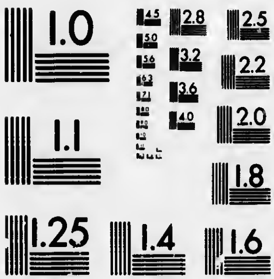
IMAGE EVALUATION
TEST TARGET (MT-3)



Technical and Bibliographic Notes/Notes techniques et bibliographiques

The Institute has attempted to obtain the best original copy available for filming. Features of this copy which may be bibliographically unique, which may alter any of the images in the reproduction, or which may significantly change the usual method of filming, are checked below.

L'Institut a microfilmé le meilleur exemplaire qu'il lui a été possible de se procurer. Les détails de cet exemplaire qui sont peut-être uniques du point de vue bibliographique, qui peuvent modifier une image reproduite, ou qui peuvent exiger une modification dans la méthode normale de filmage sont indiqués ci-dessous.

- [ ] Coloured covers/ Couverture de couleur
- [ ] Covers damaged/ Couverture endommagée
- [ ] Covers restored and/or laminated/ Couverture restaurée et/ou pelliculée
- [ ] Cover title missing/ Le titre de couverture manque
- [ ] Coloured maps/ Cartes géographiques en couleur
- [ ] Coloured ink (i.e. other than blue or black)/ Encre de couleur (i.e. autre que bleue ou noire)
- [ ] Coloured plates and/or illustrations/ Planches et/ou illustrations en couleur
- [x] Bound with other material/ Relié avec d'autres documents
- [ ] Tight binding may cause shadows or distortion along interior margin/ La reliure serrée peut causer de l'ombre ou de la distorsion le long de la marge intérieure
- [ ] Blank leaves added during restoration may appear within the text. Whenever possible, these have been omitted from filming/ Il se peut que certaines pages blanches ajoutées lors d'une restauration apparaissent dans le texte, mais, lorsque cela était possible, ces pages n'ont pas été filmées.

- [ ] Coloured pages/ Pages de couleur
- [ ] Pages damaged/ Pages endommagées
- [ ] Pages restored and/or laminated/ Pages restaurées et/ou pelliculées
- [x] Pages discoloured, stained or foxed/ Pages décolorées, tachetées ou piquées
- [ ] Pages detached/ Pages détachées
- [x] Showthrough/ Transparence
- [ ] Quality of print varies/ Qualité inégale de l'impression
- [ ] Includes supplementary material/ Comprend du matériel supplémentaire
- [ ] Only edition available/ Seule édition disponible
- [ ] Pages wholly or partially obscured by errata slips, tissues, etc., have been refilmed to ensure the best possible image/ Les pages totalement ou partiellement obscurcies par un feuillet d'errata, une pelure, etc., ont été filmées à nouveau de façon à obtenir la meilleure image possible.

- [x] Additional comments:/ Commentaires supplémentaires: Original includes calculation (table and protractor) but has not been filmed.

This item is filmed at the reduction ratio checked below/
Ce document est filmé au taux de réduction indiqué ci-dessous.

10X 12X 14X 16X 18X 20X 22X 24X 26X 28X [x] 30X 32X

The copy filmed here has been reproduced thanks to the generosity of:

Seminary of Quebec
Library

The images appearing here are the best quality possible considering the condition and legibility of the original copy and in keeping with the filming contract specifications.

Original copies in printed paper covers are filmed beginning with the front cover and ending on the last page with a printed or illustrated impression, or the back cover when appropriate. All other original copies are filmed beginning on the first page with a printed or illustrated impression, and ending on the last page with a printed or illustrated impression.

The last recorded frame on each microfiche shall contain the symbol —▶ (meaning "CONTINUED"), or the symbol ▼ (meaning "END"), whichever applies.

Maps, plates, charts, etc., may be filmed at different reduction ratios. Those too large to be entirely included in one exposure are filmed beginning in the upper left hand corner, left to right and top to bottom, as many frames as required. The following diagrams illustrate the method:

L'exemplaire filmé fut reproduit grâce à la générosité de:

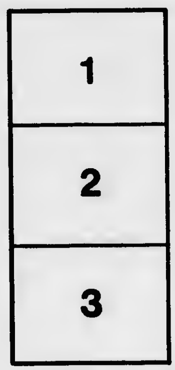
Séminaire de Québec
Bibliothèque

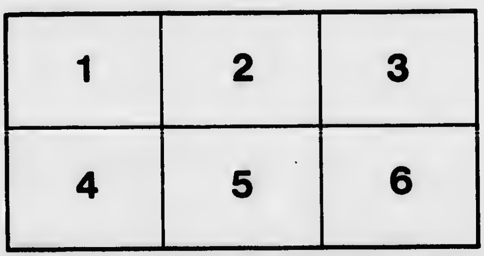
Les images suivantes ont été reproduites avec le plus grand soin, compte tenu de la condition et de la netteté de l'exemplaire filmé, et en conformité avec les conditions du contrat de filmage.

Les exemplaires originaux dont la couverture en papier est imprimée sont filmés en commençant par le premier plat et en terminant soit par la dernière page qui comporte une empreinte d'impression ou d'illustration, soit par le second plat, selon le cas. Tous les autres exemplaires originaux sont filmés en commençant par la première page qui comporte une empreinte d'impression ou d'illustration et en terminant par la dernière page qui comporte une telle empreinte.

Un des symboles suivants apparaîtra sur la dernière image de chaque microfiche, selon le cas: le symbole —▶ signifie "A SUIVRE", le symbole ▼ signifie "FIN".

Les cartes, planches, tableaux, etc., peuvent être filmés à des taux de réduction différents. Lorsque le document est trop grand pour être reproduit en un seul cliché, il est filmé à partir de l'angle supérieur gauche, de gauche à droite, et de haut en bas, en prenant le nombre d'images nécessaire. Les diagrammes suivants illustrent la méthode.





18

180 Rev. J. B. J. Balduc the Quebec
with Compliments of Rev J. J. McDonell PP.

KEY

TO

Chisholm's Mathematical Mechanical Scale:

AN INSTRUMENT FOR

SOLVING ALL PROBLEMS IN ARITHMETIC, GEOMETRY, AND TRIGONOMETRY,

RIGHT-ANGLED AND OBLIQUE, PLANE AND SPHERICAL.

WITHOUT THE AID OF TABLES, EXCEPT THOSE OF LATITUDE AND LONGITUDE.

BY A. M. CHISHOLM, ESQ.

HALIFAX, N. S.
PRINTED BY JAMES BOWES & SONS, HOLLIS STREET.

RECOMMENDATIONS.

ANTIGONISH, August 5th, 1861.

HAVING had an opportunity for some time past of testing the power and accuracy of Chisholm's Mathematical Scale, I am happy to be able to state that it far exceeded my expectations.

As a labor-saving instrument, particularly in Trigonometry and Navigation, I believe it has no equal. It should be taught in every school, and no navigator should be without a copy of it.

RODK. McDONALD,
Teacher of Mathematics, St. Francis Xavier's College.

ALEXANDER CHISHOLM, Esq., of Antigonish, has just shown me a very ingenious and, I believe, novel instrument, which he has invented, and which he calls "A Mechanical and Mathematical Scale." From the brief examination of it which I have had the opportunity of making, I am satisfied that it will prove a valuable acquisition to Surveyors, Mariners, Engineers, and business men in general. If accurately graduated it must give correct result. Though exceedingly simple, the sphere of its application is very extensive. The more thoroughly it is known and understood, the more fully it will be appreciated. Its introduction into Schools and higher Seminaries of Education will greatly facilitate the study of Mathematical Science, and probably increase the number of its students. I sincerely wish the inventor much success.

JAMES ROSS.

Presbyterian College, Truro, August 6th, 1861.

TRURO, 7th August, 1861.

I HAVE examined Mr. Chisholm's Scale with Explanations, and have no hesitation in stating that I believe it will be of great utility in our Schools, provided it can be printed at a moderate price. It furnishes an impressive and admirable illustration of the various departments of Practical Mathematics, and, on this ground, I cordially recommend its publication.

ALEXANDER FORRESTER,
Superintendent of Education.

SPRING GARDEN ACADEMY,
Halifax, N. S., August 19, 1861.

HAVING had an opportunity of examining a Scale invented by Mr. Chisholm, I feel convinced from the satisfactory results which it gave after some severe tests, that it is calculated to be of great service in schools, and particularly to those engaged in navigation, who require to have a correct result, in a short space of time; in other words it is a labour-saving invention, and as such is deserving of notice. I do not hesitate in giving it my unqualified approval.

JAMES WOODS.
Principal.

HALIFAX, N. S., August 20th, 1861.

I HAVE inspected with very great pleasure the "Mathematical Scale" invented by Mr. Chisholm. Though marked by striking simplicity in its construction, it possesses a range and a precision much superior to any other scale with which I am acquainted. It is not encumbered with tables or a large array of figures, and yet it lays the results of Mathematical processes, which would involve great labour and much time if wrought out in the ordinary way, before one at a glance. The most intricate problems which I proposed were solved with a rapidity for which I was not prepared; the question was hardly proposed before the result lay full and clear upon the scale, and almost as self-evident as a simple axiom. In its simplicity lies its astonishing power; since it is equally applicable to the solution of the most difficult problems in the various departments of Practical Mathematics, and to the elucidation of the abstruser truths of pure science. It is a scale which will be more appreciated by any one in proportion to his Mathematical skill; and those most advanced will see more clearly into its unrivalled powers and be more ready to acknowledge its high capacity. I consider it as a powerful addition to the cause of Science, as abridging in a complete manner the toil of study, and the laborious calculations of professional men; and have no doubt that the talented inventor will meet that ready recognition from scientific men which his Scale really deserves.

WILLIAM GARVIE,
Teacher Dalhousie College, and
Secretary of N. S. Literary and Scientific Society.

ST. MARY'S COLLEGE,
Halifax, N. S., 21st August, 1861.

I HAVE tested "The Chisholm Scale" in the resolution of several Mathematical problems, and I found it to be sufficiently accurate for all practical numerical results in the art of navigation, surveying, engineering, &c. I have no doubt that when "The Chisholm Scale" is sufficiently brought before the scientific world, that it will be at once adopted instead of other scientific scales and tables.

JOHN WOODS,
President, &c.

KEY TO CHISHOLM'S MATHEMATICAL MECHANICAL SCALE:

AN INSTRUMENT FOR

SOLVING ALL PROBLEMS IN ARITHMETIC,* GEOMETRY, AND TRIGONOMETRY, RIGHT-ANGLED AND OBLIQUE, PLANE AND SPHERICAL,

WITHOUT THE AID OF TABLES, EXCEPT THOSE OF LATITUDE AND LONGITUDE.

By A. M. CHISHOLM, Esq.

In presenting to the Public the Scale to which the following is a Key, the Inventor and Patentee has not failed to consider the difficulties in the way.

In the first place, the Public have often been imposed upon by Scales of various kinds, which appear at first of great practical value, but which are subsequently thrown aside as useless. In the next, parties are generally slow to purchase such articles, from the apparent training required to use them.

The Scale now offered will, however, stand any test, and is confidently believed to be such as will improve on acquaintance. It requires but little training to use it satisfactorily and successfully,—for a knowledge of the use of which the following pages will be found ample.

The examples given are not as full as might be, but the Author, desirous of making his invention as generally useful as possible, has limited the Key with the view of limiting the expense, and bringing the Scale within the means of all parties; as after becoming acquainted with the Scale, the operator may use any text book. It will be found, after trial, almost indispensable in schools and to the mariner. It will prove equally valuable in the hands of the merchant, professional man, and mechanic.

A DESCRIPTION OF CHISHOLM'S SCALE.

1. This Scale is a quadrangle or square, containing one hundred square inches, ten inches being the side of the square; but it may be of any other dimensions not less than ten

2. The sides are distinguished by letters of the Alphabet. Thus the head or upper side is designated by the letter A.; the right hand side, by B.; the bottom or lower side, by C.; and the left hand side, by D.; the index by F.

3. Each side is divided into one hundred equal parts, and numbered at every tenth, as follows: 10, 20, 30, &c.; but instead of these numbers on the sides of the scale and on the index, their equimultiplies 100, 200, 300, or 1000, 2000, 3000, &c., and their equisubmultiplies 1, 2, 3, or .1, .2, .3, &c, may be used.

4. The corresponding divisions, on the opposite sides, are joined by parallel right lines, which intersect one another at right angles, and consequently divide the area of the scale into ten thousand equal spaces, which are intended to represent quantities.

5. In order to distinguish the numbers more readily, every fifth and tenth line are shaded more deeply than the intermediate ones.

6. At the distance of 60 on A. a quadrantal arc is drawn, terminating at 60 on D. On this quadrant the degrees are marked and numbered at every tenth from A. to D. Within the quadrant are marked the points, half and quarter points of the compass.

7. These divisions or lines together with a moveable index, graduated like the scale, and attached to it by a pivot at the angular point of A. and D., form the whole apparatus.

8. Although the four sides of the scale are numbered in exactly the same manner, yet it seldom becomes necessary to have recourse to more than one side and the index, in the process of solving any one problem. The sides A. and C., being opposite and parallel, are, in every respect equal; as also are B. and D.; but, in practice, the operator will find the sides A. and B. more convenient than C. and D.

9. The few figures marked on the scale, combined with the simplicity of its construction, render a more detailed description unnecessary. It will suffice, therefore, to make a few remarks on its powers, comprehensiveness, and the labor it saves in calculation. It will readily solve any problem in Arithmetic, Geometry, Trigonometry, and Navigation, without the aid of any tables whatsoever, except those of latitudes and longitudes. In Trigonometry and Navigation especially, the branches in which its uses are particularly important, the despatch with which the most difficult problems can be solved by an expert operator, is, to say the least, incredible unless witnessed. Another advantage in using it is, that, in any problem in the four branches already referred to, it is in no case necessary to deviate from the rules now in use in the schools.

10. The rules for solving Arithmetical problems by the scale will now be given, premising, however, that in the two first elementary rules, Addition and Subtraction, the scale, like Logarithms, is not available. Attention to the following rules will obviate every difficulty.

MULTIPLICATION.

11. CASE I.—To multiply by any number from 1 to 10.

Rule.—Set 100 on index to the perpendicular of the multiplier, taken on A.. then opposite the multiplicand on index will be found the product on A.

Example 1.—Multiply 80 by 9.

Set 100 on index to 9 on A., then opposite 80 on index will be found the product 720 on A.

Although the scale shews 72 instead of 720 in the product, it will be seen by reference to Article 3, that this number may be 720, 7200, 72,000, &c. A little consideration will, therefore, enable the operator to arrive at the correct result.

Ex. 2.—Multiply 15 by 8.

Set 100 on index to 8 on A., then opposite 15 on index is the product 120 on A.

CASE II.—To multiply by any number exceeding ten.

Rule.—Set multiplier on index opposite 10 on A., then opposite multiplicand on A. is the product on index.

Ex.—Multiply 40 by 12.

Set 12 on index opposite ten on A., then opposite 40 on A. will be found the product 480 on index.

Note.—If side B. instead of A. were used, the result would be the same. The operator can use that which he finds most convenient.

CASE III.—To multiply by a Vulgar Fraction.

Rule.—Set denominator on index to perpendicular of numerator on A., then opposite the multiplicand on index will be found the product on A.

Ex.—Multiply 8 by $\frac{3}{4}$.

Set 4 on index to 3 on A., then opposite 8 on index will be found the product 6 on A.

Without moving the index the product of any other number by the same fraction may be found.

This method, although apparently different from Cases I. and II., is yet identical with them; for when the index is set for a vulgar fraction according to the directions given, 100 on index will be opposite to the corresponding value of the given vulgar fraction, in decimals, on A. or B., according as A. or B. is used; thus, if the scale be set for $\frac{3}{4}$, it will be found that 75 on A. is opposite to 100 on index, and hence $\frac{3}{4}$ is equal to 75-100 or .75, which, being multiplied by 8, gives 6.

DIVISION.

12. CASE I.—To divide by any number not exceeding ten.

Rule.—Set 100 on index to the perpendicular of divisor on A., then opposite the dividend on A. will be found the quotient on index.

Ex.—Divide 48 by 6.

Set 100 on index to the perpendicular of 6 on A., then opposite 48 on A. will be found the quotient 8 on index.

CASE II.—To divide by any number not less than ten.

Rule.—Set divisor on index to perpendicular of 100 on A., then opposite the dividend on index is the quotient on A.

Ex.—Divide 60 by 12.

Set 12 or 120 (Art. 3) on index to 100 on A., then opposite 60 on index is the quotient 5 on A.

CASE III.—To divide by a Vulgar Fraction.

Rule.—Set the denominator on index to the perpendicular of numerator on A., then opposite to the dividend on A. is the quotient on index.

Ex.—Divide 60 by 5-6.

Set 6 on index to 5 on A., then opposite 60 on A. is the quotient 72 on index.

As in multiplying, so in dividing, by a vulgar fraction, the rule is identical with that above given; for, when the index is set for a regular fraction, the perpendicular from 100 on index to side A., will shew on A. the value of that vulgar fraction in decimals. From this also may be seen how well adapted the scale is for converting vulgar fractions into decimals, and *vice versa.*

REDUCTION.

13. As this rule depends altogether on Multiplication and Division, enough has been said in Articles 11 and 12 to enable the learner to work without further instructions.

* Unavailable, like Logarithms, for Addition or Subtraction.

PROPORTION

14. RULE 1.—Set the first term on index to the second term on A. or B., then the third term on index will shew the fourth term or answer on A. or B., according as A. or B. has been used.

RULE 2.—Set the second term on index to the first term on A. or B., then the third term on A. or B. will shew the fourth term or answer on index.

Note.—The first and third are generally taken on the same side, as are also the second and fourth.

Ex. 1.—If 3 yards of cloth cost 4 shillings, what will 9 yards cost?

This may be stated either of the two following ways:—

Yds.		Sh.		Yds.		Sh.
3	:	4	: :	9	:	12
Yds.		Yds.		Sh.		Sh.
3	:	9	: :	4	:	12

Then, by the first rule, set 3 on index to 4 on A. or B., then 9 on index will shew 12 on A. or B.

Or, set 3 on index to 9 on A. or B., then 4 on index will shew 12 on A. or B.

By Rule 2, set 4 on index to 3 on side A. or B., then 9 on side A. or B. shews 12 on index.

Or, set 9 on index to 3 on A. or B., and 4 on A. or B. shews 12 on index.

Ex. 2.—If 14 men perform a piece of work in 6 days, in what time will 24 men perform it?

Here the statement is:—

24 : 14 : : 6 : $3\frac{1}{2}$ days.

Set 24 on index to 14 on A. or B., then 6 on index will shew $3\frac{1}{2}$ on A. or B.

Or, by Rule 2, set 14 on index to 24 on A. or B., then 6 on A. or B. shews $3\frac{1}{2}$ on index.

15. When remainders or fractions occur, their values may be read on the scale, by an expert operator, with almost perfect accuracy. By persons unacquainted with the scale, however, recourse must be had, either to the diagonal on side B. for decimals, or in the following manner for regular fractions:

Set the index one division down on the perpendicular of the divisor (the first term in proportion) on A.; take the remainder on a pair of dividers, move the dividers along the index from the pivot towards side B. till they exactly coincide with the space between the index and side A., then will side A. shew the numerator of the regular fraction.

Ex. 1.—Divide 700 by 9.

Set 100 on index to 9 on A., then 700 on A. will shew on index 77 with a remainder.

To find the value of the remainder, take the remainder on dividers and set the index one division below 9 on A.; then if the dividers be moved along the index, it will be found to coincide with the space between the index and side A. at 7, which is therefore the numerator of the fraction, and hence the quotient is 77 7-9.

If the same extent on the dividers be applied to the space between side B. and the diagonal, it will be found to coincide at 77, and the whole length of B. being 100, this number will be 77-100 or .77, and in this case, therefore, the quotient is 77.77 +.

The small diagonal between the third and fourth divisions on D. may be employed in the same manner.

SIMPLE INTEREST.

16. To calculate the interest of any principal, at any rate per cent., for one year.

RULE.—Set 100 on index to the rate per cent. on A. or B., then opposite the principal on index is the answer on A. or B.

Note.—If the large divisions on the scale be assumed as £1, each of the smaller divisions will become one-tenth or 2 shillings.

Ex.—What is the interest of £80 for one year, at 6 per cent.?

Set 100 on index to 6 on A. or B., then opposite 80 on index is £4 16s. on A. or B., that is four large divisions, each £1, and eight small divisions, each two shillings.

As interest, partnership, profit and loss, discount, commission and brokerage, &c., are simply variations of the Rule of Three, the learner will have no difficulty in solving any problems in them, with the aid of the rules given in Article 14. Duodecimals can be performed by the rule given in Article 11.

EXTRACTION OF SQUARE ROOT.

17. The square root of a number is that number which, multiplied by itself, gives the proposed number.

RULE.—Let the number, whose root is required, be taken on A. or B., and let £100 on index be set to a trial divisor on A. or B.; then, if the trial divisor or index show the given number on A. or B., the trial divisor is the root required: if not, vary the trial divisor by moving the index either way, according as the trial divisor shows a result greater or less than the given number; and continue this until the trial divisor taken on index show the given number on A. or B.

Ex.—Required the square root of 600?

If 20 be assumed as a trial divisor, set 100 on index to 20 on A., then 20 on index shows only 400 on A., which is less than the given number 600; hence the trial divisor 20 is less than the root required. If 30 be assumed, A. will be found to be greater than the root required: hence the root must lie between 20 and 30. By moving the index, the operator will find that, when 100 on index is set to 24.5 nearly, or 24.49 on A., 24.49 on index will shew 600 on A.

Note.—The square root of any number, not exceeding 1000, is extracted more conveniently on side B. than A.

EXTRACTION OF THE CUBE ROOT.

18. RULE.—Set 100 on index to trial divisor, or assumed root on A. or B., then opposite trial divisor on index is its square on A. or B., and opposite this square taken on index is the given number on A. or B., if the assumed root be the correct one: if otherwise, the index must be moved, as in square root until the correct root be found. When the index is set for any trial root it is not necessary to move it until the correctness or incorrectness of the trial root is determined.

Ex.—Required the cube root of 46,000.

Here the given number can be divided only into two periods, hence there can be only two figures and a decimal fraction in the root. The cube root of the first period 46 is 3 +. 100 on index, therefore, must be set to a number between 3 and 4.

Let it be set on index to 35 on A., and 35 on index will shew 1225 on A., and 1225 on index will shew nearly 43,000 on A., which is less than the given number. Hence 35 is less than the required root. By a similar process 36 will be found to be greater. The correct root must therefore lie between 35 and 36, and by setting 100 on index to 35.8, and proceeding as before, the result is found to be 46,000 nearly. Hence 35.8 is nearly the root required.

PART II.

PLANE TRIGONOMETRY.

REMARKS, &c.

19. The lines joining the corresponding divisions on the opposite sides, D. and B., are called *parallels*, in order to distinguish them from those joining A. and C., which are called *perpendiculars*.

20. The perpendicular on the 60th division, being a tangent to the arc, is called a line of tangents; and whenever the word "tangent" is used in the rules for calculation, it must be understood to mean some portion of this line.

21. The perpendicular from any degree on the quadrant to side A., is the sine of that degree, and its numerical value to radius 60 is reckoned on B.; the parallel on side D. is the cosine, and its numerical value is reckoned on A.; and if the index be set to any degree on the quadrant its intersection with the line of tangents will show, on the index, the numerical value of the secant, and on the line of tangents, the numerical value of the tangent to the same radius 60. These values being divided by 60 give the natural sines, cosines, &c.

22. If 100 or 1 on side A. or index be considered as radius, and a quadrant conceived to be described from 100 on A. to 100 on D., the side B. becomes the tangent to the arc which was conceived to be thus formed, and by placing the index to any degree on the quadrant, the perpendicular from 100 or 1 on index to side A. will be the natural sine of that degree; the parallel on side D. the natural cosine, and the intersection of index with side B. will show on index the natural secant, and on side B. the natural tangent.

23. If the natural tangent of any degree above 59° be required, it will be necessary to use the semi or quarter tangent as found on the perpendiculars of 30 and 15 respectively.

24. In the solution of problems by the scale, when the words sines, cosines, tangents, cotangents, &c., are used, they must be understood to mean their numerical values to radius 60.

25. The division of radius into sixty equal parts agrees with the division of a degree of longitude on the equator into sixty minutes; and thus affords an easy way of finding the length of a degree of longitude in any parallel of latitude. The parallel from any degree on the quadrant to side D. will be the length of a degree of longitude in the parallel of that degree. For example, the length of a degree of longitude in the parallel of 30° is the measure from 30° on quadrant to side D., which being reckoned on side A. shows 52, which is the length of a degree of longitude in the parallel of 30°.

26. The meridional difference of latitude can be readily found without the aid of any tables. Thus, set the index to middle latitude on quadrant, and the intersection of the tangent with the index shows on the index the length of a meridional degree in that parallel (assuming the middle of the degree as the parallel); and if this be multiplied by the difference of latitude, in degrees, the product is the meridional difference of latitude.

27. The principles on which the calculations in Trigonometry are founded, are certain relations or proportions existing between the sides of triangles and certain lines connected with the angles, called *trigonometrical lines or ratios*, and the principles on which the use of the scale, in Trigonometry, is based, may be thus explained:—

Let A B C be a triangle, and let D E or any other line be drawn parallel to B C, one of the sides of the triangle; then A C : A B : A D or A E : A C = A D : A B. See figure 1.

Now, by means of the index, an indefinite number of triangles, with lines parallel to some of the sides, can be formed; and hence an indefinite number of proportions.

RIGHT ANGLED TRIGONOMETRY.

DEFINITIONS AND PRINCIPLES.

28. Every triangle consists of six parts, viz., three sides and three angles; and when any three of these are given, unless it be the three angles, the other three can be found.

29. The sum of the three angles of any plane triangle is equal to two right angles or 180°.

30. The greatest side of every triangle is opposite to the greatest angle.

31. The complement of an arc is its difference from a quadrant.

32. The supplement of an arc is its difference from a semicircle.

33. The complement of an angle is its difference from a right angle.

34. The supplement of an angle is its difference from two right angles.

35. The sine of an arc is a straight line drawn from one extremity of the arc, perpendicular, to the radius passing through the other extremity.

36. The tangent of an arc is a straight line touching the arc

at one extremity, and limited by the radius produced through the other extremity.

37. The secant of an arc is the straight line joining the centre of the circle and the further extremity of the tangent drawn from the origin of the arc.

38. The sine, tangent, and secant of the complement of an arc are called the cosine, cotangent and cosecant of that arc.

Thus B C is the complement of the arc A B; B M D is the supplement of A B; angle B O C is the complement of A O B, and B O D is the supplement of A O B; B E is the sine of A B; A F is the tangent of A B; O F is the secant of A B; so B G is the sine of B C, or the cosine of A B; C H is the tangent of B C, or the cotangent of A B; and O H is the secant of B C, or the cosecant of A B. See figure 2.

39. The sine, tangent, and secant of an arc, are the sine, tangent, and secant of the angle measured by the arc.

Thus, the arc A B measures A O B and B E; A F, O F; the sine, tangent, and secant of the arc A B are also the sine, tangent, and secant of the angle A O B.

40. The sine, tangent, and secant of an angle, are the cosine, cotangent and cosecant of the complement of that angle.

Thus, B G or its equal O E the sine of angle B O C, is the cosine of A O B; C H and O H the tangent and secant of B O C, are the cotangent and cosecant of A O B.

41. The sine, tangent, and secant of an arc, are equal to the sine, tangent, and secant of its supplement.

Thus, B E, the sine of A B, is also the sine of B M D; A F, the tangent of A B, is equal to the tangent of A I L, which is equal to B M D.

Hence, when an angle is obtuse, its supplement must be used.

Propositions.

42. When the hypothenuse of a right-angled triangle is made radius, its sides become the sines of the opposite angles, or the cosines of the adjacent angles.

Thus, if A C be considered as radius, it is evident, by completing the figure (Art. 38), that B C is sine angle A or cosine angle C, and that A B is sine angle C or cosine angle A. See figure 3.

43. When the base is made radius, the perpendicular becomes the tangent of its opposite angle, and the hypothenuse the secant of the same; or the perpendicular becomes the cotangent of the adjacent angle, and the hypothenuse the cosecant of the same.

Thus, when A B is made radius, B C becomes tangent angle A or cotangent angle C, and A C becomes secant angle A or cosecant angle C. See figure 4.

44. When the perpendicular is made radius, the base is tangent of its opposite angle, and the hypothenuse secant of the same; or the base is cotangent of its adjacent angle, and the hypothenuse the cosecant of the same.

Thus, when B C is made radius, A B becomes tangent angle C or cotangent angle A, and A C becomes secant angle C or cosecant angle A. See figure 5.

Rules for Computation.

45. CASE I.—When a side and one of the oblique angles are given to find a side.

Rule.—Make any side radius: then
As the name of the given side
Is to the given side,
So is the name of the side required
To the side required.

46. CASE II.—When two sides are given to find an angle.

Rule.—Make one of the given sides radius: then
As the side made radius
Is to radius,
So is the other given side
To sine, tangent, or secant of the angle by it represented.

Note.—In working by the scale, let it be remembered that for sine, tangent, and secant, their numerical values to radius 60 must be used.

The examples in Trigonometry, Navigation, &c., are taken out of Norie's Navigation.

47. *Ex.*—Given the hypothenuse 370 miles, the angle A 56° 30′ and consequently the angle C 33° 30′ required the base A B and the perpendicular B C.

Making A C radius, A B will be sine angle C or cosine angle A, and B C will be sine angle A or cosine angle C. See figure 6.

Radius : Side A C 370 = Sine angle A 56° 30 : Side B C.
— — = Sine angle C 33° 30 : Side A B.

By the scale:

First radius is equal to 60, and sine 56° 30 is equal to 50. Then by Art. 14, set radius or 60 on index to 370 on side A, then opposite sine angle A 50 on index is found B C, 308.5 on A. And if the sine of angle C 33° 30 which is 33.3, be taken on index, by the one setting we find opposite it on A the side A B 204.2.

Or, set the index to 33° 30′ on quadrant, then opposite 370 on index is found on A the side B C 308.5, and on B the side A B 204.2.

48. Given the base A B 625 and angle A 48° 45, to find the hypothenuse A C and the perpendicular B C; *making A B radius, B C becomes tangent angle A or cotangent angle C, and A C becomes secant angle A or cosecant angle C.* See figure 7.

Then radius : A B 625 = Tangent angle A 48° 45′ : B C.
— — = Secant angle A 48° 45′ : A C.

By the scale:

Set 625 on index to radius 60 on A, then opposite tangent 48° 45′ or 68.4 on A is B C 713 nearly on index, and opposite secant 48° 45′ or 91 on A is found A C 948 nearly on index.

Or, set index to 48° 45′ on quadrant, and opposite 625 on A is 948 on index; and if the parallel from 948 on index be traced to B, it shows on B the perpendicular 713.

49. Given the hypothenuse A C 400, and the base B A 236; required the angles A and C, and the perpendicalar B C.

Making the hypothenuse radius, B C becomes sine angle A or cosine angle C, and A B sine angle C or cosine angle A. See figure 8.

Then, A C 400 : Radius = A B 236 : Sine angle C.

By the scale:

Set radius 60 on index to 400 on A, then opposite 236 on A will be found sine angle C 35.4 on index: if the parallel of 35.4 on B be traced to quadrant, it will show on quadrant angle C 36° 9′, which, being subtracted from 90°, gives angle A 53° 51′.

Or, set 400 on index to 236 on side B, then the perpendicular from 400 on index to ..., shows on A the perpendicular B C 323, and the index cuts the quadrant in 36° 9′, which is the angle required.

Note.—The angle can be found more easily by making either of the sides about the right angle radius, when possible, as will be seen by next problem.

50. Given the base B A 35.5 and the perpendicular B C 41.6; required the angles A and C and the hypothenuse A C.

Making A B radius, B C will be the tangent of angle A or cotangent of angle C, and A C will be secant of angle A or cosecant of angle C. See figure 9.

Then A B 35.5 : Radius 60 = B C 41.6 : Tangent angle A.

By the scale:

Set 60 on index to 35.5 on A, then opposite 41.6 on A is found on index tangent angle A 70.2; then, if index be set to 70.2 on the line of tangents, it will cut the quadrant at 49° 31′, which is the number of degrees on angle A; and if the perpendicular from 35.5 on A be traced to index, it will show on index the hypothenuse A C 54.7, nearly.

Or, trace the perpendicular of 41.6 on side A 'till it will intersect the parallel of 35.5 on side B; set the index to the point of intersection: at this point will be found the hypothenuse 54.7 on index, and on the quadrant will be found the angle C 41° 29′, which, being subtracted from 90°, leaves angle A 49° 31′.

Examples for Exercise.

1. Given the hypothenuse 108 and the angle opposite the perpendicular 25° 36′; required the base and the perpendicular.

Ans.—The base is 97.4 and the perpendicular 46.66.

2. Given the base 96 and its opposite angle 71° 45′; required the perpendicular and the hypothenuse.

Ans.—The perpendicular is 31.66 and the hypothenuse 101.1.

3. Given the base 360 and the perpendicular 480; required the angles and the hypothenuse.

Ans.—The angles are 53° 8′ and 36° 52′, and the hypothenuse 600.

OBLIQUE-ANGLED TRIGONOMETRY.

51. CASE I.—When two angles and a side opposite to one of them are given.

Rule.—As the sine of the angle opposite to the given side is to the given side, so is the sine of the angle opposite to the required side, to the required side.

52. CASE II.—When two sides and an angle opposite to one of them are given.

Rule.—As the side opposite to the given angle is to the given angle, so is the side opposite to the required angle, to the required angle.

Note.—When two of the angles are known, the third is found by subtracting their sum from 180°.

Ex. 1.—Given angle A 36° 15′, and the angle B 105° 30′, and the side A B 53; required the sides A C and B C. See figure 10.

As sine angle 38° 15′	=	37.2 on B.
Is to its opposite side	"	53 on F.
So is sine angle 105° 30′	"	57.8 on B.
To its opposite side	"	82.5 on F.
And so is sine of sup. 36° 15°	"	35.5 on B.
To its opposite side	"	50.6 on F.

Set 53 taken on index to 37.1 on B; then opposite 57.8 on B is found on index A C 82.5, and opposite 35.5 on B is found on index side B C 50.6.

Ex. 2.—Given the side A B 336, the side B C 355, and the angle A 49° 26′; required the angles B and C and the side A C. See figure 11.

As side given 355	=	355 on B.
Is to sine of its opposite angle 49° 26′	"	45.5 on F.
So is the other given side	"	336 on B.
To sine of its opposite angle 45° 58′	"	43.1 on F.
And without a move so is		
Sine of supplement 84° 36′	"	59.8 on F.
To its opposite side	"	466 + on B.

Set 45.5 on index to 355 on side B, then opposite 336 on B is found sine angle C 43.1 on index: trace the parallel of 43.1 on B, till it intersect the quadrant, and at the point of intersection is found on the quadrant 45° 58′, the angle at C.

If the angles A and C be now added, and the sum subtracted from 180°, the remainder is angle B 84° 36′: then, by the first case, A C can be found.

53. CASE III.—When two sides and the angle contained between them are given.

Rule.—As the sum of the two given sides is to their difference, so is tangent of half the sum of the unknown angles to the tangent of half their difference. This half difference, added to half their sum, gives the greater angle, and subtracted, leaves the less. The angles being thus all known, the remaining side is found by Rule to Case I.

Ex.—Given the side A B 85, the side A C 47, and the angle A 52° 40′; required the angles C and B and the side B C. See figure 12.

$$\begin{array}{lcl} & & 180^\circ \\ \text{Angle A} & = & 52^\circ\ 40 \\ \hline \text{B} + \text{C} & = & 127^\circ\ 20' \\ \hline \tfrac{1}{2}\,(\text{B} + \text{C} & = & 63^\circ\ 40' \end{array}$$

(A B + A C) 132 : (A B ∽ A C) 38 = Tang. $\frac{1}{2}$ (C + B) 63° 40′ : Tang. $\frac{1}{2}$ (C ∽ B).

Here, we must use the semi-tangent, found (Art. 23) on the perpendicular of 30 on A, to be 60.6; and on trial the operator will find it necessary to employ 66 and 19 in the first two terms of the proportion, instead of 132 and 38.

Thus, set 66 on index to 19 on B, then opposite 60.6 on index is found semi-tangent of half the difference of the unknown angles 17.5 on B: if 17.5 be now taken on the line of semi-tangents, viz., the perpendicular of 30 and the index set to it, the quadrant will be cut by the index at 30° 11', half the difference of the unknown angles. Then,

63° 40 + 30° 11' = 93° 51' greater angle C.
63° 40 — 30° 11' = 33° 22' less angle B.

B C is readily found by first case.

Or, set the index to the given angle 52° 40 on quadrant; take 47 on index and 85 on side A, imagine a right line drawn from 47 on index to 85 on side A, and the triangle is complete. (See figure 13.) The perpendicular from 47 on index to side A is 37.5, and divides the base into two segments, 28.5 and 56.5, and the triangle into two right-angled triangles. If the perpendicular of 56.5, segment D B, adjacent to the required angle, taken on A, and the parallel of 37.5 taken on B, be traced till they intersect, and the index set to the point of intersection, this point shows on index the side B C 67.7, and the intersection of index with the arc of the quadrant, shows on the quadrant angle B 33° 29.

The same worked with the aid of the assisting index.

RULE.—Set the attached index F to the given angle 52° 40' on the quadrant, and while in this position, set the centre of the assisting index H to 47 on F: bring the graduated edge in contact with the other given side 85 on A: then the circular part will indicate the angle at meeting of indices to be 93° 51', and the side sought to be 67.7. Two angles being known, the third can easily be found by note to Art. 52, or thus: reverse the assisting index by placing its centre on 85 taken on A, and its graduated edge on 47 taken on attached index, the circular part will indicate the angle to be 33° 29', and the side 67.7.

54. CASE IV.—Given the three sides to find the angles.

RULE.—Draw a perpendicular from one of the angles upon the opposite side or this side produced; then calling this side base, say as base is to the sum of the other two sides, so is the difference of these sides to the difference of the segments of the base.

Then half this difference added to half the sum gives the greater segment, and subtracted from half the sum—that is half the base—gives the less. Then the triangle will be divided into two right-angled triangles, the angles of which can be found by Art. 46.

Ex.—Given the side A B 157, the side B C 110 and the side A C 88, to find the angles A B and C. (See figure 14.)

A B : A C+C B = A C-C B : A D-D B
157 : 198 = 22 : 27.74 C
or 78.5 : 99 = 22 : 27.74

Set 99 on index to 78.5 on B; then opposite 22 on B is found on index 27.74 difference of the segments of the base

Then 27.74

13.87 half difference of the segments.
78.5 half the sum or half the base.

92.37 sum gives greater segment D B.

64.63 diff. gives less segment A D.

Set side A C 88 taken on index to its adjacent segment A D 64.63 taken on A; the index will shew on quadrant the angle A 42° 44'.

Again set side C B 110 taken on index, to its adjacent segment D B 92.37 taken on A; the index will show on quadrant, in like manner the angle B 32° 53'.

The same worked with the aid of the detached index H.

First take the halves of the sides, namely 78.5, 44 and 55. On the attached index F take 44, and to it set the centre of the detached index. Move the indices till you get 55 on detached index in contact with 78.5 on side A; the attached index will be found to intersect the quadrant at 42° 44' angle A, and the circular part will indicate the angle at the meeting of the indices to be 104° 23' angle C.

EXAMPLES FOR EXERCISE.

1. Given one side 129, an adjacent angle 56° 30' and the opposite angle 81° 36': required the third angle and the remaining sides.

Ans.—The third angle is 41° 54', and the remaining sides are 108.7 and 87.08.

2. Given one side 110, another side 102, and the contained angle 113° 36': required the remaining angles and the third side.

Ans.—The remaining angles are 34° 37' and 31° 47', and the third side is 177.5.

3. Given the three sides respectively 120.6, 125.5, and 146.7: required the angles.

Ans.—The angles are 51° 53', 54° 58', and 73° 9'.

PLANE SAILING.

55. In plane sailing, the earth is supposed to be an extended plane, and the meridians are, therefore, considered as being parallel to each other, the parallels of latitude at right angles to the meridians, and the length of a degree on the meridian, equator, and parallels of latitude every where equal.

56. The *course* is the angle which the ship's track makes with the meridian..

The *distance* is the number of miles, &c., between any two places, reckoned on the rhumb line of the course.

57. The *difference of latitude* is the distance which a ship makes North or South of the place sailed from, and is reckoned on a meridian.

58. The *departure* is the distance which a ship makes East or West, and is reckoned on a parallel of latitude.

Note.—As the course is generally taken on the arc of the quadrant, the operator will find it more convenient to take the difference of latitude on side A and the departure on side B.

Ex. 1.—A ship from latitude 48° 40 N., sails N. E. by N. 296 miles, required her present latitude, and the departure made good. (See figure 15.)

Then, by Trigonometry:

Radius : Dist. 296 = Cosine cou. 3 pts. : diff lat.
Sine course 3 pts. : dep.

By the Scale:

Set radius 60 on index to 296 on B, then opposite cosine 3 pts 49.9 on index is diff. lat. 246.1 on B and opposite sine 3 pts. 33.2 on index is dep. 164.4 on B. Or, set the index to the course 3 pts, then the distance 296 on index will cut the perpendicular of the difference of latitude 246.4 on side A, and at the same time will cut the parallel of the departure 164.4 on side B. Then the proportion will be—

As radius 60 on F is to cosine 3 pts 49.9 on A, so is distance 296 or 29.6 on F to 24.64 or 246.4 on A; and so is distance 296 or 29.6 on F to departure 16.44 or 164.4 on A.

59. The operator cannot fail to see that all the exercises in Navigation can be solved by the scale in various ways; but as a work of this kind must necessarily be short, we will after this confine ourselves to the easiest methods; and for this purpose we must reserve the usual position of the figure, by drawing the difference of latitude across the page, and the departure in a direction from top to bottom.

Ex. 2.—A ship sails S. E. ½ E. from St. Helena, in latitude 15° 55' S., until by observation she is in latitude 18° 49' S., require her distance run and departure made good. (See figure 16.)

Latitude St. Helena	15° 55'
Latitude come to	18° 49
Difference latitude	2° 54
	60
In miles	174

RULE.—Set the index to the course 4½ pts., then opposite the difference of latitude 174 on A will appear the distance 274.3 on index, and the parallel traced from this point on index to side B, will show on B the departure 212.

Ex. 3.—A ship from latitude 3° 16' N., sails S. W. by W. ¼ W. until she has made 356 miles of departure: required her present latitude and distance sailed. (See figure 17.)

RULE.—Set the index to the course 5¼ points, then opposite the departure 356 on B will appear the distance 415.1 on index, and the perpendicular traced from this point to side A, will show on A the difference of latitude 213.4.

Lat. left	3° 16' N.
Diff. lat. 213 miles or	3° 33' S.
Lat in	0° 17' S.

Ex. 4.—A ship from Cape St. Vincent in latitude 37° 3' N., sails between the North and West 430 miles, until her difference of latitude is 214 miles: required her course steered and departure made good. (See figure 18.)

RULE.—Set the distance 430 on index to the perpendicular of the latitude 214 on A, then opposite 430 on index is departure 373 on B, and the intersection of index with the arc of the quadrant shows on the arc the course 60° 9'.

Ex. 5.—A ship from latitude 1° 32' S., sails between the North and East 250 miles, and finds she has made 126 miles departure: required the course steered and her latitude in. (See figure 18.)

RULE.—Set the distance 250 on index to the departure 126 on side B, then opposite 250 on index is found the difference of latitude 215.9 on side A, and the intersection of index with the arc of the quadrant shows on the arc the course 30° 16'.

Lat. left..................	1° 32' S.
Diff. lat. 216 miles, or........	3° 36' N.
Lat. in	2° 4' N.

Ex. 6.—A ship from Funchal, in Madeira, in latitude 32° 38' N., sails a direct course between the south and west until she is in latitude 31° 13' N., by observation, having made 72 miles of departure; required her course steered and distance run. (See figure 19.)

Lat. of Funchal............	32° 38' N.
Lat. in, by observation......	31° 13' N.
Difference of lat.	1° 25
	60
In miles	85

RULE.—Trace the perpendicular of the latitude 85 taken on A, till it will intersect the parallel of the departure 72 taken on B; set the index to the point of intersection, and this point will show on index the distance 111.4, and the index will show on quadrant the course 40° 16'.

EXAMPLES FOR EXERCISE.

1. A ship from latitude 36° 30 N. sails SW. by W. 420 miles: what is her present latitude, and what departure has she made?

Ans.—Latitude in 32° 37 N., and departure 349.3 miles.

2. A ship from latitude 3° 54 S. has sailed NW. ¾ W. till she arrives at latitude 2° 14 N.: required her distance run, and departure made good?

Ans.—Distance 617.8, and departure 496.2 miles.

3.—A ship sails between the north and west 170 leagues, from a port in latitude 38° 42 N., until her departure is 98 leagues: required her course and latitude in?

Ans.—Course N. 35° 12 W., and latitude in 45° 31 N.

TRAVERSE SAILING.

60. RULE.—Find by the scale the difference of latitude and departure corresponding to each course and distance, as in plane sailing; set these down opposite the distance in the proper column, observing that the difference of latitude must be placed in the north column, if the course be northerly, and

in the south column, if the course be southerly; and that the departure must be placed in the east column if the course be easterly, and in the west column if it be westerly. Add up the columns of northing, southing, easting and westing, and set down the sum of each at the bottom; then the difference between the sums of the north and south columns will be the whole difference of latitude made good of the same name with the greater; and the difference between the sums of the east and west columns is the whole departure made good of the same name with the greater sum.

With this whole difference of latitude and departure, find the direct course and distance, as in the sixth example plane sailing.

Ex.—Suppose a ship from the start in latitude 50° 13′ N. sail W. S. W. 51 miles, W. by N. 35 miles, S. by E. 45 miles, S. W. by W. 55 miles, and S. S. E. 41 miles: required her direct course and distance sailed, and her latitude in?

TRAVERSE TABLE.

Courses.	Dist.	Diff. of Latitude.		Departure.	
		N.	S.	E.	W.
W. S. W.	51		19.5		47.1
W. by N.	35	6.8			34.3
S. by E.	45		44.1	8.8	
S. W. by W.	55		30.6		45.7
S. S. E.	41		37.9	15.7	
		6.8	132.1	24.5	127.1
			6.8		24.5
		Diff. of lat.	125.3	Departure,	102.6

Then, trace the perpendicular of the difference of latitude 125.3 on A till it intersect the parallel of the departure 102.6 on side B; set the index to the point of intersection, and this point will show on index the distance 162, and the index will show on the arc of the quadrant the course 39° 19′ (See figure 20.)

Note.—If the halves of the difference of latitude and departure be taken, their intersection on the scale will be more evident.

PARALLEL SAILING.

61. *Parallel sailing* is the method of finding the distance between two places in the same latitude, when their difference of longitude is known; or of finding the difference of longitude answering to the meridian distance or departure made good when a ship sails due east or west.

For the principles on which parallel sailing depends, the learner is referred to Norie's Navigation, page 80.

RULES.

As radius is to difference of longitude, so is cosine latitude to the meridian distance or departure.

Or, as radius is to any given portion of the equator, so is cosine of latitude to a similar portion of a given parallel.

As cosine of latitude is to meridian distance or departure, so is radius to difference of longitude.

As difference of longitude is to radius, so is meridian distance to cosine of latitude.

Ex.—A ship in latitude 36° 58′ N., and longitude 20° 25′ W., is bound to St. Mary's, one of the Western Islands, in the same latitude, and in longitude 25° 13′ W., what distance must she run to arrive at the island?

Longitude of ship,	20° 25′ W.	
Longitude of St. Mary's,	25 13 W.	
Difference of longitude,	4° 48′	= 288 miles.

RULE.—Set the index to co. lat. 53° 2′ on quadrant, and opposite the difference of longitude 288 (See figure 21.)

on index will be found the distance or departure 230.1 on side B.

Or, set the index to the latitude 36° 58′ on quadrant, and opposite the difference of longitude 288 on index, will appear the distance on side A.

MIDDLE LATITUDE AND MERCATOR SAILING.

62.—With the directions already given, the operator will have little difficulty in solving problems in Middle Latitude Sailing, by the scale, according to the rules laid down in Norie's Navigation; and although Mercator sailing can readily be worked with the aid of tables according to the usual rules, yet by combining the two rules the operator can solve any problem in these sailings without the aid of any tables whatsoever.

63. In finding the meridional difference of latitude, if there be minutes, take the number representing the minutes on side A, and set the index to middle latitude; the perpendicular from the minutes on A will show the proportional part for meridional difference of latitude on index.

Ex.—Required the course and distance from the Cape of Good Hope, in latitude 34° 22′ S., and longitude 18° 21′ E., to the Island of St. Helena, in latitude 15° 55′ S., and longitude 5° 45′ W.

Lat. Cape Good Hope,	34° 22′ S.	34° 22′ S.	Long.	18° 21′ E.
Lat. St. Helena......	15 55 S.	15 55 S.	Long.	5 45 W.
Difference of latitude,	18° 27	50° 17	Diff. long.	24° 9
	60			60
		Mid. lat. 25° 8′		
In miles,	1107		In miles,	1449

RULE.—Set the index as directed (Art. 26) to middle latitude 25° 8′, then the intersection of tangent (Art. 20) with the index shows on the index the length of a meridional degree in that parallel to be 66.3; and to find the meridional difference of latitude, first, for degrees, multiply 66.3 by 18 and the product is 1193.4; then, for the minutes, set the index as above directed, and the perpendicular of the 27 minutes on A will cut the index in 30, the proportional part for 27 minutes. The former result 1193.4 added to the latter 30, gives the meridional difference of latitude 1223.4 nearly. Take half the meridional difference of latitude 611.7 on A, and trace its perpendicular till it intersect the parallel of half the difference of longitude 724.5 on B; set the index to the point of intersection and it will show on the quadrant the course 49° 50, and the perpendicular of half the proper difference of latitude 553.5 on A, traced to index, will show on index half the distance 858. Hence the distance is 1716 miles, and the course 49° 50′. (See figure 22.)

64. When the difference of latitude is large, especially in high latitudes, the above method, like middle latitude sailing, is not strictly accurate.

A correct result may, however, be obtained by taking the meridian difference of latitude in parts not exceeding four degrees; thus:—

			Diff. Lat.		Mer. Diff. Lat.
15° 55′	to 56°	=	0° 5′	=	5.2
16	" 18	"	2	"	125.6
18	" 20	"	2	"	126.6
20	" 22	"	2	"	128.4
22	" 24	"	2	"	130.4
24	" 26	"	2	"	132.4
26	" 28	"	2	"	134.4
28	" 30	"	2	"	137.4
30	" 32	"	2	"	140.
32	" 34	"	2	"	143.
34	" 34 22	"	0 22	"	26.8
			D. L. 18° 27′		Mer. D. L. 1230.0

The meridional difference of latitude thus found agrees exactly with the tables, and if operated with as in the preceding example, it will give the same result as that found by Mercator Sailing.

Ex. 2.—Required the bearing and distance of Pernambuco, in latitude 8° 4′ S. and longitude 31° 53′ W. from Cape Verd, in latitude 14° 45′ N. and longitude 17° 32′ W.

Lat. Pernambuco,	8° 4′ S.	8° 4′ S.	Long.	31° 53′ W.
Lat. Cape Verd,	14 45 N.	14 45 N.	Long.	17 32 W.
Diff. latitude,	22° 49′	6° 41′		17° 21′
	60			60
		Mid. lat. 3° 20′		
In miles,	1369		Diff. long.	1041

Set the index to middle latitude 3° 20′ on quadrant, and the line of tangents will cut the index at 60.2, the length of a meridional degree; then to find the meridian difference of latitude: first, for degrees, multiply 60.2 by 22 and the product is 1324.4; again, for the minutes, set the index as above directed and the perpendicular of 49′ on A will show on index 49.1 the proportional part for 49′. The latter result 49.1 added to the former 1324.4, will give 1373.5, the meridian difference of latitude nearly.

Take half the meridian difference latitude 686.75 on side A, and trace its perpendicular till it intersect the parallel of half the difference longitude 520.5 taken on B; set the index to the point of intersection and it will show on quadrant the course 37° 12′. For the distance, while the index is thus set, take on A half the proper difference latitude 684.5, and trace its perpendicular to index, and it will show on index half the distance 859 nearly. Hence the distance is 1718 and the course S. 37° 12′ W. (See figure 23.)

Ex. 3.—A ship from latitude 29° 47′ N. and longitude 24° 36′ W., sails S. S. W. ¾ W. 320 leagues: required her present latitude and longitude.

RULE.—Set the index to course 2¾ points, then opposite distance 320 leagues, or rather 960 miles, on index, will appear on A the difference of latitude 823 miles, and on B the departure 493 miles. (See figure 24.)

Lat. left		29° 47′ N.
Diff. lat. 823 m............	=	13 43 S.
Lat. in		16° 4′
Sum..........................	=	45 51
Mid. lat.......................	=	22 55

Set the index to middle latitude 22° 55′, then opposite the departure 493 on A will appear on index difference of longitude 537 miles.

Long. left		24° 36′ W.
Diff. long. 537 m	=	8 57 W.
Long. in		33° 33′

Hence latitude in is 16° 4′ N., and longitude 33° 33′ W.

Ex. 4.—Suppose a ship from latitude 9° 10′ N. and longitude 19° 32′ W., sails in the south-east quarter till she has made 415 miles departure, and is by observation in latitude 2° 19′ S.: required her course steered, distance run, and longitude in.

Lat. left......	9° 10′ N.	9° 10′ N.
Lat. in.......	2 19 S.	2 19 S.
Diff. lat......	11° 29′	6° 51′
	60	
		Mid. lat. 3° 25′
In miles....	689	

RULE.—Find the point in which the perpendicular of 689 on A, and the parallel of 415 on B intersect each other; set the index to this point, and on the quadrant will be indicated the course 31° 4′, and the above mentioned point will show on index the distance 804.2: if the index be (See figure 25.)

set to the middle latitude 3° 25′, and the departure 115 taken on A, opposite it on index will be seen the difference of longitude 116 miles.

Long. left....................		49° 32′ W.
Diff. long. 416 miles........	=	6 56 E.
Long. in....................		12° 36′ W.

Hence her course is S. 31° 4′ E., distance run 804.2, and longitude in 12° 36′ W.

A ship from latitude 46° 35′ N., and longitude 176° 42′ W., sails N. W. by W. ½ W. till she arrives in latitude 51° 18′ N.: required the distance run and longitude in.

Lat. left......	46° 35′ N.	46° 35′ N.
Lat. in.......	51 18 N.	51 38′ N.
Diff. lat......	4° 43′	97° 53′
	60	
		Mid. lat 48° 56′
Diff. in miles	283	

This problem can be solved by a process nearly similar to that made use of in the third example; yet, in order to show the powers of the scale, we shall adopt a different method. It may, however, be proper to remark, before beginning, that when the larger divisions are considered degrees, each of the smaller ones will represent six minutes, being the tenth part of sixty.

RULE.—Set the index to middle latitude 48° 56′. and on side A take the difference of latitude 4° 43′, that is, 4 large divisions and 7 1-6 small ones, or the division representing 47.16; opposite this will be found on index the meridional difference of latitude 7.2 or 7° 12′.

Now, set the index to the course 5½ points, and opposite the difference of latitude 4° 43′ on A, will appear on index the distance 10° or 600 miles; and if half * the meridional difference of latitude 3° 36, or the division representing 36, be taken on A and its perpendicular traced to index, then opposite this point on index will be found on B half the difference of longitude 6° 72 or 6° 41′. Hence the difference of longitude is 13° 28′, and longitude in 169° 50′ E. See figure 26.

The object of this work being to teach the application of the scale to Navigational purposes, and not to throw any additional light on Navigation, it is not thought necessary to treat on oblique and current sailings here. If the operator thoroughly understands Trigonometry and its application to Traverse Sailing, any cases that may occur in these, however, will not cost him a moment's thought when in possession of the scale.

SPHERICAL TRIGONOMETRY.

65. In treating on Spherical Trigonometry at all, our object is merely to show that the scale is adapted as well to Spherical as to Plane Trigonometry. We shall therefore give only a few examples.

Ex. 1.—In the spherical triangle A B C, right-angled at B, the hypothenuse A C is 64°, and the angle C 46°: find B C. See figure 27.

To find B C:

Cot. A C 64° : Rad. = Cosine C 46° : Tang. B C.

The cot. of 64° (Art. 21) is 29.3, radius is 60, and cosine 46° is 43.6. Set radius 60 on index to 29.3 on A, then opposite 43.6 on A is tangent B C 85 on index; take 85 on line of tangents, set the index to it, and on quadrant will appear the number of degrees in the arc B C 54° 55′.

Ex. 2.—Let the hypotheuuse A C and side B C of the figure A B (Ex. 1) be given, equal to 70° 21′ and 65° 10′ respectively: find angle C.

* Because the perpendicular of the meridional difference of latitude will not intersect the index, its half is used.

Rad. : Cot. A C 70° 21′ = Tang. B C 65° 10′ : Cos. C.

Radius is 60, cotangent 70° 21′ is 21.2, and the semi-tangent of 65° 10′ is 65; then, set 60 on index to 21.2 on B, and opposite 65 on index is half the cosine C 23.1 on B; therefore cosine angle C is 46.2. The perpendicular of 46.2 taken on A, being traced to the arc of the quadrant, will indicate on it the number of degrees 39° 42′.

ASTRONOMICAL PROBLEMS.

66. To find the sun's longitude on a given day.

RULE.—Count the number of days from the nearest equinoctial point; and if the sun is on the south side of the equator, their number will very nearly agree with the sun's longitude taken in degrees on the quadrant of the scale. If the declination be north, count the number of days as before, and subtract one day for every thirty, and in proportion for a less number, and the remainder will agree with the sun's longitude in degrees and minutes on the quadrant.

Note.—The sun's longitude is often useful to discover data for the solution of problems in Astronomy.

Ex. 1.—Required the sun's longitude on the 25th day of November, 1860.

The number of days from the 22d September (the day on which the sun was on the equator) to the 25th day of November, is 64; hence the sun's longitude on that day was 64°.

Ex. 2.—Required the sun's longitude on the 25th day of May, 1860.

From the 20th March (the day on which the sun was on the equator) to the 25th May, are 66 days; and subtracting a day for every 30, that is 2 1-5 or 2.2 days, leaves 63.8 or 63° 48′, the sun's longitude.

67. To find the sun's declination on a given day.

Ex.—Required the sun's declination on the 25th day of November.

The sun's longitude by (Art. 66) is 64°.

Then, as radius........................	=	60 on F
Is to sine 64° (the sun's decl.)...........		54 on B
So is sine of 23° 28′ (greatest decl.).....	=	24 on F
To sine present decl. 20° 55′		21.36 on B

68. The greatest declination and the present declination given to find the sun's longitude.

Ex.—Given the greatest declination 23° 28′, and the present declination 20° 55′: to find the sun's longitude.

RULE.—As sine of 23° 28′ (greatest decl.)	24	on F
Is to sine 20° 55′ (present decl.).........	21.36	on B
So is radius..........................	60	on F
To sine of sun's longitude 64°...........	54	on B

69. The latitude and declination given to find the sun's amplitude, or the distance in degrees the sun is from the east or west at its rising or setting.

Ex.—Given the latitude 40° N. and the declination 22° 30′ N.: required the sun's amplitude at rising.

RULE.—As cosine lat. 40°.............	46	on F
Is to sine decl. 22° 30′................	23	on B
So is radius.........................	60	on F
To sine amplitude 29° 50′ nearly.........	29.8	on B

70. To find the time of the sun's rising and setting on a given day in any latitude.

Note.—If the declination is not given, find it by Art. 67.

Ex. 1.—Required the time of the sun's rising and setting in latitude 50° N., declination being 23° 88′ N.

As radius..............................		60 on B
Is to tang. lat. 50°.....................		71.2 on F
So is tang. decl. 23° 28′................		26 on B
To sine ascensional difference 31°	=	34.1 on F

The ascensional difference converted into time (allowing 15° to hour and 1° to 4 minutes of time), gives the time that the sun rises before, or sets after, 6 o'clock in summer, and the reverse in winter, in north latitude. The above ascensional difference 31°, converted into time, gives 2 hours 4 minutes, which being added to 6 o'clock, gives the time of the sun's setting 8 hours 4 minutes, and being subtracted from 6 o'clock gives 3 hours 56 minutes; therefore the sun sets at 4 minutes past 8 and rises at 56 minutes past 3.

Ex. 2.—Required the time of the sun's rising in lat. 40° N., the declination being 15° N.

As radius..............................	60 on F
Is to tang. lat. 40°......................	50.2 on B
So is tang. decl. 15....................	16.2 on F
To sine ascensional difference 13°........	13.6 on B

13 degrees converted into time gives 52 minutes, which, being subtracted from 6 o'clock, gives 5 hours 8 minutes; hence the sun rises at 8 minutes past 5 o'clock.

71. To find the length of the longest day in any latitude under 66° 32′.

The longest day will happen when the sun is in the solstice, at which time the declination is 23° 28′.

Ex.—Required the longest day in latitude 58°.

As radius............................		60 on B
Is to tang. lat. 58°		95 on F
So is tan. of decl. 23° 28′...		26 on B
To sine ascensional difference 43°	=	41 on F

43 degrees converted into time is equal to 2 hours 52 minutes, and this added to 6 o'clock (Art. 70) gives the time of the sun's setting 8 hours 52 minutes, which, being doubled, gives the length of the day 17 hours 44 minutes.

72. To find the length of the longest day in any latitude above 66° 32′.

Ex.—What is the length of the longest day at the North Cape, in the Island of Maygeroe, in latitude 71° 30′ N.?

RULE.—Set the index to lat. 71° 30′ on quadrant, and on the perpendicular of 30 taken on A will be found the semi-tangent of the latitude 89.3.

Then take 89.1 on index and set it to the parallel of half radius 30 on B, and opposite 60 (sine of ascensional difference for 6 hours) on index will be found on B 20.2, the tangent of declination on the day on which the sun ceases to set in the given latitude. Set the index to 20.2 on the line of tangents, and the declination will appear on the arc to be 18° 35′, and its sine will be found on side B to be 19.1.

Set 21 (sine of greatest declination 23° 28′) on index to 19.1 (sine of aforesaid decl.) on B, and on the arc of the quadrant will appear the sun's longitude when it ceases to set 53° 35′. Subtract 51° 35′ from 90°, and the remainder 38° 25′ doubled gives 76° 50′, which, being taken in time, is equal to 76 days 20 hours.

The operation may be more easily understood by the following proportions:—

As semi-tangent lat. 71° 30′	89.1 on index
Is to half radius....................	30 on B
So is sine ascensional diff. for 6 hours..	60 on index
To tangent decl. when the sun ceases *to set* in the given latitude 18° 35′.....	20.2 on B
The sine of 18° 35′ is equal to........	19.1
Then, as sine of greatest decl. 23° 28′.	24 on F
Is to sine of above decl. 18° 35.......	19.1 on B
So is radius.	60 on F
Sine of sun's long. 51° 35′.	47 + on B

l into time (allowing
), gives the time that
ock in summer, and
. The above ascen-
me, gives 2 hours 4
k, gives the time of
nd being subtracted
s; therefore the sun
minutes past 3.
rising in lat. 40° N.,

.....	60 on F
.....	50.2 on B
.....	16.2 on F
.....	13.6 on B

52 minutes, which,
5 hours 8 minutes;
o'clock.
day in any latitude

e sun is in the sol-
° 28′.
tude 58°.

.....	60 on B
.....	95 on F
.....	26 on B
=	41 on F

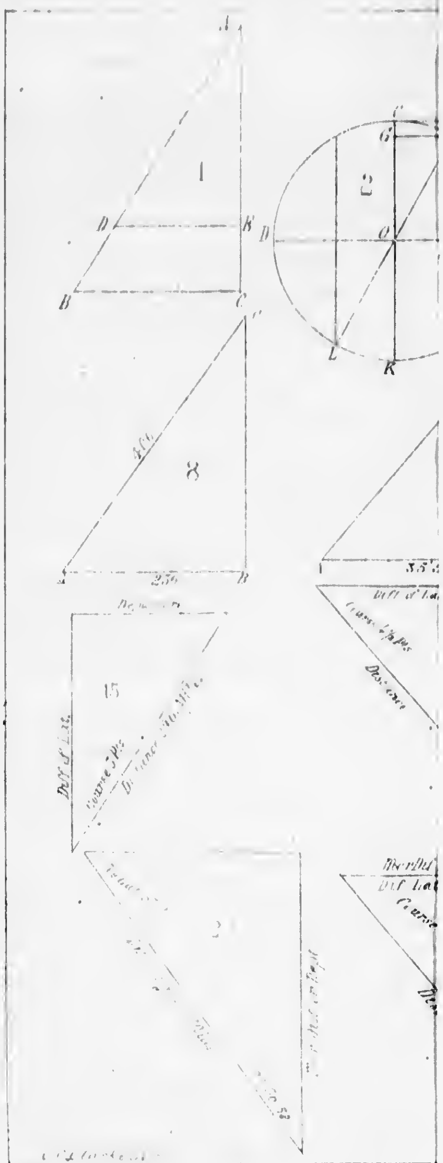
l to 2 hours 52 min-
70) gives the time of
hich, being doubled,
minutes.
day in any latitude

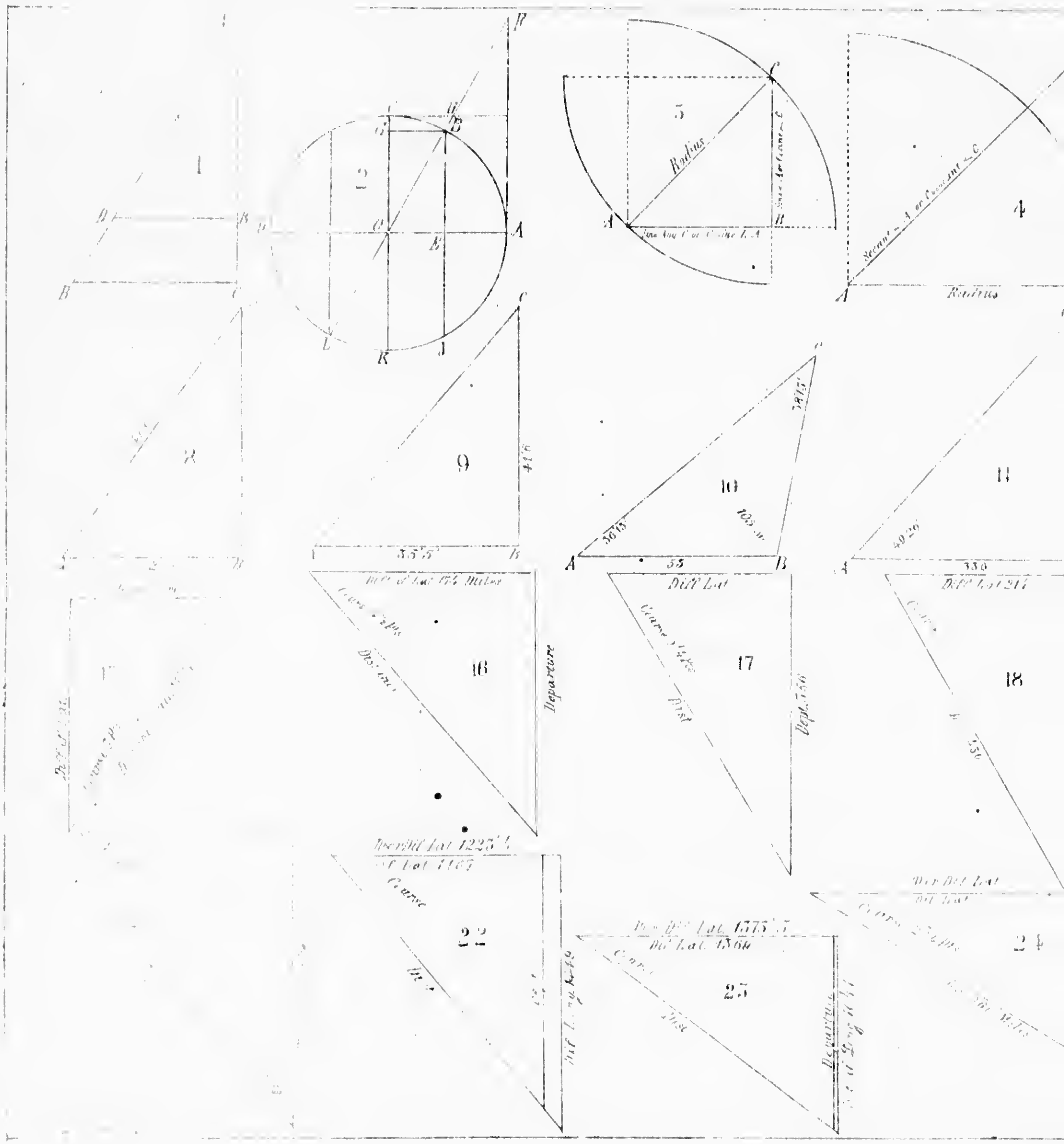
st day at the North
itude 71° 30′ N.?
n quadrant, and on
be found the semi-

the parallel of half
f ascensional differ-
l on B 20.2, the tan-
the sun ceases to set
20.2 on the line of
ar on the arc to be
e B to be 19.1.
3° 28′) on index to
l on the arc of the
when it ceases to set
l the remainder 38°
aken in time, is equal

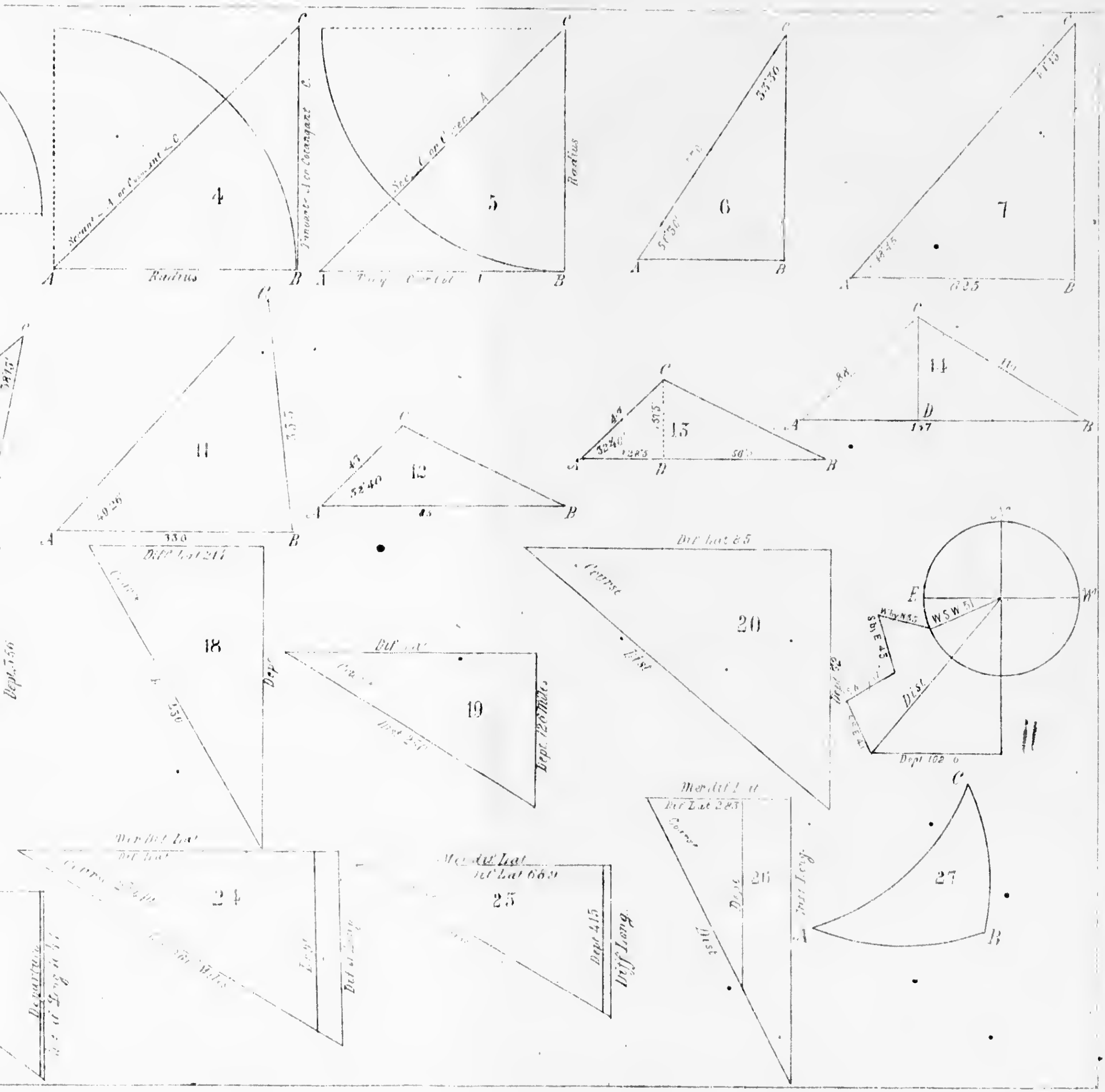
nderstood by the fol-

..	89.1 on index
..	30 on B
..	60 on index
to	
..	20.2 on B
..	19.1
8′.	24 on F
..	19.1 on B
..	60 on F
..	47 + on B





1
2
3
4
Radius
Radius
8
9
10
11
16
17
18
22
23
24
Departure
Diff Lat
Dist
Course


4
5
6
7
11
12
13
14
18
19
20
24
25
26
27
Radius
Dist
Course
Diff Lang
WSW 51

If 51° 35′ be sub
doubled as above, the
to be 76 days 20 hour

M

73. To find the sup
1st.—When the br
Rule.—Set 12 on
length in feet on 1 wi
Ex.—How many s
long and 9 inches wid
Set 12 on F to 9 o
F will show 18 feet o
Ex. 2.—Required
inches broad.
12 on F : 10 on A
12 on F : 10 on B
Ex. 3.—In a plan
many square feet?
12 on F : 8 on B :
tent required.
2d.—When the bre
Rule.—Set breadt
length on A or B wil
The rule may be st
12 on A or B : bre
A or B : : content on
Ex.—Required the
long and 18 inches br
12 on A : 18 on
feet.
74. When the brea
Rule.—Set 10 on
F shows content on A
The rule may be st
10 on F : breadth
The learner can cl
75. To find the so
timber or stone.
Rule.—By two op
or board measure.
The method is easi
Ex. 1.—Required
9 inches broad, and 8
12 in. on F : 9 in.
12 on F : 8 on B :
quired.
Note.—When the
tions are performed b
Ex.—Required th
inches the side of the
12 in. on F : 6 in.
12 on F : 6 in. on
required.
Ex. 2.—Required
and 15 inches the sid
12 on A : 15 on F
12 on A : 15 on F
required.
76. Round and tap
ing the rules in any t
Ex.—How many s
and the girt 42 inche
The rule is, to con
the square. Applie
lows:—
4 or 40 on A : 42
6 s. on F.
12 on F : 10.5 on

If 51° 35′ be subtracted from 90, and the remainder doubled as above, the length of the longest day will be found to be 76 days 20 hours.

PART III.

MENSURATION.

73. To find the superficial contents of a board or plank.

1st.—When the breadth is less than 12 inches.

RULE.—Set 12 on I to breadth in inches on A or B, then length in feet on I will show content in feet on A or B.

Ex.—How many square feet are there in a board 24 feet long and 9 inches wide?

Set 12 on F to 9 on B, or 120 on F to 90 on B, then 24 on F will show 18 feet on B.

Ex. 2.—Required the area of a deal 18 feet long and 10 inches broad.

12 on F : 10 on A : : 18 on F : 15 on A = the answer.

12 on F : 10 on B : : 18 on F : 15 on B = answer.

Ex. 3.—In a plank 7 ft. 6 in. long and 8 in. broad, how many square feet?

12 on F : 8 on B : : $7\frac{1}{2}$ or 7.5 on F : 5 on B = the content required.

2d.—When the breadth is greater than 12 inches.

RULE.—Set breadth in inches on F to 12 on A or B, then length on A or B will show content in feet on F.

The rule may be stated thus:

12 on A or B : breadth in inches on F : : length in feet on A or B : : content on F.

Ex.—Required the superficial content of a board 22 feet long and 18 inches broad.

12 on A : 18 on F : : 22 on A : 33 on F. Answer in feet.

74. When the breadth is given in feet.

RULE.—Set 10 on F to breadth on A or B, then length on F shows content on A or B.

The rule may be stated thus:

10 on F : breadth on A : length on F : : content on A.

The learner can choose exercises out of any text book.

75. To find the solid content of square or unequal sided timber or stone.

RULE.—By two operations similar to those in superficial or board measure.

The method is easily illustrated by example.

Ex. 1.—Required the solid content of a log 50 feet long, 9 inches broad, and 8 inches deep.

12 in. on F : 9 in. on B : 50 ft. on F : : 37.5 ft. on B.

12 on F : 8 on B : 37.5 on F : : 25 on B: the content required.

Note.—When the timber or stone is square, both operations are performed by one move of the index.

Ex.—Required the content of a log 72 feet long, and 6 inches the side of the square.

12 in. on F : 6 in. on B : : 72 ft. on F : : 36 ft. on B.

12 on F : 6 in. on B : : 36 on F : : 18 on B: the content required.

Ex. 2.—Required the solid content of a tree 18 feet long, and 15 inches the side of the square.

12 on A : 15 on F : 18 on A : : $22\frac{1}{2}$ or 22.5 on F.

12 on A : 15 on F : 22.6 on A : : 28.1+ on F : content required.

76. Round and tapering timber can be measured by applying the rules in any text book to the scale.

Ex.—How many solid feet in a round tree 30 feet long, and the girt 42 inches?

The rule is, to consider quarter of the girt as the side of the square. Applied to the scale the operation is as follows:—

4 or 40 on A : 42 on F : 10 on A : : $10\frac{1}{2}$, 10.5 or 10 in. 6 s. on F.

12 on F : 10.5 on A : 30 on F : : 26 ft. 3 in. on A.

12 on F : 10.5 on A : 26 ft. 3 in. on F : : 22 ft. 11 in. + or 23 ft. nearly on A, which is the content required.

A shorter rule is to assume the diameter as if it were the side of the square—say the diameter is 15 inches and log 20 feet long.

Then, as 12 on A is to 15 on F, so is 20 on A to 25 on F, and (without a move) so is 25 on A to 31 on F. Then, as 100 on F to 7854 on A, so is 31 on F to 24.4 on A: the required solidity in feet.

77. To find the area of a parallelogram; whether it be a square, a rectangle, a rhombus, or a rhomboid.

RULE.—Multiply the length by the perpendicular height, according to the directions given for Multiplication.

Ex. 1.—Required the area of a square whose side is 8 feet 6 inches.

As 100 on F : 8 ft. 6 in. or 8.5 on B : : 8.5 on F : : 72.25 or 72 ft. 3 in., the area required on B.

Ex. 2.—Required the area of a rhombus, whose length is 12, and breadth or height 6.5.

100 on F : 6.5 on A : : 12 on F : : 78 on A. Answer.

78. To find the area of a triangle, when the base and perpendicular are given.

RULE.—Set half the base on F to 10 on A, then perpendicular on A will show area on F.

Ex. 1.—Required the area of a triangle, whose base is 60 and perpendicular height 20.

As 30 on F : 10 on A or B : : 20 on A or B : : 600 on F = the area.

Or, set the base on F to 20 on A or B, then perpendicular on A or B will show area on F.

Ex. 2.—Required the area of a triangle, whose base is 80 and perpendicular height 6.

As 80 on F : 20 on B : : 6 on B : : 240 on F = area required.

In some cases the operation can be performed the more readily by taking the base on A or B to 20 on F, then opposite the perpendicular on F is the area on A or B.

Ex. 3.—What is the area of a triangle, whose base is 120 and height 40?

As 20 on F : 120 on A or B : : 40 on F : : 240 on A or B = the area required.

79. Given any two sides of a right-angled triangle, to find the third side.

CASE I.—When the base and perpendicular are given, to find the hypothenuse.

RULE.—Move the index so that the same point or number on F will at the same time be opposite one of the sides on A, and opposite the other side on B, then the said number on F is the hypothenuse required.

Ex. 1.—In a right-angled triangle the base is 42, and the perpendicular 56; what is the length of the hypothenuse?

Move the index until the working edge is at the point of intersection of the lines from 56 on A and 42 on B, which shows 70 on F = the length of the hypothenuse.

CASE II.—When the hypothenuse and one of the sides are given, to find the remaining side.

RULE.—Set hypothenuse on F to the given side on A, then hypothenuse on F will show the remaining side on B.

Or, set hypothenuse on F to the given side on B, then hypothenuse on F will show the remaining side on A.

Ex.—The hypothenuse of a right-angled triangle is 53, and the base 45: required the perpendicular.

As 53 on F : 45 on A : : 53 on F : : 28 on B.

Or, as 53 on F : 45 on B : : 53 on F : : 28 on A = the length of the perpendicular.

80. To find the area of a trapezium, the diagonal and the two perpendiculars being given.

RULE.—Set 100 on F to the sum of the perpendiculars on A or B, then opposite half the diagonal on F is the required area on A or B.

Ex.—Required the area of a trapezium, whose diagonal is 60, the perpendiculars being 36 and 44 respectively.

As 100 on F : 80 on A or B : : 30 on F : : 2400 on A or B = the required area.

The area of a trapezoid can be determined in nearly the same manner, the only variation in the operation being that the sum of the parallel sides and half the perpendicular are used, instead of the sum of the perpendiculars and half the diagonal, as in the preceding article.

The area of a regular polygon can be found by the directions given for triangles, that is when the side and the perpendicular drawn to it from the centre are given; for a regular polygon can always be divided into as many equal triangles as it has sides.

81. To find the circumference of a circle, when the diameter is given.

RULE.—Set 100 on F to 3.1416 on A or B; or, set 70 on F to 22 on A or B, then opposite diameter on F is circumference on A or B.

Ex.—What is the circumference of a circle, whose diameter is 8?

As 7 on F : 22 on B : : 8 on F : : 25.13 on B = circumference.

Or, as 100 on F : 3.1416 on B : : 8 on F : : 25.13 on B.

Another method:—

As 100 on F : : diameter on A or B : : 3.1416 on F : : circumference on A or B.

Or, as diameter on F : 100 on A or B : : 3.1416 on A or B : circumference on F.

Note.—The diameter of a circle, whose circumference is given, may be found by reversing the operation described in either of the preceding methods.

89. To find the area of a circle.

1st.—When the diameter is given.

RULE.—Set 100 on F to 7854 or $78\frac{1}{2}$ on A or B, then the square of the diameter on F will show the area on A or B.

Ex.—What is the are of a circle, whose diameter is 9?

As 100 on F : 7854 on A : 81 on F : 6.36 + on A the area.

2d.—When the circumference is given.

RULE.—Set 100 on F to .07958 or 79 6-10 on A or B, then the square of the circumference on F will show the area on A or B.

Ex.—Required the area of a circle, whose circumference is 8.

As 100 on F : .07958 on B : : 64 on F : : 5. on B = area.

90. To find the area of a regular polygon, when only a side is given.

RULE.—Set the index to half the angle at the centre, contained by the two equal sides of any one of the equal triangles into which the polygon can be divided; then 25 on B traced to the index, and thence to A, will show a quantity on A, which, if multiplied by the number of sides the polygon contains, will give a constant multiplier. The product of the square of the side and this multiplier is the area of the polygon. Half the angle at the centre is always determined by dividing 180 degrees by the number of sides. Thus, for a nonagon it is 20°, for an octagon $22\frac{1}{2}$°, for a hexagon 30°, &c.

Ex.—Required the area of a regular pentagon, whose side is 10.

Here, evidently, half the angle at the centre is 36°. Then set the working edge of the index to 36° on the quadrant, and 25 on B traced to F will cut .344 + on A, which, being multiplied by (the number of sides) 5, gives 1.720 + the constant multiplier for pentagons. Consequently the square of the side or 100 × 1.720 + = 172. + the area required.

In computing the areas of regular polygons, the learner can also find the constant multipliers on the scale by means of cotangents; but this properly belongs to Trigonometry, and requires no explanation here. The method already described will be found to answer all purposes without having recourse to any other, so that the learner can at any time form a table of multipliers for polygons in the space of a few minutes.

91. To find the side of a polygon, to contain a given quantity.

RULE.—Find the multiplier for the regular polygon by the last article. Set 100 on F to the multiplier on A or B, then opposite the area on A or B is the square of the side on F.

Ex.—What is the side of a regular nonagon, whose area is 395 feet?

The multiplier for nonagons will be found to be 6.18 + .

Then, as 100 on F : 6.18 on B : : 395 on B : : 64 on F : the square root of 64 = 8 feet the length of the side required.

92. The area of a circle given to find the diameter.

RULE.—Set 100 on F to .7854 on A or B, then opposite the area on A or B is the square of the diameter on F.

Ex.—What is the diameter of a circle, whose area is 38.5?

As 100 on F : 7854 on B : : 38.5 on B : : 49 on F : the square root of 49 = 7 = the diameter.

When the circumference is required it may be determined in the same manner, using .07958 instead of .7854; or it may be found from the diameter. In the example given the circumference may be thus found:—

As 100 on F : .07958 on B : : 38.5 on B : : 484 on F : the square root of 484 = 22, which is the circumference.

93. To find the area of a sector of a circle, the chord and diameter being given.

RULE.—Find the area of the circle by Art. 89. Set the diameter on F to the chord on B, or set the radius on F to half the chord on B, and the index will show half the number of degrees in the sector on the quadrant.

Then the area of the sector can be determined by the following proportion:—

As 180 : area of circle : : half the number of degrees in the sector : : area of sector.

Ex.—What is the area of a sector, whose diameter is 18, and the chord of whose arc is 6?

The area of the circle by Art. 89 is 254. Setting 18 on F to 6 on B, or 9 on F to 3 on B, the index cuts 19° 45′ on the quadrant, which is half the number of degrees in the sector.

Then, as 180 on B : 254 on F : : $19\frac{3}{4}$ on B : : 27.5 on F = the area of the sector.

94. To find the area of a segment of a circle, the chord and diameter being given.

RULE.—Find the area of the sector as in the last article, and the area of the triangle as in Art. 78, and the sum or difference of these areas, according as the segment is less or greater than a semi circle, shall be the area of the segment.

Examples are unnecessary.

In calculating the area of a sector of a circle, when the chord and versed sine are given, the diameter is easily found by dividing the sum of the square of half the chord and of the versed sine by the versed sine.

OF SOLIDS.

95. To find the solid content of a cube.

RULE.—Set 100 on F to the side on A or B; opposite the side on F is a certain quantity on A or B; and opposite this last quantity on F is the solid content on A or B.

Or, multiply the given side by itself, and that product again by the side.

Ex.—What is the solidity of a cube, whose side is 9?

By setting 100 on F to 90 on A or B,
90 on F shows 81 on A or B,
81 on F shows 729 on A or B, = the solidity required.

The content of a parallelopiped is found on the scale by multiplying the length by the breadth, and that product by the altitude.

96. To find the solidity of a cylinder.

RULE.—Find the area of the base by Art. 89. Multiply the area of the base by the perpendicular height of the cylinder.

Ex.—What is the solidity of a cylinder, whose diameter is 3 and height 8 inches?

The area of the base (Art. 89, mensuration of surfaces) is 28.2.

As 100 on F : 28.2 on B : : 8 on F : : 225 on B = solidity required.

97. To find the convex surface of a sphere.

RULE.—Set 100 on F to the diameter on A or B, then opposite 3.1416 on F is a certain quantity on A or B, and opposite this quantity on F is the convex surface of the sphere on A or B.

Ex.—What is the convex surface of a sphere, whose diameter is 9 inches?

Set 100 on F to 9 on A or B.
3.1416 on F is opposite 28.2 + on A B.
28.2 on F is opposite 254 + on A or B, which is the convex surface required.

Note.—In this, as in many cases, it may be sometimes more convent to have the quotient on the index,

98. To find the solidity of a sphere.

RULE.—The cube of the diameter multiplied by .5236 will be the solidity.

Ex.—What is the solidity of a sphere, whose diameter is 2 inches?

$2^3 = 8$, and $8 \times .5236 = 4.1888$.

On the scale the operation is performed by the directions given for multiplication.

99. To find the convex surface of a right cone.

RULE.—Set 100 on F to the circumference of the base on A or B, and the slant height on F will show double the convex surface on A or B.

Ex.—What is the convex surface of a cone whose slant height is 5, and the circumference of whose base is 9.42.

As 100 on F : 9.42 on B : : 5 on F : : 47 on B ∴ 47 ÷ 2 = 23.5 the convex surface.

100. To find the solidity of a cone or pyramid.

RULE.—Multiply the area of the base by the altitude, and one-third of the product will be the solidity.

Ex.—What is the solidity of a cone, the diameter of whose base is 2 and altitude 50 feet?

$2 \times 2 \times .7854 = 3.1416$ = area of the base.

$$\frac{3.1416 \times 50}{3} = 52.03 + = \text{solidity.}$$

On the scale the operation is performed thus:—

As 100 on F : 50 on B : : 3.1416 on F : : 157 + on B ∴ 100 on F : 3 on B : : 157 + on F : : 52.03 on B = solidity required.

TABLE OF LATITUDES AND LONGITUDES.

Name of Place.	Lat. N. °	′	Lon. W. °	′
Coast of Great Britain and adjacent Islands.				
London, St. Paul's, var. 24° 9′, 1824	51	30	0	5
Greenwich Observatory	51	28	0	0
Deal Castle	51	13	1	23
Dover Castle	51	7	1	19
Dungeness Light	50	55	0	57
Portsmouth Church	50	47	1	5
Needles Light, Isle of Wight	50	39	1	33
Weymouth	50	36	2	27
Eddystone Light	50	10	4	13
Plymouth, New Church	50	22	4	7
" Old Church	50	22	4	7
Falmouth, St. Anthony's Head	50	8	4	59
Land's End Stone	50	4	5	41
Cape Cornwall	50	8	5	41
Bristol Cathedral	51	27	2	35
Holyhead Signal Tower	53	19	4	39
Liverpool, St. Paul's	53	24	2	58
W. and N. Coasts of Scotland.				
Greenock	55	57	4	44
Glasgow	55	51	4	16
Dunnet Head	58	42	3	26
Stromness, Orkney Islands	58	56	3	24
E. Coast of Scotland.				
Inverness	57	31	4	12
Dundee	56	28	2	57
Edinburgh, Coll	55	56	3	10
E. Coast of England.				
Berwick	55	46	1	59
Tynemouth Light	55	1	1	24
			E.	
Yarmouth	52	36	1	43
W. Coast of Ireland.				
Cape Clear Light	51	24	9	29
Fastnet Rock	51	22	9	37
Bantry Bay.				
Sheep's Head	51	31	9	52
Limerick	52	39	8	33
Galway	53	17	8	51
N. Coast of Ireland.				
Sligo	54	16	8	26
Londonderry	54	59	7	20
Giant's Causeway	55	14	6	31
E. Coast of Ireland.				
Belfast	54	36	5	36
Dundalk, Soldier's Point	54	15	6	21
Dublin Light	53	21	6	4
" Observatory	53	23	6	20

Name of Place.	Lat. N. °	′	Lon. W. °	′
Wexford Harbor, entrance	52	22	6	18
Waterford	52	12	7	6
Cork	51	53	8	28
Baltimore	51	28	9	20
N. Coast of France.				
			E.	
Calais	50	57	1	51
Boulogne	50	43	1	36
Havre	49	29	0	6
			W.	
Cherbourg	49	38	1	37
N. and W. Coasts of Spain.				
	N.		W.	
Bilboa	43	14	2	43
Cape Finisterre	42	56	9	16
Coast of Portugal.				
Oporto	41	8	8	38
Lisbon Observatory	38	42	9	8
The East Coast of America, from Cape Horn to Cape Sable.				
	S.			
Cape Horn	55	58	67	12
Montevideo Light	34	53	56	16
St. Catherine's Island, S. Point	27	51	48	41
Rio Janeiro	22	54	43	15
Santa Cruz	16	18	39	1
St. Salvador or Bahia	12	55	38	30
Cape St. Roque	5	28	35	17
Para or Belim	1	28	48	30
	N.			
Berbice, R. entrance	6	20	57	11
Cape Nassau	7	32	58	49
Coast of Colombia.	Lat.	N.		
Cumana	10	27	64	15
Barcelona Castle	10	13	64	48
Cape St. Roman	12	11	70	8
Cartagena Papo	10	26	75	37
Porto Bello	9	34	79	43
Bay of Honduras.				
Belize, Fort George	17	29	88	11
Cape Catoche	21	34	86	57
Vera Cruz	19	12	96	7
Mexico	19	25	99	5
West India Islands.				
Bermuda, St. George's Town	32	22	64	37
Kingston, Jam.	17	49	76	46
Port Royal Point	17	46	76	49
Cuba.				
St. Iago de Cuba	19	57	76	2
Matanzas Castle	23	2	81	32
Havana Light	23	8	82	22

Name of Place.	Lat. N. °	′	Lon. W. °	′
Port au Prince	18	33	72	23
Cape Tiburon	18	19	74	27
St. Domingo	18	28	69	59
Cape Haytien City	19	46	72	14
United States.				
New Orleans	29	57	90	6
Mobile	30	39	88	12
Savanna, S. C.	32	4	81	1
Charleston	32	45	79	50
Cape Hatteras Light	35	15	75	30
Norfolk, Va.	36	50	76	15
Baltimore	39	18	76	36
Washington	38	52	77	0
Philadelphia	39	56	75	10
New York	40	42	74	2
Providence, R. I.	41	50	71	25
New Bedford Light	41	35	70	56
Cape Cod Light	42	3	70	4
Boston	42	22	71	4
Portland Light	43	36	70	11
B. N. A. Provinces.				
St. Andrew's, N. B.	45	3	67	1
St. John, N. B.	45	15	66	2
Digby Light, N. S.	44	40	65	46
Cape Fourchu	43	48	66	9
Cape Sable, S. Point	43	23	65	34
Shelburne, N. S.	43	44	65	17
Liverpool, N. S.	44	1	64	36
Lunenburg, N. S.	44	19	64	8
Halifax	44	39	63	33
White Head Island	45	10	61	5
Cape Canso	45	16	60	56
Gut of Canso, Eddy Point	45	30	61	12
Ship Harbor	45	36	61	17
Louisbourg	45	53	59	55
Port Hood	45	58	61	30
Cape St. George	45	51	61	51
Pictou	45	41	62	38
Tatamagouche	45	49	63	8
Richibucto, N. B.	46	44	64	47
Bay Chaleur, Point Moscow	48	1	64	31
Gaspe Bay, Douglas Town	48	46	64	21
Anticosta Island	49	8	61	42
P. E. Island.				
George Town	46	10	62	30
Charlottetown	46	16	63	10
New London	46	33	63	32
Newfoundland.				
Cape Ray	47	37	59	17
Fortune Harbor	49	32	55	10
Fleur de Lis Harbor	50	6	56	2
Trinity Harbor	48	21	53	16
St. John's	47	34	52	38
Cape Race	46	39	52	59
Trepassey Bay, Cape Pine	46	37	53	30
Placentia Harbor	47	15	53	55
Little Miquelon	46	47	56	19
Bordeaux Harbor	47	45	53	58