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THE DISTRIBUTION OF STRESS IN RIVETED CONNECTIONS

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(Read before the General Section, April 19th, 1906.)

PREVAILING CONCEPTIONS OF STRESS-DISTRIBUTION.

The inequality of distribution of stress among the rivets of a riveted connection is a matter of general agreement among engineers. In most cases, however, whatever inequality of stress-distribution exists, is attributed to the imperfect matching of holes, and the want of close fit of the rivets to the walls of the holes, thus placing the rivets under different conditions for the resistance of shear. It is further agreed that these conditions render it impossible to tell anything definite about the distribution of stress in riveted connections, and that more refined methods of design than are at present in use are out of the question.

ACTUAL CONDITIONS AFFECTING STRESS-DISTRIBUTION.

Two erroneous assumptions underlie the above conclusions:

First. It pre-supposes that if a connection containing a number of rivets in the line of the stress-producing force, as shown in Figure 1,

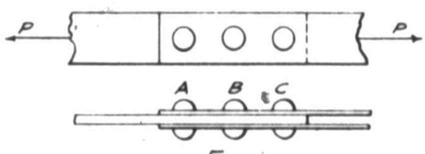

Fig. 1

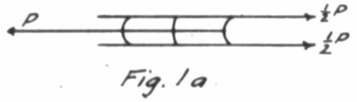

Fig. 1a

were constructed so that the rivet holes were perfectly concentric and the rivets all fitted perfectly, and a load were applied to the connection, the rivets would be stressed equally. This could not be the case, however, because of the elastic properties of the material of the connection. Professor William H. Burr, of Columbia University, speaking of the influence of the elastic properties of the plates in riveted connections on the distribution of stress among the rivets, says: * "In the case of lap joints with three or more rows of rivets (frequently found in truss work), or in similar work when two rows of rivets join a small plate to a much larger one, the outside rows, or row, in consequence of the stretching of the material at the joint, must take far more than their portion of stress, if, indeed, they do not carry nearly all. The same condition of things will exist in butt joints if two or more rows are found, under similar circumstances, on the same side of the joint."

A very neat demonstration of this necessary inequality of stress-distribution among the rivets of a perfect connection is given by Professor W. H. Boughton, in the Proceedings of the Ohio Society of Surveyors and Civil Engineers, for 1902, page 17. A connection of the form shown in Fig. 1, composed of one 3" × ½" bar and two 3" × ¼" bars connected together by three rivets, is considered. A pull P is supplied in the direction of the line of the rivets. Let it be assumed, as is done in practice, that the stress is distributed equally among the three rivets. If this be true $\frac{1}{3}P$ is taken out of the main bar by the rivet A into the two side bars, $\frac{1}{3}P$ by the rivet B, and $\frac{1}{3}P$ by the rivet C, leaving a stress in the main bar between A and B of $\frac{2}{3}P$, and between B and C of $\frac{1}{3}P$. At the same time the stress in the *two* side bars between A and B is $\frac{1}{3}P$, and between B and C it is $\frac{2}{3}P$. But the area of the main bar is the same as the area of the two side bars together, hence the extension of the main bar between A and B is twice the extension of the two side bars, and the extension of the main bar between B and C is one-half the extension of the two side bars. Assuming, then, that the rivets fit their holes perfectly, the distance between their centres in the side bars is less than in the main bar for rivets A and B, and greater for the rivets B and C. From this necessarily follows an inequality of distortion of the rivets, which is shown diagrammatically in Fig. 1 (a). Rivets A and C are more distorted than rivet B, and are consequently more highly stressed, although we set out with the assumption that all the rivets were equally stressed. It is manifest that with the plates of the connection of the relative areas assumed, the rivets A and C would be equally stressed and each more highly than B. With these relative

* "Elasticity and Resistance of the Materials of Engineering," p. 700.

sections the inequality of stress disappears with two rivets, but it is more and more pronounced as the number is increased above three. The end rivets would, with the assumed relative plate sections, be equally stressed and rivets on opposite sides of the centre of the connection and equidistant from the centre would be equally stressed.

The second erroneous assumption is that the faulty matching of holes and the imperfect fitting of rivets to the walls of the holes in themselves influence the distribution of stress among the rivets of the connection. This is based on the further assumption that rivets under ordinary working loads resist by shear, which has been conclusively disproved by many able investigators, among whom are Considère, Bach, Dupuy, and Van der Kolk. A concise paper describing the results obtained by the last two authorities was read before this Society by Professor J. T. Nicholson, a few years ago, and further reference need not be made to the matter other than to say that experimental enquiry all goes to show that within ordinary working loads rivets resist entirely by tension in the shaft which grips the assembled plates together and never come into shear until the safe working load has become exceeded, when a more or less sudden slipping of the assembled plates occurs. It is evident, therefore, that slight mis-matching of holes, which, has heretofore been considered sufficient to vitiate any theoretical calculation of the distribution of stress among the rivets of a connection, can have little effect. It is only when the mis-matching is great enough to produce sensible differences of sectional area of the rivets, thus causing differences in their gripping power, that theoretical calculations would be offset. Of course, in all good work the mis-matching would not be great enough to produce this effect to any extent, and in the case of drilled and reamed work it would not happen at all.

With these facts before us, we can enter upon the derivation of a method of calculating the distribution of stress in riveted connections with a feeling that the calculated stresses will, in general, agree pretty closely with the actual ones. The normal tendency is for this agreement, and the nearer the workmanship approaches perfection, the nearer will the agreement be to complete realization. The theory which will be developed can be applied to any joint, given certain experimental quantities, of which more will be said. These quantities have as yet been determined for only one form of connection, the multiple-riveted butt-jointed with two cover plates.

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* Transactions Canadian Society of Civil Engineers, Vol. 12, Part 2.

Before proceeding further we must now enquire into the slippings which occur in riveted connections under ordinary working loads.

SLIPPINGS IN RIVETED CONNECTIONS.

The researches of Van der Kolk, to which reference has already been made, threw great light on the slipping in riveted connections.* He measured the slippings at the various rivets in a great number of connections, as well as the extensions between consecutive rivets, for loads gradually varying from zero to past the maximum allowable working load. In doing so, he found that from the very commencing of the loading at a joint there is a gradual relative slipping of the plates at each rivet, part of which disappears when the load is removed, and the remainder of which is permanent. The former is called the elastic slipping and the latter the permanent slipping.

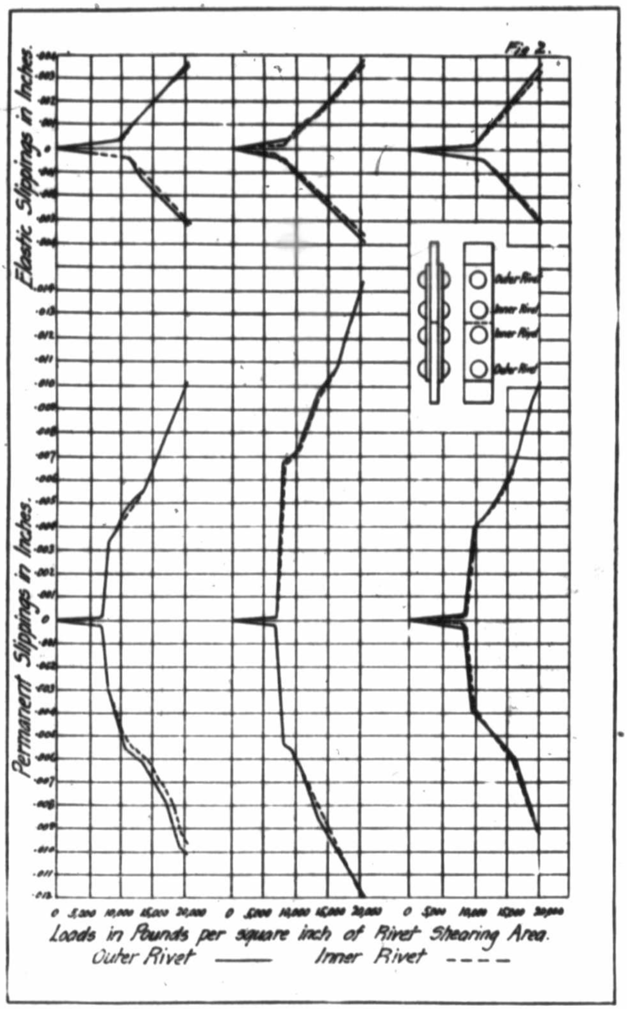
The test pieces employed were all butt joints with two cover plates, there being a single line of rivets in the direction of the stress-producing force. In some cases there were two rivets on each side of the splice, and in other cases three. The cover plates for all the specimens but those of one series were each one-half the thickness of the main plates, while for the specimens of that series the plates were all of the same thickness. The thickness of the cover plates for the first group of specimens was 0".493 and of the main plates 0".985, while for the second group the plates were all 0".61 thick. In every case the plates were carefully machined to a width of 2".76. With four rivets in a joint the spacing was 3".15, while for six rivets it was 2".60. The rivets were $\frac{3}{4}$ inch in diameter.

The slippings at each rivet of every joint were embodied in a very complete series of diagrams, from which typical ones have been prepared for Figs. 2, 3 and 4. The upper diagrams in each figure give the elastic slippings for the various applied loads, and the lower ones the permanent slippings for the same loads. The upper part of each diagram refers to the rivets above the splice (the specimen being considered as standing vertically), and the lower part to the rivets below the splice. Distinctive lines are used for the outer, middle and inner rivets. The plotted slippings at any rivet are the averages of the observed slippings on the right and left sides of the test piece.

Fig. 2 gives the load slip diagrams of three typical machine-riveted specimens with cover plates, each one-half the thickness

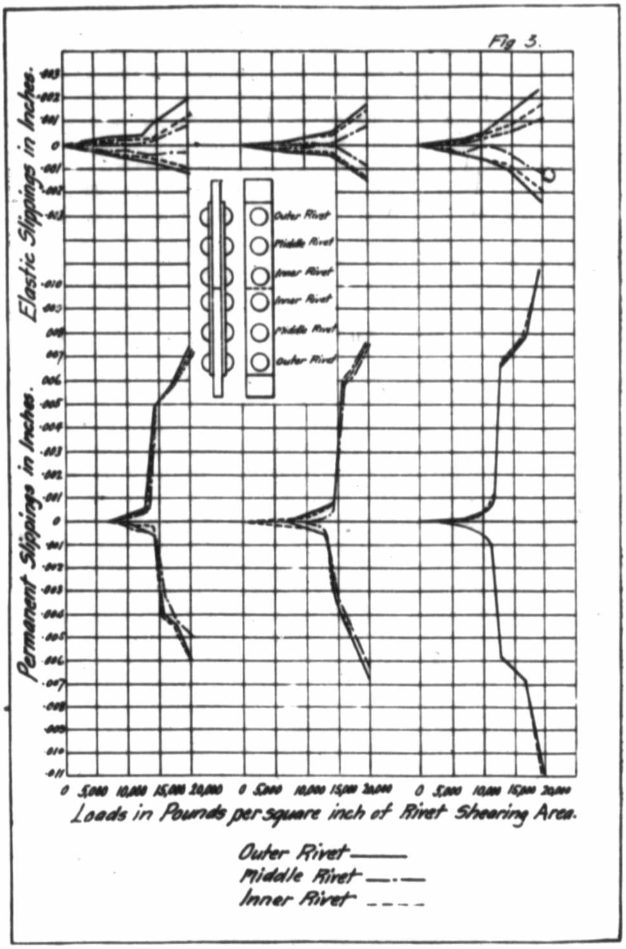
* See Zeitscrift des Vereines Deutcher Ingenieure, 1897, p. 739.

of the main plate and with four rivets. Upon examination of the plotted results we see that both elastic and permanent slippings increase uniformly up to a well-defined point of sudden slipping.



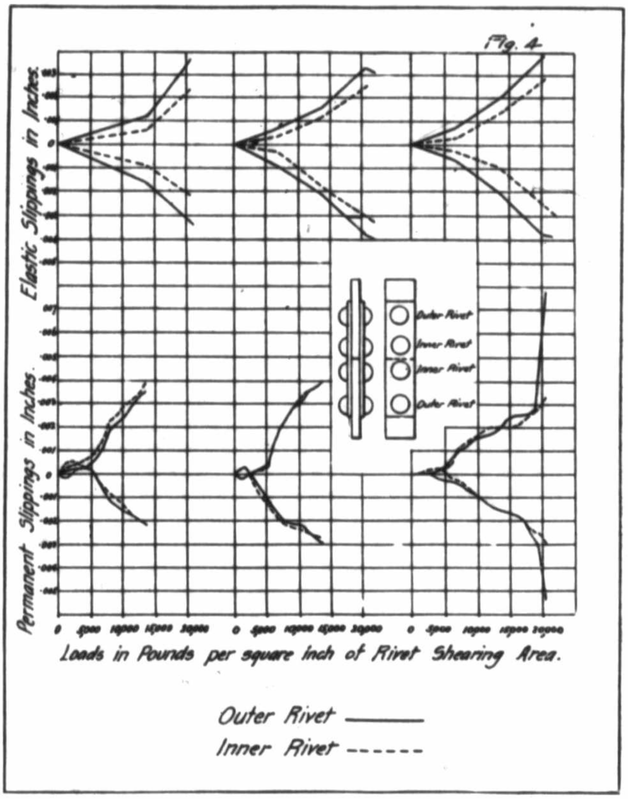
Both the elastic and permanent slippings of the outer and inner rivets are practicallly equal.

Fig. 3 shows the slippings in three machine-riveted specimens with cover plates each one-half the thickness of the main plate, and with six rivets. The elastic and permanent slippings are both



fairly uniform up to a point of sudden slipping, as for the preceding specimens. The permanent slippings are practically the same for all rivets, but there is a significant inequality of the elastic slippings. The elastic slipping of the inner rivet is shown, within the ordinary limits of loading, to be about the same as for the outer rivet, but that for the middle rivet is much less than for either of the others.

In Fig. 4 are given the plotted results for three typical hand-riveted specimens with cover plates the same thickness as the



main plate and with four rivets. The elastic slipping is seen to increase uniformly with the load for both outer and inner rivets up to a fairly well-defined point for each rivet, while the permanent slippings are somewhat irregular. Taking the average of all the specimens tested in this series, however, the permanent slippings are fairly uniform up to a well-defined point and about equal for the outer and inner rivets. Of course, the hand-riveting is responsible for the irregularity.

From this examination of Van der Kolk's results we see that, in the average case, the elastic and the permanent slippings are both directly proportioned to the load on the specimen up to the point of more or less sudden slipping. Also we may say, that the sum of the elastic and permanent slippings for any rivet, which we shall call the *total slipping*, is directly proportional to the load on the joint up to the point of sudden slipping. This point is at about the limit of shearing stress on the rivet generally allowed in design.

The results of this enquiry into the question of slipping may be summed up as follows:

(1) The relative slipping of the plates at a rivet in a butt joint with two cover plates is directly proportional to the load on the rivet within the ordinary limits of working stress assigned to it.

(2) In butt joints with two rivets on either side of the splice, and cover plates one-half the thickness of the main plate, the total slipping is the same at both rivets.

(3) In butt joints with three rivets on either side of the splice, and cover plates one-half the thickness of the main-plate, the total slipping at the outer and inner rivets is equal, but the total slipping at the middle rivet is less than at the outer and inner rivets.

(4) In butt joints with two rivets on either side of the splice, and cover plates the same thickness as the main plate, the total slipping at the inner rivet is less than at the outer rivet.

SIGNIFICANCE OF FACTS ESTABLISHED FROM VAN DER KOLK'S RESULTS.

In view of fact (1), the other three become of special significance. Clearly, the only interpretation which can be put on them is that in any given connection the stresses on the rivets are as the relative slippings of the plates at them. Thus, in butt joints with two rivets on either side of the splice, and cover plates one-half the thickness of the main plate, the rivets must be stressed equally. In the case of similar joints, with three rivets on either side of the splice, the outer and inner rivets are stressed equally, and the middle rivet is stressed less than either. With butt joints, with two rivets on either side of the splice, and cover plates the same thickness as the main plate, the inner rivet must be stressed to a smaller extent than the outer one.

Thus it is seen that we are not without experimental reasons for suspecting a definite law of stress distribution. The matter will now be approached theoretically, using data obtained from

Van der Kolk's results for the numerical calculations which will arise.

A HYPOTHETICAL CASE DISCUSSED.

The conditions of stress-distribution in actual riveted connections are elucidated by a consideration of the hypothetical case where the cover plates are inextensible and incompressible and the main plate retains its elastic properties. Suppose that the latter is provided with cylindrical projections on its sides, as shown in Fig. 5, and let it be gripped between the two cover plates by

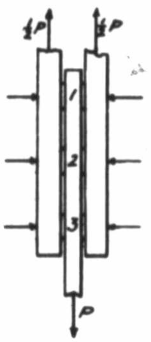


Fig. 5

forces applied as shown, the cylindrical projections slipping between the cover plates amount directly proportional to the vertical loads on the projections. Since riveted connections resist loading entirely by friction of the plates within ordinary loads, and since the pressure exerted by a rivet in drawing the plates together is probably confined to a small circular area immediately around the rivet, this hypothetical method of connecting the plates together would be exactly analogous to riveting.

Suppose, now, a load P be applied to the joint. What part of it is taken into the cover plates at the points 1, 2 and 3? If the main plate were inextensible and incompressible, like the cover plates, there could be no doubt in the matter—the same part of P would be transferred at 1, 2 and 3, because whatever amount of slipping occurred at 3 must also occur at 1 and 2, since the main plate is considered as inextensible. The equal slippings, of course, mean equal loads, a principle which we have already established for actual connections. Further, suppose that while the cover plates remain perfectly rigid the main plate becomes highly extensible. It is self-evident that under these conditions nearly all the load would be transferred to the cover plates at 3. For a degree of extensibility of the main plate between these two extremes it would seem as if the distribution of the loads would be somewhere between the distributions in the other two cases.

This was confirmed by an experiment devised by the author, which, though not attempting to realize the conditions of this hypothetical case, still involved exactly analogous ones. Three white pine strips, $\frac{1}{2}'' \times \frac{1}{4}'' \times 2'\ 0'$ were supported flatwise, directly above one another, on heavy wire nails driven into a wall, as shown in Fig. 6.

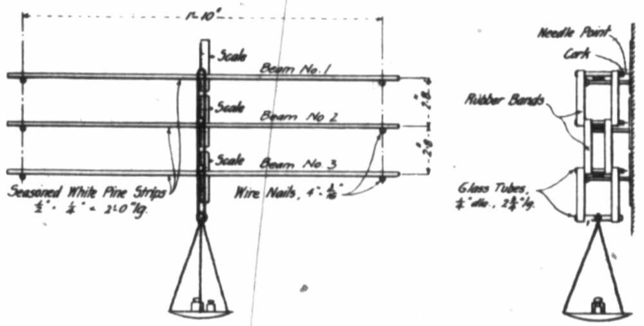


Fig. 6

These strips were 2.8" apart and the spans were all 1′ 10″. The strips were connected to each other at the centre by rubber bands looping around glass tubes $\frac{1}{4}''$ in outside diameter, which rested across the strips. In the ends of these tubes next the wall were placed corks, which carried needles as shown in the figure. Opposite these needles, and fixed to the wall, were scales graduated to one-twentieth of an inch, so that the deflection of each strip under load could be estimated by eye to the one-hundredth part of an inch. To the tube resting on the bottom strip, a sling arrangement, consisting of two more bands and another tube, was attached, carrying a scale pan. Into this pan were put weights, varying from 1 pound to $1\frac{3}{4}$ pounds, and the scale readings taken for each loading. From these readings the increments in deflection of each strip for various additions of load to the scale pan could be derived. Care was taken to see that the initial load, due to the pan, the bands, and the tubes, caused all the tubes to bear fully over the strips for which they were intended. The loads were always kept within the elastic limit of the material of the strips so that the increments of deflection due to any addition of load to the scale pan were direct measures of the amount of the added load sustained by each strip.

The parts of the various loads in the scale pan carried by each beam were computed from the deflections. For example, when 1½ lbs. had been added, the parts of it sustained by beams 1, 2 and 3 were as 1 : 1.92 : 4.62. The results of this experiment were

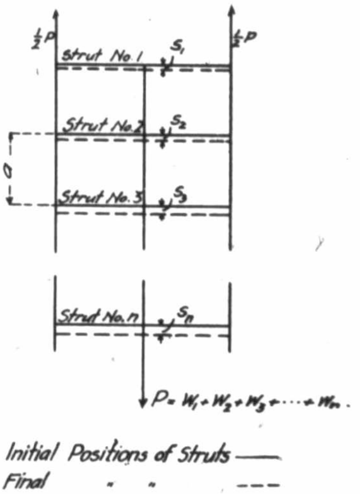


Fig 7

rather closely calculated beforehand from a knowledge of the moduli of elasticity of the pine and rubber. It was estimated that within the elastic limit of the materials the parts of any added load which would be sustained by 1, 2 and 3 would be in the proportion of 1 : 2.2 : 6.04, which agreed very well with the experimental results, considering the variations likely to occur in such a material as rubber.

But let us return to the hypothetical case. In view of the results of the experiment with the three pine strips, it is clear that when the main plate in the joint shown in Fig. 5 is subjected to a pull P, and it is not of inextensible material, the distribution of the load among the supports 1, 2 and 3 should proceed according to some law analogous to that existing for the case of the simple beams just discussed. This is true, because the case of the beams and the hypothetical case are exactly parallel. In both cases an extensible tie is attached to certain parts of a system which moves parallel to the axis of the tie amounts directly proportional to the load applied. The distribution of loads among these parts must, therefore, be according to a similar law.

It will be of value to us in the consideration of practical cases to establish the general law of distribution of load in the hypothetical case under discussion. The cylindrical projections on the sides of the main plate may be regarded as rigid struts, fitting tightly between two rigid walls, and having the property of slipping between these walls amounts directly proportional to the loads applied to them by the extensible main plate. Let there be any number, n, of these struts placed at equal distances apart in the same plane, as shown in Fig. 7. They may be of any lengths, not necessarily equal, as they are considered as non-flexible, moving bodily in the sense of the applied forces.

Let P = load at bottom of tie.

$W_1, W_2, W_3 \ldots W_n$ = loads supported by struts 1, 2 3 . . . n due to application of P.

$S_1, S_2, S_3 \ldots S_n$ = slippings of these struts between the walls due to the loads $W_1, W_2, W_3 \ldots Wn$.

a = initial distance apart of struts.

K = ratio of total load on any strut to the slipping of strut caused by that load.

A = sectional area of tie.

F = ratio of stress per unit area of tie to unit elongation between struts (not necessarily the same as the modulus of elasticity of the material of the tie).

G = ratio of total load in a tie between two struts to the total extension of such part of tie. It therefore $= \frac{A}{a} F$.

Now, from the assumed conditions,

$$S_1 = \frac{W_1}{K}$$

$$S_2 = \frac{W_2}{K}$$

$$S_n = \frac{W_n}{K}$$

and within the elastic limit of the material of the tie

$$W_1 = G\,(S_2 - S_1)$$
$$W_1 + W_2 = G\,(S_3 - S_2)$$

$$W_1 + W_2 + W_3 + \ldots + W_{n-1} = G\,(S_n - S_{n-1})$$

Putting in these latter equations the values of S obtained from the upper set, we are able to express all the loads in terms of W_1. After the loads on several struts are thus expressed, the general expression can be written down, giving the load on the n-th strut from the top. The loads in terms of W_1 are:

$$W_1 = W_1$$

$$W_2 = W_1\left[1 + \frac{K}{G}\right]$$

$$W_3 = W_1\left[1 + 3\frac{K}{G} + \left(\frac{K}{G}\right)^2\right]$$

$$W_4 = W_1\left[1 + 6\frac{K}{G} + 5\left(\frac{K}{G}\right)^2 + \left(\frac{K}{G}\right)^3\right]$$

$$W_n = W_1\left[1 + \frac{(n-1)\,n}{\underline{|2}}\,\frac{K}{G} + \frac{(n-2)(n-1)\,n\,(n+1)}{\underline{|4}}\left(\frac{K}{G}\right)^2 + \frac{(n-3)(n-2)(n-1)\,n\,(n+1)(n+2)}{\underline{|6}}\left(\frac{K}{G}\right)^3 + \ldots\ldots\left(\frac{K}{G}\right)^{n-1}\right]$$

This law of distribution of load over the strut in such a system may be shown graphically by assigning fixed values to $\frac{K}{G}$. From the nature of K and G, it is evident that they are always positive and may be of such values as to cause $\frac{K}{G}$ to vary from zero to infinity. The diagram given in Fig. 8 has been constructed, giving $\frac{K}{G}$ the values of .1, .2, .3, .4 and .5. The numbers along the vertical axis are the numbers of the struts counting from the top, while the numbers along the horizontal axis represent loads proportional to W_1. The relative loads on the struts are calculated for the various values of $\frac{K}{G}$, and horizontally out from the numbers representing these struts points are located which are vertically opposite numbers on the horizontal scale equal to the loads on the struts in terms of W_1. For any value of $\frac{K}{G}$ the points thus located are connected by straight lines to make it easier to see the law of variation of load. These curves are not algebraic, since they are not continuous. The only points having any meaning are those horizontally opposite strut numbers.

From this diagram may be seen at a glance the truth of certain statements already referred to as self-evident. These were that if the main plate were inextensible and incompressible, like the cover plates, the load would be equally distributed among the

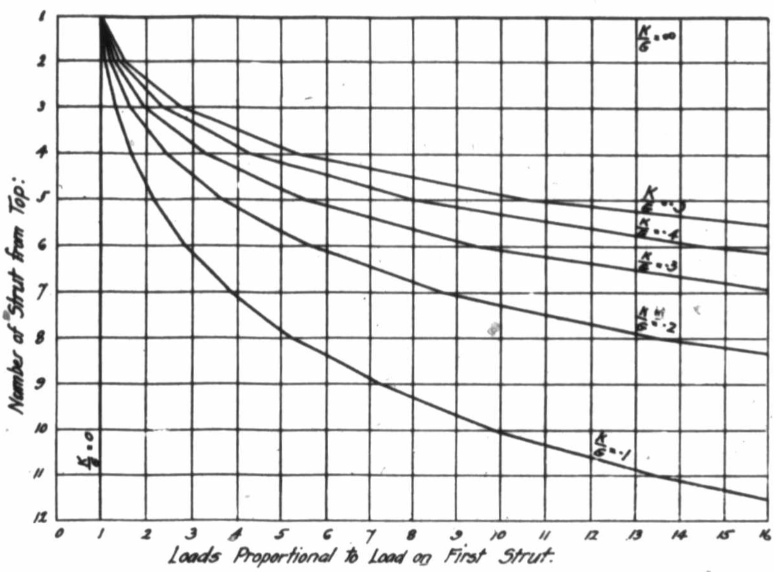


Fig. 8.

struts, and if the main plate were infinitely extensible the bottom one would get all the load. These conditions would render $\frac{K}{G} = 0$ and $\frac{K}{G} = \infty$ respectively. In the former case the curve becomes a straight line coinciding with the vertical line through 1, showing that on all struts the load is the same. In the latter case, the curve becomes a horizontal line through 1, showing that the load on any strut below the first one is infinitely greater than the load on the first one.

These same results would occur if, while the main plate retained its ordinary elastic properties, the resistance of the struts to slipping became extremely small, or if, on the other hand, it became infinitely great.

There is another factor which influences the distribution of load in this hypothetical case, and that is the distance between the struts. As has been shown already, the load-distribution depends entirely upon the value of $\frac{K}{G}$, G being equal to $\frac{AF}{a}$. The effects of variations in K, A and F have been seen, and the effect of variation in A remains to be noted. Evidently if a were increased in value indefinitely, G would become very small, and hence $\frac{K}{G}$ would approach infinity. The bottom strut would then get practically all the load. If the reverse change take place, that is, if a become very small, the struts would all sustain practically the same load.

From this enquiry we may make the following statements of the effect of variations in K, A, F and a on the distribution of loads in such a hypothetical system, assuming that all other variables except the one under consideration are constant for the time being:

(1) An increase in K produces further inequality of load-distribution among the struts, while a decrease in K produces further equality.

(2) An increase in A produces further equality, while a decrease produces further inequality.

(3) An increase in F produces further equality, while a decrease produces further inequality.

(4) An increase in a produces further inequality, while a decrease produces further equality.

ANALYSIS OF ACTUAL CONNECTIONS.

The hypothetical case just discussed enables us to form a clear idea of how the distribution of stress in a riveted connection should proceed, although it cannot be used to calculate the stress on any rivet. We may suppose that the slipping in a joint is made up of two parts, occurring at two different times. Consider, first, that the cover plates are of perfectly rigid material, and that the main plate possesses ordinary elastic properties. The joint may be of any form, such as shown in Fig. 9. Let the first slipping at any rivet be the slipping which occurs under these hypothetical conditions when the load comes on the connection. The slippings at the various rivets would proceed according to the law established in the discussion of the hypothetical case, or they would be to each other as the abscissas at 1, 2, 3, . . . n of some curve similar to that shown in the right hand part of Fig 10. But now, when the main plate has adjusted itself, suppose the cover plates regain their normal elastic properties. Here the second part of the slipping takes place and the slipping at any rivet takes its final value. The second part of the slipping evidently would be according to a law similar to that governing the first part, for when the second part takes place the main plate may be arbitrarily assumed as rigid and the cover plates as extensible. These secondary slippings at the various rivets would then be to each

Fig. 9.

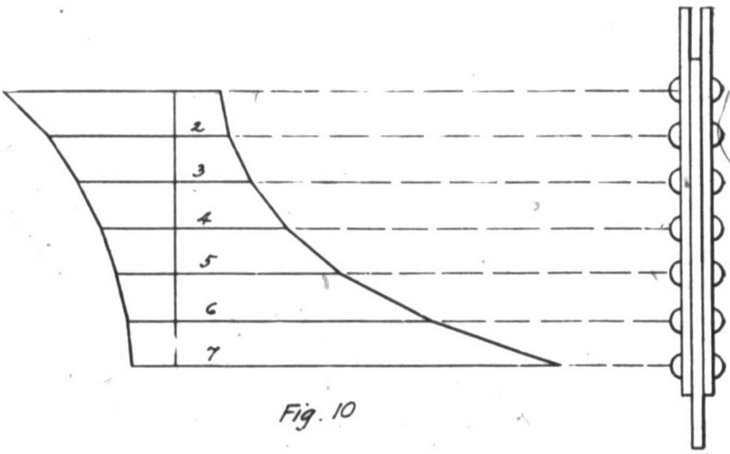


Fig. 10

other as the abscissas at 1, 2, 3 . . . n of some curve similar to that shown in the left hand part of Fig. 10. Now, the actual slipping at any rivet is composed of the sum of the two slippings, and if the horizontal scales be the same in the two parts of Fig. 10, then the stresses in the various rivets must be proportional to the horizontal distances between the two curves at these rivets.

The actual stresses in riveted connections cannot be analyzed by this method, because we do not know the ratio of the load on a rivet connecting extensible and inextensible plates to the slipping occurring at the rivet. We do know, however, something concerning the slipping at a rivet connecting extensible plates, and it will be shown that formulas may be derived involving this slipping, which are capable of being used to calculate the load on any rivet in a connection.

Let us consider the simplest case of multiple riveting, where two cover plates of equal section are connected to a main plate of any section by two rivets, as shown in the sketch, Fig. 11.

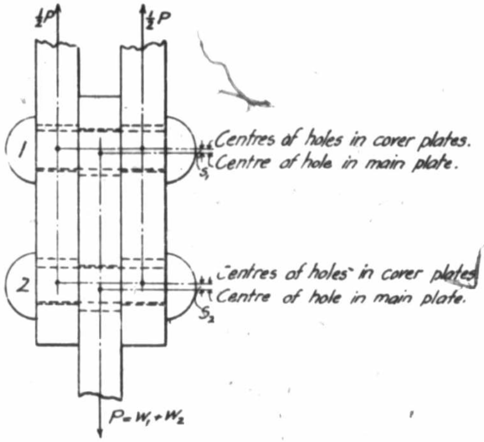


Fig. 11

Let P = load on connection.

W_1, W_2 = loads on rivets 1 and 2.

S_1, S_2 = relative slippings of plates at rivets 1 and 2, due to loads W_1 and W_2.

K = ratio of total load on a rivet to the slipping at that rivet.

A_1 = sectional area of main plate.

A_2 = sectional area of the two cover plates together.

F = ratio of stress per unit area of plates to elongation per unit of length between the rivets (not necessarily the same as the modulus of elasticity of the material of the plates).

a = pitch of rivets.

G_1 = ratio of the load in the main plate between the rivets to the total extension of this part of the plate under plates under such load. It, therefore, $= \frac{A_1}{a} F$.

G_2 = ratio of the load in the two cover plates between the rivets to the total extension of this part of the plates under such load. It, therefore, $= \frac{A_2}{a} F$.

Now

$$S_1 = \frac{W_1}{K}$$

and

$$S_2 = \frac{W_2}{K}$$

But the difference between the extension in the main plate of the length a and of the extension in the cover plates of the same length is equal to the difference between S_1 and S_2. Suppose S_2 is greater than S_1, then the extension of the length a in the main plate is evidently greater than the extension of the same length in the cover plates, and we may write:

$$S_2 - S_1 = \frac{W_1}{G_1} - \frac{W_2}{G_2}$$

or

$$\frac{W_2}{K} - \frac{W_1}{K} = \frac{W_1}{G_1} - \frac{W_2}{G_2}$$

It is clear that it makes no difference whether S_1 or S_2 is the greater as far as this equation is concerned, so for purposes of demonstration we will assume that the slipping gets greater the farther down we go from the first rivet.

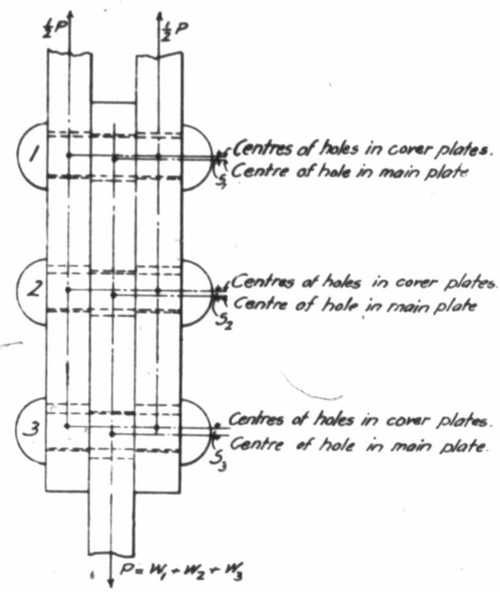


Fig. 12

Solving this equation for W_2 in terms of W_1 we get:

$$W_2 = W_1 \left(\frac{1 + \frac{K}{G_1}}{1 + \frac{K}{G_2}} \right)$$

From this we can find the relative stresses on the rivets if we are given K, A_1, A_2, F and a.

Let us now examine the case of a similar connection with three rivets as shown in the sketch, Fig. 12.

The quantities involved in this case will be similar to those in the case just discussed, W_1, W_2, W_3 being the loads on the rivets and S_1, S_2, S_3 being the slippings at them due to these loads.

Now, pursuing the same reasoning as in the last case, we may write two equations similar to the one first written down for it. They are:

$$S_2 - S_1 = \frac{W_1}{G_1} - \frac{W_2 + W_3}{G_2}$$

or

$$\frac{W_2}{K} - \frac{W_1}{K} = \frac{W_1}{G_1} - \frac{W_2 + W_3}{G_2}$$

and

$$S_3 - S_2 = \frac{W_1 + W_2}{G_1} - \frac{W_3}{G_2}$$

or

$$\frac{W_3}{K} - \frac{W_2}{K} = \frac{W_1 + W_2}{G_1} - \frac{W_3}{G_2}$$

Solving these two equations for W_2 and W_3 in terms of W_1, we get

$$W_2 = W_1 \left(\frac{1 + \frac{K}{G_1} + \frac{K}{G_2}}{1 + 3\frac{K}{G_2} + \frac{K^2}{G_2^2} + \frac{K^2}{G_1 G_2}} \right)$$

$$W_3 = W_1 \left(\frac{1 + 3\frac{K}{G_1} + \frac{K}{G_2} + 4\frac{K^2}{G_1 G_2} + \frac{K^2}{G_1^2} + \frac{K^3}{G_1 G_2^2} + \frac{K^3}{G_1^2 G_2}}{1 + 4\frac{K}{G_2} + 4\frac{K^2}{G_2^2} + \frac{K^2}{G_1 G_2} + \frac{K^3}{G_2^3} + \frac{K^3}{G_1 G_2^2}} \right)$$

On account of the great amount of labour required to derive formulas for joints containing more than three rivets, they were not carried further, the law being shown by the consideration of these short connections.

In order to show the application of this theory to the determination of the stress in any rivet of a connection, and to render clearer the law of distribution of stress, several numerical examples will be given, involving values of K and G, which are known approximately from experiment.

K, the ratio of the load on a rivet to the slipping at it, may for joints about 2 inches thick, assembled by $\frac{3}{4}$-inch rivets, be arrived at from the experiments of Van der Kolk. An examination of results for 24 of the most represented specimens with two rivets on each side of the splice, and cover plates one-half the thickness of the main plates, showed, for an average load per rivet of 4,855 pounds, an average slipping of .000305 inches. This gives a value to K of about 16,000,000.

G, as has already been stated, is a quantity algebraically defined by the relation $G = \frac{A}{a} F$. Values of G could readily be computed if we knew the value of F. This latter quantity in the case of an ordinary riveted connection is the ratio of the stress per gross unit area of a perforated plate to the extension per unit of length centre to centre of holes, the piece being gripped tightly around the holes within certain areas enclosed by the dotted lines in Fig. 13.

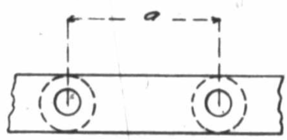


Fig. 13

Information concerning the extension of such portions of plates may be obtained from the researches of Van der Kolk. The Author has carefully examined all the information available and finds that F does not differ greatly in the general case from the modulus of elasticity of the material, and this value will be given to F for the purposes of the present calculation.

Having now determined values of K and F, we shall proceed to the solution of numerical examples, care being taken to select these so that they are consistent with the values of K and F already adopted. Six cases of joints with two rivets and six cases with three rivets will be considered as follows:

2 Rivets — ¾" diameter.

Example.	*Material of Joint.*	*Rivet Spacing.*
I a	1 Main Plate — 3" × 1½" 2 Cover Plates — 3" × ¼"	
I b	1 Main Plate — 3" × 1" 2 Cover Plates — 3" × ½"	3"
I c	1 Main Plate — 3" × ½" 2 Cover Plates — 3" × ¾"	
II *a*	Same material as I *a*	
II *b*	" " " I *b*	6"
II *c*	" " " I *c*	

3 Rivets—¾" diameter.

III *a*	Same material as for I *a*	
III *b*	" " " " I *b*	3"
III *c*	" " " " I *c*	
IV *a*	Same material as for I *a*	
IV *b*	" " " " I *b*	6"
IV *c*	" " " " I *c*	

It should be noted that care has been taken to select joints which will give a thickness of 2 inches, the thickness of the specimens from which the values of K and F have been determined. The rivets are chosen as ¾-inch for a similar reason. In all the solutions K is taken as 16,000,000, and F as 30,000,000.

The solutions were all made by inserting the correct numerical values of K and G in the general formulas derived above. The results, in terms of W_1, are as follows:—

2 Rivets.

Example.	W_1	W_2
I *a*	1	.654
I *b*	1	1
I *c*	1	1.525
II a	1	.546
II *b*	1	1
II *c*	1	1.830

3 Rivets.

Example.	W_1	W_2	W_3
III *a*	1	.424	.451
III *b*	1	.652	1
III *c*	1	.941	2.215
IV *a*	1	.286	.385
IV *b*	1	.484	1
IV *c*	1	.745	2.595

These results are embodied in Figs. 14 and 15. The loads proportional to those on rivet No. 1 are plotted on horizontal lines at regular distances apart opposite the different rivets.

Let us consider the results for the specimens with two rivets, Fig. 14. Where the area of the two cover plates is half

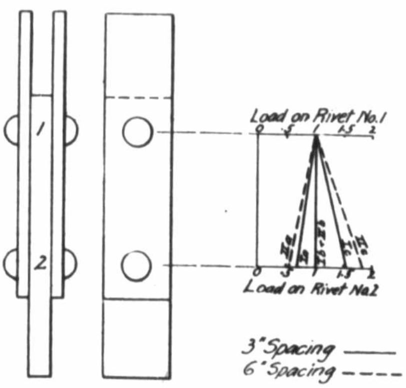


Fig. 14

the area of the main plate the load on rivet No. 2 is less than on rivet No. 1; where these areas are equal the loads on the two rivets are equal; where the area of the two cover plates is 3 times the area of the main plate, rivet No. 2 is loaded more than rivet No. 1. The effect of the increase in spacing is to further increase the inequality of loading in the first and third cases, while it does not influence the distribution of loads at all in the second case.

Now, consider the results for the specimens with three rivets,

Fig. 15. For both spacings, and for all sections, the loads on rivet No. 2 are less than on either Nos. 1 or 3. In the case where the cover plates are the area of the main plate, the load on rivet No. 3 is less than on No. 1; where the cover plates are of the same area as the main plate the loads on these two rivets are equal; where the cover plates are three times the area of the main plate, the load on rivet No. 3 is much greater than on rivet No. 1. The effect of the increase in spacing is to increase the inequality of loading on rivets Nos. 2 and 3 with respect to rivet No. 1, in the

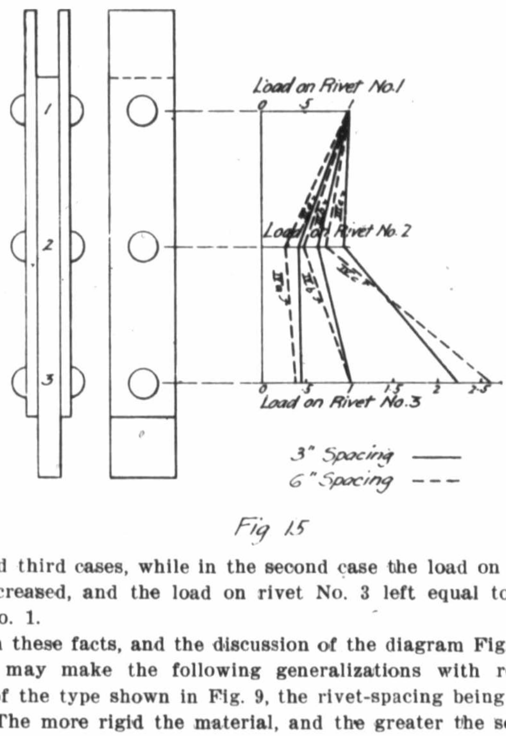


Fig 15

first and third cases, while in the second case the load on rivet No. 2 is decreased, and the load on rivet No. 3 left equal to that on rivet No. 1.

With these facts, and the discussion of the diagram Fig. 8 before us, we may make the following generalizations with respect to joints of the type shown in Fig. 9, the rivet-spacing being uniform.

(1) The more rigid the material, and the greater the sections of both main plate and cover plates, the more nearly are the rivets stressed equally.

(2) An increase of sectional area of the two cover plates relatively to the sectional area of the main plate causes an increase in the proportion of the stress which the bottom rivets (see Fig. 9) carry, while a decrease causes an increase in the proportion of the stress which the top rivets carry.

(3) An increase of the sectional area of the main plate relatively to the sectional area of the two cover plates causes an increase in the proportion of the stress which the top rivets carry, while a decrease causes an increase in the proportion of the stress which the bottom rivets carry.

(4) In joints in which the area of the two cover plates equals the area of the main plate, the rivets equidistant from the centre of the riveting are stressed equally, the end rivets being stressed more than the intermediate ones. In the case of joints with two rivets only there is an equality of stress on the two rivets.

(5) The effect of increasing the rivet spacing, if it be kept uniform, is to render the distribution of stress among the rivets more unequal.

Although these generalizations have been made with reference to connections with two cover plates, they hold equally well for lap joints, one of the plates taking the place of the two cover plates in the above discussion.

CONCLUSION.

What modifications should we make, then, in our designs of riveted connections if this normal law of the distribution of stress substantially holds? Evidently something must be done to throw a greater proportion of stress on the intermediate rivets than they carry under the conditions assumed in the foregoing discussion. There are two practical methods of doing this:

First, we might in some way increase the resistance of the plates to slipping at the intermediate rivets. This would throw additional load on these rivets because of the principle that when a load has to travel over several paths it divides itself in direct proportion to the rigidities of these paths. This extra resistance of the intermediate rivets to slipping might be secured by using larger rivets or by maintaining the pressure of the riveting tool on these rivets until they become black, instead of releasing it immediately after driving, as in the case of the end rivets.

The second method is to decrease the spacing of the rivets near the centre. The reason of this is evident from conclusion (5). The shortening of the spaces over the entire connection will also tend to further equalization of stress on the various rivets.