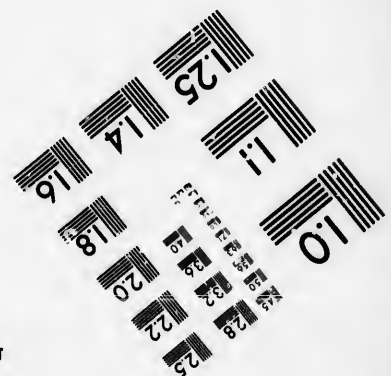
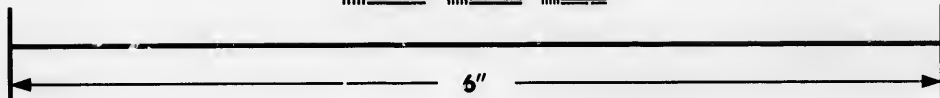
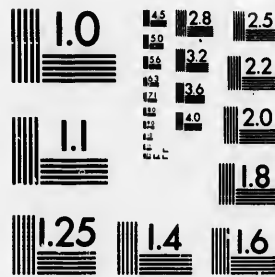


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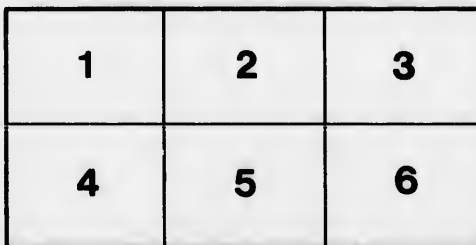
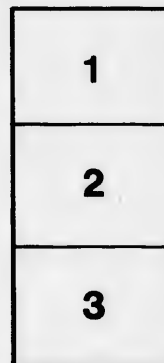
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LOVELL'S SERIES OF SCHOOL BOOKS.

KEY

TO THE

NATIONAL ARITHMETIC;

CONTAINING

Full Solutions to nearly all the Problems,

DESIGNED FOR THE

USE OF TEACHERS AND PRIVATE STUDENTS.

BY JOHN HERBERT SANGSTER, M.A., M.D.,
MATHEMATICAL MASTER AND LECTURER IN CHEMISTRY AND NATURAL
PHILOSOPHY IN THE NORMAL SCHOOL FOR ONTARIO.

THIRD EDITION—CAREFULLY REVISED.

Montreal:

PRINTED AND PUBLISHED BY JOHN LOVELL,
AND FOR SALE AT THE BOOKSTORES.

1889.

Entered, according to the Act of the Provincial Parliament, in
the year one thousand eight hundred and sixty-one, by JOHN
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• Canada.

PREFACE.

It was the original intention of the Author to give, in the Key, merely a series of brief hints upon the Solutions of the more difficult Problems. He was led to modify this plan, and to issue the work in its present form, chiefly from the consideration that as there are in the country many young persons who, from various causes, are unable to avail themselves of the advice and assistance of a teacher, it would be a great boon to these to have access to a book to which they might refer with the certainty of having every doubt removed as to the correctness of their work and methods of solution. He offers the work to his fellow-teachers with the hope that they will accord it the same favorable reception that they have so kindly given to the National Arithmetic.

TORONTO, *May*, 1861.

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one, by JOHN
Province of

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CONTENTS.

	PAGE
Reduction Descending.....	7
Reduction Ascending.....	8
Reduction of Canadian Currency to Dollars and Cents...	10
Multiplication.....	11
Division.....	16
Miscellaneous Exercises, Arith. page 118.....	22
Resolution of Numbers into their prime factors.....	29
Divisors of Numbers.....	29
Number of Divisors.....	30
Common measure of two or more Numbers.....	31
Greatest Common Measure.....	32
Least Common Multiple.....	34
Reduction of Scales of Notation.....	36
Fundamental Rules in different scales.....	41
Duodecimal Multiplication.. ..	42
Miscellaneous Exercises, Arith. page 149.....	43
Reduction of Fractions.....	50
Addition of Fractions.....	60
Subtraction of Fractions.....	65
Multiplication of Fractions.	67
Division of Fractions.....	71
Miscellaneous Exercises in Vulgar Fractions.....	76
Reduction of Decimals.....	79
Addition of Decimals.....	85
Subtraction of Decimals.....	87
Multiplication and Division of Decimals.....	88
Miscellaneous Exercises in Decimals.....	89
Miscellaneous Exercises, Arith. page 198.....	93
Simple Proportion.....	104
Compound Proportion.....	111

	PAGE
Conjoined Proportion.....	114
Miscellaneous Exercises, Arith. page 222.....	116
Practice.....	123
Bills of Parcels.....	129
Miscellaneous Exercises, Arith. page 231.....	131
Percentage.....	137
Commission.....	138
Brokerage.....	139
Insurance.....	141
Custom House Business.....	142
Assessment of Taxes.....	143
Simple Interest.....	144
Compound Interest.....	156
Discount.....	160
Bank Discount.....	163
Equation of Payments.....	164
Partnership.....	165
Profit and Loss.....	171
Barter.....	175
Alligation.....	177
Reduction of Currencies.....	182
Exchange.....	184
Arbitration of Exchange.....	185
Involution.....	186
Square Root.....	187
Cube Root.....	192
Extraction of other Roots.....	201
Logarithms.....	202
Logarithmic Arithmetic.....	210
Arithmetical Progression.....	217
Geometrical Progression.....	222
Single Position.....	228
Double Position.....	231
Compound Interest.....	235
Annuities at Simple Interest.....	237
Annuities at Compound Interest.....	238
Examination Problems.....	241
Arithmetical Recreations.....	286

K

(1)
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2332

9331

6

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119

1

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5729

...	114
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...	137
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...	171
...	175
...	177
...	182
...	184
...	185
...	186
...	187
...	192
...	201
...	202
...	210
...	217
...	222
...	228
...	231
...	235
...	237
...	238
...	241
...	286

KEY TO NATIONAL ARITHMETIC.

EXERCISE 5—Page 50.

(1)	(2)	(3)	(4)	(5)
d	£	£ s.	£ s.	£ s.
23328	348	38 10	58 13	58 13
4	20	20	20	20
93312 f.	6960 s.	770 s.	1173 s.	1173 s.
		12	12	12
		9240 d.	14076 d.	14076 d.
				4
				56304 f.

(6)	(7)	(8)	(9)
£ s. d.	£ s. d.	cwt. qrs. lbs.	cwt. qrs. lbs.
59 13 6½	63 0 9	16 2 16	14 3 16
20	20	4	4
1193 s.	1260 s.	66 qrs.	59 qrs.
12	12	25	25
14322 d.	15129 d.	346	311
4		132	118
57291 f.		1666 lbs.	1491 lbs.

(10)		(11)			(12)	(13)	(14)			
lbs.	oz. dwt. grs.	lbs.	oz. dwt. grs.	yrs.	mile.	yrs.	d.	h.	m.	
3	5 12 16	7	11 15 14	20		1	46	21	8 56	
12		12		365	8	365				
<hr/>		<hr/>			<hr/>		<hr/>			
41	oz.	95		7300	dys.	8	fur.	251		
20		20		24		40		276		
<hr/>		<hr/>			<hr/>		<hr/>			
832	dwt.	1915	dwt.	29200		320	per.	138		
24		24		14600		5½		16811	days.	
<hr/>		<hr/>			<hr/>		<hr/>			
3344		7674		175200	hrs	1600		24		
1664		3830				160		67252		
<hr/>		<hr/>			<hr/>		<hr/>			
19984	grs.	45974	grs.			1760	yds.	33622		
<hr/>		<hr/>			<hr/>		<hr/>			
						3		403472	hrs.	
<hr/>		<hr/>			<hr/>		<hr/>			
						5280	ft.	60		
<hr/>		<hr/>			<hr/>		<hr/>			
						24208370 min.				
(15)		(16)		(17)	(18)	(19)	(20)			
sq.	per.	a.	r. per.	sq. miles.	cub. ft.	pks.	pks.			
74		46	3 12	767	767	767	797			
30½		4		640	1728	2	2			
<hr/>		<hr/>		<hr/>		<hr/>		<hr/>		
2220		187	r.	30680	6136	1534	gals.	1594	gals.	
18½		40		4602	1534	4		4		
<hr/>		<hr/>		<hr/>		<hr/>		<hr/>		
2238½	sq. yds.	7492	per.	490880	sq. a.	767	6136	qts.	6376	qts.
<hr/>		<hr/>			<hr/>		<hr/>			
		30½						2		
<hr/>		<hr/>			<hr/>		<hr/>			
		224760				1325376	cub. in.			
<hr/>		<hr/>			<hr/>		<hr/>			
		1873						12752	pts.	
<hr/>		<hr/>			<hr/>		<hr/>			
		226633	sq. yds.							

EXERCISE 6—Page 51.

(1)	(2)	(3)
f.	grs.	yds.
4)32756	24)3547	5½)397024
<hr/>	<hr/>	<hr/>
12)8189 d.	20)981 dwt. 3 grs.	2 2
<hr/>	<hr/>	<hr/>
20)682s. 5d.	12)49 oz. 1 dwt. 3 grs.	11)794048
<hr/>	<hr/>	<hr/>
£34 2s. 5d.	4 lbs. 1 oz. 1 dwt. 3 g. 8)	40)72186r. 2hf-yds. = 1 yd.
<hr/>		<hr/>
		1804 fur. 26 r. 1 yd.
<hr/>		<hr/>
		225 m. 4 f. 26 r. 1 y.

(14)
yrs. d. h. m.
46 21 8 56
385

251
16

11 days.
24

52
2

72 hrs.

80
70 min.

(20)
pks.
797
2

1594 gals.
4

6376 qts.
2

12752 pts.

ds.=1 yd.
3 r. 1 yd.

26 r. 1 y.

(4)
sec.
60)28635

60)477 m. 15 sec.

7 hrs. 57 m. 15 sec. 16 cwt. 2 q. 16 lbs. 14 cwt. 3 q. 16 lbs.

(5)
lbs.
25)1666

4)66 qrs. 16 lbs.

(6)
lbs.
25)1491

4)59 qrs. 16 lbs.

(7)
grs.
24)115200

20)4800 dwt.

12)240 oz.

20 lbs.

(8)
oz.
16)107520

6720 lbs.

(9)
cub. in.
1728)1674674

969 ft. 242 in.

(10)
Fl. e.
767
3

4)2301 qrs.

575 yds. 1 qr.

(11)
ft.
3)183810

51)61270 yds.
2) 2

11)122540

40)11140 per.

8)278 fur. 20 per.

3)34 m. 6 fur. 20 per.

11 lea. 1 m. 6 fur. 20 per.

(12)
cub. in.
1728)138297

27)80 ft. 57 in.

2 c. yds. 28 c. ft. 57 c. in.

(13)
cub. ft.
128)67893

530 cords 53 c. ft.

(14)
sec.
60)3561829

60)59363 m. 49 sec.

(15)
qts.
4)1597

2)399 gals. 1 qt.

(16)
c. ft.
8)1000

125 cords.

24)989 h. 23 m. 49 s. 4)199 pks. 1 gal. 1 qt.

7)41 d. 5 h. 23 m. 49 s. 49 bush. 3 pecks 1 gal. 1 qt.

5 wks. 6 days 5 hrs. 23 min. 49 sec.

(17)
seconds.
60)10000

(18)
sq. links.
10030)70000

(19)
grs.
20)11621

60)168' 40" 7 sq. ch. 3)576 scr. 1 gr.

2° 46' 40" 8)192 dr. 1 gr.

12)24 oz. 1 gr.

2 lbs. 1 gr.

(20)
sq. ft.
9)28025

30)2891 yds. 6 ft.
4) 4

121)11564 quarter yards.
95 per. 69 quar! yds. 6 ft. =
40)95 per. 17 yds. 8 ft. 36 in.

2 r. 15 sq. p. 17 sq. y. 8 sq. ft. 36 sq. in.

EXERCISE 7—Page 53.

(1) £3 × 400 = 1200 cents. (2) £29 × 400 = \$116·00
7s. × 20 = 140 " 18s. × 20 = 3·60
1½d. = 5 far. × 5 ÷ 12 = 2½ " 3½d. = 14 far. × 5 ÷ 12 = .05½
£ 7s. 1½d. = 1342½ cts. £29 18s. 3½d. = \$119.65½

(3) 11½d. = 45 far. × 5 ÷ 12 = 18½ cts.

(4) £69 × 400 = \$276·00 (5) 18s. × 20 = \$3·60
15s. × 20 = 3·00 8½d. = 34 far. × 5 ÷ 12 = .14½
6d. = 24 far. × 5 ÷ 12 = .10
£69 15s. 6d. = \$279·10 18s. 8½d. = \$3·74½

cr. 1 gr.

lr. 1 gr.

z. 1 gr.

bs. 1 gr.

(6) £17 × 400 = \$68·00
 16s. × 20 = 3·20
 $\frac{1}{2}d. = 23 \text{ far.} \times 5 \div 12 = .09 \frac{7}{12}$

(7) £87 × 400 = \$348·00
 15s. × 20 = \$3·00
 $11 \frac{1}{2}d. = 47 \text{ far.} \times 5 \div 12 = .19 \frac{7}{12}$

(8) 15s. 11½d. = \$3·19½

(9) £16 × 400 = \$64·00
 6s. × 20 = 1·20
 2d. = 8 far. × 5 ÷ 12 = .03½

(10) £2 × 400 = \$8·00
 9s. × 20 = 1·80
 11d. = 44 far. × 5 ÷ 12 = .18½

£16 Cs. 2d. = \$65·23½ £2 9s. 11d. = \$9·98½

EXERCISE 13—Page 90.

36 sq. in.

= \$116·00

= 3·60

= .05½

= \$119·65½

= \$3·60

= .14½

= \$3·74½

(1)	(2)	(3)	(4)
6 = 12 ÷ 2	121 = 11 × 11	144 = 12 × 12	648 = 12 × 9 × 6
$\frac{\$169 \cdot 78}{12}$	$\frac{796342 \cdot 3}{11}$	$\frac{\$33460}{12}$	$\frac{735}{12}$
<u>2037·33</u>	<u>8759765·3</u>	<u>401520</u>	<u>8820</u>
3	11	12	9
<u>\$6112·08</u>	<u>96357418·3</u>	<u>\$4818240</u>	<u>79380</u>
			6
			<u>476280</u>

(5)	(6)	(7)	(8)
18 = 6 × 3	22 = 11 × 2	810 = 10 × 9 × 9	54 = 9 × 6
£ s. d.	£ s. d.	£ s. d.	cwt. qrs. lbs. oz.
3 7 6	5 14 6½	3 4 7	11 3 14 7
<u>0 7 6</u>	<u>0 11</u>	<u>0 10</u>	<u>0 9</u>
<u>20 5 0</u>	<u>62 19 11½</u>	<u>32 5 10</u>	<u>107 0 4 15</u>
3	2	9	16
<u>60 15 0</u>	<u>125 19 11</u>	<u>290 12 6</u>	<u>642 1 4 10</u>
			<u>2615 12 16</u>

(9)
 $49 = 7 \times 7$
 bush. pks. gal. qt. pt.
 26 3 1 1 1
 7

 188 1 1 2 1
 7

 1319 0 1 1 1

(10)
 $63 = 9 \times 7$
 yds. qrs. na. in.
 2 2 2 2
 9

 24 0 2 0
 7

 168 3 2 0

(11)
 $288 = 12 \times 12 \times 2$
 dys. hrs. min. sec.
 5 17 33 11
 12

 68 18 33 12
 12

 825 7 38 24
 2

 1650 15 16 48

EXERCISE 14—Page 92.

(1)
 $83 = 3 \times 10 \times 8$
 £ s. d. £ s. d.
 12 2 4 $\times 3 =$ 36 7 0
 10

 121 3 4 $\times 8 =$ 969 6 8
 1005 13 8

(2)
 $999 = 10 \times 10 \times 10 - 1$
 £ s. d.
 963 0 0½
 10

 9630 0 7½
 10

 96300 6 3
 10

 963003 2 6
 963 0 0½

 962040 2 5½

(3)
 $8178 = 8 + 10 \times 7 + 10 \times 10 \times 1 + 10 \times 10 \times 10 \times 3$
 £ s. d. £ s. d.
 3 6 5½ $\times 8 =$ 28 11 6
 10

 33 4 4½ $\times 7 =$ 232 10 7½
 10

 332 3 9 $\times 1 =$ 332 3 9
 10

 3321 17 6 $\times 3 =$ 9965 12 6
 10556 18 4½

(4)
 $678 = 8 + 10 \times 7 + 10 \times 10 \times 6$
 bush. pk. gal. bush. pk. gal.
 16 3 1 $\times 8 =$ 135 0 0
 10

 168 3 0 $\times 7 =$ 1181 1 0
 10

 1687 2 0 $\times 6 =$ 10125 0 0
 11441 1 0

393
 353
 393
 (4)
 3.25
 .02
 975
 5503
 7478

[NAT. ARITH.

(11)
 $3 = 12 \times 12 \times 2$
 lys. hrs. min. sec.
 5 17 33 11
 12

38 18 33 12
 12

5 7 38 24
 2

0 15 16 48

(2)
 $0 \times 10 \times 10 - 1$
 £ s. d.
 963 0 0½
 10

9630 0 7½
 10

6300 6 3
 10

3003 2 6
 963 0 0½

2040 2 5½

(4)
 $< 7 + 10 \times 10 \times 6$
 bush. pk. gal.
 = 135 0 0

= 1181 1 0

= 10125 0 0

11441 1 0

(5)
 $247 = 7 + 10 \times 4 + 10 \times 10 \times 2.$
 m. fur. rds. yds. m. fur. rds. yds.
 23 6 33 4 × 7 = 166 7 36 0½
 10

 238 4 17 1½ × 4 = 954 1 29 0½
 10

 2385 4 12 4 × 2 = 4771 0 25 2½
 5892 2 10 3½

(6)
 $721 = 1 + 10 \times 2 + 10 \times 10 \times 7$
 S. deg. min. sec. S. deg. min. sec.
 3 16 30 45 × 1 = 3 16 30 45
 10

 35 15 7 30 × 2 = 71 0 15 0
 10

 355 1 15 0 × 7 = 2485 8 45 0
 2559 25 30 45

EXERCISE 15—Page 93.

(6)	(7)	(8)	(9)
7071	15607	39948123	2778588
556	3094	6007	9867
42426	62428	279636861	19450116
35355	140463	23968873800	16671528
35355	468210	239968374861	22228704
3931476	48288058	25007292	27416327796

EXERCISE 16—Page 95.

(4)	(5)	(6)	(7)	(8)
3.2517	64.001	482000	3782.4	87.96
.023	340	.37	.00917	220
97551	2560040	3374000	264768	175920
65034	192003	1446000	37824	17592
747891	21760.340	178340.00	340416	19351.20
			34.684603	

EXERCISE 17—Page 100.

(1)	(2)	(3)	(4)
$216=6 \times 6 \times 6$	\$61135.37	255226	$176=11 \times 8 \times 2$
\$83469	229	143	203736
6	55021833	765678	11
500814	12227074	1020904	2241996
6	12227074	255226	8
3004884	\$13999999.73	36497318	17928768
6			2
\$18029304			35857536

(5)	(6)	(7)	(8)
116700	3721	$297=11 \times 9 \times 3$	$35=7 \times 5$
235	73	32000	9344000
583500	11163	11	7
350100	26047	352000	65408000
233400	271633	9	5
27424500		3168000	327040000
		3	
		9504000	

(9)	(10)
$749=9+10 \times 4+10 \times 10 \times 7$	$999998=1000000-2$
lbs. oz. drs. scr. gr.	lbs. oz. drs. scrs. grs.
123 4 7 2 $17 \times 9=$	1110 8 7 1 13
10	1698732
1234 1 7 1 $10 \times 4=$	1000000
10	1698732000000
12341 7 3 0 $0 \times 7=$	3397464
	1698728602536
	92438 8 2 1 13

(4)
 176 = 11 × 8 × 2
 203736
 11
 2241996
 8
 17928768
 2
 35857536

(11)
 640 = 10 × 8 × 8
 bush. pk. gal. qt. pt.
 123 1 1 1 1
 10
 1234 0 1 3 0
 8
 9873 3 0 0 0
 8
 78990 0 0 0 0

(12)
 89
 .73
 267
 623
 \$64.97

(8)
 3 35 = 7 × 5
 9344000
 7
 65408000
 5
 327040000

(13)
 1143 = 3 + 10 × 4 + 10 × 10 × 1 + 10 × 10 × 10 × 1
 yds. qrs. na. in. yds. qrs. na. in.
 7 3 2 1 × 3 = 23 2 3 0½
 10
 79 0 0 1 × 4 = 316 0 1 ¼
 10
 790 1 0 1 × 1 = 790 1 0 1
 10
 7902 3 0 1 × 1 = 7902 3 0 1
 10
 9032 3 2 0

(14)
 1634-5789
 635000
 81728945000
 49037367
 98074734
 1037957601.5

(10)
 98 = 1000000 - 2
 1698732
 1000000
 1698732000000
 3397464
 1698728602536

\$968.49
 3.4
 387396
 290547
 \$3292.866

(15)
 \$12183.6042
 3292.866
 968.49
 \$16444.9602

\$3292.866
 3.7
 23050062
 9878598
 \$12183.6042

EXERCISE 18—Page 110.

(9)

6423)798965(124111
6423
 15666
12846
 28205
25692
 2513

10

£ s. d.
 12)176 14 6
14 14 6

(11)

741)56789(7611
5187
 4919
4446
 473

(12)

7894)6785158(859111
63152
 46995
39470
 75258
71046
 4212

(13)

£ s. d. £ s. d.
 317)4728 16 2(14 18 451
317
 1558
1268
 290
20
 5816
317
 2646
2536
 110
12
 1322
1268
 54

(15)

6)970763
161793 8333+

(16)

9)71234
7914

(14)

429)9789664(\$228.1911
858
 1209
858
 3516
3432
 84.6
42.9
 41.74
38.61
 3.13

(17)

47600)977076(2047608
95200
 25076

5)3
 5)7
 1
 3x5

10
 £ s. d.
 6 14 7 6

 14 14 7 6½

(12)
 58(8594112

 19118

5
 0

 58
 6

 2

 7076(2044878
 200

 076

(18)
 lbs. oz. drs. scr. grs. lbs. oz. drs. scr. grs.
 498)7289 6 4 2 13(14 7 5 0 12487

 498

 2309

 1992

(19)
 £ s. d. s. d.
 487)157 16 7(6 6½..48

 20

3810 3156
 3486 2922

 324 234
 8 12

 2596 2815
 2490 2435

 105 380
 3 4

 320 1520
 20 1461

 6413 59
 5976

 437

(20) 9712)7867674(810 9712

 77696

 9807

 9712

 984

(21)
 m. fur. rds. m. fur. rds.
 37)422 3 38(11 3 14

 407

 15
 8

 123
 111

 12
 40

 518
 37

 148
 148

EXERCISE 19—Page 112.

(1) 25=5×5	(2) 42=7×6	(3) 96=12×8	(4) 24=12×2
5)3766	7)26406	12)25431	£ s. d. 12)24 17 6
5)753... 1	6)3772... 2	8)2119... 3	2)2 1 5½
150... 3	628... 4	264... 7	1 0 8½
3×5+1=16	4×7+2=30	7×12+3=87	
150½	628½	264½	

(7)
 $35 = 7 \times 5$
 $7)6789436$

 $5)969919...3$

 $193983...4$
 $4 \times 7 + 3 = 31$
 $193983\frac{1}{2}$
(9)
 $1 = 9 \times 9$
dwt. grs.
 $3 \ 11 \ 9$

 $10 \ 1 \ 6...3$

 $2 \ 9 \ 0...6$
 $3 = 57$
dwt. $0\frac{1}{2}$ grs.

	£	s.	d.	(3)	£	s.	d.	dwt	grs.	(4)	lbs.	oz.	dwt.	grs.
	57	0	7 $\frac{1}{2}$)	171	1	10 $\frac{1}{2}$	5	9)	9	9	3	12
	20				20			24			12			
	1140				3421			129			117			
	12				12						20			
	13687				41062						2343			
	4				4						24			
	54750)			164250	(3)					9384			
					164250						4686			
								129			50244	(436		
											516			
											484			
											387			
											774			
											774			

EXERCISE 21—Page 115.

(2)
m. fur. rds.
 $) 1027 \ 1 \ 6$

 8

 8217
 40

 328686 (58)
 28335

 45336
 45336

	a.	r.	per.	a.	r.	per.	(5)	(6)
	91	0	6)	2366	3	36	$47 \cdot 655 \div 1 \cdot 5 =$
	4				4			$45)476 \cdot 55(10 \cdot 59$
	364				9467			45
	40				40			26·5
	14566)			378716	(26		22·5
					29132			4·05
					87396			4·05
					87396			

	(7)	(8)
	$756 \cdot 98 \div 76 \cdot 73612 =$	$47 \cdot 5782975 \div 26 \cdot 175 =$
	$7673612)75698000$	$26175)47578 \cdot 2975(1 \cdot 8177$
	69062508	26175
	$6635492 \cdot 0$	$21403 \cdot 2$
	$6138889 \cdot 6$	$20940 \cdot 0$
	$496602 \cdot 40$	$463 \cdot 29$
	$460416 \cdot 72$	$261 \cdot 75$
	$36185 \cdot 620$	$201 \cdot 547$
	$30694 \cdot 448$	$183 \cdot 225$
	$5491 \cdot 232$	3225
		$18 \cdot 3236$

$$\begin{array}{r}
 (9) \\
 1 \div 76345 = \\
 76345 \overline{) 100000} \cdot 0 \cdot 1309 + \\
 \underline{76345} \\
 236550 \\
 \underline{229035} \\
 751500 \\
 687105
 \end{array}$$

$$\begin{array}{r}
 (10) \\
 75347 \div 0.3829 = \\
 3829 \overline{) 753470} (196 \cdot 7798 + \\
 \underline{3829} \\
 37057 \\
 \underline{34461} \\
 25960 \\
 \underline{22974} \\
 29860 \\
 \underline{26803} \\
 30570 \\
 \underline{26803} \\
 37670 \\
 \underline{34461} \\
 32090 \\
 \underline{30632} \\
 1458
 \end{array}$$

$$\begin{array}{r}
 (11) \\
 \cdot 0002 \div 0.000000008 = \\
 8) 200000 \\
 \underline{160000} \\
 40000 \\
 \underline{32000} \\
 80000 \\
 \underline{80000} \\
 25000
 \end{array}$$

EXERCISE 22—Page 116.

(1)		(2)
95) \$3300000	(\$34736.8421	126) \$3860000
285		378
<u>450</u>	28800) 95270400	800
380	(3308	756
<u>700</u>	86400	440
665	88704	378
<u>350</u>	86400	620
285	230400	504
<u>650</u>	230400	116.0
570	days.	113.4
	365) 3308	
	4 4	
	yr. days.	
80.0	1461) 13232	2.60
76.0	(9 20)	2.52
<u>4.00</u>	13149	800
3.80	4) 83	756
	20	044
.200		
.190		
.100		
.095		
.005		

(4)
35781628) \$1145012096
(\$32
107344884
<u>71563256</u>
71563256

+
 (11)
 $2 \div 000000008 =$
 8)200000
25000

 (2)
 $00(\$30634 \cdot 9206$

 0
 6
 40
 78
 620
 504
 116.0
 113.4
 2.60
 2.52
 .800
 .756
 .044
 (4)
 $5012096(\$32$
 344884
 71563258
 71563258

	(5)	(6)	(7)
	27475271)	9)972	108)972(\$9
	\$3764112127(\$137	<u>972</u>	972
	27475271	\$108	
	<u>101658502</u>		(10)
	82425813	(9)	1728)1000(.578 oz.
	<u>192326897</u>	792)340480(429 $\frac{2}{3}$ oz.	864.0
	192326897	3168	<u>136.00</u>
		2368	120.96
	(8)	1584	
294)	\$8526(\$29	(12)	19)4750(250 lbs.
	588	38	15.040
	m. fur.	7840	13.824
	2646 33 2	7128	
	2646 8	712	95
		712 = $\frac{2}{3}$	95
	266		
	40		
	<u>10640</u>	bush. pk. gal. qt. pt.	bush. pk. gal. qt. pt.
	54	297)729 1 1 1 1	(2 1 1 2 $\frac{1}{2}$)
	<u>53200</u>	594	
	5320	135	
		4	
	1155)58520(50 $\frac{7}{10}$	541	
	5775	297	
	<u>770</u>	244	
	50 $\frac{7}{10}$ = 50 $\frac{3}{5}$	2	
	(13)	489	
	978.634.96.34762 =	297	
	9634762)97863400(10.157		
	9634762	192	
		4	
	1515780.0	769	
	963476.2	594	
	<u>552303.80</u>	175	
	481738.10	2	
	70565.700	351	
	67443.334	297	
	<u>3122.366</u>	54	
		297 = $\frac{2}{11}$	

	(15)						
lbs. oz. dr.	owt. qr. lbs. oz. dr.						
9 7 8)	179 3 4 16 0						
16	4			(16)			
—	—			m. fur. rds.	m.		
151	719			93 4 7	25000		
16	25			8	8		
—	—			—	—		
914	3599			748	200000		
151	1438			40	40		
—	—			—	—		
2424	17979			29927	29927)	8000000	dys. hrs.
	16					(267	718888
	—					59854	
	107890					—	
	17979					201460	
	—					179562	
	287680					—	
	16					218980	
	—					209489	
	1726080					—	
	287680					9491	
	—					24	
	2424)4602880	(1898888				—	
	2424					37964	
	—					18982	
	21788					—	
	19392					227784	
	—					209489	
	23968					—	
	21816					18295	
	—					—	
	21520					—	
	19392					—	
	—					—	
	2128			1111=111.		—	

EXERCISE 23—Page 118.

(3)

DCCIX, M̄V̄CC̄LXXVI, M̄X̄CC̄M̄XCIX, L̄XXX̄VM̄IV,
 MMM̄CC̄M̄XL̄VM̄MD̄XC̄VI.

(4)

$$72 = 8 \times 9$$

lbs. oz.

$$\begin{array}{r} 749 \text{ 10} \\ 8 \\ \hline 5997 \text{ 0} \\ 9 \\ \hline 53973 \text{ 0} \end{array}$$

(5)

$$17 = 7 + 10 \times 1$$

s. d. £ s. d.

$$\begin{array}{r} 4 \text{ 7} \frac{1}{2} \times 7 = 1 \text{ 12 } 6 \frac{1}{2} \\ \phantom{4 \text{ 7} \frac{1}{2}} 10 \\ \hline \text{£} 2.6 \text{ 5} \frac{1}{2} + 1 = 3 \text{ 6 } 5 \frac{1}{2} \\ \hline \phantom{\text{£}} 3 \text{ 18 } 11 \frac{1}{2} \end{array}$$

-dys. hrs.
0(267 718884

(6)

$$3Q)285000000Q$$

days. hrs.

$$\begin{array}{r} 24)95000000(3958333 \text{ 8} \\ 72 \\ \hline 230 \text{ 365} \frac{1}{4} 3958333(10837 \\ 216 4 4 \\ \hline 140 \text{ 1461} 1583332 \\ 120 1461 \\ \hline 200 12233 \\ 192 11688 \\ \hline 80 5453 \\ 72 4383 \\ \hline 80 10702 \\ 72 10227 \\ \hline 80 4)475 \text{ quarter days.} \\ 72 \text{ days. hrs.} \\ 118 \frac{1}{2} = 118 \text{ 18} \\ 8 \text{ rem. Add } 8 \\ \hline 119 \text{ 2} \end{array}$$

10837 yrs. 119 days, 2 hrs.

(7)

$$\begin{array}{r} \text{£} 729 \times 400 = \$2916 \cdot 00 \\ 17s. \times 20 = 3 \cdot 40 \\ 6 \frac{1}{2} d. = 25 \text{ far.} \times 5 \div 12 = 10 \frac{1}{4} \\ \hline \$2919 \cdot 50 \frac{1}{4} \end{array}$$

(8)

$$\begin{array}{r} \$10000 \\ 9876 \cdot 23 \\ \hline \$123 \cdot 77 \end{array}$$

XXVMIV,

(15)

	tons.
	324
	20
cwt. qr. lbs. —	
13 2 14 6480	
4	4
—	—
54	25920
25	25
—	—
284	129600
108	51840
—	—
1364	648000
(14)	475 100
78-96	475 100
.00042	475 100
—	—
15792	6920
31584	6820
—	—
.0331662	100

(16)

	\$136
\$136 × 4 = 544 - 95 = 449	
	1902
—	—
	2487
—	—
\$9297 - \$2847 = \$6750	

(17)

yds. qrs. na.	yds. qrs. na.
3 1 2) 39 2 3	
4	4
—	—
13	158
4	4
—	—
54) 635 (11 1/4	
	54
—	—
	95
	54
—	—
	41

(18)

(19)

a.	a.	a. r. per.
25	732	96 3 17
197	674	4
156	—	—
97	58	387
199	.	40
—	—	—
574	15497	\$7764.0 (\$0.501
		7748.5
		—
		15.500
		15.497
		—
		3

(21)

lbs. oz. dwt. grs.
12) 36 8 14 16
—
3 0 14 13 1/2

(20)

\$	
20	\$312
75	275
97	—
83	\$ 37
—	—
275	

(22)

a. r. per.
6 3 12
7 2 0
9 0 13
5 2 36
—
29 21

36 in.
119 in
11 in.
5 ft. 11 in.
yds. 5 ft. 11 in.

mln.
33

40

20
33

min.

26

KEY.

[NAT. ARITH.]

(23)

(24)

(25)

5
 7
 9
 —
 21)294(14
 21
 —
 84
 84

lbs. oz. dwt. grs.
 5 9 8 0
 3 2 16 16
 4 6 17 0
 1 8 19 22
 —————
 15 4 1 14

£972 × 400 = \$3888-00
 11s. × 20 = 2-20
 11½d. = 45 far. × 5 ÷ 12 = .18½
 —————
 \$3890-38½

(26)

(27)

(28)

lbs. oz. drs. scr. grs.
 179 3 3 1 14
 12
 ———
 2151 oz.
 8
 ———
 17211 drs.
 3
 ———
 51634 scr.
 20
 ———
 1032694 grs.

56
 25
 ———
 280
 112
 ———
 1400
 2
 ———
 2800 sq. ft. in roof.
 6
 ———
 16800

cwt. qr. lbs.
 6 2 11
 5 3 16
 8 0 7
 3 1 17
 ————— lbs.
 24 0 1 = 2401
 .15
 —————
 12005
 2401
 —————
 \$360-15

(29)

(30)

29
 57
 ———
 203
 145
 ———
 1653
 .15
 ———
 8265
 1653
 ———
 \$247.95

\$
 139468
 98579
 ———
 \$238047

370129
 238047
 ———
 \$132082

00 = \$3888.00
 20 = 2.20
 12 = .18½

 \$3890.38½

lbs.
 11
 16
 7
 17
 lbs.
 1 = 2401
 .15

 12005
 2401

 \$360.15

(31)
 £ s. d. £ s. d.
 9 19 11½ 1694 16 0½
 20 20
 — ½ = ½ — ½ = ½
 199 33896
 12 12

 2399 406752
 84 84

 9659 1627030
 19192 3254016

 201579) 34167190 (169.49
 201579

 1400929
 1209474

 1914550
 1814211

 100339.0
 80631.6

 19707.40
 18142.11

 1565.29

(32)
 £19 × 400 = \$76.00
 19s. × 20 = 3.80
 11½d = 47far × 5 ÷ 12 = .19½

 \$79.99½

(33)
 cwt. qr. lbs. cwt. qr. lbs.
 3 2 11 12 0 0
 4 1 15 8 0 1
 lbs.
 8 0 1 3 3 24 = 399
 .15

 1995
 399

 \$59.85

(34)
 cwt. qr. lbs.
 2 0 17
 3 2 15
 2 1 20
 5 3 17
 lbs.
 14 0 19 = 1419
 .37½

 9933
 4257
 709½

 \$532.12½

(36)
 43.2 ÷ 76.8437 =
 768437)432000.0 (0.562
 384218.5

 47781.50
 46108.22

 1675.280
 1536.874

 138.406

(37)

$123 \cdot 4 \div 000000066 =$
 $123400000000 \div 66$
6)123400000000

11)205666666666
1869696969
69

(38)

$\$63 \cdot 29$
17
 $\$2789 \cdot 27$
1075 \cdot 93

 44303
 6329

 $\$1075 \cdot 93$

(39)

$\pounds 29 \times 400 = \$116 \cdot 00$
 $6s \times 20 = 1 \cdot 20$
 $11 \frac{1}{2}d. = 47 \text{ far.} \times 5 \div 12 = 19 \frac{1}{2}$

 $\$278 \cdot 43$
 $417 \cdot 16$
 $11 \cdot 27$
2110 \cdot 40
 $723 \cdot 15$
117 \cdot 39 \frac{1}{2}

 $173) 3657 \cdot 80 \frac{1}{2}$
12
12

(40)

$2076) 491544(236 \frac{1}{2}$
4152

 7634
6228

 14064
12456

 1608

 $\frac{1}{2} \frac{1}{2} = 1 \frac{1}{4}$

 $2076) 43893 \cdot 67 (\$21 \cdot 1433$
4152

 2373
2076

 $297 \cdot 6$
207 \cdot 6

 $90 \cdot 07$
83 \cdot 04

 $7 \cdot 030$
6 \cdot 228

 $\cdot 8020$
 $\cdot 6228$

 $\cdot 1792$

Exercise 24—Page 127.

(38)
 \$2789.27
 1075.93

 \$1713.34

(1)	(2)	(3)	(4)
2)11368	2)2934	3)1011	2)1000
-----	-----	-----	-----
2)5684	3)1467	337	2)500
-----	-----		-----
2)2842	3)489	3 × 337	2)250
-----	-----		-----
7)1421	163		5)125
-----			-----
7)203	2 × 3 ² × 163		5)25
-----			-----
29			5
2 ³ × 7 ² × 29			2 ³ × 5 ³
(5)	(6)	(7)	(8)
2)1024	2)32320	7)707	2)1118
-----	-----	-----	-----
2)512	2)16160	101	13)559
-----	-----		-----
2)256	2)8080	7 × 101	43
-----	-----		
2)128	2)4040		2 × 13 × 43
-----	-----		
2)64	2)2020		
-----	-----		
2)32	2)1010		
-----	-----		
2)16	5)505		
-----	-----		
2)8	101		

2)4	2 ⁶ × 5 × 101		

2			
2 ¹⁰			

78.43
 17.16
 11.27
 10.40
 3.15
 17.39 ¹/₂
 7.80 ¹/₂
 12
 3.67 (\$21.1433
 .6
 .6
 .07
 .04
 .030
 .228
 .8020
 .6228
 .1792

EXERCISE 25—Page 128.

(1)

100 = 2² × 5²

1..2..4
 1..5..25
 1..2..4..5..10..20..25..50..100

(2)

$$810 = 3^4 \times 2 \times 5.$$

1..3..9..27..81

1..2

 1..3..9..27..81..2..6..18..54..162

1..5

 1..3..9..27..81..2..6..18..54..162..5..15..45..135..405..

10..30..90..270..810 =

 1..2..3..5..6..9..10..15..18..27..30..45..54..81..90..135..

162..270..405..810.

(3)

$$920 = 2^3 \times 5 \times 23.$$

1..2..4..8

1..5

 1..2..4..8..5..10..20..40

1..23

 1..2..4..8..5..10..20..40..23..46..92..184..115..230..460..920 =

 1..2..4..5..8..10..20..23..40..46..92..115..184..230..460..920.

(4)

$$25000 = 5^5 \times 2^3$$

1..5..25..125..625..3125

1..2..4..8

 1..5..25..125..625..3125..2..10..50..250..1250..6250..4..20..100..

500..2500..12500..8..40..200..1000..5000..25000 =

 1..2..4..5..8..10..20..25..40..50..100..125..200..250..500..625..

 1000..1250..2500..3125..5000..6250..12500..25000.

EXERCISE 26—Page 128.

(1)

$$88200 = 2^3 \times 3^2 \times 5^2 \times 7^2$$

$$3+1=4$$

$$2+1=3$$

$$2+1=3$$

$$2+1=3$$

$$4 \times 3 \times 3 \times 8 = 108$$

(2)

$$3500 = 2^3 \times 5^3 \times 7$$

$$2+1=3$$

$$3+1=4$$

$$1+1=2$$

$$3 \times 4 \times 2 = 24$$

(3)

$$6336 = 2^6 \times 3^3 \times 11$$

$$6 + 1 = 7$$

$$2 + 1 = 3$$

$$1 + 1 = 2$$

$$7 \times 3 \times 2 = 42$$

(4)

$$824 = 2^3 \times 103$$

$$3 + 1 = 4$$

$$1 + 1 = 2$$

$$4 \times 2 = 8$$

.45..135..405..

4..81..90..135..

(5)

$$49000 = 2^3 \times 5^3 \times 7^2$$

$$3 + 1 = 4$$

$$3 + 1 = 4$$

$$2 + 1 = 3$$

$$4 \times 4 \times 3 = 48$$

(6)

$$81000 = 2^3 \times 3^4 \times 5^3$$

$$3 + 1 = 4$$

$$4 + 1 = 5$$

$$3 + 1 = 4$$

$$4 \times 5 \times 4 = 80$$

.230..460..920=
..230..460..920.

(7)

$$75600 = 2^4 \times 3^3 \times 5^2 \times 7$$

$$4 + 1 = 5$$

$$3 + 1 = 4$$

$$2 + 1 = 3$$

$$1 + 1 = 2$$

$$5 \times 4 \times 3 \times 2 = 120$$

(8)

$$25600 = 2^{10} \times 5^2$$

$$10 + 1 = 11$$

$$2 + 1 = 3$$

$$11 \times 3 = 33$$

50..4..20..100..
0..25000 =
.250..500..625..
25000.

EXERCISE 27—Page 129.

(1)

$$21 = 7 \times 3$$

$$18 = 2 \times 3 \times 3$$

$$27 = 3 \times 3 \times 3$$

$$36 = 4 \times 3 \times 3$$

(2)

$$21 = 3 \times 7$$

$$77 = 11 \times 7$$

$$42 = 2 \times 3 \times 7$$

$$35 = 5 \times 7$$

(3)

$$26 = 2 \times 13$$

$$52 = 2 \times 2 \times 13$$

$$91 = 7 \times 13$$

$$143 = 11 \times 13$$

3 is common to all. 7 is common to all. 13 is common to all.

$\times 5^3 \times 7$
 $= 3$
 $= 4$
 $= 2$
 $2 = 24$

(4)

$$82 = 41 \times 2$$

$$118 = 59 \times 2$$

$$146 = 73 \times 2$$

2 is common to all.

EXERCISE 28—Page 130.

(1)

$$\begin{array}{r} 296)407(1 \\ \underline{296} \end{array}$$

$$\begin{array}{r} 111)296(2 \\ \underline{222} \end{array}$$

$$\begin{array}{r} 74)111(1 \\ \underline{74} \end{array}$$

$$\begin{array}{r} 37)74(2 \\ \underline{74} \end{array}$$

G. C. M. = 37.

(2)

$$\begin{array}{r} 308)506(1 \\ \underline{308} \end{array}$$

$$\begin{array}{r} 198)308(1 \\ \underline{198} \end{array}$$

$$\begin{array}{r} 110)198(1 \\ \underline{110} \end{array}$$

$$\begin{array}{r} 88)110(1 \\ \underline{88} \end{array}$$

G. C. M. = 22.

(3)

$$\begin{array}{r} 74)84(1 \\ \underline{74} \end{array}$$

$$\begin{array}{r} 10)74(7 \\ \underline{70} \end{array}$$

$$\begin{array}{r} 4)10(2 \\ \underline{8} \end{array}$$

$$\begin{array}{r} 2)4 \\ \underline{2} \end{array}$$

G. C. M. = 2.

(4)

$$\begin{array}{r} 1825)2555(1 \\ \underline{1825} \end{array}$$

$$\begin{array}{r} 730)1825(2 \\ \underline{1460} \end{array}$$

$$\begin{array}{r} 365)730(2 \\ \underline{730} \end{array}$$

G. C. M. = 365.

(5)

$$\begin{array}{r} 556)672(1 \\ \underline{556} \end{array}$$

$$\begin{array}{r} 116)556(4 \\ \underline{464} \end{array}$$

$$\begin{array}{r} 92)116(1 \\ \underline{92} \end{array}$$

$$\begin{array}{r} 24)92(3 \\ \underline{72} \end{array}$$

$$\begin{array}{r} 20)24(1 \\ \underline{20} \end{array}$$

$$\begin{array}{r} 4)20(5 \\ \underline{20} \end{array}$$

G. C. M. = 4.

EXERCISE 29—Page 131.

$$\begin{array}{r}
 (3) \\
 74)84(1 \\
 \underline{74} \\
 10)74(7 \\
 \underline{70} \\
 4)10(2 \\
 \underline{8} \\
 2)4 \\
 \underline{2}
 \end{array}$$

$$\begin{array}{r}
 (1) \\
 110)140(1 \\
 \underline{110} \\
 30)110(3 \\
 \underline{90} \\
 20)30(1 \\
 \underline{20} \\
 10)680 \quad 10)20 \\
 \underline{68} \quad \underline{2}
 \end{array}$$

$$\begin{array}{r}
 (2) \\
 1326)3094(2 \\
 \underline{2652} \\
 442)1326(3 \\
 \underline{1326}
 \end{array}$$

Also 4420 is divisible by 442 ; therefore it is their G. C. M.

Therefore 10 is their G. C. M.

$$\begin{array}{r}
 (1) \\
 88(4 \text{ G. C. M.} = 2. \\
 \underline{88} \\
 M. = 22.
 \end{array}$$

$$\begin{array}{r}
 (3) \\
 468)922(1 \\
 \underline{468} \\
 454)468(1 \\
 \underline{454}
 \end{array}$$

$$\begin{array}{r}
 (4) \\
 204)1190(5 \\
 \underline{1020}
 \end{array}$$

$$\begin{array}{r}
 34)1445(42 \\
 \underline{136} \\
 85 \\
 \underline{68}
 \end{array}$$

$$\begin{array}{r}
 170)204(1 \\
 \underline{170}
 \end{array}$$

$$\begin{array}{r}
 34)170(5 \\
 \underline{170}
 \end{array}$$

$$\begin{array}{r}
 17)34(2 \\
 \underline{34}
 \end{array}$$

$$\begin{array}{r}
 14)454(32 \\
 \underline{42} \\
 34 \\
 \underline{28}
 \end{array}$$

$$\begin{array}{r}
 17)2006(118 \\
 \underline{17}
 \end{array}$$

$$\begin{array}{r}
 30 \\
 \underline{17} \\
 136 \\
 \underline{136}
 \end{array}$$

G. C. M. = 17.

375 is not divisible by 2, and therefore their G. C. M. is 1.

$$\begin{array}{r}
 2(1 \\
 6 \\
 3)556(4 \\
 \underline{464} \\
 92)116(1 \\
 \underline{92} \\
 24)92(3 \\
 \underline{72} \\
 20)24(1 \\
 \underline{20} \\
 4)20(5 \\
 \underline{20} \\
 \text{G. C. M.} = 4.
 \end{array}$$

EXERCISE 30—Page 132.

(2)

$$\begin{array}{l}
 56 = 2^3 \times 7 \\
 84 = 2^2 \times 3 \times 7 \\
 140 = 2^2 \times 5 \times 7 \\
 168 = 2^3 \times 3 \times 7
 \end{array}$$

The greatest factors which are common are 2^2 and 7 ; therefore the G. C. M. = $2^2 \times 7 = 28$.

0

(3)

$$241920 = 2^5 \times 3^3 \times 5 \times 7$$

$$380160 = 2^5 \times 3^3 \times 5 \times 11$$

$$69120 = 2^9 \times 3^3 \times 5$$

$$103680 = 2^8 \times 3^4 \times 5$$

The greatest factors which are common are 2^3 , 3^3 and 5 ;
therefore the G. C. M. $= 2^3 \times 3^3 \times 5 = 34560$.

(4)

$$10800 = 2^4 \times 3^3 \times 5^2$$

$$28040 = 2^3 \times 5 \times 701$$

$$2160 = 2^4 \times 3^3 \times 5$$

The greatest factors which are common are 2^3 and 5 ;
therefore the G. C. M. $= 2^3 \times 5 = 40$.

EXERCISE 31—Page 133.

(2)

$$6 = 2 \times 3$$

$$7 = 7$$

$$42 = 2 \times 3 \times 7$$

$$9 = 3^2$$

$$10 = 2 \times 5$$

$$630 = 2 \times 3^2 \times 5 \times 7$$

$$2 \times 3^2 \times 5 \times 7 = 630.$$

(3)

$$1 = 1$$

$$2 = 2$$

$$3 = 3$$

$$4 = 2^2$$

$$5 = 5$$

$$6 = 2 \times 3$$

$$7 = 7$$

$$8 = 2^3$$

$$9 = 3^2$$

$$3^2 \times 2^3 \times 5 \times 7 = 2520.$$

(4)

$$6 = 2 \times 3$$

$$9 = 3^2$$

$$12 = 2^2 \times 3$$

$$15 = 3 \times 5$$

$$18 = 2 \times 3^2$$

$$21 = 3 \times 7$$

$$30 = 2 \times 3 \times 5$$

$$2^2 \times 3^2 \times 5 \times 7 = 1260.$$

(5)

$$670 = 2 \times 5 \times 67$$

$$100 = 2^2 \times 5^2$$

$$335 = 5 \times 67$$

$$25 = 5^2$$

$$2^2 \times 5^2 \times 67 = 6700$$

(6)

$$8 = 2^3$$

$$10 = 2 \times 5$$

$$18 = 2 \times 3^2$$

$$27 = 3^3$$

$$36 = 2^2 \times 3^2$$

$$44 = 2^2 \times 11$$

$$396 = 2^2 \times 3^3 \times 11$$

$$2^3 \times 3^3 \times 5 \times 11 = 11880.$$

EXERCISE 32—Page 134.

- (1) 2)12..10..24 2)14..21..3..2 .63 2)18..12..39..
- (2) 2) 6.. 5..12 3) 7..21..3..1..63 2) 9.. 6..39..
- (3) 3) 3.. 5.. 6 7) 7.. 7..1..1..21 3) 9.. 3..39..
- 1.. 5.. 2 1.. 1..1..1.. 3 3) 3.. 1..13..

2x2x3x5x2=120 2x3x7x3=126 13) 1.. 1..13..

1.. 1.. 1..
2x2x3x3x13+

(4) 2)8..18..15..20..70 2)24..16..18..20

2)4.. 9..15..10..35 2)12.. 8.. 9..10

3)2.. 9..15.. 5..35 2) 6.. 4.. 9.. 5

5)2.. 3.. 5.. 5..35 3) 3.. 2.. 9.. 5

2.. 3.. 1.. 1.. 7 1.. 2.. 3.. 5
2x2x3x5x2x3x7=2520 2x2x2x3x2x3x5=720

(6) 2)60..50..144..35..18 2)27..54..81..14..63

2)30..25.. 72..35.. 9 3)27..27..81.. 7..63

3)15..25.. 36..35.. 9 3) 9.. 9..27.. 7..21

3) 5..25.. 12..35.. 3 3) 3.. 3.. 9.. 7.. 7

5) 5..25.. 4..35.. 1 7) 1.. 1.. 3.. 7.. 7

1.. 5.. 4.. 7.. 1 1.. 1.. 3.. 1.. 1
2x2x3x3x5x5x4x7=25200 2x3x3x3x7x3=1134

EXERCISE 33—Page 136.

- (1) 300|300..200..150..50..50..75..125 165| 20..60..15..155..210..63..27
- 10| 2 5 21 4.. 4 14..21.. 9
- 300x10=3000. 12| 4.. 4 14..21.. 9
- 165x21x12=41580.

2³, 3³ and 5;
=34560.

are 2³ and 5;
40.

(4)

6=2x3
9=3²

12=2²x3

15=3x5

18=2x3²

21=3x7

30=2x3x5

3²x5x7=1260.

x5

x3²

x3²

x11

x3²x11

x11=11880.

(3)

$$\begin{array}{r} 144 \mid 12 \dots 132 \dots 144 \dots 60 \dots 96 \dots 1728 \\ 12 \mid 11 5 2 13 \\ 55 \mid 11 5 \\ \hline 144 \times 12 \times 55 = 95040. \end{array}$$

EXERCISE 34—Page 138.

(1) 12)592835	(2) 5)3700	(3) 11)10000	(4) 6)1000000
12)49402..e	5)740..0	11)909..1	6)166666..4
12)4116..t	5)148..0	11)82..7	6)27777..4
12)343..0	5)29..3	7..5	6)4629..3
12)28..7	5)5..4	7571.	6)771..3
2..4	1..0		6)128..3
2470te	104300.		6)21..2
			3..3
			33233344
(5) 8)10000	(6) 12)12345654321	(7) 9)10000	(8) 2)300
8)1250..0	12)1028804526..9	9)1111..1	2)150..0
8)156..2	12)85733710..6	9)123..4	2)75..0
8)19..4	12)7144475..t	9)13..6	2)37..1
2..3	12)595372..e	1..4	2)18..1
23420	12)49614..4	14641	2)9..0
	12)4134..6		2)4..1
	12)344..6		2)2..0
	12)28..8		1..0
	2..4		100101100
	248664et69		

EXERCISE 35—Page 139.

(4)
 6)1000000

 6)166666...4

 6)27777...4

 6)4629...3

 6)771...3

 6)128...3

 6)21...2

 3...3
 33233344
 (8)
 2)300

 1 2)150...0

 4 2)75...0

 6 2)37...1

 4 2)18...1

 1 2)9...0

 2)4...1

 2)2...0

 1...0

 100101100

(1)	(2)	(3)	(4)
IX	V	V	IV
8)37704	7)444	7)4321	9)1212201
<u> </u>	<u> </u>	<u> </u>	<u> </u>
8)4311...5	7)32...5	7)313...5	9)23121...0
<u> </u>	<u> </u>	<u> </u>	<u> </u>
8)480...1	2..3	7)21...6	9)1101...0
<u> </u>	235.	<u> </u>	<u> </u>
8)54...4		1..4	9)21...0
<u> </u>		1465.	<u> </u>
6..1			1..0
61415.			10000.

EXERCISE 36—Page 140.

(1)	(2)	(3)	(4)
IV	III	IX	VI
20212331	101202220	1522365	33233344
4	3	9	6
-	-	-	-
8	3	14	21
4	3	9	6
-	-	-	-
34	10	128	128
4	3	9	6
-	-	-	-
137	32	1154	771
4	3	9	6
-	-	-	-
550	96	10389	4629
4	3	9	6
-	-	-	-
2203	290	93507	27777
4	3	9	6
-	-	-	-
8815	872	841568	166666
4	3		6
-	-	-	-
35261	2618		1000000
	3		
	7854		

(6)

IX
3)132713
3)40834..0
3)13271..1
3)4083..1
3)1327..0
3)408..1
3)132..2
3)40..2
3)13..0
3)4..0
1..1

IX
12)132713
12)10207..9
12)682..t
12)51..8
3. t

IX
8)132713
8)14757..1
8)1652..0
8)184..6
8)21..5
2..3

IX	III	XII	VIII
132713 =	11002210110 =	31819 =	235601
<u>9</u>	<u>3</u>	<u>12</u>	<u>8</u>
12	4	46	19
<u>9</u>	<u>3</u>	<u>12</u>	<u>8</u>
110	12	560	157
<u>9</u>	<u>3</u>	<u>12</u>	<u>8</u>
997	36	6730	1262
<u>9</u>	<u>3</u>	<u>12</u>	<u>8</u>
8974	110	80769 den.	10096
<u>9</u>	<u>3</u>		<u>8</u>
80769 denary.	332	26923	80769 denary.
		<u>3</u>	
		80769 denary.	

(7)

IX
3)132713
 8)14757..1
8)1652..0
 8)184..6
8)21..5
 2..3

XII	XII	XII	XII
9) <u>t2t290</u>	6) <u>t2t290</u>	4) <u>t2t290</u>	2) <u>t2t290</u>
9) <u>117978..0</u>	6) <u>185856..0</u>	4) <u>268683..0</u>	2) <u>515146..0</u>
9) <u>1624t..2</u>	6) <u>34e4e..0</u>	4) <u>78180..3</u>	2) <u>268683..0</u>
9) <u>2032..4</u>	6) <u>69 t9..5</u>	4) <u>1e050..0</u>	2) <u>134341..1</u>
9) <u>284..2</u>	6) <u>1179..3</u>	4) <u>5913..0</u>	2) <u>78180..1</u>
9) <u>37..1</u>	6) <u>233..3</u>	4) <u>1533..3</u>	2) <u>3t0 t0..0</u>
4..7	6) <u>46..3</u>	4) <u>439..3</u>	2) <u>1e050..0</u>
	6) <u>9..0</u>	4) <u>10e..1</u>	2) <u>e626..0</u>
	1..3	4) <u>32..3</u>	2) <u>5913..0</u>
		4) <u>9..2</u>	2) <u>2t67..1</u>
		2..1	2) <u>1533..1</u>
			2) <u>877..1</u>
			2) <u>439..1</u>
			2) <u>21 t..1</u>
			2) <u>10 e..0</u>
			2) <u>65..1</u>
			2) <u>32..1</u>
			2) <u>17..0</u>
			2) <u>9..1</u>
			2) <u>4..1</u>
			2) <u>2..0</u>
			1..0

VIII
235601
 8
19
 8
157
 8
262
 8
096
 8
 769 denary.

(Continued on next page)

(7 continued.)

XII	IX	VI	IV	II	
<u>22290=4712420=130333500=21231330030=100110110111100001100</u>					
12	9	6	4	2	
<u>122</u>	<u>43</u>	9	9	2	1243
12	9	6	4	2	2
<u>1474</u>	<u>388</u>	54	38	4	2487
12	9	6	4	2	2
<u>17690</u>	<u>3494</u>	327	155	9	4975
12	9	6	4	2	2
<u>212289</u>	<u>31450</u>	1965	621	19	9951
12	9	6	4	2	2
<u>2547468</u>	<u>283052</u>	11793	2487	38	19902
	9	6	4	2	2
<u>2547468</u>	<u>70763</u>	9951	77	39804	2
	6	4	2	2	2
	<u>424578</u>	<u>39804</u>	155	79608	2
	6	4	2	2	2
	<u>2547468</u>	<u>159216</u>	310	159216	2
		4	2	2	2
		<u>636867</u>	621	318433	2
		4	2	2	2
		<u>2547468</u>	1243	636867	2
				2	2
				<u>1273734</u>	2
				2	2
				<u>2547468</u>	2

EXERCISE 37—Page 142.

		(1)	(2)	(3)	(4)
II		VI	XII	III	VIII
0001101101111100001100		252	62te)32e75721(62te	201210	57264
2		252	31556	102221	675
-		544	161e7	21212	354604
2	1243	2224	1059t		513354
2	2	544			434070
-			58192		
4	2487	122024	52512		51117344
2	2				
-			58801		
9	4975		58801		
2	2				
-		(5)	(6)	(7)	(8)
19	9951	II	VII	VII	XII
2	2	101	2143)142613(50·5254+	65432	7t348
-		1001	14111	43210	5e6t4
38	19902	1111		1444	
2	2	1011	1503·0	65001	1t864
-		1000	1411·1	54321	
77	39804	1111			
2	2	10101	61·60	326041	
-			43·16		
155	79608	010100			
2	2		15·410		
-			14·111		
310	159216				
2	2		1·2660		
-			1·1635		
321	3184·3				
2	2				
-					
343	636867	(9)	(10)		
	2	XII	II		
		347	100101)1010100001(10010	10010	10010
		6666	100101		
		18536		101000	
		18536		100101	
		18536			
		18536			111
		1t36e296			

EXERCISE 38—Page 146.

(1)					(2)						
4 ft. 7'	6"	10'''			19 ft. 10'	3"					
9	7	11	11		11	2	7				
<hr/>					<hr/>						
	4	2	11'''	3''''	2''''	11	6	11'''	9''''		
	4	2	11	3	2	3	3	8	6		
2	8	4	11	10		218	4	9			
41	8	1	6								
<hr/>					<hr/>						
44	9	1	8	0	5	2	222	8	0	5	9

(3)					(4)						
9'	7'''	4''''			9½ in.	=	9'	9''			
7	3	11''''			9'	9''					
<hr/>					<hr/>						
			8''''	9''''	8''''	8''''					
	2	4	10	0				3	3''''	0''''	
5	7	3	4					5	8	3	
<hr/>					<hr/>						
5	10	4	11	8	8			4	0	9	
<hr/>					<hr/>						
								4	6	8	6

(5)				
7 ft. 4'	11''			
3	2	2		
<hr/>				
	1	2	9'''	10''''
1	2	9	.10	
22	2	9		
<hr/>				
23	6	9	7	10

EXERCISE 39—Page 147.

(1)		(2)		(3)	
15 ft.		xii		10 ft.	
1	2'	45·6		5	
<hr/>		t·3		<hr/>	
2	6			50 sq. ft.	
15				7	
<hr/>		1146		<hr/>	
17	6	3870		---c'rds. c. ft.	
	8			128)350(2	94
<hr/>		398·46		256	
11	8 0''=	2		<hr/>	
11½ cu. ft.=11cu. ft.1152 cu. in.		_____ cu. ft.		94 cub. ft.	
		774·90=1096 9'			
		xii			
		774=1096 com. scale.			

(2)	3"	
2	7	
<hr/>		
1	6	11''' 9'''
3	8	6
4	9	
<hr/>		
	0	5 9
<hr/>		
(4)	in.=9' 9"	
	9"	
7	4'''	
<hr/>		
3	3''' 0'''	
5	8 3	
0	9	
<hr/>		
3	8 6	

(4)	4 ft.	(5)	25 ft.=300 in.
	5½	xii	20 " =240 "
	—	4.78	2 ft. 6 in.= 30 "
	20	9.6	
	1	—	8
	—	2310	4
	21 sq. ft.	3590	—
	70	—	32
	—	38.0t	2
	—	2.e	—
8)1470(11¾ cords	128	—	64=8×8
	—	34492	72000
	190	7418	30
	128	—	—
	—	— cub.ft.	8)2160000
	62	t8.652=128 6' 5" 2"	—
½s=¾.	—	t8 duoden.= 128 den.	8)270000
			—
			33750

EXERCISE 40—Page 149.

(3)	10 ft.
	5
<hr/>	
	50 sq. ft.
	7
<hr/>	
	— c'rds. c. ft.
8)350(2 94	
256	
<hr/>	
	94 cub. ft.

(1)	3×400 = \$372.00	£276×400 = \$1104.00
	.×20 = 2.80	19s.×20 = 3.80
	1.=30f.×5÷12 = .12½	10½d.=42f.×5÷12 = .17½
<hr/>		
	3 14s. 7½d. = \$374.92½	£276 19s. 10½d. = \$1107.97½
	5×400 = \$1100.00	\$729.18
	×20 = .80	710.50
	d.=47f.×5÷12 = .19½	166.78
	—	374.92½
	75 4s. 11½d. = \$1100.99½	1107.97½
		497.81
		1100.99½
		<hr/>
		\$4688.16½

(2)	576=6+10×7+10×10×5
	m.fur.per.yds.ft.in
	47 6 17 4 2 7×6= 286 6 27 1 2 0
	10
	<hr/>
	478 0 18 4 1 10×7= 3346 3 11 4 2 4
	10
	<hr/>
	4780 4 28 2 0.4×5 = 23902 7 21 4 3 2
	<hr/>
	27536 1 21 0 1 6

(3)
 $243000 = 2^3 \times 3^5 \times 5^3$
 $3 + 1 = 4$
 $4 + 1 = 5$
 $3 + 1 = 4$
 $4 \times 6 \times 4 = 96$

(4)

(5)

v
 8)4234434

viii
 5)713427

79:342 ÷ 00006378 =
 6378)7934200000(1243994.98278

8)241110..4

5)133721..2

6378

15562

8)13423..1

5)22303..2

12756

8)1024..1

5)3532..1

28060

8)32..3

5)570..2

25512

2..1

5)113..1

25480

19134

5)17..0

63460

57402

3..0

60580

57402

viii

v

713427

30012122

31780

213114

4234434

25512

500313

20222133

6268.0

5740.2

527.80

510.24

17.560

12.756

4.8040

4.4646

33940

31890

02050

(7)

5) 5. 7. 9. 11. 13. 15. 17. 19. 21. 23. 25. 27. 29. 31. 33. 35. 37. 39. 41. 43. 45. 47. 49. 51.
 1) 7. 9. 11. 13. 15. 17. 19. 21. 23. 25. 27. 29. 31. 33. 35. 37. 39. 41. 43. 45. 47. 49. 51.
 3) 9. 11. 13. 15. 17. 19. 21. 23. 25. 27. 29. 31. 33. 35. 37. 39. 41. 43. 45. 47. 49. 51.
 0) 11. 13. 15. 17. 19. 21. 23. 25. 27. 29. 31. 33. 35. 37. 39. 41. 43. 45. 47. 49. 51.

$40 \times 21 \times 33 \times 10 = 277200.$

(9)

$9999993000 = 10000000000 - 7000.$
 $64276 \cdot 3427 \times 10000000000 = 642763427000000$
 $64276 \cdot 3427 \times 7000 = 449934398 \cdot 9$

 $642762977065601 \cdot 1$

(10)

IX
 5) 78263

 5) 15230..3

 5) 2760..0

 5) 511..4

 5) 102..0

 5) 17..3

 3..1

IX
 11) 78263

 11) 6430..3

 11) 526..6

 11) 43..0

 3..6

| | | |
|-------------|--------------|------------|
| IX | V | XI |
| 7) 78263 = | 7) 3130403 = | 7) 36063 |
| <hr/> | <hr/> | <hr/> |
| 7) 11160..3 | 7) 214200..3 | 7) 5640..3 |
| <hr/> | <hr/> | <hr/> |
| 7) 1407..5 | 7) 13220..5 | 7) 884..5 |
| <hr/> | <hr/> | <hr/> |
| 7) 177..3 | 7) 1101..3 | 7) 128..3 |
| <hr/> | <hr/> | <hr/> |
| 7) 23..4 | 7) 41..4 | 7) 17..4 |
| <hr/> | <hr/> | <hr/> |
| 3..0 | 3..0 | 3..0 |

5)
 $42 \div 00006378 =$
 $0000(1243994 \cdot 9827)$

0
 4
 60
 02
 580
 402
 1780
 5512
 268·0
 740·2
 527·80
 510·24
 17·560
 12·756
 4·8040
 4·4646
 33940
 31890
 02050

(12)

(13)

£672 × 400 = \$2688-00
 7s. × 20 = 1-40
 7d. = 28 f. × 5 ÷ 12 = 11½
 £672 7s. 7d. = \$2689-51½

891)243000(272
 1782
 6480
 6237

(13 continued.)

81)37800(466
 324
 540
 486
 540
 486

27)35100
 1300

2430
 1782
 648)891(1
 648
 243)648(2
 486
 162)243(1
 162
 81)162(2
 162

54)81(1
 54

27)54(2
 54

Therefore G. C. M. = 27.

(17)

(18)

(19)

| | | | |
|------------|----------|------------------------------|--|
| £ s. d. | 2)276000 | 6 ft. 2' 7" 9''' 10'''' | |
| 178 16 4½ | 2)138000 | 13 11 11 11 7 | |
| 97 15 11½ | | | |
| 693 19 11½ | 2)69000 | 3 7 6'''' 6'''' 8'''' 10'''' | |
| 216 11 9¼ | | 5 8 5 2 0 2 | |
| 678 14 7½ | 2)34500 | 5 8 5 2 0 2 | |
| 197 13 11½ | | | |
| 117 6 5 | 2)17250 | 80 10 5 7 10 | |
| 91 1 1¼ | | | |
| 2272 0 3¼ | 3)8625 | 87 1 1 3 0 10 8 10 10 | |
| | 5)2875 | | |
| | 5)575 | | |
| | 5)115 | | |

23

2³ × 3 × 5³ × 23

(272

891(1
48

48(2
86

62)243)1
162

81)162(2
162

6'' 8'' 10''
0 2
2

8 10 10

| (20) | (21) | (22) |
|-------------------------|---------|-------|
| XII | IV | VIII |
| 713196)7te9·047(·011436 | 3333333 | 10000 |
| 713196 | 4 | 8 |
| <hr/> | <hr/> | <hr/> |
| 971·217 | 15 | 8 |
| 713196 | 4 | 8 |
| <hr/> | <hr/> | <hr/> |
| 266·4110 | 63 | 64 |
| 245·3720 | 4 | 8 |
| <hr/> | <hr/> | <hr/> |
| 21·05e00 | 255 | 512 |
| 19·3e846 | 4 | 8 |
| <hr/> | <hr/> | <hr/> |
| 3·862760 | 1023 | 4096 |
| 3·67e490 | 4 | |
| <hr/> | <hr/> | |
| ·113290 | 4095 | |
| | 4 | |
| | <hr/> | |
| | 16383 | |

(23)

74002702 ÷ 144 = 513907 ft. 94 in.
 513907 ft. ÷ 9 = 57100 yards 7 ft.
 57100 yds. ÷ 30½ = 1887 per. 18½ yds.
 1887 per. 18 yds. 2 ft. 36 in.
 Add 7 ft. 94 in.

40)1887 per. 19 yds. 0 ft. 130 in.

4)47 r. 7 per. 19 yds. 0 ft. 130 in.

11 a. 3 r. 7 per. 19 yds. 0 ft. 130 in.

(24)

1728 | 240.. 780.. 1820.. 1728
 65 | 5.. 85.. 15
 3 | 3
 1728 × 65 × 3 = 338960

(25)

6 children will have 6 children's shares
 4 women will have $4 \times 2 = 8$ " "
 3 men will have $3 \times 5 \times 2 = 30$ " "

3 men 4 w'n & 6 chi'n will have 44 children's sha.

4) \$7894.16

11) \$1973.54

\$179.41 $\frac{3}{11}$ = child's share.

\$179.41 $\frac{3}{11} \times 2 = \$358.82 \frac{6}{11}$ = woman's share.

\$358.82 $\frac{6}{11} \times 5 = \$1794.12 \frac{30}{11}$ = man's share.

(26)

(27)

| II | | II | | yds. qrs. na. in. | | | |
|------------|------------|------|-------|-------------------|----------|------|---|
| 1111111111 | 1000000000 | 7 | 1 1 1 | 1) 729 | 3 | 3 | 1 |
| 2 | 2 | 4 | | 4 | | | |
| - | - | | | | | | |
| 3 | 2 | 29 | | 2919 | | | |
| 2 | 2 | 4 | | 4 | | | |
| - | - | | | | | | |
| 7 | 4 | 117 | | 11679 | | | |
| 2 | 2 | 24 | | 24 | | | |
| - | - | | | | | | |
| 15 | 8 | 235 | | 23359 | | | |
| 2 | 2 | 294 | | 29194 | | | |
| - | - | | | | | | |
| 31 | 16 | 2644 | | 262784 | | | |
| 2 | 2 | 4 | | 4 | | | |
| - | - | | | | | | |
| 63 | 32 | 1057 |) | 105115 | (99-472) | 1057 | |
| 2 | 2 | | | 9513 | | | |
| - | - | | | | | | |
| 127 | 64 | | | 9985 | | | |
| 2 | 2 | | | 9513 | | | |
| - | - | | | | | | |
| 255 | 128 | | | 472 | | | |
| 2 | 2 | | | | | | |
| - | - | | | | | | |
| 511 | 256 | | | | | | |
| 2 | 2 | | | | | | |
| - | - | | | | | | |
| 1023 | 512 | | | | | | |

shares
"
"
children's sha.

(28)
762·4978
63·423

22874934
15249956
30499912
22874934
45749868

48359·8979694

(29)
723426
938·9126141

722487·0873859

(30)
lbs. oz. drs. scr.
129 0 0 0
63 4 7 2

65 7 0 1

's share.
hare.

(31)

1064 = 2^3 x 7 x 19.

1..2..4..8

. yds. qrs. na. in.
) 729 3 3 1
4

1..7

1..2..4..8..7..14..28..56

2919
4

1..19

1..2..4..8..7..14..28..56..19..38..76..152..133..266..532..1064==

11679
2 1/2

1..2..4..7..8..14..19..28..38..56..76..133..152..266..532..1064

23359
2919 1/2

(32)

30 ft. 6 in. = 366 in. 366
20 ft. 11 in. = 251 in. 251
2 ft. 7 in. = 31 in. 31

26278 1/2
4

366
1830
732

05115 (99 472 / 1057)
9513

in.
31) 91866 (2963 1/3
62

9985
9513

298
279

472

2963 1/3 ÷ 36 = 82 2/3 yds.

196
186

106
93

13

EXERCISE 46—Page 158.

(1)

$$\frac{2}{3}, \frac{4}{5}, \frac{6}{7}, \frac{8}{9}, \frac{10}{11} = \frac{2 \times 7 \times 9 \times 5 \times 18}{5 \times 7 \times 9 \times 5 \times 18}, \frac{5 \times 5 \times 9 \times 5 \times 18}{5 \times 7 \times 9 \times 5 \times 18}, \frac{8 \times 5 \times 7 \times 5 \times 18}{5 \times 7 \times 9 \times 5 \times 18}, \frac{11340}{28350}, \frac{20250}{28350}, \frac{25200}{28350}, \frac{17010}{28350}, \frac{7875}{28350}$$

(2)

$$\frac{1}{11}, \frac{13}{11}, \frac{14}{11} = \frac{8 \times 13 \times 14}{11 \times 13 \times 14}, \frac{12 \times 11 \times 14}{11 \times 13 \times 14}, \frac{5 \times 11 \times 13}{11 \times 13 \times 14} = \frac{1456}{2002}, \frac{1848}{2002}, \frac{715}{2002}$$

(3)

$$\frac{2}{7}, \frac{4}{11}, \frac{5}{13}, \frac{7}{7}, \frac{1}{2} = \frac{6 \times 11 \times 13 \times 7 \times 2}{7 \times 11 \times 13 \times 7 \times 2}, \frac{4 \times 7 \times 13 \times 7 \times 2}{7 \times 11 \times 13 \times 7 \times 2}, \frac{5 \times 7 \times 11 \times 7 \times 2}{7 \times 11 \times 13 \times 7 \times 2}, \frac{4 \times 7 \times 11 \times 13 \times 2}{7 \times 11 \times 13 \times 7 \times 2}, \frac{1 \times 7 \times 11 \times 13 \times 7}{7 \times 11 \times 13 \times 7 \times 2} = \frac{12012}{14014}, \frac{5096}{14014}, \frac{5390}{14014}, \frac{8008}{14014}, \frac{7007}{14014}$$

(4)

$$\frac{6}{11}, \frac{7}{7}, \frac{8}{13} = \frac{6 \times 7 \times 13}{11 \times 7 \times 13}, \frac{4 \times 11 \times 13}{11 \times 7 \times 13}, \frac{8 \times 11 \times 7}{11 \times 7 \times 13} = \frac{546}{1001}, \frac{572}{1001}, \frac{616}{1001}$$

(5)

$$\frac{5}{6}, \frac{7}{7}, \frac{8}{8}, \frac{9}{9} = \frac{5 \times 7 \times 3 \times 11}{6 \times 7 \times 5 \times 11}, \frac{4 \times 6 \times 5 \times 11}{6 \times 7 \times 5 \times 11}, \frac{4 \times 6 \times 7 \times 11}{6 \times 7 \times 5 \times 11}, \frac{2 \times 6 \times 7 \times 5}{6 \times 7 \times 5 \times 11} = \frac{1925}{2310}, \frac{1320}{2310}, \frac{1848}{2310}, \frac{420}{2310}$$

(6)

$$\frac{8 \times 5 \times 7 \times 5 \times 18}{5 \times 7 \times 9 \times 5 \times 18} = \frac{1 \times 3 \times 5 \times 7}{2 \times 3 \times 5 \times 7}, \frac{2 \times 2 \times 5 \times 7}{2 \times 3 \times 5 \times 7}, \frac{3 \times 2 \times 3 \times 7}{2 \times 3 \times 5 \times 7}$$

$$\frac{2 \times 2 \times 3 \times 5}{2 \times 3 \times 5 \times 7} = \frac{105}{210}, \frac{140}{210}, \frac{126}{210}, \frac{60}{210}$$

$$\frac{0}{210}, \frac{0}{210}, \frac{0}{210}, \frac{0}{210}$$

$$\frac{850}{28350} \quad \frac{1845}{28350} \quad \frac{315}{28350}$$

EXERCISE 47—Page 159.

(1)

$$\frac{1456}{2002}, \frac{1848}{2002}, \frac{715}{2002}$$

$\frac{1}{5}, \frac{2}{8}, \frac{3}{6}, \frac{4}{4}, \frac{5}{15}$

The least common multiple of 5, 8, 6, 4, 15 is 120.

The multiplier for both terms of the first fraction is $1 \frac{2}{5} = 24$; for the second $1 \frac{2}{8} = 15$; for the third $1 \frac{2}{6} = 20$; for the fourth $1 \frac{2}{4} = 30$; for the fifth $1 \frac{2}{15} = 8$.

Multiplying by these numbers, we obtain $\frac{26}{120}, \frac{15}{120}, \frac{80}{120}, \frac{90}{120}$, and $\frac{56}{120}$.

(2)

$$\frac{7 \times 13 \times 7 \times 2}{1 \times 13 \times 7 \times 2} = \frac{11 \times 13 \times 7}{13 \times 7 \times 2}$$

$$\frac{7007}{14014}$$

$\frac{1}{11}, \frac{2}{3}, \frac{3}{7}, \frac{4}{77}, \frac{5}{33}$

The least common multiple of 11, 3, 7, 77 and 33 is 231.

The multiplier for both terms of the first fraction is $2 \frac{1}{11} = 21$; for the second, $2 \frac{1}{3} = 77$; for the third, $2 \frac{1}{7} = 33$; for the fourth, $2 \frac{1}{77} = 3$; and for the fifth, $2 \frac{1}{33} = 7$.

Multiplying by these numbers, we obtain $\frac{136}{231}, \frac{154}{231}, \frac{133}{231}, \frac{54}{231}$, and $\frac{137}{231}$.

(3)

$$\frac{546}{1001}, \frac{572}{1001}, \frac{616}{1001}$$

$\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{4}{6}, \frac{5}{10}, \frac{6}{8}, \frac{7}{16}, \frac{8}{16}$

The least common multiple of 2, 3, 5, 6, 8, 10, 15, 16 and 80 is 240.

The multiplier for both terms of the first fraction is $2 \frac{1}{2} = 120$; for the second, $2 \frac{2}{3} = 80$; for the third, $2 \frac{3}{5} = 48$; for the fourth, $2 \frac{4}{6} = 40$; for the fifth, $2 \frac{5}{10} = 30$; for the sixth, $2 \frac{6}{8} = 24$; for the seventh, $2 \frac{7}{16} = 16$; for the eighth, $2 \frac{8}{16} = 15$; and for the ninth, $2 \frac{8}{16} = 3$.

Multiplying by these numbers, we obtain $\frac{120}{240}, \frac{160}{240}, \frac{144}{240}, \frac{300}{240}, \frac{210}{240}, \frac{216}{240}, \frac{192}{240}, \frac{144}{240}$, and $\frac{111}{240}$.

$$\frac{4 \times 6 \times 7 \times 11}{6 \times 7 \times 5 \times 11}$$

(4)

$$3, 15, 25, 35, 45, 55.$$

The least common multiple of 5, 10, 25, 30, 45, and 60 is 900.

The multiplier for both terms of the first fraction is $\frac{200}{5} = 180$; for the second, $\frac{200}{10} = 90$; for the third, $\frac{200}{25} = 36$; for the fourth, $\frac{200}{30} = 30$; for the fifth, $\frac{200}{45} = 20$; and for the sixth, $\frac{200}{60} = 15$.

Multiplying by these numbers, we obtain $540, 630, 312, 330, 380,$ and 345 .

(5)

$$10, 15, 20, 25.$$

The least common multiple of 20, 30, 40 and 50 is 600.

The multiplier for both terms of the first fraction is $\frac{600}{20} = 30$; for the second, $\frac{600}{30} = 20$; for the third, $\frac{600}{40} = 15$; and for the fourth, $\frac{600}{50} = 12$.

Multiplying by these numbers, we obtain $670, 140, 165$ and 120 .

(6)

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}, \frac{10}{11}, \frac{11}{12}, \frac{12}{13}, \frac{13}{14}, \frac{14}{15}, \frac{15}{16}, \frac{16}{17}, \frac{17}{18}, \frac{18}{19}, \frac{19}{20}.$$

The least common multiple of 2, 3, 4, 6, 8, 12, 16, and 24 is 48.

The multiplier for both terms of the first fraction is $\frac{48}{2} = 24$; for the second, $\frac{48}{3} = 16$; for the third, $\frac{48}{4} = 12$; for the fourth, $\frac{48}{6} = 8$; for the fifth, $\frac{48}{8} = 6$; for the sixth, $\frac{48}{12} = 4$; for the seventh, $\frac{48}{16} = 3$; and for the eighth, $\frac{48}{24} = 2$.

Multiplying by these numbers, we obtain $24, 32, 36, 40, 42, 44, 46, 48,$ and 49 .

(7)

$$7, 12, 15, 27, 35, 40.$$

The least common multiple of 7, 12, 15, 27, 35 and 40 is 7560.

The multiplier for both terms of the first fraction is $\frac{7560}{7} = 1080$; for the second, $\frac{7560}{12} = 630$; for the third, $\frac{7560}{15} = 504$; for the fourth, $\frac{7560}{27} = 280$; for the fifth, $\frac{7560}{35} = 216$; for the sixth, $\frac{7560}{40} = 189$.

Multiplying by these numbers, we obtain $5400, 9280, 4980,$ and 3880 .

(8)

$1\frac{1}{2}, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35$.

The least common multiple of 15, 8, 3, 12, 11, 20, 7, and 35 is 9240.

The multiplier for both terms of the first fraction is $2\frac{2}{3} = 616$; for the second, $2\frac{2}{5} = 1155$; for the third, $2\frac{2}{7} = 3080$; for the fourth, $2\frac{2}{9} = 770$; for the fifth, $2\frac{2}{11} = 840$; for the sixth, $2\frac{2}{13} = 462$; for the seventh, $2\frac{2}{17} = 1320$; for the eighth, $2\frac{2}{19} = 264$.

Multiplying by these numbers, we obtain $\frac{8624}{9240}, \frac{8925}{9240}, \frac{12220}{9240}, \frac{8278}{9240}, \frac{5248}{9240}, \frac{8278}{9240}, \frac{7320}{9240}$, and $\frac{7656}{9240}$.

EXERCISE 48—Page 160.

(1)

$$\frac{1}{4} \text{ of } \frac{2}{3} \text{ of } \frac{6}{11} \text{ of } \frac{3}{5} = \frac{4 \times 3 \times 6 \times 35}{7 \times 5 \times 11 \times 72} = \frac{2520}{27720} = \frac{1}{11}$$

(2)

$$\frac{2}{3} \text{ of } \frac{1}{9} \text{ of } \frac{6}{7} \text{ of } \frac{8}{100} \text{ of } \frac{2}{4} = \frac{2 \times 4 \times 6 \times 81 \times 25}{3 \times 9 \times 7 \times 100 \times 24} = \frac{97200}{453600} = \frac{1}{4}$$

(3)

$$\frac{2}{35} \text{ of } \frac{6}{11} \text{ of } \frac{7}{36} = \frac{21 \times 6 \times 77}{35 \times 11 \times 36} = \frac{7}{16}$$

(4)

$$\frac{2}{5} \text{ of } \frac{1}{7} \text{ of } \frac{3}{11} \text{ of } \frac{1}{3} = \frac{2 \times 4 \times 3 \times 13}{5 \times 7 \times 11 \times 17} = \frac{312}{6545}$$

EXERCISE 49—Page 161.

(1)

$$\frac{5}{8} \text{ of } \frac{7}{9} \text{ of } \frac{2}{3} \text{ of } \frac{3}{16} = \frac{5 \times 6 \times 2 \times 3}{9 \times 7 \times 3 \times 16} = \frac{5 \times \overset{2}{\cancel{6}} \times 2 \times \cancel{3}}{\overset{3}{\cancel{9}} \times 7 \times \overset{8}{\cancel{3}} \times \overset{4}{\cancel{16}}} = \frac{5}{3 \times 7 \times 4} = \frac{5}{84}.$$

(2)

$$\frac{3}{5} \text{ of } \frac{5}{8} \text{ of } \frac{11}{13} \text{ of } \frac{6}{17} \text{ of } \frac{11}{13} \text{ of } \frac{11}{17} = \frac{2 \times 5 \times 18 \times 6 \times 11 \times 13}{3 \times 9 \times 132 \times 11 \times 13 \times 17} =$$

$$\frac{2 \times 5 \times \overset{2}{\cancel{18}} \times \overset{2}{\cancel{6}} \times 11 \times 13}{\overset{3}{\cancel{9}} \times 11 \times 13 \times 17} = \frac{2 \times 5}{33 \times 17} = \frac{10}{561}.$$

(3)

$$\frac{2}{7} \text{ of } \frac{4}{11} \text{ of } 5\frac{1}{2} = \frac{2 \times 4 \times 11}{7 \times 11 \times 2} = \frac{2 \times 4 \times \cancel{11}}{7 \times \cancel{11} \times 2} = 1.$$

(4)

$$\frac{1}{9} \text{ of } \frac{8}{13} \text{ of } \frac{117}{200} \text{ of } \frac{50}{169} \text{ of } \frac{17}{13} \text{ of } \frac{13}{6} = \frac{1 \times 8 \times 117 \times 50 \times 13 \times 13}{9 \times 13 \times 200 \times 169 \times 17 \times 6} =$$

$$\frac{1 \times \overset{2}{\cancel{8}} \times \overset{9}{\cancel{117}} \times \overset{5}{\cancel{50}} \times \overset{13}{\cancel{13}} \times \overset{13}{\cancel{13}}}{\overset{3}{\cancel{9}} \times \overset{4}{\cancel{13}} \times \overset{13}{\cancel{169}} \times 17 \times \overset{2}{\cancel{6}}} = \frac{1}{17 \times 3} = \frac{1}{51}.$$

(5)

$$\frac{3}{7} \text{ of } \frac{4}{11} \text{ of } \frac{9}{13} \text{ of } \frac{33}{17} \text{ of } \frac{33}{17} \text{ of } 47 = \frac{3 \times 4 \times 9 \times 33 \times 38 \times 47}{11 \times 7 \times 19 \times 47 \times 72 \times 7} =$$

$$\frac{3 \times 4 \times \overset{3}{\cancel{9}} \times \overset{2}{\cancel{33}} \times \overset{2}{\cancel{33}} \times 47}{\overset{11}{\cancel{11}} \times 7 \times 19 \times \overset{18}{\cancel{47}} \times \overset{2}{\cancel{72}} \times 7} = \frac{3 \times 3}{7 \times 7} = \frac{9}{49}.$$

(6)

$$\frac{5}{3 \times 7 \times 4} = \frac{5}{84}$$

$$\frac{1}{7} \text{ of } \frac{3}{11} \text{ of } 1\frac{1}{4} = \frac{2 \times 3 \times 154}{7 \times 11 \times 1} = \frac{4 \times 3 \times 154}{7 \times 11 \times 1} = \frac{2 \times 4 \times 3}{1} = 24.$$

EXERCISE 50—Page 162.

(1)

$$\frac{6 \times 11 \times 13}{11 \times 13 \times 17} =$$

$$\frac{14}{17} = \frac{14}{17} = \frac{14 \times 25}{45 \times 42} = \frac{14 \times 25}{9 \times 3} = \frac{5}{3}$$

(2)

$$= 7.$$

$$\frac{1\frac{1}{2}}{7\frac{1}{8}} = \frac{1\frac{1}{2}}{7\frac{1}{8}} = \frac{11 \times 18}{12 \times 143} = \frac{11 \times 18}{2 \times 13} = \frac{3}{26}$$

(3)

$$\frac{50 \times 13 \times 13}{169 \times 17 \times 6} =$$

$$\frac{15\frac{3}{4}}{7\frac{1}{2}} = \frac{7\frac{3}{4}}{3\frac{1}{2}} = \frac{78 \times 5}{5 \times 39} = \frac{78 \times 5}{5 \times 39} = 2.$$

(4)

$$\frac{11\frac{3}{8}}{12\frac{3}{8}}, \frac{3\frac{1}{9}}{9}, \frac{7}{3} = \frac{3\frac{3}{8}}{6\frac{3}{8}}, \frac{1\frac{3}{4}}{2}, \frac{7}{3} = \frac{35 \times 5}{3 \times 68}, \frac{13 \times 1}{9 \times 4}, \frac{2 \times 5}{7 \times 3} = \frac{175}{274}, \frac{13}{36}, \frac{10}{21}$$

(5)

$$\frac{33 \times 38 \times 47}{7 \times 72 \times 7} =$$

$$\frac{7\frac{1}{2}}{15\frac{1}{2}}, \frac{57}{16}, \frac{2\frac{1}{3}}{3\frac{1}{3}} = \frac{7\frac{1}{2}}{6\frac{3}{4}}, \frac{47}{16}, \frac{1\frac{1}{3}}{2\frac{1}{3}} = \frac{7 \times 4}{12 \times 68}, \frac{47 \times 16}{8 \times 3}, \frac{12 \times 7}{5 \times 24} =$$

$$\frac{1}{3 \times 9}, \frac{47 \times 2}{3}, \frac{7}{5 \times 2} = \frac{1}{27}, 31\frac{1}{3}, \frac{7}{10}$$

(6)

$$\frac{16\frac{1}{2}}{11\frac{1}{2}}, \frac{6\frac{1}{2}}{13}, \frac{17}{18\frac{1}{2}}, \frac{21\frac{1}{2}}{10\frac{1}{2}}, \frac{1}{4\frac{1}{2}} = \frac{40}{2\frac{1}{2}}, \frac{3\frac{1}{2}}{1\frac{1}{2}}, \frac{17}{4\frac{1}{2}}, \frac{10\frac{1}{2}}{7\frac{1}{2}}, \frac{1}{2\frac{1}{2}} = \frac{10}{50 \times 8},$$

$$\frac{10}{8 \times 35}$$

$$\frac{31 \times 1}{5 \times 13}, \frac{17 \times 3}{55 \times 1}, \frac{108 \times 7}{72 \times 5}, \frac{1 \times 5}{2 \times 23} = \frac{10}{7}, \frac{31}{65}, \frac{51}{55}, \frac{21}{10}, \frac{5}{46} = 1\frac{1}{2}, 3\frac{1}{5}, 3\frac{1}{5}, 2\frac{1}{10}, \frac{5}{46}$$

EXERCISE 51—Page 163.

(1)

$$\frac{2}{3} \text{ of } \frac{1}{16} = \frac{1}{80} \text{ of a lb.}$$

(2)

$$\frac{2}{3} \text{ of } \frac{3}{7} \text{ of } \frac{1}{12} \text{ of } \frac{1}{20} = \frac{1}{7 \times 6 \times 20} = \frac{1}{840}$$

(3)

$$\frac{2}{9} \text{ of } \frac{35}{4} \text{ of } \frac{1}{7} = \frac{5}{9 \times 2} = \frac{5}{18} \text{ wk.}$$

(4)

$$\frac{5}{11} \text{ of } \frac{81}{5} \text{ of } \frac{1}{4} \text{ of } \frac{1}{5} = \frac{81}{11 \times 4 \times 5} = \frac{81}{220} \text{ Eng. Ell.}$$

(5)

$$\frac{3}{7} \text{ of } \frac{4}{11} \text{ of } \frac{1}{5\frac{1}{2}} = \frac{3}{7} \text{ of } \frac{4}{11} \text{ of } \frac{2}{11} = \frac{24}{847} \text{ per.}$$

$$\frac{1}{2^2} = \frac{10}{50 \times 3} = \frac{1}{3 \times 35} = \frac{1}{7}$$

$$\frac{2}{3} \text{ of } \frac{4}{7} \text{ of } \frac{1}{21} \text{ of } \frac{1}{8} = \frac{2 \times 4 \times 295 \times 1}{3 \times 7 \times 14 \times 8} = \frac{295}{294} = 1\frac{1}{98} \text{ c.}$$

(7)

$$= 17,888,882,10,46.$$

$$\frac{3}{19} \text{ of } \frac{4}{17} \text{ of } \frac{1}{9} \text{ of } \frac{1}{40} \text{ of } \frac{1}{4} = \frac{3 \times 4 \times 19 \times 1 \times 1}{19 \times 17 \times 2 \times 40 \times 4} = \frac{3}{17 \times 2 \times 40} = \frac{3}{1360} \text{ a.}$$

EXERCISE 52—Page 164.

(1)

$$\frac{14}{79} \text{ of } \frac{4}{1} \text{ of } \frac{2}{1} \text{ of } \frac{4}{1} = 4\frac{1}{8} \text{ qt.}$$

(2)

$$\frac{2}{3} \text{ of } \frac{4}{1} \times \frac{2}{1} \times \frac{4}{1} \times \frac{5}{1} \times \frac{3}{2} = \frac{2 \times 4 \times 4 \times 5}{3} = 19\frac{2}{3}$$

£840.

(3)

$$\frac{7}{9} \times \frac{2}{1} \times \frac{2}{1} \times \frac{4}{1} \times \frac{2}{1} \times \frac{2}{1} \times \frac{3}{2} = \frac{7 \times 2 \times 2 \times 4 \times 2}{3} = 22\frac{2}{3}$$

wk.

(4)

$$\frac{17}{22} \times \frac{6}{1} \times \frac{8}{1} \times \frac{3}{1} = \frac{17 \times 6 \times 8 \times 3}{11} = 244\frac{8}{11} \text{ scr.}$$

Eng. Ell.

(5)

$$\frac{1}{5000} \times \frac{2}{3} \times \frac{6}{4} \times \frac{22}{11} \times \frac{2}{7} \times \frac{2}{1} \times \frac{4}{1} = \frac{2 \times 6 \times 2 \times 2 \times 4}{625 \times 7} = \frac{192}{4375} \text{ dr.}$$

per.

625

EXERCISE 53—Page 164.

(1)

bush. pk. gal. qt. pt.
 11)3 0 0 0 0
 —————
 1 0 0 1½

lbs. oz. dr.
 7)6 0 0
 —————
 13 11½

(2)
 yds. qr. na. in.
 113) 7 (2 0 1½
 4
 —————
 28 qrs.
 26
 —————
 2
 4
 —————
 8 na.
 2½
 —————
 18
 13
 —————
 5

(4)
 fur. per. yds. ft. in.
 9)8 0 0 0 0
 —————
 35 3 0 2

£ s. d.
 7)4 0 0
 —————
 11 5½

(2)

lbs. oz. dwt. grs.
 9)8 0 0 0
 —————
 10 13 8

sq. m. a. r. pr. yds. ft. in.
 113) 11 (62 1 8 4 2 79½

640
 —————
 7040 a.
 678

260 484 yds
 226 452
 —————
 34 32
 4 9

136 r. 288 ft.
 113 226
 —————
 23 62
 40 144

920 per. 248
 904 248
 —————
 16 62
 30½ 8928 in.
 —————
 791

480
 4 1018
 —————
 484 yds. 1017
 —————
 1

EXERCISE 54—Page 165.

(2)

oz. dwt. grs.
 0 0 0

 0 13 8

r. pr. yds. ft. in.
 1 8 4 2 79 $\frac{1}{2}$

(1)

6 bus. 1 pk. 1 gal. 1 qt. 1 pt. = 411 pts.
 50 bush. = 3200 pts.
 And the required fraction is $\frac{411}{3200}$.

(2)

35 per. 9 ft. 2 in. = 7040 in.
 1 fur. = 7920 in.
 The required fraction is $\frac{7040}{7920} = \frac{88}{99} = \frac{8}{9}$.

(3)

7 hrs. 12 min. = 432 min.
 1 day = 1440 min.
 Therefore the fraction is $\frac{432}{1440} = \frac{3}{10}$.

(4)

2 sq. yds. 2 ft. 120 in. = 3000 in.
 3 sq. per. $13\frac{1}{2}$ yds. 1 ft. 72 in. = 135000 in.
 And the fraction is $\frac{3000}{135000} = \frac{1}{45}$.

(5)

7 oz. 7 drs. 2 scr. 14 grs. = 3834 grs.
 21 lbs. = 120960 grs.
 The fraction is $\frac{3834}{120960} = \frac{426}{14400} = \frac{71}{2400}$.

(6)

9 min. 48 sec. = 588 sec.
 1 day = 86400 sec.
 The required fraction is $\frac{588}{86400} = \frac{77}{10800}$.

(7)

16 bush. 1 pk. 1 pt. = 1041 pts.
 69 bush. = 4416 pts.
 Therefore the fraction is $\frac{1041}{4416} = \frac{347}{1472}$.

484 yds
 452

 32
 9

 288 ft.
 226

 62
 144

 248
 248
 62

 8928 in.
 791

 1018
 1017

 1

(8)

$$3 \text{ qrs. } 3\frac{1}{2} \text{ na.} = 15\frac{1}{2} = \frac{132}{8} \text{ na.}$$

$$1 \text{ Eng. ell} = 20 \text{ na.}$$

$$\text{And the fraction is } \frac{132}{20} = 6\frac{3}{5} = 3\frac{3}{5}.$$

(9)

$$13 \text{ dwt. } 7 \text{ grs.} = 319 \text{ grs.}$$

$$1 \text{ lb. Troy} = 5760 \text{ grs.}$$

$$\text{The required fraction is } \frac{319}{5760}.$$

(10)

$$4800 \text{ cub. ft.}$$

$$54 \text{ cords} = 6912 \text{ cub. ft.}$$

$$\text{Therefore the fraction is } \frac{4800}{6912} = \frac{100}{144} = \frac{25}{36}.$$

EXERCISE 55--Page 167.

(1)

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3}{6} + \frac{2}{6} + \frac{1}{6} = \frac{6}{6} = 1.$$

(2)

$$\frac{1}{2} + \frac{6}{12} + \frac{7}{12} + \frac{9}{12} + \frac{11}{12} + \frac{5}{12} = \frac{32}{12} = 2\frac{8}{12} = 2\frac{2}{3}.$$

(3)

$$4\frac{3}{4} + 11\frac{1}{4} + 16\frac{3}{4} + 21\frac{1}{4} + 19\frac{3}{4} = 4 + 11 + 16 + 21 + 19 + (\frac{3}{4} + \frac{1}{4} + \frac{3}{4} + \frac{1}{4} + \frac{3}{4}) = 71 + \frac{12}{4} = 73\frac{3}{4}.$$

(4)

$$16\frac{1}{2} + 11\frac{1}{2} + 18\frac{1}{2} + 17\frac{1}{2} + 112\frac{1}{2} = 16 + 11 + 18 + 17 + 112 + (\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}) = 174 + \frac{5}{2} = 174 + 2\frac{1}{2} = 176\frac{1}{2}.$$

(5)

$$4\frac{1}{4} + 1\frac{1}{4} + 7\frac{1}{4} = 4 + 1 + 7 + (\frac{1}{4} + \frac{1}{4} + \frac{1}{4}) = 12 + \frac{3}{4} = 12\frac{3}{4}.$$

(6)

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \frac{7}{8} + \frac{8}{9}.$$

These fractions reduced to their least common denominator become $\frac{1260}{1260} + \frac{1680}{1260} + \frac{1575}{1260} + \frac{2520}{1260} + \frac{2100}{1260} + \frac{2100}{1260} + \frac{2100}{1260} + \frac{2100}{1260}$

(7)

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ when reduced to their least common denominator come $\frac{6}{12} + \frac{4}{12} + \frac{3}{12} = \frac{13}{12} = 2\frac{1}{12}$.

(8)

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$$

These fractions when reduced to their least common denominator become $\frac{30}{60} + \frac{20}{60} + \frac{15}{60} + \frac{12}{60} + \frac{10}{60} = \frac{87}{60} = 1\frac{27}{60}$.

(9)

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$$

These fractions when reduced to their least common denominator become $\frac{42}{84} + \frac{28}{84} + \frac{21}{84} + \frac{14}{84} + \frac{14}{84} + \frac{12}{84} = \frac{131}{42} = 3\frac{1}{42}$.

(10)

$16\frac{3}{4} + 47\frac{1}{2} + 21\frac{1}{3} + \frac{7}{8} + 19\frac{1}{2} = 16 + 47 + 21 + 19 \times (\frac{3}{4} + \frac{1}{2} + \frac{1}{3} + \frac{7}{8})$
 $16 + 47 + 21 + 19 = 103$
 $+\frac{3}{4} + \frac{1}{2} + \frac{1}{3} + \frac{7}{8} = \frac{18}{24} + \frac{12}{24} + \frac{8}{24} + \frac{21}{24} = \frac{59}{24}$
 $103 + 19 \times \frac{59}{24} = 104\frac{19}{24}$

(11)

$17\frac{1}{2} + 43\frac{1}{3} + 168\frac{1}{4} + 207\frac{1}{5} + 506\frac{1}{6} = 17 + 43 + 168 + 7 + 506 + (\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6})$
 $17 + 43 + 168 + 207 + 506 = 941$
 $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{30}{60} + \frac{20}{60} + \frac{15}{60} + \frac{12}{60} + \frac{10}{60} = \frac{87}{60} = 2\frac{1}{4}$
 $941 + 2\frac{1}{4} = 943\frac{1}{4}$

(12)

$6\frac{1}{2} + 11\frac{1}{3} + \frac{9}{8} + 16\frac{7}{8} + \frac{1}{2} + \frac{1}{4} + 17\frac{1}{2} = 6 + 11 + 16 + \frac{1}{2} + (\frac{1}{3} + \frac{1}{8} + \frac{7}{8} + \frac{1}{4} + \frac{1}{4} + \frac{1}{2})$
 $6 + 11 + 16 + 17 = 50$
 $\frac{1}{2} + \frac{1}{3} + \frac{1}{8} + \frac{7}{8} + \frac{1}{4} + \frac{1}{4} + \frac{1}{2} = \frac{24}{24} + \frac{8}{24} + \frac{3}{24} + \frac{21}{24} + \frac{6}{24} + \frac{12}{24} = \frac{74}{24} = 3\frac{1}{6}$
 $50 + 3\frac{1}{6} = 53\frac{1}{6}$

na.
na.
 $\frac{1}{2} = 3\frac{1}{2}$
s.
s.
o.
 $= 4\frac{1}{2} = 3\frac{1}{2}$
4.
 $= 3\frac{1}{2} = 3\frac{1}{2}$
- 16 + 21 + 19 +
A = 73\frac{1}{2}
- 11 + 18 + 17 + 112 -
174 + 3\frac{1}{2} = 177\frac{1}{2}
5 + (13\frac{3}{4} + 13\frac{1}{4})
+ \frac{1}{2}.
common denominator
 $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

(13)

$$\frac{1}{2} + \frac{3}{4} + \frac{5}{8} + 68\frac{1}{8} = 68 + (\frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \frac{1}{8}).$$

$$\frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \frac{1}{8} = \frac{4}{8} + \frac{6}{8} + \frac{5}{8} + \frac{1}{8} = \frac{16}{8} = 2.$$

$$68 + 2 = 70.$$

(14)

$$173\frac{1}{2} + 8\frac{1}{2} + 91\frac{1}{2} = 173 + 8 + 91 + (\frac{1}{2} + \frac{1}{2} + \frac{1}{2}).$$

$$173 + 8 + 91 = 272.$$

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} = 1\frac{1}{2}.$$

$$272 + 1\frac{1}{2} = 273\frac{1}{2}.$$

(15)

$$1\frac{1}{2} + 2\frac{1}{2} + 3\frac{1}{2} + 4\frac{1}{2} = 1 + 2 + 3 + 4 + (\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}).$$

$$1 + 2 + 3 + 4 = 10.$$

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{4}{2} = 2.$$

$$10 + 2 = 12.$$

(16)

$$\frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \frac{7}{8} + \frac{9}{8} + \frac{11}{8} + \frac{13}{8} = \frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \frac{13}{8}.$$

$$\frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \frac{13}{8} = \frac{4}{8} + \frac{6}{8} + \frac{5}{8} + \frac{13}{8} = \frac{28}{8} = 3\frac{4}{8} = 3\frac{1}{2}.$$

(17)

$$7 + 11\frac{1}{2} + 18 + 26\frac{1}{2} + 79\frac{1}{2} = 7 + 11 + 18 + 26 + 79 + (\frac{1}{2} + \frac{1}{2} + \frac{1}{2}).$$

$$7 + 11 + 18 + 26 + 79 = 141.$$

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} = 1\frac{1}{2}.$$

$$141 + 1\frac{1}{2} = 142\frac{1}{2}.$$

(18)

$$\frac{2}{3} \text{ of } \frac{3}{4} \text{ of } \frac{4}{5} = \frac{2}{5} = 3\frac{2}{5}.$$

$$\frac{2}{5} + 7\frac{2}{5} + 3\frac{2}{5} = 10 + (\frac{2}{5} + \frac{2}{5} + \frac{2}{5}).$$

$$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}.$$

$$10 + 1\frac{1}{5} = 11\frac{1}{5}.$$

(19)

$$\frac{4\frac{1}{2}}{\frac{1}{7}} = \frac{\frac{13}{2}}{\frac{1}{7}} = \frac{13 \times 18}{3 \times 7} = 7^3 = 11\frac{1}{2}$$

$$\frac{1}{2} \text{ of } \frac{3}{4} \text{ of } \frac{1}{5} \text{ of } \frac{1}{2} = \frac{1 \times 36 \times 4 \times 11}{2 \times 11 \times 15 \times 4} = \frac{18}{15} = 1\frac{1}{5}$$

$$\frac{20\frac{1}{2}}{\frac{1}{11}} = \frac{2^3}{\frac{1}{11}} = \frac{83 \times 11}{4 \times 83} = \frac{11}{4} = 2\frac{3}{4}$$

$$11\frac{1}{2} + 1\frac{1}{5} + 2\frac{3}{4} = 11 + 1 + 2 + (\frac{1}{2} + \frac{1}{5} + \frac{3}{4}) = 14 + (\frac{1}{2} + \frac{1}{5} + \frac{3}{4})$$

$$\frac{1}{2} + \frac{1}{5} + \frac{3}{4} = \frac{10}{20} + \frac{4}{20} + \frac{15}{20} = \frac{29}{20} = 1\frac{9}{20}$$

$$14 + 1\frac{9}{20} = 15\frac{9}{20}$$

(20)

$$3\frac{5}{8} + 11\frac{1}{6} + 14\frac{3}{4} = 3 + 11 + 14 + (\frac{5}{8} + \frac{1}{6} + \frac{3}{4}) = 28 + (\frac{5}{8} + \frac{1}{6} + \frac{3}{4})$$

$$\frac{5}{8} + \frac{1}{6} + \frac{3}{4} = \frac{15}{24} + \frac{4}{24} + \frac{18}{24} = \frac{37}{24} = 1\frac{13}{24}$$

$$28 + 1\frac{13}{24} = 29\frac{13}{24}$$

(21)

$$\frac{1}{2} \text{ of } \frac{2}{3} = \frac{2}{3}, \frac{2}{3} \text{ of } \frac{4}{5} = \frac{8}{15}, \frac{2}{3} \text{ of } \frac{1}{7} = \frac{2}{21}, \frac{2}{3} \text{ of } \frac{3}{8} = \frac{1}{4}, \frac{2}{3} \text{ of } \frac{1}{2} \text{ of } \frac{1}{2}$$

$$\frac{2}{3} + \frac{8}{15} + \frac{2}{21} + \frac{1}{4} + \frac{1}{2} = \frac{630}{1680} + \frac{960}{1680} + \frac{160}{1680} + \frac{420}{1680} + \frac{840}{1680} = \frac{2990}{1680} = 1\frac{1795}{840}$$

(22)

$$41\frac{1}{2} + 105\frac{1}{3} + 300\frac{1}{2} + 241\frac{2}{3} + 472\frac{1}{2} = 41 + 105 + 300 + 241 + 472 + (\frac{1}{2} + \frac{1}{3} + \frac{1}{2} + \frac{2}{3} + \frac{1}{2})$$

$$41 + 105 + 300 + 241 + 472 = 1159$$

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{2} + \frac{2}{3} + \frac{1}{2} = \frac{30}{60} + \frac{20}{60} + \frac{30}{60} + \frac{40}{60} + \frac{30}{60} = \frac{150}{60} = 2\frac{30}{60} = 2\frac{1}{2}$$

$$1159 + 2\frac{1}{2} = 1161\frac{1}{2}$$

(23)

$$92\frac{5}{14} + 37\frac{8}{19} + 7\frac{1}{2} = 92 + 37 + 7 + (\frac{5}{14} + \frac{8}{19} + \frac{1}{2}) = 136 + (\frac{5}{14} + \frac{8}{19} + \frac{1}{2})$$

$$\frac{5}{14} + \frac{8}{19} + \frac{1}{2} = \frac{95}{238} + \frac{96}{238} + \frac{119}{238} = \frac{310}{238} = 1\frac{155}{119}$$

$$136 + 1\frac{155}{119} = 137\frac{155}{119}$$

(24)

$$\frac{10\frac{3}{8}}{2\frac{3}{8}} = \frac{5\frac{3}{8}}{1\frac{3}{8}} = \frac{53 \times 5}{5 \times 12} = \frac{53}{12} = 4\frac{5}{12} \quad \frac{2}{3} \text{ of } \frac{3}{4} = \frac{1}{2}$$

$$21\frac{1}{2} + 35\frac{1}{2} + 4\frac{5}{12} + \frac{1}{2} = 21 + 35 + 5 + (\frac{1}{2} + \frac{1}{2}) = 61\frac{1}{2}$$

(25)

$$\frac{1}{4} \text{ of } \frac{1}{2} = \frac{1 \times 1}{2 \times 4} = \frac{1}{8} = 10\frac{1}{8}$$

$$\frac{1}{3} \text{ of } \frac{2}{5} \text{ of } \frac{1}{4} \text{ of } \frac{1}{2} = \frac{2 \times 1 \times 1}{3 \times 5 \times 4 \times 2} = \frac{2}{120} = \frac{1}{60} = 15\frac{1}{60}$$

$$10\frac{1}{8} + 6\frac{1}{8} + 15\frac{1}{60} + 1\frac{1}{2} = 10 + 6 + 15 + 1 + (\frac{1}{8} + \frac{1}{8} + \frac{1}{60} + \frac{1}{2})$$

$$= 32 + (\frac{1}{2} + \frac{1}{8} + \frac{1}{60} + \frac{1}{2})$$

$$\frac{1}{2} + \frac{1}{8} + \frac{1}{60} + \frac{1}{2} = \frac{30}{60} + \frac{7.5}{60} + \frac{1}{60} + \frac{30}{60} = \frac{68.5}{60} = 1\frac{8.5}{60} = 1\frac{17}{120}$$

$$32 + 1\frac{17}{120} = 33\frac{17}{120}$$

EXERCISE 56—Page 169.

(1)

| | oz. | dr. | scr. | grs. |
|-------------------------|----------|----------|----------|-------------------------------------|
| $\frac{1}{4}$ of a lb. | = 4 | 2 | 2 | $14\frac{4}{11}$ |
| $\frac{2}{3}$ of an oz. | = 3 | 1 | | $5\frac{4}{11}$ |
| $\frac{1}{4}$ of a dr. | = | 1 | | $1\frac{2}{11}$ |
| $\frac{1}{8}$ of a scr. | = | | | $16\frac{2}{11}$ |
| | <u>4</u> | <u>6</u> | <u>2</u> | <u>$18\frac{17}{11}$</u> |

(2)

| | qr. | na. | in. |
|-------------------------------|----------|----------|-----------------------------------|
| $\frac{3}{8}$ of a yard | = 2 | 1 | $1\frac{7}{8}$ |
| $\frac{1}{2}$ of an Eng. ell. | = | 2 | $1\frac{1}{2}$ |
| $\frac{1}{8}$ of a qr. | = | 3 | $0\frac{7}{8}$ |
| | <u>3</u> | <u>3</u> | <u>$1\frac{39}{8}$</u> |

(3)

| | in. |
|-------------------------|------------------|
| $\frac{1}{2}$ of a yd. | = $5\frac{1}{2}$ |
| $\frac{1}{2}$ of a ft. | = $1\frac{1}{2}$ |
| $\frac{1}{2}$ of an in. | = $\frac{1}{2}$ |
| | <u>7</u> |

(4)

| | fur. | per. | yds. | ft. | in. |
|-------------------------|----------|-----------|----------|----------|-----------------------------------|
| $\frac{1}{4}$ of a mile | = 5 | 3 | 8 | 1 | 6 |
| $\frac{1}{3}$ of a fur. | = | 12 | 1 | 2 | $0\frac{2}{3}$ |
| $\frac{2}{3}$ of a yd. | = | | | 1 | $2\frac{1}{3}$ |
| | <u>5</u> | <u>18</u> | <u>0</u> | <u>0</u> | <u>$3\frac{23}{3}$</u> |

(5)

| | day | hrs. | min. |
|--------------------------|----------|----------|-----------|
| $\frac{1}{2}$ of a week | = 1 | 18 | 0 |
| $\frac{1}{2}$ of a day | = | 8 | 0 |
| $\frac{1}{2}$ of an hour | = | | 12 |
| | <u>2</u> | <u>2</u> | <u>12</u> |

(6)

| | s. | d. |
|-----------------------|----------|-----------------------------------|
| $\frac{1}{2}$ of a £ | = 2 | $10\frac{1}{2}$ |
| $\frac{2}{3}$ of a s. | = | $2\frac{2}{3}$ |
| $\frac{1}{2}$ d. | = | $1\frac{1}{2}$ |
| | <u>3</u> | <u>$13\frac{1}{2}$</u> |

of $\frac{1}{2} = \frac{1}{2}$.

+ $\frac{1}{2} = 6\frac{1}{2}$.

$15\frac{1}{2} = 15\frac{1}{2}$.

$15 + 1 + (\frac{1}{2} +$

$\frac{1}{2}) = 16\frac{1}{2}$.

(7)

| | £ | s. | d. |
|------------------------------|---|-------|-------------------|
| $\frac{1}{2}$ of 21s. | = | 13 | $1\frac{1}{2}$ |
| $\frac{1}{2}$ of 5s. | = | 3 | $1\frac{1}{2}$ |
| $\frac{1}{2}$ of £3 12s. 6d. | = | 2 | 5 $3\frac{1}{2}$ |
| $\frac{1}{2}$ of a £ | = | 10 | $9\frac{1}{2}$ |
| $1\frac{1}{2}$ | = | | $4\frac{1}{2}$ |
| | | <hr/> | |
| | | 3 | 12 $4\frac{1}{2}$ |

EXERCISE 57—Page 171.

(1)

$\frac{1}{2} - \frac{1}{20} = \frac{10}{20} - \frac{1}{20} = \frac{9}{20} = \frac{9}{20}$.

(2)

$\frac{1}{7}$ of $\frac{1}{3}$ of $\frac{1}{11} = \frac{3 \times 48}{17 \times 11} = \frac{144}{187}$. $\frac{1495}{1495} + \frac{144}{187} = \frac{2056}{1495} + \frac{1152}{1495} =$

$\frac{3208}{1495} = 1\frac{1661}{1495} = 1\frac{1}{11}$.

$\frac{8\frac{1}{2}}{35 \times 11} = \frac{17}{385} = \frac{11}{385} = \frac{1}{35} = 1\frac{1}{35}$.

$\frac{6\frac{1}{2}}{1\frac{1}{2} - 1\frac{1}{2}} = \frac{13}{0} = 0$.

(3)

$982\frac{1}{2} - 29\frac{1}{2} = 982\frac{340}{1740} - 29\frac{105}{1740} = 981 + 1\frac{340}{1740} - 29\frac{105}{1740} = 981\frac{235}{1740} - 29\frac{105}{1740} = 952\frac{130}{1740}$.

(4)

$69\frac{1}{2} - 18\frac{80}{146} = 69\frac{746}{146} - 18\frac{880}{146} = 68 + 1\frac{746}{146} - 18\frac{880}{146} = 68\frac{746}{146} - 18\frac{880}{146} = 50\frac{226}{146} = 50\frac{113}{73}$.

(5)

$100\frac{1}{2} - 9\frac{1}{2} = 100\frac{1}{2} - 9\frac{1}{2} = 99 + 1\frac{1}{2} - 9\frac{1}{2} = 99\frac{1}{2} - 9\frac{1}{2} = 90\frac{1}{2}$.

(6)

$\frac{1}{2}$ of $\frac{37}{4} = \frac{37}{8} = 4\frac{5}{8}$. $6\frac{1}{2} - 4\frac{5}{8} = 6\frac{4}{8} - 4\frac{5}{8} = 5 + 1\frac{4}{8} - 4\frac{5}{8} = 5\frac{1}{2} - 4\frac{5}{8} = 1\frac{1}{8}$.

(7)

$611\frac{13}{191} - 610\frac{198}{191} = 611\frac{8557}{38009} - 610\frac{38009}{38009} = 610 + 1\frac{8557}{38009} - 610\frac{38009}{38009} = 610\frac{8557}{38009} - 610\frac{38009}{38009} = 610\frac{4750}{38009} = 610\frac{475}{38009}$.

E

(2)

| qr. | na. | in. |
|-----|-------|----------------|
| = 2 | 1 | $1\frac{1}{2}$ |
| = | 2 | $1\frac{1}{2}$ |
| = | 3 | $0\frac{1}{2}$ |
| | <hr/> | |
| 3 | 3 | $1\frac{1}{2}$ |

(4)

| r. yds. | ft. | in. |
|---------|-------|----------------|
| 8 | 1 | 6 |
| 1 | 2 | $0\frac{1}{2}$ |
| | 1 | $2\frac{1}{2}$ |
| | <hr/> | |
| 0 | 0 | $3\frac{1}{2}$ |

(6)

| s. | d. |
|-----|-----------------|
| = 2 | $10\frac{1}{2}$ |
| = | $2\frac{1}{2}$ |
| = | $1\frac{1}{2}$ |
| | <hr/> |
| 3 | $1\frac{1}{2}$ |

(8)

$$\frac{5}{9} \text{ of } \frac{2}{3} = \frac{10}{9}. \quad \frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}. \quad \frac{7}{8} \text{ of } \frac{1}{4} = \frac{7}{32}.$$

$$\frac{10}{9} - \frac{1}{8} = \frac{80}{72} - \frac{9}{72} = \frac{71}{72}.$$

(9)

| | | |
|--------------------------|-----|-----------------|
| | oz. | dr. |
| $\frac{3}{8}$ of a lb. = | 10 | $10\frac{3}{4}$ |
| $\frac{8}{9}$ of a dr. = | | $\frac{8}{9}$ |
| | 10 | $9\frac{3}{4}$ |

(10)

$$24\frac{1}{2} - 21\frac{1}{2} = 24\frac{7}{16} - 21\frac{8}{16} =$$

$$23 + \frac{1}{16} - 21\frac{8}{16} = 23\frac{17}{16} -$$

$$21\frac{8}{16} = 2\frac{67}{16}.$$

(11)

| | | | | | |
|---------------------------|------|------|------|-----|-----|
| | fur. | per. | yds. | ft. | in. |
| $\frac{3}{8}$ of a mile = | 1 | 31 | 0 | 1 | 10 |
| $\frac{1}{7}$ of a fur. = | | 25 | 2 | 1 | 6 |
| | 1 | 5 | 3 | 1 | 10 |

(12)

$$7\frac{1}{2} - 1\frac{3}{4} = 7\frac{2}{4} - 1\frac{3}{4} = 6 + \frac{1}{4} - 1\frac{3}{4} = 6\frac{1}{4} - 1\frac{3}{4} = 5\frac{1}{2}.$$

$$\frac{1}{6} \text{ of } 57 = \frac{57}{6} = 9\frac{3}{2}.$$

(13)

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } \frac{3}{4} \text{ of } \frac{3}{4} \text{ of } \frac{3}{4} = \frac{3^3}{2^3} = \frac{1 \times 3 \times 2 \times 33 \times 62 \times 5}{2 \times 7 \times 9 \times 4 \times 33 \times 6} = \frac{155}{162}.$$

$$12\frac{319}{1764} + \frac{155}{162} = 12\frac{319}{1764} + \frac{1765}{1764} = 12\frac{4954}{1764} = 12\frac{2477}{882} = 12\frac{247}{88}.$$

$$\frac{12\frac{247}{88}}{12\frac{247}{88} - 10\frac{1}{2}} = \frac{12\frac{247}{88}}{12\frac{247}{88} - 10\frac{44}{88}} = \frac{12\frac{247}{88}}{2\frac{203}{88}} = \frac{247}{203} = 10\frac{1}{2}.$$

$$12\frac{247}{88} - 10\frac{1}{2} = 12\frac{247}{88} - 10\frac{44}{88} = 2\frac{203}{88}.$$

(14)

$$3\frac{1}{2} + 8\frac{1}{2} + 5\frac{1}{2} + 6\frac{1}{2} = 3 + 8 + 5 + 6 + (\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}) =$$

$$22 + (\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}).$$

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{180}{180} + \frac{180}{180} + \frac{180}{180} + \frac{180}{180} = \frac{720}{180}.$$

$$22 + \frac{720}{180} = 22\frac{720}{180}.$$

$$3\frac{3}{10} + 2\frac{5}{10} + 16\frac{1}{10} = 3 + 2 + 16 + (\frac{3}{10} + \frac{5}{10} + \frac{1}{10}) = 21 +$$

$$(\frac{3}{10} + \frac{5}{10} + \frac{1}{10}).$$

$$\frac{3}{10} + \frac{5}{10} + \frac{1}{10} = \frac{180}{180} + \frac{540}{180} + \frac{180}{180} = \frac{900}{180} = 5.$$

$$22\frac{720}{180} - 22\frac{360}{180} = 22\frac{360}{180} - 22\frac{360}{180} = \frac{0}{180} = 0.$$

$\frac{1}{2} = 1\frac{1}{2}$.

$\frac{7}{8} - \frac{21}{16} = \frac{14}{16} - \frac{21}{16} = -\frac{7}{16}$

1.
0
6
0

$\frac{12}{13} - \frac{1}{13} = \frac{11}{13}$

$\frac{1}{2} \times \frac{5}{6} \times \frac{7}{8} = \frac{35}{48}$

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6}{12} + \frac{4}{12} + \frac{3}{12} = \frac{13}{12}$

$\frac{161}{180}$

$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

(15)

| | | | | | |
|--------------------------|----|-------|------|-----|-------|
| | r. | per. | yds. | ft. | in. |
| $\frac{1}{4}$ of an acre | = | 1 | 18 | 5 | 4 72 |
| $\frac{1}{8}$ of a per. | = | | | 13 | 4 |
| | | <hr/> | | | |
| | | 1 | 17 | 22 | 2 108 |

(16)

$16\frac{1}{2} - 9\frac{1}{4} = 16\frac{2}{4} - 9\frac{1}{4} = 15 + 1\frac{1}{4} - 9\frac{1}{4} = 15\frac{1}{2} - 9\frac{1}{4} = 6\frac{1}{4}$

$169\frac{17}{100} - 83\frac{17}{100} = 169\frac{34}{100} - 83\frac{34}{100} = 168 + \frac{34}{100} - 83\frac{34}{100} = 85\frac{34}{100}$

EXERCISE 58—Page 173.

(1)

$\frac{7}{2}$ of $\frac{5}{6} = \frac{7 \times 5}{12 \times 6} = \frac{35}{72}$

(2)

$\frac{5}{8} \times \frac{4}{5} = \frac{20}{40} = \frac{1}{2}$

(3)

$\frac{4}{15} \times \frac{5}{24} = \frac{20}{360} = \frac{1}{18}$

(4)

$\frac{7}{8} \times \frac{5}{6} \times \frac{7}{8} = \frac{245}{384}$

(5)

$\frac{14}{1} \times \frac{241}{16} \times \frac{8\frac{1}{2}}{9} = \frac{14 \times 241 \times 17}{16 \times 9} = \frac{57418}{144} = 398.736$

(6)

$\frac{3}{10} \times \frac{7}{4} \times \frac{9}{11} \times \frac{11}{12} = \frac{3 \times 7 \times 9}{2 \times 4 \times 4} = \frac{189}{32} = 5\frac{29}{32}$

(7)

$\frac{4}{5} \times \frac{3}{11} \times \frac{9}{17} \times \frac{182}{200} \times \frac{5}{2} = \frac{3 \times 182}{11 \times 17 \times 25} = \frac{546}{4675}$

(8)

$$\frac{3}{8} \times \frac{11}{8} \times \frac{3}{8} \times \frac{3}{21} \times \frac{8}{1} \times \frac{5}{5} = \frac{3 \times 3 \times 3}{2} = \frac{27}{2} = 13\frac{1}{2}$$

(9)

$$\frac{2}{8} \times \frac{8}{5} \times \frac{6}{11} \times \frac{4}{19} \times \frac{11}{209} = \frac{2 \times 6 \times 4}{5} = \frac{48}{5} = 9\frac{3}{5}$$

(10)

$$\frac{18}{2} \times \frac{80}{7} \times \frac{180}{11} \times \frac{2}{18} \times \frac{7}{80} \times \frac{1}{90} = \frac{1}{11}$$

(11)

$$\frac{4}{7} \times \frac{3}{11} \times \frac{9}{16} \times \frac{7}{17} \times \frac{8}{1} \times \frac{8}{7} \times \frac{18}{91} \times \frac{167}{1} \times \frac{3 \times 9 \times 167}{24} = \frac{4500}{4} = 1125$$

(12)

$$\frac{1}{8} \times \frac{8}{12} \times \frac{94}{8} \times \frac{10}{101} \times \frac{1}{27} \times \frac{8}{8} =$$

$$\frac{1}{7 \times 8} \times \frac{8 \times 2}{7 \times 12} \times \frac{8 \times 9}{8 \times 8} \times \frac{10 \times 14}{101 \times 4} \times \frac{1}{9} \times \frac{8}{8} = \frac{1}{7 \times 101} = \frac{1}{707}$$

(13)

$$\frac{1}{4} \times \frac{2}{8} \times \frac{2}{7} \times \frac{19}{1} = \frac{2 \times 2 \times 19}{7} = \frac{76}{7} = 10\frac{6}{7}$$

(14)

$$\frac{9}{10} \times \frac{7}{5} \times \frac{11}{15} \times \frac{32}{11} = \frac{9 \times 7 \times 32}{5} = 302\frac{4}{5} = 403\frac{4}{5}$$

(15)

$$\frac{27}{4} \times \frac{7}{8} \times \frac{4}{5} \times \frac{4}{7} = 16 = 2\frac{7}{8}$$

(16)

$$\frac{11}{8} \times \frac{13}{8} \times \frac{15}{1} = \frac{11 \times 13 \times 15}{8} = 214\frac{1}{8} = 268\frac{1}{8}$$

(17)

$$\frac{1}{8} \times \frac{7}{4} \times \frac{8}{19} \times \frac{19}{2} \times \frac{11}{11} \times \frac{16}{17} \times \frac{49}{8} \times \frac{4}{5} \times \frac{27}{31} \times \frac{31}{2} \times \frac{191}{188} = \frac{7 \times 49 \times 27 \times 191}{2 \times 11 \times 17} = 1793\frac{51}{17} = 4729\frac{9}{17}$$

(18)

$$\frac{27}{37\frac{1}{2}} \times \frac{87\frac{1}{2}}{98\frac{1}{2}} \times \frac{7}{2\frac{1}{2}} \times \frac{81\frac{1}{2}}{128} = \frac{27}{189} \times \frac{785}{9 \times 785} \times \frac{7 \times 8}{8 \times 7} \times \frac{895 \times 1}{11 \times 128} = \frac{5}{3 \times 11} = \frac{5}{33}$$

(19)

$$\frac{\$ \frac{5}{11}}{11} \times \frac{1}{7} \times \frac{3}{5} \times \frac{17}{19} = \frac{3 \times 17}{11 \times 7} = \$\frac{51}{77}$$

(20)

$$\frac{75\frac{3}{4}}{61\frac{1}{4}} \times \frac{\frac{2}{3} \text{ of } 8\frac{1}{2} \times \frac{1}{15} \text{ of } 28}{\frac{2}{11} \text{ of } 6\frac{3}{8} \times \frac{1}{17} \text{ of } 24} \times \frac{7\frac{1}{2} \times \frac{2}{5}}{15 \times \frac{4}{5}} \times 14\frac{3}{4} \times \frac{100}{121} \times$$

$$\frac{4}{5\frac{1}{2}} \times \frac{5\frac{1}{2}}{9} =$$

$$\frac{69\frac{3}{4}}{67\frac{1}{4}} \times \frac{\frac{2}{3} \times \frac{23}{4} \times \frac{1}{15} \times 2\frac{2}{3}}{\frac{2}{11} \times \frac{51}{8} \times \frac{1}{17} \times 2\frac{2}{3}} \times \frac{3\frac{3}{8}}{1\frac{1}{2}} \times \frac{2}{4} \times \frac{101}{7} \times \frac{100}{121} \times$$

$$\frac{1}{3} \times \frac{4}{2} =$$

$$\frac{69\frac{3}{4} \times 11}{3 \times 67} \times \frac{11 \times 4 \times 3 \times 3 \times 2 \times 8 \times 11 \times 8 \times 17}{7 \times 4 \times 15 \times 2 \times 51 \times 24} \times \frac{9}{5 \times 15} \times \frac{3 \times 7}{4 \times 5} \times$$

$$\frac{101}{7} \times \frac{100}{121} \times \frac{4 \times 3}{16} \times \frac{5}{7 \times 9} =$$

$$\frac{11 \times 9 \times 101}{5 \times 7 \times 16} = \frac{9999}{280} = 17\frac{179}{280}$$

EXERCISE 59—Page 174.

(1)

$$\frac{13}{36} \text{ of } 4 \text{ days, } 5 \text{ hours, } = \frac{180 \text{ d. } 23 \text{ h.}}{36} = 5 \text{ d. } 0 \text{ h. } 38 \text{ min. } 20 \text{ sec.}$$

(2)

$$\frac{1}{42} \text{ of } £29 = \frac{£29 \times 13}{42} = \frac{£377}{42} = £8 \text{ } 19\text{s. } 6\frac{1}{2}\text{d.}$$

(3)

$$\frac{100}{121} \times \frac{7}{7} \text{ of } 186 \text{ a. } 3 \text{ r.} = \frac{186 \text{ a. } 3 \text{ r.} \times 7}{9} = \frac{1307 \text{ a. } 1 \text{ r.}}{9} = 145 \text{ a. } 1 \text{ r.}$$

(4)

$$\frac{1}{17} \text{ of } \frac{2}{7} \text{ of } \frac{1}{30} \text{ of } \frac{1}{2} \text{ of } 24 \text{ h. } 30 \text{ m.} = \frac{1}{15} \text{ of } 24 \text{ h. } 30 \text{ m.} = 1 \text{ h. } 38 \text{ m.}$$

(5)

$$\frac{3}{7} \text{ of } \frac{4}{9} \text{ of } \frac{2}{10} \text{ of } \frac{1}{7} \text{ of } 33 \text{ bu. } 2 \text{ p. } 1 \text{ ga.} = \frac{3}{90} \text{ of } 33 \text{ bu. } 2 \text{ p. } 1 \text{ ga.} = 33 \text{ bu. } 2 \text{ p. } 1 \text{ ga.} \times 7 = \frac{235 \text{ b. } 1 \text{ p. } 1 \text{ g.}}{90} = 2 \text{ b. } 2 \text{ p. } 0 \text{ g. } 3 \text{ q } 1 \frac{1}{2} \text{ p}$$

$$\times \frac{100}{121} \times$$

$$\times \frac{3 \times 7}{4 \times 5} \times$$

EXERCISE 60—Page 175.

(1)

$$\frac{1}{2} \text{ of } \frac{3}{4} \div \frac{1}{2} \text{ of } \frac{3}{4} = \frac{1}{2} \times \frac{3}{5} \times \frac{4}{3} \times \frac{4}{35} = \frac{2 \times 4}{5 \times 35} = \frac{8}{175}$$

(2)

$$\frac{1}{2} \div \frac{2}{3} \div \frac{1}{11} = \frac{5}{22} \times \frac{5}{3} \times \frac{11}{5} = \frac{5}{2 \times 3} = \frac{5}{6}$$

(3)

$$82 \frac{1}{7} \div 26 \frac{5}{11} = \frac{155}{189} \times \frac{41}{119} = \frac{155 \times 41}{17 \times 119} = \frac{6355}{2023} = 3 \frac{236}{2023}$$

min. 20 sec.

(4)

$$2 \frac{1}{2} \div \frac{1}{2} + \frac{5}{8} = \frac{5}{2} \div \frac{1}{2} = \frac{5}{2} \times \frac{4}{11} = \frac{5 \times 4}{11} = \frac{20}{11} = 1 \frac{9}{11}$$

(5)

$$1\frac{1}{2} \div \frac{1}{4} \text{ of } 2\frac{1}{2} \text{ of } 16 \text{ of } \frac{3}{4} \text{ of } \frac{7}{10} = \frac{7}{2} \times \frac{7}{1} \times \frac{4}{11} \times \frac{1}{16} \times \frac{4}{25} \times$$

$$\frac{2}{70} = \frac{7 \times 7}{2 \times 11} = \frac{49}{22} = 2\frac{5}{22}.$$

(6)

$$2\frac{1}{2} \div (\frac{1}{2} \div \frac{3}{12} \text{ of } 9) = \frac{7}{2} \div (\frac{1}{2} \text{ of } \frac{3}{2} \text{ of } \frac{1}{3}) = \frac{7}{2} \times \frac{9}{5} \times \frac{2}{16} \times$$

$$\frac{3}{1} = \frac{7 \times 9 \times 3 \times 3}{5 \times 16} = \frac{567}{80} = 7\frac{7}{80}.$$

(7)

$$48\frac{1}{2} \div \frac{2}{3} + \frac{3}{4} \text{ of } 6 = \frac{97}{2} \div \frac{2}{3} + \frac{3}{4} = \frac{97}{2} \div \frac{2}{3} = \frac{97}{2} \times \frac{3}{2} =$$

$$\frac{97 \times 18}{89} = \frac{1746}{89} = 19\frac{5}{89}.$$

(8)

$$6\frac{1}{2} \div \frac{3}{4} \text{ of } \frac{9}{10} + \frac{8}{17} = \frac{13}{2} \div \frac{27}{20} + \frac{8}{17} = \frac{13}{2} \div \frac{27}{20} = \frac{13}{2} \times \frac{20}{27} =$$

$$\frac{425}{859} = \frac{13 \times 425}{859} = \frac{5525}{859} = 6\frac{375}{859}.$$

(9)

$$\frac{2}{3} \times \frac{10}{9} \div \frac{4}{3} \times \frac{2}{5} = \frac{2}{3} \times \frac{10}{3} \times \frac{4}{9} \times \frac{4}{25} = \frac{4 \times 4}{3 \times 5} = \frac{16}{15} = 1\frac{1}{15}.$$

(10)

$$\frac{67}{35} \div \frac{7}{32} = \frac{67 \times 3}{9 \times 35} \div \frac{3 \times 8}{7 \times 33} = \frac{67 \times 3}{3 \times 35} \times \frac{7 \times 33}{3 \times 8} = \frac{67 \times 11}{3 \times 5 \times 8} = \frac{737}{120} = 6\frac{17}{120}$$

(11)

$$\frac{5}{11} \text{ of } \frac{10}{11} \div \frac{7}{11} \text{ of } \frac{10}{11} = \frac{5}{11} \times \frac{10}{11} \times \frac{11}{7} \times \frac{7}{61} = \frac{5 \times 10 \times 7}{9 \times 61} = \frac{350}{61}$$

(12)

$$\frac{15}{8} \text{ of } \frac{10}{8} \text{ of } \frac{3}{4} \text{ of } \frac{7}{5} \div \frac{5}{8} \text{ of } \frac{2}{3} \text{ of } \frac{3}{4} \text{ of } \frac{4}{5} = \frac{15}{8} \times \frac{10}{8} \times \frac{3}{4} \times \frac{7}{5} \times \frac{6}{5} \div \frac{5}{8} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{3 \times 2 \times 7 \times 6}{13 \times 5} = \frac{252}{65} = 3\frac{7}{65}$$

(13)

$$\frac{7}{2} \div \frac{7}{2} = \frac{7 \times 2}{4 \times 9} \div \frac{7 \times 4}{3 \times 9} = \frac{7 \times 2}{4 \times 9} \times \frac{3 \times 9}{7 \times 4} = \frac{3}{2 \times 4} = \frac{3}{8}$$

(14)

$$\frac{3}{5} \div \frac{21}{25} = \frac{3}{25} \div \frac{21 \times 2}{5 \times 35} = \frac{3}{25} \times \frac{5 \times 35}{21 \times 2} = 1$$

(15)

$$\frac{113}{135} \times \frac{1}{9} \div \frac{7}{3} \times \frac{107}{135} \times \frac{13}{135} = \frac{113}{8} \times \frac{1}{9} \times \frac{7}{3} \times \frac{13}{107} \times \frac{17}{135} = \frac{113 \times 2 \times 17}{9 \times 3 \times 107} = \frac{3842}{2889} = 1\frac{2553}{2889}$$

$$\frac{1}{16} \times \frac{4}{35} \times$$

$$\frac{3}{8} \times \frac{32}{16} \times$$

$$\frac{18}{86} \times \frac{89}{89} =$$

$$\frac{13}{2} \times$$

$$\frac{16}{15} = 1\frac{1}{15}$$

(16)

$$\frac{3\frac{1}{2}}{2} \times \frac{\frac{7}{9}}{2 \times 9} \times \frac{\frac{1}{7}}{7 \times 3} \times \frac{\frac{10}{7}}{7} \div \frac{\frac{41}{7}}{41} \times \frac{\frac{7}{3 \times 4}}{3 \times 4} \times \frac{\frac{7}{7 \times 2}}{7 \times 2} \times \frac{\frac{11}{11 \times 7}}{11 \times 7} =$$

$$\frac{3\frac{1}{2}}{2} \times \frac{7}{18} \times \frac{1}{21} \times \frac{10}{7} \div \frac{41}{7} \times \frac{7}{12} \times \frac{7}{14} \times \frac{11}{77} =$$

$$\frac{31}{2} \times \frac{2 \times 9}{7 \times 7} \times \frac{7 \times 9}{2} \times \frac{7}{10 \times 3} \div \frac{7}{9 \times 7} \times \frac{12}{19} \times \frac{7}{8 \times 7} \times \frac{7}{4 \times 4} =$$

$$\frac{31}{2} \times \frac{2 \times 9}{7 \times 7} \times \frac{7 \times 9}{2} \times \frac{7}{10 \times 3} \times \frac{9 \times 7}{41} \times \frac{19}{8 \times 4} \times \frac{7 \times 7}{7 \times 4} \times \frac{7 \times 4}{11 \times 7} =$$

$$\frac{31 \times 3 \times 9 \times 9 \times 4}{5 \times 41 \times 11} = \frac{63612}{2255} = 28\frac{172}{2255}$$

EXERCISE 61—Page 176.

(1)

$$\frac{1\frac{1}{2}}{\frac{4}{5}} = \frac{19 \times 3}{11 \times 5} = \frac{57}{55}. \quad \text{£8 14s. 6}\frac{1}{2}\text{d.} \div \frac{57}{55} = \text{£8 14s. 6}\frac{1}{2}\text{d.} \times \frac{55}{57} =$$

$$\frac{\text{£8 14s. 6}\frac{1}{2}\text{d.} \times 55}{57} = \text{£8 8s. 5}\frac{1}{2}\text{d.}$$

(2)

$$\frac{2\frac{3}{4} \times \frac{7}{11} = \frac{11\frac{1}{2}}{11}}{5 \text{ fur. 91 yds. 2 ft.} \times \frac{11\frac{1}{2}}{11}} = \frac{1 \text{ m. 5 fur. 91 yds. 2 ft.} \div \frac{11\frac{1}{2}}{11}}{1 \text{ m. 5 fur. 91 yds. 2 ft.} \times 22} = 2 \text{ fur. 124 yds. 2 ft}$$

115

(3)

$$3 \text{ a. 3 r. 3 per.} \div \frac{2}{3} = 3 \text{ a. 3 r. 3 p.} \times \frac{3}{2} = \frac{3 \text{ a. 3 r. 3 p.} \times 5}{3} =$$

6 a. 1 r. 5 per.

(4)

$$\text{£7 16s. 2d.} \div \frac{1}{4} = \text{£7 16s. 2d.} \times 4 = \frac{\text{£7 16s. 2d.} \times 9}{4} =$$

£17 11s. 4½d.

(3)

$$\frac{12\frac{1}{2}}{5\frac{1}{2}} = \frac{2\frac{1}{2}}{1\frac{1}{2}} = \frac{20}{12} = \frac{5}{3}.$$

$$\frac{2\frac{1}{2}}{1\frac{1}{2}} = \frac{5}{3} = \frac{10}{6} = \frac{10}{6} = \frac{5}{3}.$$

$$\frac{5}{4\frac{1}{2}} = \frac{2}{3} = \frac{4}{6} = \frac{4}{6} = \frac{2}{3}.$$

$$\frac{3\frac{1}{2}}{16\frac{1}{2}} = \frac{7}{33} = \frac{7}{33} = \frac{7}{33} = \frac{7}{33}.$$

$$\frac{220}{63} \div \frac{8}{9} = \frac{220}{63} \times \frac{9}{8} = \frac{55}{14} = 3\frac{1}{2}.$$

EXERCISE 63—Page 180.

(1)

$$\frac{800}{2000} = \frac{2}{5}, \quad \frac{420}{2000} = \frac{21}{100}, \quad \frac{100}{2000} = \frac{1}{20}, \quad \frac{160}{2000} = \frac{2}{25}.$$

$$\frac{35}{2000} = \frac{7}{400}.$$

(2)

$$\frac{2}{5} \text{ of } \frac{3}{4} \text{ of } \frac{2}{3} \text{ of } \frac{1}{4} \text{ of } \frac{1}{2} \text{ of } \frac{1}{5} = \frac{2}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{4} \times \frac{1}{2} \times \frac{1}{5} = \frac{1}{100}.$$

(3)

$$6\frac{1}{2} \times 65\frac{1}{2} \text{ cts.} = \frac{55}{2} \times 29\frac{1}{2} \text{ cts.} = 14\frac{1}{2} \times 59 \text{ cts.} = \$4.52\frac{1}{2}.$$

(4)

$$\frac{3}{8} + \frac{1}{17} = \frac{51}{136} + \frac{8}{136} = \frac{59}{136}$$

(5)

$$\frac{1}{3} + \frac{1}{10} + \frac{1}{8} + \frac{1}{6} = \frac{12}{120} + \frac{12}{120} + \frac{15}{120} + \frac{20}{120} = \frac{59}{120} = \frac{23}{10}$$

1 or $\frac{10}{10} - \frac{23}{10} = \frac{1}{10}$.

(6)

$$\frac{5\frac{1}{2} - 2\frac{1}{8}}{3\frac{3}{4} + \frac{2}{20}} \text{ of } \frac{4\frac{1}{2} + 5\frac{1}{2}}{4\frac{1}{10}} \text{ of } \frac{2\frac{3}{5} + 1\frac{3}{4}}{7\frac{1}{2} - 2\frac{1}{4}} = \frac{5\frac{3}{10} - 2\frac{1}{10}}{3\frac{1}{5} + \frac{2}{20}} \text{ of } \frac{4\frac{3}{5} + 5\frac{3}{5}}{\frac{8}{10}}$$

$$\frac{2\frac{2}{10} + 1\frac{1}{10}}{7\frac{1}{4} - 2\frac{1}{4}} = \frac{3\frac{3}{10}}{4\frac{1}{2}} \text{ of } \frac{10\frac{3}{10}}{\frac{8}{10}} \text{ of } \frac{4\frac{1}{5}}{5\frac{1}{2}} = \frac{1\frac{1}{10}}{\frac{8}{10}} \text{ of } \frac{5\frac{1}{10}}{\frac{8}{10}} \text{ of } \frac{9\frac{1}{4}}{1\frac{3}{4}} =$$

$$\frac{7}{8} \times \frac{2 \times 57}{5 \times 9} \times \frac{8 \times 64}{5 \times 188} = \frac{2 \times 64}{5 \times 3 \times 5} = \frac{128}{75} = 1\frac{53}{75}$$

(7)

$$1670\frac{7}{13} \times 12\frac{3}{4} \text{ cts.} = 217\frac{17}{13} \times 5\frac{1}{4} \text{ cts.} = 1107\frac{67}{62} \text{ cts.} = \$212.99\frac{1}{2}$$

(8)

$\frac{3}{4}$ of the longer = $\frac{3}{4}$ of the shorter; therefore $\frac{1}{3}$ of the longer = $\frac{1}{2}$ of $\frac{3}{4}$ = $\frac{1}{2}$ of the shorter.

Hence the longer = $\frac{3}{4} \times 3 = \frac{9}{4}$ of the shorter.

The whole tree = longer + shorter = $\frac{9}{4} + \frac{4}{4}$ of shorter = $1\frac{1}{4}$ of the shorter.

If 136 ft. = $\frac{1}{7}$ of the shorter, $\frac{1}{7}$ of 136 = 8 = $\frac{1}{3}$ of the shorter.

Hence shorter = $8 \times 8 = 64$ ft.; and longer = $136 - 64 = 72$ ft.

(9)

$$97\frac{1}{4} + 127\frac{3}{8} + 500\frac{3}{8} + 333\frac{1}{2} = 97\frac{30}{120} + 127\frac{48}{120} + 500\frac{150}{120} + 333\frac{120}{120} = 1057\frac{153}{120} = 1058\frac{1}{8}$$

$$\$1000 + \$1375\frac{1}{2} + \$6831 + \$4013\frac{3}{8} = \$1000 + \$1375\frac{8}{16} + \$6831 + \$4013\frac{6}{16} = \$13219\frac{14}{16} = \$13219.68\frac{1}{2}$$

$$\frac{4}{45} \times \frac{4}{45} = \frac{16}{2025}$$

$$\$4.52\frac{1}{3}$$

(10)

$$12\frac{2}{5} + 1\frac{2}{5} = 13\frac{4}{5}. \quad 8\frac{3}{4} + 1\frac{1}{4} = 9\frac{4}{4} = 9\frac{1}{1}. \quad 13\frac{1}{10} - 9\frac{7}{10} = 3\frac{4}{10} = \frac{2}{5}.$$

$$7\frac{5}{12} - 6\frac{1}{12} = 1\frac{4}{12} = \frac{1}{3}. \quad \frac{211}{100} \times \frac{2}{3} \times \frac{1}{12} = \frac{222}{1800} = 14\frac{1}{150}.$$

$$\frac{2}{3} \div \frac{1}{7} = \frac{2}{3} \times \frac{7}{1} = \frac{14}{3}. \quad \frac{5}{8} \div \frac{3}{11} = \frac{5}{8} \times \frac{11}{3} = \frac{55}{24}. \quad 7\frac{5}{8} - \frac{1}{8} = 7\frac{4}{8} = \frac{29}{4}.$$

(11)

$$19\frac{1}{2} \times \$6\frac{3}{4} = \frac{19}{2} \times \$\frac{27}{4} = \$\frac{513}{8} = \$134.15\frac{1}{2}.$$

(12)

$$376\frac{1}{8} \times \$75\frac{3}{8} = \frac{6779}{8} \times \$\frac{603}{8} = \frac{4087737}{64} = \$28387.06\frac{1}{4}.$$

(13)

$$147\frac{1}{2} + 320\frac{1}{2} = 147\frac{1}{2} + 320\frac{1}{2} = 467\frac{1}{2}. \quad 467\frac{1}{2} - 156\frac{1}{2} = 311.$$

$$467\frac{1}{2} - 156\frac{1}{2} = 311.$$

(14)

$$\frac{7 \left(1\frac{1}{2} \text{ of } \frac{3}{4}\right)}{\frac{1}{8} \left(\frac{3}{3\frac{1}{2}} \text{ of } 7\right)} \div 7\frac{1}{2} = \frac{7 \times \frac{3}{4} \times \frac{1}{2}}{\frac{1}{8} \times \frac{3}{\frac{7}{2}} \times 7} \div 6\frac{1}{2} = \frac{7 \times 3 \times 3}{1 \times 2 \times 4} \times \frac{8}{6\frac{1}{2}} =$$

$$\frac{7 \times 3 \times 3}{1 \times 2 \times 4} \times \frac{2}{8} = 1. \quad \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}{1 \quad 1 \quad 1} = \frac{\frac{6}{12} + \frac{4}{12} + \frac{3}{12}}{1 \quad 1 \quad 1} =$$

$$\frac{\frac{13}{12}}{\frac{13}{12}} = \frac{\frac{13}{12}}{\frac{13}{12}} = \frac{13}{12} = \frac{2535}{2176} = 1\frac{359}{2176}.$$

(15)

$$17\frac{1}{2} \div 7\frac{1}{2} = \frac{17}{2} \div \frac{7}{2} = \frac{17}{2} \times \frac{2}{7} = \frac{17}{7} = 2\frac{3}{7}.$$

(16)

3 3/4 + 4 1/4 + 4 1/4 = 3 3/4 + 4 1/4 + 4 1/4 = 12 3/4 = 7 3/4

7 7/8 - 5 3/8 = 7 14/16 - 5 6/16 = 2 8/16 = 1 1/2

94 1/2 + 93 1/2 = 94 9/16 + 93 9/16 = 187 18/16 = 187 9/8

7 3/8 x 2 1/2 ÷ 1 3/4 = (7 3/8 x 5/2) ÷ 7/4 = (36 15/8) x 4/7 = 21 15/14 = 15 3/7

(17)

2 2/3 + 1/3 + 4 = 2 2/3 + 1 1/3 + 4 = 7 3/3 = 8

2 ÷ 1 1/2 = 2 x 2/3 = 4/3 = 1 1/3

1 1/2 + 3/4 = 3/2 + 3/4 = 3 3/4

583 x 7/10 = 408 1/10

(18)

1/2 + 1/3 = 5/6

1 1/2 + 2 1/2 = 4 1/2 = 4 2/4

2 1/4 - 1 1/2 = 1 1/4 = 1 1/4

3 1/10 - 2 = 1 1/10 = 1 1/10

1 1/2 ÷ 2 1/2 = 3/4 x 2/3 = 1/2

5 1/2 ÷ 3 1/2 = 1 1/2 x 2/3 = 1 1/3

(19)

1 - (1/3 + 1/4) = 1 - 7/12 = 5/12

2/3 of 1/4 = 1/6

1/2 - 1/3 = 1/6

1/3 + 1/6 = 1/2

EXERCISE 66—Page 183.

(1)

(2)

1/2 = 2)1

2/3 = 3)2

5/6 = 6)5

1/4 = 4)1

.5

.375

.833 = 5/6

.25 = 1/4

(3)

| | | |
|---------------|----------------|----------------|
| 75)73 (.9733+ | 123)574(4.666+ | 34)15 (.44117+ |
| <u>67.5</u> | <u>492</u> | <u>13.6</u> |
| 5.50 | 82.0 | 1.40 |
| <u>5.25</u> | <u>73.8</u> | <u>1.36</u> |
| .250 | 8.20 | 40 |
| <u>.225</u> | <u>7.38</u> | <u>34</u> |
| 250 | .820 | 60 |
| <u>225</u> | <u>.738</u> | <u>34</u> |
| 25 | 82 | 260 |
| | | <u>238</u> |
| | | 22 |

(4)

| | | |
|-----------------|---------------|----------------|
| 7)6 | 12)5 | 9)4 |
| <u>.857142+</u> | <u>.4166+</u> | <u>.44444+</u> |

(5)

| | |
|-----------------------|--------------------|
| 112)17 (.15178571428+ | 1296)718 (.554012+ |
| <u>11.2</u> | <u>648.0</u> |
| 5.80 | 800 |
| <u>5.60</u> | <u>784</u> |
| .200 | 160 |
| <u>.112</u> | <u>112</u> |
| 880 | 480 |
| <u>784</u> | <u>448</u> |
| 960 | 320 |
| <u>896</u> | <u>224</u> |
| 640 | 960 |
| <u>560</u> | <u>896</u> |
| 800 | 64 |

EXERCISE 67—Page 184.

(1)

12)1·0 in.

3)2·083333 ft.

5½)3·694444 yd.
2 2

11) 7·388888

40)·671717 per.

·01679+ fur.

(2)

12)17·0 grs.

2)1·41666666

20)3·70833333 dwt.

12)·18541666 oz.

·01545138+ lb.

(3)

20)7·0 grs.

3)2·35 scr.

8)·7833333 dr.

12)·0979166 oz.

·0081597+ lb.

(4)

12)9·0 in

3)2·75 ft.

5½)2·91666
2 2

11)5·83333

40)35·53030 per.

8)5·88825 fur.

·73603+ mile.

(5)

4)2·0 na.

4)3·5 qr.

·875 yd.

(7)

60)21·0 sec.

60)55·35 min.

12)12·9225 hr.

2)1·076875

·5384375 day.

(6)

13s. 4d. = 160d.

5s. = 60d.

$\frac{60}{160} = \frac{3}{8} = \cdot375$

(8)

$\frac{7}{8}$ of $\frac{1}{2}$ of $6\frac{1}{2}$ d. = $\frac{7}{16}$ d. and $\pounds\frac{1}{3}$ = 80d. $\frac{2}{3}$ of $\frac{1}{2}$ of 1 mil. = 12672 in.

$\frac{2}{3}$ d. = $\frac{1}{150}$ of $\frac{2}{3}$ of $\pounds\frac{1}{3}$ = $\frac{27}{2240}$ of $\pounds\frac{1}{3}$.

$27 \div 2240 = 0 \cdot 012053.$

3½)12672

2 2

7)25344

3620·571428+

(10)

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } 1^3 \text{ lbs.} = \frac{1}{3} \text{ lb.} = 110\frac{1}{2} \text{ drs.} = 1664 \text{ drs.}$$

$$\frac{2}{3} \text{ of an oz.} = 12 \text{ drs. } 1664 \div 12 = 138\frac{4}{3}$$

$$180)1664(9 \cdot 2444+$$

1620

440

360

800

720

800

720

800

720

80

(11)

2)1·0 pts.

4)1·5 qt.

2)1·375 gal.

4)3·6875 pk.

·921875 bush.

EXERCISE 68—Page 186.

(1)

·3945
2415780
78909·4680 hrs.
6028·0800 min.
60

4·8000 sec.

(2)

·3965
83·1720 fur.
406·8800 per.
5½44000
44004·8400 yds.
32·5200 ft.
12

6·2400 in.

(3)

·309153
206·183060 dwt.
24732240
366120

4·393440 grs.

EXERCISE 71—Page 191.

(1)

$$\cdot\dot{8} = \frac{8}{9}$$

$$\cdot\ddot{05} = \frac{5}{99}$$

$$\cdot\dot{342} = \frac{342}{999} = \frac{38}{111}$$

$$\cdot\dot{7004} = \frac{7004}{9999}$$

$$\cdot\dot{002003} = \frac{2003}{999999}$$

(2)

$$\cdot\ddot{19} = \frac{19}{99}$$

$$\cdot\dot{1067} = \frac{1067}{9999} = \frac{97}{909}$$

$$\cdot\dot{11115} = \frac{11115}{99999} = \frac{1235}{11111}$$

$$\cdot\dot{704103} = \frac{704103}{999999} = \frac{234791}{333333}$$

(3)

$$\cdot\dot{102} = \frac{102}{999} = \frac{34}{333}$$

$$\cdot\dot{0013} = \frac{13}{9999}$$

$$\cdot\dot{00007103} = \frac{7103}{99999999}$$

$$\cdot\dot{01020304} = \frac{1020304}{99999999}$$

$$\cdot\dot{987654321} = \frac{987654321}{999999999} = \frac{109739369}{111111111}$$

EXERCISE 72—Page 192.

(1)

$$\begin{array}{r} \cdot\dot{8325} \\ \underline{83} \\ 8323 \\ 9988 \end{array} = \frac{4121}{4988}$$

$$\begin{array}{r} \cdot\dot{147658} \\ \underline{147} \\ 147511 \\ 999558 \end{array}$$

$$\begin{array}{r} \cdot\dot{4320075} \\ \underline{432} \\ 4319643 \\ 9999000 \end{array} = \frac{1439381}{3333000}$$

(2)

$$\begin{array}{r} 875 \cdot \dot{4965} \\ \underline{49} \\ 8754915 \\ 9990 \end{array} = 875\frac{1229}{2475}$$

$$\begin{array}{r} 301 \cdot \dot{82756} \\ \underline{82} \\ 30182374 \\ 99900 \end{array} = 301\frac{9186}{1100} = 301\frac{1531}{200}$$

(3)

$$\begin{array}{r} \cdot 083 \\ 8 \\ \hline \end{array} \quad \begin{array}{l} \cdot 714285 \\ 9999990 \\ \hline \end{array} = \begin{array}{l} \cdot 79365 \\ 1111110 \\ \hline \end{array} = \begin{array}{l} \cdot 7215 \\ 1010 \\ \hline \end{array} = \begin{array}{l} \cdot 1443 \\ 2020 \\ \hline \end{array} =$$

$$\frac{75}{900} = \frac{1}{12}$$

$$\begin{array}{r} \cdot 123456 \\ 123 \\ \hline \end{array} \quad \begin{array}{l} \cdot 123333 \\ 999000 \\ \hline \end{array} = \begin{array}{l} \cdot 41111 \\ 333000 \\ \hline \end{array}$$

(4)

$$\begin{array}{r} \cdot 7034 \\ 703 \\ \hline \end{array} \quad \begin{array}{r} \cdot 96432 \\ 96 \\ \hline \end{array} \quad \begin{array}{l} \cdot 96336 \\ 99900 \\ \hline \end{array} = \begin{array}{l} \cdot 10704 \\ 11100 \\ \hline \end{array} = \begin{array}{l} \cdot 3568 \\ 3700 \\ \hline \end{array} = \begin{array}{l} \cdot 892 \\ 900 \\ \hline \end{array}$$

$$\begin{array}{r} \cdot 00207 \\ 2 \\ \hline \end{array} \quad \begin{array}{l} \cdot 143271 \\ 1432 \\ \hline \end{array} \quad \begin{array}{l} \cdot 141839 \\ 99000 \\ \hline \end{array}$$

EXERCISE 73—Page 194.

(1)

Dissimilar. Similar. Similar and Coterminous.

$$\begin{array}{l} \cdot 9 \\ 6 \cdot 327 \\ 19 \cdot 43 \\ 27 \cdot 0278 \\ \cdot 0347123 \end{array} = \begin{array}{l} \cdot 99999 \\ 6 \cdot 327272 \\ 19 \cdot 43000 \\ 27 \cdot 027878 \\ \cdot 0347123 \end{array} = \begin{array}{l} \cdot 999999999 \\ 6 \cdot 3272727272 \\ 19 \cdot 430000000 \\ 27 \cdot 0278787878 \\ \cdot 0347123123 \end{array}$$

2 carried.

$$\text{Sum,} = 53 \cdot 8198638274$$

(2)

| Dissimilar. | Similar. | Similar and Coterminous. |
|---------------|---------------|--------------------------|
| 7·427̇ = | 7·42727̇ = | 7·4272727272727̇ |
| 9·1234̇ = | 9·123423̇ = | 9·123423423423423̇ |
| 17·2987643̇ = | 17·2987643̇ = | 17·298764376437643̇ |
| 18·67̇ = | 18·6767̇ = | 18·67676767676767̇ |
| | | 2 carried |
| | Sum, = | 52·526228203901471 |

(3)

| Dissimilar. | Similar. | Similar and Coterminous. |
|-------------|--------------|--------------------------|
| 4·95̇ = | 4·959595̇ = | 4·9595959595̇ |
| 7·164̇ = | 7·1641641̇ = | 7·1641641641̇ |
| 4·7123̇ = | 4·7123123̇ = | 4·7123123123̇ |
| ·97317̇ = | ·97317̇ = | ·9731777777̇ |
| | | 2 carried. |
| | Sum, = | 17·8092502138 |

(4)

| Dissimilar. | Similar. | Similar and Coterminous |
|-------------|----------------|-------------------------|
| 1·5 = | 1·5000̇ = | 1·500000000̇ |
| 99·083 = | 99·0830̇ = | 99·083000000̇ |
| ·162 = | ·162162̇ = | ·162162162̇ |
| ·814 = | ·814814̇ = | ·814814814̇ |
| 2·93 = | 2·93939̇ = | 2·939393939̇ |
| 3·769230̇ = | 3·769230769̇ = | 3·769230769̇ |
| 97·26 = | 97·2666̇ = | 97·26666666̇ |
| 134·09̇ = | 134·09090̇ = | 134·090909090̇ |
| | | 3 carried. |
| | Sum, = | 339·626177443 |

oterminous.

2727

3423

7643

7676

2 carried

1471

erminous.

carried.

oterminous

00

00

32

4

9

9

6

0

3 carried.

3

EXERCISE 74—Page 195.

(1)

| Dissimilar. | | Similar. | | Similar and Coterminous. |
|-------------|---|-------------|---|--------------------------|
| 729·3427̇ | = | 729·342742̇ | = | 729·342742̇ |
| 93·126 | = | 93·1260̇ | = | <u>93·126000̇</u> |
| | | | | <u>636·216742̇</u> |

(2)

| Dissimilar. | | Similar. | | Similar and Coterminous. |
|-------------|---|-------------|---|--------------------------|
| 1·437291̇ | = | 1·43729137̇ | = | 1·4372913729137̇ |
| ·00713̇ | = | ·00713̇ | = | <u>·0071313131313̇</u> |
| | | | | 1·4301600597824̇ |

(3)

| Dissimilar. | | Similar. | | Similar and Coterminous |
|-------------|---|----------|---|-------------------------|
| 1·12754̇ | = | 1·12754̇ | = | 1·12754754754754̇ |
| ·47384̇ | = | ·473847̇ | = | <u>·47384738473847̇</u> |
| | | | | ·65370016280907̇ |

(4)

| Dissimilar. | | Similar. | | Similar and Coterminous. |
|----------------|---|----------------|---|--------------------------|
| 42·18763̇ | = | 42·1876333̇ | = | 42·1876333333̇ |
| 17·0000008432̇ | = | 17·0000008432̇ | = | <u>17·0000008432̇</u> |
| | | | | 25·1876324900̇ |

EXERCISE 75—Page 196.

(1)

$$2\dot{9} = 2\frac{9}{9} = 3. \quad 7\cdot 25 \times 3 = 21\cdot 75.$$

(2)

$$\dot{2}97 = \frac{297}{999} = \frac{1}{3} \text{ and } 7\cdot 72 = 7\frac{72}{100} = 7\frac{18}{25} = \frac{193}{25}.$$

$$\frac{1}{3} \times \frac{193}{25} = \frac{2123}{925} = 2\cdot 29513.$$

(3)

$$\dot{8}18 = \frac{818}{999} = \frac{9}{11} \text{ and } \cdot 77 = \frac{77}{100}. \quad \frac{9}{11} \times \frac{77}{100} = \frac{63}{100} = \cdot 63$$

(4)

$$1\cdot 735 = \frac{1735}{999} = \frac{1364}{999} = \frac{352}{250} \text{ and } \cdot 47053 = \frac{42348}{9999} = \frac{3529}{2500}.$$

$$\frac{352}{250} \times \frac{3529}{2500} = \frac{3031411}{3712500} = \cdot 81654168350$$

(5)

$$4\cdot 722 = \frac{4650}{999} = 4\frac{13}{99} = \frac{85}{18} \text{ and } \cdot 198 = \frac{198}{999} = \frac{22}{111}.$$

$$\frac{85}{18} \times \frac{22}{111} = \frac{935}{999} = \cdot 935.$$

Exercise 76—Page 196.

(1)

$$\dot{0}82 = \frac{82}{999} \text{ and } \dot{1}23 = \frac{123}{999} = \frac{41}{333}.$$

$$\frac{82}{999} \div \frac{41}{333} = \frac{82}{999} \times \frac{333}{41} = \frac{2}{3} = \cdot 6.$$

(2)

$$389\cdot 185 = \frac{389185}{999} = \frac{388796}{999} \text{ and } 15\cdot 7 = 15\frac{7}{10} = \frac{142}{10}.$$

$$\frac{388796}{999} \div \frac{142}{10} = \frac{388796}{999} \times \frac{10}{142} = \frac{2738}{111} = 24\cdot 6.$$

(3)

$$\cdot 81654168350 = \frac{81654086696}{99999990000} = \frac{10206760837}{12499987500}.$$

$$\cdot 47053 = \frac{42348}{90000} = \frac{10587}{22500}.$$

$$\frac{10206760837}{12499987500} \div \frac{10587}{22500} = \frac{10206760837}{12499987500} \times \frac{22500}{10587} = \frac{10206760837}{6881660785} =$$

1\cdot 735.

(4)

$$\ddot{.45} = \frac{45}{100} = \frac{9}{20} \text{ and } \dot{.118881} = \frac{118881}{1000000} = \frac{17}{143}$$

$$\frac{9}{20} \div \frac{17}{143} = \frac{9}{20} \times \frac{143}{17} = \frac{64}{19} = 3.\dot{8}235294117647058.$$

EXERCISE 77.

(1)

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } \frac{1}{4} \text{ of } 14 = \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} \times 14 = \frac{7}{3} = 2\frac{1}{3}$$

(2)

$$\begin{aligned} \cdot 67 &= \frac{67}{100} \text{ and } 2.\dot{13} = \frac{213}{100} = \frac{211}{99} \\ \frac{67}{100} \times \frac{211}{99} &= \frac{14137}{9900} = 1.4445566778 + \end{aligned}$$

(3)

wk.

$$\cdot 678125 = 4 \text{ days } 17 \text{ hours } 55 \text{ minutes } 30 \text{ seconds.}$$

7

$$4 \cdot 746875 \text{ days.}$$

24

$$2987500$$

$$1493750$$

$$17 \cdot 925000 \text{ hours.}$$

60

$$55 \cdot 500000 \text{ minutes.}$$

60

$$30 \cdot 000000 \text{ seconds.}$$

(4)

$$\cdot 92437$$

92

$$\cdot 92437 = \frac{92437}{100000} = \frac{19499}{10000}$$

(5)

| Dissimilar. | Similar. | Similar and Coterminous. |
|-------------|----------|--------------------------|
|-------------|----------|--------------------------|

$$67\cdot23\dot{4} = 67\cdot234343\dot{4} = 67\cdot234343434\dot{4}$$

$$98\cdot71\dot{3} = 98\cdot713713\dot{7} = 98\cdot713713713\dot{7}$$

$$91\cdot0347123\dot{4} = 91\cdot03471234 = 91\cdot0347123423\dot{4}$$

$$\text{Sum,} = 256\cdot9827694903\dot{9}$$

| Dissimilar. | Similar and Coterminous. |
|-------------|--------------------------|
|-------------|--------------------------|

$$256\cdot9827694903\dot{9} = 256\cdot9827694903\dot{9}$$

$$100\cdot12345678\dot{9} = 100\cdot1234567894\dot{5}$$

$$\text{Difference} = 156\cdot8593127009\dot{4}$$

(6)

$$12) 9 \text{ in.}$$

$$3) 2\cdot75 \text{ ft.}$$

$$5\frac{1}{2}) 2\cdot916 \text{ yds.}$$

$$\underline{2} \quad \underline{2}$$

$$11) 5\cdot833$$

$$40) 36\cdot5303 \text{ rds.}$$

$$8) 5\cdot913257 \text{ fur.}$$

$$\cdot739157196 \text{ miles.}$$

(7)

$$17\cdot428571 \text{ sq. ft.} = 17\frac{3}{7} \text{ sq. ft.} = 17\frac{3}{7} \text{ sq. ft.} = 17 \text{ sq. ft. } 61\frac{1}{7} \text{ in}$$

$$100\cdot8 \text{ sq. in.} = 100\frac{4}{5}$$

$$\text{Difference,} = 16 \text{ sq. ft. } 104\frac{2}{3} \text{ in.}$$

Coterminous.

43434

71371

44234

9039

inuous.

9

5

4

$$\begin{array}{r} \textcircled{9} \\ \cdot 91789772 \\ \underline{917897} \\ \hline \end{array}$$

$$\cdot 91789772 \text{ of } 2 \text{ a.} = \frac{99871875}{99000000} \times \frac{2}{1} \text{ a.} = \frac{99871875}{49500000} = \frac{3231}{1760} = 1 \text{ a. } 3 \text{ r. } 13 \text{ per. } 22 \text{ yds.}$$

$$\begin{array}{r} \textcircled{9} \\ 11 \cdot 287 \\ \underline{2} \end{array}$$

$$11 \cdot 287 = \frac{11287}{990} = \frac{1119}{88} \quad 1 \cdot 0428571 = \frac{1428571}{9999990} = \frac{17}{90}$$

(10)

$$47 \cdot 345 = \frac{47345}{1000} \text{ and } 1 \cdot 76 = \frac{176}{99} = \frac{175}{99}$$

$$\frac{47345}{1000} \div \frac{175}{99} = \frac{47345}{1000} \times \frac{99}{176} = \frac{937431}{980000} = 26 \cdot 7837428571$$

(11)

Dissimilar. Similar. Similar and Coterminous.

$$85 \cdot 62 = 85 \cdot 626 = 85 \cdot 62626$$

$$13 \cdot 76432 = 13 \cdot 76432 = \underline{13 \cdot 76432}$$

$$\text{Difference,} = 71 \cdot 86193$$

(12)

(13)

$$\cdot 734 \text{ of a lb.} = 11 \cdot 744 \text{ oz.} \quad 2 \text{ ft. } 5\frac{1}{2} \text{ in.} = 29\frac{1}{2} \text{ in.} = \frac{59}{2} \text{ in.}$$

$$\cdot 198 \text{ of an oz.} = .198 \text{ oz.}$$

$$27 \cdot 3 \text{ ft.} = 27\frac{3}{4} \text{ ft.} = 328 \text{ in}$$

$$\text{Difference,} = 11 \cdot 546 \text{ oz.}$$

$$20 \cdot 16 \text{ ft.} = 20\frac{1}{6} \text{ ft.} = 242 \text{ in.}$$

$$328 \times 242 \div \frac{59}{2} = \frac{328}{1} \times \frac{242}{1} \times \frac{2}{59} = 2706 \text{ in.} = 75\frac{1}{2} \text{ yds.}$$

(14)

$$3 \cdot 145 = \frac{3145}{990} = \frac{38}{99} = \frac{173}{66} \text{ and } 4 \cdot 297 = \frac{4297}{999} = \frac{411}{87} = \frac{159}{37}$$

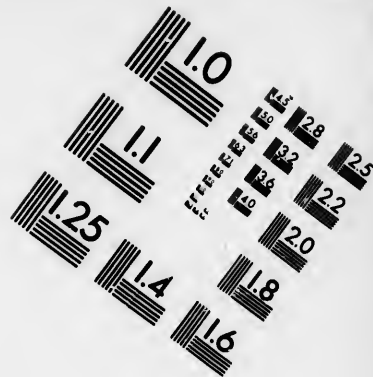
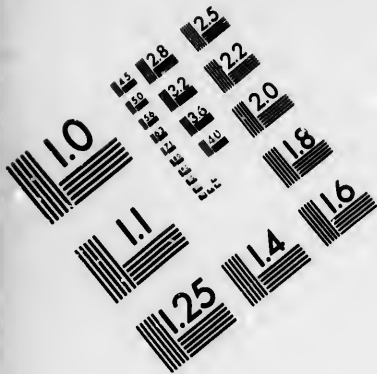
$$\frac{173}{66} \times \frac{159}{37} = \frac{27507}{2442} = 13 \cdot 5169533$$

ft. 61 $\frac{1}{2}$ in

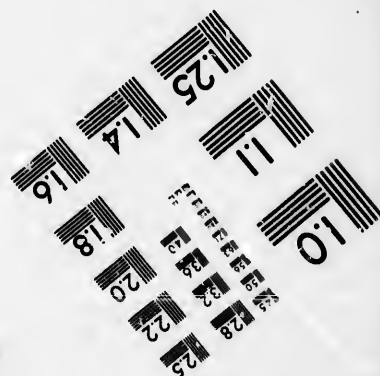
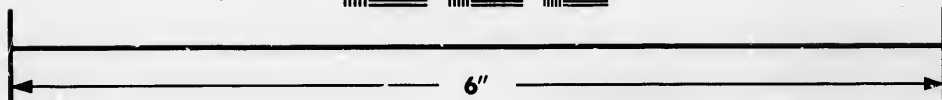
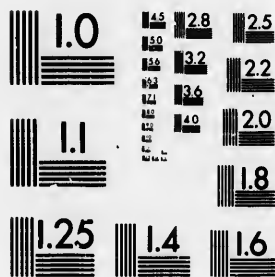
100 $\frac{8}{9}$

t. 104 $\frac{2}{3}$ in.





**IMAGE EVALUATION
TEST TARGET (MT-3)**



**Photographic
Sciences
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WEBSTER, N.Y. 14580
(716) 872-4503



(15)

$40 = 2^3 \times 5$. Therefore the equivalent decimal will contain 3 places

| | | | | | | | | | |
|--------------------------------|---|------------------------|---|---|---|---|---|---|---|
| $\frac{7}{40}$ | " | $24 = 2^3 \times 3$ | " | " | " | " | " | 3 | " |
| $\frac{8}{40}$ | " | $15 = 5 \times 3$ | " | " | " | " | " | 1 | " |
| $\frac{11}{40}$ | " | $144 = 2^4 \times 3^2$ | " | " | " | " | " | 4 | " |
| $\frac{11}{90} = \frac{1}{15}$ | " | $15 = 5 \times 3$ | " | " | " | " | " | 1 | " |
| $\frac{119}{3584}$ | " | $3584 = 2^9 \times 7$ | " | " | " | " | " | 9 | " |

(16)

$81\dot{3} = 81\cdot\dot{6}$ and $328\dot{3}\dot{3} = 328\cdot\dot{2}\dot{3}$.

| Dissimilar. | | Similar | | Similar and Coterminous |
|--------------------------------|---|---|---|---|
| $81\cdot\dot{6}$ | = | $81\cdot\dot{6}\dot{6}\dot{6}$ | = | $81\cdot\dot{6}\dot{6}\dot{6}\dot{6}\dot{6}\dot{6}$ |
| $61\cdot\dot{1}\dot{2}\dot{6}$ | = | $61\cdot\dot{1}\dot{2}\dot{6}$ | = | $61\cdot\dot{1}\dot{2}\dot{3}\dot{6}\dot{6}\dot{6}\dot{6}$ |
| $328\cdot\dot{2}\dot{3}$ | = | $328\cdot\dot{2}\dot{3}\dot{2}\dot{3}$ | = | $328\cdot\dot{2}\dot{3}\dot{2}\dot{3}\dot{2}\dot{3}\dot{2}\dot{3}$ |
| $5\cdot\dot{6}\dot{2}\dot{4}$ | = | $5\cdot\dot{6}\dot{2}\dot{4}\dot{6}\dot{2}$ | = | $5\cdot\dot{6}\dot{2}\dot{4}\dot{6}\dot{2}\dot{4}\dot{6}\dot{2}$
2 carried |
| Sum, | | | | $= 476\cdot\dot{6}\dot{5}\dot{0}\dot{2}\dot{8}\dot{1}\dot{1}\dot{9}$ |

(17)

$$\begin{aligned} & \left(\frac{4\cdot\dot{4} - 2\cdot\dot{8}\dot{3}}{1\cdot\dot{6} + 2\cdot\dot{6}\dot{2}\dot{9}} \times \frac{6\cdot\dot{8} \times 3}{2\cdot\dot{2}\dot{5}} \right) + \frac{2\cdot\dot{8} \times 2\cdot\dot{2}\dot{7}}{1\cdot\dot{1}\dot{3}\dot{6}} \\ &= \left(\frac{1\cdot\dot{6}\dot{1}}{4\cdot\dot{2}\dot{9}\dot{6}} \times \frac{20\cdot\dot{4}}{2\cdot\dot{2}\dot{5}} \right) + \frac{2\dot{1}\dot{6} \times 2\dot{2}\dot{7}}{1\dot{3}\dot{6}\dot{6}} \\ &= \left(\frac{1\dot{5}\dot{6}}{4\dot{2}\dot{9}\dot{6}} \times \frac{20\dot{4}}{2\dot{4}} \right) + \frac{2\dot{1}\dot{6} \times 2\cdot\dot{2}\dot{7}}{1\dot{3}\dot{6}} \\ &= \left(\frac{1\dot{1}\dot{1}}{4\dot{2}\dot{9}\dot{6}} \times \frac{10\dot{2}}{2} \right) + \frac{1\dot{1} \times 2\dot{1}\dot{7}}{2\dot{2}} \\ &= \left(\frac{2\dot{2}}{4\dot{2}\dot{9}\dot{6}} \times \frac{3\dot{4}}{2} \right) + \frac{1\dot{1} \times 2\dot{1}\dot{7}}{2\dot{2}} \\ &= \left(\frac{1}{1\dot{1}\dot{1}} \times \frac{3\dot{4}}{2} \right) + \frac{2\dot{1}\dot{7}}{2\dot{2}} = \left(\frac{1\dot{1}\dot{1}}{2\dot{2}\dot{2}} \times \frac{1\dot{3}\dot{6}}{1\dot{3}\dot{6}} \right) + \frac{2\dot{1}\dot{7}}{2\dot{2}} \\ &= \left(\frac{1}{3} \times \frac{1\dot{3}\dot{6}}{1\dot{3}\dot{6}} \right) + \frac{2\dot{1}\dot{7}}{2\dot{2}} = \frac{1\dot{7}}{2} + \frac{2\dot{1}\dot{7}}{2\dot{2}} = \frac{4\dot{5}}{2} = 9. \end{aligned}$$

EXERCISE 78—Page 198.

(1)

valent decimal
 contain 3 places
 " 3 "
 " 1 "
 " 4 "
 " 1 "
 " 9 "

| | | |
|-------------|--------------|-------------|
| ▼ | ▼ | ▼ |
| 9)4312131 | 3)4312131 | 8)4312131 |
| 9)224322..8 | 3)1234023..2 | 8)242343..7 |
| 9)12043..5 | 3)224322..2 | 8)14022..2 |
| 9)344..7 | 3)41240..2 | 8)1032..1 |
| 9)21..0 | 3)12043..1 | 8)32..6 |
| 1..2 | 3)2144..1 | 2..1 |
| | 3)344..2 | |
| | 3)113..0 | |
| | 3)21..0 | |
| | 3)3..2 | |
| | 1..0 | |

d Cotermious
 666666
 366666
 232323
 462462
 2 carried
 028119

2:27
 136
 7
 9
 3
 11
 4
 1
) + 2/5
 4/5 = 9.

| | | | |
|----------------|------------|---------------|--------|
| ▼ | IX | III | VIII |
| 4312131 = | 120758 = | 10200211222 = | 216127 |
| 5 | 9 | 3 | 8 |
| 23 | 11 | 3 | 299 |
| 5 | 9 | 3 | 3 |
| 116 | 99 | 11 | 898 |
| 5 | 9 | 3 | 3 |
| 582 | 898 | 33 | 2695 |
| 5 | 9 | 3 | 3 |
| 2911 | 8087 | 99 | 8087 |
| 5 | 9 | 3 | 3 |
| 14558 | 72791 dec. | 299 | 24263 |
| 5 | | | 3 |
| 72791 decimal. | | 72791 dec. | |

(3)

$$976 \cdot 432 \div -00000096 = 97643200000 \div 96 \text{ and } 96 = 12 \times 8.$$

$$\underline{12)97643200000}$$

$$\underline{8)8136933333 \cdot 3}$$

$$1017116666 \cdot 6$$

(4)

$$(2\frac{7}{8} + \cdot 5625 - 15 + \frac{1}{16}) \div \frac{1}{11} =$$

$$\underline{(1\frac{14}{16} \times \frac{1}{2} \times 296 \times \frac{1}{101} \div \frac{1}{8}) \div \cdot 9472947} =$$

$$\frac{(2\frac{7}{8} + \frac{9}{16} - 1\frac{1}{2} + \frac{1}{16}) \times \frac{1}{11}}{\frac{1}{2} \times \frac{1}{11}}$$

$$\frac{(1\frac{14}{16} \times \frac{1}{2} \times 296 \times \frac{1}{101} \times \frac{1}{11}) \div \frac{1}{8}}{1\frac{14}{16} \times \frac{1}{2} \times 296 \times \frac{1}{101} \times \frac{1}{11} \times \frac{1}{8}}$$

$$\frac{\frac{11}{16}}{\frac{11}{16}} = \frac{11}{11} = \frac{16}{16} = 2\frac{3}{4}$$

(5)

| lbs. | oz. | dr. | scr. | lbs. | oz. | dr. | scr. | grs. |
|-------|-----|-----|------|--------|----------|-----|------|-------|
| 9 | 7 | 7 | 2 | 97 | 3 | 4 | 1 | 17 |
| 12 | | | | 12 | | | | |
| 115 | | | | 1167 | | | | |
| 8 | | | | 8 | | | | |
| 927 | | | | 9340 | | | | |
| 3 | | | | 3 | | | | |
| 2783 | | | | 28021 | | | | |
| 20 | | | | 20 | | | | |
| 55660 |) | | | 560437 | (10,3837 | | | 55660 |
| | | | | 55660 | 55660 | | | |
| | | | | 3837 | | | | |

(6)

$196 = 12 \times 8.$

$15 \text{ yds.} = 540 \text{ in. and } 7 \text{ ft.} = 84 \text{ in.}$

$6 \text{ ft.} = 72 \text{ in. and } 4 \text{ ft.} = 48 \text{ in.}$

$540 \times 84 \times 13) - (72 \times 48 \times 13) = 589680 - 44928 = 544752.$

$544752 \div 108 = 5044.$

(7)

| | | | | | |
|----------|-----|------|--------|---------|-----------|
| 9 ft. 6' | 4'' | 7''' | | | |
| 11 | 7 | 9 | 11 | | |
| | | | | | |
| | 8 | 8 | 10'''' | 2'''''' | 5'''''''' |
| | 7 | 1 | 9 | 5 | 3 |
| | 5 | 6 | 8 | 8 | 1 |
| 104 | 10 | 2 | 5 | | |
| | | | | | |
| 111 | 0 | 9 | 7 | 4 | 5 5 |

(8)

$$\frac{47 + \frac{8}{13} - \frac{7}{13}}{\frac{2}{3} \text{ of } \frac{8}{13} + \frac{1}{6} \text{ of } \frac{8}{13}} = \frac{47\frac{8}{13} - \frac{7}{13}}{\frac{8}{13} + \frac{8}{24}} = \frac{\frac{1157}{13} - \frac{7}{13}}{\frac{262}{13}} = \frac{1150}{262} = \frac{575}{131} = 8\frac{54}{131}$$

(9)

(10)

| | | | | | | | | | | | | | | | | | | | | |
|------------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| pts. | 77 | 77 | .. | 42 | .. | 27 | .. | 21 | .. | 33 | .. | 14 | .. | 7 | .. | 11 | .. | 63 | .. | 30 |
| 2) 782436 | 27 | 6 | .. | 27 | .. | 3 | .. | 3 | .. | 2 | .. | 9 | .. | 30 | | | | | | |
| 4) 391218..0 pt. | 10 | 2 | | | | | | | | | | | | | 2 | | | 19 | | |
| 2) 97804..2 qt. | | | | | | | | | | | | | | | | | | | | |
| 4) 48902..0 gal. | | | | | | | | | | | | | | | | | | | | |

$77 \times 27 \times 10 = 20790 = \text{L. c. m.}$

- 12225..2 pks.
- 12225 bush. 2 pks. 0 gal. 2 qts.

(11)

| | |
|------------------------|----------|
| XII | IX |
| 28e4)36t87942(1375t·12 | 3762814 |
| <u>28e4</u> | <u>9</u> |
| 9e47 | 34 |
| <u>82t0</u> | <u>9</u> |
| 18679 | 312 |
| <u>17274</u> | <u>9</u> |
| 14054 | 2810 |
| <u>11888</u> | <u>9</u> |
| 23882 | 25298 |
| <u>23554</u> | <u>9</u> |
| 32t·0 | 227683 |
| <u>28e·4</u> | <u>9</u> |
| 5t·80 | 2049151 |
| <u>55·t8</u> | |
| 4·94 | |

(12)

150528 = 2¹⁰ × 3 × 7².

10 + 1 = 11.
 1 + 1 = 2
 2 + 1 = 3
 11 × 3 × 2 = 66.

(13)

2 wks. 2 dys. = 16 dys.

1234625
 16

7407750
 1234625

1·9754000 dys.
 24

39016000
 19508000

23·4096000 hrs.
 60

24·5760000 min.
 60

34·5600000 = 34 1/2 sec.

(14)

728 1/2 = 8 1/2 + 2 × 10 + 7 × 10 × 10.

lbs. oz. dr.
 27 4 3 × 8 1/2 =

272 9 14 × 2 =
 10

2726 2 12 × 7 =

lbs. oz. dr.
 231 11 9 1/2

545 3 12

19083 3 4

19860 2 9 1/2

(15)

$$\begin{aligned} \text{£}16\ 3\text{s.}\ 8\text{d.} &= \$64.74\frac{1}{2}, \text{ and } \text{£}67\ 17\text{s.}\ 7\text{d.} = \$271.52\frac{1}{2}. \\ \$98.17 + \$42.29 + \$64.74\frac{1}{2} + \$97.19 + \$127.87\frac{1}{2} &= \$430.27\frac{1}{2} \\ \$430.27\frac{1}{2} - \$271.52\frac{1}{2} &= \$158.75. \end{aligned}$$

(16)

$$\begin{aligned} .8 = \frac{8}{10}, \quad .76 = \frac{76}{100}, \quad .9123 &= \frac{9123-91}{9900} = \frac{9032}{9900} = \frac{2258}{2475}. \\ .003327 &= \frac{3327-3}{999000} = \frac{3324}{999000} = \frac{277}{83250}. \end{aligned}$$

(18)

$$\begin{aligned} & \left[(2\frac{1}{2} \times .5 \text{ of } 1\frac{1}{2}) + 9\frac{1}{2} + .09 + \frac{23}{31} \right] - 11\frac{1}{7} \div (\frac{1}{2} \text{ of } .16) \\ & \left[(.7632763 \times 11) \times \frac{1}{2} \text{ of } \frac{10}{11} \right] \times (\frac{1}{2} \text{ of } 2 \text{ of } 3 \text{ of } 25 \text{ of } 96) \div 2 \\ & \frac{1}{2} \text{ of } .6732467 \div \frac{1}{2} \\ & \left[\left(\frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} \right) + 9\frac{1}{2} + \frac{1}{7} + \frac{23}{31} \right] - 11\frac{1}{7} \div (\frac{1}{2} \text{ of } \frac{1}{2}) \\ & \left(\frac{1}{9999} \times \frac{1}{4} \times \frac{1}{2} \times \frac{1}{108} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right) \div \frac{1}{2} \\ & \frac{1}{2} \times \frac{1}{99999} \div \frac{1}{2} \\ & \left(2 + 9\frac{1}{2} + \frac{1}{7} + \frac{23}{31} \right) - 11\frac{1}{7} \div (\frac{1}{2} \times \frac{1}{2}) \\ & \frac{1}{9999} \times \frac{1}{4} \times \frac{1}{2} \times \frac{1}{108} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ & \frac{1}{2} \times \frac{1}{99999} \times \frac{1}{2} \\ & \frac{(12 - 11\frac{1}{7}) \div (\frac{1}{2} \times \frac{1}{2})}{\frac{1}{6}} = \frac{\frac{1}{7} \times \frac{1}{2} \times \frac{1}{2}}{\frac{1}{6}} = \frac{\frac{1}{6}}{\frac{1}{6}} = \frac{\frac{1}{6}}{\frac{1}{6}} = \frac{1}{6} \\ & \frac{11111}{11111} = \frac{11111}{11111} = \frac{11111}{11111} = \frac{11111}{11111} \\ & \frac{11111}{11111} = 3\frac{1}{6} \end{aligned}$$

(19)

8 children will have 8 children's shares.

One woman will have 3 children's shares \therefore 6 women will have
 $6 \times 3 = 18$ children's shares.

One man will have 6 children's shares \therefore 4 men will have
 $4 \times 6 = 24$ children's shares.

4 men, 6 women, and 8 child. will therefore have 50 child. shares.

$\text{£}550 \text{ 3s. } 1\frac{1}{2}\text{d.} \div 50 = \text{£}11 \text{ 0s. } 0\frac{3}{4}\text{d.} = \text{child's share.}$

$\text{£} 11 \text{ 0s. } 0\frac{3}{4}\text{d.} \times 3 = \text{£}33 \text{ 0s. } 2\frac{1}{4}\text{d.} = \text{woman's share.}$

$\text{£} 33 \text{ 0s. } 2\frac{1}{4}\text{d.} \times 2 = \text{£}66 \text{ 0s. } 4\frac{1}{2}\text{d.} = \text{man's share.}$

(20)

$$16\frac{7}{7} + 19\frac{1}{1} + 23\frac{7}{7} + 129\frac{9}{9} = 16 + 19 + 23 + 129 + \\ (\frac{7}{7} + \frac{1}{1} + \frac{7}{7} + \frac{9}{9}) = 187 + 3\frac{19}{9} = 190\frac{19}{9}$$

(21)

$$8100 = 2^2 \times 3^4 \times 5^2.$$

1..3..9..27..81

1..2..4

1..3..9..27..81..2..6..18..54..162..4..12..36..108..324

1..5..25

1..3..9..27..81..2..6..18..54..162..4..12..36..108..324..

5..15..45..135..405..10..30..90..270..810..20..60..180..

540..1620..25..75..225..675..2025..50..150..450..1350..

4050..100..300..900..2700..8100.

Therefore the divisors of 8100 are 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 25, 27, 30, 36, 45, 50, 54, 60, 75, 81, 90, 100, 108, 135, 150, 162, 180, 225, 270, 300, 324, 405, 450, 540, 675, 810, 900, 1350, 1620, 2025, 2700, 4050, 8100.

(22)

$$\begin{array}{r}
 2691)11817(4 \\
 \underline{10764} \\
 1053)2691(2 \\
 \underline{2106} \\
 585)1053(1 \\
 \underline{585} \\
 468)585(1 \\
 \underline{468} \\
 117)468(4 \\
 \underline{468}
 \end{array}$$

(23)

$$\begin{array}{r}
 \text{sec.} \\
 60)2551443 \\
 \underline{60)42524..} 3 \\
 \underline{24)708..} 44 \\
 \underline{29..} 12 \\
 29\text{d., } 12\text{ h., } 44\text{ m., } 3\text{ sec.} \\
 \text{sec.} \\
 60)31556928 \\
 \underline{60)525948..} 48 \\
 \underline{24)8765..} 48 \\
 \underline{365..} 5 \\
 365\text{ d., } 5\text{ h., } 48\text{ m., } 48\text{ sec.}
 \end{array}$$

9828 is divisible by 117 \therefore 117 is the G. C. M.

(24)

$$\begin{array}{l}
 14\text{ ft. } 11\text{ in.} = 179\text{ in.} \\
 38\text{ miles} = 2407680\text{ in.} \\
 2407680 \div 179 = 13450\frac{1}{3}
 \end{array}$$

(25)

$$\begin{array}{l}
 11\text{ ft.} \times 13\text{ ft.} \times 15\text{ ft.} = 2145\text{ cub. ft.} \\
 \text{One cubic foot weighs } 62\frac{1}{2}\text{ lbs. } 2145 \times 62\frac{1}{2} = 134062\frac{1}{2} = \text{weight} \\
 \text{of } 2145\text{ cub. ft.} \\
 \text{One gallon weighs } 10\text{ lbs. } 134062\frac{1}{2} \div 10 = 13406\frac{1}{4} = \text{gals. in} \\
 134062\frac{1}{2}\text{ lbs.}
 \end{array}$$

(26)

$$\begin{array}{r}
 £73 \times 400 = \$292.00 \\
 17\text{s.} \times 20 = 3.40 \\
 11\frac{1}{2}\text{d.} = 47\text{ far.} \times 5 \div 12 = 19\frac{1}{2}
 \end{array}$$

$$£73\ 17\text{s.}\ 11\frac{1}{2}\text{d.} = \$295.59\frac{1}{2}$$

(27)

$$\begin{array}{l}
 93\frac{1}{11} - 76\frac{1}{13} = 92\frac{1}{11} - 76\frac{1}{13} = 16\frac{15}{143} = \frac{4206}{258} \\
 \frac{4206}{258} \div \frac{17}{258} = \frac{4206}{258} \times \frac{258}{17} = \frac{4206}{17} = 247\frac{7}{17}
 \end{array}$$

...
 omen will have
 omen will have
 50 child. shares.
 ild's share.
 oman's share.
 n's share.

$$23 + 129 + 190\frac{19}{308}$$

$$..36.. 108 .. 324$$

$$\begin{array}{l}
 ..36.. 108.. 324.. \\
 ..20.. 60.. 180.. \\
 0.. 450.. 1350 ..
 \end{array}$$

$$\begin{array}{l}
 4, 5, 6, 9, 10, 12, \\
 90, 100, 108, 135, \\
 0, 675, 810, 900,
 \end{array}$$

(28)

$$\frac{5\frac{1}{2} \div \frac{2}{3}}{1\frac{1}{2} \text{ of } \frac{5}{9} \div 10\frac{1}{2}} \times \frac{\frac{2}{3} \text{ of } \frac{1\frac{1}{2} \text{ of } 4\frac{1}{2}}{13\frac{1}{2} \text{ of } 5\frac{1}{2}}}{\frac{4\frac{1}{2} \times 3}{1 \times 3\frac{1}{2}}} = \frac{1\frac{1}{2} \times \frac{2}{3}}{\frac{5}{9} \times \frac{5}{9} \times \frac{3\frac{1}{2}}{3\frac{1}{2}}} \times \frac{2}{3} \times \frac{\frac{2}{3} \times \frac{3\frac{1}{2}}{1\frac{1}{2}} \times \frac{3\frac{1}{2}}{1\frac{1}{2}}}{1\frac{1}{2} \times 1\frac{1}{2}} =$$

$$\frac{1 \times 3\frac{1}{2}}{1 \times 3\frac{1}{2}} \times \frac{2}{3} \times \frac{3\frac{1}{2} \times 2}{3\frac{1}{2} \times 2} =$$

$$\frac{45 \times 3 \times 31}{16 \times 2} \times \frac{3}{5} \times \frac{37}{2 \times 3 \times 37 \times 2} = \frac{3 \times 9 \times 31}{16 \times 2 \times 2 \times 2} =$$

$$\frac{837}{128} = 6\frac{69}{128}$$

(29)

| |
|------------|
| XI |
| 5)91342 |
| 5)19074..4 |
| 5)4015..1 |
| 5)891..0 |
| 5)184..3 |
| 5)39..3 |
| 5)8..2 |
| 1..3 |

| |
|------------|
| XI |
| 12)91342 |
| 12)8314..9 |
| 12)773..1 |
| 12)70..3 |
| 6..5 |

| |
|------------|
| XI |
| 2)91342 |
| 2)46176..1 |
| 2)23093..0 |
| 2)11571..1 |
| 2)6276..0 |
| 2)3153..0 |
| 2)1627..0 |
| 2)869..0 |
| 2)434..1 |
| 2)217..1 |
| 2)109..0 |
| 2)57..0 |
| 2)27..1 |
| 2)15..0 |
| 2)8..0 |
| 2)4..0 |
| 2)2..0 |
| 1..0 |

(29 continued.)

$$\frac{\frac{3}{2} \times \frac{37}{9}}{\frac{11}{3} \times \frac{19}{3}} =$$

$$3 \times 9 \times 31 =$$

$$\times 2 \times 2 \times 2 =$$

XI

- 2)91342
- 2)46176..1
- 2)23093..0
- 2)11511..1
- 2)6216..0
- 2)3153..0
- 2)1627..0
- 2)869..0
- 2)434..1
- 2)217..1
- 2)109..0
- 2)5t..0
- 2)2t..1
- 2)15..0
- 2)8..0
- 2)4..0
- 2)2..0
- 1..0

| | XI | V | XII | II |
|--|-------------|-------------|-------------|--------------------|
| | 91342 | 13233014 | 65319 | 100000100110000101 |
| | 11 | 5 | 12 | 2 |
| | 100 | 8 | 77 | 2 |
| | 11 | 5 | 12 | 2 |
| | 1103 | 42 | 927 | 4 |
| | 11 | 5 | 12 | 2 |
| | 12137 | 213 | 11125 | 8 |
| | 11 | 5 | 12 | 2 |
| | 133509 dec. | 1068 | 133509 dec. | 16 |
| | | 5 | | 2 |
| | | 5340 | | 32 |
| | | 5 | | 2 |
| | | 26701 | | 65 |
| | | 5 | | 2 |
| | | 133509 dec. | | 130 |
| | | | | 2 |
| | | | | 200 |
| | | | | 31377 |
| | | | | 2 |
| | | | | 66754 |
| | | | | 3 |
| | | | | 133509 dec. |

(30)

$$\begin{array}{r} 2)7680 \\ \hline 2)3840 \\ \hline 2)1920 \\ \hline 2)960 \\ \hline 2)480 \\ \hline 2)240 \\ \hline 2)120 \\ \hline 2)60 \\ \hline 2)30 \\ \hline 3)15 \\ \hline 5 \end{array}$$

(31)

m. f. p. y. ft. in.

$$\begin{array}{r} 72\ 3\ 7\ 2\ 1\ 7 \\ \hline 8 \\ \hline 579\ \text{fur.} \\ \hline 40 \\ \hline 23167\ \text{per.} \\ \hline 5\frac{1}{2} \\ \hline 115837 \\ \hline 11583\frac{1}{2} \\ \hline 1274'0\frac{1}{2}\ \text{yds.} \\ \hline 382262\frac{1}{2}\ \text{ft.} \\ \hline 12 \\ \hline 4587157\ \text{in.} \\ \hline 12 \\ \hline 55045884\ \text{lines} \end{array}$$

(32)

$$\$47 \times 97 = \$4559.$$

(33)

$$(73 \times 4 \times 11) \div 128 = 25\frac{3}{2}.$$

$$\$3.62\frac{1}{2} \times 25\frac{3}{2} = \$90.96\frac{3}{4}.$$

(34)

$$93 \cdot 723 = 93\frac{723}{1000} = \frac{92786}{1000} \text{ and } 29 \cdot 4173 = 29\frac{4173}{1000} = \frac{293879}{1000}.$$

$$\frac{92786}{1000} \div \frac{293879}{1000} = \frac{92786}{293879} \times \frac{111}{11} = \frac{92786 \times 111}{11 \times 293879} =$$

$$\frac{10309246}{3232669} = 3.185988 +$$

(35)

One bushel of oats weighs 34 lbs. ∴ in 73429 lbs. there are $73429 \div 34 = 2159\frac{3}{4}$ bushels.

(36)

In 719630 lbs. of wheat there are $719630 \div 60 = 11993\frac{5}{6}$ bus.
 $\$1.80 \times 11993\frac{5}{6} = \$21588.90,$
 Or $\$1.80$ per bushel = 3 cents per lb.
 $719630 \times 3 = 2158890$ cents. = $\$21588.90.$

(32)

$97 = \$45.50.$

(33)

$11 \div 128 = 25\frac{3}{2}.$

$25\frac{3}{2} = \$90.36\frac{3}{4}.$

21389)180781(8

171112

(37)

9669)21389(2

19338

$\$72.14 + \$93.76 = \$165.90$

2051)9669(4

$\$165.90 \times 9.47 = \1571.0730

8204

$\$1571.0730 \div 11 = \$142.8248+$

1465)2051(1

1465

586)1465(2

1172

293)586(2

Last divisor 293 = G. C. M.

(39)

$\frac{7}{11}, \frac{1}{2}, \frac{2}{7}, \frac{2}{33}, \frac{1}{14}, \frac{7}{10}, \frac{1}{2}.$

The least common multiple of 11, 5, 7, 33, 14, 10 and 2 is 2310.

The multiplier for both terms of the first fraction is $\frac{2310}{11} = 210$; for the second, $\frac{2310}{2} = 1155$; for the third, $\frac{2310}{7} = 330$; for the fourth, $\frac{2310}{33} = 70$; for the fifth, $\frac{2310}{14} = 165$; for the sixth, $\frac{2310}{10} = 231$; for the seventh, $\frac{2310}{2} = 1155$.

Multiplying by these numbers, we obtain $\frac{1470}{2310}, \frac{1155}{2310}, \frac{330}{2310}, \frac{70}{2310}, \frac{165}{2310}, \frac{2310}{2310},$ and $\frac{1155}{2310}$ for the required fractions.

(40)

$\$11 \times 17 = \$1.87. \quad \$37\frac{1}{2} \times 19 = \$7.12\frac{1}{2}. \quad \$2.17 \times 14\frac{1}{2} =$

$\$31.46\frac{1}{2}. \quad \$27 \times 67 = \$18.09. \quad \$1.37\frac{1}{2} \times 15 = \$20.62\frac{1}{2}.$

$\$1.87 + \$7.12\frac{1}{2} + \$31.46\frac{1}{2} + \$4.75 + \$11.50 + \$18.09 +$

$\$20.62\frac{1}{2} + \$7.93 = \$103.35\frac{1}{2}.$

29 lbs. there are

60 = 11993 $\frac{1}{2}$ bus.

1 lb.

1588.90,

EXERCISE 84—Page 210.

(1)

$$\begin{array}{r} \text{Baskets.} \\ 11 : 87 :: \$13.42 : \frac{1.22}{11} \times 87 = \$106.14. \end{array}$$

(2)

$$\begin{array}{r} \text{Jords.} \\ 28 : 25 :: \$266 : \frac{19}{3} \times 25 = \$237.50. \end{array}$$

(3)

$$\begin{array}{r} \text{days} \\ \$29.20 : \$83.60 :: 16 : \frac{4}{7.3} \times 83.60 = 45\frac{2}{3} \text{ days.} \end{array}$$

(4)

$$\begin{array}{r} \text{Bags.} \\ 16 : 156 :: \$12.80 : \frac{.8}{16} \times 156 = \$124.80. \end{array}$$

(5)

$$\begin{array}{r} \text{Feet.} \quad \text{ft.} \\ 5 : 112 :: 7 : \frac{7 \times 112}{5} = 156\frac{2}{5} \text{ ft.} \end{array}$$

(6)

$$\begin{array}{r} \text{Cows.} \quad \text{days.} \\ 55 : 27 :: 99 : \frac{9}{5} \times 27 = 48\frac{3}{5} \text{ days.} \end{array}$$

(7)

$$\begin{array}{r} \text{Acres.} \quad \text{bus.} \\ 5 : 48 :: 9 : \frac{9 \times 48}{5} = 86\frac{2}{5} \text{ bush.} \end{array}$$

(8)

$$\begin{array}{r} 11 \\ \text{Perches. days. } 2 \times 808 \\ 73 : 803 :: 2 : \frac{\quad}{88} = 22 \text{ days.} \end{array}$$

6-14.

(9)

$$\begin{array}{r} 141 \\ \text{Pails. lbs. } 100 \times 1128 \\ 176 : 1128 :: 100 : \frac{\quad}{22} = 640\frac{1}{2} \text{ lbs.} \end{array}$$

0.

(10)

$$\begin{array}{r} .58 \quad 155 \\ \$20.88 \times 465 \\ 103 : 465 :: \$20.88 : \frac{\quad}{36} = \$89.90. \end{array}$$

59 $\frac{2}{3}$ days.

(11)

$$\begin{array}{r} 9 \quad 639 \\ \$ \quad \$ \quad \text{brls. } 72 \times 1278 \\ 16 : 1278 :: 72 : \frac{\quad}{2} = 5751 \text{ barrels.} \end{array}$$

\$124.80.

(12)

$$\begin{array}{r} 15 \\ \text{Men. Acres } 165 \times 3 \\ 11 : 3 :: 165 : \frac{\quad}{11} = 45 \text{ acres.} \end{array}$$

ft.

(13)

$$\begin{array}{r} 125 \\ \text{Barrels. loaves } 250 \times 67 \\ 4 : 67 :: 250 : \frac{\quad}{2} = 4187\frac{1}{2} \text{ loaves.} \end{array}$$

days.

(14)

$$\begin{array}{r} 16 \times 88 \\ \text{Bushels. brls. } \\ 190 : 38 :: 16 : \frac{\quad}{5} = 3\frac{1}{5} \text{ barrels.} \end{array}$$

(15)

$$\begin{array}{l} \text{Days.} \quad \text{men} \quad 90 \times 12 \\ 15 : 12 :: 90 : \frac{1080}{15} = 72 \text{ men.} \end{array}$$

(16)

$$\begin{array}{l} \text{D. work. brls.} \quad 2 \times 279 \\ 17 : 279 :: 2 : \frac{558}{17} = 32\frac{1}{7} \text{ barrels.} \end{array}$$

(17)

$$\begin{array}{l} \text{Hours.} \quad \text{miles.} \\ 1 : 24 :: 27 : 27 \times 24 = 648 \text{ miles.} \end{array}$$

(18)

$$\begin{array}{l} \text{Cows.} \quad \text{lbs.} \quad 30 \times 23 \\ 7 : 23 :: 30 : \frac{690}{7} = 98\frac{1}{7} \text{ lbs.} \end{array}$$

EXERCISE 85—Page 211.

(1)

$$\frac{1}{8} : \frac{3}{16} :: \$9750 : \frac{375}{1} \times \frac{7}{28} \times \frac{16}{8} = \$42000.$$

(2)

$$\begin{array}{l} \text{Yard.} \quad \text{s.} \quad 5 \quad 1 \quad 8 \quad \text{s.} \\ \frac{1}{3} : \frac{1}{4} :: \frac{5}{6} : \frac{1}{3} \times \frac{8}{4} \times \frac{3}{7} = \frac{2}{7} = 2\frac{2}{7} \text{d.} \end{array}$$

(3)

$$\begin{array}{l} \text{Tons.} \\ \frac{1}{5} : 8\frac{1}{2} :: \$7.49 : \frac{7.49 \times 8\frac{1}{2}}{5} = \frac{1.07}{1} \times \frac{25}{8} \times \frac{3}{7} = \$80.25. \end{array}$$

(4)

$$\begin{array}{l} \text{Yards.} \\ 5\frac{1}{2} : \frac{1}{4} :: \$28.42 : \frac{.14}{1} \times \frac{4}{7} \times \frac{5}{28} = \$2.80. \end{array}$$

(5)

$$\text{Dollar. bag } \frac{4}{12} : \frac{7}{20} :: \frac{4}{5} : - \times \frac{7}{20} \times \frac{5}{12} = 1\frac{7}{8} \text{ of a bag.}$$

(6)

$$\begin{matrix} \$ & \$ & \$ \\ 100 : 472\frac{1}{2} :: 98\frac{7}{8} : \frac{98\frac{7}{8} \times 472\frac{1}{2}}{100} = \frac{98 \cdot 875 \times 472 \cdot 44}{100} = \$467 \cdot 12\frac{1}{2}. \end{matrix}$$

(7)

$$\begin{matrix} \text{Tons.} & \text{days.} \\ 17\frac{3}{8} : 11\frac{1}{4} :: 107\frac{3}{4} : \frac{107\frac{3}{4} \times 11\frac{1}{4} \times \frac{295}{11} \times \frac{198}{17} \times \frac{5}{88}}{17\frac{3}{8}} = 70\frac{187}{8} \text{ dys.} \end{matrix}$$

(8)

$$\begin{matrix} \text{Tons.} & \text{cords.} \\ 15\frac{7}{8} : 11\frac{3}{8} :: 22\frac{1}{2} : \frac{22\frac{1}{2} \times 11\frac{3}{8} \times \frac{295}{9} \times \frac{18}{26} \times \frac{18}{202}}{15\frac{7}{8}} = 16\frac{7}{8} \text{ cords.} \end{matrix}$$

(9)

$$\begin{matrix} \text{Yds.} & \text{yds} & \$ \\ \frac{1}{2} \text{ of } \frac{2}{3} \text{ of } 3\frac{1}{2} : \frac{2}{3} \text{ of } \frac{1}{2} \text{ of } \frac{5}{8} :: \frac{2}{7} \text{ of } \frac{3}{4} \text{ of } 4\frac{3}{4} : \frac{\frac{2}{7} \text{ of } \frac{3}{4} \text{ of } 4\frac{3}{4} \times \frac{2}{3} \text{ of } \frac{1}{2} \text{ of } \frac{5}{8}}{\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } 3\frac{1}{2}} = \end{matrix}$$

$$\frac{4}{11} \times \frac{15}{165} = \frac{60}{1815} = \frac{4}{224}$$

EXERCISE 86—Page 212.

(1)

37 sq. yds. 4 ft. 120 in. = 48648 in., and 9 sq. yds. 2 ft. = 11952 in.

$$\begin{matrix} \text{Inches.} & & \\ 11952 : 48648 :: \$3 \cdot 50 ; \frac{3 \cdot 50 \times 48648}{11952} = \$14 \cdot 245 + \end{matrix}$$

= \$80.25.

2.80.

(2)

$$12 \text{ lbs. } 10 \text{ oz.} = 154 \text{ oz.}$$

Ounces.

$$1 : 154 :: \$1.25 : 1.25 \times 154 = \$192.50.$$

(3)

$$10 \text{ yds.} = 40 \text{ qrs., and } 3 \text{ yds. } 2 \text{ qrs.} = 14 \text{ qrs.}$$

.17 7

Quarters.

$$\$.40 \times 14$$

$$40 : 14 :: \$3.40 : \frac{\$3.40 \times 14}{40} = \$1.19.$$

40

20

(4)

$$15 \text{ oz. } 12 \text{ dwt. } 16 \text{ grs.} = 7504 \text{ grs., and } 13 \text{ oz. } 14 \text{ grs.} = 6254 \text{ grs.}$$

.95 3127

Grains.

$$\$.80 \times 6254$$

$$7504 : 6254 :: \$3.80 : \frac{\$3.80 \times 6254}{7504} = \$3.167 +$$

7504

1876

938

(5)

$$3 \text{ lbs. } 1 \text{ oz. } 11 \text{ dwt.} = 751 \text{ dwt. and } 12 \text{ lbs. } 6 \text{ oz. } 4 \text{ dwt.} = 3004 \text{ dwt.}$$

150

Dwt.

$$\$.600 \times 751$$

$$3004 : 751 :: 600 : \frac{600 \times 751}{3004} = \$150.$$

3004

4

(6)

$$\text{Barrels. h. m. s. } 2 \text{ h. } 46 \text{ m. } 39 \text{ s.} \times \frac{4}{24}$$

$$54 : 24 :: 2 \text{ } 46 \text{ } 30 : \frac{2 \text{ } 46 \text{ } 30 \times 4}{24} = 1 \text{ hr. } 14 \text{ min.}$$

54

9

(7)

73 yds. 3 qrs. 2 na. 1 in = 2660½ in. 3 Fl. e. 2 qrs. 1 na. = 101½ in.
 And £4 17s. 8½d. = 1172½d.

$$\begin{array}{l} \text{Inches.} \quad \text{d.} \quad 1172\frac{1}{2} \times 2660\frac{1}{2} = \frac{521}{4} \times \frac{5321}{2} \times \frac{4}{45} = \\ 101\frac{1}{2} : 2660\frac{1}{2} :: 1172\frac{1}{2} : \frac{2773241}{90} \text{ d.} = \text{£}128 \text{ 6s. } 10\frac{11}{10} \text{ d.} \end{array}$$

(8)

$$\begin{array}{l} 8\frac{1}{2} \text{ lbs.} = 136\frac{3}{4} \text{ oz.} \\ \text{Ounces.} \quad \text{s.} \quad 49 : 136\frac{3}{4} :: 8\frac{3}{4} : \frac{287}{16} \times \frac{410}{8} \times \frac{9}{41} = \frac{4704}{19} = \text{£}13 \text{ 9s. } 0\frac{1}{4} \text{ d.} \end{array}$$

(9)

$$\begin{array}{l} \text{Pages.} \quad \text{52} \\ 327 : 400 :: 156 : \frac{156 \times 400}{109} = 190\frac{20}{109}, \text{ i. e. on the 191st p.} \end{array}$$

(10)

$$\begin{array}{l} 46 \text{ a., } 3 \text{ r., } 14 \text{ p.} = 7494 \text{ p., and } 35 \text{ a., } 2 \text{ r., } 10 \text{ p.} = 5690 \text{ p.} \\ \text{Perches.} \quad \text{£} \quad \frac{100 \times 5690}{3747} = \text{£}75 \text{ 18s. } 6\frac{3}{4} \text{ d.} \end{array}$$

(11)

$$\begin{array}{l} \text{Days.} \quad \text{miles.} \quad \frac{12 \times 68}{48} = 17 \text{ miles per day.} \\ 48 : 68 :: 12 : \frac{48}{4} \end{array}$$

(12)

$$\begin{array}{r} \text{Shillings. lbs. } 113 \\ 21\frac{1}{2} : 32\frac{1}{2} :: 16\frac{1}{2} : \frac{113}{7} \times \frac{226}{7} \times \frac{3}{64} = \frac{28307}{1568} = 24\frac{675}{1568} \text{ lbs} \end{array}$$

(13)

$$\begin{array}{l} 17493 \times 1000 \times 5 \text{ cub. ft.} = 87465000 \text{ cub. ft.} \\ 192724 \times 1000 \times 4 \text{ cub. ft.} = 770896000 \text{ cub. ft.} \\ 87465000 + 770896000 = 858361000 \text{ cub. ft.} \\ \text{Cubic feet. ton. } 858361000 \\ 9000 : 858361000 :: 1 : \frac{858361000}{9000} = 95373\frac{1}{3} \text{ tons.} \end{array}$$

(14)

$$\begin{array}{l} 50000 \times 9000 = 450000000 = \text{cub. ft. of gas in 50000 tons of coal} \\ \text{Cubic feet. hour.} \\ 4 : 450000000 :: 1 : \frac{450000000}{4} = 112500000 \text{ h.} = 12842 \text{ y. } 170 \text{ d.} \end{array}$$

(15)

$$\begin{array}{l} \text{lbs. lbs. lbs. lb. lb.} \\ 4 + 3 + 2 + 1 + \frac{1}{2} = 10\frac{1}{2} \text{ lbs.} \\ \text{lbs. } 11270 \\ 10\frac{1}{2} : 11270 :: 1 : \frac{11270}{10\frac{1}{2}} = 1073, \text{ and } 3\frac{1}{2} \text{ lbs. remaining.} \end{array}$$

(16)

$$\begin{array}{l} 180 \text{ miles} = 180 \times 1760 = 316800 \text{ yards.} \\ \text{Yards. day.} \\ 100 : 316800 :: 1 : \frac{316800}{100} \times 1 = 3168 \text{ dys. or about } 8\frac{2}{3} \text{ yrs.} \end{array}$$

EXERCISE 87—Page 216.

(1)

$$\left. \begin{array}{l} 120 : 90 \text{ bush.} \\ 6 : 14 \text{ horses.} \end{array} \right\} :: 56 \text{ days} : \frac{7 \ 15}{56 \times 90 \times 14} = 7 \times 14 = 98 \text{ days.}$$

$$\frac{120 \times 6}{8}$$

(2)

$$\left. \begin{array}{l} 28 : 32 \text{ ft. high.} \\ 8 : 15 \text{ days.} \end{array} \right\} :: 63 \text{ men} : \frac{9 \ 4}{63 \times 32 \times 15} = 9 \times 15 = 135 \text{ men.}$$

$$\frac{8 \times 28}{7}$$

(3)

$$\left. \begin{array}{l} 3 : 45 \text{ length.} \\ 1\frac{1}{2} : 1 \text{ width.} \end{array} \right\} :: 1 \text{ lb.} : \frac{45}{3 \times 1\frac{1}{2}} = \frac{45}{\frac{3}{2}} = \frac{45 \times 2}{3} = \frac{3}{15} \times 4 = 3 \times 4 = 12 \text{ lbs.}$$

(4)

$$\left. \begin{array}{l} 10 : 100 \text{ length.} \\ 1\frac{1}{2} : 1\frac{1}{2} \text{ width.} \end{array} \right\} :: 3 \text{ lbs.} : \frac{3 \times 1\frac{1}{2} \times 100}{1\frac{1}{2} \times 10} = 2 \times 1\frac{1}{2} \times 10 = 25 \text{ lbs.}$$

(5)

$$\left. \begin{array}{l} 44 : 132 \text{ tons.} \\ 18 : 5 \text{ days.} \end{array} \right\} :: 12 \text{ horses} : \frac{2}{12 \times 5 \times 18} = 2 \times 5 = 10 \text{ horses.}$$

$$\frac{44 \times 18}{6}$$

(6)

$$\left. \begin{array}{l} 4 : 14 \text{ men.} \\ 7 : 10 \text{ days.} \end{array} \right\} :: 27s. : \frac{2 \ 5}{27 \times 14 \times 10} = 27 \times 5 = 135s. = £6 \ 15s.$$

$$\frac{4 \times 7}{2}$$

(7)

$$\left. \begin{array}{l} 3:5 \text{ masters.} \\ 8:10 \text{ apprentices.} \\ 5:8 \text{ weeks} \\ 6:5\frac{1}{2} \text{ days per wk.} \end{array} \right\} :: \$144 : \frac{8}{24} \frac{144 \times 5\frac{1}{2} \times 8 \times 10 \times 5}{8 \times 8 \times 5 \times 6} = \$440.$$

(8)

$$\left. \begin{array}{l} 6 : 18 \text{ s.mak.} \\ 4 : 5 \text{ weeks.} \end{array} \right\} :: 36 \text{ pairs of men's shoes : } \frac{9}{36} \frac{3}{18} \times 18 \times 5 = 135 \text{ pairs men's and the women's} = \frac{2}{3} \frac{1}{2} = \frac{1}{3} \text{ of } 135 = 90 \text{ pairs.}$$

(9)

$$\left. \begin{array}{l} 9 : 18 \text{ feet high.} \\ 4 : 6 \text{ days.} \end{array} \right\} :: 12 \text{ men : } \frac{3}{12} \frac{2}{18} \times 18 \times 6 = 3 \times 2 \times 6 = 36 \text{ men.}$$

(10)

$$\left. \begin{array}{l} 130 : 390 \text{ miles.} \\ 7 : 14 \text{ hours.} \end{array} \right\} :: 3 \text{ days : } \frac{2}{3} \frac{3}{14} \times 14 \times 390 = 3 \times 2 \times 3 = 18 \text{ days.}$$

(11)

$$\left. \begin{array}{l} 10 : 60 \text{ oz.} \\ 22\frac{1}{2} : 30 \text{ d.} \end{array} \right\} :: 1 \text{ d. : } \frac{60 \times 30}{10 \times 22\frac{1}{2}} = \frac{60}{1} \times \frac{30}{22\frac{1}{2}} = \frac{60}{1} \times \frac{1}{1} \times \frac{1}{10} \times \frac{2}{45} = 4 \times 2 = 8 \text{d.}$$

(12)

$$\left. \begin{array}{l} 10 : 5 \text{ compositors} \\ 7 : 14 \text{ hours.} \\ 20 : 40 \text{ sheets.} \\ 24 : 16 \text{ pages.} \\ 50 : 60 \text{ lines.} \\ 40 : 50 \text{ letters.} \end{array} \right\} :: 16 \text{ days : } \frac{2}{16} \frac{2}{40} \frac{5}{60} \times 5 \times 14 \times 40 \times 16 \times 60 \times 50 = 10 \times 7 \times 20 \times 24 \times 50 \times 40 = 2 \times 16 = 32 \text{ days.}$$

$$\frac{0 \times 5}{1} = \$440.$$

$$\frac{3 \times 18 \times 5}{8 \times 4} =$$

$$f 135 = 90 \text{ pairs.}$$

$$\times 2 \times 6 = 36 \text{ men.}$$

$$\times 2 \times 3 = 18 \text{ days.}$$

$$\frac{2}{45} = 4 \times 2 = 8d.$$

$$\frac{8 \times 16 \times 60 \times 50}{24 \times 50 \times 40} =$$

336 : 240 men.
 5 : 9 days.
 10 : 12 hours.
 6 : 5 degrees.
 5 : 3 yards wide
 2 : 2 yards deep

(13)

$$\left. \begin{array}{l} 336 : 240 \text{ men.} \\ 5 : 9 \text{ days.} \\ 10 : 12 \text{ hours.} \\ 6 : 5 \text{ degrees.} \\ 5 : 3 \text{ yards wide} \\ 2 : 2 \text{ yards deep} \end{array} \right\} :: 70 \text{ yards : } \frac{70 \times 240 \times 9 \times 12 \times 5 \times 3 \times 2}{336 \times 5 \times 10 \times 6 \times 5 \times 3} =$$

$$9 \times 2 \times 2 = 36 \text{ yards.}$$

(14)

$$\left. \begin{array}{l} 6 : 12 \text{ horses.} \\ 4 : 9 \text{ months.} \end{array} \right\} :: 16 \text{ acres : } \frac{16 \times 12 \times 9}{6 \times 4} = 4 \times 2 \times 9 = 72 \text{ acres.}$$

(15)

$$\left. \begin{array}{l} 25 : 139 \text{ persons} \\ 1 : 7 \text{ years.} \end{array} \right\} :: 300 \text{ bush. : } \frac{300 \times 139 \times 7}{25} = 11676 \text{ bushels.}$$

(16)

$$\left. \begin{array}{l} 48 : 32 \text{ men.} \\ 36 : 864 \text{ feet long.} \\ 8 : 5 \text{ feet high.} \\ 4 : 3 \text{ feet wide.} \end{array} \right\} :: 4 \text{ days : } \frac{4 \times 32 \times 864 \times 5 \times 3}{48 \times 36 \times 8 \times 4} = 30 \text{ days.}$$

(17)

$$\left. \begin{array}{l} 679 : 22407 \text{ sold's.} \\ 536 : 112 \text{ days.} \end{array} \right\} :: 702 \text{ bushels : } \frac{234 \times 33}{536 \times 112} =$$

$$234 \times 33 = 7722 \text{ bushels.}$$

(18)

$$\left. \begin{array}{l} 13 : 494 \text{ suits.} \\ 19 : 27 \text{ days.} \end{array} \right\} :: 12 \text{ tailors : } \frac{12 \times 494 \times 27}{13 \times 19} = 648 \text{ tailors.}$$

(19)

$$\begin{array}{l}
 17:40 \text{ head of cattle} \\
 30:51 \text{ days.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 17:40 \\ 30:51 \end{array}} \right\} :: 5 \text{ a. } 2 \text{ r. } 10 \text{ p.} : \frac{5 \text{ a. } 2 \text{ r. } 10 \text{ p.} \times \overset{4}{20} \times \overset{8}{51}}{17 \times \overset{8}{30}} =$$

$$5 \text{ a. } 2 \text{ r. } 10 \text{ p.} \times 4 = 22 \text{ a. } 1 \text{ r.}$$

(20)

$$\begin{array}{l}
 20 : 100 \text{ ft. long} \\
 6 : 4 \text{ feet wide.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 20 \\ 6 \end{array}} \right\} :: 180 \text{ bricks} : \frac{30 \times 5 \times 180 \times 100 \times 4}{20 \times 6} =$$

$$30 \times 5 \times 4 = 600 \text{ bricks.}$$

EXERCISE 88.—Page 21

(1)

- 17 cords = 116 lbs.
- 187 lbs. = 23 barrels
- 19 barrels = 34 days' work
- 92 days' work = 57 baskets peaches
- 31 baskets peaches = 24 dollars
- 12 dollars = 2 tons
- 35 tons = x cords

$$\frac{17 \times \overset{8}{87} \times 19 \times \overset{23}{92} \times 31 \times 12 \times 35}{116 \times \overset{2}{23} \times \overset{34}{92} \times \overset{57}{24} \times \overset{2}{12} \times 2} = \frac{31 \times 35}{2 \times 2 \times 2} = \frac{1085}{8} = 135\frac{5}{8}.$$

(2)

- 6 lbs. tea = 29 lbs. sugar
- 17 lbs. sugar = 1 bushel
- 27 bushels = 4 tons
- 34 tons = 15 cows
- 29 cows = 1160 dollars
- 20 dollars = x lbs. tea.

$$\frac{6 \times \overset{3}{17} \times \overset{9}{27} \times \overset{17}{34} \times \overset{15}{29} \times 20}{29 \times 1 \times 4 \times 15 \times 1160} = \frac{17 \times 17 \times 27}{5 \times 58} = \frac{7803}{290} = 26\frac{23}{290}$$

$$.10 p. \times 40 \times 51 =$$

$$\frac{17 \times 80}{10} =$$

(3)

| | | | | |
|-----------------|---|-------------------|---|---|
| 11 bush. barley | = | 21 bush. potatoes | } | = |
| 19 " potatoes | = | 29 " oats | | |
| 115 " oats | = | 44 " wheat | | |
| 14½ " wheat | = | 38 " peas | | |
| 60 " peas | = | 55 " rye | | |
| 75 " rye | = | 11½ " clover sd. | | |
| 36 " clover sd. | = | x " barley | | |

$$\frac{11 \times 19 \times 115 \times 14\frac{1}{2} \times 60 \times 75 \times 36}{7 \times 2 \times 11 \times 2 \times 5} = \frac{5 \times 75 \times 18}{7 \times 11} = \frac{6750}{77} = 87\frac{9}{77}$$

(4)

| | | | | |
|------------------|---|------------------|---|---|
| 16 baskets pears | = | 29 turkeys | } | = |
| 17 turkeys | = | 7 days' work | | |
| 7½ days' work | = | 187 loaves | | |
| 3½ loaves | = | 4 lbs. veal | | |
| 1 lb. veal | = | 11 cents | | |
| 792 cents | = | 63 lbs. sugar | | |
| x lbs. sugar | = | 21 baskets pears | | |

$$\frac{29 \times 7 \times 187 \times 4 \times 11 \times 63 \times 21}{16 \times 17 \times 7\frac{1}{2} \times 3\frac{1}{2} \times 1 \times 792} = \frac{11 \times 7 \times 21}{4} = \frac{1617}{4} = 404\frac{3}{4}$$

(5)

| | | | |
|-------------|---|---|---|
| 7 A = 11 B | } | = | $\frac{7 \times 5 \times 15 \times 11 \times 42}{11 \times 8 \times 21 \times 5} = \frac{7 \times 15}{4} = \frac{105}{4} = 26\frac{3}{4}$ |
| 5 B = 8 C | | | |
| 15 C = 21 D | | | |
| 11 D = 5 E | | | |
| 42 E = x A | | | |

$$\frac{7803}{990} = 26\frac{3}{10}$$

(6)

7 barrels flour = 23 cords
 6 cords = 11 cwt.
 46 cwt. = £28
 £77 = 9 sheep
 5 sheep = 8 tons
 9 tons = x barrels flour

} =

$$\frac{7 \times 6 \times 46 \times 77 \times 5 \times 9}{23 \times 11 \times 28 \times 9 \times 8} = \frac{3 \times 7 \times 5}{8} = \frac{105}{8} = 13\frac{1}{8}$$

(7)

15 N. England = 20 New York
 24 New York = 22½ N. Jersey
 30 New Jersey = 20 Canada
 4807½ Canada = x N. England

} =

$$\frac{15 \times 24 \times 30 \times 4807\frac{1}{2}}{20 \times 22\frac{1}{2} \times 20} = 961\frac{1}{2} \times 6 = 5769 \text{ s.} = \text{£}288 \text{ 9s.}$$

EXERCISE 89.—Page 222.

(1)

$$\frac{7}{8} \times \frac{17}{11} \times \frac{23}{29} \times \frac{11}{17} \times \frac{2}{16} = \frac{2}{3} = 2 : 3$$

(2)

$$\begin{aligned} \text{£}119 \times 400 &= \$476 \cdot 00 \\ 16\text{s.} \times 20 &= 3 \cdot 20 \\ 6\frac{1}{2}\text{d.} \times 26 \text{ far.} \times 5 \div 12 &= 10\frac{5}{6} \\ \hline \text{£}119 \text{ 16s. } 6\frac{1}{2}\text{d.} &= \$479 \text{ } 30\frac{5}{6} \end{aligned}$$

(4)

$$\left. \begin{array}{l} 9 : 13 = 9 \div 13 = \cdot 692 \\ 21 : 27 = 21 \div 27 = \cdot 777 \\ 7 : 10 = 7 \div 10 = \cdot 7 \\ 11 : 15 = 11 \div 15 = \cdot 733 \end{array} \right\}$$

Hence 21 : 27 is the greatest,
and 9 : 13 the least.

(5)

Dissimilar. Similar. Similar and Coterminous.

$$76 \cdot 23478 = 76 \cdot 234784 = 76 \cdot 234784784784784$$

$$19 \cdot 1342291 = 19 \cdot 1342291 = 19 \cdot 134229122912291$$

$$\text{Difference,} = 57 \cdot 100555661872493$$

(6)

71324t undenary = 1146287 denary, 23421 quinary = 1736 denary, and 4e7 duodenary = 17995 denary.

1146287 × 1736 = 1989954232 ÷ 17995 = 110583¹⁷⁹⁹⁵₁₇₉₉₅ denary* = 53ee3¹⁷⁹⁹⁵₁₇₉₉₅ duodenary, 12014313¹⁷⁹⁹⁵₁₇₉₉₅ quinary, and 76010¹⁷⁹⁹⁵₁₇₉₉₅ undenary.

(7)

$$\left. \begin{array}{l} 5 \cdot 63 : 7 \cdot 9 \text{ cubic inches.} \\ 1 : 1 \cdot 220 \text{ spec. grav.} \\ 31 \cdot 362052 \end{array} \right\} \begin{array}{l} \text{oz.} \\ \therefore 3 \cdot 254 : \\ \hline 5 \cdot 63 \end{array} = \frac{3 \cdot 254 \times 7 \cdot 9 \times 1 \cdot 220}{5 \cdot 63} = 5 \cdot 57052 \text{ oz. Ans.}$$

* To reduce the fractional part, reduce both numerator and denominator separately.

(8)

| | | | | | | | | | |
|-----------------|------|-----|-----|-----|------|------|-----|-----------------|-----|
| yds. | qrs. | na. | in. | (8) | yds. | qrs. | na. | in. | (9) |
| 17)63 | 3 | 2 | 1 | (| 3 | 3 | 0 | 0 $\frac{1}{4}$ | |
| 51 | | | | | | | | | |
| — | | | | | | | | | |
| 12 | | | | | | | | | |
| 4 | | | | | | | | | |
| — | | | | | | | | | |
| 51 | | | | | | | | | |
| 51 | | | | | | | | | |
| — | | | | | | | | | |
| 0 | | | | | | | | | |
| 4 | | | | | | | | | |
| — | | | | | | | | | |
| 2 | | | | | | | | | |
| 2 $\frac{1}{4}$ | | | | | | | | | |
| — | | | | | | | | | |

·913625 of an acre = ·913625 × 4840 =
 4421·945 sq. yds.
 4421·945 × \$.67 = \$2962·70+

(10)

$\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{1}{4}$ of 20 bush. × .5 × .6 × $\frac{7}{8}$ =
 $\frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} \times 20 \times \frac{1}{2} \times \frac{3}{4} \times \frac{7}{8}$ =
 $\frac{49}{32}$ bush. = 1 bush. 2 pks. 0 gal. 1 qt.

$5\frac{1}{2} = \frac{11}{2} \div 17 = \frac{11}{34}$.

(12)

Whole amount of increase = 2571437 - 1842265 = 729172.
 $1842265 : 100 :: 729172 : \frac{729172 \times 100}{1842265} = 39$ per cent.

(13)

$\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{18}{29}$ - $\frac{1}{8}$ of $\frac{2}{3}$ of $\frac{1}{4}$ = $\frac{6}{29}$ - $\frac{1}{84}$ = $\frac{359}{2436}$.

(14)

100 : 7 :: 11 : $\frac{11 \times 7}{100} = \frac{77}{100}$. 11 - $\frac{77}{100}$ = $10\frac{23}{100}$.

(15)

$79 \times 16 \times £.00163 = £2.06032 = £2$ 1s. $2\frac{3}{8}$ d.

(16)

4:3 men }
 10:12 hours } :: $3\frac{1}{2}$ days : $\frac{2\frac{1}{2} \times 3 \times 12 \times 25}{4 \times 10 \times 20} = \frac{43}{16} = 3\frac{1}{2}$ days.
 20:35 acres }

(9)

13625 × 4340 =

2962 · 70 +

0)

·5 × ·6 × $\frac{7}{8}$ =

× $\frac{3}{4}$ × $\frac{7}{8}$ =

0.0 gal. 1 qt.

5 = 729172.

39 per cent.

359.
2436.

100.

232 $\frac{1}{2}$ d.

= 3 $\frac{1}{8}$ days.

$(\frac{1}{2} \text{ of } \frac{2}{3} \times .02 \times 456) \div (\frac{1}{17} \text{ of } \frac{3}{4} \text{ of } \frac{1}{2} \text{ of } 51) =$

$$\frac{\frac{2}{4} \times \frac{3}{5} \times \frac{1}{25} \times \frac{152}{37} \times \frac{17}{8} \times \frac{3}{2} \times \frac{1}{8}}{\frac{5}{11} \times \frac{50}{25} \times \frac{333}{111} \times \frac{16}{8} \times \frac{2}{2} \times \frac{1}{51} \times \frac{3}{8}} = \frac{2 \times 38}{5 \times 11 \times 25 \times 37} = \frac{76}{60875}$$

(18)

$$\frac{2}{1} \times \frac{4}{7} \times \frac{13}{5} \times \frac{7}{2} \times \frac{5}{2} = 4 \times 13 = 52.$$

(19)

50 barrels = 125 yards
 80 yards = 6 bales,
 13 bales = 3 $\frac{1}{2}$ hogsheads =
 x hogsheads = 1000 barrels

$$\frac{5 \times 3 \times 125}{125 \times 6 \times 3 \frac{1}{2} \times 1000} = \frac{125 \times 3 \times 3 \frac{1}{2}}{50 \times 80 \times 13} = \frac{125 \times 3 \times 3 \frac{1}{2}}{2 \times 13} = 50 \frac{3}{2}$$

(20)

$$\frac{73 \cdot 47 \times .0063 \div 17 \cdot 2345}{7347} = \frac{7347 \times .63}{10000} \div \frac{57391}{3330} = \frac{7347}{100} \times \frac{63}{10000} \times \frac{3330}{57391} = \frac{154133713}{5739100000} = .026856599989 +$$

(21)

2 roods 7 per. 4 yds. 3 ft. 117 in. = 3416481 in. and 7 acres =
 43908480 inches.

3416481 ÷ 43908480 = .0778 +

(22)

$$\begin{aligned} \frac{7}{8} \text{ of } \frac{1}{6} \text{ of } \frac{1}{3} \text{ of } 70 \text{ miles} &= \frac{1}{6} \text{ miles} = 5.33333+ \text{ miles.} \\ \cdot 73 \text{ of } 11 \text{ fur.} &= 8.03 \text{ fur.} = 1.00375 \text{ mile.} \\ 5.33333 - 1.00375 &= 4.32958 \text{ miles.} \end{aligned}$$

(23)

$$\begin{aligned} 274312 \text{ nonary} &= 167195 \text{ denary, } 1101011010 = 858 \text{ denary, and} \\ \cdot 5555 \text{ septenary} &= 2000 \text{ denary.} \\ 167195 - 858 &= 166337 \times 2000 = 332674000. \\ 332674000 \text{ denary} &= 764876837 \text{ nonary.} \\ &= 10011110101000011001111010000 \text{ binary,} \\ &= 11146453021 \text{ septenary.} \end{aligned}$$

(24)

$$\begin{array}{r|l} 275 & 44 \cdot 275 \cdot 18 \cdot 180 \cdot 200 \cdot 225 \\ \cdot 38 & 4 \\ 18 & 2 \\ \hline & 9 \\ & 9 \\ & 9 \end{array}$$

$$275 \times 38 \times 18 = 188100 = \text{l. c. m.}$$

(25)

10:6 weeks
6:5 days
11:10 hours
2400:8742 feet long
18:20 feet wide
11:8 feet high

$$\begin{array}{r} \text{men } 6 \qquad \qquad \qquad 2914 \qquad \qquad \qquad 2 \\ 60 \times 6 \times 5 \times 10 \times 8742 \times 20 \times 8 \\ \hline 10 \times 6 \times 11 \times 2400 \times 18 \times 11 \\ \qquad \qquad \qquad 240 \qquad \qquad \qquad 8 \\ \qquad \qquad \qquad 12 \qquad \qquad \qquad 3 \end{array}$$

$$\therefore 60: \frac{60 \times 6 \times 5 \times 10 \times 8742 \times 20 \times 8}{10 \times 6 \times 11 \times 2400 \times 18 \times 11} =$$

$$\frac{5 \times 2914 \times 2}{11 \times 3 \times 11} = \frac{29140}{363} = 80 \frac{100}{363}$$

(26)

172000 = 2⁵ × 5³ × 43. Increasing each exponent by 1 and multiplying them together we obtain 6 × 4 × 2 = 48.

3333+miles.
5 mile.
miles.

=858 denary, and

674000.

11010000 binary,

25
9
9
n.

2914 2
8742 x 20 x 8

10 x 18 x 11
0 8
2
3

ment by 1 and
<2=48.

42 7 = 42 $\frac{7}{9}$ = $\frac{388}{9}$ and $9 \cdot 7123 = 9 \frac{7123}{999} = 9 \frac{1186}{1000} = \frac{16171}{1000}$
 $\frac{388}{9} \times \frac{16171}{1000} = \frac{6226832}{10000} = 415.471137804$

(27)

100 : 27 :: \$73.42 : $\frac{73.42 \times 27}{100} = \19.8234
 $\$73.42 - \$19.8234 = \$53.5966$

(28)

6300 = 2² x 3² x 5² x 7.

- 1..5..25
- 1..2.. 4
- 1..5..25..2..10..50..4..20..100
- 1..3.. 9
- 1..5..25..2..10..50..4..20..100..3..15..75..6..30..150..
- 12..60..300..9..45..225..18..90..450..36..180..900
- 1..7
- 1..5..25..2..10..50..4..20..100..3..15..75..6..30..150..
- 12..60..300..9..45..225..18..90..450..36..180..900..7..
- 35..175..14..70..350..28..140..700..21..105..525..42..
- 210..1050..84..420..2100..63..315..1575..126..630..3150
- ..252..1260..6300.

Therefore the divisors of 6300 are 1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 14, 15, 18, 20, 21, 25, 28, 30, 35, 36, 42, 45, 50, 60, 63, 70, 75, 84, 90, 100, 105, 126, 140, 150, 175, 180, 210, 225, 252, 300, 315, 350, 420, 450, 525, 630, 700, 900, 1050, 1260, 1575, 2100, 3150, 6300

(29)

$\frac{2}{7}$ of $\frac{3}{8}$ of $3\frac{1}{2}$ lbs. = $\frac{3}{8}$ lbs., $\frac{2}{7}$ of $\frac{2}{3}$ of $\frac{2}{7}$ of $\frac{1}{4}$ of \$1 = $\frac{2}{3}$,
and $\frac{3}{8}$ of $\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{2}{10}$ of 90 lbs. = $\frac{1323}{200}$ lbs.

lbs. 63
 $\frac{3}{8} : \frac{1323}{200} :: \frac{2}{7} : \frac{\$ \frac{2}{7} \times \frac{1323}{200}}{\frac{3}{8}} = \frac{2}{7} \times \frac{441}{1323} \times \frac{8}{3} = \frac{126}{25} = \5.04

(31)

7 men will have 7 men's shares.

One woman has $\frac{2}{7}$ of a man's share; \therefore 2 women will have 2 $\times \frac{2}{7} = \frac{4}{7}$ of a man's share.

One child has $\frac{1}{7}$ of $\frac{2}{7} = \frac{2}{49}$ of a man's share; \therefore 11 children will have $11 \times \frac{2}{49} = \frac{22}{49}$ of a man's share.

7 men, 2 women, and 11 children will have $7 + \frac{4}{7} + \frac{22}{49} = 8\frac{37}{49}$ men's shares.

$$\$2739 \cdot 18 \div 8\frac{37}{49} = \$325 \cdot 99\frac{133}{49} = \text{a man's share.}$$

$$\frac{2}{7} \text{ of } \$325 \cdot 99\frac{133}{49} = \$88 \cdot 90\frac{118}{49} = \text{a woman's share.}$$

$$\frac{1}{7} \text{ of } \$88 \cdot 90\frac{118}{49} = \$25 \cdot 40\frac{128}{49} = \text{a child's share.}$$

(33)

(34)

| | | | | | |
|---|---------------------------|-----------------------------------|---|--------------|--|
| | yds. ft. in. | | | | |
| $\frac{1}{2}$ of $6\frac{1}{2}$ yds. | $= 2\ 2\ 8$ | $2\ 28 : 7\ 2$ | } | $= 104 : 5.$ | |
| $\frac{3}{8}$ of $\frac{7}{8}$ of $8\frac{1}{2}$ ft. | $= 1\ 0\ 0$ | $4 : 11$ | | | |
| $\frac{2}{7}$ of $\frac{3}{7}$ of $7\frac{1}{10}$ in. | $= \frac{3}{8}\ 2\ 6 : 5$ | $13 : 11\frac{1}{2}$ | | | |
| Sum | $= 3\ 2\ 8\frac{3}{8}$ | $2\frac{1}{2}\ 28\frac{1}{2} : 2$ | | | |

(35)

23 bush. 2 pks. 1 gal. 1 qt. 1 pt. = 1515 pts.

$$1515 \times 9000 \times \frac{1}{4} = 4545000 \text{ in.} = 71 \text{ miles } 5 \text{ fur. } 34 \text{ per. } 3 \text{ yds.}$$

(36)

$$\frac{4158}{10395} = \frac{462}{1155} = \frac{66}{165} = \frac{22}{55} = \frac{2}{5}.$$

(37)

VIII.

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$. Here the common denominator is $2 \times 3 \times 5 \times 7 = 322$. The numerators of the fractions are, for the first, $1 \times 3 \times 5 \times 7 = 151$; for the second, $2 \times 2 \times 5 \times 7 = 214$; for the third, $4 \times 2 \times 3 \times 7 = 250$; for the fourth, $2 \times 2 \times 3 \times 5 = 74$; and the equivalent fractions are, $\frac{151}{322}, \frac{214}{322}, \frac{250}{322}$, and $\frac{74}{322}$, which when added together $= \frac{721}{322} = 2\frac{65}{322}$, the numbers all through being in the octenary scale.

women will have 2
 ∴ 11 children
 $7 + 1\frac{6}{7} + \frac{6}{7} = 8\frac{3}{7}$

s share.
 an's share.
 's share.
 (34)

$\left. \begin{array}{l} K \ 2 \\ 1 \\ 5 \\ 1\frac{1}{2} \\ 3 \end{array} \right\} = 104 : 5.$

515 pts.
 ur. 34 per. 3 yds.

$2 \times 3 \times 5 \times 7$
 for the first, $1 \times$
 $\times 7 = 214$; for
 h, $2 \times 2 \times 3 \times$
 $\frac{1}{2}, \frac{3}{2}, \frac{5}{2},$ and
 the numbers all

(38)

$$\left. \begin{array}{l} 17 \text{ sheep} = 6 \text{ cows} \\ 26 \text{ cows} = 27\frac{1}{2} \text{ acres} \\ 12 \text{ acres} = 13 \text{ horses} \\ 11 \text{ horses} = 28 \text{ goats} \\ x \text{ goats} = 68 \text{ sheep} \end{array} \right\} = \frac{3}{6} \times \frac{2\frac{1}{2}}{27\frac{1}{2}} \times \frac{13}{12} \times \frac{28}{11} \times \frac{17}{68} =$$

$$2\frac{1}{2} \times 28 = 70 \text{ goats.}$$

(39)

$$\left. \begin{array}{l} 27 : 54 \text{ days} \\ 24 : 18 \text{ cel.} \\ 36 : 48 \text{ ft. l.} \\ 21 : 28 \text{ ft. w.} \\ 10 : 9 \text{ ft. d.} \\ 3 : 5 \text{ hrs.} \end{array} \right\} :: 50 \text{ men} : \frac{50 \times 54 \times 18 \times 48 \times 28 \times 9 \times 5}{27 \times 24 \times 36 \times 21 \times 10 \times 3} = 200 \text{ men.}$$

EXERCISE 90.—Page 226.

(1)
 $\$.35 \times 92647 = \$32426.45.$

(2)

| | £ | s | d. | |
|---------------------|-------|----|----|----------------------------------|
| 4d. $\frac{1}{4}$ | 4746 | 17 | 0 | = cost of 94937 pails at 1s. |
| 1d. $\frac{1}{4}$ | 1582 | 5 | 8 | = " " " at 4d. |
| | 395 | 11 | 5 | = " " " at 1d. |
| <hr/> | | | | |
| | £6724 | 14 | 1 | = cost of 94937 pails at 1s. 5d. |

(3) $\$.07\frac{1}{2} \times 95972 = \7197.90

(4) $\$28.80 \times 62 = \$1785.60.$

(5) $\$.32\frac{1}{2} \times 2310 = \$750.75.$

(6) $\$.37\frac{1}{2} \times 2117 = \$793.87\frac{1}{2}.$

(7)

| | £ | s | d. | |
|---------------------|------|----|-----------------|---|
| 6d. $\frac{1}{4}$ | 375 | 6 | 0 | = price of 7506 pairs at 1s. |
| 3d. $\frac{1}{4}$ | 187 | 13 | 0 | = " " at 6d. |
| 1d. $\frac{1}{4}$ | 93 | 16 | 6 | = " " at 3d. |
| | 23 | 9 | 1 $\frac{1}{2}$ | = " " at $\frac{1}{2}$ d. |
| <hr/> | | | | |
| | £680 | 4 | 7 $\frac{1}{2}$ | = price of 7506 pairs at 1s. 9 $\frac{1}{2}$ d. |

$$(8) \quad \$ \cdot 17\frac{1}{2} \times 1217 = \$212.97\frac{1}{2}.$$

$$(9) \quad \$3 \cdot 07\frac{1}{2} \times 2103 = \$6466 \cdot 72\frac{1}{2}.$$

(10)

| | | | | |
|---------|----------------|-------|-----|---------------------------|
| 10s. | $\frac{1}{2}$ | 2096 | | |
| | | 3 | | |
| | | £6288 | 0 0 | = cost of 2096 oz. at £3. |
| 5s. | $\frac{1}{2}$ | 1048 | 0 0 | = " " at 0 10s. |
| 2s. 6d. | $\frac{1}{2}$ | 524 | 0 0 | = " " at 0 5s. |
| 1s. 3d. | $\frac{1}{2}$ | 262 | 0 0 | = " " at 0 2s. 6d. |
| 1½d. | $\frac{1}{10}$ | 131 | 0 0 | = " " at 0 1s. 3d. |
| | | 13 | 2 0 | = " " at 0 0 1½d. |
| | | £8266 | 2 0 | = " " at £3 18s. 10½d. |

(11)

| | | | | |
|----------------|----------------|--------|--------------------|--|
| 10 dwt. | $\frac{1}{2}$ | \$1.55 | | |
| | | 6 | | |
| | | \$9.30 | = cost of 6 oz. | |
| 5 dwt. | $\frac{1}{2}$ | .77½ | = " 10 dwt. | |
| 2 dwt. 12 grs. | $\frac{1}{2}$ | .38½ | = " 5 dwt. | |
| 1 dwt. 6 grs. | $\frac{1}{2}$ | .19½ | = " 2 dwt. 12 grs. | |
| 2 grs. | $\frac{1}{15}$ | .09½ | = " 1 dwt. 6 grs. | |
| | | .00¾ | = " 2 grs. | |

$$\$10 \cdot 75\frac{3}{4} = \text{cost of 6 oz. 18 dwt. 20 grs.}$$

(12)

| | | | | |
|--------|---------------|----------------|--|--|
| 10s. | $\frac{1}{2}$ | £98 0 0 | = cost of 98 yards at £1. | |
| 5s. | $\frac{1}{2}$ | 49 0 0 | = " " 0 10s. | |
| | | 24 10 0 | = " " 0 5s. | |
| | | £171 10 0 | = cost of 98 yards at £1 15s. | |
| 2 qrs. | $\frac{1}{2}$ | £1 15 | | |
| | | 17 6 | = cost of 2 qrs. | |
| 1 qr. | $\frac{1}{2}$ | 8 9 | = " 1 qr. | |
| 1 na. | $\frac{1}{4}$ | 2 2½ | = " 1 na. | |
| | | £1 8 5½ | = cost of 3 qrs. 1 na. | |
| | | Then £171 10 0 | = cost of 98 yards at £1 15s. | |
| | | 1 8 5½ | = cost of 3 qrs. 1 na. at £1 15s. per yard. | |
| | | £172 18 5½ | = cost of 98 yds. 3 qrs. 1 na. at £1 15s. per yd | |

(9)
03 = \$6466.72½.

£3.
0 10s.
0 5s.
0 2s. 6d.
0 1s. 3d.
0 0 1½d.
£3 18s. 10½d.

2 grs.
6 grs.
dwt. 20 grs.

1.
0 10s.
0 5s.
1 15s.
s. per yard.
£1 15s. per yd

(13)

| | | | | | |
|-----|----------------|-------|----|------------------------------------|--|
| 1s. | $\frac{1}{20}$ | 344 | | | |
| | | 4 | | | |
| | | <hr/> | | | |
| 1d. | $\frac{1}{12}$ | £1376 | 0 | 0 = rent of 344 acres at £4. | |
| | | 17 | 4 | 0 = " " at 0 ls. | |
| | | 1 | 8 | 8 = " " at 0 0 ld. | |
| | | <hr/> | | | |
| | | £1394 | 12 | 8 = rent of 344 acres at £4 1s. 1d | |

| | | | | | |
|---------|---------------|-------|---|----------------------------|--|
| 2 r. | $\frac{1}{2}$ | £4 | 1 | 1 | |
| | | <hr/> | | | |
| 1 r. | $\frac{1}{2}$ | 2 | 0 | 6½ = rent of 2 roods. | |
| 10 per. | $\frac{1}{4}$ | 1 | 0 | 3¼ = " 1 rood. | |
| 5 per. | $\frac{1}{2}$ | 5 | 0 | 1½ = " 10 perches. | |
| | | 2 | 6 | ½ = " 5 perches. | |
| | | <hr/> | | | |
| | | £3 | 8 | 4¾ = " 3 roods 15 perches. | |

£1394 12 8 = rent of 344 acres at £4 1s. 1d.
3 8 4¾ = " 3 roods 15 per. at £4 1s. 1d. per ac.
£1398 1 0¾ = " 344 a. 3 r. 15 per. at £4 1s. 1d.

(14)

| | | | | | |
|---------|---------------|--------------------|-------------------|---|--|
| 5 dwt. | $\frac{1}{4}$ | 5 | 10 | | |
| | | 5 | | | |
| | | <hr/> | | | |
| | | £1 | 9 | 2 = price of 5 oz. at 5s. 10d. per oz | |
| 1 dwt. | $\frac{1}{8}$ | 1 | 5½ = " 5 dwt. " " | | |
| 12 grs. | $\frac{1}{2}$ | 3½ = " 1 dwt. " " | | | |
| 4 grs. | $\frac{1}{3}$ | 1¾ = " 12 grs. " " | | | |
| 1 gr. | $\frac{1}{4}$ | 0¾ = " 4 grs. " " | | | |
| | | 0¾ = " 1 gr. " " | | | |
| | | <hr/> | | | |
| | | £1 | 11 | 1¾ = " 5 oz. 6 dwt. 17 grs. at 5s. 10d. per oz. | |

(15)

| | | | | |
|--------|---------------|----------------------|---------------------------------------|-----|
| 2 qrs. | $\frac{1}{2}$ | £1 2 4 | | |
| | | | 4 | |
| | | <hr/> | | |
| 2 na. | $\frac{1}{4}$ | £4 9 4 | = price of 4 yards at £1 2 4 per yard | |
| 1 na. | $\frac{1}{8}$ | 11 2 | = " 2 qrs. | " " |
| | | 2 9 $\frac{1}{2}$ | = " 2 na. | " " |
| | | 1 4 $\frac{3}{4}$ | = " 1 na. | " " |
| | | <hr/> | | |
| | | £5 4 8 $\frac{1}{2}$ | = price of 4 yds. 2 qrs. 3 na. | " " |

(16)

| | | | | |
|---------|---------------|-----------------------|------------------------------------|-----|
| 1 rood. | $\frac{1}{4}$ | £1 16 | | |
| | | | 32 | |
| | | <hr/> | | |
| | | £57 12 0 | = price of 32 acres at £1 16s. | |
| 10 per. | $\frac{1}{4}$ | 9 0 | = " 1 rood. | " " |
| 2 per. | $\frac{1}{8}$ | 2 3 | = " 10 per. | " " |
| 2 per. | $\frac{1}{8}$ | 5 $\frac{3}{4}$ | = " 2 per. | " " |
| | | 5 $\frac{3}{4}$ | = " 2 per. | " " |
| | | <hr/> | | |
| | | £58 4 1 $\frac{3}{8}$ | = price of 32 acres 1 rood 14 per. | |

(17)

| | | | | |
|--------|---------------|----------------------|--|-----|
| 4 pts. | $\frac{1}{2}$ | 7 6 | | |
| | | | 3 | |
| | | <hr/> | | |
| | | £1 2 6 | = price of 3 gals. at 7s. 6d. per gal. | |
| 1 pt. | $\frac{1}{4}$ | 3 9 | = " 4 pts. | " " |
| | | 11 $\frac{1}{4}$ | = " 1 pt. | " " |
| | | <hr/> | | |
| | | £1 7 2 $\frac{1}{4}$ | = price of 3 gals. 5 pts. | |

(18)

$$\$1.67\frac{1}{2} \times 724 = \$1212.70.$$

(19)

$$\$1.93\frac{1}{4} \times 721 = \$1396.93\frac{1}{4}.$$

(20)

| | | | | | |
|------------------|----------------|-------------|----------------------------|---|----------------------------|
| 10s. | $\frac{1}{2}$ | 4514 | | | |
| | | 2 | | | |
| | | <hr/> | | | |
| | | £9028 0 0 | = cost of 4514 rods at £2. | | |
| 6s. 8d. | $\frac{1}{2}$ | 2257 0 0 | = | " | " at 0 10 |
| 10d. | $\frac{1}{2}$ | 1504 13 4 | = | " | " at 0 6 8 |
| 1d. | $\frac{1}{10}$ | 188 1 8 | = | " | " at 0 0 10 |
| $\frac{1}{2}$ d. | $\frac{1}{2}$ | 18 16 2 | = | " | " at 0 0 1 |
| | | 9 8 1 | = | " | " at 0 0 0 $\frac{1}{2}$ |
| | | <hr/> | | | |
| | | £13005 19 3 | = | " | " at £2 17 7 $\frac{1}{2}$ |

(21)

| | | | | | |
|------|----------------|---------------------------|--|---|------------|
| 10s. | $\frac{1}{2}$ | £3749 7 6 | | | |
| | | 3 | | | |
| | | <hr/> | | | |
| | | £11248 2 6 | = price of 3749 $\frac{3}{4}$ acres at £3 | | |
| 5s. | $\frac{1}{2}$ | 1874 13 9 | = | " | " at 0 10 |
| 6d. | $\frac{1}{10}$ | 937 6 10 $\frac{1}{2}$ | = | " | " at 0 5 |
| | | 93 14 8 $\frac{1}{2}$ | = | " | " at 0 0 6 |
| | | <hr/> | | | |
| | | £14153 17 9 $\frac{1}{2}$ | = price of 3749 $\frac{3}{4}$ acres at £3 15 6 | | |

(22)

| | | | | | |
|-----|---------------|---------|-----------------------------|---|------------|
| 4s. | $\frac{1}{5}$ | £17 0 0 | = cost of 17 cwt. at £1 | | |
| 8d. | $\frac{1}{5}$ | 3 8 0 | = | " | " at 0 4 |
| 1d. | $\frac{1}{5}$ | 11 4 | = | " | " at 0 0 8 |
| | | 1 5 | = | " | " at 0 0 1 |
| | | <hr/> | | | |
| | | £21 0 9 | = cost of 17 cwt. at £1 4 9 | | |

| | | | | | |
|--------------------------|----------------|----------------------|---------------------|-----------------------|---------------|
| 1 qr. | $\frac{1}{4}$ | £1 4 9 | | | |
| | | <hr/> | | | |
| 16 lbs. | $\frac{1}{4}$ | 6 2 $\frac{1}{2}$ | = cost of 1 qr. | | |
| 1 lb. | $\frac{1}{16}$ | 3 6 $\frac{3}{4}$ | = | " | 16 lbs. |
| | | 0 2 $\frac{7}{16}$ | = | " | 1 lb. |
| | | <hr/> | | | |
| | | 9 11 $\frac{37}{16}$ | = | " | 1 qr. 17 lbs. |
| £21 0 9 | | = cost of 17 cwt. at | £1 4s. 9d. per cwt. | | |
| 9 11 $\frac{37}{16}$ | | = | " | 1 qr. 17 lbs. | " " |
| <hr/> | | | | | |
| £21 10 8 $\frac{37}{16}$ | | = | " | 17 cwt. 1 qr. 17 lbs. | " " |

(23)

| | | | | | |
|--------|------|------------|---------------------------------------|--------|-----|
| 2 qrs. | 1 | \$11.55 | | | |
| | | 78 | | | |
| | | 9240 | | | |
| | | 8085 | | | |
| | | \$900.90 | = cost of 78 cwt. at \$11.55 per cwt. | | |
| 1 qr. | 1/2 | 5.77 1/2 = | " | 2 qrs. | " " |
| 7 lbs. | 1/4 | 2.88 1/4 = | " | 1 qr. | " " |
| 4 lbs. | 1/8 | .72 1/6 = | " | 7 lbs. | " " |
| 1 lb. | 1/16 | .41 1/4 = | " | 4 lbs. | " " |
| | | .10 5/16 = | " | 1 lb. | " " |
| | | \$910.80 | = cost of 78 cwt. 3 qrs. 12 lbs. | | |

(24)

£10 10
20

£210 0 = price of 20 tons at £10 10s.

19 cwt. 3 qrs. 27 1/2 lbs. = 1 ton. - 1/2 lb. The price of 1 ton is £10 10s., and the price of 1/2 lb. = 1/4480 of £10 10s. = 11 3/4 d. ∴ the price of 19 cwt. 3 qrs. 27 1/2 lbs. = £10 10s. - 11 3/4 d. = £10 9s. 11 1/2 d.

£210 0 0 = price of 20 tons at £10 10s.

10 9 11 1/2 = " 19 cwt. 3 qrs. 27 1/2 lbs.

£220 9 11 1/2 = " 20 tons 19 cwt. 3 qrs. 27 1/2 lbs. at £10 10s. per ton.

(25)

| | | | | | |
|---------|-----|----------------|---|---|---------|
| 10 cwt. | 1/2 | \$45.50 | | | |
| | | 219 | | | |
| | | <hr/> | | | |
| | | 40950 | | | |
| | | 4550 | | | |
| | | 9100 | | | |
| | | <hr/> | | | |
| | | \$9964.50 | = price of 219 tons at \$45.50 per ton. | | |
| 5 cwt. | 1/2 | 22.75 | = | " | 10 cwt. |
| 1 cwt. | 1/2 | 11.37 1/2 | = | " | 5 cwt. |
| 2 qrs. | 1/2 | 2.27 1/2 | = | " | 1 cwt. |
| 1 qr. | 1/2 | 1.13 3/4 | = | " | 2 qrs. |
| | | 56 7/8 | = | " | 1 qr. |
| | | <hr/> | | | |
| | | \$10002.60 1/2 | = price of 219 tons 16 cwt. 3 qrs. | | |

EXERCISE 91—Page 228.

BILLS OF PARCELS.

(No. 2.)

| | s. | d. | £ | s. | d. |
|---|----|-------|----------|----|-----------|
| 9 pair of worsted stockings, at..... | 4 | 6 | 2 | 0 | 6 |
| 6 pair of silk ditto, at..... | 15 | 9 | 4 | 14 | 6 |
| 17 pair of thread ditto, at..... | 5 | 4 | 4 | 10 | 3 |
| 23 pair of cotton ditto, at..... | 4 | 10 | 5 | 11 | 2 |
| 14 pair of yarn ditto, at..... | 2 | 4 | 1 | 12 | 8 |
| 18 pair of women's silk gloves, at... 4 | 2 | " | 3 | 15 | 0 |
| 19 yards of flannel, at..... | 1 | 7 1/2 | per yard | 1 | 10 10 1/2 |

Ans. £23 15 4 1/2

(No. 3.)

| | | |
|---------------------------------|---------------------|------------|
| 75 1/2 lbs. of sugar, at..... | 7 1/2 cents per lb. | \$5.85 1/2 |
| 63 lbs. of tea, at..... | 93 | " 58.59 |
| 126 lbs. of butter, at..... | 13 | " 16.38 |
| 35 1/2 lbs. of raisins, at..... | 18 1/2 | " 6.71 1/2 |
| 17 lbs. of sago, at..... | 15 | " 2.55 |
| 23 lbs. of rice, at..... | 9 | " 2.07 |
| 58 1/2 lbs. of starch, at..... | 22 | " 12.87 |

Ans. \$105.02 1/2

(No. 4.)

| | | |
|---|--------|----------|
| 198 Sangster's National Arithmetic, at..... | \$0.60 | \$118.80 |
| 197 Robertson's Philosophy of Grammar, at... | 0.50 | 98.50 |
| 83 Hodgins' Geography, at..... | 1.00 | 83.00 |
| 57 Sangster's Algebraic Formula, at..... | 0.12½ | 7.12½ |
| 217 Strachan's Canadian Penmanship, at..... | 0.37½ | 81.37½ |
| 143 Hodgins' Geography of British Provinces, at | 0.45 | 64.35 |
| 227 Sangster's First Arithmetic, at..... | 0.30 | 68.10 |

Ans. \$521.25

(No. 5.)

| | s. | d. | £ | s. | d. |
|-----------------------------------|----|------------|----|----|----|
| 9½ yards of silk, at..... | 12 | 9 per yard | 6 | 1 | 1½ |
| 13 yards of flowered ditto, at... | 15 | 6 " | 10 | 1 | 6 |
| 11½ yards of lustring, at..... | 6 | 10 " | 4 | 0 | 3½ |
| 14 yards of brocade, at..... | 11 | 3 " | 7 | 17 | 6 |
| 12½ yards of satin, at..... | 10 | 8 " | 6 | 10 | 8 |
| 11½ yards of velvet, at..... | 18 | 0 " | 10 | 4 | 9 |

Ans. £44 15 10

(No. 6.)

| | | |
|-----------------------------|--------|--------|
| 14 oz. ipecacuanha, at..... | \$0.67 | \$9.38 |
| 23 " laudanum, at..... | 0.89 | 20.47 |
| 17 " emetic tartar, at..... | 1.25 | 21.25 |
| 25 " cantharides, at..... | 2.17 | 54.25 |
| 27 " gum mastic, at..... | 0.61 | 16.47 |
| 56 " gum camphor, at..... | 0.27 | 15.12 |

Ans. \$136.94

(No. 7.)

| | s. | d. | £ | s. | d. |
|--|----|-----------|----|-----|----|
| 15½ lbs. of currants, at..... | 0 | 4 per lb. | 5 | 2 | |
| 17½ lbs. of Malaga raisins, at..... | 0 | 5½ " | 7 | 10½ | |
| 19½ lbs. of sun raisins, at..... | 0 | 6 " | 9 | 10½ | |
| 17 lbs. of rice, at..... | 0 | 3½ " | 4 | 11½ | |
| 8½ lbs. of pepper, at..... | 1 | 6 " | 12 | 9 | |
| 3 loaves of sugar, weight 32½ lbs. at. | 0 | 8½ " | 1 | 3 | 0½ |
| 13 oz. of cloves, at..... | 0 | 9 per oz. | 9 | 9 | |

Ans. £3 13 5½

...\$0.60 \$118.80
 ... 0.50 98.50
 ... 1.00 83.00
 ... 0.12½ 7.12½
 ... 0.37½ 81.37½
 at 0.45 64.35
 ... 0.30 68.10

Ans. \$521.25

£ s. d.
 6 1 1½
 10 1 6
 4 0 3½
 7 17 6
 6 10 8
 10 4 9

Ans. £44 15 10
 \$9.38
 20.47
 21.25
 54.25
 16.47
 15.12

Ans. \$136.94
 £ s. d.
 5 2
 7 10½
 9 10½
 4 11½
 12 9
 1 3 0½
 oz. 9 9
 s. £3 13 5½

EXERCISE 92—Page 231.

(2)

$$427.1 \div .0000637 = 427100000 \div 637 = 6704866.561 +.$$

(3)

| | | | | | | |
|------|---|------|----|---|--------------------------|--|
| 10s. | ½ | £19 | | | | |
| | | 19 | | | | |
| | | 171 | | | | |
| | | 19 | | | | |
| | | £361 | 0 | 6 | = cost of 19 tons at £19 | |
| 5s. | ½ | 9 | 10 | 0 | = " " at 0 10 | |
| 4s. | ½ | 4 | 15 | 0 | = " " at 0 5 | |
| 6d. | ½ | 3 | 16 | 0 | = " " at 0 4 | |
| 3d. | ½ | | 9 | 6 | = " " at 0 0 6 | |
| 2d. | ½ | | 4 | 9 | = " " at 0 0 3 | |
| ½d. | ½ | | 3 | 2 | = " " at 0 0 2 | |
| ¼d. | ½ | | 9½ | | = " " at 0 0 0½ | |
| | | | 4½ | | = " " at 0 0 0¼ | |

£379 19 7½ = cost of 19 tons at £19 19 11½

19 cwt. 3 qrs. 27½ lbs. = 1 ton — ½ lb. The price of 1 ton is
 19 19s. 11½d., and the cost of ½ lb. = $\frac{1}{4180}$ of £19 19s. 11½d.
 = $1\frac{1279}{17920}$ d.; ∴ the cost of 19 cwt. 3 qrs. 27½ lbs. = £19 19s.
 11½d. — $1\frac{1279}{17920}$ d. = £19 19s. $10\frac{2161}{17920}$ d.
 £379 19 7½ = cost of 19 tons.
 19 19 $10\frac{2161}{17920}$ = " 19 cwt. 3 qrs. 27½ lbs.
 £399 19 $5\frac{1661}{17920}$ = " 19 tons 19 cwt. 3 qrs. 27½ lbs.

(4)

| Dissimilar. | | Similar. | | Similar and Coterminous. |
|-------------|---|--------------|---|----------------------------|
| 73.723 | = | 73.723723 | = | 73.723723723 |
| 11.342 | = | 11.3422 | = | 11.342222222 |
| 16.713 | = | 16.7130 | = | 16.713000000 |
| 19.034 | = | 19.034034 | = | 19.034034034 |
| 713.213437 | = | 713.213437 | = | 713.213437437 |
| 12.345678 | = | 12.345678345 | = | 12.345678345
2 carried. |
| Sum | | | = | 846.372095763 |

(5)

$5 : 7 = 5 \div 7 = .714+$
 $9 : 13 = 9 \div 13 = .692+$
 $12 : 17 = 12 \div 17 = .705+$
 $7 : 10 = 7 \div 10 = .7$

Hence 5 : 7 is the greatest,
and 9 : 13 least.

$$\frac{5}{7} \times \frac{9}{13} \times \frac{12}{17} \times \frac{7}{10} = \frac{54}{221} = 54 : 221.$$

(6)

1 acre = 160 rods, and 25 acres 2 rods 35 rods = 4115 rods.

$$160 : 4115 :: \$80.50 : \frac{40 \cdot 25 \quad 823}{80 \cdot 50 \times 4115} = \$2070.3593.$$

(8)

$$\$3.682 \times 7439 = \$27431.314.$$

and Coterminous.

723723723
 342222222
 713000000
 34634034
 13437437
 45678345
 2 carried.
 72095763

(9)

$\frac{135795}{222210}$. The G. C. M. of 135795 and 222210 is 12345; when both terms of the fraction are divided by 12345, it becomes $\frac{11}{18}$.

$\frac{714235}{999999}$. Here 714235 and 999999 have no G. C. M.; \therefore the fraction cannot be reduced.

$\frac{109375}{100000}$. The G. C. M. of 109375 and 100000 is 3125; when both terms of the fraction are divided by 3125, it becomes reduced to $\frac{34}{32}$.

$\frac{20301}{33633}$. The G. C. M. of 20301 and 33633 is 303; when both terms of the fraction are divided by 303, it is reduced to its lowest terms, viz., $\frac{67}{111}$.

(10)

| | | | |
|---------------------|---|---------------------|-----|
| 34½ bushels turnips | = | 17 bushels potatoes | } = |
| 9 " potatoes | = | 59½ lbs. tea | |
| 6 lbs. tea | = | 11½ stone flour | |
| 13 stone flour | = | 360 cents | |
| 38 cents | = | 12 lbs. bread | |
| 119 lbs bread | = | x bushels turnips | |

$$\frac{34\frac{1}{2}}{17} \times \frac{9}{59\frac{1}{2}} \times \frac{6}{11\frac{1}{2}} \times \frac{13}{360} \times \frac{19}{12} \times \frac{7}{1} = \frac{3 \times 13 \times 19}{8\frac{1}{2} \times 40} = 2\frac{61}{40}$$

ls = 4115 rods.

(11)

| | | |
|-------------------|---------------|--|
| 54 : 27 men | } :: 7 days : | $\frac{7 \times 27 \times 8 \times 7 \times 24 \times 22 \times 5}{54 \times 11 \times 22 \times 20 \times 16 \times 3}$ |
| 11 : 8 hours | | |
| 42 : 77 floors | | |
| 20 : 24 feet long | | |
| 16 : 22 feet wide | | |
| 3 : 5 coats paint | | |

$$= \frac{7 \times 11}{2 \times 3} = 12\frac{2}{3} \text{ days.}$$

070-3593.

(13)

IX.
12)72342
12)5403..2
12)407..0
12)30..7
2..3

IX.
6)72342
6)11806..2
6)1731..0
6)264..4
6)40..4
6)6..0
1..0

IX.
3)72342
3)23713..2
3)7234..0
3)2371..1
3)723..1
3)237..0
3)72..1
3)23..2
3)7..0
2..1

| | | | | | | | |
|--------------|---|--------------|---|----------------|---|-------------------|-------|
| IX. | = | XII. | = | VI. | = | III | |
| <u>72342</u> | | <u>23702</u> | | <u>1004402</u> | | <u>2102101102</u> | |
| 9 | | 12 | | 6 | | 3 | |
| --- | | --- | | --- | | --- | |
| 65 | | 27 | | 6 | | 7 | |
| 9 | | 12 | | 6 | | 3 | |
| --- | | --- | | --- | | --- | |
| 588 | | 331 | | 36 | | 21 | |
| 9 | | 12 | | 6 | | 3 | |
| --- | | --- | | --- | | --- | |
| 5296 | | 3972 | | 220 | | 65 | 1765 |
| 9 | | 12 | | 6 | | 3 | 3 |
| --- | | --- | | --- | | --- | --- |
| 47666 | | 47666 | | 1324 | | 196 | 5296 |
| | | | | 6 | | 3 | 3 |
| | | | | --- | | --- | --- |
| | | | | 7944 | | 588 | 15888 |
| | | | | 6 | | 3 | 3 |
| | | | | --- | | --- | --- |
| | | | | 47666 | | 1765 | 47666 |

(14)

IX.
3)72342
3)23713..2
3)7234..0
3)2371..1
3)723..1
3)237..0
3)72..1
3)23..2
3)7..0
2..1

| II. | III. | IV. | V. |
|--------------|-----------|----------------|-------------|
| 111111 | 100000 | 333333 | 100000 |
| 2 | 2 | 4 | 4 |
| — | — | — | — |
| 3 | 2 | 15 | 4 |
| 2 | 2 | 4 | 4 |
| — | — | — | — |
| 7 | 4 | 63 | 16 |
| 2 | 2 | 4 | 4 |
| — | — | — | — |
| 15 | 8 | 255 | — |
| 2 | 2 | 4 | 64 |
| — | — | — | 4 |
| 31 | 16 | 1023 | — |
| 2 | 2 | 4 | 256 |
| — | — | — | 4 |
| 63 Greatest. | 32 Least. | 4095 Greatest. | 1024 Least. |

III
2102101102
3
—
7
3
—
21
3
—
65 1765
3 3
—
196 5296
3 3
—
588 15888
3 3
—
765 47666

| VI. | VII. | VIII. | IX. |
|-----------------|-------------|------------------|--------|
| 555555 | 100000 | 777777 | 100000 |
| 6 | 6 | 8 | 8 |
| — | — | — | — |
| 35 | 6 | 63 | 8 |
| 6 | 6 | 8 | 8 |
| — | — | — | — |
| 215 | 36 | 511 | 64 |
| 6 | 6 | 8 | 8 |
| — | — | — | — |
| 1295 | 216 | 4095 | 512 |
| 6 | 6 | 8 | 8 |
| — | — | — | — |
| 7775 | 1296 | 32767 | 4096 |
| 6 | 6 | 8 | 8 |
| — | — | — | — |
| 46655 Greatest. | 7776 Least. | 262143 Greatest. | 32768 |

(Continued on next page.)

(17)

| Dissimilar. | | Similar. | | Similar and Coterminous. |
|-------------|---|------------|---|--------------------------|
| 97-91342 | = | 97-913423 | = | 97-913423423423423 |
| 18-1234567 | = | 19-1234567 | = | 18-123456745674567 |
| | | Difference | = | 79-789966677748855 |

(18)

| | |
|-------------|----------|
| 20 ft. 7' | |
| 19 ft. 5 7" | |
| <hr/> | |
| 1 | 0 0 1''' |
| 8 | 6 11 |
| 391 | 1 |
| <hr/> | |

400 7 11 1 = 44 sq. yds. + $\frac{1}{8}$ + $\frac{7}{16}$ + $\frac{11}{32}$ + $\frac{1}{64}$ =
 $44\frac{8053}{16384}$ sq. yds. = 44-517 + sq. yds.
 $\$2-87\frac{1}{2} \times 44-517 = \$127-98 +.$

(19)

916 acres 3 roods 17 per. 7 yds. = 4437591 $\frac{1}{4}$ sq. yds., and 43
 acres 1 rood 2 per. 17 yds. = 209407 $\frac{1}{4}$ sq. yds.
 $4437591\frac{1}{4} \div 209407\frac{1}{4} = 4437591.25 \div 207407.5 = 21-19117+.$

EXERCISE 94--Page 233

(1)

$\$742-10 \times .05 = \$37-10\frac{1}{2}.$

(2)

$\$1000 \times .11 = \$110.$

(3)

$\$734-19 \times .10 = \$73-419.$

(4)

$\$1624-50 \times .875 = \$1421-4375.$

5)

$$\$994.70 \times .125 = \$124.3375.$$

(6)

$$\$777.50 \times .0875 = \$68.03125, \text{ or } \$68.03\frac{1}{2}.$$

(7)

$$\$7135.80 \times .0225 = \$160.5555.$$

(8)

$$2740 \times .2 = 548.$$

(9)

$$\$7490 \times .10 = \$749$$

$$\$7490 \times .17 = \$1273.30$$

$$\$7490 \times .27 = \$2022.30$$

$$\$7490 \times .46 = \$3445.40$$

(10)

$$\$740 \times .045 = \$33.30$$

$$\$1680 \times .025 = \$42.00$$

$$\$42.00 - \$33.30 = \$8.70$$

(11)

$$729 \times .11 = 80.19$$

$$729 - 80.19 = 648.81 = 648\frac{81}{100}$$

$$\$763.22 \times .25 = \$190.8050$$

$$\$847.16 \times .16 = 135.5456$$

$$\$1234.17 \times .0625 = 77.135625$$

(12)

$$\text{Sum} = \$403.486225$$

(13)

$$\$17429.40 \times .43 = \$7494.64\frac{1}{2}$$

$$\$17429.40 \times .37 = 6448.87\frac{1}{2}$$

$$\$13943.52$$

$$\$17429.40 - \$13943.52 = \$3485.88.$$

(14)

$$68978 \times .36 = 24832.08.$$

(15)

$$29800 \times .17 = 5066$$

$$29800 - 5066 = 24734$$

EXERCISE 95—Page 235.

(1)

$$\$1000 \times .045 = \$45.$$

(2)

$$\$1678.30 \times .0225 = \$37.76175,$$

(3)

(4)

$$\$7531.19 \times .0375 = \$282.419625. \quad \$508.60 \times .0125 = \$6.3575$$

(5)

(6)

$$\$7863.50 \times .0175 = \$137.61125. \quad \$878.30 \times .025 = \$21.9575$$

(7)

(8)

$$\$7193.16 \times .03125 = \$224.78625. \quad \$6734.10 \times .17 = \$1144.797.$$

(9)

$$\$7.13 \times 718 \times .0425 = \$217.57195.$$

(10)

$$\$1.85 \times 8243 \times .05625 = \$857.7871875.$$

 EXERCISE 96—Page 236.

(1)

(2)

$$\$7893.87 \times .02 = \$157.8774. \quad \$8000 \times .00875 = \$70.$$

(3)

$$\$8643.22 \times .0125 = \$108.04025.$$

(4)

$$\$78963.80 \times .00875 = \$690.93325.$$

(5)

$$\$1987.27 \times .0375 = \$74.522625.$$

EXERCISE 97—Page 237.

(1)

$$\begin{aligned} \$4000 \div 1.0125 &= \$3950.61728 + = \text{sum to be invested.} \\ \$4000 - 3950.61728 &= \$49.38271 = \text{commission.} \end{aligned}$$

(2)

$$\begin{aligned} \$7500 \div 1.045 &= \$7177.03349 = \text{sum to be expended in laces.} \\ \$7500 - \$7177.03349 &= \$322.96651 = \text{commission.} \end{aligned}$$

(3)

$$\begin{aligned} \$8470 \div 1.05 &= \$8066.66\frac{2}{3} = \text{sum to be invested.} \\ \$8066.66\frac{2}{3} \div \$6.40 &= 1260\frac{1}{2} \text{ barrels.} \end{aligned}$$

(4)

$$\$11000 \div 1.00875 = \$10904.584882 = \text{sum to be invested.}$$

(5)

$$\begin{aligned} \$13000 \div 1.045 &= \$12440.1913 + = \text{sum to be invested.} \\ \$13000 - \$12440.1913 &= \$559.8086 + = \text{commission.} \\ \$12440.1913 + \div \$3.63 &= 3427.0499 \text{ yds.} \end{aligned}$$

EXERCISE 98—Page 238.

(1)

$$\$9000 \div 0.83 = \$10843.373.$$

(2)

$$\$8500 \div 1.11 = \$7657.6576.$$

(3)

$$\begin{aligned} \$17500 \div 1.0125 &= \$17283.951 = \text{amount to be invested.} \\ \$17283.951 \div 1.07 &= \$16153.22 = \text{stock.} \end{aligned}$$

(4)

$$\begin{aligned} \$20000 \div 1.0175 &= \$19656.01965 = \text{amount to be invested.} \\ \$19656.01965 \div 0.97 &= \$20263.937 = \text{stock remitted.} \end{aligned}$$

(5)

$$\begin{aligned} \$200 \times 100 &= \$20000 = \text{par value of 200 shares.} \\ \$1 \text{ stock costs } \$1.055. & \quad \$1.055 \times 20000 = \$21100 = \text{cost of stock.} \\ \$21100 \times .00875 &= \$184.625 = \text{brokerage.} \\ \$21100 + \$184.625 &= \$21284.625 = \text{whole cost.} \end{aligned}$$

EXERCISE 99—Page 240.

(1)

$$\$7500 \times .0175 = \$131.25.$$

(2)

$$\$8375 \times .0075 = \$62.8125.$$

(3)

$$\$6000 \times .01875 = \$112.50$$

(4)

$$\$5000 \times .0117 = \$58.50.$$

(5)

$$\$6400 \times .0090 = \$57.60$$

(6)

$$\$4500 \times .0035 = \$15.75.$$

(7)

$$\$36000 \times .03 = \$1080.$$

(8)

$$\$27000 \times .0482 \times 4 = \$5205.60.$$

(9)

$$\$39000 \times .022 = \$858.$$

(10)

$$\$17800 \times .005 = \$89.$$

(11)

$$\$12350 \times .01\frac{1}{2} \times 7 = \$1235.$$

EXERCISE 100—Page 241.

(1)

$$\$17000 \div 965 = \$17616.58.$$

(2)

$$\$22750 \div .94 = \$24202.127.$$

(3)

$$\$15000 \div .9775 = \$15345.2685.$$

(4)

$$\$33000 \div .9425 = \$35013.2625.$$

EXERCISE 101—Page 243.

(1)

$$1347 \times 5 = 6735 \text{ lbs.} = \text{gross weight.}$$

$$6735 \times .06 = 404.1 \text{ lbs.} = \text{tare.}$$

$$\begin{array}{r} 6330.9 \text{ lbs.} = \text{net at } 3\frac{1}{2} \text{ cents per lb.} = 6330.9 \\ \times .035 = \$221.58. \end{array}$$

(2)

$$127 \times 11 = 1397 \text{ lbs.} = \text{gross weight.}$$

$$1397 \times .03 = 41.91 \text{ lbs.} = \text{tare.}$$

$$\begin{array}{r} 1355.09 \text{ lbs.} = \text{net at } \$.012 \text{ per lb.} = 1355.09 \\ \times .012 = \$16.26. \end{array}$$

(3)

$$.29 \times .13 = \$16.77.$$

(4)

$$31 \times 207 = 6417 \text{ lbs.} = \text{gross weight.}$$

$$207 \times 2\frac{1}{4} = 465\frac{1}{4} \text{ lbs.} = \text{tare.}$$

$$\begin{array}{r} 5951\frac{1}{4} \text{ lbs.} = \text{net at } 5\frac{1}{2} \text{ cents per lb.} = 5951\frac{1}{4} \times \\ .0575 = \$342.1968. \end{array}$$

(5)

$$214 \times .47 = \$100.58.$$

EXERCISE 102—Page 243.

(1)

$$\$17429.80 \times .21 = \$3660.2580. \quad \$2920.16 \times .075 = \$219.012.$$

(2)

(3)

$$\$71342.90 \times .25 = \$17835.725. \quad \$913.73 \times .2 = \$182.746.$$

(4)

(5)

$$\$14713.19 \times .33 = \$4855.3527.$$

EXERCISE 103—Page 244.

(1)

$$\begin{aligned} \$23900 \div 7142300 &= \$0.0033462 = \text{rate per dollar.} \\ \$0.0033462 \times 14729.50 &= \$49.2878 +. \end{aligned}$$

(2)

$$\begin{aligned} \$100000 \div 5793000 &= \$0.017262 = \text{rate per dollar.} \\ \$0.017262 \times 18600 &= \$321.0732. \end{aligned}$$

(3)

$$\begin{aligned} \$100000 \div 5793000 &= \$0.017262 = \text{rate per dollar.} \\ \$0.017262 \times 7500 &= \$129.465. \end{aligned}$$

(4)

$$\begin{aligned} \$100000 \div 5793000 &= \$0.017262 = \text{rate per dollar.} \\ \$0.017262 \times 11400 &= \$196.7868. \end{aligned}$$

EXERCISE 104—Page 252.

(1)

Here $P = \$723.19$, $r = .067$, and $t = 7.32$.Then $I = Prt = 723.19 \times .067 \times 7.32 = \354.6813036 .

(2)

Here $P = 857.19$, $r = .065$, and $t = 6\frac{1}{2}$ or 6.5.Then $A = P(1 + rt) = \$857.19 \times 1.4225 = \1219.352775 .

(3)

Here $t = 11$, and $r = .725$.Then $n = tr + 1 = 11 \times .725 + 1 = 8.975$.

(4)

Here $P = \$654.32$, $I = \$234.56$, and $r = .07$.Then $t = \frac{I}{Pr} = \frac{234.56}{654.32 \times .07} = 5.12112$ or 5 years 1 m. 13 d.

(5)

Here $A = \$1200$, $P = \$700$, and $t = 5$.Then $r = \frac{A-P}{Pt} = \frac{1200-700}{700 \times 5} = \frac{1}{7} = \text{rate per unit} \therefore 14\frac{2}{7} = \text{rate per cent.}$

(6)

Here $n = 4$, and $r = .23$.Then $t = \frac{n-1}{r} = \frac{4-1}{.23} = 13$ years 15 days.

(7)

Here $P = \$270$, $I = \$87$ and $r = .07$.Then $t = \frac{I}{Pr} = \frac{87}{270 \times .07} = 4$ years $7\frac{1}{2}$ months.

(8)

Here $P = \$680$, $t = 11\frac{1}{2}$, and $r = .11$.

$$\text{Then } A = P(1 + rt) = 680 \times 2.265 = \$1540.20$$

(9)

Here $A = \$2000$, $t = 20$, and $r = .08$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2000}{2.6} = \$769.23\frac{1}{3}$$

(10)

Here $n = 21$, and $t = 24$.

$$\text{Then } r = \frac{n-1}{t} = \frac{21-1}{24} = .83\frac{1}{3} = \text{rate per unit. } \therefore 83\frac{1}{3} = \text{rate per cent.}$$

(11)

Here $n = 23$, and $r = .16$.

$$\text{Then } t = \frac{n-1}{r} = \frac{23-1}{.16} = 137\frac{1}{2} \text{ years}$$

(12)

Here $P = \$679.18$, $r = .0775$, and $t = 11.73$.

$$\text{Then } I = Prt = 679.18 \times .0775 \times 11.73 = \$617.4255.$$

(13)

Here $P = \$950$, $A = \$1763.42$, and $t = 10$.

$$\text{Then } r = \frac{A - P}{Pt} = \frac{1763.42 - 950}{950 \times 10} = .08562 = \text{rate per unit}$$

$$\therefore 8.562 = \text{rate per cent.}$$

K

(14)

Here $P = \$666$, $A = \$1347.50$, and $r = .06$.

$$\text{Then } t = \frac{A - P}{Pr} = \frac{1347.50 - 666}{666 \times .06} = 17.054 + \text{ years, or 17 years 19 days.}$$

(15)

Here $P = \$273$, $I = \$100$, and $r = .09$

$$\text{Then } t = \frac{I}{Pr} = \frac{100}{273 \times .09} = 4.07 \text{ years} = 4 \text{ years 25 days.}$$

(16)

Here $P = \$476.30$, $A = \$500$, and $t = 2$.

$$\text{Then } r = \frac{A - P}{Pt} = \frac{500 - 476.30}{476.30 \times 2} = .0248 = \text{rate per unit.}$$

$\therefore 2\frac{1}{2}\% = \text{rate per cent.}$

(17)

Here $P = \$749.49$, $I = \$257$, and $t = 7$.

$$\text{Then } r = \frac{I}{Pt} = \frac{257}{749.49 \times 7} = .04898 = \text{rate per unit.}$$

$\therefore 4.898 = \text{rate per cent.}$

(18)

Here $A = \$1111.11$, $t = 11$, and $r = .11$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1111.11}{2.21} = \$502.7647.$$

(19)

$P = £167.47$, $r = .11$, and $t = 9$.

$$I = Prt = 167.47 \times .11 \times 9 = £165.7953 = £165 \text{ 15s. } 10\frac{1}{2}\text{d.}$$

EXERCISE 105—Page 253.

(1)

$$11 \div 2 = 5\frac{1}{2} \text{ cents.}$$

(2)

$$16 \div 2 = 8 \text{ cents} = \$0.08.$$

(3)

$$9 \text{ years and } 8 \text{ months} = 116 \text{ months, and } 116 \div 2 = 58 \text{ cents} \\ = \$0.58.$$

(4)

$$16 \text{ years and } 3 \text{ months} = 195 \text{ months, and } 195 \div 2 = 97\frac{1}{2} \text{ cents} \\ = \$0.97\frac{1}{2}.$$

(5)

$$11 \text{ years and } 7 \text{ months} = 139 \text{ months, and } 139 \div 2 = 69\frac{1}{2} \text{ cents} \\ = \$0.695.$$

(6)

$$12 \text{ years and } 5 \text{ months} = 149 \text{ months, and } 149 \div 2 = 74\frac{1}{2} \text{ cents} \\ = \$0.745.$$

(7)

$$3 \text{ years and } 2 \text{ months} = 38 \text{ months, and } 38 \div 2 = 19 \text{ cents} = \\ \text{interest of } \$1 \text{ for given rate and time.} \\ \$0.19 \times 279.40 = \$53.086.$$

(8)

$$6 \text{ years and } 7 \text{ months} = 79 \text{ months, and } 79 \div 2 = 39\frac{1}{2} \text{ cents} = \\ \text{interest of } \$1 \text{ for given rate and time.} \\ \$0.395 \times 189.70 = \$74.9315.$$

(9)

3 years and 11 months = 47 months, and $47 \div 2 = 23\frac{1}{2}$ cents =
interest of \$1 for given rate and time.

$$\$0.235 \times 1463 = \$343.805.$$

(10)

11 years and 1 month = 133 months, and $133 \div 2 = 66\frac{1}{2}$ cents =
interest of \$1 for given rate and time.

$$\$0.665 \times 28967.50 = \$19263.3875.$$

EXERCISE 106—Page 254.

(1)

$$2 \div 6 = \frac{1}{3} \text{ mill} = \$0.0003.$$

(2)

$$7 \div 6 = 1\frac{1}{6} \text{ mills} = \$0.001\frac{1}{6}.$$

(3)

$$11 \div 6 = 1\frac{5}{6} \text{ mills} = \$0.001\frac{5}{6}.$$

(4)

$$27 \div 6 = 4\frac{1}{2} \text{ mills} = \$0.004\frac{1}{2}.$$

(5)

$$47 \div 6 = 7\frac{5}{6} \text{ mills} = \$0.007\frac{5}{6}.$$

(6)

$$8 \div 2 = 4 \text{ cents} = \$0.04.$$

$$12 \div 6 = 2 \text{ mills} = \$0.002 \text{ and } \$0.04 + \$0.002 = \$0.042$$

(7)

$$66 \div 6 = 11 \text{ mills} = \$0.011.$$

(8)

2 years 2 m'ths = 26 months, and $26 \div 2 = 13$ cents = \$0.13.
 $19 \div 6 = 3\frac{1}{6}$ mills = $\$0.003\frac{1}{6}$ and $\$0.13 + \$0.003\frac{1}{6} = \$0.133\frac{1}{6}$.

(9)

7 years 8 m'ths = 92 months, and $92 \div 2 = 46$ cents = \$0.46.
 $9 \div 6 = 1\frac{1}{2}$ mills = \$0.001 $\frac{1}{2}$ and $\$0.46 + \$0.001\frac{1}{2} = \$0.461\frac{1}{2}$.

(10)

17 years 11 months = 215 months, and $215 \div 2 = 107\frac{1}{2}$ cents = \$1.075.
 $23 \div 6 = 3\frac{5}{6}$ mills = \$0.003 $\frac{5}{6}$, and $\$1.075 + \$0.003\frac{5}{6} = \$1.078\frac{5}{6}$.

(11)

12 years 7 months = 151 months, and $151 \div 2 = 75\frac{1}{2}$ cents = \$0.755.
 $17 \div 6 = 2\frac{5}{6}$ mills = \$0.002 $\frac{5}{6}$, and $\$0.755 + \$0.002\frac{5}{6} = \$0.757\frac{5}{6}$.

EXERCISE 107—Page 255.

(1)

Interest on \$1 for 7 months = \$0.035
 Interest on \$1 for 17 days = $2\frac{5}{6}$

Therefore interest on \$1 for 7 months 17 days, = \$0.037 $\frac{5}{6}$
 Then $\$0.037\frac{5}{6} \times 917.30 = \34.704516 .

(2)

Interest on \$1 for 3 months = \$0.015
 Interest on \$1 for 13 days = $2\frac{1}{6}$

Therefore interest on \$1 for 3 months 13 days = \$0.017 $\frac{1}{6}$
 Then $\$0.017\frac{1}{6} \times 842.50 = \14.462916 .

(3)

Interest on \$1 for 2 years 11 months = \$0.175
 Interest on \$1 for 10 days = $1\frac{3}{4}$

Therefore interest on \$1 for 2 years 11 m'ths 10 days = \$0.176 $\frac{3}{4}$
 Then $\$0.176\frac{3}{4} \times 573.83 = \101.3766 .

(4)

Interest on \$1 for 6 years 9 months = \$0.405

Interest on \$1 for 19 days = $3\frac{1}{6}$ Therefore interest on \$1 for 6 years 9 m'ths 19 days = \$0.408 $\frac{1}{6}$ Then $\$0.408\frac{1}{6} \times 642.30 = \$262.16545.$

(5)

Interest on \$1 for 5 years 5 months = \$0.325

Interest on \$1 for 7 days = $1\frac{1}{6}$ Therefore interest on \$1 for 5 years 5 months 7 days = \$0.326 $\frac{1}{6}$ Then $\$0.326\frac{1}{6} \times 1427.875 = \$465.7252.$

(6)

Interest on \$1 for 4 years 7 months = \$0.275

Interest on \$1 for 16 days = $2\frac{3}{4}$ Therefore interest on \$1 for 4 years 7 m'ths 16 days = \$0.277 $\frac{3}{4}$ Then $\$0.277\frac{3}{4} \times 709.63 = 197.040596.$

(7)

Interest on \$1 for 7 years 7 months = \$0.455

Interest on \$1 for 22 days = $3\frac{3}{4}$ Therefore interest on \$1 for 7 years 7 m'ths 22 days = \$0.458 $\frac{3}{4}$ Then $\$0.458\frac{3}{4} \times 2463.20 = \$1129.7877 + \$2463.20 = \$3592.9877.$

(8)

Interest on \$1 for 9 years 9 months = \$0.585

Interest on \$1 for 9 days = $1\frac{1}{2}$ Therefore interest on \$1 for 9 years 9 months 9 days = \$0.586 $\frac{1}{2}$ Then $\$0.586\frac{1}{2} \times 999.99 = \$586.494135.$

(9)

$$\begin{aligned} \text{Interest on \$1 for 3 years 4 months} &= \$0.20 \\ \text{Interest on \$1 for 27 days} &= 4\frac{1}{2} \end{aligned}$$

Therefore interest on \$1 for 3 years 4 m'ths 27 days = $\$0.204\frac{1}{2}$
 Then $\$0.2045 \times 68.70 = \14.04915 .

(10)

$$\begin{aligned} \text{Interest on \$1 for 3 years} &= \$0.18 \\ \text{Interest on \$1 for 28 days} &= 4\frac{3}{4} \end{aligned}$$

Therefore interest on \$1 for 3 years 28 days = $\$0.184\frac{3}{4}$
 Then $\$0.184\frac{3}{4} \times 742.63 = \137.139 .

(11)

$$\begin{aligned} \text{Interest on \$1 for 7 years 4 months} &= \$0.44 \\ \text{Interest on \$1 for 11 days} &= 1\frac{5}{8} \end{aligned}$$

Therefore interest on \$1 for 7 years 4 m'ths 11 days = $\$0.441\frac{5}{8}$
 Then $\$0.441\frac{5}{8} \times 200 = \$88.366 + \$200 = \288.366 .

(12)

$$\begin{aligned} \text{Interest on \$1 for 9 years 3 months} &= \$0.555 \\ \text{Interest on \$1 for 9 days} &= 1\frac{1}{2} \end{aligned}$$

Therefore interest on \$1 for 9 years 3 months 9 days = $\$0.556\frac{1}{2}$
 Then $\$0.5565 \times 743.63 = \$413.830095 + \$743.63 = \1157.460095 .

EXERCISE 108—Page 256.

(1)

Interest on \$1 at 6 per cent. for given time = $\$0.526\frac{2}{3}$.
 Interest on \$1234.56 at 6 per cent. for given time = $\$0.526\frac{2}{3} \times 1234.56 = \650.2016 .
 Hence interest on \$1234.56 at 7 per cent. for given time = $\$650.2016 + \text{one sixth of } \$650.2016 = \$758.5685$.

(2)

Interest on \$1 at 6 per cent. for given time = $\$0.126\frac{2}{3}$.

Interest on \$9876.54 at 6 per cent. for given time = $\$0.126\frac{2}{3} \times 9876.54 = \1252.67449 .

Hence interest on \$9876.54 at 3 per cent. for given time = $\$1252.67449 \div 2 = \626.337245 .

(3)

Interest on \$1 at 6 per cent. for given time = $\$0.216\frac{2}{3}$.

Interest on \$715.30 at 6 per cent. for given time = $\$0.216\frac{2}{3} \times 715.30 = \154.98166 .

Hence interest on \$715.30 at 8 per cent. for given time = $\$154.98166 + \text{one third of } \$154.98166 = \$206.6422$.

(4)

Interest on \$1 at 6 per cent. for given time = $\$0.141\frac{1}{3}$.

Interest on \$555.55 at 6 per cent. for given time = $\$0.141\frac{1}{3} \times 555.55 = \78.51773 .

Hence interest on \$555.55 at 12 per cent. for given time = $\$78.51773 \times 2 = \157.03546 ; amount = $\$157.03546 + \$555.55 = \$712.58546$.

(5)

Interest on \$1 at 6 per cent. for given time = $\$0.016\frac{2}{3}$.

Interest on \$7766.55 at 6 per cent. for given time = $\$0.016\frac{2}{3} \times 7766.55 = \129.4425 .

Hence interest on \$7766.55 at 5 per cent. for given time = $\$129.4425 - \text{one sixth of } \$129.4425 = \$107.86875$.

Amount = $\$107.86875 + \$7766.55 = \$7874.41875$.

(6)

Interest on \$1 at 6 per cent. for given time = $\$0.521\frac{1}{3}$.

Interest on \$500 at 6 per cent. for given time = $\$0.521\frac{1}{3} \times 500 = \$260.666\frac{2}{3}$.

Hence interest on \$500 at 16 per cent. for given time = $\$260.666\frac{2}{3} \times 2\frac{2}{3} = \695.111 ; amount = $\$695.111 + \$500 = \$1195.111$.

(7)

Interest on \$1 at 6 per cent. for given time = \$0.206 $\frac{1}{6}$.Interest on \$576 at 6 per cent. for given time = \$0.206 $\frac{1}{6}$ \times 576
= \$118.752.Hence interest on \$576 at 5 per cent. for given time = \$118.752
—one sixth of \$118.752 = \$98.96.

(8)

Interest on \$1 at 6 per cent. for given time = \$0.151 $\frac{1}{6}$.Interest on \$2478.91 at 6 per cent. for given time = \$0.151 $\frac{1}{6}$ \times
2478.91 = \$376.38116.Hence interest on \$2478.91 at 4 $\frac{1}{2}$ per cent. for given time =
\$376.38116 — one fourth of \$376.38116 = \$282.285.

(9)

From May 9th to December 11th = 216 days. Interest on \$1 at
6 per cent. for 216 days = \$0.036.Interest on \$780 at 6 per cent. for 216 days = \$0.036 \times 780 =
\$28.08.

(10)

From August 16th 1851 to June 19th 1852 = 308 days.

Interest on \$1 at 6 per cent. for given time = \$0.051 $\frac{1}{3}$.Interest on \$1830.63 at 6 per cent. for given time = \$0.051 $\frac{1}{3}$ \times
1830.63 = \$93.97234.Hence interest on \$1830.63 at 7 per cent. for given time =
\$93.97234 + one sixth of \$93.97234 = \$109.63439.

(11)

From September 3rd 1858 to January 9th 1859 = 128 days.

Interest on \$1 at 6 per cent. for given time = \$0.021 $\frac{1}{3}$.Interest on \$6200 at 6 per cent. for given time = \$0.021 $\frac{1}{3}$ \times 6200
= \$132.266.

Amount = \$132.266 + \$6200 = \$6332.266.

EXERCISE 109.—Page 258.

(1)

| | | |
|------------|-----------------|-------------------------|
| From June | 2nd to July | 17th there are 45 days. |
| “ July | 17th to October | 6th “ 81 “ |
| “ October | 6th to December | 11th “ 66 “ |
| “ December | 11th to March | 29th “ 109 “ |
| “ March | 29th to October | 7th “ 192 “ |

Whole sum \$1217·30 for 45 days = \$54778·50 for 1 day.
 1st endorsement 207·80

Balance \$1009·50 for 81 days = \$81769·50 for 1 day.
 2nd endorsement 209·60

Balance[†] \$799·90 for 66 days = \$52793·40 for 1 day.
 3rd endorsement 320·90

Balance \$479·00 for 109 days = \$52211·00 for 1 day.
 4th endorsement 421·83

Balance \$57·17 for 192 days = \$10976·64 for 1 day.

Whole interest = that of \$252529·04 for 1 day.

Interest on \$252529·04 at 6 per cent. for 1 year = \$15151·7424.

Hence interest for 1 day = $\$15151·7424 \div 365 = \$41·5116$.

Then interest due = \$41·5116

Balance on Note = \$57·17

Principal and interest due = \$98·6816

(2)

| | | |
|-----------------|------------------|--------------------|
| From 17th June | to 5th September | there are 80 days. |
| " 5th September | to 7th December | " 93 " |
| " 7th December | to 11th June | " 186 " |
| " 11th June | to 7th February | " 241 " |
| " 7th February | to 19th December | " 315 " |
| " 19th December | to 1st May | " 133 " |

Whole sum \$7348.25 for 80 days = \$587860.00 for 1 day.
 1st endorsement 2463.80

Balance \$4884.45 for 93 days = \$454253.85 for 1 day.
 2nd endorsement 392.20

Balance \$4492.25 for 186 days = \$835558.50 for 1 day.
 3rd endorsement 982.20

Balance \$3510.05 for 241 days = \$845922.05 for 1 day.
 4th endorsement 2842.90

Balance \$667.15 for 315 days = \$210152.25 for 1 day.
 5th endorsement 317.23

Balance \$349.92 for 133 days = \$46539.36 for 1 day.

Whole interest = that of \$2980286.01 for 1 day.

Interest on \$2980286.01 at 8 per cent. for 1 year = \$238422.8808.

Hence interest for 1 day = \$238422.8808 ÷ 365 = \$653.2133.

Then interest due = \$653.2133

Balance on Note = \$349.92

Principal and interest due = \$1003.1333

EXERCISE 110.—Page 259.

| | | | |
|---------------|-----------------------------|--|-----------------------------|
| \$1800 | | Principal. | (1) |
| 108 | | | |
| | | Interest for 1st year. | |
| \$1908 | | Amount for 1 year = principal for 2nd year. | Interest for 2nd year. |
| 114.48 | | | |
| \$2022.48 | | Amount for 2 years = principal for 3rd year. | Interest for 3rd year. |
| 121.3488 | | | |
| \$2143.8288 | | Amount for 3 years = principal for 4th year. | Interest for 4th year. |
| 128.629728 | | | |
| \$2272.458528 | | Amount for 4 years = principal for 5th year. | Interest for 5th year. |
| 136.347511 | | | |
| \$2408.806039 | | Amount for 5 years = principal for 5th year. | Given Principal. |
| 1800 | | | |
| \$608.806 = | Compound interest required. | | |
| (2) | | | |
| \$700 | | Principal. | (2) |
| 49 | | | |
| | | Interest for 1st half year. | |
| \$749 | | Amount for 1 half y. = principal for 2nd half y. | Interest for 2nd half year. |
| 52.43 | | | |
| \$801.43 | | Amount for 1 year = principal for 3rd half y. | Interest for 3rd half year. |
| 56.1001 | | | |
| \$857.5301 | | Amount for 1½ years = principal for 4th half y | Interest for 4th half year. |
| 60.027107 | | | |
| \$917.557207 | | Amount for 2 years = principal for 5th half y | Interest for 5th half year. |
| 64.229004 | | | |
| \$981.786211 | | Amount for 2½ years = principal for 6th half y. | Interest for 6th half year. |
| 68.725034 | | | |
| \$1050.511245 | | Amount for 3 years = principal for 7th half y | Interest for 7th half year. |
| 73.535787 | | | |
| \$1124.047032 | | Amount for 3½ years | Given Principal. |
| 700 | | | |
| \$424.047 = | Compound interest required. | | |

\$673
20\$693
20\$714
21\$735
22\$757
22\$780
23\$804
24\$828
24\$853
673

\$179

\$860
34\$894
35\$930
37\$967
38\$1006
40\$1046
41\$1088
860

\$22

(3)

| | |
|-------------------|---|
| \$673·40 | Principal. |
| <u>20·202</u> | Interest for 1st quarter. |
| \$693·602 | Amount for 1 quar. = principal for 2nd quarter. |
| <u>20·80806</u> | Interest for 2nd quarter. |
| \$714·41006 | Amount for 1 half y. = principal for 3rd quart. |
| <u>21·4323018</u> | Interest for 3rd quarter. |
| \$735·8423618 | Amount for 3 quarters = principal for 4th quar. |
| <u>22·0752708</u> | Interest for 4th quarter. |
| \$757·9176326 | Amount for 1 year = principal for 5th quarter. |
| <u>22·7375289</u> | Interest for 5th quarter. |
| \$780·6551615 | Amount for 5 quarters = principal for 6th quar. |
| <u>23·4196548</u> | Interest for 6th quarter. |
| \$804·0748163 | Amount for 3 half y. = principal for 7th quarter. |
| <u>24·1222444</u> | Interest for 7th quarter. |
| \$828·1970607 | Amount for 7 quarters = principal for 8th qr. |
| <u>24·8459118</u> | Interest for 8th quarter. |
| \$853·0429 = | Amount for 2 years required. |
| <u>673·40</u> | Given Principal. |
| \$179·6429 = | Compound Interest required. |

(4)

| | |
|-----------------|---|
| \$860 | Principal. |
| <u>34·4</u> | Interest for 1st half year. |
| \$894·4 | Amount for 1 half year = principal for 2nd half y. |
| <u>35·776</u> | Interest for 2nd half year. |
| \$930·176 | Amount for 1 year = principal for 3rd half year. |
| <u>37·20704</u> | Interest for 3rd half year. |
| \$967·38304 | Amount for 3 half years = principal for 4th half y. |
| <u>38·69532</u> | Interest for 4th half year. |
| \$1006·07836 | Amount for 2 years = principal for 5th half year. |
| <u>40·24313</u> | Interest for 5th half year. |
| \$1046·32149 | Amount for 5 half years = principal for 6th half y. |
| <u>41·85285</u> | Interest for 6th half year. |
| \$1088·17434 = | Amount for 3 years required. |
| <u>860</u> | Given Principal. |
| \$228·1743 = | Compound Interest required. |

EXERCISE 111—Page 261.

(1)

By the table the am't of \$1 at 6 per cent. for 11 years = \$1·8983.

Then $\$1\cdot8983 \times 875 = \$1661\cdot0125 = \text{Amount.}$

| | |
|-----|------------|
| 875 | Principal. |
|-----|------------|

 $\$786\cdot0125 = \text{Interest.}$

(2)

By the table the am't of \$1 for the given time and rate = \$2·77247.

Then $\$2\cdot77247 \times 643\cdot98 = \$1785\cdot41523 = \text{Amount.}$

| | |
|--------|------------|
| 643·98 | Principal. |
|--------|------------|

 $\$1141\cdot43523 = \text{Interest.}$

(3)

By the table the am't of \$1 at 6 per cent. for 45 years = \$13·76461.

Then $\$13\cdot76461 \times \cdot01 = \$1\cdot37646 = \text{Amount.}$

| | |
|-----|------------|
| ·01 | Principal. |
|-----|------------|

 $\$.127646 = \text{Interest.}$

(4)

By the table the am't of \$1 for the given time and rate = \$2·28793.

Then $\$2\cdot28793 \times 78\cdot2 = \$178\cdot916 = \text{Amount.}$

| | |
|------|------------|
| 78 2 | Principal. |
|------|------------|

 $\$100\cdot716 = \text{Interest.}$

(5)

By the table the am't of \$1 for the given rate and time = \$2·40662

Then $\$2\cdot40662 \times 777\cdot77 = \$1871\cdot7968 = \text{Amount.}$

| | |
|--------|------------|
| 777·77 | Principal. |
|--------|------------|

 $\$1094\cdot0268 = \text{Interest.}$

(6)

$$£44 \text{ 5s. 9d.} = £44.2875.$$

By the table the am't of £1 at 6 per cent. for 11 years = £1.8983.

$$\text{Then } £1.8983 \times 44.2875 = £84.07096 = £84 \text{ 1 5} = \text{Amount.}$$

$$\begin{array}{r} 44 \text{ 5 9} \\ \hline \end{array} \text{ Principal.}$$

$$£39 \text{ 15 8} = \text{Interest.}$$

(7)

$$£32 \text{ 4s. 9}\frac{1}{2}\text{d.} = £32.240625.$$

By the table the amount of £1 for the given time and rate =

$$£1.26532. \text{ Then } £1.26532 \times 32.240625 =$$

$$£40.7947076 = £40 \text{ 15 10}\frac{3}{4} \text{ nearly} = \text{Amount.}$$

$$\begin{array}{r} 32 \text{ 4 9}\frac{1}{2} \\ \hline \end{array} \text{ Principal.}$$

$$£8 \text{ 11 1} = \text{Interest.}$$

EXERCISE 112—Page 262.

(1)

$$\text{Amount of } \$1 \text{ for 7 years at 4 per cent.} = \$1.31593.$$

$$\$7439.87 \div 1.31593 = \$5653.697.$$

(2)

$$\text{Amount of } \$1 \text{ at 5 per cent. for 20 years} = \$2.6533.$$

$$\$9193.90 \div 2.6533 = \$3465.081.$$

(3)

$$£595 \text{ 10s. 2}\frac{1}{2}\text{d.} = £595.51$$

$$\text{Amount of } £1 \text{ at 6 per cent. for 3 years} = £1.19102.$$

$$£595.51 \div 1.19102 = £500.$$

(4)

$$\text{Amount of } \$1 \text{ at 6 per cent. for 7 years} = \$1.50363.$$

$$\$7111.11 \div 1.50363 = \$4729.295.$$

(5)

£268 Os. 4½d. = £268.02.

Amount of £1 at 5 per cent for 6 years = £1.3401.

£268.02 ÷ 1.3401 = £200.

EXERCISE 113—Page 263.

(1)

Here $A = \$962$, $r = .04$, and $t = 1$. Whence $1 + rt = 1.04$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{962}{1.04} = \$925.$$

(2)

Here $A = \$2202$, $r = .06$, and $t = 5.75$. Whence $1 + rt = 1.345$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2202}{1.345} = \$1637.174.$$

(3)

Here $A = \$1003.50$, $r = .06$, and $t = \frac{3}{4}$ year. Whence $1 + rt = 1.04$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1003.50}{1.04} = \$964.9038.$$

(4)

Here $A = \$716$, $r = .08$, and $t = \frac{7}{8}$ year. Whence $1 + rt = 1.04\frac{3}{4}$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{716}{1.04\frac{3}{4}} = \$684.0764.$$

(5)

Here $A = \$1342.50$, $r = .065$, and $t = \frac{11}{12}$ year. Whence $1 + rt = 1.022\frac{11}{12}$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1342.50}{1.022\frac{11}{12}} = \$1313.266.$$

(6)

Here $A = \$2400$, $r = .05$, and $t = \frac{336}{12}$ year. Whence $1 + rt = 1.03\frac{1}{2}$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2400}{1.03\frac{1}{2}} = \$2324.84.$$

(7)

Here $A = \$2202$, $r = .05$, and $t = .75$ year. Whence $1 + rt = 1.0375$.

$$\begin{aligned} \$2202 \div 1.0375 &= \$2122.40963+ = \text{Present worth.} \\ \$2202 - \$2122.40963+ &= \$79.59036 = \text{Discount.} \end{aligned}$$

(8)

Here $A = \$4360$, $r = .06$, and $t = 1\frac{5}{12}$. Whence $1 + rt = 1.085$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{4360}{1.085} = \$4018.43317.$$

(9)

Here $A = \$1647$, $r = .06$, and $t = 1\frac{1}{12}$ year. Whence $1 + rt = 1.055$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1647}{1.055} = \$1561.13744.$$

(10)

Here $A = \$2000$, $r = .06$, and $t = 3\frac{7}{12}$. Whence $1 + rt = 1.215$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2000}{1.215} = \$1646.09053.$$

L

(11)

Here $A = \$2070.90$, $r = .05$, and $t = 1\frac{1}{2}$. Whence $1 + rt = 1.07\frac{1}{2}$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2070.90}{1.07\frac{1}{2}} = \$1918.9806.$$

$\$2070 - \$1918.9806 = \$151.019 = \text{Discount required.}$

(12)

Here $A = \$970.63$, $r = .08$, and $t = \frac{1}{2}$ year. Whence $1 + rt = 1.07\frac{1}{2}$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{970.63}{1.07\frac{1}{2}} = \$904.313.$$

(13)

Here in first case $A = \$1512$, $r = .07$, and $t = .5$ year. Whence $1 + rt = 1.035$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1512}{1.035} = \$1460.8695.$$

Also $A = 1512$, $r = .07$, and $t = 1$. Whence $1 + rt = 1.07$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1512}{1.07} = \$1413.0841.$$

$\$1460.8695 + \$1413.0841 = \$2873.9536 = \text{Present worth of whole amount.}$

$\$3024 - \$2873.9536 = \$150.0464 = \text{Discount required.}$

(14)

Here in first case $A = \$440$, $r = .08$, and $t = 1.25$. Whence $1 + rt = 1.1$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{440}{1.1} = \$400.$$

In second case $A = \$896$, $r = .08$, and $t = 1.5$. Whence $1 + rt = 1.12$.

$$\text{Then } P = \frac{A}{1 + rt} = \frac{896}{1.12} = \$800.$$

$\$400 + \$800 = \$1200.$

EXERCISE 114—Page 265.

Whence $1 + rt =$

\$1918.9806.

Amount required.

(1)

Here the time the note has to run is 2 years 3 months 3 days.

Interest of \$1 at 7 per cent. for 2 yrs., 3 m., 3-days = \$0.1580 $\frac{1}{2}$.Interest of \$986 at 7 per cent. for 2 years, 3 months, 3 days =
\$0.1580 $\frac{1}{2}$ \times 986 = \$155.8701.Whence $1 + rt$

\$904.313.

5 year. Whence

(2)

Here the time the note has to run is 103 days = 3 months 13 days.

Interest of \$1 at 8 per cent. for 3 months 13 days = \$0.022 $\frac{1}{2}$.Interest of \$640 at 8 per cent. for 3 months, 13 days =
\$0.022 $\frac{1}{2}$ \times 640 = \$14.6488.

\$1460.8695.

 $1 + rt = 1.07$.

13.0841.

Present worth of

Amount required.

1.25. Whence

0.

5. Whence $1 +$

0.

(3)

Here the time the note has to run is 94 days = 3 months 4 days.

Interest of \$1 at 6 per cent. for 3 months 4 days = \$0.015 $\frac{1}{2}$.Interest of \$563.80 at 6 per cent. for 3 months 4 days =
\$0.015 $\frac{1}{2}$ \times 563.80 = \$8.8328 and \$563.80 - \$8.8328 = \$554.967.

EXERCISE 115—Page 266.

(1)

Interest on \$1 for 93 days at 7 p. c. = \$0.0180 $\frac{1}{2}$, and this taken
from \$1 gives a remainder of \$0.9819 $\frac{1}{2}$ = present worth of \$1.Then \$3755 \div 0.9819 $\frac{1}{2}$ = \$3824.15.

(2)

Interest on \$1 for 6 months 3 days at 5 per cent. = \$0.0254 $\frac{1}{2}$
and this taken from \$1 gives a remainder \$0.9745 $\frac{1}{2}$ = present
worth of \$1.Then \$1147.80 \div 0.9745 $\frac{1}{2}$ = \$1177.734.

(3)

Interest on \$1 for 48 days at $3\frac{1}{2}$ per cent. = $\$0.004\frac{1}{2}$, and this taken from \$1 gives a remainder $\$0.995\frac{1}{2}$ = present worth of \$1.

$$\text{Then } \$713.90 \div 0.995\frac{1}{2} = \$717.2471.$$

EXERCISE 116—Page 268.

(1)

$$\begin{array}{r} \$200 \times 3 = 600 \\ 150 \times 4 = 600 \\ 250 \times 6 = 1500 \\ \hline 600 \quad 600)2700(4\frac{1}{2} \text{ months.} \\ \quad \quad \quad 2400 \\ \quad \quad \quad \hline \quad \quad \quad 300 \} \\ \quad \quad \quad \hline \quad \quad \quad 600 \} = \frac{1}{2} \end{array}$$

(2)

$$\begin{array}{r} \frac{1}{2} \times 0 = 0 \\ \frac{1}{2} \times 3 = \frac{3}{2} \\ \frac{1}{2} \times 6 = 1\frac{1}{2} \\ \frac{1}{2} \times 9 = 2\frac{1}{2} \\ \hline 1 \quad 1)4\frac{1}{2} \\ \hline \quad \quad 4\frac{1}{2} \text{ months.} \end{array}$$

(3)

$$\begin{array}{r} \$50 \times 2 = 100 \\ 40 \times 5 = 200 \\ 30 \times 7 = 210 \\ \hline 120 \quad 120)510(4\frac{1}{4} \text{ months.} \\ \quad \quad \quad 480 \\ \quad \quad \quad \hline \quad \quad \quad 30 \} \\ \quad \quad \quad \hline \quad \quad \quad 120 \} = \frac{1}{4} \end{array}$$

(4)

$$\begin{array}{r} \$1000 \times 0 = 0 \\ 1500 \times 1 = 1500 \\ 600 \times 3 = 1800 \\ 700 \times 5 = 3500 \\ 1400 \times 7 = 9800 \\ \hline 5200 \quad 5200)16600(3\frac{5}{8} \text{ months.} \\ \quad \quad \quad \hline \quad \quad \quad 15600 \\ \quad \quad \quad \hline \quad \quad \quad 1000 \} \\ \quad \quad \quad \hline \quad \quad \quad 5200 \} = \frac{5}{8} \end{array}$$

(5)

Six months from 15th January = 15th July, and from 1st July to 15th July there are 14 days.

Six months from 10th February = 10th August, and from 1st July to 10th August there are 40 days.

Six months from 6th March = 6th September, and from 1st July to 6th September there are 67 days.

Six months from 8th June = 8th December, and from 1st July to 8th December there are 160 days.

$$\begin{array}{r}
 \$3750 \times 14 = 52500 \\
 3000 \times 40 = 120000 \\
 2400 \times 67 = 160800 \\
 2250 \times 160 = 360000 \\
 \hline
 11400 \quad 11400) 693300 (60\frac{3}{8} \text{ days.} \\
 \underline{684000} \\
 9300 \\
 \underline{11400} \quad \left. \vphantom{\begin{array}{r} 9300 \\ 11400 \end{array}} \right\} = \frac{3}{8}
 \end{array}$$

Therefore the note must be made payable on the 61st day from the 1st of July, which is the 31st of August.

EXERCISE 117.—Page 269.

(1)

• Whole stock : A's stock :: whole profit : A's profit.

$$1117 \times 3000$$

That is, \$4300:\$3000::\$1117: $\frac{1117 \times 3000}{4300} = \$779.302 + = \text{A's sh.}$

\$1117 - \$779.302 + = \$337.697 = B's share,

(2)

Whole stock = \$6470 + \$3780 + \$9860 = \$20110.

Whole stock : A's stock :: whole profit : A's profit.

$$7890 \times 6470$$

That is, \$20110 : \$6470 :: \$7890 : ————— = \$2538·453+ = A's sh.

$$20110$$

Again, whole stock : B's stock :: whole profit : B's profit.

$$7890 \times 3780$$

That is, \$20110 : \$3780 :: \$7890 : ————— = \$1483·053+ = B's sh.

$$20110$$

Lastly, whole stock : C's stock :: whole profit : C's profit.

$$7890 \times 9860$$

That is, \$20110 : \$9860 :: \$7890 : ————— = \$3868·493+ = C's sh.

$$20110$$

(3)

Whole stock : B's stock :: whole gain : B's gain.

$$89 \times 120$$

That is, \$320 : \$120 :: \$80 : ————— = \$30 = B's gain.

$$320$$

Again, whole stock : C's stock :: whole gain : C's gain.

$$80 \times 200$$

That is, \$320 : \$200 :: \$80 : ————— = \$50 = C's share.

$$320$$

(4)

Whole stock : B's stock :: whole gain : B's gain.

$$728 \times 1200$$

That is, \$2800 : \$1200 :: \$728 : ————— = \$312 = B's gain.

$$2800$$

Again, whole stock : C's stock :: whole gain : C's gain.

$$728 \times 1600$$

That is, \$2800 : \$1600 :: \$728 : ————— = \$416 = C's gain.

$$2800$$

(5)

Whole stock : B's stock :: whole amount to be divided : B's share

That is, \$3 : \$2 :: \$100 : $\frac{100 \times 2}{3} = \$66\cdot66\frac{2}{3} = \text{B's share.}$

Again, whole st'k : C's st'k :: whole amo't to be divided : C's sh'e

That is, \$3 : \$1 :: \$100 : $\frac{100 \times 1}{3} = \$33\cdot33\frac{1}{3} = \text{C's share.}$

(6)

$\pounds 1400 : \pounds 500 :: \pounds 1100 : \frac{1100 \times 500}{1400} = \pounds 392\frac{2}{7} = \text{B's share.}$

$\pounds 1100 - \pounds 392\frac{2}{7} = \pounds 707\frac{1}{7} = \text{C's share.}$

(7)

casks. casks. 180×200
 $900 : 200 :: 180 : \frac{900}{900} = 40 \text{ casks} = \text{B's loss.}$

$900 : 300 :: 180 : \frac{180 \times 300}{900} = 60 \text{ casks} = \text{C's loss.}$

$180 - (40 + 60) = 80 \text{ casks} = \text{D's loss.}$

(8)

$\$1800 : \$800 :: \$100 : \frac{100 \times 800}{1800} = \$44\cdot44\frac{4}{9} = \text{B's share.}$

$\$1800 : \$600 :: \$100 : \frac{100 \times 600}{1800} = \$33\cdot33\frac{1}{3} = \text{C's share.}$

$\$44\cdot44\frac{4}{9} + \$33\cdot33\frac{1}{3} = \$77\cdot77\frac{7}{9}$, and $\$100 - \$77\cdot77\frac{7}{9} =$
 $\$22\cdot22\frac{2}{9} = \text{D's share.}$

(9)

$$6 : 1 :: 120 : \frac{120 \times 1}{6} = 20$$

$$6 : 2 :: 120 : \frac{120 \times 2}{6} = 40$$

$$6 : 3 :: 120 : \frac{120 \times 3}{6} = 60$$

(10)

$$\text{Whole loss} = \$900 - \$540 = \$360.$$

$$8 : 1 :: \$360 : \frac{360}{8} = \$45 = \text{B's loss.}$$

$$8 : 2 :: \$360 : \frac{360 \times 2}{8} = \$90 = \text{C's loss.}$$

$$\$45 + 90 = \$135, \text{ and } \$360 - 135 = \$225 = \text{D's loss.}$$

(11)

$$\$12 : \$6 :: \$1320 : \frac{1320 \times 6}{12} = \$660 = \text{B's gain.}$$

$$\$12 : \$4 :: \$1320 : \frac{1320 \times 4}{12} = \$440 = \text{C's gain.}$$

$$\$12 : \$2 :: \$1320 : \frac{1320 \times 2}{12} = \$220 = \text{D's gain.}$$

(12)

$$£75 + £29 = £104, \text{ and } £110 - £104 = £6 = \text{D's profit.}$$

$$\text{D's profit} : \text{B's profit} :: \text{D's stock} : \text{B's stock.}$$

$$\text{That is, } £46 : £35 :: £1090 : \frac{1090 \times 35}{46} = £829 \text{ 6s. } 11\frac{1}{2}\text{d} = \text{B's st.}$$

$$\text{Again, D's profit} : \text{C's profit} :: \text{D's stock} : \text{C's stock.}$$

$$\text{That is, } £46 : £29 :: £1090 : \frac{1090 \times 29}{46} = £687 \text{ 3s. } 5\frac{1}{2}\text{d} = \text{C's st.}$$

EXERCISE 118.—Page 271.

(1)

$$\left. \begin{array}{l} \$357 \times 5 = \$1785 \text{ for one month} \\ 371 \times 7 = 2597 \text{ for one month} \\ 154 \times 11 = 1694 \text{ for one month} \end{array} \right\} = \$6076 \text{ for one month.}$$

$$\$6076 : \$1785 :: \$347 \cdot 20 : \frac{347 \cdot 20 \times 1785}{6076} = \$102$$

$$\$6076 : \$2597 :: \$347 \cdot 20 : \frac{347 \cdot 20 \times 2597}{6076} = \$148 \cdot 40.$$

$$\$6076 : \$1694 :: \$347 \cdot 20 : \frac{347 \cdot 20 \times 1694}{6076} = \$96 \cdot 80$$

(2)

$$\left. \begin{array}{l} 40 \times 6 = 240 \text{ for one month} \\ 30 \times 5 = 150 \text{ for one month} \\ 50 \times 1 = 50 \text{ for one month} \end{array} \right\} = 440 \text{ for one month.}$$

$$440 : 240 :: \$160 : \frac{160 \times 240}{440} = \$87 \cdot 27_{11}^3; \text{ B's share.}$$

$$440 : 150 :: \$160 : \frac{160 \times 150}{440} = \$54 \cdot 54_{11}^6; \text{ C's share.}$$

$$440 : 50 :: \$160 : \frac{160 \times 50}{440} = \$18 \cdot 18_{11}^2; \text{ D's share.}$$

(3)

$$\left. \begin{array}{l} £150 \times 6 = £900 \text{ for one month} \\ 200 \times 3 = 600 \text{ for one month} \\ 125 \times 16 = 2000 \text{ for one month} \end{array} \right\} = £3500 \text{ for one month.}$$

$$£3500 : £900 :: £291 \text{ 13s. 4d.} : \frac{£291 \text{ 13s. 4d.} \times 900}{3500} = £75.$$

$$£3500 : £600 :: £291 \text{ 13s. 4d.} : \frac{£291 \text{ 13s. 4d.} \times 600}{3500} = £50.$$

$$£3500 : £2000 :: £291 \text{ 13s. 4d.} : \frac{£291 \text{ 13s. 4d.} \times 2000}{3500} = £166 \text{ 13s. 4d.}$$

(4)

$$\left. \begin{array}{l} \$4000 \times 12 = \$48000 \text{ for one month} \\ 3000 \times 15 = 45000 \text{ for one month} \\ 5000 \times 8 = 40000 \text{ for one month} \end{array} \right\} = \$133000 \text{ for one month}$$

$$\$133000 : \$48000 :: \$665 : \frac{665 \times 48000}{133000} = \$240; \text{ B's share.}$$

$$\$133000 : \$45000 :: \$665 : \frac{665 \times 45000}{133000} = \$225; \text{ C's share.}$$

$$\$133000 : \$40000 :: \$665 : \frac{665 \times 40000}{133000} = \$200; \text{ D's share.}$$

(5)

$$\left. \begin{array}{l} 56 \times 12 = 672 \text{ for one day} \\ 64 \times 15 = 960 \text{ for one day} \\ 80 \times 18 = 1440 \text{ for one day} \end{array} \right\} = 3072 \text{ for one day.}$$

$$3072 : 672 :: \$320 : \frac{320 \times 672}{3072} = \$70 = \text{rent to be paid by 1st troop.}$$

$$3072 : 960 :: \$320 : \frac{320 \times 960}{3072} = \$100 = \text{ " " " 2nd "}$$

$$3072 : 1440 :: \$320 : \frac{320 \times 1440}{3072} = \$150 = \text{ " " " 3rd "}$$

(7)

$$\text{Sum of profits} = 240 + 800 + 400 = \$1440.$$

$$\text{Whole profit} : \text{A's profit} :: \text{Whole stock for 1 m.} : \text{A's st. for 1 m.}$$

$$\text{That is, } 1440 : 240 :: 34560 : \frac{34560 \times 240}{1440} = 5760 = \text{A's stock}$$

for one month. Hence, since A's stock was in for 6 months, it will be $\$5760 \div 6 = \960 .

(Continued on next page.)

(7 Continued.)

Whole profit : B's profit :: Whole stock for 1 m. : B's st. for 1 m.

$$1440 : 800 :: 34560 : \frac{34560 \times 800}{1440} = 19200 = \text{B's stock for one}$$

month. And, since B's stock was in for 12 months, $19200 \div 12 = \$1600$ will be his stock.

Whole profit : C's profit :: whole stock for 1 m. : C's st. for 1 m.

$$1440 : 400 :: 34560 : \frac{34560 \times 400}{1440} = \$9600 = \text{C's stock for one}$$

month, and hence his stock will be $\$9600 \div 15 = \640 .

(8)

A's profit was \$240 for 6 months = \$40 for 1 month.

B's profit was \$800 for 12 months = \$66 $\frac{2}{3}$ for 1 month.

C's profit was \$400 for 15 months = \$26 $\frac{2}{3}$ for 1 month.

Sum of profits for 1 month = \$133 $\frac{1}{3}$

Whole profit for 1 m. : A's profit for 1 m. :: whole stock : A's st.

$$133\frac{1}{3} : 40 :: 3200 : \frac{3200 \times 40}{133\frac{1}{3}} = \$960 = \text{A's stock.}$$

$$133\frac{1}{3} : 66\frac{2}{3} :: 3200 : \frac{3200 \times 66\frac{2}{3}}{133\frac{1}{3}} = \$1600 = \text{B's stock.}$$

$$133\frac{1}{3} : 26\frac{2}{3} :: 3200 : \frac{3200 \times 26\frac{2}{3}}{133\frac{1}{3}} = \$640 = \text{C's stock.}$$

EXERCISE 119—Page 275.

(1)

\$0.12 $\frac{1}{2}$ = selling price.

\$0.09 = buying price.

$$\begin{aligned} & \$0.03\frac{1}{2} = \text{gain per lb.} \\ & \$0.03\frac{1}{2} \times 317 = \$11.095, \end{aligned}$$

(2)

\$1.20 = selling price

\$0.87 $\frac{1}{2}$ = buying price.

$$\begin{aligned} & \$0.32\frac{1}{2} = \text{gain per bushel} \\ & \$0.32\frac{1}{2} \times 2138 = \$694.85, \end{aligned}$$

(3)

$\$0.15 \times 317 \times 13 = \$618.15 = \text{cost of 13 barrels at } \0.15 per lb.
 $\$735 - 618.15 = \116.85 gain.

(4)

$\$3.15 \times 22 \times 17 = \$1178.10 = \text{price of 17 kegs at } \3.15 per gal.
 $\$0.37\frac{1}{2} \times 1178.1 = \$441.7875 = \text{ad valorem duty.}$
 $\$1178.10 + \$441.7875 + \$26.33 = \$1646.2175 = \text{whole cost.}$
 $\$1646.2175 - \$1625 = \$21.2175 \text{ loss.}$

EXERCISE 120—Page 276.

(1)

Here for every \$1 I expend I wish to receive \$1.30, and hence the selling price will be $\$3.25 \times 1.30 = \$4.22\frac{1}{2}$.

(2)

Here for every \$1 I expend I wish to receive \$1.05, and hence the selling price will be $\$1.05 \times 13420 = \14091 .

(3)

Here for every \$1 I expend I desire to receive \$1.15, and hence the selling price will be $\$1.15 \times .11 = \$0.1265 = 12\frac{1}{2}\frac{3}{10} \text{ cents.}$

(4)

Here for every \$1 I expend I wish to receive \$1.23, and hence the selling price will be $\$1.23 \times 15.25 = \$18.75\frac{3}{4}$.

(5)

Here for every \$1 I expend I am willing to receive \$0.89, and hence the selling price will be $\$0.89 \times 7890 = \7022.10 ,

EXERCISE 121—Page 277.

(1)

Here the whole gain is $\$0.87\frac{1}{2} - \$0.60 = \$0.27\frac{1}{2}$.

That is, $\$0.60$ gains $\$0.27\frac{1}{2}$, and therefore 1 cent gains $\frac{27\frac{1}{2}}{60} =$

$$\frac{1}{2} \text{¢} = \frac{1}{4} \text{ of a cent.}$$

And hence, the gain per cent $= \frac{1}{4} \times 100 = 25 = 25\frac{1}{2}$ per cent.

(2)

Here the loss on each lb. is 2 cents.

That is, every 13 cents invested gives a loss of 2 cents.

Therefore every cent invested loses $\frac{1}{13}$ of 2 $= \frac{2}{13}$ cents.

And hence, the loss per cent $= \frac{2}{13} \times 100 = \frac{200}{13} = 15\frac{5}{13}$ per c.

(3)

Here the gain on each barrel is $\$1.60$.

That is, every $\$6.20$ invested gives a gain of $\$1.60$.

Therefore every $\$1$ invested gains $\frac{1}{6.20}$ of 160 $= \frac{8}{31}$ of a \$.

And hence, the gain per cent $= \frac{8}{31} \times 100 = \frac{800}{31} = 25.8 = 25\frac{1}{2}$ p. c.

(4)

Here the gain on each yard is 35 cents.

That is, every $\$2.75$ invested gives a gain of 35 cents.

Therefore every $\$1$ invested gains $\frac{1}{2.75}$ of 35 $= \frac{35}{27.5} = \frac{7}{5.5}$ of a dollar.

And hence the gain per cent $= \frac{7}{5.5} \times 100 = \frac{700}{5.5} = 127\frac{1}{11}$ p. c.

(5)

Here the gain on every bushel is 9 cents.

That is, every 47 cents invested gives a gain of 9 cents.

Therefore every cent invested gains $\frac{1}{47}$ of 9 $= \frac{9}{47}$ cents.

And hence the gain per cent $= \frac{9}{47} \times 100 = \frac{900}{47} = 19\frac{1}{47}$ p. c.

(6)

Here the loss on each lb. is $1\frac{1}{2}$ cents.

That is every 12 cents invested gives a loss of $1\frac{1}{2}$ cents.

Therefore every cent invested loses $\frac{1}{2}$ of $1\frac{1}{2} = \frac{1}{4}$ of a cent.

And hence, the loss per cent $= \frac{1}{4} \times 100 = \frac{100}{4} = 25\frac{1}{2}$ p. c.

(7)

Here the whole gain is $\$127 - \$93 = \$34$.

That is, $\$93$ gain $\$34$, and therefore $\$1$ gains $\frac{34}{93}$ of a dollar.

Hence, gain per cent $= \frac{\$34}{\$93} \times 100 = \frac{3400}{93} = 36\frac{2}{3}$ per cent.

(8)

Here the loss is $\$6742.50 - \$6000 = \$742.50$.

That is, $\$6742.50$ loses $\$742.50$, and therefore $\$1$ loses $\frac{742.50}{6742.50}$ of a dollar.

Hence loss per cent $= \frac{\$742.50}{\$6742.50} \times 100 = \frac{74250}{67425} = 11\frac{1}{3}$ per cent.

(9)

Here $\$5700 + \$275 + \$1987.32 = \$7962.32 =$ whole sum expended.

Whole gain $= \$8750 - \$7962.32 = \$787.68$.

That is, $\$7962.32$ gains $\$787.68$, and therefore $\$1$ gains $\frac{787.68}{7962.32}$ of a dollar.

Hence gain per cent $= \frac{\$787.68}{\$7962.32} \times 100 = \frac{78768}{79623.2} = 9.89$ or nearly 10 per cent.

(10)

$\$4.25 \times 723 = \$3072.75 =$ price of 723 yds. @ $\$4.25$.

$\$3072.75 \times .07 = \$215.0925 =$ amount for Insurance.

$\$3072.75 \times .22 = \$676.005 =$ amount for ad valorem duty.

Then whole cost $= \$3072.75 + \$215.0925 + \$23.70 + \$2.70 + \$3.16 + \$676.005 = \$3993.4075$.

Whole gain $= \$5270 - \$3993.4075 = \$1276.5925$.

That is, $\$3993.4075$ gains $\$1276.5925$ \therefore $\$1$ gains $\frac{1276.5925}{3993.4075}$ of a dollar.

Hence gain per cent $= \frac{\$1276.5925}{\$3993.4075} \times 100 = 31.96749$ or nearly 32 per cent.

EXERCISE 122—Page 278.

(1)

Loss on \$1 is 4 cents, or for every \$1 paid I receive \$0.96.
Hence cost = $\$24.60 \div 0.96 = \25.625 .

(2)

Loss on \$1 is 10 cents, or for every \$1 paid he receives \$0.90.
Hence cost = $\$2360 \div .90 = \2622.22 .

(3)

Gain on \$1 is 11 cents, or for every \$1 paid he receives \$1.11.
Hence cost = $\$7400 \div 1.11 = \6666.666 .

(4)

Gain on \$1 is 17 cents, or for every \$1 paid he receives \$1.17.
 $\$117 : \$100 :: \$3789.40 : \frac{3789.40 \times 100}{117} = \3238.803 .

(5)

Loss on \$1 is 13 cents, or for every \$1 paid I receive \$0.87.
 $\$87 : \$100 :: \$2740 : \frac{2740 \times 100}{87} = \3149.425 .

EXERCISE 123—Page 279.

(1)

\$2 gains 50 cents.
Hence $\$0.50 : \$0.10 :: \$2.00 : \frac{2.00 \times 10}{50} = 40$ cents.

(2)

$$\$2.00 : \$2.80 :: \$2.50 : \frac{2.50 \times 2.80}{2.00} = \$3.50.$$

(3)

8 cents gain 5 cents in 9 months.

$$\text{Hence 9 mo's : 6 mo's :: 5 cents : } \frac{5 \times 6}{9} = 3\frac{1}{3} = \text{gain for 6 mo's.}$$

$$8 \text{ cts. : 12 cts. :: } 3\frac{1}{3} : \frac{3\frac{1}{3} \times 12}{8} = 5 \text{ cts. gain on 12 cts. for 6 mo's.}$$

Therefore $12 + 5 = 17$ cents = his selling price.

(4)

$$\$1.60 : \$1.85 :: \$0.55 : \frac{1.85 \times .55}{1.60} = \$0.6359375 = \text{what L}$$

ought to get in order to sell at the same profit as K.

But L only gets 60 cents, therefore K has the advantage.

$$70 \text{ yds. of cloth at } \$1.85 = \$1.85 \times 70 = \$129.50.$$

$$\$129.50 \div \$.60 = 215\frac{1}{2}.$$

(5)

$$5 \text{ tons of butter at } \$102 = \$102 \times 5 = \$510$$

$$10\frac{1}{2} \text{ tons of tallow at } \$135 = \$135 \times 10\frac{1}{2} = \$1417.50$$

$$\text{Total value} = \$1927.50$$

$$\text{Deduct ready money, } \$600.30$$

$$\underline{\underline{\$1327.20}}$$

$$\$1327.20 \div \$4.20 = 316 \text{ barrels.}$$

EXERCISE 124--Page 281.

(1)

(2)

$$\begin{array}{r}
 7 \text{ oz.} \times 22 = 154 \text{ carats.} \\
 12\frac{1}{2} \text{ " } \times 21 = 262\frac{1}{2} \text{ " } \\
 17 \text{ " } \times 9 = 153 \text{ " } \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 2 \text{ gallons @ } 14\text{s.} = 28\text{s.} \\
 1 \text{ " @ } 12\text{s.} = 12\text{s.} \\
 2 \text{ " @ } 9\text{s.} = 18\text{s.} \\
 4 \text{ " @ } 8\text{s.} = 32\text{s.} \\
 \hline
 9 \qquad \qquad \qquad 9)90\text{s.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 36\frac{1}{2} \qquad \qquad 36\frac{1}{2})569\frac{1}{2} \text{ " } \\
 \qquad \qquad \qquad 2 \quad 2 \text{ " } \\
 \hline
 \end{array}$$

$$73)1139(15\frac{4}{3} \text{ carats.}$$

73

409

365

44

(3)

$$\begin{array}{r}
 15 \text{ bushels @ } \$1.20 = \$18.00 \\
 30 \text{ " @ } \$1.50 = \$45.00 \\
 60 \text{ " @ } \$1.10 = \$66.00 \\
 83 \text{ " @ } \$1.75 = \$145.25 \\
 \hline
 188 \qquad \qquad \qquad 188) \$274.25 (\$1.458
 \end{array}$$

188

86.2

75.2

11.05

9.40

1.650

1.504

.146

(4)

| | | | |
|----------------------------|------------|---|------------|
| 12 lbs. | @ 50 cents | = | 600 cents. |
| 16 " | @ 72 " | = | 1152 " |
| 22 " | @ 65 " | = | 1430 " |
| 18 " | @ 85 " | = | 1530 " |
| 100 " | @ 42 " | = | 4200 " |
| <hr style="width: 100%;"/> | | | |

| | |
|-----|--|
| 168 | 168)8912 cents (53 $\frac{1}{2}$ cents. |
| | 840 |
| | <hr style="width: 50px; margin-left: 0;"/> |
| | 512 |
| | 504 |
| | <hr style="width: 50px; margin-left: 0;"/> |
| | 8 |
| | } = $\frac{1}{2}$. |
| | 168 |

EXERCISE 125—Page 283.

(1)

Prices. Differences. Prices.

$$125 = \left\{ \begin{array}{l} 160 - 35 \text{ --- } 15 + 110 \\ 140 - 15 \text{ --- } 25 + 100 \end{array} \right\} = 125$$

Prices. Differences. Prices.

$$125 = \left\{ \begin{array}{l} 160 - 35 \text{ --- } 15 + 110 \\ 140 - 15 \text{ --- } 25 + 100 \end{array} \right\} = 125.$$

Ans. 35 bush. @ \$1.10, 15 @ \$1.60, 15 @ \$1, and 25 @ \$1.40.
 25 bush. @ \$1.00, 15 @ \$1.40, 15 @ \$1.10, and 25 @ \$1.60.

EXERCISE 126—Page 284.

(1)

By Case I we find that 17 quarts @ 31 cents, 6 @ 16 cents, 6 @ 19 cents, and 6 @ 23 cents will make a mixture worth 25 cents per quart.

Therefore 17 qts. : 87 qts. :: 6 qts. : $\frac{6 \times 87}{17} = 30\frac{1}{2}$ quarts @ 16 cents, and as there are 6 lbs. at each of the other prices, the same statement may be used, and the answer is therefore $30\frac{1}{2}$ quarts @ each price.

(2)

To produce a mixture worth 75 cents per bushel, we require 45 bushels @ 80 cents, 5 @ 37 cents, and 5 @ 68 cents.

Therefore 45 bush. : 70 bush. :: 5 bush. : $\frac{5 \times 70}{45} = 7\frac{2}{3}$ bush.

oats @ 37 cents.

45 bush. : 70 bush. :: 5 bush. : $\frac{5 \times 70}{45} = 7\frac{2}{3}$ bush.

barley @ 68 cents.

(3)

To produce a mixture worth 1s. per lb., we require $1\frac{1}{2}$ lbs. @ 16d., $1\frac{1}{2}$ @ 14d., and 6 @ 10 $\frac{1}{2}$ d.

Then $1\frac{1}{2}$ lbs. : 50 lbs. :: $1\frac{1}{2}$ lbs. : 50 lbs. brass @ 14d.

$1\frac{1}{2}$ lbs. : 50 lbs. :: 6 lbs. : 200 lbs. pewter @ 10 $\frac{1}{2}$ d.

(4)

By Case I we find that 1 oz. of 20 carats fine, 1 of 21 carats fine and 3 of 23 carats fine, will make a mixture 22 carats fine.

Then 1 oz. : 30 oz. :: 1 oz. : 30 oz. of 21 carats fine.

1 oz. : 30 oz. :: 3 oz. : 90 oz. of 23 carats fine.

EXERCISE 127—Page 285.

(1)

To produce a mixture worth \$1.40 per lb., we require 20 lbs. @ \$1.00, 40 @ \$1.20, 40 @ \$1.60, and 20 @ \$1.80. But all of these added together, will make 120 lbs.

$$\begin{array}{cccc} \text{lbs.} & \text{lbs.} & \text{lbs.} & \text{lbs.} \\ \text{Therefore } 120 : 20 :: 168 : \frac{168 \times 20}{120} = 28 \text{ lbs., the required} \\ & & & \text{quantity @ } \$1.00. \end{array}$$

$$\begin{array}{cccc} & & & \text{lbs.} \\ 120 : 40 :: 168 : \frac{168 \times 40}{120} = 56 \text{ lbs., the required} \\ & & & \text{quantity @ } \$1.20. \end{array}$$

$$\begin{array}{cccc} & & & \text{lbs.} \\ 120 : 40 :: 168 : \frac{168 \times 40}{120} = 56 \text{ lbs., the required} \\ & & & \text{quantity @ } \$1.60. \end{array}$$

$$\begin{array}{cccc} & & & \text{lbs.} \\ 120 : 20 :: 168 : \frac{168 \times 20}{120} = 28 \text{ lbs., the required} \\ & & & \text{quantity @ } \$1.80. \end{array}$$

(2)

To produce a mixture worth 4s. 4d. per lb., we require 10 lbs. @ 5s. and 8 @ 3s. 6d. But these added together make 18 lbs.

$$\begin{array}{cccc} \text{lbs.} & \text{lbs.} & \text{lbs.} & \text{lbs.} \\ \text{Therefore } 18 : 10 :: 27 : \frac{27 \times 10}{18} = 15 \text{ lbs., the required} \\ & & & \text{quantity of tea @ } 5s. \end{array}$$

$$\begin{array}{cccc} & & & \text{lbs.} \\ 18 : 8 :: 27 : \frac{27 \times 8}{18} = 12 \text{ lbs., the required} \\ & & & \text{quantity of tea @ } 3s. 6d. \end{array}$$

(3)

To produce a mixture worth \$2.70 per gallon, we require 20 gallons @ \$2.40, 10 @ \$2.60, 10 @ \$2.80, and 30 @ \$2.90. But all of these added together will make 70 gallons. Therefore

$$70 : 20 :: 63 : \frac{63 \times 20}{70} = 18 \text{ gallons, the required quantity of brandy @ } \$2.40.$$

$$70 : 10 :: 63 : \frac{63 \times 10}{70} = 9 \text{ gallons, the required quantity of brandy @ } \$2.60.$$

$$70 : 10 :: 63 : \frac{63 \times 10}{70} = 9 \text{ gallons, the required quantity of brandy @ } \$2.80.$$

$$70 : 30 :: 63 : \frac{63 \times 30}{70} = 27 \text{ gallons, the required quantity of brandy @ } \$2.90.$$

EXERCISE 128—Page 289.

(1)

$$1974.80 \times \frac{2}{3} = £740.55 = £740 \text{ 11s.}$$

(2)

$$765.43 \times \frac{2}{3} = £306.172 = £306 \text{ 3s. } 5\frac{7}{8}\text{d.}$$

(3)

$$8172.19 \times \frac{1}{4} = £2043.0475 = £2043 \text{ 0s. } 11\frac{1}{2}\text{d.}$$

EXERCISE 129—Page 289.

(1)

$$£743 \text{ 18s. } 11\text{d.} = £743.94583 \text{ and } 743.94583 \div \frac{3}{10} = \$2479.8194.$$

(2)

$$£119 \text{ 9s. } 8\frac{1}{2}\text{d.} = £119.484375 \text{ and } 119.484375 \div \frac{3}{8} = \$318.625.$$

(3)

$$£473 \text{ 17s. } 1\frac{1}{2}\text{d.} = £473.8572916, \text{ and } 473.8572916 \div \frac{7}{10} = \\ \$2030.816964.$$

EXERCISE 130—Page 290.

(1)

$$1006.90 \div 4.867 = £206.88309 = £206 \text{ 17s. } 7\frac{1}{2}\text{d.}$$

(2)

$$916.87 \div 4.867 = £188.38504 = £188 \text{ 7s. } 8\frac{1}{2}\text{d.}$$

(3)

$$2114.81 \div 4.867 = £434.52023 = £434 \text{ 10s. } 4\frac{1}{2}\text{d.}$$

EXERCISE 131—Page 290.

(1)

$$£2043 \text{ 11s. } 3\text{d.} = £2043.5625 \text{ and } 2043.5625 \times 4.867 = \\ \$9946.01868.$$

(2)

$$£777 \text{ 7s. } 7\text{d.} = £777.37916 \text{ and } 777.37916 \times 4.867 = \\ \$3783.50437.$$

(3)

$$£557 \text{ 19s. } 5\frac{1}{2}\text{d.} = £557.972916 \text{ and } 557.972916 \times 4.867 = \\ \$2715.65418.$$

EXERCISE 132—Page 294.

(1)

$$\$16785.25 \times 5.04 = 84597 \text{ francs } 66 \text{ centimes.}$$

(2)

| | | |
|--------------------------------------|---------------------------------------|--------|
| Commercial value of the marc banco = | 35 | cents. |
| Add 1 per cent | 35 | |
| | <hr style="width: 50px; margin: 0;"/> | |
| | 3535 | |

$$\text{Then } \$0.3535 \times 4000 = \$1414.$$

(3)

$$\$35678 \times 1.0225 = \$36480.755.$$

(4)

| | | |
|----------------------------|---------------------------------------|--------|
| The par value of 1 ruble = | 75 | cents. |
| Deduct 2 per cent | 15 | |
| | <hr style="width: 50px; margin: 0;"/> | |
| | 735 | |

$$\text{Then } \$0.735 \times 2560 = \$1881.60.$$

(5)

| | | |
|-------------------------------------|---------------------------------------|-------------|
| Old commercial par of £1 sterling = | \$4.444 | = \$4.44444 |
| Add 8 per cent | .35555 | |
| | <hr style="width: 50px; margin: 0;"/> | |
| | \$4.79999 | |

$$\text{Then } \$4.79999 \times 800 = \$3839.999 = \$3840.00.$$

EXERCISE 133—Page 295.

(1)

£1 = 420d.

19½d. = 1 franc.

300 francs = 60 ducats.

1 ducat = 360 maravedis.

x = £1000.

$$x = \frac{84 \times 300 \times 1 \times 360 \times 1000}{19\frac{1}{2} \times 300} = 1564138 \text{ maravedis by cir. ex.}$$

$$42\frac{1}{2}d. : £1000 :: 272 \text{ maravedis} : \frac{272 \times 1000 \times 20 \times 12}{42\frac{1}{2}}$$

$$\frac{16 \times 272 \times 1000 \times 8 \times 12}{17} = 1536000 \text{ maravedis by direct exchange.}$$

Difference = 1564138 - 1536000 = 28138 maravedis.

34)28138

8)827 reals 20 maravedis

103 piastres 3 reals 20 maravedis.

(2)

Old commercial par of £1 sterling = \$4.444

To which add 10 per cent. of itself = .4444

Gives price of £1 sterling = \$4.8884

\$4888.40 ÷ \$4.8884 = £1000 = amount of bill he receives if he remits direct to London.

\$1 = 515 centimes. } x = $\frac{515 \times 4888.40}{2580}$ = £975.78526.

2580 cen. = £1 sterling. } x = $\frac{515 \times 4888.40}{2580}$ = £975 15s. 8½d. = amount of bill he receives if he remits through Paris.

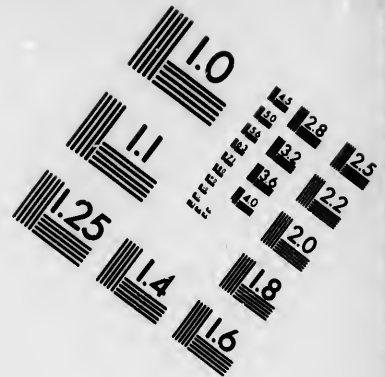
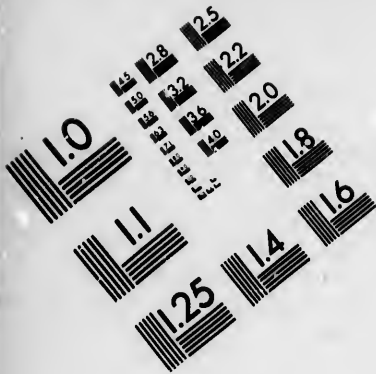
35 cents = 1 marc.

3½ marcs = £1 sterling.

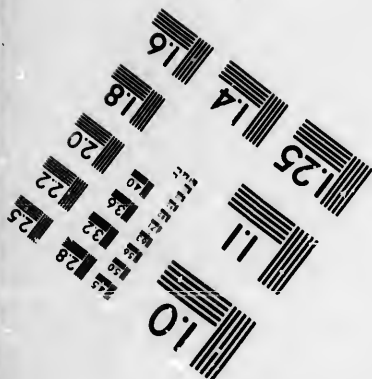
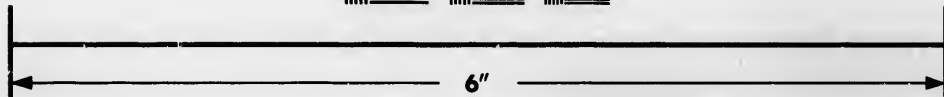
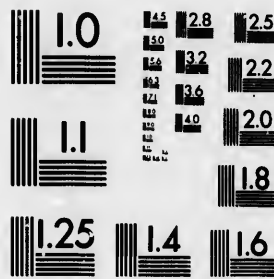
x = \$4888.40.

$$x = \frac{4888.40 \times 391072}{35 \times 13\frac{3}{4} \times 385} = £1015.77142 = £1015 15s. 5d. = \text{amount of bill he receives by remitting through Hamburg.}$$





**IMAGE EVALUATION
TEST TARGET (MT-3)**



**Photographic
Sciences
Corporation**

23 WEST MAIN STREET
WEBSTER, N.Y. 14580
(716) 872-4503

0
11
15 28
18 32
22 36
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(3)

$$\left. \begin{array}{l} 18 \text{ cents.} = 1 \text{ franc.} \\ 25 \text{ francs.} = 240 \text{d.} \\ 180 \text{d.} = 3 \text{ milrees,} \\ 5 \text{ milrees.} = 18 \text{ marcs ban.} \\ 1200 \text{ marcs ban.} = x \end{array} \right\} x = \frac{18 \times 25 \times 180 \times 1200 \times 5}{240 \times 3 \times 18}$$

= \$375 = circuitous exchange or sum he pays for 1200 marks.

1200 × .35 = \$420 = direct exchange or sum paid for 1200 marks, \$420 - \$375 = \$45 = gain by circuitous exchange.

EXERCISE 134—Page 298.

(1)

$$(3)^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243.$$

(2)

$$(20)^{10} = 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 = 10240000000000.$$

(3)

$$(1.05)^6 = 1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 = 1.340095640625.$$

(4)

$$(3)^7 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2187.$$

(5)

$$(3)^8 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 6561.$$

(6)

$$11\frac{1}{2} = \frac{23}{2}. \quad \left(\frac{23}{2}\right)^3 = \frac{23}{2} \times \frac{23}{2} \times \frac{23}{2} = \frac{12167}{8} = 1520\frac{7}{8}.$$

EXERCISE 135—Page 299.

(1)

$$4^2 \times 4^4 \times 4^5 \times 4^7 = 4^{2+4+5+7} = 4^{18}.$$

(2)

(3)

$$13^{11} \div 13^2 = 13^{11-2} = 13^9. \quad (3^3)^5 = 3^{3 \times 5} = 3^{15}.$$

(4)

$$\{(7^4 \times 7^3) \div (7^2 \times 7^2)\}^6 = \{(7^4 + 3) \div (7^2 + 2)\}^6 = \{7^7 \div 7^4\}^6 = (7^{7-4})^6 = (7^3)^6 = 7^{3 \times 6} = 7^{18}.$$

(5)

$$\{(5^3 \times 5^4 \times 5^{11} \times 5^9) \div (5^3 \times 5^2 \times 5^7 \times 5^5)\}^3 = \{(5^{3+4+11+9}) \div (5^{3+2+7+5})\}^3 = \{5^{27} \div 5^{17}\}^3 = (5^{27-17})^3 = (5^{10})^3 = 5^{10 \times 3} = 5^{30}.$$

EXERCISE 137—Page 304.

(1)

(2)

(3)

$$\begin{array}{r} \dots \\ 195364(442 \\ 16 \\ \hline 84)353 \\ 336 \\ \hline 882)1764 \\ 1764 \end{array}$$

$$\begin{array}{r} \dots \\ -0676(-26 \\ 4 \\ \hline 46)276 \\ 276 \end{array}$$

$$\begin{array}{r} \dots \\ 984064(992 \\ 81 \\ \hline 189)1740 \\ 1701 \\ \hline 1982)3964 \\ 3964 \end{array}$$

60 5
1800 1200 5
1800 1800
for 1200 marks.
n. paid for 1200
ous exchange.

20 20 20
5 x 1.05 =

1481.88

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(5)

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 —
 42)100
 84
 —
 443)1600
 1329
 —
 4466)27100
 26796
 —
 447206)3040000
 2683236
 —
 356764

500000000000(-707106
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 —
 1407)10000
 9849
 —
 14141)15100
 14141
 —
 1414206)9590000
 8485236
 —
 1104764

(6)

(7)

60.487129(7-777
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 —
 147)1148
 1029
 —
 1547)11971
 10829
 —
 15547)114229
 108829
 —
 5400

79792266297612001(282475249
 4
 —
 48)397
 384
 —
 562)1392
 1124
 —
 5644)26826
 22576
 —
 56487)425062
 395409
 —

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0000012321(-00111
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 21)23
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 —
 221)221
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564945)2965397
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 —
 5649502)14067261
 11299004
 —
 56495044)276825720
 225980176
 —
 564950489)5084554401
 5084554401

(5)

EXERCISE 138—Page 304.

(1)

$$i = \frac{1}{5} \text{ and } \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$$

(3)

$$5\frac{1}{2} = 5.142857142857 \text{ and } \sqrt{5.142857142857} = 2.267786.$$

(4)

$$\frac{217}{25} = .4033457249 \text{ and } \sqrt{.4033457249} = .63509.$$

(5)

$$13\frac{1}{2} = 13.2 \text{ and } \sqrt{13.2} = 3.63318$$



EXERCISE 139—Page 305.

(1)

(2)

```

    . . . . .
  11333311(2626
    4
  —
46)433
   411
  —
552)2233
   1434
  —
5546)46611
   46611
  —

```

```

    . . . . .
  33233344(4344
    24
  —
123)523
   413
  —
1304)11033
   10024
  —
13124)100544
   100544
  —

```

00(-707106

01(282475249

(2)

$$50^2 = 2500$$

$$80^2 = 6400$$

$$\text{Sum} = 8900 \text{ and } \sqrt{8900} = 94.34 \text{ nearly}$$

(3)

$$24^2 = 576 \div 2 = 288 \text{ and } \sqrt{288} = 16.97.$$

(4)

$$36^2 = 1296$$

$$20^2 = 400$$

$$\text{Difference} = 896 \text{ and } \sqrt{896} = 29.933.$$

(5)

$$40^2 = 1600$$

$$14^2 = 196$$

$$\text{Difference} = 1404 \text{ and } \sqrt{1404} = 37.469.$$

$$40^2 = 1600$$

$$26^2 = 676$$

$$\text{Difference} = 924 \text{ and } \sqrt{924} = 30.397.$$

$$37.469 + 30.397 = 67.866 \text{ and } 67.866 \div 3 = 22.622.$$

(6)

$$1760 \text{ sq. yds.} = 15840 \text{ sq. ft. and } \sqrt{15840} = 125.857.$$

(7)

$$\sqrt{141376} = 376.$$

(8)

$$3^2 = 9$$

$$3^2 = 9$$

$$\text{Sum} = 18 \text{ and } \sqrt{18} = 4.24264.$$

(9)

$16^2 = 256$

$12^2 = 144$

$$\text{Sum} = 400 \text{ and } \sqrt{400} = 20$$

(10)

$3^2 + 3^2 + 3^2 = 27 \text{ and } \sqrt{27} = 5.196.$

(11)

$(\frac{1}{100})^2 = \frac{1}{10000} \text{ and } (1)^2 = 1.$

$$\text{Then } \frac{1}{100} : 1 :: 450 : \frac{450}{.00} = 45000.$$

(12)

$$1 \text{ sq. acre} = 160 \text{ sq. perches. } 160 \div 3.1416 = 50.929462 \text{ and } \sqrt{50.929462} = 7.136.$$

EXERCISE 141—Page 311.

(1)

| | | |
|----------------------------|---|---|
| | | 62712728317(3973 |
| | | 27 |
| | | <hr style="width: 100px; margin-left: 0;"/> |
| | | 35712 |
| $3^2 \times 300 =$ | 2700 | |
| $3 \times 9 \times 30 =$ | 810 | |
| $9^2 =$ | 81 | |
| | <hr style="width: 100px; margin-left: 0;"/> | |
| | 3591 | 32319 |
| | <hr style="width: 100px; margin-left: 0;"/> | |
| $39^2 \times 300 =$ | 456300 | 3393728 |
| $39 \times 7 \times 30 =$ | 8190 | |
| $7^2 =$ | 49 | |
| | <hr style="width: 100px; margin-left: 0;"/> | |
| | 464539 | 3251773 |
| | <hr style="width: 100px; margin-left: 0;"/> | |
| $397^2 \times 300 =$ | 47282700 | 141955317 |
| $397 \times 3 \times 30 =$ | 35730 | |
| $3^2 =$ | 9 | |
| | <hr style="width: 100px; margin-left: 0;"/> | |
| | 47318439 | 141955317 |

(3)

| | | |
|---------------------------|-------|-------------|
| | | 1953125(125 |
| | | 1 |
| | | - |
| $1^2 \times 300 =$ | 300 | 953 |
| $1 \times 2 \times 30 =$ | 60 | |
| $2^2 =$ | 4 | |
| | <hr/> | |
| | 364 | 728 |
| | <hr/> | |
| $12^2 \times 300 =$ | 43200 | 225125 |
| $12 \times 5 \times 30 =$ | 1800 | |
| $5^2 =$ | 25 | |
| | <hr/> | |
| | 45025 | 225125 |

50-929462 and

(3)

| | | |
|----------------------------|---------|-----------------|
| | | 1076890625(1025 |
| | | 1 |
| | | - |
| $1^2 \times 300 =$ | 300 | 76 |
| $10^2 \times 300 =$ | 30000 | 76890 |
| $10 \times 2 \times 30 =$ | 600 | |
| $2^2 =$ | 4 | |
| | <hr/> | |
| | 30604 | 61208 |
| | <hr/> | |
| $102^2 \times 300 =$ | 3121200 | 15682625 |
| $102 \times 5 \times 30 =$ | 15300 | |
| $5^2 =$ | 25 | |
| | <hr/> | |
| | 3136525 | 15682625 |

73

(4)

| | | |
|---------------------------|---------|----------------|
| | | 697864103(·887 |
| | | 512 |
| | | — |
| $8^2 \times 300 =$ | 19200 | 185864 |
| $8 \times 8 \times 30 =$ | 1920 | |
| $8^2 =$ | 64 | |
| | — | |
| | 21184 | 139472 |
| | — | — |
| $88^2 \times 300 =$ | 2323200 | 16392103 |
| $88 \times 7 \times 30 =$ | 18480 | |
| $7^2 =$ | 49 | |
| | — | |
| | 2341729 | 16392103 |

(5)

| | | |
|---------------------------|--------|-----------------|
| | | 102503·232(46·8 |
| | | 64 |
| | | — |
| $4^2 \times 300 =$ | 4800 | 38503 |
| $4 \times 6 \times 30 =$ | 720 | |
| $6^2 =$ | 36 | |
| | — | |
| | 5556 | 33336 |
| | — | — |
| $46^2 \times 300 =$ | 634800 | 5167·232 |
| $46 \times 8 \times 30 =$ | 11040 | |
| $8^2 =$ | 64 | |
| | — | |
| | 645904 | 5167·232 |

(6)

$$\begin{array}{r} 179597 \cdot 069288(56 \cdot 42 \\ 125 \end{array}$$

 54597

$$5^2 \times 300 = 7500$$

$$5 \times 6 \times 30 = 900$$

$$6^2 = 36$$

 8436

 50616

$$56^2 \times 300 = 940800$$

$$56 \times 4 \times 30 = 6720$$

$$4^2 = 16$$

 947536

 3931 \cdot 069

$$564^2 \times 300 = 95428800$$

$$564 \times 2 \times 30 = 33840$$

$$2^2 = 4$$

 95462644

 3790 \cdot 144

 190 \cdot 925288

 190 \cdot 925288

(7)

$$\begin{array}{r} 483 \cdot 736625(7 \cdot 85 \\ 343 \end{array}$$

 140 \cdot 736

$$7^2 \times 300 = 14700$$

$$7 \times 8 \times 30 = 1680$$

$$8^2 = 64$$

 16444

 131 \cdot 552

$$78^2 \times 300 = 1825200$$

$$78 \times 5 \times 30 = 11700$$

$$5^2 = 25$$

 1836925

 9 \cdot 184625

 9 \cdot 184625

(-887)

(46 \cdot 8)

(8)

$$\begin{array}{r}
 \cdot 636056 \overline{) 86} \\
 \underline{512} \\
 124056 \\
 \underline{} \\
 20676 \\
 \underline{} \\
 124056
 \end{array}$$

$$8^3 \times 300 = 19200$$

$$8 \times 6 \times 30 = 1440$$

$$6^2 = 36$$

$$20676$$

$$124056$$

$$124056$$

EXERCISE 147.—Page 312.

(1)

$$\sqrt[3]{.105263157894} = .4721 \text{ and } \sqrt[3]{.105263157894} = .4721.$$

(2)

$$\sqrt[3]{.176470588235} = .5609 \text{ and } \sqrt[3]{.176470588235} = .5609.$$

(3)

$$\frac{1}{3} \text{ of } 2\frac{1}{2} = \frac{5}{6} = .83333333 \text{ and } \sqrt[3]{.83333333} = .941.$$

(4)

$$28\frac{3}{4} = 28.75 \text{ and } \sqrt[3]{28.75} = 3.063$$

(5)

$$32\frac{1}{4} = 32.72 \text{ and } \sqrt[3]{32.72} = 3.198.$$

EXERCISE 143.—Page 313.

(1)

One million = 33233344 senary.

33233344(244.
12

$$\begin{array}{r}
 2^2=4 \times 300= 2000 \\
 3 \times 30=100 \times 4= 400 \\
 4^2= 24
 \end{array}$$

21233

2424

14544

$$\begin{array}{r}
 24^2=1104 \times 300=332000 \\
 24 \times 30=1200 \times 4= 5200 \\
 4^2= 24
 \end{array}$$

2245344

341224

2245344

(2)

6131271·000000(165·32.
1

= .4721.

$$\begin{array}{r}
 1^2 \times 300= 300 \\
 1 \times 30 \times 6= 220 \\
 6^2= 44
 \end{array}$$

5131

564

4270

= .5609.

$$\begin{array}{r}
 16^2=304 \times 300= 111400 \\
 16 \times 30=520 \times 6= 3220 \\
 5^2= 31
 \end{array}$$

641271

114651

600115

33 = .941.

$$\begin{array}{r}
 165^2=32571 \times 300= 12015300 \\
 165 \times 30=5370 \times 3= 20350 \\
 3^2= 11
 \end{array}$$

41154·000

12035681

36131·423

$$\begin{array}{r}
 1653^2=3272071 \times 300=1205625300 \\
 1653 \times 30=54010 \times 2= 130020 \\
 2^2= 4
 \end{array}$$

3022·355000

1205755324

2413·732650

406·422130

(3)

10221012 · 10200000

1 | 112 · 012 = root.

| | | |
|---|--------------|------------------|
| | 1000 | 2221 |
| $1 \times 1000 =$ | 1000 | |
| $1 \times 1 \times 100 =$ | 100 | |
| $1^2 =$ | 1 | |
| | 1101 | 1101 |
| $11^2 = 121 \times 1000 =$ | 121000 | 1120012 |
| $11 \times 100 = 1100 \times 2 =$ | 2200 | |
| $2^2 =$ | 11 | |
| | 200211 | 1101122 |
| $112^2 = 21021 \times 1000 =$ | 21021000 | 11120 · 102 |
| $1120^2 = 2102100 \times 1000 =$ | 2102100000 | 11120 · 102000 |
| $1120 \times 100 = 112000 \times 1 =$ | 112000 | |
| $1^2 =$ | 1 | |
| | 2102212001 | 2102 · 212001 |
| $11201^2 = 211010101 \times 1000 =$ | 211010101000 | 2010 · 112222000 |
| $11201 \times 100 = 1120100 \times 2 =$ | 10010200 | |
| $2^2 =$ | 11 | |
| | 211020111211 | 1122 · 111000122 |
| | | 111 · 001221101 |

440
440

(4)

$teteet \cdot 000000(e7 \cdot t2)$
 $92e$

$$\begin{aligned} e^2 &= t1 \times 360 = & 26300 \\ e \times 30 &= 290 \times 7 = & 1730 \\ & 7^2 = & 41 \end{aligned}$$

18 e e e t

27t71

167217

$$\begin{aligned} e7^2 &= e221 \times 300 = & 2966300 \\ e7 \times 30 &= 2t90 \times t = & 24e60 \\ & t^2 = & 84 \end{aligned}$$

249 t3 \cdot 000

298e324

24154 \cdot 7e4

$$\begin{aligned} e7t^2 &= e39544 \times 300 = & 29e441000 \\ e7t \times 30 &= 2te60 \times 2 = & 59e00 \\ & 2^2 = & 4 \end{aligned}$$

84 t \cdot 408000

29e49 t e 04

57 t \cdot 979 t08

28 e \cdot 64 t1e4

(5)

$421030 \cdot 441200000(44 \cdot 004)$
 224

$$\begin{aligned} 4^2 &= 31 \times 300 = & 14300 \\ 4 \times 30 &= 220 \times 4 = & 1430 \\ & 4^2 = & 31 \end{aligned}$$

142030

21311

141244

$$44^2 = 4301 \times 300 = 2340300$$

231 \cdot 441

$$440^2 = 430100 \times 300 = 234030000$$

231 \cdot 441200

$$4400^2 = 43010000 \times 300 = 23403000000$$

231 \cdot 441200000

$$\begin{aligned} 4400 \times 30 &= 242000 \times 4 = & 2123000 \\ & 4^2 = & 31 \end{aligned}$$

23410123031

210 \cdot 141102224

21 \cdot 300042221

EXERCISE 144.—Page 314.

(2)

$$3^3 : 6^3 :: 4 \text{ lb.} : \text{Ans.} = 32 \text{ lbs.}$$

(3)

$$1^3 : \left(\frac{1}{3}\right)^3 :: \$120 : \text{Ans.} = \$5145.$$

(4)

$$\begin{aligned} (70)^3 & : \left(\frac{1}{2}\right)^3 :: 180 \text{ lbs.} : \text{Ans.} \\ 243000 & : \frac{1}{8} :: 180 : \text{Ans.} = \\ 180 \times \frac{243000}{1} & \times \frac{1}{8} = 1015.1 \text{ lbs.} \end{aligned}$$

(5)

$$973^3 = 921167317$$

$$45^3 = 91125$$

$$62^3 = 238328$$

$$30^3 = 27000$$

$$80^3 = 512000$$

$$20^3 = 8000$$

$$\begin{aligned} 921167317 - (91125 + 238328 + 27000 + 512000 + 8000) = \\ 920290864 \text{ and } \sqrt[3]{920290864} = 972.69. \end{aligned}$$

(6)

8 feet 3 inches = 99 inches, 3 feet = 36 inches, and 2 feet 7 inches = 31 inches.

$$99 \times 36 \times 31 = 110484 \text{ and } \sqrt[3]{110484} = 47.9843.$$

(7)

After the first has wound off her portion, there will remain $\frac{1}{2}$ of the thread.

Then the whole ball : part remaining :: cube of diameter of whole ball : cube of diameter of part remaining.

That is, $1 : \frac{1}{4} :: 3^3 : x^3$, and hence $x = 3 \times \sqrt[3]{\frac{1}{4}} = 3 \times \sqrt[3]{.75} = .90856 \times 3 = 2.72568 =$ diameter of the ball after the first has wound off her portion.

Similarly after the second has wound off her portion, there will remain $\frac{1}{4}$ of the ball, and after the third has taken her portion, $\frac{1}{4}$ of the ball.

Hence $1 : \frac{1}{4} :: 3^3 : x^3$, whence $x = 3 \times \sqrt[3]{\frac{1}{4}} = 3 \times \sqrt[3]{.5} = 3 \times .79370 = 2.38110 =$ diameter after the second has taken her portion.

$1 : \frac{1}{4} :: 3^3 : x^3$, whence $x = 3 \times \sqrt[3]{\frac{1}{4}} = 3 \times \sqrt[3]{.25} = 3 \times .62996 = 1.88988 =$ diameter after the third has taken her portion.

| | | |
|-----------------------|---------------------|----------------|
| Hence 1st takes off 3 | — 2.72568 = | .27432 inches. |
| 2nd " " | 2.72568 — 2.38110 = | .34458 " |
| 3rd " " | 2.38110 — 1.88988 = | .49122 " |
| 4th " " | remaining | 1.88988 " |

EXERCISE 145—Page 315.

(1)

$$\sqrt{19987173376} = 141376, \text{ and } \sqrt{141376} = 376.$$

(2)

$$\sqrt{308915776} = 676, \text{ and } \sqrt{676} = 26.$$

(3)

$$\sqrt{40353607} = 343, \text{ and } \sqrt{343} = 7.$$

(4)

$$\sqrt{387420489} = 730, \sqrt{729} = 9, \text{ and } \sqrt{9} = 3.$$

(5)

$$\sqrt[3]{134217728} = 512, \sqrt[3]{512} = 8, \text{ and } \sqrt[3]{8} = 2.$$

EXERCISE 148—Page 321.

(1)

The mantissa of the logarithm of 8193 (the first four digits) = .913443, and the next following mantissa is .913496.

Then from .913496

Subtract.. .913443

Difference, 53; and 53×217 (remaining digits of given number) = 11501, from which we cut off three digits, since we multiplied by a number of three digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which gives us 12.

Then mantissa of logarithm of first four digits .913443

Add, 12

Mantissa of logarithm of given number, .913455

To which attach the characteristic 6 and required logarithm = 6.913455.

The mantissa of the logarithm of 7392 (the first four digits) = .868762, and the next following mantissa is .868821.

Then from .868821

Subtract.. .868762

Difference, 59; and 59×45 (remaining digits of given number) = 2655, from which we cut off two digits, since we multiplied by a number of two digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which gives us 27.

Then mantissa of logarithm of first four digits, .868762

Add, 27

Mantissa of logarithm of given number, .868789

(Continued on next page.)

(1 continued.)

To which attach the characteristic 1 and required logarithm =
1.868789.

The mantissa of the logarithm of 8437 (the first four digits)
= .926188, and the next following mantissa is .926240.

Then from .926240

Subtract.. .926188

Difference, 52; and 52×42 (remaining digits of given number) = 2184, from which we cut off two digits, since we multiplied by a number of two digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which becomes 22.

Then mantissa of logarithm of first four digits .926188

Add, 22

Mantissa of logarithm of given number, .926210

To which attach the characteristic $\bar{1}$ and required logarithm =
 $\bar{1}.926210$.

(2)

The mantissa of the logarithm of 2345 = .370143, and the next following mantissa is .370328.

Then from .370328

Subtract.. .370143

Difference, 185; and $185 \times 64 = 11840$, from which we cut off two digits, since we multiplied by a number of two digits, which gives us 118.

Then mantissa of logarithm of 2345 = .370143

Add, 118

Mantissa of logarithm of given number = .370261

To which attach the characteristic $\bar{4}$ and required logarithm =
 $\bar{4}.370261$.

(Continued on next page.)

(2 continued.)

The mantissa of the logarithm of 1007 = .003029, and the next following mantissa is .003461.

Then from .003461

Subtract.. .003029

—————

Difference, 432; and $432 \times 013 = 5616$, from which we cut off three digits, since we multiplied by a number of three digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which gives us 6.

Then mantissa of logarithm of 1007 = .003029

Add, 6

—————

Mantissa of logarithm of given number .003035

To which attach the characteristic 3, and required logarithm = $\bar{3}.003035$.

(3)

Mantissa of logarithm of 5237719083
 Difference from column D = 83; and $83 \times 6 = 498$
 from which we cut off 1 digit and add..... 50

—————

And also attach the characteristic 1, and required
 logarithm = 1.719133

Mantissa of logarithm of 1294..... .111934
 Difference from column D = 335; and $335 \times 76 =$
 25460 from which we cut off two digits and add, 255

—————

And also attach the characteristic 2 and required
 logarithm = 2.112189

(4)

| | | | |
|--------------------------|------------|--------|----------|
| Mantissa of logarithm of | ·0004713 | = | ·673297 |
| P. P. corresponding to | ·00000009 | = | 83 |
| P. P. " to | ·000000008 | = | 74 |
| | | | <hr/> |
| | | Sum, = | ·6733874 |

Therefore required mantissa = ·673387 and required logarithm
= 4·673387.

| | | | |
|--------------------------|---------|--------|-----------|
| Mantissa of logarithm of | 9136000 | = | ·960756 |
| P. P. corresponding to | 700 | = | 33 |
| P. P. " to | 10 | = | 5 |
| P. P. " to | 2 | = | 9 |
| | | | <hr/> |
| | | Sum, = | ·96078959 |

Therefore required mantissa = ·960790 and required logarithm
= 6·960790.

(5)

| | | | |
|--------------------------|---------|--------|----------|
| Mantissa of logarithm of | 4·23400 | = | ·626751 |
| P. P. corresponding to | 20 | = | 20 |
| P. P. " to | 9 | = | 92 |
| | | | <hr/> |
| | | Sum, = | ·6267802 |

Therefore required logarithm is 0·626780.

| | | | |
|--------------------------|--------|--------|------------|
| Mantissa of logarithm of | 763·1 | = | ·882581 |
| P. P. corresponding to | ·02 | = | 11 |
| P. P. " to | ·009 | = | 51 |
| P. P. " to | ·0008 | = | 46 |
| P. P. " to | ·00009 | = | 40 |
| | | | <hr/> |
| | | Sum, = | ·882597600 |

Therefore required logarithm is 2·882598.

EXERCISE 149.—Page 323.

(1)

Given logarithm, $\cdot 137139$ Next lower in table, $\cdot 137037 = \log.$ of 1371.

| | | |
|------------|--|--------------------------------|
| Difference | | 102, Tabular difference = 316. |
|------------|--|--------------------------------|

Then $1020000 \div 316$ gives 3227 for digits in 5th, 6th, 7th, and 8th places.

Hence the digits of the natural number are 13713227; and since the characteristic is 4, i.e., one less than the number of digits to the left of the decimal point the required number is 13713·227.

Given logarithm, $\cdot 718134$ Next lower in table, $\cdot 718086 = \log.$ of 5225.

| | | |
|-------------|--|------------------------------|
| Difference, | | 48, Tabular difference = 83. |
|-------------|--|------------------------------|

Then $48000 \div 83$ gives 578 for digits in 5th, 6th, and 7th places.

Hence the digits of the natural number are 5225578, and since the characteristic is 0, i.e., one less than the number of digits to the left of the decimal point, the required number is 5·225578.

Given logarithm, $\cdot 635421$ Next lower in table, $\cdot 635383 = \log.$ of 4319.

| | | |
|-------------|--|-------------------------------|
| Difference, | | 38, Tabular difference = 101. |
|-------------|--|-------------------------------|

Then $38000 \div$ gives 376 for digits in 5th, 6th, and 7th places.

Hence the digits of the natural number are 4319376, and since the characteristic is $\bar{4}$, i.e., one more than the number of ciphers between the decimal point and the first figure to the right, the required number is $\cdot 0004319376$.

(2)

Given log. $\cdot 921686 = \log.$ of 8350.

And since the characteristic is 2, i.e., one less than the number of digits to the left of the decimal point, the required number is 835.

Given logarithm, $\cdot 922165$

Next lower in table, $\cdot 922154 = \log.$ of 8359.

Difference = 11, Tabular difference = 52.

Then $11000 \div 52$ gives 211 for digits in 5th, 6th, and 7th places.

Hence the digits of the natural number are 8359211; and since the characteristic is $\bar{1}$, i.e., one more than the number of ciphers between the decimal point and first figure to the right, the required number is $\cdot 8359211$.

(3)

Given logarithm, $\cdot 407968$

Next lower in table, $\cdot 407901 = \log.$ of 2558.

Difference, = 67

Highest P. P. not greater than 67 = 51 corresponds to 3
for 5th place.

Highest P. P. not greater than 160 = 153 corresponds to 9
for 6th place.

Highest P. P. not greater than 70 = 68 corresponds to 4
for 7th place.

Therefore digits of required number are 2558394; and since the characteristic is 5, there must be six digits to the left of the decimal point.

Hence required number is 2558394.

(Continued on next page.)

(3 continued.)

| | | |
|-------------------------------------|---------|-----------------------------------|
| Given logarithm, | ·408386 | |
| Next lower in table, | ·408240 | = log. of 2560. |
| Difference, = | 146 | |
| Highest P.P. not greater than 146 = | 136 | corresponds to 8
in 5th place. |
| | 100 | |
| Highest P.P. not greater than 100 = | 85 | corresponds to 5
in 6th place. |
| | 150 | |
| Highest P.P. not greater than 150 = | 136 | corresponds to 8
in 7th place. |
| | 140 | |
| Highest P.P. not greater than 140 = | 136 | corresponds to 8
in 8th place. |

Therefore digits of required number are 25608588; and since the characteristic is 7, there must be eight digits to the left of the decimal point.

Hence required number is 25608588.

| | | |
|------------------------------------|---------|-----------------------------------|
| Given logarithm, | ·416369 | |
| Next lower in table, | ·416308 | = log. of 2608. |
| Difference, = | 61 | |
| Highest P.P. not greater than 61 = | 49 | corresponds to 3
in 5th place. |
| | 12 | |

Therefore digits of required number are 26083; and since the characteristic is 3, there must be two ciphers between the decimal point and first figure.

Hence required number is ·0026083.

(4)

| | | |
|-------------------------------------|-------------------------|-----------------------------------|
| Given logarithm, | .877777 | |
| Next lower in table, | .877774 = log. of 7547. | |
| Difference, = | <u>3</u> | |
| There is no P.P. not greater than 3 | | 0 corresponds to 0 in 5th place. |
| | <u>30</u> | |
| Highest P.P. not greater than 30 = | | 29 corresponds to 5 in 6th place. |
| | <u>10</u> | |
| Highest P.P. not greater than 10 = | | 6 corresponds to 1 in 7th place. |
| | <u>40</u> | |
| Highest P.P. not greater than 40 = | | 35 corresponds to 6 in 8th place. |
| | <u>50</u> | |
| Highest P.P. not greater than 50 = | | 46 corresponds to 8 in 9th place. |
| | <u>4</u> | |

Therefore digits of required number are 754705168 ; and since the characteristic is 4, there must be five digits to the left of the decimal point.

Hence required number is 75470.5168.

| | | |
|------------------------------------|-------------------------|-----------------------------------|
| Given logarithm, | .555555 | |
| Next lower in table, | .555457 = log. of 3593. | |
| Difference, = | <u>98</u> | |
| Highest P.P. not greater than 98 = | | 98 corresponds to 8 in 5th place. |

Therefore digits of required number are 35938 ; and since the characteristic is 0, there must be one digit to the left of the decimal point.

Hence required number is 3.5938.

EXERCISE 150.—Page 324.

(1)

$10 - 5.631642 = 4.368358.$

$10 - 0.714000 = 9.286000.$

(2)

$10 - \overline{3}123456 = 12.876544.$

$10 - \overline{7}213149 = 16.786851.$

(3)

$10 - 6.124357 = 3.875643 \text{ and } 10 - \overline{2}000837 = 11.999163.$

EXERCISE 151.—Page 325.

(1)

$\text{Logarithm of } 61 = 1.785330$

$\text{" } 22 = 1.342423$

$\text{" } 65 = 1.812913$

 $\text{Sum} = 4.940666 = \text{logarithm of } 87230.$

(2)

$\text{Logarithm of } 52 = 1.716003$

$\text{" } 734 = 2.865696$

$\text{" } 6 = 0.778151$

 $\text{Sum} = 5.359850$

$5.359835 = \text{logarithm of } 229000$

 8

 $\text{Ans. } 229008$

(3)

$$\begin{array}{r}
 \text{Logarithm of } 35.86 = 1.554610 \\
 \text{" } 2.1046 = 0.323169 \\
 \text{" } .8372 = \bar{1}.922829 \\
 \text{" } .00294 = \bar{3}.468347 \\
 \hline
 \text{Sum} = \bar{1}.268955 \\
 \bar{1}.268812 = \text{logarithm of } .185706 \\
 \hline
 143 = \qquad \qquad \qquad 61 \\
 \hline
 \text{Ans. } .185761
 \end{array}$$

(4)

$$\begin{array}{r}
 \text{Log. of } .00008764 = \bar{5}.942702 \\
 \text{" } .86359 = \bar{1}.936308 \\
 \hline
 \text{Sum} = \bar{5}.879010 \\
 \bar{5}.878981 = \text{logarithm of } .000075680 \\
 \hline
 29 = \qquad \qquad \qquad 5 \\
 \hline
 \text{Ans. } .000075685
 \end{array}$$

EXERCISE 152.—Page 326.

(1)

$$\begin{array}{r}
 \text{Logarithm of } .6734 = \bar{1}.828273 \\
 \text{" } .0009278 = \bar{4}.967454 \\
 \hline
 \text{Difference} = 2.860819 \\
 2.860817 = \text{logarithm of } 725.8000 \\
 \hline
 2 = \qquad \qquad \qquad 33 \\
 \hline
 \text{Ans. } 725.8033
 \end{array}$$

(2)

$$\text{Logarithm of } 437.89 = 2.641365$$

$$\text{" } 62.735 = 1.797510$$

$$\text{Difference} = \underline{.843855} = \text{logarithm of } 6.98$$

(3)

$$\text{Logarithm of } 93.217 = 1.969495$$

$$\text{" } .0007132 = \underline{4.853211}$$

$$\text{Difference} = 5.116284$$

$$\underline{5.116276} = \text{logarithm of } 130700.0$$

$$8 =$$

$$\underline{2.4}$$

$$\text{Ans. } 130702.4$$

(4)

$$\text{Logarithm of } 23 = 1.361728$$

$$\text{" } 189 = 2.276462$$

$$\text{" } 2.748 = \underline{0.439017}$$

$$\text{Sum} = \underline{4.077207}$$

$$\text{Logarithm of } 9835267 = 6.992786$$

$$\underline{4.077207}$$

$$\text{Difference} = 2.915579$$

$$\underline{2.915558} = \text{logarithm of } 823.300$$

$$21 =$$

$$\underline{39}$$

$$\text{Ans. } 823.339$$

EXERCISE 153.—Page 326.

(1)

$$\text{Logarithm of } 5 = 0.698970.$$

$$\text{Then } 0.698970 \times 5 = 3.494850 = \text{logarithm of } 3125.$$

(2)

Logarithm of 1.073 = .030600.

Then $\cdot 030600 \times 6 = \cdot 183600 = \text{logarithm of } 1\cdot 5261.$

(3)

Logarithm of .0279 = $\bar{2}\cdot 445604.$ Then $\bar{2}\cdot 445604 \times 4 = \bar{7}\cdot 782416 = \text{logarithm of } \cdot 00000060592.$

(4)

Logarithm of 1.111 = .045714.

Then $\cdot 045714 \times 11 = \cdot 502854 = \text{logarithm of } 3\cdot 1831.$

EXERCISE 154.—Page 327.

(1)

Logarithm of 913426000 = 8.960673.

 $8\cdot 960673 \div 7 = 1\cdot 2800961 = \text{logarithm of } 19\cdot 0588.$

(2)

Logarithm of 1.61342 = .207747.

 $\cdot 207747 \div 11 = \cdot 01888609 = \text{logarithm of } 1\cdot 0444.$

(3)

Logarithm of .000007139 = $\bar{6}\cdot 853637 = \bar{10} + 4\cdot 853637.$ $(\bar{10} + 4\cdot 853637) \div 5 = \bar{2}\cdot 970727 = \text{logarithm of } \cdot 0934817.$

(4)

Logarithm of .002147 = $\bar{3}\cdot 331832 = \bar{7} + 4\cdot 331832.$ $(\bar{7} + 4\cdot 331832) \div 7 = \bar{1}\cdot 6188331 = \text{logarithm of } \cdot 41575.$

EXERCISE 155.—Page 328.

(1)

$$14000 = 7 \times 2 \times 1000 \therefore \log. 14000 = (\log. 7) + (\log. 2) + (\log. 1000).$$

$$\text{Log. } 7 = 0.845098$$

$$\text{Log. } 2 = 0.301030$$

$$\text{Log. } 1000 = 3$$

$$\text{Sum,} \quad \underline{\hspace{1.5cm}} = 4.146128 = \log. 14000$$

$$4.9 = 7^2 \div 10 \therefore \log. 4.9 = (\log. 7) \times 2 - (\log. 10).$$

$$\text{Log. } 7 = 0.845098 \times 2 = 1.690196$$

$$\text{Log. } 10 = \quad \quad \quad \underline{1}$$

$$\text{Difference} = .690196 = \log. 4.9$$

$$.00196 = 49 \times 4 \div 100000 = 7^2 \times 2^2 \div 100000$$

$$\therefore \log .00196 = (\log. 7) \times 2 + (\log. 2) \times 2 - (\log. 100000).$$

$$\text{Log. } 7 = 0.845098 \times 2 = 1.690196$$

$$\text{Log. } 2 = 0.301030 \times 2 = 0.602060$$

$$\text{Sum} = \underline{2.292256}$$

$$\text{Log. of } 100000 = 5 \text{ and } 2.292256 - 5 = \bar{3}.292256 = \log \text{ of } .00196.$$

$$\text{Since } 5 = 10 \div 2, \text{ the logarithm of } 5 = \log. 10 - \log. 2 = 1 - 0.301030 = 0.698970.$$

$$1750 = 5^2 \times 7 \times 10 \therefore \log. 1750 = (\log. 5) \times 2 + (\log. 7) + (\log. 10)$$

$$\text{Log. } 5 = 0.698970 \times 2 = 1.397940$$

$$\text{Log. } 7 = \quad \quad \quad .845098$$

$$\text{Log. } 10 = \quad \quad \quad \underline{1}$$

$$\text{Sum,} = 3.243038 = \log. \text{ of } 1750.$$

$$1428.571428 = \frac{1}{7} \times 10000 \therefore \log. 1428.571428 = (\log. \frac{1}{7}) + \log. 10000.$$

(1 continued.)

$$\text{Log. } \frac{1}{7} = (\text{log. } 1) - (\text{log. } 7) = 0 - 0.845098 = \overline{1.154902}$$

$$\text{Log. } 10000 = 4$$

$$\therefore \text{log. of } 1428.571428 = \text{sum} = \overline{3.154902}$$

$$\cdot 00000112 = 2^4 \times 7 \div 100000000 \therefore \text{log. } \cdot 00000112 =$$

$$(\text{log. } 2) \times 4 + (\text{log. } 7) - (\text{log. } 100000000).$$

$$\text{Log. } 2 = 0.301030 \times 4 = 1.204120$$

$$\text{Log. } 7 = 0.845098 = 0.845098$$

$$\text{Sum} = 2.049218 = \text{and log. } 100000000 = 8$$

$$2.049218 - 8 = \overline{6.049218} = \text{log. } \cdot 00000112$$

$$3.0625 = \frac{1}{8} \therefore \text{log. } 3.0625 = (\text{log. } 49) - (\text{log. } 16) =$$

$$(\text{log. } 7) \times 2 - (\text{log. } 2) \times 4.$$

$$\text{Log. } 7 = 0.845098 \times 2 = 1.690196$$

$$\text{Log. } 2 = 0.301030 \times 4 = 1.204120$$

$$\text{Difference} = 0.486076 = \text{log. of } 3.0625.$$

(2)

$$49\frac{1}{2} = \frac{99}{2} = 3^2 \times 11 \times \frac{1}{2} \therefore \text{log. } 49\frac{1}{2} = (\text{log. } 3) \times 2 + (\text{log. } 11)$$

$$+ (\text{log. } \frac{1}{2}).$$

$$\text{Log. } 3 = 0.477121 \times 2 = 0.954242$$

$$\text{Log. } 11 = 1.041393$$

$$\text{Log. } \frac{1}{2} = \overline{1.698970}$$

$$\text{Sum} = 1.694605 = \text{log. of } 49\frac{1}{2}.$$

$$363 = 11^2 \times 3 \therefore \text{log. } 363 = (\text{log. } 11) \times 2 + (\text{log. } 3).$$

$$\text{Log. } 11 = 1.041393 \times 2 = 2.082786$$

$$\text{Log. } 3 = 0.477121$$

$$\text{Sum} = 2.559907 = \text{log. of } 363.$$

Log. $\cdot 5$ or $\frac{1}{2} = 1.698970$, and by altering the characteristic we get 0.698970 for log. of 5 .

(Continued on next page.)

(2 continued.)

$$4.09 = 4_{11} = \frac{44}{11} = 3^2 \times 5 \div 11 \therefore \log. 4.09 = (\log. 3) \times 2 + (\log. 5) - (\log. 11).$$

$$\begin{array}{r} \text{Log. 3} = .477121 \times 2 = 0.954242 \\ \text{Log. 5} \qquad \qquad \qquad = .698970 \end{array}$$

$$1.653212$$

$$\text{Log. 11} = 1.041393 \text{ and } 1.653212 - 1.041393 = 0.611819 = \text{log. of } 4.09.$$

$$2.4 = 2_{\frac{4}{9}} = \frac{2^2}{9} = 11 \times 2 \div 9 \therefore \log. 2.4 = (\log. 11) + (\log. 2) - (\log. 3) \times 2.$$

$$\text{Log. 2} = (\log. 10) - (\log. 5) = 1 - 0.698970 = 0.301030.$$

$$\text{Log. 11} = 1.041393$$

$$\text{Log. 2} = 0.301030$$

$$1.342423$$

$$\text{Log. 3} = 0.477121 \times 2 = 0.954242 \text{ and } 1.342423 - 0.954242 = 0.388181 = \text{log. of } 2.4.$$

$$392.72 = 392_{11} = \frac{4320}{11} = 2^4 \times 3^3 \times 10 \div 11 \therefore \log. 392.72 = (\log. 2) \times 4 + (\log. 3) \times 3 + (\log. 10) - (\log. 11).$$

$$\text{Log. 2} = 0.301030 \times 4 = 1.204120$$

$$\text{Log. 3} = 0.477121 \times 3 = 1.431363$$

$$\text{Log. 10} \qquad \qquad \qquad = 1$$

$$\text{Sum} = 3.635483$$

$$\text{Log. 11} = 1.041393 \text{ and } 3.635483 - 1.041393 = 2.594090 = \text{log. of } 392.72.$$

$$293333\frac{1}{3} = \frac{880000}{3} = 2^3 \times 11 \times 10000 \div 3 \therefore \log. 293333\frac{1}{3} = (\log. 2) \times 3 + (\log. 11) + (\log. 10000) - (\log. 3).$$

$$\text{Log. 2} = 0.301030 \times 3 = 0.903090$$

$$\text{Log. 11} \qquad \qquad \qquad = 1.041393$$

$$\text{Log. 10000} \qquad \qquad \qquad = 4$$

$$\text{Sum} = 5.944483$$

(Continued on next page.)

(2 continued.)

$$\text{Log. } 3 = 0.477121 \text{ and } 5.944483 - 0.477121 = 5.467362 = \text{log. of } 293333\frac{1}{3}$$

$$19.965 = 11^3 \times 5 \times 3 \div 1000 \therefore \text{log. } 19.965 = (\text{log. } 11) \times 3 + (\text{log. } 5) + (\text{log. } 3) - (\text{log. } 1000)$$

$$\text{Log. } 11 = 1.041393 \times 3 = 3.124179$$

$$\text{Log. } 5 = 0.698970$$

$$\text{Log. } 3 = 0.477121$$

$$\text{Sum} = 4.300270$$

$$\text{Log. } 1000 = 3 \text{ and } 4.300270 - 3 = 1.300270 = \text{log. of } 19.965$$

$$= (\text{log. } 3) \times 2$$

$$\begin{array}{r} 242 \\ 970 \\ \hline \end{array}$$

$$212$$

$$= 0.611819 =$$

$$\text{g. } 11) + (\text{log.}$$

$$= 0.301030$$

EXERCISE 156—Page 336.

(1)

Here we have given the first term 4, the number of terms 17 and the sum of the series 884, to find l , the last term.

$$\text{Then } l = \frac{2r}{n} - a = \frac{884 \times 2}{17} - 4 = 104 - 4 = 100$$

(2)

Here we have given the first term 21, the last term 497 and the number of terms 41, to find the common difference.

$$\text{Then } d = \frac{l - a}{n - 1} = \frac{497 - 21}{41 - 1} = \frac{476}{40} = \frac{119}{10} = 11\frac{9}{10}$$

(3)

Here we have given a , l , and d , to find n , and since $a = 12$, $l = 96$, and $d = 6$, we have

$$n = \frac{l - a}{d} + 1 = \frac{96 - 12}{6} + 1 = 14 + 1 = 15$$

(4)

Here we have given l , d , and s , to find n , and since $l = 14$, $d = 1$, and $s = 105$, we have

$$n = \frac{2l + d}{2d} + \sqrt{\left(\frac{2l + d}{2d}\right)^2 - \frac{2s}{d}} = \frac{2 \times 14 + 1}{2 \times 1} + \sqrt{\left(\frac{2 \times 14 + 1}{2 \times 1}\right)^2 - \frac{2 \times 105}{1}} = 14\frac{1}{2} + \sqrt{\left(\frac{29}{2}\right)^2 - 210} = 14\frac{1}{2} + \sqrt{\frac{841}{4} - 210} = 14\frac{1}{2} + \sqrt{\frac{841 - 840}{4}} = 14\frac{1}{2} + \sqrt{\frac{1}{4}} = 14\frac{1}{2} + \frac{1}{2} = 15.$$

(5)

Here we have given a , d , and s , to find l , and since $a = \frac{2}{3}$, $d = \frac{2}{3}$, and $s = 1180$, we have

$$l = -\frac{1}{2}d + \sqrt{2ds + \left(a - \frac{1}{2}d\right)^2} = -\frac{1}{2} \text{ of } \frac{2}{3} + \sqrt{2 \times \frac{2}{3} \times 1180 + \left(\frac{2}{3} - \frac{1}{2} \times \frac{2}{3}\right)^2} = -\frac{1}{3} + \sqrt{\frac{47320}{3} + \left(\frac{1}{3}\right)^2} = -\frac{1}{3} + \sqrt{\frac{47320}{3} + \frac{1}{9}} = -\frac{1}{3} + \sqrt{\frac{141961}{9}} = -\frac{1}{3} + \frac{119}{3} = \frac{118}{3} = 39\frac{2}{3}.$$

(6)

Here we have given a , l , and s , to find d , and since $a = 8$, $l = 170$, and $s = 4895$, we have

$$d = \frac{(l + a)(l - a)}{2s - l - a} = \frac{(170 + 8)(170 - 8)}{2 \times 4895 - 170 - 8} = \frac{178 \times 162}{9790 - 178} = \frac{28836}{9612} = 3.$$

(7)

Here we have given a , l , and d , to find n , and since $a = 5$, $l = 27\frac{1}{2}$, and $d = 2\frac{1}{4}$, we have

$$n = \frac{l - a}{d} + 1 = \frac{27\frac{1}{2} - 5}{2\frac{1}{4}} + 1 = \frac{22\frac{1}{2}}{2\frac{1}{4}} + 1 = \frac{\frac{45}{2}}{\frac{5}{2}} + 1 = 10 + 1 = 11.$$

(8)

Here we have given a , l , and n , to find s , and since $a = 2$, $l = 478$, and $n = 86$, we have

$$s = (a + l) \frac{n}{2} = (2 + 478) \frac{86}{2} = 480 \times 43 = 20640.$$

(9)

Here we have given a , l , and d , to find s , and since $a = 2$, $l = 998$, and $d = 6$, we have

$$s = \frac{(l+a)(l-a)}{2d} + \frac{l+a}{2} = \frac{(998+2)(998-2)}{2 \times 6} + \frac{998+2}{2} = \frac{1000 \times 996}{12} + \frac{1000}{2} = 83000 + 500 = 83500.$$

(10)

Here we have given a , n , and d , to find l , and since $a = 5$, $n = 11$, and $d = 2\frac{1}{4}$, we have

$$l = a + (n-1)d = 5 + (11-1)2\frac{1}{4} = 5 + (10 \times 2\frac{1}{4}) = 5 + 4\frac{5}{2} = 5\frac{5}{2} = 27\frac{1}{4}.$$

(11)

Here we have given l , d , and n , to find s , and since $l = 199$, $d = 11$, and $n = 19$, we have

$$s = \{2l - (n-1)d\} \frac{n}{2} = \{2 \times 199 - (19-1)11\} \frac{19}{2} = \{398 - (18 \times 11)\} \frac{19}{2} = 200 \times \frac{19}{2} = 1900.$$

(12)

Here we have given s , a , and l , to find n , and since $s = 39840$, $a = 2$, and $l = 478$, we have

$$n = \frac{2s}{l+a} = \frac{2 \times 39840}{478+2} = \frac{79680}{480} = 166.$$

(13)

Here we have given s , l , and a , to find d , and since $s = 83500$, $l = 998$, and $a = 2$, we have

$$l = \frac{(l+a)(l-a)}{2s-l-a} = \frac{(998+2)(998-2)}{(2 \times 83500) - 998 - 2} = \frac{1000 \times 996}{167000 - 1000} = \frac{996000}{166000} = 6$$

(14)

Here we have given s , a , and d , to find n , and since $s = 360$, $a = 2$, and $d = 2$, we have

$$n = \frac{d-2a}{2d} + \sqrt{\frac{2s}{d} + \left(\frac{2a-d}{2d}\right)^2} = \frac{2-(2 \times 2)}{2 \times 2} + \sqrt{\frac{2 \times 360}{2} + \left(\frac{(2 \times 2) - 2}{2 \times 2}\right)^2} = -\frac{1}{2} + \sqrt{360 + \left(\frac{1}{2}\right)^2} = -\frac{1}{2} + \sqrt{360\frac{1}{4}} = -\frac{1}{2} + 16.13226 = 15.63226 \text{ days} = 15 \text{ days, } 15 \text{ hours, } 10 \text{ minutes, } 27.264 \text{ seconds.}$$

(15)

Here we have given s , a , and d , to find l , and since $s = 83500$, $a = 2$, and $d = 6$, we have

$$l = \frac{-\frac{1}{2}d + \sqrt{2ds + (a - \frac{1}{2}d)^2}}{2} = \frac{-\frac{1}{2} \times 6 + \sqrt{2 \times 6 \times 83500 + (2 - \frac{1}{2} \times 6)^2}}{2} = \frac{-3 + \sqrt{1002000 + (2-3)^2}}{2} = \frac{-3 + \sqrt{1002001}}{2} = \frac{-3 + 1001}{2} = 998.$$

(16)

Here we have given s , n , and l , to find a , and since $s = \$1125$, $n = 18$, and $l = 120$, we have

$$a = \frac{2s}{n} - l = \frac{2 \times 1125}{18} - 120 = 125 - 120 = 5.$$

(17)

Here we have given a , l , and n , to find d , and since $a = 5$, $l = 27\frac{1}{2}$, and $n = 11$ we have

$$d = \frac{l - a}{n - 1} = \frac{27\frac{1}{2} - 5}{11 - 1} = \frac{22\frac{1}{2}}{10} = 2\frac{1}{4}.$$

(18)

Here we have a , d , and n given, to find s , and since to deposit one stone he must walk 5 yards, and the distance travelled for each succeeding stone is 5 yards, therefore $a = 5$, $d = 5$, and $n = 220$.

$$\begin{aligned} \text{Then } s &= \left\{ 2a + (n - 1)d \right\} \frac{n}{2} = \left\{ 2 \times 5 + (220 - 1)5 \right\} \frac{220}{2} \\ &= \left\{ 10 + (219 \times 5) \right\} 110 = \\ &1105 \times 110 = 121550 \text{ yards} = 69\frac{1}{8} \text{ miles.} \end{aligned}$$

(19)

Here we have s , n , and l given, to find a , and since $s = 39840$, $n = 166$, and $l = 478$, we have

$$a = \frac{2s}{n} - l = \frac{2 \times 39840}{166} - 478 = 480 - 478 = 2.$$

(20)

Here we have n , a , and d given, to find s , and since $n = 12$, $a = 4$, and $d = 2$, we have

$$s = \left\{ 2a + (n - 1)d \right\} \frac{n}{2} = \left\{ 2 \times 4 + (12 - 1)2 \right\} \frac{12}{2} = \left\{ 8 + (11 \times 2) \right\} 6 = 30 \times 6 = 180.$$

(21)

Here we have given a , l , and n , to find s , and $a = 1$, $l = 24$, and $n = 24$.

$$\text{Then } s = (a + l) \frac{n}{2} = (1 + 24) \frac{24}{2} = 25 \times 12 = 300,$$

and since $s = 83500$

$$\frac{1000 \times 996}{167000 - 1000} =$$

and since $s = 360$,

$$\frac{2 - (2 \times 2)}{2 \times 2} +$$

$$60 + \left(\frac{1}{2}\right)^2 =$$

days = 15 days,
ds.

since $s = 83500$,

$$\frac{6 +}{0000 + (2 - 3)^2} = 998.$$

since $s = \$1125$,

$$120 = 5.$$

EXERCISE 157—Page 342.

(1)

Here $n = 11$, $a = £1024$, and $r = 1\frac{1}{2}$.Then $l = ar^n - 1 = 1024 \times (\frac{3}{2})^{10} = 1024 \times \frac{59049}{1024} = £59049$

$$s = \frac{rl - a}{r - 1} = \frac{\frac{3}{2} \times 59049 - 1024}{\frac{3}{2} - 1} = \frac{177147 - 1024}{\frac{1}{2}} = \frac{176123}{\frac{1}{2}} =$$

£175099 = whole fortune.

(2)

Here $a = 7$, $l = 1240029$ and $s = 1860040$.

$$\text{Then } r = \frac{s - a}{s - l} = \frac{1860040 - 7}{1860040 - 1240029} = \frac{1860033}{620011} = 3.$$

(3)

Here $n = 12$, $a = £1$, and $l = £2048$.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{2048}{1}\right)^{\frac{1}{12-1}} = \sqrt[11]{2048} = 2.$$

$$s = \frac{rl - a}{r - 1} = \frac{(2 \times 2048) - 1}{2 - 1} = 4096 - 1 = £4095.$$

(4)

Here $r = \frac{3}{2}$, $n = 8$, and $l = 106\frac{2}{3}$.

$$\text{Then } s = \frac{l(r^n - 1)}{(r - 1)r^{n-1}} = \frac{106\frac{2}{3} \times [(\frac{3}{2})^8 - 1]}{(\frac{3}{2} - 1)(\frac{3}{2})^7} = \frac{4197\frac{1}{2} \times 6305}{\frac{1}{2} \times \frac{2187}{128}} =$$

$$\frac{25 \times 6305}{1} = 307\frac{1}{2}.$$

(5)

Here $a = 1$, $n = 7$, and $r = 3$.

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (3^7 - 1)}{3 - 1} = \frac{2187 - 1}{2} = 1093.$$

(6)

Here $a = 1$, $l = 10077696$, and $n = 10$.

$$\text{Then } s = \frac{l^{\frac{n}{r}} - a^{\frac{n}{r}}}{l^{\frac{1}{r}} - a^{\frac{1}{r}}} = \frac{(10077696)^{\frac{10}{6}} - 1}{(10077696)^{\frac{1}{6}} - 1} = \frac{\sqrt[6]{(10077696)^{10}} - 1}{\sqrt[6]{10077696} - 1} = \frac{\sqrt[6]{(216)^{10}} - 1}{\sqrt[6]{216} - 1} = \frac{6^{10} - 1}{6 - 1} = \frac{60466176 - 1}{5} = 12093235.$$

(7)

Here $a = 6$, $l = 3072$, and $s = 6138$.

$$\text{Then } r = \frac{s - a}{s - l} = \frac{6138 - 6}{6138 - 3072} = \frac{6132}{3066} = 2.$$

(8)

Here $r = 2$, $n = 11$, and $s = 20470$.

$$\text{Then } l = \frac{(r-1)sr^{n-1}}{r^n - 1} = \frac{(2-1) \times 20470 \times 2^{10}}{2^{11} - 1} = \frac{20470 \times 1024}{2047} = 10240.$$

(9)

Here $a = 1s.$, $n = 12$, and $r = 2$.

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (2^{12} - 1)}{2 - 1} = 4095 = 4095s. \\ = \text{£}204 \text{ } 15s.$$

(10)

Here $a = 1$ farthing, $r = 2$ and $n = 32$.

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (2^{32} - 1)}{2 - 1} = 4294967295 \text{ far.} = \\ \text{£}4473924 \text{ } 5s. \text{ } 3\frac{1}{2}d.$$

(11)

Here $a = 4$, $l = 78732$, and $n = 10$.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{78732}{4}\right)^{\frac{1}{10-1}} = \sqrt[9]{19683} = 3.$$

(12)

Here $a = 5$, $r = 2$, and $n = 7$.

$$\text{Then } l = ar^{n-1} = 5 \times 2^{7-1} = 5 \times 2^6 = 5 \times 64 = 320.$$

(13)

Here $a = 5$, $l = 327680$, and $r = 4$.

$$\text{Then } s = \frac{rl - a}{r - 1} = \frac{(327680 \times 4) - 5}{4 - 1} = 1310715 = 436905.$$

(14)

Here $a = 1$, $r = 2$, and $n = 64$.

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (2^{64} - 1)}{2 - 1} = 18446744073709551615 \text{ gr.}$$

$$18446744073709551615 \div (7680 \times 64) = 37529996894754 \text{ bush.}$$

$$\$1.50 \times 37529996894754 = \$56294995342131$$

(15)

Here $r = 3$, $n = 10$, and $s = 295240$.

$$\text{Then } l = \frac{(r-1)sr^{n-1}}{r^n - 1} = \frac{(3-1) \times 295240 \times 3^9}{3^{10} - 1} = \frac{2 \times 295240 \times 19683}{59048} = 196830.$$

(16)

Here $a = 1$, $l = 2048$, and $n = 12$.

$$\text{Then } s = \frac{l \frac{n}{n-1} - a \frac{n}{n-1}}{l \frac{1}{n-1} - a \frac{1}{n-1}} = \frac{2048 \frac{12}{12-1} - 1 \frac{12}{12-1}}{2048 \frac{1}{12-1} - 1 \frac{1}{12-1}} =$$

$$\frac{\sqrt[11]{(2048)^{12} - 1}}{\sqrt[11]{2048 - 1}} = \frac{2^{12} - 1}{2 - 1} = 2^{12} - 1 = 4095.$$

17)

Here $a = 5$, $r = 4$, and $n = 9$.

$$\text{Then } l = ar^{n-1} = 5 \times 4^{9-1} = 5 \times 4^8 = 5 \times 65536 = 327680.$$

P

EXERCISE 156.—Page 344.

(1)

Here $a = \frac{2}{7}$, and $r = \frac{3}{8}$.

$$\text{Then } s = \frac{a}{1-r} = \frac{\frac{2}{7}}{1-\frac{3}{8}} = \frac{\frac{2}{7}}{\frac{5}{8}} = \frac{2}{7} \times \frac{8}{5} = \frac{16}{35}.$$

(2)

Here $a = 4$, and $r = \frac{1}{2}$,

$$\text{Then } s = \frac{a}{1-r} = \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = 4 \times 2 = 8.$$

(3)

Here $a = \frac{72}{100}$, and $r = \frac{1}{100}$.

$$\text{Then } s = \frac{a}{1-r} = \frac{\frac{72}{100}}{1-\frac{1}{100}} = \frac{\frac{72}{100}}{\frac{99}{100}} = \frac{72}{99} = \frac{8}{11}.$$

(4)

Here $a = \frac{1234}{10000}$, and $r = \frac{1}{10000}$.

$$\text{Then } s = \frac{a}{1-r} = \frac{\frac{1234}{10000}}{1-\frac{1}{10000}} = \frac{\frac{1234}{10000}}{\frac{9999}{10000}} = \frac{1234}{9999} = \frac{1234}{9999}.$$

EXERCISE 159.—Page 345.

(1)

Since there are 9 means and 2 extremes the number of terms is 11.

$$\text{Then } d = \frac{l-a}{n-1} = \frac{92-2}{11-1} = \frac{90}{10} = 9.$$

1st term = 2; 2nd = 2 + 9 = 11; 3rd = 11 + 9 = 20; 4th = 20 + 9 = 29; 5th = 29 + 9 = 38; 6th = 38 + 9 = 47; and so on.

And series is 2, 11, 20, 29, 38, 47, 56, 65, 74, 83, 92.

(2)

Since there are 4 means and 2 extremes the number of terms is 6.

$$\text{Then } d = \frac{l-a}{n-1} = \frac{50-7}{6-1} = \frac{43}{5} = 8\frac{3}{5}.$$

1st term = 7; 2nd = 7 + 8 $\frac{3}{5}$ = 15 $\frac{3}{5}$; 3rd = 15 $\frac{3}{5}$ + 8 $\frac{3}{5}$ = 24 $\frac{1}{5}$; 4th = 24 $\frac{1}{5}$ + 8 $\frac{3}{5}$ = 32 $\frac{4}{5}$; 5th = 32 $\frac{4}{5}$ + 8 $\frac{3}{5}$ = 41 $\frac{2}{5}$; and 6th = 41 $\frac{2}{5}$ + 8 $\frac{3}{5}$ = 50.

And series is 7, 15 $\frac{3}{5}$, 24 $\frac{1}{5}$, 32 $\frac{4}{5}$, 41 $\frac{2}{5}$, 50.

(3)

Since there are 8 means and 2 extremes the number of terms is 10.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{4096}{4096}\right)^{\frac{1}{10-1}} = \left(\frac{1}{1}\right)^{\frac{1}{9}} = 1.$$

1st term = 4096; 2nd = 4096 × $\frac{1}{2}$ = 2048; 3rd = 2048 × $\frac{1}{2}$ = 1024; 4th = 1024 × $\frac{1}{2}$ = 512; 5th = 512 × $\frac{1}{2}$ = 256, and so on.

And the means are 2048, 1024, 512, 256, 128, 64, 32, and 16.

(4)

Since there are 7 means and 2 extremes the number of terms is 9.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = (23514624)^{\frac{1}{8}} = (1679616)^{\frac{1}{8}} = 6.$$

1st term = 14; 2nd = $14 \times 6 = 84$; 3rd = $84 \times 6 = 504$;
4th = $504 \times 6 = 3024$; 5th = $3024 \times 6 = 18144$; and so on.

And the means are 84, 504, 3024, 18144, 108864, 653184, and 3919104.

EXERCISE 160.—Page 347

(1)

Assume 4 to be the number of men.

Then $2 \times 4 = 8 =$ number of women.

And $8 \times 3 = 24 =$ number of children.

$6d. \times 4 = 24d. =$ amount received by the men.

$4d. \times 8 = 32d. =$ " " " women.

$2d. \times 24 = 48d. =$ " " " children.

Sum, = 104d., but it should, by question, = 78d.

$$\text{Then } 104 : 78 :: 4 : \frac{78 \times 4}{104} = 3 = \text{number of men.}$$

$3 \times 2 = 6 =$ number of women, and $6 \times 3 = 18 =$ number of children.
[ren.]

(2)

Assume £8 to be the price of the harness.

Then $£8 \times 2 = 16 =$ price of horse.

And $£8 + £16 = £24 \times 2 = 48 =$ " chaise.

Sum, = £ 72, but it should by question = £60.

$$\text{Then } £72 : £60 :: £8 : \frac{8 \times 60}{72} = £6 \ 13 \ 4 = \text{price of harness.}$$

$£6 \ 13 \ 4 \times 2 = 13 \ 6 \ 8 =$ " horse.

$£6 \ 13 \ 4 + £13 \ 6 \ 8 = £20 \times 2 = 40 \ 0 \ 0 =$ " chaise.

(3)

Assume 20 as C's age.

Then $20 \times 3 = 60 =$ B's age.And $60 \times 2 = 120 =$ A's age.

Sum = 200, but by question it should = 140.

 20×140 Then $200 : 140 :: 20 : \frac{20 \times 140}{200} = 14 =$ C's age. $14 \times 3 = 42 =$ B's age, and $42 \times 2 = 84 =$ A's age.

(4)

Assume 100.

One fourth of 100 = 25 and remainder = $100 - 25 = 75$.One fifth of 75 = 15 and remainder = $75 - 15 = 60$, but it should by the question = 72.Then $60 : 72 :: 100 : \frac{100 \times 72}{60} = 120$.

(5)

A can do the work in 7 days \therefore he will do $\frac{1}{7}$ of it in 1 day.B " " 5 " \therefore " $\frac{1}{5}$ " "C " " 6 " \therefore " $\frac{1}{6}$ " "Then all working together will do $\frac{1}{7} + \frac{1}{5} + \frac{1}{6} = \frac{17}{105}$ in 1 day.Therefore to do the whole work it will take them $\frac{1}{\frac{17}{105}} = \frac{105}{17} =$ $1\frac{10}{17}$ days.

(6)*

A and B working together can do it in 10 days \therefore they will do $\frac{1}{10}$ of it in 1 day.A can do it in 15 days \therefore he will do $\frac{1}{15}$ of it in 1 day.Therefore $\frac{1}{10} - \frac{1}{15} = \frac{1}{30} =$ amount done by B in 1 day.Then if he does $\frac{1}{30}$ in 1 day, it will take him 30 days to do the whole.

*The mode of working these questions by position is so simple that they cannot trouble any one; it has therefore been thought advisable to work them by simple analysis.

(7)*

The first pipe empties the whole of it in 1 hour.

The second pipe empties $\frac{1}{2}$ of it in 1 hour.

The third pipe empties $\frac{1}{3}$ of it in 1 hour.

Then all these pipes running together will empty $1 + \frac{1}{2} + \frac{1}{3}$
 $= \frac{11}{6}$ in 1 hour.

Therefore to empty the cistern it will take $1 \div \frac{11}{6} = \frac{6}{11}$ hours.

(8)

Assume 84

One third of 84 = 28

One sixth of 84 = 14

One seventh of 84 = 12

—

Sum = 54, but by question it should = 27.

Then $54 : 27 :: 84 : \frac{84 \times 27}{54} = 42$.

(9)

All 5 mills working together will grind $7 + 5 + 4 + 3 + 1$
 $= 20$ bushels in 1 hour.

Therefore to grind 500 bushels it will take them $500 \div 20 =$
 25 hours.

(10)*

One pipe fills $\frac{1}{2}$ of the cistern in 1 hour, and the other
 empties $\frac{1}{8}$ of it in 1 hour.

Then $\frac{1}{2} - \frac{1}{8} = \frac{3}{8} =$ part of the cistern filled in 1 hour
 when both are left open.

And if $\frac{3}{8}$ of it is filled in 1 hour, the whole will be filled in

$\frac{1}{\frac{3}{8}} = 36$ hours.

$\frac{1}{36}$

* See note on page 227.

EXERCISE 161.—Page 352.

(1)

Assume 60 for father's age, then 15 = son's.

| | |
|------|----|
| 5 | 5 |
| — | — |
| 5)55 | 10 |
| — | — |
| 11 | |
| 10 | |
| — | |
| — 1 | |

Assume 100 for father's age, then 25 = son's.

| | |
|------|----|
| 5 | 5 |
| — | — |
| 5)95 | 20 |
| — | — |
| 19 | |
| 20 | |
| — | |
| + 1 | |

Errors. Assumed numbers.

| | | | | |
|-----|---|-----|---|-----|
| — 1 | × | 100 | = | 100 |
| + 1 | × | 60 | = | 60 |
| — | | | | — |

Sum of errors = 2 Sum of products = 160

Therefore result required = $160 \div 2 = 80 =$ father's age, and
 $\frac{1}{2}$ of 80 = 20 = son's age.

(2)

Assume 80

Assume 44

| | |
|--------------------------|--------------------------|
| 34 | 34 |
| — | — |
| 46 | 10 |
| 3 | 3 |
| — | — |
| 138 | 30 |
| 80 | 44 |
| — | — |
| 58 | — 14 |
| $\frac{1}{2}$ of 80 = 20 | $\frac{1}{2}$ of 44 = 11 |
| — | — |
| + 38 | — 25 |

(Continued on next page,)

(2 continued.)

| Errors. | Assumed numbers. | | |
|--------------------|------------------|---|------|
| - 25 | × 80 | = | 2000 |
| + 38 | × 44 | = | 1672 |
| Sum of errors = 63 | | | 3672 |

Therefore result required = $3672 \div 63 = 58\frac{2}{3}$.

(4)

| | |
|---------------------------------|-------------------|
| Assume 18 | and 7 |
| One half of 18 = $\frac{9}{14}$ | $2 \times 7 = 14$ |
| - 5 | |

| | |
|---------------------------------|------------------|
| Assume 22 | and 3 |
| One half of 22 = $\frac{11}{6}$ | $2 \times 3 = 6$ |
| + 5 | |

| Errors. | Assumed numbers. | | |
|---------|------------------|---|-----|
| - 5 | × 22 | = | 110 |
| + 5 | × 18 | = | 90 |

Sum of errors = 10 Sum of products = 200

Then $200 \div 10 = 20 =$ one number, and $25 - 20 = 5$
= other number.

(5)

| A. | B. |
|-----------|-----|
| Suppose 8 | 6 |
| 22½ | 9 |
| 180 | 12 |
| 132 | 15 |
| 8)48 | 18 |
| 21 | 21 |
| + 6 | 24 |
| 6 | 27 |
| 36 | 132 |
| 72 | |
| 3)36 | |
| 12 | |

$9 - 6 = 3 =$ difference of errors.

(7)

Assume 30.

$$\begin{aligned} \frac{1}{2} \text{ of } 30 &= 15; \quad \frac{1}{4} \text{ of } 30 = 7\frac{1}{2}; \\ \frac{1}{6} \text{ of } 30 &= 5; \quad \text{and } \frac{1}{8} \text{ of } 30 = 3\frac{3}{8}; \\ 15 \times 7\frac{1}{2} \times 6 \times 5 &= 3375; \\ 3375 - 6998\frac{3}{8} &= -3623\frac{1}{4} = \text{error.} \end{aligned}$$

Assume 60.

$$\begin{aligned} \frac{1}{2} \text{ of } 60 &= 30; \quad \frac{1}{4} \text{ of } 60 = 15; \\ \frac{1}{6} \text{ of } 60 &= 10; \quad \text{and } \frac{1}{8} \text{ of } 60 = 7\frac{1}{2}. \\ 30 \times 15 \times 12 \times 10 &= 54000. \\ 54000 - 6998\frac{3}{8} &= +47001\frac{1}{8} = \text{error.} \\ 30^4 &= 810000, \quad \text{and } 60^4 = 12960000 \\ -3623\frac{1}{4} \times 12960000 &= 46959264000 \\ +47001\frac{1}{8} \times 810000 &= 38071296000 \end{aligned}$$

$$\begin{array}{r} \text{Sum} = 50625 \qquad \text{Sum} = 85030560000 \\ 85030560000 \div 50625 = 1679616 \end{array}$$

4th root = square root of square root.

$$\sqrt{1679616} = 1296, \quad \text{and } \sqrt{1296} = 36 = \text{required number.}$$

NOTE.—For reason why we multiply by the 4th powers of the assumed numbers and then take the 4th root of the quotient, see Arith. page 353, Example 11.

It may, however, perhaps be clearer from the following illustration:

Let x = the number required.

$$\begin{array}{r} x \quad x \quad x \quad x \quad x^4 \\ \text{Then } \frac{x}{2} \times \frac{x}{4} \times \frac{x}{5} \times \frac{x}{6} = \frac{x^4}{240} = 6998\frac{3}{8} \\ \therefore x^4 = 1679616 \\ \therefore x = \sqrt[4]{1679616} = 36. \end{array}$$

(8)

Suppose A had 9s. at first.

Then $9 + 1 = 10$; $10 \div 2 = 5$; $5 + 1 = 6 =$ what B had at first.

$6 + 1 = 7$, but should = $9 - 1 = 8$.

$$\text{Error} = 7 - 8 = -1.$$

Suppose A had 11s. at first.

Then $11 + 1 = 12$; $12 \div 2 = 6$; $6 + 1 = 7 =$ what B had at first.

$7 + 1 = 8$, but should = $11 - 1 = 10$.

$$\text{Error} = 8 - 10 = -2.$$

(Continued on next page.)

(8 continued.)

Errors.

$$- 2 \times 9 = 18$$

$$- 1 \times 11 = 11$$

$$\text{Diff.} = 1 \quad \text{diff.} = 7$$

$$7 \div 1 = 7 = \text{shillings A had at first.}$$

$$7 + 1 = 8; 8 \div 2 = 4; 4 + 1 = 5 = \text{shillings B had at first}$$

(9)

Assume 24 and 6.

$$\frac{2^4}{2} + \frac{2^4}{3} + \frac{2^4}{6} = 24.$$

$$\frac{6}{2} + \frac{6}{3} \text{ of } 6 + \frac{6}{6} = 9.$$

$$24 - 9 = + 15 = \text{error.}$$

Assume 20 and 10.

$$\frac{2^0}{2} + \frac{2^0}{3} + \frac{2^0}{6} = 20.$$

$$\frac{10}{2} + \frac{10}{3} \text{ of } 10 + \frac{10}{6} = 15.$$

$$20 - 15 = + 5 = \text{error.}$$

Errors.

$$+ 15 \times 20 = 300$$

$$+ 5 \times 24 = 120$$

$$\text{Diff.} = 10 \quad \text{diff.} = 180$$

$$180 \div 10 = 18 = \text{one number.}$$

$$30 - 18 = 12 = \text{other number.}$$

(10)

Suppose 1st horse to be worth £20.

$$20 + 50 = 70; 70 \div 2 = \text{£}35 = \text{value of 2nd horse.}$$

$$35 + 50 = 85, \text{ but it should equal } 60, \text{ i. e. } (20 \times 3).$$

$$\text{Then } 60 - 85 = - 25 = \text{error.}$$

Suppose 1st horse to be worth £60.

$$\text{£}60 + \text{£}50 = \text{£}110; \text{£}110 \div 2 = \text{£}55 = \text{worth of 2nd horse.}$$

$$55 + 50 = 105, \text{ but it should equal } 180, \text{ i. e. } (60 \times 3).$$

$$180 - 105 = + 75 = \text{error.}$$

Errors.

$$+ 75 \times 20 = 1500$$

$$- 25 \times 60 = 1500$$

$$\text{Sum} = 100$$

$$\text{Sum} = 3000$$

$$3000 \div 100 = \text{£}30 = \text{value of 1st horse.}$$

$$\text{£}30 + \text{£}50 = \text{£}80; \text{£}80 \div 2 = \text{£}40 = \text{value of 2nd horse,}$$

(11)

Suppose there were 11 beggars.

$$11 \times 4 = 44; 44 + 6 = 50 = \text{number of pence he had.}$$

$$11 \times 6 = 66; 66 - 12 = 54 = \text{ " " " "}$$

$$54 - 50 = + 4 = \text{error.}$$

Suppose there were 12 beggars.

$$12 \times 4 = 48; 48 + 6 = 54 = \text{pence he had.}$$

$$12 \times 6 = 72; 72 \div 12 = 60 = \text{pence he had.}$$

$$60 - 54 = + 6 = \text{error.}$$

Errors.

$$+ 6 \times 11 = 66$$

$$+ 4 \times 12 = 48$$

$$\text{Diff.} = 2 \quad \text{diff.} = 18, \text{ and } 18 \div 2 = 9 = \text{number of beggars.}$$

EXERCISE 162.—Page 357.

(1)

Here $P = \$713.29$, $r = .045$, and $t = 14$.

$$\text{Then } A = P(1+r)^t, \text{ or } \log. A = \log. P + \log. (1+r) \times t$$

$$= 2.853267 + (.019116 \times 14) = 3.120891 = \log. \text{ of } Ans.$$

Hence amount = \$1320.96.

(2)

Here $n = 7$, $r = .015$.

$$\log. n. \quad .845098$$

$$\text{Then } t = \frac{\log. n.}{\log. (1+r)} = \frac{.845098}{.006466} = 130.698 \text{ payments, and}$$

$$\log. (1+r) \quad .006466$$

$$130.698 \div 4 = 32.674 \text{ years} = 32 \text{ years } 8 \text{ months } 2 \text{ days.}$$

(3)

Here $A = \$1111.11$, $P = 111.11$, and $r = .08$.

$$\log. A - \log. P \quad 3.045757 - 2.045753 \quad 1.000004$$

$$\text{Then } t = \frac{\log. A - \log. P}{\log. (1+r)} = \frac{1.000004}{.033424} =$$

$$\log. (1+r) \quad .033424 \quad .033424$$

$$= 29.918 \text{ years} = 29 \text{ years } 11 \text{ months.}$$

(4)

Here $A = \$3333.33$, $P = \$222.22$, and $t = 120$.

$$\text{Then } r = \sqrt[t]{\frac{A}{P}} - 1; \text{ or } \log. (1+r) = \frac{\log. A - \log. P}{t} =$$

$$\frac{3.522878 - 2.346783}{120} = \frac{1.176095}{120} = .0098007. \text{ Hence } 1+r$$

$$= 1.0228, r = .0228, \text{ and rate per cent.} = 2\frac{7}{10}\%$$

(5)

Here $n = 2$ and $r = .07$.

$$\text{Then } t = \frac{\log. n.}{\log. (1+r)} = \frac{0.301030}{0.029234} = 10.2446 \text{ years} = 10 \text{ yrs.}$$

2 months 28 days.

(6)

Here $A = \$100$, $r = .0225$, and $t = 28$.

$$\text{Then } P = \frac{A}{(1+r)^t}, \text{ or } \log. P = \log. A - \log. (1+r) \times t.$$

$$\text{Log. } P = 2 - (0.009664 \times 28) = 2 - 0.270592 = 1.729408.$$

Hence $P = \$53.63$.

(7)

Here $P = \$2468.13$, $r = .0375$, and $t = 26$.

$$\text{Then } A = P(1+r)^t, \text{ or } \log. A = \log. P + \log. (1+r) \times t.$$

$$\text{Log. } A = 3.392368 + (0.015988 \times 26) = 3.392368 + 0.415688$$

$$= 3.808056.$$

Hence $A = \$6427.705$.

(8)

Here $A = \$7137.40$, $r = .0425$, and $t = 22$.

$$\text{Then } P = \frac{A}{(1+r)^t}, \text{ or } \log. P = \log. A - \log. (1+r) \times t.$$

$$\text{Log. } P = 3.853540 - (0.018076 \times 22) = 3.853540 - 0.397672$$

$$= 3.455868.$$

Hence $P = \$2856.723$.

(16)

Here $n = 19$, and $r = .0525$.

$$\text{Then } t = \frac{\log. P}{\log. (1 + r)} = \frac{1.278754}{0.022222} = 57.5445 \text{ payments} = 28.7722 \text{ years} = 28 \text{ years } 9 \text{ months } 8 \text{ days.}$$

$$\frac{\log. P}{\log. (1 + r)} =$$

Hence $1 + r$

EXERCISE 163.—Page 360.

(1)

Here $r = .03$, $a = 500$, $A = 8365$.

ars = 10 yrs.

$$\sqrt{\left\{ \frac{8rA}{a} + (2-r)^2 \right\}} - (2-r)$$

$$\text{Formula IV. } t = \frac{\sqrt{\left\{ \frac{8rA}{a} + (2-r)^2 \right\}} - (2-r)}{2r}$$

$$= \frac{\sqrt{\left\{ \frac{8 \times .03 \times 8365}{500} + (2 - .03)^2 \right\}} - (2 - .03)}{2 \times .03}$$

$(1 + r) \times t$

= 1.729408.

$$= \frac{\sqrt{\left\{ \frac{2007.6}{500} + 3.8809 \right\}} - 1.97}{.06}$$

$(1 + r) \times t$

= 38 + 0.415688

$$= \frac{\sqrt{4.0152 + 3.8809} - 1.97}{.06} = \frac{\sqrt{7.8961} - 1.97}{.06}$$

$$= \frac{2.81 - 1.97}{.06} = \frac{.84}{.06} = \frac{84}{6} = 14 \text{ payments} = 7 \text{ years.}$$

(2)

Here $a = 112.50$, $r = .015$, $t = 44$.

$(1 + r) \times t$

= 0.397672

$$\text{Formula I. } A = at \left(1 + \frac{(t-1)r}{2} \right)$$

$$= 112.50 \times 44 \left(1 + \frac{(44-1) \times .015}{2} \right) = 4950 \times 1.3225 = \$6546.375.$$

(3)

Here $a = 300$, $A = 1680$, and $t = 5$.

$$\begin{aligned} \text{Formula III. } r &= \frac{2(A - at)}{at(t-1)} = \frac{2\{1680 - (300 \times 5)\}}{300 \times 5(5-1)} \\ &= \frac{2(1680 - 1500)}{300 \times 5 \times 4} = \frac{2 \times 180}{6000} = \frac{360}{6000} = .06 \\ \therefore \text{Rate per cent} &= .06 \times 100 = 6. \end{aligned}$$

(4)

Here $A = 2080$, $r = .04$, and $t = 16$.

$$\begin{aligned} \text{Formula II. } a &= \frac{2A}{t\{2 + (t-1)r\}} = \frac{2 \times 2080}{16\{2 + (16-1) \cdot 04\}} \\ &= \frac{4160}{4160} = \frac{4160}{4160} = \frac{4160}{4160} = 1 \\ &= \frac{16 \times \{2 + (15 \times .04)\}}{16 \times 2.6} = \frac{41.6}{41.6} = 1 \\ &= \$100 = 1 \text{ payment or rent for half a year, hence yearly} \\ &\quad \text{rent} = \$100 \times 2 = \$200. \end{aligned}$$

EXERCISE 164.—Page 366.

(1)

Here $r = .04$, and $v = \$3000$.Then $a = vr = 3000 \times .04 = \120 .

(2)

Here $a = 563$, and $v = 11260$

$$\begin{aligned} \text{Then } r &= \frac{a}{v} = \frac{563}{11260} = \frac{1}{20} = .05, \text{ and hence rate} \\ &\quad \text{per cent.} = 5. \end{aligned}$$

(3)

Here $a = 75$, $r = .05$, and $s = 14$.

$$\begin{aligned} \text{Then } v &= \frac{a}{r(1+r)^s} = \frac{75}{.05 \times (1.05)^{14}} \\ \log. v &= \log. 75 - \{(\log. 1.05 \times 14) + \log. .05\} \\ &= 1.875061 - (0.021189 \times 14 + \log. .05) \\ &= 1.875061 - (0.296646 + \overline{2}.698970.) \\ &= 2.879445. \end{aligned}$$

$\therefore v =$ nat. number corresponding to the logarithm 2.879445, which is \$757.608.

(4)

Here $a = \$90$, $r = .04$, $t = 12$, $s = 7$, and $\therefore s + t = 19$.

$$\begin{aligned} \text{Formula VIII. } v &= \frac{a}{r} \left\{ \frac{1}{(1+r)^t} - \frac{1}{(1+r)^{s+t}} \right\} \\ &= \frac{90}{.04} \left\{ \frac{1}{(1.04)^{12}} - \frac{1}{(1.04)^{19}} \right\} = \frac{9000}{4} \left\{ \frac{1}{1.60101} - \frac{1}{2.10682} \right\} \\ &= 2250 \times (.624605 - .474649) = 2250 \times .149956 \\ &= \$337.401. \end{aligned}$$

(5)

Here $a = 1500$, and $r = .05$.

$$\begin{aligned} \text{Formula IX. } v &= \frac{a}{r} = \frac{1500}{.05} = \frac{150000}{5} = \$30000, \\ &= 20 \times 1500 \text{ or 20 years' purchase.} \end{aligned}$$

(6)

Here $a = 22$, $v = 308.64166$, and $r = .04$.

$$\begin{aligned} \text{Then Formula VII. } t &= \frac{\log. a - \log. (a - vr)}{\log. (1 + r)} \\ &= \frac{\log. 22 - \log. (22 - 308.6416 \times .04)}{\log. (1.04)} \\ &= \frac{1.342423 - \log. (9.65425)}{.017033} = \frac{1.342423 - 0.984707}{.017033} \\ &= \frac{0.357716}{.017033} = \frac{357716}{17033} = 21 + \end{aligned}$$

(7)

Here $a = 154$, $t = 19$, and $r = .05$.

$$\begin{aligned} \text{Formula V. } v &= \frac{a}{r} \left\{ 1 - \frac{1}{(1+r)^t} \right\} \\ &= \frac{154}{.05} \times \left\{ 1 - \frac{1}{(1.05)^{19}} \right\} = \frac{15400}{5} \times \left\{ 1 - \frac{1}{2.5269} \right\} \\ &= 3080 \times (1 - .39574) = 3080 \times .60426 = \$1861.12 + \end{aligned}$$

(8)

Here $A = 600$, $t = 40$, and $r = .0375$.

$$\begin{aligned} \text{Formula II. } a &= \frac{Ar}{(1+r)^t - 1} = \frac{600 \times .0375}{(1.0375)^{40} - 1} \\ &= \frac{22.5}{22.5} = \frac{2250000}{4.36034 - 1} = \frac{336034}{3.36034} \\ &= £6.6957 = £6 \text{ 13s. } 10\frac{1}{2}\text{d} +. \end{aligned}$$

(9)

Here $a = 8$, $A = 187.315625$, and $r = .03$:

$$\begin{aligned} \text{Formula IV. } t &= \frac{\log.(Ar + a) - \log. a}{\log.(1+r)} \\ &= \frac{\log.(187.315625 \times .03 + 8) - \log 8}{\log. 1.03} \\ &= \frac{\log 5.61946875 + 8 - \log 8}{\log 1.03} \\ &= \frac{1.34160 - 0.903090}{0.012837} = \frac{0.43851}{0.012837} = 34.16 \approx 18. \end{aligned}$$

(10)

Here $a = 74$, $r = .04$, and $t = 30$

$$\begin{aligned} \text{Formula I. } A &= a \left\{ (1+r)^t - 1 \right\} / r = 74 \times \left\{ (1.04)^{30} - 1 \right\} / .04 \\ &= \frac{74}{.04} \times (3.24332 - 1) = \frac{7400}{4} \times 2.24332 = \$4150.142 \end{aligned}$$

By Table, page 362. Amount of \$1 for 30 years, at 4 per cent.

$$\begin{aligned} &= \$56.08494 \\ \text{Then } & \$56.08494 \times 74 = \$4150.28. \end{aligned}$$

EXERCISE 165—Page 367.

EXAMINATION PROBLEMS.

FIRST SERIES.

(2)

$\$7580 \times .19 = \1440.20 , and $\$7580 - \$1440.20 = \$6139.80$.
 D is to have one third as much as A, B, and C together, therefore he will have one-fourth of the whole. $\frac{1}{4}$ of $\$6139.80 = \$1534.95 =$ D's share.

$\$6139.80 - \$1534.95 = \$4604.85 =$ amount to be divided among A, B, and C.

B is to have $\$90.90$ more than C.

A is to have $\$111.11 + \$90.90 = 202.01$ " " "

$\$292.91$

$\$4604.85 - \$292.91 = \$4311.94 =$ three times C's share.

$\$4311.94 \div 3 = \$1437.31\frac{1}{3} =$ C's share.

$\$1437.31\frac{1}{3} + \$90.90 = \$1528.21\frac{1}{3} =$ B's share.

$\$1528.21\frac{1}{3} + \$111.11 = \$1639.32\frac{1}{3} =$ A's share.

(3)

A and B working together can do the work in 96 hours, therefore in one hour they will do $\frac{1}{96}$ of it.

A by himself can do the work in 192 hours; therefore in 1 hour he can do $\frac{1}{192}$ of it. $\frac{1}{96} - \frac{1}{192} = \frac{1}{192} =$ part B can do in one hour. Therefore he will require as many hours to finish it as $\frac{1}{192}$ is contained times in the whole, i. e. $1 \div \frac{1}{192} = 192$ hours. Then $192 \div 14 = 13\frac{2}{7}$ days.

(4)

$\pounds 179 \text{ 14s. } 8\frac{1}{2}\text{d.} = \$718.94\frac{1}{2} = \$718.94583.$
 $\$718.94583 \div .00000048 = \$71894583333.3 \div 48 =$
 $\$1497803819.4444.$

(5)

| | | |
|----|--|------------------------|
| 77 | | 44..18..30..77..55..27 |
| 30 | | 4..18..30 |
| 36 | | 2..8 |

$77 \times 30 \times 36 = 83160 =$ l. c. m.

Q

(6)

Here $n = 20$, and $r = .0525$.

$$\text{Then } t = \frac{n-1}{r} = \frac{20-1}{.0525} = \frac{19}{.0525} = 361.9048 \text{ years} =$$

361 years 10 months 25 days.

(7)

7342163 octenary = 70e57 duodenary, and 61351 nonary = 1e454 duodenary.

70e57 ÷ 1e454 = 40.38 duodenary.

(8)

$$783\frac{1}{2} = 3\frac{1}{2} + 10 \times 8 + 10 \times 10 \times 7.$$

| lbs. oz. dwt. grs. | × | $3\frac{1}{2}$ | = | lbs. oz. dwt. grs. |
|--------------------|---|----------------|---|--------------------|
| 43 3 17 11 | | | | 151 7 11 2½ |
| 10 | | | | |
| <hr/> | | | | |
| 433 2 14 14 | | | | 3465 9 16 16 |
| 10 | | | | |
| <hr/> | | | | |
| 4332 3 5 20 | | | | 30325 11 0 20 |
| | | | | <hr/> |
| | | | | 33943 4 8 14½ |

(9)

Here $a = 1$, and $r = \frac{1}{2}$.

$$\text{Then } S = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2.$$

(10)

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } 192 \div \frac{3}{4} = 64 \div \frac{3}{4} = 64 \div \frac{3}{2} = 64 \div \frac{3}{8} = 64 \div \frac{3}{16}$$

| | | | |
|--------------------------|---------------|---------------|---------------|
| $\frac{2\frac{1}{2}}{3}$ | $\frac{4}{3}$ | $\frac{4}{3}$ | $\frac{8}{3}$ |
| <hr/> | <hr/> | <hr/> | <hr/> |
| 4½ | 2 | 2 | 2 |
| <hr/> | <hr/> | <hr/> | <hr/> |
| 2 | 2 | 2 | 2 |
| <hr/> | <hr/> | <hr/> | <hr/> |
| ¾ | ¾ | ¾ | ¾ |
| <hr/> | <hr/> | <hr/> | <hr/> |
| = 64 | × | ¾ | = 129½. |

(11)

48 years =

Logarithm of 129140163 = 8.111061.
 $8.111061 \div 17 = .477121 = \text{logarithm of } 3.$

(12)

nonary =

| | |
|--|---|
| Suppose 48
18
<hr style="width: 50%; margin-left: 0;"/> 66
84
<hr style="width: 50%; margin-left: 0;"/> - 18 | Suppose 36
18
<hr style="width: 50%; margin-left: 0;"/> 54
63
<hr style="width: 50%; margin-left: 0;"/> - 9 |
|--|---|

Errors. Assumed numbers.

- 18 × 36 = 648
 - 9 × 48 = 432

grs.
2½

Difference of errors = 9 9)216 = sum of products.

24

16

SECOND SERIES.

(13)

20

B is to have \$69.18 more than C.

14½

A is to have \$69.18 + \$93.40 = \$162.58 " " "

\$231.76

\$897.43 - \$231.76 = \$665.67 = Amount to be divided
 equally amongst A, B, and C.

\$665.67 ÷ 3 = \$221.89 = C's share.

\$221.89 + \$69.18 = \$291.07 = B's "

\$291.07 + \$93.40 = \$384.47 = A's "

(14)

64 ÷ $\frac{5}{16}$

| | | | |
|----------------|------------------|-----------|---|
| 7 lbs. wheat | = 9 lbs. rye | 7 = 9 | |
| 5 " rye | = 8 " oats | 5 = 8 | |
| 13 " oats | = 21 " buckwheat | 13 = 21 | 7 |
| 27 " buckwheat | = 20 " barley | 3 27 = 20 | 4 |
| 24 " barley | = 26 " peas | 3 24 = 26 | 2 |
| 11 " peas | = 35 " potatoes | 11 = 35 | |
| x " potatoes | = 16 " wheat | x = 16 | |

Ans. $\frac{4 \times 2 \times 35 \times 16}{3 \times 11} = \frac{4480}{33} = 135\frac{5}{33}$

(21)

| | | |
|---------------|----------------|--|
| | 838)171347(204 | |
| | 1676 | |
| | ----- | |
| | 3747 | |
| | 3352 | |
| | ----- | |
| 17598)46090(2 | 395)838(2 | |
| 35196 | 790 | |
| ----- | ----- | |
| 10894)17598(1 | 48)395)8 | |
| 10894 | 384 | |
| ----- | ----- | |
| 6704)10894(1 | 11)48(4 | |
| 6704 | 44 | |
| ----- | ----- | |
| 4190)6704(1 | 4)11(2 | |
| 4190 | 8 | |
| ----- | ----- | |
| 2514)4190(1 | 3)4(1 | |
| 2514 | 3 | |
| ----- | ----- | |
| 1676)2514(1 | 1)3 | |
| 1676 | 3 | |
| ----- | ----- | |
| | 838)1676(2 | |
| | 1676 | |
| | ----- | |

As no number greater than unity will divide all of them without a remainder, they have no G. C. M.

(22)

$$\begin{aligned}
 & \$12000 \times 4 = \$48000 \\
 & \$12000 + \$8000 = \$20000 \times 2 = \$40000 \\
 & \qquad \qquad \qquad \underline{\qquad \qquad \qquad} \\
 & \qquad \qquad \qquad \$88000 = \text{product of A's} \\
 & \qquad \qquad \qquad \qquad \qquad \text{stock and time.} \\
 & \qquad \qquad \qquad \$25000 \times 3 = \$75000 \\
 & \$25000 - \$10000 = \$15000 \times 3 = \$45000 \\
 & \qquad \qquad \qquad \underline{\qquad \qquad \qquad} \\
 & \qquad \qquad \qquad \$120000 = \text{product of B's} \\
 & \qquad \qquad \qquad \qquad \qquad \text{stock and time.} \\
 & \qquad \qquad \qquad \$35000 \times 2 = \$70000
 \end{aligned}$$

Continued on next page.)

(22 continued.)

$$\frac{2}{3} \text{ of } \$35000 = \$10000. \quad \$35000 - \$10000 = \$25000 \times 4 = \frac{100000}{170000}$$

= product of C's stock and time.

$$\$88000 + \$120000 + \$170000 = \$378000 = \text{sum of the products of stocks and times.}$$

$$\text{Then } \$378000 : \$88000 :: \$15000 : \frac{15000 \times 88000}{378000} = \$3492.06$$

= A's share.

$$\$378000 : \$170000 :: \$15000 : \frac{15000 \times 170000}{378000} = \$6746.03$$

= C's share.

$$\$15000 - (\$3492.06 + \$6746.03) = \$4761.91 = \text{B's share.}$$

(23)

| | |
|--|---------------------------|
| A's gain in 5 months = \$125 ∴ his gain for 9 months | |
| = $1\frac{1}{3} \times \$125$ | = \$225 |
| B's gain in 6 months = \$125 ∴ his gain for 9 months | |
| = $1\frac{1}{2} \times \$125$ | = \$187 $\frac{1}{2}$ |
| C's gain in 9 months..... | = \$125 |
| | Sum = \$537 $\frac{1}{2}$ |

$$\text{Then } \$537\frac{1}{2} : \$225 :: \$400 : \frac{400 \times 225}{537\frac{1}{2}} = \$167\frac{1}{3} = \text{A's stock.}$$

$$\$537\frac{1}{2} : \$187\frac{1}{2} :: \$400 : \frac{400 \times 187\frac{1}{2}}{537\frac{1}{2}} = \$139\frac{1}{3} = \text{B's stock.}$$

$$\$537\frac{1}{2} : \$125 :: \$400 : \frac{400 \times 125}{537\frac{1}{2}} = \$93\frac{1}{3} = \text{C's stock.}$$

(24)

$$\frac{1}{8} + \frac{1}{8} + \frac{1}{10} + \frac{1}{12} = \frac{57}{240} = \frac{19}{80} = \text{part of the cistern filled in one hour when the four pipes are left open.}$$

$$\frac{1}{6} + \frac{1}{8} + \frac{1}{4} + \frac{1}{3} = \frac{11}{24} = \frac{38}{80} = \text{part of the cistern emptied in one hour when the four are left open.}$$

$$\frac{19}{80} - \frac{38}{80} = \frac{19}{80} = \text{part of the cistern which remains filled after the eight pipes have been left open for one hour. And if } \frac{19}{80} \text{ of the cistern are emptied in one hour, it will take } 1 \div \frac{19}{80} = 2\frac{2}{19} \text{ hours to empty the whole of it.}$$

$$\begin{array}{r} 4=100000 \\ \hline \$170000 \end{array}$$

of the pro-

$$= \$3492.06$$

$$= \$6746.03$$

share.

$$= \$225$$

$$= \$187\frac{1}{2}$$

$$= \$125$$

$$= \$537\frac{1}{2}$$

= A's stock.

= B's stock.

= C's stock.

rn filled in

emptied in

filled after

ar. And if

take $1 \div$

THIRD SERIES.

(26)

As often as the first receives 4 the second receives 3, therefore as often as the first receives 6 the second receives $4\frac{1}{2}$. Then $6 + 4\frac{1}{2} + 7 = 17\frac{1}{2}$ loaves.

$$17\frac{1}{2}:6 :: 2310: \frac{2310 \times 6}{17\frac{1}{2}} = 792 \text{ loaves} = \text{number the first receives.}$$

$$17\frac{1}{2}:4\frac{1}{2} :: 2310: \frac{2310 \times 4\frac{1}{2}}{17\frac{1}{2}} = 594 \text{ " = " second "}$$

$$17\frac{1}{2}:7 :: 2310: \frac{2310 \times 7}{17\frac{1}{2}} = 924 \text{ " = " third "}$$

(27)

To produce a mixture worth 8 cents a pound, we require 4 lbs. @ 12 cents, 4 @ 4 cents, 1 @ 5 cents, and 3 @ 9 cents, or 3 lbs @ 12 cents, 1 @ 4 cents, 4 @ 5 cents, and 4 @ 9 cents, lbs.lbs.lbs. lbs.lbs.lbs.

$$\begin{array}{l} \text{Then } 4:72::4:72 \text{ lbs. @ 4 cts. or } 3:72::1:24 \text{ lbs. @ 4 cts.} \\ 4:72::1:18 \text{ lbs. @ 5 cts. } 3:72::4:96 \text{ lbs. @ 5 cts.} \\ 4:72::3:54 \text{ lbs. @ 9 cts. } 3:72::4:96 \text{ lbs. @ 9 cts.} \end{array}$$

(28)

$$\text{Here } A = \$4444.44, r = .0444, \text{ and } t = 4.3\frac{1}{3}$$

$$\text{Then } P = \frac{A}{1+rt} = \frac{\$4444.44}{1+(.0444 \times 4.3\frac{1}{3})} = \frac{\$4444.44}{1.19289\frac{1}{3}} = \$3725.764.$$

(29)

$$\$1.00 - \$0.0225 = \$0.9775. \quad \$23470 \div 0.9775 = \$24010.23.$$

(30)

$$\text{Here } A = \$7493.47, r = .07, \text{ and } t = 8.$$

$$\text{Then } P = \frac{A}{1+rt} = \frac{\$7493.47}{1+(.07 \times 8)} = \frac{\$7493.47}{1.56} = \$4803.5064.$$

(31)

\$17460 ÷ 1.03125 = \$16930.909 = sum to be invested.
 16930.909 ÷ 2.95 = 5739.29 yds. cloth.
 16930.909 × .02½ = \$423.27272 = ad valorem duty.
 \$17460 + \$1347.90 + \$479.40 + \$169.83 + \$423.27272 =
 \$19880.40272 = whole cost.

\$25000 - \$19880.40272 = \$5119.59728 = whole gain.

Then \$19880.40272 : \$100 :: \$5119.59728 : $\frac{5119.59728 \times 100}{19880.40272}$ =
 25.75 = 25½ per cent.

(32)

| V. | = | III. | = | VIII. | = | XII. |
|-----------|---|-----------|---|-----------|---|-----------|
| 134234 | | 21122021 | | 12701 | | 3281 |
| 5 | | 3 | | 8 | | 12 |
| 8 | | 7 | | 10 | | 38 |
| 5 | | 3 | | 8 | | 12 |
| 44 | | 22 | | 87 | | 464 |
| 5 | | 3 | | 8 | | 12 |
| 222 | | 68 | | 696 | | 5569 den. |
| 5 | | 3 | | 8 | | |
| 1113 | | 206 | | 5569 den. | | |
| 5 | | 3 | | | | |
| 5569 den. | | 618 | | | | |
| | | 3 | | | | |
| | | 1856 | | | | |
| | | 3 | | | | |
| | | 5569 den. | | | | |

(33)

$\frac{9}{7}$ of $4\frac{1}{2}$ of $\frac{1}{10}$ of $\frac{1}{3}$ of £43 18s. 11½d. £43 18s. 11½d. =
 $\frac{13}{20}$
 \$175.79½.

(Continued on next page.)

(33 continued.)

$$\frac{3}{7} \text{ of } \frac{2}{3} \text{ of } \frac{3^2}{13} \text{ of } \frac{1}{16} \text{ of } \frac{1}{3} \text{ of } \$175.79\frac{1}{2} = \frac{3}{7} \text{ of } \frac{2}{2} \text{ of } \frac{15}{1} \text{ of } \frac{1}{15} \text{ of } \frac{7}{9}$$

$$\text{of } \$175.79\frac{1}{2} = \frac{2}{3} \text{ of } \$175.79\frac{1}{2} = \$263.6875.$$

$$3\frac{2}{3} \text{ of } \frac{1}{17\frac{1}{2}} \text{ of } .56 \text{ of } 1.75 \text{ of } 6\frac{1}{2} \text{ times } \$97.18 =$$

$$\frac{1}{3\frac{2}{3}} \text{ of } \frac{1}{100} \text{ of } \frac{56}{100} \text{ of } \frac{17\frac{1}{2}}{100} \text{ of } 6\frac{1}{2} \text{ times } \$97.18; 6\frac{1}{2} \text{ times } \$97.18$$

$$= \$631.67.$$

$$\frac{25}{9} \text{ of } \frac{2}{35} \text{ of } \frac{14}{100} \text{ of } \frac{7}{175} \text{ of } \$631.67 = \frac{49}{9 \times 25} \text{ of } \$631.67$$

$$= \frac{49}{225} \text{ of } \$631.67$$

$$\frac{49}{225} \text{ of } \$631.67 = \$137.5636.$$

$$\text{Then } \$263.6875 - \$137.5636 = \$126.1239 = \text{difference.}$$

(34)

$$\frac{1}{13} = \frac{1}{13} \div 13 \therefore \log. \frac{1}{13} = \log. 1 - \log. 13 = 0 - 1.113943 = -2.886057.$$

$$19.5 = 3 \times 13 \times 5 \div 10 \therefore \log. 19.5 = \log. 3 + \log. 13 + \log. 5 - \log. 10.$$

$$\log. 3 = 0.477121$$

$$\log. 13 = 1.113943$$

$$\log. 5 = \log. 10 - \log. 2 = 1 - 0.301030 \therefore \log. 5 = 0.698970$$

$$\text{Sum} = 2.290034$$

$$\text{From which take } \log. 10 = 1$$

$$\text{Rem.} = 1.290034$$

$$= \log. 19.5.$$

$$1125 = 5^3 \times 3^2 \therefore \log. 1125 = (\log. 5) \times 3 + (\log. 3) \times 2.$$

$$\log. 5 = 0.698970 \times 3 = 2.096910$$

$$\log. 3 = 0.477121 \times 2 = 0.954242$$

$$\text{Sum} = 3.051152 = \log. \text{ of } 1125.$$

(Continued on next page.)

(34 continued.)

$$28 \cdot 16 = 28 \frac{1}{2} = 14^2 = 13^2 \div 6 \therefore \log. 28 \cdot 16 = (\log. 13) \times 2 \\ - (\log. 2 + \log. 3.)$$

$$\log. 13 = 1 \cdot 113943 \times 2 = 2 \cdot 227886$$

$$(\log. 2 + \log. 3) = (0 \cdot 301030 + 0 \cdot 477121) = 0 \cdot 778151$$

$$\text{Diff.} = \underline{1 \cdot 449735}$$

$$= \log. 28 \cdot 16.$$

$$65000 = 13 \times 5 \times 1000 \therefore \log. 65000 = \log. 13 + \log. 5 \\ + \log. 1000.$$

$$\log. 13 = 1 \cdot 113943$$

$$\log. 5 = 0 \cdot 698970$$

$$\log. 1000 = 3$$

$$\text{Sum} = \underline{4 \cdot 812913} = \log. \text{ of } 65000.$$

$$\log. \cdot 0005 = \log. 5 \text{ with characteristic changed to } -4 \\ = \underline{4 \cdot 698970}.$$

$$152 \cdot 1 = 3^2 \times 13^2 \div 10 \therefore \log. 152 \cdot 1 = (\log. 3) \times 2 \\ + (\log. 13) \times 2 - \log. 10.$$

$$\log. 3 = 0 \cdot 477121 \times 2 = 0 \cdot 954242$$

$$\log. 13 = 1 \cdot 113943 \times 2 = 2 \cdot 227886$$

$$\text{Sum} = \underline{3 \cdot 182128}$$

$$\text{From which take } \log. 10 = 1$$

$$\text{Diff.} = \underline{2 \cdot 182128} = \log. 152 \cdot 1$$

$$8 \cdot 112 = 2^4 \times 13^2 \times 3 \div 1000 \therefore \log. 8 \cdot 112 = (\log. 2) \times 4 \\ - (\log. 13) \times 2 + \log. 3 - \log. 1000.$$

$$\log. 2 = 0 \cdot 301030 \times 4 = 1 \cdot 204120$$

$$\log. 13 = 1 \cdot 113943 \times 2 = 2 \cdot 227886$$

$$\log. 3 = \quad \quad \quad 0 \cdot 477121$$

$$\text{Sum} = \underline{3 \cdot 909127}$$

$$\text{From which take } \log. 1000 = 3$$

$$\text{Diff.} = \underline{0 \cdot 909127} = \log. 8 \cdot 112.$$

t 8 t²

t 8 t

 $\frac{1}{6} + \frac{1}{12}$ $\frac{3}{8} -$

a

 $\frac{17}{8}$ of

fa

 $\frac{17}{8}$ of t $\therefore 9$ y $\therefore 9$ y

If 9 y

If $\frac{1}{28}$ i

Assum

 $\frac{1}{6} + \frac{1}{12}$

21

 \therefore he l

(35)

XII.

| | |
|---|--|
| $t^2 \times 300 = 21000$ $t \times 8 \times 30 = 1800$ $8^2 = 54$ <hr style="width: 100px; margin-left: auto; margin-right: 0;"/> 22854
$t^2 \times 300 = 2454000$ $t \times 8 \times t \times 30 = 22800$ $t^2 = 84$ <hr style="width: 100px; margin-left: auto; margin-right: 0;"/> 2476884
$t \times 8 \times t^2 \times 300 = 249961000$ $t \times 8 \times t \times 2 \times 30 = 54500$ $2^2 = 4$ <hr style="width: 100px; margin-left: auto; margin-right: 0;"/> 249965504 | $871tet-72 (t8-t2)$ $6e4$ <hr style="width: 100px; margin-left: 0; margin-right: auto;"/> $179tet$ 159768 <hr style="width: 100px; margin-left: 0; margin-right: auto;"/> 20352720
$1et372e4$ <hr style="width: 100px; margin-left: 0; margin-right: auto;"/> 517428000
$4977ttt08$ <hr style="width: 100px; margin-left: 0; margin-right: auto;"/> $3e8391e4$ |
|---|--|

(36)

$\frac{1}{6} + \frac{1}{12} + \frac{1}{7} + 5$ years = $\frac{11}{8}$ of life time + 5 years = age at birth of son.
 $\frac{32}{8} - (\frac{11}{8} + 5) = \frac{1}{8}$ of his life time - 5 years = time he lived after birth of son.

$\frac{17}{8}$ of father's life time - 5 years - 4 years = age of son = $\frac{1}{2}$ father's age.

$\frac{17}{8}$ of father's life time - 9 years = $\frac{1}{2}$ father's age.

\therefore 9 years is the difference between $\frac{17}{8}$ and $\frac{1}{2}$ of father's age.

\therefore 9 years is equal to $\frac{3}{8}$ of father's age.

If 9 years is $\frac{3}{8}$ of his age, $\frac{1}{8}$ will be the $\frac{1}{3}$ of 9 which is 3 years.

If $\frac{1}{8}$ is 3 years, $\frac{28}{8}$ or the whole age will be $3 \times 28 = 84$ years.

Or by Position.

Assume 42 for father's age at death, the son's age = 21.

$\frac{1}{6} + \frac{1}{12} + \frac{1}{7} + 5 = \frac{11}{8} + 5$; $\frac{11}{8}$ of 42 = $16\frac{1}{2}$ and $16\frac{1}{2} + 5 =$

$21\frac{1}{2}$ = age of father when son was born.

\therefore he lived after birth of his son $42 - 21\frac{1}{2} = 20\frac{1}{2}$ years.

(Continued on next page.)

(36 continued.)

By the question he lived $21 + 4 = 25$ years.

The error $25 - 20\frac{1}{2} = -4\frac{1}{2}$.

Assume 98 for father's age, then son's age = $\frac{1}{2}$ of 98 = 49.

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + 5 = \frac{3}{2} + 5$; $\frac{1}{2}$ of 98 = $38\frac{1}{2}$, and $38\frac{1}{2} + 5 = 43\frac{1}{2}$
= age of father at birth of son.

\therefore he lived after birth of his son $98 - 43\frac{1}{2} = 54\frac{1}{2}$ years.

But by the question he lived $49 + 4$ years = 53 years.

Then $53 - 54\frac{1}{2} = +1\frac{1}{2}$ = error.

Errors.

$$-4\frac{1}{2} \times 98 = 441$$

$$+1\frac{1}{2} \times 42 = 63$$

$$\text{Sum} = \begin{array}{r} 441 \\ 63 \\ \hline 504 \end{array}$$

$$504 \div 6 = 84 = \text{father's age.}$$

(37)

| m. | fur. | per. | yds. | ft. | in. | ÷ | fur. | per. | yds. |
|----------------------|------|------|------|-----|-----|---|--------------------|------|-----------------|
| 63 | 3 | 7 | 3 | 2 | 7 | ÷ | 7 | 23 | 3 $\frac{1}{2}$ |
| 8 | | | | | | | 40 | | |
| 507 | | | | | | | 303 | | |
| 40 | | | | | | | 5 $\frac{1}{2}$ | | |
| 20287 | | | | | | | 1518 $\frac{1}{2}$ | | |
| 5 $\frac{1}{2}$ | | | | | | | 151 $\frac{1}{2}$ | | |
| 101438 | | | | | | | 1670 $\frac{1}{2}$ | | |
| 10143 $\frac{1}{2}$ | | | | | | | 3 | | |
| 111581 $\frac{1}{2}$ | | | | | | | 5010 $\frac{1}{2}$ | | |
| 3 | | | | | | | 12 | | |
| 334746 $\frac{1}{2}$ | | | | | | | 60129 | | |
| 12 | | | | | | | | | |
| 4016965 | | | | | | | | | |

(Continued on next page.)

6·3 ÷

(37 continued.)

60129)4016965(66·80578 times

360774

409225

360774

484510

481032

347800

300645

471550

420903

506470

481032

(38)

6·3 ÷ ·000000274

274)6300000000(22992700·72992700

548

820

548

2720

2466

2540

2466

740

548

1920

1918

2000

1918

820

548

2720

2466

2540

2466

740

548

1920

1918

200 remainder.

8 = 49.
1 + 5 = 43

ears.
ars.

yds.
3

(39)

$$\frac{1}{4} \text{ yds.} : 6\frac{1}{4} \text{ yds.} :: \$17 : 4 \times 11 \times 17 = \frac{1178}{100} = \$5.482.$$

(40)

$$I = Prt. = \$4237.71 \times .065 \times 1.67 = \$460.0034205.$$

(41)

$$t = \frac{A - P}{Pr} = \frac{\$1000 - \$674.30}{\$674.30 \times .085} = \frac{325.70}{57.3155} = 5.68258 \text{ years} = 5 \text{ years } 8 \text{ months } 5.7288 \text{ days.}$$

(42)

By Table, page 260, the amount of \$1 for 14 payments at 4 per cent. is \$1.73168.

$$\text{Then } \$1.73168 \times 813.71 = \$1409.0853328 = \text{Amount.}$$

$$\text{Subtract } 813.71$$

$$\text{Difference} = 595.3753328 = \text{Interest.}$$

(43)

| | | | | |
|-------|---|----|---|---|
| \$300 | × | 0 | = | 0 |
| 700 | × | 4 | = | 2800 |
| 750 | × | 7 | = | 5250 |
| 850 | × | 9 | = | 7650 |
| 400 | × | 13 | = | 5200 |
| 1300 | × | 19 | = | 24700 |
| 4300 | | | |) |
| | | | | 45600 (10 months 18 $\frac{1}{3}$ days. |
| | | | | 4300 |
| | | | | 2600 |
| | | | | 30 |
| | | | | 78000 = days. |
| | | | | 4300 |
| | | | | 35000 |
| | | | | 34400 |
| | | | | 4138 |

23
 — \$9
 D tog
 \$1078
 Ded
 be div
 and D
 than
 gets \$
 get, th
 + \$78
 \$181
 Tha
 to \$21
 Hen
 Then \$

P =
 1
 { (37 -
 { (-
 { (-
 { (-

(44)

23 per cent. of \$4200 = $\frac{23}{100}$ of 4200 = \$966.00, and \$4200 - \$966.00 = \$3234.00. E has half as much as A, B, C, and D together; therefore E has *one-third* of \$3234.00, which is \$1078.00.

Deducting E's share, \$1078, from \$3234, the whole sum to be divided, there remains \$2156 to be divided among A, B, C and D. Now D gets a certain amount; C gets \$42.11 more than D; B gets \$61.34 (42.11 + 19.23) more than D; and A gets \$78.44 (61.34 + 17.10) more than D. Together they get, then, *four times* D's share, together with \$42.11 + \$61.34 + \$78.44, or, in other words, four times D's share, together with \$181.89.

That is, four times D's share, together with \$181.89 is equal to \$2156.

Hence \$2156.00 - \$181.89 = \$1974.11 = four times D's share. Then \$1974.11 ÷ 4 = \$493.5275 = D's share.

$$\begin{array}{r} \text{Add} \quad 42.11 \\ \hline \text{Sum} \quad \$535.6375 = \text{C's share.} \\ \text{Add} \quad 19.23 \\ \hline \text{Sum} \quad \$554.8675 = \text{B's share.} \\ \text{Add} \quad 17.10 \\ \hline \text{Sum} \quad \$571.9675 = \text{A's share.} \end{array}$$

(45)

$$P = \frac{A}{1+rt} = \frac{\$3786.80}{1+1.76} = \frac{3786.80}{2.76} = \frac{378680}{276} = \$1372.02898 +$$

(46)

$$\left\{ (3\frac{3}{4} - 2\frac{7}{10}) \times .46 \div \frac{2}{3} \text{ of } .142857 \right\} \div 8\frac{1}{2} \text{ times } (\frac{1}{2} + \frac{1}{3} + \frac{1}{6} - \frac{337}{2310})$$

$$\left\{ (.73 \times .12345 \div \frac{978}{10}) + \frac{2}{3} + 9\frac{3}{4} + 17\frac{1}{11} \right\} \div 27.4922077$$

$$\left\{ (3\frac{3}{4} - 2\frac{7}{10}) \times \frac{46}{100} \div \frac{2}{3} \text{ of } \frac{1}{7} \right\} \div \frac{1}{2} \times (\frac{35}{10} + \frac{10}{10} + \frac{10}{10} - \frac{337}{2310})$$

$$\left\{ (.88 \times \frac{12345}{99888} \div \frac{978}{10}) + \frac{2}{3} + 9\frac{3}{4} + 17\frac{1}{11} \right\} \div 27.4922077$$

(Continued on next page.)

$$*rt = .16 \times 11 = 1.76.$$

(46 continued.)

$$\begin{aligned}
 &= \frac{(4\frac{1}{10} \times 3\frac{2}{10} \times 2 \times 7) \times \frac{2}{17} \times \frac{3310}{1610}}{\{(1\frac{1}{10} \times \frac{670}{3000}) \times 750 + 27\frac{151}{388}\} \div 27\cdot4922077} \\
 &= \frac{\frac{391}{66} \times \frac{2}{17} \times \frac{3310}{1610}}{1} \\
 &= \frac{(1\frac{1}{10} + 27\frac{151}{388}) \div 27\cdot4922077}{1} = \frac{27\frac{778}{388} \div 27\cdot4922077}{1} \\
 &= \frac{1}{27\cdot4922077 \div 27\cdot4922077} = \frac{1}{1} = 1
 \end{aligned}$$

(47)

312312302 quaternary = 224690 decimal scale.

2312132 quaternary = 11678 decimal scale.

Sum = 236368

4234 quinary = 569 decimal, and 569 × 23011 = 13093259.

236368 × 13093259 = 3094827443312.

555 + 444 + 333 + 222 + 111 senary = 2553 senary = 645 decimal.

3094827443312 - 645 = 3094827442667.

6542 septenary = 2333 decimal.

3094827442667 ÷ 2333 = 1326544124 $\frac{3375}{333}$ den.

X. VIII.

1326544124 = 11704272374

X. VIII.

1375 = 2537

X. VIII.

2333 = 4435

X. VIII.

∴ 1326544124 $\frac{3375}{333}$ = 11704272374 $\frac{4435}{33}$.

(48)

·1 = $\frac{1}{10}$ and $(\frac{1}{10})^2 = \frac{1}{100} = 01$

·1 = $\frac{1}{9}$ and $(\frac{1}{9})^2 = \frac{1}{81} = \cdot012345679.$

FIFTH SERIES.

(50)

Assume 27 | 2..9..16..27..48 and 81, strike out 2, 9 and 16 since they are contained as factors in the others.

$$\text{The l. c. m.} = 27 \times 16 \times 3 = 1296.$$

(51)

$$t = \frac{\log. n}{\log. (1+r)} = \frac{\log. 7}{\log. (1.06)} = \frac{0.845098}{0.025306} = 33.395 \text{ years.}$$

(52)

20 miles = 1267200 inches ; and 14 ft. 10 in. = 178 inches.
 $1267200 \div 178 = 7119\frac{2}{3}$ times.

(53)

$1749600 = 2^5 \times 3^7 \times 5^2$; increasing each index by unity and multiplying, we have $6 \times 8 \times 3 = 144$.

(54)

$$\begin{aligned} \frac{2}{3} \text{ of } \frac{96}{\frac{5}{8}} \div \frac{\frac{1}{2} \text{ of } 7}{3\frac{1}{4}} &= \frac{2}{3} \times \frac{2\frac{2}{3}}{\frac{5}{8}} \div \frac{\frac{7}{2}}{3\frac{1}{4}} = \frac{2}{3} \times 2\frac{2}{3} \div \frac{7}{1\frac{1}{4}} \\ &= \frac{2}{3} \times 2\frac{2}{3} \div \frac{7}{1\frac{1}{4}} = \frac{2}{3} \times 2\frac{2}{3} \times \frac{1\frac{1}{4}}{7} = 35\frac{1}{3}. \end{aligned}$$

(55)

A can do the whole work in 12 days, therefore he can do $\frac{1}{12}$ in 1 day. A and B together can do the work in 5 days, therefore they can do $\frac{1}{5}$ in 1 day. Therefore B can do $\frac{1}{5} - \frac{1}{12} = \frac{7}{60}$ in 1 day, and he will require as many times 1 day to do the whole work as $\frac{7}{60}$ is contained times in 1, i. e. $1 \div \frac{7}{60} = \frac{60}{7} = 8\frac{4}{7}$ days.

R

(56)

$$P = \frac{A}{(1+r)^t}; \log. P = \log. A - \log. (1+r) \times t = \log. 8899.77$$

$$- \log. (1.06) \times 22 = 3.949378 - 0.025306 \times 22$$

$$= 3.949378 - 0.556732 = 3.392646, \text{ and } \log. 3.392646$$

$$= \$2469.71.$$

By Table, page 260, amount of \$1 at 6 per cent. for 22 payments = 3.60354.

$$\text{Then } \$8899.77 \div 3.60354 = \$2469.73 \text{ nearly.}$$

(57)

Let the 1st number be 2. Then $2 \times 2 = 4$

$$1\frac{1}{2} \times 3 = 4\frac{1}{2}$$

$$10 - (2 + 1\frac{1}{2}) = 10 - 3\frac{1}{2} = 6\frac{1}{2} \times 4 = 26\frac{1}{2}, \text{ but it should equal 4.}$$

$$\text{Therefore } 26\frac{1}{2} - 4 = + 22\frac{1}{2} = \text{error.}$$

Let $1\frac{1}{2}$ be the 1st number; then $1\frac{1}{2} \times 2 = 3$

$$1 \times 3 = 3$$

$$10 - (1\frac{1}{2} + 1) = 10 - 2\frac{1}{2} = 7\frac{1}{2} \times 4 = 30, \text{ but it should } = 3.$$

$$\text{Therefore } 30 - 3 = + 27 = \text{error.}$$

Errors.

$$+ 27 \times 2 = 54$$

$$+ 22\frac{1}{2} \times 1\frac{1}{2} = 34$$

$$\text{Diff.} = 4\frac{1}{2} \text{ diff.} = 20, \text{ and } 20 \div 4\frac{1}{2} = 4\frac{2}{3} = \text{1st number.}$$

$$4\frac{2}{3} \times 2 = 9\frac{2}{3} = \text{1st product.}$$

$$\text{Second number} = 9\frac{2}{3} \div 3 = 3\frac{1}{3} \times 3 = 9\frac{2}{3} = \text{2nd product.}$$

$$10 - 7\frac{2}{3} = 2\frac{4}{3} \times 4 = 9\frac{2}{3} = \text{3rd product.}$$

(58)

Suppose A has 40; then B has $110 - 40 = 70$, and C has $130 - 70 = 60$.

A and C together have $40 + 60 = 100$, but it should be 120.

$$\text{Therefore } 100 - 120 = - 20 = \text{error.}$$

Suppose A has 80; then B has $110 - 80 = 30$, and C has $130 - 30 = 100$.

A and C together have $80 + 100 = 180$, but they should have 120.

$$\text{Therefore } 180 - 120 = + 60 = \text{error.}$$

(Continued on next page.)

(58 continued.)

$$\begin{array}{r} \text{Errors.} \\ + 60 \times 40 = 2400 \\ - 20 \times 80 = 1600 \\ \hline \end{array}$$

$$\text{Sum} = 80 \qquad \text{Sum} = 4000$$

4000 \div 80 = 50 = number A has.

Then B has 110 - 50 = 60, and C has 130 - 60 = 70.

$$\frac{50 + 60 + 70}{3} = 60 = \text{each man's share when equally divided.}$$

(59)

Formula I, p. 333. $l = a + (n - 1)d = 7 + (47 - 1) \times 4$
 $= 7 + (46 \times 4) = 7 + 184 = 191.$

Formula VI, p. 333. $s = \left\{ 2a + (n - 1)d \right\} \frac{n}{2}$
 $= \left\{ 2 \times 7 + (93 - 1) \times 4 \right\} \frac{93}{2} = \left\{ 14 + (92 \times 4) \right\} \frac{93}{2}$
 $= (14 + 368) \times \frac{93}{2} = \frac{382 \times 93}{2} = 17763.$

(60)

$$t = \frac{\log. n}{\log. (1 + r)} = \frac{\log. 21}{\log. (1.07)} = \frac{1.322219}{0.029384} = 44.997 \text{ years.}$$

SIXTH SERIES.

(61)

B gets \$196.87 more than C, and A gets \$387 + \$196.87 = \$583.87 more than C, therefore together they get three times C's share, together with \$196.87 + \$583.87, i. e. three times C's share, together with \$780.74; but together they get \$3700.

Therefore \$3700 = three times C's share, together with \$780.74, or \$3700 - \$780.74 = \$2919.26 = three times C's share.

Hence \$2919.26 \div 3 = \$973.083 = C's share.

$$\text{Add} \quad 196.87$$

$$\text{Sum} = \$1169.953 = \text{B's share.}$$

$$\text{Add} \quad 387.00$$

$$\text{Sum} = \$1556.953 = \text{A's share.}$$

(62)

$$5716 = 2^3 \times 1429$$

$$1 \dots 2 \dots 4$$

$$1 \dots 1429$$

$$1 \dots 2 \dots 4 \dots 1429 \dots 2858 \dots 5716$$

(63)

$$\left\{ (17\frac{1}{2} - 10\frac{1}{3}) - (4 + \frac{1}{2} + .9 - \frac{1}{3}) \right\} \div (.8378 \div \frac{1}{2} \text{ of } 31)$$

$$.6322632 \times \frac{1}{2} \text{ of } 9\frac{1}{2} \div (\frac{1}{2} \text{ of } 4\frac{1}{2} \text{ of } \frac{1}{11} \text{ of } 85\frac{1}{2} \div 101)$$

$$6\frac{1}{2} - 1 \div (8\frac{1}{3} \times \frac{1}{2})$$

$$= \frac{8\frac{1}{3} \times \frac{1}{2} \times \frac{27}{4} \div (\frac{1}{2} \times \frac{27}{4} \times \frac{1}{11} \times \frac{21}{101} \times \frac{1}{101})}{5\frac{1}{2} \times 8\frac{1}{3} \times \frac{1}{2}}$$

$$= \frac{8\frac{1}{3} \times \frac{1}{2} \times \frac{27}{4} \times \frac{1}{2} \times \frac{27}{4} \times \frac{1}{11} \times \frac{1}{2} \times \frac{27}{101} \times \frac{1}{101}}{\frac{27}{2} \times \frac{27}{4} \times \frac{1}{2}}$$

$$= \frac{\frac{27}{101} \times \frac{27}{4} \times \frac{1}{2} \times \frac{1}{2} \times \frac{27}{101} \times \frac{1}{2}}{14 \times 37} \quad \frac{5}{14 \times 37}$$

$$= \frac{5}{\frac{1}{2} \times \frac{1}{2} \times \frac{27}{2}} = \frac{5}{5 \times 37} = \frac{5}{185} = 2\frac{1}{2}\%$$

4

(64)

Each child gets 1 child's share, \therefore 17 children get 17 shares.

Each woman gets *three* times a child's share, \therefore 4 women get 12 shares.

Each man gets *six* times a child's share, \therefore 3 men get 18 shares.

And together they get 47 times a child's share.

Therefore $\$7200 \div 47 = \$153.19\frac{1}{7}$ = a child's share.

$$\$153.19\frac{1}{7} \times 3 = \$459.57\frac{1}{7}$$

$$\$153.19\frac{1}{7} \times 6 = \$919.14\frac{1}{7}$$

(65)

$25400 = 2^3 \times 5^2 \times 127$. Adding unity to each index and multiplying the results, we get $4 \times 3 \times 2 = 24$.

(66)

$$\frac{2}{3} \text{ of } 4\frac{1}{2} \text{ of } \frac{9\frac{7}{11}}{1\frac{1}{11}} \text{ of } \frac{1}{8} \text{ of } \text{£}3 \text{ 16s. } 11\frac{1}{2}\text{d.} = \frac{2}{3} \times \frac{9}{2} \times \frac{66 \times 14}{7 \times 11} \times \frac{1}{8}$$

$$\times \$15.39\frac{1}{2} = 6 \text{ times } \$15.39\frac{1}{2} = \$92.35.$$

$$\frac{1}{11} \text{ of } 4\frac{3}{8} \text{ of } \frac{19\frac{1}{2}}{3\frac{1}{2}} \text{ of } \frac{1}{11} \text{ of } \frac{1}{11} \text{ of } \frac{1}{11} \text{ of } .85 \text{ of } \frac{1}{42\frac{1}{2}} \text{ of } \$1783$$

$$= \frac{1}{11} \times \frac{22}{3} \times \frac{32}{12} \times \frac{1}{11} \times \frac{1}{11} \times \frac{1}{11} \times \frac{1}{11} \times \frac{1}{11} \times \frac{1}{11} \text{ of } \$1783.$$

$$= \frac{8}{11} \times \frac{22}{5} \times \frac{78}{12} \times \frac{95}{117} \times \frac{11}{22} \times \frac{85}{100} \times \frac{2}{85} \times \frac{1783}{1}$$

$$= \$17.83 \times 4 = \$71.32. \quad \$92.35 - \$71.32 = \$21.03.$$

(67)

$7 : 13 = 7 \div 13 = .538$
 $9 : 16 = 9 \div 16 = .562$
 $8 : 15 = 8 \div 15 = .533$
 $10 : 19 = 10 \div 19 = .526$

Therefore 9 : 16 is the greatest, and 10 : 19 is the least.

$$\text{Compound ratio} = \frac{7}{13} \times \frac{9}{16} \times \frac{8}{15} \times \frac{10}{19} = \frac{21}{247} = 21.247,$$

(68)

$$67.432 = 67 \frac{432}{1000} = \frac{66752}{1000} \text{ and } 7.9036 = 7 \frac{9036}{10000} = \frac{79996}{10000}$$

$$\frac{66752}{990} \div \frac{78957}{9990} = \frac{66752}{990} \times \frac{111}{9990} = \frac{7410138}{868527} = 8.5318452.$$

(69)

9 per. 9 yds. 7 ft. 120 in. = 365628 inches
 $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of 35 acres 2 roods = $\frac{1}{8}$ of 35 acres 2 roods = $\frac{1}{8}$ of 222678720 inches

$$\frac{365628}{\frac{1}{8} \text{ of } 222678720} = \frac{2559396}{133607232} = 0.019156118.$$

(70)

Dissimilar.

Similar.

| | |
|----------------|----------------|
| 17-0342 | 17-03424242 |
| 27-06357 | 27-06357575 |
| 98-123456 | 98-123456456 |
| 829-6423 | 829-642342342 |
| 986-1234298 | 986-1234298429 |
| 9-876342 | 9-876342876342 |
| 813-9864234567 | 813-9864234567 |

Similar and Coterminous.

| |
|------------------------|
| 17-034242424242424242 |
| 27-063575757575757575 |
| 98-123456456456456456 |
| 829-642342342342342342 |
| 986-123429842984298429 |
| 9-876342876342876342 |
| 813-986423456745674567 |
| 4 carried |

2781-849813156689829957

Exe



Height

CF =

GH =

GI =

The m

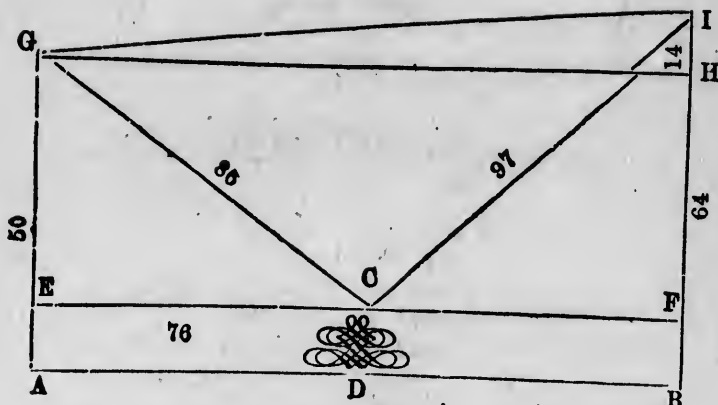
m

20

ta

Ther

(71)



$$EG = \sqrt{86^2 - 76^2} = \sqrt{1620} = 40.249 \text{ feet}$$

Height of Statue $CD = AG - EG = 50 - 40.249 = 9.751 \text{ ft.} = BF$

$$FI = BI - BF = 64 - 9.751 = 54.249 \text{ feet}$$

$$CF = \sqrt{CI^2 - FI^2} = \sqrt{97^2 - 54.249^2} = \sqrt{6466.45999} = 80.411 \text{ feet}$$

$$GH = EF = EC + CF = 76 + 80.411 = 156.411 \text{ feet and}$$

$$HI = 64 - 50 = 14 \text{ feet}$$

$$GI = \sqrt{GH^2 + HI^2} = \sqrt{156.411^2 + 14^2} = \sqrt{24660.400921} = 157.036 \text{ feet.}$$

(72)

The mixture = spirits + water = $\frac{1}{3}$ of mixture + 25 gal. + $\frac{1}{3}$ of mixture - 5 gal. = $\frac{1}{3} + \frac{1}{3} + 20 \text{ gal.} = \frac{2}{3} + 20 \text{ gal.}$ Then 20 gal. = $\frac{1}{3}$ of the mixture, and therefore the mixture contained $6 \times 20 = 120 \text{ gal.}$

Then $\frac{1}{3}$ of 120 = 60 + 25 = 85 gal. = spirits }
 $\frac{1}{3}$ of 120 = 40 - 5 = 35 gal. = water }

SEVENTH SERIES.

(73)

$$\begin{array}{r}
 401241 \cdot 3424 \text{ (422} \cdot 82 \\
 31 \\
 \hline
 132 \quad) \quad 412 \\
 \quad \quad 314 \\
 \hline
 1342 \quad) \quad 4341 \\
 \quad \quad 3234 \\
 \hline
 13443 \quad) \quad 110234 \\
 \quad \quad 101434 \\
 \hline
 140012) \quad 330024 \\
 \quad \quad 330024 \\
 \hline
 \end{array}$$

(74)

Suppose father's age = 60, the son's age now = $60 \div 5 = 12$,
 and son's age four years ago = $12 - 4 = 8$. But the son's
 age four years ago should, by the question, have been $60 \div$
 $7 = 8\frac{4}{7}$.

Therefore $8 - 8\frac{4}{7} = -\frac{4}{7} = \text{error}$.

Suppose father's age = 35; then son's age now = $35 \div 5 = 7$,
 and age four years ago = $7 - 4 = 3$.

But son's age four years ago should, by question, have been 35
 $\div 7 = 5$.

Therefore $3 - 5 = -2 = \text{error}$.

Errors.

$$-2 \times 60 = 120$$

$$-\frac{4}{7} \times 35 = 20$$

$$\text{diff. } 1\frac{3}{7} \text{ diff.} = 100$$

$$100 \div 1\frac{3}{7} = 70 = \text{father's and son's age} = 70 \div 5 = 14.$$

(75)

$$\cdot 72347 \div \cdot 0032 = \frac{72275}{99900} \div \frac{32}{9900} =$$

$$\frac{72275}{99900} \times \frac{11}{32} = \frac{795025}{3552} = 223 \cdot 82460585$$

(76)

Logarithm of 97294764.372 is 7.988089
 $7.988089 \div 11 = 0.726189$
 Log. 0.726189 = 5.32341 = 11th root of 97294764.372.

(77)

Assume $43\frac{1}{2}$ for the greater number

$$7\frac{1}{2} : 3\frac{1}{2} :: 43\frac{1}{2} : \frac{43\frac{1}{2} \times 3\frac{1}{2}}{7\frac{1}{2}} = 21 \text{ the less}$$

$$43\frac{1}{2} - 21 = 22\frac{1}{2} \text{ but it should} = 30$$

$$\text{Therefore error} = 22\frac{1}{2} - 30 = -7\frac{1}{2}.$$

Assume $72\frac{1}{2}$ for the greater number

$$7\frac{1}{2} : 3\frac{1}{2} :: 72\frac{1}{2} : \frac{72\frac{1}{2} \times 3\frac{1}{2}}{7\frac{1}{2}} = 35 = \text{the less}$$

$$72\frac{1}{2} - 35 = 37\frac{1}{2}, \text{ but it should} = 30$$

$$\text{Therefore error} = 37\frac{1}{2} - 30 = +7\frac{1}{2}.$$

Errors.

$$+ 7\frac{1}{2} \times 43\frac{1}{2} = 326\frac{1}{4}$$

$$- 7\frac{1}{2} \times 72\frac{1}{2} = 543\frac{3}{4}$$

$$\text{Sum} = 15 \quad \text{Sum} = 870$$

$$870 \div 15 = 58 \text{ greater}$$

$$58 \times 3\frac{1}{2}$$

$$7\frac{1}{2} : 3\frac{1}{2} :: 58 : \frac{58}{7\frac{1}{2}} = 28 \text{ less.}$$

(78)

Assume 35 | 35, 16, 18, 28, 62, 63, 40

Assume 16 | 16, 18, 28, 62, 63, 40

Assume 9 | 9, 31, 9

31

$$l. c. m. = 35 \times 16 \times 9 \times 31 = 156240.$$

(79)

Here $a = 1, d = 6, n = 101,$

$$s = \left\{ 2a + (n-1)d \right\} \frac{n}{2} = \left\{ 2 \times 1 + (101 - 1) \times 6 \right\} \frac{101}{2}$$

$$= (2 + 600) \frac{101}{2} = \frac{602 \times 101}{2} = 30401.$$

(80)

$$\frac{19}{7} \times \frac{11}{58} \times \frac{5}{121} \times \frac{117}{29} \times \frac{8}{44} \times \frac{47}{3} = \frac{117 \times 4 \times 5}{7 \times 7 \times 11 \times 3} \times \frac{2287}{1617} = 2287 : 1617.$$

(82)

$$\frac{\left\{ (9\frac{1}{2} + 4\frac{1}{2} + 3\frac{1}{2} - 16\frac{3}{4}) \times 54 \right\} \div 14}{\left\{ .97 \times .24378 \times (1\frac{1}{4} \times 4\frac{1}{2}) \right\} \times (4\frac{1}{7} - 2\frac{1}{7})}$$

$$= \frac{\left\{ (16\frac{3}{4} - 16\frac{3}{4}) \times 54 \right\} \div 14}{\frac{88 \times 3355 \times 11 \times 1850 \times (4\frac{5}{7} - 2\frac{1}{7})}{128 \times 71 \times 71 \times 3^2 \times 7} \times \frac{1}{187}}$$

$$= \frac{11 \times 151 \times 14 \times 1850 \times 387}{128 \times 187} = 187$$

(83)

Suppose the *hour* hand moves over 4 minutes, then since the minute hand moves 12 times as fast, it will have travelled over 48 minutes. But in order to overtake the hour hand, the minute hand must traverse the entire circle, 60 minutes, plus the 4 minutes we have supposed the hour hand to have moved forward, *i. e.* 64 minutes. Then 48 should equal 64, for we should find the same number by each process; $48 - 64 = -16$ error.

Suppose hour hand moves over 6 minutes, the minute hand moves over $6 \times 12 = 72$ minutes. But minute hand moves over $60 + 6 = 66$ minutes.

Then $72 - 66 = +6$ error.

(Continued on next page.)

120 +
he
=

Log

∴ Log

∴ Log

∴ Log

Log

Log

9

∴ Log

Simple

Amount

= 9

Int.

\$98,814

(83 continued.)

Errors.

$$- 16 \times 6 = 96$$

$$+ 6 \times 4 = 24$$

$$\text{Sum } 22 \quad \text{Sum } 120$$

$120 \div 22 = 5\frac{1}{11}$ min. = minutes passed over by the hour hand,
hence space passed over by the minute hand = $5\frac{1}{11} \times 12$
= $65\frac{5}{11}$ min. = 1 hour $5\frac{5}{11}$ min. = time.

(84)

$$\text{Log. } 5 = \text{log. } 10 - \text{log. } 2 = 1 - 0.301030 = 0.698970$$

$$3850000 = 5 \times 7 \times 11 \times 10000.$$

$$\therefore \text{Log. } 3850000 = \text{log. } 5 + \text{log. } 7 + \text{log. } 11 + \text{log. } 10000$$

$$= 0.698970 + 0.845098 + 1.041393 + 4 = 6.585461.$$

$$3181.81 = 31.81 \times 100 = 31\frac{81}{100} \times 100 = 31\frac{81}{100} \times 100 = 31\frac{81}{100} \times 100.$$

$$\therefore \text{Log. } 3181.81 = \text{log. } 5 + \text{log. } 7 + \text{log. } 1000 - \text{log. } 11$$

$$= 0.698970 + 0.845098 + 3 - 1.041393 = 3.502675$$

$$.0000154 = 2 \times 7 \times 11 \div 10000000$$

$$\therefore \text{Log. } .0000154 = \text{log. } 2 + \text{log. } 7 + \text{log. } 11 - \text{log. } 10000000$$

$$= 0.301030 + 0.845098 + 1.041393 - 7 = 5.187521.$$

$$\text{Log. } \frac{1}{7} = \text{log. } 1 - (\text{log. } 7 + \text{log. } 11) = 0 - (0.845098$$

$$+ 1.041393) = 0 - 1.886491 = 2.113509.$$

$$1.571428 = 1\frac{4}{7} = \frac{11}{7}.$$

$$\text{Log. } 1.571428 = \text{log. } 11 - \text{log. } 7 = 1.041393 - 0.845098$$

$$= 0.196295$$

$$93.17 = 9317 \div 100 = 11^2 \times 7 \div 100.$$

$$\therefore \text{Log. } 9317 = 3 \text{ times log. } 11 + \text{log. } 7 - \text{log. } 100 = 1.041393$$

$$\times 3 + 0.845098 - 2 = 1.969277.$$

EIGHTH SERIES.

(85)

$$\text{Simple Interest} = Prt = \$700 \times .045 \times 3 = \$94.50.$$

$$\text{Amount Compound Interest} = P(1+r)^t = \$700 \times (1.045)^3$$

$$= \$700 \times 1.14116 = \$798.814 - \$700 = \$98.814 = \text{Comp}$$

Int.

$$\$98.814 - \$94.50 = \$4.314.$$

(86)

X's gain = $\frac{1}{12}$, and Z's = $\frac{1}{12}$; \therefore Y's gain = $1 - (\frac{1}{12} + \frac{1}{12})$
 $= 1 - \frac{1}{6} = \frac{5}{6}$.

X's gain is $\frac{1}{12}$ for 3 months, therefore for 1 month it is $\frac{1}{36}$.

Y's gain is $\frac{5}{6}$ for 9 months, " " " $\frac{5}{108}$.

Z's gain is $\frac{1}{12}$ for 4 months, " " " $\frac{1}{36}$.

$\frac{1}{36} : \frac{1}{108} :: \$3024 : \$3024 \times \frac{1}{36} \times \frac{1}{3} = \$672 =$ X's stock.

$\frac{1}{36} : \frac{1}{108} :: \$3024 : \$3024 \times \frac{1}{36} \times \frac{1}{3} = \$1120 =$ Y's stock.

(87)

$$\frac{1}{2} \times \sqrt{17} \div (1\frac{1}{2})^3 = \frac{1}{2} \times \sqrt{17} \div (\frac{3}{2})^3 = \frac{1}{2} \times \frac{2}{3} \times \frac{2}{3} = \frac{2}{9}$$

(88)

| | | | |
|--------------------------------|---|----------|--------------------------|
| $4^2 = 16 \times 300$ | = | 4800 | 80677568161 (4321 cubert |
| $4 \times 3 = 12 \times 30$ | = | 360 | 64 |
| 3^2 | = | 9 | 16677 |
| | | 5169 | 15507 |
|
 | | | |
| $43^2 = 1849 \times 300$ | = | 554700 | 1170568 |
| $43 \times 2 = 86 \times 30$ | = | 2580 | |
| 2^2 | = | 4 | |
| | | 557284 | 1114568 |
|
 | | | |
| $432^2 = 186624 \times 300$ | = | 55987200 | 56000161 |
| $432 \times 1 = 432 \times 30$ | = | 12960 | |
| 1^2 | = | 1 | |
| | | 56000161 | 56000161 |

(89)

$$7 = \left\{ \begin{array}{l} 8 - 1 \\ \left. \begin{array}{l} 3+4 \\ 1+6 \end{array} \right\} \end{array} \right\} = 7$$

4 lbs. at 8d. }
 1 lb. at 4d. } Make a mixture of 6 lbs. at 7d.
 1 lb. at 6d. }

$$6 : 112 :: 4 : \frac{112 \times 4}{6} = 74\frac{2}{3} \text{ at 8d.}$$

(Continued on next page.)

Assum
 Since
 And 1

And 2
 +
 And 3
 +

Assum
 Since 1
 And 1s
 +
 And 2s
 +
 And 3s
 +

Diff

(89 continued.)

$$6 : 112 :: 1 : \frac{112 \times 1}{6} = 18\frac{2}{3} \text{ at 4d.}$$

$$6 : 112 :: 1 : \frac{112 \times 1}{6} = 18\frac{2}{3} \text{ at 6d.}$$

(90)

Assume 40 as the sum of the three numbers.

Since $1st + 2nd + 3rd = 40$,

And $1st + \frac{1}{2}(2nd + 3rd) = 34 \therefore \frac{1}{2}(2nd + 3rd) = 6 \dots \therefore 2nd + 3rd = 12$

And $2nd + \frac{1}{3}(1st + 3rd) = 34 \therefore \frac{2}{3}(1st + 3rd) = 6 \dots \therefore 1st + 3rd = 9$

And $3rd + \frac{1}{4}(1st + 2nd) = 34 \therefore \frac{3}{4}(1st + 2nd) = 6 \dots \therefore 1st + 2nd = 8$

Adding, $2 \times (1st + 2nd + 3rd) = 29$
 $\therefore 1st + 2nd + 3rd = 14\frac{1}{2}$.

But the sum should equal 40.
Hence $14\frac{1}{2} - 40 = -25\frac{1}{2}$.

Assume 48 as the sum of the three numbers.

Since $1st + 2nd + 3rd = 48$.

And $1st + \frac{1}{2}(2nd + 3rd) = 34 \therefore \frac{1}{2}(2nd + 3rd) = 14 \dots \therefore 2nd + 3rd = 28$

And $2nd + \frac{1}{3}(1st + 3rd) = 34 \therefore \frac{2}{3}(1st + 3rd) = 14 \dots \therefore 1st + 3rd = 21$

And $3rd + \frac{1}{4}(1st + 2nd) = 34 \therefore \frac{3}{4}(1st + 2nd) = 14 \dots \therefore 1st + 2nd = 18\frac{2}{3}$

Adding, $2 \times (1st + 2nd + 3rd) = 67\frac{1}{3}$
 $\therefore 1st + 2nd + 3rd = 33\frac{2}{3}$

But the sum should equal 48.
Hence $33\frac{2}{3} - 48 = -14\frac{1}{3} = \text{error}$.

Errors.

$$-25\frac{1}{2} \times 48 = 1224$$

$$-14\frac{1}{3} \times 40 = 566\frac{2}{3}$$

$$\text{Diff.} = 11\frac{1}{3} \quad \text{Diff.} = 657\frac{1}{3}$$

$$657\frac{1}{3} \div 11\frac{1}{3} = 58 = \text{the sum of the three numbers.}$$

(Continued on next page.)

(90 continued.)

$$\begin{aligned} 1st + \frac{1}{2}(2nd + 3rd) &= 34 \therefore \frac{1}{2}(2nd + 3rd) = 58 - 34 = 24 \\ \therefore 2nd + 3rd &= 48. \\ 2nd + \frac{1}{3}(1st + 3rd) &= 34 \therefore \frac{2}{3}(1st + 3rd) = 58 - 34 = 24 \\ \therefore 1st + 3rd &= 36. \\ 1st + 2nd + 3rd &= 58, \text{ and } 2nd + 3rd = 48 \therefore 1st = 10. \\ 1st + 2nd + 3rd &= 58, \text{ and } 1st + 3rd = 36 \therefore 2nd = 22. \\ 2nd + 3rd &= 48, \text{ and } 2nd = 22 \therefore 3rd = 26. \end{aligned}$$

(91)

4 means + 2 extremes = 6 terms.

$$\text{Formula IX, p. 333. } d = \frac{l - a}{n - 1} = \frac{40 - 1}{6 - 1} = \frac{39}{5} = 7\frac{4}{5}.$$

1, $8\frac{4}{5}$, $16\frac{8}{5}$, $24\frac{12}{5}$, $32\frac{16}{5}$, 40.

(92)

 $s = 1860040$, $l = 1240029$, and $r = 3$.

$$\begin{aligned} \text{Formula XI, p. 340. } a &= rl - (r - 1)s = 1240029 \times 3 \\ &- (2 \times 1860040) = 3720087 - 3720080 = 7. \end{aligned}$$

(93)

6 apples + 7 pears cost 33 pence \therefore 2 apples + $2\frac{1}{2}$ pears cost 11 pence.10 apples + 8 pears cost 44 pence \therefore 2 apples + $1\frac{1}{2}$ pears cost $8\frac{1}{2}$ pence.Subtract, and $2\frac{1}{2} - 1\frac{1}{2}$ pears cost 11d. - $8\frac{1}{2}$ d.That is, $\frac{1}{6}$ of a pear costs $2\frac{1}{2}$ d.If $\frac{1}{6}$ cost $\frac{1}{2}$ d., $\frac{1}{18}$ will cost $\frac{1}{18}$ of $\frac{1}{2}$ d., which is $\frac{1}{36}$ d.If $\frac{1}{18}$ cost $\frac{1}{36}$ d., $\frac{1}{9}$ will cost $\frac{1}{9}$ d. = 3d.6 apples + 7 pears cost 33 pence, and 7 pears cost 21d. \therefore 6 apples cost 12d. and 1 apple costs 2d.

(94)

$$\begin{aligned} & \frac{1}{4} \times \frac{1}{2} \times \frac{3}{5} \times \frac{57}{4} \times \frac{2}{3} \times \frac{4}{5} \times \frac{3}{4} \\ &= \frac{1}{4} \times \frac{3}{4} \times \frac{3}{5} \times \frac{57}{12} \times \frac{2}{3} \times \frac{4}{5} \times \frac{3}{4} = \frac{19}{2 \times 4 \times 3 \times 2} = \frac{19}{48} \end{aligned}$$

(95)

- 34 = 24

\$10 = $\frac{1}{2}$ of 2nd rem. - \$20 \therefore $\frac{1}{2}$ of 2nd rem. = \$30 \therefore 2nd rem. = \$40.

- 34 = 24

\$40 = $\frac{1}{2}$ of 1st rem. - \$30 \therefore $\frac{1}{2}$ of 1st rem. = \$70 \therefore 1st rem. = \$140.

1st = 10.

\$87.50 = $\frac{1}{2}$ of original sum - \$50 \therefore $\frac{1}{2}$ of original sum = \$137.50 \therefore original sum = \$137.50 \times 2 = \$275.

2nd = 22.

26.

(96)

$a = 60, n = 17,$ and $d = 4.$

Formula VI, p. 333. $s = \left\{ 2a + (n - 1)d \right\} \frac{n}{2}$

$= \left\{ 2 \times 60 + (17 - 1) \times 4 \right\} \frac{17}{2} = (120 + 64) \times \frac{17}{2}$

$= \frac{184 \times 17}{2} = \$1564 =$ sum received for 17 years.

Formula I, p. 333. $l = a + (n - 1)d = 60 + (17 - 1) \times 4 = 60 + 64 = \$124 =$ wages for 17th year.

40029 \times 3

NINTH SERIES.

(98)

£749 16s. 5 $\frac{1}{2}$ d. = £749.823958 ; £1 sterling = \$4 867
£749.823958 \times 4.867 = \$3549.3932.

pears cost

pears cost

(99)

2)177408

2)88704

2)44352

2)22176

2)11088

2)5544

2)2772

2)1386

3)693

3)231

7)77

11

$2^3 \times 3^2 \times 7 \times 11.$

21d. \therefore 6

$\frac{18}{2} = 9$

(100)

Formula III, page 354, $r = \sqrt[t]{\frac{A}{P}} - 1 \therefore r + 1 = \sqrt[t]{\frac{A}{P}}$

$\text{Log. } (r + 1) = (\text{log. } A - \text{log. } P) \div t$

That is, $\text{log. } (r + 1) = (\text{log. } 11111 \cdot 11 - \text{log. } 704) \div 11$
 $= (4 \cdot 045757 - 2 \cdot 847573) \div 11$
 $= 1 \cdot 198184 \div 11 = 0 \cdot 108925$

Therefore $r + 1 =$ natural number corresponding to the logarithm $0 \cdot 108925$ which is $1 \cdot 285$.

Since $r + 1 = 1 \cdot 285$, $r = \cdot 285 =$ rate per unit and rate per cent. $= \cdot 285 \times 100 = 28\frac{1}{2}$.

(101)

If 9 be $\frac{1}{3}$, $\frac{1}{3}$ or the whole will equal $9 \times 13 = 117$.

(102)

3 gal. + 4 gal. + 7 gal. = 14 gal.

Hence 14 gal. : 292 gal. :: 3 gal. : $\frac{292 \times 3}{14} = 62\frac{1}{2}$ of 1st kind.

14 gal. : 292 gal. :: 4 gal. : $\frac{292 \times 4}{14} = 83\frac{1}{2}$ gal. of 2d. "

14 gal. : 292 gal. :: 7 gal. : $\frac{292 \times 7}{14} = 146$ gal. of 3d. "

(103)

$\pounds\frac{1}{2} + \pounds\frac{1}{2} + \pounds\frac{1}{2} + \pounds\frac{1}{2} = \pounds 1\frac{1}{2}$

Then $\pounds 1\frac{1}{2} : \pounds 500 :: \pounds\frac{1}{2} : \pounds 500 \times \frac{1}{2} \times \frac{99}{77} = \frac{\pounds 15000}{77}$

$= \pounds 194 \text{ 16s. } 1\frac{1}{2}\text{d.}$

$\pounds 1\frac{1}{2} : \pounds 500 :: \pounds\frac{1}{3} : \pounds 500 \times \frac{1}{3} \times \frac{99}{77} = \frac{\pounds 10000}{77}$

$= \pounds 129 \text{ 17s. } 4\frac{1}{2}\text{d.}$

$\pounds 1\frac{1}{2} : \pounds 500 :: \pounds\frac{1}{4} : \pounds 500 \times \frac{1}{4} \times \frac{99}{77} = \frac{\pounds 7500}{77}$

$= \pounds 97 \text{ 8s. } 0\frac{1}{2}\text{d.}$

$\pounds 1\frac{1}{2} : \pounds 500 :: \pounds\frac{1}{5} : \pounds 500 \times \frac{1}{5} \times \frac{99}{77} = \frac{\pounds 6000}{77}$

$= \pounds 77 \text{ 18s. } 5\frac{1}{2}\text{d.}$

(104)

By Table, page 363, present value of annuity of \$1 at 6 per cent. for 23 payments = \$12.30338.

Hence present value of \$100 = \$12.30338 × 100 = \$1230.338.

By Formula V, page 361, $v = \frac{a}{r} \left\{ 1 - \frac{1}{(1+r)^t} \right\}$

$$= \frac{100}{.06} \times \left(1 - \frac{1}{(1.06)^{23}} \right) = \frac{10000}{6} \times (1 - 0.261795)$$

$$= \frac{10000}{6} \times 0.738205 = \frac{738205}{6} = \$1230.34$$

(105)

Since each loses 1 hour per day for 24 days, the whole hours lost = 24 × 25.

Also, 5 men working 1 hour per day for 12 days make up 5 × 12 × 1 = 60 hours.

Hence they will each have to work as many hours per day as 60 hours is contained times in 24 × 25 hours, i.e. $\frac{24 \times 25}{60} = 10$ hours.

(106)

$$a = 5, s = 161 \text{ and } d = 6$$

Then Formula II, p. 333. $l = -\frac{1}{2}d + \sqrt{2ds + (a - \frac{1}{2}d)^2} = -\frac{1}{2} \text{ of } 6 + \sqrt{2 \times 6 \times 161 + (5 - \frac{1}{2} \text{ of } 6)^2} = -3 + \sqrt{1932 + 4} = -3 + \sqrt{1936} = -3 + 44 = 41 \text{ years.}$

(107)

$$6^3 : 10^3 :: 1 \text{ day} : \frac{10^3 \times 1}{6^3} = \frac{1000}{216} = 4.629 \text{ days.}$$

* Log. $\frac{1}{(1.06)^{23}} = \text{log. } 1 - \text{log. } 1.06 \times 23 = 0 - 0.025306 \times 23 = 0 - 0.582038 = \bar{1}.417962$

$\frac{1}{(1.06)^{23}} =$ natural number corresponding to the logarithm

$\bar{1}.417962$, which is 0.261795

(108)

For 12 months he was to receive £8 and a suit of clothes; for 7 months he received £2 13s. 4d. and the suit of clothes; ∴ for 5 months he would have received the difference between £8 and £2 13s. 4d., which is £5 6s. 8d.

Hence for 1 month he would have received £5 6s. 8d. \div 5, which is £1 1s. 4d., and hence his wages for the year would have been, in money alone, £1 1s. 4d. \times 12, *i.e.*, £12 16s. Therefore the suit of clothes was valued at £12 16s. — £8 = £4 16s.

TENTH SERIES.

(109)

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}$; if $\frac{13}{12}$ of a number = 48, $\frac{1}{12}$ will = $48 \div 13 = 3\frac{2}{13}$. If $3\frac{2}{13} = \frac{1}{12}$, $\frac{13}{12}$, or the whole number = $3\frac{2}{13} \times 12 = 44\frac{1}{3}$.

(110)

$$6^3 : 8^3 :: 600 : \frac{600 \times 8^3}{6^3} = \frac{600 \times 512}{216} = 1422.2 \text{ lbs.}$$

(See Art. 33, sec. X.)

(111)

Part of ball remaining after 1st has taken off her share = $\frac{1}{4}$
Then whole ball : remainder :: cube of diameter of whole : cube of diameter of remainder

$$1 : \frac{1}{4} :: 5^3 : x^3 \text{ hence } x = \sqrt[3]{\frac{1}{4} \times 125} = \sqrt[3]{\frac{125}{4}} = \sqrt[3]{93.75} = 4.542$$

∴ Part taken off by 1st = 5 in. — 4.542 in. = 0.458 in.

After 2nd had taken off her portion $\frac{1}{2}$ of the ball remained.

$$1 : \frac{1}{2} :: 5^3 : x^3, \text{ hence } x = \sqrt[3]{\frac{1}{2} \times 125} = \sqrt[3]{\frac{125}{2}} = \sqrt[3]{62.5} = 3.968 \text{ in.}$$

∴ Part taken off by 2nd = 4.542 — 3.968 = 0.574 in.

After 3rd had taken off her share there remained $\frac{1}{4}$ of the ball.

$$1 : \frac{1}{4} :: 5^3 : x^3, \text{ hence } x = \sqrt[3]{\frac{1}{4} \times 125} = \sqrt[3]{31.25} = 3.149 \text{ in.}$$

∴ Part taken off by 3rd = 3.968 — 3.149 = 0.819 inches

Remainder = 3.149 = part taken off by 4th.

(113 continued).

| | | | |
|-----|---|----------------------|---------------------------|
| 3rd | } | \$400 × 48 = \$19200 | } = \$103200 for 1 month. |
| | | \$500 × 42 = 21000 | |
| | | \$500 × 36 = 18000 | |
| | | \$500 × 30 = 15000 | |
| | | \$500 × 24 = 12000 | |
| | | \$500 × 18 = 9000 | |
| | | \$500 × 12 = 6000 | |
| | | \$500 × 6 = 3000 | |

Sum = \$103200

| | | | |
|-----|---|----------------------|---------------------------|
| 4th | } | \$900 × 40 = \$36000 | } = \$138600 for 1 month. |
| | | \$900 × 34 = 30600 | |
| | | \$900 × 28 = 25200 | |
| | | \$900 × 22 = 19800 | |
| | | \$900 × 16 = 14400 | |
| | | \$900 × 10 = 9000 | |
| | | \$900 × 4 = 3600 | |

Sum = \$138600

\$43280

104400

103200

138600

4 years at \$1.25 per day

= \$1.25 × 4 × 365 = \$1825 = share of 5th.

\$389480 for one month.

\$20000 — \$1825 = \$18175 = sum to be divided among the four.

\$389480 : \$18175 :: \$43280 : \$2019.651 = share of 1st.

\$389480 : \$18175 :: \$104400 : \$4871.803 = " 2nd.

\$389480 : \$18175 :: \$103200 : \$4815.805 = " 3rd.

\$389480 : \$18175 :: \$138600 : \$6467.739 = " 4th.

(114)

Simple Interest, formula IX, p. 248. $t = \frac{n-1}{r} = \frac{16-1}{.05} = \frac{15}{.05}$
 $= \frac{1500}{5} = 300$ years.

Compound Interest, formula V, p. 354. $t = \frac{\log. n}{\log. (1+r)}$
 $= \frac{\log. 16}{\log. 1.05} = \frac{1.204120}{0.021189} = \frac{1204120}{21189} = 56.827$ years.

(115)

For every \$1 the first gave, the second gave \$3, and the third \$6. $\$1 + \$3 + \$6 = \10 .

Hence the 1st gave \$1, the second \$3, and the third \$6 as often as \$10 is contained times in \$9202, which is $920\frac{1}{2}$ times.

month.

$$\begin{aligned} \$1 \times 920\frac{1}{2} &= \$920 \cdot 20 = \text{payment of 1st person.} \\ \$3 \times 920\frac{1}{2} &= \$2760 \cdot 60 = \quad \quad \quad \text{2nd "} \\ \$6 \times 920\frac{1}{2} &= \$5521 \cdot 20 = \quad \quad \quad \text{3rd "} \end{aligned}$$

(116)

$25 + 22 = 47 =$ whole number of men.

$165 \div 47 = 3\frac{1}{4} =$ acres cleared by each man.

$3\frac{1}{4} \times 22 = 77\frac{1}{4}$ acres = acres cleared by company of 22 men.

165 acres — $77\frac{1}{4}$ acres = $87\frac{3}{4}$ acres = acres cleared by company of 25 men.

month.

1st company contains 3 more men than 2nd company and receives \$86 more.

Therefore \$86 pays 3 men. Hence each man gets $\$86 \div 3 = \$28 \cdot 66\frac{2}{3}$.

Each man clears $3\frac{1}{4}$ acres, and receives $\$28 \cdot 66\frac{2}{3}$ for it; therefore cost of 1 acre = $\$28 \cdot 66\frac{2}{3} \div 3\frac{1}{4} = \$84\frac{3}{4}$.

re of 5th.

(117)

$15^2 = 225$; $346 - 225 = 121 =$ square of the less.

Hence less = $\sqrt{121} = 11$.

g the four.

1st.

2nd.

3rd.

4th.

(118)

Formula V, page 248, $A = P(1 + rt) = \$1200 \times 1.95 = \$2340 \cdot 00$.

(119)

$$\begin{array}{l|l} 24 : 496 \\ 9 : 11 \\ 7 : 4 \\ 465 : 337\frac{1}{2} \\ 3\frac{3}{4} : 5\frac{3}{4} \\ 2\frac{1}{2} : 3\frac{1}{2} \end{array} \quad \begin{array}{l} \\ \\ \\ \therefore 5\frac{1}{2} \cdot x \\ \\ \end{array}$$

(Continued on next page.)

- r)

rs.

ELEVENTH SERIES.

(121)

$$\begin{aligned}
 \cdot 7 = \frac{7}{1} ; \cdot 83 = \frac{83}{100} ; \cdot 727 = \frac{727}{1000} ; \cdot 91325 = \frac{91325}{100000} = \frac{3653}{4000} = \frac{45667}{50000} \\
 8 \cdot 671347 = \frac{8671347}{1000000} = \frac{8671347}{1000000} = 8 \frac{111887}{125000}
 \end{aligned}$$

(122)

713 *unden.* = 861 *den.* ; 291 *unden.* = 342 *den.* ; 3f1 *unden.*
= 474 *den.*

291

Then 713 $\frac{291}{3f1}$ *unden.* = 861 $\frac{342}{3f1}$ *den.* = 861 $\frac{7}{3}$ *den.*

12123 *quat.* = 411 *den.* ; 11223 *quat.* = 363 *den.* ; 100000 *quat.*
= 1024 *den.*

Then 12123 $\frac{11223}{100000}$ = 411 $\frac{363}{1024}$ *den.*

(123)

3 $\frac{3}{4}$ of 2 $\frac{1}{2}$ of 7 $\frac{1}{2}$ of £1 = 2 $\frac{1}{2}$ of 1 $\frac{1}{2}$ of 1 $\frac{1}{2}$ of £1
= £1 $\frac{48817}{10000}$ = £56 1 2 $\frac{1}{10}$

9 $\frac{2}{3}$ of 3 $\frac{2}{3}$ of 1s. = 4 $\frac{2}{3}$ of 3 $\frac{2}{3}$ of 1s. = 13 $\frac{2}{3}$ s. = 1 16 8

8 $\frac{1}{2}$ of 4 $\frac{1}{2}$ of 1d. = 3 $\frac{3}{4}$ of 3 $\frac{3}{4}$ of 1d. = 10 $\frac{3}{4}$ d. = 0 2 10 $\frac{1}{2}$

Sum = £58 0 8 $\frac{21}{100}$

1 $\frac{1}{2}$ of $\frac{5}{4}$ of $\frac{3}{2}$ of 3 $\frac{1}{2}$ d. = 1 $\frac{1}{2}$ × $\frac{5}{4}$ × $\frac{3}{2}$ × $\frac{7}{2}$ = 7 $\frac{7}{8}$ d.

£58 0s. 8 $\frac{21}{100}$ d. = 2228501d.

$\frac{2228501}{100} \div \frac{55}{128} = \frac{2228501}{100} \times \frac{128}{55} = 202521 \times \frac{1}{2} = \frac{210364}{2}$
= 32414.56.

(124)

| | | | |
|------------------|-------------------|----|-----------------------|
| 24 | : 90 | | |
| 2 $\frac{1}{2}$ | : 4 $\frac{1}{5}$ | | |
| 12 $\frac{1}{2}$ | : 9 $\frac{2}{3}$ | :: | 139 $\frac{1}{2}$: x |
| 4 $\frac{7}{8}$ | : 4 $\frac{1}{2}$ | | |
| 3 $\frac{1}{2}$ | : 2 $\frac{1}{2}$ | | |

(Continued on next page.)

1 1
- × $\frac{1}{1}$
x 465
98
81

87
47
80 = $\frac{1}{2}$

87
27
80 = $\frac{1}{2}$

87
43
80 = $\frac{1}{2}$

87
15
80 = $\frac{1}{2}$

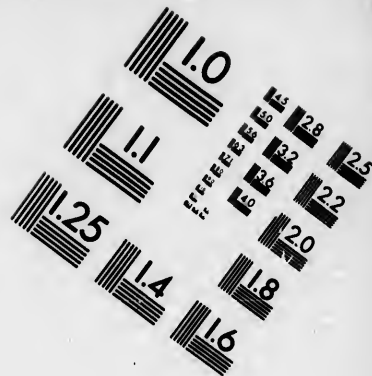
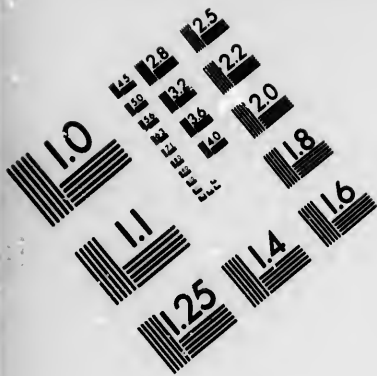
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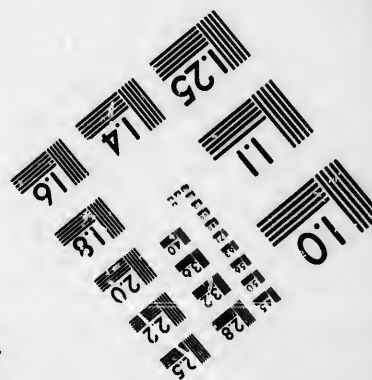
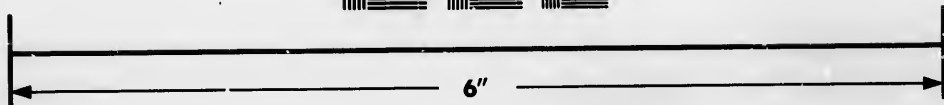
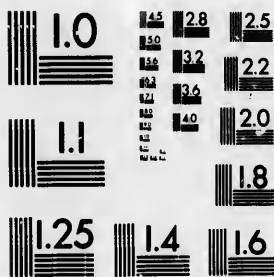
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(124 continued.)

$$\frac{9}{18} \times \frac{7}{21} \times \frac{3}{29} \times \frac{3}{2} \times \frac{5}{2} \times \frac{43}{552} \times \frac{1}{24} \times \frac{2}{5} \times \frac{2}{25} \times \frac{3}{32}$$

$$\times \frac{5}{16} = \frac{9 \times 7 \times 29 \times 43}{2 \times 4 \times 5 \times 4} = \frac{78561}{160} = 491\frac{1}{160}$$

(125)

\$182 is $\frac{91}{100}$ of buying price \therefore \$182 \div 91 = \$2 = $\frac{100}{100}$ of buying price \therefore buying price = \$2 \times 100 = \$200.
 To realize a profit of 7 per cent., he must receive \$1.07 for every \$1 the goods cost; but they cost him \$200, therefore he must sell for \$1.07 \times 200 = \$214.

(126)

Simple Interest $t = \frac{n-1}{r} = \frac{11\frac{1}{2} - 1}{.06} = \frac{10.5}{.06} = \frac{1050}{6}$
 $= 175$ years.

Compound Interest $t = \frac{\log. n}{\log. (1+r)} = \frac{\log. 11\frac{1}{2}}{\log. 1.06} = \frac{1.060698}{0.025306}$
 $= \frac{1060698}{25306} = 41.914$ years.

(127)

An acre contains 4 roods = 160 sq. perches.
 $\therefore 160 \div 15\frac{1}{2} = 10\frac{1}{11}$ perches = length.

(128)

35 yards = 32 metres \therefore 1 yd. = $\frac{32}{35}$ of a metre.
 $69\frac{1}{2}$ miles = $69\frac{1}{2} \times \frac{1760}{1}$ yards = $69\frac{1}{2} \times \frac{1760}{1} \times \frac{32}{35}$ metres

$$= \frac{217}{35} \times \frac{1760}{1} \times \frac{32}{35} = 217 \times 16 \times 32 = 111104$$
 metres.

(129)

r means + 2 extremes = 9 terms.

Formula XIII, p. 340. $r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{19683}{3}\right)^{\frac{1}{8}} = (6561)^{\frac{1}{8}} = 3$

Hence means are 9, 27, 81, 243, 729, 2187, and 6561.

(130)

Formula XXI, p. 344. $s = \frac{a}{1-r} = \frac{7}{1-\frac{1}{2}} = \frac{7}{\frac{1}{2}} = \frac{7}{1} \times \frac{2}{1} = 14$

(131)

Part remaining after 1st has received his share = $\frac{1}{4}$.

$1 : \frac{1}{4} :: 60^2 : x^2$; whence $x = \sqrt{3600 \times \frac{1}{4}} = \sqrt{900 \times 3}$
 $= 30\sqrt{3} = 1.732 \times 30 = 51.96$ inches.

Hence 1st ground off 60 - 51.96 = 8.04 inches.

Part remaining after 2nd had taken off his share = $\frac{1}{4}$.

$1 : \frac{1}{4} :: 60^2 : x^2$; whence $x = \sqrt{3600 \times \frac{1}{4}} = 30\sqrt{2}$
 $= 1.4142 \times 30 = 42.426$.

Hence 2nd ground off 51.96 - 42.426 = 9.534 inches.

Part remaining after the 3rd had taken off his share = $\frac{1}{4}$.

$1 : \frac{1}{4} :: 60^2 : x^2$; whence $x = \sqrt{3600 \times \frac{1}{4}} = \sqrt{900} = 30$ inches.

Hence 3rd ground off 42.426 - 30 inches = 12.426 inches,
 and the 4th ground off remaining 30 inches.

(132)

1 guinea = 21s.

1 half guinea = 10½s.

1 crown = 5s.

1 half crown = 2½s.

1 shilling = 1s.

Sixpence = ½s.

100 guineas = 2100 shillings.

2100 ÷ 40½ = 51 times and remainder, 69 half-shillings.

69 half-shil. = 34s. = £17 = 17s.

Sum = 40½s.

8
 × $\frac{8}{88}$
 18

of buy-

1.07 for
 0, there-

1050
 = $\frac{1050}{6}$

.060698

.025306

metres

metres.

TWELFTH SERIES.

(133)

$$\frac{3}{11} \text{ of } \frac{2}{9} \text{ of } \frac{4}{17} = \frac{8}{561}; \quad \frac{2\frac{1}{2}}{4\frac{1}{4}} \text{ of } \frac{2}{5} = \frac{10}{17} \text{ of } \frac{2}{5} = \frac{4}{17}$$

$$\frac{8}{561} : \frac{4}{17} :: \$12\frac{4}{3} : \$12\frac{4}{3} \times \frac{4}{17} \times \frac{561}{8} = \frac{200}{33} \times \frac{4}{17} \times \frac{561}{8}$$

$$= \$200.$$

(134)

By Formula III, page 354, $r = \sqrt[t]{\frac{A}{P}} - 1 \therefore r + 1 = \sqrt[t]{\frac{A}{P}}$

$$\therefore \text{Log. } (1 + r) = (\text{log. } A - \text{log. } P) \div t$$

$$= (\text{log. } 1679.40 - \text{log. } 700.90) \div 5$$

$$= (3.225154 - 2.845656) \div 5.$$

$$= 0.379498 \div 5 = 0.075899.$$

$\therefore 1 + r = \text{nat. num. corresponding to the logarithm } 0.075899$
which is 1.19, $\therefore r = .19 = \text{rate per unit, and hence rate}$
per cent. = 19.

(135)

Having paid 10 per cent. he had 90 per cent. remaining.

$$\frac{90}{100} \text{ or } \frac{9}{10} \text{ of his salary} = \$1250, \therefore \frac{1}{10} = \frac{1250}{10} = \$138\frac{2}{3}.$$

$$\text{If } \$138\frac{2}{3} = \frac{1}{10}, \text{ the whole} = \$138\frac{2}{3} \times 10 = \$1388.888.$$

(136)

21 children receive 21 times a child's share

21 women " 42 " "

21 men " 63 " "

Together they receive 126 " "

$$£3 \text{ } 13\text{s. } 6\text{d.} \div 126 = 7\text{d.} = \text{a child's share.}$$

$$7\text{d.} \times 2 = 1\text{s. } 2\text{d.} = \text{a woman's share.}$$

$$7\text{d.} + 1\text{s. } 2\text{d.} = 1\text{s. } 9\text{d.} = \text{a man's share.}$$

(137)

- A gets 1 time A's share
- B " 1 " A's "
- C " 2 " A's "
- D " 4 " A's "

Together they get 8 times A's share.

$$\$200 \div 8 = \$25 = \text{A's share}; \$25 = \text{B's share.}$$

$$\$25 + \$25 = \$50 = \text{C's share}; \$25 + \$25 + \$50 = \$100 = \text{D's share.}$$

(138)

$$\sqrt[3]{\frac{1}{2}} = \frac{1}{2} \sqrt[3]{12} = \frac{1}{2} \text{ of } 2.62074 = .87358$$

$$\sqrt[3]{\frac{1}{3}} = \frac{1}{3} \sqrt[3]{6} = \frac{1}{3} \text{ of } 2.44948 = .81649$$

$$\text{Difference} = .05709$$

(139)

92807 when each term is divided by 121, becomes 767 .

$$17\frac{5}{12} + \frac{1}{8} + 144\frac{1}{12} = 161 + \frac{5}{12} + \frac{1}{8} + \frac{1}{12} = 161 + \frac{11}{24} + \frac{1}{12}$$

$$+ \frac{1}{12} = 161 + \frac{11}{24} = 161 + \frac{11}{24} = 162\frac{11}{24} = 162\frac{11}{24}$$

$$2\frac{1}{3} - \frac{1}{12} = 2\frac{4}{12} - \frac{1}{12} = 1\frac{3}{12} = 1\frac{1}{4}$$

$$\frac{3}{4} \text{ of } \frac{6}{15} \text{ of } \frac{4}{11} \text{ of } \frac{15}{23} \text{ of } \frac{21}{253} = \frac{54}{253}$$

$$6347 \div 2\frac{1}{4} = 2539 \div \frac{1}{4} = 2539 \times 4 = 2308.$$

(140)

$$884736 \text{ (96 = cube root.)}$$

$$729$$

$$155736$$

$$9^2 = 81 \times 300 = 24300$$

$$9 \times 6 = 54 \times 30 = 1620$$

$$6^2 = 36$$

$$25956$$

$$155736$$

$$95951161 = 95951 \cdot 2576.$$

(Continued on next page.)

(140 continued.)

95951.2576 (309.76 = square root.

9

609) 5951
5481

309.76 (17.6 = 17 1/2 = fourth root.
1

618.7)470.25
433.09

27)209
189

619.46)37.1676
37.1676

34.6)2076
2076

(141)

250
300
400
500

$$1450:250::\$520:\frac{\$520 \times 250}{1450} = \$89\frac{1}{3} = \text{contrib. on 1st village.}$$

$$1450:300::\$520:\frac{\$520 \times 300}{1450} = \$107\frac{1}{3} = \text{ " 2nd "}$$

$$1450:400::\$520:\frac{\$520 \times 400}{1450} = \$143\frac{1}{3} = \text{ " 3rd "}$$

$$1450:500::\$520:\frac{\$520 \times 500}{1450} = \$179\frac{1}{3} = \text{ " 4th "}$$

(142)

By Table on p. 362, the amount of \$1 for 34 payments at 3 per cent. = \$57.73018.

$$\$57.73018 \times 260 = \$15009.84.$$

By Formula I, page 361, $A = \frac{a \{ (1+r)^t - 1 \}}{r}$

$$= \frac{a}{r} \{ (1+r)^t - 1 \} = \frac{260}{.03} \{ (1.03)^{34} - 1 \}$$

$$= \frac{26000}{3} \times (2.731855 - 1) = \frac{26000 \times 1.731855}{3} = \$15009.41$$

(143)

$$\text{By Formula IX, p. 333, } d = \frac{l-a}{n-1} = \frac{79-2}{6-1} = \frac{77}{5} = 15\frac{2}{5}.$$

Hence the series is 2, $17\frac{2}{5}$, $32\frac{4}{5}$, $48\frac{6}{5}$, $63\frac{8}{5}$, and 79.

$$\text{Formula I, p. 333. } l = a + (n-1)d = 3 + (9-1) \times 4 \\ = 3 + (8 \times 4) = 3 + 32 = 35.$$

$$\text{Formula VI, p. 333. } s = \left\{ 2a + (n-1)d \right\} \frac{n}{2} \\ = \left\{ 2 \times 3 + (207-1) \times 4 \right\} \frac{207}{2} = \left\{ 6 + (206 \times 4) \right\} \frac{207}{2} \\ = (6 + 824) \times \frac{207}{2} = \frac{830 \times 207}{2} = 85905.$$

(144)

B-travels 4 miles per-day faster than A, and will therefore gain the circumference of the island in $7\frac{3}{4} = 18\frac{1}{4}$ days.

C-travels 10 miles per day faster than A, and will therefore gain the whole circumference of the island in $7\frac{3}{10} = 7\frac{3}{10}$ days.

Now B cannot be with A except at the end of $18\frac{1}{4}$ days or twice $18\frac{1}{4}$ days, or three times $18\frac{1}{4}$ days, or some other multiple of $18\frac{1}{4}$ days.

Similarly C cannot be with A except at the end of $7\frac{3}{10}$ days, or of some other multiple of $7\frac{3}{10}$ days.

Therefore C and B will both be with A for the first time after the lapse of a number of days expressed by the least common multiple of $18\frac{1}{4}$ and $7\frac{3}{10}$.

The greatest common factor of $18\frac{1}{4}$ and $7\frac{3}{10}$ is $3\frac{3}{20}$.

Hence the l. c. m. of $7\frac{3}{10}$ and $18\frac{1}{4}$ is $\frac{7\frac{3}{10} \times 18\frac{1}{4}}{3\frac{3}{20}} = 36\frac{1}{2} = \text{number}$

of days when A, B, and C will first be together.

ARITHMETICAL RECREATIONS.

1. The third of 6 = 2, and the fourth of 20 = 5.
Then if 2 becomes 3, what should 5 become? Evidently
7½. *Ans.*

or

$$\left. \begin{array}{l} 6 : 20 \\ \frac{1}{3} : \frac{1}{4} \end{array} \right\} :: 3 : x = \frac{3 \times 20 \times \frac{1}{4}}{6 \times \frac{1}{3}} = 7\frac{1}{2}.$$

2. The half of 5 = 2½; then if 7 becomes 2½, what will 11 become?

$$\frac{2\frac{1}{2} \times 11}{7} = 4\frac{1}{4}. \text{ Lastly, what part of 9 is } 4\frac{1}{4}?$$

$$\frac{4\frac{1}{4}}{9} = \frac{55}{126}. \text{ *Ans.*}$$

or

$$\left. \begin{array}{l} 9 : 5 \\ 7 : 11 \end{array} \right\} :: \frac{1}{2} : x = \frac{\frac{1}{2} \times 5 \times 11}{9 \times 7} = \frac{55}{126} = 4\frac{1}{4}. \text{ *Ans.*}$$

3. 99½.

4. ½ of 2d. = ½d. Then ½d. is what part of 3d.? *Ans.* ⅓.
5. 1½d. for a herring and a half is at the rate of 1d. per herring; hence 11 herrings will cost 11d.
6. 12 apples = 21 pears = 7 cents.

If 12 apples cost 7 cents, what will 100 apples cost?

$$12 : 100 :: 7 : \frac{100 \times 7}{12} = 58\frac{1}{3} \text{ cents.}$$

7. If 5 is ⅔ of a certain number, ⅓ will be ⅓ of 5, which is ⅕.
If ⅕ is ⅓ of a certain number, the whole number will be
⅕ × 7 = 7⅕ = 11⅓. *Ans.*

8. The hurdles are arranged so as to form a rectangular enclosure having 49 hurdles on each side and one on each end. Two additional hurdles will give two hurdles to each end, and will thus double the size of the enclosure.

9. The mode of dividing the plot may be learned from the following figure:—

vidently

will 11

Ans.

ns. 3.

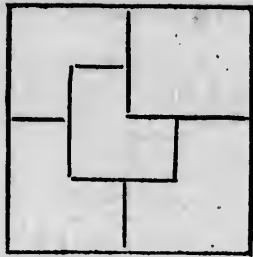
er her-

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ch is 3.
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lar en-
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dles to
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om the



- 10. $33\frac{1}{3}$
- 11. XIII; rub out the lower half, and there remains the expression VIII = 8.
- 12. 1st Step: Fill the 3-gallon cask and empty it into the 5-gallon cask.
- 2nd Step: Again fill the 3-gallon cask out of the 8-gallon cask.
- 3rd Step: Fill up the 5-gallon cask out of the 3-gallon cask. This will leave one gallon in the latter.
- 4th Step: Empty the 5-gallon cask into the 8-gallon cask.
- 5th Step: Pour the one gallon out of the 3-gallon cask into the 5-gallon cask.
- 6th Step: Fill the 3-gallon cask out of the 8-gallon cask, and empty it into the 5-gallon cask.

The following diagrams show this more clearly :

1st Step.

2nd Step.



3rd Step.

4th Step.

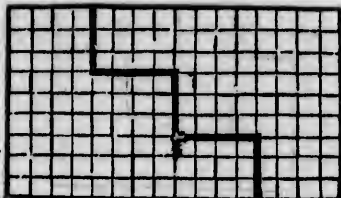


5th Step.

6th Step.



13. The heavy line in the accompanying figure shows how the board is to be cut.



| | | |
|---|---|---|
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

15. Weigh out 7 lbs. as often as possible and there will remain 2 lbs.; add two four pounds and one seven pounds to this, and the sum will be 17 lbs., the share of one. Weigh 7 lbs. as often as possible out of the remaining 34 lbs. and there will remain 6 lbs., to which add 7 lbs. and 4 lbs., and the sum will be 17 lbs., the share of the second. The remaining 17 lbs. will be the share of the third.
16. The hurdles are, in the first case, placed 12 on a side and one on each end, and then they inclose a space represented by 12 squares whose area is, by the question, 40 square yards. If two hurdles be taken away there will remain 24, and if these be placed in the form of a square, each side containing 6 hurdles, they will enclose a space represented by 36 squares of the same size as the former. Hence they now inclose three times as much space as before, *i. e.* three times 40 square yards, or 120 square yards.
17. He takes the goose to the remote bank and leaves it there, returning, he next carries over the fox, which he leaves, but takes the goose back with him. He now leaves the goose on the first bank, and carries over the oats which he allows to remain on the remote bank with the fox, and returns for the goose.
18. The following diagrams exhibit the solution of this problem:

s how the

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|-----|------|-----|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| I. | II. | III. | IV. | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>3</td><td>3</td><td>3</td></tr><tr><td>3</td><td>P</td><td>3</td></tr><tr><td>3</td><td>3</td><td>3</td></tr></table> | 3 | 3 | 3 | 3 | P | 3 | 3 | 3 | 3 | <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>4</td><td>1</td><td>4</td></tr><tr><td>1</td><td>P</td><td>1</td></tr><tr><td>4</td><td>1</td><td>4</td></tr></table> | 4 | 1 | 4 | 1 | P | 1 | 4 | 1 | 4 | <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>2</td><td>5</td><td>2</td></tr><tr><td>5</td><td>P</td><td>5</td></tr><tr><td>2</td><td>5</td><td>2</td></tr></table> | 2 | 5 | 2 | 5 | P | 5 | 2 | 5 | 2 | <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>1</td><td>7</td><td>1</td></tr><tr><td>7</td><td>P</td><td>7</td></tr><tr><td>1</td><td>7</td><td>1</td></tr></table> | 1 | 7 | 1 | 7 | P | 7 | 1 | 7 | 1 |
| 3 | 3 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | P | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 3 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 1 | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | P | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 1 | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | 5 | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | P | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | 5 | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 7 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 7 | P | 7 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 7 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 24 | 20 | 28 | .. | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

| | | | | | | | | | | | | | | | | | | | |
|---|-----|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| V. | VI. | | | | | | | | | | | | | | | | | | |
| <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>0</td><td>9</td><td>0</td></tr><tr><td>9</td><td>P</td><td>9</td></tr><tr><td>0</td><td>9</td><td>0</td></tr></table> | 0 | 9 | 0 | 9 | P | 9 | 0 | 9 | 0 | <table border="1" style="display: inline-table; border-collapse: collapse;"><tr><td>5</td><td>0</td><td>4</td></tr><tr><td>0</td><td>P</td><td>0</td></tr><tr><td>4</td><td>0</td><td>5</td></tr></table> | 5 | 0 | 4 | 0 | P | 0 | 4 | 0 | 5 |
| 0 | 9 | 0 | | | | | | | | | | | | | | | | | |
| 9 | P | 9 | | | | | | | | | | | | | | | | | |
| 0 | 9 | 0 | | | | | | | | | | | | | | | | | |
| 5 | 0 | 4 | | | | | | | | | | | | | | | | | |
| 0 | P | 0 | | | | | | | | | | | | | | | | | |
| 4 | 0 | 5 | | | | | | | | | | | | | | | | | |
| 36 | 18 | | | | | | | | | | | | | | | | | | |

19. XII; rub out the lower half, and VII remains;
 20.

| | | | | |
|----|----|----|----|----|
| 17 | 24 | 1 | 8 | 15 |
| 23 | 5 | 7 | 14 | 16 |
| 4 | 6 | 13 | 20 | 22 |
| 10 | 12 | 19 | 21 | 3 |
| 11 | 18 | 25 | 2 | 9 |

RULE FOR FILLING MAGIC SQUARES OF ODD NUMBER OF CELLS.

Begin in centre cell of top horizontal row by placing 1 in it; ascend diagonally to the right, and where this carries us beyond the square, transport the next number to the cell at the remote end of the vertical or horizontal band to which it belongs. When in ascending we come to a cell already filled, we place the number in the cell next below the cell last filled. The following is a square of 7 cells in a side filled after this method :

(Continued on next page.)

problem:

| | | | | | | |
|----|----|----|----|----|----|----|
| 30 | 39 | 48 | 1 | 10 | 19 | 28 |
| 38 | 47 | 7 | 9 | 18 | 27 | 29 |
| 46 | 6 | 8 | 17 | 26 | 35 | 37 |
| 5 | 14 | 16 | 25 | 34 | 36 | 45 |
| 13 | 15 | 24 | 33 | 42 | 44 | 4 |
| 21 | 23 | 32 | 41 | 43 | 3 | 12 |
| 22 | 31 | 40 | 49 | 2 | 11 | 20 |

21. Half-a-dozen dozen = $6 \times 12 = 72$.
 Six dozen dozen = $6 \times 12 \times 12 = 864$.
 $864 - 72 = 792$. *Ans.*

22. The following shows the mode of performing this.
 It will be observed that the two side counters are merely moved one counter higher when the other two are taken away.

0
 00
 0
 0
 0
 0
 0
 0

23. This problem admits of the following two solutions:

1ST SOLUTION.

| Persons. | Full bottles. | Hf.-full bottles. | Empty bottles. |
|----------|---------------|-------------------|----------------|
| 1st | 2 | 3 | 2 |
| 2nd | 2 | 3 | 2 |
| 3rd | 3 | 1 | 3 |
| | <hr/> | <hr/> | <hr/> |
| | 7 | 7 | 7 |

Each person has $3\frac{1}{2}$ bottles of wine and 7 bottles.

2ND SOLUTION.

| | | | |
|-----|-------|-------|-------|
| 1st | 3 | 1 | 3 |
| 2nd | 3 | 1 | 3 |
| 3rd | 1 | 5 | 1 |
| | <hr/> | <hr/> | <hr/> |
| | 7 | 7 | 7 |

Each person, as before, has 7 bottles and $3\frac{1}{2}$ bottles of wine.

- 24. There were in all 8 bottles of wine, of which each drank $\frac{1}{4}$, which is $2\frac{1}{2}$. The third person, therefore, drank $\frac{1}{4}$ of a bottle belonging to him who had but 3 bottles, and $\frac{1}{4}$ of a bottle belonging to him who owned the 5 bottles. Hence the latter should have *seven* times as much of the money as the former, or, in other words, the latter gets 7 shillings, and the former 1 shilling.
- 25. This problem is merely to find some number between 50 and 100 which is exactly divisible by 2 and by 3, but which divided by 5 leaves a remainder 3.

The only numbers between 50 and 100 that are divisible by both 2 and 3, are 54, 60, 66, 72, 78, 84, 90, and 96, and by inspection the only one of these which gives a remainder 3 when divided by 5 is 78; therefore the basket contained 78 eggs.

- 26. *Ans.* 1 lb., 3 lbs., 9 lbs., and 27 lbs.
 For 1 lb. = 1 lb.; 2 lbs. = 3 lbs. --- 1 lb., i. e. 3 lbs. in one scale and 1 lb. in the other; 3 lbs. = 3 lbs.; 4 lbs. = 3 lbs. + 1 lb.; 5 lbs. = 9 lbs. --- (3 lbs. + 1 lb.); 6 lbs. = 9 lbs. --- 3 lbs.; 7 lbs. = 9 lbs. + 1 lb. --- 3 lbs.; 8 lbs. = 9 lbs. --- 1 lb.; 9 lbs. = 9 lbs.; 10 lbs. = 9 + 1 lb.; 11 lbs. = 9 lbs. + 3 lbs. --- 1 lb.; 12 lbs. = 9 lbs. + 3 lbs.; 13 lbs. = 9 lbs. + 3 lbs. + 1 lb.; 14 lbs. = 27 lbs. --- (9 lbs. + 3 lbs. + 1 lb.); 15 lbs. = 27 lbs. --- (9 lbs. + 3 lbs.); 16 lbs. = 27 lbs. + 1 lb. --- (9 lbs. + 3 lbs.); 17 lbs. = 27 lbs. --- (9 lbs. + 1 lb.); 18 lbs. = 27 lbs. --- 9 lbs.; &c., &c.

- 27. In order to fill seven out of the eight points, it is merely requisite to remember that the second counter must be carried to the point from which the first *started*, the third to the point from which the second started, &c.

Thus if the first counter is carried from 1 to 4 and there deposited, the second must be taken from 6 to 1 and there deposited; the third from 3 to 6; the fourth from 8 to 3; the fifth from 5 to 8; the sixth from 2 to 5; and the seventh either from 7 to 2 or from 2 to 7.

- 28. The mouth fills the reservoir in 6 hours, therefore it fills $\frac{1}{6}$ in 1 hour; the right eye fills it in 38 hours, therefore it fills

0
000
is. 0
are 0
er 0
0
0
0
s: 0

y bottles.
2
2
3
7
les.

3
3
1
7
s of wine.

$\frac{1}{48}$ in 1 hour; the left eye fills it in 72 hours, therefore it fills $\frac{1}{72}$ in 1 hour; the foot fills it in 96 hours, therefore it fills $\frac{1}{96}$ in 1 hour. Hence together they fill $\frac{1}{48} + \frac{1}{72} + \frac{1}{96}$ in 1 hour, and to fill the reservoir they require $1 \div \frac{1}{288} = \frac{288}{1} = 4$ hours 43 min. $16\frac{2}{3}$ sec.

29. The person who thinks of the numbers must proceed as follows: He must multiply the 1st by 2 and add 5 to the product; he must next multiply this sum by 5 and add the second number to the product; he must next multiply this result by 10 and add the third number to the product; lastly, he must subtract 250 and name the remainder.

The three digits of the remainder will be the three numbers thought of, and will be in the order in which they were thought of.

The reason is obvious: let $a = 1$ st, $b = 2$ nd, and $c = 3$ rd number thought of.

$$a \times 2 + 5 = 2a + 5.$$

$$(2a + 5) \times 5 + b = 10a + b + 25.$$

$$(10a + b + 25) \times 10 + c = 100a + 10b + c + 250.$$

$$(100a + 10b + c + 250) - 250 = 100a + 10b + c = a \text{ in hundreds' place, } b \text{ in tens' place, and } c \text{ in units' place.}$$

30. Since each man possesses 63 square rods of land more than his son, we must form three pairs of numbers, such that the difference of their squares shall be 63.

The difference of the squares of two numbers is equal to their sum multiplied by their difference, and hence 63 must be divided into two factors in three distinct ways, thus:

$$63 = 63 \times 1 = 21 \times 3 = 9 \times 7.$$

If sum = 63 and difference = 1, the numbers are 32 and 31.

If sum = 21 and difference = 3, the numbers are 12 and 9.

If sum = 9 and difference = 7, the numbers are 8 and 1.

Hence the squares of Jones, Brown, and Smith, are respectively 32 rods, 12 rods, and 8 rods on the side, and the son's squares are respectively 31, 9, and 1 yards on the side.

Jones' piece was 23 rods longer on each side than Tom's, and since the difference between 32 and 9 is 23, we may conclude that Jones' square was 32 rods to the side, and Tom's 9 rods on a side.

before it
before it
 $\frac{1}{2} + \frac{1}{2}$
require

red as
to the
add the
only this
product;
r.
numbers
y were
= 3rd

50.
+ c =
place.
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o their
ust be
:
and 31.
and 9.
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e son's
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s, and
y con-
Tom's

Brown's piece was 11 rods longer on a side than Harry's, and since if the above numbers 12 and 1 have 11 for their difference, we may conclude that Brown's piece was 12 rods on a side, and Harry's piece 1 rod.

Hence Tom was Brown's son, Harry was Smith's son, and Ned was Jones' son

31. The mode of arranging the crew may be remembered by attention to the vowels in the following line.

Populeam virgam mater regina ferebat.

The vowels refer to the crew as follows, $a = 1$, $e = 2$, $i = 3$, $o = 4$, and $u = 5$.

We begin with 4 whites because the first vowel is o , next $u = 5$ blacks, next $e = 2$ whites, next $a = 1$ black, next $i = 3$ whites, next $a = 1$ black, next $a = 1$ white, next $e = 2$ blacks, next $e = 2$ whites, next $i = 3$ blacks, &c., as follows, o standing for a white and $+$ for a black.

oooo++++oo+oo+o++++o++++

32. You select the multiplier or the multiplicand, such that the sum of its digits shall be exactly divisible by nine. Hence upon the principle of the proof by casting out the nines, the product has the sum of its digits exactly divisible by nine. By subtracting the sum of the digits of the remainder from the next higher multiple of 9 you determine the digit crossed out.

Thus suppose you select 117, and he takes for multiplicand 21613. Then $21613 \times 117 = 2528721$. Now, suppose he crosses out the 7; upon reading you the remaining digits 252821, you find that their sum = 20, which taken from 27 the next higher multiple of 9 leaves 7 the digit he crossed out.

If he crosses out a 0 or a 9, you cannot determine which, but in all other cases you can tell the exact figure.

33. You write the second, fourth, sixth, &c. lines in such a manner as to make the sum of the first pair, the sum of the second pair, &c. an exact number of 9's. Then having settled the number of pairs, you get the answer by multiplying by that number a row of 9's containing as many digits as there are to be figures in the line.

Thus suppose you agree to write 5 lines each, and that each line is to contain 5 digits, or not more than 5 digits. Then $99999 \times 5 = 499995$ will be the answer. This is shown as follows :

| | | | | | | |
|---|-------|---|---|--------|---|--------------|
| Suppose he writes | 41113 | } | = | 99999 | } | |
| You write | 53886 | } | | | | |
| Suppose he writes | 61451 | } | = | 99999 | } | |
| You write | 38548 | } | | | | |
| Suppose he writes | 6500 | } | = | 99999 | } | = 99999 × 5. |
| You write | 93499 | } | | | | |
| Suppose he writes | 1 | } | = | 99999 | } | |
| You write | 99998 | } | | | | |
| Suppose he writes | 99999 | } | = | 99999 | } | |
| You write | 00000 | } | | | | |
| <hr style="width: 100px; margin: 0 auto;"/> | | | | | | |
| Sum = | | | | 499995 | | |

THE END.

ARITH

t each
Then
wn as

x 5.

