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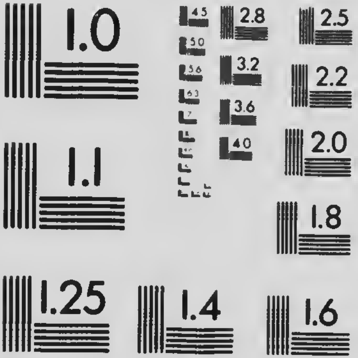

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MECHANICS

OF

MACHINERY

BY

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CHAPTER I.

THE DETERMINATION OF VELOCITIES IN MECHANISMS.

In many cases it is desirable to know the velocities of various parts of a machine during operation, as for example the velocity of an engine piston in a given position, or the velocity of a slide valve at the point of cut-off; or it may be desired to study the motion of the cutting tool in a shaper using a quick return motion, or the examination of the advantages of the triplex pump over the simple pump in the matter of uniformity of discharge. Then, again, it is often necessary to determine the turning effect produced on the crank shaft of an engine by the steam pressure on the piston, or to study the advantage in the way of producing uniform motion of placing four cylinders on an automobile engine, etc.

All of these problems may be solved very directly by the determination of the velocities of various points in the machine under consideration, and as such problems are of very frequent occurrence in the experience of the designing engineer, it is desirable that as simple a process as possible be employed in solving them. The problems may be solved by graphical methods most conveniently, as the motions in most machines are so complex that algebraic solutions are too tedious and difficult.

In all machines there is one part which has a known motion, and generally this motion is one of uniform angular velocity about a fixed axis, e.g., the flywheel in an engine, the belt wheel in a shaper, the belt wheel in a stone crusher, etc.

In most cases in machines all parts have plane motion, and in what follows it is to be understood that all parts referred to remain in one plane, unless the contrary is expressly stated. The solutions may in general be applied to non-plane motion with proper modifications.

The method of determining the velocities of parts of machines to be explained here is called the phorograph* method, and gives a convenient graphical method for finding the desired velocities.

THE PHOROGRAPH

Let us consider any body having plane motion, such as the connecting rod of a steam engine. It is well known that any point in this rod can move relatively to any other point in it only at right angles to the line joining these points. Thus

* So named by its discoverer, Professor T. R. Rosebrugh, of the University of Toronto, who gave the method to his students twenty years ago, but so far as the writer knows the method has not been discovered or used elsewhere.

let A, Fig. 1, represent a part of this rod, in which are two points B and C, which we shall for convenience assume are in the plane of the paper. Let the body A move to the new position A', B and C taking the positions B' and C' respectively, and although we are uncertain as to the actual history of the motion during the change of position, it is quite evident that it may have been accomplished by (a) a motion of translation of the body A through the distance CC', during which C reaches its new position C' and B arrives at B_1. During this motion B and C have moved through the same distance in the same direction and sense, and hence have had no relative motion. The second part of the motion consists of (b) a motion of rotation of the whole body A about an axis normal to the paper through the point C', rotation taking place through the angle $B_1C'B'$.

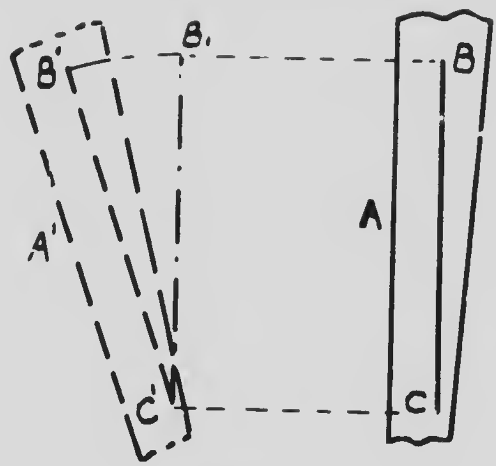

Fig. 1.

During the motion of A therefore B has had only one motion not shared by C, or B has moved relatively to C through the arc B_1B', and at each stage of the motion the direction of this arc was evidently at right angles to the radius from C', or at right angles to the line joining B and C.

Thus when a body has plane motion any point in the body can move relatively to any other point in the body only at right angles to the line joining the two points. It follows from this that if the line joining the two points should lie normal to the plane of motion the two points could have no relative motion.

We shall now employ this principle to the determination of velocities. Let Fig. 2 represent diagrammatically a machine having four links a, b, c, d, joined together by four turning pairs at O, P, Q, R. If the link d is nearly vertical and the length of a be decreased this could be taken to represent one-half of a beam engine, in which a is the crank, b the connecting rod, and c is one half of the walking beam, the other end of which would be connected with the piston rod.

In all such machines one link is fixed and forms the frame, here indicated by d. Thus O and R are fixed bearings and P and Q move in arcs of circles about O and R respectively. Let us now choose one of the moving links as the link of reference, either a or c will be the most convenient, as they have one fixed bearing, and a will be selected. Imagine that to a an immense sheet of cardboard is attached which extends indefinitely in all directions from O, and let us for brevity refer to this whole sheet as the link a.

A consideration of the matter will show that on the link a there are points having all conceivable velocities in magnitude, direction and sense, thus if a circle be drawn on a with centre at O all points on the circle will have velocities of the same magnitude, but of different direction and sense; or if a vertical line

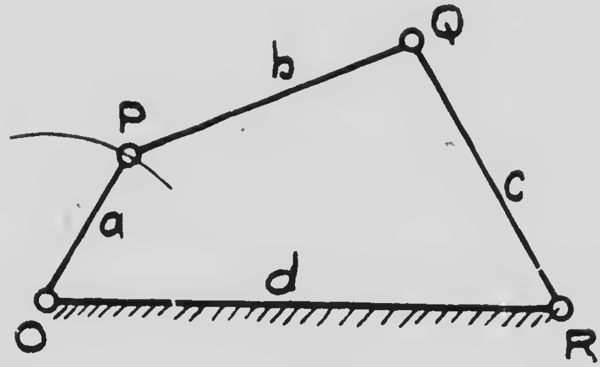

Fig. 2.

be drawn through O all points on this line will move in the same direction, i.e., horizontal, those above O moving in opposite sense to those below O, and the magnitudes of all the velocities being different. Thus, if any point be chosen on a the magnitude of its velocity will depend upon the distance from O, the direction of its velocity will be normal to the radial line joining it to O, and its sense will depend upon the relative positions of the point and O on the radial line. It must be remembered that the above statements are true whether a has constant angular velocity or not and are also true although O is moving.

From the foregoing it follows at once that it will be possible to find a point on a having the same motion as that of any point, such as Q, in the machine, which motion it is desired to study; and thus we can collect on a a set of points, each representing the motion of a given point in the machine, and since this set of points is all on the one link their relative velocities is at once known completely. This collection of points on a will be of great assistance in studying the motions of points in the machine, because if the motion of a is known, as is usual, that of any other point is known; whereas if the motion of a is unknown only the relative motions of the different points are known. This collection of points on the link of reference is called the *Phorograph*, as it represents graphically the motions of all points in the machine.

The method of determining the phorograph for a given machine may be explained as follows: Let any body K, Fig. 3, have plane motion, and let us choose in it two points E and F. We are, however, given no information about the nature of the motion of K. On some other body there is a point G, and we are told the direction only of the motions of G ~ E*, viz.,

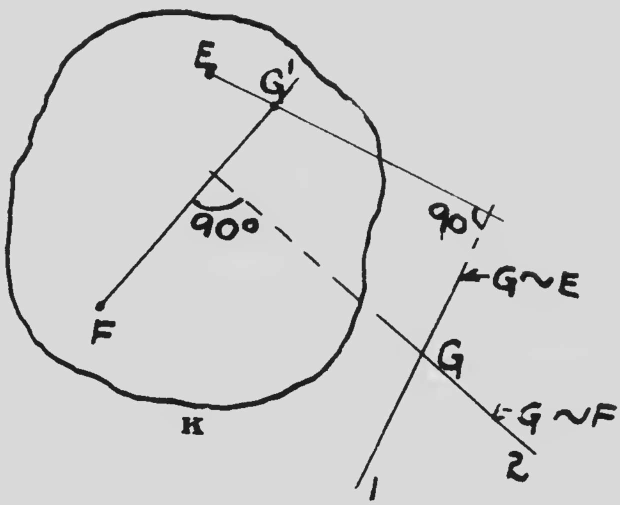

Fig. 3.

G — 1, and of G ~ F, viz., G — 2: it is required to find a point on K having the same motion as G.

Referring to our preliminary proposition we see that the motion of any point in K ~ E is perpendicular to the line joining it to E, e.g., the motion of F ~ E is $\perp$ to FE. But a point is to be found having the same motion as G, and as the direction of G ~ E is given we are at once told the direction of the line joining E to the required point, it must be $\perp$ to $G1$ and pass through E as it is only points on EG^1 which have the desired direction G ~ E. If we call the point to be found G^1 then G^1 lies on EG^1 $\perp$ to G-1. Similarly it may be shown that G^1 must lie on a line through F $\perp$ to G — 2, and hence it must lie at the intersection of the lines through E and F or at G^1, as shown

* The sign ~ is used to mean "with regard to."

in Fig. 3. Thus G^1 is a point on K having the same motion as the point G in some external body.

It is to be noted that we cannot assume the sense of the motions nor the magnitude, only the two directions. We could, however, assume the magnitude, direction and sense of $G \sim E$ and find G^1, provided the angular velocity of K were known. If K turns in the clockwise sense then the senses of the lines representing the motion of G are $G \sim 1$ and $G \sim 2$, and if the angular velocity of K is ω radians per second the magnitude of the velocity of $G \sim E$ is $G^1E \cdot \omega$ and of $G \sim F$ is $G^1F \cdot \omega$.

We shall now apply these principles to the solution of problems connected with machinery, first calling particular attention to the fact that the usual information given us is such as we have chosen above, viz., the directions of motion of an external

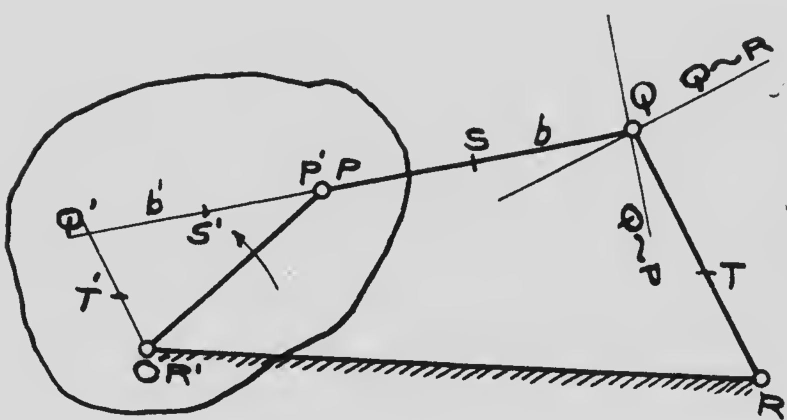

Fig. 4.

point relatively to two points in the link of reference. The simple mechanism with four links and four turning pairs will be chosen as the first example, and is shown in Fig. 4, the letters a, b, c, d, O, P, Q, R having the same significance as before, and a being chosen as the link of reference, and a rough outline of this link is shown to indicate its large extent. It is required to find the linear velocity of the point Q. Points will first be found on a having the same motions as Q and R, which are external to a, and the points so found shall be referred to as the **images** of Q and R and indicated by accents, thus Q^1 is a point on a having the same motion in every respect as Q and similarly with R^1.

Inspection will at once show that since P is a point on a, P^1 will coincide with P, and if we call ω the angular velocity of a in radians per second (which may be constant or variable), then the linear velocity of P is $OP \cdot \omega = a\omega$ ft. per sec., and is in the direction $\perp$ to OP and in the sense indicated by ω. Such being the case the length OP or a represents $a\,\omega$ ft. per sec., and the scale is thus $\omega : 1$. Further inspection will also show that since R is stationary, R^1 will lie at the only stationary point in a_1, viz., at O.

The remaining point Q^1 may be found thus: The direction of motion of $Q \frown P$ is $\perp$ to OP or b, and hence, from the proposition already given, Q^1 must lie in a line through P^1 (or P) $\perp$ to the direction of $Q \frown P$, i.e., on the line through P^1 in the direction of b or on b produced. Again, the direction of motion of $Q \frown R$ is $\perp$ to QR or c, and since R^1 (at O) has the same motion as R this is also the direction of motion of $Q \frown R^1$, so that Q^1 lies on a line through R^1 $\perp$ to the motion of $Q \frown R$, i.e., on a line through R^1 in the direction of c, and thus Q^1 is fixed. The velocity of Q is then $Q^1O \cdot \omega$, the direction in space is $\perp$ to OQ^1 and the sense is fixed by that of ω.

Since P^1 and Q^1 are the images of P and Q on b, we may regard P^1Q^1 as the image of b, and shall in future denote it by b^1, similarly R^1Q^1 (OQ^1) will be denoted by c^1. By a similar

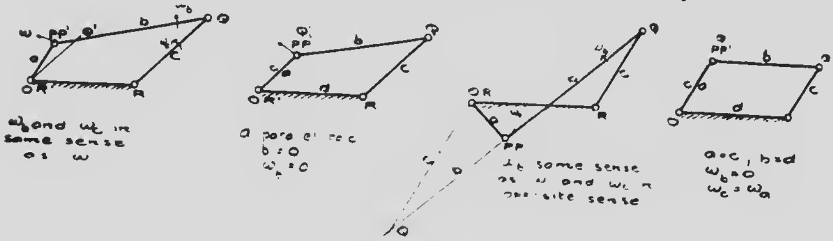

Fig. 5.

process of reasoning it may readily be shown that if S is a point on b midway between P and Q then S^1 will divide P^1Q^1 equally, and also T^1 may be found by the relation $R^1T^1 : T^1Q^1 = RT : TQ$.

The diagram may be put to further use in determining the magnitude and sense of the angular velocities of b and c when that of a is known. Let ω_b and ω_c denote respectively the angular velocities of the links b and c in space, the angular velocity of the link of reference being ω. Now since Q and P are on one link b, which has an angular velocity ω_b, therefore the velocity of $Q \frown P$ is $QP \cdot \omega_b$ or $b \cdot \omega_b$, and since Q^1 and P^1 are points on a, whose angular velocity is ω, therefore the velocity of $Q^1 \frown P^1$ is $Q^1P^1\omega$ or $b^1\omega$. But Q^1 has the same motion as Q, and P^1 has the same motion as P, and therefore the velocity of $Q \frown P$ is the same as that of $Q^1 \frown P^1$ or $b\,\omega_b =$

$$b^1\omega, \text{ i.e., } \omega_b = \frac{b^1}{b}\,\omega = b^1 \cdot \frac{\omega}{b}. \quad \text{Similarly } \omega_c = \frac{c^1}{c} \cdot \omega = c^1 \cdot \frac{\omega}{c}$$

and since b and c are fixed in length the length of the images of the links b and c are a measure of their velocities. The sense of these velocities is readily determined from the signs of the ratios $\frac{b^1}{b}$ and $\frac{c^1}{c}$ thus b^1 and b are opposite in sign, hence ω, is of opposite sense to ω, and by a similar process of reasoning it may be shown that ω is of the same sense as ω.

The figure $O^1P^1Q^1R^1$ is evidently a vector diagram for the mechanism, the distance of any point on this diagram from the pole O being a measure of the velocity of the corresponding point in the mechanism. The direction of motion is normal to the line joining the point on the vector diagram to O and the sense of motion is also known from that of the angular velocity of the primary link. Further, the lengths of the sides of this figure b^1 (P^1Q^1), d^1 or (OR^1) etc., are measures of the angular velocities of the links, the sense of each angular velocity being readily determined. (Note that the length d^1 or OR^1 is infinitely

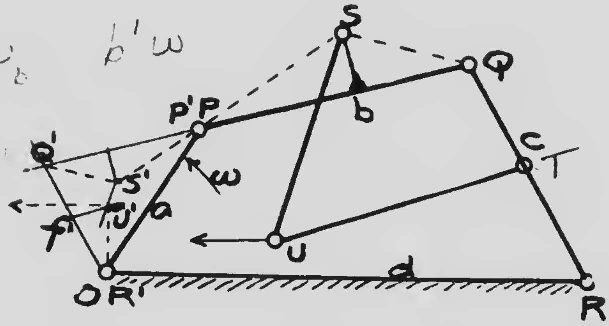

Fig. 6.

short denoting that d has no angular velocity, since it is fixed in space.)

In Fig. 5 other positions and proportions of a similar mechanism are shown in which the solution is given and various relations marked below. It is to be noted that if the image of any link reduces to a single point two causes are possible, (a) if this point falls at O the link is stationary for the instant, as at d^1, but if the point be not at O the inference is that all points in the link move in exactly the same way, or the link has a motion of translation at the given instant.

The method will now be employed to solve a few typical cases.

Fig. 6 is taken as a simple example, not because it illustrates any practical mechanism.

Here we find Q^1P^1 and R^1 as before, and since we know the motions of S — Q and S — P to be — respectively to SQ and SP, hence we draw S^1P^1 parallel to SP and S^1Q^1 parallel to SQ, which determines S^1; also $R^1T^1 : T^1Q^1 = RT : TQ$ determines T^1. Next, since we know the motions of U ⁀ S and U ⁀ T, we draw U^1T^1 parallel to UT and S^1U^1 parallel to SU, and thus U^1 is determined. If a be assumed to turn in the sense shown with angular velocity ω, then the angular velocity of SU is $\dfrac{S^1U^1}{SU} \cdot \omega$, and is in the same sense as a, and the angular velocity of UT is $\dfrac{U^1T^1}{UT} \cdot \omega$ in opposite sense to a. The linear velocity of U is $OU^1 \cdot \omega$, the direction is $\perp$ to OU^1, and the sense is to the left.

Fig. 7 gives a further example in which a sliding pair is introduced. OP is again the link of reference and P^1, Q^1, R^1 and S^1 are found as before. The direction of T is given in space by construction. It slides in the directions shown. Hence T^1 will

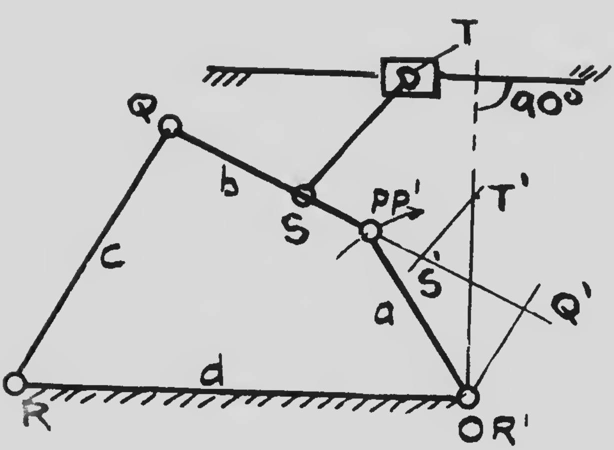

Fig. 7.

lie on a line through O $\perp$ to the direction of T, and as T^1 lies on S^1T^1 parallel to ST, T^1 becomes fixed. The velocity of T is $OT^1 \cdot \omega$ its direction $\perp$ to OT^1, and its sense is to the right.

Fig. 8 shows the engine mechanism in two forms, (a) where the piston direction passes through the crank shaft, (b) where the cylinder is offset. The same letters and description apply to both. Evidently Q^1 lies on P^1Q^1 through P^1, parallel to PQ (here on QP produced), and also since the motion of Q in space is horizontal, Q^1 will lie in the vertical through O. Thus the velocity of the piston Q is $OQ^1 \cdot \omega$ in the direction and sense shown, and offsetting the cylinder evidently decreases the piston velocity in this position, and it may be shown that there will be a corresponding increase in the return stroke. The angular velocity of the rod is $\dfrac{P^1Q^1}{PQ} \omega$. Inspection shows that in the upper diagram the piston velocity is zero at the dead points, is equal to that of the crank pin when the crank is ver-

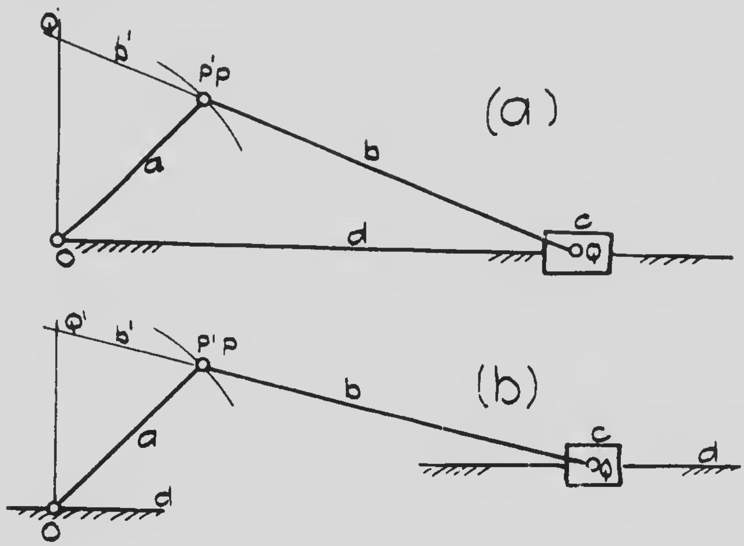

Fig. 8.

tical, and has a maximum val e when the crank pin is slightly to the right of the vertical th ough O. For the lower diagram

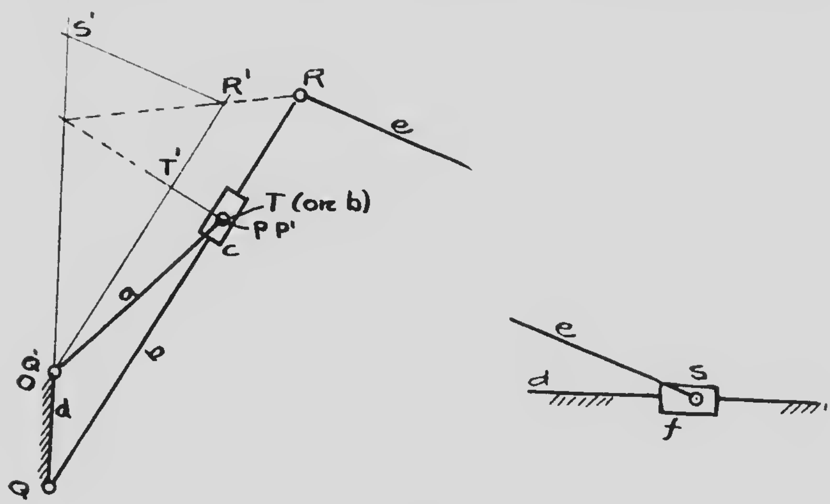

Fig. 9.

the piston velocity is also that of the crank pin when the crank is vertical.

Fig. 9 shows the Whitworth quick-return motion, which is slightly more difficult. There are here four links *a*, *b*, *d* and *e* and two sliding blocks, *c* and *f*, *d* being fixed and *a* being the driving link, which rotates at constant angular velocity ω in the clockwise sense. P^1 and Q^1 are found by inspection. Further, S^1 lies on a vertical line through *O*, and R^1 on a line through Q^1 parallel to *QR*. Now, *P* is a point on both *a* and *c*. Choose *T* on *b* exactly below *P* on *a* and *c*, and it will be evident that since *a*, *b* and *c* all have plane motion, the only motion which *T* can have relative to *P* is sliding in the direction of *b*, or the motion of *T* — *P* is in the direction of *b*, hence T^1 is on a line through P^1 ⊥ to *b*, and since it is also on a line through *O* par-

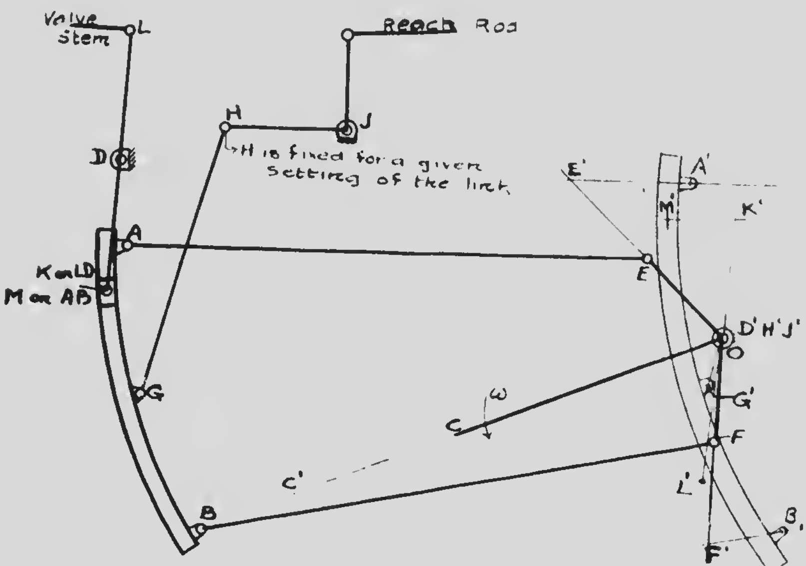

Fig. 10.

allel to *b* it is found at T^1. Again, R^1 may be found since $\frac{QT^1}{T^1R^1} = \frac{QT}{TR}$. The dotted lines show a simple geometrical method for obtaining this ratio. S^1 is on R^1S^1 parallel to *RS* and also on the vertical through *O*.

It will be understood that *T* is not a fixed point on *b*, but will change for each position of the mechanism. The linear velocity of the tool holder *S* will be $OS^1 \cdot \omega$ and the angular velocity of *RS* will be $\frac{R^1S^1}{RS} \omega$ in opposite sense to *a*.

Note that although *P* and *T* coincide, their images do not, for *T* has a sliding motion with regard to *P*, and hence both could not have the same velocity. If P^1 and T^1 coincided then both *P* and *T* would have the same velocity.

The Stephenson link motion shown in Fig. 10 involves a slightly different method of attack and is worked out in full here, but is not drawn correctly to scale, so as to avoid confusion of the diagram. In this case the link of reference is the crank shaft containing the crank C and the eccentrics E and F, and instead of making C^1, E^1 and F^1 coincide with C, E and F, as in the previous examples, we have made $OC^1 = 2OC$, etc. The scale will then be $OC^1 = OC \times \omega$ ft. per sec. We locate C^1, E^1, F^1, H^1, D^1 and I^1 at once. Further, we choose M on the link AB directly below K on LDK, and we also know that E^1A^1, F^1B^1, H^1G^1, and D^1K^1 are parallel respectively to EA, FB, HG and DK. Now, we have already seen that the image of each link is similar and similarly divided to the link itself, and we see that the link AGB has the points G, A and B. We also know the lines along which G^1, A^1 and B^1 lie, so that the problem is simply one of locating a curved line similar to AGB, with its ends on the lines A^1E^1 and F^1B^1, and divided at G^1 by the line through O parallel to GH, so that $\frac{A^1G^1}{B^1G^1} = \frac{AG}{BG}$. (There are simple geometrical methods of accomplishing this result, but these are omitted here.) Thus $A^1G^1B^1$ is located and the whole link may be drawn in similar to AGB, but to a larger scale, and on it the point M^1 may be found from the relation $\frac{M^1A^1}{B^1M^1} = \frac{MA}{BM}$. Since K slides with regard to M we have K^1M^1 normal to A^1B^1 at M^1, which locates K^1, and we may readily locate L^1 from the relation $\frac{L^1D^1}{D^1K^1} = \frac{LD}{DK}$.

The linear velocity of the slide valve is $OL^1 \cdot \omega$, and it moves to the right.

Note.—The images of all links are similar to and similarly divided to the links themselves, and are always parallel to the links, of which they are the images.

Lack of space prevents further illustrations, of which very many useful ones exist, but enough cases have been given to show the method of procedure in any mechanism, and to show that by this method the velocity of any point in a mechanism may readily be found by means of a drafting board. Those using the photograph will no doubt invent geometrical methods for getting the desired ratios between the image and the link in any case which occurs.

CHAPTER II.

TOOTHED GEARING

In many cases in machinery it is necessary to transmit power from one shaft to another, the ratio of the angular velocities of the shafts being known, and in very many cases this ratio is constant; thus it may be desired to transmit power from a shaft running at 120 revs. per min. to another running at, say 200 revs. per min. Various methods are possible, for example, pulleys of proper size may be attached to the shafts and connected by a belt, or sprocket wheels may be used connected by a chain, as in a bicycle, or pulleys may be placed on the shafts and the faces of the pulleys pressed together, so that the friction between them may be sufficient to transmit the power, a drive used sometimes in auto wagons, or, again, toothed wheels called gear wheels may be used on the two shafts, as in street cars and most automobiles.

Any of these methods is possible in some cases, but usually the location of the shafts, their speeds, etc., make some one of the methods the more preferable. Thus, if the shafts are not very close together, a belt and pulleys may be used, but as the drive is not positive the belt may slip, and thus the relative speeds may change, the speed of the driven wheel often being five per cent. lower than the diameters of the pulleys would indicate. Where the shafts are fairly close together a belt does not work with satisfaction, and then a chain and sprockets are sometimes used which cannot slip, and hence the speed ratio required may be maintained. For shafts which are still closer together either friction gears or toothed gears are generally used. Thus the nature of the drive will depend upon various circumstances, one of the most important being the distance apart of the shafts concerned in it.

We shall deal here only with drives of the latter class or toothed gears, which, broadly speaking, are used between shafts which are not far apart, and for which the ratio of the angular velocities must be fixed and known at any instant. We shall first deal with parallel shafts which turn in opposite senses, the gear wheels connected with which are called *spur wheels*, the larger one commonly called the *gear*, and the smaller one the *pinion*. Kinematically, spur gears are the exact equivalent of a pair of smooth round wheels of the same mean diameter, and which are pressed together so as to drive one another by friction. Thus if two shafts 15 in. apart are to rotate at 200 revs. and 100 revs. per min., respectively, they may be connected by two smooth wheels 10 in. and 20 in. in diameter, one on each shaft, which are pressed together so they will not slip, or by a pair of spur wheels of the same mean diameter, both methods producing the desired results. But if the power to be transmitted is great the friction wheels are inadmissible on account of the great pressure between them necessary to prevent slipping. If slipping occurs the velocity ratio is variable and such an arrangement would be of no value in such a drive, as is used on a street car, for instance, on account of the jerky motion it would produce on the latter.

In order to begin the problem in the simplest possible way we shall first take the most general case of a pair of spur gears connecting two shafts which are to have a constant velocity ratio. That is, the ratio between the speeds n_1 and n_2 is to be constant at every instant that the shafts are revolving. Let l be the distance from centre to centre of the shafts. Then, if friction wheels were used, we would have the velocities at the rim of each $\pi d_1 n_1$ and $\pi d_2 n_2$ in inches per minute, where d_1 and d_2 are the diameters of the wheels in inches, and it will be clear that the velocity of the rim of each will be the same since there is to be no slipping. Thus $d_1 n_1 = d_2 n_2$ or $r_1 n_1 = r_2 n_2$, where r_1 and r_2 are the radii of the wheels. But $r_1 + r_2 = l$. Therefore since $r_1 = r_2 \cdot \frac{n_2}{n_1}$ we get $r_2 \cdot \frac{n_2}{n_1} + r_2 = l$,

$$\text{or } r_2 \left[\frac{n_2}{n_1} + 1 \right] = l \text{ or } r_2 = \frac{n_1 l}{n_1 + n_2} \text{ and } r_1 = \frac{n_2 l}{n_1 + n_2}$$

Now, whatever actual shape we give to these wheels the motion of the shafts must be the same as if two smooth wheels, of sizes as determined above, rolled together without slipping. In other words, whatever shape the wheels actually have the resulting motion must be equivalent to the rolling together of two circles centred on the shafts. In gear wheels these circles are called the *pitch circles*, and they evidently touch at a point on the line joining the centres of the wheels, which point is called the *pitch point*. Now, let the actual outlines of these wheels be as shown on Fig. 11, the projections being placed there in order that the slipping of the pitch lines may be prevented. It is desired to find the necessary shape which these projections must have. Let the wheels touch at any point P and join P to the pitch point C.

It has already been shown that these pitch circles must always roll upon one another without slipping. Now P is a point which is common to both wheels. As a point in the gear it moves with regard to C on the pinion at right angles to PC, and as a point in the pinion it must move at right angles to PC with regard to C on the gear, thus, whether P is a point on the gear or pinion its motion must be normal to the line joining it to C. Some consideration will show that in order that P may have this direction of motion in each wheel, the shape of the wheels at P must be perpendicular to PC.

In order to see this let us examine the case shown in the lower figure, where the projections are not normal to P_1C at the point P_1, where they touch. It is at once evident that sliding must occur at P_1, from the very nature of the case, and where two bodies slide upon one another the direction of sliding must always be along the common tangent to their surfaces at the point of contact, hence the direction of sliding here must be P_1P^1. But P_1 is the point of contact and is therefore a point in each wheel, and the motion of the two wheels must be the same as if the two pitch circles rolled together, having contact at C. Such being the case, if we place two projections, as shown on the wheels, the direction of motion at their point of contact should be perpendicular to P_1C, whereas here it is perpendicular to P_1C^1. This would cause slipping at C, and would give the

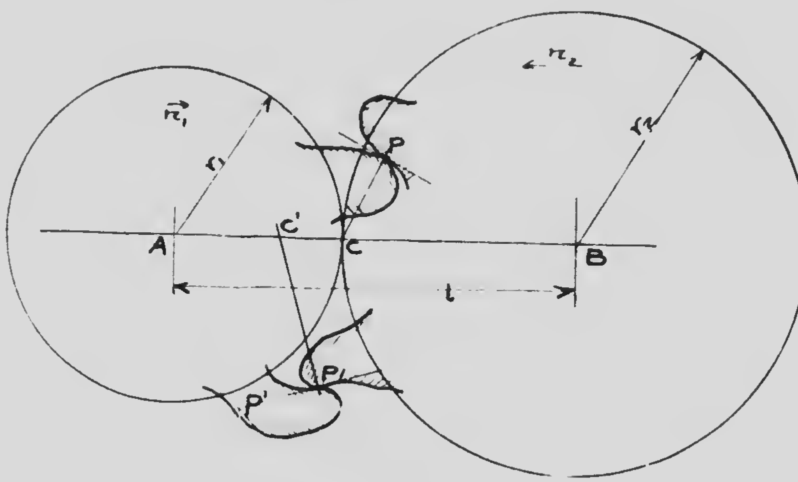

Fig. 11.

proper shape for pitch circles of radii AC^1 and BC^1, which would correspond to a different velocity ratio, thus C^1 should lie at C and P_1P^1 should be normal to P_1C.

From the foregoing we may make the following important statements: **The shapes of the projections on the wheels must be such that at any point of contact they will have a common normal passing through the fixed pitch point, and that while the pitch circles roll on one another the projections will have a sliding motion.** These projections on gear wheels are called *teeth*, and for convenience in manufacturing, all the teeth on each gear have the same shape, although this is not at all necessary to the motion. The teeth on the pinion are not the same shape as those on the gear with which it *meshes*.

There are a great many shapes of teeth, which will satisfy the necessary condition set forth in the previous paragraph, but by far the most common of these are the cycloidal and the involute teeth, so called because the curves forming them are cycloids and involutes respectively.

CYCLOIDAL TEETH

Select two circles PC and $P'C$, Fig. 12, and suppose these to be mounted on fixed shafts, so that the centres A and B of the pitch circles, and the centres of the *describing circles* PC and $P'C$, as well as the pitch point C, all lie in the same straight line, which means that the four circles are tangent at C. Now place a pencil at P on the circle PC and let all four circles run in contact without slipping, i.e., the circumferential velocity of

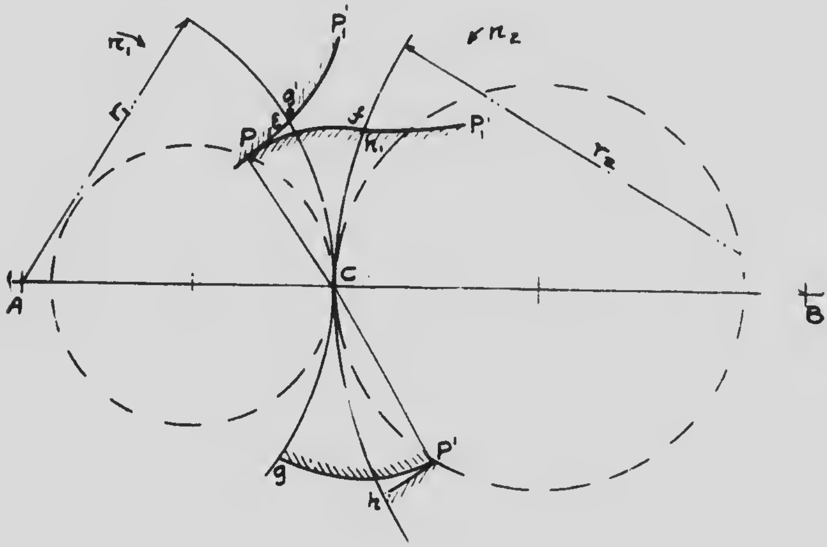

Fig. 12.

all circles at any instant is the same. As the motion continues P approaches the pitch circles ec and fc, and if the right hand body be extended beyond the circle fch, the pencil at P will describe two curves, a shorter one Pe on the body ecg and a longer one Pf on the body fch, the points e and f being reached when P reaches the point c, and from the conditions of motion arc PC = arc ec = arc fc.

Now P is a common point on the curves Pe and Pf and also a point on the circle PC, which has the common point C with the remaining three circles. Hence the motion of P with regard to ecg is perpendicular to PC, and of P with regard to fch is perpendicular to PC: that is, the tangents to Pe and Pf at P are normal to PC, or the two curves have a common tangent and hence a common normal PC at their point of contact, and this normal will pass through the pitch point C. Thus Pe and Pf fulfill the necessary conditions for the shapes of gear teeth. Evidently the points of contact of these two curves lie along PC, since both curves are described simultaneously by a point which always remains on the circle PC. Since these curves are first in contact at P and then again at C, when P, c, e and f coincide, it is evident that during the motion from P to C the curve Pe slips on the curve Pf through the distance $Pf - Pe$. Below C the pencil at P would simply describe the same curves over again, only reversed.

To further extend these curves, we place a second pencil at P^1, which will draw the curves P^1g and P^1h in the same way as before, these curves having the same properties as Pe and Pf, the amount of slipping in this case being $P^1g - P^1h$, and the points of contact always lying on the circle CP^1.

Now join the two curves formed on ecg, that is, join gP^1 to Pe, as shown at $Peg^1P_1^1$, and then the two curves on fch, as shown at $Pfh_1P_1^1$, and we have a pair of curves which will remain in contact from P to P_1, which always have a point of contact on the curve PCP^1, and which always have a common normal at their point of contact passing through C. The relative amount of slipping is $Pfh_1P_1^1 - Peg^1P_1^1$. If, now, we cut out two pieces of wood, one having its side shaped like the curve PeP_1^1 and pivoted at A, while the other is shaped like PfP_1^1 and pivoted at B; then from what has been said, the former may be used to drive the latter, and the motion will be the same as that produced by the rolling of the two pitch circles together, hence these shapes will be the proper ones for the *profiles* of gear teeth.

The curves Pe, Pf, P^1g and P^1h, which are produced by the rolling of one circle inside or outside of another are called *cycloidal* curves, the two Pe and P^1h being known as hypocycloids, since they are formed by the describing circle rolling inside the pitch circle, while the two curves Pf and P^1g are known as epicycloidal curves, in this case lying outside the pitch circles. Gears having these curves as the profiles of the teeth are said to have cycloidal teeth (sometimes erroneously called epicycloidal teeth), a form which is in very common use. So far we have only drawn one side of the tooth, but it will be evident that the other side is simply obtained by making a tracing of the curve PeP^1_1 on a piece of tracing cloth, with centre A also marked; then by turning the tracing over and bringing the point A to the original centre A, the other side of the tooth on the wheel ecg may be pricked through with a needle. The same method is employed for the teeth on wheel fch.

Nothing has so far been said of the sizes of the describing circles, and, indeed, it is evident that any size of describing circle, so long as it is somewhat smaller than the pitch circle, may be used, and will produce a curve fulfilling the desired conditions, but it may be shown that when the describing circle is one-half the diameter of the corresponding pitch circle the hypocycloid becomes a radial line in the pitch circle, and for reasons to be explained later this is undesirable. The maximum size of the describing circle is thus one-half that of the corresponding pitch circle, and for convenience the two describing circles are frequently of the same size, although this is not a necessity.

The proof that the hypocycloid is a radial line if the describing circle is half the size of the pitch circle, may be given as follows: Let *ABC*, Fig. 13, be the pitch circle, and *DPC* the describing circle, *P* being the pencil, and *BP* the line described by *P*, as *P* and *B* approach *C*. The arc *BC* is

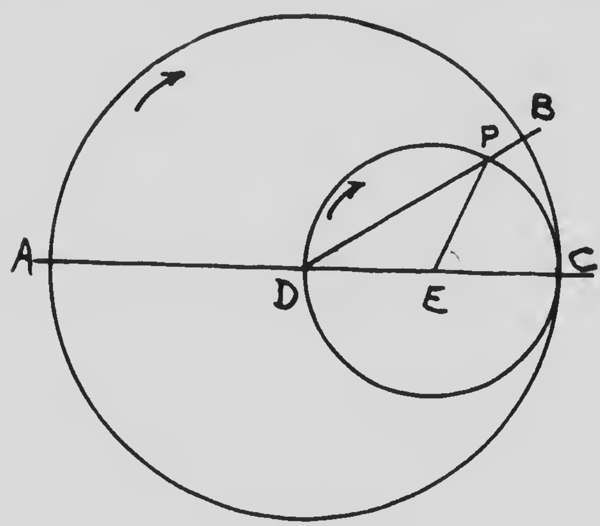

Fig. 13.

equal to the arc *PC* by construction, and hence the angle *PEC* at the centre *E* of *DPC* is twice the angle *BDC*, because the radius in the latter case is twice that in the former. But the angles *BDC* and *PEC* are both in the one circle, the one at the circumference and the other at the centre, and since the latter is double the former they must stand on the same arc *PC*. In other words *BP* is a radial line.

In the actual gear the tooth profiles are not very long, but are limited between two circles concentric with the pitch circles in each gear, and called the *addendum* and *root circles*, as indicated in Fig. 14, the path of contact being evidently *PCP*, and the amount of slipping on each pair of teeth is $PR - PD + P_1E - P_1F$, or $PR + P_1E - (PD + P_1F)$. Further, since the common normal to the teeth pass through *C* then the direction of pressure between a given pair of teeth is always the line joining their point of contact to C, friction being neglected.

The arc PC is called the *arc of approach*, being the location of the points of contact down to the pitch point C, and CP_1 is called the *arc of recess*, P being the last point of contact. The angles DAC and CAE are called respectively the angles of approach and recess. As will be explained later, the distance between the addendum and root circles and the pitch circle depends upon the number of teeth in the gear, so that with these circles fixed the length of the *arc of contact* PCP_1 will depend upon the diameters of the describing circles being longer as the describing circles become larger. If this arc of contact is shorter than the distance between two teeth on the one gear, then only

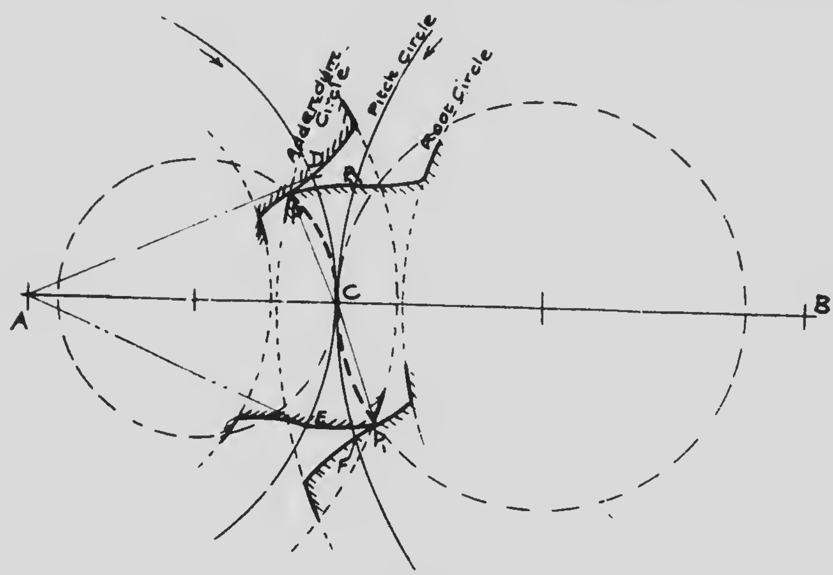

Fig. 14.

one pair of teeth can be in contact at once, and the running is uneven, while, if this arc is just equal to the distance between the centres of a given pair of teeth on one gear, or the *pitch*, as it called (See Fig. 17) one pair of teeth will just be going out of contact as the second pair is coming in, which will also cause jarring. It is usual to make PCP^1 at least 1.5 times the pitch of the teeth. This will, of course, increase the amount of slipping of the teeth.

With the usual proportions it is found that when the number of teeth in a wheel is less than 12 the teeth are not well shaped for strength or wear, and hence, although they will fulfil the kinematic conditions, they are not to be commended in practice.

INVOLUTE TEETH

The second and perhaps the most common method of forming the curves for gear teeth is by means of involute curves. Let A and B, Fig. 15, represent the axes of the gears, the pitch circles of which touch at C, and through C draw a secant DCE at any angle θ to the normal to AB, and with centres A and B respectively draw circles to touch the secant in D and E. Now $\frac{n}{n_2} = \frac{BC}{AC} = \frac{BD}{AE}$ so that if a string be run from D to E and used as a belt between the two dotted *base circles* at D and E, we would have exactly the same velocity ratio as if the original pitch circles rolled together having contact at C.

Now, choose any point P on the belt DE and attach at this

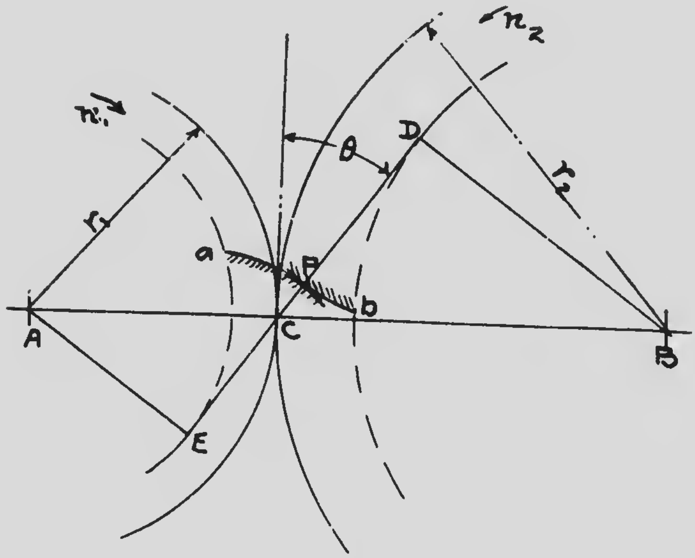

Fig. 15.

point a pencil, and as the wheels revolve it will evidently mark on the original wheels, having centres at A and B, two curves Pa and Pb respectively, a being reached when the pencil gets down to E and b being the starting point just as the pencil leaves D, and since the point P traces the curves simultaneously they will always be in contact at some point along DE, the point of contact traveling downward with the pencil at P. Since P can only have a motion with regard to the wheel aE normal to the string PE, and its motion with regard to the wheel Db is at right angles to PD, it will be at once evident that these two curves have a common normal at the point where they are in contact.

and this normal evidently passes through C, hence the curves may be used as the profiles of gear teeth.

The curves Pa and Pb are called involute curves, and when they are used as the profiles of gear teeth the latter are called involute teeth. The angle θ is called the *angle of obliquity*, and evidently gives the direction of pressure between the teeth, so that the smaller this angle becomes the less will be the pressure between the teeth for a given amount of power transmitted. If, on the other hand, this angle is unduly small, the base circles approach so nearly to the pitch circles in size that the curves Pa and Pb have very short lengths below the pitch circles. Many firms adopt for θ the angle $14\frac{1}{2}°$, in which case the diameter of the base circle is .968 (about 31/32) that of the pitch circle. If the teeth are to be extended below the base circles, as is usual, the lower part is made radial. With teeth of this form the distance between the centres A and B may be somewhat increased without affecting in any way the regularity of the motion. Involute teeth are also stronger in general than the corresponding cycloidal teeth.

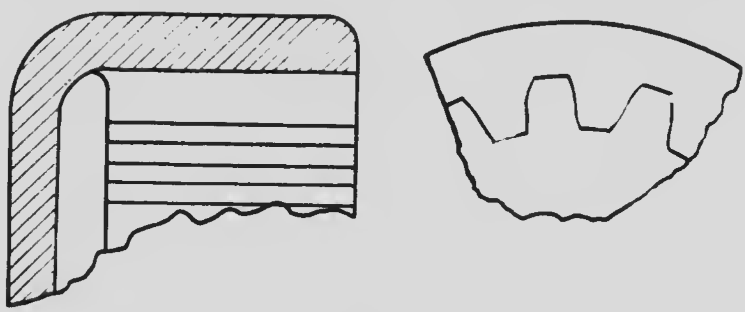

Fig. 16

The arc of contact in these teeth is usually about twice the pitch, and the number of teeth in a gear should not be less than 12, as the teeth will be weak at the root unless the angle of obliquity is increased.

Gears are sometimes made with the teeth on the inside instead of the outside of the rim, Fig. 16. Such gears are called *annular gears*, and they are always made to mesh with a spur pinion, the property being that both gear and pinion rotate in the same sense. The teeth on the annular gear are made in exactly the same way as those for the spur gear, and are involute or cycloidal.

When one gear of the pair has an infinite radius the pitch line becomes a straight line, and it is then called a *rack*, the teeth being cycloids in one case, and in the involute system being straight lines, forming an angle $90 - \theta$ with the pitch line, the gear meshing with the rack being called the pinion.

Gear teeth are formed in various ways, such as casting, cutting from solid casting, etc., and as it is only possible to make the teeth accurately by the latter method, we shall speak hereafter of *cut* teeth. In this case an accurately turned casting is taken of the same diameter as the outside of the teeth, and the metal forming the spaces between the teeth is carefully cut out, leaving accurate shapes if the work be properly done. The various terms applied to gear teeth will appear from Fig. 17. The *addendum* line is a circle whose diameter is that of the outside of the gear. The *root* or *dedendum line* is a circle whose diameter is that at the bottoms of the teeth. The difference between the radii of these two circles gives the *height* of the teeth. The dimension of the teeth parallel with the shaft is the *width of face* or often the *face* of the tooth, although the word *face* is also

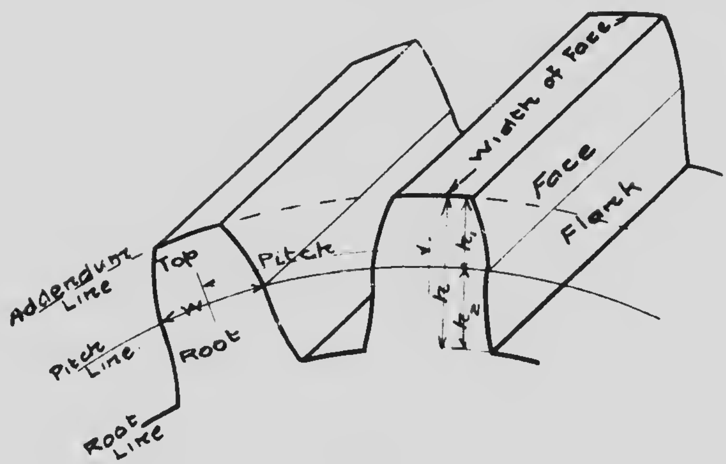

Fig. 17

used to denote the surface of the tooth outside the pitch line, the part of the surface of the tooth below the pitch line being the *flank*. The solid part of the tooth above the pitch line is the *point*, and the solid part below this line is the *root*.

Let d be the pitch diameter of a gear having t teeth, h_1 be the height of the tooth above the pitch line, and h_2 the depth below the pitch line, the total height $h = h_1 + h_2$; further, let w be the thickness of the tooth measured along the pitch line. The distance from centre to centre of teeth measured along the pitch line is the *circular pitch* or pitch p, and this definition at once gives $tp = \pi d$. As a matter of convenience Brown and Sharpe have introduced a second pitch, now also commonly adopted, called the *diametral* pitch and defined as $q = \frac{t}{d}$. It would naturally be expected that the diametral pitch would be the number of inches of diameter per tooth, since the circular pitch is the number of inches of circumference per tooth. The diametral pitch is, however, the inverse and is not a number of inches. The following formulas are adopted by Brown and Sharpe: $p = \frac{\pi d}{t}$; $q = \frac{t}{d}$; $h_1 = \frac{1}{q}$; $h_2 = \frac{1}{q} + \frac{p}{20}$; $w = \frac{p}{2}$ these dimensions being used for cut teeth. For cast teeth $w = .48p$, and hence there is a *back lash* $= .04p$ between any pair of teeth which are in mesh. In cut gears there is no back lash. Notice that since $h_2 - h_1 = \frac{p}{20}$ there is always a *clearance* space of $.05p$ between the top of one tooth and the root line of the other.

It will be evident at once that if a pair of gears are to work together it is necessary that they have the same pitch p, and also that in the cycloidal system the same describing circle must have been used in both, or if in the involute system the same obliquity should be used in both. Wheels so constructed that any pair of them may work together correctly are called *set wheels*. Let d_1 and d_2 be the pitch diameters and r_1 and r_2 the radii of two wheels which are to work together, the shafts being l inches between centres, and the wheels turning at n_1 and n_2 revolutions per minute. Then from page 15

$$r_1 = \frac{n_2}{n_1 + n_2} \cdot l, \text{ and } r_2 = \frac{n_1}{n_1 + n_2} l$$

this formula applying to spur gears only, not to annular gears. Further $\frac{r_1}{r_2} = \frac{t_1}{t_2} = \frac{n_2}{n_1}$.

Example:—Two spur wheels are to be placed between shafts running at 100 and 200 revs. per min. respectively, the shafts being 9 in. centres, and the diametral pitch being 3.

Then $r_1 = \frac{200}{100 + 200} \times 9 = 6$ in. while $r_2 = \frac{100}{100 + 200} \times 9 = 3$ in. Thus $d_1 = 12$ in., $d_2 = 6$ in. Again, $t_1 = qd_1 = 3 \times 12 = 36$ teeth, and $t_2 = 3 \times 6 = 18$ teeth. The outside diameter of the gears are $d_1 + \frac{2}{q} = 12 + \frac{2}{3} = 12\frac{2}{3}$ and $d_2 + \frac{2}{q} = 6 + \frac{2}{3}$ or $6\frac{2}{3}$ in. The circular pitch p is $\frac{\pi d}{t} = \pi \frac{1}{\frac{t}{d}} = \pi \frac{1}{q}$, or $p = \frac{\pi}{q} = \frac{3.1416}{3} = 1.047$ in. The height $h_2 = \frac{1}{q} \times \frac{p}{20} = \frac{1}{3} + \frac{1.047}{20} = .385$ in. $\therefore h = .719$ in.

The student should practice solving problems on gears, assuming different quantities, and also working on questions involving annular gears. On being told that the outside diameter of a gear is 4 in. and the diametral pitch 8, he should at once know that it has 30 teeth, and he should become very familiar with such calculations.

Bevel Gearing

There are many cases in practice where gears must drive between two shafts which are not parallel, and these shafts may or may not lie in one plane. If they do lie in one plane their axes will intersect, and this latter case is the only one with which we shall deal here. The axes of the shafts may be in-

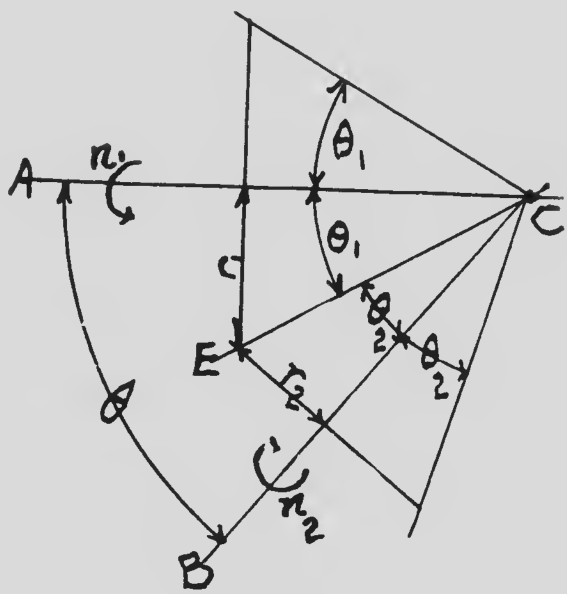

Fig. 18.

clined at any angle to one another, the most common case being where they are at right angles, although they frequently intersect at other angles. The gears used to drive between two such shafts are called *bevel gears*, and in the case where the shafts are at right angles and both turn at the same speed, the two bevel wheels would be exactly equal in all respects, and are then called *mitre gears*. Bevel gears are rarely made annular.

Let *A* and *B*, Fig. 18, represent the axes of two shafts intersecting at *C*, and let their speeds be n_1 and n_2 respectively. To find the sizes of the gears necessary to drive between them. Let *E* be a point of contact between the gears, the radii to it being r_1 and r_2. Then we have $r_1 n_1 = r_2 n_2$, as in the case of the spur gear, or $\frac{r_1}{r_2} = \frac{n_2}{n_1} =$ constant, and hence at any point where these gears would touch we should have the ratio $\frac{r_1}{r_2} =$ const., a condition which can only be fulfilled by points lying on the line EC. In the case of bevel gears, therefore, contact is along a straight line passing through the intersection of their shafts, and it may be shown that we can only get the desired motion by rolling together two cones, each having its apex at C, and an angle at the apex of $2\theta_1$ or $2\theta_2$, as marked. If $\theta = 90°$ and $n_1 = n_2$, then $\theta_1 = \theta_2 = 45°$. It is to be observed here that the angles θ_1 and θ_2 are fixed, when θ, n_1 and n_2 are known, but

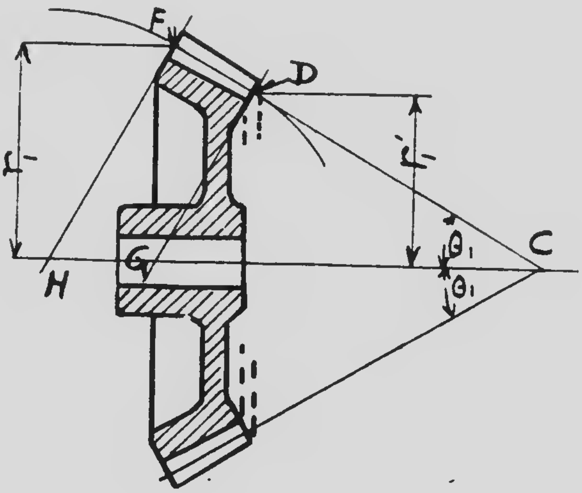

Fig. 10.

one of the radii r_1 or r_2 may be selected at will by the designer.

It is not considered advisable in this discussion to enter into the exact form the teeth should have in such a case, and the method of finding the proper shapes will merely be described. Let Fig. 10 represent one of the wheels, with angle $2\theta_1$ at the vertex of the cone, and let the radii r_1 and $r_1{}^1$ be selected to suit external conditions. Through D and E draw lines DG and EH normal to CDE, to intersect the axis of the shaft at G and H respectively. Then at D the teeth will have the same shape as if constructed for a spur gear of radius GD, and at E the teeth should be constructed for a spur gear of radius HE, and so for any intermediate point. The teeth are, of course, tapering from E to D, and either the involute or cycloidal system may be used.

HELICAL GEARS

A study of Fig. 14 shows that the less the height of the teeth the more nearly the lines PC and P_1C become normal to the line of centres ACB, and hence, under such a condition, the less the pressure between the teeth (which is in the direction PC) for a given amount of power transmitted, thus for a given pressure between the teeth the maximum power would be transmitted if the line of pressures were tangent to the pitch circles at C. If the height of the teeth is decreased, however, the arc of approach is decreased, and hence, for a given pitch, the smaller will be the number of teeth in contact at once, and the more uneven will be the motion. If now, instead of making the teeth directly across the gear parallel with the axis, they be run across it diagonally, so as to form parts of a helix, then, instead of a whole tooth on the gear coming suddenly into contact with a whole tooth on the pinion, we would have a pair of teeth coming gradually into contact, the contact beginning at one end and gradually working across the gear, till the other end is touching its mate. In such a case the teeth need not be high, and yet there will be no unevenness in the motion.

Wheels with the teeth made in this way are called *helical gears*, and it is to be remembered that if we pass a plane through the wheel normal to its axis the profile of the tooth so shown should be involute or cycloidal.

Helical wheels are used in the De Laval steam turbine, where the pinions run at 400 revs. per sec. without noise. They are also used in mills and other places, where steady motion is desired or the power transmitted is large.

CHAPTER III.

TRAINS OF GEARING.

In ordinary use gears are frequently arranged in a series on several separate axles, a series being called a *train* of gearing, so that a train of gearing consists of two or more wheels, which all turn at the same time, the angular velocities of all wheels in the train being known when that of any one is given. A train of gearing may always be replaced by a single pair of wheels of proper diameter, but in many cases the diameters of the two gears would be such as to make the arrangement undesirable.

When the train consists of four or more wheels, and when any two of these of different sizes are keyed to the same intermediate shaft, the arrangement is said to be a *compound train*. These trains are very common. If the gears are so arranged that the axis of the last gear lies in the same straight line as that of the first gear, as in the train between the minute and hour

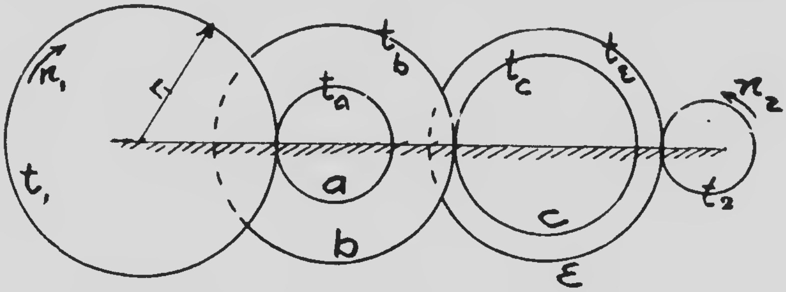

Fig. 20.

hands of a clock, the train is said to be *reverted*. If one of the gears in the train is held stationary and some or all of the other gears revolve about it, as in the case of the differential gear of an automobile, where one back wheel stops, the scheme is called an *epicyclic train*. It is quite common to have a reverted, epicyclic, compound train or a simple epicyclic train, thus in the epicyclic train the axis of the last gear may coincide with that of the first, although this is not at all necessary.

The *velocity ratio* of any train of gearing is the number of revolutions made by the last wheel divided by the number of revolutions in the same time of the first wheel in the ordinary train, or of the frame in the epicyclic train. Thus, let n_2 be the number of revolutions per minute made by the last gear, and let n_1 be the revolutions per minute of the first gear in the ordinary train or of the frame in the epicyclic train, then the ratio of the train $R = \frac{n_2}{n_1}$. Taking first a train in which the frame is fixed and all wheels revolve, let it consist of six gears 1, *a*, *b*, *c*, *e*, and

2, Fig. 20, having speeds n_1, n_a, etc., etc., radii r_1, r_a, etc., and numbers of teeth, t_1, t_a ...... t_2, respectively, the gears a and b being keyed to one shaft, as also the gears c and e. Thus, this is a compound train. Evidently any pair meshing together, such as b and c, must have the same pitch, and also the same type of teeth (i.e., involute or cycloidal), but any other gear in the train may have a different system and pitch, provided only that it suits the gear with which it meshes. Then, it at once follows that

$$\frac{n_a}{n_1} = \frac{r_1}{r_a} = \frac{t_1}{t_a} \text{ and } \frac{n_c}{n_b} = \frac{r_b}{r_c} = \frac{t_b}{t_c} \text{ and } \frac{n_2}{n_e} = \frac{r_e}{r_2} = \frac{t_e}{t_2}$$

and hence that

$$R = \frac{n_2}{n_1} = \frac{n_a}{n_1} \cdot \frac{n_c}{n_b} \cdot \frac{n_2}{n_e} = \frac{r_1}{r_a} \cdot \frac{r_b}{r_c} \cdot \frac{r_e}{r_2} = \frac{t_1}{t_a} \cdot \frac{t_b}{t_c} \cdot \frac{t_e}{t_2}.$$

Calling now the first wheel in each pair the driver and second wheel the driven, we at once get the rule: The ratio of a train R is the product of the radii of the drivers divided by the product of the radii of the driven wheels, or the ratio is the product of the teeth in the drivers divided by the product of the teeth in the driven wheels.

Should any of the wheels in the above train be annular, exactly the same law holds; and, in fact, the same law will hold if some of the gears are replaced by belts and pulleys, so that the determination of the ratio is quite simple in any case. Thus, in the above case, let $n_1 = 50$ revs. per min., $n_a = 80$ revs., $n_c = 120$ revs., and $n_2 = 200$ revs, and let $r_1 = 6$ in., $r_b = 4$ in., and $r_e = 5$ in.; also let the diametral pitches be 4, 6 and 8 for the pairs 1 and a, b and c and e and 2 respectively. Then we have $r_a = 3\frac{3}{4}$ in., $r_c = 2\frac{2}{3}$ in., $r_2 = 3$ in., or $d_1 = 12$ in., $d_a = 7\frac{1}{2}$ in., $d_b = 8$ in., $d_c = 5\frac{1}{3}$ in., $d_e = 10$ in., and $d_2 = 6$ in.; also $t_1 = 48$ teeth, $t_a = 30$ teeth, $t_b = 48$ teeth, $t_c = 32$ teeth, $t_e = 80$ teeth, and $t_2 = 48$ teeth. The velocity ratio

$$R \text{ of the train} = \frac{r_1 \cdot r_b \cdot r_e}{r_a \cdot r_c \cdot r_2} = \frac{6 \cdot 4 \cdot 5}{3\frac{3}{4} \cdot 2\frac{2}{3} \cdot 3} = 4$$

$$\text{or } R = \frac{t_1 \cdot t_b \cdot t_e}{t_a \cdot t_c \cdot t_2} = \frac{48 \cdot 48 \cdot 80}{30 \cdot 32 \cdot 48} = 4$$

It may be observed that the whole train might be replaced by a gear 30.07 in dia., meshing with a pinion 9.76 in dia., without changing the distance between the first and last shafts, but the sizes of these gears would in many cases be prohibitive.

As to the sense of rotation, it will be evident that for one contact (two wheels) between spur gears, the sense is reversed; for two contacts it is the same, for three reversed, etc., i.e., if the number of contacts is odd the first and last wheels turn in opposite sense and vice versa. Each contact with an annular gear neutralizes a contact with spur gears in respect to the sense of rotation, and if at any place between gears of the train a belt and pulley are used, then an open belt produces the same effect as an annular gear and pinion, and a crossed belt the same effect as a spur gear and pinion.

It not infrequently happens that in a compound train the two wheels on an intermediate axle are made the same diameter and combined into one. Thus we may make $r_a = r_b$, i.e., $t_a = t_b$. Such a wheel is then called an *idler*, and inspection of the formula shows that such an idler has no effect upon R, and is used solely to change the sense of rotation or to increase the distance between the axes of the other wheels without at the same time increasing their diameters.

We shall now solve a few problems illustrating the use of the formulas:

(1.) A wheel of 144 teeth drives one of 12 teeth, on a shaft which makes one revolution in 12 secs., while a second shaft driven by it makes a revolution in 5 secs. On the latter shaft is a 40 in. pulley connected by a crossed belt with a 12 in. pulley, this latter pulley making 2 revs., while one geared to it makes 3 revs. Show that the ratio of the train is 144, and that the first and last wheels turn in the same sense where no annular gears are used.

(2.) It is required to arrange a train of gears giving a ratio $\frac{13}{250}$. Remembering that $R = \frac{\text{product of teeth in drivers}}{\text{product of teeth in driven wheels}}$ we might use directly a gear of 250 teeth to drive a pinion of 13 teeth; but if this gear is too large we may break up the ratio R thus: $R = \frac{250}{13} = \frac{5 \times 5 \times 10}{13} = \frac{5}{1} \times \frac{5}{4} \times \frac{40}{13}$ or $t_1 = 60$, $t_a = 12$, $t_b = 20$, $t_c = 16$, $t_d = 40$, and $t_2 = 13$, giving gears without large numbers of teeth. If the distances between centres are given, then we must either arrange the diametral pitch to suit, or we must select some of the large number of other values of t_1, t_a, etc., which will fit the above case. The above solution gives six wheels, but we might use eight or four as well.

(3.) To design a train of gears which would be suitable for connecting the second hand of a watch to the hour hand. Here $R = 720$, and the last wheel must turn in the same sense as the first one, and hence the number of contacts must be even, requiring 4 or 8 or 12, etc., wheels in the train. As before, many solutions are possible, thus

$$R = \frac{720}{1} = \frac{4 \times 4 \times 5 \times 9}{1} = \frac{56}{14} \times \frac{48}{12} \times \frac{50}{10} \times \frac{108}{12} \text{ or}$$

$$R = \frac{720}{1} = \frac{6 \times 6 \times 4 \times 5}{1} = \frac{72}{12} \times \frac{60}{10} \times \frac{52}{13} \times \frac{60}{12}$$

with 8 wheels. The solution may also be worked for 12 wheels if desired.

(4.) The train of gears for connecting the minute and hour hands of a clock is required. Here the train is reverted with $R = 12$, and we must have an even number of contacts, so that we shall select four wheels. In addition to obtaining the desired ratio, we must have $r_1 + r_a = r_b + r_2$, and if all the wheels have the same pitch, $t_1 + t_a = t_b + t_2$.

Now, $R = \frac{12}{1} = \frac{4}{1} \times \frac{3}{1} = \frac{48}{12} \times \frac{45}{15}$, thus the intermediate shaft would have the gears with 12 and 45 teeth, while the 48-toothed wheel would be connected to the hour hand, and the 15-toothed wheel to the minute hand.

THE SCREW-CUTTING LATHE

Most lathes are arranged for the cutting of threads on a piece of work, and as these form an interesting application of the principles already described, we shall use it as an illustration. The general arrangement of the head-stock of a lathe is shown in

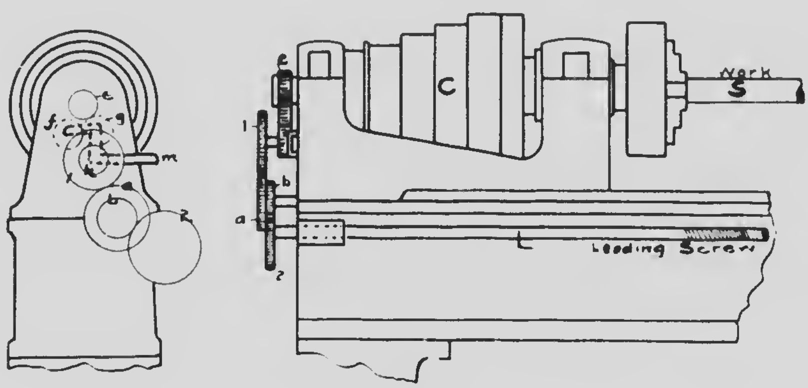

Fig. 21.

Fig. 21, in which the back gear is omitted to avoid complication. The cone C is connected by belt to the source of power, and is secured to the spindle S, which carries the live centre and also the chuck for driving the work, so that S turns at the same rate as C. To the end of S is a gear e, which drives a gear h through one idler g or two idlers f and g, the shaft which carries h also has a gear i, which is keyed to it, and must turn with the shaft at the same speed as h. The gear i meshes with a pinion a on a separate shaft, this pinion being also rigidly connected to and revolving with gear b, which latter gear meshes with the wheel 2, which is keyed to the leading screw L. Thus the spindle S is geared to the leading screw L through the wheels e, f, g, h, 1, a, b, 2, of which the first four are permanent, and the latter four may be changed to suit conditions, and are called *change gears*.

The work is attached between the centre on S and the centre on the tail stock, and is attached to S so that it rotates with it. The leading screw L passes thr ugh a nut in the carriage carrying the cutting tool, and it will be evident that for given gears on 1, a, b, 2 a definite number of turns of S correspond to a definite number of turns of L, and hence to a certain horizontal travel of the carriage and cutting tool. Suppose now that we wish to cut a screw on the work having s threads per inch the number of threads per inch l on the leading screw being given, then it will be clear that while the tool travels one inch horizontally corresponding to l turns of the leading screw L, the work must revolve s times, or if n_1 represents the revs. per min. of the work and n_2 of the leading screw we have

$$R = \frac{n_2}{n_1} = \frac{l}{s} = \frac{t_c}{t_h} \cdot \frac{t_1}{t_a} \cdot \frac{t_b}{t_2} = \frac{t_1}{t_a} \cdot \frac{t_b}{t_2}$$

where t_c, t_h, t_1, t_a, t_b and t_2 are the teeth in the wheels c, h, 1, a, b and 2 respectively, the idlers f and g having no effect on the velocity ratio, and we are considering the common case where $t_c = t_h$. If, further, L and S turn in the same sense the thread cut on the work will be right hand, that on the leading screw being right hand, and vice-versa.

The idlers f and g are provided to facilitate this matter, and if a right hand thread is to be cut, the handle m carrying the axes of f and g is moved so that g alone connects c and h, while, if a left hand thread is to be cut the handle is depressed so that f meshes with c and g with h. The figure shows the setting for a right hand thread.

An illustration will show the method of setting the gears to do a given piece of work. Suppose that a lathe has a leading screw cut with 4 threads per inch, and the change gears have respectively 20, 40, 45, 50, 55, 60, 65, 70, 75, 80 and 115 teeth.

(1) It is required to cut a right hand screw with 20 threads per inch. We have $\frac{l}{s} = \frac{t_1}{t_a} \cdot \frac{t_b}{t_2}$ where $l = 4$ and s is to be 20.

$$\text{Thus} \quad \frac{t_1}{t_a} \cdot \frac{t_b}{t_2} = \frac{4}{20} = \frac{1}{5}$$

This ratio may be satisfied by using the following gears $t_1 = 20$, $t_a = 50$, $t_b = 40$ and $t_2 = 80$. Only the one idler g would be used to give the right hand thread.

(2) To cut a standard thread on a 2 in, gas pipe in the lathe. The proper number of threads here would be 11½ per in. and hence $l = 4$, $s = 11\frac{1}{2}$ and $\frac{t_1}{t_a} \cdot \frac{t_b}{t_2} = \frac{4}{11\frac{1}{2}} = \frac{8}{23}$. Here we could make it $t_1 = 40$ and $t_2 = 115$, if we made $t_b = t_a$ or replaced both by an idler.

(3) If we required to cut 100 threads per inch then $l = 4$, $s = 100$ and $\frac{t_1}{t_a} \cdot \frac{t_b}{t_2} = \frac{4}{100} = \frac{1}{25}$, and we may divide this into two parts, thus, $\frac{1}{25} = \frac{1}{4} \times \frac{1}{6\frac{1}{4}}$, so that if we make $t_1 = 20$, $t_a = 80$, $t_2 = 75$, we should have to have an extra gear of 12 teeth to take the place of b, as $t_b = 12$.

The axle holding the gears a and b may be changed in position so that to make these gears fit in all cases between 1 and 2. The details of the method of doing this are omitted in the drawing.

When odd numbers of threads are to be cut various artifices are resorted to, sometimes only approximations being employed,— for example, the number of threads per inch commonly used on a $1\frac{1}{2}$ in. gas pipe is $11\frac{1}{2}$, but no serious trouble would ordinarily result if we had to cut it in a lathe in which the nearest number of threads would be $11\frac{1}{4}$ per in. There are cases, however, in which certain exact threads of very odd pitches must be cut, and one example will be given to show how such a case may be solved.

Let it be required to cut a screw with an exact pitch of one millimeter, the leading screw on the lathe having 8 threads per in. (1 mm. = .0393708 in.). This is worked out by a series of approximations by the method of continued fractions, the exact value of R for the case being $\frac{1}{R} = \frac{1}{8} \times \frac{1}{.0393708}$.

The first approximation is 3, the real value being $3\frac{68876}{393708}$.

The second approximation is $3 + \frac{1}{5}$, the real value being

$$3 + \cfrac{1}{5 + \cfrac{49328}{68876}}$$

and proceeding in this way we find the third, fourth, fifth, sixth, etc. approximations, the sixth being

$$3 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + 1}}}} = 3\frac{7}{40} \text{ or } \frac{127}{40}$$

Thus the sixth approximation gives $\frac{1}{R} = \frac{127}{40}$ or $R = \frac{40}{127}$. (It is worthy of note that $\frac{1}{8} \times \frac{1}{.0393708} = 3.17494$ while $\frac{127}{40} = 3.175$) so that this screw could be cut with great exactness by the use of the ratio $\frac{40}{127}$ between the work and leading screw.

Many problems of similar nature occur in practice, all of which may be solved by this method.

Hunting tooth gears have now almost disappeared, but were formerly much used by millwrights who thought that more evenness of wear resulted when a given pair of teeth in two gears came in contact the least number of times. Suppose we had a velocity ratio $R = 1$ and a pair of gears had 80 teeth each, then a given tooth on one gear would come in contact with the same tooth on the other gear at each revolution, but if we place 81 teeth in one gear, leaving the other with 80 teeth, then the ratio R is $\frac{81}{80}$ which differs very little from the vlaue desired, but a given tooth on one gear will only come in contact with a certain tooth on the other when one of the wheels ha made 80 revs. and the other 81 revs. This may be compared with the case where the numbers of teeth are 12 and 13.

Epicyclic Gearing

An epicyclic train of gears has already been defined as one in which one of the wheels is held stationary and at least one other gear revolves about it, the frame carrying the revolving gear must also revolve. The train is called epicyclic because a point on the revolving gear describes epicyclic curves. This arrangement is in very common use where a very low velocity ratio is to be obtained without an unduly large number of gears; thus a ratio of $\frac{1}{10,000}$ may readily be obtained by the use of four gears, the largest one

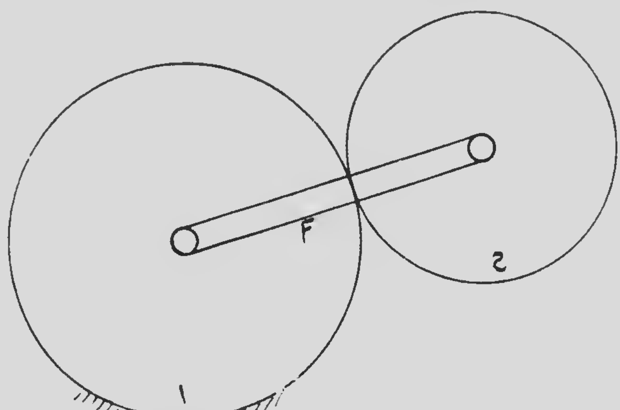

Fig. 22.

having only 101 teeth. The train may have many or few wheels, but it usually contains not over four. We shall start with the simplest case of two wheels.

In Fig. 22 let 1 and 2 represent gear wheels of radii r_1 and r_2 and teeth t_1 and t_2 respectively, F being the frame which is attached by a pin bearing to both wheels. Now if we hold the frame stationary the ratio of the train would be $R = \frac{r_1}{r_2} = \frac{t_1}{t_2}$, and is negative i. e., the first and last wheels turn in opposite sense. If now we fix the wheel 1 so that it cannot revolve, and turn the frame F abou the pin connection to 1, we would have the gear 2 revolving about its bearing on the frame and the train would be called epicycli and the ratio, E, of the train would be the number of turns of th last wheel 2, per turn of the frame F. To find E we may first assum that the frame and both wheels are rigidly connected togethe ike one solid body, then turn the whole machine about the axis between F and 1, that is, wheel 1 gets one revolution as do also the frame F and wheel 2. But in the operation the wheel 1 is to remain at rest, we therefore revolve it *back* one revolution without disturbing the frame, and during this operation the wheel 2 turns *forward* R revolutions because the wheels revolve in the opposite sense.

During the complete motion above described, the frame has revolved one revolution, the first wheel has revolved one revolution and then back again, *i. e.*, the net result is that it has not moved at all while the last wheel has turned $1 + R$ revolutions, so that the ratio of the train $E = \frac{1 + R}{1} = 1 + R$. If the train had three wheels or if the number of contacts between the toothed wheels were even then R would be positive and the ratio would be $E = 1 - R$. In fact this latter formula is the general one and R is positive or negative according to whether the last wheel in the train would revolve in the same or opposite sense of the first wheel if the frame were fixed.

The following method for obtaining E may appeal to some, the ratio R being here taken as positive, *i. e.*, the number of contacts are even. Let us first assume that the frame is fixed and all wheels revolve as in the ordinary train, then we may set down the results as follows:

Frame fixed.	1 rev. added to each part.
Turns made by frame 0 revs.	Turns made by frame $0 - 1 = -1$ rev.
First wheel turned through 1 rev.	First wheel turns $1 - 1 = 0$ revs.
Last wheel must turn R revs.	Last wheel turns $R - 1$ revs.

That is after the last operation the first wheel has been returned to its position of rest, the frame has made -1 rev. and the last wheel $R - 1$ revs., or the ratio $E = \frac{R - 1}{-1} = 1 - R$.

A few examples will illustrate the case.

1. Let the frame have a wheel 1 with 60 teeth, an idler and a wheel 2 with 59 teeth to find the ratio of the train when wheel 1 is fixed.

$$\text{Here } R = \frac{t_1}{t_2} = \frac{60}{59} \quad \therefore E = 1 - R = 1 - \frac{60}{59} = -\frac{1}{59}$$

or the wheel 2 will revolve in opposite sense to the frame and at $\frac{1}{59}$ the speed.

$$\text{If wheel 2 had been fixed } R = \frac{59}{60} \quad \therefore E = 1 - \frac{59}{60} = +\frac{1}{60}$$

or the wheel 1 would turn in the same sense to the frame and at $\frac{1}{60}$ of the speed.

2. Design an epicycle train giving a ratio of $\frac{1}{10000}$ the last wheel to turn in the same sense as the frame. Here $E = + \frac{1}{10000} = 1 - R$ if there are an even number of contacts. Hence $R = 1 - \frac{1}{10000} = \left(1 - \frac{1}{100}\right)\left(1 + \frac{1}{100}\right) = \frac{99}{100} \times \frac{101}{100}$ so that fixed wheel should have 99 teeth, the two wheels on the intermediate shaft 100 (gears with the fixed wheel) and 101 and the last wheel would have 100 teeth.

In practice such a train could be readily reverted because the

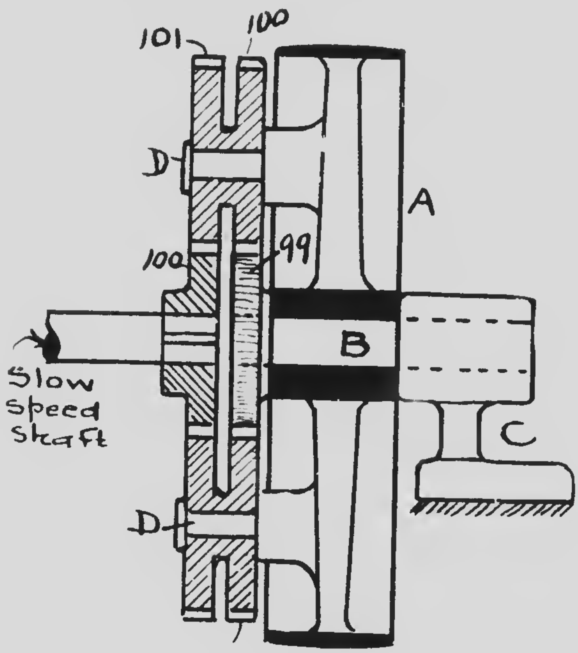

Fig. 23.

diameters of all gears could be made equal withour seriously affecting the teeth and we should then have the arrangement sketched in Fig. 23 which shows a practical form of the drive, the belt wheel being the frame and running 10000 times as fast as the slow speed shaft. The pulley A is a running fit on the shaft B which shaft is keyed to the support C and also to the gear with 99 teeth. The gears are loose on the pins D, while the 100 toothed gear is keyed to the slow speed shaft.

3. The Weston triplex pulley block contains a further example of the epicyclic train, and for the sake of simplicity only the essential parts are illustrated in Fig. 24. The frame D contains bearings which carry the hoisting sprocket wheel F and to the casting carrying

the hoisting sprocket are axles each carrying a pair of compound gears *BC* the smaller one *C* of which gears with an annular wheel made as part of the frame *D* while the other and larger gear of the pair mesheses with a pinion *A* attached to the end of the shaft *S* carrying the hand chain sprocket *H*. When a workman pulls on the hand chain he revolves correspondingly the sprocket *F* and hence the pinion *A* on the end of the shaft which in turn sets the compound gears *BC* in motion. As one of the compound gears C meshes with the annular wheel in the frame, the latter wheel being stationary, the only possible action is for the axles of the compound gears to revolve in a circle carrying the hoisting sprocket with them.

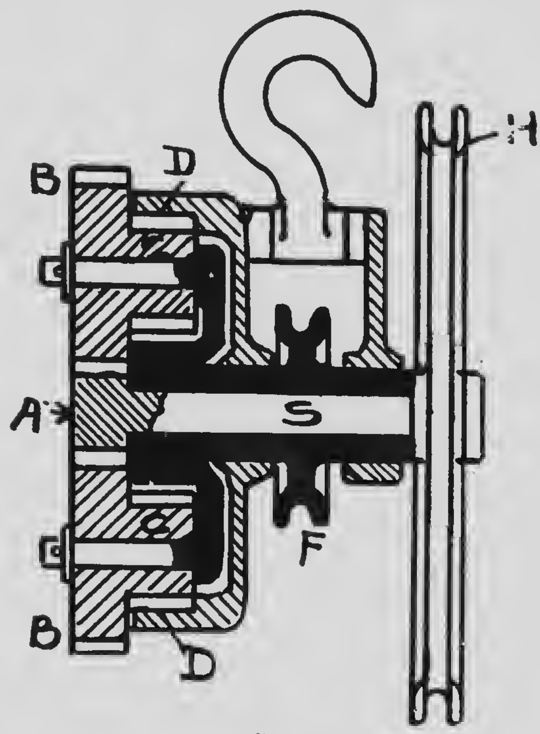

Fig. 24.

In a one ton Weston triplex block the annular gear on the frame has 49 teeth, while the two gears, *B* and *C* have respectively 31 teeth and 12 teeth there being 13 teeth in thepinion *A* on the hand wheel shaft. The hoisting wheel is 3¼ in diam., while the hand wheel is 9¾ in. diam., to find the pull on the hand chain to lift one ton, neglecting friction.

In this case $R = \frac{49}{12} \cdot \frac{31}{13} = 9.73$ and is negative as one of the wheels is annular Hence $E = 1 - R = 1 - (-9.73) = 10.73$, so that the hand wheel turns 10.73 revs. for one rev. of the hoisting wheel, and hence for each foot the load is lifted the hand chain must be moved $10.73 \times \frac{9\frac{3}{4}}{3\frac{1}{8}} = 33.2$ ft. Or the pull on the hand chain to lift one ton, neglecting friction would be $\frac{2000}{33.2} = 60$ pounds. (Note— In the actual case friction would raise this probably to 80 pounds or more)

4. A form of motor driven portable drill is shown at Fig. 25 in which the gears are worked on this principle. Here, again, only the barest outlines arse shown as the actual construction is rather complicated, the machine is very well made and fitted with ball bearings throughout instead of the plain bearings shown. The tool is called the Duntley drill and is made by the Chicago Pneumatic Tool Co.

The outer casing of the machine, Fig. 25, is held stationary

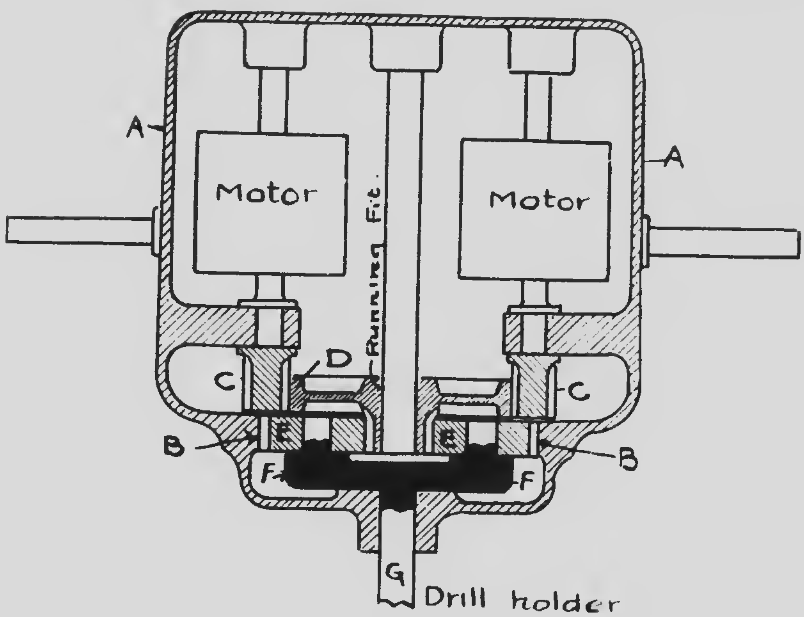

Fig. 25.

by the two handles shown and contains two motors driven by current brought in through one of the handles. Each of the motors has a pinion C attached to it which meshes with the larger gear D of a pair of compound gears which latter rotate freely on a central shaft as indicated. The smaller gear of the pair meshes with two gears E carried on axles on which they run freely mounted on the piece F shown black in the figure, this latter piece carrying the socket for the drill which is to be driven. The two gears also mesh with an annular wheel B, forming part of the frame and thus remaining stationary whether the motors run or not. When the motors are driven the compound gear is driven by the pinion attached to each motor the compound gear drives the gear E and hence the part shown in black is caused to rotate.

CHAPTER IV.

GOVERNORS

In all prime movers, which we will briefly call engines, there must be a continual balance between the energy supplied to the engine by the working fluid and the energy delivered by the machine to some other which it is driving, e.g. a dynamo, lathe, etc., allowance being made for the friction of the prime mover. Thus, if the energy delivered by the working fluid (steam, water or gas) in a given time exceeds the sum of the energies delivered to the dynamo and the friction of the engine, then there will be some energy left to accelerate the latter, and it will go on increasing in speed, the friction also increasing till a balance is reached or the machine is destroyed. The opposite result happens if the energy coming in is insufficient, the result being that the machine will decrease in speed and may eventually stop.

In all cases in actual practice, the output of an engine is continually varying because if a dynamo is being driven by it for lighting purposes the number of lights in use varies from time to time, the same is true if the engine drives a lathe or drill, the demands of these continually changing.

The output thus varying very frequently, the energy put in by the working fluid must be varied in the same way if the desired balance is to be maintained, and hence if the prime mover is to run at constant speed some means of controlling the energy admitted to it during a given time must be provided.

Various methods are employed, such as adjusting the weight of fluid admitted, adjusting the energy admitted per pound of fluid, or doing both of these at one time, and this adjustment may be made by hand as in the locomotive or automobile, or it may be automatic as in the case of the stationary engine or the water turbine where the adjustment is made by a contrivance called a *governor*.

A governor may thus be defined as a device used in connection with prime movers for so adjusting the energy admitted with the working fluid that the speed of the prime mover will be constant under all conditions. The complete governor consists essentially of two parts, the first part consisting of certain masses which rotate at a speed proportional to that of the prime mover, and the second part a valve or similar device controlled by the part already described and operating directly on the working fluid.

It is not the intention here to discuss the second part, or valve, because this takes various forms, according to circumstances and forms a subject of study by itself for each given case. Suffice it to say that this device is usually made to act in one of the following ways:

(*a*) To partly close off the working fluid and thus reduce the weight admitted in a given time; e. g., the water in a water turbine or the length of cut off in a steam engine.

(*b*) To reduce the energy per pound of working fluid admitted.

e.g., to throttle the steam and thus reduce its pressure as it enters an engine.

(c) Various combinations of the above methods.

The part of the governor which has masses revolving at a speed proportional to that of the engine will now be considered in detail, and for convenience will be referred to in future as the governor One of the simplest forms of this device, also shown diagrammatically in connection with the valve as required for a throttling steam engine is shown at Fig. 26. It consists of a vertical spindle, driven from the engine shaft at a speed which bears a fixed ratio to that of the crank shaft. To this spindle balls, are attached by arms, as shown, and these balls are again connected to a sleeve, which is free to slide up and down the spindle. To this sleeve the throttling valve or valve gear is attached by suitable mechan-

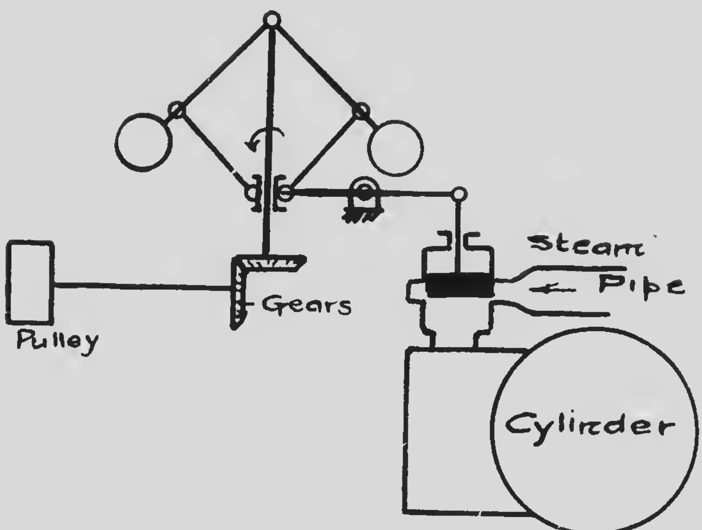

Fig. 26.

ism such as that indicated in the figure. The action is evidently as follows: assume the engine to be running at normal speed, then the balls will rotate in a given plane the height of which will be fixed by the resultant of the centrifugal force on the balls, the weight of the latter and the pull produced by the collar. If now part of the load be suddenly thrown off the engine the latter will tend to speed up, the centrifugal force will increase and the balls will rise, lifting the collar and closing the supply of steam until the equilibrium is again restored, but in general the balls will rotate in a higher plane than before. The converse is true for decrease of load.

Let us examine the problem first of all without considering the effect of friction or the resistance offered by the sleeve. Let each ball have a weight $\frac{w}{2}$ and rotate in a circle of radius r ft. Fig

27, and let the spindle rotate with angular velocity ω radius per second. Each ball is held in equilibrium by three forces; (*a*) the attraction of gravity parallel to the spindle of amount $\frac{w}{2}$ pds., (*b*) the centrifugal force acting normal to the spindle and of amount $\frac{w}{2g} r\omega^2$ pds., (*c*) the resultant of these two forces must be in the direction of the arm.

Now if we take l to represent length of the arm, and h the vertical height from the plane of rotation of the balls to the place where the ball arms (or the arms produced, see figures) intersect the spindle the following relation is at once evident:

$$\frac{w}{\frac{w}{g} r\omega^2} = \frac{h}{r} \quad \text{or } h = \frac{g}{\omega^2}$$

or the *height h* varies inversely as the square of the angular velocity and is independent of the dimensions of the parts of the governor.

An examination of this governor will show at once that it possesses

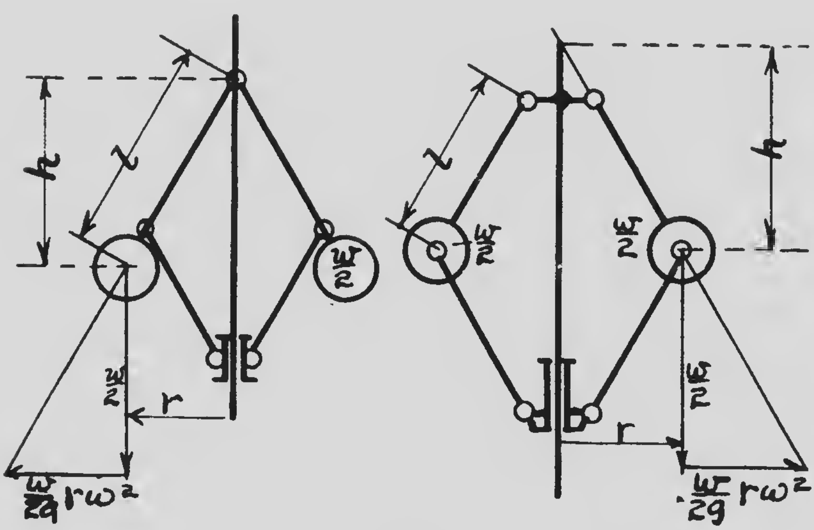

Fig. 27.

certain serious defects: (1) That from the very construction of the governor a change in adjustment of the collar will correspond to a change in the height h and hence a change in ω, a condition which it is the purpose of a governor to prevent, for a governor is designed essentially to keep the speed constant for all loads on the engine, and

(2) That for any reasonable value of ω, h is very small. Thus let the spindle turn at 120 revs. per min., then $\omega = \frac{2\pi \times 120}{60} = 4\pi$ radians per sec, and $h = \frac{g}{\omega^2} = \frac{32.16}{(4\pi)^2} = .2036$ ft. or 2.44 in.; a dimension which is so small as to be difficult to work with in practice.

The first defect is described by saying that the governor is not *isochronous*, the meaning of isochronism being that the speed of the governor will not vary during the entire range of travel of the collar, or in other words the valve of the engine may be moved to any position to suit the load, and yet the engine will run at the same speed. It will at once be recognized that if isochronism had no counterbalancing disadvantages it would be very desirable and we shall see how it may be accomplished.

From the formula $\omega^2 h = g$ it is evident that if ω is to remain

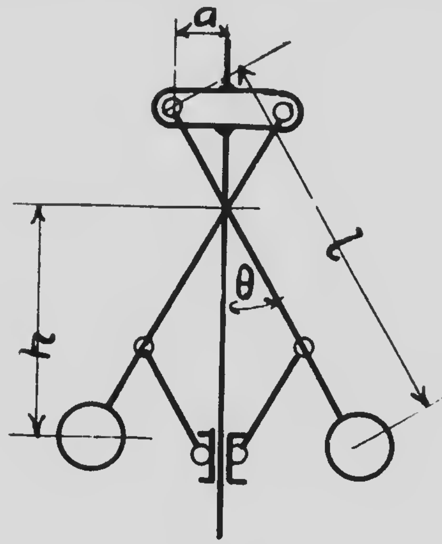

Fig. 28.

constant the height h must also remain constant, and it is evident that the crossed arm arrangement sketched in Fig. 28 will, under certain conditions, give approximately constant heights for different positions of the balls.

Inspection of the figure gives the relation

$$h = l \cos\theta - a \cot\theta.$$

If now h is to remain constant during variations in θ we have

$$\frac{dh}{d\theta} = 0 = -l \sin\theta + a \operatorname{cosec}^2\theta$$

or $a = l \sin^3\theta$

and $h = l \cos^3\theta$

$$\therefore a = l \sin^3\theta = h \tan^3\theta = \frac{g}{\omega^2} \tan^3\theta$$

Ex. Let $\omega = 10$ radians per sec. (97 revs. per min.), $\theta = 30°$.

Then $a = .062$ ft. $= .74$ in., and $l = \frac{a}{\sin\theta} = .494$ ft $= 5.92$ in.

The value of h corresponding to $\theta = 30°$ is .322 ft., and when through a changed load the balls move out till θ becomes 35° then h becomes .316 ft. a decrease of about 2% and the change in speed corresponding to this is slightly less than l%.

In the case of the governor without the crossed arms taking $\omega = 10$ as before, a change of θ from 30° to 35° means a change in speed of 3%.

It has been suggested that a governor of this type could be made isochronous for a large range of positions, provided the centres of the balls are made to move so that they always lie in a paraboloid of revolution which has the spindle for its axis, and it may be shown that for such a design h and therefore ω will be constant for all positions.

The defect of an isochronous governor, however, is that it will alter its position enormously for the slightest momentary change in speed of the engine and the balls will race out and in producing corresponding changes on the engine, and there is considerable *hunting* for the correct position, *i.e.*, such a governor is not *stable*. The condition of instability is not admissible in practice and designers always must sacrifice isochronism to some extent to the very necessary feature of stability, because this hunting of the balls for their final position means that the valve is being opened and closed too much and hence that the prime mover is changing its speed continually or is *racing*. Reverting to our original example, it will at once appear that a definite position of the balls will correspond to each speed because for each position there is a definite value of h, and therefore of ω.

We shall next consider the modification introduced by Mr. C. T. Porter, which consists in placing a heavy weight on the collar or sleeve of the governor, either with or without crossed arms. Fig. 29 shows such an arrangement. The conditions of equilibrium are readily solved by the phorograph considering OP as the link of reference, thus the images of Q and P are as shown, and by taking moments about O we get

$$\frac{W}{2}\,.\,2\,l\sin\theta + \frac{w}{2}\,l\sin\theta - \frac{w}{2\,g}\,r\,\omega^2\,l\cos\theta = 0$$

From which it follows that

$$h = \frac{2\,W + w}{w}\,.\,\frac{g}{\omega^2}$$

Ex. Given $l = .75$ ft. (9 in.) $\omega = 20$ radians per sec. (194 revs. per min.) $w = 8$ lbs. $\theta = 45°$, $a = 0$, we find $h = .53$ ft. and hence $W = 2.8\,w = 22.4$ lbs.

This governor possesses the following important advantages over the type already described:—

(*a*) The height h may be adjusted to suit any proportions required in practice merely by altering W to suit.

(*b*) The variation in height h corresponding to a given change in ω is very much increased in this case. Thus for a given alteration in speed the change in position of the sleeve is much greater than formerly, or a smaller range in speed will be necessary to correspond to the two extreme positions of the throttling valve, that is this governor is more sensitive than the former one.

To illustrate this take a simple unweighted governor for which the relation is $h = \frac{g}{\omega^2}$ or $g = h\,\omega^2$

By differentiation we obtain the result

$$\frac{\delta\,h}{h} = -\frac{2\,\delta\,\omega}{\omega}$$

or the proportional change in height is twice the proportional change

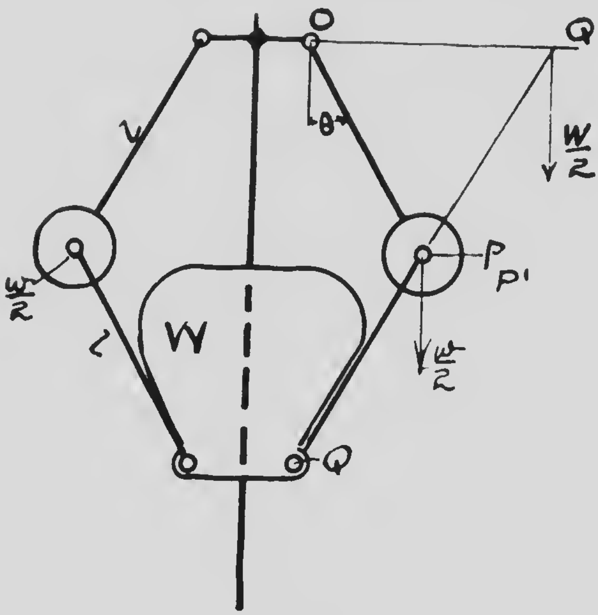

Fig. 29.

in speed and an increase in the former means a decrease in the latter.

The ratio $\frac{\delta\,\omega}{\omega}$ is called the *sensitiveness* where $\delta\,\omega$ is the change of speed corresponding to the extreme range of travel of the sleeve and ω is the mean angular velocity. Now let $\omega = 10$ radians per sec., and suppose the total range of height of the sleeve is ½ in., the height h for $\omega = 10$ being 3.86 in.

Here $\frac{\delta\,h}{h} = \frac{\frac{1}{2}}{3.86} = .129 = -2\,\frac{\delta\,\omega}{\omega}$ or $-\frac{\delta\,\omega}{\omega} = .064$

or for this range the variation of speed or sensitiveness is 6.4%.

For the weighted governor let $W = 60$, $w = 8$ and taking $\omega = 10$ as before, we get

$$h = \frac{2W + w}{w} \quad 3.86 \ldots \quad \frac{\delta h}{h} = \frac{w}{2W + w} = 129 = .008$$

hence $\frac{\delta \omega}{\omega} = \frac{.008}{2}$ or the sensitiveness here is .4%.

It is evident that as it is not possible to produce exact isochron-

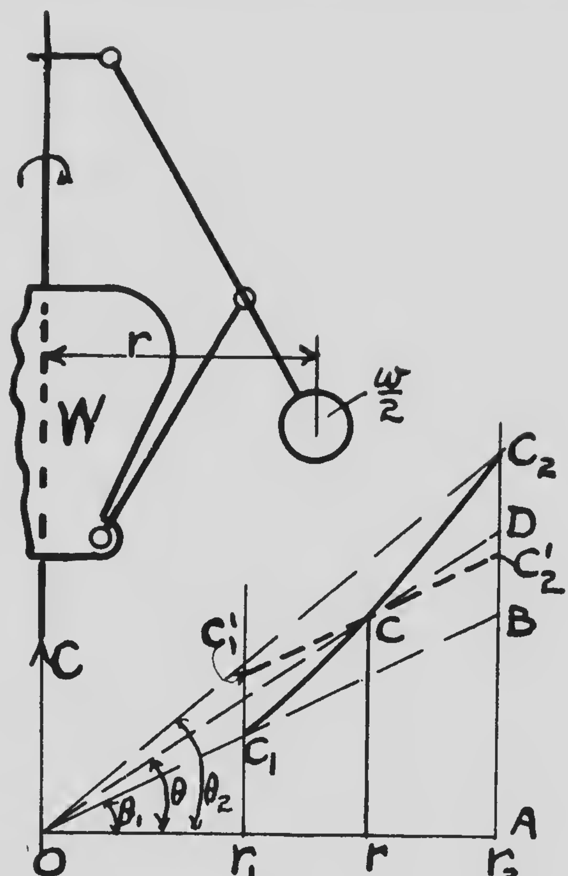

Fig. 30.

ism in a governor it will be very desirable to have it as sensitive as possible, and thus decrease the variation in speed.

(*c*) Since the sleeve already has a very heavy weight attached to it, therefore the governor is less effected by friction of the sleeve on the spindle or the pull necessary to operate the valve gear and have such a governor is said to be *powerful*. Powerfulness is also a very desirable feature, for it is well known in practice that the force necessary to operate the valve gear is not constant and therefore produces a variable effect upon the rotating parts, it is thus important that this variable effect be made as small proportionately as possible.

Thus the Porter governor may be arranged to suit any speed, the variation in speed corresponding to the extreme positions of the valve gear may be made as small as required and the governor is not greatly affected by the external forces produced by the connection to the valve gear.

Having now generally defined and explained the terms employed in connection with governors we shall choose one or two types and study them more in detail. Let us consider the weighted or Porter governor illustrated in Fig. 30.

THE CHARACTERISTIC CURVE. A curve showing the relation between the radius of rotation of the balls and the centrifugal force is of very great value in studying governors and as its shape shows very many things connected with the action of the governor it is called the characteristic curve, or we shall simply call it the C curve. Let r_1 and r_2 represent the inner and outer radii of rotation of the balls, the corresponding angular velocities being ω_1 and ω_2, and let r be the radius of rotation corresponding to the mean speed of rotation ω, defined by the formula $\omega = \frac{\omega_1 + \omega_2}{2}$. Now at any radius the centrifugal force $C = \frac{w}{g} r \omega^2$ where w is the total weight of the two balls and r is in ft., ω in radians per sec.

If now it were possible to make the governor isochronous we would have $\omega_1 = \omega = \omega_2$ a constant, and hence C would depend on r only, *i.e.*, the C curve would be a striaght line passing through O as shown at OC and here the ball may occupy any position at the same speed, such an arrangement is not stable as has been said already. If we plot the C curve for the case shown in figure, however, it takes the form $C_1 C C_2$ crossing the curve OC and being steeper than OC where they intersect. It will be evident that the curve $C_1 C C_2$ means that the speeds are not the same for the three positions of the balls and from the formula $C = \frac{w}{g} r \omega^2$ it is seen that $\omega_1 < \omega < \omega_2$.

This curve $C_1 C C_2$ corresponds to a stable arrangemnet because to each position only one speed corresponds and such speed increases as the balls move out.

Further, the shape of this curve is a measure of the sensitiveness of the governor as is shown below. Calling S the sensitiveness, we have

$$S = \frac{\omega_2 - \omega_1}{\omega}\;;\text{ now } \frac{\omega_2^2 - \omega_1^2}{\omega^2} = \frac{(\omega_2 - \omega_1)(\omega_2 + \omega_1)}{\omega\,(\omega_1 + \omega_2)}$$

$$\therefore S = \frac{1}{2}\,\frac{\omega_2^2 - \omega_1^2}{\omega^2} \quad \text{since } \omega = \frac{\omega_1 + \omega_2}{2}$$

but $C \quad \frac{w}{g} r \omega \qquad C \quad \frac{w}{g} r \omega$ and $C \quad \frac{w}{g} r \omega$

$$\therefore S \quad \frac{1}{2}\left[\frac{\frac{C}{\frac{w}{g} r} \qquad \frac{C}{\frac{w}{g} r}}{\frac{C}{\frac{w}{g} r}}\right] \quad \frac{1}{2}\left[\frac{\frac{C}{r} - \frac{C}{r}}{\frac{C}{r}}\right] \text{ where}$$

C the centrifugal force corresponding to the mean speed ω

Again $\frac{C}{r} \quad \tan\theta \quad \frac{BA}{OA}$ also $\frac{C}{r} \quad \frac{C_2A}{OA}$ and $\frac{C}{r} \quad \frac{DA}{OA}$

$$\text{Hence } S \quad \frac{1}{2}\left[\frac{\tan\theta \qquad \tan\theta}{\tan\theta}\right] \quad \frac{1}{2}\left[\frac{CA \qquad BA}{DA}\right]$$

$$\frac{1}{2} \quad \frac{CB}{DA}$$

This curve shows that an increase in the stability of the governor means a decrease in the sensitiveness. If at any part of its length the C curve is radial from O at that part S O and the governor is isochronous and therefore not stable so that if stability is desired the curve must make as great an angle as possible with the line joining it at any point to O, but on the other hand this angle must not make too large an angle on account of a decreased sensitiveness. For example at C the governor is as nearly isochronous and unstable so that we would get most uniform results by making the curve $C\ C\ C_2$ as nearly a straight line as possible.

It is desirable here to point out that if the curve takes the dotted form $C\ C\ C$ then the sensitiveness is very much improved and may be made almost perfect, but here since $C \quad \frac{w}{g} r\omega^2$ the outer position corresponds to the lowest speed and the inner position to the highest speed since it is evident from the figure that C does not increase as rapidly as r. Such an arrangement is evidently unstable since by an increase in speed more energy is imparted to the balls and the weights are being lowered thus further increasing the energy supplied to the system, instead of balancing it, so that if the ball begins to move inward it will fly to its inmost position under the combined action of the two forces. Thus we get stability only when the C curve is steeper than the line from O which cuts it. In Fig. 31 curve $C\ C\ C_2$ denotes stability, $C\ CC_2'$ instability, $C\ C\ C_2$ stability of the part $C\ C$ and instability for the part CC_2.

This C curve may be used for a further purpose of showing the powerfulness of the governor, since on the curve horizontal distances denote the space through which the ball moves and vertical heights the corresponding forces acting while the ball is moving.

Thus any elementary area $C.\delta r$ on this diagram represents a product of force and distance or work done hence the area $C_1 r_1 r_2 C_2 C_1$ is the work done on the balls while they move out, and further represents the work which can be done by the balls on the weights and valve gear.

This total work $= \int Cdr$ is expended in lifting the weights w and W, and in overcoming friction and resistance offered by the valve gear. We shall neglect the effect of friction (although in the actual case it must be considered) and shall further assume that the resistance in the valve gear may be included in W, it should, however, be stated that this latter resistance is variable and these variations should be considered in any design, but are omitted here on account of the complication of the cases. The effect of this variation on the governor's action will depend largely on the magnitude of it as compared with the total weight W and the mean resistance offered by the gear.

It will be noted that the force C is balanced by the sum of

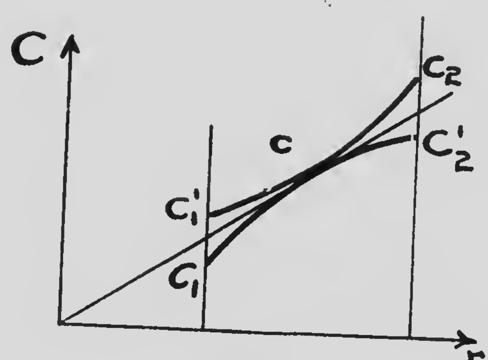

Fig. 31.

several other forces, viz.: (*a*) That due to the weight of the balls w; (*b*) That due to the central weight W; (*c*) The resistance at the collar, and (*d*) The friction of the parts. We shall assume that the force required to move the valve gear, (*c*) is included in (*b*), *i.e.*, that the force necessary to move the valve gear is constant and that this force plus the central weight amounts to W pds, we shall also neglect friction. Fig. 32 shows a Porter governor with the corresponding C curve. To find the part of C necessary to lift the weight w we resolve w in the directions of C and of the arm, then C_w is the part in the direction of C and which must be overcome by the latter while the remaining part produces a pull on the upper pin A. Lay off above the axis of r the values of C_w thus found for each position of the balls getting the C_w curve. Next find the part of C necessary to balance W as follows— Draw a $\triangle CDE$, making $CD = W$ and making BCE a straight line, then CE is the resolved part of W in the direction of the arm CB. Now resolve CE into two parts, one horizontal C_W and one passing through the pin A, these forces are at once found by the method shown above A, where $FA = W$ and GA is parallel to CB, HG is then C_w the part of C necessary to balance W. Lay off C_w, for each position of the

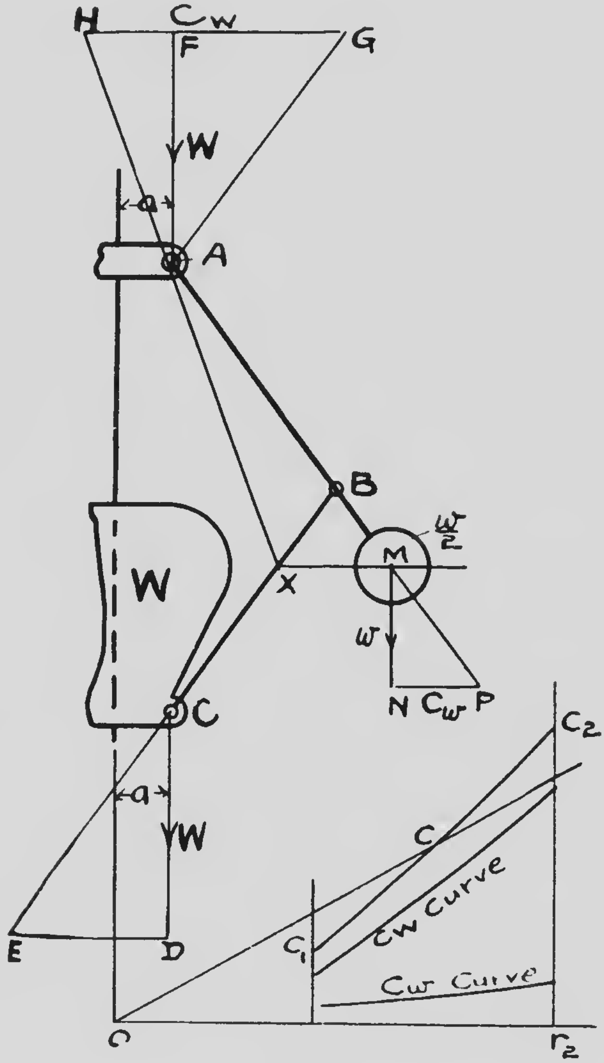

Fig. 32.

balls, above Or obtaining the C_w curve. Since $C = C_u + C_w$ the ordinates between the curves of C and C_w must represent the corresponding C_u.

It will be at once evident that for the unloaded governor the power is only the area below the C_u curve, since, neglecting friction and the pull of the valve gear, the whole of C is spent in overcoming the effect of the weight of the balls or $C = C_w$, while for the loaded governor it is very much increased, being the area below the C curve. The work represented by the area below the C_W curve must be employed in lifting the central weight and overcoming any force necessary at the clutch to operate the valve gear.

Before leaving this matter the C curve will be applied to the solution of one problem in the design of a governor. Suppose it is required to design a governor of the Porter type to operate at a mean speed Nn revs. per min. and the maximum and minimum speeds n_2 and n_1 are given. The work to be performed (or the power) is also given, to find the dimensions of the various parts. From general experience certain proportions will be known and only one

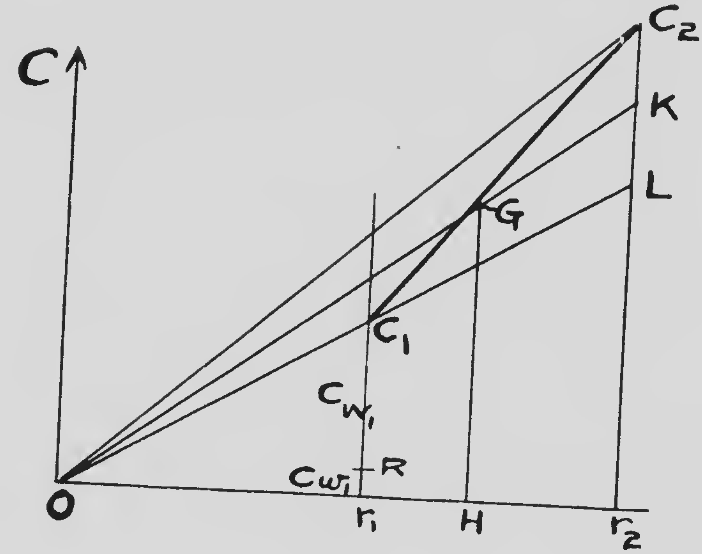

Fig. 3

or two points remain to be determined. We shall assume that r_1 and r_2 are given, also the lengths L AB BC of the arms. In Fig. 33 lay off r_1 and r_2 to scale, then from the work which the governor is to do lay off GH equal the mean height up to the C curve (note the area GH $r_1 r_2$ is the total work of the governor including that required to lift the balls, the available work at the clutch will be correspondingly decreased). Now the sensitiveness S $\dfrac{n_2 \quad n_1}{n}$ which is given, hence we lay off the distance (see page 48) S Kr_2 (KL KC_2) both above and below K along the line $r_2 K$, then joining LO we at once get C_1 and C_2 and without serious error the C curve is the line $C_1 G C_2$.

Next $C_1 = \frac{w}{g} r_1 \omega_1^2$ gives at once w the weight of the two balls since C_1 r_1 and ω are known. This may be also computed from $C_2 = \frac{w}{g} r_2 \omega_2^2$. We may now finish the problem in one of several ways depending on which quantities we assume and which we leave to determine. Probably we could best assume the angle ABC (which gives us the ABC) and also W. Assuming angle ABC at once enables us to draw the MNP. See Fig. 32, from which we find C_{w_1} which we lay off along r_1 C_1 and we then get $C_{w_1} = RC_1$. Now above A lay off $AF = W$ and draw AG parallel to CB and from G lay off GH horizontally through F so that $GH = C_w$

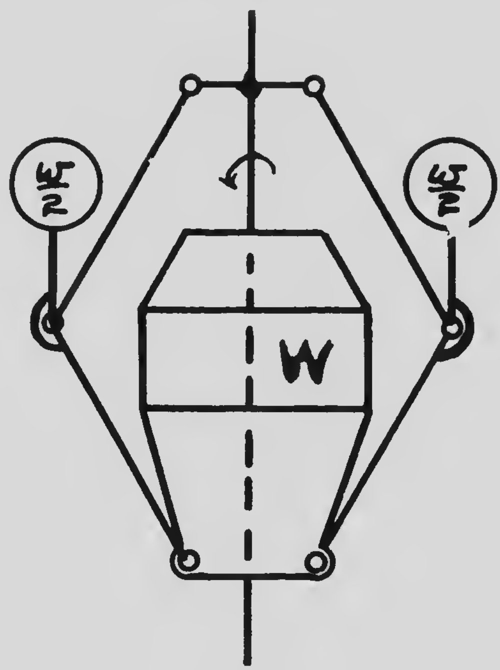

Fig. 34.

and then HA produced intersects CB at N and the horizontal line through N intersects AB at M the centre of the ball. The radius r_1 then locates the spindle and the design is complete.

The design should be checked at the outer radius r_2 and also the exact form of the C curve should be found and if it does not agree with that assumed, some of the assumed quantities must be adjusted and the calculation made over again.

Before leaving the matter it must be stated that the design of a governor is a very complicated piece of work in the actual case because the effect of friction is very serious and must in all cases be taken into account and further the exact forces at the clutch necessary to operate the valve gear must always be determined, and these are not constant. The determination of these forces is too complicated and lengthy to be introduced here and must be left to be considered by the designer, but when these forces have been determined the work may be carried out by a method similar to that described.

Fig. 34 shows an outline of a weighted governor by Proell possessing advantages over the Porter type which it would be of considerable value for the student to work out for himself.

Spring Governors

The modern tendency is to replace the weight W by a spring and as this usually means a rather different disposition of the parts,

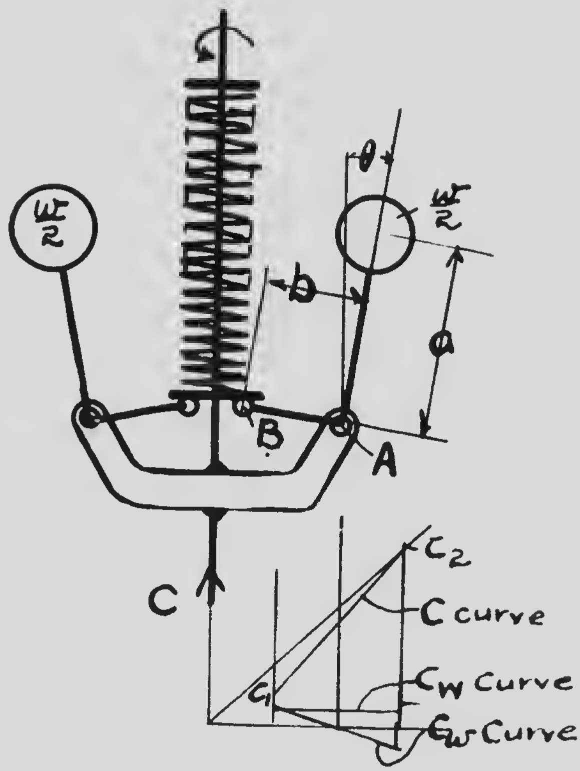

Fig. 35.

some consideration will be given to it here. Fig. 35 shows one of the very simplest governors of this type which is shown mounted on a vertical shaft although it is quite as frequently used on a horizontal shaft, it generally runs at a fairly high speed. Now let W be the weight on the spindle including the spring weight, F be the force produced by the spring, and w be the combined weight of the two balls. Then taking moments about A for the effect of W we get $W\,b\cos\theta = C_W\,a\cos\theta$ or $C_W = \frac{b}{a}W$ = const. and further taking moments about A for the ball weight w we get $C_w\,a\cos\theta = -wa\sin\theta$ or $C_w = -w\tan\theta$. So that C_w is positive or negative according to the value of θ. From a knowledge of these curves for C_W, C_w and C we are at once able to draw the C_F curve showing the resistance which must be offered by the spring, together with the force required to move the valve gear. Such governors are evidently powerful and may be made as sensitive as desired.

Calling F the force exerted by the spring assuming the curve C_F to be a straight line, we may readily obtain the necessary data for the design of spring. Thus $F\,b\cos\theta = C_F\,a\cos\theta$ or

$F = \frac{a}{b}C_F = C_F \times$ a constant so that the C_F curve may be also taken to represent a curve of F on a different scale, thus at radius

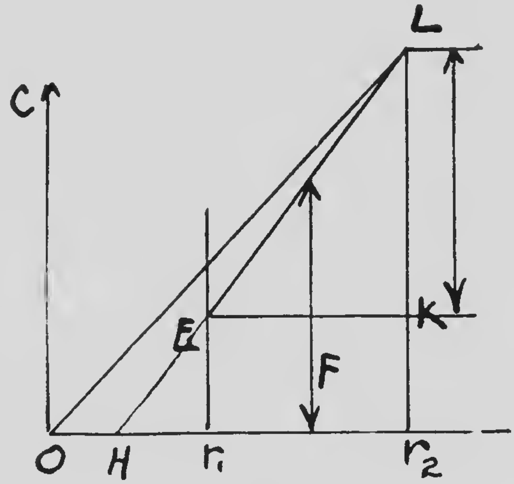

Fig. 36.

r_1 the value of F is rerpesented by $r_1 E$. Now produce the curve for C_F till it meets the axis of r at H. Then at radius OH, $F = O$ and hence the spring must be so designed that its zero compression corresponds to OH and the compression force S which it must produce per unit of compression will be

$$LK \times \frac{a}{b} \times \frac{1}{r_2 - r_1}. \quad \text{(Fig. 36.)}$$

An arrangement of this kind is not to be recommended because of the very great pressure and the corresponding friction produced on the pin A. A governor of the form already described is used on the small Leonard engines amongst others, but its shaft there is horizontal so that C_w and C_W are zero.

The form of governor used by Belliss and Morcom on their high speed engines is shown at Fig. 37, the governor being on the crank shaft and therefore horizontal so that C_a and C_w become zero and the centrifugal force of the balls is balanced by the pull of the spring and the resistance offered by the valve gear. Taking W as the weight of the two balls and a and b as sketched and calling F the spring pull we may find this force by the formula below provided the resistance offered by the valve gear is neglected. We have

$$F \cdot b \cos(\theta - \alpha) = C\, a \cos\alpha \;\therefore\; F = C\,\frac{a}{b} \times \frac{\cos\alpha}{\cos(\alpha - \theta)}$$

where α is the angle between the axis of rotation and the ball arm, or $F = C\,\dfrac{a}{b} \times \dfrac{1}{\cos\theta + \sin\theta\tan\alpha}$ and in this formula a, b, θ are constants, the only variables being C and α, so that in a given

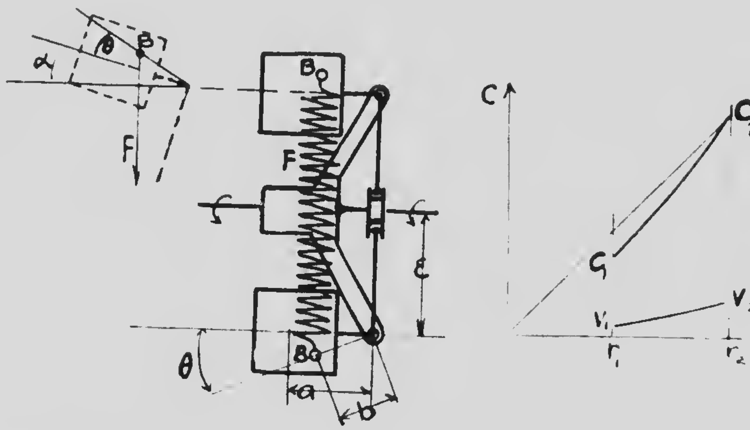

Fig. 37.

case the spring pull F is readily found and the proper design and location of the spring pins B to produce best results may be directly determined.

The spring pull in the actual case should be less than the force F as determined above because of the force necessary to move the valve gear, and of course this difference can only be determined when we know the exact construction and operation of the gear so that no general solution will here be attempted. In any case, however, this may be determined and plotted below the C curve at V_1, V_2 the force V at any time being the part of C necessary to operate the valve gear and the distance from the V curve to the C curve representing the part of C which must be balanced by the spring pull. Having determined the spring pull for each radius, the corresponding value of S can readily be found as in the last example. Then since the spring pull $F = eS$ where e is the extension of the spring and S the force necessary to stretch it one inch, the location of the pin connections must be so chosen that the elongation e is proportional to the force F acting at any instant.

As has already been pointed out, all spring governors may be made very powerful because the spring may be made to offer great resistance without being unduly large or heavy, and hence the angular velocity of these governors may be great. High angular velocity a means large power because the height of the C curve on which the power depends is $C = \frac{w}{g} r a^2$ which evidently increases as the square of a, so that doubling the speed makes the power roughly four times as great.

Certain firms are now undertaking the manufacture of complete governors for specified duty, and the student is recommended to get catalogues from these makers and study the forms adopted by them. The advantage of any form may readily be determined by the methods given

The Shaft Governor

In modern practice it has been found desirable in many cases to connect the governor directly to the main shaft of the machine, such as the crank shaft of an engine or else to the main lay shaft, as the cam shaft in a gas engine. In general in such a case the revolving weights are pivoted to a wheel keyed on the shaft, the weights thus always revolving in one plane instead of in planes of varying position as in the governors already described. Such governors are commonly called shaft governors and possess numerous points of excellence, so that it will be an advantage to study them with some care.

The shaft governor is used most commonly on steam engines and also finds considerable favor with builders of large gas engines. In the case of the steam engine the revolving weights are usually connected directly to the eccentric which operates the slide valve, the eccentric eye not being fixed to the shaft, but its position controlled by the governor. In most cases the governor alters the eccentricity as well as the angular advance of the eccentric, thus changing all the events of the stroke for a given change in load

A little thought will show that such governors should be made very powerful because the weights must be able of themselves to hold the eccentric in position against the force necessary to move the slide valve and although the latter always is of special construction in this type of engine yet this force is not inconsiderable; to make such a governor powerful the centrifugal force must be large or the revolving weights must be heavy and we must have high rotative speeds or especially adapted high-speed engines. It is not the purpose here to enter into a discussion of the steam distribution as affected by such governors.

Consider the conditions existing on a disk A, Fig. 58, which is revolving about a fixed centre O at n revs. per min., and we shall neglect the effect of gravity because in most governors it is balanced, although in this case no arrangement is shown for this purpose, To this ball let a spring D be attached, which is also attached to the disk at p and let the ball be free to move radially along the rod B. When the ball is at any distance r ft. from the centre of rotation O, the centrifugal force C acting on it is $C = \frac{w}{g} r \omega^2$ where w is the weight of the ball and ω is the angular velocity in radians per second corresponding to n.

Now let S denote the spring pull per foot of extension and let the spring have no extension when the ball is at O, thus for this position of the weight the extension of the spring will be r ft. Then the pull exerted by the spring will be $s' \times r$ pds., and as there must be equilibrium between the pull of the spring and the centrifugal force we have $s'r = \frac{w}{g} r \omega^2$ or $s' = \frac{w}{g} \omega^2$. We shall find it convenient to use S to denote the force required to change the length of the spring

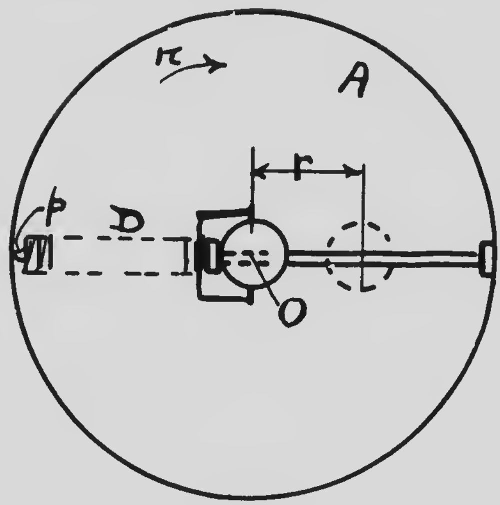

Fig. 38.

one inch so that $s' = 12S$. And if r be also measured in inches then we get by supplying the constants $Sr = .0000284\, w r n^2$ for the inch unit. Suppose now we wish to have n constant for all values of r, *i.e.*, an *isochronous* arrangement, we would then make $S = .0000284\, w n^2$, or if we take $w = 25$ lbs., $n = 200$ revs. per min. $S = 28.4$ lbs. *i. e.*, if to this ball we attach a spring so designed that a force of 28.4 pounds will change its length by one inch, and if further the spring be so connected with the ball that the extension of the former is always equal to the radius of rotation of the latter, then the arrangement is isochronous, or the ball will remain at any radius from the centre so long as the speed is 200 r. p. m. It will be evident, however, that the least external force would send the ball to the extreme end of its travel, or it is not stable.

Now let us examine the effect of altering S and let us take two cases (1) $S = 50$ pounds, and (2) $S = 24$ pounds. Taking the first case, let us assume as before a condition of equilibrium at

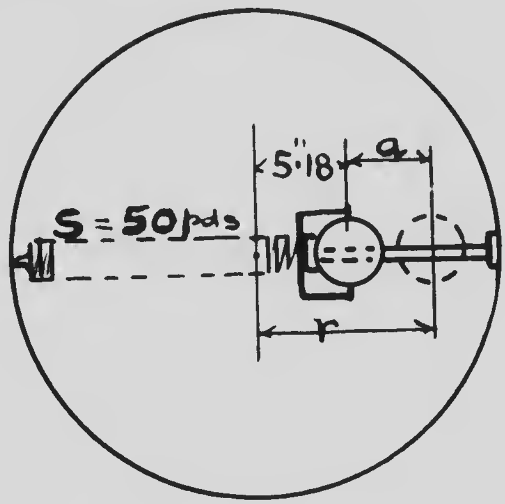

Fig. 39.

200 revs. per min. when the ball is 12 in. from the centre of rotation. Then $C = .0000284\, w\, r\, n^2 = 340.8$ pounds, and hence the extension of the spring must now be $\frac{340.8}{50} = 6.82$ in. instead of 12 in., in other words the extension of the spring will be less than the radius of

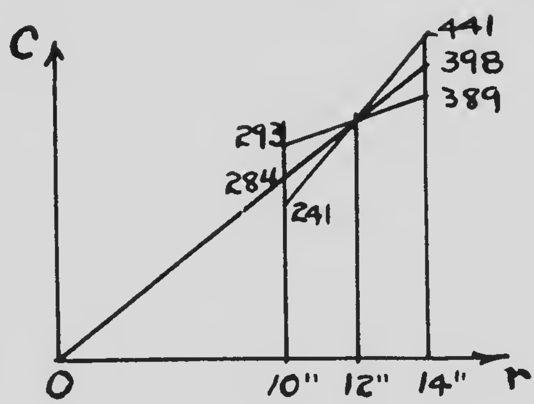

Fig. 40.

rotation of the ball or the spring will have its free length when the ball is $12 - 6.82 = 5.18$ in. from O and the arrangement is sketched in Fig. 39 in which the extension of the spring is denoted by a.

Now let the ball move out 2 in. n being still 200 revs. per min., a is then 8.82 in., $r = 14$ in. and hence $Sa = 441$, which tends to draw the ball inward while $C = .0000284\,w\,r\,n^2 = 397.6$ pounds tending to force the ball outward and hence the ball will return to its original position at 12 in. radius unless a force of 43.4 pounds be interposed to prevent this. On the other hand if the ball is rotated in a circle of 10 in. radius we would have $Sa = 241$ pounds and $C = 284$ pounds, so that a force of 43 pounds is urging the ball outward and hence there is only one position at this speed in which it can remain or the arrangement is *stable*.

Now let $S = 24$ pounds, then if equilibrium is to be maintained at $r = 12$ as before we find $a = 14.2$ in. or when the ball is at O the spring will have an elongation of 2.2 in. At 14 in. from the centre $C = 398$ pounds and the spring pull $Sa = 388.9$ or the ball will stay at the outer radius whereas if $r = 10$ in. $C = 284$ and

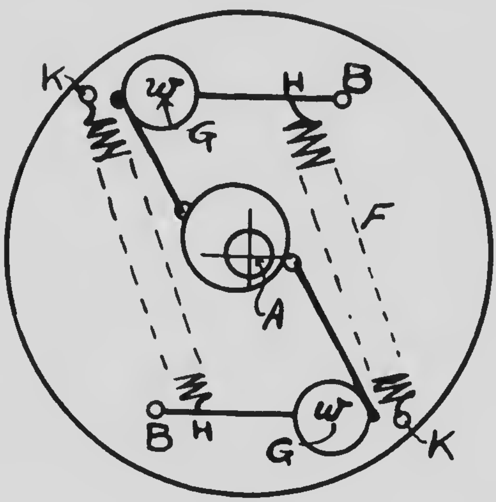

Fig. 41.

$Sa = 292.8$ pounds or the ball will stay at the inner radius. Hence, if in this case the ball be disturbed at all it will immediately fly outward or inward having no tendency to return to its proper position at 12 in. radius, in other words the equilibrium is *unstable*.

This is very nicely illustrated by a study of the C curves, Fig. 40, in each case.

It is further to be noted that with $S = 50$ we could only have the ball remaining at 14 in. from O when $n = 211$ revs., and at 10 in. when $n = 184$ revs. Hence, if this represents the necessary range of travel of the ball the sensitiveness is $2\left[\dfrac{211-184}{211+184}\right] = 15\%$. Where, however, $S = 24$, the corresponding speeds will be 198 revs. for $r = 14$ ins., and 203 revs. at $r = 10$, with the curious result that the speed increases as the balls move inward. Here the sensitiveness is 1.24% as compared with 15% in the previous case, thus, while the sensitiveness is very much improved in the case where $S = 24$ pounds, yet on account of the instability the arrangement is an impossible one.

In the shaft governor, however, the weights cannot be arranged as above, but must be mounted so that they may act directly on the eccentric and, consequently, the forces which they can exert must in some way be controlled. A very common arrangement is shown at Fig. 41, in which two weights are used attached to the rotating

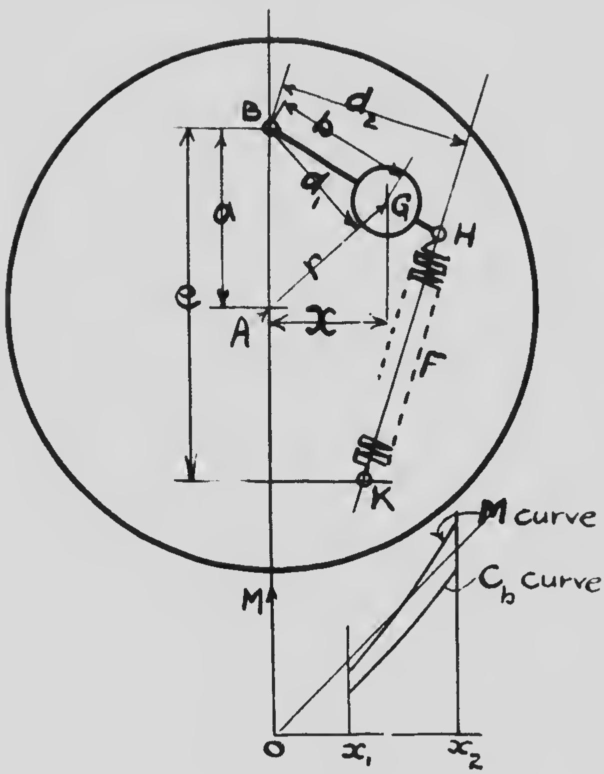

Fig. 42.

disk, or wheel by pins B, the centrifugal force of the balls G being balanced by the springs F and links shown connecting the ball arms to the eccentric. (Note—This form of governor is not much used now, but for the purpose of instruction it is chosen as an illustration, the modern form following later on).

Let Fig. 42 represent one half of a typical shaft governor, the other half being similar and the two parts being so connected that gravity effect is neutralized. A is the centre of rotation, B the point of connection of the weight with the fly wheel, G the centre of gravity of the weight H, and K the points of connection of the spring to the weight and wheel respectively, and F is the force in the spring. The letters indicate the following $a = AB$, $r = AG$, $b = BG$, d_1 is the shortest distance from B to AG, and d_2 is the shortest distance from B to HK, the direction of the force F.

Now let w be the weight of each revolving mass and F the force produced by one spring, then we have at once $C = \frac{w}{g} r \omega^2 = m r \omega^2$ where $m = \frac{w}{g}$ and the moment of C about the pivot B is $M = m r \omega^2 d_1$ and if we let x represent the shortest distance from G to AB it is at once evident from similar triangles that $r d_1 = a x$ and hence that $M = m r \omega^2 d_1 = m \omega^2 a x$. From this it will be seen that M depends entirely on ω and x, and if we choose M and x as axes of co-ordinates, we may plot upon the sheet curves similar to the C curves already taken up. If ω is constant or the governor is isochronous then, evidently M varies directly with x only and the "M" curve will be a straight line passing through O and we have again the case of neutral equilibrium. From what has already been said, it will be evident that if the M curve is steeper than the line from any point on it to O, the arrangement is stable, and on the other hand if the curve is less steep the arrangement will be unstable, the stable condition again corresponding to greater variations in speed than the unstable case, exactly as in the case of the fly ball governor already discussed. Thus the M curve is the characteristic curve for this type of governor.

Now through K draw a line perpendicular to AB, cutting the latter line at distance c from the pin B. Let C_2 be the resolved part of F such that the moment of the spring about $B = C_2 c = F d_2$ and then we have $M = m \omega^2 a x = C_2 c$ provided we neglect the effect of the valve gear. Thus $C_2 = m \omega^2 x \frac{a}{c} = const \times m \omega^2 x$, or the C_2 curve may also be drawn on the same axes as before, and this curve shows the effect of the spring. From the curve thus drawn the spring pull F may be found and the spring designed to suit the given conditions.

If, in addition to the two curves already described, a C curve on an r base be drawn the power of the governor may be obtained by integrating the quantity $C \cdot dr$ between r_1 and r_2.

While the investigations already made enable one to determine the conditions of equilibrium of the parts, they give no information as to the rapidity of the adjustment to new conditions of load, and this point will now be discussed. So far we have only been dealing with the centrifugal force on the balls, *i.e.*, the force due to the acceleration of the weights along a radius, and this force acts continuously during the running of the governor. When, however, the speed of the wheel is changing during the adjustment for new load, we must accelerate the wheel as well as all masses connected with it, each mass having an angular acceleration $\alpha = \frac{\delta \omega}{\delta t}$ where

$\delta\omega$ is the change in the velocity of the wheel in time δt., and further an acceleration in the direction of motion or tangential to the circle in which it is travelling, we may call this the tangential acceleration. These accelerations of the weights, which only come into play when the speed changes, may be made to oppose or assist the effect due to centrifugal force, and thus may be made to cause slow or rapid change of adjustment.

The diagrams in Fig. 43 will show the meaning of this very nicely where in all cases A is the centre of rotation, B the point of connection of the weight to the disk and G is the centre of gravity of the weight. The centrifugal force due to radial acceleration of the ball is always in the direction AG. At (a) the tangential acceleration produces no effect since the tangent to the path of G passes through the pin B and the force necessary to accelerate the

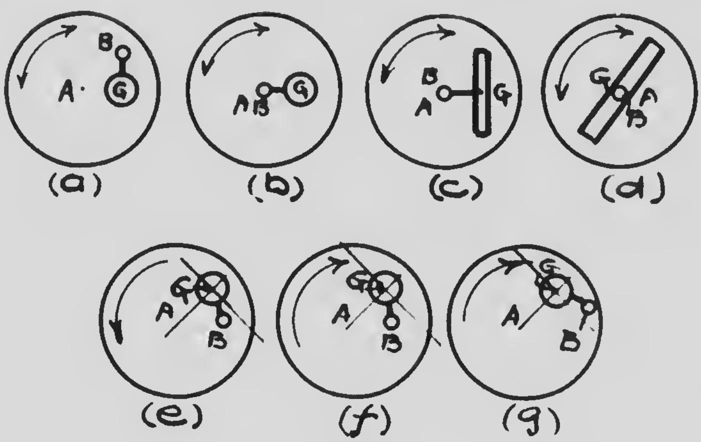

Fig. 43.

weight is borne directly by the pin B. At (b) the centrifugal effect is zero, the tangential acceleration producing a very decided turning moment about the pin B; but in both of these cases the angular acceleration is small since the weight is concentrated about its centre of gravity or its moment of inertia about its centre of gravity is small. (c) and (d) show a different distribution of the mass, and in both cases the angular acceleration produces considerable effect, and when we have a change of speed $\delta\omega$ we must not only accelerate the centre of gravity G, but also the whole weight undergoes an angular acceleration, and in (d) the angular acceleration is the only active force.

In the figures (e), (f) and (g) the sense of rotation is marked, and we shall suppose that in each case there is a sudden increase in speed corresponding to a decreased load. In fig. (e) the tangential acceleration *assists* the centrifugal force in producing rapid adjustment, while in (f) these oppose one another resulting in slower adjustment merely due to change of sense of rotation and in (g) rapid adjustment is again realized. In these three cases the angular effect is small.

The distribution of the weights for a Rites governor is shown in Fig. 44, and it will be readily seen that the centrifugal effect

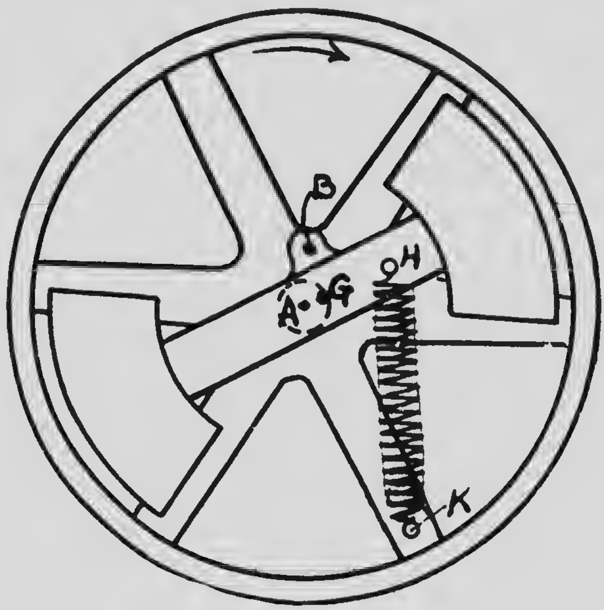

Fig. 44.

is not large, comparatively, the tangential effect is also decreased and the angular acceleration produces a very decided effect. Such governors as these adjust themselves very rapidly and may be made as stable as desired, without undue variation in speed for varying loads and positions.