 431649 | 25.6320112 | 717 | 514089 | 26.7768557 |
| 598 | 357604 | 24.4540385 | 658 | 432964 | 25.6515107 | 718 | 515524 | 26.7955220 |
| 599 | 358801 | 24.4744765 | 659 | 434281 | 25.6709953 | 719 | 516961 | 26.8141754 |
| 600 | 360000 | 24.4948974 | 660 | 435600 | 25.6904652 | 720 | 518400 | 26.8328157 |

| No. | Square. | Sqre. root. | No. | Square. | Sqre. root. | No. | Square. | Sqre. root. |
|---|---|---|---|---|---|---|---|---|
| 721 | 519841 | 26.8514442 | 781 | 609961 | 27.9463772 | 841 | 707281 | 29.0000000 |
| 722 | 521284 | 26.8700577 | 782 | 611524 | 27.9642629 | 842 | 708964 | 29.0172363 |
| 723 | 522729 | 26.8886593 | 783 | 613089 | 27.9821372 | 843 | 710649 | 29.0344623 |
| 724 | 524176 | 26.9072481 | 784 | 614656 | 28.0000000 | 844 | 712336 | 29.0516781 |
| 725 | 525625 | 26.9258240 | 785 | 616225 | 28.0178515 | 845 | 714025 | 29.0688837 |
| 726 | 527076 | 26.9443872 | 786 | 617796 | 28.0356915 | 846 | 715716 | 29.0860791 |
| 727 | 528529 | 26.9629375 | 787 | 619369 | 28.0535203 | 847 | 717409 | 29.1032644 |
| 728 | 529984 | 26.9814751 | 788 | 620944 | 28.0713377 | 848 | 719104 | 29.1204396 |
| 729 | 531441 | 27.0000000 | 789 | 622521 | 28.0891438 | 849 | 720801 | 29.1376046 |
| 730 | 532900 | 27.0185122 | 790 | 624100 | 28.1069386 | 850 | 722500 | 29.1547595 |
| 731 | 534361 | 27.0370117 | 791 | 625681 | 28.1247222 | 851 | 724201 | 29.1719043 |
| 732 | 535824 | 27.0554985 | 792 | 627264 | 28.1424946 | 852 | 725904 | 29.1890390 |
| 733 | 537289 | 27.0739727 | 793 | 628849 | 28.1602557 | 853 | 727609 | 29.2061637 |
| 734 | 538756 | 27.0924344 | 794 | 630436 | 28.1780056 | 854 | 729316 | 29.2232784 |
| 735 | 540225 | 27.1108834 | 795 | 632025 | 28.1957444 | 855 | 731025 | 29.2403830 |
| 736 | 541696 | 27.1293199 | 796 | 633616 | 28.2134720 | 856 | 732736 | 29.2574777 |
| 737 | 543169 | 27.1477439 | 797 | 635209 | 28.2311884 | 857 | 734449 | 29.2745623 |
| 738 | 544644 | 27.1661554 | 798 | 636804 | 28.2488938 | 858 | 736164 | 29.2916370 |
| 739 | 546121 | 27.1845544 | 799 | 638401 | 28.2661881 | 859 | 737881 | 29.3087018 |
| 740 | 547600 | 27.2029410 | 800 | 640000 | 28.2842712 | 860 | 739600 | 29.3257566 |
| 741 | 549081 | 27.2213152 | 801 | 641601 | 28.3019434 | 861 | 741321 | 29.3428015 |
| 742 | 550564 | 27.2396769 | 802 | 643204 | 28.3196045 | 862 | 743044 | 29.3598365 |
| 743 | 552049 | 27.2580263 | 803 | 644809 | 28.3372546 | 863 | 744769 | 29.3764616 |
| 744 | 553536 | 27.2763634 | 804 | 646416 | 28.3548938 | 864 | 746496 | 29.3938769 |
| 745 | 555025 | 27.2946881 | 805 | 648025 | 28.3725219 | 865 | 748225 | 29.4108823 |
| 746 | 566516 | 27.3130006 | 806 | 649636 | 28.3901391 | 866 | 749956 | 29.4278779 |
| 747 | 558009 | 27.3313007 | 807 | 651249 | 28.4077454 | 867 | 751689 | 29.4448637 |
| 748 | 559504 | 27.3495887 | 808 | 652864 | 28.4253408 | 868 | 753424 | 29.4618397 |
| 749 | 561001 | 27.3678644 | 809 | 654481 | 28.4429253 | 869 | 755161 | 29.4788059 |
| 750 | 562500 | 27.3861279 | 810 | 656100 | 28.4604989 | 870 | 756900 | 29.4957624 |
| 751 | 564001 | 27.4043792 | 811 | 657721 | 28.4780617 | 871 | 758641 | 29.5127091 |
| 752 | 565504 | 27.4226184 | 812 | 659344 | 28.4956137 | 872 | 760384 | 29.5296461 |
| 753 | 567009 | 27.4408455 | 813 | 660969 | 28.5131549 | 873 | 762129 | 29.5465734 |
| 754 | 568516 | 27.4590604 | 814 | 662596 | 28.5306852 | 874 | 763876 | 29.5634910 |
| 755 | 570025 | 27.4772633 | 815 | 664225 | 28.5482048 | 875 | 765625 | 29.5803989 |
| 756 | 571536 | 27.4954542 | 816 | 665856 | 28.5657137 | 876 | 767376 | 29.5972972 |
| 757 | 573049 | 27.5136330 | 817 | 667489 | 28.5832119 | 877 | 769129 | 29.6141858 |
| 758 | 574564 | 27.5317998 | 818 | 669124 | 28.6006993 | 878 | 770884 | 29.6310648 |
| 759 | 576081 | 27.5499546 | 819 | 670761 | 28.6181760 | 879 | 772641 | 29.6479342 |
| 760 | 577600 | 27.5680975 | 820 | 672400 | 28.6356421 | 880 | 774400 | 29.6647939 |
| 761 | 579121 | 27.6862284 | 821 | 674041 | 28.6530976 | 881 | 776161 | 29.6816442 |
| 762 | 580644 | 27.6043475 | 822 | 675684 | 28.6705424 | 882 | 777924 | 29.6984848 |
| 763 | 582169 | 27.6224546 | 823 | 677329 | 28.6879766 | 883 | 779689 | 29.7153159 |
| 764 | 583696 | 27.6405499 | 824 | 678976 | 28.7054002 | 884 | 781456 | 29.7321375 |
| 765 | 585225 | 27.6586334 | 825 | 680625 | 28.7228130 | 885 | 783225 | 29.7488496 |
| 766 | 585756 | 27.6767050 | 826 | 682276 | 28.7402157 | 886 | 784996 | 29.7657521 |
| 767 | 588289 | 27.6947648 | 827 | 683929 | 28.7507677 | 887 | 786769 | 29.7825452 |
| 768 | 589824 | 27.7128129 | 828 | 685584 | 28.7749891 | 888 | 788544 | 29.7993289 |
| 769 | 591361 | 27.7308492 | 829 | 687241 | 28.7923601 | 889 | 790321 | 29.8161030 |
| 770 | 582900 | 27.7488739 | 830 | 688900 | 28.8097206 | 890 | 792100 | 29.8328678 |
| 771 | 594441 | 27.7668868 | 831 | 690561 | 28.8270706 | 891 | 793881 | 29.8496231 |
| 772 | 595984 | 27.7849880 | 832 | 692224 | 28.8444102 | 892 | 795664 | 29.8663690 |
| 773 | 597529 | 27.8020775 | 833 | 693889 | 28.8617394 | 893 | 797449 | 29.8831056 |
| 774 | 599076 | 27.8208555 | 834 | 695556 | 28.8790582 | 894 | 799236 | 29.8998328 |
| 775 | 600625 | 27.8388218 | 835 | 697225 | 28.8963666 | 895 | 801025 | 29.9165506 |
| 776 | 602176 | 27.8567766 | 836 | 698896 | 28.9136646 | 896 | 802816 | 29.9332591 |
| 777 | 603729 | 27.8747197 | 837 | 700569 | 28.9309523 | 897 | 804609 | 29.9499583 |
| 778 | 605284 | 27.8926514 | 838 | 702244 | 28.9482297 | 898 | 806404 | 29.9666481 |
| 779 | 606841 | 27.9105715 | 839 | 703921 | 28.9654967 | 899 | 808201 | 29.9833687 |
| 780 | 608400 | 27.9284801 | 840 | 705600 | 28.9827535 | 900 | 810000 | 30.0000000 |

| No. | Square. | Sqre. root. | No. | Square. | Sqre. root. | No. | Square. | Sqre. root. |
|---|---|---|---|---|---|---|---|---|
| 901 | 811801 | 30.0166621 | 961 | 923521 | 31.0000000 | 1021 | 1042441 | 31.9530906 |
| 902 | 813604 | 30.0333148 | 962 | 925444 | 31.0161248 | 1022 | 1044484 | 31.9687347 |
| 903 | 815409 | 30.0499584 | 963 | 927369 | 31.0322413 | 1023 | 1046529 | 31.9843712 |
| 904 | 817216 | 30.0665928 | 964 | 929296 | 31.0483494 | 1024 | 1048576 | 32.0000000 |
| 905 | 819025 | 30.0832179 | 965 | 931225 | 31.0644491 | 1025 | 1050625 | 32.0156212 |
| 906 | 820836 | 30.0998339 | 966 | 933156 | 31.0805405 | 1026 | 1052676 | 32.0312348 |
| 907 | 822649 | 30.1164407 | 967 | 935089 | 31.0966236 | 1027 | 1054729 | 32.0468407 |
| 908 | 824464 | 30.1330383 | 968 | 937024 | 31.1126984 | 1028 | 1056784 | 32.0624391 |
| 909 | 826281 | 30.1496269 | 969 | 938961 | 31.1287648 | 1029 | 1058841 | 32.0780298 |
| 910 | 828100 | 30.1662063 | 970 | 940900 | 31.1448230 | 1030 | 1060900 | 32.0936131 |
| 911 | 829921 | 30.1827765 | 971 | 942841 | 31.1608729 | 1031 | 1069[illegible]61 | 32.1091877 |
| 912 | 831744 | 30.1993377 | 972 | 944784 | 31.1769145 | 1032 | 1065024 | 32.1247568 |
| 913 | 833569 | 30.2158899 | 973 | 946729 | 31.1929479 | 1033 | 1067089 | 32.1403173 |
| 914 | 835397 | 30.2324329 | 974 | 948676 | 31.2089731 | 1034 | 1069156 | 32.1558704 |
| 915 | 837225 | 30.2489669 | 975 | 950625 | 31.2249900 | 1035 | 1071225 | 32.1714159 |
| 916 | 839056 | 30.2654919 | 976 | 952576 | 31.2409987 | 1036 | 1073296 | 32.1869539 |
| 917 | 840889 | 30.2820079 | 977 | 954529 | 31.2569992 | 1037 | 1075369 | 32.2024844 |
| 918 | 842724 | 30.2985148 | 978 | 956484 | 31.2729915 | 1038 | 1077444 | 32.2180074 |
| 919 | 844561 | 30.3150128 | 979 | 958441 | 31.2889757 | 1039 | 1079521 | 32.2335229 |
| 920 | 846400 | 30.3315018 | 980 | 960400 | 31.3049517 | 1040 | 1081600 | 32.2490310 |
| 921 | 848241 | 30.3479818 | 981 | 962361 | 31.3209195 | 1041 | 1083681 | 32.2645316 |
| 922 | 850084 | 30.3644529 | 982 | 964324 | 31.3368792 | 1042 | 1085764 | 32.2800248 |
| 923 | 851929 | 30.3809151 | 983 | 966289 | 31.3528308 | 1043 | 1087849 | 32.2955105 |
| 924 | 853776 | 30.3973683 | 984 | 968256 | 31.3687743 | 1044 | 1089836 | 32.3109888 |
| 925 | 855625 | 30.4138127 | 985 | 970225 | 31.3847097 | 1045 | 1092025 | 32.3264598 |
| 926 | 857476 | 30.4302481 | 986 | 972196 | 31.4006369 | 1046 | 1094116 | 32.3419233 |
| 927 | 859329 | 30.4466747 | 987 | 974169 | 31.4165561 | 1047 | 1096209 | 32.3573794 |
| 928 | 861184 | 30.4630924 | 988 | 976144 | 31.4324673 | 1048 | 1098304 | 32.3728281 |
| 929 | 863041 | 30.4795013 | 989 | 978121 | 31.4483704 | 1049 | 1100401 | 32.3882695 |
| 930 | 864900 | 30.4959014 | 990 | 980100 | 31.4642654 | 1050 | 1102500 | 32.4037035 |
| 931 | 866761 | 30.5122926 | 991 | 982081 | 31.4901525 | 1051 | 1104601 | 32.4191301 |
| 932 | 868624 | 30.5286750 | 992 | 984064 | 31.4960315 | 1052 | 1106704 | 32.4345495 |
| 933 | 870489 | 30.5450487 | 993 | 986049 | 31.5119025 | 1053 | 1108899 | 32.4499615 |
| 934 | 872356 | 30.5614136 | 994 | 988036 | 31.5277655 | 1054 | 1110916 | 32.4653662 |
| 935 | 874225 | 30.5777697 | 995 | 990025 | 31.5436206 | 1055 | 1113025 | 32.4807635 |
| 936 | 876096 | 30.5941171 | 996 | 992016 | 31.5594577 | 1056 | 1115136 | 32.4961536 |
| 937 | 877969 | 30.6104557 | 997 | 994009 | 31.5753068 | 1057 | 1117249 | 32.5115364 |
| 938 | 879844 | 30.6267856 | 998 | 996004 | 31.5911380 | 1058 | 1119364 | 32.5269119 |
| 939 | 881721 | 30.6431069 | 999 | 1998001 | 31.6059613 | 1059 | 1121481 | 32.5422802 |
| 940 | 883600 | 30.6594194 | 1000 | 1000000 | 31.6227766 | 1060 | 1123600 | 32.5576412 |
| 941 | 885481 | 30.6757233 | 1001 | 1000201 | 31.6385840 | 1061 | 1125721 | 32.5729949 |
| 942 | 887364 | 30.6920185 | 1002 | 1004004 | 31.6543836 | 1062 | 1127844 | 32.5883415 |
| 943 | 889249 | 30.7083051 | 1003 | 1006009 | 31.6701752 | 1063 | 1129969 | 32.6036807 |
| 944 | 891136 | 30.7245830 | 1004 | 1008016 | 31.6859590 | 1064 | 1132096 | 32.6190129 |
| 945 | 893025 | 30.7408523 | 1005 | 1010025 | 31.7017349 | 1065 | 1134225 | 32.6343377 |
| 946 | 894916 | 30.7571130 | 1006 | 1010036 | 31.7175030 | 1066 | 1136356 | 32.6496554 |
| 947 | 896808 | 30.7733651 | 1007 | 1014049 | 31.7332633 | 1067 | 1138489 | 32.6649659 |
| 948 | 898704 | 30.7896086 | 1008 | 1016064 | 31.7490157 | 1068 | 1140624 | 32.6802693 |
| 949 | 900601 | 30.8058436 | 1009 | 1018081 | 31.7647603 | 1069 | 1142761 | 32.6955654 |
| 950 | 902500 | 30.8220700 | 1010 | 1020100 | 31.7804972 | 1070 | 1144900 | 32.7108544 |
| 951 | 904401 | 30.8382879 | 1011 | 1020121 | 31.7962262 | 1071 | 1147041 | 32.7261363 |
| 952 | 906304 | 30.8544972 | 1012 | 1024144 | 31.8119474 | 1072 | 1149184 | 32.7414111 |
| 953 | 908209 | 30.8706981 | 1013 | 1026169 | 31.8276609 | 1073 | 1151329 | 32.7566787 |
| 954 | 910116 | 30.8868904 | 1014 | 1028196 | 31.8433666 | 1074 | 1153476 | 32.7719392 |
| 955 | 912025 | 30.9030743 | 1015 | 1030225 | 31.8590646 | 1075 | 1155625 | 32.7871926 |
| 956 | 913936 | 30.9192477 | 1016 | 1032256 | 31.8747549 | 1076 | 1157776 | 32.8024398 |
| 957 | 915849 | 30.9354166 | 1017 | 1034289 | 31.8904374 | 1077 | 1159929 | 32.8172782 |
| 958 | 917764 | 30.9515751 | 1018 | 1036324 | 31.9061123 | 1078 | 1162084 | 32.8329103 |
| 959 | 919681 | 30.9677251 | 1019 | 1038361 | 31.9217794 | 1079 | 1164241 | 32.8481354 |
| 960 | 921600 | 30.9838668 | 1020 | 1040400 | 31.9374388 | 1080 | 1166400 | 32.8633535 |

| No. | Square. | Sqre. root. | No. | Square. | Sqre. root. | No. | Square. | Sqre. root. |
|---|---|---|---|---|---|---|---|---|
| 1081 | 1168561 | 32.8785644 | 1141 | 1301881 | 33.7786915 | 1201 | 1442401 | 34.6554469 |
| 1082 | 1170724 | 32.8937684 | 1142 | 1304614 | 33.7934905 | 1202 | 1444804 | 34.6698716 |
| 1083 | 1172889 | 32.9089653 | 1143 | 1306449 | 33.8082830 | 1203 | 1447209 | 34.6842904 |
| 1084 | 1175056 | 32.9241553 | 1144 | 1308736 | 33.8230691 | 1204 | 1449616 | 34.6987031 |
| 1085 | 1177225 | 32.9393382 | 1145 | 1311025 | 33.8378486 | 1205 | 1452025 | 34.7131099 |
| 1086 | 1179396 | 32.9545141 | 1146 | 1313316 | 33.8526218 | 1206 | 1454436 | 34.7275107 |
| 1087 | 1181569 | 32.9696830 | 1147 | 1315609 | 33.8673884 | 1207 | 1456849 | 34.7419055 |
| 1088 | 1183744 | 32.9848450 | 1148 | 1317904 | 33.8821487 | 1208 | 1459264 | 34.7562944 |
| 1089 | 1185921 | 33.0000000 | 1149 | 1320201 | 33.8969025 | 1209 | 1461681 | 34.7706773 |
| 1090 | 1188100 | 33.0151480 | 1150 | 1322500 | 33.9116499 | 1210 | 1464100 | 34.7850543 |
| 1091 | 1190281 | 33.0302891 | 1151 | 1324801 | 33.9263909 | 1211 | 1466521 | 34.7994253 |
| 1092 | 1192464 | 33.0454233 | 1152 | 1327104 | 33.9411255 | 1212 | 1468944 | 34.8137904 |
| 1093 | 1194649 | 33.0605505 | 1153 | 1329409 | 33.9558537 | 1213 | 1471369 | 34.8281495 |
| 1094 | 1196836 | 33.0756708 | 1154 | 1331716 | 33.9705755 | 1214 | 1473796 | 34.8425028 |
| 1095 | 1199025 | 33.0907842 | 1155 | 1334025 | 33.9852910 | 1215 | 1476225 | 34.8568501 |
| 1096 | 1201216 | 33 1058907 | 1156 | 1336336 | 34.0000000 | 1216 | 1478656 | 34.8711915 |
| 1097 | 1203409 | 33.1209903 | 1157 | 1338649 | 34.0147027 | 1217 | 1481089 | 34.8855271 |
| 1098 | 1205604 | 33.1360830 | 1158 | 1340964 | 34.0293990 | 1218 | 1483524 | 34.8998567 |
| 1099 | 1207801 | 33.1511689 | 1159 | 1343281 | 34.0440890 | 1219 | 1485961 | 34.9141805 |
| 1100 | 1210000 | 33.1662479 | 1160 | 1345600 | 34.0587727 | 1220 | 1488400 | 34.9284984 |
| 1101 | 1212201 | 33.1813200 | 1161 | 1347921 | 34.0734501 | 1221 | 1490841 | 34.9428984 |
| 1102 | 1214404 | 33.1963853 | 1162 | 1350244 | 34.0881211 | 1222 | 1493284 | 34.9428104 |
| 1103 | 1216609 | 33.2114438 | 1163 | 1352569 | 34.1027858 | 1223 | 1495729 | 34.9571166 |
| 1104 | 1218816 | 33.2266955 | 1164 | 1354896 | 34.1174442 | 1224 | 1498176 | 34.9714169 |
| 1105 | 1221025 | 33.2415403 | 1165 | 1357225 | 34.1320963 | 1225 | 1500625 | 34.9857114 |
| 1106 | 1223236 | 33.2565783 | 1166 | 1359556 | 34.1467422 | 1226 | 1503076 | 35.0000000 |
| 1107 | 1225449 | 33.2716095 | 1167 | 1361889 | 34.1613817 | 1227 | 1505529 | 35.0142828 |
| 1108 | 1227664 | 33.2866339 | 1168 | 1364224 | 34.1760150 | 1228 | 1507984 | 35.0285598 |
| 1109 | 1229881 | 33.3016616 | 1169 | 1366561 | 34.1906420 | 1229 | 1510441 | 35.0428309 |
| 1110 | 1232100 | 33.3166625 | 1170 | 1368900 | 34.2052627 | 1230 | 1512900 | 35.0570963 |
| 1111 | 1234321 | 33.3316666 | 1171 | 1371241 | 34.2298773 | 1231 | 1515361 | 35.0713558 |
| 1112 | 1236544 | 33.3466640 | 1172 | 1373584 | 34.2344855 | 1232 | 1517824 | 35.0856096 |
| 1113 | 1238769 | 33.3616546 | 1173 | 1375929 | 34.2490875 | 1233 | 1520289 | 35.0998575 |
| 1114 | 1240996 | 33.3766385 | 1174 | 1378276 | 34.2636834 | 1234 | 1522756 | 35.1140997 |
| 1115 | 1243225 | 33.3916157 | 1175 | 1380625 | 34.2782730 | 1235 | 1525225 | 35.1283361 |
| 1116 | 1245456 | 33.4065862 | 1176 | 1382976 | 34.2928564 | 1236 | 1527696 | 35.1425568 |
| 1117 | 1247689 | 33.4215499 | 1177 | 1385329 | 34.3074336 | 1237 | 1530169 | 35.1567917 |
| 1118 | 1249924 | 33.4365070 | 1178 | 1387684 | 34.3220046 | 1238 | 1532644 | 35.1710108 |
| 1119 | 1252161 | 33.4514573 | 1179 | 1390041 | 34.3365694 | 1239 | 1535121 | 35.1852242 |
| 1120 | 1254400 | 33.4664011 | 1180 | 1392400 | 34.3511281 | 1240 | 1537600 | 35.1994318 |
| 1121 | 1256641 | 33.4913381 | 1181 | 1394761 | 34.3656805 | 1241 | 1540081 | 35.2136337 |
| 1122 | 1258884 | 33.4962684 | 1182 | 1397124 | 34.3802268 | 1242 | 1542564 | 35.2278299 |
| 1123 | 1261129 | 33.5111821 | 1183 | 1399489 | 34.3947670 | 1243 | 1545049 | 35.2420204 |
| 1124 | 1263376 | 33.5261092 | 1184 | 1401856 | 34.4093011 | 1244 | 1547536 | 35.2561501 |
| 1125 | 1265625 | 33.5410196 | 1185 | 1404225 | 34.4238289 | 1245 | 1550025 | 35.2703842 |
| 1126 | 1267876 | 33.5559234 | 1186 | 1406596 | 34.4383507 | 1246 | 1552516 | 35.2845575 |
| 1127 | 1270129 | 33.5708206 | 1187 | 1408969 | 34.4528663 | 1247 | 1555009 | 35.2987252 |
| 1128 | 1272384 | 33.5857112 | 1188 | 1411344 | 34.4673759 | 1248 | 1557509 | 35.3128872 |
| 1129 | 1274641 | 33.6005952 | 1189 | 1413721 | 34.4818793 | 1249 | 1550001 | 35.3270435 |
| 1130 | 1276900 | 33.6154726 | 1190 | 1416100 | 34.4963766 | 1250 | 1562500 | 35.3411941 |
| 1131 | 1279161 | 33.6303434 | 1191 | 1418481 | 34.5108678 | 1251 | 1565001 | 35.3553391 |
| 1132 | 1281424 | 33.6452077 | 1192 | 1420864 | 34.5253530 | 1252 | 1567504 | 35.3694784 |
| 1133 | 1283689 | 33.6600653 | 1193 | 1423249 | 34.5398321 | 1253 | 1570009 | 35.3836120 |
| 1134 | 1285956 | 33.6749165 | 1194 | 1425636 | 34.5543051 | 1254 | 1572516 | 35.3977400 |
| 1135 | 1288225 | 33.6897610 | 1195 | 1428025 | 34.5687720 | 1255 | 1575025 | 35.4118624 |
| 1136 | 1290496 | 33.7045991 | 1196 | 1430416 | 34.5832329 | 1256 | 1577536 | 35.4259792 |
| 1137 | 1292769 | 33.7194306 | 1197 | 1432809 | 34.5976879 | 1257 | 1580049 | 35.4400903 |
| 1138 | 1295044 | 33.7340556 | 1198 | 1435204 | 34.6121366 | 1258 | 1582564 | 35.4541958 |
| 1139 | 1297321 | 33.7490741 | 1199 | 1437601 | 34.6265794 | 1259 | 1585081 | 35.4682957 |
| 1140 | 1299600 | 33.7638860 | 1200 | 1440000 | 34.6410162 | 1260 | 1587600 | 35.4823900 |

| No. | Square. | Sqre. root. | No. | Square. | Sqre. root. | No. | Square. | Sqre. root. |
|---|---|---|---|---|---|---|---|---|
| 1261 | 1590121 | 35.5105618 | 1321 | 1745041 | 36.3455637 | 1381 | 1907161 | 37.1618084 |
| 1262 | 1592644 | 35.5246393 | 1322 | 1747684 | 36.3593179 | 1382 | 1909924 | 37.1752606 |
| 1263 | 1595166 | 35.5387113 | 1323 | 1759329 | 36.3730670 | 1383 | 1912689 | 37.1887079 |
| 1264 | 1597696 | 35.5527777 | 1324 | 1752976 | 36.3868108 | 1384 | 1915456 | 37.2021505 |
| 1265 | 1600225 | 35.5668385 | 1325 | 1755625 | 36.4005494 | 1385 | 1918225 | 37.2155881 |
| 1266 | 1602756 | 35.5808937 | 1326 | 1758276 | 36.4142829 | 1386 | 1920996 | 37.2290209 |
| 1267 | 1605289 | 35.5949434 | 1327 | 1760929 | 36.4280112 | 1387 | 1923769 | 37.2224489 |
| 1268 | 1607824 | 35.6089876 | 1328 | 1763584 | 36.4417343 | 1388 | 1926544 | 37.2558720 |
| 1269 | 1610361 | 35.6230262 | 1329 | 1766241 | 36.4554523 | 1389 | 1929321 | 37.2692903 |
| 1270 | 1612900 | 35.6370593 | 1330 | 1768900 | 36.4691650 | 1390 | 1932100 | 37.2827037 |
| 1271 | 1615441 | 35.6510869 | 1331 | 1771561 | 36.4828727 | 1391 | 1934881 | 37.2961124 |
| 1272 | 1617984 | 35.6651090 | 1332 | 1774224 | 36.4965752 | 1392 | 1937664 | 37.3095162 |
| 1273 | 1620529 | 35.6791255 | 1333 | 1776889 | 36.5102725 | 1393 | 1940419 | 37.3229152 |
| 1274 | 1623076 | 35.6931366 | 1334 | 1779556 | 36.5239647 | 1394 | 1943236 | 37.3363094 |
| 1275 | 1625625 | 35.7071421 | 1335 | 1782225 | 36.5376548 | 1395 | 1946025 | 37.3496988 |
| 1276 | 1628176 | 35.7211422 | 1336 | 1784896 | 36.5513388 | 1396 | 1948816 | 37.3630834 |
| 1277 | 1630729 | 35.7351367 | 1337 | 1787569 | 36.5650106 | 1397 | 1951609 | 37.3764632 |
| 1278 | 1633284 | 35.7491258 | 1338 | 1790244 | 36.5786823 | 1398 | 1954404 | 37.3898382 |
| 1279 | 1635841 | 35.7631095 | 1339 | 1792921 | 36.5923489 | 1399 | 1957201 | 37.4032084 |
| 1280 | 1638400 | 35.7770876 | 1340 | 1795600 | 36.6060104 | 1400 | 1960000 | 37.4165738 |
| 1281 | 1640961 | 35.7910603 | 1341 | 1798281 | 36.6196668 | 1401 | 1962801 | 37.4299345 |
| 1282 | 1643524 | 35.8050276 | 1342 | 1800964 | 36.6333181 | 1402 | 1965604 | 37.4432904 |
| 1283 | 1646089 | 35.8189894 | 1343 | 1803649 | 36.6469644 | 1403 | 1968409 | 37.4566416 |
| 1284 | 1648656 | 35.8329457 | 1344 | 1806336 | 36.6606056 | 1404 | 1971216 | 37.4699880 |
| 1285 | 1651225 | 35.8468966 | 1345 | 1809025 | 36.6742416 | 1405 | 1974025 | 37.4833296 |
| 1286 | 1653796 | 35.8608421 | 1346 | 1811716 | 36.6878726 | 1406 | 1976836 | 37.4966665 |
| 1287 | 1656369 | 35.8747822 | 1347 | 1814409 | 36.7014986 | 1407 | 1979649 | 37.5099987 |
| 1288 | 1658944 | 35.8887169 | 1348 | 1817104 | 36.7151195 | 1408 | 1982464 | 37.5233261 |
| 1289 | 1661521 | 35.9026461 | 1349 | 1819801 | 36.7287353 | 1409 | 1985281 | 37.5366487 |
| 1290 | 1664100 | 35.9165699 | 1350 | 1822500 | 36.7423461 | 1410 | 1988100 | 37.5499667 |
| 1291 | 1666681 | 35.9304884 | 1351 | 1825201 | 36.7559519 | 1411 | 1990921 | 37.5632799 |
| 1292 | 1669264 | 35.9444015 | 1352 | 1827904 | 36.7695526 | 1412 | 1993744 | 37.5765855 |
| 1293 | 1671849 | 35.9583092 | 1353 | 1830609 | 36.7831483 | 1413 | 1996569 | 37.5898922 |
| 1294 | 1674436 | 35.9722115 | 1354 | 1833316 | 36.7967390 | 1414 | 1999396 | 37.6031913 |
| 1295 | 1677025 | 35.9861084 | 1355 | 1836025 | 36.8103246 | 1415 | 2002225 | 37.6164857 |
| 1296 | 1679616 | 36.0000000 | 1356 | 1838736 | 36.8239053 | 1416 | 2005056 | 37.6297754 |
| 1297 | 1682209 | 36.0138862 | 1357 | 1841449 | 36.8374809 | 1417 | 2007889 | 37.6430604 |
| 1298 | 1684804 | 36.0277671 | 1358 | 1844164 | 36.8510515 | 1418 | 2010724 | 37.6563407 |
| 1299 | 1687401 | 36.0416426 | 1359 | 1846881 | 36.8646172 | 1419 | 2013561 | 37.6696164 |
| 1300 | 1690000 | 36.0555128 | 1360 | 1849600 | 36.8781778 | 1420 | 2016400 | 37.6828874 |
| 1301 | 1692601 | 36.0693776 | 1361 | 1852321 | 36.8917335 | 1421 | 2019241 | 37.6961536 |
| 1302 | 1695204 | 36.0832371 | 1362 | 1855044 | 36.9052842 | 1422 | 2022084 | 37.7094153 |
| 1303 | 1694809 | 36.0970913 | 1363 | 1857769 | 36.9188299 | 1423 | 2024929 | 37.7226722 |
| 1304 | 1700416 | 36.1109402 | 1364 | 1860496 | 36.9323706 | 1424 | 2027776 | 37.7359245 |
| 1305 | 1703025 | 36.1247837 | 1365 | 1863225 | 36.9459064 | 1425 | 2030625 | 37.7491722 |
| 1306 | 1705636 | 36.1386220 | 1366 | 1865956 | 36.9594372 | 1426 | 2033476 | 37.7624152 |
| 1307 | 1708249 | 36.1524550 | 1367 | 1868689 | 36.9729631 | 1427 | 2036329 | 37.7756535 |
| 1308 | 1710864 | 36.1662826 | 1368 | 1871424 | 36.9864840 | 1428 | 2039184 | 37.7888873 |
| 1309 | 1713481 | 36.1801050 | 1369 | 1874161 | 37.0000000 | 1429 | 2042041 | 37.8021163 |
| 1310 | 1716100 | 36.1939221 | 1370 | 1876900 | 37.0135110 | 1430 | 2044900 | 37.8153408 |
| 1311 | 1718721 | 36.2077340 | 1371 | 1879641 | 37.0270172 | 1431 | 2047761 | 37.8285606 |
| 1312 | 1721344 | 36.2215406 | 1372 | 1882384 | 37.0405184 | 1432 | 2050624 | 37.8417759 |
| 1313 | 1723969 | 36.2353419 | 1373 | 1885129 | 37.0540146 | 1433 | 2053489 | 37.8549864 |
| 1314 | 1726596 | 36.2491379 | 1374 | 1887876 | 37.0675060 | 1434 | 2056356 | 37.8681924 |
| 1315 | 1729225 | 36.2626287 | 1375 | 1890625 | 37.0899924 | 1435 | 2059225 | 37.8813938 |
| 1316 | 1731856 | 36.2767143 | 1376 | 1893376 | 37.0944740 | 1436 | 2062096 | 37.8945906 |
| 1317 | 1734489 | 36.2904946 | 1377 | 1896129 | 37.1079506 | 1437 | 2064959 | 37.9077828 |
| 1318 | 1737124 | 36.3042697 | 1378 | 1898884 | 37.1214224 | 1438 | 2067844 | 37.9209704 |
| 1319 | 1739761 | 36.3180396 | 1379 | 1901641 | 37.1348893 | 1439 | 2070721 | 37.9341535 |
| 1320 | 1742400 | 36.3318042 | 1380 | 1904400 | 37.1483512 | 1440 | 2073600 | 37.9473319 |

| No. | Square. | Sqre. root. | No. | Square. | Sqre. root. | No. | Square. | Sqre. root. |
|---|---|---|---|---|---|---|---|---|
| 1441 | 2076481 | 37.9605058 | 1495 | 2235025 | 38.6652299 | 1548 | 2396304 | 39.3446311 |
| 1442 | 2079364 | 37.9736751 | 1496 | 2238016 | 38.6781593 | 1549 | 2399401 | 39.3573373 |
| 1443 | 2082249 | 37.9868938 | 1497 | 2241009 | 38.6910843 | 1550 | 2402500 | 39.3700394 |
| 1444 | 2085136 | 38.0000000 | 1498 | 2244004 | 38.7040050 | 1551 | 2405601 | 39.3827373 |
| 1445 | 2088025 | 38.0131556 | 1499 | 2247001 | 38.7169214 | 1552 | 2408704 | 39.3954312 |
| 1446 | 2090915 | 38.0263067 | 1500 | 2250000 | 38.7298335 | 1553 | 2411809 | 39.4081210 |
| 1447 | 2093809 | 38.0394532 | 1501 | 2253001 | 38.7427412 | 1554 | 2414916 | 39.4208067 |
| 1448 | 2096704 | 38.0525952 | 1502 | 2256004 | 38.7556447 | 1555 | 2418025 | 39.4334883 |
| 1449 | 2099601 | 38.0657326 | 1503 | 2259009 | 38.7685439 | 1556 | 2421136 | 39.4461658 |
| 1450 | 2102500 | 38.0788655 | 1504 | 2262016 | 38.7814389 | 1557 | 2424249 | 39.4588393 |
| 1451 | 2105401 | 38.0919939 | 1505 | 2265025 | 38.7943294 | 1558 | 2427364 | 39.4715087 |
| 1452 | 2108304 | 38.1051178 | 1506 | 2268034 | 38.8032158 | 1559 | 2430481 | 39.4841740 |
| 1453 | 2111209 | 38.1183371 | 1507 | 2271049 | 38.8200978 | 1560 | 2433600 | 39.4968353 |
| 1454 | 2114116 | 38.1313519 | 1508 | 2274064 | 38.8329757 | 1561 | 2436721 | 39.5094925 |
| 1455 | 2117025 | 38.1444622 | 1509 | 2277081 | 38.8458491 | 1562 | 2439844 | 39.5221457 |
| 1456 | 2119936 | 38.1575681 | 1510 | 2280100 | 38.8587184 | 1563 | 2442969 | 39.5347948 |
| 1457 | 2122849 | 38.1706693 | 1511 | 2283121 | 38.8715834 | 1564 | 2446096 | 39.5474399 |
| 1458 | 2125764 | 38.1837662 | 1512 | 2286144 | 38.8844442 | 1565 | 2449225 | 39.5600809 |
| 1459 | 2128681 | 38.1968585 | 1513 | 2289169 | 38.8973006 | 1566 | 2452356 | 39.5727179 |
| 1460 | 2131600 | 38.2099463 | 1514 | 2292196 | 38.9101529 | 1567 | 2455489 | 39.5853508 |
| 1461 | 2134521 | 38.2220297 | 1515 | 2295225 | 38.9230009 | 1568 | 2458624 | 39.5979797 |
| 1462 | 2137444 | 38.2361085 | 1516 | 2298256 | 38.9358447 | 1569 | 2461761 | 39.6106046 |
| 1463 | 2140369 | 38.2491829 | 1517 | 2301289 | 38.9486841 | 1570 | 2464900 | 39.6232255 |
| 1464 | 2143296 | 38.2622529 | 1518 | 2304334 | 38.9615194 | 1571 | 2468041 | 39.6358424 |
| 1465 | 2146225 | 38.2753184 | 1519 | 2307361 | 38.9743505 | 1572 | 2471184 | 39.6484552 |
| 1466 | 2149156 | 38.2883794 | 1520 | 2310400 | 38.9871774 | 1573 | 2474319 | 39.6610640 |
| 1467 | 2152089 | 38.3014360 | 1521 | 2313441 | 39.0000000 | 1574 | 2477475 | 39.6736688 |
| 1468 | 2155024 | 38.3144881 | 1522 | 2316484 | 39.0128184 | 1575 | 2480625 | 39.6862696 |
| 1469 | 2157961 | 38.3275358 | 1523 | 2319529 | 39.0256326 | 1576 | 2483776 | 39.6988665 |
| 1470 | 2160900 | 38.3405790 | 1524 | 2322576 | 39.0384426 | 1577 | 2486929 | 39.7114593 |
| 1471 | 2163841 | 38.3536178 | 1525 | 2325625 | 39.0512483 | 1578 | 2490084 | 39.7240481 |
| 1472 | 2166784 | 38.3666522 | 1526 | 2328676 | 39.0640499 | 1579 | 2493241 | 39.7365329 |
| 1473 | 2169729 | 38.3796821 | 1527 | 2331729 | 39.0768473 | 1580 | 2496400 | 39.7492138 |
| 1474 | 2172676 | 38.3927076 | 1528 | 2334784 | 39.0896406 | 1581 | 2499561 | 39.7617907 |
| 1475 | 2175625 | 38.4057287 | 1529 | 2337841 | 39.1024296 | 1582 | 2502724 | 39.7743636 |
| 1476 | 2178576 | 38.4187454 | 1530 | 2340900 | 39.1152144 | 1583 | 2505889 | 39.7669325 |
| 1477 | 2181529 | 38.4317577 | 1531 | 2343961 | 39.1279951 | 1584 | 2509056 | 39.7994976 |
| 1478 | 2184484 | 38.4447656 | 1532 | 2347024 | 39.1407716 | 1585 | 2512225 | 39.8120585 |
| 1479 | 2187441 | 38.4577691 | 1533 | 2350089 | 39.1535439 | 1586 | 2515396 | 39.8241155 |
| 1480 | 2190400 | 38.4707681 | 1534 | 2353156 | 39.1663120 | 1587 | 2518569 | 39.8376646 |
| 1481 | 2193361 | 38.4837627 | 1535 | 2356225 | 39.1790760 | 1588 | 2521744 | 39.8497177 |
| 1482 | 2196324 | 38.4967530 | 1536 | 2359296 | 39.1918359 | 1589 | 2524921 | 39.3622628 |
| 1483 | 2199289 | 38.5097390 | 1537 | 2362369 | 39.2045915 | 1590 | 2528100 | 39.8748040 |
| 1484 | 2202256 | 38.5227206 | 1538 | 2365444 | 39.2173431 | 1591 | 2531281 | 39.8873413 |
| 1485 | 2205225 | 38.5356977 | 1539 | 2368521 | 39.2300905 | 1592 | 2534464 | 39.8998747 |
| 1486 | 2208196 | 38.5486705 | 1540 | 2371600 | 39.2428337 | 1593 | 2537649 | 39.9124041 |
| 1487 | 2211169 | 38.5616389 | 1541 | 2374681 | 39.2555728 | 1594 | 2540836 | 39.9249295 |
| 1488 | 2214144 | 38.5746030 | 1542 | 2377764 | 39.2683078 | 1595 | 2544025 | 39.9374511 |
| 1489 | 2217121 | 38.5875627 | 1543 | 2380849 | 39.2810387 | 1596 | 2547216 | 39.7498687 |
| 1490 | 2220100 | 38.6005181 | 1544 | 2383936 | 39.2937654 | 1597 | 2550409 | 39.9624824 |
| 1491 | 2223081 | 38.6134691 | 1545 | 2387025 | 39.3064880 | 1598 | 2553604 | 39.9749922 |
| 1492 | 2226004 | 38.6264158 | 1546 | 2390116 | 39.3192065 | 1599 | 2556841 | 39.9874980 |
| 1493 | 2229049 | 38.6393582 | 1547 | 2393209 | 39.3319208 | 1600 | 2560000 | 40.0000000 |
| 1494 | 2232036 | 38.6522962 | | | | | | |

# TABLE II. a.

## AREAS OF CIRCLES, FROM 1/64 TO 150.

*[Advancing by an Eighth.]*

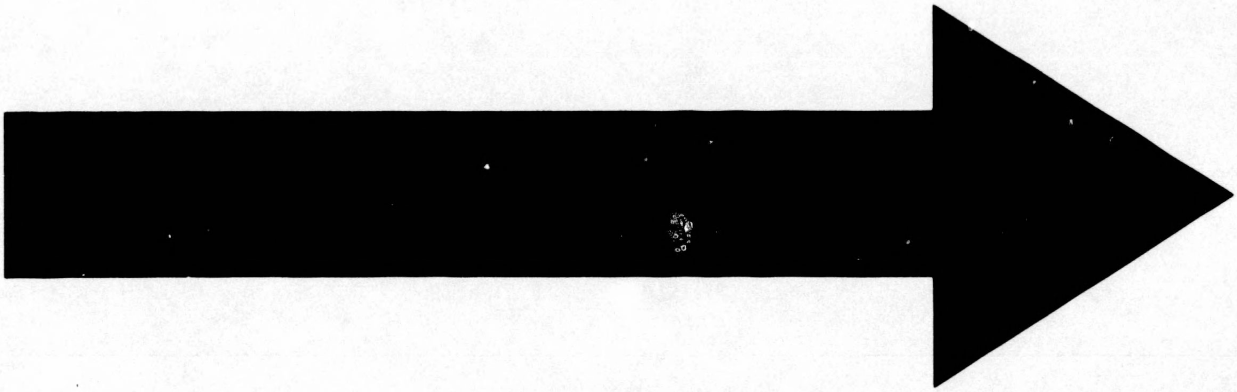
| Diam. | Area. | Diam. | Area. | Diam. | Area. | Diam. | Area. | Diam. | Area. |
|---|---|---|---|---|---|---|---|---|---|
| 1/64 | .00019 | 4. | 12.5664 | 10. | 78.54 | 16. | 201.062 | 22. | 380.134 |
| 1/32 | .00077 | .1/8 | 13.364 | .1/8 | 80.5157 | .1/8 | 204.216 | .1/8 | 384.465 |
| 1/16 | .00307 | .1/4 | 14.1862 | .1/4 | 82.5161 | .1/4 | 207.394 | .1/4 | 388.822 |
| 1/8 | .01227 | .3/8 | 15.0331 | .3/8 | 84.5409 | .3/8 | 210.597 | .3/8 | 393.203 |
| 3/16 | .02761 | .1/2 | 15.9043 | .1/2 | 86.59 | .1/2 | 213.825 | .1/2 | 397.608 |
| 1/4 | .04909 | .5/8 | 16.8001 | .5/8 | 88.6643 | .5/8 | 217.073 | .5/8 | 402.038 |
| 5/16 | .0767 | .3/4 | 17.7205 | .3/4 | 90.7628 | .3/4 | 220.353 | .3/4 | 406.493 |
| 3/8 | .11045 | .7/8 | 18.6655 | .7/8 | 92.8858 | .7/8 | 223.654 | .7/8 | 440.972 |
| 7/16 | .15033 | 5. | 19.635 | 11. | 95.0334 | 17. | 226.981 | 23. | 416.477 |
| 1/2 | .19635 | .1/8 | 20.629 | .1/8 | 97.2055 | .1/8 | 230.33 | .1/8 | 420.004 |
| 9/16 | .2485 | .1/4 | 21.6475 | .1/4 | 99.4022 | .1/4 | 233.705 | .1/4 | 424.557 |
| 5/8 | .30679 | .3/8 | 22.6907 | .3/8 | 101.6234 | .3/8 | 237.104 | .3/8 | 429.135 |
| 11/16 | .37122 | .1/2 | 23.7583 | .1/2 | 103.8691 | .1/2 | 240.528 | .1/2 | 433.731 |
| 3/4 | .44178 | .5/8 | 24.8505 | .5/8 | 106.1394 | .5/8 | 243.977 | .5/8 | 438.363 |
| 13/16 | .51848 | .3/4 | 25.9672 | .3/4 | 108.4343 | .3/4 | 247.45 | .3/4 | 443.014 |
| 7/8 | .60132 | .7/8 | 27.1085 | .7/8 | 110.7536 | .7/8 | 250.947 | .7/8 | 447.699 |
| 15/16 | .69029 | 6. | 28.2744 | 12. | 113.098 | 18. | 254.467 | 24. | 452.39 |
| 1. | .7854 | .1/8 | 29.4647 | .1/8 | 115.466 | .1/8 | 258.016 | .1/8 | 457.115 |
| .1/8 | .99402 | .1/4 | 30.6796 | .1/4 | 117.859 | .1/4 | 261.587 | .1/4 | 461.864 |
| .1/4 | 1.2271 | .3/8 | 31.9192 | .3/8 | 120.276 | .3/8 | 265.182 | .3/8 | 466.638 |
| .3/8 | 1.4848 | .1/2 | 33.1831 | .1/2 | 122.718 | .1/2 | 268.803 | .1/2 | 471.436 |
| .1/2 | 1.7671 | .5/8 | 34.4717 | .5/8 | 125.184 | .5/8 | 272.447 | .5/8 | 476.259 |
| .5/8 | 2.0739 | .3/4 | 35.7847 | .3/4 | 127.676 | .3/4 | 276.117 | .3/4 | 481.106 |
| .3/4 | 2.4052 | .7/8 | 37.1224 | .7/8 | 130.192 | .7/8 | 279.811 | .7/8 | 485.978 |
| .7/8 | 2.7611 | 7. | 38.4846 | 13. | 132.733 | 19. | 283.529 | 25. | 490.875 |
| 2. | 3.1416 | .1/8 | 39.8713 | .1/8 | 135.297 | .1/8 | 287.272 | .1/8 | 495.796 |
| .1/8 | 3.5465 | .1/4 | 41.2825 | .1/4 | 137.886 | .1/4 | 291.039 | .1/4 | 500.741 |
| .1/4 | 3.976 | .3/8 | 42.7184 | .3/8 | 140.5 | .3/8 | 294.831 | .3/8 | 505.711 |
| .3/8 | 4.4302 | .1/2 | 44.1787 | .1/2 | 143.139 | .1/2 | 298.648 | .1/2 | 510.706 |
| .1/2 | 4.9087 | .5/8 | 45.6636 | .5/8 | 145.802 | .5/8 | 302.489 | .5/8 | 515.725 |
| .5/8 | 5.4159 | .3/4 | 47.173 | .3/4 | 148.489 | .3/4 | 306.355 | .3/4 | 520.769 |
| .3/4 | 5.9395 | .7/8 | 48.707 | .7/8 | 151.201 | .7/8 | 310.245 | .7/8 | 525.837 |
| .7/8 | 6.4918 | 8. | 50.2656 | 14. | 153.938 | 20. | 314.16 | 26. | 530.93 |
| 3. | 7.0686 | .1/8 | 51.8486 | .1/8 | 156.699 | .1/8 | 318.099 | .1/8 | 536.047 |
| .1/8 | 7.6699 | .1/4 | 53.4562 | .1/4 | 159.485 | .1/4 | 322.063 | .1/4 | 541.189 |
| .1/4 | 8.2957 | .3/8 | 55.0885 | .3/8 | 162.295 | .3/8 | 326.051 | .3/8 | 546.356 |
| .3/8 | 8.9462 | .1/2 | 56.7451 | .1/2 | 165.13 | .1/2 | 330.064 | .1/2 | 551.547 |
| .1/2 | 9.6211 | .5/8 | 58.4264 | .5/8 | 167.989 | .5/8 | 334.101 | .5/8 | 556.762 |
| .5/8 | 10.3206 | .3/4 | 60.1321 | .3/4 | 170.873 | .3/4 | 338.163 | .3/4 | 562.002 |
| .3/4 | 11.0446 | .7/8 | 61.8625 | .7/8 | 173.782 | .7/8 | 342.25 | .7/8 | 567.267 |
| .7/8 | 11.7932 | 9. | 63.6174 | 15. | 176.715 | 21. | 346.361 | 27. | 572.557 |
| | | .1/8 | 65.3968 | .1/8 | 179.672 | .1/8 | 350.497 | .1/8 | 577.87 |
| | | .1/4 | 67.2007 | .1/4 | 182.654 | .1/4 | 354.657 | .1/4 | 583.208 |
| | | .3/8 | 69.0293 | .3/8 | 185.661 | .3/8 | 358.841 | .3/8 | 588.571 |
| | | .1/2 | 70.8823 | .1/2 | 188.692 | .1/2 | 363.051 | .1/2 | 593.958 |
| | | .5/8 | 72.7599 | .5/8 | 191.748 | .5/8 | 367.284 | .5/8 | 599.376 |
| | | .3/4 | 74.662 | .3/4 | 194.828 | .3/4 | 371.543 | .3/4 | 604.807 |
| | | .7/8 | 76.5887 | .7/8 | 197.933 | .7/8 | 375.826 | .7/8 | 610.268 |

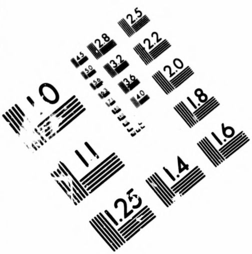
TABLE.—(*Continued.*)

| Diam. | Area. | Diam. | Area. | Diam. | Area. | Diam. | Area. | Diam. | Area. |
|---|---|---|---|---|---|---|---|---|---|
| 28. | 615.754 | 35. | 962.115 | 42. | 1385.44 | 49. | 1885.74 | 56. | 2463.01 |
| .1/8 | 621.263 | .1/8 | 968.999 | .1/8 | 1393.7 | .1/8 | 1895.37 | .1/8 | 2474.02 |
| .1/4 | 626.798 | .1/4 | 975.908 | .1/4 | 1405.98 | .1/4 | 1905.03 | .1/4 | 2485.05 |
| .3/8 | 632.357 | .3/8 | 982.842 | .3/8 | 1410.29 | .3/8 | 1914.7 | .3/8 | 2496.11 |
| .1/2 | 637.941 | .1/2 | 989.8 | .1/2 | 1418.63 | .1/2 | 1924.42 | .1/2 | 2507.19 |
| .5/8 | 643.549 | .5/8 | 996.783 | .5/8 | 1426.98 | .5/8 | 1934.15 | .5/8 | 2418.3 |
| .3/4 | 649.182 | .1/4 | 1003.79 | .3/4 | 1435.36 | .3/4 | 1943.91 | .3/4 | 2529.43 |
| .7/8 | 654.839 | .7/8 | 1010.822 | .7/8 | 1443.77 | .7/8 | 1953.69 | .7/8 | 2540.54 |
| 29. | 660.521 | 36. | 1017.878 | 43. | 1452.21 | 50. | 1963.5 | 57. | 2551.76 |
| .1/8 | 666.227 | .1/8 | 1024.959 | .1/8 | 1460.65 | .1/8 | 1973.33 | .1/8 | 2562.97 |
| .1/4 | 671.958 | .1/4 | 1032.065 | .1/4 | 1469.13 | .1/4 | 1983.18 | .1/4 | 2574.2 |
| .3/8 | 677.714 | .3/8 | 1039.195 | .3/8 | 1477.63 | .3/8 | 1993.05 | .3/8 | 2585.45 |
| .1/2 | 683.494 | .1/2 | 1046.349 | .1/2 | 1486.17 | .1/2 | 2002.97 | .1/2 | 2596.73 |
| .5/8 | 689.298 | .5/8 | 1053.528 | .5/8 | 1494.72 | .5/8 | 2012.89 | .5/8 | 2608.83 |
| .3/4 | 695.128 | .3/4 | 1060.732 | .3/4 | 1503.3 | .3/4 | 2022.85 | .3/4 | 2619.36 |
| .7/8 | 700.981 | .7/8 | 1067.96 | .7/8 | 1511.9 | .7/8 | 2032.82 | .7/8 | 2630.71 |
| 30. | 706.86 | 37. | 1075.213 | 44. | 1520.53 | 51. | 2042.82 | 58. | 2642.09 |
| .1/8 | 712.762 | .1/8 | 1082.49 | .1/8 | 1529.18 | .1/8 | 2052.85 | .1/8 | 2653.49 |
| .1/4 | 718.69 | .1/4 | 1089.792 | .1/4 | 1537.86 | .1/4 | 2062.9 | .1/4 | 2664.91 |
| .3/8 | 724.641 | .3/8 | 1097.118 | .3/8 | 1546.55 | .3/8 | 2072.98 | .3/8 | 2676.36 |
| .1/2 | 730.618 | .1/2 | 1104.469 | .1/2 | 1555.28 | .1/2 | 2083.08 | .1/2 | 2687.[illegible] |
| .5/8 | 736.619 | .5/8 | 1111.844 | .5/8 | 1564.03 | .5/8 | 2093.2 | .5/8 | 2699.33 |
| .3/4 | 742.644 | .3/4 | 1119.244 | .3/4 | 1572.81 | .3/4 | 2103.35 | .3/4 | 2710.86 |
| .7/8 | 748.694 | .7/8 | 1126.668 | .7/8 | 1581.61 | .7/8 | 2113.52 | .7/8 | 2722.4 |
| 31. | 754.769 | 38. | 1134.118 | 45. | 1590.43 | 52. | 2123.72 | 59. | 2733.98 |
| .1/8 | 760.868 | .1/8 | 1141.591 | .1/8 | 1599.28 | .1/8 | 2133.94 | .1/8 | 2745.57 |
| .1/4 | 766.992 | .1/4 | 1149.089 | .1/4 | 1608.15 | .1/4 | 2144.19 | .1/4 | 2757.2 |
| .3/8 | 773.14 | .3/8 | 1156.612 | .3/8 | 1617.04 | .3/8 | 2154.46 | .3/8 | 2768.84 |
| .1/2 | 779.313 | .1/2 | 1164.159 | .1/2 | 1625.97 | .1/2 | 2164.76 | .1/2 | 2780.51 |
| .5/8 | 785.51 | .5/8 | 1171.731 | .5/8 | 1634.92 | .5/8 | 2175.08 | .5/8 | 2792.21 |
| .3/4 | 791.732 | .3/4 | 1179.327 | .3/4 | 1643.89 | .3/4 | 2185.42 | .3/4 | 2803.93 |
| .7/8 | 797.978 | .7/8 | 1186.948 | .7/8 | 1652.88 | .7/8 | 2195.79 | .7/8 | 2815.67 |
| 32. | 804.545 | 39. | 1194.593 | 46. | 1661.91 | 53. | 2206.19 | 60. | 2827.44 |
| .1/8 | 810.545 | .1/8 | 1202.263 | .1/8 | 1670.95 | .1/8 | 2216.61 | .1/8 | 2839.23 |
| .1/4 | 816.865 | .1/4 | 1209.958 | .1/4 | 1680.01 | .1/4 | 2227.05 | .1/4 | 2851.05 |
| .3/8 | 823.209 | .3/8 | 1217.677 | .3/8 | 1689.1 | .3/8 | 2237.52 | .3/8 | 2862.89 |
| .1/2 | 829.578 | .1/2 | 1225.42 | .1/2 | 1698.23 | .1/2 | 2248.01 | .1/2 | 2874.76 |
| .5/8 | 835.972 | .5/8 | 1233.188 | .5/8 | 1707.37 | .5/8 | 2258.53 | .5/8 | 2886.65 |
| .3/4 | 842.390 | .3/4 | 1240.981 | .3/4 | 1716.54 | .3/4 | 2269.07 | .3/4 | 2898.57 |
| .7/8 | 848.833 | .7/8 | 1248.798 | .7/8 | 1725.73 | .7/8 | 2279.64 | .7/8 | 2910.51 |
| 33. | 855.301 | 40. | 1256.64 | 47. | 1734.95 | 54. | 2290.23 | 61. | 2922.47 |
| .1/8 | 861.792 | .1/8 | 1264.5 | .1/8 | 1744.18 | .1/8 | 2300.84 | .1/8 | 2934.46 |
| .1/4 | 868.309 | .1/4 | 1272.39 | .1/4 | 1753.45 | .1/4 | 2311.48 | .1/4 | 2946.48 |
| .3/8 | 874.85 | .3/8 | 1280.31 | .3/8 | 1762.73 | .3/8 | 2322.14 | .3/8 | 2958.52 |
| .1/2 | 881.415 | .1/2 | 1288.25 | .1/2 | 1772.05 | .1/2 | 2332.83 | .1/2 | 2970.58 |
| .5/8 | 888.605 | .5/8 | 1296.21 | .5/8 | 1781.39 | .5/8 | 2343.55 | .5/8 | 2982.67 |
| .3/4 | 894.62 | .3/4 | 1304.2 | .3/4 | 1790.76 | .3/4 | 2354.28 | .3/4 | 2994.78 |
| .7/8 | 901.259 | .7/8 | 1312.21 | .7/8 | 1800.14 | .7/8 | 2365.05 | .7/8 | 3006.92 |
| 34. | 907.922 | 41. | 1320.26 | 48. | 1809.56 | 55. | 2375.83 | 62. | 3019.08 |
| .1/8 | 914.61 | .1/8 | 1328.32 | .1/8 | 1818.99 | .1/8 | 2386.65 | .1/8 | 3031.26 |
| .1/4 | 921.323 | .1/4 | 1336.4 | .1/4 | 1828.46 | .1/4 | 2397.48 | .1/4 | 3043.47 |
| .3/8 | 928.06 | .3/8 | 1344.51 | .3/8 | 1837.93 | .3/8 | 2408.34 | .3/8 | 3055.71 |
| .1/2 | 934.822 | .1/2 | 1352.65 | .1/2 | 1847.45 | .1/2 | 2419.22 | .1/2 | 3067.97 |
| .5/8 | 941.609 | .5/8 | 1360.81 | .5/8 | 1856.99 | .5/8 | 2430.18 | .5/8 | 3080.25 |
| .3/4 | 948.419 | .3/4 | 1369. | .3/4 | 1866.55 | .3/4 | 2441.07 | .3/4 | 3092.56 |
| .7/8 | 955.255 | .7/8 | 1377.21 | .7/8 | 1876.13 | .7/8 | 2452.03 | .7/8 | 3104.89 |

TABLE.—(*Continued.*)

| Diam. | Area. | Diam. | Area. | Diam. | Area. | Diam. | Area. | Diam. | Area. |
|---|---|---|---|---|---|---|---|---|---|
| 63. | 3117.25 | 70. | 3848.46 | 77. | 4656.64 | 84. | 5541.78 | 91. | 6503.9 |
| .1/8 | 3129.63 | .1/8 | 3862.23 | .1/8 | 4671.77 | .1/8 | 5558.29 | .1/8 | 6521.78 |
| .1/4 | 3142.04 | .1/4 | 3876. | .1/4 | 4686.92 | .1/4 | 5574.82 | .1/4 | 6539.68 |
| .3/8 | 3154.47 | .3/8 | 3889.8 | .3/8 | 4702.1 | .3/8 | 5591.37 | .3/8 | 6557.61 |
| .1/2 | 3166.93 | .1/2 | 3903.63 | .1/2 | 4717.31 | .1/2 | 5607.95 | .1/2 | 6575.56 |
| .5/8 | 3179.41 | .5/8 | 3917.49 | .5/8 | 4732.54 | .5/8 | 5624.56 | .5/8 | 6593.54 |
| .3/4 | 3191.91 | .3/4 | 3931.37 | .3/4 | 4747.79 | .3/4 | 5641.18 | .3/4 | 6611.55 |
| .7/8 | 3204.44 | .7/8 | 3945.27 | .7/8 | 4763.07 | .7/8 | 5657.84 | .7/8 | 6629.57 |
| 64. | 3217. | 71. | 3959.2 | 78. | 4778.37 | 85. | 5674.51 | 92. | 6647.63 |
| .1/8 | 3229.58 | .1/8 | 3973.15 | .1/8 | 4793.7 | .1/8 | 5691.22 | .1/8 | 6665.7 |
| .1/4 | 3242.18 | .1/4 | 3987.13 | .1/4 | 4809.05 | .1/4 | 5707.94 | .1/4 | 6683.8 |
| .3/8 | 3254.81 | .3/8 | 4001.13 | .3/8 | 4824.43 | .3/8 | 5724.69 | .3/8 | 6701.93 |
| .1/2 | 3267.46 | .1/2 | 4015.16 | .1/2 | 4839.83 | .1/2 | 5741.47 | .1/2 | 6720.08 |
| .5/8 | 3280.18 | .5/8 | 4029.21 | .5/8 | 4855.26 | .5/8 | 5758.27 | .5/8 | 6738.25 |
| .3/4 | 3292.84 | .3/4 | 4043.29 | .3/4 | 4870.71 | .3/4 | 5775.1 | .3/4 | 6756.45 |
| .7/8 | 3305.56 | .7/8 | 4057.39 | .7/8 | 4886.18 | .7/8 | 5791.94 | .7/8 | 6774.68 |
| 65. | 3319.31 | 72. | 4071.51 | 79. | 4901.68 | 86. | 5808.82 | 93. | 6792.92 |
| .1/8 | 3331.09 | .1/8 | 4085.66 | .1/8 | 4917.21 | .1/8 | 5825.72 | .1/8 | 6811.2 |
| .1/4 | 3343.89 | .1/4 | 4099.83 | .1/4 | 4932.75 | .1/4 | 5842.64 | .1/4 | 6829.49 |
| .3/8 | 3356.71 | .3/8 | 4114.04 | .3/8 | 4948.33 | .3/8 | 5859.59 | .3/8 | 6847.82 |
| .1/2 | 3369.56 | .1/2 | 4128.26 | .1/2 | 4963.92 | .1/2 | 5876.56 | .1/2 | 6866.16 |
| .5/8 | 3382.43 | .5/8 | 4142.51 | .5/8 | 4979.55 | .5/8 | 5893.55 | .5/8 | 6884.53 |
| .3/4 | 3395.33 | .3/4 | 4156.78 | .3/4 | 4995.19 | .3/4 | 5910.58 | .3/4 | 6902.93 |
| .7/8 | 3408.26 | .7/8 | 4171.08 | .7/8 | 5010.87 | .7/8 | 5927.62 | .7/8 | 6921.35 |
| 66. | 3421.2 | 73. | 4185.4 | 80. | 5026.56 | 87. | 5944.69 | 94. | 6939.79 |
| .1/8 | 3434.17 | .1/8 | 4199.74 | .1/8 | 5042.28 | .1/8 | 5961.[illegible] | .1/8 | 6958.26 |
| .1/4 | 3447.17 | .1/4 | 4214.11 | .1/4 | 5058.02 | .1/4 | 5978.9 | .1/4 | 6976.76 |
| .3/8 | 3460.19 | .3/8 | 4228.51 | .3/8 | 5073.79 | .3/8 | 5996.05 | .3/8 | 6995.28 |
| .1/2 | 3473.24 | .1/2 | 4242.93 | .1/2 | 5089.59 | .1/2 | 6013.22 | .1/2 | 7013.82 |
| .5/8 | 3486.3 | .5/8 | 4257.37 | .5/8 | 5105.41 | .5/8 | 6030.41 | .5/8 | 7032.39 |
| .3/4 | 3499.4 | .3/4 | 4271.84 | .3/4 | 5121.25 | .3/4 | 6047.63 | .3/4 | 7050.98 |
| .7/8 | 3512.52 | .7/8 | 4286.33 | .7/8 | 5137.12 | .7/8 | 6064.87 | .7/8 | 7069.59 |
| 67. | 3525.66 | 74. | 4300.85 | 81. | 5153.01 | 88. | 6082.14 | 95. | 7088.24 |
| .1/8 | 3538.83 | .1/8 | 4315.39 | .1/8 | 5168.93 | .1/8 | 6099.43 | .1/8 | 7106.9 |
| .1/4 | 3552.02 | .1/4 | 4329.96 | .1/4 | 5184.87 | .1/4 | 6116.74 | .1/4 | 7125.59 |
| .3/8 | 3565.24 | .3/8 | 4344.55 | .3/8 | 5200.83 | .3/8 | 6134.08 | .3/8 | 7144.31 |
| .1/2 | 3578.48 | .1/2 | 4359.17 | .1/2 | 5216.82 | .1/2 | 6151.45 | .1/2 | 7163.04 |
| .5/8 | 3591.74 | .5/8 | 4373.81 | .5/8 | 5232.84 | .5/8 | 6168.84 | .5/8 | 7181.81 |
| .3/4 | 3605.03 | .3/4 | 4388.47 | .3/4 | 5248.88 | .3/4 | 6186.25 | .3/4 | 7200.6 |
| .7/8 | 3618.35 | .7/8 | 4403.16 | .7/8 | 5264.94 | .7/8 | 6203.69 | .7/8 | 7219.41 |
| 68. | 3631.69 | 75. | 4417.87 | 82. | 5281.03 | 89. | 6221.15 | 96. | 7238.25 |
| .1/8 | 3645.05 | .1/8 | 4432.16 | .1/8 | 5297.14 | .1/8 | 6238.64 | .1/8 | 7257.11 |
| .1/4 | 3658.44 | .1/4 | 4447.37 | .1/4 | 5313.28 | .1/4 | 6256.15 | .1/4 | 7275.99 |
| .3/8 | 3671.85 | .3/8 | 4462.16 | .3/8 | 5329.44 | .3/8 | 6273.69 | .3/8 | 7294.91 |
| .1/2 | 3685.29 | .1/2 | 4476.98 | .1/2 | 5345.63 | .1/2 | 6291.25 | .1/2 | 7313.84 |
| .5/8 | 3698.76 | .5/8 | 4491.81 | .5/8 | 5361.84 | .5/8 | 6308.84 | .5/8 | 7332.8 |
| .3/4 | 3712.24 | .3/4 | 4506.67 | .3/4 | 5378.08 | .3/4 | 6326.44 | .3/4 | 7351.79 |
| .7/8 | 3625.75 | .7/8 | 4521.56 | .7/8 | 5394.34 | .7/8 | 6344.08 | .7/8 | 7370.79 |
| 69. | 3739.29 | 76. | 4536.47 | 83. | 5410.62 | 90. | 6361.74 | 97. | 7389.83 |
| .1/8 | 3752.85 | .1/8 | 4551.4 | .1/8 | 5426.93 | .1/8 | 6379.42 | .1/8 | 7408.89 |
| .1/4 | 3766.43 | .1/4 | 4566.36 | .1/4 | 5443.26 | .1/4 | 6397.13 | .1/4 | 7427.97 |
| .3/8 | 3780.04 | .3/8 | 4581.35 | .3/8 | 5459.62 | .3/8 | 6414.86 | .3/8 | 7447.08 |
| .1/2 | 3793.68 | .1/2 | 4596.36 | .1/2 | 5476.01 | .1/2 | 6432.62 | .1/2 | 7466.21 |
| .5/8 | 3807.34 | .5/8 | 4611.39 | .5/8 | 5492.41 | .5/8 | 6450.4 | .5/8 | 7485.36 |
| .3/4 | 3821.02 | .3/4 | 4626.45 | .3/4 | 5508.84 | .3/4 | 6468.21 | .3/4 | 7504.55 |
| .7/8 | 3834.73 | .7/8 | 4641.53 | .7/8 | 5525.3 | .7/8 | 6486.04 | .7/8 | 7523.75 |





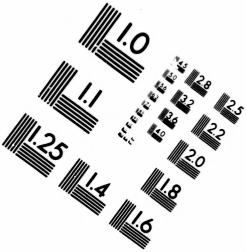
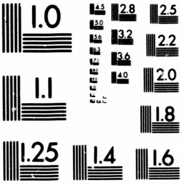
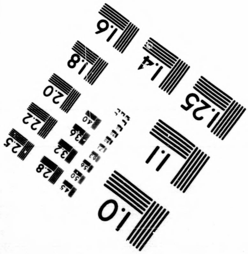
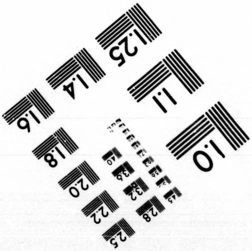


**IMAGE EVALUATION**
**TEST TARGET (MT-3)**







TABLE—(*Continued*).—[*Advancing by a Quarter and a half.*]

| Diam. | Area. | Diam. | Area. | Diam. | Area. | Diam. | Area. | Diam. | Area. |
|---|---|---|---|---|---|---|---|---|---|
| 98. | 7542.98 | 105. | 8659.03 | 114. | 10207.06 | 123. | 11882.32 | 139. | 15174.71 |
| .⅛ | 7562.24 | .¼ | 8700.32 | .¼ | 10251.88 | .¼ | 11930.67 | .½ | 15284.08 |
| .¼ | 7581.51 | .½ | 8741.7 | .½ | 10296.79 | .½ | 11979.2 | 140. | 15393.84 |
| .⅜ | 7600.8[illegible] | .¾ | 8783.18 | .¾ | 10341.8 | .¾ | 12027.66 | .½ | 15503.98 |
| .½ | 7620.15 | 106. | 8824.75 | 115. | 10386.91 | 124. | 12076.31 | 141. | 15614.53 |
| .⅝ | 7639.5 | .¼ | 8866.43 | .¼ | 10432.12 | .¼ | 12125.05 | .½ | 15725.47 |
| .¾ | 7658.88 | .½ | 8908.2 | .½ | 10477.43 | .½ | 12173.9 | 142. | 15836.8 |
| .⅞ | 7678.28 | .¾ | 8950.07 | .¾ | 10522.84 | .¾ | 12222.84 | .½ | 15948.52 |
| 99. | 7697.71 | 107. | 8992.04 | 116. | 10568.34 | 125. | 12271.87 | 143. | 16060.54 |
| .⅛ | 7717.16 | .¼ | 9034.11 | .¼ | 10613.94 | .½ | 12370.25 | .½ | 16173.15 |
| .¼ | 7736.63 | .½ | 9076.28 | .½ | 10659.64 | 126. | 12469.01 | 144. | 16286.05 |
| .⅜ | 7756.13 | .¾ | 9118.53 | .¾ | 10705.44 | .½ | 12568.17 | .½ | 16399.34 |
| .½ | 7775.66 | 108. | 9160.91 | 117. | 10751.34 | 127. | 12667.72 | 145. | 16513.03 |
| .⅝ | 7795.2 | .¼ | 9203.37 | .¼ | 10797.34 | .½ | 12767.66 | .½ | 16627.11 |
| .¾ | 7814.78 | .½ | 9245.92 | .½ | 10843.43 | 128. | 12867.99 | 146. | 16741.59 |
| .⅞ | 7834.38 | .¾ | 9288.58 | .¾ | 10889.62 | .½ | 12968.71 | .½ | 16856.44 |
| 100. | 7854. | 109. | 9331.34 | 118. | 10935.9 | 129. | 13069.84 | 147. | 16971.71 |
| .¼ | 7993.32 | .¼ | 9374.19 | .¼ | 10982.3 | .½ | 13171.35 | .½ | 17087.36 |
| .½ | 7932.74 | .½ | 9417.14 | .½ | 11028.78 | 130. | 13273.26 | 148. | 17203.4 |
| .¾ | 7972.21 | .¾ | 9460.19 | .¾ | 11075.37 | .½ | 13371.55 | .½ | 17319.83 |
| 101. | 8011.87 | 110. | 9503.34 | 119. | 11122.06 | 131. | 13478.25 | 149. | 17436.67 |
| .¼ | 8051.58 | .¼ | 9546.69 | .¼ | 11168.83 | .½ | 13581.33 | .½ | 17553.89 |
| .½ | 8091.39 | .½ | 9589.93 | .½ | 11215.71 | 132. | 13684.81 | 150. | 17671.5 |
| .¾ | 8131.3 | .¾ | 9633.37 | .¾ | 11262.69 | .½ | 13788.67 | .½ | 17789.51 |
| 102. | 8171.3 | 111. | 9676.91 | 120. | 11309.76 | 133. | 13992.94 | | |
| .¼ | 8211.41 | .¼ | 9720.73 | .¼ | 11356.93 | .½ | 13997.54 | | |
| .½ | 8251.61 | .½ | 9764.29 | .½ | 11404.2 | 134. | 14102.64 | | |
| .¾ | 8291.91 | .¾ | 9808.12 | .¾ | 11451.57 | .½ | 14208.07 | | |
| 103. | 8332.31 | 112. | 9852.06 | 121. | 11499.04 | 135. | 14313.91 | | |
| .¼ | 8372.81 | .¼ | 9896.09 | .¼ | 11546.61 | .½ | 14420.14 | | |
| .½ | 8413.4 | .½ | 9940.22 | .½ | 11594.27 | 136. | 14526.76 | | |
| .¾ | 8464.09 | .¾ | 9984.45 | .¾ | 11642.09 | .½ | 14633.76 | | |
| 104. | 8494.89 | 113. | 10028.77 | 122. | 11689.89 | 137. | 14741.17 | | |
| .¼ | 8535.78 | .¼ | 10073.2 | .¼ | 11737.85 | .½ | 14848.96 | | |
| .½ | 8576.77 | .½ | 10117.72 | .½ | 11785.9[illegible] | 138. | 14957.16 | | |
| .¾ | 8617.85 | .¾ | 10162.34 | .¾ | 11834.06 | .½ | 15065.73 | | |

## To Compute the Area of a Diameter greater than any in the preceding Table.

RULE.—Divide the dimension by two, three, four, etc., if practicable to do so, until it is reduced to a diameter to be found in the table.

Take the tabular area for the diameter, multiply it by the square of the diviser, and the product will give the area required.

EXAMPLE.—What is the area for a diameter of 1050 ?

$1050 \div 7 = 150$ ; tab. area, $150 = 17671.5$, which $\times 7^2 = 865903.5$, *area required.*

## To Compute the Area of an Integer and a Fraction not given in the Table.

RULE.—Double, treble, or quadruple the dimension given, until the fraction is increased to a whole number, or to one of those in the table, as $\frac{1}{8}$, $\frac{1}{4}$, etc., provided it is practicable to do so.

Take the area for this diameter; and if it is double of that for which the area is required, take one fourth of it ; if treble, take one 9th. of it and if quadruple, take one sixteenth of it, etc., etc.

EXAMPLE.—Required the area for a circle of 2. $\frac{3}{16}$ inches.

2. $\frac{3}{16} \times 2 = 4\frac{3}{8}$, area for which $= 15.0331$, which $\div 4 = 3{,}758$ *ins.*

# TABLE II. b.

## CIRCUMFERENCES OF CIRCLES, FROM 1/64 TO 150.

*[Advancing by an Eighth.]*

| Diam. | Circum. | Diam. | Circum. | Diam. | Circum. | Diam. | Circum. | Diam. | Circum. |
|---|---|---|---|---|---|---|---|---|---|
| 1/64 | .04909 | 4. | 12.5664 | 10. | 31.416 | 16. | 50.2656 | 22. | 69.1152 |
| 1/32 | .09817 | 1/8 | 12.9591 | 1/8 | 31.8087 | 1/8 | 50.6583 | 1/8 | 69.5079 |
| 1/16 | .19635 | 1/4 | 13.3518 | 1/4 | 32.2014 | 1/4 | 51.051 | 1/4 | 69.9066 |
| 1/8 | .3927 | 3/8 | 13.7445 | 3/8 | 32.5941 | 3/8 | 51.4437 | 3/8 | 70.2933 |
| 3/16 | .589 | 1/2 | 14.1372 | 1/2 | 32.9868 | 1/2 | 51.8364 | 1/2 | 70.686 |
| 1/4 | .7854 | 5/8 | 14.5299 | 5/8 | 33.3795 | 5/8 | 52.2291 | 5/8 | 71.0787 |
| 5/16 | .98175 | 3/4 | 14.9226 | 3/4 | 33.7722 | 3/4 | 52.6218 | 3/4 | 71.4714 |
| 3/8 | 1.1781 | 7/8 | 15.3153 | 7/8 | 34.1649 | 7/8 | 53.0145 | 7/8 | 71.8641 |
| 7/16 | 1.37445 | 5. | 15.708 | 11. | 34.5576 | 17. | 53.4072 | 23. | 72.2568 |
| 1/2 | 1.5708 | 1/8 | 16.1007 | 1/8 | 34.9503 | 1/8 | 53.7999 | 1/8 | 72.6495 |
| 9/16 | 1.76715 | 1/4 | 16.4934 | 1/4 | 35.343 | 1/4 | 54.1926 | 1/4 | 73.0422 |
| 5/8 | 1.9635 | 3/8 | 16.8861 | 3/8 | 35.7353 | 3/8 | 54.5853 | 3/8 | 73.4349 |
| 11/16 | 2.15985 | 1/2 | 17.2788 | 1/2 | 36.1284 | 1/2 | 54.978 | 1/2 | 73.8276 |
| 3/4 | 2.3562 | 5/8 | 17.6715 | 5/8 | 36.5211 | 5/8 | 55.3707 | 5/8 | 74.2203 |
| 13/16 | 2.55255 | 3/4 | 18.0642 | 3/4 | 36.9138 | 3/4 | 55.7634 | 3/4 | 74.613 |
| 7/8 | 2.7489 | 7/8 | 18.4569 | 7/8 | 37.3065 | 7/8 | 56.1561 | 7/8 | 75.0057 |
| 15/16 | 2.94525 | 6. | 18.8496 | 12. | 37.6992 | 18. | 56.5488 | 24. | 75.3984 |
| 1. | 3.1416 | 1/8 | 19.2423 | 1/8 | 38.0919 | 1/8 | 56.9415 | 1/8 | 75.7911 |
| 1/8 | 3.5343 | 1/4 | 19.635 | 1/4 | 38.4846 | 1/4 | 57.3342 | 1/4 | 76.1838 |
| 1/4 | 3.927 | 3/8 | 20.0277 | 3/8 | 38.8773 | 3/8 | 57.7269 | 3/8 | 76.5765 |
| 3/8 | 4.3197 | 1/2 | 20.4204 | 1/2 | 39.27 | 1/2 | 58.1196 | 1/2 | 76.9692 |
| 1/2 | 4.7124 | 5/8 | 20.8131 | 5/8 | 39.6627 | 5/8 | 58.5123 | 5/8 | 77.3619 |
| 5/8 | 5.1051 | 3/4 | 21.2058 | 3/4 | 40.0554 | 3/4 | 58.905 | 3/4 | 77.7546 |
| 3/4 | 5.4978 | 7/8 | 21.5985 | 7/8 | 40.4481 | 7/8 | 59.2977 | 7/8 | 78.1473 |
| 7/8 | 5.8905 | 7. | 21.9912 | 13. | 40.8408 | 19. | 59.6904 | 25. | 78.54 |
| 2. | 6.2832 | 1/8 | 22.3839 | 1/8 | 41.2335 | 1/8 | 60.0831 | 1/8 | 78.9327 |
| 1/8 | 6.6759 | 1/4 | 22.7766 | 1/4 | 41.6262 | 1/4 | 60.4758 | 1/4 | 79.3254 |
| 1/4 | 7.0686 | 3/8 | 23.1693 | 3/8 | 42.0189 | 3/8 | 60.8685 | 3/8 | 79.7181 |
| 3/8 | 7.4613 | 1/2 | 23.562 | 1/2 | 42.4116 | 1/2 | 61.2612 | 1/2 | 80.1102 |
| 1/2 | 7.854 | 5/8 | 23.9547 | 5/8 | 42.8043 | 5/8 | 61.6539 | 5/8 | 80.5035 |
| 5/8 | 8.2467 | 3/4 | 24.3474 | 3/4 | 43.197 | 3/4 | 62.0466 | 3/4 | 80.8962 |
| 3/4 | 8.6394 | 7/8 | 24.7401 | 7/8 | 43.5897 | 7/8 | 62.4393 | 7/8 | 81.2889 |
| 7/8 | 9.0321 | 8. | 25.1328 | 14. | 43.9824 | 20. | 62.832 | 26. | 81.6816 |
| 3. | 9.4248 | 1/8 | 25.5255 | 1/8 | 44.3751 | 1/8 | 63.2247 | 1/8 | 82.0743 |
| 1/8 | 9.8175 | 1/4 | 25.9182 | 1/4 | 44.7678 | 1/4 | 63.6174 | 1/4 | 82.467 |
| 1/4 | 10.2102 | 3/8 | 26.3109 | 3/8 | 45.1605 | 3/8 | 64.0101 | 3/8 | 82.8597 |
| 3/8 | 10.6029 | 1/2 | 26.7036 | 1/2 | 45.5532 | 1/2 | 64.4028 | 1/2 | 83.2524 |
| 1/2 | 10.9956 | 5/8 | 27.0963 | 5/8 | 45.9459 | 5/8 | 64.7955 | 5/8 | 83.6451 |
| 5/8 | 11.3883 | 3/4 | 27.489 | 3/4 | 46.3386 | 3/4 | 65.1882 | 3/4 | 84.0378 |
| 3/4 | 11.781 | 7/8 | 27.8817 | 7/8 | 46.7313 | 7/8 | 65.5809 | 7/8 | 84.4305 |
| 7/8 | 12.1737 | 9. | 28.2744 | 15. | 47.124 | 21. | 65.9736 | 27. | 84.8232 |
| | | 1/8 | 28.6671 | 1/8 | 47.5167 | 1/8 | 66.3663 | 1/8 | 85.2159 |
| | | 1/4 | 29.0598 | 1/4 | 47.9094 | 1/4 | 66.759 | 1/4 | 85.6086 |
| | | 3/8 | 29.4525 | 3/8 | 48.3021 | 3/8 | 67.1517 | 3/8 | 86.0013 |
| | | 1/2 | 29.8452 | 1/2 | 48.6948 | 1/2 | 67.5444 | 1/2 | 86.394 |
| | | 5/8 | 30.2379 | 5/8 | 49.0875 | 5/8 | 67.9371 | 5/8 | 86.7867 |
| | | 3/4 | 30.6306 | 3/4 | 49.4802 | 3/4 | 68.3298 | 3/4 | 87.1794 |
| | | 7/8 | 31.6233 | 7/8 | 49.8729 | 7/8 | 68.7225 | 7/8 | 87.5721 |

TABLE.—(*Continued.*)

| Diam. | Circum. | Diam. | Circum. | Diam. | Circum. | Diam. | Circum. | Diam. | Circum. |
|---|---|---|---|---|---|---|---|---|---|
| 28. | 87.9648 | 35. | 109.956 | 42. | 131.947 | 49. | 153.938 | 56. | 175.93 |
| .⅛ | 88.3575 | .⅛ | 110.349 | .⅛ | 132.34 | .⅛ | 154.331 | .⅛ | 176.322 |
| .¼ | 88.7502 | .¼ | 110.741 | .¼ | 132.733 | .¼ | 154.724 | .¼ | 176.715 |
| .⅜ | 89.1429 | .⅜ | 111.134 | .⅜ | 133.125 | .⅜ | 155.117 | .⅜ | 177.108 |
| .½ | 89.5356 | .½ | 111.527 | .½ | 133.518 | .½ | 155.509 | .½ | 177.5 |
| .⅝ | 89.9283 | .⅝ | 111.919 | .⅝ | 133.911 | .⅝ | 155.902 | .⅝ | 177.893 |
| .¾ | 90.321 | .¾ | 112.312 | .¾ | 134.303 | .¾ | 156.295 | .¾ | 178.286 |
| .⅞ | 90.7137 | .⅞ | 112.705 | .⅞ | 134.696 | .⅞ | 156.687 | .⅞ | 178.679 |
| 29. | 91.1064 | 36. | 113.098 | 43. | 135.089 | 50. | 157.08 | 57. | 179.071 |
| .⅛ | 91.4991 | .⅛ | 113.49 | .⅛ | 135.481 | .⅛ | 157.473 | .⅛ | 179.464 |
| .¼ | 91.8918 | .¼ | 113.883 | .¼ | 135.874 | .¼ | 157.865 | .¼ | 179.857 |
| .⅜ | 92.2845 | .⅜ | 114.276 | .⅜ | 136.267 | .⅜ | 158.258 | .⅜ | 180.249 |
| .½ | 92.6772 | .½ | 114.668 | .½ | 136.66 | .½ | 158.651 | .½ | 180.642 |
| .⅝ | 93.0699 | .⅝ | 115.061 | .⅝ | 137.052 | .⅝ | 159.044 | .⅝ | 181.035 |
| .¾ | 93.4626 | .¾ | 115.454 | .¾ | 137.445 | .¾ | 159.436 | .¾ | 181.427 |
| .⅞ | 93.8553 | .⅞ | 115.846 | .⅞ | 137.838 | .⅞ | 159.823 | .⅞ | 181.82 |
| 30. | 94.248 | 37. | 116.239 | 44. | 138.23 | 51. | 160.222 | 58. | 182.213 |
| .⅛ | 94.6407 | .⅛ | 116.632 | .⅛ | 138.623 | .⅛ | 160.614 | .⅛ | 182.606 |
| .¼ | 95.0334 | .¼ | 117.025 | .¼ | 139.016 | .¼ | 161.007 | .¼ | 182.998 |
| .⅜ | 95.4261 | .⅜ | 117.417 | .⅜ | 139.408 | .⅜ | 161.4 | .⅜ | 183.391 |
| .½ | 95.8188 | .½ | 117.81 | .½ | 139.801 | .½ | 161.792 | .½ | 183.784 |
| .⅝ | 96.2115 | .⅝ | 118.203 | .⅝ | 140.194 | .⅝ | 162.185 | .⅝ | 184.176 |
| .¾ | 96.6042 | .¾ | 118.595 | .¾ | 140.587 | .¾ | 162.578 | .¾ | 184.569 |
| .⅞ | 96.9969 | .⅞ | 118.988 | .⅞ | 140.979 | .⅞ | 162.971 | .⅞ | 184.962 |
| 31. | 97.3896 | 38. | 119.381 | 45. | 141.372 | 52. | 163.363 | 59. | 185.354 |
| .⅛ | 97.7823 | .⅛ | 119.774 | .⅛ | 141.765 | .⅛ | 163.756 | .⅛ | 185.747 |
| .¼ | 98.175 | .¼ | 120.166 | .¼ | 142.157 | .¼ | 164.149 | .¼ | 186.14 |
| .⅜ | 98.5677 | .⅜ | 120.559 | .⅜ | 142.55 | .⅜ | 164.541 | .⅜ | 186.533 |
| .½ | 98.9604 | .½ | 120.952 | .½ | 142.943 | .½ | 164.934 | .½ | 186.925 |
| .⅝ | 99.3531 | .⅝ | 121.344 | .⅝ | 143.336 | .⅝ | 165.327 | .⅝ | 187.318 |
| .¾ | 99.7458 | .¾ | 121.737 | .¾ | 143.728 | .¾ | 165.719 | .¾ | 187.711 |
| .⅞ | 100.1385 | .⅞ | 122.13 | .⅞ | 144.121 | .⅞ | 166.112 | .⅞ | 188.103 |
| 32. | 100.5312 | 39. | 122.522 | 46. | 144.514 | 53. | 166.505 | 60. | 188.496 |
| .⅛ | 100.9239 | .⅛ | 122.915 | .⅛ | 144.906 | .⅛ | 166.898 | .⅛ | 188.889 |
| .¼ | 101.3166 | .¼ | 123.308 | .¼ | 145.299 | .¼ | 167.29 | .¼ | 189.281 |
| .⅜ | 101.7093 | .⅜ | 123.701 | .⅜ | 145.692 | .⅜ | 167.683 | .⅜ | 189.674 |
| .½ | 102.102 | .½ | 124.093 | .½ | 146.084 | .½ | 168.076 | .½ | 190.067 |
| .⅝ | 102.4947 | .⅝ | 124.486 | .⅝ | 146.477 | .⅝ | 168.468 | .⅝ | 190.46 |
| .¾ | 102.8874 | .¾ | 124.879 | .¾ | 146.87 | .¾ | 168.861 | .¾ | 190.852 |
| .⅞ | 103.2801 | .⅞ | 125.271 | .⅞ | 147.263 | .⅞ | 169.254 | .⅞ | 191.245 |
| 33. | 103.673 | 40. | 125.664 | 47. | 147.655 | 54. | 169.646 | 61. | 191.538 |
| .⅛ | 104.066 | .⅛ | 126.057 | .⅛ | 148.048 | .⅛ | 170.039 | .⅛ | 192.03 |
| .¼ | 104.458 | .¼ | 126.449 | .¼ | 148.441 | .¼ | 170.432 | .¼ | 192.423 |
| .⅜ | 104.851 | .⅜ | 126.842 | .⅜ | 148.833 | .⅜ | 170.825 | .⅜ | 192.816 |
| .½ | 105.244 | .½ | 127.235 | .½ | 149.226 | .½ | 171.217 | .½ | 193.208 |
| .⅝ | 105.636 | .⅝ | 127.627 | .⅝ | 149.619 | .⅝ | 171.61 | .⅝ | 193.601 |
| .¾ | 106.029 | .¾ | 128.02 | .¾ | 150.011 | .¾ | 172.003 | .¾ | 193.994 |
| .⅞ | 106.422 | .⅞ | 128.413 | .⅞ | 150.404 | .⅞ | 172.396 | .⅞ | 194.387 |
| 34. | 106.814 | 41. | 128.806 | 48. | 150.797 | 55. | 172.788 | 62. | 194.779 |
| .⅛ | 107.207 | .⅛ | 129.198 | .⅛ | 151.19 | .⅛ | 173.181 | .⅛ | 195.172 |
| .¼ | 107.6 | .¼ | 129.591 | .¼ | 151.582 | .¼ | 173.573 | .¼ | 195.565 |
| .⅜ | 107.993 | .⅜ | 129.984 | .⅜ | 151.975 | .⅜ | 173.966 | .⅜ | 195.957 |
| .½ | 108.385 | .½ | 130.376 | .½ | 152.368 | .½ | 174.359 | .½ | 196.35 |
| .⅝ | 108.778 | .⅝ | 130.769 | .⅝ | 152.76 | .⅝ | 174.752 | .⅝ | 196.743 |
| .¾ | 109.171 | .¾ | 131.162 | .¾ | 153.153 | .¾ | 175.144 | .¾ | 197.135 |
| .⅞ | 109.563 | .⅞ | 131.554 | .⅞ | 153.546 | .⅞ | 175.537 | .⅞ | 197.528 |

| Diam. | Circum. |
|---|---|
| 56. | 175.93 |
| .1/8 | 176.322 |
| .1/4 | 176.715 |
| .3/8 | 177.108 |
| .1/2 | 177.5 |
| .5/8 | 177.893 |
| .3/4 | 178.286 |
| .7/8 | 178.679 |
| 57. | 179.071 |
| .1/8 | 179.464 |
| .1/4 | 179.857 |
| .3/8 | 180.249 |
| .1/2 | 180.642 |
| .5/8 | 181.035 |
| .3/4 | 181.427 |
| .7/8 | 181.82 |
| 58. | 182.213 |
| .1/8 | 182.606 |
| .1/4 | 182.998 |
| .3/8 | 183.391 |
| .1/2 | 183.784 |
| .5/8 | 184.176 |
| .3/4 | 184.569 |
| .7/8 | 184.962 |
| 59. | 185.354 |
| .1/8 | 185.747 |
| .1/4 | 186.14 |
| .3/8 | 186.533 |
| .1/2 | 186.925 |
| .5/8 | 187.318 |
| .3/4 | 187.711 |
| .7/8 | 188.103 |
| 60. | 188.496 |
| .1/8 | 188.889 |
| .1/4 | 189.281 |
| .3/8 | 189.674 |
| .1/2 | 190.067 |
| .5/8 | 190.46 |
| .3/4 | 190.852 |
| .7/8 | 191.245 |
| 61. | 191.538 |
| .1/8 | 192.03 |
| .1/4 | 192.423 |
| .3/8 | 192.816 |
| .1/2 | 193.208 |
| .5/8 | 193.601 |
| .3/4 | 193.994 |
| .7/8 | 194.387 |
| 62. | 194.779 |
| .1/8 | 195.172 |
| .1/4 | 195.565 |
| .3/8 | 195.957 |
| .1/2 | 196.35 |
| .5/8 | 196.743 |
| .3/4 | 197.135 |
| .7/8 | 197.528 |



TABLE.—(*Continued.*)

| Diam. | Circum. | Diam. | Circum. | Diam. | Circum. | Diam. | Circum. | Diam. | Circum. |
|---|---|---|---|---|---|---|---|---|---|
| 63. | 197.921 | 70. | 219.912 | 77 | 241.903 | 84. | 263.894 | 91. | 285.886 |
| .1/8 | 198.314 | .1/8 | 220.305 | 1/8 | 242.296 | .1/8 | 264.287 | .1/8 | 286.278 |
| .1/4 | 198.706 | .1/4 | 220.697 | .1/4 | 242.689 | .1/4 | 264.68 | .1/4 | 286.671 |
| .3/8 | 199.099 | .3/8 | 221.09 | .3/8 | 243.081 | .3/8 | 265 073 | .3/8 | 287.064 |
| .1/2 | 199.492 | .1/2 | 221.483 | .1/2 | 243.474 | .1/2 | 265.465 | .1/2 | 287.456 |
| .5/8 | 199.884 | .5/8 | 221.876 | .5/8 | 243.867 | .5/8 | 265.858 | .5/8 | 287.849 |
| .3/4 | 200.277 | .3/4 | 222.268 | .3/4 | 244 259 | .3/4 | 266.251 | .3/4 | 288.242 |
| .7/8 | 200.67 | .7/8 | 222.661 | .7/8 | 244.652 | .7/8 | 266.643 | .7/8 | 288.634 |
| 64. | 241.062 | 71. | 223.054 | 78. | 245.045 | 85. | 267.036 | 92. | 289.027 |
| .1/8 | 201.455 | .1/8 | 223.446 | .1/8 | 245.438 | .1/8 | 267.429 | .1/8 | 289.42 |
| .1/4 | 201.848 | .1/4 | 223.839 | 1/4 | 245.83 | .1/4 | 267.821 | .1/4 | 289.813 |
| .3/8 | 202.241 | .3/8 | 224.232 | .3/8 | 246.223 | .3/8 | 268.214 | .3/8 | 290.205 |
| .1/2 | 202.633 | .1/2 | 224.624 | .1/2 | 246.616 | .1/2 | 268.607 | .1/2 | 290.598 |
| .5/8 | 203.026 | .5/8 | 325.017 | .5/8 | 247.008 | .5/8 | 268.999 | .5/8 | 290.991 |
| .3/4 | 203.419 | .3/4 | 225.41 | .3/4 | 247.401 | .3/4 | 269.392 | .3/4 | 281.383 |
| .7/8 | 203.811 | .7/8 | 225.803 | .7/8 | 247.794 | .7/8 | 269.785 | .7/8 | 281.776 |
| 65. | 204.204 | 72. | 226.195 | 79. | 248.186 | 86. | 270.178 | 93. | 292.169 |
| .1/8 | 204.597 | .1/8 | 226.588 | .1/8 | 248.579 | .1/8 | 270.57 | .1/8 | 292.562 |
| .1/4 | 204.989 | .1/4 | 226.981 | .1/4 | 248.972 | .1/4 | 270.963 | .1/4 | 292.954 |
| .3/8 | 205.382 | .3/8 | 227.373 | 3/8 | 249.365 | .3/8 | 271.356 | .3/8 | 293.347 |
| .1/2 | 205.775 | .1/2 | 227.766 | .1/2 | 249.757 | .1/2 | 271.748 | .1/2 | 293.74 |
| .5/8 | 206.168 | .5/8 | 228.159 | .5/8 | 250.15 | .5/8 | 272.141 | .5/8 | 294.132 |
| .3/4 | 206.56 | .3/4 | 228.551 | .3/4 | 250.543 | 3/4 | 272.534 | .3/4 | 294.525 |
| .7/8 | 206.953 | .7/8 | 228.944 | .7/8 | 250.935 | .7/8 | 272.926 | .7/8 | 294.918 |
| 66. | 207.346 | 73 | 229.337 | 80. | 251.328 | 87. | 273.319 | 94. | 291.31 |
| .1/8 | 207.738 | .1/8 | 229.73 | .1/8 | 251.721 | .1/8 | 273.712 | .1/8 | 295.703 |
| .1/4 | 208.131 | .1/4 | 230.122 | 1/4 | 252.113 | 1/4 | 274.105 | .1/4 | 296.096 |
| .3/8 | 208.524 | .3/8 | 230.515 | .3/8 | 252.506 | 3/8 | 374.497 | .3/8 | 296.489 |
| .1/2 | 208.916 | .1/2 | 230.908 | .1/2 | 252.899 | .1/2 | 274.89 | .1/2 | 296.881 |
| .5/8 | 209.309 | .5/8 | 231.3 | .5/8 | 252.292 | .5/8 | 275.283 | .5/8 | 297.274 |
| .3/4 | 209.702 | .3/4 | 231.693 | .3/4 | 253.684 | 3/4 | 275.675 | .3/4 | 297.667 |
| .7/8 | 210.095 | .7/8 | 232.086 | .7/8 | 254.077 | .7/8 | 276.068 | .7/8 | 299.059 |
| 67. | 210.487 | 74. | 232.478 | 81. | 254.47 | 88. | 276.461 | 95. | 299.452 |
| .1/8 | 210.88 | .1/8 | 232.871 | .1/8 | 254.862 | .1/8 | 276.853 | .1/8 | 298.845 |
| .1/4 | 211.273 | .1/4 | 233.264 | .1/4 | 255.255 | .1/4 | 277.246 | .1/4 | 299.237 |
| .3/8 | 211.665 | .3/8 | 233.657 | .3/8 | 255 648 | 3/8 | 277.629 | .3/8 | 299.63 |
| .1/2 | 212.058 | .1/2 | 234.049 | 1/2 | 256.04 | .1/2 | 278.032 | .1/2 | 300.023 |
| .5/8 | 212.451 | .5/8 | 234.443 | .5/8 | 256.433 | .5/8 | 278.424 | .5/8 | 300.416 |
| .1/4 | 212.843 | .3/4 | 234.835 | .3/4 | 256.826 | .3/4 | 278.817 | .3/4 | 300.808 |
| .7/8 | 213.236 | .7/8 | 235.227 | .7/8 | 257.219 | .7/8 | 279.21 | .7/8 | 301.201 |
| 68. | 213.629 | 75. | 235.62 | 82. | 257.611 | 89. | 279.602 | 96. | 301.594 |
| .1/8 | 214.022 | .1/8 | 236.013 | 1/8 | 258.004 | .1/8 | 279.995 | .1/8 | 301.986 |
| .1/4 | 214.414 | .1/4 | 236.405 | .1/4 | 258.397 | .1/4 | 280.388 | .1/4 | 302.379 |
| .3/8 | 214.807 | .3/8 | 236.798 | .3/8 | 258.789 | .3/8 | 280.781 | .3/8 | 302.772 |
| .1/2 | 215.2 | .1/2 | 237.191 | .1/2 | 259.182 | .1/2 | 281.173 | .1/2 | 303.164 |
| .5/8 | 215.592 | .5/8 | 237.584 | .5/8 | 259.575 | .5/8 | 281.566 | .5/8 | 303.557 |
| .3/4 | 215.985 | .3/4 | 237.976 | .3/4 | 259.967 | .3/4 | 281.959 | .3/4 | 303.95 |
| .7/8 | 216.378 | .7/8 | 238.369 | .7/8 | 260.36 | .7/8 | 282.351 | .7/8 | 304.343 |
| 69. | 216.77 | 76. | 238.762 | 83. | 260.753 | 90. | 282.744 | 97. | 304.735 |
| .1/8 | 217.163 | .1/8 | 239.154 | .1/8 | 261.146 | .1/8 | 283.137 | .1/8 | 305.128 |
| .1/4 | 217.556 | .1/4 | 239.547 | .1/4 | 261.538 | .1/4 | 283.529 | .1/4 | 305.521 |
| .3/8 | 217.948 | 3/8 | 239.94 | .3/8 | 261.931 | .3/8 | 283.922 | 3/8 | 305.913 |
| .1/2 | 218.341 | .1/2 | 240.332 | .1/2 | 262.324 | .1/2 | 284.315 | .1/2 | 306.306 |
| .5/8 | 218.734 | .5/8 | 240.725 | .5/8 | 262.716 | .5/8 | 284.708 | .5/8 | 306.699 |
| .3/4 | 219.127 | .3/4 | 241.118 | .3/4 | 263.109 | .3/4 | 285.1 | .3/4 | 307.091 |
| .7/8 | 219.519 | .7/8 | 241.511 | .7/8 | 263.502 | .7/8 | 285.493 | .7/8 | 307.484 |

TABLE.—(*Continued.*)

| Diam. | Circum. | Diam. | Circum. | Diam. | Circum. | Diam. | Circum. | Diam. | Circum. |
|---|---|---|---|---|---|---|---|---|---|
| 98. | 307.877 | 105. | 329.868 | 114. | 358.142 | 123. | 386.417 | 139. | 436.682 |
| .1/8 | 308.27 | .1/4 | 330.653 | .1/4 | 358.928 | .1/4 | 387.202 | .1/2 | 438.253 |
| .1/4 | 308.662 | .1/2 | 331.439 | .1/2 | 359.713 | .1/2 | 387.988 | 140. | 439.824 |
| .3/8 | 309.055 | .3/4 | 332.224 | .3/4 | 360.499 | .3/4 | 399.773 | .1/2 | 441.395 |
| .1/2 | 309.448 | 106. | 333.01 | 115. | 361.284 | 124. | 389.558 | 141. | 442.966 |
| .5/8 | 309.84 | .1/4 | 333.795 | .1/4 | 362.069 | .1/4 | 390.344 | .1/2 | 444.536 |
| .3/4 | 310.233 | .1/2 | 334.58 | .1/2 | 362.855 | .1/2 | 391.129 | 142. | 446.107 |
| .7/8 | 310.626 | .3/4 | 335.366 | .3/4 | 363.64 | .3/4 | 391.915 | .1/2 | 447.678 |
| 99. | 311.018 | 107. | 336.151 | 116. | 364.426 | 125. | 392.7 | 143. | 449.249 |
| .1/8 | 311.411 | .1/4 | 336.937 | .1/4 | 365.211 | .1/2 | 394.271 | .1/2 | 450.82 |
| .1/4 | 311.804 | .1/2 | 337.722 | .1/2 | 365.996 | 126. | 395.842 | 144. | 452.39 |
| .3/8 | 312.196 | .3/4 | 338.507 | .3/4 | 366.782 | .1/2 | 397.412 | .1/2 | 453.961 |
| .1/2 | 312.589 | 108. | 339.293 | 117. | 367.567 | 127. | 398.983 | 145. | 455.532 |
| .5/8 | 312.982 | .1/4 | 340.078 | .1/4 | 368.353 | .1/2 | 400.554 | .1/2 | 457.103 |
| .3/4 | 313.375 | .1/2 | 340.864 | .1/2 | 369.138 | 128. | 402.125 | 146. | 458.674 |
| .7/8 | 313.767 | .3/4 | 341.649 | .3/4 | 369.923 | .1/2 | 403.696 | .1/2 | 460.244 |
| 100. | 314.16 | 109. | 342.434 | 118. | 370.709 | 129. | 405.266 | 147. | 461.815 |
| .1/4 | 314.945 | .1/4 | 343.22 | .1/4 | 371.494 | .1/2 | 406.837 | .1/2 | 463.386 |
| .1/2 | 315.731 | .1/2 | 344.005 | .1/2 | 372.28 | 130. | 408.408 | 148. | 464.957 |
| .3/4 | 316.516 | .3/4 | 344.791 | .3/4 | 373.065 | .1/2 | 409.979 | .1/2 | 466.528 |
| 101. | 317.302 | 110. | 345.576 | 119. | 373.85 | 131. | 411.55 | 149. | 468.098 |
| .1/4 | 318.087 | .1/4 | 346.361 | .1/4 | 374.636 | .1/2 | 413.12 | .1/2 | 469.669 |
| .1/2 | 318.872 | .1/2 | 347.147 | .1/2 | 375.421 | 132. | 414.691 | 150. | 471.24 |
| .3/4 | 319.658 | .3/4 | 347.932 | .3/4 | 376.207 | .1/2 | 416.262 | .1/2 | 472.811 |
| 102. | 320.443 | 111. | 348.718 | 120. | 376.992 | 133 | 417.833 | | |
| .1/4 | 321.229 | .1/4 | 349.503 | .1/4 | 377.777 | .1/2 | 419.404 | | |
| .1/2 | 322.014 | .1/2 | 350.288 | .1/2 | 378.563 | 134. | 420.974 | | |
| .3/4 | 322.799 | .3/4 | 350.074 | .3/4 | 379.348 | .1/2 | 422.545 | | |
| 103. | 323.585 | 112. | 351.859 | 121. | 380.134 | 135. | 424.116 | | |
| .1/4 | 324.37 | .1/4 | 352.645 | .1/4 | 380.919 | .1/2 | 425.687 | | |
| .1/2 | 325.156 | .1/2 | 353.43 | .1/2 | 381.704 | 136. | 427.258 | | |
| .3/4 | 325.941 | .3/4 | 354.215 | .3/4 | 382.49 | .1/2 | 428.828 | | |
| 104. | 326.726 | 113. | 355.001 | 122. | 383.275 | 137. | 430.399 | | |
| .1/4 | 327.512 | .1/4 | 355.786 | .1/4 | 384.061 | .1/2 | 431.97 | | |
| .1/2 | 328.297 | .1/2 | 356.572 | .1/2 | 384.846 | 138 | 433.541 | | |
| .3/4 | 329.083 | .3/4 | 357.357 | .3/4 | 385.631 | .1/2 | 435.112 | | |

## To Compute the Circum. of a Diameter greater than any in the preceding Table.

RULE.—Divide the dimention by two, three, four, etc., if practicable to do so, until it is reduced to a diameter to be found in the table.

Take the tabular circumference for this dimention, multiply by 2, 3, 4, 5, etc, according as it was divided, and the product will give the circumference required.

EXAMPLE.—What is the circumference for a diameter of 1050?

1050÷7=150; tab. circum. 150=471,239, which × 7=3299.073, *circum. required.*

## To Compute the Circumference for an Integer and Fraction not given in the Table.

RULE.—Double, treple, or quadruple the dimention given, until the fraction is increased to a whole number or to one of those in the table, as $\frac{1}{8}$, $\frac{1}{4}$, etc., provided it is practical to do so.

Take the circumferences for this diameter; and if it is double of that for which the circumference is required, take one half of it; if treble, take one third of it; and if quadruple, one fourth of it.

EXAMPLE.—Required the circumference of 2.21875 inches.

2.21875 × 2 =4.4375 =4$\frac{7}{16}$, which × 2 =8$\frac{7}{8}$; tab. circum. =27.8817, which÷4=6.9704 *ins.*

## To Compute the Circum. of a Diameter in Feet and Inches, etc. by the preceding Table.

RULE.—Reduce the dimention to inches or eighths, as the case may be, and take the circumference in that term from the table for that number.

Divide this number by 8 if it is in eighths, and by 12 if in inches, and the quotient will give the area in feet.

EXAMPLE.—Required the circumference of a circle of 1 foot 6$\frac{3}{8}$ inches.

1 foot 6$\frac{3}{8}$ ins.=18$\frac{3}{8}$ ins.=147 eighths. Circum. of 147=461.815, which÷8=57.727 *inches*; and by 12=4.81 *feet.*

| Diam. | Circum. |
|---|---|
| 139. | 436.682 |
| ½ | 438.253 |
| 140. | 439.824 |
| ½ | 441.395 |
| 141. | 442.966 |
| ½ | 444.536 |
| 142. | 446.107 |
| ½ | 447.678 |
| 143. | 449.249 |
| ½ | 450.82 |
| 144. | 452.39 |
| ½ | 453.961 |
| 145. | 455.532 |
| ½ | 457.103 |
| 146. | 458.674 |
| ½ | 460.244 |
| 147. | 461.815 |
| ½ | 463.386 |
| 148. | 464.957 |
| ½ | 466.528 |
| 149. | 468.098 |
| ½ | 469.669 |
| 150. | 471.24 |
| ½ | 472.811 |

prec d ng Table.
so. until it is reduced
5, etc, according as it
quired.
ve in the Table.
ction is increased to a
tical to do so.
hich the circumference
ple, one fourth of it.
÷4=6 9704 *ins.*
the preceding Table.
and take the circumfe-
quot ent will give the
8=57.727 *inches* ; and

# TABLE III.

## AREAS AND CIRCUMFERENCES OF CIRCLES, FROM $\frac{1}{10}$ TO 100.

[*Advancing by Tenths.*]

| Diam. | Area. | Circum. | Diam. | Area. | Circum. | Diam. | Area. | Circum. |
|---|---|---|---|---|---|---|---|---|
| | | | 5. | 19.635 | 15.708 | 10. | 78.54 | 31.416 |
| .1 | .007854 | .31416 | .1 | 20.4282 | 16.0221 | .1 | 80.1186 | 31.7301 |
| .2 | .031416 | .62832 | .2 | 21.2372 | 16.3363 | .2 | 81.713 | 32.0443 |
| .3 | .070686 | .94248 | .3 | 22.0618 | 16.6504 | .3 | 83.323 | 32.358 |
| .4 | .12566 | 1.2566 | .4 | 22.9022 | 16.9646 | .4 | 84.9488 | 32.6726 |
| .5 | .19635 | 1.5708 | .5 | 23.7583 | 17.2788 | .5 | 86.5903 | 32.9868 |
| .6 | .28274 | 1.885 | .6 | 24.6301 | 17.5929 | .6 | 88.2475 | 33.3009 |
| .7 | .38485 | 2.1991 | .7 | 25.5176 | 17.9071 | .7 | 89.9204 | 33.6151 |
| .8 | .50266 | 2.5133 | .8 | 26.4208 | 18.2212 | .8 | 91.609 | 33.9292 |
| .9 | .63617 | 2.8274 | .9 | 27.3397 | 18.5354 | .9 | 93.3133 | 34.2434 |
| 1. | .7854 | 3.1416 | 6. | 28.2744 | 18.8496 | 11. | 95.0334 | 34.5576 |
| .1 | .9503 | 3.4557 | .1 | 29.2247 | 19.1637 | .1 | 96.7691 | 34.8717 |
| .2 | 1.1309 | 3.7699 | .2 | 30.1907 | 19.4779 | .2 | 98.5205 | 35.1859 |
| .3 | 1.3273 | 4.084 | .3 | 31.1725 | 19.792 | .3 | 100.2877 | 35.501 |
| .4 | 1.5393 | 4.3982 | .4 | 32.1699 | 20.1062 | .4 | 102.0705 | 35.8142 |
| .5 | 1.7671 | 4.7124 | .5 | 33.1831 | 20.4204 | .5 | 103.8691 | 36.1284 |
| .6 | 2.0106 | 5.0265 | .6 | 34.212 | 20.7345 | .6 | 105.6834 | 36.4425 |
| .7 | 2.2698 | 5.3407 | .7 | 35.2566 | 21.0487 | .7 | 107.5134 | 36.7567 |
| .8 | 2.5446 | 5.6548 | .8 | 36.3168 | 21.3628 | .8 | 109.359 | 37.0708 |
| .9 | 2.8352 | 5.969 | .9 | 37.3928 | 21.677 | .9 | 111.2204 | 37.384 |
| 2. | 3.1416 | 6.2832 | 7. | 38.4846 | 21.9912 | 12. | 113.0976 | 37.6992 |
| .1 | 3.4636 | 6.5973 | .1 | 39.592 | 22.3053 | .1 | 114.9904 | 38.0133 |
| .2 | 3.8013 | 6.9115 | .2 | 40.7151 | 22.6195 | .2 | 116.8989 | 38.3275 |
| .3 | 4.1547 | 7.2256 | .3 | 41.8539 | 22.9336 | .3 | 118.8231 | 38.6416 |
| .4 | 4.5239 | 7.5398 | .4 | 43.0085 | 23.2478 | .4 | 124.7631 | 38.9558 |
| .5 | 4.9087 | 7.854 | .5 | 44.1787 | 23.562 | .5 | 122.7187 | 39.27 |
| .6 | 5.3093 | 8.1681 | .6 | 45.3647 | 23.8761 | .6 | 124.6901 | 39.5841 |
| .7 | 5.7255 | 8.4823 | .7 | 46.5663 | 24.1903 | .7 | 126.6771 | 39.8983 |
| .8 | 6.1575 | 8.7964 | .8 | 47.7837 | 24.5044 | .8 | 128.6799 | 40.2124 |
| .9 | 6.6052 | 9.1105 | .9 | 49.0168 | 24.8186 | .9 | 130.6984 | 40.5266 |
| 3. | 7.0686 | 9.4248 | 8. | 50.2656 | 25.1328 | 13. | 132.7326 | 40.8408 |
| .1 | 7.5476 | 9.7389 | .1 | 51.53 | 25.4469 | .1 | 134.7824 | 41.1549 |
| .2 | 8.0424 | 10.0531 | .2 | 52.8102 | 25.7611 | .2 | 136.848 | 41.4691 |
| .3 | 8.553 | 10.3672 | .3 | 54.1062 | 26.0752 | .3 | 138.9294 | 41.7832 |
| .4 | 9.0792 | 10.6814 | .4 | 55.4178 | 26.3894 | .4 | 141.0264 | 41.0974 |
| .5 | 9.6211 | 10.9956 | .5 | 56.7451 | 26.7036 | .5 | 143.1391 | 42.4116 |
| .6 | 10.1787 | 11.3097 | .6 | 58.0881 | 27.0177 | .6 | 145.2675 | 42.7257 |
| .7 | 10.7521 | 11.6239 | .7 | 59.4469 | 27.3319 | .7 | 147.4117 | 43.0399 |
| .8 | 11.3411 | 11.938 | .8 | 60.8213 | 27.646 | .8 | 149.5715 | 43.354 |
| .9 | 11.9459 | 12.2522 | .9 | 62.2115 | 27.9602 | .9 | 151.7471 | 43.6682 |
| 4. | 12.5664 | 12.5664 | 9. | 63.6174 | 28.2744 | 14. | 153.9384 | 43.9824 |
| .1 | 13.2025 | 12.8805 | .1 | 65.0389 | 28.5885 | .1 | 156.1453 | 44.2965 |
| .2 | 13.8544 | 13.1947 | .2 | 66.4762 | 28.9027 | .2 | 158.368 | 44.6107 |
| .3 | 14.522 | 13.5088 | .3 | 67.9292 | 29.2168 | .3 | 160.6064 | 44.9248 |
| .4 | 15.2053 | 13.823 | .4 | 69.3979 | 29.531 | .4 | 162.8605 | 45.239 |
| .5 | 15.9043 | 14.1372 | .5 | 70.8823 | 29.8452 | .5 | 165.1303 | 45.5532 |
| .6 | 16.619 | 14.4513 | .6 | 72.3824 | 30.1593 | .6 | 167.4158 | 45.8673 |
| .7 | 17.3494 | 14.7655 | .7 | 73.3982 | 30.4735 | .7 | 169.717 | 46.1815 |
| .8 | 18.0956 | 15.0796 | .8 | 75.4298 | 30.7876 | .8 | 172.034 | 46.4956 |
| .9 | 18.8574 | 15.3938 | .9 | 76.977 | 31.1018 | .9 | 174.3666 | 46.8098 |

TABLE.—(*Continued.*)

| Diam. | Area. | Circum. | Diam. | Area. | Circum. | Diam. | Area. | Circum. |
|---|---|---|---|---|---|---|---|---|
| 15. | 176.715 | 47.124 | .6 | 333.2923 | 64.7161 | .2 | 539.1299 | 82.3099 |
| .1 | 179.079 | 47.4381 | .7 | 336.536 | 65.0311 | .3 | 543.2533 | 82.624 |
| .2 | 181.4588 | 47.7523 | .8 | 339.7954 | 65.3452 | .4 | 547.3923 | 82.9382 |
| .3 | 183.8542 | 48.0664 | .9 | 343.0705 | 65.6594 | .5 | 541.5471 | 83.2524 |
| .4 | 186.2654 | 48.3806 | 21. | 346.3614 | 65.9736 | .6 | 555.7176 | 83.5665 |
| .5 | 188.6923 | 48.6948 | .1 | 349.6679 | 66.2877 | .7 | 559.9038 | 83.8807 |
| .6 | 191.1349 | 49.0089 | 2 | 352.9901 | 66.6019 | .8 | 564.1056 | 84.1948 |
| .7 | 193.5932 | 49.3231 | .3 | 356.3281 | 66.916 | .9 | 568.3232 | 84.509 |
| .8 | 196.0672 | 49.6372 | .4 | 359.6817 | 67.2302 | 27. | 572.5566 | 84.8232 |
| .9 | 198.5569 | 49.9514 | .5 | 363.0511 | 67.5444 | .1 | 576.8056 | 85.1373 |
| 16. | 201.0624 | 50.2656 | .6 | 366.4362 | 67.8585 | .2 | 581.0703 | 85.4515 |
| .1 | 203.5835 | 50.5797 | .7 | 369.837 | 68.1727 | .3 | 585.3403 | 85.7656 |
| .2 | 206.1203 | 50.8939 | .8 | 373.2534 | 68.4868 | .4 | 589.6469 | 86.0798 |
| .3 | 208.6729 | 51.208 | .9 | 376.6856 | 68.801 | .5 | 593.9587 | 86.394 |
| .4 | 211.2411 | 51.5224 | 22. | 380.1336 | 69.1152 | .6 | 598.2863 | 86.7081 |
| .5 | 213.8251 | 51.8364 | .1 | 383.5972 | 69.4293 | .7 | 602.6295 | 87.0223 |
| .6 | 216.4248 | 52.1505 | .2 | 387.0765 | 69.7435 | .8 | 606.9885 | 87.3364 |
| .7 | 219.0402 | 52.4647 | .3 | 390.5751 | 70.0576 | .9 | 611.3632 | 87.6506 |
| .8 | 221.6712 | 52.7788 | .4 | 394.0823 | 70.3718 | 28. | 615.7536 | 87.9648 |
| .9 | 224.318 | 53.093 | .5 | 397.6087 | 70.686 | .1 | 620.1596 | 88.2789 |
| 17. | 226.9806 | 53.4072 | .6 | 401.1509 | 71.0001 | .2 | 624.5814 | 88.5931 |
| .1 | 229.6588 | 53.7213 | .7 | 404.7087 | 71.3143 | .3 | 628.019 | 88.9072 |
| .2 | 232.3527 | 54.0355 | .8 | 408.2823 | 71.6284 | .4 | 633.4722 | 89.2214 |
| .3 | 235.0623 | 54.3496 | .9 | 411.8716 | 71.9426 | .5 | 637.9411 | 89.5356 |
| .4 | 237.7877 | 54.6038 | 23. | 415.4766 | 72.2568 | .6 | 642.4257 | 89.8497 |
| .5 | 240.5287 | 54.978 | .1 | 418.0972 | 72.5709 | .7 | 646.9261 | 90.1639 |
| .6 | 243.2855 | 55.2921 | .2 | 422.7336 | 72.8851 | .8 | 651.4421 | 90.478 |
| .7 | 246.0579 | 55.6063 | .3 | 426.3858 | 73.1992 | .9 | 655.8739 | 90.7922 |
| .8 | 248.8461 | 55.9204 | .4 | 430.0536 | 73.5134 | 29. | 660.5214 | 91.1064 |
| .9 | 251.65 | 56.2346 | .5 | 433.7371 | 73.8276 | .1 | 965.0845 | 91.4205 |
| 18. | 254.4696 | 56.5488 | .6 | 437.4363 | 74.1417 | .2 | 669.6634 | 91.7347 |
| .1 | 257.3048 | 56.8629 | .7 | 441.1511 | 74.4559 | .3 | 674.258 | 92.0488 |
| .2 | 260.1558 | 57.1771 | .8 | 444.8819 | 74.768 | .4 | 678.8683 | 92.363 |
| .3 | 263.0226 | 57.4912 | .9 | 448.6283 | 75.0882 | .5 | 683.4943 | 92.3772 |
| .4 | 265.905 | 57.8054 | 24. | 452.3904 | 75.3984 | .6 | 688.136 | 92.9913 |
| .5 | 268.8031 | 58.1196 | .1 | 456.1681 | 75.7125 | .7 | 692.7934 | 93.3055 |
| .6 | 271.7169 | 58.4337 | .2 | 459.9916 | 76.0267 | .8 | 697.4666 | 93.6196 |
| .7 | 274.6465 | 58.7479 | .3 | 463.7708 | 76.3408 | .9 | 702.1554 | 93.9338 |
| .8 | 277.5917 | 59.062 | .4 | 467.5957 | 76.6523 | 30. | 706.86 | 94.248 |
| .9 | 280.5527 | 59.3762 | .5 | 471.4363 | 76.9692 | .1 | 711.5802 | 94.5621 |
| 19. | 283.5294 | 59.6904 | .6 | 475.2926 | 77.2833 | .2 | 716.3162 | 94.8763 |
| .1 | 286.5217 | 60.0645 | .7 | 479.1646 | 77.5975 | .3 | 721.0678 | 95.1904 |
| .2 | 289.5298 | 60.3187 | .8 | 483.0524 | 77.9116 | .4 | 725.8352 | 95.5046 |
| .3 | 292.5536 | 60.6328 | .9 | 486.9558 | 78.2258 | .5 | 730.6183 | 95.8188 |
| .4 | 295.5931 | 60.947 | 25. | 490.875 | 78.54 | .6 | 735.4171 | 96.1329 |
| .5 | 298.6483 | 61.2612 | .1 | 494.8098 | 78.8541 | .7 | 740.2316 | 96.1471 |
| .6 | 301.7192 | 61.5753 | .2 | 498.7604 | 78.1693 | .8 | 715.0618 | 96.7612 |
| .7 | 301.806 | 61.8895 | .3 | 502.7266 | 79.4824 | .9 | 749.9077 | 97.0754 |
| .8 | 307.9082 | 62.2036 | .4 | 506.7086 | 79.7966 | 31. | 754.7694 | 97.3896 |
| .9 | 311.0252 | 62.5178 | .5 | 510.7063 | 80.1108 | .1 | 759.6467 | 97.7037 |
| 20. | 314.16 | 62.832 | .6 | 514.7196 | 80.4248 | .2 | 764.5397 | 98.0179 |
| .1 | 317.3094 | 63.1461 | .7 | 518.7488 | 80.7391 | .3 | 769.4485 | 98.332 |
| .2 | 320.4746 | 63.4603 | .8 | 522.7936 | 81.0532 | .4 | 774.3729 | 98.6452 |
| .3 | 323.6554 | 63.7744 | .9 | 526.8541 | 81.3674 | .5 | 779.3131 | 98.9604 |
| .4 | 326.852 | 64.0886 | 26. | 530.9304 | 81.6816 | .6 | 781.2689 | 99.2745 |
| .5 | 330.0643 | 64.4028 | .1 | 555.0223 | 81.9976 | .7 | 789.2406 | 99.5887 |

TABLE.—(*Continued.*)

| Diam. | Area. | Circum. | Diam. | Area. | Circum. | Diam. | Area. | Circum. |
|---|---|---|---|---|---|---|---|---|
| .8 | 794.2278 | 99.9028 | .4 | 1098.5862 | 117.4958 | 43. | 1452.2046 | 135.0888 |
| .9 | 799.2308 | 100.217 | .5 | 1104.4687 | 117.81 | .1 | 1458.9668 | 135.4029 |
| 32. | 804.2496 | 100.5312 | .6 | 1110.3671 | 118.1241 | .2 | 1465.7448 | 135.7171 |
| .1 | 809.284 | 100.8453 | .7 | 1116.2811 | 118.4383 | .3 | 1472.5385 | 136.0332 |
| .2 | 814.3341 | 101.1595 | .8 | 1122.2109 | 118.7524 | .4 | 1479.348 | 136.3454 |
| .3 | 819.3999 | 101.4736 | .9 | 1128.1564 | 119.0666 | .5 | 1486.1731 | 136.6596 |
| .4 | 824.4815 | 101.7478 | 38. | 1134.1176 | 119.3808 | .6 | 1493.0139 | 136.9737 |
| .5 | 829.5787 | 102.102 | .1 | 1140.946 | 119.6949 | .7 | 1499.8705 | 137.2879 |
| .6 | 834.6917 | 102.4161 | .2 | 1146.087 | 120.0091 | .8 | 1506.7427 | 137.602 |
| .7 | 839.8203 | 102.7303 | .3 | 1152.0954 | 120.3232 | .9 | 1513.6287 | 137.9162 |
| .8 | 844.9647 | 103.0444 | .4 | 1158.1194 | 120.6374 | 44. | 1520.5344 | 138.2304 |
| .9 | 850.1248 | 103.3586 | .5 | 1164.1591 | 120.9516 | .1 | 1527.4537 | 138.5445 |
| 33. | 855.3006 | 103.6728 | .6 | 1170.2145 | 121.2657 | .2 | 1534.3888 | 138.8587 |
| .1 | 860.492 | 103.9869 | .7 | 1176.2857 | 121.5799 | .3 | 1531.3396 | 139.1728 |
| .2 | 865.6992 | 104.3011 | .8 | 1182.3725 | 121.894 | .4 | 1548.3061 | 139.487 |
| .3 | 870.9222 | 104.6151 | .9 | 1188.4651 | 122.2082 | .5 | 1555.2883 | 139.8012 |
| .4 | 876.1608 | 104.9294 | 39. | 1194.5434 | 122.5224 | .6 | 1562.2862 | 140.1153 |
| .5 | 881.4151 | 105.2436 | .1 | 1200.7273 | 122.8365 | .7 | 1569.2998 | 140.4295 |
| .6 | 886.6851 | 105.5577 | .2 | 1206.877 | 123.1507 | .8 | 1576.3292 | 140.7436 |
| .7 | 891.9709 | 105.8719 | .3 | 1213.0424 | 123.4648 | .9 | 1583.3742 | 141.0578 |
| .8 | 897.2723 | 106.186 | .4 | 1219.2243 | 123.779 | 45. | 1590.435 | 141.372 |
| .9 | 902.5895 | 106.5002 | .5 | 1225.4203 | 124.0932 | .1 | 1597.5114 | 141.6861 |
| 34. | 907.9224 | 106.8144 | .6 | 1231.6328 | 124.4073 | .2 | 1604.6036 | 142.0003 |
| .1 | 913.2709 | 107.1285 | .7 | 1237.861 | 124.7215 | .3 | 1611.7114 | 142.3144 |
| .2 | 918.6352 | 107.4272 | .8 | 1244.121 | 125.0356 | .4 | 1613.835 | 142.6286 |
| .3 | 924.0115 | 107.7568 | .9 | 1250.3646 | 125.3498 | .5 | 1625.9743 | 142.9428 |
| .4 | 929.4109 | 108.071 | 40. | 1256.64 | 125.664 | .6 | 1633.1293 | 143.2569 |
| .5 | 934.8223 | 108.3852 | .1 | 1262.931 | 125.9781 | .7 | 1640.302 | 143.5711 |
| .6 | 940.2494 | 108.6993 | .2 | 1269.2388 | 126.2923 | .8 | 1647.4846 | 143.8852 |
| .7 | 945.6922 | 109.0352 | .3 | 1275.5602 | 126.6064 | .9 | 1654.6885 | 144.1994 |
| .8 | 951.1508 | 109.3076 | .4 | 1281.8984 | 126.9206 | 46. | 1661.9064 | 144.5136 |
| .9 | 956.625 | 109.6418 | .5 | 1288.2523 | 127.2348 | .1 | 1669.1399 | 144.8277 |
| 35. | 962.115 | 109.856 | .6 | 1294.6219 | 127.5489 | .2 | 1676.3891 | 145.1419 |
| .1 | 967.6206 | 110.2701 | .7 | 1301.0071 | 127.8631 | .3 | 1683.6541 | 145.456 |
| .2 | 973.142 | 110.5843 | .8 | 1307.4082 | 128.1772 | .4 | 1690.9347 | 145.7702 |
| .3 | 978.679 | 110.8984 | .9 | 1313.8249 | 128.4914 | .5 | 1698.2311 | 146.0844 |
| .4 | 984.2318 | 111.2126 | 41. | 1320.2574 | 128.8056 | .6 | 1705.5432 | 146.3985 |
| .5 | 989.8003 | 111.5268 | .1 | 1326.7055 | 129.1197 | .7 | 1712.871 | 146.7127 |
| .6 | 995.3845 | 111.8409 | .2 | 1333.1693 | 129.4323 | .8 | 1720.2144 | 147.0268 |
| .7 | 1000.9843 | 112.1551 | .3 | 1339.6489 | 129.748 | .9 | 1727.5736 | 147.341 |
| .8 | 1006.6 | 112.4692 | .4 | 1346.1441 | 130.0622 | 47. | 1734.9486 | 147.6552 |
| .9 | 1012.2313 | 112.7834 | .5 | 1352.6551 | 130.3764 | .1 | 1742.3392 | 147.9693 |
| 36. | 1017.8784 | 113.0976 | .6 | 1359.1818 | 130.6905 | .2 | 1749.7455 | 148.2835 |
| .1 | 1023.5411 | 113.4117 | .7 | 1365.7242 | 131.0047 | .3 | 1757.1675 | 148.5976 |
| .2 | 1029.2195 | 113.7259 | .8 | 1372.2822 | 131.3188 | .4 | 1764.6045 | 148.9118 |
| .3 | 1034.9131 | 114.04 | .9 | 1378.856 | 131.632 | .5 | 1772.0587 | 149.226 |
| .4 | 1040.6235 | 114.3542 | 42. | 1385.4456 | 131.9472 | .6 | 1779.5279 | 149.5361 |
| .5 | 1046.3491 | 114.6684 | .1 | 1392.0508 | 132.2613 | .7 | 1787.0127 | 149.8543 |
| .6 | 1052.0904 | 114.9825 | .2 | 1398.6717 | 132.5755 | .8 | 1794.5133 | 150.1684 |
| .7 | 1057.8474 | 115.2967 | .3 | 1405.3083 | 132.8896 | .9 | 1802.0296 | 150.4826 |
| .8 | 1063.62 | 115.6108 | .4 | 1411.9607 | 133.2039 | 48. | 1809.5616 | 150.7968 |
| .9 | 1069.4084 | 115.925 | .5 | 1418.6287 | 133.518 | .1 | 1817.1092 | 151.1109 |
| 37. | 1075.2126 | 116.2392 | .6 | 1425.3125 | 133.8321 | .2 | 1824.6726 | 151.4251 |
| .1 | 1081.0324 | 116.5533 | .7 | 1432.0119 | 134.1463 | .3 | 1832.2518 | 151.7392 |
| .2 | 1086.8679 | 116.8675 | .8 | 1438.7271 | 134.4604 | .4 | 1839.8466 | 152.0534 |
| .3 | 1092.7191 | 117.1816 | .9 | 1445.458 | 134.7746 | .5 | 1847.4576 | 152.3676 |

TABLE.—(*Continued.*)

| Diam. | Area. | Circum. | Diam. | Area. | Circum. | Diam. | Area. | Circum. |
|---|---|---|---|---|---|---|---|---|
| .6 | 1855.0833 | 152.6817 | .2 | 2307.2224 | 170.2747 | .8 | 2808.6218 | 187.8676 |
| .7 | 1862.7253 | 152.9959 | .3 | 2315.744 | 170.5883 | .9 | 2818.023 | 188.1818 |
| .8 | 1870.3829 | 153.31 | .4 | 2324.2813 | 170.903 | 60. | 2827.44 | 188.496 |
| .9 | 1878.0563 | 153.6242 | .5 | 2332.8343 | 171.2172 | .1 | 2836.8726 | 188.8101 |
| 49. | 1885.7454 | 153.9384 | .6 | 2341.403 | 171.5343 | .2 | 2846.321 | 189.1243 |
| .1 | 1893.4501 | 154.2525 | .7 | 2349.9874 | 171.8455 | .3 | 2855.785 | 189.4384 |
| .2 | 1901.1706 | 154.5667 | .8 | 2358.5876 | 172.1596 | .4 | 2865.2648 | 189.7526 |
| .3 | 1908.9068 | 154.8808 | .9 | 2367.2034 | 172.4738 | .5 | 2874.7603 | 190.0668 |
| .4 | 1916.6587 | 155.195 | 55. | 2375.835 | 172.788 | .6 | 2884.2615 | 190.3809 |
| .5 | 1924.4263 | 155.5092 | .1 | 2384.4822 | 173.1021 | .7 | 2893.7984 | 190.6951 |
| .6 | 1932.2096 | 155.8233 | .2 | 2393.1452 | 173.4163 | .8 | 2903.341 | 191.0092 |
| .7 | 1940.0086 | 156.1375 | .3 | 2401.8238 | 173.7304 | .9 | 2912.8993 | 191.3234 |
| .8 | 1947.8234 | 156.4516 | .4 | 2410.5182 | 174.0446 | 61. | 2922.4734 | 191.6376 |
| .9 | 1955.6538 | 156.7558 | .5 | 2419.2283 | 174.3588 | .1 | 2932.0631 | 191.9517 |
| 50. | 1963.5 | 157.08 | .6 | 2427.9541 | 174.6729 | .2 | 2941.6685 | 192.2659 |
| .1 | 1971.3618 | 157.3941 | .7 | 2436.6956 | 174.9771 | .3 | 2951.2897 | 192.58 |
| .2 | 1979.2304 | 157.7083 | .8 | 2445.4528 | 175.3092 | .4 | 2960.9265 | 193.8942 |
| .3 | 1987.1326 | 158.0224 | .9 | 2454.2257 | 175.6154 | .5 | 2970.5791 | 193.2084 |
| .4 | 1995.0416 | 158.3366 | 56. | 2463.0144 | 175.9296 | .6 | 2980.2474 | 193.5225 |
| .5 | 2902.9663 | 158.6508 | .1 | 2471.8187 | 176.2437 | .7 | 2989.9314 | 193.8367 |
| .6 | 2010.9067 | 158.9649 | .2 | 2480.6387 | 176.5579 | .8 | 2999.63 | 194.1508 |
| .7 | 2018.8628 | 159.2791 | .3 | 2489.4745 | 176.872 | .9 | 3009.3464 | 194.465 |
| .8 | 2026.8346 | 159.5932 | .4 | 2498.3259 | 177.1862 | 62. | 3019.0776 | 194.7792 |
| .9 | 2034.877 | 159.9074 | .5 | 2507.1931 | 177.5004 | .1 | 3028.8244 | 195.0933 |
| 51. | 2042.8254 | 160.2216 | .6 | 2515.076 | 177.8145 | .2 | 3038.5869 | 195.4075 |
| 1 | 2050.8443 | 160.5357 | .7 | 2524.9736 | 178.1287 | .3 | 3048.3651 | 195.7216 |
| .2 | 2058.8784 | 160.8499 | .8 | 2533.8888 | 178.4428 | .4 | 3058.1591 | 196.0358 |
| .3 | 2066.9293 | 161.164 | .9 | 2542.8188 | 178.757 | .5 | 3067.9687 | 196.35 |
| .4 | 2074.9953 | 161.4782 | 57. | 2551.7646 | 179.0712 | .6 | 3077.7941 | 196.6641 |
| .5 | 2083.0771 | 161.7924 | .1 | 2560.726 | 179.3853 | .7 | 3087.6341 | 196.9783 |
| .6 | 2091.1746 | 162.1065 | .2 | 2669.7031 | 179.6995 | .8 | 3097.4949 | 197.2924 |
| .7 | 2099.2878 | 162.4207 | .3 | 2578.6959 | 180.0136 | .9 | 3107.3644 | 197.6066 |
| .8 | 2107.4166 | 162.7348 | .4 | 2587.7045 | 180.3278 | 63. | 3117.2526 | 197.9208 |
| .9 | 2115.5612 | 163.049 | .5 | 2596.7287 | 180.642 | .1 | 3127.1564 | 198.2349 |
| 52. | 2123.7216 | 163.3632 | .6 | 2605.7687 | 180.9561 | .2 | 3137.0758 | 198.5491 |
| .1 | 2131.8976 | 163.6773 | .7 | 2614.8243 | 181.2803 | .3 | 3147.0114 | 198.8632 |
| .2 | 2140.0893 | 163.9935 | .8 | 2623.8957 | 181.5844 | .4 | 3156.9664 | 199.1774 |
| .3 | 2148.2967 | 164.3056 | .9 | 2632.9828 | 181.8986 | .5 | 3166.9291 | 199.4916 |
| .4 | 2156.5199 | 164.6198 | 58. | 2642.0856 | 182.2128 | .6 | 3176.9115 | 199.8057 |
| .5 | 2164.7587 | 164.934 | .1 | 2651.2046 | 182.5269 | .7 | 3186.9097 | 200.1199 |
| .6 | 2173.0133 | 165.2481 | .2 | 2664.3382 | 182.8411 | .8 | 3196.9235 | 200.434 |
| .7 | 2181.2835 | 165.5623 | .3 | 2669.4882 | 183.1552 | .9 | 3206.9531 | 200.7482 |
| .8 | 2189.5695 | 165.8764 | .4 | 2678.6538 | 183.4694 | 64. | 3216.9984 | 201.0624 |
| .9 | 2197.8712 | 166.1906 | .5 | 2687.8351 | 183.7836 | .1 | 3227.0593 | 201.3765 |
| 53. | 2206.1886 | 165.5048 | .6 | 2697.0321 | 184.0977 | .2 | 3237.136 | 201.6907 |
| .1 | 2214.5216 | 166.8189 | .7 | 2706.2449 | 184.4119 | .3 | 3247.2284 | 202.0048 |
| .2 | 2222.8704 | 167.1331 | .8 | 2715.4733 | 184.726 | .4 | 3257.3365 | 202.319 |
| .3 | 2231.235 | 167.4472 | .9 | 2724.7175 | 185.0402 | .5 | 3267.4603 | 262.6332 |
| .4 | 2239.6152 | 167.7614 | 59. | 2733.9774 | 185.3544 | .6 | 3277.5998 | 202.9473 |
| .5 | 2248.0111 | 168.0756 | .1 | 2743.2529 | 185.6685 | .7 | 3287.755 | 203.2615 |
| .6 | 2256.4227 | 168.3897 | .2 | 2752.5442 | 185.9827 | .8 | 3297.926 | 203.5756 |
| .7 | 2264.8701 | 168.7049 | .3 | 2761.8512 | 186.2696 | .9 | 3308.1126 | 203.8898 |
| .8 | 2273.2931 | 169.018 | .4 | 2771.1739 | 186.611 | 65. | 3318.315 | 204.204 |
| .9 | 2281.7519 | 169.3322 | .5 | 2780.5123 | 186.9252 | .1 | 3328.534 | 204.5181 |
| 54. | 2290.2264 | 169.6464 | .6 | 2789.8664 | 187.2393 | .2 | 3339.7668 | 204.8323 |
| .1 | 2298.7165 | 169.9605 | .7 | 2799.2362 | 187.5535 | .3 | 3349.0162 | 205.1464 |

TABLE.—(*Continued.*)

| Diam. | Area. | Circum. | Diam. | Area. | Circum. | Diam. | Area. | Circum. |
|---|---|---|---|---|---|---|---|---|
| .4 | 3359.2814 | 205.4606 | 71. | 3959.2014 | 223.0536 | .6 | 4608.3816 | 240.6465 |
| .5 | 3369.5623 | 205.7748 | .1 | 3970.3619 | 223.3677 | .7 | 4620.4218 | 240.9607 |
| .6 | 3379.8589 | 206.0889 | .2 | 3981.5381 | 223.6819 | .8 | 4632.4776 | 241.2748 |
| .7 | 3390.1712 | 206.4031 | .3 | 3992.7301 | 223.996 | .9 | 4644.5492 | 241.6987 |
| .8 | 3400.4992 | 206.7172 | .4 | 4003.9373 | 224.3102 | 77. | 4656.6366 | 241.9032 |
| .9 | 3410.8429 | 207.0314 | .5 | 4015.1611 | 224.6244 | .1 | 4668.7396 | 242.2173 |
| 66. | 3421.2024 | 207.3456 | .6 | 4026.4002 | 224.9385 | .2 | 4680.8583 | 242.5315 |
| .1 | 3431.5775 | 207.6597 | .7 | 4037.655 | 225.2527 | .3 | 4692.9927 | 242.8456 |
| .2 | 3441.9633 | 207.9739 | .8 | 4048.9254 | 225.5668 | .4 | 4705.1429 | 243.1598 |
| .3 | 3452.3749 | 208.288 | .9 | 4060.2116 | 225.881 | .5 | 4717.3087 | 243.474 |
| .4 | 3462.7971 | 208.6022 | 72. | 4071.5136 | 226.1952 | .6 | 4729.4903 | 243.7881 |
| .5 | 3473.2351 | 208.9164 | .1 | 4082.8332 | 226.5093 | .7 | 4741.6875 | 244.1023 |
| .6 | 3483.6888 | 209.2305 | .2 | 4094.1645 | 226.8235 | .8 | 4753.9605 | 244.4164 |
| .7 | 3494.164 | 209.5446 | .3 | 4105.5125 | 227.1376 | .9 | 4766.1292 | 244.7306 |
| .8 | 3504.6432 | 209.8588 | .4 | 4116.8793 | 227.4518 | 78. | 4778.3736 | 245.0448 |
| .9 | 3515.143 | 210.173 | .5 | 4128.2587 | 227.766 | .1 | 4790.6336 | 245.3589 |
| 67. | 3525.6606 | 210.4872 | .6 | 4139.6524 | 228.0801 | .2 | 4802.9094 | 245.6731 |
| .1 | 3536.1928 | 210.8013 | .7 | 4151.0667 | 228.3943 | .3 | 4815.201 | 245.9872 |
| .2 | 3546.7404 | 211.1155 | .8 | 4162.4943 | 228.7084 | .4 | 4827.5082 | 246.3014 |
| .3 | 3557.3043 | 211.4296 | .9 | 4173.9376 | 229.0226 | .5 | 4839.8311 | 246.6156 |
| .4 | 3567.8837 | 211.7438 | 73. | 4185.3966 | 229.3368 | .6 | 4852.1697 | 246.9297 |
| .5 | 3578.4787 | 212.058 | .1 | 4196.8712 | 229.6509 | .7 | 4864.5241 | 247.2439 |
| .6 | 3589.0895 | 212.3721 | .2 | 4208.3614 | 229.9651 | .8 | 4876.8973 | 247.548 |
| .7 | 3599.7159 | 212.6863 | .3 | 4219.8678 | 230.2792 | .9 | 4889.2799 | 247.8722 |
| .8 | 3610.3581 | 213.0004 | .4 | 4231.3896 | 230.5934 | 79. | 4901.6814 | 248.1864 |
| .9 | 3621.016 | 213.3146 | .5 | 4242.9271 | 230.9076 | .1 | 4914.0985 | 248.5005 |
| 68. | 3631.6896 | 213.6288 | .6 | 4254.4803 | 231.2217 | .2 | 4926.5314 | 248.8147 |
| .1 | 3642.3788 | 213.9429 | .7 | 4266.0493 | 231.5359 | .3 | 4938.982 | 249.1288 |
| .2 | 3653.0838 | 214.2571 | .8 | 4277.6339 | 231.85 | .4 | 4951.4443 | 249.443 |
| .3 | 3663.804 | 214.5712 | .9 | 4280.2343 | 232.1642 | .5 | 4963.9243 | 249.7572 |
| .4 | 3674.541 | 214.8454 | 74. | 4300.8504 | 232.4784 | .6 | 4976.484 | 250.0713 |
| .5 | 3685.2931 | 215.1996 | .1 | 4312.4821 | 232.7925 | .7 | 4988.9314 | 250.3855 |
| .6 | 3696.006 | 215.5137 | .2 | 4324.1296 | 233.1067 | .8 | 5061.4586 | 250.6996 |
| .7 | 3706.8445 | 215.8279 | .3 | 4335.7928 | 233.4208 | .9 | 5014.0014 | 251.0138 |
| .8 | 3717.6437 | 216.142 | .4 | 4347.4717 | 234.735 | 80. | 5026.56 | 251.3280 |
| .9 | 3728.4587 | 216.4562 | .5 | 4359.1663 | 234.0492 | .1 | 5039.1342 | 251.6421 |
| 69. | 3739.2894 | 216.7704 | .6 | 4370.8766 | 234.3633 | .2 | 5051.7242 | 251.9563 |
| .1 | 3750.1357 | 217.0845 | .7 | 4382.6026 | 234.6775 | .3 | 5064.3258 | 252.2704 |
| .2 | 3760.9978 | 217.3987 | .8 | 4394.3448 | 234.9916 | .4 | 5076.9552 | 252.5846 |
| .3 | 3771.8756 | 217.7128 | .9 | 4406.1018 | 235.3058 | .5 | 5089.5883 | 252.8988 |
| .4 | 3782.7691 | 218.027 | 75. | 4417.875 | 235.62 | .6 | 5102.2411 | 53.2129 |
| .5 | 3793.6783 | 218.3412 | .1 | 4429.6638 | 235.9341 | .7 | 5114.9096 | 253.5271 |
| .6 | 3804.6032 | 218.6553 | .2 | 4441.4684 | 236.2483 | .8 | 5127.5938 | 253.8412 |
| .7 | 3815.5438 | 218.9695 | .3 | 4453.2886 | 236.5624 | .9 | 5140.2937 | 254.1554 |
| .8 | 3826.5002 | 219.2836 | .4 | 4465.1246 | 236.8766 | 81. | 5153.0094 | 254.4696 |
| .9 | 3847.4722 | 219.5978 | .5 | 4476.9763 | 237.1908 | .1 | 5165.7407 | 254.7837 |
| 70. | 3848.46 | 219.912 | .6 | 4488.8437 | 237.5049 | .2 | 5178.4877 | 255.0979 |
| .1 | 3859.4952 | 220.2261 | .7 | 4500.7268 | 237.8191 | .3 | 5191.2505 | 255.412 |
| .2 | 3870.4826 | 220.5403 | .8 | 4512.6256 | 238.1332 | .4 | 5204.0285 | 255.7262 |
| .3 | 3881.5174 | 220.8544 | .9 | 4524.5401 | 238.4474 | .5 | 5216.8231 | 256.0404 |
| .4 | 3892.568 | 221.1686 | 76. | 4536.4704 | 238.7616 | .6 | 5229.633 | 256.3545 |
| .5 | 3903.6343 | 221.4828 | .1 | 4548.4163 | 239.0757 | .7 | 5242.4586 | 256.6687 |
| .6 | 3914.7163 | 221.7969 | .2 | 4560.3787 | 239.3899 | .8 | 5255.2998 | 256.9828 |
| .7 | 3925.814 | 222.1111 | .3 | 4572.3553 | 239.704 | .9 | 5268.1568 | 257.297 |
| .8 | 3936.9274 | 222.4252 | .4 | 4584.3583 | 240.0182 | 82. | 5281.0286 | 257.6112 |
| .9 | 3948.0565 | 222.7394 | .5 | 4596.3571 | 240.3324 | .1 | 5293.918 | 257.9253 |

TABLE.—(*Continued.*)

| Diam. | Area. | Circum. | Diam. | Area. | Circum. | Diam. | Area. | Circum. |
|---|---|---|---|---|---|---|---|---|
| .2 | 5306.8221 | 258.2395 | .8 | 6054.5149 | 275.8324 | .4 | 6851.4840 | 293.4254 |
| .3 | 5319.7439 | 258.5536 | .9 | 6068.3224 | 275.1466 | .5 | 6866.1631 | 293.7396 |
| .4 | 5332.6775 | 258.8646 | 88. | 6082.1376 | 276.4608 | .6 | 6880.8579 | 294.0537 |
| .5 | 5345.6287 | 259.182 | .1 | 6095.9684 | 276.7749 | .7 | 6895.5685 | 294.3679 |
| .6 | 5358.5957 | 259.4961 | .2 | 6109.815 | 277.0891 | .8 | 6908.2947 | 294.682 |
| .7 | 5371.5983 | 259.8103 | .3 | 6123.6774 | 277.4032 | .9 | 6925.0367 | 294.9962 |
| .8 | 5384.5762 | 260.1244 | .4 | 6137.5554 | 277.7174 | 94. | 6939.7944 | 295.3104 |
| .9 | 5397.5908 | 260.4386 | .5 | 6151.4491 | 278.0316 | .1 | 6954.5677 | 295.6245 |
| 83. | 5410.6206 | 260.7528 | .6 | 6165.3585 | 278.3457 | .2 | 6969.3568 | 295.9387 |
| .1 | 5423.666 | 261.0669 | .7 | 6179.2837 | 278.6599 | .3 | 6984.1614 | 296.2436 |
| .2 | 5436.7272 | 261.3811 | .8 | 6193.2245 | 278.975 | .4 | 6998.9821 | 296.567 |
| .3 | 5449.8042 | 261.6952 | .9 | 6207.1811 | 279.2882 | .5 | 7013.8183 | 296.8812 |
| .4 | 5462.8968 | 262.0094 | 89. | 6221.1534 | 279.6024 | .6 | 7028.6702 | 297.1953 |
| .5 | 5476.0051 | 262.3236 | .1 | 6235.1413 | 279.9165 | .7 | 7043.5025 | 297.5095 |
| .6 | 5489.1291 | 262.6376 | .2 | 6249.145 | 280.2307 | .8 | 7058.418 | 297.8236 |
| .7 | 5502.2689 | 262.9519 | .3 | 6263.1644 | 280.5448 | .9 | 7073.3202 | 298.1378 |
| .8 | 5515.4243 | 263.264 | .4 | 6277.1995 | 280.859 | 95. | 7088.235 | 298.452 |
| .9 | 5528.5958 | 263.5802 | .5 | 6291.2035 | 281.1732 | .1 | 7103.1654 | 298.7661 |
| 84. | 5541.7824 | 263.8944 | .6 | 6305.3168 | 281.4873 | .2 | 7118.1116 | 299.0723 |
| .1 | 5554.9847 | 264.2085 | .7 | 6319.399 | 281.8825 | .3 | 7133.0734 | 299.3944 |
| .2 | 5568.2032 | 264.5227 | .8 | 6333.497 | 282.1156 | .4 | 7148.051 | 299.7086 |
| .3 | 5581.4372 | 264.8368 | .9 | 6347.6813 | 282.4298 | .5 | 7163.0443 | 300.0228 |
| .4 | 5594.6869 | 265.151 | 90. | 6361.74 | 282.744 | .6 | 7178.0533 | 300.3369 |
| .5 | 5607.9523 | 265.4652 | .1 | 6375.885 | 283.0581 | .7 | 7193.078 | 300.6511 |
| .6 | 5621.2334 | 265.7793 | .2 | 6390.0458 | 283.3723 | .8 | 7208.1184 | 300.9652 |
| .7 | 5634.5682 | 266.0935 | .3 | 6404.2222 | 283.6864 | .9 | 7223.1745 | 301.2794 |
| .8 | 5647.8428 | 266.4076 | .4 | 6418.4144 | 284.0066 | 96. | 7238.2464 | 301.5936 |
| .9 | 5661.171 | 266.7218 | .5 | 6432.6223 | 284.3148 | .1 | 7253.3339 | 301.9077 |
| 85. | 5674.515 | 267.036 | .6 | 6446.8474 | 284.6289 | .2 | 7268.4371 | 302.2219 |
| .1 | 5687.8746 | 267.3501 | .7 | 6461.0852 | 284.9431 | .3 | 7283.5561 | 302.536 |
| .2 | 5701.25 | 267.6643 | .8 | 6475.3402 | 285.2572 | .4 | 7298.6907 | 302.8502 |
| .3 | 5714.641 | 267.9784 | .9 | 6489.6109 | 285.5714 | .5 | 7313.8411 | 303.1644 |
| .4 | 5728.0478 | 268.2926 | 91. | 6503.8674 | 285.8856 | .6 | 7329.0072 | 303.4785 |
| .5 | 5741.4703 | 268.6068 | .1 | 6518.1995 | 286.1997 | .7 | 7344.189 | 303.7927 |
| .6 | 5754.9085 | 268.9209 | .2 | 6532.5173 | 286 5139 | .8 | 7359.3864 | 304.1068 |
| .7 | 5768.3624 | 269.2351 | .3 | 6546.8909 | 286.829 | .9 | 7374.5996 | 304.421 |
| .8 | 5781.832 | 269.5492 | .4 | 6561.2081 | 287.1422 | 97. | 7389.8286 | 304.7352 |
| .9 | 5795.3173 | 269.8634 | .5 | 6575.5651 | 287.4564 | .1 | 7405.0732 | 305.0493 |
| 86. | 5808.8184 | 270.1776 | .6 | 6589.9458 | 287.7705 | .2 | 7420.3335 | 305.3635 |
| .1 | 5822.3351 | 270.4917 | .7 | 6604.3222 | 288.0847 | .3 | 7435.6095 | 305.6776 |
| .2 | 5835.8675 | 270.8059 | .8 | 6618.7542 | 288.3988 | .4 | 7450.9013 | 305.9918 |
| .3 | 5849.4157 | 271.12 | .9 | 6633.182 | 288.713 | .5 | 7466.2087 | 306.306 |
| .4 | 5862.9795 | 271.4342 | 92. | 6647.6356 | 289.0272 | .6 | 7481.5319 | 306.6201 |
| .5 | 5876.5591 | 271.7484 | .1 | 6662.0848 | 289.3413 | .7 | 7496.8707 | 306.9363 |
| .6 | 5890.1541 | 272.0665 | .2 | 6676.5597 | 289.6555 | .8 | 7512.2253 | 307.2484 |
| .7 | 5903.7654 | 272.3767 | .3 | 6691.0161 | 289.9696 | .9 | 7527.5956 | 307.5626 |
| .8 | 5917.392 | 272.6908 | .4 | 6705.5567 | 290.2838 | 98. | 7542.9816 | 307.8768 |
| .9 | 5931.0344 | 273.005 | .5 | 6720.0787 | 290.598 | .1 | 7558.3832 | 308.1909 |
| 87. | 5944.6926 | 273.3192 | .6 | 6734.6165 | 290.9121 | .2 | 7573.8006 | 308.5051 |
| .1 | 5958.3644 | 273.6333 | .7 | 6749.1699 | 291.2263 | .3 | 7589.2338 | 308.8192 |
| .2 | 5972.0559 | 273.9875 | .8 | 6763.7391 | 291.5404 | .4 | 7604.6826 | 309.1334 |
| .3 | 5985.7691 | 274.2616 | .9 | 6778.324 | 291.8546 | .5 | 7620.1471 | 309.4476 |
| .4 | 5999.4821 | 274.5758 | 93. | 6792.9246 | 292.1688 | .6 | 7635.6273 | 309.7617 |
| .5 | 6013.2187 | 274.89 | .1 | 6807.5408 | 292.4829 | .7 | 7651.1933 | 310.0769 |
| .6 | 6026.9711 | 275.2041 | .2 | 6822.173 | 292.7971 | .8 | 7666.9349 | 310.395 |
| .7 | 6040.7391 | 275.5183 | .3 | 6836.8296 | 293.1112 | .9 | 7682.1623 | 310.7072 |

TABLE.—(*Continued.*)

| Diam. | Area. | Circum. | Diam. | Area. | Circum. | Diam. | Area. | Circum. |
|---|---|---|---|---|---|---|---|---|
| 99. | 7697.7054 | 311.0184 | .4 | 7760.0347 | 312.275 | .8 | 7822.6154 | 313.5116 |
| .1 | 7713.2641 | 311.3325 | .5 | 7775.6563 | 312.5892 | .9 | 7838.2998 | 313.8458 |
| .2 | 7728.8336 | 311.6467 | .6 | 7791.2936 | 312.9033 | 100. | 7854. | 314.16 |
| .3 | 7744.4288 | 311.9608 | .7 | 7806.9466 | 313.2175 | | | |

## To Compute the Area or Circumference of a Diameter greater than any in the preceding Table.

See Rules, pages 176 and 181.

Or, *If the Diameter exceeds* 100 *and is less than* 1001.

Remove the decimal point, and take out the area or circumference as for a Whole Number by removing the decimal point, if for the area, two places to the right; and if for the circumference, one place.

ILLUSTRATION.—The area of 96.7 is 7344.189; hence for 967 it is 734418.9; and the circumference of 96.7 is 303.7927, and for 967 it is 3037.927.

## TABLE IIII.

### AREAS AND CIRCUMFERENCES OF CIRCLES

FROM 1 TO 50 FEET.

(*Advancing by an Inch.*)

| Diam. | Area. | Circum. | Diam. | Area. | Circum. | Diam. | Area. | Circum. |
|---|---|---|---|---|---|---|---|---|
| | Feet. | Feet. Ins. | | Feet. | Feet. Ins. | | Feet. | Feet. Ins. |
| 1 *ft.* | .7854 | 3 1⅝ | 3 *ft.* | 7.0686 | 9 5 | 5 *ft.* | 19.635 | 15 8⅛ |
| 1 | .9217 | 3 4⅝ | 1 | 7.4666 | 9 8¼ | 1 | 20.2947 | 15 11⅝ |
| 2 | 1.069 | 3 8 | 2 | 7.8757 | 9 11⅜ | 2 | 20.9656 | 16 2¾ |
| 3 | 1.2271 | 3 11 | 3 | 8.2957 | 10 2½ | 3 | 21.6475 | 16 5¾ |
| 4 | 1.3962 | 4 2⅛ | 4 | 8.7265 | 10 5⅝ | 4 | 22.34 | 16 9 |
| 5 | 1.5761 | 4 5⅜ | 5 | 9.1683 | 10 8¾ | 5 | 23.0437 | 17 ⅛ |
| 6 | 1.7671 | 4 8½ | 6 | 9.6211 | 10 11⅞ | 6 | 23.7583 | 17 3¼ |
| 7 | 1.9689 | 4 11⅝ | 7 | 10.0846 | 11 3 | 7 | 24.4835 | 17 6⅜ |
| 8 | 2.1816 | 5 2¾ | 8 | 10.5591 | 11 6⅛ | 8 | 25.2199 | 17 9⅝ |
| 9 | 2.4052 | 5 5⅞ | 9 | 11.0446 | 11 9⅜ | 9 | 25.9672 | 18 ¾ |
| 10 | 2.6398 | 5 9 | 10 | 11.5409 | 12 ½ | 10 | 26.7251 | 18 3⅞ |
| 11 | 2.8852 | 6 2¼ | 11 | 12.0481 | 12 3⅝ | 11 | 27.4943 | 18 7⅛ |
| 2 *ft.* | 3.1416 | 6 3⅜ | 4 *ft.* | 12.5664 | 12 6¾ | 6 *ft.* | 28.2744 | 18 10⅛ |
| 1 | 3.4087 | 6 6½ | 1 | 13.0952 | 12 9⅞ | 1 | 29.0649 | 19 1¼ |
| 2 | 3.6869 | 6 9⅝ | 2 | 13.6353 | 13 1 | 2 | 29.8668 | 19 4⅜ |
| 3 | 3.976 | 7 ¾ | 3 | 14.1862 | 13 4⅛ | 3 | 30.6796 | 19 7½ |
| 4 | 4.276 | 7 3⅞ | 4 | 14.7479 | 13 7¼ | 4 | 31.5029 | 19 10⅝ |
| 5 | 4.5869 | 7 7 | 5 | 15.3206 | 13 10½ | 5 | 32.3376 | 20 1⅞ |
| 6 | 4.9087 | 7 10¼ | 6 | 15.9043 | 14 1⅝ | 6 | 33.1831 | 20 4⅞ |
| 7 | 5.2413 | 8 1⅜ | 7 | 16.4986 | 14 4⅝ | 7 | 34.0391 | 20 8⅛ |
| 8 | 5.585 | 8 4½ | 8 | 17.1041 | 14 7⅞ | 8 | 34.9065 | 20 11½ |
| 9 | 5.9395 | 8 7⅝ | 9 | 17.7205 | 14 11 | 9 | 35.7847 | 21 2⅜ |
| 10 | 6.3049 | 8 10¾ | 10 | 18.3476 | 15 2⅛ | 10 | 36.6735 | 21 5½ |
| 11 | 6.6813 | 9 1⅞ | 11 | 18.9858 | 15 5¼ | 11 | 37.5736 | 21 8¾ |

TABLE.—(*Continued.*)

| Diam. | Area. | Circum. | Diam. | Area. | Circum. | Diam. | Area. | Circum. |
|---|---|---|---|---|---|---|---|---|
| | Feet. | Feet. Ins. | | Feet. | Feet. Ins. | | Feet. | Feet. Ins. |
| 7 *ft.* | 38.4846 | 21 11⅞ | 7 | 105.3794 | 36 4½ | 2 | 205.2726 | 50 9⅝ |
| 1 | 39.406 | 22 3 | 8 | 106.9013 | 36 7¾ | 3 | 207.3946 | 51 ½ |
| 2 | 40.3388 | 22 6⅛ | 9 | 108.4342 | 36 10⅞ | 4 | 209.5264 | 51 3¾ |
| 3 | 41.2825 | 22 9¼ | 10 | 109.9772 | 37 2¾ | 5 | 211.6703 | 51 6½ |
| 4 | 42.2367 | 23 ⅜ | 11 | 111.5319 | 37 5¼ | 6 | 213.8251 | 51 10 |
| 5 | 43.2022 | 23 2⅛ | 12 *ft.* | 113.0976 | 37 8¾ | 7 | 215.9896 | 52 1⅛ |
| 6 | 44.1787 | 23 6¾ | 1 | 114.6732 | 37 11½ | 8 | 218.1662 | 52 4¼ |
| 7 | 45.1656 | 23 9⅞ | 2 | 116.2607 | 38 2⅝ | 9 | 220.3537 | 52 7⅜ |
| 8 | 46.1638 | 24 1⅛ | 3 | 117.859 | 38 5¾ | 10 | 222.551 | 52 10½ |
| 9 | 47.173 | 24 4⅛ | 4 | 119.4674 | 38 8⅞ | 11 | 224.7603 | 53 1⅝ |
| 10 | 48.1926 | 24 7¼ | 5 | 121.0876 | 39 | 17 *ft* | 226.9806 | 53 4⅞ |
| 11 | 49.2236 | 24 10⅜ | 6 | 122.7187 | 39 3¼ | 1 | 229.2105 | 53 8 |
| 8 *ft.* | 50.2656 | 25 1½ | 7 | 124.3598 | 39 6⅜ | 2 | 231.4525 | 53 11⅛ |
| 1 | 51.3178 | 25 4⅝ | 8 | 126.0127 | 39 9½ | 3 | 233.7055 | 54 2⅛ |
| 2 | 52.3816 | 25 7⅞ | 9 | 127.6765 | 40 ⅝ | 4 | 235.9682 | 54 5⅜ |
| 3 | 53.4562 | 25 11 | 10 | 129.3504 | 40 3¾ | 5 | 238.243 | 54 8½ |
| 4 | 54.5412 | 26 2⅛ | 11 | 131.036 | 40 6⅞ | 6 | 240.5287 | 54 11⅝ |
| 5 | 55.6377 | 26 5¼ | 13 *ft.* | 132.7326 | 40 10 | 7 | 242.8241 | 55 2⅞ |
| 6 | 56.7451 | 26 8⅜ | 1 | 134.4391 | 41 1⅛ | 8 | 245.1316 | 55 6 |
| 7 | 57.8628 | 26 11½ | 2 | 136.1574 | 41 4⅜ | 9 | 247.45 | 55 9⅛ |
| 8 | 58.992 | 27 2¾ | 3 | 137.8867 | 41 7½ | 10 | 249.7781 | 56 ¼ |
| 9 | 60.1321 | 27 5¾ | 4 | 139.626 | 41 10⅝ | 11 | 252.1184 | 56 3½ |
| 10 | 61.2826 | 27 9 | 5 | 141.3771 | 42 1⅝ | 18 *ft.* | 254.4696 | 56 6½ |
| 11 | 62.4445 | 28 ⅛ | 6 | 143.1391 | 42 4⅞ | 1 | 256.8303 | 56 9⅝ |
| 9 *ft.* | 63.6174 | 28 3¼ | 7 | 144.9211 | 42 8 | 2 | 259.2033 | 57 ⅞ |
| 1 | 64.8006 | 28 6⅜ | 8 | 146.6949 | 42 11⅛ | 3 | 261.5872 | 57 4 |
| 2 | 65.9951 | 28 9½ | 9 | 148.4896 | 43 2¼ | 4 | 263.9807 | 57 7⅛ |
| 3 | 67.2007 | 29 ⅝ | 10 | 150.2943 | 43 5½ | 5 | 266.3864 | 57 10¼ |
| 4 | 68.4166 | 29 3¾ | 11 | 152.1109 | 43 8⅝ | 6 | 268.8031 | 58 1⅜ |
| 5 | 69.644 | 29 7 | 14 *ft.* | 153.9384 | 43 11¾ | 7 | 271.2293 | 58 4½ |
| 6 | 70.8823 | 29 10⅛ | 1 | 155.7758 | 44 2⅞ | 8 | 273.6678 | 58 7⅝ |
| 7 | 72.1309 | 30 1¼ | 2 | 157.625 | 44 6 | 9 | 276.1171 | 58 10¾ |
| 8 | 73.391 | 30 4⅜ | 3 | 159.4852 | 44 9⅛ | 10 | 278.5761 | 58 2 |
| 9 | 74.662 | 30 7½ | 4 | 161.3553 | 44 ¼ | 11 | 281.0472 | 59 5⅛ |
| 10 | 75.9433 | 30 11⅝ | 5 | 163.2373 | 45 3½ | 19 *ft.* | 283.5294 | 59 8¼ |
| 11 | 77.2362 | 31 1¾ | 6 | 165.1303 | 45 6⅝ | 1 | 286.021 | 59 11½ |
| 10 *ft.* | 78.54 | 31 5 | 7 | 167.0331 | 45 9¾ | 2 | 288.5249 | 60 2½ |
| 1 | 79.854 | 31 8⅛ | 8 | 168.9479 | 46 ⅞ | 3 | 291.0397 | 60 5⅝ |
| 2 | 81.1795 | 31 11¼ | 9 | 170.8735 | 46 4 | 4 | 293.5641 | 60 8¾ |
| 3 | 82.516 | 32 2⅜ | 10 | 172.8091 | 46 7⅛ | 5 | 296.1107 | 60 11⅞ |
| 4 | 83.8627 | 32 5½ | 11 | 174.7565 | 46 11¼ | 6 | 298.6483 | 60 3⅛ |
| 5 | 85.2211 | 32 8⅝ | 15 *ft.* | 176.715 | 47 1½ | 7 | 301.2054 | 61 6¼ |
| 6 | 86.5903 | 32 11¾ | 1 | 178.6832 | 47 4⅝ | 8 | 303.7747 | 61 6½ |
| 7 | 87.9697 | 33 2⅞ | 2 | 180.6634 | 47 7¾ | 9 | 306.355 | 61 ½ |
| 8 | 89.3608 | 33 6⅛ | 3 | 182.6545 | 47 10⅞ | 10 | 308.9448 | 61 3⅝ |
| 9 | 90.7627 | 33 9¼ | 4 | 184.6555 | 48 2½ | 11 | 311.5469 | 62 6¾ |
| 10 | 92.1749 | 34 ⅜ | 5 | 186.6684 | 48 5⅛ | 20 *ft.* | 314.16 | 62 9⅞ |
| 11 | 93.5986 | 34 3½ | 6 | 188.6923 | 48 8¼ | 1 | 316.7824 | 62 1⅛ |
| 11 *ft.* | 95.0334 | 34 6⅝ | 7 | 190.726 | 48 11⅜ | 2 | 319.4173 | 63 4¼ |
| 1 | 96.4783 | 34 9¾ | 8 | 192.7716 | 49 2⅝ | 3 | 322.063 | 63 7⅜ |
| 2 | 97.9347 | 35 ⅞ | 9 | 194.8282 | 49 5¾ | 4 | 324.7182 | 63 11½ |
| 3 | 99.4021 | 35 4⅛ | 10 | 196.8946 | 49 8⅞ | 5 | 327.3858 | 63 1⅝ |
| 4 | 100.8797 | 35 7¼ | 11 | 198.973 | 50 | 6 | 330.0643 | 64 4¾ |
| 5 | 102.3689 | 35 10⅝ | 16 *ft.* | 201.0624 | 50 3⅛ | 7 | 332.7522 | 64 7⅞ |
| 6 | 103.8691 | 36 1½ | 1 | 203.1615 | 50 6¼ | 8 | 335.4525 | 64 11½ |

TABLE.—(*Continued.*)

| Diam | Area. | Circum. | | Diam. | Area. | Circum | | Diam. | Area. | Circum | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Feet. | Feet. | Ins. | | Feet. | Feet. | Ins. | | Feet. | Feet. | Ins. |
| 9 | 338.1637 | 65 | 2¼ | 4 | 504.051 | 79 | 7⅛ | 11 | 702.9377 | 93 | 11⅞ |
| 10 | 340.8844 | 65 | 5⅜ | 5 | 507.3732 | 77 | 11⅛ | 30 *ft.* | 706.86 | 94 | 2⅞ |
| 11 | 343.6174 | 65 | 8¼ | 6 | 510.7063 | 80 | 1¼ | 1 | 710.7909 | 94 | 6 |
| 21 *ft.* | 346.3614 | 65 | 11⅝ | 7 | 514.0484 | 80 | 4 3/3 | 2 | 714.735 | 94 | 9¼ |
| 1 | 349.1147 | 66 | 2¾ | 8 | 517.4034 | 80 | 7⅝ | 3 | 718.69 | 95 | ⅜ |
| 2 | 351.8804 | 66 | 5⅞ | 9 | 520.7692 | 80 | 10¾ | 4 | 722.6537 | 95 | 3½ |
| 3 | 354.6571 | 66 | 9 | 10 | 524.1441 | 81 | 1⅞ | 5 | 726.6305 | 95 | 6⅝ |
| 4 | 357.4432 | 66 | ⅛ | 11 | 527.5318 | 81 | 5 | 6 | 730.6183 | 95 | 9¾ |
| 5 | 360.2417 | 67 | 3⅜ | 26 *ft.* | 530.9304 | 81 | 8⅛ | 7 | 734.6147 | 96 | ⅞ |
| 6 | 363.0511 | 67 | 6½ | 1 | 534.3379 | 81 | 11¼ | 8 | 738.6242 | 96 | 4 |
| 7 | 365.8698 | 67 | 9⅝ | 2 | 537.7583 | 82 | 2⅜ | 9 | 742.6447 | 96 | 7¼ |
| 8 | 368.7011 | 68 | ¾ | 3 | 541.1896 | 82 | 5¼ | 10 | 746.6738 | 96 | 10⅜ |
| 9 | 371.5432 | 68 | 3⅞ | 4 | 544.6299 | 82 | 8⅝ | 11 | 750.7161 | 97 | 1½ |
| 10 | 374.3947 | 68 | 7 | 5 | 548.093 | 82 | 11⅞ | 31 *ft* | 754.7694 | 97 | 4⅝ |
| 11 | 377.2587 | 68 | 10¼ | 6 | 551.5471 | 83 | 3 | 1 | 758.8311 | 97 | 7¾ |
| 22 *ft.* | 380.1336 | 69 | 1⅜ | 7 | 555.0201 | 83 | 6⅛ | 2 | 762.9062 | 97 | 10⅞ |
| 1 | 383.0177 | 69 | 4½ | 8 | 558.5059 | 83 | 9¼ | 3 | 766.9921 | 98 | 2 |
| 2 | 385.9144 | 69 | 7⅝ | 9 | 562.0027 | 84 | ⅜ | 4 | 771.0866 | 98 | 5⅛ |
| 3 | 388.822 | 69 | 10¾ | 10 | 565.5084 | 84 | 3½ | 5 | 775.1914 | 98 | 8⅜ |
| 4 | 391.7389 | 70 | 1⅞ | 11 | 569.027 | 84 | 6⅝ | 6 | 779.3131 | 98 | 11½ |
| 5 | 394.6683 | 70 | 5 | 27 *ft.* | 572.5566 | 84 | 9⅞ | 7 | 783.4403 | 99 | 2⅝ |
| 6 | 397.6087 | 70 | 8¼ | 1 | 576.0949 | 85 | 1 | 8 | 787.5803 | 99 | 5¾ |
| 7 | 400.5583 | 70 | 11⅛ | 2 | 579.6463 | 85 | 4¼ | 9 | 791.7322 | 99 | 8⅞ |
| 8 | 403.5204 | 71 | 2½ | 3 | 583.2085 | 85 | 8⅛ | 10 | 795.8922 | 100 | |
| 9 | 406.4935 | 71 | 5⅝ | 4 | 586.7796 | 85 | 11⅜ | 11 | 800.0654 | 100 | 3⅛ |
| 10 | 409.4759 | 71 | 8¾ | 5 | 590.3637 | 86 | 1½ | 32 *ft.* | 804.2496 | 100 | 6⅜ |
| 11 | 412.4707 | 71 | 11⅞ | 6 | 593.9587 | 86 | 4⅝ | 1 | 808.4422 | 100 | 9½ |
| 23 *ft.* | 415.4766 | 72 | 3 | 7 | 597.5625 | 86 | 7⅞ | 2 | 812.6481 | 101 | ⅝ |
| 1 | 418.4915 | 72 | 6⅛ | 8 | 601.1793 | 86 | 11 | 3 | 816.865 | 101 | 3¾ |
| 2 | 421.5192 | 72 | 9⅜ | 9 | 604.807 | 87 | 2⅛ | 4 | 821.0904 | 101 | 6⅞ |
| 3 | 424.5577 | 73 | ½ | 10 | 608.4436 | 87 | 5¼ | 5 | 825.3291 | 101 | 10 |
| 4 | 427.6055 | 73 | 3⅝ | 11 | 612.931 | 87 | 8¾ | 6 | 829.5787 | 102 | 1⅛ |
| 5 | 430.6658 | 73 | 6¾ | 28 *ft.* | 615.7536 | 87 | 11½ | 7 | 833.8368 | 102 | 4⅜ |
| 6 | 433.7371 | 73 | 9⅞ | 1 | 619.4228 | 88 | 2⅝ | 8 | 838.1082 | 102 | 7½ |
| 7 | 436.8175 | 74 | 1 | 2 | 623.105 | 88 | 5¾ | 9 | 842.3095 | 102 | 10⅝ |
| 8 | 439.9106 | 74 | 4⅛ | 3 | 626.7982 | 88 | 9 | 10 | 846.6813 | 103 | 1¾ |
| 9 | 443.0146 | 74 | 7¼ | 4 | 630.5002 | 89 | ⅛ | 11 | 850.9855 | 103 | 4⅞ |
| 10 | 446.1278 | 74 | 10⅝ | 5 | 634.2152 | 89 | 3¼ | 33 *ft.* | 855.3006 | 103 | 8 |
| 11 | 449.2536 | 75 | 1⅝ | 6 | 637.9411 | 89 | 6⅜ | 1 | 859.624 | 103 | 11⅛ |
| 24 *ft.* | 452.3904 | 75 | 4¾ | 7 | 641.6758 | 89 | 9½ | 2 | 863.9608 | 104 | 2¼ |
| 1 | 455.5362 | 75 | 7⅞ | 8 | 645.4235 | 90 | ⅝ | 3 | 868.3087 | 104 | 5⅜ |
| 2 | 458.6948 | 75 | 11 | 9 | 649.1821 | 90 | 3¾ | 4 | 872.6649 | 104 | 8⅝ |
| 3 | 461.8642 | 76 | 2⅛ | 10 | 652.9495 | 90 | 6⅞ | 5 | 877.0346 | 104 | 11¾ |
| 4 | 465.0428 | 76 | 5¼ | 11 | 656.73 | 90 | 11⅛ | 6 | 881.4151 | 105 | 2⅞ |
| 5 | 468.2341 | 76 | 8½ | 29 *ft.* | 660.5214 | 91 | 1¼ | 7 | 885.804 | 105 | 6 |
| 6 | 471.4363 | 76 | 11⅝ | 1 | 664.3214 | 91 | 4⅜ | 8 | 890.2064 | 105 | 9⅛ |
| 7 | 474.6476 | 77 | 2¾ | 2 | 668.1346 | 91 | 7½ | 9 | 894.6196 | 106 | ¼ |
| 8 | 477.8716 | 77 | 5⅞ | 3 | 671.9587 | 91 | 10⅝ | 10 | 899.0413 | 106 | 3⅜ |
| 9 | 481.1065 | 77 | 9 | 4 | 675.7915 | 92 | 1¾ | 11 | 903.4763 | 106 | 6⅝ |
| 10 | 484.3506 | 78 | ⅛ | 5 | 679.6375 | 92 | 4⅞ | 34 *ft.* | 907.9224 | 106 | 9¾ |
| 11 | 487.6073 | 78 | 3¼ | 6 | 683.4943 | 92 | 8⅛ | 1 | 912.3767 | 107 | 0⅞ |
| 25 *ft.* | 490.875 | 78 | 6½ | 7 | 687.3598 | 92 | 11⅛ | 2 | 916.8445 | 107 | 4 |
| 1 | 494.1516 | 78 | 9½ | 8 | 691.2385 | 93 | 2⅜ | 3 | 921.3232 | 107 | 7⅛ |
| 2 | 497.4411 | 79 | ¾ | 9 | 695.128 | 93 | 5½ | 4 | 925.8103 | 107 | 10¼ |
| 3 | 500.7415 | 79 | 3⅞ | 10 | 699.0263 | 93 | 8⅝ | 5 | 930.3108 | 108 | 1⅜ |

TABLE.—(*Continued.*)

| Diam. | Area. | Circum. | Diam. | Area. | Circum. | Diam. | Area. | Circum. |
|---|---|---|---|---|---|---|---|---|
| | Feet. | Feet. Ins. | | Feet. | Feet. Ins. | | Feet. | Feet. Ins. |
| 6 | 934.8223 | 108 4 5/8 | 1 | 1199.7195 | 122 9 1/2 | 8 | 1497.5821 | 137 2 1/8 |
| 7 | 939.3421 | 108 7 3/4 | 2 | 1204.8244 | 123 1/2 | 9 | 1503.3046 | 137 5 1/4 |
| 8 | 943.8753 | 108 10 7/8 | 3 | 1209.9577 | 123 3 5/8 | 10 | 1509.0348 | 137 8 3/8 |
| 9 | 948.4195 | 109 2 | 4 | 1215.099 | 123 6 3/4 | 11 | 1514.7791 | 137 11 5/8 |
| 10 | 952.972 | 109 5 1/8 | 5 | 1220.2542 | 123 9 7/8 | 44 *ft.* | 1520.5344 | 138 2 3/4 |
| 11 | 957.538 | 109 8 1/4 | 6 | 1225.4203 | 124 1 1/8 | 1 | 1526.2971 | 138 5 7/8 |
| 35 *ft.* | 962.115 | 109 11 3/8 | 7 | 1230.5943 | 124 4 1/4 | 2 | 1532.0742 | 138 9 |
| 1 | 966.7701 | 110 2 5/8 | 8 | 2235.7822 | 124 7 3/8 | 3 | 1537.8622 | 139 1/8 |
| 2 | 971.2989 | 110 5 3/4 | 9 | 1240.981 | 124 10 1/2 | 4 | 1543.6578 | 139 3 1/4 |
| 3 | 975.9085 | 110 8 7/8 | 10 | 1246.1878 | 125 1 5/8 | 5 | 1549.4776 | 139 6 3/8 |
| 4 | 980.5264 | 111 | 11 | 1251.4084 | 125 4 3/4 | 6 | 1555.2883 | 139 9 5/8 |
| 5 | 985.1579 | 111 3 1/8 | 40 *ft.* | 1256.64 | 125 7 7/8 | 7 | 1561.1165 | 140 3/4 |
| 6 | 989.8003 | 111 6 1/4 | 1 | 1261.8794 | 125 11 | 8 | 1566.9591 | 140 3 7/8 |
| 7 | 994.4509 | 111 9 3/8 | 2 | 1267.1327 | 126 2 1/4 | 9 | 1572.8125 | 140 7 1/2 |
| 8 | 999.1151 | 112 1/2 | 3 | 1272.397 | 126 5 3/8 | 10 | 1578.6735 | 141 10 1/8 |
| 9 | 1003.7902 | 112 3 3/4 | 4 | 1277.6692 | 126 8 1/2 | 11 | 1584.5488 | 141 1 1/4 |
| 10 | 1008.4736 | 112 6 7/8 | 5 | 1282.9553 | 126 11 5/8 | 45 *ft.* | 1590.435 | 141 4 3/8 |
| 11 | 1013.1705 | 112 10 | 6 | 1288.2523 | 127 2 3/4 | 1 | 1596.3286 | 141 7 1/2 |
| 36 *ft.* | 1017.8784 | 113 1 1/8 | 7 | 1293.5572 | 127 5 7/8 | 2 | 1602.2366 | 141 10 3/4 |
| 1 | 1022.5944 | 113 4 1/4 | 8 | 1298.876 | 127 9 | 3 | 1608.1555 | 142 1 7/8 |
| 2 | 1027.324 | 113 7 3/8 | 9 | 1304.2057 | 128 1/4 | 4 | 1614.0819 | 142 5 |
| 3 | 1032.0646 | 113 10 5/8 | 10 | 1305.5433 | 128 3 3/8 | 5 | 1620.0226 | 142 8 1/8 |
| 4 | 1036.8134 | 114 1 3/4 | 11 | 1314.8949 | 128 6 1/2 | 6 | 1625.9743 | 142 11 1/4 |
| 5 | 1041.5758 | 114 4 7/8 | 41 *ft.* | 1320.2574 | 128 9 5/8 | 7 | 1631.9334 | 143 2 3/8 |
| 6 | 1046.3491 | 114 8 | 1 | 1325.6276 | 129 3/4 | 8 | 1637.9068 | 143 5 1/2 |
| 7 | 1051.1306 | 114 11 1/8 | 2 | 1331.0119 | 129 3 7/8 | 9 | 1643.8912 | 143 8 3/4 |
| 8 | 1055.9257 | 115 2 1/4 | 3 | 1336.4071 | 129 7 | 10 | 1649.8831 | 143 11 7/8 |
| 9 | 1060.7317 | 115 5 3/8 | 4 | 1341.8101 | 129 10 1/8 | 11 | 1655.8892 | 144 3 |
| 10 | 1065.5459 | 115 9 1/4 | 5 | 1347.2271 | 130 1 3/8 | 46 *ft.* | 1661.9064 | 144 6 1/8 |
| 11 | 1070.3738 | 115 11 5/8 | 6 | 1352.6551 | 130 4 1/2 | 1 | 1667.9308 | 144 9 1/4 |
| 37 *ft.* | 1075.2126 | 116 2 7/8 | 7 | 1358.0908 | 130 7 5/8 | 2 | 1673.9698 | 145 3/8 |
| 1 | 1080.0594 | 116 6 | 8 | 1363.5406 | 130 10 3/4 | 3 | 1680.0196 | 145 3 1/2 |
| 2 | 1084.9201 | 116 9 1/8 | 9 | 1369.0012 | 131 1 7/8 | 4 | 1686.0769 | 145 6 5/8 |
| 3 | 1089.7915 | 117 1/4 | 10 | 1374.4697 | 131 5 | 5 | 1692.1485 | 145 9 7/8 |
| 4 | 1094.6711 | 117 3 1/2 | 11 | 1379.9521 | 131 8 1/8 | 6 | 1698.2311 | 146 1 1/8 |
| 5 | 1099.5644 | 117 6 1/2 | 42 *ft.* | 1385.4456 | 131 11 3/8 | 7 | 1704.321 | 146 4 1/8 |
| 6 | 1104.4687 | 117 9 5/8 | 1 | 1390.2467 | 132 2 1/2 | 8 | 1710.4254 | 146 7 1/4 |
| 7 | 1109.381 | 118 3/4 | 2 | 1396.4619 | 132 5 5/8 | 9 | 1716.5407 | 146 10 3/8 |
| 8 | 1114.3071 | 118 4 | 3 | 1401.988 | 132 8 3/4 | 10 | 1722.6634 | 147 1 1/2 |
| 9 | 1119.244 | 118 7 1/8 | 4 | 1107.5219 | 132 11 7/8 | 11 | 1728.9005 | 147 4 5/8 |
| 10 | 1124.1891 | 118 10 1/4 | 5 | 1413.6698 | 133 3 | 47 *ft.* | 1734.9486 | 147 7 3/4 |
| 11 | 1129.1478 | 119 1 3/8 | 6 | 1418.6287 | 133 6 1/8 | 1 | 1741.1039 | 147 11 |
| 38 *ft.* | 1134.1176 | 119 4 1/2 | 7 | 1424.1952 | 133 9 1/4 | 2 | 1747.2738 | 148 2 1/8 |
| 1 | 1139.0953 | 119 7 5/8 | 8 | 1429.7759 | 134 1/2 | 3 | 1753.4545 | 148 5 1/4 |
| 2 | 1144.0868 | 119 10 3/4 | 9 | 1435.3675 | 134 3 5/8 | 4 | 1759.6426 | 148 8 3/8 |
| 3 | 1149.0892 | 120 2 | 10 | 1440.9668 | 134 6 3/4 | 5 | 1765.8452 | 148 11 1/2 |
| 4 | 1154.0997 | 120 5 1/8 | 11 | 1446.5802 | 134 9 7/8 | 6 | 1772.0587 | 149 2 5/8 |
| 5 | 1159.1239 | 120 8 3/8 | 43 *ft.* | 1452.2046 | 135 1 | 7 | 1778.2795 | 149 5 7/8 |
| 6 | 1164.1591 | 120 11 3/8 | 1 | 1457.8365 | 135 4 1/8 | 8 | 1784.5148 | 149 8 7/8 |
| 7 | 1169.2023 | 121 2 1/2 | 2 | 1463.4827 | 135 7 1/4 | 9 | 1790.761 | 150 1/8 |
| 8 | 1174.2592 | 121 5 5/8 | 3 | 1469.1397 | 135 10 1/2 | 10 | 1797.0145 | 150 3 1/4 |
| 9 | 1179.3271 | 121 8 3/4 | 4 | 1474.8044 | 136 1 5/8 | 11 | 1803.2826 | 150 6 3/8 |
| 10 | 1184.403 | 121 11 7/8 | 5 | 1480.4833 | 136 4 3/4 | 48 *ft.* | 1809.5616 | 150 9 1/2 |
| 11 | 1189.4927 | 122 3 1/8 | 6 | 1486.1731 | 136 7 7/8 | 1 | 1815.8477 | 151 5/8 |
| 39 *ft.* | 1194.5934 | 122 6 1/4 | 7 | 1491.8705 | 136 11 | 2 | 1822.1485 | 151 3 3/4 |

TABLE.—(*Continued.*)

| Diam. | Area. | Circum. | | Diam. | Area. | Circum. | | Diam. | Area. | Circum. | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Feet. | Feet. | Ins. | | Feet. | Feet. | Ins. | | Feet. | Feet. | Ins. |
| 3 | 1828.4602 | 151 | 6⅞ | 11 | 1879.3355 | 153 | 8⅛ | 7 | 1930.9188 | 155 | 9¼ |
| 4 | 1834.7791 | 151 | 10⅛ | 49 *ft.* | 1885.7454 | 153 | 11¼ | 8 | 1937.3159 | 156 | ⅛ |
| 5 | 1841.1727 | 152 | 1¼ | 1 | 1892.1724 | 154 | 2⅜ | 9 | 1943.914 | 156 | 3½ |
| 6 | 1847.4571 | 152 | 4⅜ | 2 | 1898.5041 | 154 | 5½ | 10 | 1950.4392 | 156 | 6⅝ |
| 7 | 1853.8087 | 152 | 7½ | 3 | 1905.0367 | 154 | 8⅝ | 11 | 1956.9691 | 156 | 9¾ |
| 8 | 1860.175 | 152 | 10⅝ | 4 | 1911.4965 | 154 | 11⅞ | 50 *ft.* | 1963.5 | 157 | ⅞ |
| 9 | 1866.5521 | 153 | 1¾ | 5 | 1917.9609 | 155 | 2⅞ | | | | |
| 10 | 1872.9365 | 153 | 3⅞ | 6 | 1924.4263 | 155 | 6 | | | | |

# TABLE V.

## TABLE OF THE SIDES OF SQUARES-EQUAL IN AREA TO A CIRCLE OF ANY DIAMETER.

FROM 1 TO 100.

| Diam | Side of Sq. | Diam. | Side of Sq. | Diam. | Side of Sq. | Diam. | Side of Sq. | Diam. | Side of Sq. |
|---|---|---|---|---|---|---|---|---|---|
| 1. | .8862 | 8. | 7.0898 | 15. | 13.2934 | 22. | 19.497 | 29. | 25.7006 |
| .¼ | 1.1078 | .¼ | 7.3114 | .¼ | 13.515 | .¼ | 19.7185 | .¼ | 25.9221 |
| .½ | 1.3293 | .½ | 7.5329 | .½ | 13.7365 | .½ | 19.9401 | .½ | 26.1437 |
| .¾ | 1.5509 | .¾ | 7.7545 | .¾ | 13.9581 | .¾ | 20.1617 | .¾ | 26.3653 |
| 2. | 1.7724 | 9. | 7.976 | 16. | 14.1795 | 23. | 20.3832 | 30. | 26.5868 |
| .¼ | 1.994 | .¼ | 8.1976 | .¼ | 14.4012 | .¼ | 20.6048 | .¼ | 26.8084 |
| .½ | 2.2156 | .½ | 8.4192 | .½ | 14.6227 | .½ | 20.8263 | .½ | 27.0299 |
| .¾ | 2.4371 | .¾ | 8.6407 | .¾ | 14.8443 | .¾ | 21.0479 | .¾ | 27.2515 |
| 3. | 2.6587 | 10. | 8.8623 | 17. | 15.0659 | 24. | 21.2694 | 31. | 27.473 |
| .¼ | 2.8802 | .¼ | 9.0838 | .¼ | 15.2874 | .¼ | 21.491 | .¼ | 27.6947 |
| .½ | 3.1018 | .½ | 9.3054 | .½ | 15.509 | .½ | 21.7126 | .½ | 27.9161 |
| .¾ | 3.3233 | .¾ | 9.5269 | .¾ | 15.7305 | .¾ | 21.9341 | .¾ | 28.1377 |
| 4. | 3.5449 | 11. | 9.7485 | 18. | 15.9521 | 25. | 22.1557 | 32. | 28.3593 |
| .¼ | 3.7665 | .¼ | 9.97 | .¼ | 16.1736 | .¼ | 22.3772 | .¼ | 28.5808 |
| .½ | 3.988 | .½ | 10.1916 | .½ | 16.3952 | .½ | 22.5988 | .½ | 28.8024 |
| .¾ | 4.2096 | .¾ | 10.4132 | .¾ | 16.6168 | .¾ | 22.8203 | .¾ | 29.0239 |
| 5. | 4.4311 | 12. | 10.6347 | 19. | 16.8383 | 26. | 23.0419 | 33. | 29.2455 |
| .¼ | 4.6527 | .¼ | 10.8563 | .¼ | 17.0599 | .¼ | 23.2634 | .¼ | 29.467 |
| .½ | 4.8742 | .½ | 11.0778 | .½ | 17.2814 | .½ | 23.485 | .½ | 29.6886 |
| .¾ | 5.0958 | .¾ | 11.2994 | .¾ | 17.503 | .¾ | 23.7066 | .¾ | 29.9102 |
| 6. | 5.3174 | 13. | 11.5209 | 20. | 17.7245 | 27. | 23.9281 | 34. | 30.1317 |
| .¼ | 5.5389 | .¼ | 11.7425 | .¼ | 17.9461 | .¼ | 24.1497 | .¼ | 30.3533 |
| .½ | 5.7605 | .½ | 11.9641 | .½ | 18.1677 | .½ | 24.3712 | .½ | 30.5748 |
| .¾ | 5.982 | .¾ | 12.1856 | .¾ | 18.3892 | .¾ | 24.5928 | .¾ | 30.7964 |
| 7. | 6.2036 | 14. | 12.4072 | 21. | 18.6109 | 28. | 24.8144 | 35. | 31.0179 |
| .¼ | 6.4251 | .¼ | 12.6287 | .¼ | 18.8323 | .¼ | 25.0359 | .¼ | 31.2395 |
| .½ | 6.6467 | .½ | 12.8503 | .½ | 19.0539 | .½ | 25.2575 | .½ | 31.4611 |
| .¾ | 6.8683 | .¾ | 13.0718 | .¾ | 19.2754 | .¾ | 25.459 | .¾ | 31.6826 |

TABLE.—(*Continued.*)

| Diam. | Side of Sq. | Diam. | Side of Sq. | Diam. | Side of Sq. | Diam. | Side of Sq. | Diam. | Side of Sq. |
|---|---|---|---|---|---|---|---|---|---|
| 36. | 31.9042 | 49. | 43.4251 | 62. | 54.9461 | 75. | 66.467 | 88. | 77.988 |
| .¼ | 32.1257 | .¼ | 43.6467 | .¼ | 55.1676 | .¼ | 66.6886 | .¼ | 78.2095 |
| .½ | 32.3473 | .½ | 43.8682 | .½ | 55.3892 | .½ | 66.9104 | .½ | 78.4316 |
| .¾ | 32.5688 | .¾ | 44.0898 | .¾ | 55.6107 | .¾ | 67.1317 | .¾ | 78.6526 |
| 37. | 32.7904 | 50. | 44.3113 | 63. | 55.8323 | 76. | 67.3532 | 89. | 78.8742 |
| .¼ | 33.0112 | .¼ | 44.5329 | .¼ | 56.0538 | .¼ | 67.5748 | .¼ | 79.0957 |
| .½ | 33.2335 | .½ | 44.7545 | .½ | 56.2754 | .½ | 67.7964 | .½ | 79.3173 |
| .¾ | 33.4551 | .¾ | 44.976 | .¾ | 56.497 | .¾ | 68.0179 | .¾ | 79.5389 |
| 38. | 33.6766 | 51. | 45.1976 | 64. | 56.7185 | 77. | 68.2395 | 90. | 79.7604 |
| .¼ | 33.8982 | .¼ | 45.4191 | .¼ | 56.9401 | .¼ | 68.461 | .¼ | 79.982 |
| .½ | 34.1197 | .½ | 45.6407 | .½ | 57.1616 | .½ | 68.6826 | .½ | 80.2035 |
| .¾ | 34.3413 | .¾ | 45.8622 | .¾ | 57.3832 | .¾ | 68.9041 | .¾ | 80.4251 |
| 39. | 34.5628 | 52. | 46.0838 | 65. | 57.6047 | 78 | 69.1257 | 91. | 80.6467 |
| .¼ | 31.7884 | .¼ | 46.3054 | .¼ | 57.8263 | .¼ | 69.3473 | .¼ | 80.8682 |
| .½ | 35.006 | .½ | 46.5269 | .½ | 58.0479 | .½ | 69.5688 | .½ | 81.0898 |
| .¾ | 35.2275 | .¾ | 46.7485 | .¾ | 58.2694 | .¾ | 69.7904 | .¾ | 81.3113 |
| 40. | 35.4491 | 53. | 46.97 | 66. | 58.491 | 79. | 70.0119 | 92. | 81.5329 |
| .¼ | 35.6706 | .¼ | 47.1916 | .¼ | 58.7125 | .¼ | 70.2335 | .¼ | 81.7544 |
| .½ | 25.8922 | .½ | 47 4131 | .½ | 58.9341 | .½ | 70.455 | .½ | 81.976 |
| .¾ | 36.1137 | .¾ | 47.6347 | .¾ | 59.1556 | .¾ | 70.6766 | .¾ | 82.1975 |
| 41. | 36.3353 | 54. | 47.8562 | 67. | 59.3772 | 80. | 70.8981 | 93. | 82.4191 |
| .¼ | 36.5569 | .¼ | 48.0778 | .¼ | 59.5988 | .¼ | 71.1197 | .¼ | 82.6407 |
| .½ | 36.7784 | .½ | 48.2994 | .½ | 59 8203 | .½ | 71.3413 | .½ | 82.8622 |
| .¾ | 37. | .¾ | 48.5209 | .¾ | 60.0419 | .¾ | 71.5628 | .¾ | 83.0838 |
| 42. | 37.2215 | 55. | 48.7425 | 68. | 60.2634 | 81. | 71.7844 | 94. | 83.3053 |
| .¼ | 37.4431 | .¼ | 48.964 | .¼ | 60.485 | .¼ | 72.0059 | .¼ | 83.5269 |
| .½ | 37.6649 | .½ | 49.1856 | .½ | 60.7065 | .½ | 72.2275 | .½ | 83.7484 |
| .¾ | 37.8862 | .¾ | 49.4071 | .¾ | 66.9281 | .¾ | 72.4491 | .¾ | 83.970 |
| 43. | 38.1078 | 56. | 49.6287 | 69. | 61.1497 | 82. | 72.6706 | 95. | 84.1916 |
| .¼ | 38.3293 | .¼ | 49.8503 | .¼ | 61.3712 | .¼ | 72.8921 | .¼ | 84.4131 |
| .½ | 38.5509 | .½ | 50.0718 | .½ | 61.5928 | .½ | 73.1137 | .½ | 84.6347 |
| .¾ | 38.7724 | .¾ | 50.2934 | .¾ | 61.8143 | .¾ | 73.3353 | .¾ | 84.8562 |
| 44. | 38.994 | 57. | 50.5149 | 70. | 62.0359 | 83. | 73.5568 | 96. | 85.0778 |
| .¼ | 39.2155 | .¼ | 50.7365 | .¼ | 62.2574 | .¼ | 73 7784 | .¼ | 85.2993 |
| .½ | 39.4371 | .½ | 50.958 | .½ | 62.479 | .½ | 73.9999 | .½ | 85.5209 |
| .¾ | 39.6587 | .¾ | 51.1796 | .¾ | 62.7006 | .¾ | 74.2215 | .¾ | 85.7425 |
| 45. | 39.8802 | 58. | 51.4012 | 71. | 62.9221 | 84. | 74.4431 | 97. | 85.9616 |
| .¼ | 40.1018 | .¼ | 51.6227 | .¼ | 63.1437 | .¼ | 74.6647 | .¼ | 86.185 |
| .½ | 40.3233 | .½ | 51.8443 | .½ | 63.3652 | .½ | 74.8862 | .½ | 86.4071 |
| .¾ | 40.5449 | .¾ | 52.0658 | .¾ | 63.5868 | .¾ | 75.1077 | .¾ | 86.6289 |
| 46. | 40.7664 | 59. | 52.2874 | 72. | 63.8083 | 85. | 75.3293 | 98. | 86.8502 |
| .¼ | 40.988 | .¼ | 52.5089 | .¼ | 64.0299 | .¼ | 75.5508 | .¼ | 87.0718 |
| .½ | 41.2096 | .½ | 52.7305 | .½ | 64.2514 | .½ | 75.7724 | .½ | 87.2933 |
| .¾ | 41.4311 | .¾ | 52.9521 | .¾ | 64.4730 | .¾ | 75.9934 | .¾ | 87.5449 |
| 47. | 41.9527 | 60. | 53.1736 | 73. | 64.6945 | 86. | 76.2155 | 99. | 87.7364 |
| .¼ | 41.8742 | .¼ | 53.3952 | .¼ | 64.9161 | .¼ | 76.4371 | .¼ | 87.958 |
| .½ | 42.0958 | .½ | 53.6167 | .½ | 65.1377 | .½ | 76.6586 | .½ | 88.1796 |
| .¾ | 42.3173 | .¾ | 53.8383 | .¾ | 65.3592 | .¾ | 76.8802 | .¾ | 88.4011 |
| 48. | 42.5839 | 61. | 54.0598 | 74. | 65.5808 | 87. | 77.1017 | 100. | 88.6227 |
| .¼ | 42 7604 | .¼ | 54.2814 | .¼ | 65.8023 | .¼ | 77.3233 | .¼ | 88.8442 |
| .½ | 42.982 | .½ | 54.503 | .½ | 66.0239 | .½ | 77.5449 | .½ | 89.0658 |
| .¾ | 43.2036 | .¾ | 54.7245 | .¾ | 66.2455 | .¾ | 77.7664 | .¾ | 89.2874 |

# TABLE VI.

## TABLE OF THE LENGTHS OF CIRCLAR ARCS.

*The Diameter of a Circle assumed to be Unity, and divided into 1000 equal Parts.*

| H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. |
|---|---|---|---|---|---|---|---|---|---|
| .1 | 1.02645 | .148 | 1.05743 | .196 | 1.09949 | .244 | 1.15186 | .292 | 1.21381 |
| .101 | 1.02698 | .149 | 1.05819 | .197 | 1.10048 | .245 | 1.15308 | .293 | 1.2152 |
| .102 | 1.02752 | .15 | 1.05896 | .198 | 1.10147 | .246 | 1.15429 | .294 | 1.21658 |
| .103 | 1.02806 | .151 | 1.05973 | .199 | 1.10247 | .247 | 1.15549 | .295 | 1.21794 |
| .104 | 1.0286 | .152 | 1.06051 | .2 | 1.10348 | .248 | 1.1567 | .296 | 1.21926 |
| .105 | 1.02914 | .153 | 1.0613 | .201 | 1.10447 | .249 | 1.15791 | .297 | 1.22061 |
| .106 | 1.0297 | .154 | 1.06209 | .202 | 1.10548 | .25 | 1.15912 | .298 | 1.22203 |
| .107 | 1.03026 | .155 | 1.06288 | .203 | 1.1065 | .251 | 1.16033 | .299 | 1.22347 |
| .108 | 1.03082 | .156 | 1.06368 | .204 | 1.10752 | .252 | 1.16157 | .3 | 1.22495 |
| .109 | 1.03139 | .157 | 1.06449 | .205 | 1.10855 | .253 | 1.16279 | .301 | 1.22635 |
| .11 | 1.03196 | .158 | 1.0653 | .206 | 1.10958 | .254 | 1.16402 | .302 | 1.22776 |
| .111 | 1.03254 | .159 | 1.06611 | .207 | 1.11062 | .255 | 1.16526 | .303 | 1.22918 |
| .112 | 1.03312 | .16 | 1.06693 | .208 | 1.11165 | .256 | 1.16649 | .304 | 1.23061 |
| .113 | 1.03371 | .161 | 1.06775 | .209 | 1.11269 | .257 | 1.16774 | .305 | 1.23205 |
| .114 | 1.0343 | .162 | 1.06858 | .21 | 1.11374 | .258 | 1.16899 | .306 | 1.23349 |
| .115 | 1.0349 | .163 | 1.06941 | .211 | 1.11479 | .259 | 1.17024 | .307 | 1.23494 |
| .116 | 1.03551 | .164 | 1.07025 | .212 | 1.11584 | .26 | 1.1715 | .308 | 1.23636 |
| .117 | 1.03611 | .165 | 1.07109 | .213 | 1.11692 | .261 | 1.17275 | .309 | 1.2378 |
| .118 | 1.03672 | .166 | 1.07194 | .214 | 1.11796 | .262 | 1.17401 | .31 | 1.23921 |
| .119 | 1.03734 | .167 | 1.07279 | .215 | 1.11904 | .263 | 1.17527 | .311 | 1.2407 |
| .12 | 1.03797 | .168 | 1.07365 | .216 | 1.12011 | .264 | 1.17655 | .312 | 1.24216 |
| .121 | 1.0386 | .169 | 1.07451 | .217 | 1.12118 | .265 | 1.17784 | .313 | 1.2436 |
| .122 | 1.03923 | .17 | 1.07537 | .218 | 1.12225 | .266 | 1.17912 | .314 | 1.24506 |
| .123 | 1.03987 | .171 | 1.07624 | .219 | 1.12334 | .267 | 1.1804 | .315 | 1.24654 |
| .124 | 1.04051 | .172 | 1.07711 | .22 | 1.12445 | .268 | 1.18162 | .316 | 1.24801 |
| .125 | 1.04116 | .173 | 1.07799 | .221 | 1.12556 | .269 | 1.18294 | .317 | 1.24946 |
| .126 | 1.04181 | .174 | 1.07888 | .222 | 1.12663 | .27 | 1.18428 | .318 | 1.25095 |
| .127 | 1.04247 | .175 | 1.07977 | .223 | 1.12774 | .271 | 1.18557 | .319 | 1.25243 |
| .128 | 1.04313 | .176 | 1.08066 | .224 | 1.12885 | .272 | 1.18688 | .32 | 1.25391 |
| .129 | 1.0438 | .177 | 1.08156 | .225 | 1.12997 | .273 | 1.18819 | .321 | 1.25539 |
| .13 | 1.04447 | .178 | 1.08246 | .226 | 1.13108 | .274 | 1.18969 | .322 | 1.25686 |
| .131 | 1.04515 | .179 | 1.08337 | .227 | 1.13219 | .275 | 1.19082 | .323 | 1.25836 |
| .132 | 1.04584 | .18 | 1.08428 | .228 | 1.13331 | .276 | 1.19214 | .324 | 1.25987 |
| .133 | 1.04652 | .181 | 1.08519 | .229 | 1.13444 | .277 | 1.19345 | .325 | 1.26137 |
| .134 | 1.04722 | .182 | 1.08611 | .23 | 1.13557 | .278 | 1.19477 | .326 | 1.26286 |
| .135 | 1.04792 | .183 | 1.08704 | .231 | 1.13671 | .279 | 1.1961 | .327 | 1.26437 |
| .136 | 1.04862 | .184 | 1.08797 | .232 | 1.13786 | .28 | 1.19743 | .328 | 1.26588 |
| .137 | 1.04932 | .185 | 1.0889 | .233 | 1.13903 | .281 | 1.19887 | .329 | 1.2674 |
| .138 | 1.05003 | .186 | 1.08984 | .234 | 1.1402 | .282 | 1.20011 | .33 | 1.26892 |
| .139 | 1.05075 | .187 | 1.09079 | .235 | 1.14136 | .283 | 1.20146 | .331 | 1.27044 |
| .14 | 1.05147 | .188 | 1.09174 | .236 | 1.14247 | .284 | 1.20282 | .332 | 1.27196 |
| .141 | 1.0522 | .189 | 1.09269 | .237 | 1.14363 | .285 | 1.20419 | .333 | 1.27349 |
| .142 | 1.05293 | .19 | 1.09365 | .238 | 1.1448 | .286 | 1.20558 | .334 | 1.27502 |
| .143 | 1.05367 | .191 | 1.09461 | .239 | 1.14597 | .287 | 1.20696 | .335 | 1.27656 |
| .144 | 1.05441 | .192 | 1.09557 | .24 | 1.14714 | .288 | 1.20828 | .336 | 1.2781 |
| .145 | 1.05516 | .193 | 1.09654 | .241 | 1.14831 | .289 | 1.20967 | .337 | 1.27964 |
| .146 | 1.05591 | .194 | 1.09752 | .242 | 1.14949 | .29 | 1.21202 | .338 | 1.28118 |
| .147 | 1.05667 | .195 | 1.0985 | .243 | 1.15067 | .291 | 1.21239 | .339 | 1.28273 |

TABLE.—(*Continued.*)

| H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. |
|---|---|---|---|---|---|---|---|---|---|
| .34 | 1.28428 | .373 | 1.3373 | .406 | 1.39372 | .439 | .15327 | .472 | 1.51571 |
| .341 | 1.28583 | .374 | 1.33896 | .407 | 1.39548 | .44 | 1.45512 | .473 | 1.51764 |
| .342 | 1.28739 | .375 | 1.34063 | .408 | 1.39724 | .441 | 1.45697 | .474 | 1.51958 |
| .343 | 1.28895 | .376 | 1.34229 | .409 | 1.399 | .442 | 1.45883 | .475 | 1.52152 |
| .344 | 1.29052 | .377 | 1.34396 | .41 | 1.40077 | .443 | 1.46069 | .476 | 1.52346 |
| .345 | 1.29209 | .378 | 1.34563 | .411 | 1.40254 | .444 | 1.46255 | .477 | 1.52541 |
| .346 | 1.29366 | .379 | 1.34731 | .412 | 1.40432 | .445 | 1.46441 | .478 | 1.52736 |
| .347 | 1.29523 | .38 | 1.34899 | .413 | 1.406 | .446 | 1.46628 | .479 | 1.52931 |
| .348 | 1.29681 | .381 | 1.35068 | .414 | 1.40788 | .447 | 1.46815 | .48 | 1.53126 |
| .349 | 1.29839 | .382 | 1.35237 | .415 | 1.40966 | .448 | 1.47002 | .481 | 1.53322 |
| .35 | 1.29997 | .383 | 1.35406 | .416 | 1.41145 | .449 | 1.47189 | .482 | 1.53518 |
| .351 | 1.30156 | .384 | 1.35575 | .417 | 1.41324 | .45 | 1.47377 | .483 | 1.53714 |
| .352 | 1.30315 | .385 | 1.35744 | .418 | 1.41503 | .451 | 1.47565 | .484 | 1.5391 |
| .353 | 1.30474 | .386 | 1.35914 | .419 | 1.41682 | .452 | 1.47753 | .485 | 1.54106 |
| .354 | 1.30634 | .387 | 1.36084 | .42 | 1.41861 | .453 | 1.47942 | .486 | 1.54302 |
| .355 | 1.30794 | .388 | 1.36254 | .421 | 1.42041 | .454 | 1.48131 | .487 | 1.54499 |
| .356 | 1.30954 | .389 | 1.36425 | .422 | 1.42222 | .455 | 1.4832 | .488 | 1.54696 |
| .357 | 1.31115 | .39 | 1.36596 | .423 | 1.42402 | .456 | 1.48509 | .489 | 1.54893 |
| .358 | 1.31276 | .391 | 1.36767 | .424 | 1.42583 | .457 | 1.48699 | .49 | 1.5509 |
| .359 | 1.31347 | .392 | 1.36939 | .425 | 1.42764 | .458 | 1.48889 | .491 | 1.55288 |
| .36 | 1.31599 | .393 | 1.37111 | .426 | 1.42942 | .459 | 1.49079 | .492 | 1.55486 |
| .361 | 1.31761 | .394 | 1.37283 | .427 | 1.43127 | .46 | 1.49268 | .493 | 1.55685 |
| .362 | 1.31923 | .395 | 1.37455 | .428 | 1.43309 | .461 | 1.4946 | .494 | 1.55854 |
| .363 | 1.32086 | .396 | 1.37628 | .429 | 1.43491 | .462 | 1.49651 | .495 | 1.56083 |
| .364 | 1.32249 | .397 | 1.37801 | .43 | 1.43673 | .463 | 1.49842 | .496 | 1.56282 |
| .365 | 1.32413 | .398 | 1.37974 | .431 | 1.43856 | .464 | 1.50033 | .497 | 1.56481 |
| .366 | 1.32577 | .399 | 1.38148 | .432 | 1.44039 | .465 | 1.50224 | .498 | 1.5668 |
| .367 | 1.32741 | .4 | 1.38322 | .433 | 1.44222 | .466 | 1.50416 | .499 | 1.56879 |
| .368 | 1.32905 | .401 | 1.38496 | .434 | 1.44405 | .467 | 1.50608 | .5 | 1.57079 |
| .369 | 1.33069 | .402 | 1.38671 | .435 | 1.44589 | .468 | 1.508 | | |
| .37 | 1.33234 | .403 | 1.38846 | .436 | 1.44773 | .469 | 1.50992 | | |
| .371 | 1.33399 | .404 | 1.39021 | .437 | 1.44957 | .47 | 1.51185 | | |
| .372 | 1.33564 | .405 | 1.39196 | .438 | 1.45142 | .471 | 1.51378 | | |

## To Ascertain the Length of an Arc of a Circle by the preceding Table.

RULE.—Divide the height by the base, find the quotient in the column of heights, and take the length of that height from the next righthand column Multiply the length thus obtained by the base of the arc, and the product will give the lenth of the arc.

EXAMPLE.—What is the length of an arc of a circle, the base or span of it being 100 feet, and the height 25 feet ?

25÷100=.25; and .25 per table,=1.15912, *the length of the base, which, being multiply by* 100=115.912 *feet.*

NOTE.—When, in the division of a height by the base, the quotient has a remainder after the third place of decimals, and great accuracy is required

Take the length for the first three figures, subtract it from the next following length; multiply the remainder by the said fraction al remainder, add the product to the first length, and the sum will be the length for the whole quotient

EXAMPLE.—What is the length of an arc of a circle, the base of which in 35 feet, and the height or versed sine 8 feet?

8÷35=.2285714; the tabular length for .228=1.13331, and for .229=1.13444, the difference between which ls .00113. Then .5714×.00113= .000645682.

Hence .228=1.13331.

and .0005714= .000645682

1.133955682, the sum by which the base of the arc is to be multiplied; and 1.133955682×35=39.68845 *feet.*

# TABLE VII.

## TABLE OF THE LENGTHS OF SEMI-ELLIPTIC ARCS.

*The Transverse Diameter of an Ellipse assumed to be Unity, and divided into 1000 equal Parts.*

| H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. |
|---|---|---|---|---|---|---|---|---|---|
| .1 | 1.04162 | .148 | 1.09119 | .196 | 1.14531 | .244 | 1.2038 | .292 | 1.26601 |
| .101 | 1.04262 | .149 | 1.09228 | .197 | 1.14646 | .245 | 1.20506 | .293 | 1.26734 |
| .102 | 1.04362 | .15 | 1.0933 | .198 | 1.14762 | .246 | 1.20632 | .294 | 1.26867 |
| .103 | 1.04462 | .151 | 1.09448 | .199 | 1.14888 | .247 | 1.20758 | .295 | 1.27 |
| .104 | 1.04562 | .152 | 1.09558 | 2 | 1.15014 | .248 | 1.20884 | .296 | 1.27133 |
| .105 | 1.04662 | .153 | 1.09669 | .201 | 1.15131 | .249 | 1.2101 | .297 | 1.27267 |
| .106 | 1.04762 | .154 | 1.0978 | .202 | 1.15248 | .25 | 1.21136 | .298 | 1.27401 |
| .107 | 1.04862 | .155 | 1.09891 | .203 | 1.15366 | .251 | 1.21263 | .299 | 1.27535 |
| .108 | 1.04962 | .156 | 1.10002 | .204 | 1.15484 | .252 | 1.2139 | .3 | 1.27669 |
| .109 | 1.05063 | .157 | 1.10113 | .205 | 1.15602 | .253 | 1.21517 | .301 | 1.27803 |
| .11 | 1.05164 | .158 | 1.10224 | .206 | 1.1572 | .254 | 1.21644 | .302 | 1.27937 |
| .111 | 1.05265 | .159 | 1.10335 | .207 | 1.15838 | .255 | 1.21772 | .303 | 1.28071 |
| .112 | 1.05366 | .16 | 1.10447 | .208 | 1.15957 | .256 | 1.219 | .304 | 1.28205 |
| .113 | 1.05467 | .161 | 1.1056 | .209 | 1.16076 | .257 | 1.22028 | .305 | 1.28339 |
| .114 | 1.05568 | .162 | 1.10672 | .21 | 1.16196 | .258 | 1.22156 | .306 | 1.28474 |
| .115 | 1.05669 | .163 | 1.10784 | .211 | 1.16316 | .259 | 1.22284 | .307 | 1.28609 |
| .116 | 1.0577 | .164 | 1.10896 | .212 | 1.16436 | .26 | 1.22412 | .308 | 1.28744 |
| .117 | 1.05872 | .165 | 1.11008 | .213 | 1.16557 | .261 | 1.22541 | .309 | 1.28879 |
| .118 | 1.05974 | .166 | 1.1112 | .214 | 1.16678 | .262 | 1.2267 | .31 | 1.29014 |
| .119 | 1.06076 | .167 | 1.11232 | .215 | 1.16799 | .263 | 1.22799 | .311 | 1.29149 |
| .12 | 1.06178 | .168 | 1.11344 | .216 | 1.1692 | .264 | 1.22928 | .312 | 1.29285 |
| .121 | 1.0628 | .169 | 1.11456 | .217 | 1.17041 | .265 | 1.23057 | .313 | 1.29421 |
| .122 | 1.06382 | .17 | 1.11569 | .218 | 1.17163 | .266 | 1.23186 | .314 | 1.29557 |
| .123 | 1.06484 | .171 | 1.11682 | .219 | 1.17285 | .267 | 1.23315 | .315 | 1.29603 |
| .124 | 1.06586 | .172 | 1.11795 | .22 | 1.17407 | .268 | 1.23445 | .316 | 1.29829 |
| .125 | 1.06689 | .173 | 1.11908 | .221 | 1.17529 | .269 | 1.23575 | .317 | 1.29965 |
| .126 | 1.06792 | .174 | 1.12021 | .222 | 1.17651 | .27 | 1.23705 | .318 | 1.30102 |
| .127 | 1.06895 | .175 | 1.12134 | .223 | 1.17774 | .271 | 1.23835 | .319 | 1.30239 |
| .128 | 1.06998 | .176 | 1.12247 | .224 | 1.17897 | .272 | 1.23966 | .32 | 1.30376 |
| .129 | 1.07001 | .177 | 1.1236 | .225 | 1.1802 | .273 | 1.24097 | .321 | 1.30513 |
| .13 | 1.07204 | .178 | 1.12473 | .226 | 1.18143 | .274 | 1.24228 | .322 | 1.3065 |
| .131 | 1.07308 | .179 | 1.12586 | .227 | 1.18266 | .275 | 1.24359 | .323 | 1.30787 |
| .132 | 1.07412 | .18 | 1.12699 | .228 | 1.1839 | .276 | 1.2448 | .324 | 1.30924 |
| .133 | 1.07516 | .181 | 1.12813 | .229 | 1.18514 | .277 | 1.24612 | .325 | 1.31061 |
| .134 | 1.07221 | .182 | 1.12927 | .23 | 1.18638 | .278 | 1.24744 | .326 | 1.31198 |
| .135 | 1.07726 | .183 | 1.13041 | .231 | 1.18762 | .279 | 1.24876 | .327 | 1.31335 |
| .136 | 1.07831 | .184 | 1.13155 | .232 | 1.18886 | .28 | 1.2501 | .328 | 1.31472 |
| .137 | 1.07937 | .185 | 1.13269 | .233 | 1.1901 | .281 | 1.25142 | .329 | 1.3161 |
| .138 | 1.08043 | .186 | 1.13383 | .234 | 1.19134 | .282 | 1.25274 | .33 | 1.31748 |
| .139 | 1.08149 | .187 | 1.13497 | .235 | 1.19258 | .283 | 1.25406 | .331 | 1.31886 |
| .14 | 1.08255 | .188 | 1.13611 | .236 | 1.19382 | .284 | 1.25538 | .332 | 1.32024 |
| .141 | 1.08362 | .189 | 1.13726 | .237 | 1.19506 | .285 | 1.2567 | .333 | 1.32162 |
| .142 | 1.08469 | .19 | 1.13841 | .238 | 1.1963 | .286 | 1.25803 | .334 | 1.323 |
| .143 | 1.08576 | .191 | 1.13956 | .239 | 1.19755 | .287 | 1.25936 | .335 | 1.32438 |
| .144 | 1.08684 | .192 | 1.14071 | .24 | 1.1988 | .288 | 1.26069 | .336 | 1.32576 |
| .145 | 1.08792 | .193 | 1.14186 | .241 | 1.20005 | .289 | 1.26202 | .337 | 1.32715 |
| .146 | 1.08901 | .194 | 1.14301 | .242 | 1.2013 | .29 | 1.26335 | .338 | 1.32854 |
| .147 | 1.0901 | .195 | 1.14416 | .243 | 1.20255 | .291 | 1.26468 | .339 | 1.32993 |

## TABLE.—(*Continued.*)

| H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. |
|---|---|---|---|---|---|---|---|---|---|
| .34 | 1.33132 | .396 | 1.41211 | .452 | 1.49618 | .508 | 1.58319 | .564 | 1.67087 |
| .341 | 1.33272 | .397 | 1.41357 | .453 | 1.49771 | .509 | 1.58474 | .565 | 1.67245 |
| .342 | 1.33412 | .398 | 1.41504 | 454 | 1.49924 | .51 | 1.58629 | .566 | 1.67403 |
| .343 | 1.33552 | .399 | 1.41651 | .455 | 1.50077 | .511 | 1.58784 | .567 | 1.67561 |
| .344 | 1.33692 | .4 | 1.41798 | .456 | 1.5023 | .512 | 1.5894 | .568 | 1.67719 |
| .345 | 1.33833 | .401 | 1.41945 | .457 | 1.50383 | .513 | 1.59096 | .569 | 1.67877 |
| .346 | 1.33974 | .402 | 1.42092 | .458 | 1.50536 | .514 | 1.59252 | .57 | 1.68036 |
| .347 | 1.34115 | .403 | 1.42239 | .459 | 1.50689 | .515 | 1.59408 | .571 | 1.68195 |
| .348 | 1.34256 | .404 | 1.42386 | .46 | 1.50842 | .516 | 1.59564 | .572 | 1.68354 |
| .349 | 1.34397 | .405 | 1.42533 | .461 | 1.50996 | .517 | 1.5972 | .573 | 1.68513 |
| .3 | 1.34539 | .406 | 1.42681 | .462 | 1.5115 | .518 | 1.59876 | .574 | 1.68672 |
| .351 | 1.34681 | .407 | 1.42829 | 463 | 1.51304 | .519 | 1.60032 | .575 | 1.68831 |
| .352 | 1.34823 | .408 | 1.42977 | .464 | 1.51458 | .52 | 1.60188 | .576 | 1.6899 |
| .353 | 1.34965 | .409 | 1.43125 | 465 | 1.51612 | .521 | 1.60344 | .577 | 1.69149 |
| .354 | 1.35108 | .41 | 1.43273 | .466 | 1.51766 | .522 | 1.605 | .578 | 1.69308 |
| .355 | 1.35251 | .411 | 1.42421 | .467 | 1.5192 | .523 | 1.60656 | .579 | 1.69467 |
| .356 | 1.35394 | .412 | 1.42569 | .468 | 1.52074 | .524 | 1.60812 | .58 | 1.69626 |
| .357 | 1.35537 | .413 | 1.43718 | 469 | 1.52229 | .525 | 1.60968 | .581 | 1.69785 |
| .358 | 1.3568 | .414 | 1.43867 | .47 | 1.52384 | .526 | 1.61124 | .582 | 1.69945 |
| .359 | 1.35823 | .415 | 1.44016 | .471 | 1.52539 | .527 | 1.6128 | .583 | 1.70105 |
| .36 | 1.35967 | .416 | 1.44165 | .472 | 1.52691 | .528 | 1.61436 | .584 | 1.70264 |
| .361 | 1.36111 | .417 | 1.44314 | .473 | 1.52849 | .529 | 1.61592 | .585 | 1.70424 |
| .362 | 1.36255 | .418 | 1.44463 | .474 | 1.53004 | .53 | 1.61748 | .586 | 1.70584 |
| .363 | 1.36399 | .419 | 1.44613 | .475 | 1.53159 | .531 | 1.61904 | .587 | 1.70745 |
| .364 | 1.36543 | .42 | 1.44763 | .476 | 1.53314 | .532 | 1.6206 | .588 | 1.70905 |
| .365 | 1.36688 | .421 | 1.44913 | .477 | 1.53469 | .533 | 1.62216 | .589 | 1.71065 |
| .366 | 1.36833 | .422 | 1.45064 | .478 | 1.53625 | .534 | 1.62372 | .59 | 1.71225 |
| .367 | 1.36978 | .423 | 1.45214 | .479 | 1.53781 | .535 | 1.62528 | .591 | 1.71286 |
| .368 | 1.37123 | .424 | 1.45364 | .48 | 1.53937 | .536 | 1.62684 | .592 | 1.71546 |
| .369 | 1.37268 | .425 | 1.45515 | .481 | 1.54093 | .537 | 1.6284 | .593 | 1.71707 |
| .37 | 1.37414 | .426 | 1.45665 | .482 | 1.54249 | .538 | 1.62996 | .594 | 1.71868 |
| .371 | 1.37662 | .427 | 1.45815 | .483 | 1.54405 | .539 | 1.63152 | .595 | 1.72029 |
| .372 | 1.37708 | .428 | 1.45966 | .484 | 1.54561 | .54 | 1.63309 | .596 | 1.7219 |
| .373 | 1.37854 | .429 | 1.46167 | .485 | 1.54718 | .541 | 1.63465 | .597 | 1.7235 |
| .374 | 1.38 | .43 | 1.46268 | .486 | 1.54875 | .542 | 1.63623 | .598 | 1.72511 |
| .375 | 1.38146 | .431 | 1.46419 | .487 | 1.55032 | .543 | 1.6378 | .599 | 1.72672 |
| .376 | 1.38292 | .432 | 1.4657 | .488 | 1.55189 | .544 | 1.63937 | .6 | 1.72833 |
| .377 | 1.38439 | .433 | 1.46721 | .489 | 1.55346 | .545 | 1.64094 | .601 | 1.72994 |
| .378 | 1.38585 | .434 | 1.46872 | .49 | 1.55503 | .546 | 1.64251 | .602 | 1.73155 |
| .379 | 1.38732 | .435 | 1.47023 | .491 | 1.5566 | .547 | 1.64408 | .603 | 1.73316 |
| .38 | 1.38879 | .436 | 1.47174 | .492 | 1.55817 | .548 | 1.64565 | .604 | 1.73477 |
| .381 | 1.39024 | .437 | 1.47326 | .493 | 1.55974 | .549 | 1.64722 | .605 | 1.73638 |
| .382 | 1.39169 | .438 | 1.47478 | .494 | 1.56131 | .5 | 1.64879 | .606 | 1.73799 |
| .383 | 1.39314 | .439 | 1.4763 | .495 | 1.56289 | .551 | 1.65036 | .607 | 1.7396 |
| .384 | 1.39459 | .44 | 1.47782 | .496 | 1.56447 | .552 | 1.65193 | .608 | 1.74121 |
| .385 | 1.39605 | .441 | 1.47934 | 497 | 1.56605 | .553 | 1.6535 | .609 | 1.74283 |
| .386 | 1.29751 | .442 | 1.48086 | .498 | 1.56763 | .554 | 1.65507 | 6 | 1.74444 |
| .387 | 1.39897 | .443 | 1.48238 | .499 | 1.56921 | .555 | 1.65665 | .611 | 1.74605 |
| .388 | 1.40043 | .444 | 1.48391 | .5 | 1.57089 | .556 | 1.65823 | .612 | 1.74767 |
| .389 | 1.40189 | .445 | 1.48544 | .501 | 1.57234 | .557 | 1.65981 | .613 | 1.74929 |
| .39 | 1.40335 | .446 | 1.48697 | .502 | 1.57389 | .558 | 1.66139 | .614 | 1.75091 |
| .391 | 1.40481 | .447 | 1.4885 | .503 | 1.57544 | .559 | 1.66297 | .615 | 1.75252 |
| .392 | 1.40627 | .448 | 1.49003 | .504 | 1.57699 | .56 | 1.66455 | .616 | 1.75414 |
| .393 | 1.40773 | .449 | 1.49154 | .505 | 1.57854 | .561 | 1.66613 | .617 | 1.75576 |
| .394 | 1.40919 | .45 | 1.49311 | .506 | 1.58009 | .562 | 1.66771 | .618 | 1.75738 |
| .395 | 1.41065 | .451 | 1.49465 | .507 | 1.58164 | .563 | 1.66929 | .619 | 1.759 |

TABLE.—(*Continued.*)

| H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. |
|---|---|---|---|---|---|---|---|---|---|
| .62 | 1.76062 | .676 | 1.85215 | .732 | 1.94552 | .788 | 2.04117 | .844 | 2.13976 |
| .621 | 1.76224 | .677 | 1.85379 | .733 | 1.94721 | .789 | 2.0429 | .845 | 2.14155 |
| .622 | 1.76386 | .678 | 1.85544 | .734 | 1.9489 | .79 | 2.04462 | .846 | 2.14334 |
| .623 | 1.76548 | .679 | 1.85709 | .735 | 1.95059 | .791 | 2.04635 | .847 | 2.14513 |
| .624 | 1.7671 | .68 | 1.85874 | .736 | 1.95228 | .792 | 2.04809 | .848 | 2.14692 |
| .625 | 1.76872 | .681 | 1.86039 | .737 | 1.95397 | .793 | 2.04983 | .849 | 2.14871 |
| .626 | 1.77034 | .682 | 1.86205 | .738 | 1.95566 | .794 | 2.05157 | .85 | 2.1505 |
| .627 | 1.77197 | .683 | 1.8637 | .739 | 1.95735 | .795 | 2.05331 | .851 | 2.15229 |
| .628 | 1.77359 | .684 | 1.86535 | .74 | 1.95994 | .796 | 2.05505 | .852 | 2.15409 |
| .629 | 1.77521 | .685 | 1.867 | .741 | 1.96074 | .797 | 2.05679 | .853 | 2.15589 |
| .63 | 1.77684 | .686 | 1.86866 | .742 | 1.96244 | .798 | 2.05853 | .854 | 2.1577 |
| .631 | 1.77847 | .687 | 1.87031 | .743 | 1.96414 | .799 | 2.06027 | .855 | 2.1595 |
| .632 | 1.78009 | .688 | 1.87196 | .744 | 1.96583 | .8 | 2.06202 | .856 | 2.1613 |
| .633 | 1.78172 | .689 | 1.87362 | .745 | 1.96753 | .801 | 2.06377 | .857 | 2.16309 |
| .634 | 1.78335 | .69 | 1.87527 | .746 | 1.96923 | .802 | 2.06552 | .858 | 2.16489 |
| .635 | 1.78498 | .691 | 1.87693 | .747 | 1.97093 | .803 | 2.06727 | .859 | 2.16668 |
| .636 | 1.7866 | .692 | 1.87859 | .748 | 1.97262 | .804 | 2.06901 | .86 | 2.16848 |
| .637 | 1.78823 | .693 | 1.88024 | .749 | 1.97432 | .805 | 2.07076 | .861 | 2.17028 |
| .638 | 1.78986 | .694 | 1.8819 | .75 | 1.97602 | .806 | 2.07251 | .862 | 2.17209 |
| .639 | 1.79149 | .695 | 1.88356 | .751 | 1.97772 | .807 | 2.07427 | .863 | 2.17389 |
| .64 | 1.79312 | .696 | 1.88522 | .752 | 1.97943 | .808 | 2.07602 | .864 | 2.1757 |
| .641 | 1.79475 | .697 | 1.88688 | .753 | 1.98113 | .809 | 2.07777 | .865 | 2.17751 |
| .642 | 1.79638 | .698 | 1.88854 | .754 | 1.98283 | .81 | 2.07953 | .866 | 2.17932 |
| .643 | 1.79801 | .699 | 1.8902 | .755 | 1.98453 | .811 | 2.08128 | .867 | 2.18113 |
| .644 | 1.79964 | .7 | 1.89186 | .756 | 1.98623 | .812 | 2.08304 | .868 | 2.18294 |
| .645 | 1.80127 | .701 | 1.89352 | .757 | 1.98794 | .813 | 2.0848 | .869 | 2.18475 |
| .646 | 1 8029 | .702 | 1.89519 | .758 | 1.98964 | .814 | 2.08656 | .87 | 2.18656 |
| .647 | 1.80454 | .703 | 1.89685 | .759 | 1.99134 | .815 | 2.08832 | .871 | 2.18837 |
| .648 | 1.80617 | .704 | 1.89851 | .76 | 1.99305 | .816 | 2 09008 | .872 | 2.19018 |
| .649 | 1.8078 | .705 | 1.90017 | .761 | 1.99476 | .817 | 2.09198 | .873 | 2.192 |
| .65 | 1.80943 | .706 | 1.90184 | .762 | 1.99647 | .818 | 2.0936 | .874 | 2.19382 |
| .651 | 1.81107 | .707 | 1.9035 | .763 | 1.99818 | .819 | 2.09536 | .875 | 2.19564 |
| .652 | 1.81271 | .708 | 1.90517 | .764 | 1.99989 | .82 | 2.09712 | .876 | 2.19746 |
| .653 | 1.81435 | .709 | 1.90684 | .765 | 2.0016 | .821 | 2.09888 | .877 | 2.19928 |
| .654 | 1.81599 | .71 | 1.90852 | .766 | 2.00331 | .822 | 2.10065 | .878 | 2.2011 |
| .655 | 1.81763 | .711 | 1.91019 | .767 | 2.00502 | .823 | 2.10242 | .879 | 2.20292 |
| .656 | 1.81928 | .712 | 1.91189 | .768 | 2.00673 | .824 | 2 10419 | .88 | 2.20474 |
| .657 | 1.82091 | .713 | 1.91355 | .769 | 2.00844 | .825 | 2.10596 | .881 | 2.20656 |
| .658 | 1.82255 | .714 | 1.91523 | .77 | 2.01016 | .826 | 2.10773 | .882 | 2.20839 |
| .659 | 1.82419 | .715 | 1.91691 | .771 | 2.01187 | .827 | 2.1095 | .883 | 2.21022 |
| .66 | 1.82583 | .716 | 1.91859 | .772 | 2.01359 | .828 | 2.11127 | .884 | 2.21205 |
| .661 | 1.82747 | .717 | 1.92027 | .773 | 2.01531 | .829 | 2.11304 | .885 | 2.21388 |
| .662 | 1.82911 | .718 | 1.92195 | .774 | 2.01702 | .83 | 2.11481 | .886 | 2.21571 |
| .663 | 1.83075 | .719 | 1.92363 | .775 | 2.01874 | .831 | 2.11659 | .887 | 2.21754 |
| .664 | 1.8324 | .72 | 1.92531 | .776 | 2.02045 | .832 | 2.11837 | .888 | 2.21937 |
| .665 | 1.83404 | .721 | 1.927 | .777 | 2.02217 | .833 | 2.12015 | .889 | 2.2212 |
| .666 | 1.83568 | .722 | 1.92868 | .778 | 2.02389 | .834 | 2.12193 | .89 | 2.22303 |
| .667 | 1.83733 | .723 | 1.93036 | .779 | 2.02561 | .835 | 2.12371 | .891 | 2.22486 |
| .668 | 1.83897 | .724 | 1.93204 | .78 | 2.02733 | .836 | 2.12549 | .892 | 2.2267 |
| .669 | 1.84061 | .725 | 1.93373 | .781 | 2.02907 | .837 | 2.12727 | .893 | 2.22854 |
| .67 | 1.84226 | .726 | 1.93541 | .782 | 2.0308 | .838 | 2.12905 | .894 | 2.23038 |
| .671 | 1.84391 | .727 | 1.9371 | .783 | 2.03252 | .839 | 2.13083 | .895 | 2.23222 |
| .672 | 1.84556 | .728 | 1.93678 | .784 | 2.03425 | .84 | 2.13261 | .896 | 2.23406 |
| .673 | 1.8472 | .729 | 1.94046 | .785 | 2.03598 | .841 | 2.13439 | .897 | 2.2359 |
| .674 | 1.84885 | .73 | 1.94215 | .786 | 2.03771 | .842 | 2.13618 | .898 | 2.23774 |
| .675 | 1.8505 | .731 | 1.94383 | .787 | 2.03944 | .843 | 2.13797 | .899 | 2.23958 |

TABLE.—(*Continued.*)

| H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. |
|---|---|---|---|---|---|---|---|---|---|
| .9 | 2.24142 | .921 | 2.27987 | .942 | 2.31852 | .963 | 2.3581 | .984 | 2.39823 |
| .901 | 2.24325 | .922 | 2.2817 | .943 | 2.32038 | .964 | 2.36 | .985 | 2.40016 |
| .902 | 2.24508 | .923 | 2.28354 | .944 | 2.32224 | .965 | 2.36191 | .986 | 2.40208 |
| .903 | 2.24691 | .924 | 2.28537 | .945 | 2.32411 | .966 | 2.36381 | .987 | 2.404 |
| .904 | 2.24874 | .925 | 2.2872 | .946 | 2.32598 | .967 | 2.36571 | .988 | 2.40592 |
| .905 | 2.25057 | .926 | 2.28903 | .947 | 2.32785 | .968 | 2.36762 | .989 | 2.40784 |
| .906 | 2.2524 | .927 | 2.29086 | .948 | 2.32972 | .969 | 2.36952 | .99 | 2.40976 |
| .907 | 2.25423 | .928 | 2.2927 | .949 | 2.3316 | .97 | 2.37143 | .991 | 2.41169 |
| .908 | 2.25606 | .929 | 2.29453 | .95 | 2.33348 | .971 | 2.37334 | .992 | 2.41362 |
| .909 | 2.25789 | .93 | 2.29636 | .951 | 2.33537 | .972 | 2.37525 | .993 | 2.41556 |
| .91 | 2.25972 | .931 | 2.2982 | .952 | 2.33726 | .973 | 2.37716 | .994 | 2.41749 |
| .911 | 2.26155 | .932 | 2.30004 | .953 | 2.33915 | .974 | 2.37908 | .995 | 2.41943 |
| .912 | 2.26338 | .933 | 2.30188 | .954 | 2.34104 | .975 | 2.381 | .996 | 2.42136 |
| .913 | 2.26521 | .934 | 2.30373 | .955 | 2.34293 | .976 | 2.38291 | .997 | 2.42329 |
| .914 | 2.26704 | .935 | 2.30557 | .956 | 2.34483 | .977 | 2.38482 | .998 | 2.42522 |
| .915 | 2.26888 | .936 | 2.30741 | .957 | 2.34673 | .978 | 2.38673 | .999 | 2.42715 |
| .916 | 2.27071 | .937 | 2.30926 | .958 | 2.34862 | .979 | 2.38864 | 1. | 2.42908 |
| .917 | 2.27254 | .938 | 2.31111 | .959 | 2.35051 | .98 | 2.39055 | | |
| .918 | 2.27437 | .939 | 2.31295 | .96 | 2.35241 | .981 | 2.39247 | | |
| .919 | 2.2762 | .94 | 2.31479 | .961 | 2.35431 | .982 | 2.39439 | | |
| .92 | 2.27803 | .941 | 2.31666 | .962 | 2.35621 | .983 | 2.39631 | | |

## To Ascertain the Length of a Semi-Elliptic Arc (right Semi-Ellipse) by the preceding Table.

RULE.—Divide the height by the base, find the quotient in the column of heights, and take the length of that height from the next righthand column. Multiply the length thus obtained by the base of the arc, and the product will be the length of the arc.

EXAMPLE.—What is the length of the arc of a semi-ellipse, the base being 70 feet, and the height 30.10 feet.

$30.10 \div 70 = .43$; and .43 per table, $=1.46268$.

Then $1.46268 \times 70 = 102.3876$ *feet.*

## *When the Curve is not that of a Right Semi-Ellipse, the Height being half of the Tranverse Diameter.*

RULE.—Divide half the base by twice the height, then proceed as in the preceding example; multiply the tabular length by twice the height, and the product will be the length required

EXAMPLE.—What is the length of the arc of a semi-ellipse, the height being 35 feet, and the base 60 feet?

$60 \div 2 = 30$, and $30 \div \overline{35 \times 2} = .428$ the tabular length of which is 1.45966.

Then $1.45966 \times \overline{35 \times 2} = 102.1762$ *feet.*

NOTE.—If in the division of a height by the base there is a remainder, proceed in the manner given for the Lengths of Circular Arcs, page 32.

| H'ght. | Length. |
|---|---|
| .984 | 2.39823 |
| .985 | 2.40016 |
| .986 | 2.40208 |
| .987 | 2.404 |
| .988 | 2.40592 |
| .989 | 2.40784 |
| .99 | 2.40976 |
| .991 | 2.41169 |
| .992 | 2.41362 |
| .993 | 2.41556 |
| .994 | 2.41749 |
| .995 | 2.41943 |
| .996 | 2.42136 |
| .997 | 2.42329 |
| .998 | 2.42522 |
| .999 | 2.42715 |
| 1. | 2.42908 |

-Ellipse)

heights, and take the
thus obtained by the

eing 70 feet, and the

*ht being half*

preceding example;
ength required
eing 35 feet, and the

roceed in the manner

# TABLE VIII.

## TABLE OF THE AREAS OF THE SEGMENTS OF A CIRCLE.

*The Diameter of a Circle assumed to be Unity, and divided into 1000 equal Parts.*

| Versed Sine. | Seg. Area. | Versed Sine. | Seg. Area. | Versed Sine. | Seg. Area. | Versed Sine. | Seg. Area. | Versed Sine. | Seg. Area. |
|---|---|---|---|---|---|---|---|---|---|
| .001 | .00004 | .048 | .01382 | .095 | .0379 | .142 | .06822 | .189 | .10312 |
| .002 | .00012 | .049 | .01425 | .096 | .03849 | .143 | .06892 | .19 | .1039 |
| .003 | .00022 | .05 | .01468 | .097 | .03908 | .144 | .06962 | .191 | .10468 |
| .004 | .00034 | .051 | .01512 | .098 | .03968 | .145 | .07033 | .192 | .10547 |
| .005 | .00047 | .052 | .01556 | .099 | .04027 | .146 | .07103 | .193 | .10626 |
| .006 | .00062 | .053 | .01601 | .1 | .04087 | .147 | .07174 | .194 | .10705 |
| .007 | .00078 | .054 | .01646 | .101 | .04148 | .148 | .07245 | .195 | .10784 |
| .008 | .00095 | .055 | .01691 | .102 | .04208 | .149 | .07316 | .196 | .10864 |
| .009 | .00113 | .056 | .01737 | .103 | .04269 | .15 | .07387 | .197 | .10943 |
| .01 | .00133 | .057 | .01783 | .104 | .0431 | .151 | .07459 | .198 | .11023 |
| .011 | .00153 | .058 | .0183 | .105 | .04391 | .152 | .07531 | .199 | .11102 |
| .012 | .00175 | .059 | .01877 | .106 | .04452 | .153 | .07603 | .2 | .11182 |
| .013 | .00197 | .06 | .01924 | .107 | .04514 | .154 | .07675 | .201 | .11262 |
| .014 | .0022 | .061 | .01972 | .108 | .04575 | .155 | .07747 | .202 | .11343 |
| .015 | .00244 | .062 | .0202 | .109 | .04638 | .156 | .0782 | .203 | .11423 |
| .016 | .00268 | .063 | .02068 | .11 | .047 | .157 | .07892 | .204 | .11503 |
| .017 | .00294 | .064 | .02117 | .111 | .04763 | .158 | .07965 | .205 | .11584 |
| .018 | .0032 | .065 | .02165 | .112 | .04826 | .159 | .08038 | .206 | .11665 |
| .019 | .00347 | .066 | .02215 | .113 | .04889 | .16 | .08111 | .207 | .11746 |
| .02 | .00375 | .067 | .02265 | .114 | .04953 | .161 | .08185 | .208 | .11827 |
| .021 | .00403 | .068 | .02315 | .115 | .05016 | .162 | .08258 | .209 | .11908 |
| .022 | .00432 | .069 | .02336 | .116 | .0508 | .163 | .08332 | .21 | .1199 |
| .023 | .00462 | .07 | .02417 | .117 | .05145 | .164 | .08406 | .211 | .12071 |
| .024 | .00492 | .071 | .02468 | .118 | .05209 | .165 | .0848 | .212 | .12153 |
| .025 | .00523 | .072 | .02519 | .119 | .05274 | .166 | .08554 | .213 | .12235 |
| .026 | .00555 | .073 | .02571 | .12 | .05338 | .167 | .08629 | .214 | .12317 |
| .027 | .00587 | .074 | .02624 | .121 | .05404 | .168 | .08704 | .215 | .12399 |
| .028 | .00619 | .075 | .02676 | .122 | .05469 | .169 | .03779 | .216 | .12481 |
| .029 | .00653 | .076 | .02729 | .123 | .05534 | .17 | .08853 | .217 | .12563 |
| .03 | .00686 | .077 | .02782 | .124 | .056 | .171 | .08929 | .218 | .12646 |
| .031 | .00721 | .078 | .02835 | .125 | .05666 | .172 | .09004 | .219 | .12728 |
| .032 | .00756 | .079 | .02889 | .126 | .05733 | .173 | .0908 | .22 | .12811 |
| .033 | .00791 | .08 | .02943 | .127 | .05799 | .174 | .09155 | .221 | .12894 |
| .034 | .00827 | .081 | .02997 | .128 | .05866 | .175 | .09231 | .222 | .12977 |
| .035 | .00864 | .082 | .03052 | .129 | .05933 | .176 | .09307 | .223 | .1306 |
| .036 | .00901 | .083 | .03107 | .13 | .06 | .177 | .09384 | .224 | .13144 |
| .037 | .00938 | .084 | .03162 | .131 | .06067 | .178 | .0946 | .225 | .13227 |
| .038 | .00976 | .085 | .03218 | .132 | .06135 | .179 | .09537 | .226 | .13311 |
| .039 | .01015 | .086 | .03274 | .133 | .06203 | .18 | .09613 | .227 | .13394 |
| .04 | .01054 | .087 | .0333 | .134 | .06271 | .181 | .0969 | .228 | .13478 |
| .041 | .01093 | .088 | .03387 | .135 | .06339 | .182 | .09767 | .229 | .13562 |
| .042 | .01133 | .089 | .03444 | .136 | .06407 | .183 | .09845 | .23 | .13646 |
| .043 | .01173 | .09 | .03501 | .137 | .06476 | .184 | .09922 | .231 | .13731 |
| .044 | .01214 | .091 | .03558 | .138 | .06545 | .185 | .1 | .232 | .13815 |
| .045 | .01255 | .092 | .03616 | .139 | .06614 | .186 | .10077 | .233 | .139 |
| .046 | .01297 | .093 | .03674 | .14 | .06683 | .187 | .10155 | .234 | .13984 |
| .047 | .01339 | .094 | .03732 | .141 | .06753 | .188 | .10233 | .235 | .14069 |

TABLE.—(*Continued.*)

| Versed Sine. | Seg. Area. | Versed Sine. | Seg. Area. | Versed Sine. | Seg. Area. | Versed Sine. | Seg. Area. | Versed Sine. | Seg. Area. |
|---|---|---|---|---|---|---|---|---|---|
| .236 | .14154 | .289 | .18814 | .342 | .23737 | .395 | .28848 | .448 | .34079 |
| .237 | .14239 | .29 | .18905 | .343 | .23832 | .396 | .28945 | .449 | .34179 |
| .238 | .14324 | .291 | .18995 | .344 | .23927 | .397 | .29043 | .45 | .34278 |
| .239 | .14409 | .292 | .19086 | .345 | .24022 | .398 | .29141 | .451 | .34378 |
| .24 | .14494 | .293 | .19177 | .346 | .24117 | .399 | .29239 | .452 | .34477 |
| .241 | .1458 | .294 | .19268 | .347 | .24212 | .4 | .29337 | .453 | .34557 |
| .242 | .14665 | .295 | .1936 | .348 | .24307 | .401 | .29435 | .454 | .34676 |
| .243 | .14751 | .296 | .19451 | .349 | .24403 | .402 | .29533 | .455 | .34776 |
| .244 | .14837 | .297 | .19542 | .35 | .24498 | .403 | .29631 | .456 | .34875 |
| .245 | .14923 | .298 | .19634 | .351 | .24593 | .404 | .29729 | .457 | .34975 |
| .246 | .15009 | .299 | .19725 | .352 | .24689 | .405 | .29827 | .458 | .35075 |
| .247 | .15095 | .3 | .19817 | .353 | .24784 | .406 | .29925 | .459 | .35174 |
| .248 | .15182 | .301 | .19908 | .354 | .2488 | .407 | .30024 | .46 | .35274 |
| .249 | .15268 | .302 | .2 | .355 | .24976 | .408 | .30122 | .461 | .35374 |
| .25 | .15355 | .303 | .20092 | .356 | .25071 | .409 | .3022 | .462 | .35474 |
| .251 | .15441 | .304 | .20184 | .357 | .25167 | .41 | .30319 | .463 | .35573 |
| .252 | .15528 | .305 | .20276 | .358 | .25263 | .411 | .30417 | .464 | .35673 |
| .253 | .15615 | .306 | .20368 | .359 | .25359 | .412 | .30515 | .465 | .35773 |
| .254 | .15702 | .307 | .2046 | .36 | .25455 | .413 | .30614 | .466 | .35872 |
| .255 | .15789 | .308 | .20553 | .361 | .25551 | .414 | .30712 | .467 | .35972 |
| .256 | .15876 | .309 | .20645 | .362 | .25647 | .415 | .30811 | .468 | .36072 |
| .257 | .15964 | .31 | .20738 | .363 | .25743 | .416 | .30909 | .469 | .36172 |
| .258 | .16051 | .311 | .2083 | .364 | .25839 | .417 | .31008 | .47 | .36272 |
| .259 | .16139 | .312 | .20923 | .365 | .25936 | .418 | .31107 | .471 | .36371 |
| .26 | .16226 | .313 | .21015 | .366 | .26032 | .419 | .31205 | .472 | .36471 |
| .261 | .16314 | .314 | .21108 | .367 | .26128 | .42 | .31394 | .473 | .36571 |
| .262 | .16402 | .315 | .21201 | .368 | .26225 | .421 | .31403 | .474 | .36671 |
| .263 | .1649 | .316 | .21294 | .369 | .26321 | .422 | .31502 | .475 | .36771 |
| .264 | .16578 | .317 | .21387 | .37 | .26418 | .423 | .316 | .476 | .36871 |
| .265 | .16666 | .318 | .2148 | .371 | .26514 | .424 | .31699 | .477 | .36971 |
| .266 | .16755 | .319 | .21573 | .372 | .26611 | .425 | .31798 | .478 | .37071 |
| .267 | .16844 | .32 | .21667 | .373 | .26708 | .426 | .31897 | .479 | .3717 |
| .268 | .16931 | .321 | .2176 | .374 | .26804 | .427 | .31996 | .48 | .3727 |
| .269 | .1702 | .322 | .21853 | .375 | .26901 | .428 | .32095 | .481 | .3737 |
| .27 | .17109 | .323 | .21947 | .376 | .26998 | .429 | .32194 | .482 | .3747 |
| .271 | .17197 | .324 | .2204 | .377 | .27095 | .43 | .32293 | .483 | .3757 |
| .272 | .17287 | .325 | .22134 | .378 | .27192 | .431 | .32391 | .484 | .3767 |
| .273 | .17376 | .326 | .22228 | .379 | .27289 | .432 | .3249 | .485 | .3777 |
| .274 | .17465 | .327 | .22321 | .38 | .27386 | .433 | .3259 | .486 | .3787 |
| .275 | .17554 | .328 | .22415 | .381 | .27483 | .434 | .32689 | .487 | .3797 |
| .276 | .17643 | .329 | .22509 | .382 | .27580 | .435 | .32788 | .488 | .3807 |
| .277 | .17733 | .33 | .22603 | .383 | .27677 | .436 | .32887 | .489 | .3817 |
| .278 | .17822 | .331 | .22697 | .384 | .27775 | .437 | .32987 | .49 | .3827 |
| .279 | .17912 | .332 | .22791 | .385 | .27872 | .438 | .33086 | .491 | .3837 |
| .28 | .18002 | .333 | .22886 | .386 | .27969 | .439 | .33185 | .492 | .3847 |
| .281 | .18092 | .334 | .2298 | .387 | .28067 | .44 | .33284 | .493 | .3857 |
| .282 | .18182 | .335 | .23074 | .388 | .28164 | .441 | .33384 | .494 | .3867 |
| .283 | .18272 | .336 | .23169 | .389 | .28262 | .442 | .33483 | .495 | .3877 |
| .284 | .18361 | .337 | .23263 | .39 | .28359 | .443 | .33582 | .496 | .3887 |
| .285 | .18452 | .338 | .23359 | .391 | .28457 | .444 | .33682 | .497 | .3897 |
| .286 | .18542 | .339 | .23453 | .392 | .28554 | .445 | .33781 | .498 | .3907 |
| .287 | .18633 | .34 | .23547 | .393 | .28552 | .446 | .3388 | .499 | .3917 |
| .288 | .18723 | .341 | .23642 | .394 | .2878 | .447 | .3398 | .5 | .3927 |

| Versed Sine. | Seg. Area. |
|---|---|
| .448 | .34079 |
| .449 | .34179 |
| .45 | .34278 |
| .451 | .34378 |
| .452 | .34477 |
| .453 | .34557 |
| .454 | .34676 |
| .455 | .34776 |
| .456 | .34875 |
| .457 | .34975 |
| .458 | .35075 |
| .459 | .35174 |
| .46 | .35274 |
| .461 | .35374 |
| .462 | .35474 |
| .463 | .35573 |
| .464 | .35673 |
| .465 | .35773 |
| .466 | .35872 |
| .467 | .35972 |
| .468 | .36072 |
| .469 | .36172 |
| .47 | .36272 |
| .471 | .36371 |
| .472 | .36471 |
| .473 | .36571 |
| .474 | .36671 |
| .475 | .36771 |
| .476 | .36871 |
| .477 | .36971 |
| .478 | .37071 |
| .479 | .3717 |
| .48 | .3727 |
| .481 | .3737 |
| .482 | .3747 |
| .483 | .3757 |
| .484 | .3767 |
| .485 | .3777 |
| .486 | ·3787 |
| .487 | .3797 |
| .488 | .3807 |
| .489 | .3817 |
| .49 | .3827 |
| .491 | .3837 |
| .492 | .3847 |
| .493 | .3857 |
| .494 | .3867 |
| .495 | .3877 |
| .496 | .3887 |
| .497 | .3897 |
| .498 | .3907 |
| .499 | .3917 |
| .5 | .3927 |



## To Ascertain the Area of a Segment of a Circle by the preceding Table.

Rule.—Divide the height or versed sine by the diameter of the circle; find the quotient in the column of versed sines. Take the area noted in the next column, multiply it by the square of the diameter, and it will give the area

Example.—Required the area of a segment, its height being 10, and the diameter of the circle 50 feet.

$10 \div 50 = .2$, and .2, per table, $= .11182$; then $.11182 \times 50^2 = 279.55$ *feet.*

Note.—If in the division of a height by the base, the quotient has remainder after the third place of decimals, and great accuracy is required.

Take the area for the first three figures, subtract it from the next following area, multiply the remainder by the said fraction, and add the product to the first area; the sum will be the area for the whole quotient.

2 What is the area of a segment of a circle, the diameter of which is 10 feet, and the height of it 1.575 feet.

$1.575 \div 10 = .1575$; the tabular area for $.157 = .07892$, and for $.158 = 07965$, the difference between which is .00073.

Then $.5 \times .00073 = 000365$.

Hence $.157 = .07892$

$.0005 = .000365$

.079275, the sum by which the square of the diameter of the circle is to be multiplied; and $.079285 \times 10^2 = 7.9286$ *feet.*

# TABLE IX.

## TABLE OF THE AREAS OF THE ZONES OF A CIRCLE.

***The Diameter of a Circle assumed to be Unity, and divided into 1000 equal Parts.***

| H'ght. | Area. | H'ght. | Area. | H'ght. | Area. | H'ght. | Area. | H'ght. | Area. |
|---|---|---|---|---|---|---|---|---|---|
| .001 | .001 | .029 | .02898 | .057 | .05688 | .085 | .08459 | .113 | .11203 |
| .002 | .002 | .03 | .02998 | .058 | .05787 | .086 | .08557 | .114 | .113 |
| .003 | .003 | .031 | .03093 | .059 | .05886 | .087 | .08656 | .115 | .11398 |
| .004 | .004 | .032 | .03198 | .06 | .05986 | .088 | .08754 | .116 | .11495 |
| .005 | .005 | .033 | .03298 | .061 | .06085 | .089 | .08853 | .117 | .11592 |
| .006 | .006 | .034 | .03397 | .062 | .06184 | .09 | .08951 | .118 | .1169 |
| .007 | .007 | .035 | .03497 | .063 | .06283 | .091 | .0905 | .119 | .11787 |
| .008 | .008 | .036 | .03597 | .064 | .06382 | .092 | .09148 | .12 | .11884 |
| .009 | .009 | .037 | .03697 | .065 | .06482 | .093 | .09246 | .121 | .11981 |
| .01 | .01 | .038 | .03796 | .066 | .0658 | .094 | .09344 | .122 | .12078 |
| .011 | .011 | .039 | .03896 | .067 | .0668 | .095 | .09443 | .123 | .12175 |
| .012 | .012 | .04 | .03996 | .068 | .0678 | .096 | .0954 | .124 | .12272 |
| .013 | .013 | .041 | .04095 | .069 | .06878 | .097 | .09639 | .125 | .12369 |
| .914 | .014 | .042 | .04195 | .07 | .06977 | .098 | .09737 | .126 | .12469 |
| .015 | .015 | .043 | .04295 | .071 | .07076 | .099 | .09835 | .127 | .12562 |
| .016 | .016 | .044 | .04394 | .072 | .07175 | .1 | .09933 | .128 | .12659 |
| .017 | .017 | .045 | .04494 | .073 | .07274 | .101 | .10031 | .129 | .12755 |
| .018 | .018 | .046 | .04593 | .074 | .07373 | .102 | .10129 | .13 | .12852 |
| .019 | .019 | .047 | .04693 | .075 | .07472 | .103 | .10227 | .131 | .12949 |
| .02 | .02 | .048 | .04793 | .076 | .0755 | .104 | .10325 | .132 | .13045 |
| .021 | .021 | .049 | .04892 | .077 | .07669 | .105 | .10422 | .133 | .13141 |
| .022 | .022 | .05 | .04992 | .078 | .07768 | .106 | .1052 | .134 | .13238 |
| .023 | .023 | .051 | .05091 | .079 | .07867 | .107 | .10618 | .135 | .13334 |
| .024 | .024 | .052 | .0519 | .08 | .07966 | .108 | .10715 | .136 | .1343 |
| .025 | .025 | .053 | .0529 | .081 | .08064 | .109 | .10813 | .137 | .13527 |
| .026 | .02599 | .054 | .05389 | .082 | .08163 | .11 | .10911 | .138 | .13623 |
| .027 | .02699 | .055 | .05489 | .083 | .08262 | .111 | .11008 | .139 | .13719 |
| .028 | .02799 | .056 | .05588 | .084 | .0836 | .112 | .11106 | .14 | .13815 |

## TABLE.—(*Continued.*)

| H'ght. | Area. | H'ght. | Area. | H'ght. | Area. | H'ght. | Area. | H'ght. | Area. |
|---|---|---|---|---|---|---|---|---|---|
| .141 | .13911 | .197 | .19178 | .253 | .24175 | .309 | .28801 | .365 | .32931 |
| .142 | .14007 | .198 | .1927 | .254 | .24261 | .31 | .2888 | .366 | .32999 |
| .143 | .14103 | .199 | .19361 | .255 | .24347 | .311 | .28958 | .367 | .33067 |
| .144 | .14198 | .2 | .19453 | .256 | .24433 | .312 | .29036 | .368 | .33135 |
| .145 | .14294 | .201 | .19545 | .257 | .24519 | .313 | .29115 | .369 | .33203 |
| .146 | .1439 | .202 | .19636 | .258 | .24604 | .314 | .29192 | .37 | .3327 |
| .147 | .14485 | .203 | .19728 | .259 | .2469 | .315 | .2927 | .371 | .33337 |
| .148 | .14581 | .204 | .19819 | .26 | .24775 | .316 | .28348 | .372 | .33404 |
| .149 | .14677 | .205 | .1991 | .261 | .24861 | .317 | .29425 | .373 | .33471 |
| .15 | .14772 | .206 | .20001 | .262 | .24946 | .318 | .29502 | .374 | .33537 |
| .151 | .14867 | .207 | .20092 | .263 | .25021 | .319 | .2958 | .375 | .33604 |
| .152 | .14962 | .208 | .20183 | .264 | .25116 | .32 | .29656 | .376 | .3367 |
| .153 | .15058 | .209 | .20274 | .265 | .21201 | .321 | .29733 | .377 | .33735 |
| .154 | .15153 | .21 | .20365 | .266 | .25285 | .322 | .2981 | .378 | .33801 |
| .155 | .15248 | .211 | .20156 | .267 | .2537 | .323 | .29886 | .379 | .33866 |
| .156 | .15343 | .212 | .20546 | .268 | .25455 | .324 | .29962 | .38 | .33931 |
| .157 | .15438 | .213 | .20637 | .269 | .25539 | .325 | .30039 | .381 | .33996 |
| .158 | .15533 | .214 | .20727 | .27 | .25623 | .326 | .30114 | .382 | .34061 |
| .159 | .15628 | .215 | .20818 | .271 | .25707 | .327 | .3019 | .383 | .34125 |
| .16 | .15723 | .216 | .20908 | .272 | .25791 | .328 | .30266 | .384 | .3419 |
| .161 | .15817 | .217 | .20998 | .273 | .25875 | .329 | .30341 | .385 | .34253 |
| .162 | .15912 | .218 | .21088 | .274 | .25959 | .33 | .30416 | .386 | .34317 |
| .163 | .16006 | .219 | .21178 | .275 | .26043 | .331 | .30491 | .387 | .3438 |
| .164 | .16101 | .22 | .21268 | .276 | .26126 | .332 | .30566 | .388 | .34444 |
| .165 | .16195 | .221 | .21358 | .277 | .26209 | .333 | .30641 | .389 | .34507 |
| .166 | .1629 | .222 | .21447 | .278 | .26293 | .334 | .30715 | .39 | .34569 |
| .167 | .16384 | .223 | .21537 | .279 | .26376 | .335 | .3079 | .391 | .34632 |
| .168 | .16478 | .224 | .21626 | .28 | .26459 | .336 | .30864 | .392 | .34694 |
| .169 | .16572 | .225 | .21716 | .281 | .26541 | .337 | .30938 | .393 | .34756 |
| .17 | .16667 | .226 | .21805 | .282 | .26624 | .338 | .31012 | .394 | .34818 |
| .171 | .16761 | .227 | .21894 | .283 | .26706 | .339 | .31085 | .395 | .34879 |
| .172 | .16855 | .228 | .21983 | .284 | .26789 | .34 | .31159 | .396 | .3494 |
| .173 | .16948 | .229 | .22072 | .285 | .26871 | .341 | .31232 | .397 | .35001 |
| .174 | .17042 | .23 | .22161 | .286 | .26953 | .342 | .31305 | .398 | .35062 |
| .175 | .17136 | .231 | .2225 | .287 | .27035 | .343 | .31378 | .399 | .35122 |
| .176 | .1723 | .232 | .22335 | .288 | .27117 | .344 | .3145 | .4 | .35182 |
| .177 | .17323 | .233 | .22427 | .289 | .27199 | .345 | .31523 | .401 | .35242 |
| .178 | .17417 | .234 | .22515 | .29 | .2728 | .346 | .31595 | .402 | .35302 |
| .179 | .1751 | .235 | .22604 | .291 | .27362 | .347 | .31667 | .403 | .35361 |
| .18 | .17603 | .236 | .22692 | .292 | .27443 | .348 | .31639 | .404 | .3542 |
| .181 | .17697 | .237 | .2278 | .293 | .27524 | .349 | .31811 | .405 | .35479 |
| .182 | .1779 | .238 | .22868 | .294 | .27605 | .35 | .31882 | .406 | .35538 |
| .183 | .17883 | .239 | .22956 | .295 | .27686 | .351 | .31954 | .407 | .35596 |
| .184 | .17976 | .24 | .23044 | .296 | .27766 | .352 | .32025 | .408 | .35654 |
| .185 | .18069 | .241 | .23131 | .297 | .27847 | .353 | .32096 | .409 | .35711 |
| .186 | .18162 | .242 | .23219 | .298 | .27927 | .354 | .32167 | .41 | .35769 |
| .187 | .18254 | .243 | .23306 | .299 | .28007 | .355 | .32237 | .411 | .35826 |
| .188 | .18347 | .244 | .23394 | .3 | .28088 | .356 | .32307 | .412 | .35883 |
| .189 | .1844 | .245 | .23481 | .301 | .28167 | .357 | .32377 | .413 | .35939 |
| .19 | .18532 | .246 | .23568 | .302 | .28247 | .358 | .32147 | .414 | .35995 |
| .191 | .18625 | .247 | .23655 | .303 | .28327 | .359 | .32517 | .415 | .36051 |
| .192 | .18717 | .248 | .23742 | .304 | .28406 | .36 | .32587 | .416 | .36107 |
| .193 | .18809 | .249 | .23829 | .305 | .28486 | .361 | .32656 | .417 | .36162 |
| .194 | .18902 | .25 | .23915 | .306 | .28565 | .362 | .32725 | .418 | .36217 |
| .195 | .18994 | .251 | .24002 | .307 | .28644 | .363 | .32794 | .419 | .36272 |
| .196 | .19086 | .252 | .24089 | .308 | .28723 | .364 | .32862 | .42 | .36326 |

## TABLE.—*(Continued.)*

| H'ght. | Area. | H'ght. | Area. | H'ght. | Area. | H'ght. | Area. | H'ght. | Area. |
|---|---|---|---|---|---|---|---|---|---|
| .421 | .3638 | .437 | .37202 | .453 | .37931 | .469 | .38549 | .485 | .39026 |
| .422 | .36434 | .438 | .3725 | .454 | .37973 | .47 | .38583 | .486 | .3905 |
| .423 | .36488 | .439 | .37298 | .455 | .38014 | .471 | .38617 | .487 | .39073 |
| .424 | .36541 | .44 | .37346 | .456 | .38056 | .472 | .3865 | .488 | .39095 |
| .425 | .36594 | .441 | .37393 | .457 | .38096 | .473 | .38683 | .489 | .39117 |
| .426 | .36646 | .442 | .3744 | .458 | .38137 | .474 | .38715 | .49 | .39137 |
| .427 | .36698 | .443 | .37487 | .459 | .38177 | .475 | .38747 | .491 | .39156 |
| .428 | .3675 | .444 | .37533 | .46 | .38216 | .476 | .38778 | .492 | .39175 |
| .429 | .36802 | .445 | .37579 | .461 | .38255 | .477 | .38808 | .493 | .39192 |
| .43 | .36853 | .446 | .37624 | .462 | .38294 | .478 | .38838 | .494 | .39208 |
| .431 | .36904 | .447 | .37669 | .463 | .38332 | .479 | .38667 | .495 | .39223 |
| .432 | .36954 | .448 | .37714 | .464 | .38369 | .48 | .38895 | .496 | .39236 |
| .433 | .37005 | .449 | .37758 | .465 | .38406 | .481 | .38923 | .497 | .39248 |
| .434 | .37054 | .45 | .37802 | .466 | .38443 | .482 | .3895 | .498 | .39258 |
| .435 | .37104 | .451 | .37845 | .467 | .38479 | .483 | .38976 | .499 | .39266 |
| .436 | .37153 | .452 | .37888 | .468 | .38514 | .484 | .39001 | .5 | .3927 |

*This Table is computed only for Zones, the longest chord of which is diameter.*

## To Ascertain the Area of a Zone by the preceding Table.

RULE 1.—*When the Zone is Less than a Semicircle,* Divide the height by the diameter, and find the quotient in the column of height. Take out the area opposite to it in the next column on the right hand and multiply it by the square of the longest chord; the product will be the area of the zone.

EXAMPLE.—Required the area of a zone the diameter of which is 50, and its height 15.

$15 \div 50 = .3$; and $.3$; as per table, $= .28088$.

Hence $.28088 \times 50^2 = 702.2$ *area.*

RULE 2.—*When the Zone is Greater than a Semicircle:* Take the height on each side of the diameter of the circle, and ascertain, by Rule I, their respective areas; add the areas of these two portions together, and the sum will be the area of the zone.

EXAMPLE.—Required the area of a zone, the diameter of the circle being 50, and the heights of the zone on each side of the diameter of the circle 20 and 15 respectively.

$20 \div 50 = .4$; $.4$, as per table, $= .35182$; and $.35182 \times 50^2 = 879.55$.

$15 \div 50 = .3$; $.3$, as per table, $= .28088$; and $.28088 \times 50^2 = 702.2$.

Hence $879.55 + 702.2 = 1581.75$ *area.*

RULE 3.—*When the longest chord of the zone is less than diameter,* Take the height or distance from the diam. to each of the chords respectively; find the area corresponding to each height and deduct the lesser from the greater area; the result will be the area required.

NOTE.—When, in the division of a height by the chord, the quotient has a remainder after the third place of decimals, and great accuracy is required.

Take the area for the first three figures, subtract it from the next following area, multiply the remainder by the said fraction, and add the product to the first area; the sum will be the area for the whole quotient.

EXAMPLE.—What is the area of a zone of a circle, the greater chord being 100 feet, and the breadth of it 14 feet 3 inches?

14 feet 3 inches $= 14.25$ and $14.25 \div 100 = .1425$; the tabular area for $.142 = 14007$, and for $143 =$ $.14103$, the difference between which is $.00096$.

Then $.5 \times .00096 = .00048$.

Hence $.142 = .14007$

$.0005 = .00048$

$.14055$, the sum by which the square of the greater chord is to be multiplied; and $.14055 \times 100^2 = 1405.5$ *feet.*

## TABLE X.

# SPECIFIC GRAVITIES.

The Specific Gravity of a body is the proportion it bears to the weight of another body of known density.

If a body float on a fluid, the part immersed is to the whole body as the specific gravity of the body is to the specific gravity of the fluid.

When a body is immersed in a fluid, it loses such a portion of its own weight as is equal to that of the fluid it displaces.

An immersed body, ascending or descending in a fluid, has a force equal to the difference between its own weight and the weight of its bulk of the fluid, less the resistance of the fluid to its passage.

Water is well adapted for the standard of gravity; and as a cubic foot of it weights 1000 ounces avoirdupois, its weight is taken as the unit, *viz*: 1000.

### To Ascertain the Specific Gravity of a Body heavier than Water.

**Rule.**—Weight it both in and out of water, and note the difference; then, as the weight lost in water is to the whole weight, so is 1000 to the specific gravity of the body. Or, $\frac{W \times 1000}{W - w} = G$, *$w$ representing the weight in water, and G the specific gravity.*

**Example.**—What is the specific gravity of a stone which weighs in air 15 lbs., in water 10 lbs.?

15—10=5; then 5 : 15 : : 1000 : : 3000 *spec. grav.*

### To Ascertain the Specific Gravity of a Body lighter than Water.

**Rule.**—Annex to the lighter body another that is heavier than water, or the fluid used; weigh the piece added and the compound mass separately, both in and out of the water. or the fluid; ascertain how much each loses in water, or the fluid, by subtracting its weight in water, or the fluid, from its weight in air, and subtract the less of these differences from the greater; then,

As the last remainder is to the weight of the light body in air, so is 1000 to the specific gravity of the body.

**Example.**—What is the specific gravity of a piece of wood that weighs 20 lbs. in air; annexed to it is a piece of metal that weighs 24 lbs. in air and 24 lbs. in water, and the two pieces in water weigh 8 lbs.?

20+24—8=44—8=36=*loss of compound mass in water;*
24—21 = 3=*loss of heavy body in water.*
$\overline{33}$ : 20 : : 1000 : 606=24 *spec. grav.*

### To Ascertain the Specific Gravity of Fluid.

**Rule.**—Take a body of known specific gravity, weigh it in and out of the fluid; then, as the weight of the body is to the loss of weight, so is the specific gravity of the body to that of the fluid.

EXAMPLE.—What is the specific gravity of a fluid in which a piece of copper (*spec. grav.*=9000) weighs 70 lbs. in, and 80 lbs. out of it?

80 : 80—70=10 : : 9000 1125 *spec. grav.*

## To Compute the Proportions of two Ingredients in a Compound, or to discover Adulteration in Metals.

RULE.—Take the differences of each specific gravity of the ingredients and the specific gravity of the compound, then multiply the gravity of the one by the difference of the other; and, as the sum of the products is to the respective products, so is the specific gravity of the body to the proportions of the ingredients.

EXAMPLE.—A compound of gold (*spec. grav.*=18.888) and silver (*spec. grav.*= 10.535) has a specific gravity of 14; what is the proportion of each metal.

18.888—14=4.888×10.535=51.495

14 —10.535=3.465×18.888=65.447

65.447+51.495 : 65.447 : : 14 : 7.835 *gold.*

65.447+51.495 : 51.495 : : 14 : 6.165 *silver.*

## To compute the Weights of the Ingredients, that of the compound being given.

RULE.— As the specific gravity of the compound is to the weight of the compound, so are each of the proportions to the weight of its material.

EXAMPLE.—The weight, as above, being 28 lbs., what are the weights of the ingredients?

$$14 : 28 : : \begin{cases} 7.835 : 15.67 \textit{ gold}, \\ 6.165 : 12.33 \textit{ silver}. \end{cases}$$

## Proof of Spirituous Liquors.

A cubic inch of *proof spirits* weighs 234 grains; than, ıı an immersed cubic inch of any heavy body weighs 234 grains less in spirits than air, it shows that the spirit in which it was weighed is *proof*.

If it lose less of its weight, the spirit is above proof; and if it lose more, it is below proof.

ILLUSTRATION.—A cubic inch of glass weighing 700 grains weighs 500 grains when weighed in a certain spirit; what is the proof of it?

700—500=200=*grains=weight lost in the spirit.*

Then 200 : 234 : : 1. : 1.17= *ratio of proof of spirits compared to proof spirits, or* 1.=.17 *above proof.*

## Solids.

RULE.—Divide the specific gravity of the substance by 16, and the quotient will give the weight of a cubic foot of it in pounds.

## OF DIFFERENT BODIES AND SUBSTANCES.

| METALS. | Specific gravity. | Weight of a cubic inch. |
|---|---|---|
| Alluminu... | 2560 | .0926 |
| Antimony | 6712 | .2428 |
| Arsenic | 5763 | .2084 |
| Barium | 470 | .017 |
| Bismuth | 9823 | .3553 |
| Brass, copper 84 } " tin 16 } | 8832 | 3194 |
| " copper 67 } " zinc 33 } | 7820 | 2828 |
| " plate | 8380 | .3031 |
| " wire | 8214 | .2972 |
| Bronze, gun metal | 8700 | .3147 |
| Boron | 2000 | .0723 |
| Bromine | 3000 | 1085 |
| Cadmium | 8650 | .3129 |
| Calcium | 1580 | .057 |
| Chromium | 5900 | .2134 |
| Cinnabar | 8098 | .2929 |
| Cobalt | 8600 | .3111 |
| Columbium | 6000 | .217 |
| Gold, pure, cast | 19258 | .6965 |
| " hammered | 19361 | .7003 |
| " 22 carats fine | 17486 | .6325 |
| " 20 carats fine | 15709 | .5682 |
| Copper, cast | 8788 | .3179 |
| " plates | 8698 | .3146 |
| " wire | 8880 | .3212 |
| Iridium | 18680 | .6756 |
| " hammered | 23000 | .8319 |
| Iron, cast | 7207 | .2607 |
| " " gun metal | 7308 | .264 |
| " hot blast | 7065 | .2555 |
| " cold " | 7218 | .2611 |
| " wrought bars | 7788 | .2817 |
| " " wire | 7774 | .2811 |
| " rolled plate | 7704 | .2787 |
| Lead, cast | 11352 | .4106 |
| " rolled | 11388 | .4119 |
| Lithium | 590 | 0213 |
| Manganese | 8000 | .2894 |
| Magnesium | 1750 | .0633 |
| Mercury—40° | 15632 | .5661 |
| " +32° | 13598 | .4918 |
| " 60° | 13580 | .4912 |
| " 212° | 13370 | .4836 |
| Molybdenum | 8600 | .3111 |
| Nickel | 8800 | .3183 |
| " cast | 8279 | .2994 |
| Osmium | 10000 | .3613 |

| METALS. | Specific gravity. | Weight of a cubic inch. |
|---|---|---|
| Palladium | 11350 | .4105 |
| Platinum, hammered | 20337 | .7356 |
| " native | 16000 | .5787 |
| " rolled | 22069 | .7982 |
| Potassium, 59° | 865 | .0313 |
| Red-lead | 8940 | .3241 |
| Rhodium | 10650 | .3852 |
| Ruthenium | 8600 | .3111 |
| elenium | 4500 | .1627 |
| Silicium | | |
| Silver, pure, cast | 10474 | .3788 |
| " " hammered | 10511 | .3902 |
| Sodium | 970 | .0351 |
| Steel, plates | 7806 | .2823 |
| " soft | 7833 | .2833 |
| " tempered and hardened | 7818 | .2828 |
| " wire | 7847 | .2838 |
| Strontium | 2540 | .0918 |
| Tin, Cornish, hammerd | 7390 | .2673 |
| " " pure | 7291 | .2637 |
| Tellurium | 6110 | .221 |
| Thalium | 11850 | .4286 |
| Titanium | 5300 | .1917 |
| Tungsten | 17000 | .6149 |
| Uranium | 10150 | .3671 |
| Wolfram | 7119 | .2575 |
| Zinc, cast | 6861 | .2482 |
| " rolled | 7191 | .26 |

| WOODS (DRY.) | | Cubic foot. |
|---|---|---|
| Alder | 800 | 50 |
| Apple | 793 | 49.562 |
| Ash } | 845 | 52.812 |
| | 600 | 43.125 |
| Bamboo | 400 | 25. |
| Bay | 822 | 51.375 |
| Beech | 852 | 53.25 |
| " | 690 | 43.125 |
| Birch | 567 | 35.437 |
| Box, Brazilian | 1031 | 64.437 |
| " Dutch | 912 | 57. |
| " French | 1328 | 83. |
| Bullet-wood | 928 | 58. |
| Butternut | 376 | 23.5 |
| Campeachy | 913 | 57.062 |
| Cedar | 561 | 35.062 |
| " Indian | 1315 | 82.157 |

| WOODS, (Dry.) (*Continued.*) | Specific gravity. | Weight of a cubic foot. |
|---|---|---|
| Charcoal, pine | 441 | 27 562 |
| " fresh burned | 380 | 23.75 |
| " oak | 1573 | 98 312 |
| " soft wood | 280 | 17.5 |
| " triturated | 1380 | 86.25 |
| Cherry | 715 | 44 687 |
| Chesnut, sweet | 610 | 38 125 |
| Citron | 726 | 45.375 |
| Cocoa | 1040 | 65 |
| Cork | 240 | 15 |
| Cypress, Spanish | 644 | 40.25 |
| Dog-wood | 756 | 47 25 |
| Ebony, American | 1331 | 83.187 |
| " Indian | 1209 | 75 562 |
| Elder | 695 | 43 437 |
| Elm | 570 | 35 625 |
| | 671 | 41 937 |
| Filbert | 600 | 37.5 |
| Fir (Norway Space) | 512 | 32 |
| Gum, blue | 843 | 52 687 |
| " water | 1000 | 62 5 |
| Hackmatack | 592 | 37. |
| Hazel | 860 | 53.75 |
| Hawthorn | 910 | 56.875 |
| Hemlock | 368 | 23. |
| Hickory, pig-nut | 792 | 49.5 |
| " shell-bark | 690 | 43 125 |
| Holly | 760 | 47.5 |
| Jasmine | 770 | 48 125 |
| Juniper | 566 | 35 375 |
| Lance-wood | 720 | 45. |
| Larch | 544 | 34. |
| | 560 | 35. |
| Lemon | 703 | 43.937 |
| Lignum-vitæ | 1333 | 83.312 |
| Lime | 804 | 50.25 |
| Linden | 604 | 37.75 |
| Locust | 728 | 45.5 |
| Logwood | 913 | 57.062 |
| Mahogany | 720 | 45. |
| | 1063 | 66.437 |
| " Honduras | 560 | 35. |
| " Spanish | 852 | 53.25 |
| Maple | 750 | 46.875 |
| " bird's eye | 576 | 36. |
| Mastic | 849 | 53.062 |
| Mulberry | 561 | 35.062 |
| | 897 | 56.062 |
| Oak, African | 823 | 51.437 |
| " Canadian | 872 | 54.5 |

| WOODS, (Dry.) (*Continued.*) | Specific gravity. | Weight of a cubic foot. |
|---|---|---|
| Oak, Dantzic | 759 | 47.437 |
| " English | 932 | 58 25 |
| " green | 1446 | 71.625 |
| " heart, 60 years | 1170 | 73.125 |
| " live, green | 1260 | 78.75 |
| " " seasoned | 1068 | 66.75 |
| " white | 860 | 53.75 |
| Orange | 705 | 44.062 |
| Pear | 661 | 42 312 |
| Persimmon | 710 | 44.375 |
| Plum | 785 | 49.062 |
| Pine, pitch | 660 | 41.25 |
| " red | 590 | 36 875 |
| " white | 554 | 34.625 |
| " yellow | 461 | 28.812 |
| Pomegranate | 1354 | 84 625 |
| Poon | 580 | 36.25 |
| Poplar | 383 | 23.937 |
| " white | 529 | 33.062 |
| Quince | 705 | 44.062 |
| Rose-wood | 728 | 45.5 |
| Sassafras | 482 | 30.125 |
| Satin-wood | 885 | 55.312 |
| Spruce | 500 | 31.25 |
| Sycamore | 523 | 38.937 |
| Tamarack | 383 | 23.937 |
| Teak (African oak) | 657 | 41.062 |
| | 745 | 46.562 |
| Walnut | 671 | 41.937 |
| " black | 500 | 31.25 |
| Willow | 486 | 30.375 |
| | 585 | 36.562 |
| Yew, Dutch | 788 | 49.25 |
| " Spanish | 807 | 50.437 |
| ***(Well Seasoned.*)*** | | |
| Ash | 722 | 45.125 |
| Beech | 624 | 39. |
| Cherry | 606 | 37.875 |
| Cypress | 441 | 27.562 |
| Hickory, red | 838 | 52.375 |
| Mahogany, St. Domg. | 720 | 45. |
| Pine, white | 473 | 29.562 |
| " yellow | 541 | 33.812 |
| Poplar | 587 | 36.687 |
| White Oak, upland | 687 | 42.937 |
| " James River | 759 | 42.437 |

*Ordnance manual 1841.

| Stones, Earths, &c | Specific gravity. | Weight of a cubic foot. | Stones, Earths, &c | Specific gravity. | Weight of a cubic foot. |
|---|---|---|---|---|---|
| Agate | 2590 | — | " white | 2550 | — |
| Alabaster, white | 2730 | 170.625 | Cornelian | 2613 | — |
| " yellow | 2699 | 168.687 | Diamond, Oriental | 3521 | — |
| Alum | 1714 | 107.125 | " Brazilian | 3444 | — |
| Amber | 1078 | 67.375 | Earth, * common soil | 2194 | 137.125 |
| Ambergris | 866 | — | " loose | 1500 | 93.75 |
| Asbestos, starry | 3073 | 192.062 | " moist sand | 2050 | 128.125 |
| Asphaltum | 905 | 56.562 | " mould, fresh | 2050 | 128 125 |
| | 1650 | 103.125 | " rammed | 1600 | 100. |
| Barytes, sulphate | 4000 | 250. | " rough sand | 1920 | 120. |
| | 4865 | 304.062 | " with gravel | 2020 | 126.25 |
| Basalts | 2740 | 171.25 | Emery | 4000 | 250. |
| | 2864 | 179 | Flint, black | 2582 | 161.375 |
| Borax | 1714 | 107.125 | " white | 2594 | 162.125 |
| Brick | 1900 | 118.75 | Fluorine | 1320 | 82.5 |
| | 1367 | 85.437 | Glass, bottle | 2732 | 170.75 |
| " fire | 2201 | 137.562 | " Crown | 2487 | 155.437 |
| " work in cement | 1800 | 112.50 | " flint | 2933 | 183.312 |
| " " " mortar | 1600 | 100. | | 3200 | 196. |
| | 2000 | 125. | " green | 2642 | 165.125 |
| Carbon | 3500 | 218.75 | " optical | 3450 | 215.625 |
| Cement, Portland | 1300 | 81.25 | " white | 2892 | 180.75 |
| " Roman | 1560 | 97.25 | " window | 2642 | 165.125 |
| Chalk | 1520 | 95. | Garnet | 4189 | — |
| | 2784 | 174. | " black | 3750 | — |
| Chrysolite | 2782 | — | Granite, Egyptian red | 2654 | 165.875 |
| Clay | 1930 | 120.625 | " Patapsco | 2640 | 165. |
| " with gravel | 2480 | 155. | " Quincy | 2652 | 165.75 |
| Coal, Anthracite | 1436 | 89 75 | " Scotch | 2625 | 164.062 |
| | 1640 | 102.5 | " Susquehanna | 2704 | 169. |
| " Borneo | 1290 | 80.625 | Gravel, common | 1749 | 109.312 |
| " Cannel | 1238 | 77.375 | Grindstone | 2143 | 133.937 |
| | 1318 | 82.375 | Gypsum, opaque | 2168 | 135.5 |
| " Caking | 1277 | 79.812 | Hone, white, razor | 2876 | 179.75 |
| " Cherry | 1276 | 79.75 | Hornblende | 3540 | 221.25 |
| " Chili | 1290 | 80.625 | Iodine | 4940 | — |
| " Derbyshire | 1292 | 80.75 | Jet | 1300 | — |
| " Lancaster | 1273 | 79.562 | Lime, hydraulic | 2745 | 171.562 |
| " Maryland | 1355 | 84.687 | " quick | 804 | 50.25 |
| " Newcastle | 1270 | 79.375 | Limestone, green | 3180 | 198.75 |
| " Rive de Gier | 1300 | 81.25 | " white | 3156 | 197.25 |
| " Scotch | 1259 | 78 687 | Magnesia, carbonate | 2400 | 150 |
| | 1300 | 81.25 | Marble, Adelaide | 2715 | 169.687 |
| " Splint | 1302 | 81.375 | " Africain | 2708 | 169 25 |
| " Wales, mean | 1315 | 82.187 | " Biscayan, black | 2695 | 168.437 |
| Coke | 1000 | 62.5 | " Carara | 2716 | 169 75 |
| " Nat'l, Va | 746 | 46.64 | " common | 2686 | 167.875 |
| Concrete, mean | 2000 | 125. | " Egyptian | 2668 | 166.75 |
| Copal | 1045 | 65.312 | " French | 2649 | 165.562 |
| Coral, red | 2700 | — | " Italian, white | 2708 | 169.25 |

* Spec. grav. of the earth in variously estimated at from 5,450 to 5,600.

| Stones, Earths, &c | Specific gravity. | Weight of a cubic foot. |
|---|---|---|
| Marble Parian.. | 2838 | 177.375 |
| " Vermont, white | 2650 | 165.57 |
| Marl, mean | 1750 | 109.375 |
| Mica.. | 2800 | 175. |
| Mortar.. | 1384 | 86.5 |
| | 1750 | 109.375 |
| Millstone.. | 2484 | 155.25 |
| Mud | 1630 | 101.875 |
| Nitre | 1900 | 118.75 |
| Opal | 2114 | — |
| Oyster shell.. | 2092 | 130.75 |
| Paving-stone. | 2416 | 151. |
| Peal, Oriental. | 2650 | — |
| Peat. | 600 | 37.5 |
| | 1329 | 83.062 |
| Phosphorus | 1770 | 110.625 |
| Plaster of Paris. | 1176 | 73.5 |
| Plumbago | 2100 | 131.25 |
| Porphyry, red. | 2765 | 172.812 |
| Porcelain, China | 2300 | 143 75 |
| Pumice-stone | 915 | 57.187 |
| Quartz | 2660 | 166.25 |
| Rotten-stone | 1981 | 123.812 |
| Red-lead | 8940 | 558.75 |
| Resin | 1089 | 68.062 |
| Rock, crystal | 2735 | 170.937 |
| Ruby. | 4283 | — |
| Salt, common | 2130 | 133.125 |
| Saltpetre | 2090 | 130.625 |
| Sand, coarse. | 1800 | 112 5 |
| " common.. | 1670 | 104.375 |
| " damp and loose.. | 1392 | 87. |
| " dried and loose. | 1560 | 97.5 |
| " dry | 1420 | 88.75 |
| " mortar, Ft. Rich. | 1659 | 103.66 |
| " " Brooklyn | 1716 | 107.25 |
| " sillicious | 1701 | 106.33 |
| Sapphire | 3994 | — |
| Shale | 2600 | 162.5 |
| Slate | 2900 | 181 25 |
| | 2672 | 167. |
| Slate, purple. | 2784 | 174. |
| Smalt. | 2440 | 152.5 |
| Stone, Bath......Engl.. | 1961 | 122.562 |
| " Blue Hill | 2640 | 165. |
| " Bluestone (basalt) | 2625 | 164.062 |
| " Breakneck..N.Y. | 2704 | 169. |
| " Bristol...... Engl. | 2510 | 156 875 |
| " Caen, Normandy | 2076 | 129.75 |
| " Common | 2520 | 157.5 |
| Stone, Craigleth..Engl. | 2316 | 144.75 |
| " Kentish rag " | 2651 | 165.687 |
| " Kip's Bay...N.Y. | 2759 | 172. |
| " Norfolk (Parliament House). | 2304 | 744. |
| " Portland...Engl. | 2368 | 148. |
| " Sandstone, mean | 2200 | 137.5 |
| " " Sydney | 2237 | 139.812 |
| " Staten Isl'd. N.Y | 2976 | 186. |
| " Sullivan Co. " | 2688 | 168. |
| Schorl | 3170 | 198.125 |
| Spar, calcareous | 2735 | 170.937 |
| " Feld, blue | 2693 | 168.312 |
| " " green. | 2704 | 169. |
| " " Fluor | 3400 | 215.5 |
| Stalactite | 2415 | 150 937 |
| Sulphur, native | 2033 | 127.062 |
| Talc, mean | 2500 | 156.25 |
| Ta'e, black. | 2900 | 181.25 |
| Tile | 1815 | 113.437 |
| Topaz, Oriental | 4011 | — |
| Trap. | 2720 | 170. |
| Turquoise | 2750 | — |
| **Miscellaneous.** | | |
| Asphaltum | 905 | 56.562 |
| | 1650 | 103.125 |
| Atmospheric Air.. | * | .07529 |
| Beeswax | 965 | 60.312 |
| Butter | 942 | 58.875 |
| Camphor.. | 988 | 61.75 |
| Caoutchouc | 903 | 56.437 |
| Egg | 1090 | — |
| Fat of Beef | 923 | 57.688 |
| " Hogs | 936 | 58.5 |
| " Mutton | 923 | 57.687 |
| Gamboge | 1222 | — |
| Gum Arabic. | 1452 | 90.75 |
| Gunpowder, loose | 900 | 56.25 |
| " shaken... | 1000 | 62.5 |
| " solid.. | 1550 | 96.875 |
| | 1800 | 112.5 |
| Gutta-percha | 980 | 61.25 |
| Horn. | 1689 | 105.562 |
| Ice, at 32° | 920 | 57.5 |
| Indigo | 1009 | 63.062 |
| Isinglass | 1111 | 69.437 |
| Ivory | 1825 | 114.062 |
| Lard | 947 | 59.187 |

(*) .001205.

| Miscellaneous. | Specific gravity. | Weight of a cubic foot. |
|---|---|---|
| Mastic | 1074 | 67.125 |
| Myrrh | 1360 | 85. |
| Opium | 1336 | 83.5 |
| Soap, Castile | 1071 | 56.937 |
| Spermaceti | 943 | 58.937 |
| Starch | 950 | 59.375 |
| Sugar | 1606 | 100.375 |
| " .66 | 1326 | 82.875 |
| | 972 | 60.25 |
| Tallow | 941 | 58.812 |
| Wax | 964 | 60.25 |
| | 970 | 60.625 |
| **Liquids.** | | |
| Acid, Acetic | 1062 | 66.375 |
| " Benzoic | 667 | 41 687 |
| " Citric | 1034 | 64.625 |
| " Concentrated | 1521 | 95.062 |
| " Fluoric | 1500 | 93.75 |
| " Muriatic | 1200 | 75. |
| " Nitric | 1217 | 76.062 |
| " Phosphoric | 1558 | 97.375 |
| " " solid | 2800 | 175. |
| " Sulphuric | 1849 | 115.562 |
| Alcohol, pure, 60° | 794 | 49.622 |
| " 95 per cent | 816 | 51. |
| " 80 " | 863 | 53.937 |
| " 50 " | 934 | 58.375 |
| " 40 " | 951 | 59 437 |
| " 25 " | 970 | 60.625 |
| " 10 " | 986 | 61 625 |
| " 5 " | 992 | 62. |
| " proof spirit, *50 per cent 60°. | 934 | 58.375 |
| " proof spirit, 50 per cent 80°. | 875 | 54.687 |
| Ammonia, 27.9 per ct. | 891 | 55 687 |

| Liquids. | Specific gravity. | Weight of a cubic foot. |
|---|---|---|
| Aquafortis, double | 1300 | 81.25 |
| " single | 1200 | 75. |
| Beer | 1034 | 64.625 |
| Bitumen, liquid | 848 | 53. |
| Blood (human) | 1054 | 65.875 |
| Brandy, $\frac{5}{8}$ or 5 of spirit | 924 | 57.75 |
| Cider | 1018 | 63.625 |
| Ether, acetic | 866 | 54.125 |
| " muriatic | 845 | 52.812 |
| " sulphuric | 715 | 44.687 |
| Honey | 1450 | 90 625 |
| Milk | 1032 | 64.5 |
| Oil, Anise-seed | 986 | 61.625 |
| " Codfish | 923 | 57 687 |
| " Cotton-seed | — | — |
| " Linseed | 940 | 58.75 |
| " Naphta | 848 | 53. |
| " Olive | 915 | 57 187 |
| " Palm | 969 | 60.562 |
| " Petroleum | 878 | 54.875 |
| " Rape | 914 | 57.125 |
| " Sunflower | 926 | 57.875 |
| " Turpentine | 870 | 54.375 |
| " Whale | 923 | 57.687 |
| Spirit, rectified | 824 | 51 5 |
| Tar | 1015 | 63.437 |
| Vinegar | 1080 | 67.5 |
| Water, Dead Sea | 1240 | 77.5 |
| " 60° | 999 | 62.449 |
| " 212° | 957 | 59.812 |
| " distilled, 39°† | 998 | 62 379 |
| " Mediterranean | 1029 | 64.312 |
| " rain | 1009 | 62.5 |
| " sea | 1026 | 64.125 |
| Wine, Burgundy | 992 | 62. |
| " Champagne | 997 | 64.375 |
| " Madeira | 1038 | 62.312 |
| " Port | 997 | 62.312 |

Compression of the following fluids under a pressure of 15 lbs. per square inch:

| | | | |
|---|---|---|---|
| Alcohol | 0000216 | Mercury | 00000265 |
| Ether | 0000158 | Water | 00004663 |

* Specific gravity of proof spirit according to Ure's Table for Sykes's Hydrometer, 920.

† 1 cubic inch = .252.69 Troy grains.

## Elastic Fluids.

**1† *Cubic Foot of Atmospheric Air weighs 527.04 Troy Grains.***
***Its assumed Gravity of 1 is the Unit for Elastic Fluids.***

| | | | |
|---|---|---|---|
| Atmospheric air, 34° | 1. | Phosphureted hydrogen | 1.77 |
| Ammonia | .589 | Sulphureted " | 1.17 |
| Azote | .976 | Sulphurous acid | 2.21 |
| Carbonic acid | 1.52 | Steam, * 212° | .4883 |
| " oxyd | .972 | Smoke, of bituminous coal | .102 |
| Carbureted hydrogen | .559 | " coke | .105 |
| Chlorine | 2.47 | " wood | .09 |
| Chloro-carbonic | 3.389 | Vapor of alcohol | 1.613 |
| Cyanogen | 1.815 | " bisulphuret of carbon | 2.64 |
| Gas, coal | .4<br>.752 | Vapor of bromine | 5.1 |
| | | " chloric ether | 3.44 |
| Hydrogen | .07 | " ether | 2.586 |
| Hydrochloric acid | 1.278 | " hydrochloric ether | 2.255 |
| Hydrocyanic " | .942 | " iodine | 8.675 |
| Muriatic acid | 1.247 | " nitric acid | 3.75 |
| Nitrogen | .972 | " spirits of turpentine | 4.763 |
| Nitric oxyd | 1.094 | " sulphuric acid | 2.7 |
| Nitrous acid | 2 638 | " " ether | 2.586 |
| Nitrous oxyd | 1.527 | " sulphur | 2.214 |
| Oxygen | 1.102 | " water | .623 |

## Weights and Volumes of various Substances in Ordinary Use.

| Substances. | Cubic Foot. | Cubic Inch. | Substances. | Cubic Foot. | Cubic Inch. |
|---|---|---|---|---|---|
| **Metals.** | **Lbs** | **Lbs.** | **Metals.** | **Lbs.** | **Lbs.** |
| Brass { copper 67 | 488.75 | 2829 | Tin | 455.687 | .2637 |
| { zinc 33 } | | | Zinc, cast | 428.812 | .2482 |
| " gun metal | 543.75 | .3147 | " rolled | 449.437 | .2601 |
| " sheets | 513.6 | .297 | | | |
| " wire | 524.16 | .3033 | **Woods.** | | Cub. Feet in a Ton. |
| Copper, cast | 547.25 | .3179 | | | |
| " plates | 543.625 | 3167 | Ash | 52.812 | 42.414 |
| Iron, cast | 450.437 | .2607 | Bay | 51.375 | 43.601 |
| " gun metal | 466.5 | .27 | Cork | 15. | 149.333 |
| " heavy forging | 479.5 | .2775 | Cedar | 35.062 | 63.886 |
| " plates | 481.5 | .2787 | Chestnut | 38.125 | 58.754 |
| " wrought bars | 486.75 | 2816 | Hickory, pig nut | 49.5 | 45.252 |
| Lead, cast | 709.5 | .4106 | " shell-bark | 43.125 | 51.942 |
| " rolled | 711.75 | .4119 | Lignum-vitæ | 83.312 | 56.886 |
| Mercury, 60° | 848.7487 | .491174 | Logwood | 57.062 | 39.255 |
| Steel, plates | 487.75 | .2823 | Mahogany, Hon- | 35. | 64. |
| " soft | 489.562 | .2833 | duras | 66.437 | 33.714 |

† Equal to .07529143 lbs. avoirdupois. * Weight of a cubic foot, 257,333 Troy grains.

| Substances. | Cubic Foot. | Cub. Feet in a Ton. | Substances. | Cubic Foot. | Cub. Feet in a Ton. |
|---|---|---|---|---|---|
| Oak, Canadian | 54.5 | 41.101 | Coal, Welsh, mean | 81.25 | 27.56 |
| " English | 58.25 | 38.455 | Coke | 63.5 | 35.84 |
| " live, seasoned | 66 75 | 33.558 | Cotton, bale, mean | 14.5 | 154.48 |
| " white, dry | 53.75 | 41.674 | " " pressed { | 20. | 114. |
| " " upland | 42.937 | 52.169 | | 25. | 89.6 |
| Pine, pitch | 41.25 | 54.303 | Earth, clay | 120.625 | 18.569 |
| " red | 36.875 | 60.745 | " common soil | 137.125 | 16.335 |
| " white | 34.625 | 64.693 | " " gravel | 109.312 | 20.49 |
| " well seasoned | 29.562 | 75.773 | " dry, sand | 120. | 18.667 |
| " yellow | 33 812 | 66.248 | " loose | 93.75 | 23 893 |
| Spruce | 31.25 | 71.68 | " moist, sand. | 128.125 | 17.482 |
| Walnut, black, dry | 31.25 | 71.68 | " mold | 128.125 | 17.482 |
| Willow | 36.562 | 61.265 | " mud | 101 875 | 21.987 |
| " dry | 30.375 | 73.744 | " with gravel. | 126.25 | 17.742 |
| | | | Granite, Quincy | 165.75 | 13 514 |
| **Miscellaneous** | | | " Susqueh'na | 169. | 13.254 |
| | | | Hay, bale | 9.525 | 235.17 |
| Air | 075291 | — | " pressed | 25. | 89.6 |
| Basalt, mean | 175. | 12.8 | India rubber | 56.437 | 39.69 |
| Brick, fire | 137.562 | 16.284 | " vulcanized | — | — |
| " mean | 102. | 21.961 | Limestone | 197.25 | 11.355 |
| Coal, anthracite { | 89 75 | 24.958 | Marble, mean | 167 875 | 13 343 |
| | 102.5 | 21.854 | Mortar, dry, mean | 97.98 | 22.862 |
| " bitumin., mean | 80. | 28 | Water, fresh | 62.5 | 35.84 |
| " Cannel | 94.875 | 23.609 | " salt | 64.125 | 34.931 |
| " Cumberland | 84 687 | 26.451 | Steam | .036747 | — |

## Application of the Tables.

*When the Weight of a Substance is required.* RULE. — Ascertain the volume of the substance in cubic feet; multiply it by the unit in the second column of tables, and divide the product by 16; the quotient will give the weight in pounds.

*When the Volume is given or ascertained in Inches.* RULE.—Multiply it by the unit in the third column of the tables, and the product will be the weight in pounds.

EXAMPLE.—What is the weight of a cube of Italian marble, the sides bieng 3 feet?

$3^3 \times 2708 = 73116$ *oz.*, which $\div\ 16 = 4569.75$ *lbs.*

Or of a sphere of cast iron 2 inches in diameter?

$2^3 \times .5236 \times .26$ *weight of a cubic inch* $= 1.089$ *lbs.*

## Comparative Weight of Timber in a Green and Seasoned State.

| Timber. | Weight of a Cub. Ft. Green. | Weight of a Cub. Ft. Seasoned. | Timber. | Weight of a Cub. Ft. Green. | Weight of a Cub. Ft. Seasoned. |
|---|---|---|---|---|---|
| | Lbs. Oz. | Lbs. Oz. | | Lbs. Oz. | Lbs. Oz. |
| American Pine | 42.12 | 30.11 | Cedar | 32. | 28.4 |
| Ash | 58.3 | 50. | English Oak | 71.10 | 43.8 |
| Beech | 60. | 53.6 | Riga Fir | 48.12 | 35.8 |

## To Compute the Capacity of a Balloon.

RULE.—From specific gravity of the air in grains par cubic foot substract that of the gaz with which it is inflated; multiply the remainder by the volume of the balloon in cubic feet; divide the product by 7000, and from the quotient substract the weight of the balloon and its attachments.

EXAMPLE—The diameter of a balloon is 26.6 feet, its weight in 100 lbs., and the specific gravity of the gaz with which it is inflated is .06 (air being assumed at 1); what is its capacity?

$$\frac{527.04-31.62\times\overline{26.6^3\times .5236}}{7000}-100=\frac{495.42\times 9854.726}{7000}-100=597.461\ lbs.$$

## To Compute the Diameter of a Balloon, the Weight to be raised being given.

By inversion of the preceding rule,

$\sqrt[3]{\frac{W\times 700\div\overline{s-s'}}{.5236}}=d$, *s and s' representing the weight of air and gas in grains per cubic foot, and d the diameter of the balloon in feet.*

EXAMPLE.—Given the elements in the preceeding case

$$\text{Then}\ \frac{\sqrt[3]{597\ 46+100\times 7000\div\overline{527.04-31.62}}}{.5236}=\sqrt[3]{18821.09}=26.6\ feet.$$

## To Compute the Weight of Cast Metal by the Weight of the Pattern.

*When the Pattern is of White Pine.*

RULE.—Multiply the weight of the pattern in pounds by the following multiplier, and the product will give the weight of the casting:

Iron, 14; Brass, 15; Lead, 22; Tin, 14; Zinc, 13.5.

*When there are Circular Cores or Prints.*— Multiply the square of the diameter of the core or print by its length in inches, the product by .0175, and the result is the weight of the pattern of the core or print to be deducted from the weight of the pattern.

It is customary, in the making of patterns for castings, to allow for shrinkage per lineal foot of pattern.

Iron and Lead $\frac{1}{8}$th of an inch, Brass and Zinc $\frac{3}{16}$ths, and Tin $\frac{1}{12}$th.

## PROBLEM.

**To determine the accurate solidity of any irregular body of small dimensions or of a body composed of several elementary parts with different dimensions and forms.**

(1) **RULE. If it is the capacity of any vase or vessel which we want to measure,** *the idea generally suggest itself of arriving at the result by determining the number of times which such a vessel can give place to or contain the contenis of any other vessel of an elementary form of which we know the capacity.*

(2) **But if it is the solidity of the substance itself of the vessel, &c., which we desire to measure.** the manner of operating does not immediately present itself to the mind of any one wishing to obtain the result.

(3) **RULE. If the solidity to be measured is that of a non absorbent substance,** *we immerse it in a vessel full of water or any other liquid of which we will measure the displacement by means of another vessel of known capacity ; or if the first vessel is large enough and it form rectangular or cylindrical and of easy gauging, we will first put in it enough liquid to cover the object to be measured ; having afterwards observed the height of the level of the water in the vessel, we will immerse in it the object in question and observe again the level of the liquid ; if now we suppose that each fraction of a metre, inch line or any other unit of the height of the containing vessel corresponds to a cubic metre, foot, inch, or line, &c., we will have but to count the number of such units in the height of the displaced level of the water to obtain immediately the solidity of the proposed object.*

(4) **If the body is absorbent,** *we may for instance use sand or any other fluid substance, of the kind, that we can level the surface of by means of a rod with a rectilineal edge.*

In this manner we would arrive at the solidity of the most deversified bodies of the animal, vegetable or mineral kingdom and of the thousand and one raw or manufactured objects which we have constantly under o and of which it would often be impossible to measure the solidities by the ordinary rules of geometry.

It is well to remind also that we may arrive by a simple proportion at the solidity of a body by comparing its weight with that of another body of the same substance and of determined solidity, that is by the system of specific gravities which shows at the same time how to obtain the solidity of a body from its weight : which will form the subjects of the next problem.

**Ex. 1.** The weight of an irregular block of stone is 13 pounds 7 ounces : required to determine with the help of the given piece the weight nearly of a cubic foot of such stone.

**Ans.** First cube the block of stone ; to that effect get a rectangular vessel, say 10 inches square or 100 inches in horizontal area, and the height of which is divided into inches and hundreths of an inch ; having poured into the vessel water enough to cover the stone to be cubed, I note the height of the water which I find 8.53 inches, I then immerse the stone in the vessel and I note again the height of the water which is now 9.89 inches ; the difference of these heights is 1.36 inches. Since the vessel is $10 \times 10$ inches, it is plain that every inch of its height corresponds to 100 cubic inches and consequently, each hundredth of an inch of such a height to one cubic inch ; therefore the observed height 1.36, of the displaced level of the water corresponds to 136 cubic inches ; therefore the solidity of the stone is 136, and we will now obtain the weight of the cubic foot by making 136 : 215 ounces (weight of the stone) : : 1728 cubic inches (that is a cubic foot) : 2732 ounces, or, dividing by 16, $170\frac{3}{4}$ pounds, the required weight.

**2.** In a cylendrical vessel such that each inch of its height corresponds to 1 cubic inch of space or solidity, we have immersed a piece of silver which has displaced by 73 hundreths of an inch the level of the liquid in the vase ; required the solidity of the ingot of silver ?

**Ans.** .73 of a cubic inch.

**3.** Having filled with water any vessel, we have immersed in it an object the solidity of which we want to know ; we have gathered in another vessel, the water overflown, the quantity of which is 3 gal. 2 quarts and $\frac{1}{2}$ pint ; what is the soiidity of the proposed object, the gallon made use of being 231 cubic inches ?

**Ans.** 1 gallon + 2 quarts + $\frac{1}{2}$ pint $= 231 + 115\frac{1}{2} + 14\frac{7}{16} = \frac{15}{16}$ cubic inches.

**4.** Required the solidity of an absorbent substance placed in a vessel one foot square filled with sand ; after having removed the object to be measured, we find that the uniform height of the sand in the vessel, first levelled to that effect, is .3 of a foot, the height of the vessel being 1.5 feet ?

**Ans.** $1.5 - 3 = 1.2$ feet $=$ height of the displaced level of the sand, and as the vessel is 1 square foot in horizontal section, it follows that the solidity of the object is 1.2 cubic feet.

**5.** In a vessel having the form of the frustum of a cone is a quantity of liquid of which the diameter at the surface is 10 inches : we immerse in it an object which increases by 9 inches the height or depth of the liquid in the vessel and which gives to its displaced surface a diameter of 14 inches ; required the solidity of the proposed body ?

**Ans.** The volume of water displaced which is at the same time that of the object, is that of the frustum of a cone of which the parallel bases measure respectively 10 and 14 inches and of which the height is 9 inches ; this sol.= (**112**, **T.**) $(10^2 + 14^2 + 4 \text{ times } 12)^2 \times 7854 \times 9 \div 6 = 872 \times .7854 \times 1.5$ 684.8688 × 1.5= 1027.3032 cubic inches.

## THEOREM.

### To determine the solicity or weight of any body or substance, by comparing the volume or weight of such body with that of a body or substance of the same nature of which we know beforehand the weight and volume.

**(5) REm.** The weight of a cubic foot of water at the temperature of 40° Fahrenheit (at which water nearly reaches its greatest density) is 1000 *ounces avoir du poids* nearly, or 62½ pounds (english weight) and we denominate weight or specific gravity of any body or substance, the weight of a volume of such body or substance equal to that of the water taken for comparison ; whence it results that if in advance we know the weight of a cubic foot, for instance, of each of the different substances that we may be called on to measure or value, as stated in table X, we wi l at once determine by a simple proportion the volume of any other weight or quantity of the same substance or the weight of any other volume of such substance, by the following rules.

**(6) RULE. To determine the solidity of a body from its weight ;** *make the proportion : the specific weight of the proposed body is to (:) its weight in ounces or pounds, &c , as (: :) 1 cubic foot or 1728 cubic inches, is to (:) the solidity of the body in feet or inches, as the case may be.*

**Ex. 1.** The weight of a shell or cast iron ball or of any fragment of such a solid is 45 pounds : required the solidity of the proposed body ?

**Ans.** It is seen by table X of specific gravities that the weight of cast iron is 450 pounds nearly, per cubic foot ; we will then obtain the required solidity by making 450 pounds : 1728 cubic inches : : 45 pounds : 172.8 cubic inches.

**2**. Required the volume of a marble statue the weight of which is 1000 pounds, the specific gravity of the marble from which the statue is drawn being 170 pounds nearly to the cubic foot ?

**Ans**. 170 pounds : 1 cubic foot : : 1000 pounds : 5 9 cubic feet nearly.

**3**. A quantity of sand weighs 13 pounds : what is its solidity ?

**Ans**. From table X, the specific gravity of sand is 1.520, that is, 1.520 times the weight of an equal volume of water or 1520 ounces to the cubic foot (since the weight of a cubic foot of water is 1000 ounces) ; we will therefore make 1520 ounces : 1728 cubic inches : : (13 × 16 =) 208 ounces : x = $\frac{1728 \times 208}{1520}$ = $236\frac{1}{2}$ cubic inches.

**4**. The weight of a tusk or tooth of an elephant is 25 pounds ; what is its solidity ?

**Ans**. Ivory is 1825 ounces to the cubic foot ; we will therefore obtain the solidity of the tooth by making 1825 : 1 : : (25 pounds or) 400 ounces : .22 nearly of a cubic foot, or 1825 ounces : 1728 cubic inches : : 400 ounces : 378.74 cubic inches.

**5**. It is required to determine in advance the probable weight of a cast iron grating which must be cast according to a carved model of pine wood the weight of which is 7 pounds ?

**Ans**. We will first obtain the solidity of the pine model by making, as per rule (the pine being considered in this case as of 25 pounds to the cubic foot) 25 pounds : 1 cubic foot : : 7 pounds : .28 of a cubic foot. Now, as the solidity of the cast iron is 450 pounds per cubic foot, we will obtain the weight of the proposed grating = 450 × .28 = 126 pounds.

**(7) RULE. To determine the weight of a body from its volume** ; make the proportion : as one cubic foot is to ( : ) the volume of the proposed body, so is ( : : ) its specific gravity to ( : ) its weight.

**Ex. 1**. The volume of a heap of snow on the roof of a building is 7000 cubic feet, the weight of a cubic foot of this snow, made heavy by rain, &c. is 30 pounds required the total weight which bears on the roof ?

**Ans**. 7000 = 210,000 pounds.

**2**. What is the weight of a piece of pure cast gold the dimensions of which are 3 inches by $\frac{3}{4} \times \frac{1}{2}$ inches ?

**Ans**. The solidity = $3 \times \frac{3}{4} \times \frac{1}{2} = 2\frac{3}{4}$ cubic inches ; the specific gravity of pure gold is 19.258 ; the rule gives : 1 cubic foot or 1728 cubic inches : $2\frac{1}{4}$ cubic inches : : 19.258 : x = $\frac{19.258 \times 2.25}{1728}$ = 25.07552 ounces

**3**. One desires to know the weight of a firkin of butter the volume of which obtained from the rule to article (112), is 1830 cubic inches ?

**Ans.** The specific weight of the butter is .940 of that of water, that is, of 940 ounces to the cubic foot; we will therefore obtain the required weight $= \frac{1830 \times 940}{1728} = 995\frac{1}{2}$ ounces, $\div 16 = 62$ pounds $3\frac{1}{2}$ ounces.

**4.** What is the weight nearly of a stick of english oak half-dry, the volume of which is 150 cubic feet?

**Ans.** The half-dry oak, from the table, is 66 pounds nearly per cubic foot, whence the required weight, is $150 \times 66 = 9900$ pounds.

**5.** What is the weight nearly of a box of bound books the volume of which is 15 cubic feet?

**Ans.** 15 cubic feet $\times$ 43 pounds nearly $= 645$ pounds.

## PROBLEM.

### To determine the specific gravity of any body or substance.

(8) **RULE. I.** *Cube and weight the proposed body, and afterwards make this proportion; as the solidity of the body is to ( : ) its weight in ounces, so is ( : : ) a cubic foot of such body to ( : ) the weight of one foot of it in ounces; that is, by cutting off three figures for deci-its specific gravity.*

**Ex. 1.** What is the specific weight of seasoned black walnut, if a simple of this wood the dimensions of which are $11 \times 7 \times 9$ inches, weighs 24 ounces?

**Ans.** $11 \times 7 \times 9 = 69.3$ cubic inches = sol. of the proposed body; now, from the rule 69.3 inches : 24 ounces : : 1728 inches : 598 ounces or 37.4 pounds; the required specific gravity is therefore .598 of that of water the weight of which is 1000 ounces to the cubic foot.

**2.** An irregular piece ef chalk of which the solidity has been obtained, $=432$ cubic inches, by the method of exemple 4 of the last but one problem, weighs $43\frac{1}{2}$ pounds : required the specific gravity of that substance.

**Ans.** 432 inches : 1728 inches : : $43\frac{1}{2}$ pounds : 174 pounds : whence, the required specific gravity is $174 \times 16 = 2.784$ times the weight of an equal volume of water.

**3.** A bateau or pontoon of 100 feet by $20 \times 10$ feet and the total volume of which is consequently 20,000 cubic feet, required in its construction 5000 feet of white pine half-seasoned, the weight of which is estimated at 40 pounds for the cubic foot, 500 cubic feet of elm computed at 50 pounds to the cubic foot, and 5000 pounds weight of iron spikes: required the draught of water of the proposed body?

**Ans** The weight of the pine $= 5000 \times 40 = 200{,}000$ pounds, the weight of the elm $= 500 \times 50 = 25000$, the iron 5000 pounds; the total weight of the bateau is consequently 230,000 lbs; the average weight or the specific gravity of the pontoon is 230,000 pounds ÷ 20,000 cubic feet = 11.5 pounds to the cubic foot, that is 11.5 × 16 = 184 ounces per cubic foot, say .184 of the weight of an equal volume of water. The weight of the pontoon is 10 feet, therefore the draught will be .184 of the height of the pontoon or 1.84 feet, that is 1 foot 10 inches and .96 of an inch = 1 foot 11 inches nearly.

4. By what quantity can the bateau or pontoon of the last example be loaded without causing it to founder or sink beyond its deck or superior surface?

**Ans.** Since water weighs 62.5 pounds to the cubic foot and the total volume of the pontoon is 20,000 cubic feet, the total weight of the water which the pontoon must displace before sinking to the lever of the water is 20,000 × 62.5 = 1,250,000 pounds; now the weight of the boat is but 230,000 pounds; whence it follows that we might still without causing the bateau to founder load it with a weight equal or nearly equal to the difference between 1250,000 pounds and 230,000 that is 1020,000 pounds.

(9) **RULE II. If the body to be computed is heavier than water;** *first weigh the body in air, then in water, by means of a hydraulic balance; the difference between the results will be the weight lost in water, or the weight of a quantity of water equal in volume to that of the body. Make now the proportion: as the weight lost in water (:) is to the weight of the body in air (::) so is the specific gravity of water (:) to the specific gravity of the body.*

**Ex 1.** A piece of tin weighs 183 pounds, its weight in water is but 158 pounds: what is the specific gravity of tin?

**Ans.** 183—158=25 : 183 :: 1000 : 7320=required specific gravity.

**2.** A block of granite weighs 21 ounces in air and only 13 ounces in water: what is the specific gravity of the granite?

**Ans.** 2625

(10) **RULE III. If the body to be computed is lighter than water;** *tie to the proposed body by a thread the weight of which is relatively null, another body heavier than water, so that both of them taken together may penetrate or sink in the water; having first weighed each body in air, and the heavier in water, weigh then in water the compound body, and from the weight lost by the compound body, substract the weight lost by the heavier body as weighed alone; the remainder is the weight lost by the light body. Then: as the weight lost by the light body in water, (:) is to the weight of that body in air, (::) so is the specific gravity of water (:) to the specific gravity of the body.*

**Ex. 1.** To a piece of elm which in *air* weighs 15 grains, we have tied a piece of copper the weight of which is 18 grains *in air* and 16 grains *in water*, and the compound in water weighs but 6 grains: what is the specific gravity of the elm?

**Ans.** 18—16 = 2 = the number of grains lost by the copper *in the water.*

18 + 15—6 = 27 = the number of grains lost by the compound *in the water.*

27—2 = 25 = the number of grains lost by the elm *in the water.*

25 : 15 :: 1000 : 600 = the specific gravity of the elm.

**2.** A piece of copper, weighing in air 27 ounces and in water 24 ounces, is tied to a piece of cork weighing in air 6 ounces, and the compound weighs in water but 5 ounces : what is the specific gravity of cork?

**Ans.** 0.240.

## PROBLEM.

### To determine the quantity of each ingredient or element in a compound of two substances or elements.

(11) **RULE.** *Find first the specific weight of the compound, mixture or alloy, and of each of the component elements and multiply the difference of every two of these three specific weights by the third. Make then : the greatest product,* ( : ) *is to each of the other product,* ( :: ) *as the weight of the alloy,* ( : ) *is to the weight of each ingredient.*

**Ex. 1.** A mass of gold and silver weighs 62 ounces, and its specific gravity is 16126 ; what is the quantity of each ingredient, the specific gravity of gold being 19640, and that of silver 11091 ?

**Ans.** (19640—11091) × 16126 = 137,861,174. Alloy.
(19640—16126) × 11091 = 38,973,774. Silver.
(16126—11091) × 19640 = 98,887,400. Gold.

137,861,174 : 98,888,400 :: 63 : 45 ounces, 3 penny weights, 19 grains of gold.
137,861,174 : 38,973,774 :: 63 : 17 ounces, 16 penny weights, 5 grains of silver.

**2.** A mass of copper and gold weighs 48 ounces, and its specific gravity is 17150, the specific gravity of gold is 19640 and that of copper 9000 : what is the quantity of each element of the mixture?

**Ans.** Gold = 42 ounces 2 penny weights 2 $\frac{20170}{45619}$ grains, copper = 5 ounces, 17 penny weights 21 $\frac{25449}{45619}$ grains.

**3.** An alloy of silver and copper weighs 60 ounces, its specific gravity being 10535 : required the weight of each ingredient, their respective specific gravities being 11091 and 9000 ?

**Ans.** 46 ounces 7 penny-weights 9 $\frac{1232397}{1468579}$ grains silver, 13 ounces 12 penny-weights 14 $\frac{236182}{1468579}$ of copper.

**4.** An alloy of copper and tin weighs 112 pounds and its specific gravity is 8784, what is the quantity of each of the ingredients of the mixture, their respective specific gravities being 9000 and 7320 ?

**Ans.** 100 pounds copper, 12 pounds tin.

**5.** Required the weight of gold, in a compound of quartz and gold the specific gravity of which is 3500, that of gold being 19640 and that of quartz 3000 ?

**Ans.** $19640-3000=16640\times3500=58,240,000=$
Factor for the compound body.

$19640-3500=16140$, $16140\times3000=48,420,000=$
Factor for the quartz.

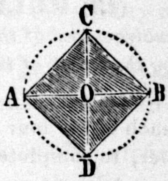
$3500-3000=500$, $500\times19640=9,820,000=$
Factor for the gold.

58240000 : 9820000 :: 100 : 16.86.8612638 – ounces of gold ; if this result be correct, the weight of the quartz must be equal to the difference between the weight of the gold and that of the alloy, and in fact 58240000 : 48420000 :: 100 : 83.1387362 + ounces of quartz ; the sum of these numbers = 100 ; there fore, &c.

## PROBLEM.

### To determine the solidity of the largest piece of squared timber that may be got out of a round log, or out of felled or standing tree.

**(12) RULE.** *Multiply the diameter of the tree or log by the half-diameter, and this product by the lengh : the result will be the required solidity.*




In fact, it is plain that the diam. AB multiplied by the half-diameter OC (or ½ AB) given for product the area of the inscribed square ABCD, that is, the area of a section, of the timber to be computed, by a plane perpendicular to its length, and that area multiplied by the length of the log gives **(78 T.)** the required solidity.

**REM.** This rule supposes that the diam. of the tree is every where the same or that we make use of a mean diameter, as taken at middle of the length, and this generally done when there is not too much difference between the diameters of the opposite ends ; but to be precise **(148, T.)** we must as already stated **(91, T.)** add to the sum of the areas of the ends of the log or tree to be measured four times the area of a section taken at the centre and multiply the whole by the sixth part of the length, or which is the same thing, multiply the sum of the areas by the whole length and take the sixth part of the result.

**Ex. I.** The circumference of a log, the length of which is 12 feet, is 6.28 feet, deduction being made of the bark if necessary : how many cubic feet of wood will there be in the stick of squared timber to be got out of the log ?

**Ans.** The circ. 6.28 corresponds to a diam. 2, the section of the timber will therefore be $2 \times 1 = 2$ square feet in area, and as the length is 12, the solidity will be 24 cubic feet.

**2**. A tree the height of which is 50 feet, has for its sup. diam. 30 inches, and for its inf. diam. 36 inches, for its interm. diam. 33 inches; what is the solidity of the piece of square timber that may be got out of it.

**Ans**. Area small end $= 2\frac{1}{2} \times 1\frac{1}{4}$ feet $= 3.125$ sup. feet, area large end $= 3 \times 1\frac{1}{2} = 4{,}5$ sup. feet, intermediat area $= 2.75 \times 1.375 = 3.78125$, 4 intermediate area $= 15.125$, the sum of the areas $= 22.75$ and that sum $\times 50 \div 6 = 189.6$ cubic feet.

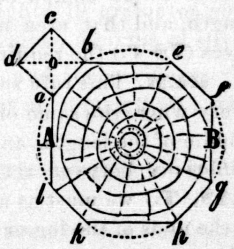
**3**. We have measured at 5 places nearly equidistant by means of a thickness compass, the diam. of an irregular tree just felled; these diameters are respectively 39, $30\frac{1}{2}$, 38, $37\frac{1}{2}$ and 36 inches, and the length of the tree 40 feet; what will its solidity be after it has been squared.

**Ans**. The sum of the diameters 190 inches $\div 5 = 38$ inches $=$ mean diam. $= 3\frac{1}{6}$ feet, $3.166 \times 1.583 = 5.012$ nearly $=$ area of the section; multiplying this latter by the length 40, we get $200\frac{1}{2}$ cubic feet.

## PROBLEM.

### To cube a stick of timber AB which is but partly squared, or of which the edges or angles are wanting, called " waney timber."

**(13) RULE.** *Square the diam.* AB *of the timber, and from such square subtract that of the diam. ab of the sapwood, the difference of these squares multiplied by the length of the timber, will be the required solidity.*




In fact, it is plain that the surface wanting at each of the four angles, corners of edges of the timber, to complete the square A B, is the triangle *abo*, or a triangle equal to *abo*, when as it is supposed, $ef = gh = kl = ab$; now the square on *ab* is worth 4 *abo*; therefore, &c.

**REM. 1**. If the sides *ab*, *ef*, &c. are not equal to each other, we may take one fourth of the sum of these four sides for a mean diameter *ab*, or for greater accuracy, we will make separately the squares of *ab*, *ef*, &c., and the fourth of the sum of those squares will be, or the sum of the fourths of those squares will be the quantity, nearly, to be subtracted from the square AB to obtain the net area of the section of the timber.

**REM. II**. Let us observe as in the last problem that if the timber is not throughout its entire length of equal size, its section must be taken at about the middle of its length, and this is generally what is done (**148 T.**) or, we will determine several sections of the timber and then take their mean, or finally we will make the sum of the areas of the opposite ends plus four times that of the intermediate section and afterwards multiply the whole by the length and take the sixth part of the result.

**REM. III.** We must also observe that we may arrive at the area of any regular or symmetrical octagon or of the kind here illustrated by subtracting from the square of the perpendicular distance AB which separates any two of its parallel sides, the square of one *ab* of the sides adjacent to the first.

**Ex. 1.** An eight sides pilar is 3 feet wide or thick AB, the side *ab* of the chamfer *aob* is 6 inches : what is the solidity of the pillar, its length or height being 10 feet ?

**Ans.** $(3+3-(.5\times .5)=8.75$ superficial feet, and $8.75\times 10=87.5$ cubic feet=required solidity.

**2.** A log of timber the edges of which are waney, measures 30 inches square and 30 feet long, the average of the sides *ab, ef,* &c., of the wane is 9 inches ; what is the solidity of the timber ?

**Ans.** $(30\times 30)$ minus $(9\times 9)=919$ square inches = area of the section of the timber=6.382 feet very nearly, and $6.382\times 30=171.46$ cubic feet.

**3.** We have reduced to 30 inches square at the large end a tree the diam, of which was at that point 36 inches ; at the small end the diam. 30 inches has been reduced to 25 inches ; the wane, sapwood or defect from a true square *ab* is from 7 to 6 inches respectively at the two ends, such as obtained by a direct measurement of the piece of wood to be cubed, or by means of a sketch made from a scale of equal parts : what is the solidity of the timber, its length being 60 feet ?

**Ans.** Area at the large end $=(30\times 30)-(7\times 7)=851$ square inches, area at small end $=(25\times 25)-(6\times 6)=589$ sq. f., the intermediate area $\left(\frac{30+25}{2}\times\frac{30+25}{2}\right)-\left(\frac{7+6}{2}\times\frac{7+6}{2}\right)=(27\frac{1}{2}\times 27\frac{1}{2})-(6\frac{1}{2}\times 6\frac{1}{2})=27.5^2-6.5^2=$ $756.25-42.25=714$ ; $851+859+4$ times $714=4296$ square inches, dividing by 144 we obtain 29.833 square feet, multiplying by $\frac{1}{6}$ of the length or by 10 we obtain 298.33 cubic feet.

**Ans.** Area section at the centre = 714 square inches, $714\div 144=4983$ square feet, $4.9583\times 60=297.498$ cubic feet, that is, equal to the accurate solidity by less than one foot nearly, or by less than one 300th nearly, or by less than one third nearly of 1 per cent, sufficient accuracy (**148. T.**) in practice.

**REM. IV.** A comparison of the two answers of the last problem indicates sufficiently that the ordinary practice of cullers, who take the dimensions of a log at the middle of its length, and afterwards multiply the area of the section at that place by the length of the timber, to obtain thus its solidity, is, considering all things, (**148 T.**) sanctioned by circumstances.

# INDEX



# TABLES