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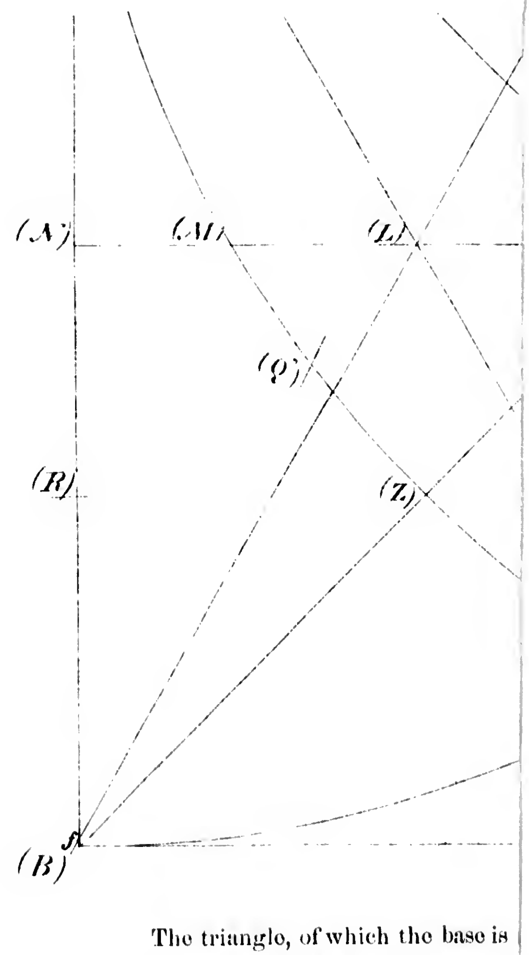

The triangle, of which the base is
are likewise equilateral).

*Because th

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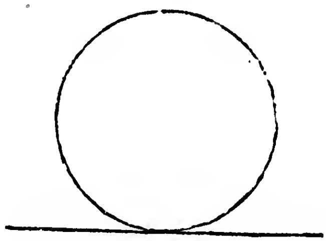
SUPPLEMENTARY ILLUSTRATIONS.



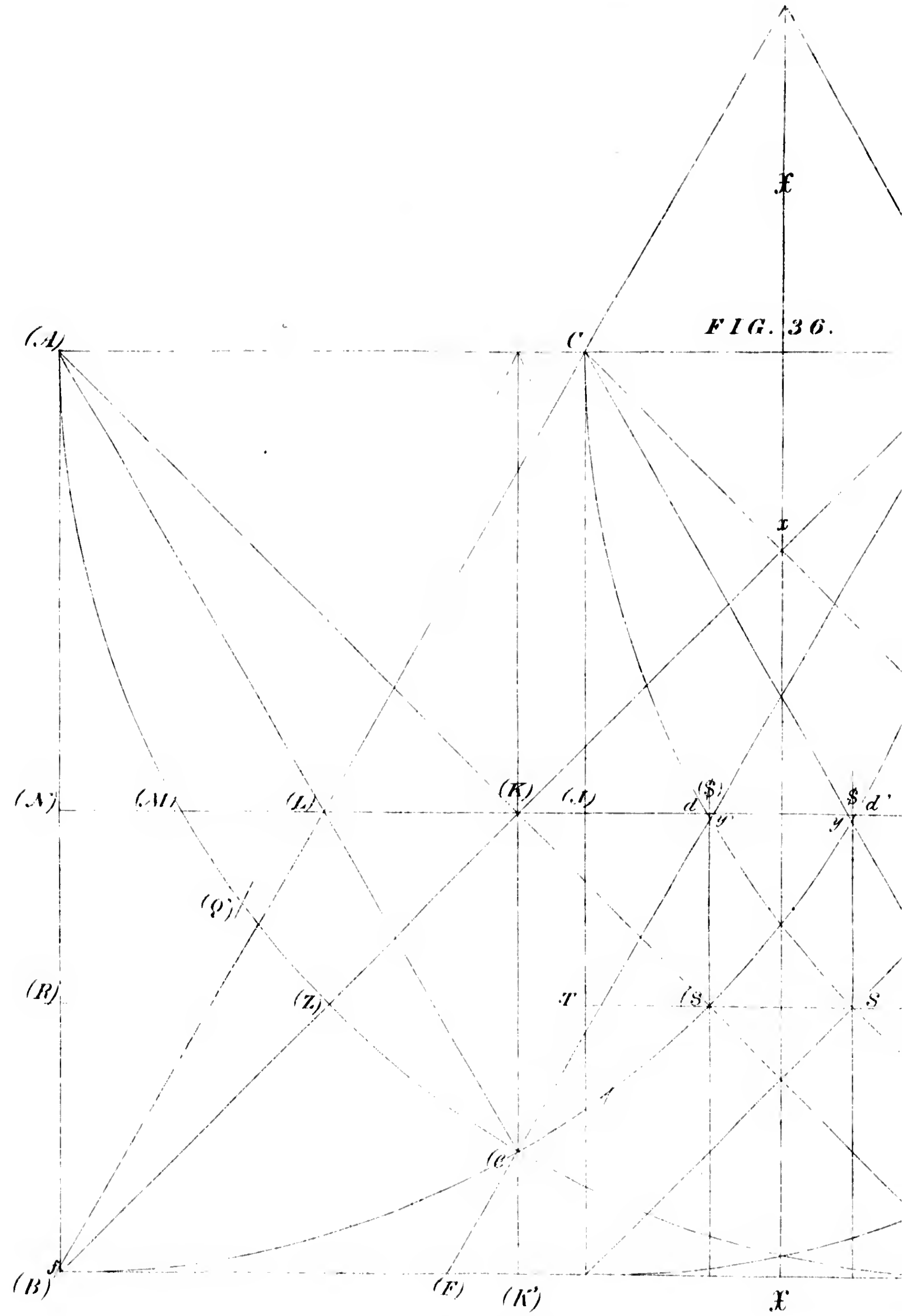
BY

JOHN HARRIS.

MONTREAL:
JOHN LOVELL, ST. NICHOLAS STREET.

MAY, 1874.

FIG. 36.

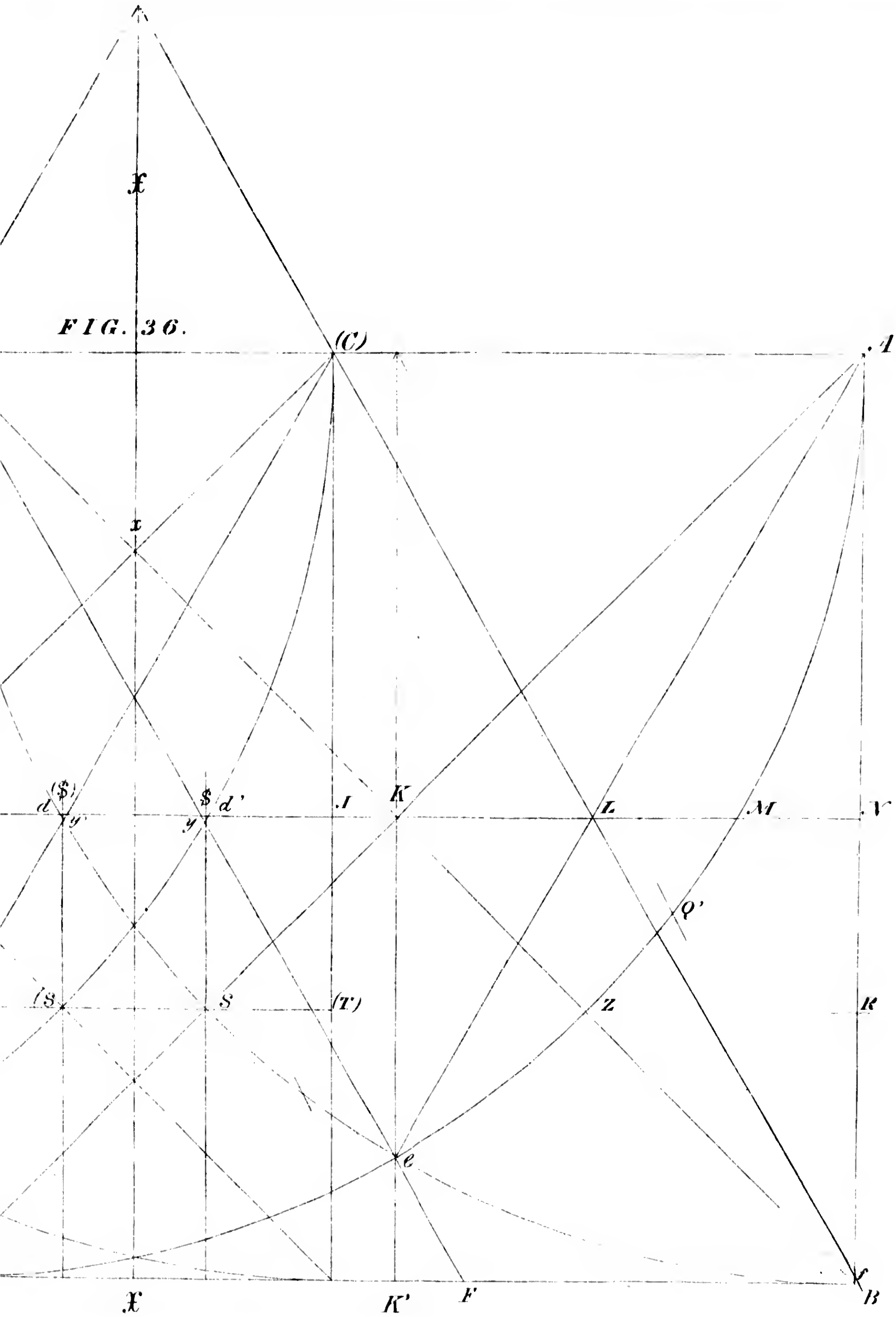


The triangle, of which the base is (F.) F., bisected by the line X. X., is an equilateral triangle, are likewise equilateral).* And the triangle C. F. *f*., is also equilateral; and the inve

See Analysis of Fig. **31**, ***pag***

*Because the sine of the arc of 30° B. *e*., equals half the radius, the triangle, of whi

FIG. 36.



. X., is an equilateral triangle, (and therefore the several triangles, which are parts thereof, also equilateral; and the inverted triangle e. C. A., is equilateral, &c., &c.

See Analysis of Fig. 31, *page* **11.**

half the radius, the triangle, of which the base is (e.) e., is manifestly equilateral.

Cop 1

MONTREAL.—JOHN LOVELL, PRINTER.

Analytical Examination by Fig. 23.–(R.)

The Arc of 30 *degrees* $(\frac{C}{12})$. *The Octuple-arc B.Q.** (*or B.K. of Fig.* 24.) *And the line B.b., which is a production of the chord of the octuple arc.*

The arc of 30 degrees B.e., † or D.e., has its sine equal to 5·0000.

If the Octuple arc B.Q. be bisected, and the secant to the half-arc be drawn, the line A.O.K.' so drawn shall intersect the line B.b. in the point bisecting the chord of the Octuple arc, and the line A.O.K.' shall be (manifestly) equal and similar to the line B.b. (Because B.b. bisects C.D.,and A.O.K. bisects B.D.)

The secant A.O.K.' to the half-octuple arc, bisects the tangential line B.D., and consequently the tangent to the half-octuple arc is also equal to 5·0000. And equal to the sine of the arc of 30 degrees.

We have therefore the arc S.e., equal to half of the arc B.e. which is one third of the quadrant B.C.; and also the sine of B.e. equal to one half of B.D. the tangent to the half-quadrant, and equal to the tangent of half the octuple arc.

The Cosecant of the arc of 30 degrees equals 20·0000 (i.e., twice the radius);

The Tangent = 5·7735...
" Cosine = 8·66025...
" Cotangent = 17·3205... (i.e., twice the Cosine.)

* We have adopted this term for the moment to distinguish this arc of which the sine is to the tangent of the half-quadrant in the ratio of 8 to 10, of which the chord is to the tangent of the half-quadrant as 'the square-root of 80' to 10, and of which the tangent is to the tangent of the half-quadrant as 8 to 6. It is to be noted that the same (octuple) arc is the particular subject of the theorem and prop. belonging to Fig. 24, page 10, Part Third, and is further considered in the following supplementary examination by Fig. 24. (R.)

† B.e. of Fig. 31.—in which B.e. = V.e. = e.d. = C.d.

A particular arithmetical relationship is at once apparent on contrasting these three quantities with each other... viz, the least is one third of the greatest, and the intermediate term is the one half of the greatest, moreover the least and the greatest of these numbers form with the number 10 a proportion of which 10 is the reciprocal or intermediate term. Now, 10 is the tangent to the half-quadrant, and the greatest of these numbers (17·32051) is the tangent to the arc of sixty degrees.

So that, herein we have a remarkable relation between a magnitudinal and arithmetical proportion. The inter-relation between the arcs contrasted with the inter-relation between the tangents is remarkable in respect to the dissimilarity in the correspondence (so to speak); for, in the arcs, the addition of one-half the first term gives the second, and the third term is twice the first (30°, 45° and 60°); but in the case of the tangent, cosine and cotangent, the third term is twice the second and three times the first.

5·7735, 8·66025, and 17·3205;

and in the tangents, the second term is greater than the first in the same ratio that the third is greater than the second,

5·7735 : 10 :: 10 : 17·3205

Scholium.—In considering the relation of the arc of 30° $(\frac{C}{12})$ to the half-quadrant $(\frac{C}{8})$, it should be noted that, if the chord of the arc of 30° be bisected, the one-half is the chord of an arc of 30° belonging to a circle of half the magnitude, (i.e., drawn on one-half the scale,) and is also the sine of the arc of 15° $(\frac{C}{24})$ cut off from the greater arc by bisection. Since the arc of 15° is one-third of the half-quadrant this relationship is of especial interest.

The definite divisions of the lines obtained by this construction are:—the line B.D. into three equal parts by perpendiculars drawn from the points *z.* and *y.* respectively. (Note.—If the tangent to the octuple arc be drawn, the difference, by which this tangent is greater than the tangent to the half-quadrant, is equal to one of the three equal divisions of B.D.)

The line B.D. divided into five equal parts by perpendiculars drawn from the points Q'.o.t.Q. respectively.

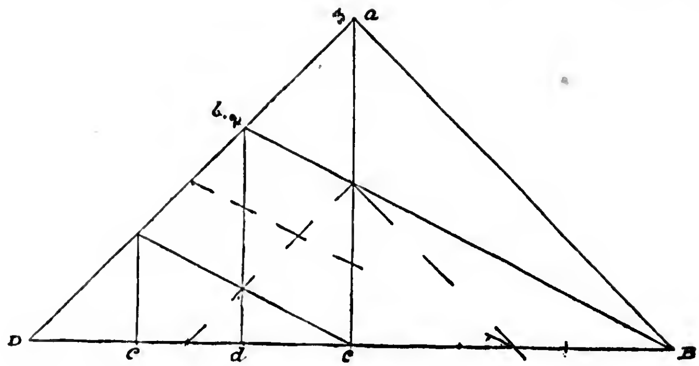


Fig. 1.

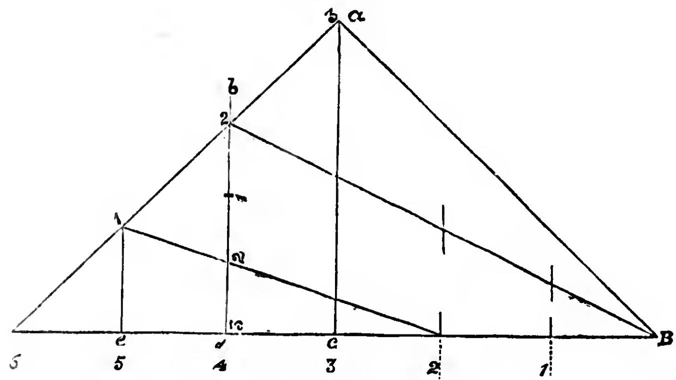


Fig. 2.

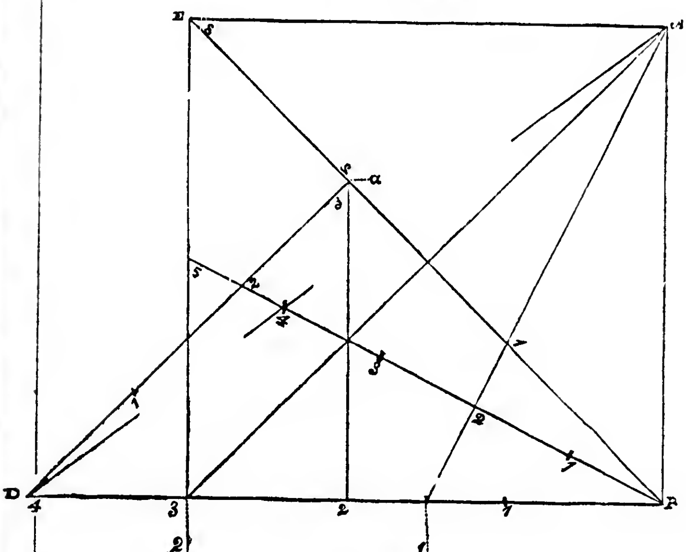


Fig. 8.

The line B.b. into fifths of which b.Q. is one fifth, Q.O. is two-fifths, and B.O. is two-fifths.

The secant A.D. into three equal parts by the points z. and *y*.

Note.—To assist the s udent in distinguishing the trigonometrical relation and values of these divisions, the following may be found useful. Figs. 1. 2. 3.—In each of those figures the line B.b. bisects the perpendicular a.C.; which line (a.C.) divides the triangle A.D.B. into two similar and equal triangles. Fig. 3., (in connection with Fig. 23. (R.), indicates the harmony of the cyclometrical and trigonometrical inter-relation of the lines and their equal divisions.

Fig. 24. (*R.*)—*The Octuple-Arc B.Q.* $\left\{ \begin{array}{l} \text{Sine} = 8{\cdot}0000. \\ \text{Co-sine} = 6{\cdot}0000. \end{array} \right.$

B.K.', the tangent to the half-octuple-arc...

B.K.' : B.D. :: 5 : 10.

B.Q. the chord of the octuple-arc...

B.Q. : B.D. :: $\sqrt{80}$: 10.*

B.Y. equals the sine of the octuple-arc...

B.Y. : B.D. :: 8 : 10.

B.E. the tangent of the octuple-arc...

B.E. : B.D. :: 8 : 6.

(Because B.E. = B.D. + B.H.')

B.E. = 13·3333, &c.—

A.P. the cosine of the octuple-arc...

A.P. : B.D. :: 6 : 10.

A.E. the secant of the octuple arc...

A.E. : B.D. :: 10 : 6.. And A.E. = $\frac{100}{6}$

Since B.Q.'z.k. bisects A.C. and B.O.y.b. bisects C.D., the part Q.'Z. of the octuple-arc D.Q.', is equal to the part O.Z. of the same arc. Now the point O. is the point of bisection of the chord of the (respondent) octuple-arc B.Q., and therefore if the octuple-arc B.Q. be described of one-half the magnitude, B.O. will be the chord thereof. But D.O. is the sextuple arc of the quadrant D.A. and responds to the

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* Hence the sine is to the chord of the octuple arc :: 8 : $\sqrt{80}$. Also—the sine of the octuple arc is to the sine of the half-quadrant :: 8 : $\sqrt{50}$. And the chord of the octuple arc is to the chord of the half-quadrant :: $\sqrt{80}$: the square-root of 'twice the versed sine of the half-quadrant multiplied by 10.' (i.e. $\sqrt{58{\cdot}5786}$.)

sextuple-arc B.t. of the quadrant B.C.; consequently the point at the extremity of the octuple-arc of half-magnitude coincides with the point at the extremity of the responding sextuple-arc.

Examination by Figs. 23. (R.), and 24. (R.)

The Sextuple-Arc—B.t. *Sine* = 6·0000.

The close relation of this arc to the octuple-arc of which the sine = 8·0000, will be made at once apparent by considering C.D. as the tangent and by taking the point C. for the original instead of the terminal extremity of the quadrant, in which case the point t. becomes the extremity of the octuple arc C.t. *

The following are the comparative elements of the two arcs. The radius equalling 10.

			Octuple-arc.	*Sextuple-arc.*
The	Sine	=	8·00000. .	6·00000 . .
"	Chord †	=	8·94427. .	6·32455 . .
"	Tangent	=	13·3333, &c.	7·50000 . .
"	Secant	=	16·6666, &c.	12·50000 . .
"	Co-sine	=	6·00000 . .	8·00000 . .

These two arcs (the Octuple and Sextuple) are also so related to the half-quadrant B.S. that the one is less by the same quantity by which the other is greater than the half-quadrant, i.e.,—the point at the terminal extremity of the half-quadrant is at an equal distance between the points at the terminal extremities of the two arcs. The relationship, therefore, is:—

B.S. + half the diff. of B.Q. and B.t. = B.Q.
B.S. − half the diff. of B.Q. and B.t. = B.t.
B·Q. + B.t. = 2 B.S. . . . . . = B.C.

From this relation the quantitive values of the two arcs are readily obtainable and we find accordingly:—

$$\left.\begin{array}{l}\text{The Octuple-arc contains } 53\tfrac{1}{8} \text{ degrees} \\ \text{The Sextuple-arc contains } \underline{36\tfrac{7}{8}} \text{ degrees}\end{array}\right\} 45 \times 2$$

Together equalling 90 degrees. = B.C.

* Evidently therefore the sextuple and the octuple arcs are so related that the one is the complement of the other; it is none the less important to clearly distinguish between them.

† Hence the chord of the Octuple arc is to the chord of the Sextuple arc :: $\sqrt{80}$: $\sqrt{40}$.

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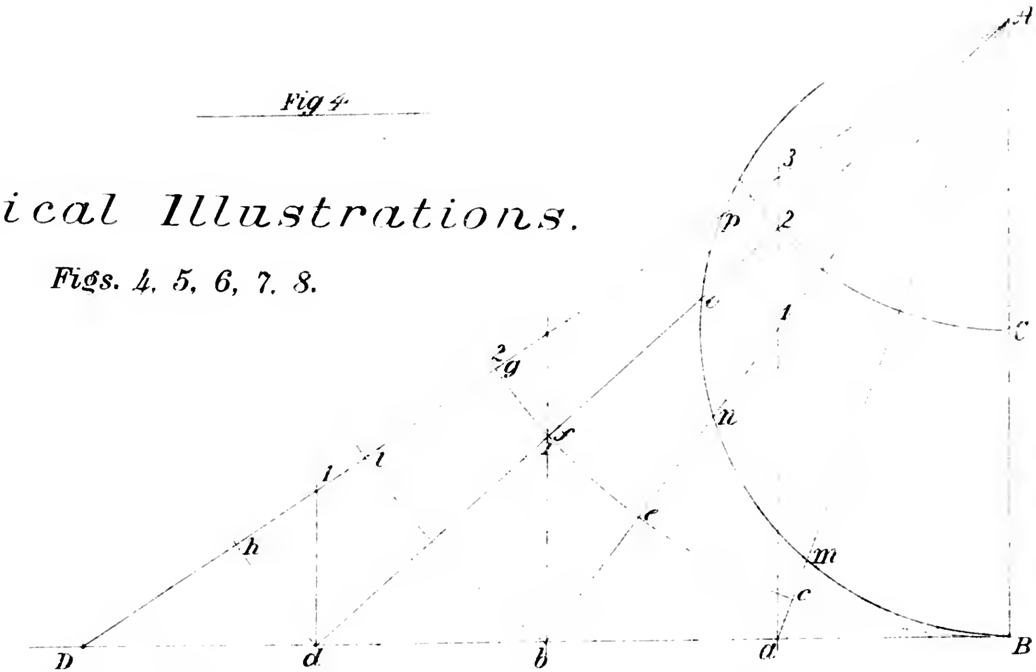
Analytical Illustrations.

Figs. 4, 5, 6, 7, 8.

Fig 4



Note.—For the purpose of assisting the examination and appreciation of the linear divisions, we furnish the following: Figs. 4. 5. 6.—In Fig. 4, the arc B.g. is described with the radius A.B., and the arc B.p. with the radius C.B. which is the one-half of A.B. The tangent B.D. of the arc B.g. is divided into four equal parts in the points a.b.d. and the lines drawn from these points respectively to the central point A. divide each of the arcs B.g. and B.p. into four (unequal) parts. It will be found that the ratios of the divisions of these arcs to each other (i.e. of each to the next) are proportionals; thus—g.f. : f.e. :: f.e. : e.c. :: e.c. : c.B. Consequently, f.c. − g.f. = c.B. (or f.c. − c.B. = g.f.), and g.f. + c.B. = f.c. That is the arc-length of g.f., together with the arc-length of c.B. equals half the length of the arc B.g. Now if the radius be reduced to the one-half and the arc B.p. described therewith be examined, the ratios of the arcs will be found proportionals as before and we have p.o. : o.n. :: o.n. : n.m. :: n.m. : m.B.; therefore the arc-lengths p.o. + m.B. = o.m., and o.m. equals the half of B.p. The Figs. 5. & 6. are, in the first place, for the purpose of showing that the same inter-relation of the divisions of an arc divided in the same manner – to wit, by division of the tangent into equal parts – holds good whether the length of the radius be increased and the extent of the arc be diminished, or whether the length of the radius be diminished and the extent of the arc be increased. The same figures serve to illustrate the trigonometrical relation of the radial lines drawn from A, which intersect the tangential line B.D. The perpendiculars at the points d.b.a. divide the line A.D. into four equal parts; and the line A.d. into three equal parts, and the line A.b. into two equal parts. The perpendiculars themselves are in arithmetical progression and are successive multiples of the least. Consequently d. + B. = b. + a.

Figs. 7. and 8. are to illustrate the conditions which necessitate the proportional inter-relations of the divisional arcs; also, in Fig. 7., we have a series of similar triangles, of which the ratios are proportionals A.c.d. : A.d.B. :: A.d.B. : A.B.D. :: A.B.D. : A.D.f.; the bases of these triangles are therefore similarly proportional, each to the next. The two sides D.e. and e.B. of the triangle D.e.B. are

Analytical Illustrations.

Figs. 4, 5, 6, 7, 8.

Fig 4

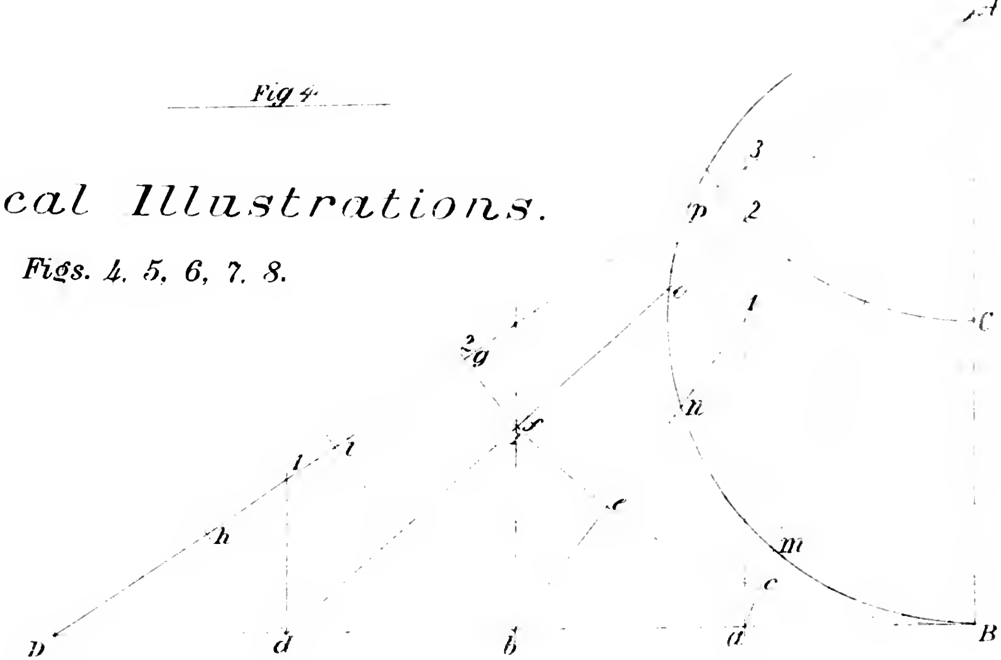
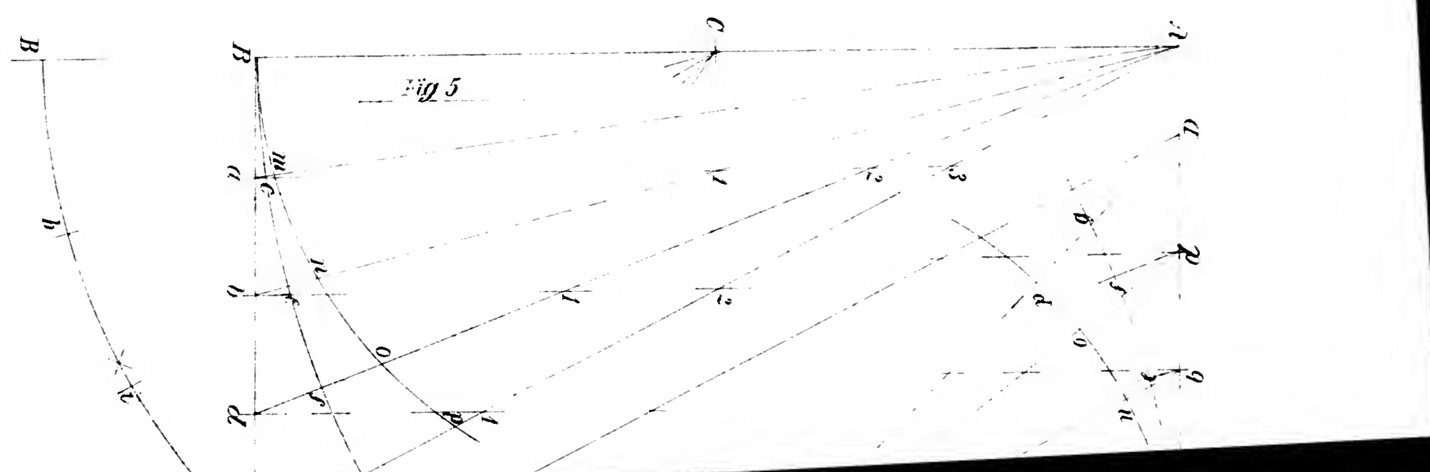


Fig 5



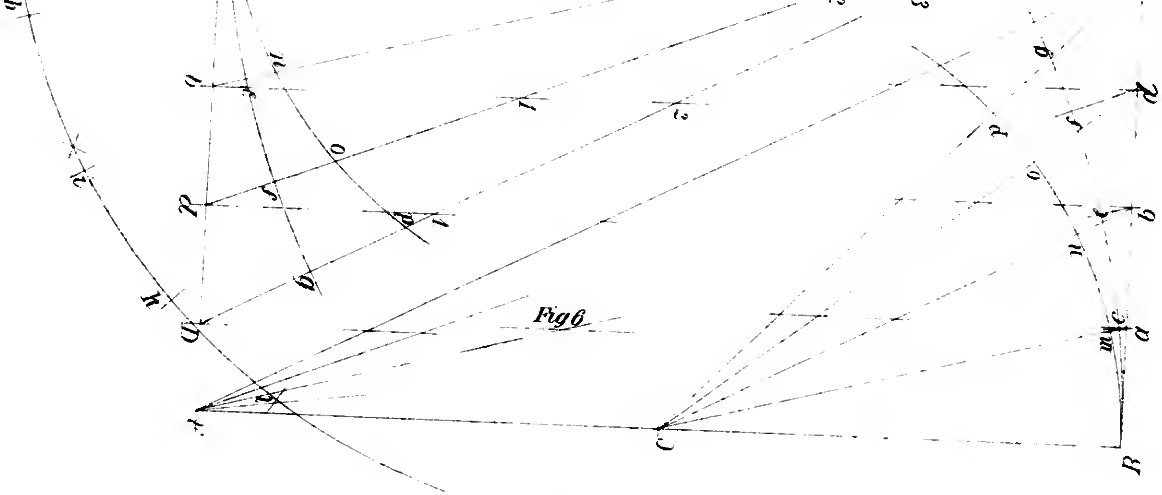


Fig 6

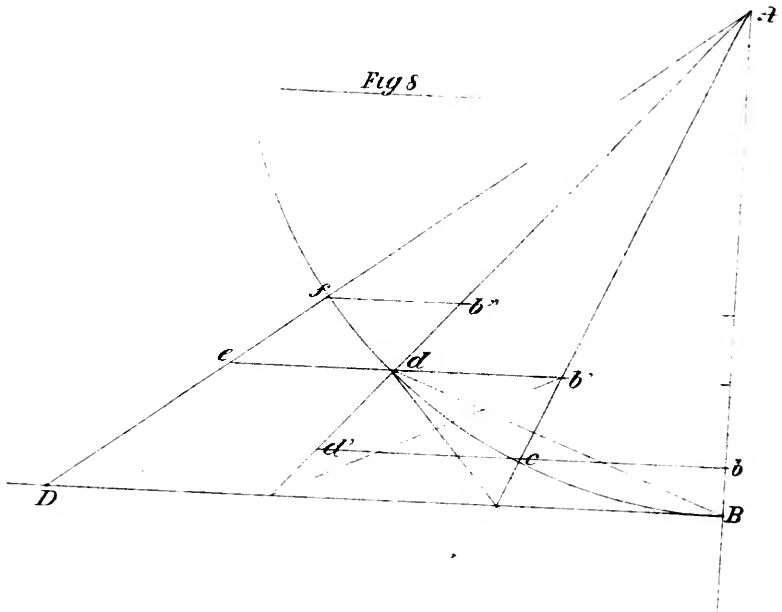


Fig 8

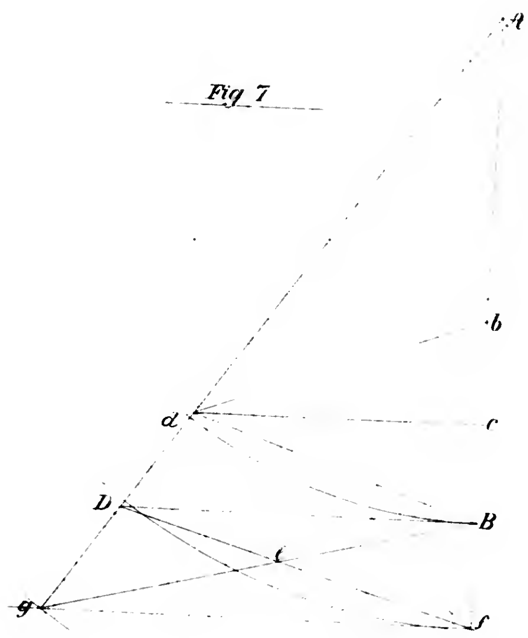


Fig 7

together, a mean proportional between D.B. and g.f., and, therefore, g.f. is greater than D.f. in the same proportion that D.e.B. is greater than D.B., &c.

Fig. 31.—*Examination of the figure.*

Of the quadrant B.C.... Q.C. is the complement of the octuple-arc B.Q.... B.d. (two thirds of the quadrant) = C.e.; ... C.d. = d.e. = e.B.; and C.d. + d.e. + e.B. = C.d. × 3 = C.B.... S.d. = S.e., and S.Q. = S.P.; therefore Q.d. = P.e.

Of the similar quadrant respondent through S.d, to wit, the quadrant (C.)(B.) the similar divisions are equal; thus (C.)(d.) + (d.)(e.) = C.d. + d.e.; also V.(e.) = B.e. = (e.)(d.) = (d.)(C.), &c.

Of the similar quadrant respondent through e., to wit, the quadrant A.D., the similar divisions are equal; thus Q.'Z. is equal to Q.S., Q.'e. is equal to Q.e., and the line K.K.' bisects the line joining Q.Q'.

Of the arcs shown in this Fig. the arc of curvature described with radius b.𝔛. intersects the central point K. of the square, which is also intersected by the arc of the sine-length described with radius B.W.

We note, in the first place, the remarkable relation between the square and the quadrant herein exhibited, and which may be thus stated as a theorem... that, if a quadrant be described in a square, and if the two adjacent sides of the square next the quadrant be bisected and a square half the magnitude of the first be formed by joining the points of bisection and the central point of the greater square, then will the quadrant be intersected by the two sides of the lesser square, and the two points of intersection shall divide the quadrant into three equal parts. This theorem may be said to be demonstrated by inspection of the figure—that is, in other words, it becomes manifest—but, moreover, we have the central part of the three into which the quadrant is divided bisected by the line K.S.D. (half the secant of the half quadrant) and we are familiar with the quantitive values of the lines K.S. and S.D. of which K.S. equals the versed sine of the half-quadrant = 2·92893... and S.D., the difference of the radius and secant of the half-quadrant, = 4·14214... (i.e., taking the side of the greater square equal to 10·0000.)

We may therefore, by taking the distance K.d. as a radius, describe a quadrant terminated by the points d. and e. which terminate the arc d.e.; or, by taking the line K.S. as a radius we may describe an arc touching the arc cut off from the greater quadrant, at the central point thereof S.; and the proportion of these arcs each to each and their quantitive values may be very readily determined.

Now if a fourth quadrant intermediate between the two last be described by making the radius equal to one-third of the radius A.B., this last quadrant will intersect the central part of the primary quadrant in two points, and if the point (K.), belonging to the primary responding quadrant (C.)(B.), be taken as a centre and with the same radius—to wit, equalling one-third of A.B.—another secondary quadrant be described, we then obtain Fig. 32.

Fig. 32.—In this figure the line $\mathcal{X}.\mathcal{X}.$ occupies the half distance between the perpendicular diameters of the two (greater) circles, and since the radius of each equals the one-third of ten (3333...) those circles intersect each other on the line $\mathcal{X}.\mathcal{X}.$ Setting aside for the moment the preceding demonstrations of the locality and characteristics of the line $\mathcal{X}.\mathcal{X}.$, it is at once apparent that the relative place of that line as shown in the figure necessarily belongs to the structural plan of the circle, because K.S. equals S.T. or (S.W.), and the distance S.$\mathcal{X}$. is included in the distance S.T. and the distance, also, between the lines (T.) V. and K.k'. is included in the side of the right-angled triangle of which K.S. is the base, therefore if S. be taken as the centre of a circle described with the radius S.K., that circle will intercept the point T., and if the point (S.), responding to S., be taken as the centre and a circle be described with the radius (S.)(K.), that circle will intercept the point (T.) The distance between the centres K. and (K.) of the two (greater) circles is therefore necessitated and determined by the actual relation of the lines. Now taking our demonstration of the quantitive value of R.$\mathcal{X}$.., as the arc-length of the half quadrant—to wit, 7·85674...—we have $\mathcal{X}$.T. = R.T. − R.$\mathcal{X}$. = 2·14326... But S.$\mathcal{X}$., by that demonstration, equals the one-tenth of R.$\mathcal{X}$. i.e., ·785674..., and S. K., by the figure, equals S.T., and also equals S.W. the versed sine of the half quadrant and of which the magnitudinal value is therefore

2·92893... *Let the fact be particularly noted that* 2·14326 + ·785674 = 2·92893.

It may be observed that, since the chord of the arc of sixty degrees equals the radius, several of the distances between the points indicated in this figure are necessarily equal each to each.

Fig. 33.—If the responding quadrant (B.) (C.) be located at a distance from the centre K. of the primary circle, a little greater than its true place as determined by the arc-length of the half quadrant—to wit, by making the distance R.X. equal 7·86565 . . instead of 7·85674... the perpendicular W.S. produced through S. will intercept the responding quadrant (B.) (C.) in the point d' (on the line J.K.), and in that case d.'K. will be equal to d'.S., and d.d.' equal to (S.) S. And also, if from K. a perpendicular be drawn through the line R.T., at E., to the point e. in the quadrant B.C., the perpendicular K.e. is manifestly equal to K.d., and K.E. equal to K.d'. Hence we obtain the formation of four squares equal each to each, and of eight parallelograms equal and similar each to each, as shown in Fig. 33.

Of these squares, each of the sides is equal to K.d'. and the diagonal equal to K.S.... Now the quantitive value of K.S. is 2·92893... and K.d.' : K.S. :: 10 : 14·14214...; therefore K.d.' = 2·07107; but in place of a diagonal each of the parallelograms, of which the two sides are each equal to K.d.' and the other two sides each equal to d.' d., or S. (S.), contains a fraction of the quadrant B.C., and which fraction is the one-sixth part of the quadrant, (an arc therefore containing 15 degrees described with radius equal to 10.)

Analytical Examination by Fig. 31.

The arc of 15 *degrees* $(\frac{C}{24})$; *and the line J.K.N.*

The arc of 30° B.e. is bisected in the point m.; B.m. is therefore an arc of 15°. (B.m. = m.e. = e.S. B.m. + m.e. + e.S. = B.S. S.d. is also an arc of 15° and equals S.e. = B.m.)

Since the line A.M.m. bisects the angle B.A.f., the line C.d.g. similarly bisects the responding angle D.C.F. and the point M. manifestly responds to the point d., therefore the line C.d. bisects an arc of 30° and J.d., on the line J.K., is the tangent to an arc of 15° described with the radius C.J. which equals one half the radius A.B., and J.d. therefore equals one-half of B.n. the tangent to the arc B.m.

But the lines A.d. and A.e., are likewise respondent each to the other, and, therefore, the point d., at the terminal extremity of the arc C.d., responds to the point e. at the terminal extremity of the arc B.e. Now the sine of the arc B.e. = (the half of B.D.) = 5. And consequently the cosine of the arc B.e. equals 8·66025. Therefore, N.d., which responds to the cosine of the arc B.e., also equals 8·66025... and J.d. = (10 — 8·66025) = 1·33975... Again N.M., at the opposite extremity of the line, responds to J.d. and also equals 1·33975... ; and N.L. : the sine of B.e. : : 2·88675 : 5.* Therefore, M.L., which is the distance between the points in which the line is intersected by the arc and by the chord of the arc respectively, equals (2·88675 – 1·33975) = 1·54700.

The magnitudinal values of these distances having thus been with certainty ascertained, the entire line J.K.N. may be analytically examined; commencing from the extremity N. we have :—

N.M.	=	1·33975...	
M.L.	=	1·54700...	
L.K.	=	2·11325...	
N.K. –	=		5·00000
K.d.'	=	2·05323...	
d.' $	=	·01784...	2·07107
$ y.	=	·04218...	
y. 𝒳.	=	·74349..	0·78567
K.𝒳.	=		7·85674
𝒳.y'	=	·74349...	
y.' ($)	=	·04218...	
($) d.	=	·01784...	
d.J.	=	1·33975...	
𝒳.J.			2·14326...
N.K.J.	=		10·00000

* Because N.L. is the sine of the same angle, and 288675 : 5 :: 5 : 866025.

Or again, by taking the (central) point $\mathcal{X}$., we have as definite parts of the line J.K. :—

$\mathcal{X}$.y. and $\mathcal{X}$.y.' each of which equals ·74349...
$\mathcal{X}$.\$. and $\mathcal{X}$.(\$).' each of which = ·785674...
$\mathcal{X}$.d. and $\mathcal{X}$.d.' each of which = ·80351..

and finally

$\mathcal{X}$.T. = 2·14326, and $\mathcal{X}$.K. = 2·85674

The definite division of the line J.K.N., thus determined evidently furnishes the means of again testing the correctness of that quantity of length which has been hitherto supposed to measure the half-quadrant, namely, ·78539... It appears almost needless to show here by the addition and subtraction of the figures that such quantity cannot be made to harmonize with those definite divisions of the line J.K.N., which directly result from trigonometrical measurement, and that, furthermore, no quantity, as representing the arc-length of the half-quadrant, can be made to harmonize therewith other than the quantity 7·85674...

As an example, let us first take this demonstrated quantity; — to wit, N.$\mathcal{X}$. = 7·85674:—

$$\begin{array}{llcl} \text{We have:} - & \text{J.K.} & = & 5.00000 \\ \left.\begin{array}{l} \text{K.}\mathcal{X}. = 2{\cdot}85674 \\ \mathcal{X}\text{.d.} = {\cdot}80351 \end{array}\right\} & & = & 3{\cdot}66025 \\ \hline & \text{J.d.} & = & 1{\cdot}33975 \end{array}$$

And 2 J.d. = 2·67950, the tangent to the arc of 15° (B.n.)

$$\begin{array}{llcl} \text{But assume} & \text{N.}\mathcal{X}. & = & 7{\cdot}8539... \\ \text{Then:} - & \text{J.K.} & = & 5{\cdot}0000 \\ \left.\begin{array}{l} \text{K.}\mathcal{X}. = 2{\cdot}8539... \\ \mathcal{X}\text{.d.} = (\mathcal{X}.\$ + \$\text{d}) = {\cdot}8007... \end{array}\right\} & & = & 3{\cdot}6546 \\ \hline & \text{J.d.} & = & 1{\cdot}3454 \end{array}$$

And 2 J.d. = (2·6908); as the tangent to $\left(\frac{C}{24}\right)$ the arc B.n.

(NOTE.—We have already shown that assigning 7·8539... as the arc-length of the half-quadrant, is in fact attributing two different lengths to the same line. See Part Second.)

Noteworthy in Fig. 31, is the foursided figure d.e. M.H. of which each of the sides is an arc of 30°. containing the central one-third of a quadrant described with radius = 10. The two longer diameters H.e. and M.d. each equals 7·3205...; of the two shorter diameters S.U. and z.u. each equals 5·85786...

Fig. 34.—If the lines A.H. and B.e. be produced through the points H. and e. respectively, until they meet each other, they will meet at a distance from the point d. equal to the radius A.B. *(or from the point J. equal to the distance N.d.) and if from the point where the two lines so meet as a centre an arc A.B. be described, the length of the radius = 18·6025... and the chord of the arc = 10.†

THE ARC OF $18\frac{7}{16}$ DEGREES. (*The half-sextuple arc.*)

Fig. 34.—Bisect the arc B.P. (i.e., the sextuple arc); and through the point of bisection c. draw the line A.Q.m.

The arc B.c. is therefore an arc containing $18\frac{7}{16}$ degrees, ‡ its linear elements are,

The Sine.......	=	3·16227...
" Co-sine......	=	9·48685...
" Tangent...	=	3·33333... &c.
" Co-tangent..	=	30·0000...
" Secant.....	=	10.5409...
" Co-secant..	=	31·6227...

(Note.)—*These figures may be compared with the elements of the arc of* 18°,

The Sine.........	=	3·09017...
" Co-sine.....	=	9·51057...
" Tangent...	=	3·24920...
" Co-tangent..	=	30·7768...
" Secant......	=	10·51462...
" Co-secant...	=	32·36068...

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* (d.A. also equals A.B..

† Having regard to the relations of the triangle thus formed the base of which (i.e. the chord of A.B.) equals 10, it is probable that the characteristics of this triangle will render it of much utility if applied in the art of computation.

‡ The sextuple arc has been shown to contain 36 $\frac{7}{8}$ degrees. (Page 6.)

Since the secant of the arc B.c. intersects the point Q., at the extremity of the quadruple arc, D.Q., the quantitive magnitudinal values of the lines belonging to the arc may be very readily computed and verified; the one-third of ten as the tangent to the arc presents itself at once... thus: —the triangle A.Q.O. is a part of the similar greater triangle A.m.B.;

Now we have A.O. = 6, and Q.O. = 2; *

(As A.O. : Q.O. :: A.B. : m.B.)

Therefore as 6 : 2 :: 10 : $\frac{10}{3}$ which represents m.B. the tangent to the arc.

We have confined these illustrations thus far to the lines and divisional arcs belonging to a circle described with a radius equal to 10. It is evident that by taking any one of these primary or more important lines as a radius, a series of quantitive magnitudes with similar inter-relations will be obtained, and which will have, through the radius common to all of them, a definite and known relationship to the lines and divisional parts of the primary circle. Since some of these secondary lines may absolutely agree in magnitude with some of the primary lines or may have some very simple (quantitive and numerical) relationship to them, it is very desirable that the relations should be investigated and classified. As a brief example we will take the line 5·85786... which equals twice the versed sine of the half-quadrant and has been now shown to be one of the most important of the primary lines belonging to the half-quadrant.

If from the point B., Fig. 34. on the line B.A., a centre $\bar{a}$. be taken at a distance = 5·85786... and from the centre $\bar{a}$. with radius $\bar{a}$.B. a quadrant B.c. be described, then if the quadrant be bisected in the point h., B.h. shall be the half-quadrant; and if one-third of the quadrant be divided off at the point $\bar{d}$. the remainder—to wit, the arc B.$\bar{d}$. – shall be an arc containing 60°. and the versed sine of this secondary

* Geometrical demonstration that the sine of the quadruple arc is to the tangent of the half-quadrant as 8 : 10 will be found at page 10, Part third, Fig. 24.

arc of 60°. (described with D.V.S. radius $\bar{a}$.B. = 5·85786), shall equal the versed sine of the half-quadrant belonging to the primary circle, (because the versed sine of the arc of 60°. equals one-half its radius).

Taking, therefore, as the example, the half-quadrant B.h. we have :—

The decimal Circle of the D.V.S. (*duplicated versed sine.*)

Radius = 5·85786.

The half-quadrant. $(\frac{C}{8})$

The Sine..... = 4·142134...
" Tangent. = 5·85786...
" Secant... = 8·28426...

Herein we have an agreement (coincidence) between certain of the principal lines belonging to one circle and certain of the principal lines belonging to another circle differing from the first in magnitude; the lines belonging to the second being dissimilar * from the lines belonging to the first, with which they agree in length.

The decimal Circle of the D.V.S.†

The arc of sixty degrees $(\frac{C}{6})$

The sine..... = 5·0730... ‡	The Co-sine..... = 2·92893
" Tangent = 10·1461...	" Co-tangent. = 3·38204
" Secant.. = 11·7157...	" Co-secant.. = 6·76408
" Chord.. = 5·8578.	

(Note.)—The chords of the secondary arcs coincide with and form a part of the chord of the primary arc if the arcs be so described that each commences from the same original point (B.) as that of the primary and agrees with the primary arc in position, which is the method adopted in the foregoing illustrations: Since the primary radius includes the radius of each of the circles, the extremity of the sine may be made the point of coincidence, as in some of the analy-

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* That is, they are constructively different in relationship.

† That is, the duplicated versed sine of the primary half-quadrant.

‡ Sine 5·073056670........ taking $\sqrt{50} = 7·07107$.

tical figures of our 'part second,' (See Fig. 12.); Or, again, the same point may be taken throughout for the centres of the circles—that is, the arcs may be described all from the one centre—in which case the secant will be the line of coincidence. In systematic analysis it will evidently be convenient to adhere to one uniform method throughout the whole or a part of the figures belonging to the series.

ANALYSIS BY DECIMAL CIRCLES.

Illustrated in Figs. 35 (a.) and 35 (b.)

In which Figures these different methods are applied to illustrate the system of decimal cyclometry.

In Fig. 35 (a.) the tangent is made the line of coincidence, in Fig. 35 (b.) the sine is the line of coincidence with reference to the radius A.B., and also the secant is indicated as the line of coincidence with reference to the radius A.C. and illustrated by the arcs intersecting that line.

The decimal system of cyclometry is illustrated in these two figures by the three decimal circles, namely, the circle of the sine; the circle of the duplicated versed sine; and the circle of the secant. These lines may be termed 'Capitals' of the system; they are all secondary to the radius A.B. which equals 10 and is the 'primary' of the system; but each of them is 'primary' to the lines and divisional arcs belonging to the circle of which it is the radius, for instance, in the example which has just been given, the circle of the D.V.S. has for its 'primary' the line A.B. which is the radius of the primary quadrant, but the line $\overline{a}$.B. equalling twice the versed sine of the primary quadrant) is the primary to the half-quadrant, to the arc of 60°, and to all other divisional arcs, belonging to its own circle, and also primary to the lines belonging to those respective arcs. In constructing a cyclometrical table it is evident that this system may be pursued by sub-divisions as far as may be found desirable. The immediate gain to the science of quantity and number from such a tabulated analysis of the circle would be the increased knowledge obtained of the inter-relations of the subjects of that division of science; for example, by taking a primary radius equal to 10, we find that the number 5 represents the sine of the arc of 30 degrees; the number 8 represents the sine of the octuple arc; and the number 6 the sine of the sextuple arc; we become thereby aware of and are able to appreciate a particular magnitudinal relationship between these numbers, and not only of each to each of these but also of each and of all of these to many other (magnitudinal quantities) numbers. Or, as another example, we may take again the number 5, which appears as the sine of the arc of 30° and the cosine of the arc of 60°, both belonging to the primary circle, and also appears as the sine and cosine of the half-quadrant belonging to the decimal-circle of the sine—namely, that which has the sine-length $\sqrt{50}$ as its (radius) 'Capital.'

Examination of these two plates, 35 (a.) and 35 (b.), together with the annexed table wtll suffice to render the cyclometrical decimal system clearly understood.

NOTE.—In Fig. 35 (b.), wherein the line of coincidence includes the sines of all the arcs, the division of the primary radius A.B. should be noted. The entire part divided off above the line of coincidence is equal to the sine-length of the primary (7·07107); from centre A., at the upper extremity of A.B., a part is divided off by the centre of the dec-circle D.V.S. equal in length to the versed sine of the primary; and, adjoining the line of coincidence—above that line—a part is divided off by the centre of the circle of the secant, also equal in length to the versed sine of the primary half-quadrant.

The method of analysis to which these figures belong will be better appreciated after consideration of Table II., which we furnish as an appendix, and in which the arc-lengths are included as elements of the circles. From that table it will immediately appear, for example, that the arc-length of the arc of nine degrees belonging to the circle of the sine equals the radius A.B. divided by nine; and that the arc-length of the half-quadrant belonging to the same circle equals five times that quantity.

In concluding these illustrations we beg to state that they may be considered generally as contributions and suggestions towards an analytical investigation of the circle; their immediate purpose however is to illustrate the proposition that the circle itself is not only a reality but is one of the primary facts of Creation, constituting, indeed, a central primary or great fundamental fact in or upon which very many of the secondary facts belonging to abstract science † have their common basis.

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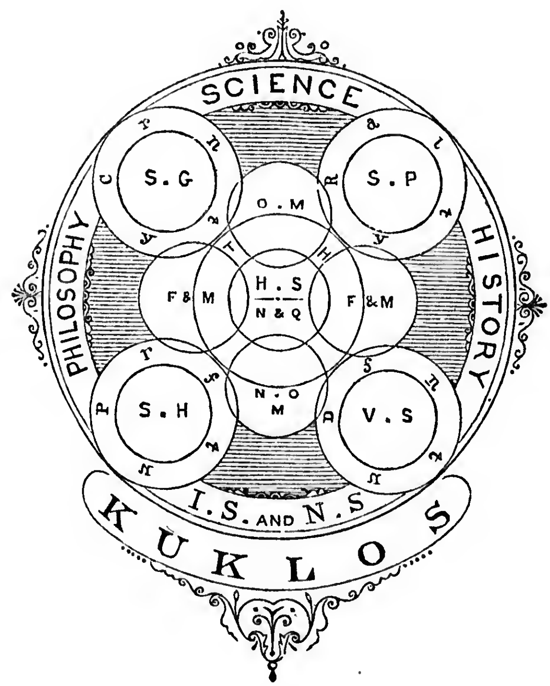
† Abstract-Science may be defined as that division of Science which treats of the inter-relation of like subjects of Science, in respect to those general properties (number, quantity, condition, form, magnitude,) which belong to those subjects as existences (things).



CYCLOMETRICAL TABLE I.

Decimal Series. Primary radius A.B. = 10.

(1) Divisional Arcs of the primary.	Half-quadrant.	Arc of 60 degrees.
Sine	7·07107	8·66025
Tangent	10·00000	17·32051
Secant	14·14214	20·00000
Co-sine	7·07107	5·00000
Chord	7·65366	10·00000
Co-tangent	10·00000	5·77350
Co-secant	14·14214	11·54701

	Arc of 30°.	Arc of 15°.
Sine	5·00000	2·58819
Tangent	5·77350	2·67949
Secant	11·54701	10·35275
Co-sine	8·66025	9·65926
Chord	5·17638	2·61052
Co-tangent	17·32051	37·32050
Co-secant	20·00000	38·63702

	Octuple Arc.	Sextuple Arc.
Sine	8·00000	6·0000
Tangent	13·33333	7·5000
Secant	16·66666	12·5000
Co-sine	6·00000	8·0000
Co-secant	12·50000	16·6666
Co-tangent	7·50000	13·3333
Chord	8·94427	6·32455

(2) Decimal Circle of the Sine. Radius = 7·07107.	Half-quadrant.	Arc of 60 degrees.
Sine	5·00000	6·12372
Tangent	7·07107	12·24744
Secant	10·00000	14·14213
Co-sine	5·00000	3·53553
Chord	5 41195	7·07106
Co-tangent	7·07107	4·08248
Co-secant	10·00000	8·16496

	Arc of 30°.	Arc of 15°.
Sine	3·53553	1·830127
Tangent	4·08248	1·894687
Secant	8·16497	7.320506
Co-sine	6·12372	6·839313
Chord	3·66025	1·845919
Co-tangent	12·24745	26·389589
Co-secant	14·14213	27·321252

	Octuple Arc.	Sextuple Arc.
Sine	5·656856	4·242642
Tangent	9·428093	5·303302
Secant	11·785116	8·838837
Co-sine	4·242642	5·656856
Co-secant	8·838837	11·785116
Co-tangent	5·303302	9·428093
Chord	6·324555	4·472133

(3) Decimal Circle of the D.V.S:* Radius = 5·85786.	Half-quadrant	Arc of 60°.
Sine	4·14213	5·073053
Tangent	5·85786	10·146106
Secant	8·28426	11·715728
Co-sine	4·142134	2·928932
Chord	4·490456	5·857864
Co-tangent	5·857864	3 382040
Co-secant	8·284268	6·764080

	Arc of 30°.	Arc of 15°.
Sine	2·928932	1·511121
Tangent	3·382040	1·569609
Secant	6·764080	6·044482
Co-sine	5·573060	5·659561
Chord	3·032252	1·529207
Co-tangent	11·146120	21·794185
Co-secant	11·715728	22·638244

* That is, of the duplicated versed-sine.

	Octuple Arc.	Sextuple Arc.
Sine..........................	4·68628	3·514716
Tangent......................	7·81048	4·393395
Secant........................	9·76310	7·322325
Co-sine......................	3·51471	4·686288
Chord.........................	5·23942	3·70841...
Co-tangent..................	4·39339	7·81048...
Co-secant....................	7·32232	9·76313...

(4) Decimal Circle of the Secant * Radius = 4·14214.	Half-quadrant.	Arc of 60°.
Sine..........................	2·92893	3·587196
Tangent......................	4·14214	7·174392
Secant........................	5·85787	8·284274
Co-sine......................	2·92893	2·071069
Chord.........................	3·17025	4·142138
Co-tangent..................	4·14214	2·391466
Co-secant....................	5·85787	4·782932

	Arc of 30°.	Arc of 15°.
Sine..........................	2·071068	1·096936
Tangent......................	2·391466	1·109883
Secant........................	4·782932	4·439532
Co-sine......................	3·587199	4·001002
Chord.........................	2·144129	1·081315
Co-tangent..................	7·174398	15·458674
Co-secant....................	8·284272	16·004008

	Octuple Arc.	Sextuple Arc.
Sine..........................	3·313712	2·485284
Tangent......................	5·522853	3·106605
Secant........................	6·903566	5·177675
Co-sine......................	2·485284	3·313712
Chord.........................	3·704841	2·61971
Co-tangent..................	3·106605	5·52285
Co-secant....................	5·177675	6·90356

* We thus apply the term 'Secant' to the difference of the Secant and Radius of half-quadrant. Since the Secant is twice the sine there is no danger of misapprehension from such application; some other term might, however, be preferable.

APPENDIX.

We have endeavoured as much as possible to avoid the unnecessary use of strange and unusual terms. It might be preferable instead of the expressions 'decimal system' and 'decimal circles' to substitute the word 'decuple' for 'decimal' in order to express the relationship, and to write 'decuple circles' belonging to the 'cyclometrical decuple system.' A decimal system in accordance with the more customary sense of the expression would be formed by making the successive 'capitals' of the system the numbers 9, 8, 7, 6, 5, 4, 3, 2, 1; that is, the radius of the first circle, to be nine-tenths the length of the 'primary,' of the second circle eight-tenths of the primary, and so on. A quadratic decimal system might also be formed by adopting the same 'primary' as the square root of 100, the first capital would then be the square root of 90, the second capital the square root of 80, and so on. In a system so framed the 'circle of the sine' would find its place with its capital as the square root of 50. For the present, however, it appears more desirable to extend the table now given by addition of other important divisional and related sub-circles; as a first addition we may take the half-octuple, the half-sextuple, the half-octant * and the arc of nine degrees; we have accordingly :—

CYCLOMETRICAL TABLE II.

The Primary.—Radius = 10. *Arc-length of Octant* = 7·85674.

	Half-Octuple $26\frac{9}{16}°$	Half-Sextuple $18\frac{7}{16}$	Half-Octant $22\frac{1}{2}°$	$\frac{C}{40}$ {Arc of 9°}
Arc..........	4·63766	3·219081	3·92837	1·571348
Sine..........	4·47213	3·162277	3·82683	1·56434
Tangent......	5.00000	3·333333	4·14214	1·58384
Chord.........	4·59507	3·203728	3·90179	1·56917
Secant........	11·18033	10·540929	10·82394	10·12465
Co-sine......	8·94427	9·486833	9·23881	9·87688
Co-tangent..	20·00000	30·000000	24·14213	63·1375
Co-secant....	22·36066	31·622787	26·13128	63·9245

* The 'Octant' is a more convenient expression than 'half-quadrant,' and, in like manner, the term 'Sextant' commends itself for general use to denote the arc of 60 degrees.

Dec-Circle of Sine—Radius=7·07107 *Arc-length of Octant*=5.55555

	Half-Octuple.	Half-Sextuple.	Half-Octant. $\frac{C}{40}$	{ Arc of 9° }
Arc.......	3·27932	2·276234	2·77777	1·111111
Sine..........	3·16227	2·236068	2·70597	1·106155
Tangent.....	3·53553	2·37023	2·92893	1·119944
Chord	3·24920	2 265379	2·75898	1·109576
Secant.......	7·91468	7·453565	7·65368	7·260210
Co-sine.	6·32455	6·708206	6·53282	6·984010
Co-tangent..	14·14213	21·21321	17·07107	44·64487
Co-secant....	15·81140	22·36068	18·49467	45·20146

Dec-Circle of D.V.S.—Radius = 5·857854... *Arc-length of Octant* = 4·60237...

	Half-Octuple.	Half-Sextuple.	Half-Octant. $\frac{C}{40}$	{ Arc of 9° }
Arc...........	2·716676	1·885689	2.301180	0·920492
Sine	2·619711	1·852417	2·241703	0·916368
Tangent......	2·928930	1·952621	2·426407	0·927781
Chord.........	2·691728	1·876683	2·285613	0·919198
Secant........	6·549278	6·174666	6·340512	5·930816
Co-sine........	5·239428	5·557212	5·411961	5·785738
Co-tangent...	11·715720	17·573580	14·122213	36·985063
Co-secant.....	13·078561	18·524182	15·321473	37·446076

Dec-Circle of Secant.—Radius = 4·14214.. *Arc-length of Octant* = 3·254371...

	Half-Octuple.	Half-Sextuple.	Half-Octant. $\frac{C}{40}$	{ Arc of 9° }
Arc...........	1·920983	1·333388	1·627185	0·650874
Sine	1·852418	1·309856	1·585164	0·647971
Tangent......	2·071068	1·380713	1·715732	0·656048
Chord.........	1·903342	1·327029	1·616176	0·649975
Secant........	4·631049	4·366201	4·483427	4·193751
Co-sine.......	3·704842	3·929579	3·826844	4·091143
Co-tangent..	8·284276	12·426414	10·000000	26·152436
Co-secant...	9·262098	13·098599	10·823942	26·478422

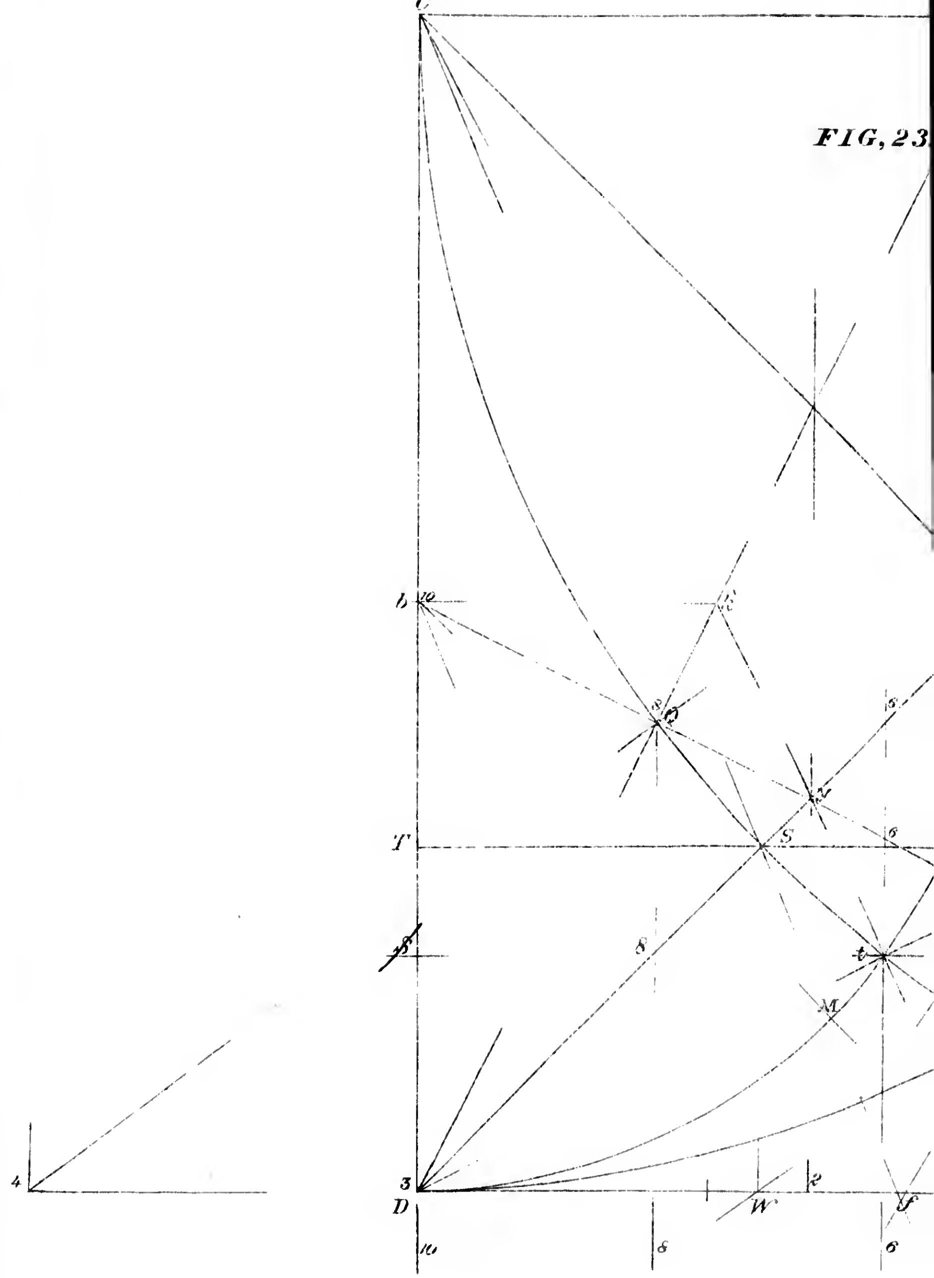
With respect to the two tables here given we have not the slightest doubt whatever as to their substantial correctness; it is, however, quite possible that some arithmetical mistakes in computation may be found in them, and as to the last decimal places the figures of many of the quantities cannot be strictly accurate because those quantities are based upon the assumption that 7·07107, is the square root of 50, and such is not exactly the square root which is 7·0710678... It is quite proper that the correction should be made and the figures be furnished with strict accuracy to the last decimal place, but the published tables of natural sines, &c., now authorized, or, at least, those most in use for general reference, are based on this assumption, and it is desirable in the first instance to show wherein our results are in perfect agreement with the results of recognized trigonometrical processes. The corrections may be very easily made and more complete tables furnished; as an example of the necessity of strict accuracy, worthy of note, we will specify the first of the elements of the half-sextuple arc belonging to the dec-circle of the secant, Table II., to wit, the arc-length thereof—which appears as 1·333388... It is most probable that strict accuracy will give the figure 3 as an interminable decimal... that is, will give the actual quantity as $10 + \frac{10}{3}$. (By correcting the equivalent of the capital (radius) in the fifth decimal place the figure 8 becomes reduced to 5 in the fifth decimal place of the example, and, moreover, the half-sextuple element of the primary, from which the corresponding element of the dec-circle is derived, is very slightly in excess from the same cause).

Fig. 36.—Is to some extent a development of Fig. 31.; it has, moreover, for its object to bring prominently under observation certain triangles and squares which, in their relation each to each and to the divisional arcs and elements of the circle, may be found to possess much utility.

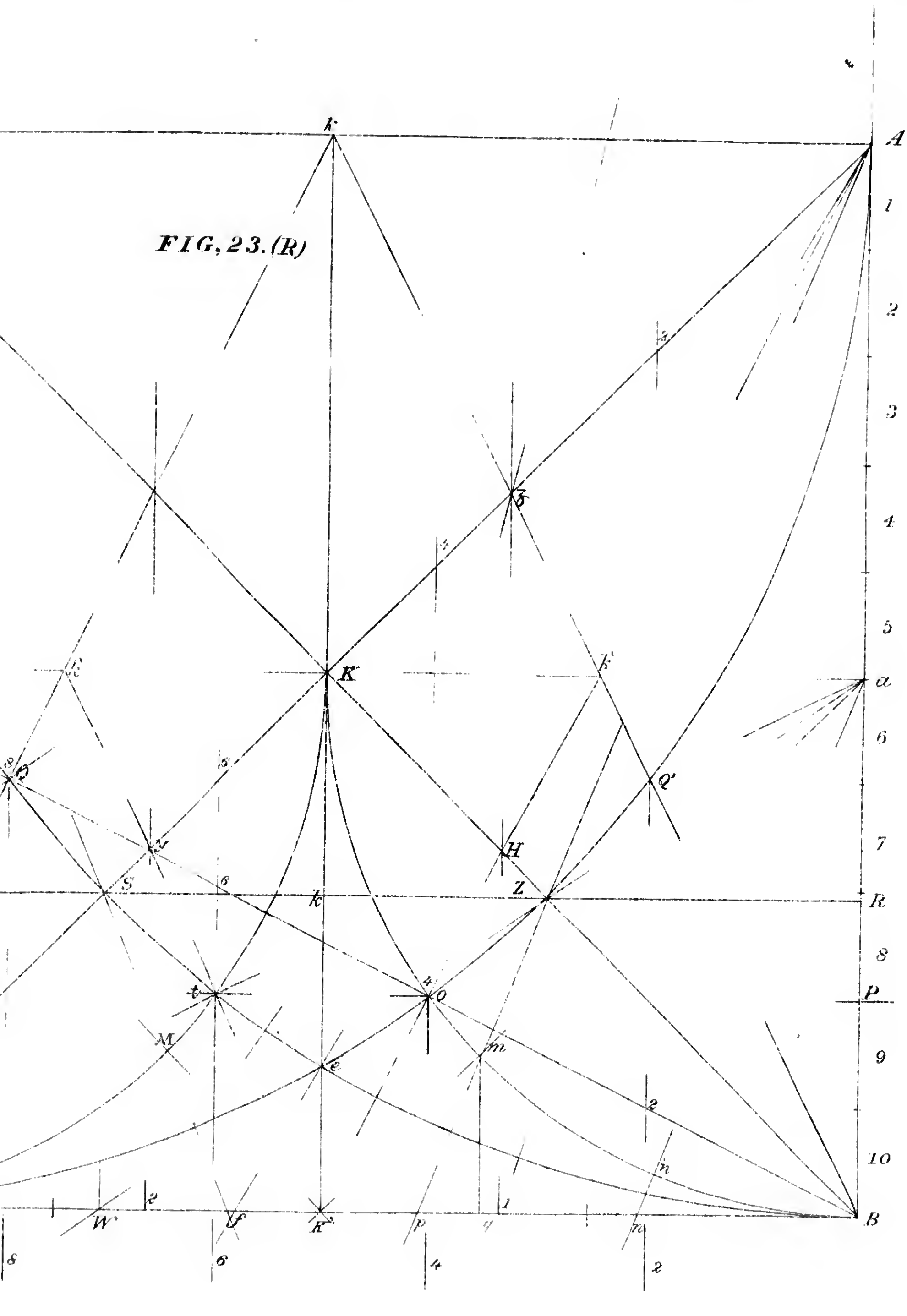
NOTE.—The arc-lengths belonging to the first cyclometrical table are as follows:—

(1) The Primary.	Half-quadrant - -	7·85674
	Arc of 60 degress - -	10·47566
	" " 30° - - -	5·23783
	" " 15° - - - -	2·61891
	Octuple arc – $53\frac{1}{8}$° - -	9·27532
	Sextuple arc – $36\frac{7}{8}$° ·	6·43816
(2) Dec-Circle of Sine.	Half-quadrant - - -	5·55555...
	Arc of 60 degrees -	7·407407...
	" " 30° - - - -	3·703703...
	" " 15° - - - - -	1·851851...
	Octuple arc – $53\frac{1}{8}$° - -	6·558642
	Sextuple arc – $36\frac{7}{8}$° -	4·552468
(3) Dec-Circle of D.V.S.	Half-quadrant - - -	4·60237
	Arc of 60 degrees -	6·13650
	" " 30° - - - -	3·06825
	" " 15° - - - -	1·53412
	Octuple arc – $53\frac{1}{8}$° - -	5·43337
	Sextuple arc – $36\frac{7}{8}$° -	3·77137
(4) Dec-Circle of Secant.	Half-quadrant - - -	3·25437
	Arc of 60 degrees -	4·33916
	" " 30° - - - -	2·16958
	" " 15° - - - -	1·08479
	Octuple arc – $53\frac{1}{8}$° - -	3·84197
	Sectuple arc – $36\frac{7}{8}$° -	2·66677

END OF APPENDIX.



FIG, 23.


FIG, 23. (R)
A
K
H
Z
R
P
B
a
Q'
S
M
W
k
m
n
o
e
t
1
2
3
4
5
6
7
8
9
10


E

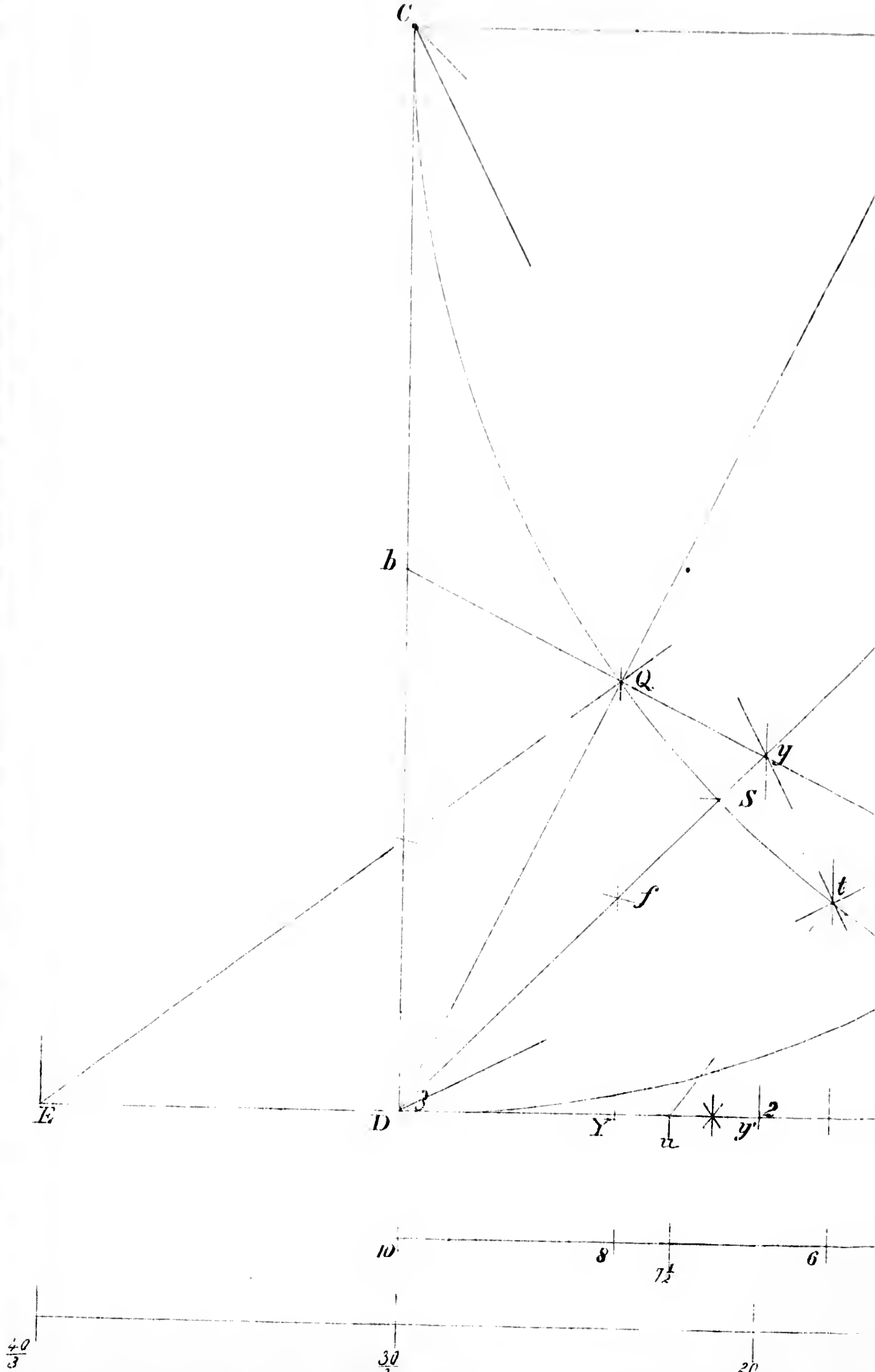
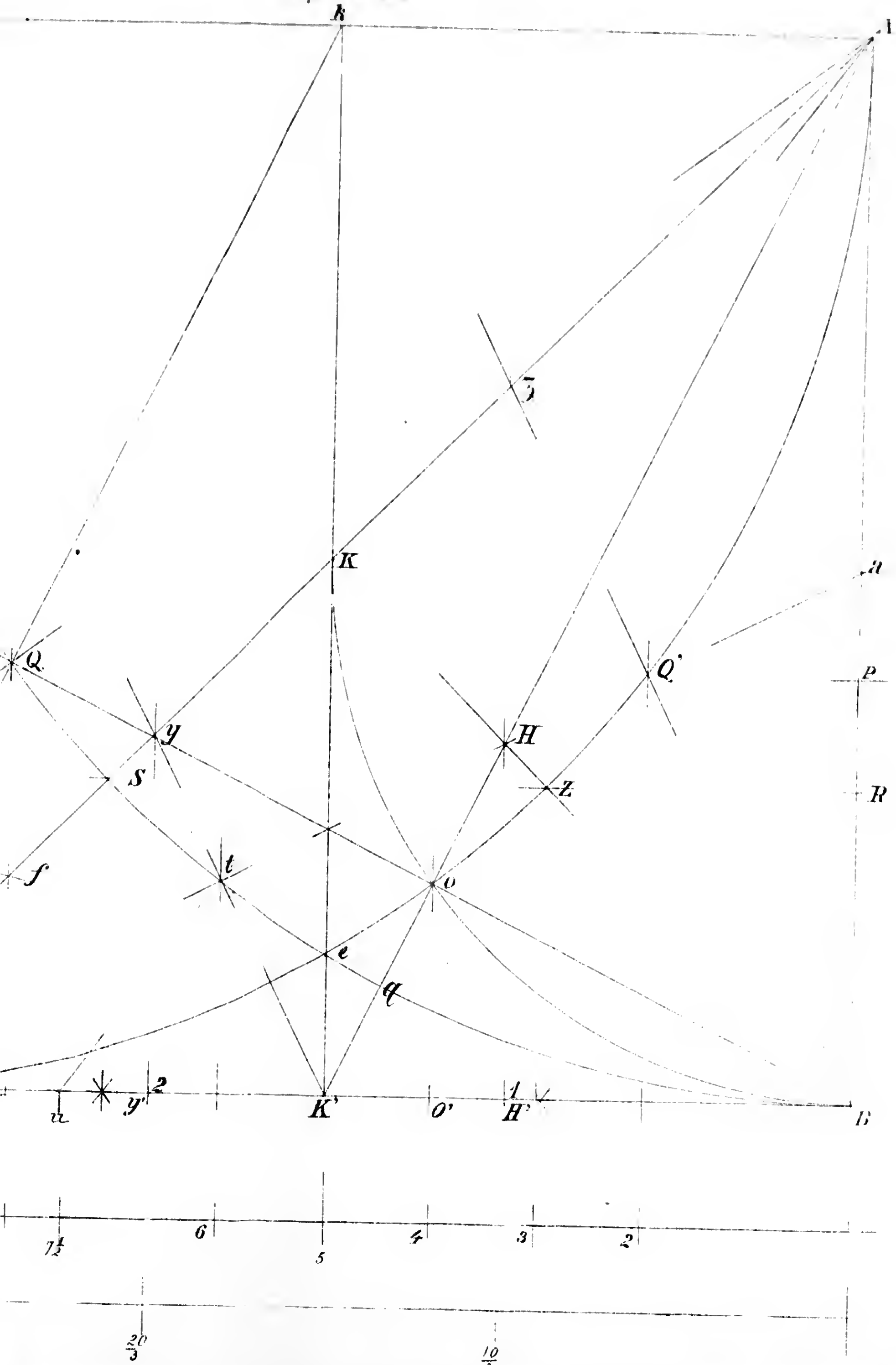
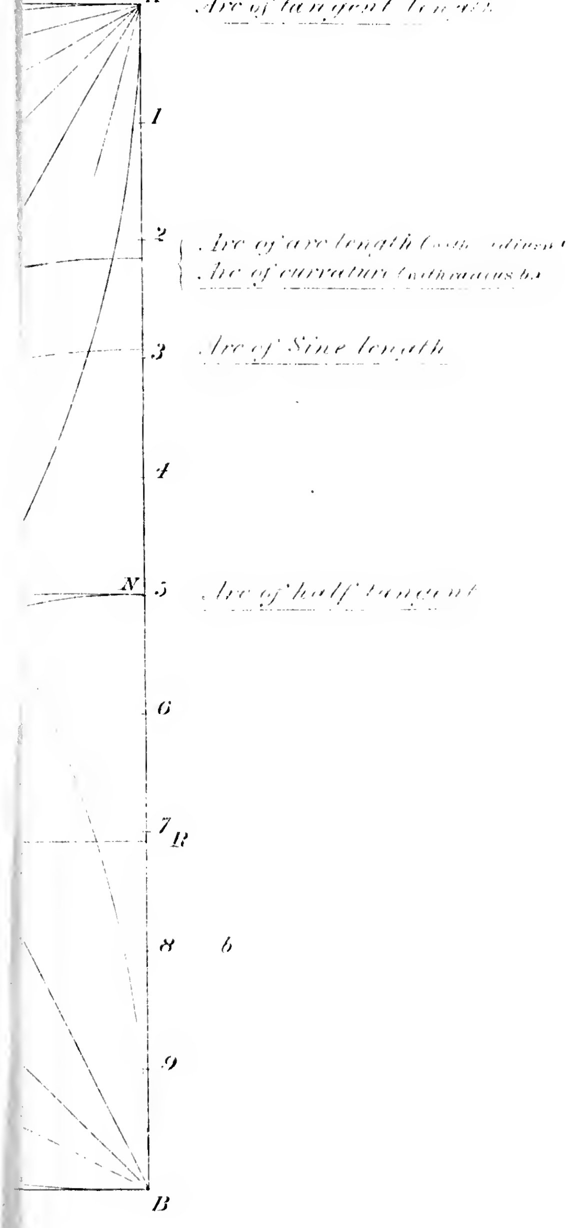

C
b
Q
y
S
t
f
E
D
3
Y
u
y
2
10
8
7½
6
40/3
30/3
20/3

Fig 24. (R)




A
Arc of tangent length
1
2
Arc of arc length
Arc of curvature
3
Arc of Sine length
4
N
5
Arc of half tangent
6
7
R
8
b
9
B

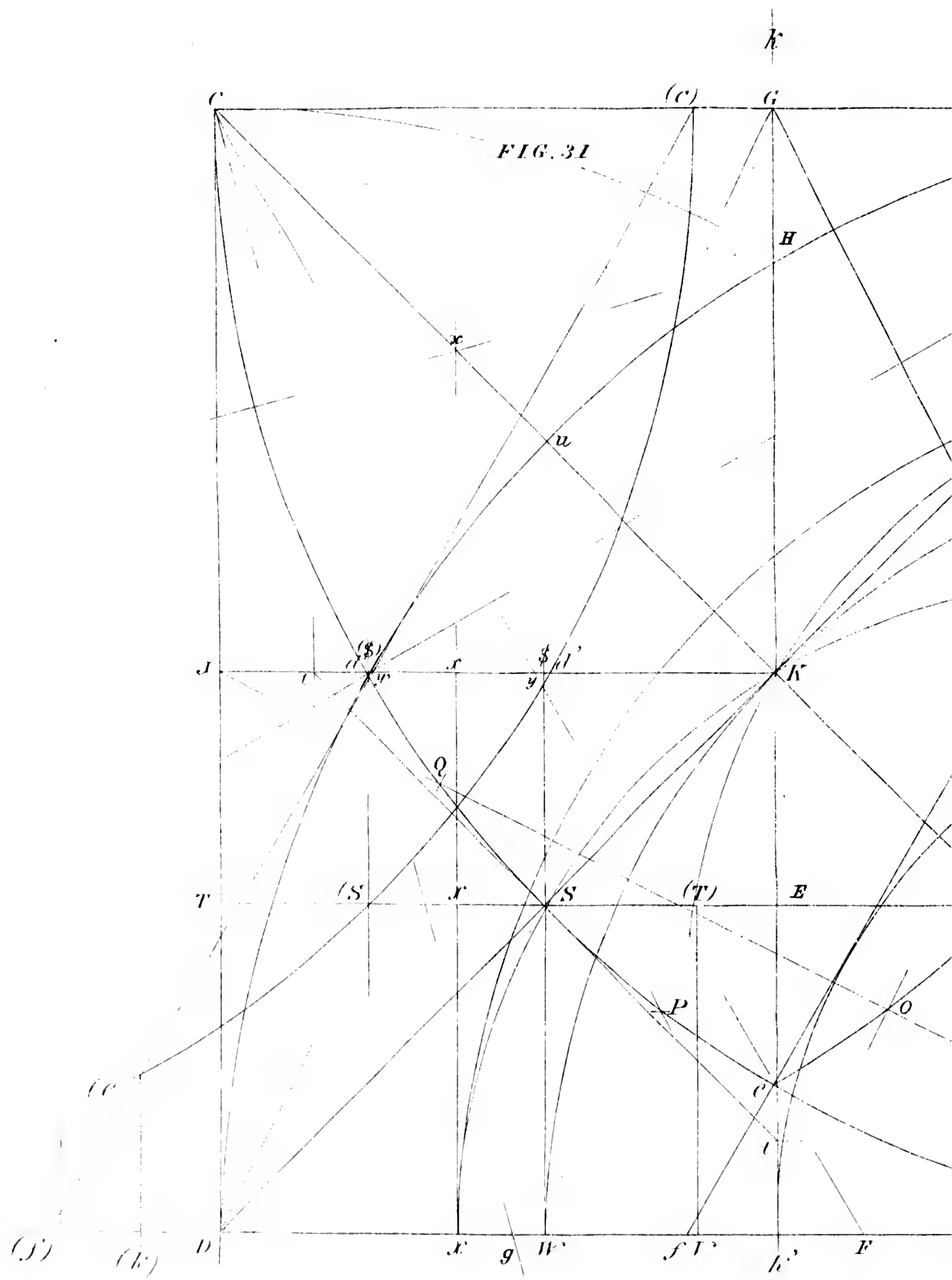
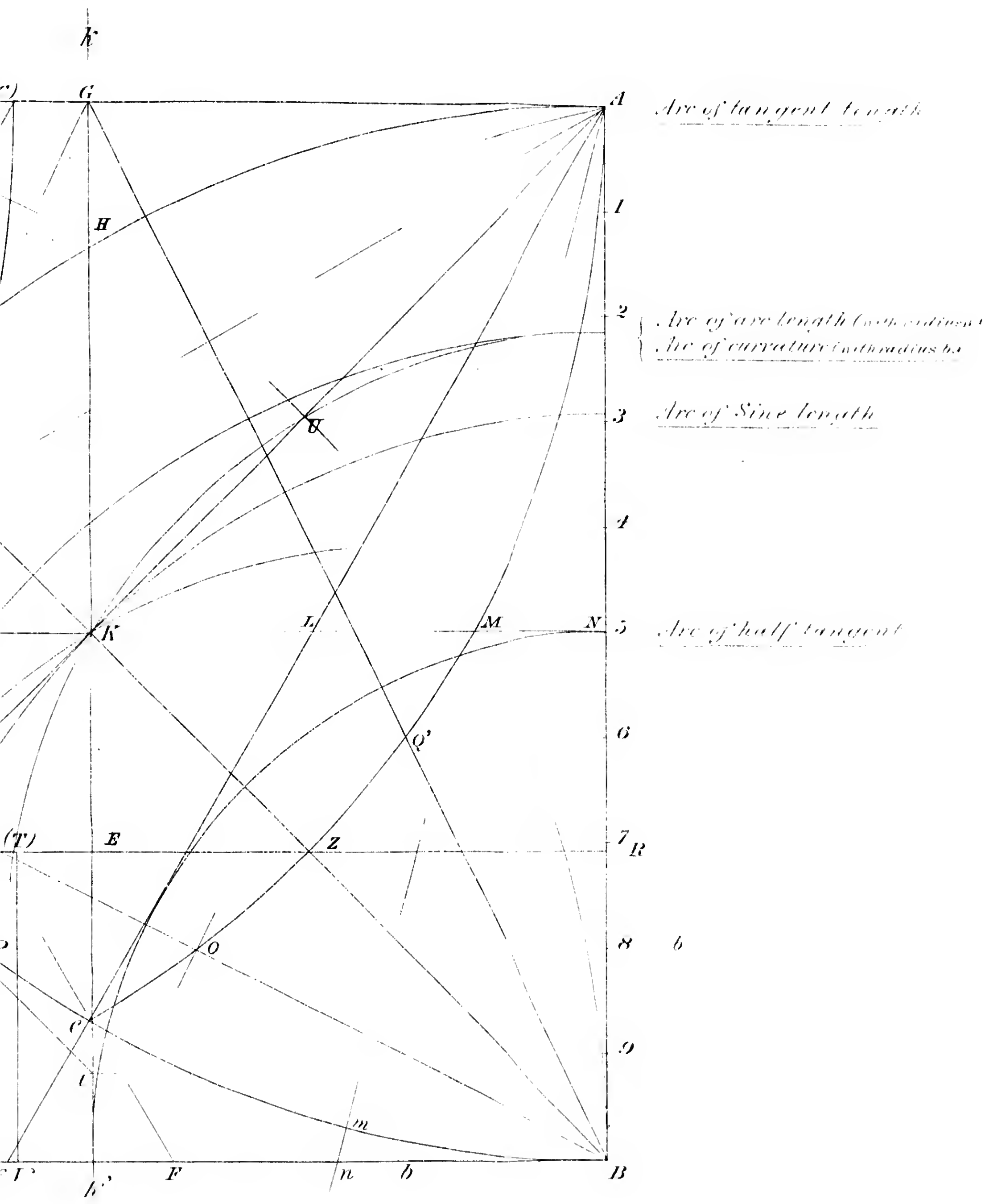


FIG. 31



k
G
A
Arc of tangent length
1
H
2
Arc of arc length
Arc of curvature
3
U
Arc of Sine length
4
L
M
N
5
K
Arc of half tangent
6
Q'
(T)
E
Z
7
R
8
b
O
C
9
t
m
F
n
b
B
k'

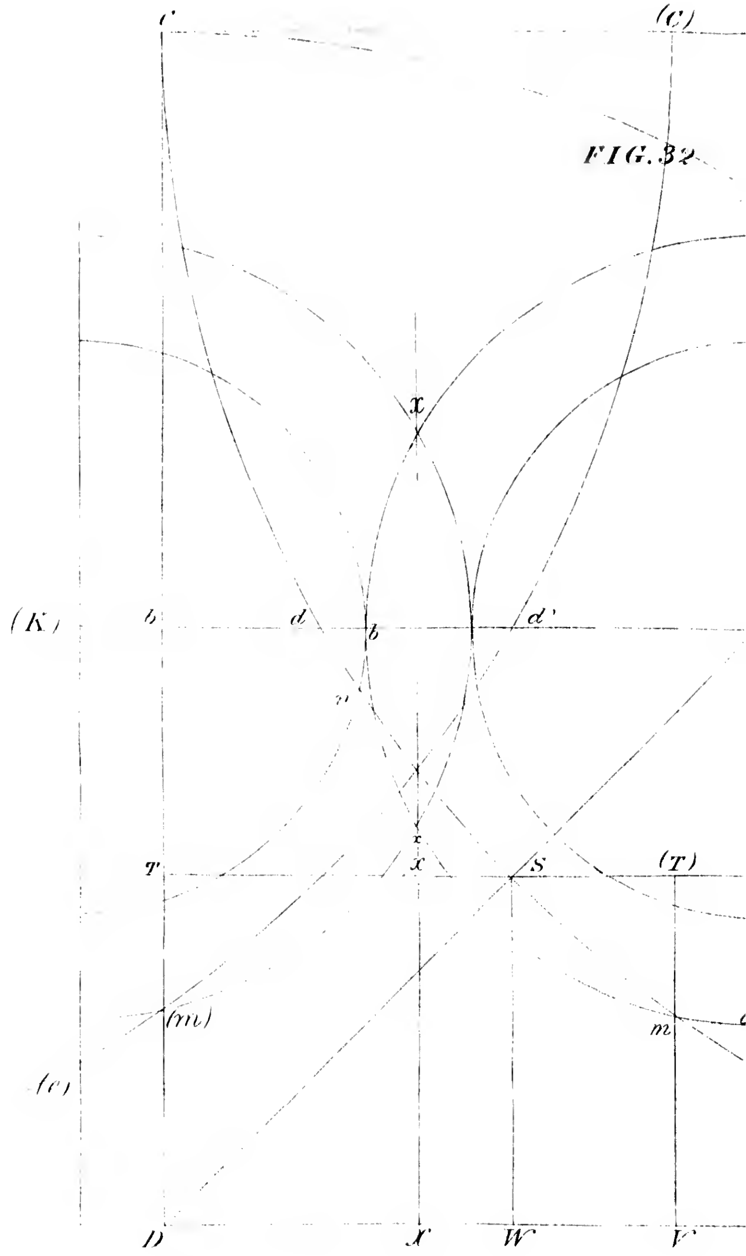


FIG. 32

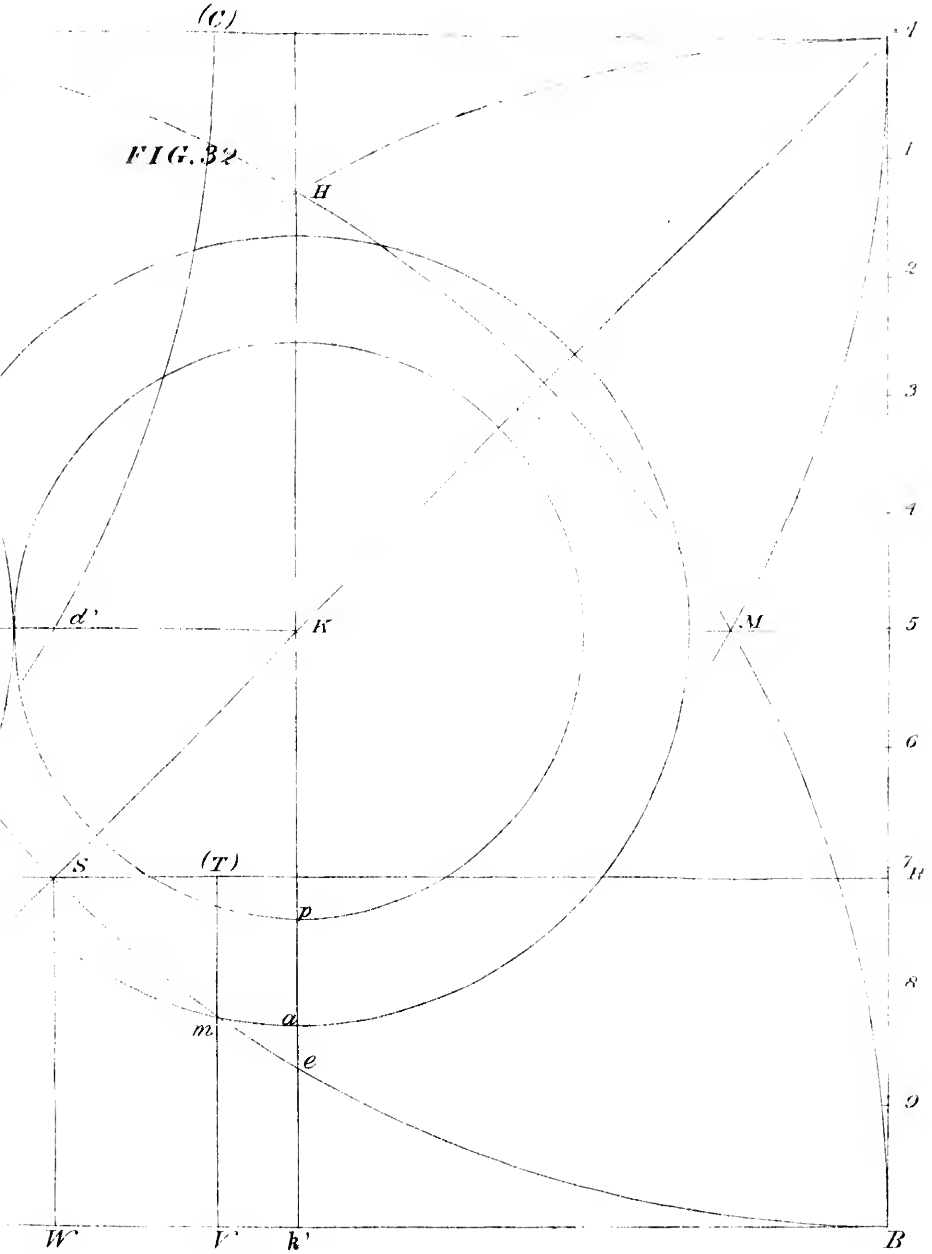


FIG. 32

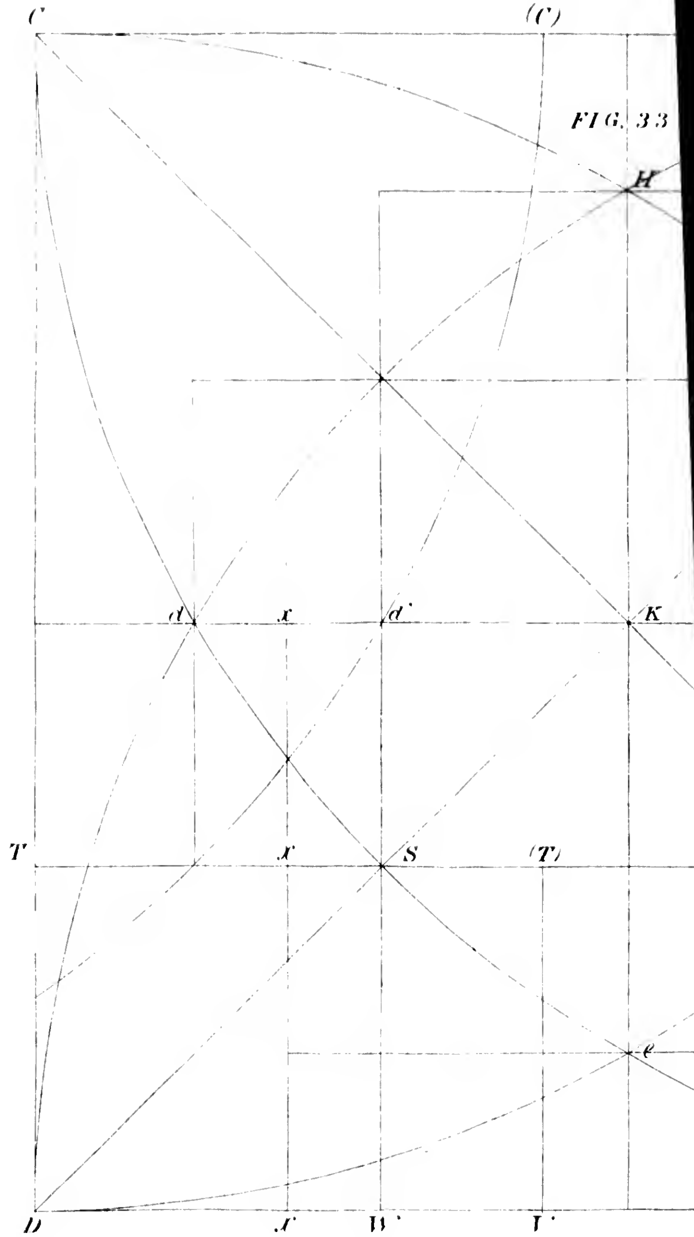
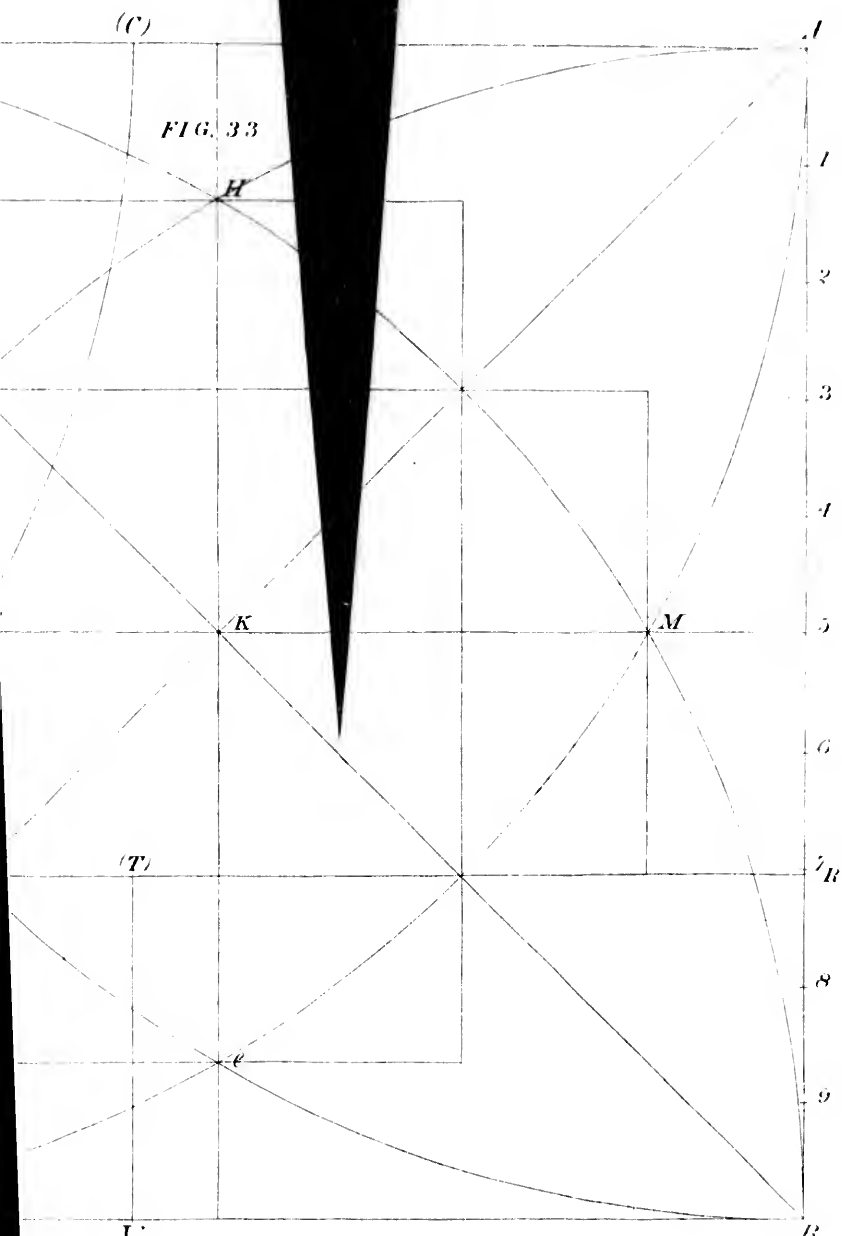
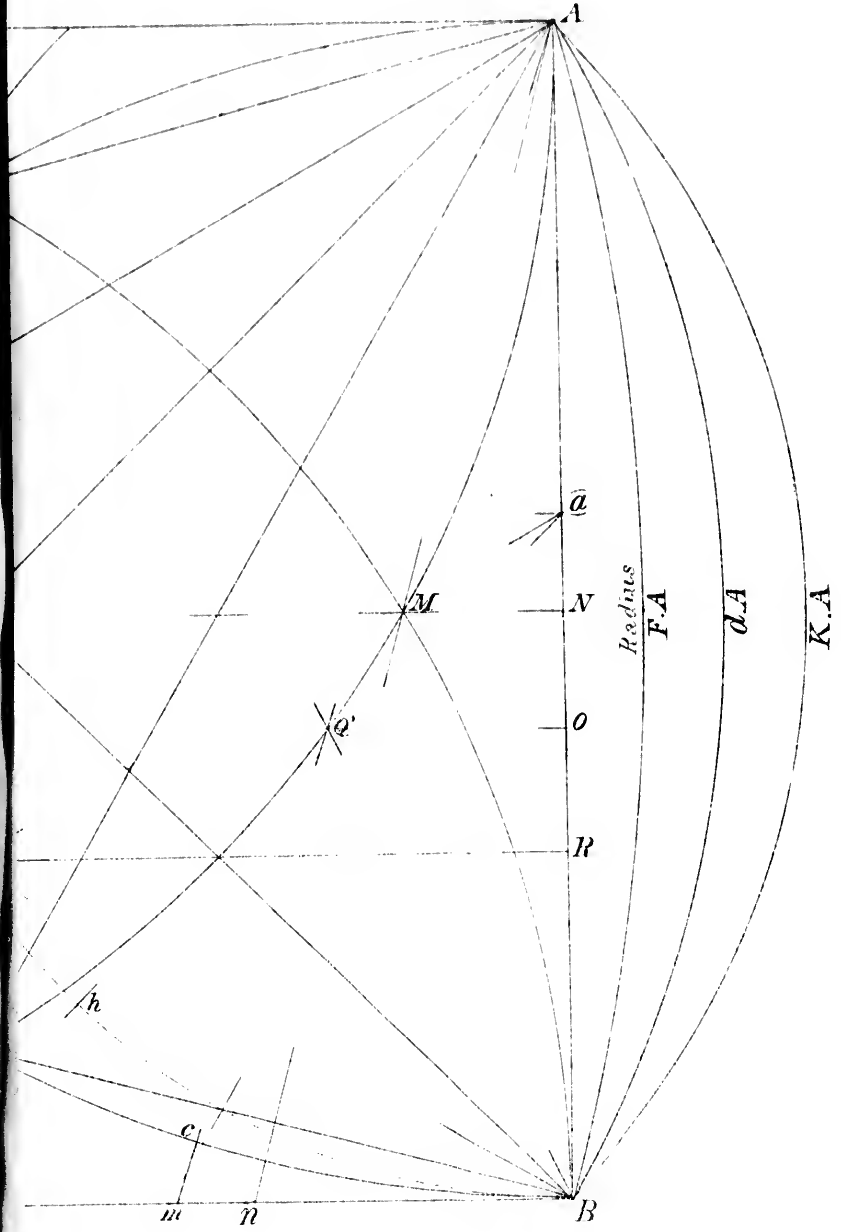


FIG. 33


(C)
A
FIG. 33
H
1
2
3
4
K
M
5
6
(T)
7
R
8
e
9
B


A
a
N
M
Q
O
R
h
c
m
n
B
Radius F.A
d.A
K.A

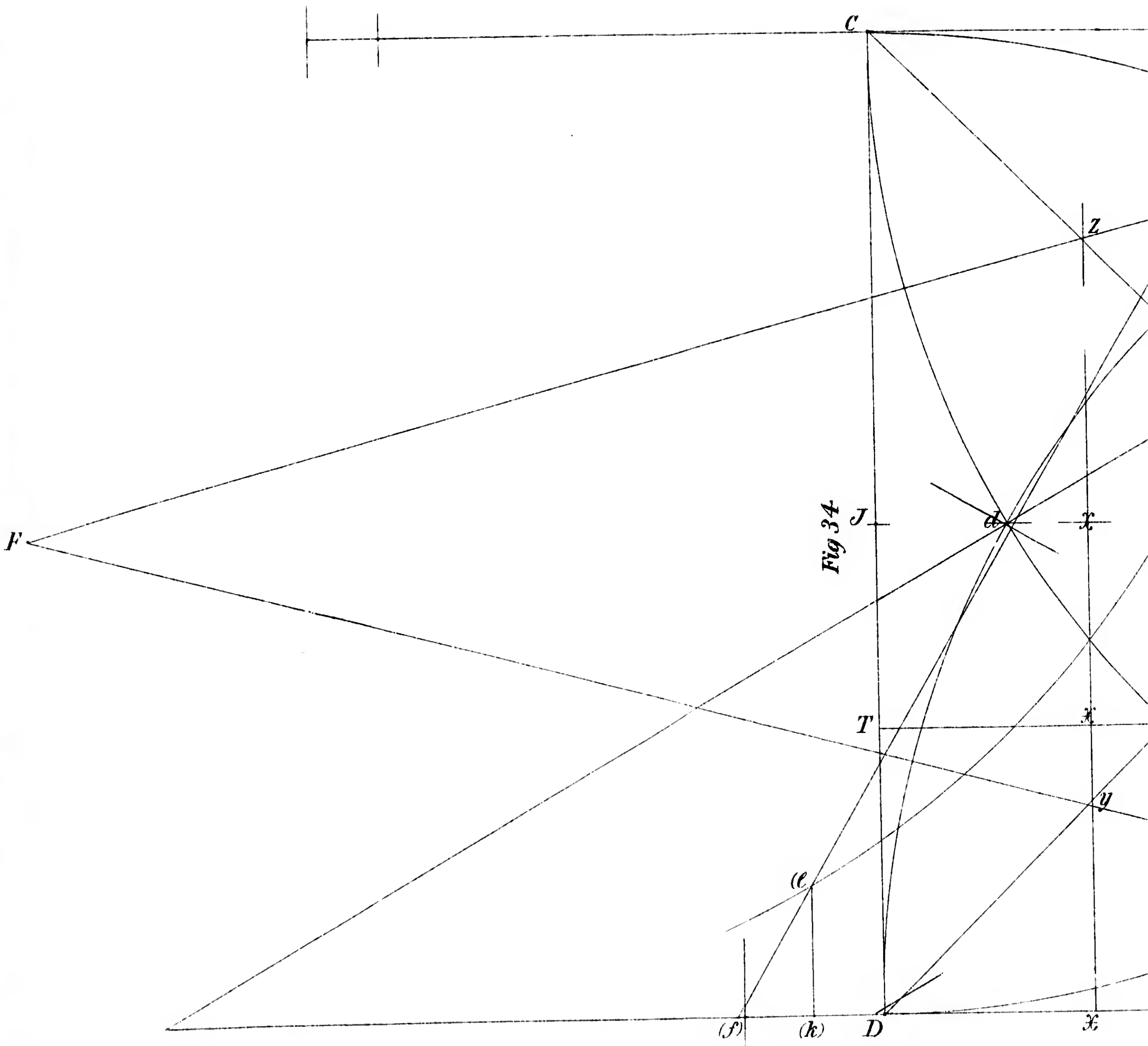


Fig 34

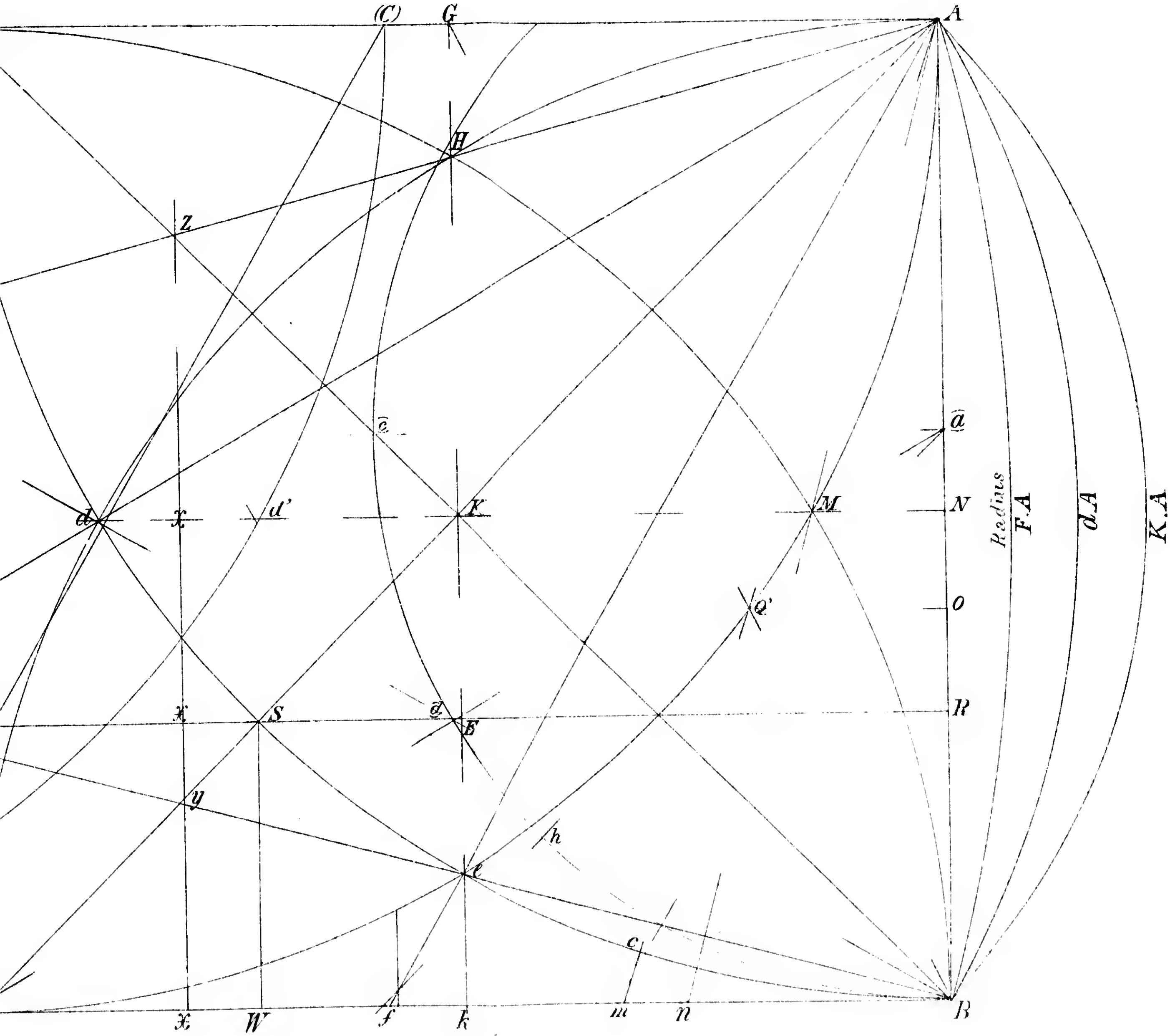

(C)
G
A
H
Z
a
d
x
d'
K
M
N
Radius
F.A
d.A
K.A
O
Q'
x
S
d
E
R
y
h
e
c
x
W
f
k
m
n
B

FIG. 35 (a)

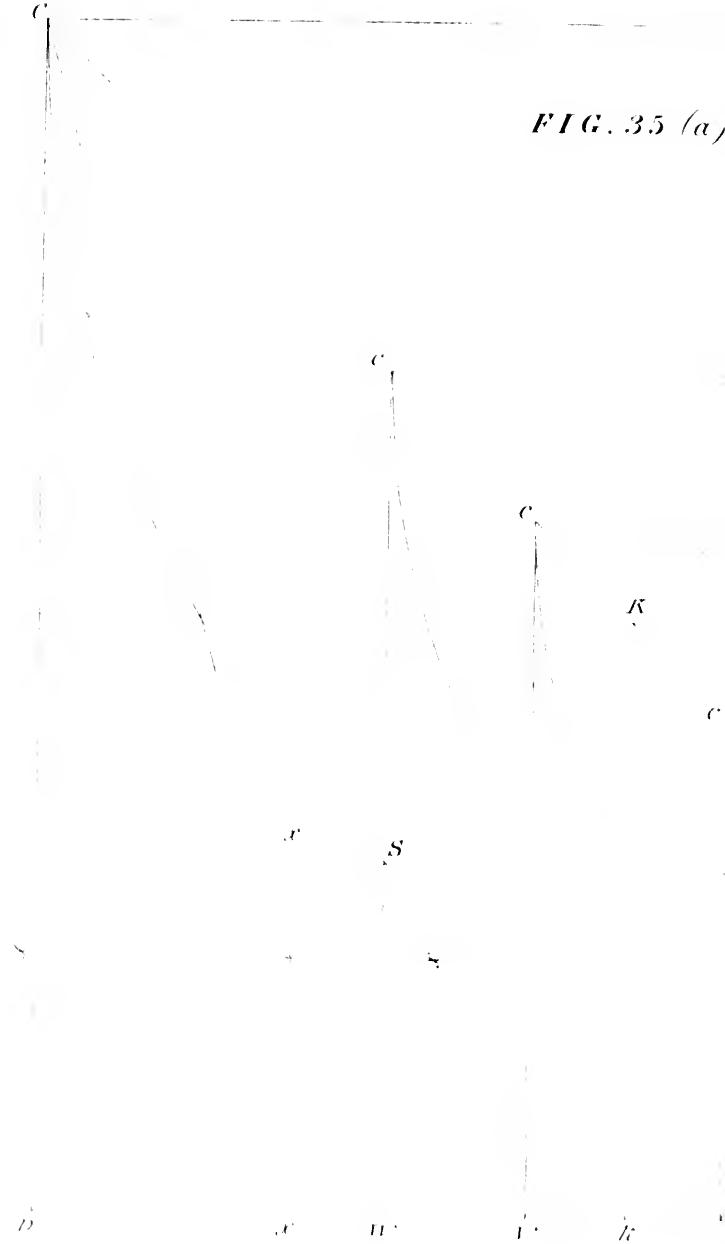
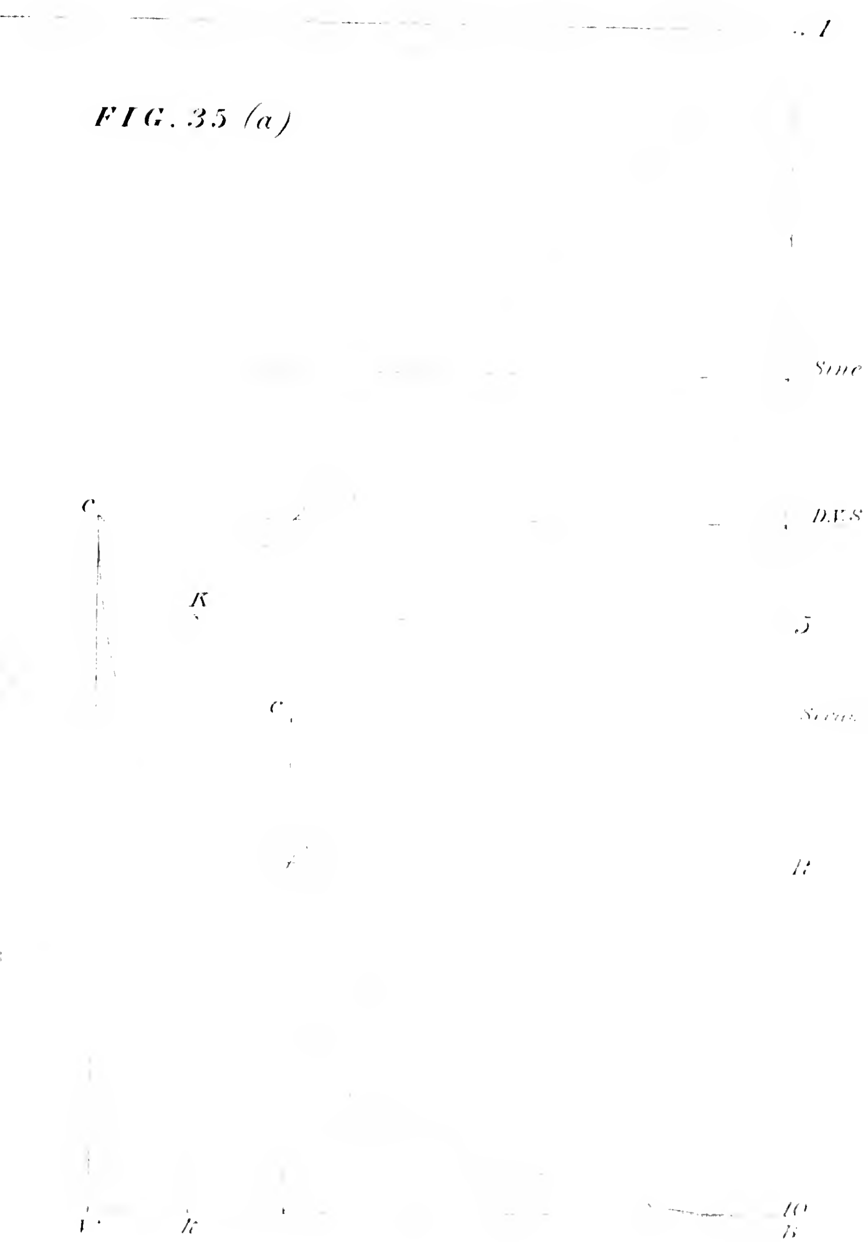


FIG. 35 (a)



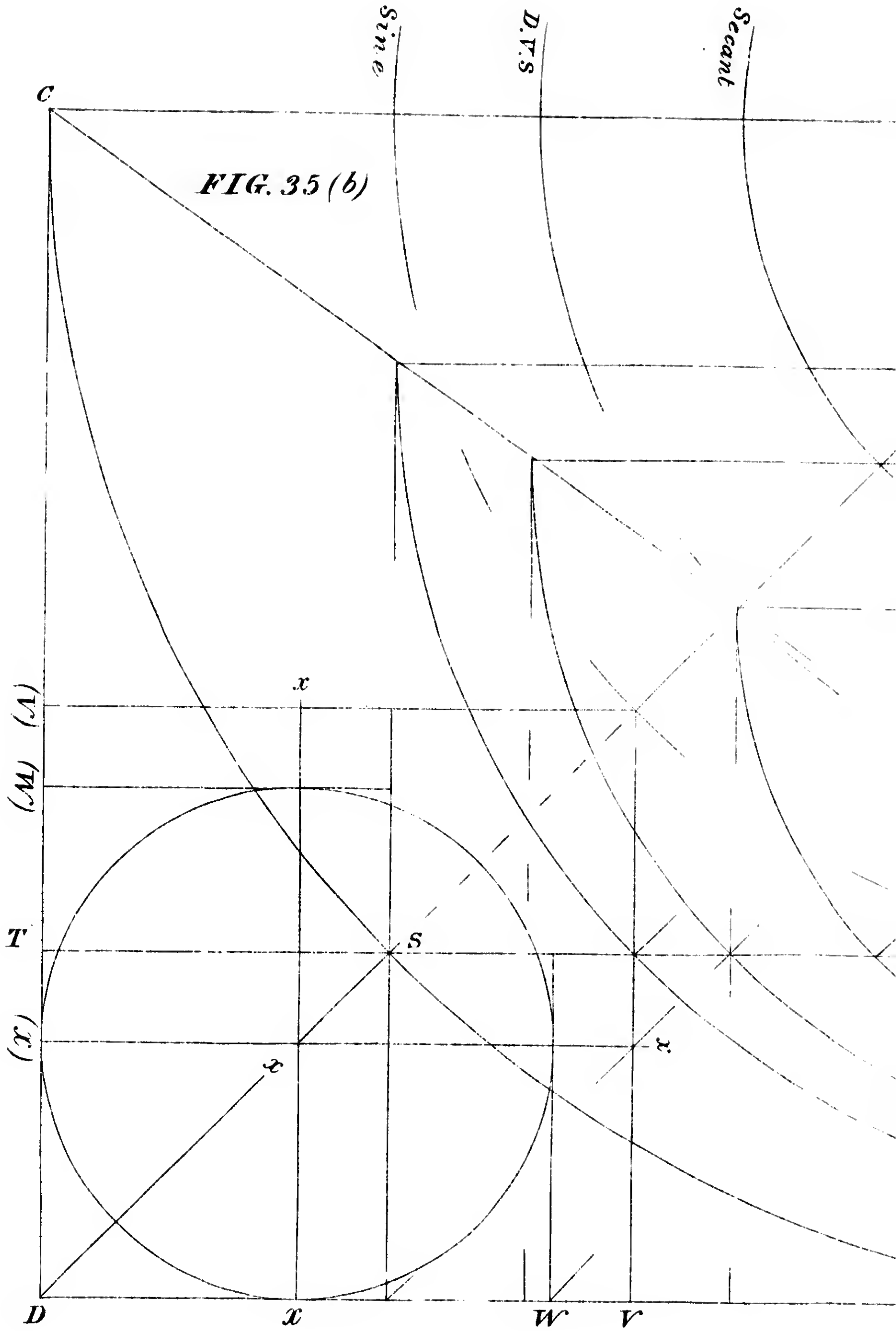
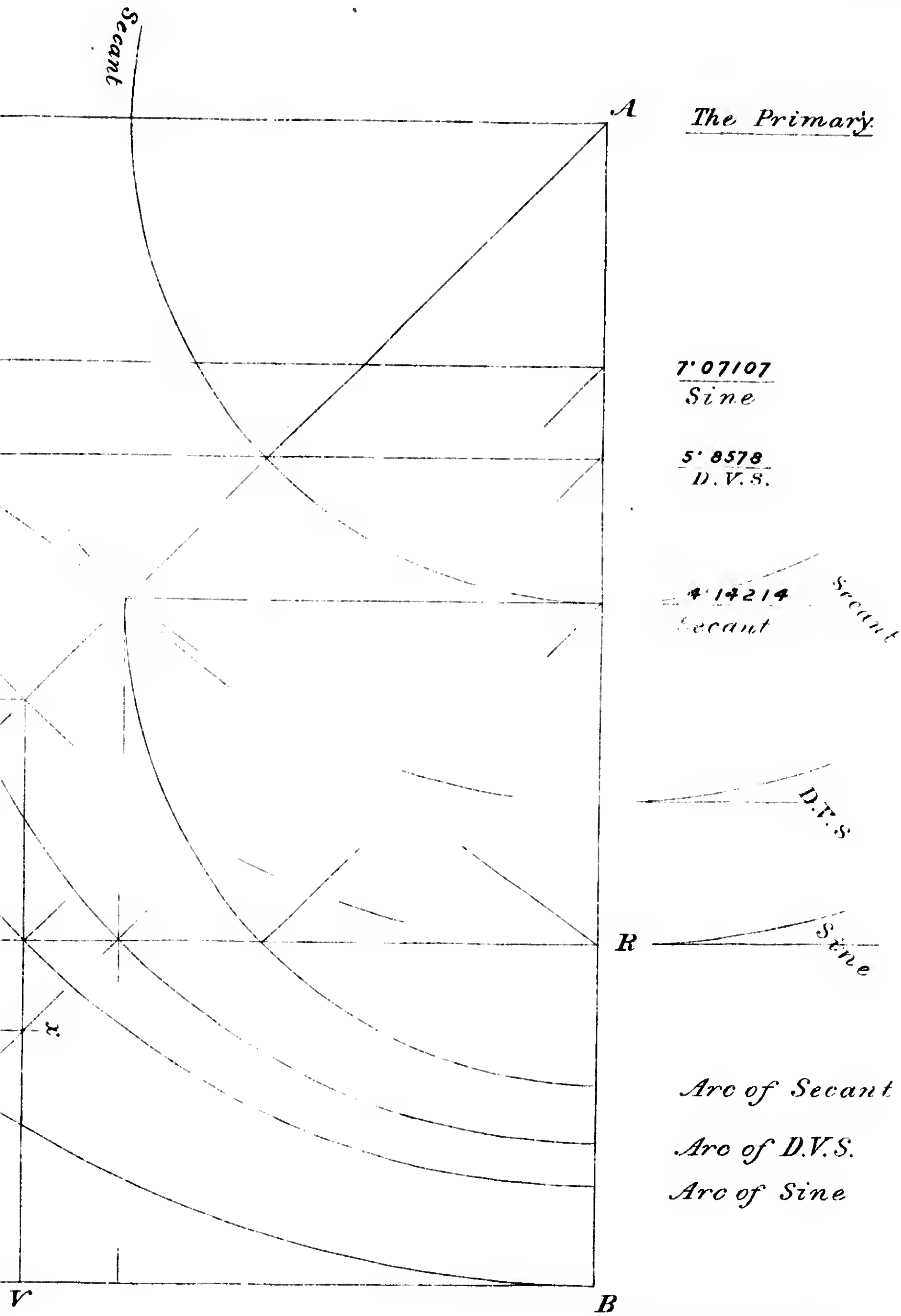


FIG. 35 (b)



Decimal Circles