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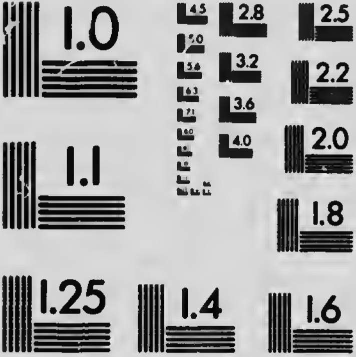

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THEORY OF MACHINES

INCLUDING

THE PRINCIPLES OF MECHANISM
AND
ELEMENTARY MECHANICS OF MACHINERY

BY

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PREFACE

In the course of a number of years of experience in the practice and teaching of engineering, the writer has had frequent occasion to deal with the general theory of machine construction and to analyze the proportions of various parts. In many cases it is largely a question of designing the parts of sufficient strength, and the principles used in such work are of the greatest importance to the engineer and student.

The science of machine design generally does not deal with the principles on which the machine is constructed, nor does it attempt to determine the stresses acting on the various parts while performing their required functions; it rather assumes that these stresses are known and assists in the proper proportioning of the parts.

In the making of machines, however, it is necessary to know the effect of changing the length and position of a link, for example, the effect of lengthening the connecting rod of a steam engine and of off-setting the cylinder. Again the effect of changing the shapes of gear teeth and also the determination of the correct shape are matters of the greatest importance.

Then again, the turning effect on the crank shaft due to the steam or gas pressure, the relative merits in this respect of two and four cycle gas engines, of tandem and cross-compound steam engines, the turning moment required on the crank of a stone crusher to crush the stone, etc., are frequently necessary in the design of the machine.

Other important problems are the design of governors, the determinat[illegible] proper weight of fly-wheels to meet given conditions, the spee[illegible]ons in various machines, the effect of the inertia of the p[illegible] effect of friction and the efficiency of machines.

Non[illegible] matters just mentioned come rightly under the head of machine design, although sometimes so treated, but form a separate study, and it is to such matters that the present treatise is devoted. These matters are not dealt with in an exhaustive manner, as the author feels that this would make the book too cumbersome, but the effort has been to make the treatise as suggestive as possible in the hope that the reader may work out his own problems with the help here given.

Some hesitation is felt about publishing this volume, partly because very much has already been written on the subject, and partly because the matter could not be dealt with as the author would have liked. The work has been illustrated throughout by numerous examples so chosen as to explain difficult although not unusual practical cases, and as far as possible the effort has been to put the material in readable form.

Some little attention has been devoted to the virtual centre because of the importance of the idea upon which it is based, and because of its usefulness in certain cases, but constructions involving its use frequently become so complicated as to render the method impracticable.

It is thought that the "phorograph" introduced in Chapter IV, is now published for the first time, and the author has found the constructions involved so simple and compact that he has used it almost exclusively. Very much of the remainder of the book is based upon a knowledge of the phorograph, and its use gives very simple methods for determining accelerations, kinetic energies of links, etc.

In the chapter on governors the author has consulted quite freely the very excellent book by M. Tolle "Die Regelung der Kraftmaschinen,"' which is a most comprehensive treatment of this very important part of machines. The characteristic curve has been employed quite broadly in his treatise and the author's regret is that space prevents its wider use in the present volume. The same author's work has also been used in other parts of this book and has been acknowledged.

The chapter on efficiency follows the treatment of Rankine and of Kennedy. The latter author's book on "The Mechanics of Machinery" was very often consulted, and certain parts of the book are based largely on Kennedy's volume.

In addition to the above authorities, the name of Prof. T. R. Rosebrugh, of the University of Toronto, should also be specially mentioned, acknowledgment of his work in the discovery of the phorograph having also been made in the body of the book.

Certain parts of the book were written under very great pressure, in order to meet the needs of the classes in the University of Toronto, so that mistakes have probably crept in, but as much care as possible has been taken to make the work accurate and it is hoped that any errors occurring will not be of such a nature as to mislead the reader.

R. W. A

University of Toronto,
Toronto, August 15, 1912.

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CHAPTER I.

THE NATURE OF THE MACHINE

In discussing any subject it is very important to know its distinguishing characteristics and also the features which it has in common with other and in many cases more fundamental matters. In the case of the machine this is particularly necessary, for the problems connected with the mechanics of machinery do not differ in many ways from those of the mechanics of free bodies, the same laws applying to both, and yet the machine has certain laws peculiar to itself which we must examine and classify. Further, machinery is now in such common use that it is well worthy of special consideration.

In order to clear our ideas let us first of all examine some of the well-known machines and see what their properties are. Take one of the most common, the reciprocating steam engine, which is known to consist essentially of the following parts:—(*a*) The frame or part which is fixed to some stationary or rigid object such as a foundation, or the frame work of a ship. This part carries the crank shaft bearing, the cylinder and steam chest and the crosshead guides, and as these are frequently cast in one piece we may treat the whole as the one part called the frame. (*b*) The piston, piston rod and crosshead which may from the point of view of mechanics be treated as the second part, which we may briefly call the piston. This part moves relatively to the frame, all points in it having a pure motion of translation or sliding; it contains the *wrist pin* to which the third part about to be mentioned is connected.

The connecting rod or third part (*c*) has a somewhat peculiar motion, one end of it being bored to receive the wrist pin, and therefore having a motion of translation or a reciprocating motion, the other end being bored to fit the crank pin which rotates in a circle. Thus this rod swings about the wrist pin similarly to a pendulum but also moves in the general direction of its length at the same time. (*d*) The fourth part consists of the crank and crank shaft, the latter rotating in the crank shaft bearing on the frame and carrying the crank pin, the axis of which is parallel with that of the shaft, and which describes a circle about the axis of the latter. Since the connecting rod is attached to the crank pin and also to the wrist pin the *stroke* of the piston will be proportional to, and it is generally equal to, the diameter of the crank pin circle. The flywheel is attached to the crank shaft. (*e*) The remaining parts consist of the valve gear and governor, and in this preliminary discussion will not be dealt with, their consideration being taken up later.

Take a second well-known machine, a lathe, which consists of the frame, the live spindle which rotates in bearings in the frame, the carriage which slides along the frame and contains a tool post having a cross motion, the change and back gears, the lead screw, belts, etc., all of which have certain definite duties to perform.

These two machines are typical of a very large number and from them we may develop the definition of the machine. Each of these machines contains *more than one part*, and if we think of any other machine we will see that it contains at least two parts, thus a crowbar is not a machine, neither is a shaft nor a pulley, if they were it would be difficult to conceive of anything which were not a machine. The so called "simple machines"—the lever, the wheel and axle, and the wedge—give confusion along this line because the complete machine is not inferred from the name; thus the bar of iron cannot be called a lever, it only serves such a purpose when along with it is a fulcrum, the wheel and axle only acts as a machine when it is mounted in a frame with proper bearings, and so with the wedge. So that we say a machine consists of a *combination of parts*.

Again these parts must offer some resistance to change of shape to be of any value in this connection. Usually the parts of a machine are *rigid*, but we very frequently find belts and ropes used, and it is well known that these are only of value when they are in tension because only when they are used in this way do they offer *resistance to change of shape*. No one ever puts a belt in a machine in a place where it is in compression. Springs are often used as in valve gears and governors, but they offer resistance wherever used. Thus the parts of a machine must be *resistant*.

Now under the preceding limitations a ship or any other *structure* could readily be included, and yet we do not call them machines, in fact, we would not call anything a machine in which the parts were incapable of motion with regard to one another. In the engine, if the frame is stationary, all the other parts are capable of moving, and when the machine is serving its true purpose they do move; in a bicycle, for example, the wheels, chain, pedals, etc., all move relatively to one another, and in all machines the parts must have *relative motion.* It is to be borne in mind that all the parts do not necessarily move, and as a matter of fact there are very few machines in which one part, which we shall refer to briefly as the frame, is not stationary, but all parts must move *relatively* to one another. If we stand on the frame of an engine the motion of the connecting rod is quite evident if it is slow enough, and if on the other hand we stood on the connecting rod of a very slow moving engine the frame would appear to us to move, that is, the frame has a motion relative to the connecting rod, and vice versa.

Now as to the nature of the motion, and it is this that especially distinguishes the machine. When a body moves in space its direction, sense and velocity depend entirely upon the forces acting on it for the time being, the path of a cannon-ball depends upon the force of the wind, the attraction of gravity, etc., and it is impossible to make two cannon balls travel over the same path, because the forces acting continually vary; a thrown ball may go in an approximately straight line until struck by the batter when its course suddenly changes, so also with a ship, etc., *i.e.*, in general, the path of a body in space varies with the external forces acting upon it. In the case of the machine, however, the matter is entirely different, for the path of each part is predetermined by the designer, and he arranges the whole machine so that each part shall act in conjunction with the others to produce in each a perfectly defined path.

Thus, in the steam engine, the piston moves in a straight line back and forth without turning at all, the crank pin describes a true circle, each point on it remaining in a definite plane, normal to the axis of the crank shaft, during the rotation, and again the motion of the connecting rod, although not so simple, is yet perfectly well known. The same is true in a lathe, the carriage for instance slides along the frame, the spindle has no longitudinal motion, but only rotation, and the gears are not free to slide along their axles. These motions are fixed by the designer and the parts are arranged so as to *constrain* them absolutely, irrespective of the external forces acting; if one presses on the side of the crosshead its motion is unchanged, and if he produces sufficient pressure to change the motion he breaks the machine and makes it useless. The carriage of the lathe can only move along the frame whether the tool which it carries is idle or subjected to considerable force due to the cutting of metal, should the carriage be pushed aside so that it does not slide on the frame, the lathe would be stopped and no work done with it till it was again properly adjusted, and so we might multiply the illustrations almost indefinitely.

This is then a distinct feature of the machine, that the relative motions of all parts are completely fixed and do not depend in any way upon the action of external forces. Or perhaps it is better to say that whatever external forces are applied, the paths of the parts are unaltered.

There remains one other matter relative to the machine, and that is its purpose. Machines are always designed for the special purpose of doing work. In a steam engine energy is supplied to the cylinder by the steam from the boiler, the object of the engine is to convert this energy into some useful form of work, such as driving a dynamo or pumping water. We deliver power to the spindle of a lathe through a belt, and the lathe in turn uses this energy in doing work on a bar by cutting a thread. We deliver energy to the crank on a windlass, and in turn, this energy is taken up by the work done in lifting a block of stone. Every machine is thus designed for the express purpose of doing *work.*

We may now sum up all these points in the form of a definition:—
A machine consists of resistant parts, which have a definitely known motion relatively to each other, and are so arranged that a given form of energy at our disposal may be made to do any desired form of work.

Many machines approach a great state of perfection, as for example the cases quoted of the steam engine and the lathe, where all parts are carefully made and the motions are all as close to those desired as one could make them. But there are many others which, although commonly and correctly classed as machines, do not come strictly under the definition. Take the case of the block and tackle which will be assumed as attached to the ceiling and lifting a weight. In the ideal case the pulling chain would always remain in a given position and the weight should travel straight up in a vertical line, and in so far as this takes place the machine may be considered as serving its purpose, but if the weight swings, then motion is lost and the machine departs from the ideal conditions. Such imperfections are not uncommon in the cases of machines *e.g.*, the rotor of an electrical motor may move endwise and thus deviate from its desired path under certain conditions, and there are many such cases occurring in machinery. We can only say that such uncertainty of motion must be avoided so far as it is possible, and the more such uncertainty is removed, the more nearly perfect is the machine, and the more nearly does it comply with the conditions for which it was designed, and the more perfectly will it do its work.

DIVISIONS OF THE SUBJECT

We may divide our study of the machine into four parts (1) A study of the motions occurring in the machine without regard to the acting forces, this may be called the *kinematics of machinery*. (2) A study of the external forces acting on different parts of a machine, treating it as a structure which is not moving, or is moving uniformly and balancing forces by the ordinary methods of statics, the problems are those of *static equilibrium*. (3) A study of the forces due to the weights and shapes of the parts as well as to the external forces. (4) A study of the proper sizes and shapes to be given to the parts to provide for them sufficient strength to carry out the motions which the designer intended, and to be able to resist the applied forces. This is called *machine design* and is of sufficient importance and magnitude to demand an entirely separate treatment so that it will not be dealt with here.

We may begin on the first division of the subject, and shall discuss the methods adopted for obtaining definite forms of motion in machines. If we study the steam engine, which we have already discussed at some length, we notice that in any moving part the path of any point always lies in one plane, *e.g.*, the path of a point on the crank pin lies on a plane normal to the crank shaft, as does also the path of any point on the connecting rod, and also the path of any point on the crosshead. Since this is the case the parts of a steam-engine mentioned are said to have *plane motion*, by this statement we simply mean that the path of any point on any part described always lies in one and the same plane. In a completed steam engine with slide valve, all parts have plane motion but the governor balls, in a lathe all parts have plane motion usually, the same is true of an electric motor, in fact, the vast majority of the motions with which we have to deal in machines are plane motions.

There are, however, cases where different motions occur, for example, we find that there are parts of machines where a point always remains at a fixed distance from another fixed point, or where the motion is such that any point will always lie on the surface of a sphere of which the fixed point is the centre, as in the universal and ball and socket joints. Such motion is called *spheric motion* and is not nearly so common as the plane motion.

A third class of motions occur where a body has a motion of rotation about an axis and also a motion of translation along the axis at the same time, the motion of translation bearing a fixed ratio to the motion of rotation. Such motion is called *helical* or *screw motion* and occurs quite frequently.

In the ordinary monkey wrench the movable jaw has a plane motion relative to the part held in the hand, the plane motion being one of translation or sliding, all points on the screw have plane motion relative to the part held, the motion being one of rotation about the axis of the screw, and the screw has a helical motion relative to the movable jaw, and vice versa.

PLANE CONSTRAINED MOTION

It has been noticed already that plane motion is frequently constrained by causing a body to rotate about a given axis or by causing the body to move along a straight line in a motion of translation, the first form of motion may be called *turning motion*, the latter form *sliding motion*.

Turning motion.—This may be constrained in many ways but Fig. 1 shows one method consisting of a shaft in a fixed bearing, this shaft carrying a pulley as shown in the upper figure, while the lower figure shows a thrust bearing for the propeller shaft of a boat. In figure (*a*) there is simply a straight shaft *S* with pulley *P*, passing through a bearing *B*, and if the construction were left in this form it would permit plane turning motion in the pulley and shaft, but would not constrain it, as the shaft might move axially through *B*. If, however, two *collars C* are secured to the shaft by screws as shown, then these collars effectually prevent the axial motion and we get only pure turning. On the propeller shaft the collars *C* are forged right on the shaft and here a number are put on on account of the great force tending to push the shaft axially. Thus in both cases the turning motion is necessitated by the two bodies, the shaft with its collars and the bearing, and these together are called a *turning pair* for obvious reasons, the pair consisting of two *elements*.

It is evident that this turning pair can be arranged in various forms as shown in Fig. 1, one form being preferred to another in certain cases. The form (*c*) is used for railroad cars, the bearing here only coming in contact with the shaft for a small part of the circumference of the latter, the two being held in contact purely because of the connection to the car which rests on top of *B*, the collars *C* are here of slightly different form. At (*d*) we have a vertical bearing which, in a somewhat better form is often used in turbines, but here again we would only get the turning motion provided the weight were on the vertical shaft and pressed it into *B*. In this case there is only one part corresponding to the collar *C*, which is the part of *B* below the shaft.

In the cases (*a*) and (*b*), turning motion will take place by construction, and is said to be secured by *chain closure*, which will be

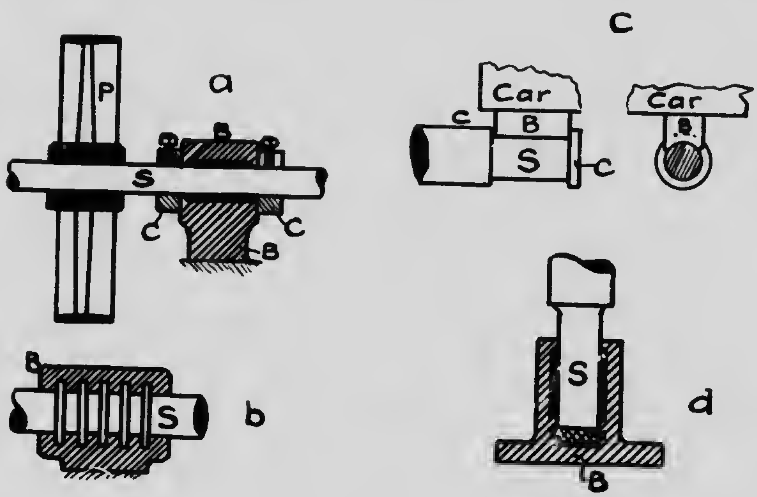


Fig. 1

referred to later, while in the cases (*c*) and (*d*) the motion is only constrained so long as the external forces act in such a way as to press the two elements of the pair together, plane motion being secured by *force closure*. In cases, such as those described, where force closure is permissible it forms the cheaper construction as a general rule.

Sliding motion.—The *sliding pair* also consists of two elements, and if a section of these elements is taken normal to the direction of sliding the elements must be non-circular. As in the previous case the sliding pair in practice has very many forms, a few of which are shown in Fig. 2, (*a*), (*b*) and (*c*) being the forms in very common use for the crossheads of steam engines, (*c*) being rather cheaper in general than the others. At (*d*) is shown a form which may be used where there is very little tendency to turning, consisting of a shaft with a long keyway cut in it while the other element has a parallel key, or "feather," fastened to it, so that the outer element may slide along the shaft but cannot rotate upon it. The student will see very many forms of this pair and should study them carefully.

In the automobile engine and in all the smaller gas and gasoline engines, the sliding pair is circular, because the crosshead is omitted and the connecting rod is directly attached to the piston, the latter

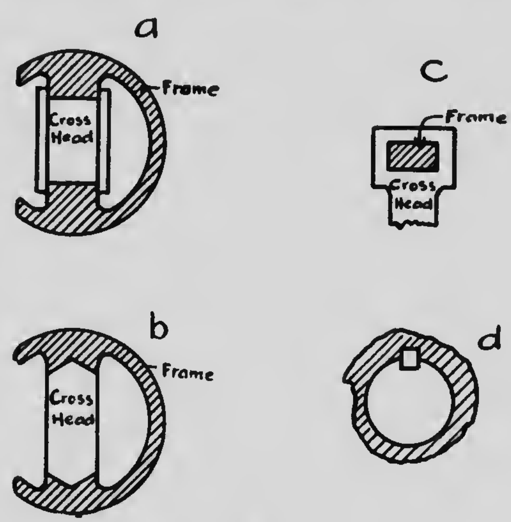


Fig. 2

being of course, circular and not constraining sliding motion. In this case the sliding motion alone is obtained because of the connecting rod, which on account of the pairing at its two ends will not permit the piston to rotate. The real sliding pair, of course, consists of the cylinder and piston, both of which are circular.

In the case of sliding pairs also we may have chain closure where constraint is due to the construction as in the cases illustrated in Fig. 2 at (*a*), (*b*), (*c*), (*d*), in these cases the motion being one of sliding irrespective of the directions of the acting forces, or we may have force closure as shown at Fig. 3, which represents a planer table, the weight of which alone keeps it in place. Occasionall·

through an accident the planer table may be pushed out of place by a pressure on the side, but of course, the planer is not again used until the table is replaced for the reason that the design is such that the table is only to have plane motion, a condition possible only if the table rests in the grooves in the frame.

The two principal forms of plane constrained motion are thus turning and sliding, these motions being controlled by turning and sliding pairs respectively, and each pair consisting of two elements. Where contact between the two elements of a pair is *over a surface* the pair is called a *lower pair*, and where the contact is only *along a line or at a point*, the pair is called a higher pair. To illustrate this we may take the ordinary bearing as a very common example of lower pairing, whereas a roller bearing has line contact and a ball bearing point contact and are examples of higher pairing, these illustrations are so familiar as to require no drawings. The contact between spur gear teeth is along a line and therefore an example of higher pairing.

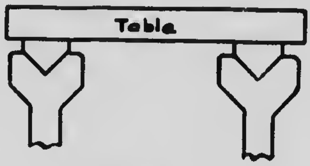


Fig. 3

In general, the lower pairs last longer than the higher, because of the greater surface exposed for wear, but the conditions of the problem settle the type of pairing. Thus lower pairing is used on

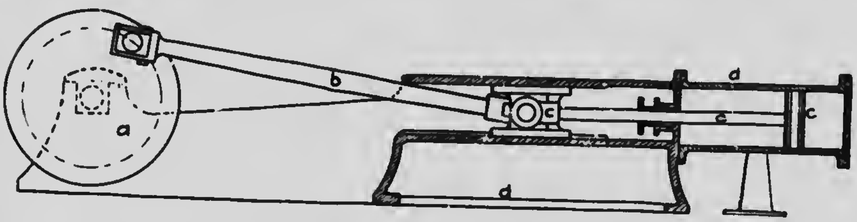


Fig. 4

the main shafts of large engines and turbines, but for automobiles and bicycles the roller and ball bearings are common.

MACHINES, MECHANISMS, ETC.

Returning now to the steam engine, Fig. 4, described at the beginning, its formation may be studied. We will omit the valve gear and governor at present and discuss the remaining parts consisting of the crank, crank shaft and fly-wheel, the connecting rod, the piston

piston rod and crosshead, and finally the frame and cylinder. Taking the connecting rod it is seen to contain two turning elements, one at either end, and that the real function of the metal in the rod is to keep these two elements at a fixed distance apart. The crank and crank shaft *a* contains two turning elements, one of which is paired with one of the elements on the connecting rod *b*, and forms the crank pin, and the other is paired with a corresponding element on the frame *d*, forming the main bearing. It is true that the main bearing may be made in two parts, both of which are made on the frame, as in centre crank engines, or one of which may be placed as an *outboard bearing*, but it will readily be understood that this divi-

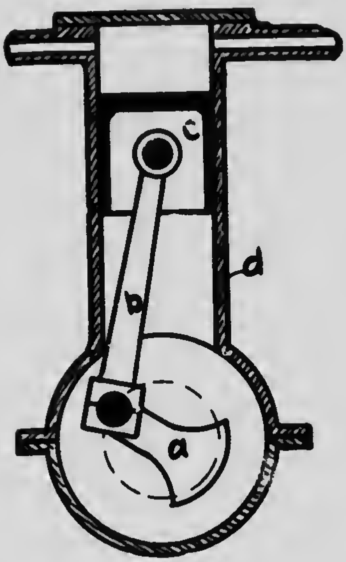


Fig. 5

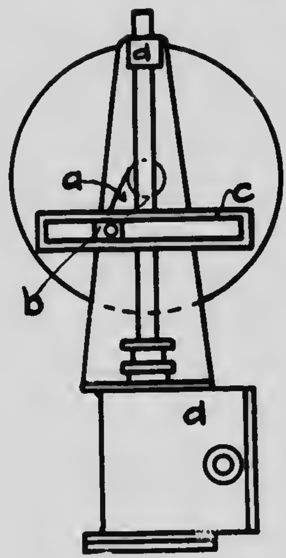


Fig. 6

sion of the bearing is only a matter of practical convenience, for it is quite conceivable that the bearing might be made in one piece, and if this piece were long enough it would serve the purpose perfectly. Thus the crank consists essentially of two turning elements properly connected.

Again the frame *d* contains the outer element of a turning pair, of which the inner element is the crank shaft, and it also contains a sliding element which is usually again divided into two parts for the purpose of convenience in construction, the parts being the cross-head guides and the cylinder. But the two parts are not absolutely essential, for in the single-acting gasoline engine the guides are omitted and the sliding element is entirely in the cylinder, and there may even be double acting machines without the crosshead guides, although they are unusual. Of course, the shape of the element depends upon the purpose to which it is put, thus in the case referred to it is round.

Then there is finally the crosshead *c*, with the turning element pairing with the connecting rod and the sliding element pairing with the sliding element on the frame. The sliding element is usually in two parts to suit those of the frame, but it may be only in one if so desired and conditions permit of it (see Fig. 2).

Thus the steam engine consists of four parts, each part containing two elements of a pair, in some cases the elements being for sliding, and in others for turning.

Again, on examining the small gasoline engine illustrated in Fig. 5, it will be seen that the same method is adopted here as in the steam engine, but the crosshead, piston and piston rod are all combined in the single piston *c*. Further in the Scotch yoke, Fig. 6, a scheme in common use for pumps of small sizes as used on fire engines at times and for other purposes, we have the crank *a* with two turning elements, the piston and crosshead *c* with two sliding elements, and the block *b*, and the frame *d*, each with one turning element and one sliding element.

The same will be found true in all machines having plane motion, each part containing at least two elements, each of which is paired with corresponding elements on the adjacent parts. For convenience we shall call each of these parts of the machine a *link*, and the series of links so connected as to give a complete machine is called a *kinematic chain*, or simply a *chain*. It must be very carefully borne in mind that if a kinematic chain is to form part of a machine or a whole machine, then all the links must be so connected as to have definite relative motions, this being an essential condition of the machine.

In Fig. 7 three cases are shown in which each link has two turning elements. Case (*a*) could not form part of a machine because the three links could have no relative motion whatever, as is evident by inspection, while at (*b*) it would be quite impossible to move any link without the others having corresponding changes of position, and for a given change in the relative positions of two of the links we have a definite change produced in the others. Lookng next at case (c), we observe at once that we could secure both DC and OD to the ground and yet move AB, BC, and OA, that · definite change in AB produces no necessary change in OA, or ·ie link may move without the others undergoing motion or relative change of position. Such an arrangement could not form part of a machine because the relative motions of the parts are not fixed but variable according to conditions. At (*d*) again we find a chain which can be used because here if we move any one link relatively to any other all the links move relatively, or if we fasten one link, say OD, to the ground and move OA then must all the other links move.

When a chain is used as a machine, usually one of the links acts as the frame and is fixed to a foundation or other stationary body,

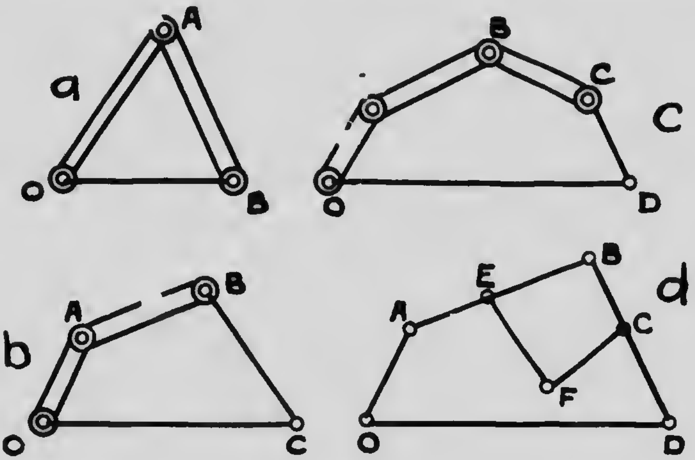


Fig. 7

In studying the motions of various links it is not necessary to know the exact shape of the links at all, for the motion is completely known if we know the location and form of the pairs of elements. Thus we may replace the actual link by a straight bar which connects the elements of the link together, and we shall always assume that this bar never changes its shape during motion. We shall thus in future represent the chain by straight lines and a chain so represented and having the relative motions of all links completely constrained and having one link fixed we shall call a *mechanism*.

If the links of a chain have only two elements each, the chain is said to be *simple*, but if any link has three or more elements as at Fig. 7 (*d*), the chain is *compound*.

Inversion of the Chain.—Since in forming a mechanism we fix one link of the chain it would appear that since any of the links may be fixed in a given chain, it may be possible to change the nature of the resulting mechanism by fixing various links successively. Take as an example the mechanism shown at (1) Fig. 8, *d* being the fixed link, here *a* would describe a circle, *c* would swing about *C* and *b* would have a pendulum motion, but with a moving pivot *B*. If we fix *b* instead of *d*, *a* still rotates, *c* swings about *b* and *d* now has the motion *b* originally had, or the mechanism is unchanged.

If *a* is fixed then the whole mechanism may rotate, *b* and *d* rotating about *A* and *O* respectively as shown, and *c* also rotating, the form of the mechanism being thus changed to one in which all

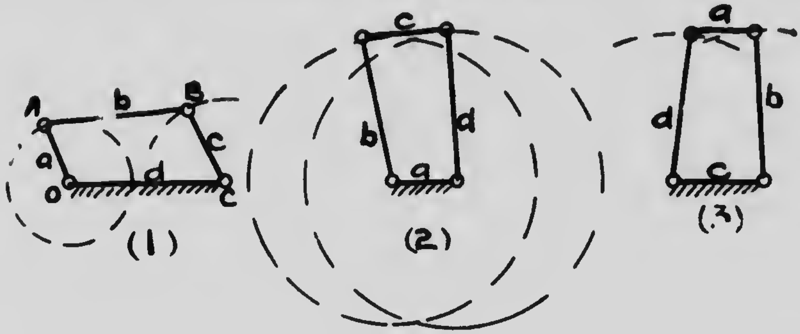


Fig. 8

the links rotate. If, on the other hand, we fix *c* then none of the links can rotate but *b* and *d* simply oscillate about *B* and *C* respectively. The student will do well to make a cardboard model to illustrate this point.

The process by which the nature of the mechanism is altered by changing the fixed link is called *inversion of the chain*, and in general, we may say that there are as many mechanisms as there are links in the chain of which it is composed, although in the above illustration there are only three for the four links.

This inversion of the chain is very well illustrated in case of the chain used in the steam engine, which we shall refer to in future as the *slider-crank chain*. The mechanism is shown in Fig. 9 with the crank *a*, connecting rod *b* and piston *c*, the latter having one sliding and one turning element and representing the reciprocating masses, *i.e.*, piston, piston-rod and crosshead. The frame *d* is here represented by a straight line (although it is common yet the line of motion of *c* does not always pass through *O*, as shown at (1), it

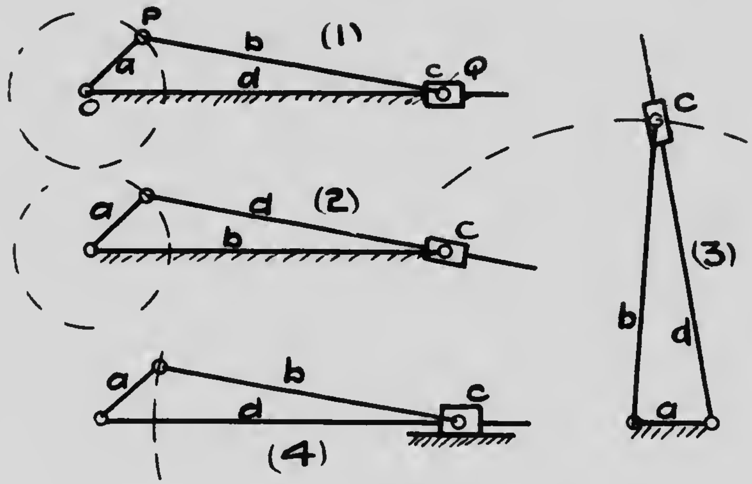


Fig. 9

represents the ordinary steam engine). If now, instead of fixing *d* we fasten *b* to the foundation, *b* being the longer of the two links containing the two turning elements, then *a* still can rotate, *c* merely swings about *Q* and *d* has a swinging and sliding motion, and if *c*

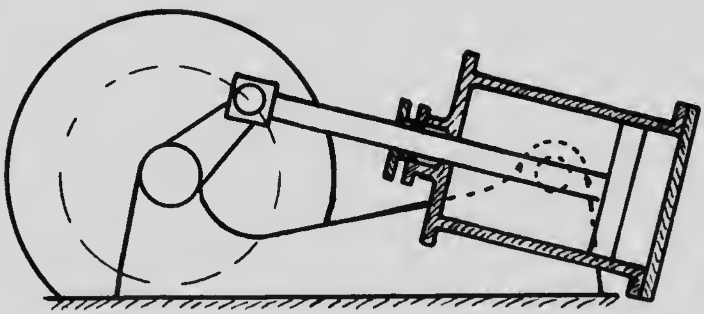


Fig. 10

is a cylinder and a piston is attached to *d* we get the oscillating engine as shown at (2) Fig. 9, and drawn in some detail in Fig. 10.

If instead of fixing the long rod *b* with the two turning elements, we fix the shorter rod *a*, then *b* and *d* revolve about *P* and *O* respectively, and *c* also revolves sliding up and down *d*. If we drive *b* by means of a belt and pulley at constant speed then the angular velocity of *d* is variable and the device may be used as a quick return motion, in fact, it is the Whitworth quick return motion and

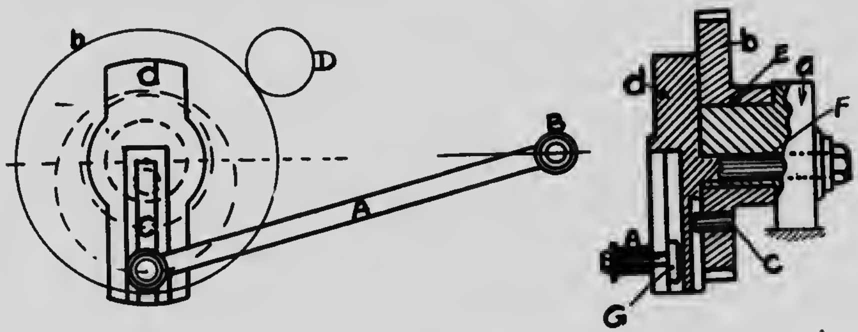


Fig. 11

is illustrated at (3). The practical form is also shown, Fig. 11, and the student should study the relation between the mechansim and the actual machine.

In the Whitworth quick-return motion. Fig. 11, *D* is the pinion driven by belt and this meshes with the gear *b*. The gear rotates on a large bearing *E* attached to the frame *a* of the machine, and through

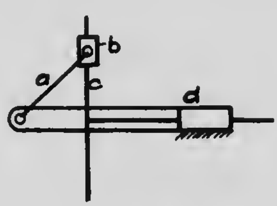


Fig. 12

Fig. 13

the bearing *E* is a pin *F*, to one side of the centre of *E*, carrying the piece *d*, the latter being driven from *b* by a pin *c* working in a slot. The arm *A* is attached to a tool holder at *B*.

The fourth inversion found by fixing *c* is rarely used though it is found occasionally. It is shown at (4) Fig. 9.

There are thus four inversions of this chain and it might be further changed slightly by placing *Q* to one side of the link *d* thus giving the scheme used in operating the sleeves in some forms of gas engines, *e.g.*, the Knight engine.

A further illustration of a chain which goes through many inversions in practice is given in Fig. 12, and contains two links, *b* and *d*, with one sliding and one turning element each, also one link

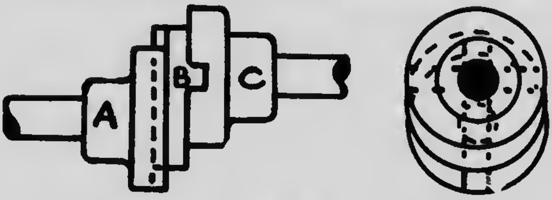


Fig. 14

a with two turning elements and one *c* with two sliding elements When the link *d* is fixed *c* has a reciprocating motion and such a setting is frequently used for small pumps driven by belt through the crank *a* (Fig. 12), *c* being the plunger. A detail of this has already been given in Fig. 6. With *a* fixed the device becomes Oldham's

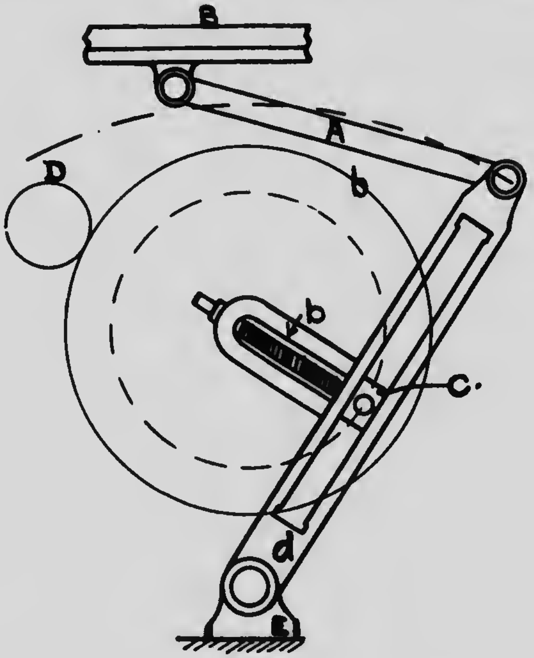


Fig. 15

coupling which is used to connect two parallel shafts which are very nearly in line, Figs 13 and 14.

In Fig. 14, *A* and *C* are the two shafts which are parallel and which rotate about fixed axes. Keyed to each shaft is a coupling with a slot running right across its face and between the couplings is a piece *B* with two keys at right angles to one another, one on each side, fitting in grooves in *A* and *C*. As *A* and *C* revolve, *B* works sideways and vertically, both shafts always turning at the same speed.

A somewhat different modification of the slider-crank chain is shown at Fig. 15, a device also used as a quick-return motion in shapers and other machines. On comparing it with the Whitworth motion shown at Fig. 11, it is seen that the nature of the mechanism may also be somewhat altered by varying the proportions of the links. The mechanism illustrated at Fig. 15 should be clear without further explanation *D* is the driving pinion working in with the large gear *b*, the tool is attached to *B* which is driven from *c* by the link *A*. It is readily seen that *B* moves faster in one direction than the other. Further an arrangement is made for varying the stroke of *B* at pleasure by moving the centre of *c* closer to, or further from, that of *b*.

CHAPTER II.

MOTION IN MACHINES

We shall find it useful to study very briefly certain of the characteristics of plane motion, and shall here again explain the term by stating that any body has plane motion when it moves in such a way that any given point in it always remains in one and the same plane, and further, that the planes of motion of two different points in the body are parallel. Thus, if any body have plane motion relative to the paper, then any point in the body must remain in a plane parallel to the plane of the paper during the motion of the body.

A little consideration will show that in the case of plane motion the location of a body is known when the location of any line in the body is known, this line being in a plane parallel to the plane of motion or else in the plane of motion itself. The explanation is that since all points in the body have plane motion, then the projection of the body on the plane is always the same for all positions and hence the line in it simply locates the body. For example, if a chair were pushed about upon the floor and had points marked *R* and *L* upon the bottoms of two of the legs, then the location of the chair is always known if we know the positions of *R* and *L*, *i.e.*, of the (imaginary) line *RL*, if, however, the chair were free to go up and down from the floor it would be necessary for us to know the position of the projection of *RL* on the floor and also the height of the line above the floor at any instant. Further, if it were possible for the chair to be tilted backwards about the (imaginary) line *RL*, the position of the latter would tell us very little about the chair, as the tips of its legs might be kept stationary while we were tilting the chair back and forth, the position of *RL* being the same for various angular positions of the chair.

If we consider the case where a body has not plane motion, then the line will tell us very little about the position of the body, in the case of an air ship, for example, the ship may stand at various angles about a given line, say the axis of the wheels, the ship dipping downward or rising at the will of the operator.

If now the location of a body having plane motion is known when the location of any line in the body is known, then the motion of the body will be completely known, if the motion of any line in the body is known. Thus let *C* Fig. 16, represent the projection on the plane of the paper of any body having plane motion, AB being any line in this body, and let AB be assumed to be in the plane of the paper, which is used as the plane of reference. Suppose now it is known that while C moves to C', the points A and B move over the paths AA' and BB', then the motion of C during the change is completely known. Thus at some intermediate position the line is at A_1B_1 and the figure of C can at once be drawn about this line, and this locates the position of the body corresponding to the location A_1B_1 of the line AB. It will therefore follow that the motion of a body is completely known provided only that the motion of any line in the body is known. This proposition is of much importance and should be carefully studied and understood.

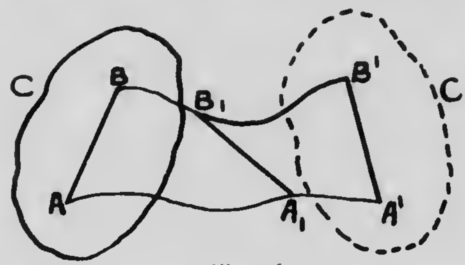


Fig. 16

RELATIVE MOTION

It will be necessary at this point to grasp some idea of the meaning of relative motion. We have practically no idea of any other kind of motion than that referred to some other body which moves in space, we say that the moon moves simply because it changes its position as we view it from the earth, or we say a train is moving as it passes us while we stand on a railroad crossing. Again, we see passengers in a railroad car as the train moves out and we say they are moving, while each in turn looks at other passengers sitting in the same car and says the latter are still. Again brakeman may walk backward on a flat car at exactly the same s the car goes forward, and a person on the ground who could jus is head would say he was stationary, while the engine driver would say he was moving at several miles per hour. If we stood on shore and saw a ship go out we would say that the funnel was moving, and yet a person on the ship would say that it was stationary.

These conflicting statements, which are, however, very common, would lead to endless confusion unless the essential differences in the various cases were grasped, and it will be seen that the real difference of view results from the fact that different persons have

entirely different standards of comparison. Standing on the ground the standard of rest is the earth, and anything that moves relative to it is said to be moving. The man on the flat car would be described as stationary because he does not move with regard to the chosen standard—the earth; but the engine driver would be thinking of the train, and he would say the man moved because he moved relative to his standard—the train. It is easy to multiply these illustrations indefinitely, but they would always lead to the same result, that whether a body moves or remains at rest depends altogether upon the standard of comparison, and it is usual to say that a body is at rest when it has the same motion as the body on which the observer stands, and that it is in motion when its motion is different to that body on which the observer stands. On a railroad train we speak of the poles flying past us, whereas a man on the ground says they are fixed.

When the standard which is used is the earth it is usual to speak of the motions of other bodies as *absolute* (although this is incorrect, for the earth itself moves) and when any standard which moves on the earth is used, the motions of the other bodies are said to be *relative*. Thus the absolute motion of a body is its motion with regard to the earth, and the relative motion is the motion as compared with another body which is itself moving on the earth. Unless these ideas are fully appreciated the reader will undoubtedly meet with much difficulty with what follows, for the notion of relative motion is difficult.

In this connection it should be pointed out that a body secured to the earth may have motion relative to another body which is not so secured. Thus when a ship is coming into port the dock appears to move toward the passengers, but to the person on shore the ship appears to come toward the shore, thus the motion of the ship relative to the dock is equal and opposite to the motion of the dock relative to the ship.

Certain propositions will now be self-evident, the first being that if two bodies have no relative motion they have the same motion relative to every other body. Thus, two passengers sitting in a train have no relative motion, or do not change their positions relative to one another, and thus they have the same motion or change of position relative to the earth, or to another train or to any other body: the converse of this proposition is also true, or two bodies which have the same change of position relative to other bodies have no relative motion.

Another very important proposition may be stated as follows; the relative motions of two bodies are not affected by any motion which they have in common. Thus the motion of the crank and connecting rod of an engine relative to the frame is the same whether the engine is a stationary one, or is on a steamboat or a locomotive, simply because in the latter cases the motion of the locomotive or ship is common to the crank, connecting rod and frame and does not effect their relative motions.

The latter proposition leads to the statement that if it be desired to study the relative motions in any machine it will not produce any effect upon them to add the same motion to all parts. For example, if a bicycle were moving along a road it would be found almost impossible to study the relative motions of the various parts, but it is known that if to all parts a motion be added sufficient to bring the frame to rest it will not in any way affect the relative motions of the parts of the bicycle. Or if it be desired to study the motions in a locomotive engine, then to all parts a common motion is added which will bring one part, usually the frame, to rest relatively to the observer, or to the observer and to all parts of the machine such a motion is added as to bring him to rest relative to them, in fact, he stands upon the engine, having added to himself as well as the engine this common motion. So that whenever it is found necessary to study the motions of machines all parts of which are moving, it will always be found convenient to add a common motion to all links which will bring one of them to rest.

To give a further illustration, let two gear wheels a and b run together and turn in opposite sense about fixed axes. Let a run at $+ 50$ r. p. m., and b at $- 80$ r. p. m., it is required to study the motion of b relative to a. To do this add to each such a motion as to bring a to rest, *i.e.*, $- 50$ r. p. m., the result being that a turns $+ 50 - 50 = 0$, while b turns $- 80 - 50 = - 130$ r. p. m. or b turns relative to a at a speed of 130 r. p. m. and in opposite sense to a. We have here simply added to each wheel the same motion, which does not affect their *relative motions* but has the effect of bringing one of the wheels to rest. To find the motion of a relative to b we bring b to rest by adding $+ 80$ r p. m., so that for a we get $+ 50 + 80 = 130$ r. p. m., or the motion of b relative to a is equal and opposite to that of a relative to b

THE INSTANTANEOUS OR VIRTUAL CENTRE

It has already been pointed out on page **26** that the motion of any body is completely known provided the motion of any line in the body in the plane of motion is known, that is, provided the motions or paths of any two points in the body are known. Now let C, Fig. 17, represent any body moving in the plane of the paper at any instant, the line AB being also in the plane of the paper, and let FA and BE represent short lengths of the paths of A and B respectively at this instant. Now the direction of motion of A is tangent to the path FA at A, and that of B is tangent to the path BE at B, or the paths of A and B give at once the direction of the motions of these points at the instant. Through A draw a normal AO to the direction of motion of A, then, if a pin is stuck through any point on the line AO into the plane of reference and C is turned very slightly about the pin it will give to A the direction of motion it has *at the instant* The same argument applies to BO and hence to the point O where AO and BO intersect, that is, if a pin be put through the point O in the body C and into the plane of reference, then at the given instant that the body is in the position shown, its motion is the same as if rotated *for an instant* about this pin. O is called the ***instantaneous*** or *virtual* centre, because it is the point in the body C about which it is virtually turning, with regard to the paper, at the instant, and it will at once appear that O will change in general from one instant to the next, see Fig. 17, unless the paths of A and B happened to be concentric circles or were parallel straight lines.

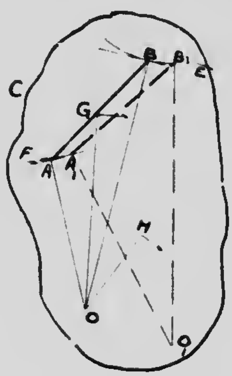


Fig. 17

Now this virtual centre gives a great deal of information about the body ***at the given instant***, thus it shows that the direction of motion of G is $\perp$ to OG and of H is $\perp$ to OH, because the direction of motion of any point in a rotating body is always perpendicular to the radius to that point; so that when the virtual centre is known the direction of motion of every point in the body is also known. We are not free to put down a path at random for G on AB because it might not agree with the paths given for A and B, but when the latter are given, the former path is determined and hence cannot be assumed.

Further, the relative linear velocities of all points in C are known. Let the body C be turning at this instant at the *rate of n* revs. per min., corresponding to ω radians per sec., so that $\omega = \frac{2 \pi n.}{60}$ Then at the instant the linear velocity v of any point B situated at $OB = r$ ft. from O will evidently be $v = 2 \pi r. \frac{n}{60} = r \omega$ ft. per sec., and since ω is the same for the whole body it is seen that the linear velocity of any point is proportional to its distance from the virtual centre. Thus $v_B = OB . \omega$; $v_A = OA . \omega$; $v_G = OG . \omega$, and so on, and the sense of the velocities v must agree with that of ω.

The virtual centre for a body may, therefore, be found, provided only that we know the *directions* (not necessarily the paths) *of motion of two points* in it,and having found this centre the directions of motion of all points in the body are known, and further their relative velocities, and the actual velocities in magnitude, sense and direction will be known if the angular velocity is known. (This should be compared with the phorograph discussed in a later chapter.) It is to be further noted that the virtual centre O is a *double point;* it is a point in the paper and also in C, and the motion of any point in C with regard to the paper being perpendicular to the radius from O to that point so also the motion of any point in the paper with regard to C is perpendicular to the line joining this point to O.

Another point is to be noticed, that if the various virtual centres O are known, then at once the relative motions of C and the paper are known. Thus the virtual centres of one body with regard to another give always the motion of the one body with regard to the other.

THE PERMANENT CENTRE

It has already been pointed out that the instantaneous or virtual centre is the centre for rotation of any one body with regard to another at a given instant, and that the location of this centre is changing from one instant to the next. There are, however, very many cases where one body is joined to another by means of a regular bearing, as in the case of the crank shaft of an engine and the frame, or a wagon wheel and the body of the axle, or the connecting-rod and crank pin of an engine. A little reflection will show that in each of these cases the one body is always turning with regard to the other, but that the centre or axis of revolution has a fixed position with regard to each of the bodies concerned, thus in these cases the virtual centre remains relatively fixed and we may apply to it the term *permanent centre.*

The term permanent centre must not be confused with the term *fixed centre*, which would be applied to a centre fixed in place on the earth, but is intended to include only the case where the virtual centre for the rotation of one body with regard to another is a point which remains at the same place in each body and does not change from one instant to another, thus the centre between the connecting rod and crank and between the crank shaft and frame are both permanent, the latter being also fixed usually.

THE THEOREM OF THE THREE CENTRES

Before applying the virtual centres in the solution of problems of various kinds, a very important property connected with them will be proved. Let *a*, *b* and *c*, Fig. 18, represent three bodies all of which have plane motion of any nature whatever, and which motion is for

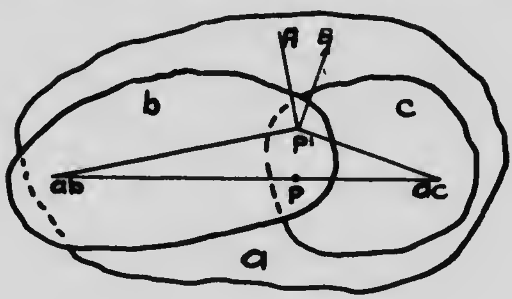


Fig. 18

the time being unknown. Now, generally *a* has motion relative to *b*, and *b* has motion relative to *c*, and similarly *c* with regard to *a*, in brief all three bodies move in different ways, hence from what has already been said, there is a virtual centre of *a* ↬ *b** which we may call *ab*, and this is of course also the centre of *b* ↬ *a*. Further, there is a virtual centre of *b* ↬ *c* which we call *bc*, and also a centre of *c* ↬ *a*, which is *ca*, thus for the three bodies there are three virtual centres. Now it will be assumed that enough information has been given about the motions of *a*, *b* and *c* to enable us to locate *ab* and *ac* only, and it is required to find *bc*.

Since *bc* is a point common to both bodies *b* and *c*, it must lie

* The sign ↬ means "with regard to."

somewhere in the area where they overlap, and let it be assumed to lie at P'. Then P' is a point in b and also in c. As a point in b its motion with regard to a will be normal to $P'-ab$, *i.e.*, in the direction $P'A$, because the motion of any point in one body with regard to another body is normal to the line joining this point to the virtual centre for the two bodies. As a point in c, the motion of P' ↩ a is normal to $P'-ac$ or in the direction $P'B$, so that P' has two different motions with regard to a at the same time, which is impossible or P' cannot be the virtual centre of b ↩ c. Since, however, this is not the point, it shows at once that the point bc is located somewhere along the line $ab-ca$, or say at P, because it is only such points as P which give the same motion with regard to a whether considered as points in b or in c; thus the centre bc must lie on the same straight line as the centres ab and ac. It is not possible to find the exact position of bc, however, without further information, all that is known is the line on which it lies.

This proposition may be thus stated:—**If in any mechanism we have any three links** a, f, g, **all having plane motion, then for the three links there are three virtual centres** af, fg **and** ag, **and these three centres must all lie on one straight line.**

Two of the centres may be permanent but not the third; in the steam engine we have the crank a, the connecting rod b and the frame d, and the centres ab and ad are permanent, but bd is not.

THE DETERMINATION OF VIRTUAL CENTRES

The chapter will be concluded by finding the virtual centres in a few mechanisms simply to illustrate the method, the application being given in the next chapter. As an example, consider the chain with four turning pairs, which is taken on account of its simplicity and its very common application. It is shown in Fig. 19, and consists of four links a, b, c and d, of different lengths, a being fixed, and by inspection the four permanent centres ab, bc, cd and ad, at the four corners of the chain are at once located. It is also seen that there are six possible centres in the mechanism, viz., ab, bc, cd, da, bd and ac, these being all the possible combinations of the links in the chain when taken in pairs, and of these six, the four permanent ones are found already, and only two others, ac and bd remain. There are two methods of finding them, the first of which is the most instructive, and will be given for that reason.

To find the centre bd. By the principle of the virtual centre we may do this at once if we know the *direction of motion* of any two points in $b \hookrightarrow d$. Now, on examining *ab* it is seen that it is a point in *a* and also in *b*; as a point in *a* it moves with regard to *d* about the centre *ad* and thus ‸ direction normal to *ad*−*ab* or to *a* itself. And as a point in *b* it must have the same motion with regard to *d* as it has when considered as a point in *a*; *i.e.*, the motion of *ab* in *b* with regard to *d* is in the direction perpendicular to *a*. Hence, from page 30, the virtual centre will lie on the line through *ab* in the direction of *a*, *i.e.*, in *a* produced. Again *bc* is a point in *b* and *c*, and as a point in *c* it moves with regard to *d* in a direction perpendicular to *cd*−*bc*, or in the direction *bc*−*F*, and this must also be the direction of *bc* as a point in $b \hookrightarrow d$, so that the virtual centre of

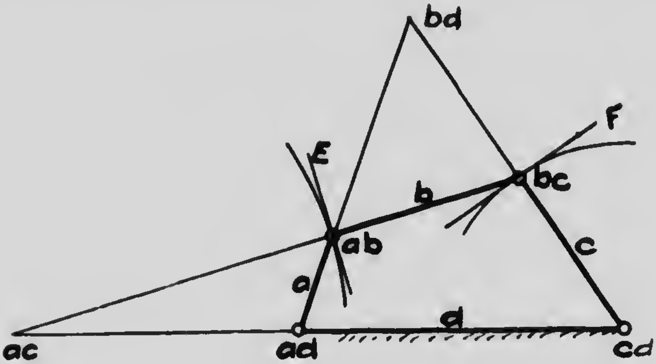


Fig. 19

$b \hookrightarrow d$ must also lie in the line through *bc* normal to *bc*−*F*, or in *c* produced. Hence, *bd* is at the intersection of *a* and *c* produced.

This could also ' been solved by the theorem of the three centres, for there w‸ centres, *ad*, *ab* and *bd*, for the three bodies *a*, *b* and *d*, anu t lie in one straight line, and as both *ad* and *ab* are known, es at once the line on which *bd* lies. Similarly, by considering the three bodies, *b*, *c* and *d*, and knowing the centres *bc* and *cd*, there is found again the line on which *bd* lies, and hence *bd* is readily found. *To find the centre ac* it is possible to proceed in either of the ways already explained, and find *ac* at the intersection of the lines *b* and *d* produced.

One other example may be solved, and in order to include a sliding pair consider the case shown in Fig. 20, in which *a* is the crank, *b* the connecting rod, *c* the crosshead, piston, etc., and *d* the fixed frame. As before there are six centres *ad*, *ab*, *bc*, *cd*, *ac*, *bd*, of which *ad*, *ab*, and *bc* are permanent and found by inspection.

To find the centre cd. The motion of *c* ↪ *d* is one of sliding in the horizontal direction, that is, *c* moves in a straight line, or what is the same thing, in a circle of infinite radius, and the centre of this

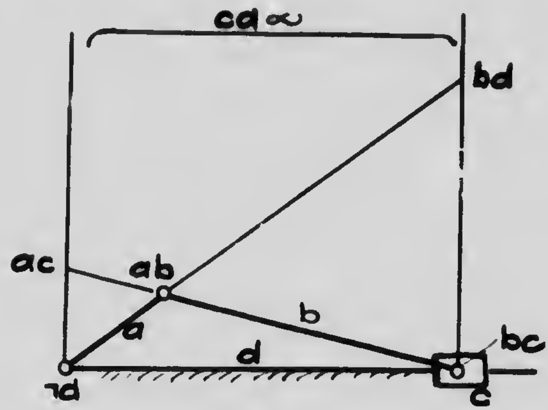


Fig. 20

circle must, as before, lie in a line normal to the direction of motion of *c* ↪ *d*. Thus *cd* lies on a vertical line through *bc*, or in a vertical line through any point in the mechanism such as *ad*, as well as *bc*.

Having found *cd*, the other centres, *ac* and *bd* may be found at once by the theorem of the three centres. Thus, *bd* lies on *bc*—*cd* and *ad*—*ab* which gives *bd*, while *ac* lies at the intersection of *bc*—*ab* and *ad*—*cd*.

CHAPTER III.

VELOCITY DIAGRAMS

In order to show some of the practical applications of the virtual centres, a few problems will be solved, and at this stage of the subject only such problems as relate to velocity, although in later chapters some further applications of these principles will be given

RELATIVE LINEAR VELOCITIES IN MECHANISMS

The method will be demonstrated by applying it in one or two of the very simplest cases, choosing first the mechanism shown in Fig. 21, and for which all the centres have already been found. Let the first problem be to compare the linear velocities of different

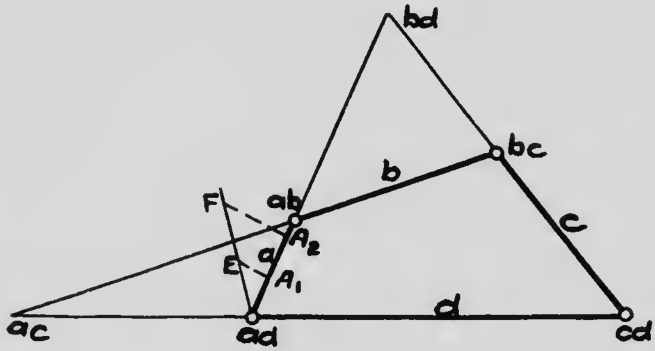


Fig. 21

points in one link, say *a*. Since the link *a* turns with regard to the earth about the permanent centre *ad*, it will be at once clear that the linear velocity of any point A_1 is to that of any point A_2 in the ratio of their distances from *ad*, so that if $A_1 E$ be drawn in any direction to represent the velocity of A_1, and if through A_2 is drawn $A_2 F$ parallel to $A_1 E$ to meet $ad-E$ in F, then will $A_2 F$ represent the linear velocity of A_2 on the same scale as $A_1 E$ represents the linear velocity of A_1. If the velocity of A_1 is unknown, then this must be written in the form $\dfrac{\text{Linear velocity of } A_1}{\text{Linear velocity of } A_2} = \dfrac{A_1 E}{A_2 F}$, giving simply the ratio between the velocities.

If it were required to compare the linear velocity of a point A_1 in *a* with that of a point B_1 in *b*, the method of procedure would be as shown in Fig. 22, which is drawn separately from the former figure for the sake of clearness. Here there are the two links a and b under consideration, and also the fixed link d, and these three links have the three centres ad, ab, bd, all on one line. ab is a point common to a and b, being a point on each link. Treating it as a point in a, proceed as in the last example to find its velocity. Thus set off $A_1 E$ in any direction to represent the linear velocity of A_1, then will $ab-F$ parallel to $A_1 E$ represent the velocity of ab to the same scale. Now treat ab as a point in b and its velocity is given as $ab-F$, so that the matter now resolves itself into finding the velocity of a point B in b, the velocity of the point ab in the same link being given.

It must here be remembered that $ab-F$ represents the absolute velocity of the point ab, *i.e.*, the velocity of this point, using the fixed

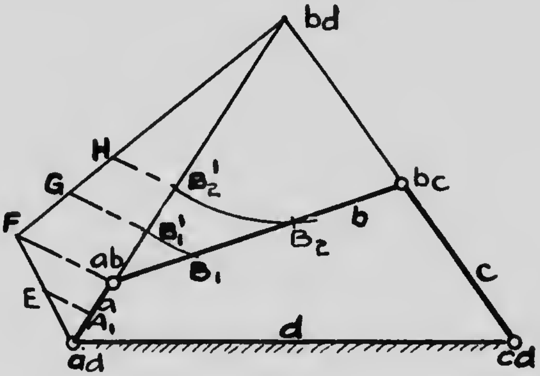


Fig. 22

frame of the machine as the standard. With regard to the frame the link is turning about the centre bd, thus for the instant b turns relative to d about bd, and the velocities of all points in it at this instant are simply proportional to their distances from bd. The velocity of B_1 is to the velocity of ab in the ratio $bd—B_1$ to $bd-ab$, and in order to get this ratio conveniently, draw the arc $B_1 B_1'$ with centre bd, then join $bd-$F and draw $B_1'G$ parallel to $ab-F$ to meet $bd-F$ in G, then $B_1 G$ represents the velocity of B_1 in the link b on the same scale that $A_1 E$ represents the velocity of A_1. If desired, it is possible also to find the relative velocities of B_1 and B_2 on b by the construction shown, the circles $B_2 B_2'$ and $B_1 B_1'$ both having the common centre bd.

Notice that in dealing with the various links in finding relative velocities, it is necessary to use the centres of the links under consideration with regard to the fixed link; thus the centres *ad* and *bd* and the common centre *ab* are used. The reason *ad* and *bd* are employed, is because the velocities under consideration are all absolute.

To compare the velocity of any point A_1 in *a* with that of C_1 in *c*, Fig. 23, it would be necessary to use the centres *ad*, *ac*, and *cd*. Proceeding as in the former case the velocity of *ac* is found by drawing the arc $A_1 L$ with centre *ad* and making *LN* represent the velocity of A_1 on any scale, then the line $ac-M$ parallel to *LN* meeting $ad-N$ produced in *M* will represent the velocity of *ac*. Join $cd-M$, draw the arc $C_1 C_1'$ with centre *cd* and then $C_1' K$ parallel to $ac - M$ will represent the linear velocity of C_1.

A general proposition may be stated as follows:—**The velocity of any point *A* in link *a* being given to find the linear velocity of *F* in *f*, the fixed link being *d*. Find the centres *ad*, *fd* and *af*, then,**

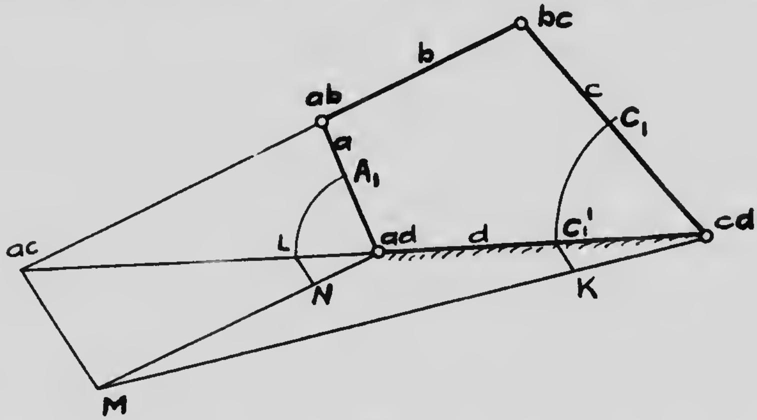


Fig. 23

using *ad* and the velocity of *A*, find the velocity of *af*, and then treating *af* as a point in *f* and using the centre *fd*, find the velocity of *F*

RELATIVE ANGULAR VELOCITIES IN MECHANISMS

Similar methods to the preceding may be employed for finding angular velocities in mechanisms.

Let any body having plane motion turn through an angle θ about any axis, either on or off the body, in time t, then the angular velocity of the body is defined by the relation $\omega = \frac{\theta}{t}$. As all links in a mechanism move except the one which we term the fixed link, there are in general as many different angular velocities as there are moving links. The angular velocities of the various links a, b, c, etc., will be designated by ω_a, ω_b, ω_c, etc., respectively, the unit being the radian per second.

The relative angular velocities of two links such as a and b may be expressed as a ratio $\frac{\omega_a}{\omega_b}$ which is a pure number, or by a difference $\omega_a - \omega_b$, which is a number of radians per second. In the former method the relation between the two velocities is obtained when both are referred to the earth as the standard, and it is this ratio which is commonly referred to in connection with pulleys, gears and other devices. Thus if a belt connects two pulleys of 20 in. and 30 in. diameter,the velocity ratio is $\frac{2}{3}$, *i.e.*, when standing on the ground and counting the revolutions with a speed counter one of the wheels will only make $\frac{2}{3}$ the number of revolutions of the other

There are, however, cases in which it is desirable to know the velocity with which one body turns relative to another; then the latter method is used. If, for example, there are two gears, one a turning at the rate of 20 revs. [illegible]n., and the other b at the rate of 30 revs. per min., in the sa[illegible]e so that $\omega_a = 2.09$ and $\omega_b = 3.14$, then the velocity of a with [illegible]gard to b will be $\omega_a - \omega_b = -1.05$ radians per second, that is, if we stood on the gear b and looked at the gear a, the latter would appear to turn backward at the rate of 10 revs. per min. $= 1.05$ radians per sec. If we stood on gear a, then b would appear to turn forward, since $\omega_b - \omega_a = 1.05$, the angular motion of $a \hookleftarrow b$ being equal and opposite to that of $b \hookleftarrow a$. This method of dealing with angular velocities is quite common, and finds many useful applications.

Given the angular velocity of a link a to find that of any other link b. Find the three centres ad, bd and ab; then as a point in a, ab has the linear velocity $(ad-ab)\,\omega_a$ and as a point in b, ab has the velocity $(bd-ab)\,\omega_b$ But as ab must have the same velocity whether considered as a point in a or in b, then $(ad - ab)\,\omega_a = (bd - ab)\,\omega_b$, or $\frac{\omega_b}{\omega_a} = \frac{ad - ab}{bd - ab}$. The illustration in Fig. 24 gives the method and will require very little explanation. Draw a circle with centre ab and radius $ab - ad$, which cuts $ab - bd$ in a_1d_1, lay off $bd - F$ in any direction to represent ω_a on any scale, then draw $a_1d_1 - E$ parallel to $bd - F$ to meet $ab - F$ in E, and $a_1d_1 - E$ will represent the angular velocity of b or ω_b.

Similar processes may be employed for the other links b and c, and as all cases may be dealt with very simply, no further discussion of the point will be given here. The general constructions are very similar to those for finding linear velocities.

THE VIRTUAL CENTRE METHOD OF FINDING VELOCITIES

Although the determination of the linear and angular velocities by means of the virtual centre is simple enough in the cases just considered, yet when it is employed in practice there is frequently much difficulty in getting convenient constructions to suit the requirements. Many of the lines locating virtual centres are nearly parallel and do not intersect within the limits of the drafting board, and hence special and often troublesome methods must be employed to bring the constructions within ordinary bounds. Further, although we are commonly given the motion of one link such as a, and often only require the motion of *one other point or link*, say f, elsewhere in the mechanism, which would only require the finding of three virtual centres, ad, af and df, yet in practice we frequently learn that all of these cannot be obtained without locating almost all the other virtual centres in the mechanism first. This involves an immense amount of labor and patience, and in some cases makes the method unworkable.

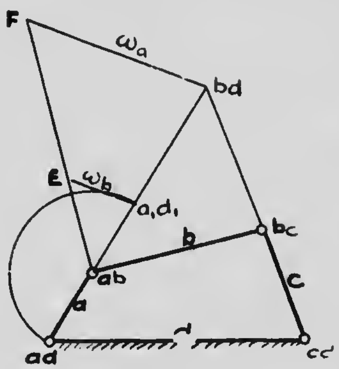


Fig. 24

A practical example of a more complicated mechanism in very common use will be worked out here to illustrate the method, no more centres being found than those absolutely necessary for the solution of the problem. Fig. 25 shows the Joy valve gear as frequently used on locomotives and other engines, more especially in England: a represents the engine crank, b the connecting rod, and c

the piston, etc., as in the ordinary case, the frame being *d*. One end of a link *e* is connected to the rod *b* and the other end to a link *f*, the latter link being also connected to the engine frame, while to the link *e* a rod *g* is jointed, which rod is also jointed to a sliding block *h*, and at its extreme upper end to the slide valve stem *V*. The part

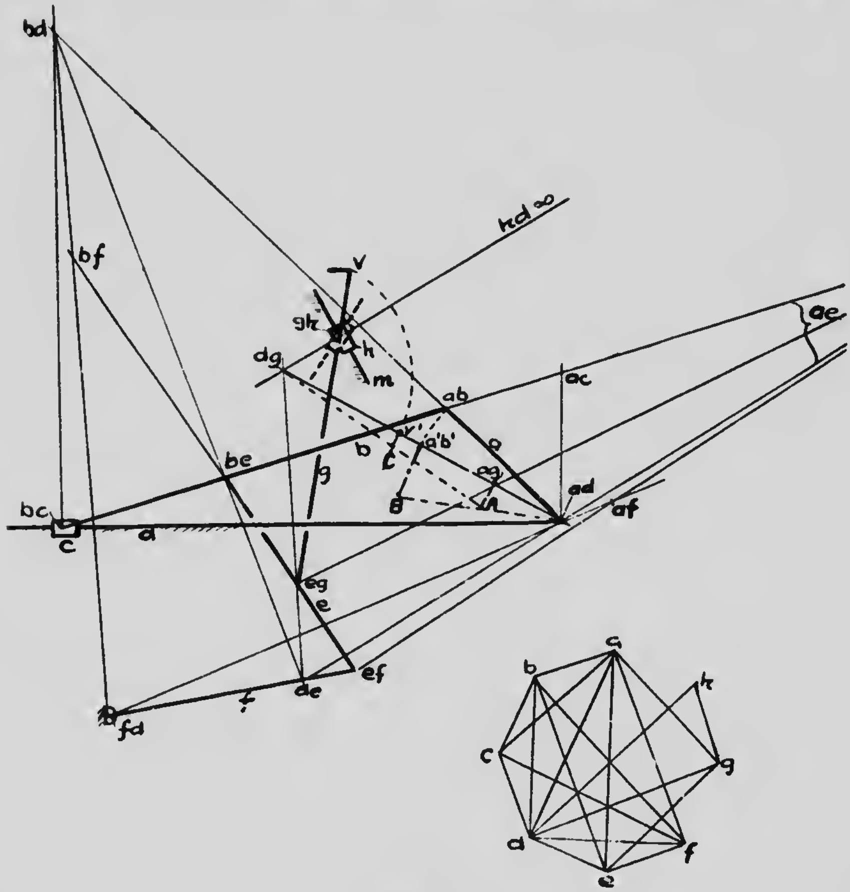


Fig. 25

m on which *h* slides is controlled in direction by the engineer who moves it into the position shown or else into the dotted position according to the sense of rotation desired in the crank shaft, but once this piece *m* is set, it is left stationary and virtually becomes fixed for the time.

A very useful problem in such a case is to find the velocity of the valve and stem *V* for a given position and speed of the crank shaft. Here we are concerned with three links, *a*, *d* and *g*, the upper end of the latter link giving the valve stem its motion, so that we need the three centres *ad*, *ag*, and *dg*. First write on all the centres which it is possible to find by inspection, such as *ad*, *ab*, *be*, *bc*, *cd*, *ac*, *ef*, etc., and then proceed to find the required centres by the theorem of three centres given on page 32. The centres *ag* and *dg* cannot be found at once and it will simplify the work to set down roughly in a circle (not necessarily accurately) anywhere on the sheet. a set of points which are approximately equidistant, there being one point for each link, in this case eight. Now letter these points *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, to correspond with the links. As a centre such as *ab* is

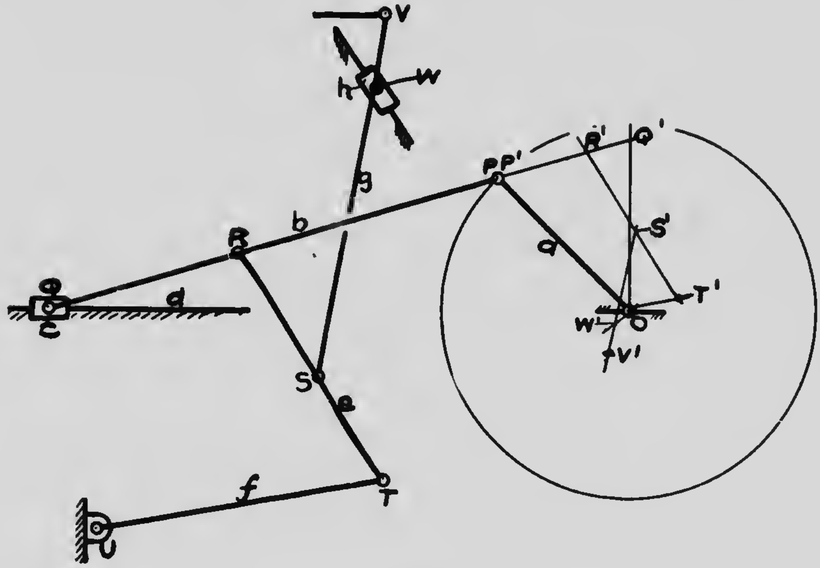


Fig. 26

found it is necessary to join the points *a* and *b* in the lower diagram and in this way it is possible to join a fairly large number of the points at once, any two points not joined representing a centre still to be found. The figure shows by the plain lines the stage of the problem after the centres *ab*, *ac*, *ad*, *ae*, *af*, *ag*, *bc*, *bd*, *bf*, *be*, *cd*, *de*, *df*, *dg*, *dh*, *ef*, *eg*, *gh*, have been found which represent the work necessary to find the above three centres *ag*, *ad* and *dg*.

When all points on the lower figure are joined, all the centres have been found, the figure showing by inspection what centres can be found at any time, for we can find any centre provided there are at any time two paths between the two points. From the figure below as shown in the plain lines, the centres joined by any line such as *ab*, *ac*, *ad*, *de*, *df*, etc., are known, but the centres *ah*, *bg*, *ch*, etc., are not known, and by inspection of the figure, it is evident that *fg* may be found if desired, because between these two points there are the paths *gd*−*df* and *ge* −*ef* and *fg* is at the intersection of these two lines. It would not be possible to get *gc* before *gd*, however, as there is only one path *ga*−*ac*, and this line is not enough to locate the point.

Having now found the centres *ad*, *dg*, and *ag*, we may proceed as in the previous cases to find the velocity of *V* or the valve, from the known velocity of *a*. If the velocity of the crank pin *ab* is given, revolve *ad*−*ab* into the line *ad*−*dg* and lay off *a′ b′ B* to represent the velocity of *ab* on any scale. Join *ad*−*B*, then *ag*−*A* parallel to *a′ b′ B* gives the velocity of *ag*. Next join *dg*−*A* and with centre *dg* draw the arc *VV″* then will *V″C* parallel to *ag*−*A* represent the velocity of the value V. The whole process is evidently very cumbersome and laborious and in general, too lengthy to be adopted very commonly.

The solution of the same problem by the method explained in Chapter IV. is shown in Fig 26, and the gain in simplicity is very noted. Here the length *OV″* represents the linear velocity of *V* on the same scale as the length *OP* = *a* represents the linear velocity of the crank pin. The method used is fully explained later on, and it is only given in this place to show that problems of this important nature can be solved without undue complexity. The figure is put in here for the purpose of direct comparison of methods; the reader is however, advised to leave a study of Fig. 26 until he has finished reading Chapter IV.

GRAPHICAL REPRESENTATION OF VELOCITIES

It is frequently desirable to have a diagram to represent the velocities of the various points in a machine for one of its complete cycles, as the study of such diagrams gives very much information about the nature of the machine and of the forces acting on it. Two methods are in fairly common use (1) by means of a polar diagram (2) by means of a diagram on a straight base. These methods may be best explained by an example.

To illustrate this a very simple and useful case, the slider-crank mechanism, Fig. 27, will be selected, and the linear velocities of the piston will be determined, a problem which may be very conveniently solved by the method of virtual centres. Let the speed of the engine be known, so that it is possible to calculate at once the linear velocity of the crank pin *ab*; for example, let *a* be 5 in. long, and let the speed be 300 revs. per min., then the velocity of the crank pin $= 2\pi \times \frac{300}{60} \times \frac{5}{12} = 13.1$ ft per second. Now *bc* is a point both on the piston *c* and on the rod *b* and clearly the velocity of *bc* is the same as that of *c* the latter link having only a motion of translation, and further the velocity of the crank pin *ab* is known, which is

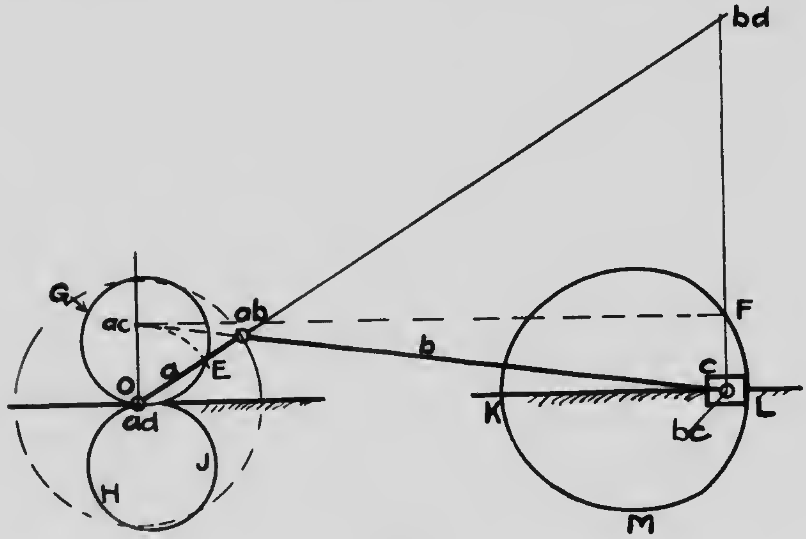


Fig. 27

also the same as that of the forward end of the connecting rod. The problem then is: given the velocity of a point *ab* in *b* to find the velocity of *bc* in the same link, and from what has already been said, the relation may be written, $\frac{\text{velocity of piston}}{\text{velocity of crank pin}} = \frac{\text{velocity of } bc}{\text{velocity of } ab}$ $= \frac{bd - bc}{bd - ab}$. But by similar triangles $\frac{bd - bc}{bd - ad} = \frac{ac - ad}{ad - ab}$, so that $\frac{\text{velocity of piston}}{\text{velocity of crank pin}} = \frac{ad - ac}{ad - ab}$, and as $ad - ab$ is constant for all positions of the machine, it is evident that $ad-ac$ represents the velocity of the piston on the same scale as the length of a represents the linear velocity of the crank pin. Or in the case chosen, if the mechanism is drawn full size then $ad-ab = 5$ in., and the scale will be 5in. = 13.1 ft. per sec. or 1 in. = 2.62 ft. per sec

Now it is convenient to plot this velocity of the piston either along a as $ad-E$ if the diagram is to show the result for the different crank positions, or vertically above the piston as $bc-F$, if we wish to represent the velocity for different positions of the piston. If this determination for the complete revolution is made, there are obtained the two diagrams shown, the one $OEGOHJO$ is called a polar diagram, O being the pole. The diagram for the piston positions is $KFLMK$. If the connecting rod is very long, the polar diagrams approach a circular form and the diagram becomes more

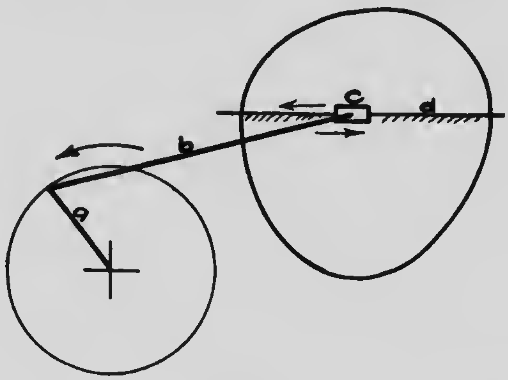


Fig. 28

nearly symmetrical above a vertical centre line, while for a shorter rod the curves become more distorted in the way indicated in the figure.

If the direction of motion of the piston does not pass through ad, then the curve $FKML$ is not symmetrical about the line of motion of the piston, but takes the form shown at Fig. 28, where the piston's direction passes above ad, a device in which it is clear from the velocity diagram that the mean velocity of the piston on its return stroke is greater than on the out stroke, and which may, therefore, be used as a quick-return motion in a shaper or other similar machine. Automobile engines are sometimes made in this way, but with the cylinder only slightly *off-set* as it is called.

One very useful application of such diagrams as those just described may be found in the case of pumping engines. Let A be the area of the pump cylinder in square feet, and let the velocity of the plunger or piston in a given position be v ft. per sec., as found by the preceding method, let Q cu. ft. per sec. be the rate at which the pump is discharging water at any instant, then evidently $Q = Av$ and as A is the same for all piston positions, Q is simply proportional to v, or the height of the piston velocity diagram represents the rate of delivery of the pump for the corresponding piston position.

If a pipe were connected directly to the cylinder, the water in it would vary in velocity in the way shown in the velocity diagram

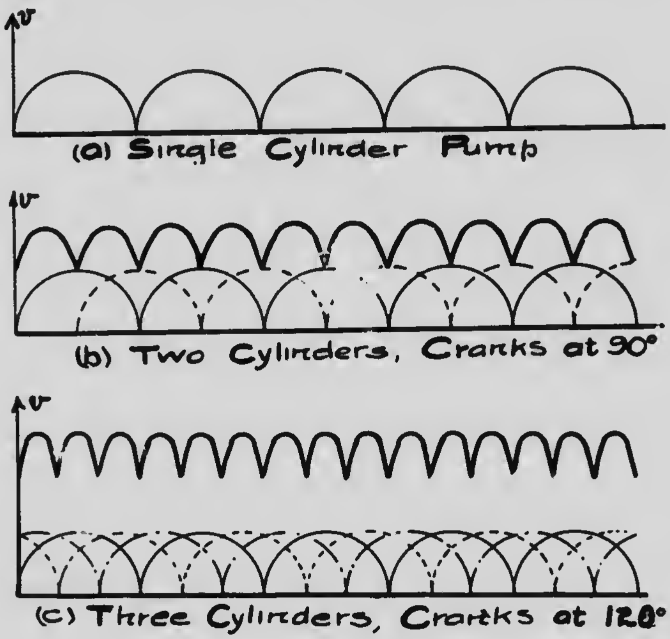


Fig. 29

(*a*), Fig. 29, the heights on this diagram representing piston velocities and hence velocities in the pipe and horizontal distances showing the *distances traversed by the crank pin*. Here we show the effect of both ends of a double-acting pump; this variation in velocity would produce so much shock on the pipe as to injure it and hence a large *air chamber* would be put on to equalize this velocity.

Curve (*b*) shows two pumps delivering into the same pipe, their cranks being 90° apart, the heavy line showing the variation of velocity in the pipe line requiring a much smaller air chamber. At (*c*) is shown a diagram corresponding to three cranks at 120° or a three-throw pump, in which case the variation in velocity in the pipe line would be very much smaller, and this velocity is represented by the height up to the heavy line, all the curves are drawn for the case of a very long connecting rod.

Thus the velocity diagram enables the study of such a problem to be made very accurately and there are very many other useful purposes to which it may be put, and which will appear in the course of the engineer's experience. Angular velocities may, of course, be plotted the same way as linear velocities.

A very useful and convenient method of finding both linear and angular velocities is described in the next chapter, and a few suggestions are made as to further uses of these velocities in practice

CHAPTER IV.

THE MOTION DIAGRAM

In the previous chapter we have pointed out methods of finding the velocities of various points in a machine, and have shown some of the uses to which these velocity determinations and the corresponding diagrams may be put. There are very many further problems which may be studied in this way, such as the relative merits of different quick-return motions or other devices, which are used for similar important work, as operating valve gears, etc. Then again it is often necessary to determine the turning effect produced on the crank shaft of an engine by the steam pressure on the piston, or to study the advantage in the way of producing uniform motion of placing four cylinders on an automobile engine, etc.

All of these problems may be solved very directly by the determination of the velocities of various points in the machine under consideration, and as such problems are of very frequent occurrence in the experience of the designing engineer, it is desirable that as simple a process as possible be employed in solving them. The problems may be solved by graphical methods most conveniently, as the motions in most machines are so complex that algebraic solutions are too tedious and difficult.

In all machines there is one part which has a known motion, and generally this motion is one of uniform angular velocity about a fixed axis, e.g., the flywheel in an engine, the belt wheel in a shaper, the belt wheel in a stone crusher, etc.

In most cases in machines all parts have plane motion, and in what follows it is to be understood that all parts referred to remain in one plane, unless the contrary is expressly stated. The solutions may in general be applied to non-plane motion with proper modifications.

The method of determining the velocities of parts of machines to be explained here is called the phorograph* method, and gives a convenient graphical method for finding the desired velocities.

THE PHOROGRAPH

Let us consider any body having plane motion, such as the connecting rod of a steam engine. It is well known that any point

* So named by its discoverer, Professor T. R. Rosebrugh, of the University of Toronto, who gave the method to his students twenty years ago, but so far as the writer knows the method has not been discovered or used elsewhere.

in this rod can move relatively to any other point in it only at right angles to the line joining these points. Thus let A, Fig. 30 represent a part of this rod, in which are two points B and C, which we shall for convenience assume are in the plane of the paper. Let the body A move to the new position A_1, B and C taking the positions B_1 and C_1 respectively, and although we are uncertain as to the actual history of the motion during the change of position, it is quite evident that it may have been accomplished by (a) a motion of translation of the body A through the distance CC_1, during which C reaches its new position C_1 and B arrives at B_1. During this motion B and C have moved through the same distance in the same direction and sense, and hence have had no relative motion. The second part of the motion consists of (b) a motion of rotation of the whole body A

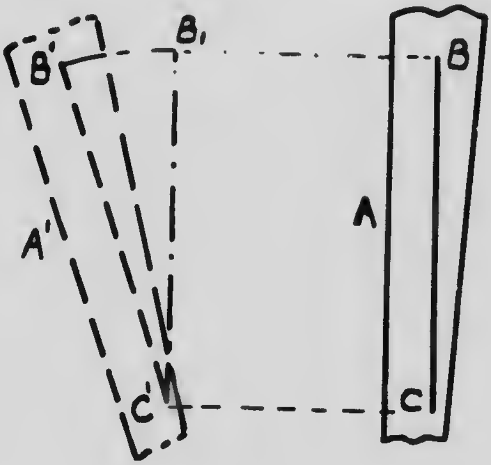


Fig. 30

about an axis normal to the paper through the point C_1, rotation taking place through the angle $B_1C'B'$.

During the motion of A therefore B has had only one motion not shared by C, or B has moved relatively to C through the arc B_1B^1, and at each stage of the motion the direction of this arc was evidently at right angles to the radius from C^1, or at right angles to the line joining B and C.

Thus when a body has plane motion any point in the body can move relatively to any other point in the body only at right

angles to the line joining the two points. It follows from this that if the line joining the two points should lie normal to the plane of motion the two points could have no relative motion.

We shall now employ this principle to the determination of velocities. Let Fig. 31 represent diagrammatically a machine having four links *a*, *b*, *c*, *d*, joined together by four turning pairs at *O*, *P*, *Q*, *R*. If the link *d* is nearly vertical and the length of *a* be decreased this could be taken to represent one-half of a beam engine, in which *a* is the crank, *b* the connecting rod, and *c* is one-half of the walking beam, the other end of which would be connected with the piston rod.

In all such machines one link is fixed and forms the frame, here indicated by *d*. Thus *O* and *R* are fixed bearings and *P* and *Q* move in arcs of circles about *O* and *R* respectively. Let us

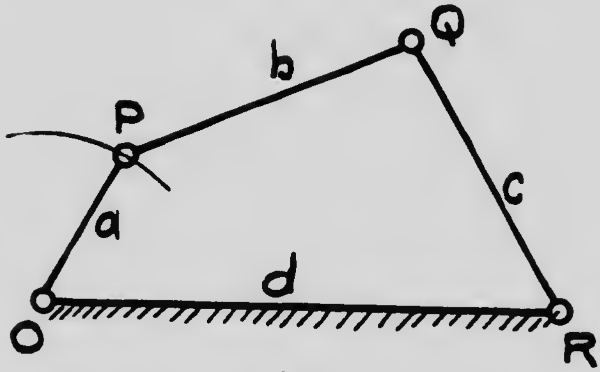


Fig. 31

now choose one of the moving links as the link of reference, either *a* or *c* will be the most convenient, as they have one fixed bearing, and *a* will be selected. Imagine that to *a* an immense sheet of cardboard is attached which extends indefinitely in all directions from *O*, and let us for brevity refer to this whole sheet as the link *a*.

A consideration of the matter will show that on the link *a* there are points having all conceivable velocities in magnitude, direction and sense, thus if a circle be drawn on *a* with centre at *O* all points on the circle will have velocities of the same magnitude, but of different direction and sense; or if a vertical line be drawn through O all points on this line will move in the same direction, i.e., horizontal, those above O moving in opposite sense to those below O, and the magnitudes of all the velocities being different. Thus, if any point be chosen on a the magnitude of its velocity will depend upon the distance from O, the direction of its velocity will be normal to the radial line joining it to O, and its sense will depend upon the relative positions of the point and O on the radial line. It must be remembered that the above statements are true whether a has constant angular velocity or not and are also true although O is moving.

From the foregoing it follows at once that it will be possible to find a point on a having the same motion as that of any point,

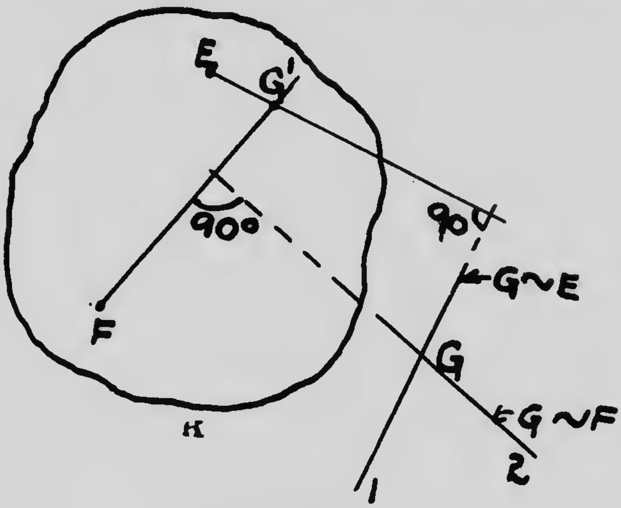


Fig. 32

such as Q, in the machine, which motion it is desired to study; and thus we can collect on a a set of points, each representing the motion of a given point in the machine, and since this set of points is all on the one link their relative velocities is at once known completely. This collection of points on a will be of great assistance in studying the motions of points in the machine, because if the motion of a is known, as is usual, that of any other point is known; whereas if the motion of a is unknown only the relative motions of the different points are known. This collection of points on the link of reference is called the *Phorograph*, as it represents graphically the motions of all points in the machine.

The method of determining the phorograph for a given machine may be explained as folows: Let any body K, Fig. 32, have plane motion, and let us choose in it two points E and F. We are, however, given no information about the nature of the motion of K. On some other body there is a point G, and we are told the direction only of the motions of $G \hookrightarrow E$, viz., $G - 1$, and of $G \hookrightarrow F$, viz., $G - 2$: it is required to find a point on K having the same motion as G.

Referring to our preliminary proposition we see that the motion of any point in $K \hookrightarrow E$ is perpendicular to the line joining it to E, e.g., the motion of $F \hookrightarrow E$ is $\perp$ to FE. But a point is to be found having the same motion at G, and as the direction of $G \hookrightarrow E$ is given we are at once told the direction of the line joining E to the required point, it must be $\perp$ to $G1$ and pass through E as it is only points on EG^1 which have the desired direction $G \hookrightarrow E$. If we call the point to be found G^1 then G^1 lies on $EG^1 \perp$ to G-1. Similarly it may be shown that G^1 must lie on a line through F $\perp$ to $G - 2$, and hence it must lie at the intersection of the lines through E and F or at G^1, as shown in Fig. 32. Thus G^1 is a point on K having the same motion as the point G in some external body.

It is to be noted that we cannot assume the sense of the motions nor the magnitude, only the two directions. We could, however, assume the magnitude, direction and sense of $G \hookrightarrow E$ and find G^1, provided the angular velocity of K were known. If K turns in the clockwise sense then the senses of the lines representing the motion of G are $G - 1$ and $G - 2$, and if the angular velocity of K is ω radians per second the magnitude of the velocity of $G \hookrightarrow E$ is $G^1 E \cdot \omega$ and of $G \hookrightarrow F$ is $G^1 F \cdot \omega$

We shall now apply these principles to the solution of problems connected with machinery, first calling particular attention to the fact that the usual information given us is such as we nave chosen above, viz., the directions of motion of an externai point relatively to two points in the link of reference. The simple mechanism with four links and four turning pairs will be chosen as the first example, and is shown in Fig. 33, the letters a, b, c, d, O, P, Q, R having the same significance as before, and *a* being chosen as the link of reference, and a rough outline of this link is shown to indicate its large extent. It is required to find the linear velocity of the point *Q*. Points will first be found on *a* having the same motions as *Q* and *R*, which are external to *a*, and the points so found shall be referred to as the **images** of *Q* and *R* and indicated by accents, thus Q^1 is a point on *a* having the same motion in every respect as *Q* and similarly with R^1.

Inspection will at once show that since *P* is a point on *a*, P^1 will coincide with *P*, and if we call ω the angular velocity of *a* in radians per second (which may be constant or variable), then the linear velocity of *P* is $OP \cdot \omega = a\omega$ ft. per sec., and is in the direction $\perp$ to *OP* and in the sense indicated by ω. Such

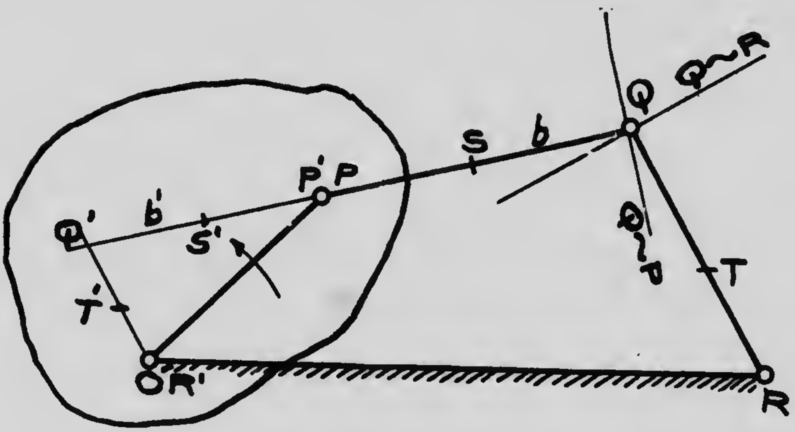
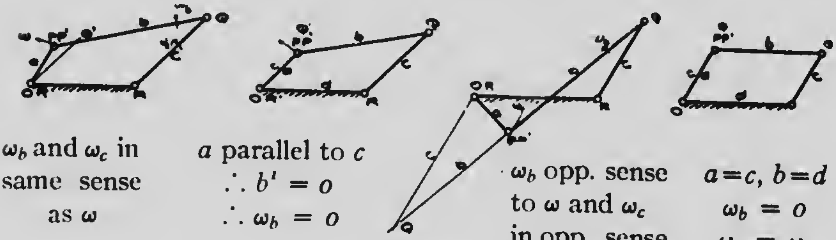


Fig. 33

being the case the length *OP* or *a* represents $a\omega$ ft. per sec., and the scale is thus $\omega : 1$. Further inspection will also show that since *R* is stationary, R^1 will lie at the only stationary point in *a*,, viz., at *O*.

The remaining point Q^1 may be found thus: The direction of motion of $Q \backsim P$ is $\perp$ to *QP* or *b*, and hence, from the proposition already given, Q^1 must lie in a line through P^1 (or *P*) $\perp$ to the direction of $Q \backsim P$, i.e., on the line through P^1 in the direction of *b* or on *b* produced. Again, the direction of motion of $Q \backsim R$ is $\perp$ to *QR* or *c*, and since R^1 (at *O*) has the same motion as *R* this is also the direction of motion of $Q \backsim R^1$, so that Q^1 lies on a line through R^1 $\perp$ to the motion of Q ↬ R, i.e., on a line through R^1 in the direction of c, and thus Q^1 is fixed. The velocity o. Q is then $Q^1O \cdot \omega$, the direction in space is $\perp$ to OQ^1 and the sense is fixed by that of ω.

Since P^1 and Q^1 are the images of P and Q on b, we may regard P^1Q^1 as the image of b, and shall in future denote it by b^1, similarly R^1Q^1 (OQ^1) will be denoted by c^1. By a similar



ω_b and ω_c in same sense as ω

a parallel to c $\therefore b^1 = o$ $\therefore \omega_b = o$

ω_b opp. sense to ω and ω_c in opp. sense

$a = c$, $b = d$ $\omega_b = o$ $\omega_c = \omega_a$

Fig. 34

process of reasoning it may readily be shown that if S is a point on b midway between P and Q then S^1 will divide P^1Q^1 equally, and also T^1 may be found by the relation $R^1T^1 : T^1Q^1 = RT : TQ$.

The diagram may be put to further use in determining the magnitude and sense of the angular velocities of b and c when that of a is known. Let ω_b and ω_c denote respectively the angular velocities of the links b and c in space, the angular velocity of the link of reference being ω. Now since Q and P are on one link b, which has an angular velocity ω_b, therefore the velocity of Q ↬ P is $QP \cdot \omega_b$ or $b \cdot \omega_b$, and since Q^1 and P^1 are points on a, whose angular velocity is ω, therefore the velocity of Q^1 ↬ P^1 is $Q^1P^1\omega$ or $b^1\omega$. But Q^1 has the same motion as Q, and P^1 has the same motion as P, and therefore the velocity of Q ↬ P is the same as that of Q^1 ↬ P^1 or $b = \omega_b$ $b^1\omega$, i.e., $\omega_b = \dfrac{b^1}{b}\,\omega = b^1 . \dfrac{\omega}{b}$ Similarly $\omega_c = \dfrac{c^1}{c} . \omega = c^1 . \dfrac{\omega}{c}$ and since b and c are fixed in length the length of the images of the links b and c are a measure of their velocities. The sense of these velocities is readily determined from the signs of the ratios $\dfrac{b^1}{b}$ and $\dfrac{c^1}{c}$ thus b^1 and b are opposite in sign, hence ω_b is of opposite sense to ω, and by a similar process of reasoning it may be shown that ω_c is of the same sense as ω.

The figure $O^1P^1Q^1R^1$ is evidently a vector diagram for the mechanism, the distance of any point on this diagram from the pole O being a measure of the velocity of the corresponding point in the mechanism. The direction of motion is normal to the line joining the point on the vector diagram to O and the sense of motion is also known from that of the angular velocity of the primary link. Further, the lengths of the sides of this figure b^1 (P^1Q^1), d^1 or (OR^1) etc., are measures of the angular velocities of the links, the sense of each angular velocity being readily determined. (Note that the length d^1 or OR^1 is infinitely

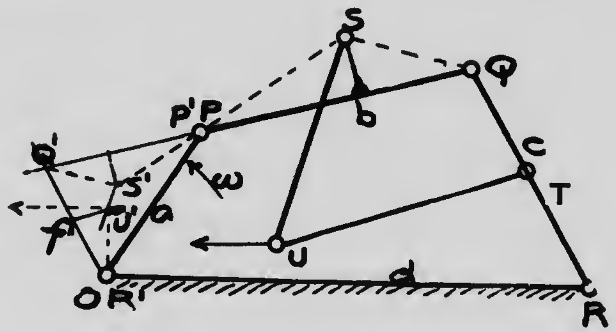


Fig. 35

short denoting that d has no angular velocity, since it is fixed in space.)

In Fig. 34 other positions and proportions of a similar mechanism are shown in which the solution is given and various relations marked below. It is to be noted that if the image of any link reduces to a single point two causes are possible, (a) if this point falls at O the link is stationary for the instant, as at d^1, but if the point be not at O the inference is that all points in the link move in exactly the same way, or the link has a motion of translation at the given instant.

The method will now be employed to solve a few typical cases.

Fig. 35 is taken as a simple example, not because it illustrates any practical mechanism.

Here we find Q^1P^1 and R^1 as before, and since we know the motions of $S \looparrowright Q$ and $S \looparrowright P$ to be $\perp$ respectively to SQ and SP, hence we draw S^1P^1 parallel to SP and S^1Q^1 parallel to SQ, which determines S^1; also $R^1T^1 : T^1Q^1 = RT : TQ$ determines T^1. Next, since we know the motions of U ↶ S and U ↶ T, we draw U^1T^1 parallel to UT and S^1U^1 parallel to SU, and thus U^1 is determined. If a be assumed to turn in the sense shown with angular velocity ω, then the angular velocity of SU is $\frac{S'U'}{SU} \cdot \omega$, and is in the same sense as a, and the angular velocity of UT is $\frac{U'T'}{UT} \cdot \omega$ in opposite sense to a. The linear velocity of U is $OU^1 \cdot \omega$ the direction is $\perp$ to OU, and the sense is to the left.

Fig. 36 gives a further example in which a sliding pair is introduced. OP is again the link of reference and P^1, Q^1, R^1 and S^1 are found as before. The direction of T is given in space by construction. It slides in the directions shown. Hence T^1 will

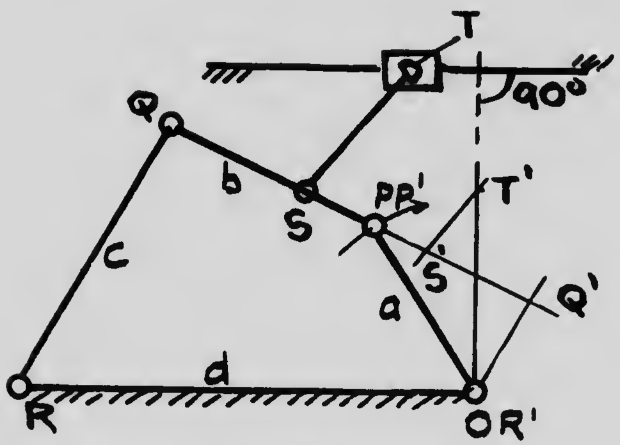


Fig. 36

lie on a line through O $\perp$ to the direction of T, and as T^1 lies on S^1T^1 parallel to ST, T^1 becomes fixed. The velocity of T is $OT^1 \cdot \omega$ its direction $\perp$ to OT^1, and its sense is to the right.

Fig. 37 shows the engine mechanism in two forms, (a) where the piston direction passes through the crank shaft, (b) where the cylinder is offset. The same letters and description apply to both. Evidently Q^1 lies on P^1Q^1 through P^1, parallel to PQ (here on QP produced), and also since the motion of

Q in space is horizontal, Q^1 will lie in the vertical through O. Thus the velocity of the piston Q is $OQ^1 \cdot \omega$ in the direction and sense shown, and offsetting the cylinder evidently decreases the piston velocity in this position, and it may be shown that there will be a corresponding increase in the return stroke. The angular velocity of the rod is $\frac{P'Q'}{PQ}\omega$. Inspection shows that in the upper diagram the piston velocity is zero at the dead points, is equal to that of the crank pin when the crank is ver-

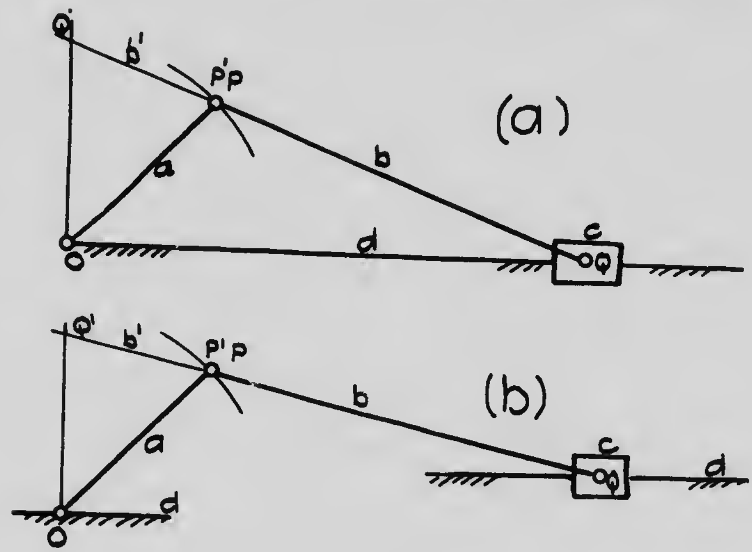


Fig. 37

tical, and has a maximum value when the crank pin is slightly to the right of the vertical through O. For the lower diagram the piston velocity is also that of the crank pin when the crank is vertical.

Fig. 38 shows the Whitworth quick-return motion, which is slightly more difficult. There are here four links a, b, d and e and two sliding blocks, c and f, d being fixed and a being the driving link, which rotates at constant angular velocity ω in the clockwise sense. P^1 and Q^1 are found by inspection. Further, S^1 lies on a vertical line through O, and R^1 on a line through Q^1 parallel to QR. Now, P is a point on both a and c. Choose T

on b exactly below P on a and c, and it will be evident that since a, b and c all have plane motion, the only motion which T can have relative to P is sliding in the direction of b, or the motion of $T \curvearrowleft P$ is in the direction of b, hence T^1 is on a line through $P^1 \perp$ to b, and since it is also on a line through O parallel to b it is found at T^1. Again, R^1 may be found since $\frac{Q'T'}{T'R'} = \frac{QT}{TR}$ The dotted lines show a simple geometrical method for obtaining this ratio. S^1 is on R^1S^1 parallel to RS and also on the vertical through O.

It will be understood that T is not a fixed point on b, but will change for each position of the mechanism. The linear

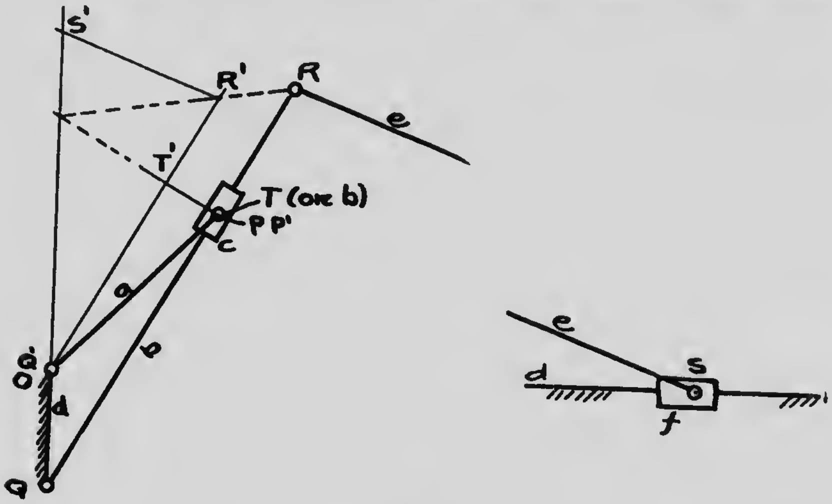


Fig. 38

velocity of the tool holder S will be $OS^1 \cdot \omega$ and the angular velocity of RS will be $\frac{R^1S^1}{RS} \cdot \omega$ in opposite sense to a.

Note that although P and T coincide, their images do not, for T has a sliding motion with regard to P, and hence both could not have the same velocity. If P^1 and T^1 coincided then both P and T would have the same velocity.

The Stephenson ling motion shown in Fig. 39 involves a slightly different method of attack and is worked out in full here, but is not drawn correctly to scale, so as to avoid confusion of the diagram. In this case the link of reference is the crank shaft containing the crank C and the eccentrics E and F, and instead of making C^1, E^1 and F^1 coincide with C, E and F, as in the previous examples, we have made $OC^1 = 2OC$, etc. The scale will then be $OC^1 = OC \times \omega$ ft per sec. We locate C^1, E^1, F^1, H^1, D^1 and J^1 at once. Further, we choose M on the link AB directly below K on LDK, and we also know that E^1A^1, F^1B^1, H^1G^1, and D^1K^1 are parallel respectively to EA, FB, HG and DK. Now, we have already seen that the image of each link is similar and similarly divided to the link itself, and we

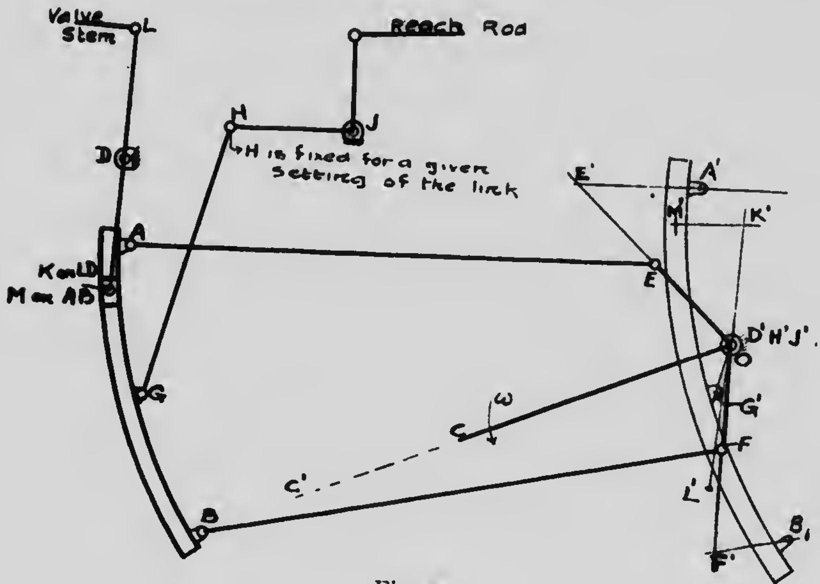


Fig. 39

see that the link AGB has the points G, A and B. We also know the lines along which G^1, A^1 and B^1 lie, so that the problem is simply one of locating a curved line similar to AGB, with its ends on the lines A^1E^1 and F^1B^1, and divided at G^1 by the line through O parallel to GH, so that $\frac{A'G'}{B'G'} = \frac{AG}{BG}$ (There are simple geometrical methods of accomplishing this result, but these are omitted here.) Thus $A^1G^1B^1$ is located and the whole link may be drawn in similar to AGB, but to a larger scale, and on it the point M^1 may be found from the relation $\frac{M'A'}{B'M'} = \frac{MB}{MA}$. Since K slides with regard to M we have K^1M^1 normal to A^1B^1 at M^1, which locates K^1, and we may readily locate L^1 from the relation $\frac{L'D'}{D'K'} = \frac{LD}{DK}$

The linear velocity of the slide valve is $OL^1 \cdot \omega$, and it moves to the right.

Note.—The images of all links are similar to and similarly divided to the links themselves, and are always parallel to the links, of which they are the images.

Lack of space prevents further illustrations, of which very many useful ones exist, but enough cases have been given to show the method of procedure in any mechanism, and to show that by this method the velocity of any point in a mechanism may readily be found by means of a drafting board. Those using the phorograph will no doubt invent geometrical methods for getting the desired ratios between the image and the link in any case which occurs.

CHAPTER V.

TOOTHED GEARING

In many cases in machinery it is necessary to transmit power from one shaft to another, the ratio of the angular velocities of the shafts being known, and in very many cases this ratio is constant; thus it may be desired to transmit power from a shaft running at 120 revs. per min. to another running at, say 200 revs. per min. Various methods are possible, for example, pulleys of proper size may be attached to the shafts and connected by a belt, or sprocket wheels may be used connected by a chain, as in a bicycle, or pulleys may be placed on the shafts and the faces of the pulleys pressed together, so that the friction between them may be sufficient to transmit the power, a drive used sometimes in auto wagons, or, again, toothed wheels called gear wheels may be used on the two shafts, as in street cars and most automobiles.

Any of these methods is possible in some cases, but usually the location of the shafts, their speeds, etc., make some one of the methods the more preferable. Thus, if the shafts are not very close together, a belt and pulleys may be used, but as the drive is not positive the belt may slip, and thus the relative speeds may change, the speed of the driven wheel often being five per cent. lower than the diameters of the pulleys would indicate. Where the shafts are fairly close together a belt does not work with satisfaction, and then a chain and sprockets are sometimes used which cannot slip, and hence the speed ratio required may be maintained. For shafts which are still closer together either friction gears or toothed gears are generally used. Thus the nature of the drive will depend upon various circumstances, one of the most important being the distance apart of the shafts concerned in it.

We shall deal here only with drives of the latter class or toothed gears, which, broadly speaking, are used between shafts which are not far apart, and for which the ratio of the angular velocities must be fixed and known at any instant. We shall first deal with parallel shafts which turn in opposite senses, the gear wheels connected with which are called *spur wheels*, the larger one commonly called the *gear*, and the smaller one the *pinion.* Kinematically, spur gears are the exact equivalent of a pair of smooth round wheels of the same mean diameter, and which are pressed together so as to drive one another by friction. Thus if two shafts 15 in. apart are to rotate at 200 revs. and 100 revs. per min., respectively, they may be connected by two smooth wheels 10 in. and 20 in. in diameter, one on each shaft, which are pressed together so they will not slip, or by a pair of spur wheels of the same mean diameter, both methods producing the desired results. But if the power to be transmitted is great the friction wheels are inadmissible on account of the great pressure between them necessary to prevent slipping. If slipping occurs the velocity ratio is variable and such an arrangement would be of no value in such a drive, as is used on a street car, for instance, on account of the jerky motion it would produce on the latter.

In order to begin the problem in the simplest possible way we shall first take the most general case of a pair of spur gears connecting two shafts which are to have a constant velocity ratio. That is, the ratio between the speeds n_1 and n_2 is to be constant at every instant that the shafts are revolving. Let l be the distance from centre to centre of the shafts. Then, if friction wheels were used, we would have the velocities at the rim of each $\pi d_1 n_1$ and $\pi d_2 n_2$ in inches per minute, where d_1 and d_2 are the diameters of the wheels in inches, and it will be clear that the velocity of the rim of each will be the same since there is to be no slipping. Thus $d_1 n_1 = d_2 n_2$ or $r_1 n_1 = r_2 n_2$, where r_1 and r_2 are the radii of the wheels. But $r_1 + r_2 = l$. Therefore since $r_1 = r_2 \cdot \frac{n_2}{n_1}$ we get $r_2 \cdot \frac{n_2}{n} + r_2 = l$,

$$\text{or } r_2\left[\frac{n_2}{n_1} + 1\right] = l \text{ or } r_2 = \frac{n_1\, l}{n_1 + n_2} \text{ and } r_1 = \frac{n_2\, l}{n_1 + n_2}.$$

Now, whatever actual shape we give to these wheels the motion of the shafts must be the same as if two smooth wheels, of sizes as determined above, rolled together without slipping. In other words, whatever shape the wheels actually have the resulting motion must be equivalent to the rolling together of two circles centred on the shafts. In gear wheels these circles are called the *pitch circles,* and they evidently touch at a point on the line joining the centres of the wheels, which point is called the *pitch point*. Now, let the actual outlines of these wheels be as shown on Fig. 40, the projections being placed there in order that the slipping of the pitch lines may be prevented. It is desired to find the necessary shape which these projections must have. Let the wheels touch at any point P and join P to the pitch point C.

It has already been shown that these pitch circles must always roll upon one another without slipping. Now P is a point which is common to both wheels. As a point in the gear it moves with regard to C on the pinion at right angles to PC, and as a point in the pinion it must move at right angles to PC with regard to C on the gear, thus, whether P is a point on the gear or pinion its motion must be normal to the line joining it

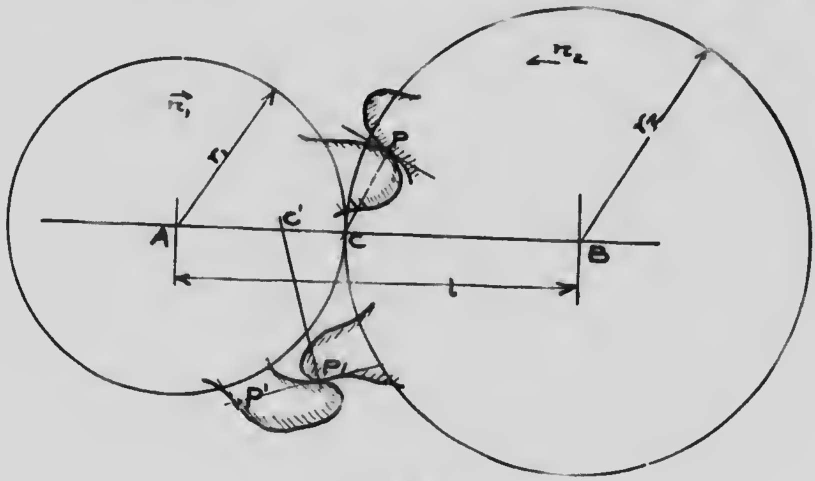


Fig. 40

to C. Some consideration will show that in order that P may have this direction of motion in each wheel, the shape of the wheels at P must be perpendicular to PC.

In order to see this let us examine the case shown in the lower figure, where the projections are not normal to P_1C at the point P_1, where they touch. It is at once evident that sliding must occur at P_1, from the very nature of the case, and where two bodies slide upon one another the direction of sliding must always be along the common tangent to their surfaces at the point of contact, hence the direction of sliding here must be P_1P^1. But P_1 is the point of contact and is therefore a point in each wheel, and the motion of the two wheels must be the same as if the two pitch circles rolled together, having contact at C. Such being the case, if we place two projections, as shown on the wheels, the direction of motion at their point of contact should be perpendicular to P_1C, whereas here it is perpendicular to P_1C^1. This would cause slipping at C, and would give the proper shape for pitch circles of radii AC^1 and BC^1, which would correspond to a different velocity ratio, thus C^1 should lie at C and P_1P^1 should be normal to P_1C.

From the foregoing we may make the following important statements: **The shapes of the projections on the wheels must be such that at any point of contact they will have a common normal passing through the fixed pitch point, and that while the pitch circles roll on one another the projections will have a sliding motion.** These projections on gear wheels are called *teeth*, and for convenience in manufacturing, all the teeth on each gear have the same shape, although this is not at all necessary to the motion. The teeth on the pinion are not the same shape as those on the gear with which it *meshes*.

There are a great many shapes of teeth, which will satisfy the necessary condition set forth in the previous paragraph, but by far the most common of these are the cycloidal and the involute teeth, so called because the curves forming them are cycloids and involutes respectively.

CYCLOIDAL TEETH

Select two circles PC and P^1C, Fig. 41, and suppose these to be mounted on fixed shafts, so that the centres A and B of the pitch circles, and the centres of the *describing circles* PC and P^1C, as well as the pitch point C, all lie in the same straight line, which means that the four circles are tangent at C. Now place a pencil at P on the circle PC and let all four circles run in contact without slipping, i.e., the circumferential velocity of all circles at any instant is the same. As the motion continues P approaches the pitch circles ec and fc, and if the right hand body be extended beyond the circle fch, the pencil at P will describe two curves, a shorter one Pe on the body ecg and a longer one Pf on the body fch, the points e and f being reached when P reaches the point c, and from the conditions of motion are $PC = \text{arc } ec = \text{arc } fc$.

Now P is a common point on the curves Pe and Pf and also a point on the circle PC, which has the common point C with the remaining three circles. Hence the motion of P with regard to ecg is perpendicular to PC, and of P with regard to fch is perpendicular to PC; that is, the tangents to Pe and Pf at P are normal to PC, or the two curves have a common tangent, and hence a common normal PC at their point of contact, and this normal will pass through the pitch point C. Thus Pe and Pf fulfil the necessary conditions for the shapes of gear teeth.

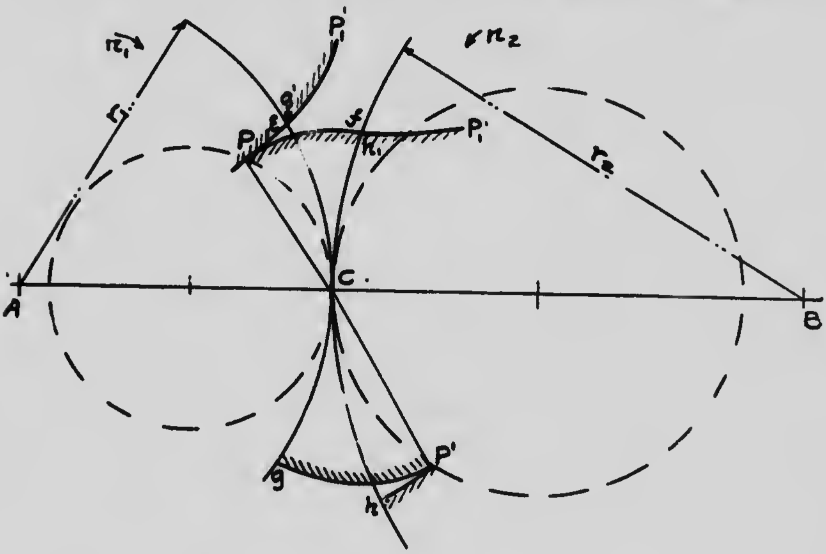


Fig. 41

Evidently the points of contact of these two curves lie along PC, since both curves are described simultaneously by a point which always remains on the circle PC. Since these curves are first in contact at P and then again at C, when P, c, e and f coincide, it is evident that during the motion from P to C the curve Pe slips on the curve Pf through the distance $Pf - Pe$. Below C the pencil at P would simply describe the same curves over again, only reversed.

To further extend these curves, we place a second pencil at P^1, which will draw the curves P^1g and P^1h in the same way as before, these curves having the same properties as Pe and Pf, the amount of slipping in this case being $P^1g - P^1h$, and the points of contact always lying on the circle CP^1.

Now join the two curves formed on ecg, that is, join gP^1 to Pe, as shown at $Peg^1P_1{}^1$, and then the two curves on fch, as shown at $Pfh_1P_1{}^1$, and we have a pair of curves which will remain in contact from P to P_1, which always have a point of contact on the curve PCP^1, and which always have a common normal at their point of contact passing through C. The relative amount of slipping is $Pfh_1P_1{}^1 - Peg^1P_1{}^1$. If, now, we cut out two pieces of wood, one having its side shaped like the curve $PeP_1{}^1$ and pivoted at A, while the other is shaped like $PfP_1{}^1$ and pivoted at B; then from what has been said, the former may be used to drive the latter, and the motion will be the same as that produced by the rolling of the two pitch circles together; hence these shapes will be the proper ones for the *profiles* of gear teeth.

The curves Pe, Pf, P^1g and P^1h, which are produced by the rolling of one circle inside or outside of another, are called *cycloidal* curves, the two Pe and P^1h being known as hypocycloids, since they are formed by the describing circle rolling inside the pitch circle, while the two curves Pf and P^1g are known as epicycloidal curves, in this case lying outside the pitch circles. Gears having these curves as the profiles of the teeth are said to have cycloidal teeth (sometimes erroneously called epicycloidal teeth), a form which is in very common use. So far we have only drawn one side of the tooth, but it will be evident that the other side is simply obtained by making a tracing of the curve $PeP^1{}_1$ on a piece of tracing cloth, with centre A also marked; then by turning the tracing over and bringing the point A to the original centre A, the other side of the tooth on the wheel ecg may be pricked through with a needle. The same method is employed for the teeth on wheel fch.

Nothing has so far been said of the sizes of the describing circles, and, indeed, it is evident that any size of describing circle, so long as it is somewhat smaller than the pitch circle, may be used, and will produce a curve fulfilling the desired conditions, but it may be shown that when the describing circle is one-half the diameter of the corresponding pitch circle the hypocyloid becomes a radial line in the pitch circle, and for reasons to be explained later this is undesirable. The maximum size of the describing circle is thus one-half that of the corresponding pitch circle, and for convenience the two describing circles are frequently of the same size, although this is not a necessity.

The proof that the hypocycloid is a radial line if the describing circle is half the size of the pitch circle, may be given as follows: Let *ABC*, Fig. 42, be the pitch circle and *DPC*

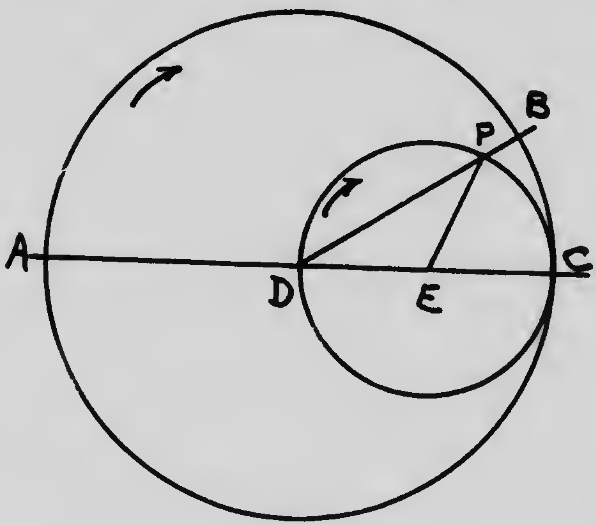


Fig. 42

the describing circle, *P* being the pencil, and *BP* the line described by *P*, as *P* and *B* approach *C*. The arc *BC* is equal to the arc *PC* by construction, and hence the angle *PEC* at the centre *E* of *DPC* is twice the angle *BDC*, because the radius in the latter case is twice that in the former. But the angles *BDC* and *PEC* are both in the one circle, the one at the circumference and the other at the centre, and since the latter is double the former they must stand on the same arc *PC*. In other words *BP* is a radial line.

In the actual gear the tooth profiles are not very long, but are limited between two circles concentric with the pitch circles in each gear, and called the *addendum* and *root circles*, as indicated in Fig. 43 the path of contact being evidently PCP, and the amount of slipping on each pair of teeth is $PR - PD + P_1E - P_1F$, or $PR + P_1E - (PD + P_1F)$. Further, since the common normal to the teeth pass through C then the direction of pressure between a given pair of teeth is always the line joining their point of contact to C, friction being neglected.

The arc PC is called the *arc of approach*, being the location of the points of contact down to the pitch point C, and CP_1 is called the *arc of recess*, P being the last point of contact. The

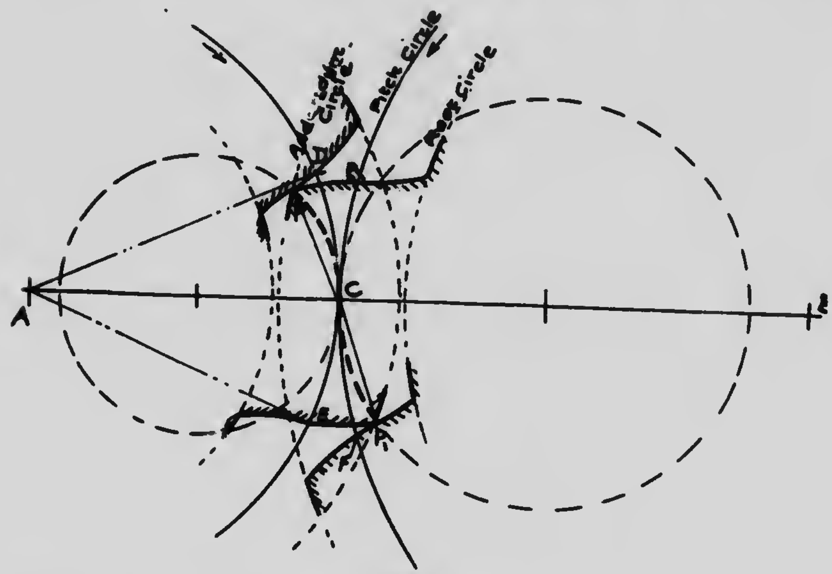


Fig. 43

angles DAC and CAE are called respectively the angles of approach and recess. As will be explained later, the distance between the addendum and root circles and the pitch circle depends upon the number of teeth in the gear, so that with these circles fixed the length of the *arc of contact* PCP_1 will depend upon the diameters of the describing circles being longer as the describing circles become larger. If this arc of contact is shorter than the distance between two teeth on the one gear, then only one pair of teeth can be in contact at once, and the running is uneven, while, if this arc is just equal to the distance between the centres of a given pair of teeth on one gear, or the *pitch*, as it called (See Fig. 46) one pair of teeth will just be going out of contact as the second pair is coming in, which will also cause jarring. It is usual to make PCP^1 at least 1.5 times the pitch of the teeth. This will, of course, increase the amount of slipping of the teeth.

With the usual proportions it is found that when the number of teeth in a wheel is less than 12 the teeth are not well shaped for strength or wear, and hence, although they will fulfil the kinematic conditions, they are not to be commended in practice.

INVOLUTE TEETH

The second and perhaps the most common method of forming the curves for gear teeth is by means of involute curves. Let *A* and *B*, Fig. 44, represent the axes of the gears, the pitch circles of which touch at *C*, and through *C* draw a secant

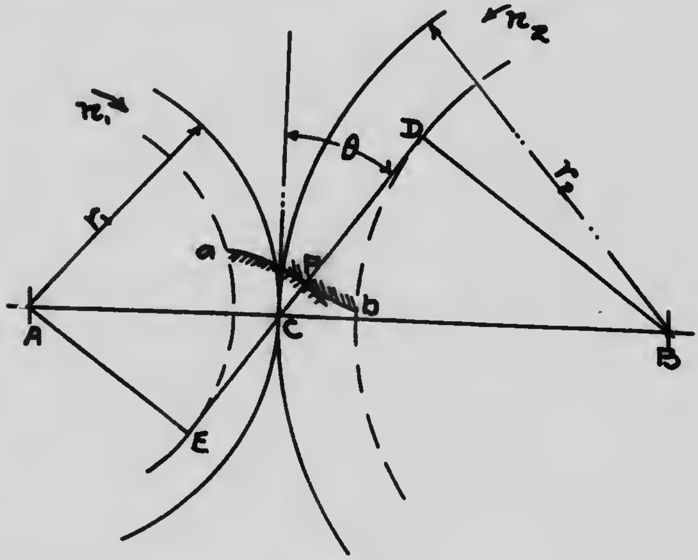


Fig. 44

DCE at any angle θ to the normal to *AB*, and with centres *A* and *B* respectively draw circles to touch the secant in *D* and *E*. Now $\frac{n_1}{n_2} = \frac{BC}{AC} = \frac{BD}{AE}$ so that if a string be run from *D* to *E* and used as a belt between the two dotted *base circles* at D and E, we would have exactly the same velocity ratio as if the original pitch circles rolled together having contact at C.

Now, choose any point P on the belt DE and attach at this point a pencil, and as the wheels revolve it will evidently mark on the original wheels, having centres at A and B, two curves Pa and Pb respectively, a being reached when the pencil gets down to E, and b being the starting point just as the pencil leaves D, and since the point P traces the curves simultaneously they will always be in contact at some point along DE, the point of contact traveling downward with the pencil at P. Since P can only have a motion with regard to the wheel aE normal to the string PE, and its motion with regard to the wheel Db is at right angles to PD, it will be at once evident that these two curves have a common normal at the point where they are in contact, and this normal evidently passes through C. Hence the curves may be used as the profiles of gear teeth.

The curves Pa and Pb are called involute curves, and when they are used as the profiles of gear teeth the latter are called involute teeth. The angle θ is called the *angle of obliquity,* and evidently gives the direction of pressure between the teeth, so that the smaller this angle becomes the less will be the pressure between the teeth for a given amount of power transmitted. If, on the other hand, this angle is unduly small, the base circles approach so nearly to the pitch circles in size that the curves Pa and Pb have very short lengths below the pitch circles. Many firms adopt for θ the angle $14\frac{1}{2}°$, in which case the diameter of the base circle is .968 (about 31/32) that of the pitch circle. If the teeth are to be extended below the base circles, as is usual, the lower part is made radial. With teeth of this form the distance between the centres A and B may be somewhat increased without affecting in any way the regularity of the motion. Involute teeth are also stronger in general than the corresponding cycloidal teeth.

The arc of contact in these teeth is usually about twice the pitch, and the number of teeth in a gear should not be less than 12, as the teeth will be weak at the root unless the angle of obliquity is increased.

Gears are sometimes made with the teeth on the inside instead of the outside of the rim, Fig. 45. Such gears are called *annular gears*, and they are always made to mesh with a spur pinion, the property being that both gear and pinion rotate in the same sense. The teeth on the annular gear are made in exactly the same way as those for the spur gear, and are involute or cycloidal.

When one gear of the pair has an infinite radius the pitch line becomes a straight line, and it is then called a *rack*, the teeth

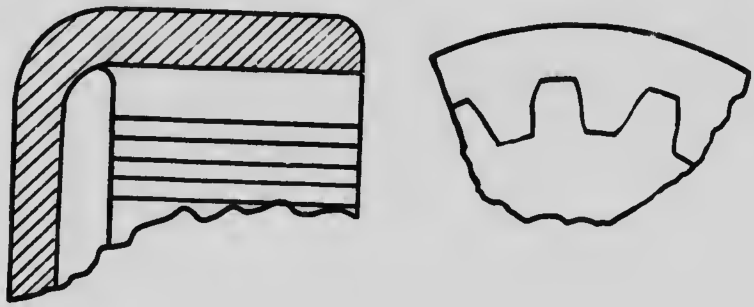


Fig. 45

being cycloids in one case, and in the involute system being straight lines, forming an angle $90 - \theta$ with the pitch line, the gear meshing with the rack being called the pinion.

Gear teeth are formed in various ways, such as casting, cutting from solid casting, etc., and as it is only possible to make the teeth accurately by the latter method, we shall speak hereafter of *cut* teeth. In this case an accurately turned casting is taken of the same diameter as the outside of the teeth, and the metal forming the spaces between the teeth is carefully cut out, leaving accurate shapes if the work be properly done. The various terms applied to gear teeth will appear from Fig. 46. The *addendum* line is a circle whose diameter is that of the outside of the gear. The *root* or *dedendum line* is a circle whose diameter is that at the bottoms of the teeth. The difference between the radii of these two circles gives the *height* of the teeth. The dimension of the teeth parallel with the shaft is the *width of face* or often the *face* of the tooth, although the word *face* is also used to denote the surface of the tooth outside the pitch line, the part of the surface of the tooth below the pitch line being the *flank*. The solid part of the tooth above the pitch line is the *point*, and the solid part below this line is the *root*.

Let d be the pitch diameter of a gear having t teeth, h_1 be the height of the tooth above the pitch line, and h_2 the depth below the pitch line, the total height $h = h_1 + h_2$; further, let w be the thickness of the tooth measured along the pitch line. The distance from centre to centre of teeth measured along the pitch line is the *circular pitch* or pitch p, and this definition at once gives $tp = \pi d$. As a matter of convenience Brown and Sharpe have introduced a second pitch, now also commonly adopted, called the *diametral* pitch and defined as $q = \frac{t}{d}$. It would naturally be expected that the diametral pitch would be the number of inches of diameter per tooth, since the circular pitch

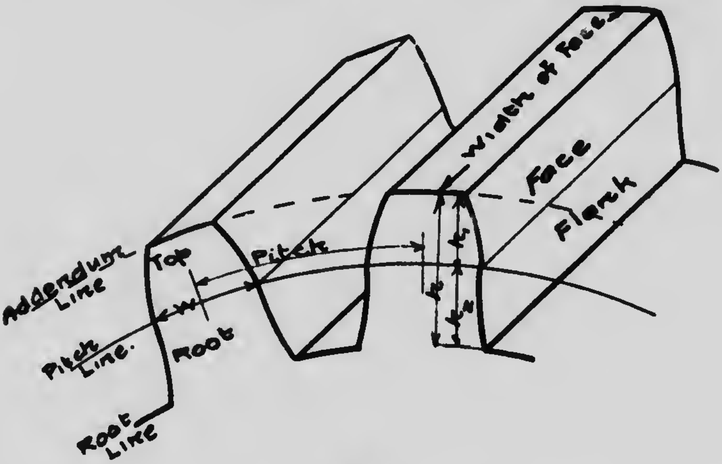


Fig. 46

is the number of inches of circumference per tooth. The diametral pitch is, however, the inverse and is not a number of inches. The following formulas are adopted by Brown and Sharpe: $p = \frac{\pi d}{\ }$; $q = \frac{t}{d}$; $h_1 = \frac{1}{q}$; $h_2 = \frac{1}{q} + \frac{p}{20}$; $w = \frac{p}{2}$, these dimensions being used for cut teeth. For cast teeth $w = .48p$, and hence there is a *back lash* $= .04p$ between any pair of teeth which are in mesh. In cut gears there is no back lash. Notice that since $h_2 - h_1 = \frac{p}{20}$ there is always a *clearance* space of $.05p$ between the top of one tooth and the root line of the other.

It will be evident at once that if a pair of gears are to work together it is necessary that they have the same pitch p, and also that in the cycloidal system the same describing circle must have been used in both, or if in the involute system, the same obliquity should be used in both. Wheels so constructed that any pair of them may work together correctly are called *set wheels*. Let d_1 and d_2 be the pitch diameters, and r_1 and r_2 the radii of two wheels which are to work together the shafts being l inches between centres, and the wheels turning at n_1 and n_2 revolutions per minute. Then from page 62,

$$r_1 = \frac{n_2}{n_1 + n_2}\, l, \quad \text{and} \quad r_2 = \frac{n_1}{n_1 + n_2}\, l,$$

this formula applying to spur gears only, not to annular gears. Further $\frac{r_1}{r_2} = \frac{t_1}{t_2} = \frac{n_2}{n_1}$.

Example:—Two spur wheels are to be placed between shafts running at 100 and 200 revs. per min. respectively, the shafts being 9 in. centres, and the diametral pitch being 3. Then $r_1 = \frac{200}{100 + 200} \times 9 = 6$ in. while $r_2 = \frac{100}{100 + 200} \times 9 = 3$ in. Thus $d_1 = 12$ in., $d_2 = 6$ in. Again, $t_1 = qd_1 = 3 \times 12 = 36$ teeth, and $t_2 = 3 \times 6 = 18$ teeth. The outside diameter of the gears are $d_1 + \frac{2}{q} = 12 + \frac{2}{3} = 12\frac{2}{3}$ and $d_2 + \frac{2}{q} = 6 + \frac{2}{3}$ or $6\frac{2}{3}$ in. The circular pitch p is $\frac{\pi d_1}{t_1} = \pi \times \frac{1}{\frac{t_1}{d_1}} = \pi \times \frac{1}{q}$, or $p = \frac{\pi}{q} = \frac{3.1416}{3} = 1.047$ in. The height $h_2 = \frac{1}{q} + \frac{p}{20} = \frac{1}{3} + \frac{1.047}{20} = .385$ in. $\therefore h = .719$ in.

The student should practice solving problems on gears, assuming different quantities, and also working on questions involving annular gears. On being told that the outside diameter of a gear is 4 in. and the diametral pitch 8, he should at once know that it has 30 teeth, and he should become very familiar with such calculations.

HELICAL GEARS

A study of Fig. 43 shows that the less the height of the teeth the more nearly the lines PC and P_1C become normal to the line of centres ACB, and hence, under such a condition, the less the pressure between the teeth (which is in the direction PC) for a given amount of power transmitted. Thus for a given pressure between the teeth the maximum power would be transmitted if the line of pressures were tangent to the pitch circles at C. If the height of the teeth is decreased, however, the arc of approach is decreased, and hence, for a given pitch, the smaller will be the number of teeth in contact at once, and the more uneven will be the motion. If now, instead of making the teeth directly across the gear parallel with the axis, they be run across it diagonally, so as to form parts of a helix, then, instead of a whole tooth on the gear coming suddenly into contact with a whole tooth on the pinion, we would have a pair of teeth coming gradually into contact, the contact beginning at one end and gradually working across the gear, till the other end is touching its mate. In such a case the teeth need not be high, and yet there will be no unevenness in the motion.

Wheels with the teeth made in this way are called *helical gears*, and it is to be remembered that if we pass a plane through the wheel normal to its axis the profile of the tooth so shown should be involute or cycloidal.

Helical wheels are used in the De Laval steam turbine, where the pinions run at 400 revs. per sec. without noise. They are also used in mills and other places, where steady motion is desired or the power transmitted is large.

CHAPTER VI.

BEVEL AND SPIRAL GEARING

It very frequently happens in practice that the shafts on which the gears are placed are not parallel, so that instead of the spur gears already described some other type must be used. If the shafts intersect, the gears used between them are known as *bevel gears*.

This class is by far the most common one of the genera' type under discussion, being used on such devices as the main transmission in automobiles, or the connection between the vertical shaft of a water turbine and the main horizontal shaft, and in many other well known machines.

On the other hand it not infrequently happens that the two shafts do not intersect, as in the case of the cam and crank shafts of a gas engine, and in such a case the bevel gear is not of value. Quite frequently the shafts are at right angles to one another, although there are cases where they are not. Gears which suit such conditions are of two classes, (a) *hyperboloidal gears*, which have *line contact* between teeth and may be used for shafts inclined at any angle, and the diameters of which depend upon the velocity ratio and location of the shafts, and (b) *spiral gears*, the teeth of which have *point contact*, and which are used most frequently for shafts at right angles, and the diameters of such gears being in general independent of the velocity ratio between the shafts. We shall now discuss these different forms in a general way.

Although a general method may be described for dealing with the problem mentioned, it will be found more simple to defer it for the present, dealing with the simplest case first, and afterward describing the general method. The case will therefore be first discussed where the two shafts intersect.

BEVEL GEARING

The axes of the shafts may be inclined at any angle to one another, the most common case being where they are at right angles, although they frequently intersect at other angles. The gears used to drive between two such shafts are called *bevel gears*, and in the case where the shafts are at right angles and both turn at the same speed, the two bevel wheels would be exactly equal in all respects, and are then called *mitre gears*. Bevel gears are rarely made annular.

Let A and B, Fig. 47, represent the axes of two shafts intersecting at C, and let their speeds be n_1 and n_2 respectively To find the sizes of the gears necessary to drive between them, let E be a point of contact between the gears, the radii to it being r_1 and r_2. Then we have $r_1 n_1 = r_2 n_2$, as in the case of the spur gear, or $\frac{r_1}{r_2} = n_h =$ constant, and hence at any point where these gears would touch we should have the ratio $\frac{r_1}{r_2}$ = const., a condition which can only be fulfilled by points lying on the line EC. In the case of bevel gears, therefore, contact is

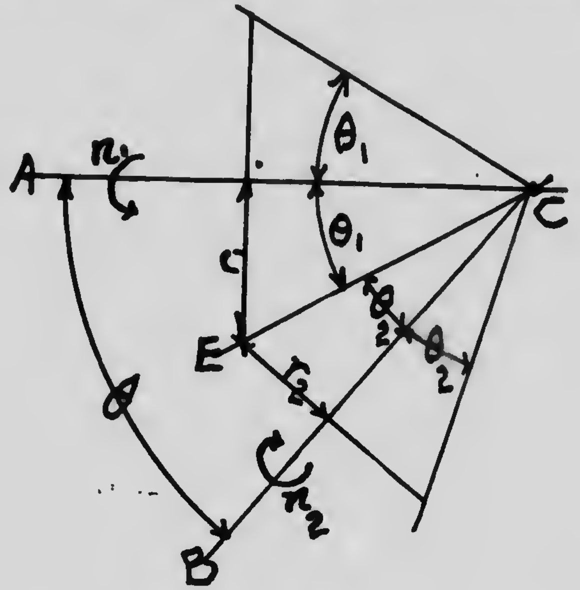


Fig. 47

along a straight line passing through the intersection of their shafts, and it may be shown that we can only get the desired motion by rolling together two cones, each having its apex at C, and an angle at the apex of $2\theta_1$ or $2\theta_2$, as marked. If $\theta = 90°$ and $n_1 = n_2$, then $\theta_1 = \theta_2 = 45°$. It is to be observed here that the angles θ_1 and θ_2 are fixed, when θ_1, n_1 and n_2 are known, but one of the radii r_1 or r_2 may be selected at will by the designer.

It is not considered advisable in this discussion to enter into the exact form the teeth should have in such a case, and the method of finding the proper shapes will merely be described. Let Fig. 48 represent one of the wheels, with angle $2\theta_1$ at the vertex of the cone, and let the radii r_1 and r_1' be selected to suit external conditions. Through D and F draw lines DG and FH normal to CDF, to intersect the axis of the shaft at G and H respectively. Then at D the teeth will have the same shape as if constructed for a spur gear of radius GD, and at F the teeth should be constructed for a spur gear of radius HF, and so for

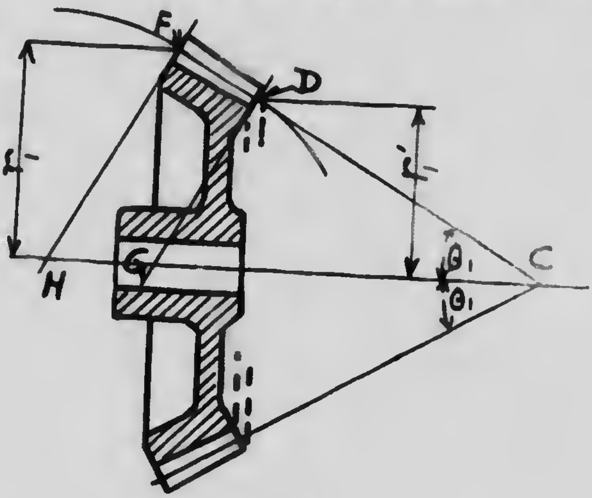


Fig. 48

any intermediate point. The teeth are, of course, tapering from F to D, and either the involute or cycloidal system may be used.

HYPERBOLOIDAL GEARING

THE TEETH OF WHICH HAVE LINE CONTACT

Having discussed the particular case of intersecting shafts, we shall now consider the general problem. Let AO and BP, Fig. 49, represent the two shafts under consideration. Then, as a rule, the axes of these shafts will be horizontal, and the lines shown will represent the ordinary plan and elevation of the centre lines of the shafts, but whether the shafts are horizontal or not we shall assume the planes of reference for the drawing chosen, the one normal to OP, the shortest distance between the shafts, while the other passes through the line OP and also the axes $O'A'$ of one of the shafts. This corresponds to the ordinary plan and elevation of the shafts that would in general appear in practice. The angle $AOB = \theta$ between the projections AO and BP on the normal plane is called ***the angle between the shafts***, while the distance $O'P' = h$ between the projections on the other plane, we call the ***distance between the shafts***, meaning by it the ***shortest distance***; we shall assume h and θ, known as well as n_1 and n_2, the angular velocities of the shafts AO and BP respectively.

The angle between the shafts may be selected as AOB or its supplement and confusion may arise as to the proper angle to select. Throughout the remainder of this discussion we

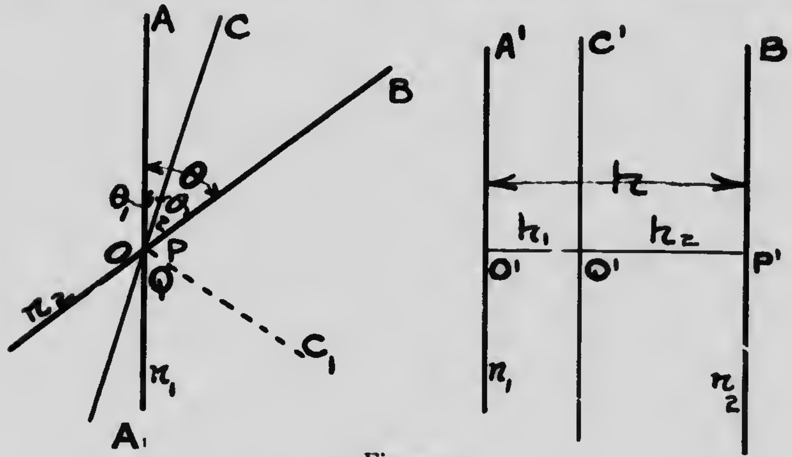


Fig. 49

have selected the angle so that the line of contact CQ falls within it, and as annular gears of this type are not used, CQ will be so located that if the gears have contact along this line, the shafts to which they are attached will turn in opposite sense. Thus in Fig. 49, θ is the angle AOB and not A_1OB, for if the line of contact were chosen, say at C_1Q in A_1OB, then it would be necessary to put on one annular gear to give the proper sense of rotation to the shafts.

The problem now is to design a pair of gears which will work between the two shafts and fulfil the given conditions, and in order to have the best possible service from the gears, it will be assumed that there is to be line contact between them. Let CQ, the line of contact pass through OP, it is required to locate CQ, i.e. to find θ_1, θ_2, h_1 and h_2, and also the proper sizes of the gears. Choose any point R, Fig. 50, in the line CQ as a point of contact, the projections of this point being R and R', and draw from R the radii RT and RV respectfively on the shafts AO and BP.

Let the components of these radii parallel to OP be ST and UV respectively, the corresponding components parallel to the normal plane to OP being RS and RU, so that $RT^2 = RS^2 + ST^2$ and $RV^2 = RU^2 + UV^2$. (To draw these radii it is only necessary

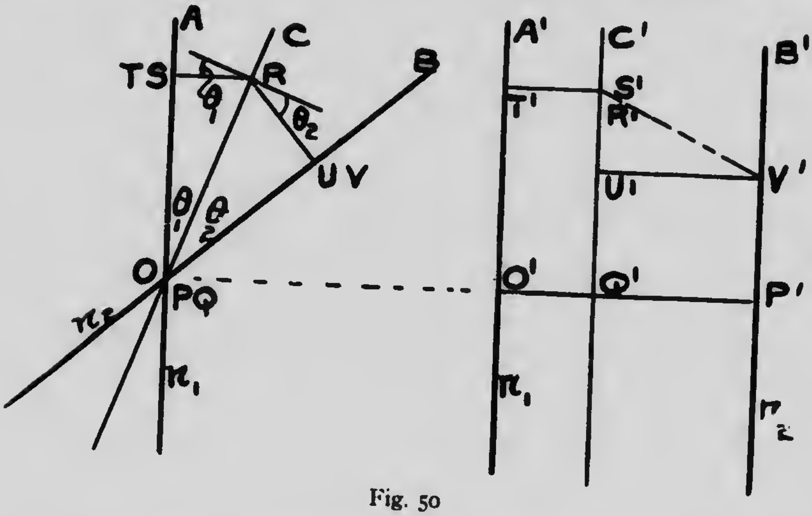


Fig. 50

to remember that RS is perpendicular to OA and RU is perpendicular to BP, further that S' and U' are found by projecting over S and U from the normal projection and also that $S'T'$ and $U'V'$ are parallel to $O'P'$.)

Now at the point of contact R, it is essential that the velocity ratio $n_1 : n_2$ must be maintained between the shafts, and since E is the point of contact, it is a common point in both gears. From what has already been said at page 63, it will be clear that, as a point in A, the motion of R *in a plane normal to the line of contact* RQ must be the same as the motion of the same point R as a point in B in the same plane, i.e., *in the plane normal to the line of contact* RQ, *the two wheels must have the same motion at the point of contact* R. It is to be kept in mind, however, that *sliding along* CQ *is permissible* for a similar reason that there is no objection to the axial motion of spur gears relative to one another while they are running; thus we shall not need to design the wheels to prevent slip *along* the line of contact.

In order to fulfil the desired conditions as simply as possible, we shall divide the motion of R in each wheel in the normal plane to RQ, into two parts, viz., those normal to each plane of reference in the drawing. Taking first the motions of R in each wheel in the

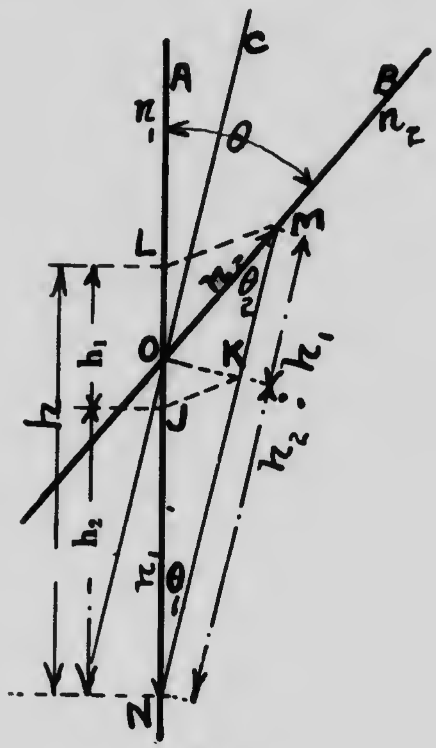


Fig. 51

direction of PO and in the plane normal to CQ, we find that (a) as a point in AO its motion is $RS \,.\, n_1$ and (b) as a point in BP its motion is $RU \,.\, n_2$, and since the motions at R are equal the components of these motions are equal or $RS \,.\, n_1 = RU \,.\, n_2$.

But from the figure it is evident that $RS = OR \sin\theta_1$ and $RU = OR \sin\theta_2$ and hence $RS \,.\, n_1 = RU \,.\, n_2$ becomes $OR \,.\, \sin\theta_1 \,.\, n_1 = OR \sin\theta_2 \,.\, n_2$ or $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

Again in the direction perpendicular to OP the actual motion of R as a point in AO in the plane RST would be $S'T'\ n_1$, and on resolving it into the plane normal to RQ we would have $S'T'.n_1\ \cos\theta_1$, similarly the motion of R as a point in BP in the same direction will be $U'V'\ .\ n_2\ .\ \cos\theta_2$. Since these two motions must be the same we get $S'T'\ .\ n_1 \cos\theta_1 = U'V' n_2 \cos\theta_2$ or $h_1 n_1 \cos\theta_1 = h_2 n_2 \cos\theta_2$.

It is now seen by combining these equations that the location of RQ is fixed by four conditions, viz.,

$$h_1 + h_2 = h, \qquad \text{(i)}$$
$$\theta_1 + \theta_2 = \theta, \qquad \text{(ii)}$$
$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \qquad \text{(iii)}$$
$$\text{and } h_2 n_1 \cos\theta_1 = h_1 n_2 \cos\theta_2. \qquad \text{(iv)}$$

and as there are only four unknowns when h, θ, n_1 and n_2 are given the values of h_1, h_2 θ_1 and θ_2 may be found. A simple graphical

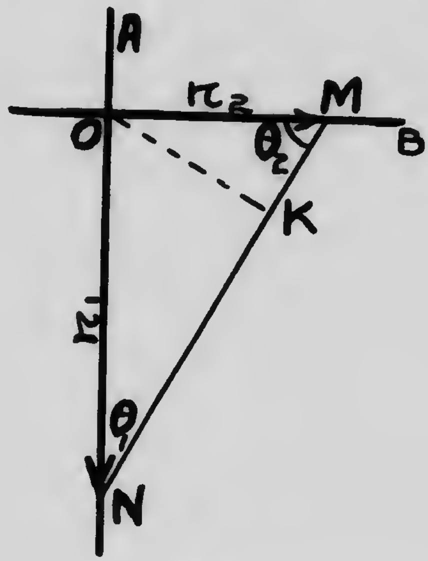


Fig. 52

method may be employed as follows: In Fig. 51 lay off $OM = n_2$ and $ON = n_1$ to any scale, these to be in opposite directions from O because the shafts turn in opposite sense. Join MN and draw OK perpendicular to MN then will MN be parallel to the required line OC, the angle $ONK = \theta_1$ and $OMK = \theta_2$ and evidently $\theta_1 + \theta_2 = \theta$. Further, $NK : KM$ is the ratio $h_2 : h_1$, so that if in the construction NL be made equal to h and M be joined to L then by drawing JK parallel to LM there is obtained $NJ = h_2$ and $LJ = h_1$.

The proof of the graphical construction is: Since $OM = n_2$ and $ON = n_1$, hence $OK = n_1 \sin ONK = n_2 \sin OMK$ from which it is evident by comparing this with equation (iii) that $ONK = \theta_1$ and $OMK = \theta_2$, also $AOB = OMK + ONK$ or $\theta = \theta_2 + \theta_1$ which satisfies

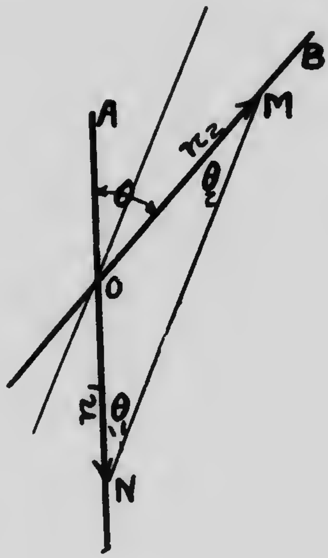


Fig. 53

also equation (ii). Further, $NK = n_1 \cos \theta_1$ and $KM = n_2 \cos \theta_2$ or $\dfrac{NK}{KM} = \dfrac{n_1 \cos \theta_1}{n_2 \cos \theta_2}$ and by comparing this with equation (iv) which may be written $\dfrac{h_1}{h_2} = \dfrac{n_2 \cos \theta_2}{n_1 \cos \theta_1}$, it is at once evident that $h_2 : h_1 = NK : KM$. The construction for finding the actual values of h_1 and h_2 presents no difficulties.

The four equations (i), (ii), (iii) and (iv) give a means of solving any problem of this nature and the application to a few cases will show the general nature of the method. n_1 and n_2 are always assumed given.

Case (1.) Shafts inclined at any angle θ and at given distance h apart. This is the general case already solved and θ_1, θ_2, h_1 and h_2 are found as indicated.

Case (2.) Shafts inclined at angle $\theta = 90°$ and at distance h apart. (Care must be taken not to confuse the method and type of gear here described with the spiral gear to be discussed later.) Choose the axes as shown in Fig. 52, lay off $ON = n_1$ and $OM = n_2$ and join MN; then draw OK perpendicular to MN from which we at once obtain $NK : KM = h_2 : h_1$. In this case $h_1\, n_1 \cos\theta_1 = h_2\, n_2 \cos\theta_2$ gives $\dfrac{h_1\, n_1}{h_2\, n_2} = \dfrac{\cos\theta_2}{\cos\theta_1} = \dfrac{\sin\theta_1}{\cos\theta_1} = \tan\theta_1 = \dfrac{n_2}{n_1}$, and hence $\dfrac{h_1}{h_2} = \left(\dfrac{n_2}{n_1}\right)^2$.

To take a definite case, suppose $n_1 = 2\, n_2$ then $\dfrac{h_1}{h_2} = \left(\dfrac{n_2}{n_1}\right)^2 = \left(\dfrac{n_2}{2\, n_2}\right)^2 = \dfrac{1}{4}$ and if the distance apart, h, of the shafts is 20 in. then $h_1 = 4$ in. and $h_2 = 16$ in., and the angle θ_1 is given by $\tan\theta_1 = \dfrac{n_2}{n_1} = \dfrac{1}{2} = .5$, or $\theta_1 = 26°\ 34'$ and $\theta_2 = 90 - \theta_1 = 63°\ 26'$, so that the line of contact is readily located.

Case (3.) Parallel shafts at distance h apart, this gives the ordinary case of the spur gear. Here $\theta = o$ and therefore $\theta_1 = o = \theta_2$, hence, $\sin\theta_1 = o = \sin\theta_2$ and $\cos\theta_1 = 1 = \cos\theta_2$, so that there are only two conditions to satisfy, viz., $h_1 + h_2 = h$ and $h_1\, n_1 = h_2\, n_2$. Solving these gives $h_2 = \dfrac{n_1}{n_2}\, h_1$ and substituting in $h_1 + h_2 = h$ gives at once $h_1 = \dfrac{n_2}{n_1 + n_2} . h$ and $h_2 = \dfrac{n_1}{n_1 + n_2}\, h$ formulas which will be found to agree exactly with those on page 62 for spur gears.

Case (4.) Intersecting shafts. Here $h = o$, therefore $h_1 = o$ and $h_2 = o$. Referring to Fig. 53, draw $OM = n_2$ and $ON = n_1$ then MN is in the direction of the line of contact OC for there are only two equations here to satisfy, viz., $\theta_1 + \theta_2 = \theta$ and $n_1 \sin\theta_1 = n_2 \sin\theta_2$, and these are satisfied by MN. Then draw OC parallel to MN (Compare this with the case of the bevel gear taken up at the beginning of the chapter).

Case (5.) Intersecting shafts at right angles. Here $\theta = 90°$. Further let $n_2 = n_1$ then $\theta_1 = \theta_2 = 45°$, thus the wheels would be equal and are called *mitre wheels*.

We shall now return to the general problem in which is given θ, h, n_1 and n_2 and have found the location of the line of contact CQ by the method described for finding h_1, h_2, θ_1 and θ_2. Now just as in the case of the spur and bevel gears, select a short part of

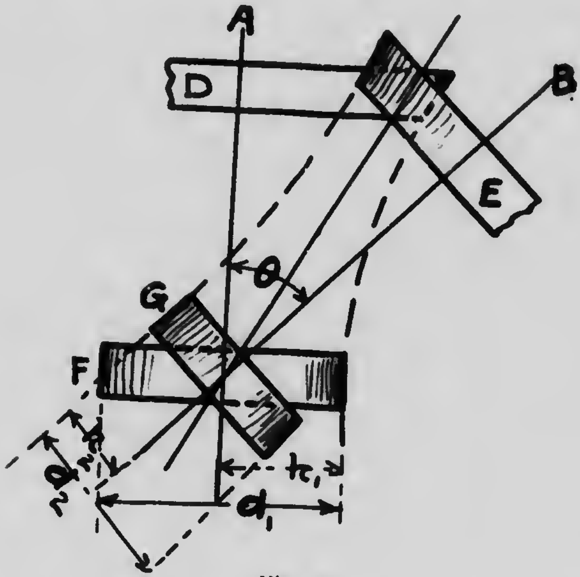


Fig. 54

the line of contact to use for the gears according to the width of face which is decided upon, the width of face largely depending upon the power to be transmitted, and therefore being rather beyond the scope of the present discussion.

Now it is known from geometry that if the line CQ were secured to AO while the latter revolved, the former line would describe a surface known as an hyperboloid of revolution and a second hyperboloid would be described by securing the line CQ to BP, the curved lines in the drawing, Fig. 54, showing sections of these hyperboloids by planes passed through the axes AO and BP. As the process of developing the hyperboloid is somewhat difficult and long, the reader is referred to books on descriptive geometry or other works for the method. Should the distance h between the shafts be small, then sections of the hyperboloids selected as shown at D and E must be employed and this can only be done by drawing the true curves, when, however, the shafts are far enough apart, as is frequently the case, the *gorges* of the hyperboloids may be used and no serious error will result by using two cylindrical wheels F and G of radii h_1 and h_2 found as before explained.

In case the wheels F and G are used the angles θ_1 and θ_2 give the inclination of the teeth across the faces of the cylinders, but if it is necessary to use D and E on account of the small value of h, the true surface is located as already explained and then the cones are selected which correspond nearest to these surfaces, the wheels being treated as ordinary bevel wheels with the teeth running diagonally across the face.

In order to explain the method, assume that the gorge wheels F and G may be used, and the angles θ_1 and θ_2 of inclination

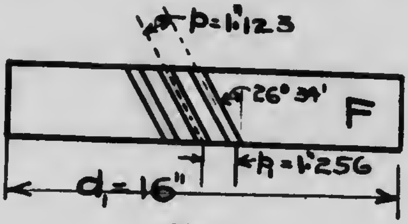


Fig. 55

of the teeth across the surface are known. As a numerical example take case (2), page 82, for which $\theta = 90°$, $n_1 = 2\,n_2$ and $h = 20$ in. and it has been found in this example that $h_1 = 4$ in., $h_2 = 16$ in., $\theta_1 = 26°\ 34'$ and $\theta_2 = 63°\ 26'$. Evidently h is sufficiently great to allow the use of the gorge wheels F and G, so that the diameter of the wheel F on OA is $d_1 = 2\,h_1 = 8$ in. and that of G is $d_2 = 2\,h_2 = 32$ in. The actual number of the teeth on each gear will depend upon the load the pair must carry so that the number of teeth will here be assumed, without calculation. Let the number of teeth on the wheel F be $t_1 = 20$ then the distance from centre to centre of teeth measured on the pitch line, Fig. 55, along the end of the gear is $p_1 = \dfrac{\pi \times 8}{20} = 1.256$ in. and this distance will not be the same as the corresponding distance in the gear G. If the gears are to work together properly, however, the normal distance from centre to centre of the teeth along the pitch line, i.e., the pitch p, must be the same in both gears and hence $p = p_1 \cos \theta_1 = 1.256 \cos 26° 34'$ or $p = 1.256 \times .8944 = 1.123$ in.

For the gear G the number of teeth must be $t_2 = 40$ since $n_1 = 2n_2$, and hence $p_2 = \frac{\pi \times 32}{40} = 2.513$ in., and $p = p_2 \cos \theta_2 = 2.513 \times \cos 63° 26' = 2.513 \times .4472 = 1.123$ in. as before.

A section of the teeth normal to their direction will have a profile like an ordinary spur gear, i.e., the section taken in that direction will have involute or cycloidal curves and may be laid out exactly the same as for spur wheel teeth of pitch p. In this type of gearing

Fig. 56

Fig. 57

there is of necessity considerable slip along the line of contact CQ, so that the frictional losses may be high and they are therefore not to be preferred in many cases. If properly made, however, they run very smoothly and if run in oil the frictional losses may be reduced.

SPIRAL GEARING

THE TEETH OF WHICH HAVE POINT CONTACT

In speaking of gears for shafts which were not parallel and did not intersect, we mentioned two classes (a) hyperboloidal gears (b) spiral gears, and having discussed the first at some length we shall now refer to the second class somewhat briefly. In this class of gearing there is no necessary relation between the diameters of the wheels and the velocity ratio $\frac{n_1}{n_2}$ between the shafts; thus one finds very frequently that, while the cam shaft of a gas engine runs at one-half the speed of the crank shaft, and is in general at right angles to the latter, the spiral gears connecting the two shafts are of practically the same size.

The most familiar form of this gearing is the well-known worm and worm wheel, which is sketched in Fig. 56, and it is to be noticed that the one wheel here takes the form of a screw, this wheel being distinguished by the name of the *worm*. The distance which any point on the worm wheel is moved by one revolution of the worm is called the pitch of the worm, and if this pitch corresponds to the distance from thread to thread along the worm, the thread is called single pitch. If the distance from one thread to the next is one-half of the pitch the thread is double pitch, and if this ratio is one-third the pitch is triple, etc. The latter two cases are illustrated at (a) and (b), Fig. 57.

Let p_1 be the axial pitch of the worm and D be the pitch diameter of the wheel measured on a plane through the axis of the worm and normal to the axis of the wheel. Then the circumference of the wheel is πD, and since, by definition of the pitch, one revolution of the worm will move the gear forward p_1 in., hence there will be $\frac{\pi D}{p_1}$ revolutions of the worm for one revolution of the wheel or this is the ratio of the gears. Let t be the number of teeth in the gear, then if the worm is single pitch $t = \frac{\pi D}{p_1}$ or the ratio of the gears is simply the number of teeth in the wheel. If the worm is double pitch, then p_1 the distance from centre to centre of teeth measured as before is given by $p_1 = 2p'$, where p' is the axial distance from the centre of one thread to the centre of the next one, and $t = \frac{\pi D}{p'}$ and as the ratio of the gears is $\frac{\pi D}{p_1}$ we get in this case the ratio equal to $\frac{t}{2}$, and for triple pitch the ratio is $\frac{t}{3}$ etc.

A brief study of the matter will show that as the velocity ratio of the gearing is fixed by the *pitch* of the worm and the diameter of the wheel, hence no matter how large the worm may be made it is possible still to retain the same pitch, and hence the same velocity ratio, for the same wheel. The only change produced by changing the diameter of the worm is that the angle of inclination of the spiral thread is altered, being decreased as the diameter increases, and vice versa. The angle made by the teeth across the face of the wheel must be the same as that made by the spiral on the worm, and if the pitch of the worm be denoted by p_1 and the mean diameter of the thread on the latter by d_1 then the inclination of the thread is given by $\tan \theta = \frac{p_1}{\pi d_1}$, and this should also properly be the inclination of the wheel teeth. From the very nature of the case there will be a great deal of slipping between the two wheels, for while the wheel moves forward only a single tooth there will be slipping of amount πd, and hence considerable frictional less, so that the diameter of the worm is made as small as possible consistent with reasonable values of θ.

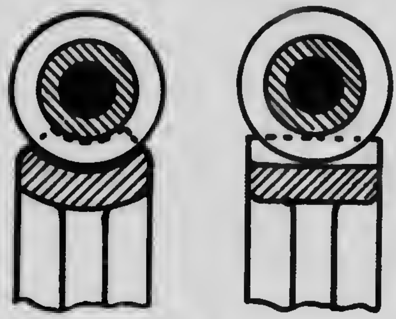


Fig. 58

When both the worm and wheel are made parts of cylinders, Fig. 58, then there would only be point contact with the worm, but as this is very unsatisfactory for power transmission, the worm and wheel are usually made as shown in section in the left-hand diagram in Fig. 58 where the construction of the teeth may be such as to approach line contact. The usual method of construction is to turn the worm up in the lathe, cutting the threads as accurately as may be desired, then to turn the wheel to the proper outside finished dimensions. The cutting of the teeth in the wheel rim may then be done in various ways of which only one will be described, that by the use of a *hob*.

A hob is constructed of steel and is an exact copy of the worm with which the wheel is to work. Grooves are cut across the threads so as to make it after the fashion of a milling cutter, as shown in Fig 59, which is taken from the Brown and Sharpe catalogue. The hob is then hardened and ground and is ready for service. The teeth on the wheel may now be roughly milled out by a cutter, after which the hob and gear are brought into contact and run at proper relative speeds, the hob milling out the teeth and gradually being forced down on the wheel till it occupies the same relative position that the worm will do. In this way teeth of the exact form required are cut out and the worm and wheel will run perfectly together having contact approximately along a line.

Space does not permit here to go further into this very interesting form of gearing, and the reader will find very much written regarding it. Only one or two points more will be mentioned. It has already been pointed out that the frictional loss in the gear is very high owing to the great amount of slipping, and hence the velocity of slipping is reduced as much as possible by reducing the size of the worm, and at the same time the latter is usually immersed in oil

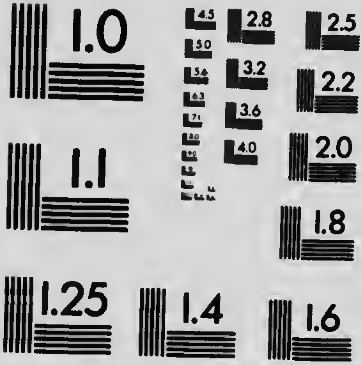
Fig. 59

while running, but for all that the frictional loss is rarely less than 25 per cent. of the power transmitted, and frequently exceeds 50 per cent.

From the point of view of velocity ratio, however, there are great advantages in being able to obtain very high ratios without excessively large wheels. Thus if a worm wheel has 40 teeth, and is geared with a single-threaded worm, the velocity ratio will $\frac{1}{40}$, while with a double-threaded worm it will be $\frac{2}{40} = \frac{1}{20}$, so that it is very convenient for such large ratios. It also finds favor because ordinarily it

MICROCOPY RESOLUTION TEST CHART

(ANSI and ISO TEST CHART No. 2)



cannot be reversed, i.e., the worm must always be used as the driver and cannot usually be driven by the wheel.

Consider now the case of the worm and wheel shown in Fig. 58, in which both are cylinders, and suppose that with a worm of given size a change is made from a single to double thread, at the same time keeping the threads of the same size. The result will be that there will be an increase in the angle θ and hence the threads will run around the worm and the teeth will run across the wheel at greater angle than before. If the pitch be further increased there is a further increase in θ which may be made as great as 45°, or even greater, and if at the same time the axial length of the worm be decreased, the threads will not run around the worm completely, but will run off the ends just in the same way as the teeth of wheels do.

By the method just described the diameter of the worm is unaltered, and yet the velocity ratio is gradually approaching unity, since we are increasing the pitch, so that keeping to a given diameter of worm and wheel, the velocity ratio may be varied in any way whatever, or the velocity ratio is independent of the diameters of the worm and wheel. When the pitch of the worm is increased and its length made quite short it changes its appearance from what it originally had and takes the form of a gear wheel with teeth running in helices across the face. A photograph of a pair of these wheels used for driving the cam shaft of a gas engine is shown in Fig. 60, and in this case the wheels give a velocity ratio of two to one between two shafts which do not intersect, but have an angle of 90° between planes passing through their axes. This form of gear is very extensively used for such purposes as the above, giving quiet steady running, but, of course, the frictional loss is quite high.

Some of the points mentioned in this discussion may be made clearer by an illustration. Let it be required to design a pair of gears of the above type to drive the cam shaft of a gas engine from the crank shaft, the velocity ratio in this case being 2 : 1, and let both gears be of the same diameter, the distance between centres being 12 in. From the data given the pitch diameter of each wheel will be 12 in. and since for one revolution of the cam shaft the crank shaft must turn twice, hence the pitch of the thread on the worm must be $\frac{1}{2} \times \pi \times 12 = 18.85$ in., so that for the gear on the crank shaft (corresponding to the worm) the "teeth" will run across its face at an angle given by $\tan \theta = \frac{18.85}{\pi \times 12} = .5$, or $\theta = 26° 34'$, and this angle is to be measured between the thread or tooth and the plane normal to the axis of rotation of the worm. The angle of the teeth of the gear on the cam shaft (corresponding to the worm wheel) will be $90 - 26° 34' = 63° 26'$ measured in the same way as before.


Fig. 60

It will be found that the number of teeth in one gear is double that in the other, also the normal pitch of both gears must be the same. The distance between adjacent teeth is made to suit the conditions of loading and will not be discussed.

Spiral gearing may be used for shafts at any angle to one another, although they are most common in practice where the angle is 90°. A more detailed discussion of the matter will not be attempted here and the reader is referred to other complete works on the subject.

GENERAL REMARKS ON HYPERBOLOIDAL AND SPIRAL GEARING

In concluding this chapter it is well to point out the differences in the two types of gearing here discussed. In appearance in many cases it is rather difficult to tell the gears apart, but a close examination will show the decided difference that in *hyperboloidal gearing contact between the gears is along a straight line.* While in *spiral gearing contact is at a point* only. A study of gears which have been in operation shows this clearly, the ordinary spiral gear as used in a gas engine wearing only over a very small surface at the centres of the teeth. It is also to be noted that the teeth of hyperboloidal gears are *straight* and run across the face of the gear while the teeth of spiral gears run across the face in helices.

Again in both classes of gears the ratio between the numbers of teeth on the gear and pinion is the velocity ratio transmitted and in the case of the spiral gears the relative diameters may be selected as desired while in the hyperboloidal gears the diameters are fixed when the angle between the shafts and the velocity ratio is given.

CHAPTER VII.

TRAINS OF GEARING

In ordinary use gears* are frequently arranged in a series on several separate axles, a series being called a *train* of gearing, so that a train of gearing consists of two or more wheels, which all turn at the same time, the angular velocities of all wheels in the train being known when that of any one is given. A train of gearing may always be replaced by a single pair of wheels of proper diameter, but in many cases the diameters of the two gears would be such as to make the arrangement undesirable.

When the train consists of four or more wheels, and when any two of these of different sizes are keyed to the same intermediate shaft, the arrangement is said to be a *compound train*. These trains are very common. If the gears are so arranged that the axis of the last gear lies in the same straight line as that of the first gear, as in the train between the minute and hour

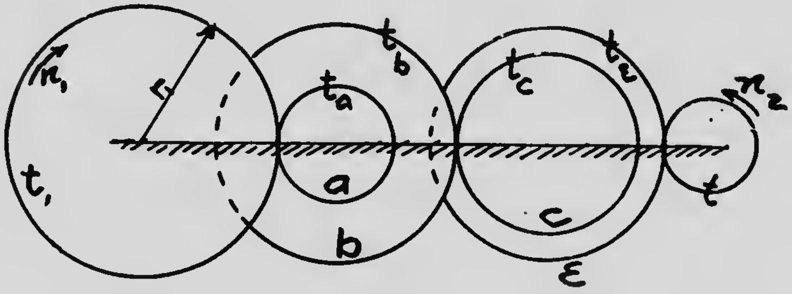


Fig. 61

hands of a clock, the train is said to be *reverted.* If one of the gears in the train is held stationary and some or all of the other gears revolve about it, as in the case of the differential gear of an automobile, where one back wheel stops, the scheme is called an *epicyclic train.* It is quite common to have a reverted, epicyclic, compound train or a simple epicyclic train. Thus in the epicyclic train the axis of the last gear may coincide with that of the first, although this is not at all necessary.

The *velocity ratio* of any train of gearing is the number of revolutions made by the last wheel divided by the number of

*The discussion in this chapter refers only to spur and bevel gears.

revolutions in the same time of the first wheel in the ordinary train, or of the frame in the epicyclic train. Thus, let n_2 be the number of revolutions per minute made by the last gear, and let n_1 be the revolutions per minute of the first gear in the ordinary train or of the frame in the epicyclic train, then the ratio of the train $R = \frac{n_2}{n_1}$. Taking first a train in which the frame is fixed and all wheels revolve, let it consist of spur gears 1, a, b, c, e, and 2, Fig. 16, having speeds n_1, n_a, etc., etc., radii r_1, r_a, etc., and numbers of teeth, t_1, t_a ...... t_2, respectively, the gears a and b being keyed to one shaft, as also the gears c and e. Thus, this is a compound train. Evidently any pair meshing together, such as b and c, must have the same pitch, and also the same type of teeth (i.e., involute or cycloidal), but any other gear in the train may have a different system and pitch, provided only that it suits the gear with which it meshes. Then, it at once follows that

$$\frac{n_a}{n_1} = \frac{r_1}{r_a} = \frac{t_1}{t_a} \text{ and } \frac{n_c}{n_b} = \frac{r_b}{r_c} = \frac{t_b}{t_c} \text{ and } \frac{n_2}{n_e} = \frac{r_e}{r_2} = \frac{t_e}{t_2}$$

and hence that

$$R = \frac{n_2}{n_1} = \frac{n_a}{n_1} \cdot \frac{n_c}{n_a} \cdot \frac{n_2}{n_c} = \frac{r_1}{r_a} \cdot \frac{r_b}{r_c} \cdot \frac{r_e}{r_2} = \frac{t_1}{t_a} \cdot \frac{t_b}{t_c} \cdot \frac{t_e}{t_2}$$

Calling now the first wheel in each pair the driver, and second wheel the driven, we at once get the rule: The ratio of a train R is the product of the radii of the drivers divided by the product of the radii of the driven wheels, or the ratio is the product of the teeth in the drivers divided by the product of the teeth in the driven wheels.

Should any of the wheels in the above train be annular, exactly the same law holds; and, in fact, the same law will hold if some of the gears are replaced by belts and pulleys, so that the determination of the ratio is quite simple in any case. Thus, in the above case, let $n_1 = 50$ revs. per min., $n_a = 80$ revs., $n_c = 120$ revs., and $n_2 = 200$ revs, and let $r_1 = 6$ in., $r_b = 4$ in., and $r_e = 5$ in.; also let the diametral pitches be 4, 6 and 8 for the pairs 1 and a, b and c and e and 2 respectively. Then we have $r_a = 3\frac{3}{4}$ in., $r_c = 2\frac{2}{3}$ in., $r_2 = 3$ in., or $d_1 = 12$ in., $d_a = 7\frac{1}{2}$ in., $d_b = 8$ in., $d_c = 5\frac{1}{3}$ in., $d_e = 10$ in., and $d_2 = 6$ in.; also $t_1 = 48$ teeth, $t_a = 30$ teeth, $t_b = 48$ teeth, $t_c = 32$ teeth, $t_e = 80$ teeth, and $t_2 = 48$ teeth.

The volocity ratio R of the train =

$$\frac{r_1 \times r_b \times r_e}{r_a \times r_c \times r_s} = \frac{6 \times 4 \times 5}{3\tfrac{3}{4} \times 2\tfrac{2}{3} \times 3} = 4$$

$$\text{or } R = \frac{t_1 \times t_b \times t_e}{t_a \times t_c \times t_s} = \frac{48 \times 48 \times 80}{30 \times 32 \times 48} = 4$$

It may be observed that the whole train might be replaced by a gear 39.07 in dia., meshing with a pinion 9.76 in dia., without changing the distance between the first and last shafts, but the sizes of these gears would in many cases be prohibitive.

As to the sense of rotation, it will be evident that for one contact (two wheels) between spur gears, the sense is reversed; for two contacts it is the same, for three reversed, etc., i.e., if the number of contacts is odd the first and last wheels turn in opposite sense and vice versa. Each contact with an annular gear neutralizes a contact with spur gears in respect to the sense of rotation, and if at any place between gears of the train a belt and pulley are used, then an open belt produces the same effect as an annular gear and pinion, and a crossed belt the same effect as a spur gear and pinion.

It not infrequently happens that in a compound train the two wheels on an intermediate axle are made the same diameter and combined into one. Thus we may make $r_a = r_b$, i.e., $t_a = t_b$. Such a wheel is then called an *idler*, and inspection of the formula shows that such an idler has no effect upon R, and is used solely to change the sense of rotation or to increase the distance between the axes of the other wheels without at the same time increasing their diameters.

We shall now solve a few problems illustrating the use of the formulas:

(1.) A wheel of 144 teeth drives one of 12 teeth, on a shaft which makes one revolution in 12 secs., while a second shaft driven by it makes a revolution in 5 secs. On the latter shaft is a 40 in. pulley connected by a crossed belt with a 12 in. pulley, this latter pulley making 2 revs., while one geared to it makes 3 revs. Show that the ratio of the train is 144, and that the first and last wheels turn in the same sense where no annular gears are used.

(2.) It is required to arrange a train of gears giving a ratio $\frac{13}{250}$. Remembering that $R = \frac{\text{product of teeth in drivers}}{\text{product of teeth in driven wheels}}$

we might use directly a gear of 250 teeth to drive a pinion of 13 teeth; but if this gear is too large we may break up the ratio R thus: $R = \frac{250}{13} = \frac{5 \times 5 \times 10}{13} = \frac{5}{1} \times \frac{5}{4} \times \frac{40}{13}$ or $t_1 = 60$, $t_a = 12$, $t_b = 20$, $t_c = 16$, $t_d = 40$, and $t_2 = 13$, giving gears without large numbers of teeth. If the distances between centres are given, then we must either arrange the diametral pitch to suit, or we must select some of the large number of other values of t_1, t_a, etc., which will fit the above case. The above solution gives six wheels, but we might use eight or four as well.

(3.) To design a train of gears which would be suitable for connecting the second hand of a watch to the hour hand. Here $R = 720$, and the last wheel must turn in the same sense as the first one, and hence the number of contacts must be even, requiring 4 or 8 or 12, etc., wheels in the train. As before, many solutions are possible, thus

$$R = \frac{720}{1} = \frac{4 \times 4 \times 5 \times 9}{1} = \frac{56}{14} \times \frac{48}{12} \times \frac{50}{10} \times \frac{108}{12} \text{ or}$$

$$R = \frac{720}{1} = \frac{6 \times 6 \times 4 \times 5}{1} = \frac{72}{12} \times \frac{60}{10} \times \frac{52}{13} \times \frac{60}{12}$$

with 8 wheels. The solution may also be worked for 12 wheels if desired.

(4.) The train of gears for connecting the minute and hour hands of a clock is required. Here the train is reverted with $R = 12$, and we must have an even number of contacts, so that we shall select four wheels. In addition to obtaining the desired ratio, we must have $r_1 + r_a = r_b + r_2$, and if all the wheels have the same pitch, $t_1 + t_a = t_b + t_2$.

Now, $R = \frac{12}{1} = \frac{4 \times 3}{1 \times 1} = \frac{48}{12} \times \frac{45}{15}$, thus the intermediate shaft would have the gears with 12 and 45 teeth, while the 48-toothed wheel would be connected to the hour hand, and the 15-toothed wheel to the minute hand.

THE SCREW-CUTTING LATHE

Most lathes are arranged for the cutting of threads on a piece of work, and as these form an interesting application of the principles already described, we shall use it as an illustration.

The general arrangement of the headstock of a lathe is shown in Fig. 62, in which the back gear is omitted to avoid complication. The cone C is connected by belt to the source of power, and is secured to the spindle S, which carries the live centre and also the chuck for driving the work, so that S turns at the same rate as C. To the end of S is a gear e, which drives a gear h through one idler g or two idlers f and g. The shaft which carries h aso has a gear i, which is keyed to it, and must turn with the shaft at the same speed as h. The gear i meshes with a pinion a on a separate shaft, this pinion being also rigidly connected to and revolving with gear b, which latter gear meshes with the wheel 2, which is keyed to the leading screw L. Thus the spindle S is geared to

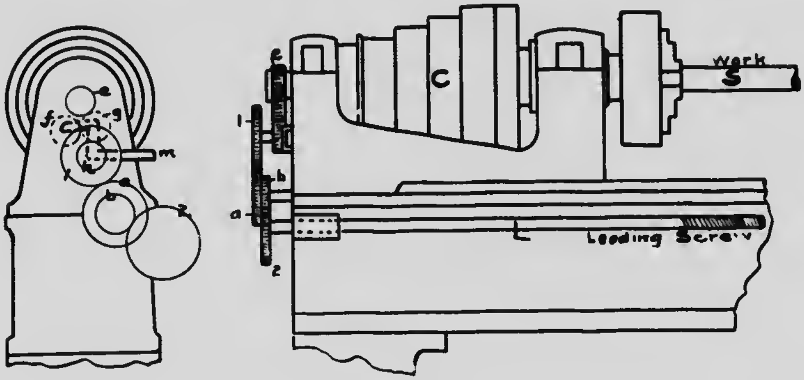


Fig. 62

the leading screw L through the wheels e, f, g, h, 1, a, b, 2, of which the first four are permanent, and the latter four may be changed to suit conditions, and are called *change gears.*

The work is attached between the centre on S and the centre on the tail stock, and is attached to S so that it rotates with it. The leading screw L passes through a nut in the carriage carrying the cutting tool, and it will be evident that for given gears on 1, a, b, 2 a definite number of turns of S correspond to a definite number of turns of L, and hence to a certain horizontal travel of the carriage and cutting tool. Suppose now that we wish to cut a screw on the work having s threads per inch, the number of threads per inch l on the leading screw being given. Then it will be clear that while the tool travels one inch horizontally corresponding to l turns of the leading screw L, the work must revolve s times, or if n_1 represents the revs. per min. of the work, and n_2 of the leading screw, we have

$$R = \frac{n_2}{n_1} = \frac{l}{s} = \frac{t_e}{t_h} \cdot \frac{t_1}{t_a} \cdot \frac{t_b}{t_2} = \frac{t_1}{t_a} \cdot \frac{t_b}{t_2}$$

where t_e, t_h, t_1, t_a, t_b, and t_2 are the teeth in the wheels *e*, *h*, 1, *a*, *b* and 2 respectively, the idlers *f* and *g* having no effect on the velocity ratio, and we are considering the common case where $t_e = t_h$. If, further, L and S turn in the same sense the thread cut on the work will be right hand, that on the leading screw being right hand, and vice-versa.

The idlers *f* and *g* are provided to facilitate this matter, and if a right hand thread is to be cut, the handle *m* carrying the axes of *f* and *g* is moved so that *g* alone connects *e* and *h*, while, if a left hand thread is to be cut the handle is depressed so that *f* meshes with *e* and *g* with *h*. The figure shows the setting for a right hand thread.

An illustration will show the method of setting the gears to do a given piece of work. Suppose that a lathe has a leading screw cut with 4 threads per inch, and the change gears have respectively 20, 40, 45, 50, 55, 60, 65, 70, 75, 80 and 115 teeth.

(1) It is required to cut a right hand screw with 20 threads per inch. We have $\frac{l}{s} = \frac{t_1}{t_a} \cdot \frac{t_b}{t_2}$ where $l = 4$ and s is to be 20.

$$\text{Thus} \quad \frac{t_1}{t_a} \cdot \frac{t_b}{t_2} = \frac{4}{20} = \frac{1}{5}$$

This ratio may be satisfied by using the following gears $t_1 = 20$, $t_a = 50$, $t_b = 40$ and $t_2 = 80$. Only the one idler *g* would be used to gi[illegible]ight hand thread.

[illegible] a standard thread on a 2 in, gas pipe in the lathe. The [illegible]umber of threads here would be 11½ per in. and hence $l =$ [illegible] 11½ and $\frac{t_1}{t_a} \cdot \frac{t_b}{t_2} = \frac{4}{11\frac{1}{2}} = \frac{8}{23}$. Here we could make it $t_1 = 40$ and $t_2 = 115$, if we made $t_b = t_a$ or replaced both by an idler.

(3) If we required to cut 100 threads per inch then $l = 4$, $s = 100$ and $\frac{t_1}{t_a} \cdot \frac{t_b}{t_2} = \frac{4}{100} = \frac{1}{25}$, and we may divide this into two parts, thus, $\frac{1}{25} = \frac{1}{4} \times \frac{1}{6\frac{1}{4}}$, so that if we make $t_1 = 20$, $t_a = 80$, $t_2 = 75$, we should have to have an extra gear of 12 teeth to take the place of *b*, as $t_b = 12$.

The axle holding the gears *a* and *b* may be changed in position so that to make these gears fit in all cases between 1 and 2. The details of the method of doing this are omitted in the drawing.

When odd numbers of threads are to be cut various artifices are resorted to, sometimes only approximations being employed. For example, the number of threads per inch commonly used on a $1\frac{1}{4}$ in., gas pipe is $11\frac{1}{2}$, but no serious trouble would ordinarily result if we had to cut it in a lathe in which the nearest number of threads would be $11\frac{1}{4}$ per in. There are cases, however, in which certain exact threads of very odd pitches must be cut, and one example will be given to show how such a case may be solved.

Let it be required to cut a screw with an exact pitch of one millimeter, the leading screw on the lathe having 8 threads per in (1 mm. = .0393708 in.). This is worked out by a series of approximations by the method of continued fractions, the exact value of R for the case being $\frac{1}{R} = \frac{1}{8} \times \frac{1}{.0393708}$.

The first approximation is 3, the real value being $3\frac{68876}{393708}$.

The second approximation is $3 + \frac{1}{5}$, the real value being

$$3 + \cfrac{1}{5\frac{49328}{68876}}$$

and proceeding in this way we find the third, fourth, fifth, sixth, etc. approximations, the sixth being

$$3 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + 1}}}} = 3\frac{7}{40} \text{ or } \frac{127}{40}$$

Thus the sixth approximation gives $\frac{1}{R} = \frac{127}{40}$ or $R = \frac{40}{127}$. (It is worthy of note that $\frac{1}{8} \times \frac{1}{.0393708} = 3.17494$ while $\frac{127}{40} = 3.175$) so that this screw could be cut with great exactness by the use of the ratio $\frac{40}{127}$ between the work and leading screw.

Many problems of similar nature occur in practice, all of which may be solved by this method.

Hunting tooth gears have now almost disappeared, but were formerly much used by millwrights who thought that more evenness of wear resulted when a given pair of teeth in two gears came in contact the least number of times. Suppose we had a velocity ratio $R = 1$, and a pair of gears had 80 teeth each, then a given tooth on one gear would come in contact with the same tooth on the other gear at each revolution, but if we place 81 teeth in one gear, leaving the other with 80 teeth, then the ratio R is $\frac{81}{80}$ which differs very little from the value desired, but a given tooth on one gear will only come in contact with a certain tooth on the other when one of the wheels has made 80 revs. and the other 81 revs. This may be compared with the case where the numbers of teeth are 12 and 13.

EPICYCLIC GEARING

An epicyclic train of gears has already been defined as one in which one of the wheels is held stationary and at least one other gear revolves about it. The frame carrying the revolving gear must also revolve. The train is called epicyclic because a point on the revolving gear describes epicyclic curves. This arrangement is in very common use where a very low velocity ratio is to be obtained without an unduly large number of gears; thus a ratio of $\frac{1}{10,000}$ may readily be obtained by the use of four gears, the largest one

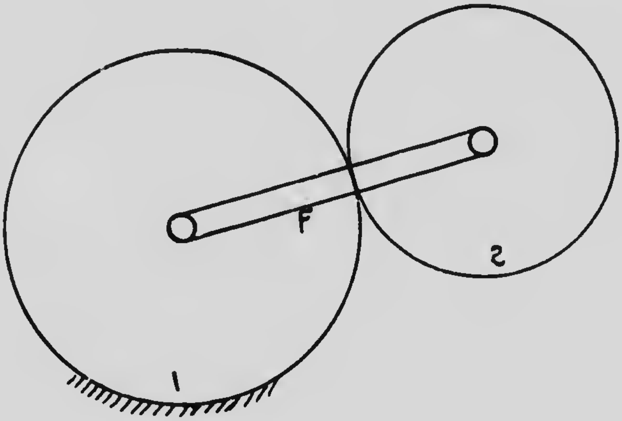


Fig. 63

having only 101 teeth. The train may have many or few wheels, but it usually contains not over four. We shall start with the simplest case of two wheels.

In Fig. 63 let 1 and 2 represent gear wheels of radii r_1 and r_2

and teeth t_1 and t_2 respectively, F being the frame which is attached by a pin bearing to both wheels. Now if we hold the frame stationary the ratio of the train would be $R = \frac{r_1}{r_2} = \frac{t_1}{t_2}$, and is negative *i. e.*, the first and last wheels turn in opposite sense. If now we fix the wheel 1 so that it cannot revolve, and turn the frame F about the pin connection to 1, we would have the gear 2 revolving about its bearing on the frame and the train would be called epicyclic, and the ratio, E, of the train would be the number of turns of the last wheel 2, per turn of the frame F. To find E we may first assume that the frame and both wheels are rigidly connected together like one solid body, then turn the whole machine about the axis between F and 1, that is, wheel 1 gets one revolution as do also the frame F and wheel 2. But in the operation the wheel 1 is to remain at rest, we therefore revolve it *back* one revolution without disturbing the frame, and during this operation the wheel 2 turns *forward* R revolutions because these wheels revolve in the opposite sense.

During the complete motion above described, the frame has revolved one revolution, the first wheel has revolved one revolution and then back again, *i.e.*, the net result is that it has not moved at all while the last wheel has turned $1 + R$ revolutions, so that the ratio of the train $E = \frac{1 + R}{1} = 1 + R$. If the train had three wheels or if the number of contacts between the toothed wheels were even, then R would be positive and the ratio would be $E = 1 - R$. In fact this latter formula is the general one and R is positive or negative according to whether the last wheel in the train would revolve in the same or opposite sense of the first wheel if the frame were fixed.

The following method for obtaining E may appeal to some, the ratio R being here taken as positive, *i. e.*, the number of contacts are even. Let us first assume that the frame is fixed and all wheels revolve as in the ordinary train, then we may set down the results as follows:

Frame fixed.	— 1 rev. added to each part.
Turns made by frame = 0 revs. First wheel turned through 1 rev. Last wheel must turn + R revs.	Turns made by frame = 0 — 1 = — 1 rev. First wheel turns 1 — 1 = 0 revs. Last wheel turns + R — 1 revs.

That is, after the last operation the first wheel has been returned to its position of rest, the frame has made — 1 rev. and the last wheel $R - 1$ revs., or the ratio $E = \frac{R-1}{-1} = 1 - R$.

A few examples will illustrate the case.

1. Let the frame have a wheel 1 with 60 teeth, an idler and a wheel 2 with 59 teeth; to find the ratio of the train when wheel 1 is fixed.

$$\text{Here } R = +\frac{t_1}{t_2} = +\frac{60}{59} \therefore E = 1 - R = 1 - \frac{60}{59} = -\frac{1}{59},$$

or the wheel 2 will revolve in opposite sense to the frame and at $\frac{1}{59}$ the speed.

$$\text{If wheel 2 had been fixed } R = +\frac{59}{60} \therefore E = 1 - \frac{59}{60} = +\frac{1}{60}$$

or the wheel 1 would turn in the same sense to the frame and at $\frac{1}{60}$ of the speed.

2. Design an epicycle train giving a ratio of $\frac{1}{10000}$, the last wheel to turn in the same sense as the frame. Here $E = +\frac{1}{10000} = 1 - R$ if there are an even number of contacts. Hence $R = 1 - \frac{1}{10000} = \left(1 - \frac{1}{100}\right)\left(1 + \frac{1}{100}\right) = \frac{99}{100} \times \frac{101}{100}$, so that fixed wheel should have 99 teeth, the two wheels on the intermediate shaft 100 (gears with the fixed wheel) and 101 and the last wheel would have 100 teeth.

In practice such a train could be readily reverted because the diameters of all gears could be made equal without seriously affecting the teeth, and we should then have the arrangement sketched in Fig. 64, which shows a practical form of the drive, the belt wheel being the frame and running 10000 times as fast as the slow speed shaft. The pulley A is a running fit on the shaft B which shaft is keyed to the support C and also to the gear with 99 teeth. The gears are loose on the pins D, while the 100 toothed gear is keyed to the slow speed shaft.

3. The Weston triplex pulley block contains a further example of the epicyclic train, and for the sake of simplicity only the essential parts are illustrated in Fig. 65. The frame D contains bearings

which carry the hoisting sprocket wheel *F* and to the casting carrying the hoisting sprocket are axles each carrying a pair of compound gears *BC*, the smaller one *C* of which gears with an annular wheel made as part of the frame *D*, while the other and larger gear of the pair meshes with a pinion *A* attached to the end of the shaft *S* carrying the hand chain sprocket *H*. When a workman pulls on the hand chain he revolves correspondingly the sprocket *F* and hence the pinion *A* on the end of the shaft, which in turn sets the compound gears *BC* in motion. As one of the compound gears C meshes

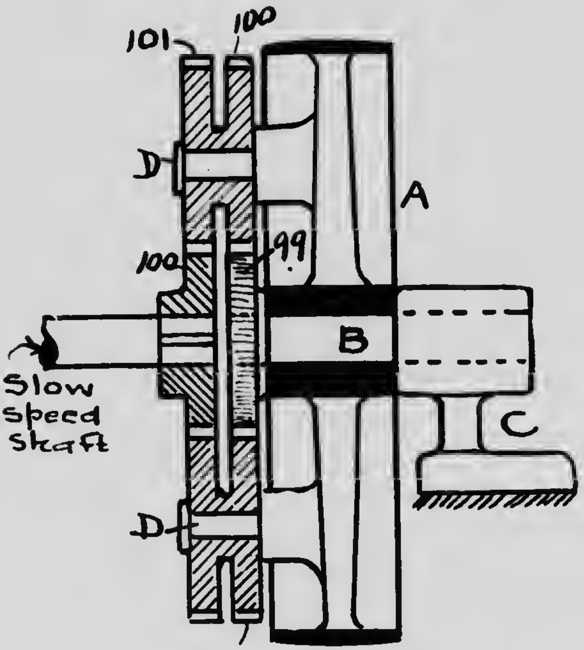


Fig. 64

with the annular wheel in the frame, the latter wheel being stationary, the only possible action is for the axles of the compound gears to revolve in a circle carrying the hoisting sprocket with them.

In a one ton Weston triplex block the annular gear on the frame has 49 teeth, while the two gears, *B* and *C* have respectively 31 teeth and 12 teeth there being 13 teeth in the pinion *A* on the hand wheel shaft. The hoisting wheel is $3\frac{1}{8}$ in. diam., while the hand wheel is $9\frac{3}{4}$ in. diam. To find the pull on the hand chain to lift one ton, neglecting friction:

In this case $R = \frac{49}{12} \cdot \frac{31}{13} = 9.73$ and is negative, as one of the wheels is annular. Hence $E = 1 - R = 1 - (-9.73) = 10.73$, so that the hand wheel turns 10.73 revs. for one rev. of the hoisting wheel, and hence for each foot the load is lifted the hand chain must be moved $10.73 \times \frac{9\frac{3}{4}}{3\frac{1}{8}} = 33.2$ ft. Or the pull on the hand chain to lift one ton, neglecting friction, would be $\frac{2000}{33.2} = 60$ pounds. (Note—In the actual case friction would raise this probably to 80 pounds or more)

4. A form of motor driven portable drill is shown at Fig. 66 in which the gears are worked on this principle. Here, again, only

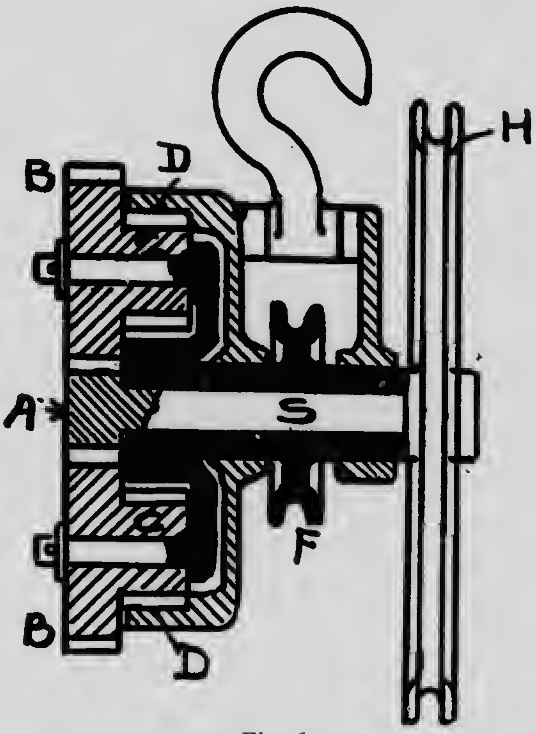


Fig. 65

the barest outlines are shown as the actual construction is rather complicated. The machine is very well made and fitted with ball bearings throughout instead of the plain bearings shown. The tool is called the Duntley drill and is made by the Chicago Pneumatic Tool Co.

The outer casing of the machine, Fig. 66, is held stationary by the two handles shown and contains two motors driven by current brought in through one of the handles. Each of the motors has a pinion *C* attached to it which meshes with the larger gear *D* of a pair of compound gears which latter rotate freely on a central

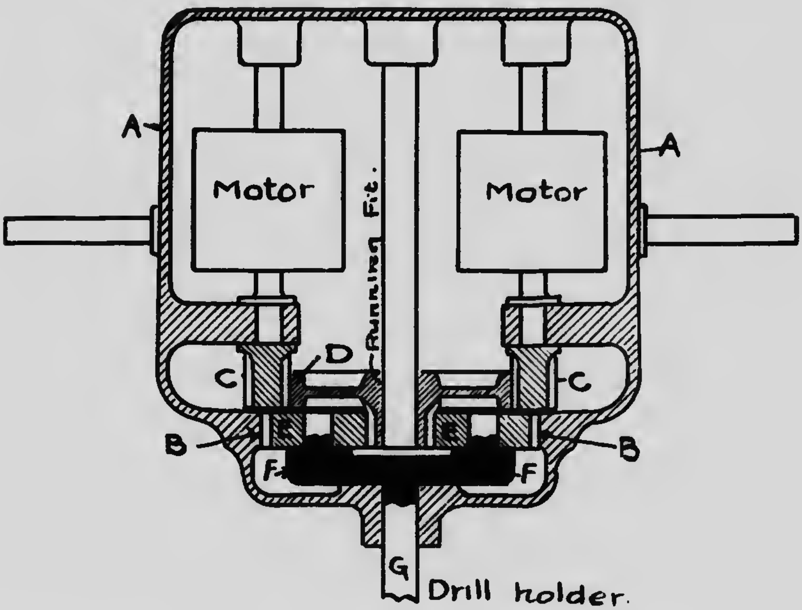


Fig. 66

shaft as indicated. The smaller gear of the pair meshes with two gears *E* carried on axles on which they run freely mounted on the piece *F* shown black in the figure, this latter piece carrying the socket for the drill which is to be driven. The two gears also mesh with an annular wheel B, forming part of the frame and thus remaining stationary whether the motors run or not. When the motors are driven the compound gear is driven by the pinion attached to each motor. The compound gear drives the gear *E* and hence the part shown in black is caused to rotate.

CHAPTER VIII.

CAMS

In machinery there are many motions which are more or less irregular and which are not uniform. Take for instance the belt shifter on a planer, which remains stationary during the main part of the stroke of the table, and then moves quickly at the end of the stroke and again comes to rest; or the exhaust valve of a gas engine which is first quickly opened, then held stationary, and then returned to its closed position, or again the needle bar of a sewing machine, the motion of which is well known and is also not uniform. In such cases of non-uniform motion as have been described, we usually have to obtain the driving power from some shaft or other link moving at uniform velocity, such as the countershaft for a planer or the lay-shaft of a gas engine, or the main shaft of a sewing machine. Now, since the one part of the machine moves non-uniformly, deriving its motion from some other part which has uniform motion, hence at least one of the connecting links between them must be more or less irregular in shape, and the whole irregularity is generally confined to one piece which is frequently fastened to the rotating shaft, or other part of the machine having uniform motion, this piece being commonly called a cam. Thus a cam is in many cases a disk of non-circular shape which is secured to a shaft running at uniform speed, the shape of the cam being such as to impart any desired non-uniform motion by suitable mechanism to any other link.

While cams are usually secured to rotating shafts, yet this is not necessarily the case, and many cams are made in the form of sliding plates, as in some forms of planers.

It will be found most simple to describe the construction and action of cams by a few illustrations so that certain typical cases will now be given.

The first illustration chosen will be the case of the stamp mill as used in mining districts for crushing ores. Let Fig. 67 represent an outline of such a machine, consisting of several stamps A which are merely heavy pieces of metal, and in the operation of the mill these stamps are to be lifted in some way to a desired height and then allowed suddenly to drop so as to crush the ore below them. The power to lift the stamps is supplied through a shaft B which is driven at constant speed by a motor or belt, and in this case much freedom is allowed in the method of lifting the stamps, the necessary condition being that they shall be lifted with the least amount of energy and then be suddenly released so that they drop freely under the action of gravity alone.

Let w be the weight of a stamp, the mass of which is $m = \frac{w}{g}$, then the force which must be applied to lift the stamp is $P = mf + wh$

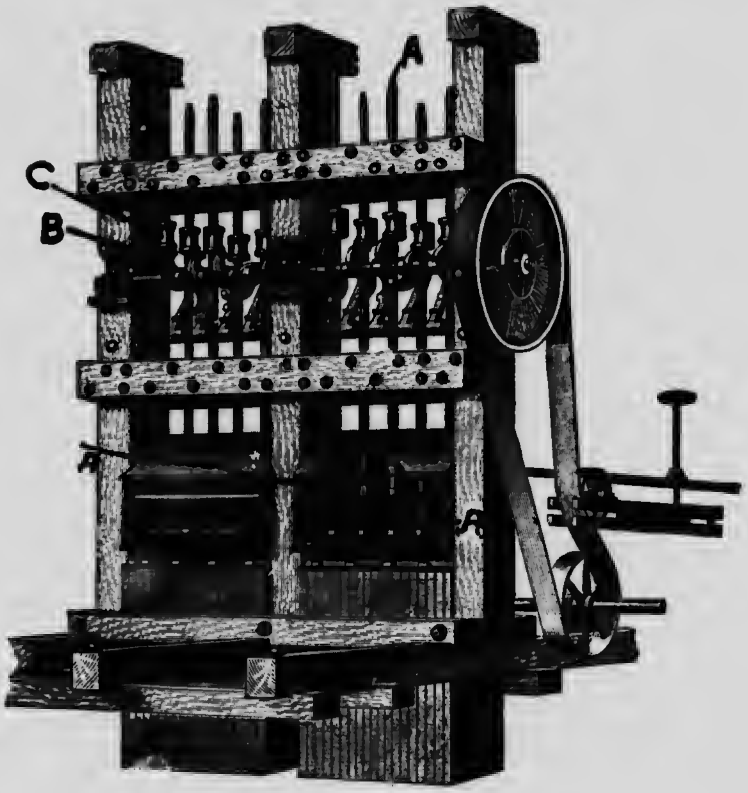


Fig. 67

where f is its acceleration, while rising, h being the height through which the stamp is lifted. and this force P will be a minimum in a given case for minimum value of the acceleration f, which will occur when the velocity is kept constant as the acceleration is then zero. Now supposing that a collar C is attached to the stamp A, and that to the shaft B is keyed a *cam* D, it is desired to find the proper shape of the cam to give A a constant velocity. In solving this problem the usual construction will be followed, in which the stamp A is lifted twice for each revolution of B, there being thus two similar cams attached on opposite sides of B, these being so designed as to lift the stamp while the shaft turns less than 180°, say 102°. Let the lift of the stamp A due to the cam be h ft.

In order to solve the problem, draw to a very much enlarged scale the limiting positions of the collar C and also the centre line of the shaft B, as shown at Fig. 68. Divide the lift h into any number of equal parts, say six, numbered, 1, 2, 3, 4, 5 and 6 in the figure, then

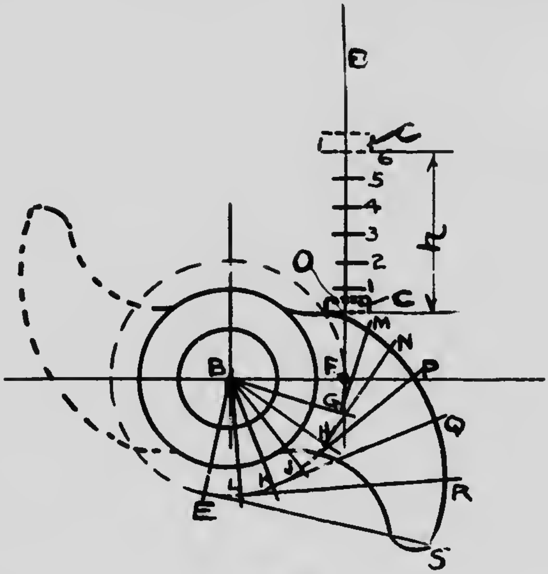


Fig. 68

since the velocity of the stamp is to be constant, each of the distances 0-1, 1-2, 2-3, etc., must be passed through in the same interval of time, i.e., the angles turned through by B will be equal for each of the spaces 0-1, 1-2, etc., each angle being equal to $\frac{102}{6} = 17°$.

Draw BF perpendicular to the line of motion of the stamp and lay off the angle $FBE = 102°$, dividing the latter into the same number of parts as the height h, viz., six, marked respectively FBG, GBH, etc. With centre B and radius BF draw a circle FGH- - - E tangent to the line of motion 0-6 of the stamp, and also draw GM, *HN*, etc., tangent to the circle at *G*, *H*, etc. Now while the stamp is being lifted from 0 to 1, the shaft *B* will have turned through the angle *FBG* and hence the line *GM* will then be vertical, so that it should be long enough to reach from *F* to 1, or *GM* should equal *F1*. Similarly, make $HN = F2$, $JP = F3$, etc., thus the points *OMNPQ RS* may be at once located and the outline of the cam drawn through them. As a guide in drawing the cam it is to be remembered that at any point such as *Q* the line *QK* is a normal to the curve of the cam.

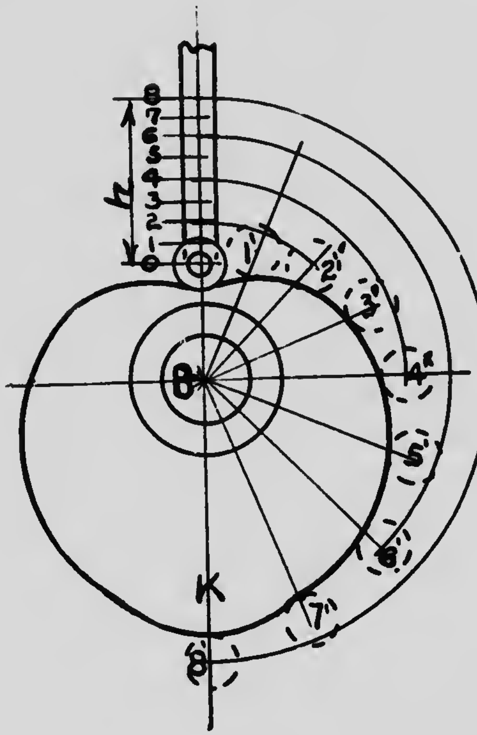


Fig. 69

A hub of suitable size is now drawn on the shaft, the dimensions of the hub being determined from the principles of machine design, and short curves drawn from *S* and *O* down to this hub, the curve from *S* must be so constructed as to let the stamp fall freely without striking it.

A little consideration will show that the curve *OMN - - - S* is an involute having the base circle *FGH - - - E*, i.e., the curve of the cam is that which would be described by a pencil attached to a cord on a drum of radius BF if this cord were unwound and the string were kept tight. The dotted line shows the other half of the cam.

It will be noticed that in this cam there is line contact, that is, there is higher pairing and the part coming in contact with the cam and receiving motion from it is called the *follower*.

As a second illustration, take a problem similar to the latter, except that the follower is to have a uniform velocity on the up and down stroke and its line of motion is to pass through the shaft B. It will be further assumed that a complete revolution of the shaft will be necessary for the complete up and down motion of the follower.

Let 0-8, Fig. 69, represent the travel of the follower, the latter being on a vertical shaft, with a roller where it comes in contact with the cam. Divide 0-8 into say eight equal parts as shown, further,

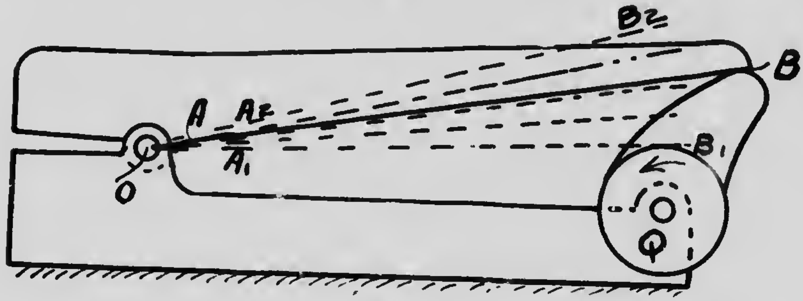


Fig. 70

divide the angle OBK (= 180°) into the same number of equal parts, eight, giving the angles $OB1'$, $1'B$, $2'$, etc. Now since the shaft F turns at uniform speed we will have the follower at 1 when $B1'$ is vertical and at 2 when $B2'$ is vertical, etc., hence it is only necessary to revolve the lengths $B1$, $B2$, etc., about B till they coincide with the lines $B1'$, $B2'$, etc., respectively, when the points $1'$, $2'$, $3'$, will be obtained on the radial lines $B1'$, $B2'$, etc., as the distances from B which the follower must have when the corresponding line is vertical. With centres $1'$, $2'$, $3'$, etc., draw circles to represent the roller and the heavy line shown, dd. Tangent to these will be the proper outline for one half of the cam, the other half being exactly the same as this about the vertical centre line. Here again we have higher pairing and some external force is supposed to keep the follower always in contact with the cam.

In many cases the position is known in which it is desired to have the follower at a certain time, and the cam may be designed to suit any conditions of this nature. Suppose it is required to design a cam for a shear of the type shown in Fig. 70, and that from the nature of the work which it has to do the various positions of the line AB, which comes in contact with the cam, are known for different positions of the revolution of the latter. For somewhat over one-half of the revolution let its position be A_1B_1 then let it rise uniformly from A_1B_1 to A_2B_2 during 120° of the cam's motion. It is required to design the cam.

In the enlarged drawing in Fig. 71, is shown the centre Q, also the extreme positions A_1B_1 and A_2B_2 of the line AB. Draw

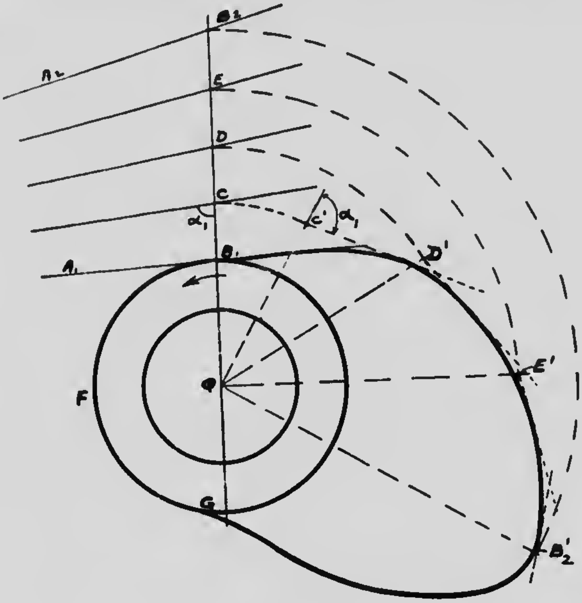


Fig. 71

for convenience the vertical line QB_1B_2 and lay off the angle $B_1QB_2{}'$ = 120°, then the semi-circle B_1FG is one-half of the cam. Now divide the angle B_1OB_2, Fig. 70, into any convenient number of parts, say four, by the lines OC, OD and OE and divide the angles B_1QB_2 into the same number of parts by the lines QC', QD' and QE'. Now while the cam turns through the angle B_1QC' the arm is rising to OC so that the line OC must be a tangent to the cam when OC' is vertical, the arm for this position moving upward. The outline of the cam may be readily drawn in as follows: Lay off $QC' = QC$ and through C' draw a line at angle α, to QC', the lines so drawn will be a tangent to the cam.

In the same way draw a line at D', making the same angle with the corresponding radius from Q that OD makes with QD, in this way obtaining another tangent to the cam. By carrying this construction out for a number of points a set of tangents are readily obtained

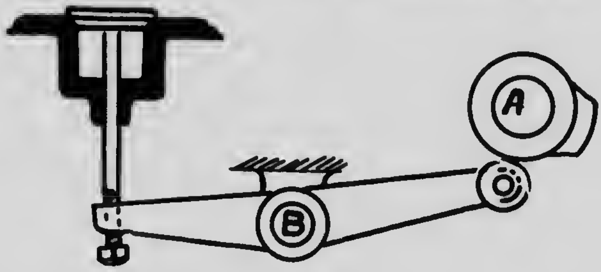


Fig. 72

and the outline of the cam may then be drawn as shown. By a similar process the back of the cam may be designed so as to again lower the arm in any desired manner.

Cams may be drawn to suit any given set of conditions by following out the method explained above.

In many cases such as in gas engines, the follower moves in the arc of a circle, a sketch of such a cam and follower as used being shown in Fig. 72, in which case allowance must be made for the deviation of the follower from a radial line. A form of cam used very frequently on screw machines is made by attaching by screws a bar of metal of any required shape to the cylindrical surface of a pulley or drum. In such a case the follower moves axially across the face of the drum and such an arrangement possesses considerable merit for the class of work on which it is used, because it is easy to alter the cams to suit new work by merely taking out the screws, removing the bar from the surface of the drum, and screwing a new piece on in its place. Very many other forms of this contrivance exist, but further details will not be given at this place as the reader will find frequent examples in the ordinary work of designing.

The work along this line will be concluded by a very general example of useful nature. In the most general case there is a certain operation to perform and it should be accomplished with the minimum expenditure of energy and shock to the parts. To take a definite example, suppose it is desired to design a cam to operate the exhaust valve of a gas engine. In such a case just before the valve is raised the pressure upon it due to the gas in the cylinder is great, and immediately after the valve is opened the pressure drops almost to that of the atmosphere. Now the desirable condition in the opening of

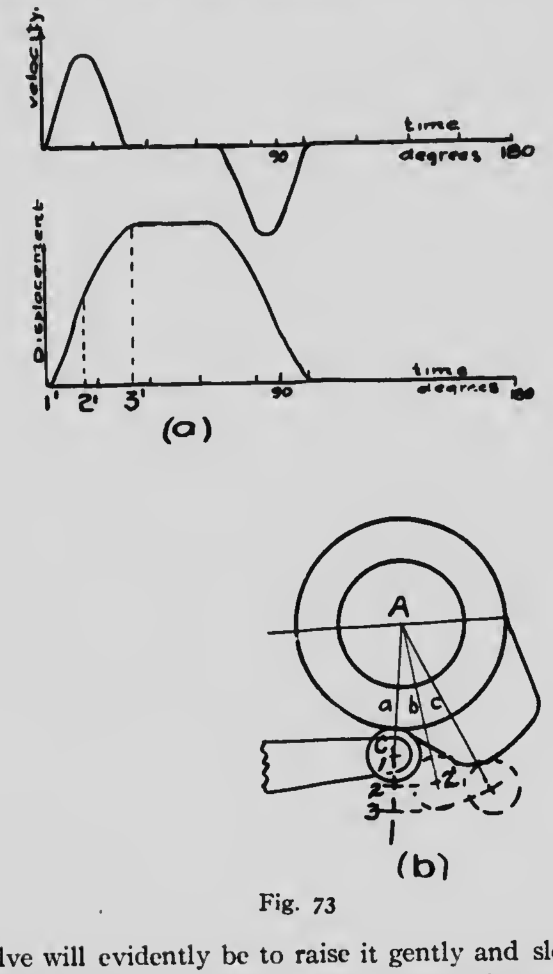


Fig. 73

such a valve will evidently be to raise it gently and slowly until it has opened sufficiently to release to some extent the pressure acting on it, the valve should then be opened very quickly to its extreme position and held in this position till the exhaust stroke is almost completed. At this point the valve should be rapidly lowered, but with decreasing velocity, upon its seat again, which final position it should attain without undue shock, i.e., its velocity should not be high at the instant just before it comes down to the valve seat.

For such a case the velocity-time curve can be assumed and will take a form similar to that shown at (a), Fig. 73, it being assumed in a given case that enough information is given about the motion to draw the velocity-time curve. Since the valve must open and close while the cam-shaft turns less than one-half a revolution the cam must occupy less than one-half the circumference.

From the velocity curve the corresponding space curve may be drawn by integration.

In Fig. 73 (b) is shown the lay shaft *A* to which the cam is attached, the fulcrum about which the exhaust valve lever swings not being shown, the follower in this case is provided with a roller *C*. The highest position of the roller is at 1, and two other positions are indicated by 2 and 3, these corresponding to the points 1', 2' and 3' on the space diagram. The remainder of the process is exactly the same as that illustrated in Figs. 70 and 71, except that in the present case the follower is a roller while in the former case it was a straight line.

The complete construction for the point 3 is shown, this point corresponding to about $\frac{1}{20}$ rev. of the cam shaft, i.e., to a turn of 17°. The angle *aAb* is made equal to 17°, and by the conditions of the problem, when the cam shaft has turned through 17° from the position *Aa*, i.e., when the radius *Ab* coincides with the line *Aa*, the follower will have moved from the position 1 to the position 2. Hence revolve *A*2 about *A* to *A*2' making *A*2' equal to *A*2, then drawing the roller about the centre 2' gives one position of the follower to which the cam must be tangent. The whole cam may be completed by a similar process.

CHAPTER IX.

FORCES ACTING IN MACHINES

When a machine is performing any useful work, or even where it is at rest there are certain forces acting on it from without, such as the steam pressure on an engine piston, the belt pull on the driving pulley, the force of gravity due to the weight of the link, the pressure of the water on a pump plunger or the pressure produced by the stone which is being crushed in a stone-crusher. These forces are called *external* because they are not due to the motion of the machine, but to outside influence, and these external forces are transmitted from link to link, producing pressures at the bearings and stresses in the links themselves. In problems in machine design we must know the effect of the external forces in producing stresses in the links, and further what the stresses are and what forces or pressures are produced at the bearings, for the dimensions of the bearings and sliding blocks depend to a very large extent upon the pressures they have to bear, and the shape and dimensions of the links are determined by these stresses.

The matter of determining the sizes of the bearings or links does not concern us at present, but we shall find it convenient to determine the stresses produced and leave to the machine designer the work of making the links, etc., of proper strength.

In most machines only one part travels with uniform motion, for example an engine crank shaft, or the belt wheel of a shaper or planer, many of the other parts moving at variable rates from moment to moment. If the links move with variable speed then they must have acceleration and a force must be exerted upon the link to overcome this. This is a very important matter as the forces required to accelerate the parts of a machine are often very great, but we shall leave the consideration of this question to a later chapter, and shall for the present neglect the weight of each link and its acceleration and deal with a mechanism as if it consisted of light, strong parts which although they require no force to accelerate them, are yet strong enough.

It will be further assumed that at any instant under consideration, the machine is in equilibrium, that is, no matter what the forces acting are, they are balanced amongst themselves, or the whole machine is not being accelerated. Thus, in case of a shaper, certain of the parts are undergoing acceleration at various times during the motion, but as the belt wheel makes a constant number of revolutions per minute, there must be a balance between the resistance due to the cutting and friction on the one hand and the power brought in by the belt on the other. In the case of the locomotive which is just starting up, the speed is steadily increasing and the locomotive is being accelerated, which simply means that more energy is being supplied through the steam than is being used up by the train, the balance of the power being free to produce the acceleration, and the forces acting are not balanced. When, however, the train is up to speed and running at a uniform rate, the input and output must be equal, or the locomotive is in equilibrium, the forces acting upon it being balanced.

The most general form of problem of this kind which comes up in practice is such as this: given the force required to crush a piece of rock in a crusher, what belt pull will be required for the purpose? or: what turning moment will be required on the driving pulley of a punch to punch a given hole in a given thickness of plate? or: given an indicator diagram for a steam engine, what is the resulting turning moment produced on the crank shaft?, and many other similar and useful problems. Such problems may be solved in two ways (a) by the use of the virtual centre (b) by the use of the phorograph, and as both methods are instructive they will both be discussed briefly.

SOLUTION OF THE PROBLEM BY THE USE OF THE VIRTUAL CENTRE

This method depends on the following fundamental principles which should be understood and proved by the reader. If any set of forces act upon a link of a machine, then there will be equilibrium, provided (1) that the resultant of all the forces is zero, (2) that the resultant is a single force passing through a fixed permanent centre, because if the force pass through a point at rest its action will simply be resisted by a stress in the frame, and (3) that, if the resultant is a single force, the latter passes through a point on the link which is for the moment at rest.

Let a set of forces act on any link b, then there will be equilibrium provided the resultant is a single force passing through bd, d being the fixed link and bd either a permanent or virtual centre. If the resultant of the forces is a couple then both forces of the couple must pass through *bd* which is only possible where *bd* is infinitely distant or where the link has a motion of translation. A few examples will explain these points fully.

Example 1. Three forces, P_3, P_2 and P_3 act on the link *b*, Fig. 74, under what condition will they be in equilibrium? Now P_1 and P_3

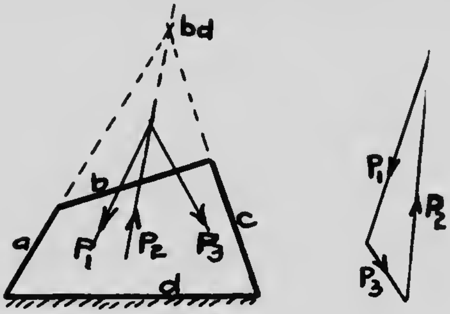


Fig. 74

intersect at *A* then if P_2 be treated as the balancing force, the latter must pass through *A* and also through the stationary point *bd*, so that its direction is known. The vector polygon on the right gives its magnitude, thus P_2 is to be as shown.

Example 2. Find the resistance P_2 which must be produced by the crank pin of an engine, Fig. 75, to balance the force P_1 on the piston,

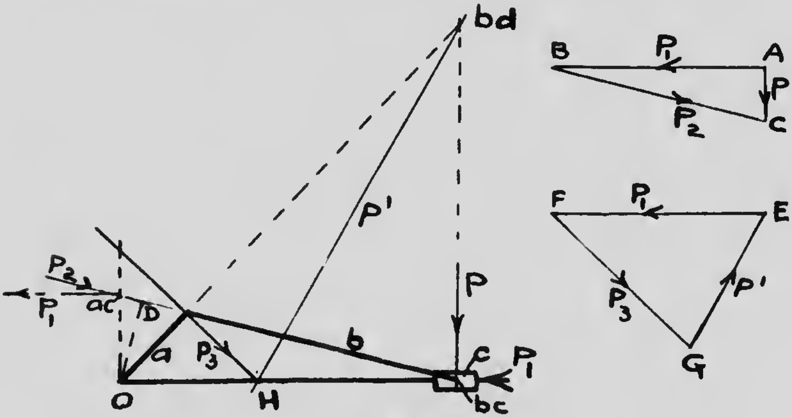


Fig. 75

In this case both P_1 and P_2 may be regarded as forces on *b* and these will intersect at *bc*, also their resultant must pass through the centre *bd* and also through *bc* and is thus known in direction and position.

In the diagram to the right draw $P_1 = A\ B$ to scale to represent the pressure on the piston, draw $A\ C$ parallel to P and $B\ C$ to P_2 then P_2 is given by the length $B\ C$. The moment of P_2 on the crank shaft is $OD \times P_2$, and it may readily be shown by geometry that this is equal to $P_1 \times Oac$, or the turning effect on the crank shaft due to P_1 is found in magnitude, direction and sense by simply transferring P_1 to ac.

Let the force P_3 act normal to a, to find its magnitude. Here P_3 and P_1 intersect at H and their resultant passes through H and bd. Draw the triangle EFG making EF equal to the known value of P_1 and FG and EG parallel respectively to P_3 D and bd — H. Then $FG = P_3$ and EG equals the resultant force P' of P_1 and P_3 and

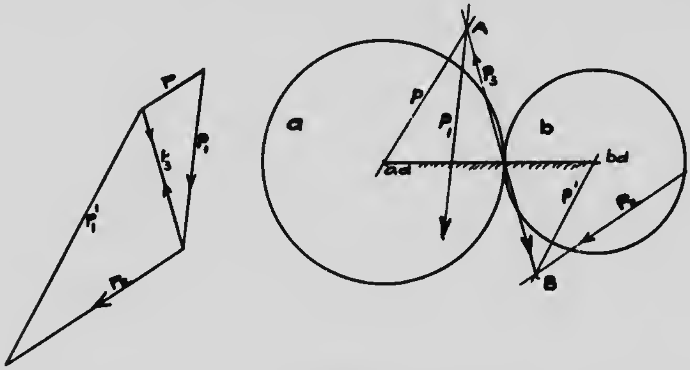


Fig. 76

the force P_3 is called the *crank effort*, being the force passing through the crank pih normal to a which just balances the steam pressure P_1.

Example 3. The direction of pressure between the teeth of a pair of gears is AB, Fig. 76, the pitch circles of which are shown, to find the relation between P_1 and P_2. In this case AB meets P_1 in A and P_2 in B, join A — ad and B — bd. The forces acting on a are now P_1 and the force P_3 due to the gear b and the resultant P of these must pass through ad so that the vector triangle gives P_3. Now for wheel b there are two forces P_3 and P_2 acting through B and their resultant P' must pass through B and bd, so that the figure on the left shows how the magnitudes of P, P_3, P' and P_2 are found by the vector triangles.

Example 4. Only one further example will be given here and for this the beam engine shown in Fig. 77 will be selected. It is required to find the turning effect produced on the crank shaft by a given pressure P_1 produced on the walking beam from the piston. Taking moments about cd, P_1 may be replaced by a corresponding force P_2 acting through bc parallel to P_1 where $P_1 \times C - cd = P_2 \times bc - cd$, P_2 and P_1 are in opposite senses and the procedure is now exactly the same as in example 1. If preferred, however, P_1 may be omitted and any point D on P_1 may be selected. Resolve the latter force into two components each passing through D, the one P_3 also passing through ac, while the other component P passes

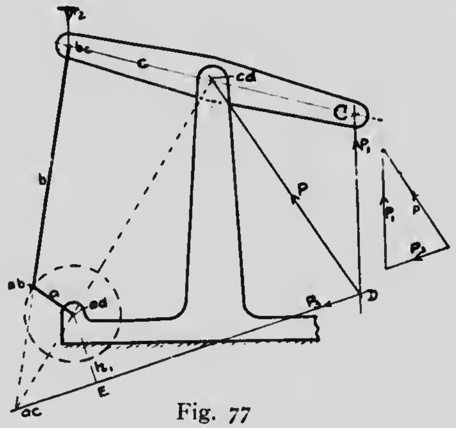


Fig. 77

through D and cd. As the point cd is stationary it is only the component P_3 which has any effect and as P_3 passes through ac a point a, on the problem is readily solved.

Draw $ad - E$ normal to P_3 from ad, then the moment of P_3 on a is $f_3 \times da - E = f_3 \times h_1$ which was required. Consideration of the matter will show that the result is independent of the position of D.

At present no further examples of this method will be given, although it will be applied later on and some further examples will be given then.

SOLUTION OF THE PROBLEM BY MEANS OF THE PHOROGRAPH

In solving such problems as are now under consideration by the use of the phorograph the matter is approached from a somewhat different standpoint, and as there is frequent occasion to use the method it will be explained in some detail.

It has already been pointed out that the present investigation deals only with the case where the machine under consideration is in equilibrium, or where it is not, on the whole, being accelerated. This is always the case where the energy put into the machine per second by the source of power is equal to that delivered by the machine, *e. g.*, where the energy per second delivered by a gas engine to a generator is equal to the energy delivered to the piston by the explosion of the gaseous mixture, friction being neglected.

Suppose now that on any mechanism there are a series of forces P_1, P_2, P_3, etc., acting on the various links, and that these forces are acting through points having the respective velocities v_1, v_2, v_3, etc., ft. per sec. in the directions of P_1, P_2, P_3. The energy which any force will impart to the mechanism per second is proportional to the magnitude of the force and the velocity with which it moves in its own direction, thus if a force of 20 pounds acts through a point moving at 4 ft. per sec. in the direction of the force, the energy imparted by the latter will be 80 ft. pds. per sec., and this will be positive or negative according to whether the sense of force and velocity are the same or different.

The above forces will impart respectively P_1v_1, P_2v_2, P_3v_3, etc., ft. pds. per sec. of energy, some of the terms being negative usually, and the direction of action of the various forces are usually different. The total energy given to the machine per second is $P_1v_1 + P_2v_2 + P_3v_3 +$ etc., ft. pds. and if this total sum is zero there will be equilibrium, since the net energy delivered to the machine is zero. This leads to the important statement that if in the machine any two points in the *same or different* links have identical motions then, as far as the equilibrium of the machine is concerned, a given force may be applied at either of the points as desired, or if at the two points we apply forces of equal magnitude and in the same direction but *opposite in sense*, then the equilibrium of the machine will be unaffected by these two forces, for the product Pv will be the same in each case, but opposite in sense, and the sum of the products Pv will be zero.

To illustrate these points further let any two points B and B' in the mechanism have the *same motion*, and let any force P act through B, then the previous paragraph asserts that without affecting the conditions of equilibrium in any way, the force P may be transferred from B to B' and such a proposition, along with the principle of the phorograph, Chapter IV., enables one to solve any problem relating to the forces acting on a machine.

From the principle of the phorograph we know that to every point in a mechanism there is a point on a link of reference (which link is usually chosen as the one with fixed centre and turning with uniform angular velocity) which has the same velocity as the original point, and which is called the image of the point. From the preceding each acting force may be transferred from its actual point of application to the image of the latter, with the result that all forces may

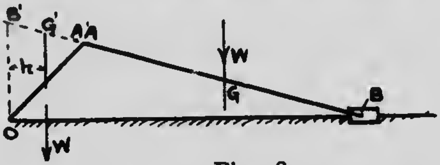


Fig. 78

thus be transferred to determined points on the one link of reference, and the whole problem simply resolves itself into finding the conditions of equilibrium of a set of forces all acting on one link. There will be equilibrium if the sum of the moments of these forces about the point of rotation is zero.

This principle is best illustrated by a series of examples, and here difficult, although usual, practical examples will be selected. It is assumed that the reader can obtain the phorograph, without explanation by the methods discussed in Chapter IV., except in the more difficult cases, in which the construction will be described.

(1) To find the turning effect produced on the crank shaft of an engine by the weight of the connecting rod. Let OA, Fig. 78, be

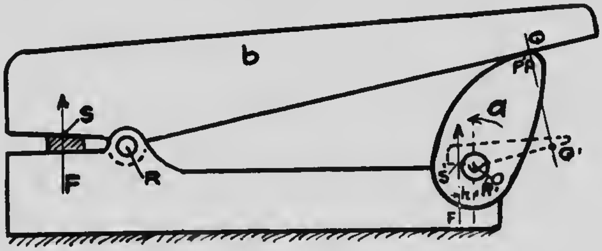


Fig. 79

the crank, and AB the connecting rod of an engine, the latter with centre of gravity G and of weight W lbs Find A', B' and G', the images of A, B and G. The weight W is assumed to act through G, and it will impart energy at the rate Wv ft. pds. per sec. where v

is the velocity of G in the direction of W. But G' has the same motion as G and is a point on the crank OA, hence W may be transferred from G to G', because in the latter case it will impart energy to the machine at the same rate as when located at G. The turning moment due to the weight is then Wh ft. pds. on the crank.

Note that W is moved from G to G', it must not be thought that the force W at G and also W at G' act at once, the dotted line shows the new position. Also G' is a point on the crank OA by the principle of the phorograph, the moment thus being Wh.

(2) A shear Fig. 79, somewhat distorted in proportions, is operated by a cam a attached to the main shaft O, the shaft being

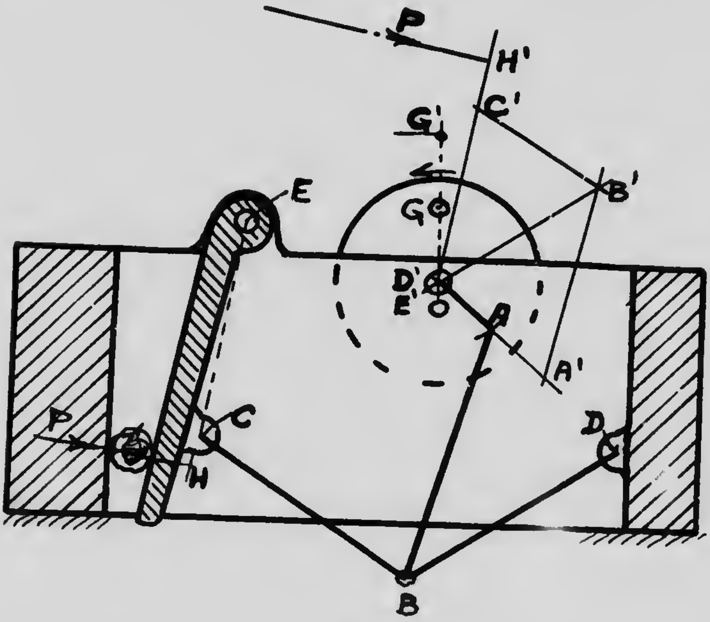


Fig. 80

driven at constant speed by a belt pulley. Knowing the force F necessary to shear the bar at S, the turning moment which must be applied at the cam shaft O is required. Let P the point on the cam a where it touches the arm b at Q, then the motion of P with regard to Q is one of sliding along the common tangent at P. Choosing a as the link of reference, P' will lie at P, R' at O, $R'Q'$ will be parallel to RQ and Q' will lie in $P'Q'$ the common normal to the surfaces at P, this locates Q' Having now two points on b', viz., R' and Q', complete the figure by drawing from Q' the line $Q'S'$ parallel to QS, also drawing $R'S'$ parallel to RS and thus locating S'. The figure shows the whole jaw drawn in, although it is quite unnecessary. Having now found S' a point on a with the same velocity at S on b, the force F may now be transferred to S' and the moment $F \cdot h$ of F about O is the moment which must be produced on the shaft in the opposite sense. By finding the moment in a number of positions it is quite easy to find the necessary power to be delivered by the belt for the complete shearing operation.

(3) Stone crusher, Fig. 80. (Not to scale.) To find the relation between the pressure on the crank and that produced by the

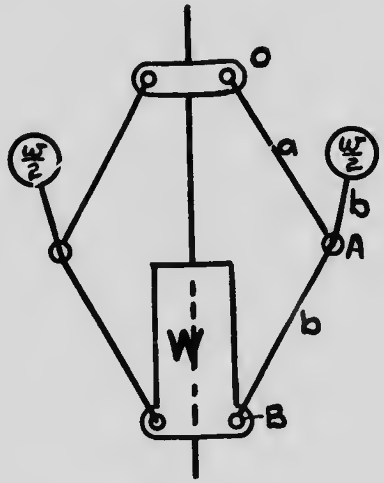


Fig. 81

jaws of the crusher. The crank is at G and it will be assumed that a stone is at H between the jaw EHC and the frame which requires a force P to crush it. The crank OA is joined through the rod AB to the toggle joint CBD, and as the crank rotates in the sense shown, B is raised, C pushed to the left and the stone crushed. Choosing OA (i.e., OG) as the link of reference find the images of all the points, and for clearness the scale of the phorograph has been doubled so that $OA_1 = 2OA$. Having found H' transfer P from H to H' and if h is used for the shortest distance from O to P, then the force necessary at G to overcome P must act through G' the image of G where $OG' = 2OG$, and the magnitude of the force through G will be $\frac{Ph}{OG'}$ it being assumed that the force through G acts normal to the radius OG. This pressure may be determined for every position of the jaw and crank, and thus the necessary pressure on the crank for different parts of the revolution may be found.

(4) Proell governor. Governors are dealt with in a later chapter but the present illustration is instructive and has not been discussed elsewhere in this book. In Fig. 81, a represents an arm pivoted to a fixed point O on the spindle and also at A to the arm b which latter

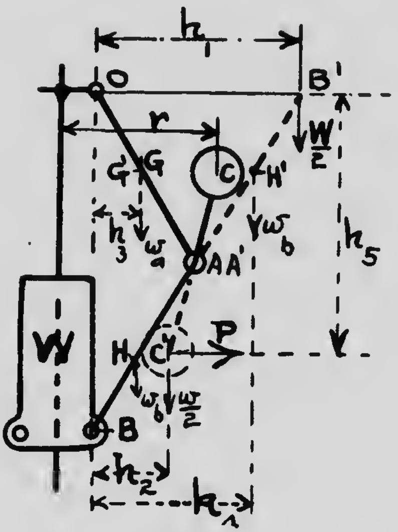


Fig. 82

arm is attached at B to a central weight W sliding on the spindle, while an extension of b carries the centrifugal weight $\frac{W}{2}$ at C. Treating a, Fig. 82, as the link of reference and O as the fixed centre, find A' at A and also B' and C', then transfer $\frac{W}{2}$ (one half the central weight acts on each side) to B' and $\frac{w}{2}$ to C', and if it is desired to allow for the weights w_a and w_b of the arms a and b the centres of gravity G and H of the latter are found and also their images G' and H', then w_b is transferred to H', but as G' is at G, w_a is not moved.

If the balls revolve with linear velocity v ft. per sec. in a circle of radius r ft. then the centrifugal force acting on each ball will be $\frac{w}{2g} \times \frac{v^2}{r}$ pds. in the horizontal direction, and this force P is transferred to C'. Let the shortest distances from the vertical line through O to B', C', G' and H' be h_1, h_2, h_3, and h_4 respectively, and let the vertical distance from C' to OB' be h_5, then for equilibrium of the parts (neglecting friction)

$$\frac{W}{2} \cdot h_1 + \frac{w}{2} \cdot h_2 + w_a h_3 + w_b h_4 = \frac{w}{2g} \times \frac{v^2}{r} \times h_5.$$

which enables the velocity v necessary to hold the governor in equilibrium in any given position to be found.

CHAPTER X.

CRANK EFFORT AND TURNING MOMENT DIAGRAMS

In the case of steam engines, gas engines, air compressors, pumps, etc., it is essential that the relation between the pressure acting on the piston by the steam, gas, etc., and the corresponding turning moment at the crank shaft be known. This is desirable in the study of the relative merits of single and double-acting engines, tandem and cross-compound engines with various crank settings, two and four cycle gas engines, and various other arrangements, in connection with the steadiness of motion which is desirable in all cases.

It will be evident that the motion of any of the above machines will be steadier the more uniform is the turning moment produced on the crank shaft by the working fluid acting on the piston. The more uniform the turning moment also the lighter the fly-wheel

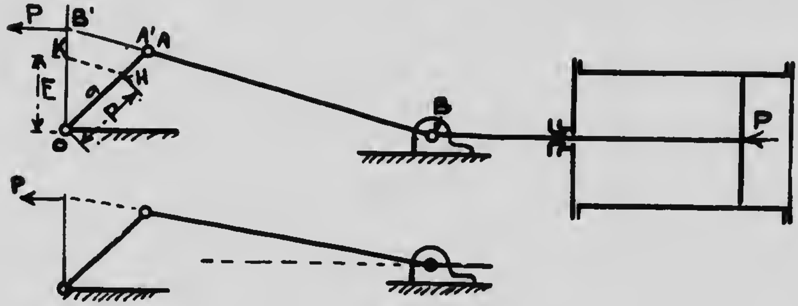


Fig. 83

becomes for any purpose, thus in the single-acting, four-cycle gas engine, where there is only one power impulse for two revolutions, it will be readily understood that there should be a much heavier fly-wheel than on the double-acting steam engine with two power impulses per revolution. One notices the extremely heavy fly-wheels used on gas engines, in comparison with steam engines, in order to reduce this unsteadiness to a minimum, and even with all this one often finds that where a gas engine drives a dynamo the lights fluctuate in a very unpleasant way, even with heavy fly-wheels, whereas, with the steam engine with much lighter wheels the fluctuation is not noticeable.

It is therefore of considerable importance to the engineer that he understands the causes of these difficulties, and also the possible remedies, and the matter will be studied in some detail.

An outline of a steam engine is shown in Fig. 83 and it is assumed that a force P is acting on the piston and transferred by it to the wrist pin B. The image of B is found at B', and since for the purposes of equilibrium a force may be transferred from its actual point of application to the image of this point, it is therefore possible to move P to B' without changing its turning effect. Now the moment produced by P on the crank shaft is evidently $P \times OB'$ ft. pds., which is properly called the *torque* T on the crank shaft, and the more uniform this torque is the more uniform will be the motion of the engine. In general, it will be found convenient to divide the torque T by the length a of the crank arm, and the quotient will be the force which, if acting through the crank pin A normal to the crank, would produce the actual torque T acting, and this force will evidently be a measure of the torque produced by the force P on the piston. The force through B acting as above is called the ***crank effort*** and is denoted by E so that $T = P \times OB' = E \times OA = E \times a$ or $E = P \cdot \dfrac{OB'}{a}$.

For any force P the corresponding crank effort E may be readily found graphically by laying off a distance OH along a to represent P on any convenient scale and drawing HK parallel to AB, then will OK represent E on the same scale.

Suppose now that the indicator diagrams are given for an engine and it is required to draw a corresponding ***crank effort diagram***, i.e., a diagram showing the crank efforts for all positions of the crank. Let the diameter of the cylinder be d in. and of the piston rod d_r in. then the area of the head end of the piston will be $A = \dfrac{\pi}{4} d^2$ sq. in. and of the crank end $A' = \dfrac{\pi}{4} d^2 - \dfrac{\pi}{4} d_r^2$ sq. in.; let the stroke of the piston be L ft.

Let the indicator diagrams be drawn with a spring of scale s in the indicator, by which is meant that each inch in height of the diagram represents a pressure of s pds. per sq. in. on the piston, thus if $s = 60$ then 1 in. in height on the diagram means 60 pds. per sq. in. on the piston. The length of the diagrams for the head and crank ends which are usually the same, is indicated by l in., and is usually much less than L, for l very rarely exceeds 4 in. for any engine while L varies from 6in. to 5 ft. or more.

The diagrams are now placed above the cylinder, Fig. 84, with the atmospheric li·· ·arallel with the line of motion of the piston, and for clearness the two diagrams for both ends are shown above one another instead of having both atmospheric lines coincide, as they usually do in taking the diagram. The indicator diagrams are shown here of the same length as that taken to represent the stroke of the piston. This may be done by changing the length of the diagram to suit the scale adopted for the mechanism or else by making the scale of the mechanism such that the stroke is the same as the length of the diagrams. Next draw on the indicator diagrams the lines of absolute zero pressure which are parallel with the atmospheric lines and at distances below them to represent the barometric pressure to scale s.

Now divide the crank pin circle into any convenient number of parts, say 24, as shown at Fig. 84, and for each position drawn in crank, connecting rod and piston, only one position is shown on the figure. Draw a vertical line above each piston position through the diagrams and also find the point B'. Looking now at the indicator diagrams it is seen that, since the piston is moving to the left, the crank end is exhausting while the head end is working under live steam, thus the point M on the head end and N on the crank end are being drawn simultaneously. (It is to be observed that the piston will again occupy the position shown on its return stroke, but at this time the piston will be driven to the right by steam in the crank end and will at the same time be exhausting from the head end, at this instant the points R in the crank end diagram and Q in the head end will be drawn).

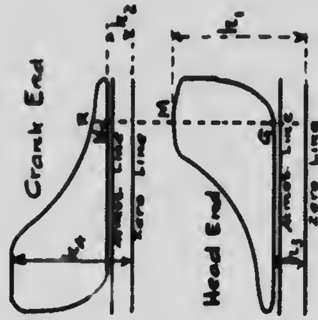
Let h_1 in. be the height of M above the zero line and h_2 the corresponding height of N, then the force urging the piston forward is $h_1 \times s \times A$, and that opposing the motion of the piston is $h_2 \times s \times A'$ so that the net force acting is $P = h_1 \times s \times A - h_2 \times s \times A'$ pds. This force P is in most cases positive, i.e., acts in the direction the piston is moving in, but at some parts of the stroke it is frequently negative, which means that the fly-wheel must at such places have enough energy in it to drag the piston along against the resistance offered by the steam. Now lay off OH to represent P on a chosen scale then OK represents the crank effort E on the same scale. This same construction is carried out for each crank position and the value of E found in each case, it will be found that the variations in E are fairly large.

Now lay off in Fig. 85 a straight line base 0 — 24 of a length to represent the circumference of the crank pin circle and divide the line into the sa:me ·her of equal parts as the crank pin circle is divided into, in this case twenty-four, and at each point draw a vertical line to represent in height the crank effort E for the corresponding crank pin position. By joining the tops of the vertical lines we obtain a *crank effort diagram*, the height of which at any point represents the crank effort for the corresponding crank pin position.

Since vertical heights on this diagram represent forces and horizontal distances represent the space travelled through by the crank pin, the area of the diagram represents foot pounds of work done and to the proper scale the area of this diagram must be the same as that of the indicator diagrams, since both must represent the same amount of work. Thus if the length of the base of the crank effort diagram is taken exactly equal to the circumference of the crank pin circle then the horizontal length of the crank effort diagram will be π times the length of each indicator diagram and as it takes two complete strokes for the two diagrams and for the crank pin to describe the complete circle therefore the mean height of the crank effort diagram in pds. will be $\frac{2}{\pi}$ times the mean total pressure acting on the piston.

By the method just described we obtain a diagram representing the pressure on the crank pin, normal to the crank, which corresponds with the pressure produced on the piston. It is to be noted that to get the turning moment or torque acting on the crank shaft in any position it is only necessary to multiply the corresponding effort by the crank radius or $T = E \times a$, and as a is a constant for all positions, the cra' ··e see that the torque is proportional to the crank effort. Th' ·am already obtained is also a torque diagram, vertical hei hich represent on a certain scale a number of foot-pounds.

Only a very brief discussion will be given here as to the effect of the shape of this diagram on the steadiness of motion, as this matter is discussed very fully in the chapter dealing with the weight of fly-wheels, but it will be helpful here to point out some features of the matter. For the sake of simplicity it will be assumed that the engine is driving a dynamo, or a turbine pump or the machinery in a machine shop or other machinery which is of such a nature as to


Crank End
Head End
Atmos. Line
Zero Line

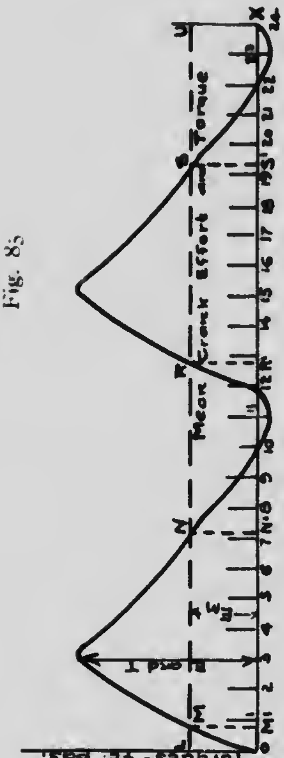

Mean Crank Effort Torque
Crank Efforts - lbs.
Torques - ft. lbs.

Fig. 85

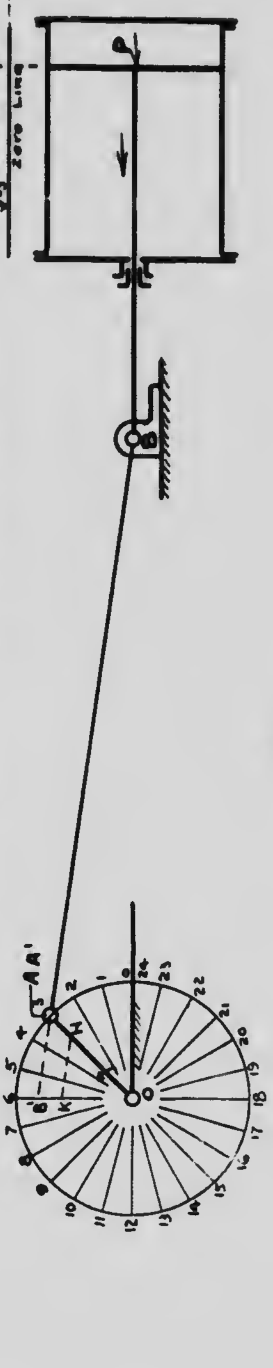


Fig. 84

offer a *constant resistance* to motion. In all the machines above mentioned, for instance, if driven by a belt the belt would for all parts of the revolution have a constant tension, in other words, the machines are such that they will require a constant torque for operation. This is one of the simplest cases of *loading* of an engine.

Now the total work delivered to the engine per revolution is represented by the area of the indicator diagrams and also by the area under the crank effort diagram as has already been explained. If now the mean height of this crank effort diagram is found and a horizontal line drawn the height of this line will represent the mean crank effort, and also the mean torque during the revolution, and this line is to be so located that the sum of the positive and negative areas between it and the crank effort curve will be equal. This mean line cuts the crank effort diagram at four points, M, N, R and S, Fig. 85.

Since friction is being neglected the work which the engine is capable of doing in the way of driving other machinery, is also equal to the area under the crank effort diagram and this discussion deals with the case where the load offers a constant resisting torque, therefore the torque diagram for the load will be a horizontal line, and if drawn on the same axes as the crank effort curve will be the line L, M, N, R, S, U, where the area under this line is equal to the work put into the engine per revolution. Further, the work put in by the working fluid will be equal to that given out in the same time if the engine is to make a constant number of revolutions per minute. Thus the mean crank effort curve may be also looked upon as the load curve for the case under consideration.

A study now of the diagram in Fig. 85 shows certain important features. During the first part of the out-stroke it is evident that the crank effort due to the steam pressure is less than that necessary to drive the load, this being the case until M is reached, at this point the effort due to the steam pressure is just equal to that necessary to drive the load, thus during the part OM' of the revolution the input to the engine being less than the output the energy of the links themselves must be drawn upon and must supply the work represented by OML. But the energy which may be obtained from the links will depend upon the mass and velocity of them, the energy being greater the larger the mass and also the greater the velocity, the result is that if the energy of the links is decreased by drawing from them for any purpose, then since the mass of the links is fixed by construction, the only other thing which may happen is that the speed of the links must decrease.

In engines the greater part of the weight in the moving parts is in the fly-wheel and hence from what has been already said, if energy is drawn from the links then the velocity of the fly-wheel will decrease and it will continue to decrease so long as energy is drawn out from it. Thus during OM' the speed of the fly-wheel will fall continually but at a decreasing rate as we approach M', and at this point the wheel will have reached its minimum speed. Having passed M' the effort supplied by the steam is greater than that necessary to do the external work and hence there is a balance left for the purpose of adding energy to the parts and speeding up the fly-wheel and other links, the effort available for this purpose in any position being that due to the height of the crank effort curve

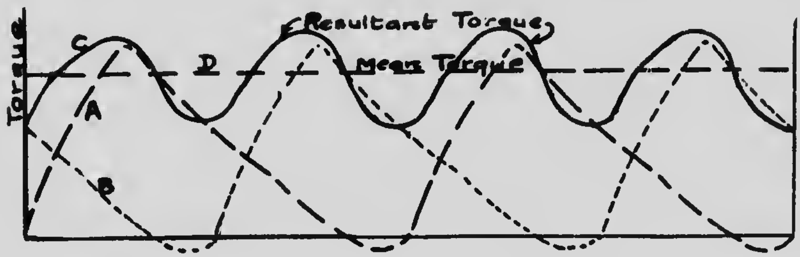


Fig. 86

above the load line. In this way the speed of the parts will increase between M' and N' reaching a maximum for this period at N'.

From N' to R' the speed will again decrease, first rapidly then more slowly, and reaching a minimum again at R' and from R' to S', there is increasing speed with a maximum at S'. The fly-wheel and other parts will, under these conditions, be continually changing their speeds from minimum to maximum and vice versa, producing much unsteadiness in the motion during the revolution. The magnitude of the unsteadiness will evidently depend upon the fluctuation in the crank effort curve, if the latter curve has large variations then the unsteadiness will be increased.

In Fig. 86, a crank effort curve is shown for two engines coupled to one shaft and with their cranks at right angles as in a locomotive. Curve A represents the crank effort curve for one cylinder and B

that for the one with crank 90° behind the first while C represents the resultant crank effort and D the mean crank effort, and if the same nature of loading as before be assumed then D will also be the load curve. It will be evident in this case that while there are a greater number of maximum and minimum speeds than before, yet the fluctuation in speeds must be much less, since the effort curve C due to the steam is closer throughout to the load curve than in the previous example.

TANDEM AND CROSS COMPOUND ENGINES

In order to show the relative merits of tandem and cross-compound engines with respect to the steadiness of motion and uniformity of torque a comparison should be made between Figs. 86 and 87. The curves in Fig. 86 are very similar to those that would be obtained

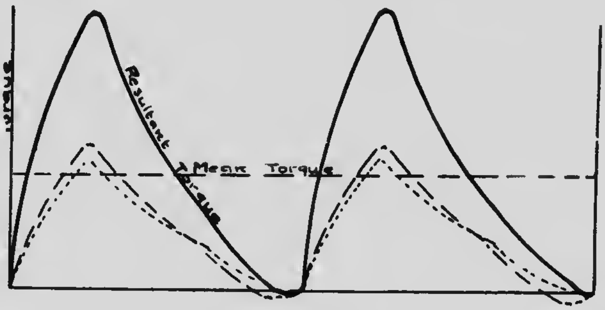


Fig. 87

from a cross-compound engine with cranks at 90° while in Fig. 87 is shown the curve for a tandem engine. A study of these diagrams at once shows the advantages of the cross-compound arrangement in securing uniform torque and it therefore makes a much preferred design from this point of view.

Engines are frequently built to work triple expansion with cranks at 120°, and if the reader will plot the curves in this case he will find that the turning moment is extremely uniform.

INTERNAL COMBUSTION ENGINES

Internal combustion engines are made to work either two or four cycle and the latter class alone will be treated here, as it is very common, at least, on larger, sizes and besides it gives a rather more instructive case.

The four cycle engine is usually made *single-acting*, and the complete cycle of the engine is finished in four strokes. Starting with the crank on the inner dead centre the inlet valve opens and as the piston makes the first out-stroke called the *suction stroke*, the charge of explosive mixture is drawn in at constant pressure slightly below that of the atmosphere, the valves in the cylinder are now closed and the piston is forced in during the *compression stroke* the mixture being compressed. Near the end of the compression stroke the charge is ignited, causing a sudden rise in pressure which drives the piston forward producing the *expansion stroke*, and then the exhaust valve is opened, and the burnt gases are swept out of

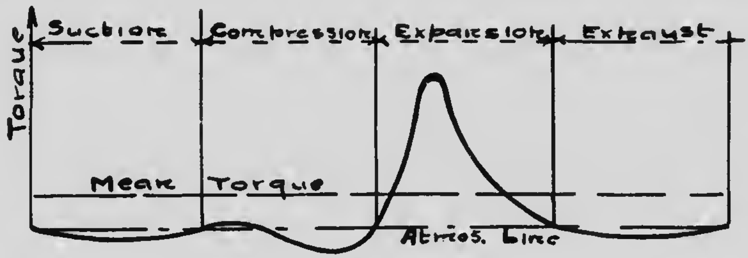


Fig. 88

the cylinder by the r[illegible]r *exhaust stroke*, the pressure in the cylinder during this stroke being nearly constant and slightly higher than that of the atmosphere.

In internal combustion engines of this type there is one stroke (the expansion stroke) of the four composing the cycle in which energy is being received from an external source of supply through the working fluid and during the remaining three strokes no energy is supplied externally (compare this with the ordinary case of the double acting steam engine) so that whatever energy is given out during these three strokes must be drawn from the moving parts.

The crank effort diagram for a single cylinder engine is shown in Fig. 88, and it will be noticed that the line of mean torque falls far below the maximum torque in the cycle so that unless the rotating masses are very heavy there will be large fluctuations in speed. We shall not here attempt further discussion of this subject but the advantages of four or six cylinders with properly arranged cranks will be very evident if the crank effort curves are drawn.

CHAPTER XI.

THE EFFICIENCY OF MACHINES

The accurate determination of the efficiency of machines and the loss by friction is extremely complicated and difficult and it is doubtful whether it is possible to deal with the matter except through fairly close approximations. All machines are constructed for the purpose of doing some specific form of work, the machine receiving energy in one form and delivering this energy, or so much of it as is not wasted, in some other form. Thus, the water turbine receives energy from the water and transforms the energy thus received into electrical energy by means of a dynamo, or a motor receives energy from the electric circuit, and changes this energy into that necessary to drive an automobile, and so for any machine, the machine receives energy in a certain form from some source and changes this energy into some useful form, delivering it again for the particular purpose desired. For convenience, the energy received by the machine will be referred to as the *input* and the energy delivered by the machine as the *output*.

Now a machine cannot create energy of itself, but is only used to change the form of the available energy into some other which is desired, so that for a complete cycle of the machine (e.g., one revolution of a steam engine, or two revolutions of a four-cycle gas engine or the forward and return stroke of a shaper) there must be some relation between the input and the output. If no energy were lost during the transformation the input and output would be equal and the machine would be perfect, as it would change the form of the energy and lose none. However, if the input per cycle were twice the output then the machine would be very inperfect for there would be a loss of one half of the energy available during the transformation, the output can, of course, never exceed the input. It is then the province of the designer to make a machine so that the output will be as nearly equal to the input as possible and the more nearly these are to being equal the more perfect will the machine be.

In dealing with machinery it is customary to use the term *mechanical efficiency* or *efficiency* to denote the ratio of the output per cycle to the input, or the efficiency $\eta = \frac{\text{output}}{\text{input}}$ per cycle. The maximum value of the efficiency is unity, which corresponds to the perfect machine, and the minimum value is zero which means that the machine is of no value in transmitting energy; the efficiency of the ordinary machine lies between these two limits, electric motors having an efficiency of .92 or over, turbine pumps usually not over .80, large steam pumping engines .95, etc., and in the case where the clutch is disconnected in an automobile engine the efficiency of the latter is zero.

The quantity $1 - \eta$ represents the proportion of the input which is lost in the bearings of the machine and in various other ways, thus in the pump above mentioned, $\eta = .80$ and $1 - \eta = .20$, or 20% of the energy is wasted in this case in the bearings and the friction of the water in the pump, similar results may be obtained for other machines. The amount of energy lost in the machine, and which helps to heat up the bearings, etc., will depend on many things such as the nature of lubricant used, the nature of the metals at the bearings and many other considerations to be discussed later.

Suppose now that on a given machine there is at any instant a force P acting through a certain point on one of the links and this point is moving at velocity v_1 in the direction and sense of P, then the energy put into the machine will be at the rate of Pv_1 ft. pds. per sec. Now at the same instant let there be a resisting force Q acting on some part of the machine and let the point of application of Q have a velocity v_2 in the direction of Q so that the energy output is at the rate of Qv_2 ft. pds. per sec. (The force P may be the pressure acting on an engine piston or the difference between the tensions on the tight and slack sides of a belt driving a lathe, while Q may represent the resistance offered by the main belt on an engine or by the metal being cut off in a lathe.) Now from what has been already stated the efficiency at the instant is $\eta = \frac{\text{output}}{\text{input}} = \frac{Q \cdot v_2}{P \cdot v_1}$ and if no losses occur then this ratio will be unity, it is, however, always less than unity in the actual case. Now, as in practice, Qv_2 is always less than Pv_1 choose a force P_o acting through the point of application of P such that $P_o v_1 = Qv_2$, then clearly P_o is the force which, if applied to a frictionless machine of given type, would just balance the resistance Q and $\eta = \frac{Qv_2}{Pv_1} = \frac{P_o v_1}{Pv_1} = \frac{P_o}{P}$ so that evidently the efficiency will be $\frac{P_o}{P}$ at any instant, P_o being always less than P.

The efficiency may also be expressed in a different form. Thus, let Q_0 be the force which could be overcome by the force P if there were no friction in the machine then $Pv_1 = Q_0v_2$ and therefore

$$\eta = \frac{Qv_2}{Pv_1} = \frac{Qv_2}{Q_0v_2} = \frac{Q}{Q_0}$$

FRICTION

Whenever two bodies touch each other there is always some resistance to their relative motion, this resistance being called friction. Suppose a pulley to be suitably mounted in a frame attached to a beam and that a rope be passed over this pulley, each end of the rope holding up a weight w lbs. Now, since each of these weights is the same they will be in equilibrium and it would be expected that if the slightest amount were added to either weight the latter would descend. Such is, however, not the case, and it is found by experiment that one weight may be considerably increased without disturbing tne conditions of rest.

It will also be found that the amount it is possible to add to one weight without producing motion will depend upon such things as (1) the amount of the original weight w being greater as w increases, (2) the kind and amount of lubricant used in the bearing of the pulley, (3) the stiffness of the rope, (4) the materials used in the bearing and the nature of the mechanical work done on it, and upon very many other considerations which the reader will readily think of for himself.

Now it is evident that there must be some force coming into play which counteracts the effect of the additional weight and keeps the pulley at rest, and it is further evident that the magnitude of this force must vary with the conditions, being zero when no addition is made to either weight and gradually increasing as the additional weight increases until motion begins. This force, which acts in such a way as to resist the motion, we call *friction* and as has been pointed out it is found that it acts in such a way as to hinder motion and is variable in amount.

Before passing on one more illustration might be given of this point. Suppose a block of iron weighing 10 lbs. is placed upon a horizontal table and that there is a wire attached to this block of iron so that a force may be produced on it parallel to the table. If now a tension were put on the wire and there were no loss of energy the block of iron should move even with the slightest tension, because no change is being made in the potential energy of the block by moving it from place to place on the table, as no alteration is taking place in its height. It will be found ,however, that the block will not begin to move until considerable force is produced in the wire, the force possibly running as high as 1.5 pd- or less. The magnitude of the force necessary will as before depend upon (1) the material of the table, (2) the nature of the surface of the table, (3) the area of the face of the block of iron touching the table, etc.

Just as in the case of the pulley, therefore, some force springs into existence to balance the pull in the wire, the greater the pull in the wire the greater will be the balancing force and vice versa, thus the latter force is variable in mangitude, being only sufficient to balance the external pull applied, and increasing as the latter increases till the limit is reached where the block begins to move. The force thus called into play is called *friction*.

Wherever motion exists friction is always acting in a sense opposed to the motion, although in many cases its very presence is essential to motion taking place. Thus it would be quite impossible to walk were it not for the friction between one's feet and the earth, a train could not run were there no friction between the wheels and rails, and a belt would be of no use in transmitting power if there were no friction between the belt and pulley. Friction therefore, acts as a resistance to motion and yet without it many motions would be impossible.

A great many experiments have been made for the purpose of finding the relation between the friction and other forces acting between two surfaces in contact, the laws of Morin stating that the frictional resistance to the sliding of one body upon another depended upon the normal pressure between the surfaces and not upon the areas in contact nor upon the velocity of slipping and that if F were the frictional resistance to slipping and N the pressure between the surfaces, then $F = \mu N$ where μ, the *coefficient of friction*, depended only upon the nature of the surfaces in contact as well as the materials composing these surfaces.

A discussion of this subject would be too lengthy to place here and the student is referred to the numerous experiments and discussions in the current engineering periodicals and in books on mechanics, such as Kennedy's "Mechanics of Machinery," and Unwin's "Machine Design," books well worthy of study. It may only be stated that the law giveh above is known to be quite untrue in the case of machines where the pressures are great, the velocities of sliding high and the methods of lubrication very variable, and a special low must be formulated in such cases. Thus before we can tell what friction there will be in the main bearing of a steam engine, we must know by experiment what laws exist for the friction in case of a similar engine ha ing similar materials in the shaft and bearing and oiled in the same way, and if we are dealing with a horizontal Corliss engine we shall not get the same laws as with a vertical high speed engine, and again the laws will depend upon whether the lubrication is forced or gravity and on a great many other things. For each type of bearing and lubrication there will be a law for determining the frictional loss and this law must in each case be determined by careful experiments. Following the method of Kennedy*, the formula $F = fN$ will be used for the friction force F in terms of the pressure N between the surfaces, and we shall employ f to denote the *friction factor*. The law for f may be very complicated, and should be found by experiments of the kind already referred to, as this factor will vary with so many different properties of the bodies in contact and the nature of the motion.

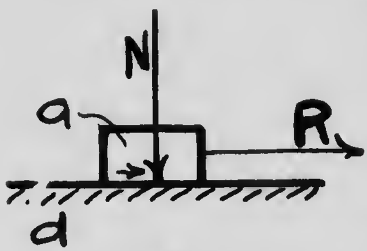


Fig. 89

In dealing with machines it has been shown that they are made up of parts united usually by sliding or turning pairs, so that it will be well at first to study the friction in these pairs separately.

FRICTION IN SLIDING PAIRS

Consider a pair of sliding elements as shown in Fig. 89 and let the normal component of the pressure between these two elements be N and let R be the resultant external force acting upon the upper element which is moving, the lower one, for the time being, considered stationary. Let the force R act parallel to the surfaces in the sense shown the tendency for the body is then to move to the right. Now, from the previous discussion, there is a certain resistance to the motion of a the amount of which is $f\,N$, where f is the friction factor, and this force must in the very nature of the case act tangent to the

* Mechanics of Machinery—Kennedy.

surfaces of contact, thus from the way in which R is chosen in this case the friction force $F = f N_{,}$, and R are in the same direction.

Now if R is small there will be no motion as is well known, for the resistance F may easily be much greater than R, this corresponds to the case of a sleigh stalled on the road where the pull of the horses is horizontal, but less than the frictional resistance. Again if R be increased sufficiently the body will move with uniform velocity, in which case the resistance F due to friction must just be equal to the force R acting, as in the case of the sleigh being drawn at uniform speed along a road, but if R be increased above this last value, then the motion becomes accelerated as frequently occurs when a sleigh is being drawn down hill, the resolved part of the force of gravitation parallel with the ground being greater than the frictional resistance. Stating these results together, we have

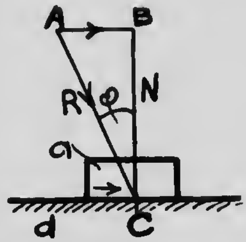


Fig. 90

(1) R less than F, i.e., $R < f N$, no relative motion.

(2) $R = F$, i.e., $R = f N$, motion with uniform velocity.

(3) R greater than F, i.e., $R > f N$, accelerated motion.

Assuming that motion is actually taking place there is only equilibrium provided $R = f N$.

Consider next the case shown in Fig. 90, where the resultant external force acts as shown, the motion of a relative to d being to the right as indicated on the figure. Let N be the resolved part of R normal to the surface of contact, then we already have seen that according to the method adopted in this discussion, the frictional resistance to slipping of a on d is $F = f N$, and this acts in such a way as to oppose the force producing motion. Resolve R into its two components AB parallel to the surfaces and BC or N normal to these surfaces, then from the argument of the preceding paragraph, AB cannot be less than F as there would then be no motion, and if $AB = F$ the motion will be of uniform velocity, while if AB is greater than F there will be acceleration of a relative to b. Let the angle ACB be denoted by ϕ so that $AB = R \sin \phi$, also $BC = N = R \cos \phi$ and $AB = N \tan \phi$. For the case in which motion takes place without acceleration, i.e., for the condition of equilibrium, $AB = F = f N$ or $\frac{AB}{N} = f = \tan \phi$ or the tangent of the angle of inclination of the resultant force R to the normal is the friction factor.

Hence for equilibrium during the relative motion of the two bodies, the resultant must be inclined at an angle $\phi = \tan^{-1} f$ to the normal to the surfaces and on such a side of this normal that the tangential component AB is in the direction of motion. The angle ϕ may be conveniently called the *angle of friction*, and will be in future denoted by the letter ϕ, frequently without using the words "angle of friction" to designate it.

In connection with this discussion it is to be borne in mind that ϕ is the limiting angle of inclination of the resultant force on the moving body, and that if the resultant acts at any angle θ to the normal then there will be no motion or acceleration according as θ is less or greater than ϕ for if θ be greater than ϕ then $\tan \theta > \tan \phi$ and $A'B' = R \sin \theta$ is greater than $AB = R \sin \phi$ or $A'B'$ is greater than the frictional resistance F and the unbalanced force will cause acceleration, similarly θ if is less than ϕ the resistance F exceeds $A'B'$ and there will be no motion, so that it is only when $\theta = \phi$ that there is equilibrium.

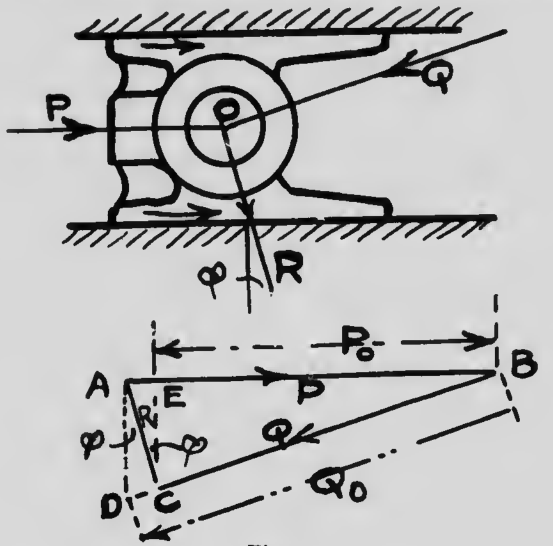


Fig. 91

A few examples should make this matter clear and in those first given, all friction is neglected except that in the sliding pair. The friction in other parts will be considered later.

(1) As an illustration, take an engine, with the crosshead moving to the right under the steam pressure P acting on the piston, Fig. 91.

The forces acting on the crosshead are the steam pressure P, the thrust Q due to the connecting rod and the resultant R of these two which also represent the pressure on the crosshead due to the guide. Now we know from the principles of statics that P, Q and R must all intersect at one point, in this case the centre of the wrist-pin O, and further that the resultant R must be inclined at an angle ϕ to the normal to the surfaces in contact, thus R has the direction shown. (Note that the side of the normal on which R lies must be so chosen that R has a component in the direction of motion.) Now draw $AB = P$, the steam pressure and draw AC and BC parallel respec-

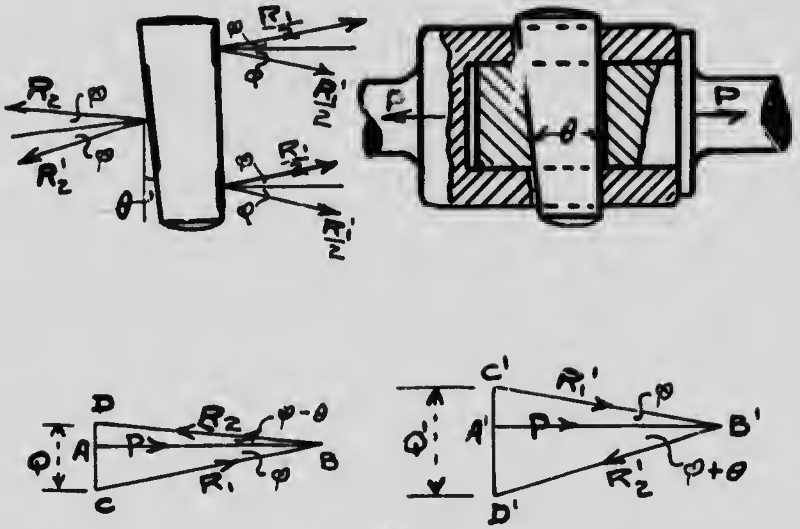


Fig. 92

tively to R and Q, then $BC = Q$, the thrust of the rod and $AC = R$ the resultant pressure on the cross-head shoe.

If there were no friction in the sliding pair then R would be normal to the surface and in the triangle ABD the angle BAD would be 90°; BD is the force in the connecting rod and AD is the pressure on the shoe. The efficiency will thus be $\eta = \frac{BC}{BD} = \frac{Q}{Q_0}$. Or we may find P_0 the force necessary to overcome Q if there were no friction by drawing CE normal to AB then $P_0 = BE$ and $\eta = \frac{P_0}{P} = \frac{BE}{BA}$

(2) A cotter is to be designed to connect two rods, Fig. 92,

it is required to find the limiting taper of the cotter to prevent it slipping out when the rod is in tension. It will be assumed that both parts of the joint have the same friction factor f, and hence the same friction angle ϕ, and that the cotter tapers only on one side with an angle θ. The sides of the cotter on which the pressure comes are marked in heavy lines and on the right hand side the total pressure R_1 is divided into two parts by the shape of the outer piece of the connection. Both the forces R_1 and R_2 act at angle ϕ to the normal to their surfaces and from what has already been said it will be readily understood that they act on the side of the normal shown when the cotter just begins to slide out, so that by drawing the vector triangle of height $AB = P$ and having CB and BD respectively parallel to

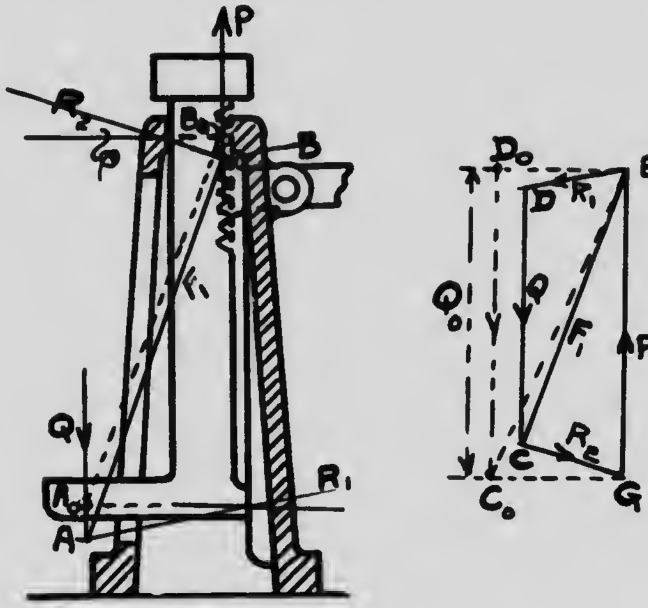


Fig. 93

R_1 and R_2, the force Q necessary to force the cotter out is at once determined.

From the figure it is seen that the angle $CBA = \phi$ and $ABD = \phi - \theta$, so that $Q = P\,[\tan\phi + \tan\,(\phi - \theta)]$ and evidently the cotter will slip out of itself if $Q = o$, i.e., if $\tan\phi + \tan\,(\phi - \theta) = o$ or $\theta = 2\phi$. This is evidently independent of P except in so far as ϕ is affected by the latter force.

If the cotter is being driven in the forces R_1 and R_2 take the directions R_1' and R_2' shown, and the corresponding vector diagram is shown in the figure. Here the force Q required is given by the relation.

$$Q_1 = P\,[\tan\phi + \tan(\theta + \phi)]$$

which is evidently increased with an increase in θ. Small values of θ make the cotter easy to drive in and hard to drive out.

(3) An interesting example of the friction in sliding pairs is illustrated in Fig. 93, which represents the very common case of a jack, which is in frequent use for various purposes. The machinery

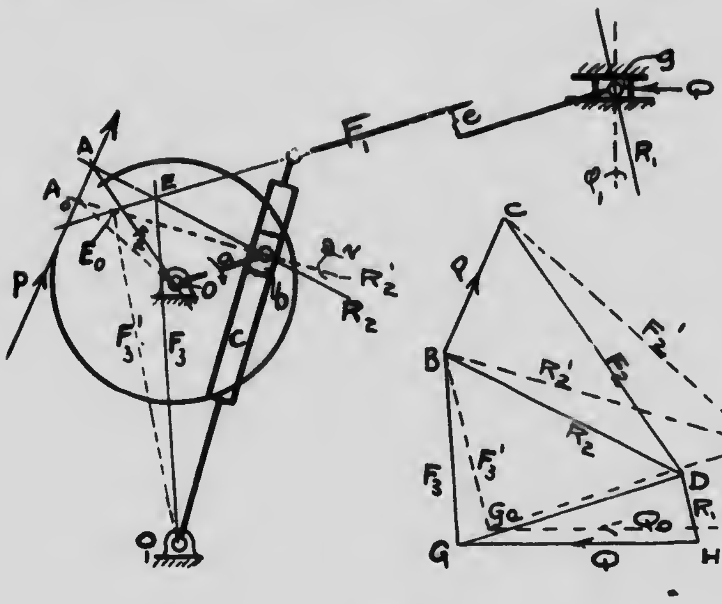


Fig 94

of the jack is not shown, the barest outline alone being drawn to explain the principles now under discussion.

In this figure the force P which is being applied to lift the jack is assumed to act vertically, although its direction will really depend upon the mechanism which is used in applying the lifting power. The position of the load Q to be lifted is also assumed and the load causes the heel of the jack to press against the frame with the force R_1 in the direction shown (the jack is assumed to be raising the load) while the force with which the top of the jack presses against the frame is R_2.

At the base of the jack are the forces Q and R_1, the resultant of which must pass through A, while at the top are the forces R_1 and P, the resultant of which must pass through B, and if there is equilibrium the resultant F_1 of Q and R_1 must balance the resultant F_1 of R_2 and P_1, which can only be the case if F_1 passes through A and B, thus the direction of F_1 is known.

Now draw the vector triangle ECG with sides parallel to F_1, R_2, and P, the latter force also being known in magnitude, so that $F_1 = EC$ and $R_2 = CG$. Next through E draw ED parallel to R' and through C draw CD parallel to Q from which $Q = CD$ is found. If there were no friction the reactions between the jack and the frame would be normal to the surfaces at the points of contact, thus A would move up to A_o and B to B_o and the vector diagram would take the form ED_oC_oG where $EG = P$ as before and $D_oC_o = Q_o$ so that $Q_o = P$.

The efficiency of the device in this position is evidently $\eta = \dfrac{Q}{Q_o}\cdot$ It is evident that the efficiency is a maximum when the jack is at its lowest position because AB is then most nearly vertical, while for the very highest positions the efficiency will be very low.

One more example of this kind will suffice to illustrate the principles. Fig. 94 shows in a very elementary form a quick return motion used on shapers and machine tools, and illustrated earlier in this book. Let Q be the resistance offered by the cutting tool which is moving to the right and let P be the net force applied by the belt to the circumference of the belt pulley. For the present problem we shall consider only the friction losses in the sliding elements, leaving the other parts till later. Here the tool holder g presses on the lower guide and the pressure on this guide is R_1, the force in the rod e is denoted by F_1. Further the pressure of b on c is to the right and as the former is moving downward for this position of the machine, the direction of pressure between the two is R_2 through the centre of the pin.

Now on the driving link a the forces acting are P and R_2, the resultant F_2 of which must pass through O and A. In the vector diagram draw BC equal and parallel to P, then CD and BD parallel respectively to F_2 and R_2 will represent these two forces, so that R_2 is determined. Again on C the forces acting are R_2 and F_1, and their resultant passes through O, and also through E, the intersection of F_1 and R_2, so that drawing BG and DG in the vector diagram parallel respectively to F_3 and F_1 gives the force $F_1 = DG$ in the rod e. Acting on the the tool holder g are the forces F_1, Q and R_1 and the directions of them are known and also the magnitude of F_1, hence complete the triangle GHD with sides parallel to the forces concerned and then $GH = Q$ and $HD = R_1$ which gives at once the resistance Q which can be overcome at the tool by a given net force P applied by the belt.

If there were no friction in these sliding pairs then the forces R_1 and R_2 would act normal to the sliding surfaces instead of at angles ϕ_1 and ϕ_2 to the normals so that A moves to A_0 and E to E_0 and the construction is shown by the dotted lines, from which we get the value of Q_0, and the efficiency for this position of the machine is $\eta = \dfrac{Q}{Q_0}$

The value of η should be found for a number of other positions of the machine, and, if desirable a curve may be plotted so that the effect of friction may be properly studied.

Before passing on to the case of sliding pairs the attention of the reader is called to the fact that the greater part of the problem

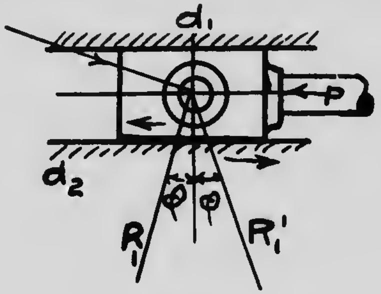


Fig. 95

is the determination of the condition of static equilibrium as described in Chapter IX, the method of solution being by means of the virtual centre, in these cases the permanent centre being used. The only difficulty here is in the determination of the direction of the pressures R between the sliding surfaces, and the following suggestions may be found helpful in this regard.

Let a crosshead a, Fig. 95, slide between the two guides d_1 and d_2, first find out, by inspection generally from the forces acting whether the pressure is on the guide d_1 or d_2. Thus if the connecting rod and piston rod are in compression the pressure is on d_2, if both are in tension it is on d_1, etc., suppose for this case that both are in compression, the heavy line showing the surface bearing the pressure.

Next find the *relative* direction of sliding. It does not matter whether both surfaces are moving or not, we simply wish the relative direction and shall assume it in the sense shown, i.e., the sense of motion of a relative to d_2 is to the left (and, of course, the sense of motion of d_2 relative to a is to the right). Now the resultant pressure between the surfaces is inclined at angle ϕ to the normal where $\phi = \tan^{-1} f$, f being the friction factor, so that the resultant must be either in the direction of R_1 or R_1'.

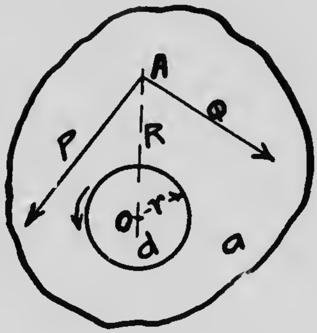


Fig. 96

Now R_1 is the pressure of a upon d_2, then R_1 acts downward, and in order that it may have a resolved part in the direction of motion, then R_1 and not R_1' is the correct direction. If R_1 is treated as the pressure of d_2 upon a then R_1 acts upward, but the sense of motion of d_2 relative to a is the opposite of that of a relative to d_2, and hence from this point of view also R_1 is correct.

The reader may readily remember the direction of R_1 by the following simple rule: Imagine either of the sliding pieces to be an ordinary carpenter's wood plane, the other sliding piece being the wood to be dressed, then the force will have the same direction as the tongue of the plane when the latter is being pushed in the given direction on the cutting stroke, the angle to the normal to the surfaces being ϕ.

FRICTION IN TURNING PAIRS

In dealing with turning pairs the same principles are adopted as are used with the sliding pairs and should not cause any difficulty.

Let a, Fig. 96, represent the outer element of a turning pair, for example, a loose pulley, which is turning in the sense indicated upon a fixed shaft d of radius r, and let P be the applied force, Q being the resistance. If there were no friction then the resultant of P and Q would act through the intersection A of these forces and also through the centre O of the bearing, so that under these circumstances it would be simple to find Q for a given value of P by drawing the vector triangle.

There is, however, frictional resistance offered to motion at the surface of contact, hence if the resultant R of P and Q acted through O, there could be no motion. In order that motion may exist it is necessary that the resultant produce a turning moment about the centre of the bearing equal and opposite to the resistance offered by

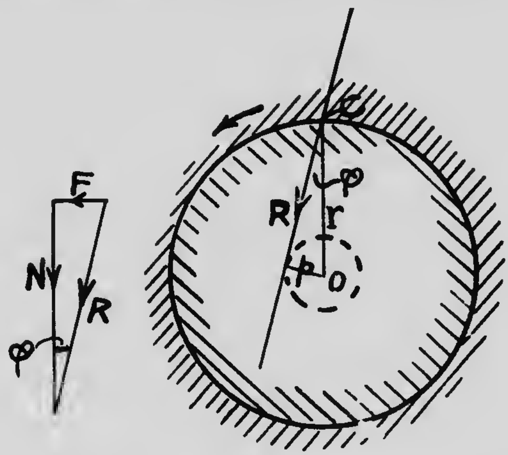


Fig. 97

the friction between the surfaces. It is known already that the frictional resistance is of such a nature as to oppose motion, and hence the resultant force must act in such a way as to produce a turning moment *in the direction* (*sense*) *of motion* equal to that offered by friction in the opposite sense. Thus in the case shown in the figure the resultant must pass through A and lie to the left of O. Fig. 97 shows an enlarged view of the bearing.

In Fig. 97 let p be the perpendicular distance from O to R, so that the moment of R about O is Rp. The point C may be conveniently called the centre of pressure, being the point of intersection of R and the surfaces under pressure. Join CO. Now resolve R into two components F tangent to the surf ces at C and N normal to the surfaces at C, i.e., radial; then clearly F will be the force necessary to overcome the friction, and following the method already adopted $F = fN$ where f is the friction factor and $f = \tan\phi$ where ϕ is the angle of friction. Thus $F = fN = N\tan\phi$, so that the angle between N and R is ϕ, and ϕ is therefore also the angle at C between R and the radial line CO.

With centre O draw a circle tangent to R as shown dotted, this circle will have a radius p where $p = r\sin\phi$, and is usually designated as the *friction circle*. Thus in the case of the turning pairs the resultant must also make an angle ϕ with the normal to the surfaces in contact, and this is accomplished by drawing the resultant tangent

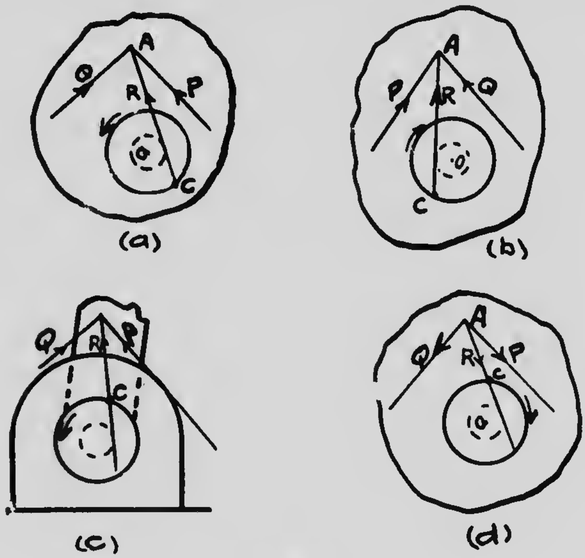


Fig. 98

to the friction circle which latter is concentric with the bearing and has a radius $p = r\sin\phi$. The side of the circle on which the resultant lies is determined by the fact that there must be a turning moment in the sense of motion.

In practice, f is always small and therefore $\tan\phi$ and ϕ are also small, so that no serious error will result in assuming $\tan\phi = \phi = \sin\phi$ and approximately $p = r\tan\phi = rf$.

Four different arrangements of forces on a turning pair are shown correctly drawn on Fig. 98, the same letters being used as in the previous figure, and the reader will be able to check the constructions. The first case (a) shows P and Q acting in interchanged sense from the position shown in Fig. 96, the sense of rotation being the same, and both P and Q act on the outer element, (c) shows a case similar to (a), in all respects except that P and Q act on the inner element, and it will be noticed that this does not change the position of R but moves C from the lower to the upper side. The cases shown at (b) and (d) are similar in that the sense of rotation is the same and the forces act on the outer element, but since the sense of R is reversed,

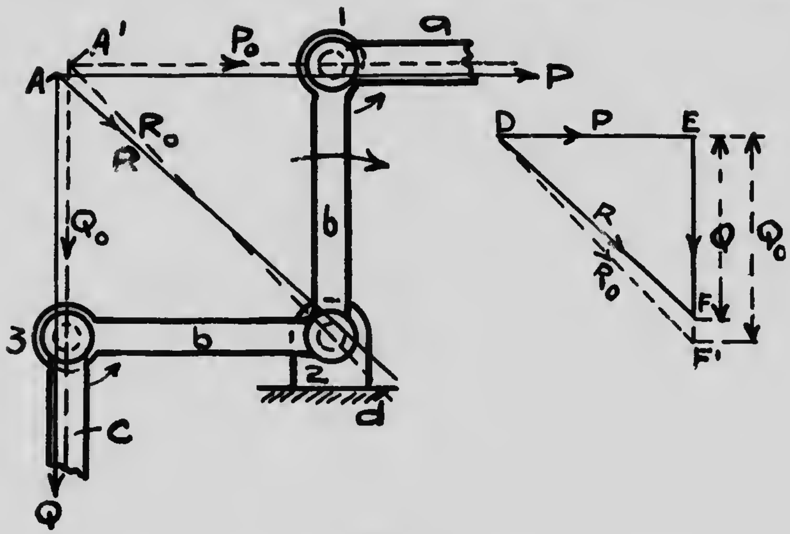


Fig. 99

the latter changes from the left to the right hand side of the friction circle.

The construction already shown will be applied in a few practical cases.

(1) The first case considered will be an ordinary bell-crank lever, Fig. 99, on which the force P acts horizontally and Q vertically on the links a and c respectively, the whole lever turning in the clockwise sense. An examination of the figure shows that the sense of motion of a relative to b is counter-clockwise as is also the motion of c relative to b, therefore P will be tangent to the lower side of the friction circle at bearing 1, and Q will be tangent to the left-hand side of the friction circle at bearing 2, and the resultant of P and Q

must pass through A and must be tangent to the upper side of the friction circle on the pair 3 so that the direction of R becomes thus fixed. Now draw DE in the direction of P to represent this force and then draw EF and DF parallel respectively to Q and R and intersecting at F, then $EF = Q$ and $DF = R$.

In case there were no friction and assuming the *directions* of P and Q to remain unchanged (this would be unusual in practice)' then P, Q and their resultant, would act through the centres of the joints 1, 3 and 2 respectively. Assuming the magnitude of P to be unchanged, then the vector triangle DEF' has its sides EF' and DF' parallel respectively to the resistance Q_o and the resultant R_o so that there is at once obtained the force $Q_o = EF'$. Then the efficiency of the lever in this position is $\eta = \frac{Q}{Q_o}$ and for any other position may be similarly found.

The friction circles are not drawn to scale but are made larger than they should be in order to make the drawing clear.

(2) Let it be required to find the line of action of the force in the connecting rod of a steam engine taking into account friction at the crank and wrist pins. To avoid confusion we shall omit the details of the rod and simply represent it by a line, drawing in the friction circles to a very much exaggerated scale. Let Fig. 100 represent the rod in the position under consideration, the direction of the crank is also shown and the piston rod is assumed to be in compression, this being the usual condition for this position of the crank.

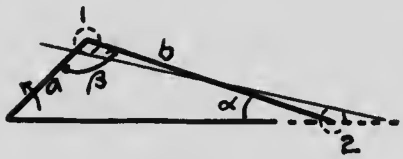


Fig. 100

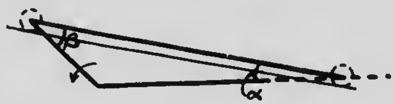


Fig. 101

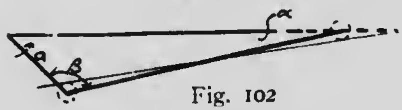


Fig. 102

Inspection of the figure shows that the angle α is increasing and the angle β is decreasing, so that the line of action of the force in the connecting rod must be tangent to the top of the friction circle at 2 and also to the bottom of the friction circle at 1, hence it takes the position shown in the light line and crosses the line of the rod.

This position of the line of action of the force is seen on examination to be correct, because in both cases the force acts on such a side of the centre of the bearing as to produce a turning moment in the direction of relative motion.

Two other positions of the engine are shown in Fig. 101 and 102, the direction of revolution being the same as before and the line of action of the force in the rod is shown dotted. In the case Fig. 101, the rod is assumed in compression and evidently both the angles α and β are decreasing so that the line of action of the force lies below the axis of the rod, while in the position shown in Fig. 102, the connecting rod is assumed in tension, α is decreasing, and β is increasing so that the force intersects the rod. In all cases the determining factor is that the force must lie on such a side of the centre of the pin as to produce a turning moment in the direction of relative motion.

EFFICIENCY OF MACHINES CONTAINING TURNING AND SLIDING PAIRS

The first machine to which we shall apply the methods already described will be the steam engine, as it is a very common and useful

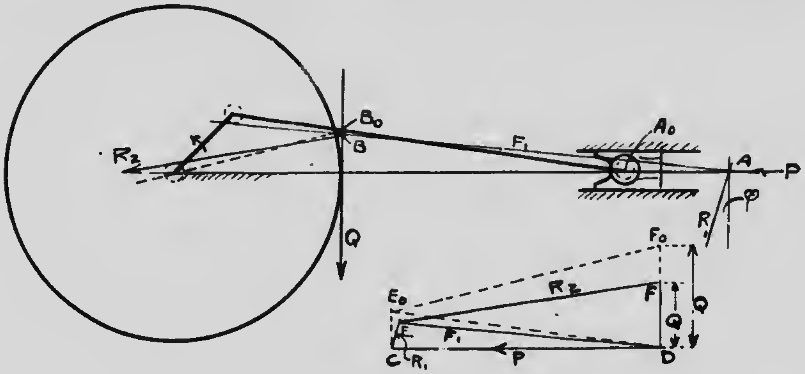


Fig. 103

illustration. Fig. 103 shows simply the outlines of an engine, which is being used for lifting a weight from a pit, the resistance, therefore, is a vertical force acting at the circumference of the drum. No account will be taken of the friction in the rope.

From the principles already laid down, the direction of R_1 is known, also the line of action of F_1 and of R_2. For equilibrium the forces F_1, R_1 and P must intersect at one point which is evidently A, as P, the force due to the steam pressure, is taken to act along the centre of the piston rod. On the crank shaft there is the force F_1 from the connecting rod, the force Q due to the weight lifted, and if there were no friction, their resultant would pass through their point of intersection B and also through O the centre of the crank shaft. To allow for friction, however, R_2 must be tangent to the friction circle at the crank shaft and must touch the top of the latter, hence the position of R_2 is fixed. Thus the locations of the five forces, P, F_1, R_1, R_2, Q are known.

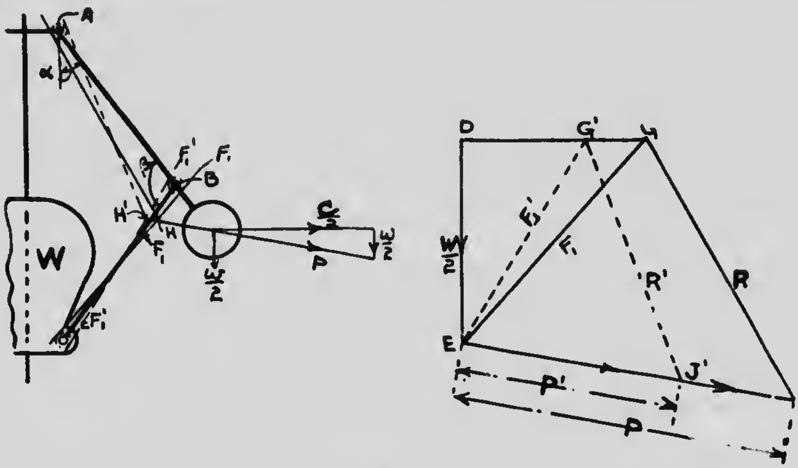


Fig. 104

Now draw the vector diagram, laying off $CD = P$ and drawing CE and DE parallel respectively to R_1 and F_1, which gives these two forces, next draw EF parallel to R_2 and DF parallel to Q which thus determines the magnitude of Q.

If there were no friction, F_1 would be along the axis of the rod, and R_1 normal to the guides, both forces passing through A_o the centre of the wrist pin. Further, R_2 would pass through B_o the intersection of F_1 and Q, it would also pass through O, so that we know the lines of action of all of the forces and may again draw the vector diagram CE_oF_oD, obtaining the resistance $Q_o = DF_o$, which could be overcome by the pressure P on the piston if there were no friction. The efficiency of the machine in this position is then $\eta = \frac{Q}{Q_o}$, and may be found in a similar way for other positions.

A further illustration may be given in the case of the governor illustrated in Fig. 104, which is a copy of Fig. 111, as found in the chapter in this work dealing with governors. Only one half of the governor is shown, and as generally constructed it will be safe to neglect the friction of the weight W on the spindle, also for simplicity the same assumption as before is made, that W includes the pull of the valve gear on the sliding weight and also the weight itself. In the problem solved in Chapter XII, no account is taken of the friction nor of the pressures on the pins, in this case, however, these pressures must be known, because the frictional resistance depends directly on them, so that a somewhat different method of treatment will be adopted here. The friction circles at A, B and C are not drawn to scale, being much larger on the drawing than they should be.

On the hall there is the centrifugal force $\frac{C}{2} = \frac{w}{2g} r\omega^2$ and also the action of gravity $\frac{w}{2}$, the two acting at right angles to one another, these forces produce the resultant force P acting on the ball. Now the arms AB and BC are evidently both in tension also, when the balls are moving outward, α increases and β decreases, so that the direction of the force F_1 in the arm BC crosses the axis of the latter as shown. On the ball arm the forces acting are P and F_1, and if there were no friction at A their resultant R would have to pass through the centre of A. On account of friction, however, the force R passes tangent to the friction circle and on the left side, this force must also pass through the intersection of F_1 and P at H.

The weight W is held up by the pull F_1 in BC and a corresponding pull in the arm of the other half of the governor, thus draw $DE = \frac{W}{2}$ and DG horizontal and GE parallel to F_1, the latter line will evidently represent the force F_1. Next draw GJ and EJ parallel respectively to R and P, P is thus known, and if the weight w is known, the corresponding speed may be easily computed by the methods given in the following chapter.

Next consider the case where the ball moves inward from the same position, W remaining unchanged. Here the arms are still in tension and will decrease while B increases, which causes F_1 to change to the position F_1' and R to the dotted position and H to H'.

Drawing the vector diagram starting with $DE = \frac{W}{2}$ as before we find $P' = EJ'$ as the force which must now be applied to the ball, and since w will usually be small compared with C, it is approximately true that C has decreased in the same ratio as P or $C' = C\frac{P'}{P} = C'\frac{EJ'}{EJ}$. But $\frac{C'}{C} = \left(\frac{\omega'}{\omega}\right)^2 \therefore \frac{\omega'}{\omega} = \sqrt{\frac{C'}{C}} = \sqrt{\frac{EJ'}{EJ}}$, which shows the proportional falling off in speed before the balls begin to move inward. Or to explain more fully, suppose that the balls are slowly moving outward, having reached the position shown, and that there is then a load thrown on the engine causing the speed to decrease, the solution of this problem shows that the speed would have to decrease in the ratio $\sqrt{\frac{EJ_1}{EJ}}$ before the balls would begin to move inward.

We have assumed P' and P to be in the same direction, which is not strictly correct, but the error introduced in this assumption is negligible in practice.

It must be understood that no allowance has been made for friction in the valve gear, W being taken as the known force necessary to move the weight and valve gear. Further, the friction circles are all drawn to a very much enlarged scale to make the work clear, and no governor would be of much practical value in which the ratio $\sqrt{\frac{EJ'}{EJ}}$ was at all large. The reader would probably do well to study this problem after having read the chapter on governors.

CHAPTER XII

GOVERNORS

In all prime movers, which we will briefly call engines, there must be a continual balance between the energy supplied to the engine by the working fluid and the energy delivered by the machine to some other which it is driving, *e.g.*, a dynamo, lathe, etc., allowance being made for the friction of the prime mover. Thus, if the energy delivered by the working fluid (steam, water or gas) in a given time exceeds the sum of the energies delivered to the dynamo and the friction of the engine, then there will be some energy left to accelerate the latter, and it will go on increasing in speed, the friction also increasing till a balance is reached or the machine is destroyed. The opposite result happens if the energy coming in is insufficient, the result being that the machine will decrease in speed and may eventually stop.

In all cases in actual practice, the output of an engine is continually varying because if a dynamo is being driven by it for lighting purposes the number of lights in use varies from time to time, the same is true if the engine drives a lathe or drill, the demands of these continually changing.

The output thus varying very frequently, the energy put in by the working fluid must be varied in the same way if the desired balance is to be maintained, and hence if the prime mover is to run at constant speed some means of controlling the energy admitted to it during a given time must be provided.

Various methods are employed, such as adjusting the weight of fluid admitted, adjusting the energy admitted per pound of fluid, or doing both of these at one time, and this adjustment may be made by hand as in the locomotive or automobile, or it may be automatic as in the case of the stationary engine or the water turbine where the adjustment is made by a contrivance called a *governor*.

A governor may thus be defined as a device used in connection with prime movers for so adjusting the energy admitted with the working fluid that the speed of the prime mover will be constant under all conditions. The complete governor consists essentially of two parts, the first part consisting of certain masses which rotate at a speed proportional to that of the prime mover, and the second part a valve or similar device controlled by the part already described and operating directly on the working fluid.

It is not the intention here to discuss the second part, or valve, because this takes various forms, according to circumstances and forms a subject of study by itself for each given case. Suffice it to say that this device is usually made to act in one of the following ways:

(*a*) To partly close off the working fluid and thus reduce the weight admitted in a given time; *e. g.* the water in a water turbine or the length of cut off in a steam engine.

(*b*) To reduce the energy per pound of working fluid admitted. *e.g.*, to throttle the steam and thus reduce its pressure as it enters an engine.

(*c*) Various combinations of the above methods.

The part of the governor which has masses revolving at a speed proportional to that of the engine will now be considered in detail,

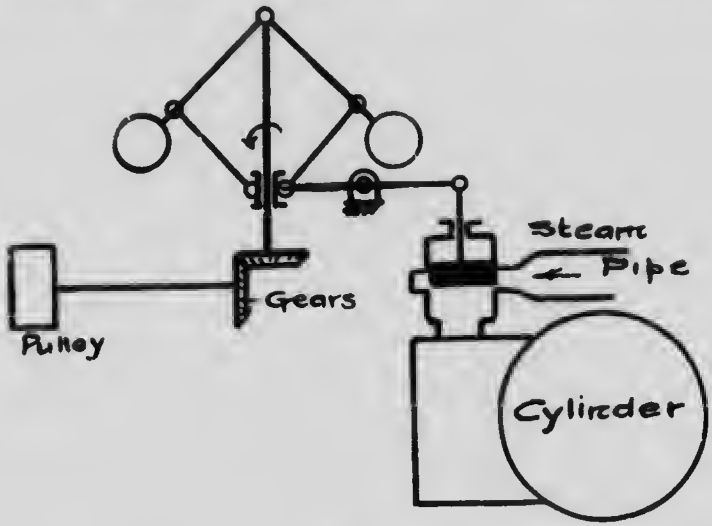


Fig. 105

and for convenience will be referred to in future as the governor. One of the simplest forms of this device, also shown diagrammatically in connection with the valve as required for a throttling steam engine is shown at Fig. 105. It consists of a vertical spindle, driven from the engine shaft at a speed which bears a fixed ratio to that of the crank shaft. To this spindle balls, are attached by arms, as shown, and these balls are again connected to a sleeve, which is free to slide up and down the spindle. To this sleeve the throttling valve or valve gear is attached by suitable mechanism such as that indicated in the figure. The action is evidently as follows: assume the engine to be running at normal speed, then the balls will rotate in a given plane the height of which will be fixed by the resultant of the centrifugal force on the balls, the weight of the latter and the pull produced by the collar. If now part of the load be suddenly thrown off the engine the latter will tend to speed up, the centrifugal force will increase and the ballf will rise, lifting the collar and closing the supply of steam until the equilibrium

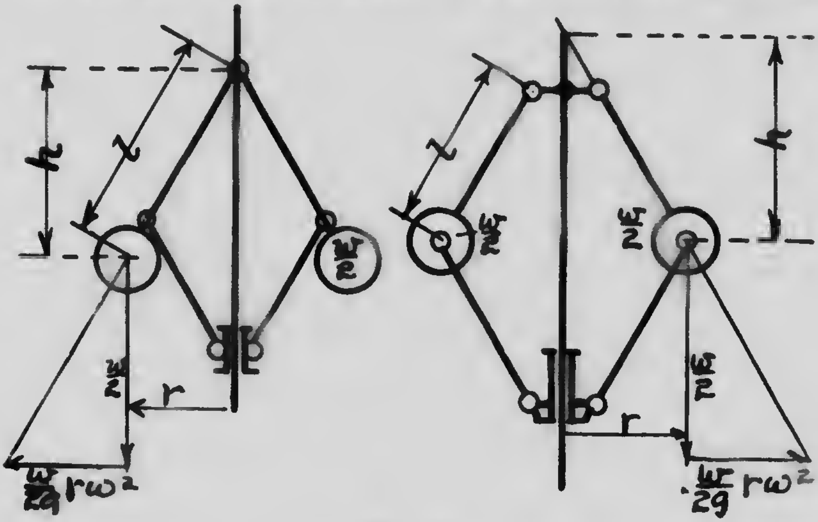


Fig. 106

is again restored, but in general the balls will rotate in a higher plane than before. The converse is true for decrease of load.

Let us examine the problem first of all without considering the effect of friction or the resistance offered by the sleeve. Let each ball have a weight $\frac{w}{2}$ and rotate in a circle of radius r ... Fig. 106, and let the spindle rotate with angular velocity ω radius per second. Each ball is held in equilibrium by three forces; (*a*) the attraction of gravity parallel to the spindle of amount $\frac{w}{2}$ pds., (*b*) the centrifugal force acting normal to the spindle and of amount $\frac{w}{2g} r\omega^2$ pds, (*c*) the resultant of these two forces must be in the direction of the arm.

Now if we take l to represent length of the arm, and h the vertical height from the plane of rotation of the halls to the place where the ball arms (or the arms produced, see figures) intersect the spindle the following relation is at once evident:

$$\frac{w}{\frac{w}{g} r \omega^2} = \frac{h}{r} \text{ or } h = \frac{g}{\omega^2}$$

or the *height h* varies inversely as the square of the angular velocity and is independent of the dimensions of the parts of the governor.

An examination of thisgovernor will show at once that it possesses

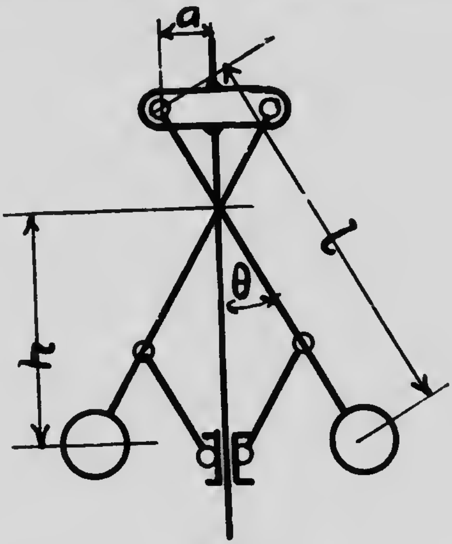


Fig. 107

certain serious defects: (1) That from the very construction of the governor a change in adjustment of the collar will correspond to a change in the height h and hence a change in ω, a condition which it is the purpose of a governor to prevent, for a governor is designed essentially to keep the speed constant for all loads on the engine, and (2) That for any reasonable value of ω, h is very small. Thus let the spindle turn at 120 revs. per min., then $\omega = \frac{2\pi \times 120}{60} = 4\pi$ radians per sec, and $h = \frac{g}{\omega^2} = \frac{32.16}{(4\pi)^2} = .2036$ ft. or 2.44 in. a dimension which is so small as to be difficult to work with in practice.

The first defect is described by saying that the governor is not *isochronous*, the meaning of isochronism being that the speed of the governor will not vary during the entire range of travel of the collar, or in other words the valve of the engine may be moved to any position to suit the load, and yet the engine will run at the same speed. It will at once be recognized that if isochronism had no counterbalancing disadvantages it would be very desirable and we shall see how it may be accomplished.

From the formula $\omega^2 h = g$ it is evident that if ω is to remain constant the height h must also remain constant, and it is evident that the crossed arm arrangement sketched in Fig. 107 will, under certain conditions, give approximately constant heights for different positions of the balls.

Inspection of the figure gives the relation

$$h = l \cos \theta - a \cot \theta.$$

If now h is to remain constant during variations in θ we have

$$\frac{dh}{d\theta} = O = -l \sin \theta + a \operatorname{cosec}^2 \theta$$

$$\text{or} \quad a = l \sin^3 \theta$$

$$\text{and } h = l \cos^3 \theta$$

$$\therefore \ a = l \sin^3 \theta = h \tan^3 \theta = \frac{g}{\omega^2} \tan^3 \theta$$

Ex.—Let $\omega = 10$ radians per sec. (97 revs. per min.), $\theta = 30°$

Then $a = .062$ ft. $= .74$ in., and $l = \dfrac{a}{\sin^3 \theta} = .494$ ft. $= 5.92$ in.

The value of h corresponding to $\theta = 30°$ is .322 ft., and when through a changed load the balls move out till θ becomes 35° then h becomes .316 ft. a decrease of about 2% and the change in speed corresponding to this is slightly less than 1%.

In the case of the governor without the crossed arms taking $\omega = 10$ as before, a change of θ from 30° to 35° means a change in speed of 3%.

It has been suggested that a governor of this type could be made isochronous for a large range of positions, provided the centres of the balls are made to move so that they always lie in a paraboloid of revolution which has the spindle for its axis, and it may be shown that for such a design h and therefore ω will be constant for all positions.

The defect of an isochronous governor, however, is that it will alter its position enormously for the slightest momentary change in speed of the engine and the balls will race out and in producing corresponding changes on the engine, and there is considerable *hunting* for the correct position, *i.e.*, such a governor is not *stable*. The condition of instability is not admissible in practice and designers always must sacrifice isochronism to some extent to the very necessary feature of stability, because this hunting of the balls for their final position means that the valve is being opened and closed too much and hence that the prime mover is changing its speed continually or is *racing*. Reverting to our original example, it will at once appear that a definite position of the balls will correspond to each speed because for each position there is a definite value of h, and therefore of ω.

We shall next consider the modification introduced by Mr. C. T. Porter, which consists in placing a heavy weight on the collar or sleeve of the governor, either with or without crossed arms, Fig. 108 shows such an arrangement. The conditions of equilibrium are readily solved by the phorogr.ph considering OP as the link of reference, thus the images of Q and P are as shown, and by taking moments about O we get

$$\frac{W}{2} . 2\, l \sin\theta + \frac{w}{2}\, l \sin\theta - \frac{w}{2\,g}\, r\, \omega^2\, l \cos\theta = O$$

From which it follows that

$$h = \frac{2\,W + w}{w} . \frac{g}{\omega^2}$$

Ex. Given $l = .75$ ft. (9 in.) $\omega = 20$ radians per sec. (194 revs. per min.) $w = 8$ lbs. $\theta = 45°$, $a = O$, we find $h = .53$ ft. and hence $W = 2.8\, w = 22.4$ lbs.

This governor possesses the following important advantages over the type already described:—

(*a*) The height h may be adjusted to suit any proportions required in practice merely by altering W to suit.

(*b*) The variation in height h corresponding to a given change in ω is very much increased in this case. Thus for a given alteration in speed the change in position of the sleeve is much greater than ofrmerly, or a smaller range in speed will be necessary to correspond to the two extreme positions of the throttling valve, that is this governor is more sensitive than the former one.

To illustrate this take a simple unweighted governor for which the relation is $h = \frac{g}{\omega^2}$ or $g = h\,\omega^2$

By differentiation we obtain the result

$$\frac{\delta h}{h} = -\frac{2\,\delta \omega}{\omega}$$

or the proportional change in height is twice the proportional change

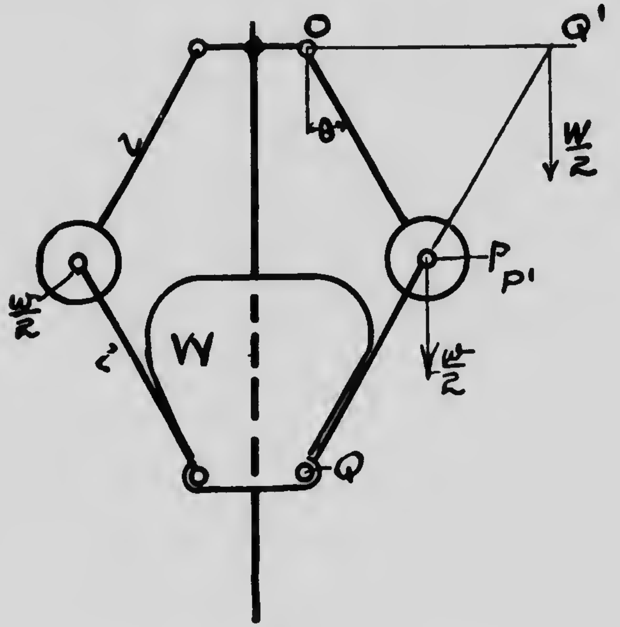


Fig. 108

in speed and an increase in the former means a decrease in the latter.

The ratio $\frac{\delta \omega}{\omega}$ is called the *sensitiveness** where $\delta \omega$ is the change of speed corresponding to the extreme range of travel of the sleeve and ω is the mean angular velocity. Now let $\omega = 10$ radians per sec., and suppose the total range of height of the sleeve is ½ in., the height h for $\omega = 10$ being 3.86 in.

Here $\frac{\delta h}{h} = \frac{\frac{1}{2}}{3.86} = .129 = -2\,\frac{\delta \omega}{\omega}$ or $-\frac{\delta \omega}{\omega} = .064$

or for this range the variation of speed or sensitiveness is 6.4%.

*This method of defining sensitiveness is a little misleading because the governor having the least value of this ratio is said to be the most sensitive. For want of a better designation the above definition has been adopted in this book.

For the weighted governor let $W = 60$, $w = 8$ and taking $\omega = 10$ as before, we get

$$h = \frac{2W + w}{w} \times 3.86 \therefore \frac{\delta h}{h} = \frac{w}{2W + w} \times .129 = .008$$

hence $\frac{\delta \omega}{\omega} = \frac{.008}{2}$ or the sensitiveness here is .4%.

It is evident that as it is not possible to produce exact isochron-

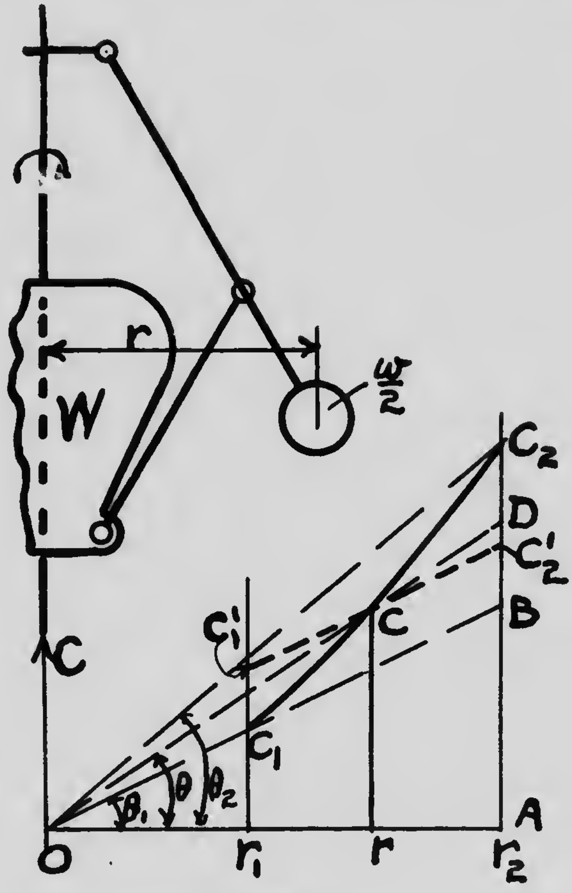


Fig. 109

ism in a governor it will be very desirable to have it as sensitive as possible, and thus decrease the variation in speed.

(*c*) Since the sleeve already has a very heavy weight attached to it, therefore the governor is less effected by friction of the sleeve on the spindle or the pull necessary to operate the valve gear and have such a governor is said to be *powerful.* Powerfulness is also a very desirable feature, for it is well known in practice that the force necessary to operate the valve gear is not constant and, therefore produces a variable effect upon the rotating parts, it is thus important that this variable effect be made as small proportionately as possible.

Thus the Porter governor may be arranged to suit any speed, the variation in speed corresponding to the extreme positions of the valve gear may be made as small as required and the governor is not greatly affected by the external forces produced by the connection to the valve gear.

Having now generally defined and explained the terms employed in connection with governors we shall choose one or two types and study them more in detail. Let us consider the weighted or Porter governor illustrated in Fig. 109.

THE CHARACTERISTIC CURVE. A curve showing the relation between the radius of rotation of the balls and the centrifugal force is of very great value in studying governors and as its shape shows, very many things connected with the action of the governor it is called the characteristic curve, or we shall simply call it the C curve. Let r_1 and r_2 represent the inner and outer radii of rotation of the balls, the corresponding angular velocities being ω_1 and ω_2, and let r be the radius of rotation corresponding to the mean speed of rotation ω, defined by the formula $\omega = \frac{\omega_1 + \omega_2}{2}$. Now at any radius the centrifugal force $C = \frac{w}{g} r \omega^2$ where w is the total weight of the two balls and r is in ft., ω in radians per sec.

If now it were possible to make the governor isochronous we would have $\omega_1 = \omega_2 = \omega$ a constant, and hence C would depend on r only, *i.e.*, the C curve would be a striaght line passing through O as shown at OC and here the ball may occupy any position at the same speed, such an arrangement is not stable as has been said already. If we plot the C curve for the case shown in figure, however, it takes the form $C_1 C C_2$ crossing the curve OC and being steeper than OC where they intersect. It will be evident that the curve $C_1 C C_2$ means that the speeds are not the same for the three positions of the balls and from the formula $C = \frac{w}{g} r \omega^2$ it is seen that $\omega_1 < \omega < \omega_2$.

This curve $C_1 C C_2$ corresponds to a stable arrangenmet because to each position only one speed corresponds and such speed increases as the balls move out.

Further, the shape of this curve is a measure of the sensitiveness of the governor as is shown below. Calling S the sensitiveness, we have

$$S = \frac{\omega_2 - \omega_1}{\omega}; \text{now } \frac{\omega_2 - \omega_1}{\omega} = \frac{(\omega_2 - \omega_1)(\omega_2 + \omega_1)}{\omega\ (\omega_2 + \omega_1)}$$

$$\therefore S = \frac{1}{2} \frac{\omega_2^2 - \omega_1^2}{\omega^2} \text{ since } \omega = \frac{\omega_1 + \omega_2}{2}$$

$$\text{but } C = \frac{w}{g} r \omega^2 \quad \therefore C_1 = \frac{w}{g} r_1 \omega_1^2 \text{ and } C_2 = \frac{w}{g} r_2 \omega_2^2$$

$$\therefore S = \frac{1}{2} \left[\frac{\frac{C_2}{\frac{w}{g} r_2} - \frac{C_1}{\frac{w}{g} r_1}}{\frac{C}{\frac{w}{g} r}} \right] = \frac{1}{2} \left[\frac{\frac{C_2}{r_2} - \frac{C_1}{r_1}}{\frac{C}{r}} \right] \text{ where}$$

C is the centrifugal force corresponding to the mean speed ω.

$$\text{Again } \frac{C_1}{r_1} = \tan \theta_1 = \frac{BA}{OA} \quad \text{also } \frac{C_2}{r_2} = \frac{C_2 A}{OA} \text{ and } \frac{C}{r} = \frac{DA}{OA}$$

$$\text{Hence } S = \frac{1}{2} \left[\frac{\tan \theta_2 - \tan \theta_1}{\tan \theta} \right] = \frac{1}{2} \left[\frac{C_2 A - BA}{DA} \right]$$

$$= \frac{1}{2} \frac{C_2 B}{DA}$$

This curve shows that an increase in the stability of the governor means a decrease in the sensitiveness. If at any part of its length the C curve is radial from O at that part $S = O$ and the governor is isochronous and therefore not stable so that if stability is desired the curve must make as great an angle as possible with the line joining it at any point to O, but on the other hand this angle must not make too large an angle on account of a decreased sensitiveness. For example at C_1 the governor is as nearly isochronous and unstable so that we would get most uniform results by making the curve $C_1 C C_2$ as nearly a straight line as possible.

It is desirable here to point out that if the curve takes the dotted form $C_1' C C_2'$ then the sensitiveness is very much improved and may be made almost perfect, but here since $C = \frac{w}{g} r \omega^2$ the outer position corresponds to the lowest speed and the inner position to the highest speed since it is evident from the figure that C does not increase as rapidly as r. Such an arrangement is evidently unstable since by an increase in speed more energy is imparted to the balls and the weights are being lowered thus further increasing the energy supplied to the system, instead of balancing it, so that if the ball begins to move inward it will fly to its inmost position under the combined action of the two forces. Thus we get stability only when the C curve is steeper than the line from O which cuts it. In Fig. 110 curve $C_1\,C\,C_2$ denotes stability, $C_1'\,CC_2'$ instability, $C_1\,C\,C_2'$ stability of the part $C_1\,C$ and instability for the part CC_2.

This C curve may be used for a further purpose of showing the powerfulness of the governor, since on the curve horizontal distances denote the space through which the ball moves and vertical

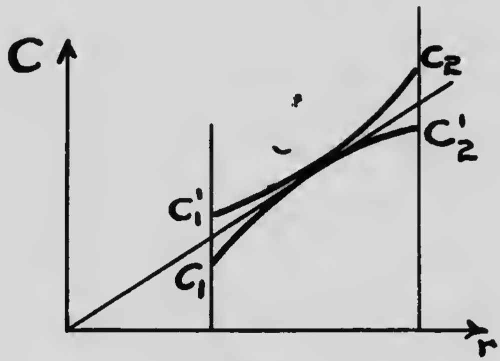


Fig. 110

heights the corresponding forces acting while the ball is moving. Thus any elementary area $C.\delta r$ on this diagram represents a produes of force and distance or work done hence the area $C_1\,r_1\,r_2\,C_2\,C_1$ it the work done on the balls while they move out, and further represents the work which can be done by the halls on the weights and valve gear.

This total work $= \int Cdr$ is expended in lifting the weigh *w*st and W', and in overcoming friction and resistance offered by the valve gear. We shall neglect the effect of friction (although in the actual case it must be considered) and shall further assume that the resistance in the valve gear may be included in W, it should, however, be stated that this latter resistance is variable and these variations

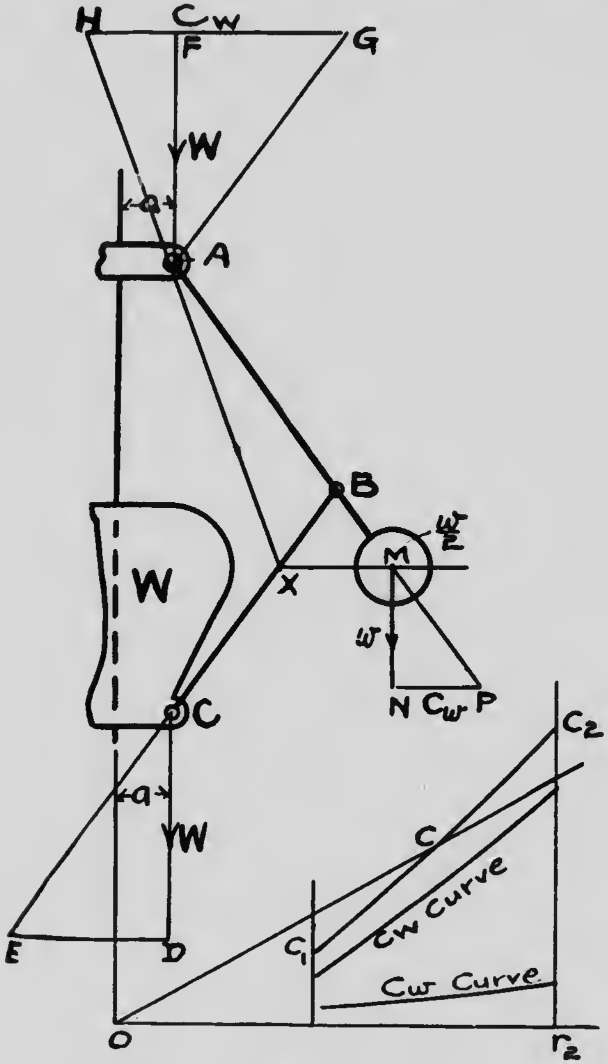


Fig. 111

should be considered in any design, but are omitted here on account of the complication of the cases. The effect of this variation on the governor's action will depend largely on the magnitude of it as compared with the total weight W and the mean resistance offered by the gear.

It will be noted that the force C is balanced by the sum of several other forces, viz.: (*a*) That due to the weight of the balls w; (*b*) That due to the central weight W; (*c*) The resistance at the collar, and (*d*) The friction of the parts. We shall assume that the force required to move the valve gear, (*c*) is included in (*b*), *i.e.*, that the force necessary to move the valve gear is constant and that this force plus the central weight amounts to Wpds, we shall also neglect friction. Fig. 111 shows a Porter governor with the corresponding C curve. To find the part of C necessary to lift the weight w we resolve w in the directions of C and of the arm, then C_w is the part in the direction of C and which

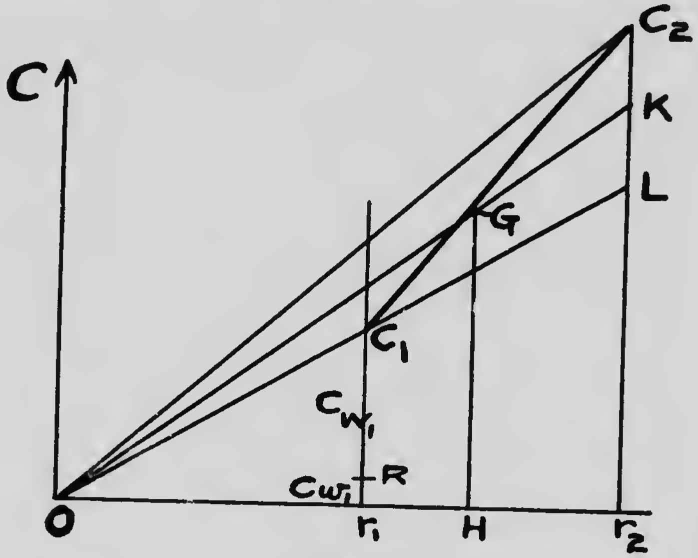


Fig. 112

must be overcome by the latter while the remaining part produces a pull on the upper pin A. Lay off above the axis of r the values of C_w thus found for each position of the balls getting the C_w curve. Next find the part of C necessary to balance W as follows—Draw a $\triangle CDE$, making $CD = W$ and making BCE a straight line, then CE is the resolved part of W in the direction of the arm CB. Now oreslve CE into two parts, one horizontal C_W and one passing through the pin A, these forces are at once found by the method shown above A, where $FA = W$ and GA is parallel to CB, HG is then C_W the part of C necessary to balance W. Lay off C_w, for each position of the balls, above Or obtaining the C_w curve. Since $C = C_w + C_W$ the ordinates between the curves of C and C_W must represent the corresponding C_w.

It will be at once evident that for the unloaded governor the power is only the area below the C_w curve, since, neglecting friction and the pull of the valve gear, the whole of C is spent in overcoming the effect of the weight of the balls or $C = C_w$, while for the loaded governor it is very much increased, being the area below the C curve. The work represented by the area below the C_W curve

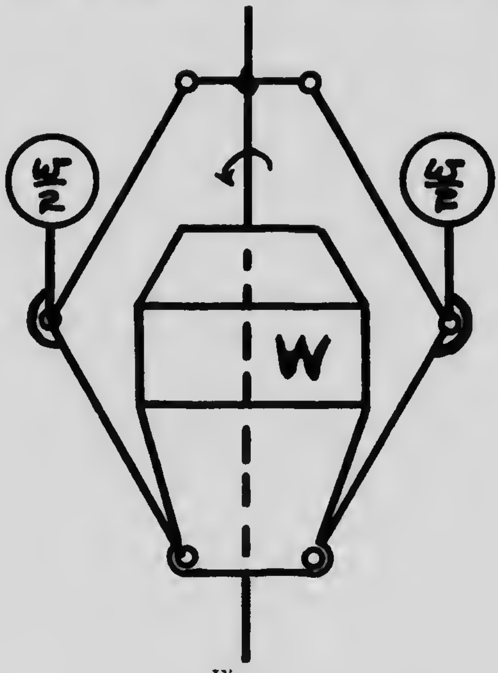


Fig. 113

must be employed in lifting the central weight and overcoming any force necessary at the clutch to operate the valve gear.

Before leaving this matter the C curve will be applied to the solution of one problem in the design of a governor. Suppose it is required to design a governor of the Porter type to operate at a mean speed Nn revs. per min. and the maximum and minimum speeds n_2 and n_1 are given. The work to be performed (or the power) is also given, to find the dimensions of the various parts. From general experience certain proportions will be known and only one or two points remain to be determined. We shall assume that r_1 and r_2 are given, also the lengths $L = AB = BC$ of the arms. In Fig. 112 lay off r_1 and r_2 to scale, then from the work which the governor is to do lay off GH equal the mean height up to the C curve (note the area $GH \times r_1 r_2$ is the total work of the governor including that required to lift the balls, the available work at the clutch will be correspondingly decreased). Now the sensitiveness $S = \frac{n_2 - n_1}{n}$ which is given, hence we lay off the distance (see page 164) $S \times Kr_2$ $(= KL = KC_2)$ both above and below K along the line $r_2 K$, then joining LO we at once get C_1 and C_2 and without serious error the C curve is the line $C_1 G C_2$.

Next $C_1 = \frac{w}{g} r_1 \omega_1^2$ gives at once w the weight of the two balls since C_1 r_1 and ω_1 are known. This may be also computed from $C_2 = \frac{w}{g} r_2 \omega_2^2$. We may now finish the problem in one of several ways depending on which quantities we assume and which we leave to determine. Probably we could best assume the angle ABC (which gives us the $\angle ABC$) and also W. Assuming angle ABC at once enables us to draw the $\triangle MNP$, see Fig. 111, from which we find C_{W_1} which we lay off along $r_1 C_1$ and we then get $C_{W_1} = RC_1$. Now above A lay off $AF = W$ and draw AG parallel to CB and from G lay off GH horizontally through F so that $GH = C_W$ and then HA produced intersects CB at X and the horizontal line through X intersects AB at M the centre of the ball. The radius r_1 then locates the spindle and the design is complete.

The design should be checked at the outer radius r_2 and also the exact form of the C curve should be found and if it does not agree with that assumed, some of the assumed quantities must be adjusted and the calculation made over again.

Before leaving the matter it must be stated that the design of a governor is a very complicated piece of work in the actual case because the effect of friction is very serious and must in all cases be taken into account and further the exact forces at the clutch necessary to operate the valve gear must always be determined, and these are not constant. The determination of these forces is too complicated and lengthy to be introduced here and must be left to be considered by the designer, but when these forces have been determined the work may be carried out by a method similar to that described.

Fig. 113 shows an outline of a weighted governor by Proell possessing advantages over the Porter type which it would be of considerable value for the student to work out for himself.

SPRING GOVERNORS

The modern tendency is to replace the weight W' by a spring and as this usually means a rather different disposition of the parts,

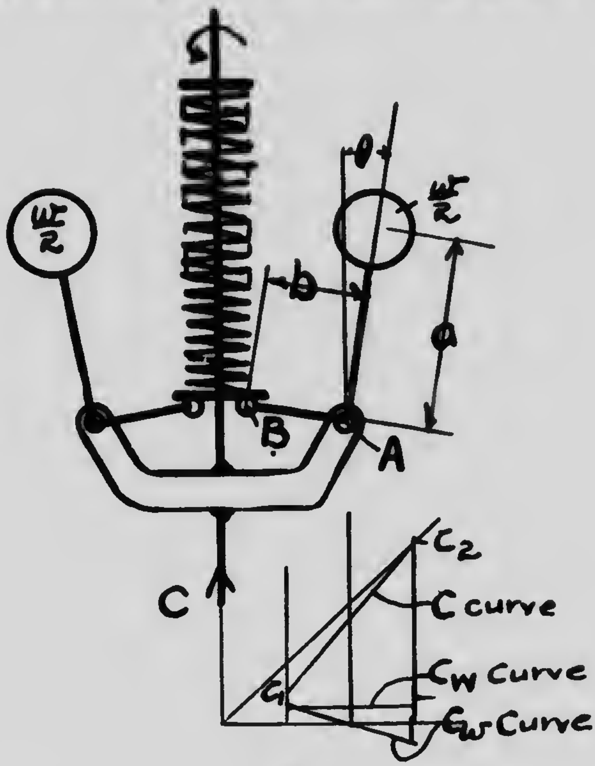


Fig. 114

some consideration will be given to it here. Fig. 114 shows one of the very simplest governors of this type which is shown mounted on a vertical shaft although it is quite as frequently used on a horizontal shaft, it generally runs at a fairly high speed. Now let W' be the weight on the spindle including the spring weight, F be the force produced by the spring, and w be the combined weight of the two balls. Then taking moments about A for the effect of W' we get $W' b \cos\theta = C_W a \cos\theta$ or $C_W = \frac{b}{a} W' =$ const. and further taking moments about A for the ball weight w we get $C_w a \cos\theta = - wa \sin\theta$ or $C_w = - w \tan\theta$. So that C_w is positive or negative according to the value of θ. From a knowledge of these curves for C_W, C_w and C we are at once able to draw the C_v curve showing the resistance which must be offered by the spring together with the force required to move the valve gear. Such governors are evidently powerful and may be made as sensitive as desired.

Calling F the force exerted by the spring assuming the curve C_v to be a straight line, we may readily obtain the necessary data for the design of spring. Thus $F b \cos\theta = C_v a \cos\theta$ or

$F = \frac{a}{b} C_v = C_v \times$ a constant so that the C_v curve may be also taken to represent a curve of F on a different scale, thus at radius

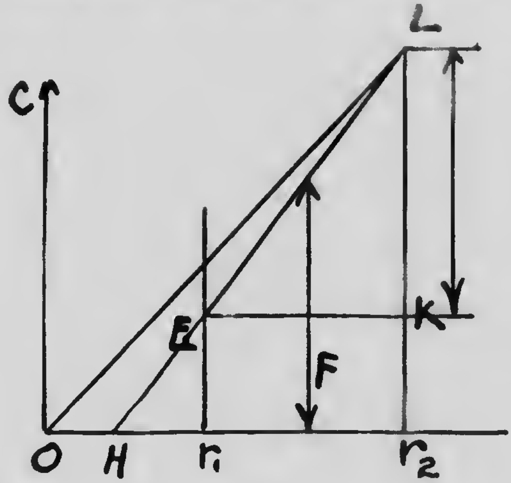


Fig. 115

r_1 the value of F is rerpesented by $r_1 E$. Now produce the curve for C_v till it meets the axis of r at H. Then at radius $O H$, $F = O$ and hence the spring must be so designed that its zero compression corresponds to OH and the compression force S which it must produce per unit of compression will be

$$L K \times \frac{a}{b} \times \frac{1}{r_2 - r_1} . \quad \text{(Fig. 115.)}$$

An arrangement of this kind is not to be recommended because of the very great pressure and the corresponding friction produced on the pin A. A governor of the form already described is used on the small Leonard engines amongst others, but its shaft there is horizontal so that C_r and C_w are zero.

The form of governor used by Belliss and Morcom on their high speed engines is shown at Fig. 116, the governor being on the crank shaft and therefore horizontal so that C_r and C_w become zero and the centrifugal force of the balls is balanced by the pull of the spring and the resistance offered by the valve gear. Taking

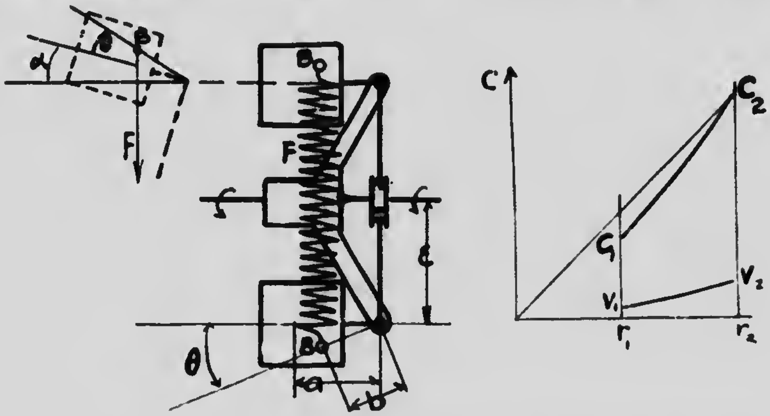


Fig. 116

W as the weight of the two balls and a and b as sketched and calling F the spring pull we may find this force by the formula below provided the resistance offered by the valve gear is neglected. We have

$$F \cdot b \cos(\theta + \alpha) = C\, a \cos \alpha \;\therefore\; F = C\,\frac{a}{b} \cdot \frac{\cos \alpha}{\cos(\alpha + \theta)}$$

where α is the angle between the axis of rotation and the ball arm,

or $F = C\,\dfrac{a}{b} \cdot \dfrac{1}{\cos\theta - \sin\theta \tan\alpha}$ and in this formula a, b, θ are constants, the only variables being C and α, so that in a given case the spring pull F is readily found and the proper design and location of the spring pins B to produce best results may be directly determined.

The spring pull in the actual case should be less than the force F as determined above because of the force necessary to move the valve gear, and of course this difference can only be determined when we know the exact construction and operation of the gear so that no general solution will here be attempted. In any case, however, this may be determined and plotted below the C curve at V_1 V_2 the force V at any time being the part of C necessary to operate the valve gear and the distance from the V curve to the C curve representing the part of C which must be balanced by the spring pull. Having determined the spring pull for each radius, the corresponding value of S can readily be found as in the last example. Then since the spring pull $F = eS$ where e is the extension of the spring and S the force necessary to stretch it one inch, the location of the pin connections must be so chosen that the elongation e is proportional to the force F acting at any instant.

As has already been pointed out, all spring governors may be made very powerful because the spring may be made to offer great resistance without being unduly large or heavy, and hence the angular velocity of these governors may be great. High angular velocity w means large power because the height of the C curve on which the power depends is $C = \frac{w}{g} r w^2$ which evidently increases as the square of w, so that doubling the speed makes the power roughly four times as great.

Certain firms are now undertaking the manufacture of complete governors for specified duty, and the student is recommended to get catalogues from these makers and study the forms adopted by them. The advantage of any form may readily be determined by the methods given.

THE SHAFT GOVERNOR

In modern practice it has been found desirable in many cases to connect the governor directly to the main shaft of the machine, such as the crank shaft of an engine or else to the main lay shaft, as the cam shaft in a gas engine. In general in such a case the revolving weights are pivoted to a wheel keyed on the shaft, the weights thus always revolving in one plane instead of in planes of varying position as in the governors already described. Such governors are commonly called shaft governors and possess numerous points of excellence, so that it will be an advantage to study them with some care.

The shaft governor is used most commonly on steam engines and also finds considerable favor with builders of large gas engines. In the case of the steam engine the revolving weights are usually connected directly to the eccentric which operates the slide valve, the eccentric eye not being fixed to the shaft, but its position controlled by the governor. In most cases the governor alters the eccentricity as well as the angular advance of the eccentric, thus changing all the events of the stroke for a given change in load.

A little thought will show that such governors should be made very powerful because the weights must be able of themselves to hold the eccentric in position against the force necessary to move the slide valve and although the latter always is of special construction in this type of engine yet this force is not inconsiderable; to make such a governor powerful the centrifugal force must be large or the revolving weights must be heavy and we must have high rotative speeds or especially adapted high-speed engines. It

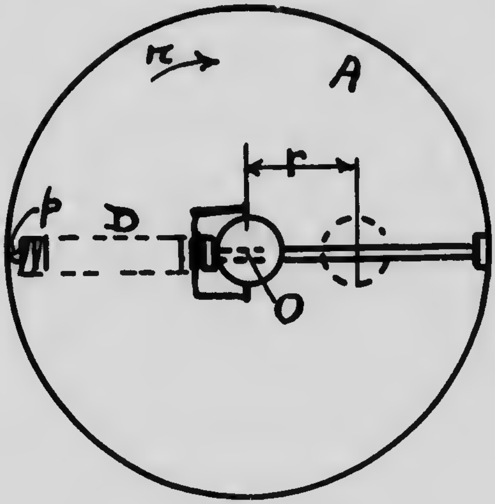


Fig. 117

is not the purpose here to enter into a discussion of the steam distribution as affected by such governors.

Consider the conditions existing on a disk A, Fig. 117, which is revolving about a fixed centre O at n revs. per min., and we shall neglect the effect of gravity because in most governors it is balanced, although in this case no arrangement is shown for this purpose. To this ball let a spring D be attached, which is also attached to the disk at p and let the ball be free to move radially along the rod B. When the ball is at any distance r ft. from the centre of rotation O, the centrifugal force C acting on it is $C = \frac{w}{g} r w^2$ where w is the weight of the ball and ω is the angular velocity in radians per second corresponding to n.

Now let S denote the spring pull per foot of extension and let the spring have no extension when the ball is at O, thus for this position of the weight the extension of the spring will be r ft. Then the pull exerted by the spring will be $s \times r$ pds., and as there must be equilibrium between the pull of the spring and the centrifugal force we have $s^1 r = \frac{w}{g} r \omega^2$ or $s^1 = \frac{w}{g} \omega^2$. We shall find it convenient to use S to denote the force required to change the length of the spring one inch so that $s^1 = 12S$. And if r be also measured in inches then we get by supplying the constants $Sr = .0000284 w r n^2$ for the

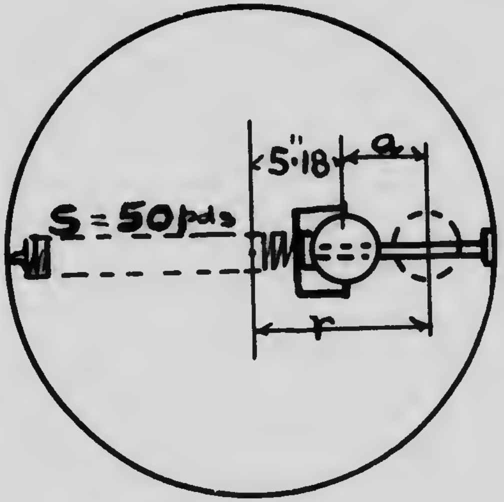


Fig. 118

inch unit. Suppose now we wish to have n constant for all values of r, *i.e.*, an *isochronous* arrangement, we would then make $S = .0000284 w n^2$, or if we take $w = 25$ lbs, $n = 200$ revs. per mi., $S = 28.4$ lbs. *i. e.*, if to this ball we attach a spring so designed that a force of 28.4 pounds will change its length by one inch, and if further the spring be so connected with the ball that the extension of the former is always equal to the radius of rotation of the latter, then the arrangement is isochronous, or the ball will remain at any radius from the centre so long as the speed is 200 r. p. m. It will be evident, however, that the least external force would send the ball to the extreme end of its travel, or it is not stable.

Now let us examine the effect of altering S and let us take two cases (1) $S = 50$ pounds, and (2) $S = 24$ pounds. Taking the first case, let us assume as before a condition of equilibrium at 200 revs. per min. when the ball is 12 in. from the centre of rotation. Then $C = .0000284\, w\, r\, n^2 = 340.8$ pounds, and hence the extension of the spring must now be $\frac{340.8}{50} = 6.82$ in. instead of 12 in., in other words the extension of the spring will be less than the radius of rotation of the ball or the spring will have its free length when the ball is 12 — 6.82 = 5.18 in. from O and the arrangement is sketched in Fig. 118 in which the extension of the spring is denoted by a.

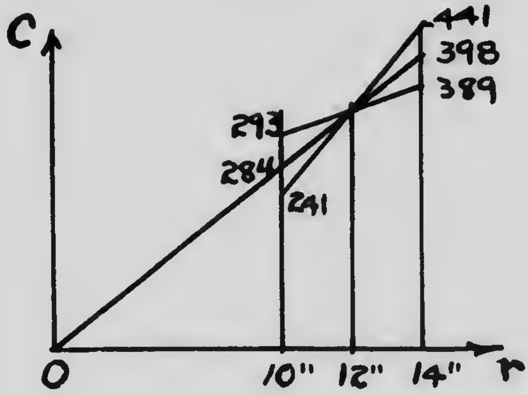


Fig. 119

Now let the ball move out 2 in. n being still 200 revs. per min., a is then 8.82 in, $r = 14$ in. and hence $Sa = 441$, which tends to draw the ball inward while $C = .0000284\, w\, r\, n^2 = 397.6$ pounds tending to force the ball outward and hence the ball will return to its original position at 12 in. radius unless a force of 43.4 pounds be interposed to prevent this. On the other hand if the ball is rotated in a circle of 10 in. radius we would have $Sa = 241$ pounds and C = 284 pounds, so that a force of 43 pounds is urging the ball outward and hence there is only one position at this speed in which it can remain or the arrangement is *stable*.

Now let $S = 24$ pounds, then if equilibrium is to be maintained at $r = 12$ as before we find $a = 14.2$ in. or when the ball is at O the spring will have an elongation of 2.2 in. At 14 in. from the centre $C = 398$ pounds and the spring pull $Sa = 388.9$ or the ball will stay at the outer radius whereas if $r = 10$ in. $C = 284$ and $Sa = 292.8$ pounds or the ball will stay at the inner radius. Hence, f in this case the ball be disturbed at all it will immediately fly outward or inward having no tendency to return to its proper position at 12 in. radius, in other words the equilibrium is *unstable*.

This is very nicely illustrated by a study of the C curves, Fig. 119, in each case.

It is further to be noted that with $S = 50$ we could only have the ball remaining at 14 in. from O when $n = 211$ revs., and at

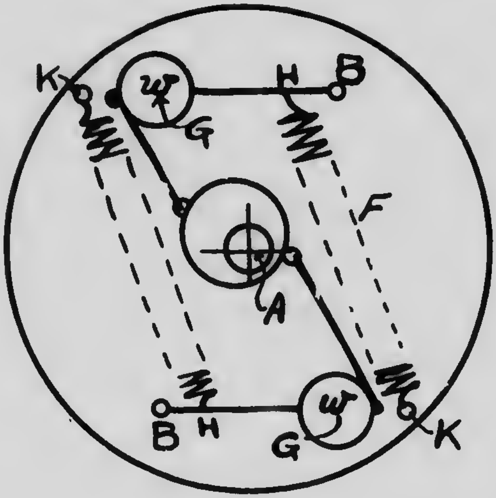


Fig. 120

10 in. when $n = 184$ revs. Hence, if this represents the necessary range of travel of the ball the sensitiveness is $2\left[\frac{211 - 184}{211 + 184}\right] = 15\%$. Where, however, $S = 24$, the corresponding speeds will be 198 revs. for $r = 14$ ins., and 203 revs. at $r = 10$, with the curious result that the speed increases as the balls move inward. Here the sensitiveness is 1.24% as compared with 15% in the previous case, thus, while the sensitiveness is very much improved in the case where $S = 24$ pounds, yet on account of the instability the arrangement is an impossible one.

In the shaft governor, however, the weights cannot be arranged as above, but must be mounted so that they may act directly on the eccentric and, consequently, the forces which they can exert must in some way be controlled. A very common arrangement is shown at Fig. 120, in which two weights are used attached to the rotating disk, or wheel by pins B_1 the centrifugal force of the balls w being balanced by the springs F and links shown connecting the ball arms to the eccentric. (Note—This form of governor is not much used now, but for the purpose of instruction it is chosen as an illustration, the modern form following later on).

Let Fig. 121 represent one half of a typical shaft governor, the other half being similar and the two parts being so connected that

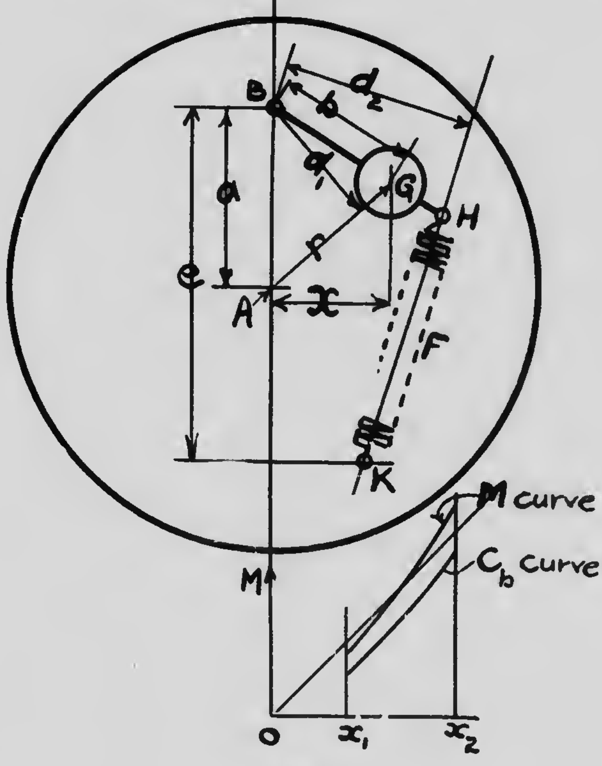


Fig. 121

gravity effect is neutralized. A is the centre of rotation, B the point of connection of the weight with the fly wheel, G the centre of gravity of the weight H, and K the points of connection of the spring to the weight and wheel respectively, and F is the force in the spring. The letters indicate the following $a = AB$, $r = AG$, $b = BG$, d_1 is the shortest distance from B to AG, and d_2 is the shortest distance from B to HK, the direction of the force F.

Now let w be the weight of each revolving mass and F the force produced by one spring, then we have at once $C = \frac{w}{g} r \omega^2 = m r \omega^2$ where $m = \frac{w}{g}$ and the moment of C about the pivot B is $M = m r \omega^2 d_1$ and if we let x represent the shortest distance from G to AB it is at once evident from similar triangles that $r.d_1 = a.x$ and hence that $M = m r \omega^2 d_1 = m \omega^2 a x$. From this it will be seen that M depends entirely on ω and x, and if we choose M and x as axes of co-ordinates, we may plot upon the sheet curves similar to the C curves already taken up. If ω is constant or the governor is isochronous then, evidently M varies directly with x only and the "M" curve will be a straight line passing through O and we have again the case of neutral equilibrium. From what has already been said, it will be evident that if the M curve is steeper than the line from any point on it to O, the arrangement is stable, and on the other hand if the curve is less steep the arrangement will be unstable, the stable condition again corresponding to greater variations in speed than the unstable case, exactly as in the case of the fly ball governor already discussed. Thus the M curve is the characteristic curve for this type of governor.*

Now through K draw a line perpendicular to AB, cutting the latter line at distance e from the pin B. Let C_k be the resolved part of F such that the moment of the spring about $B = C_k . e = F d_2$ and then we have $M = m \omega^2 a x = C_k . e$ provided we neglect the effect of the valve gear. Thus $C_k = m \omega^2 x \frac{a}{e} = const \times m \omega^2 x$, or the C_k curve may also be drawn on the same axes as before, and this curve shows the effect of the spring. From the curve thus drawn the spring pull F may be found and the spring designed to suit the given conditions.

If, in addition to the two curves already described, a C curve on an r base be drawn the power of the governor may be obtained by integrating the quantity $C . dr$ between r_1 and r_2.

* This method is explained very fully in "Die Regelung der Kraftmaschinen," by M. Tolle, a book which the designer of governors will find of very great value.

While the investigations already made enable one to determine the conditions of equilibrium of the parts, they give no information as to the rapidity of the adjustment to new conditions of load, and this point will now be discussed. So far we have only been dealing with the centrifrugal force on the balls, *i. e.*, the force due to the acceleration of the weights along a radius, and this force acts continuously during the running of the governor. When, however, the speed of the wheel is changing during the adjustment for new load, we must accelerate the wheel as well as all masses connected with it, each mass having an angular acceleration $\alpha = \frac{\delta\omega}{\delta t}$ where $\delta\omega$ is the change in the velocity of the wheel in time δt., and further an acceleration in the direction of motion or tangential to the circle

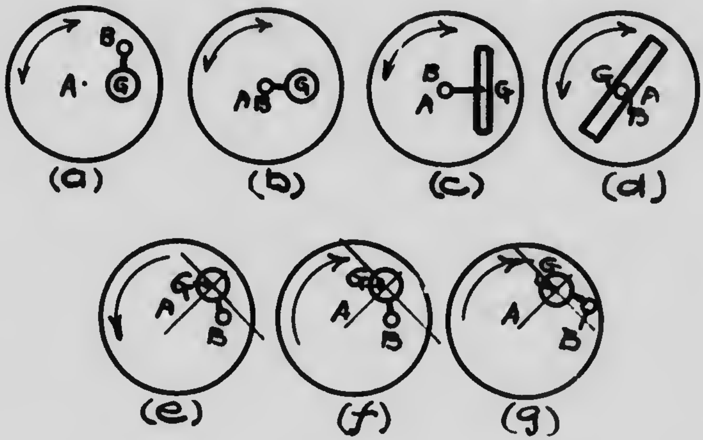


Fig. 122

in which it is travelling, we may call this the tangential acceleration These accelerations of the weights, which only come into play when the speed changes, may be made to oppose or assist the effect due to centrifugal force, and thus may be made to cause slow or rapid change of adjustment.

The diagrams in Fig. 122 will show the meaning of this very nicely where in all cases A is the centre of rotation, B the point of connection of the weight to the disk and G is the centre of gravity of the weight. The centrifugal force due to radial acceleration of the ball is always in the direction AG. At (a) the tangential acceleration produces no effect since the tangent to the path of G

passes through the pin B and the force necessary to accelerate the weight is borne directly by the pin B. At (*b*) the centrifugal effect is zero, the tangential acceleration producing a very decided turning moment about the pin B· but in both of these cases the angular acceleration is small since the weight is concentrated about its centre of gravity or its moment of inertia about its centre of gravity is small. (*c*) and (*d*) show a different distribution of the mass, and in both cases the angular acceleration produces considerable effect, and when we have a change of speed $\delta\omega$ we must not only accelerate the centre of gravity G, but also the whole weight undergoes an angular

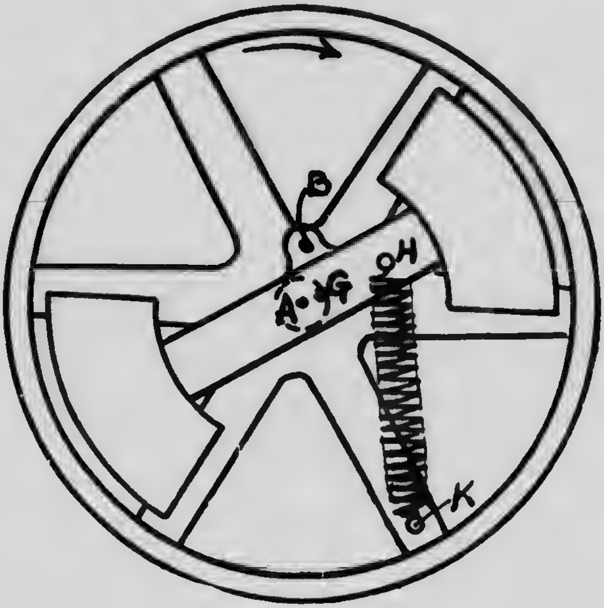


Fig. 123

acceleration, and in (*d*) the angular acceleration is the only active force.

In the figures (*e*), (*f*) and (*g*) the sense of rotation is marked, and we shall suppose that in each case there is a sudden increase in speed corresponding to a decreased load. In fig. (*e*) the tangential acceleration *assists* the centrifugal force in producing rapid adjustment, while in (*f*) these oppose one another resulting in slower adjustment merely due to change of sense of rotation and in (*g*) rapid adjustment is again realized. In these three cases the angular effect is small.

The distribution of the weights for a Rites governor is shown in Fig. 123, and it will be readily seen that the centrifugal effect is not large, comparatively, the tangential effect is also decreased and the angular acceleration produces a very decided effect. Such governors as these adjust themselves very rapidly and may be made as stable as desired, without undue variation in speed for varying loads and positions.

CHAPTER XIII.

SPEED FLUCTUATIONS IN MACHINERY

The flywheel of an engine or punch or other similar machine is used to store and restore energy to the machine according to the conditions. Thus, in an engine the energy supplied by the steam or gas per second is not constant, but varies from time to time, at the dead centres the piston is stationary and hence no energy is delivered by the working fluid, whereas when the piston has covered possibly a third of its stroke, the energy being delivered by the steam to an engine is about its maximum because the piston is moving at high speed and yet the pressure of the steam is high if cut-off has not taken place. Toward the end of the outward stroke of the piston the energy delivered per second is small, because the piston is moving with decreasing velocity, and also the pressure of the steam has very much decreased due to expansion, and in the return stroke the piston must supply energy to the working fluid to drive it out through the exhaust ports.

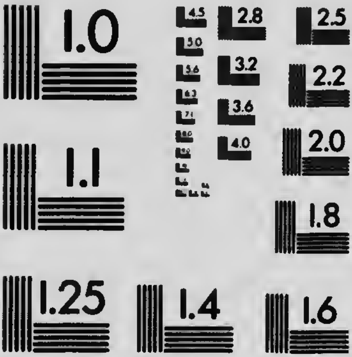
In the case of the belt-driven punch, we supply to the main shaft a constant quantity of energy per second through the belt.

Now the engine may be used to drive a pump, an air compressor, a dynamo, or other machine, but the simplest case will here be considered, viz., where the engine drives a dynamo. The resistance which such a machine will offer to the crank shaft will be constant, or the torque at the crank shaft necessary to drive this dynamo will be constant, that is, with such a load the energy given out by the engine per second is constant. The energy supplied by the working fluid varies from time to time as has already been explained, at the beginning of the stroke it is much less than that necessary to drive the dynamo, and a little further on it is much greater than required, while still farther on in the stroke, and for the entire return stroke the energy supplied by the working fluid is altogether too small to drive the load, and is, in fact, negative for certain periods.

There must, therefore, be some means of adjusting these inequalities, the usual plan being to place on the crank shaft a wheel with a very heavy rim, and of large diameter, so that when the energy supplied to the engine is greater than that given out by it the excess energy may be used in speeding up the fly-wheel and increasing its kinetic energy, and the energy thus stored up must be sufficient

MICROCOPY RESOLUTION TEST CHART

(ANSI and ISO TEST CHART No. 2)



to supply the deficit toward the end of the forward stroke and during the entire return stroke of the piston. The fly-wheel is therefore, continually storing up and restoring energy, in the storing up process it is increasing its kinetic energy by increasing its speed, and in the restoring process its speed is decreasing, thus the speed of the fly-wheel of the engine is of necessity variable.

In the case of the punch, the condition is somewhat similar, although in this case, the energy supplied by the belt is quite constant, but that given out is variable. While the punch runs light, no energy is given out (neglecting friction), but when a hole is being punched the energy supplied by the belt is not sufficient, and the fly-wheel is drawn upon (with a corresponding decrease in speed) to supply the extra energy, and then after the hole is punched, the belt gradually speeds the wheel up again to normal, after which another hole may be punched. Evidently the fly-wheel should have a heavy rim and run at high speed to be most effective.

Now it will be noticed that a fly-wheel is required if the supply of energy or if the delivery of energy (or load) is variable, so that a fly-wheel is required on an engine driving a dynamo or a reciprocating pump, or a compressor, or a turbine pump, also a fly-wheel is necessary on a punch or a sheet metal press. It is not, however, in general, necessary to have a fly-wheel on a steam turbine-driven generator, or on a motor driven turbine pumping set, or on a water turbine-driven generator working at constant head of water, because in these latter cases, both the energy supplied per second, and the load are constant, the supply being always equal to the energy given out.

The speed variations in the fly-wheel here referred to are those accruing during a given revolution or complete cycle, and no reference is made to a permanent change of speed, which may be due to a heavy load coming on the machine, it is the business of the governor to keep the *mean speed* of any machine constant.

The present investigation deals with the proper weight of the fly-wheel for a given machine and takes into account the inertia of the different parts of the machine itself.

THE KINETIC ENERGY OF MACHINES

If a body have plane motion at any instant this motion may be divided into two parts, viz.: A motion of translation of its centre of gravity and a motion of rotation about its centre of gravity. Let a body of weight w and of mass $m = \frac{w}{g}$ be moving in a plane and at any instant let the velocity of its centre of gravity be v ft. per sec., and let the angular velocity of the body be ω radians per sec. Further, let the moment of inertia of the body about its centre of gravity be I and the corresponding radius of gyration k, so that $I = m k^2$. Then from the principles of mechanics it may be shown that the total kinetic energy of the body at the given instant is

$E = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} m v_i^2 + \frac{1}{2} m k^2 \omega^2$, hence, in order to find the kinetic energy of a body we have only to find v and ω and from the other known properties E may readily be computed.

Let Fig. 124 represent a machine with four links connected by four turning pairs, the links being a, b, c, and d, of which the latter

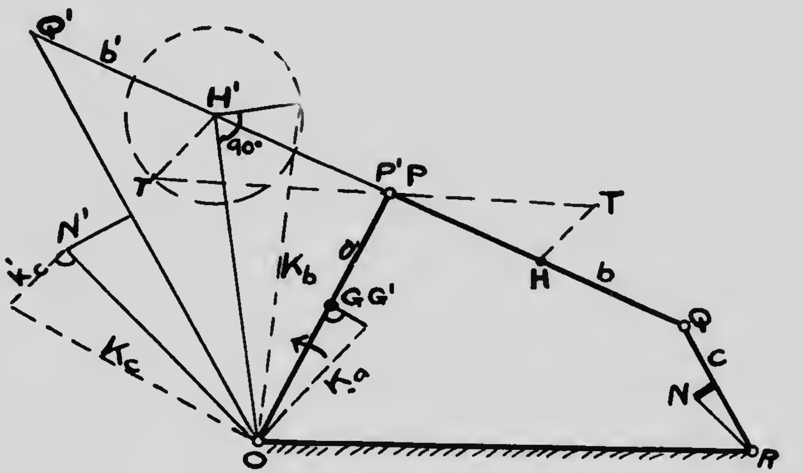


Fig. 124

is fixed, and let I_a, I_b, I_c represent respectively the moments of inertia about the centres of gravity of the links, the masses of the links being m_a, m_b and m_c. Given the angular velocity of the link a, it is required to find the kinetic energy of the machine while passing through a given position.

Find the images of a, b, c, d, P, Q, and of G, H, and N, the latter points being the centres of gravity of the links a, b and c, respectively, and let ω be the angular velocity of a, which is given, he angular velocities of the links b and c being represented by ω_b and ω_c. From the principles of the phorograph, $v_G = OG'.\omega$, $v_H = OH'.\omega$ and $v_N = ON'.\omega$ where v_G, v_H and v_N represent respectively the velocities of G, H and N, the centres of gravity of the links a, b and c, also $\omega_b = \dfrac{b'}{b}\,\omega$ and $\omega_c = \dfrac{c'}{c}\omega$, so that all the necessary linear and angular velocities are known from the drawings. The determination of the kinetic energy will be made for b anu the construction for it will also apply to the other links.

Let E_b be the kinetic energy of b at a given instant I_b, being its moment of inertia, about its centre of gravity, then from the general statement already made, $E_b = \dfrac{1}{2}\,m_b\,v_H^2 + \dfrac{1}{2}\,I_b\,\omega_b^2$, since H is the centre of gravity of b. Also $v_H = OH'\,.\,\omega$ and $I_b\,\omega_b^2 = m_b\,k_b^2\,\omega_b^2 = m_b\,k_b^2.\dfrac{b'^2}{b^2}\,\omega^2 = m_b\left(\dfrac{b'}{b}.\,k_b\right)^2\omega^2$. Now, following the notation already adopted, let $\dfrac{b'}{b}\,k_b$ be represented by k_b', as it corresponds exactly with the image of k_b on the phorograph. The magnitude of k_b' is found by drawing a line $HT = k_b$ in any

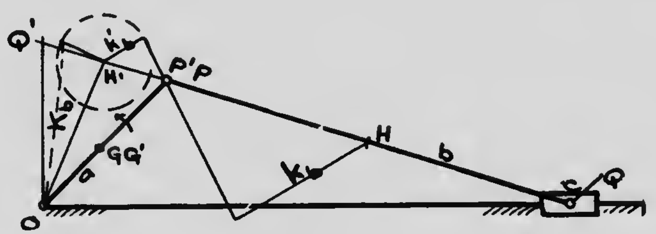


Fig. 125

direction from H and finding T' by the method indicated, in Fig. 124, $H'T'$ being the corresponding value of k_b' and is drawn parallel to HT. Then $E_b = \dfrac{1}{2}\,m_b v_H^2 + \dfrac{1}{2}\,I_b\,\omega_b^2 = \dfrac{1}{2}\,m_b\left[OH'^2 + k_b'^2\right]\omega^2$ and le. the quantity in the square bracket be denoted by K_b^2, then evidently K_b may be considered as the radius of gyration of a body, which, if secured to the link a, and having a mass m_b, would have the same kinetic energy as the link b actually has at this instant.

It is evident that the graphical construction for K_b is quite simple, it is the hypothenuse of the right-angled triangle of which one side is OH' and the other k'_b, and the method of finding it is shown dotted on Fig. 124. Employing this method gives $E_b = \frac{1}{2} m_b K_b^2 \omega^2$ and similarly, it is possible to write $E_a = \frac{1}{2} m_a K_a^2 \omega^2$ and $E_c = \frac{1}{2} m_c K_c^2 \omega^2$, the constructions for these being also shown on the figure.

For the whole machine the kinetic energy E is thus given by

$$E = \frac{1}{2} m_a K_a^2 \omega^2 + \frac{1}{2} m_b K_b^2 \omega^2 + \frac{1}{2} m_c K_c^2 \omega^2$$

$$= \frac{1}{2} \left[m_a K_a^2 + m_b K_b^2 + m_c K_c^2 \right] \omega^2$$

$$= \frac{1}{2} \left[I'_a + I'_b + I'_c \right] \omega^2$$

$$= \frac{1}{2} J \omega^2$$

where I'_a, I'_b and I'_c may be looked upon as the moments of inertia of masses which, if placed on a, the link which is rotating with angular velocity ω, would have the same kinetic energies in the given position as the actual links have, and J may be properly called a reduced moment of inertia for the machine, or it is the moment of inertia of a single mass which, if pivoted at O and rotated at the angular velocity ω of the link a, would have the same kinetic energy in this position as the whole machine has. J, of course, varies from one position to another of the machine and is a function of the position of the machine and of the form and specific gravity of the links.

This proposition enables one at any time to reduce any whole machine, no matter how complex, down to a single mass, rotating with the same speed as the selected primary link, and in this way to find the kinetic energy of the machine very easily.

An important application of this construction may be made to the steam engine, this being a well-known machine and the solution of this problem is shown in Fig. 125. The lettering and method employed in the preceding machine may be used here, the only difference being that the link c has only a motion of translation and hence $\omega_c = 0$ and for it the kinetic energy is $E_c = \frac{1}{2} m_c v_Q^2$ or $E_c = \frac{1}{2} m_c \cdot \overline{OQ'}^2 \cdot \omega^2$, and for this link $I'_c = m_c \overline{OQ'}^2$. The solutions for the links a and b are made precisely as before.

Any other machine may be treated in a similar manner so that it is convenient to determine the total kinetic energy of any machine in a given position by this very simple method.

SPEED FLUCTUATIONS

One of the most useful applications of the foregoing theory is to the determination of the proper weight of fly-wheel to suit given running conditions and to prevent undue fluctuations in speed of the main shaft of a prime mover. Usually the allowable speed variations are set by the machine which the engine or turbine or other motor is driving and these variations must be kept within very narrow limits in order to make the engine of value, because when a dynamo is being driven, for example, variations in speed affect the lights, causing them to become alternately bright and dim and spoiling their usefulness. Further, where alternators are to work in parallel, the speed variations must be very small and the same is true in many other cases of loading.

Again in many rolling mills motors are being used to drive the rolls and in such cases the rolls run light until a bar of metal is put in, when the maximum work has to be done in rolling the bar. Thus, in such a case the load rises suddenly from zero to a maximum and then falls off again suddenly to zero. Without some storage of energy this would cause probable damage to the motor and hence it is usual to attach a heavy fly-wheel somewhere between the motor and the rolls, this fly-wheel storing up energy as it is being accelerated after a bar has passed through the rolls, and again giving out part of its stored up energy as the bar enters and passes through the rolls.

The electrical conditions determine the allowable variations in speed, but when this is known, and also the work required to roll the bar and the torque which the motor is capable of exerting under given conditions, then it is necessary to determine the proper weight of fly-wheel to keep the speed variation within the set limits.

In the case of a punch already mentioned, the machine runs light for some time until a plate is pushed in suddenly and the full load is thrown on the punch. If power is being supplied by a belt a fly-wheel is also placed on the machine, usually on the shaft holding the belt pulley, this fly-wheel storing up energy while the machine is light and assisting the belt to drive the punch through the plate when a hole is being punched. The allowable percentage of slip of the belt is usually known and the wheel must be heavy enough to prevent this amount of slip being exceeded.

The general factors on which speed fluctuations depend have been mentioned at the beginning of the chapter and need not be again discussed here. Let E_1 be the kinetic energy, determined by the process just explained, of a machine at the beginning of any interval of time, and E_2 the kinetic energy at the end of this interval, then, neglecting friction, the gain in energy during the interval is $E_2 - E_1$, which may be positive or negative, according as E_2 is greater or less than E_1. In the case of an engine, $E_2 - E_1$ will represent the difference between the work done by the working fluid on the piston and the work done at the crank shaft on some external machine and the friction during the interval before mentioned, because the kinetic energy of the machine can only increase if the work done by the engine is less than the energy received by it from the working fluid. In order to simplify the problem, friction will be neglected.

A little consideration will show that $E_2 - E_1$ will be alternately positive and negative, that is, for part of the revolution E will increase till it reaches a maximum value and then again it will decrease to a miminum and so on. As long as E increases, the speed of the machine must increase in general, and thus the speed will be a maximum at the place where E just begins to decrease and conversely the speed will be a minimum at the place where the energy E just begins to increase. But E must increase just so long as the energy put into the machine is greater than that given out by the machine in a given interval, hence, also we get the maximum speed at the end of any period in which the work input to the machine exceeds the work output, the opposite is true for the minimum speed.

Suppose now that E_1, ω_1, J_1, and E_2, ω_2, and J_2 represent respectively the kinetic energies, the speeds of the primary link and the reduced inertias of the machine at the beginning and end of a certain interval of time, then

$$E_1 = \frac{1}{2} J_1 \omega_1^2 \text{ and } E_2 = \frac{1}{2} J_2 \omega_2^2$$

$$\text{and } E_2 - E_1 = \frac{1}{2} \left[J_2 \omega_2^2 - J_1 \omega_1^2 \right].$$

An approximate method may now be employed without introducing a very serious error in many cases, by taking $J = \frac{J_1 + J_2}{2}$ as being approximately equal to J_1 or J_2, because in most cases, if the time interval is small the change in J is small and the difference between J, J_1 and J_2 may be neglected, especially when these quantities are used as multipliers. Thus:

$E_2 - E_1 = \frac{1}{2} J \left(\omega_2^2 - \omega_1^2\right)$ or $\frac{E_2 - E_1}{J} = \frac{(\omega_2 - \omega_1)(\omega_2 + \omega_1)}{2}$ $= (\omega_2 - \omega_1)\,\omega$. where $\omega = \frac{\omega_1 + \omega_2}{2}$; if the machine is such as an engine or motor, and is constructed to operate under small speed variations imposed in practice, this value of ω cannot much differ from the mean speed of the main shaft and may be regarded as constant for all positions of the machine.

Then $\frac{E_2 - E_1}{J} = (\omega_2 - \omega_1)\,\omega$ or $\omega_2 - \omega_1 = \frac{E_2 - E_1}{J\omega}$

To make the case general however, it will be desirable to take account of the variations in J in which case the following method is to be adopted.

Since $E = \frac{1}{2} J \omega^2$ there is obtained by differentiation

$$\delta E = \tfrac{1}{2} [\, 2\omega J \,.\, \delta\omega + \omega^2 \,\delta J]$$

and solving for $\delta\omega$ gives $\delta\omega = \frac{\delta E - \frac{1}{2}\omega^2 \delta J}{J\omega}$ where $\delta\omega$ is the change of speed in a given short interval of time, and δJ and δE the corresponding changes in the reduced inertia and the kinetic energy of the machine, the primary link of which rotates at mean speed ω and has a mean reduced inertia J during the given time interval.

If in the case of an engine, for example, the effect of the connecting rod piston, etc., is neglected and the fly-wheel only is considered, then $\delta J = o$, as the moment of inertia of the fly-wheel is constant, so that $\delta\omega = \frac{\delta E}{J\omega}$. In any case, if δE and ω are given for any engine $\delta\omega$ can be computed, or if the allowable variation $\delta\omega$ in the speed is given, the equation may be solved, for J and the necessary moment of inertia of the fly-wheel may be found.

The meanings and application of these quantities may be best illustrated by an example which will now be discussed, and as the steam engine involves all the principles used, and is so common, it will be selected for the illustration. Moreover, the method of selecting the data in this case is very readily explained and understood.

Consider the double-acting steam engine shown in Fig. 126, with the corresponding indicator diagrams for the head and crank ends. It is required to find the change of speed while the engine moves from position A to B. It will be assumed for simplicity that the engine drives a turbine pump which offers a uniform resisting

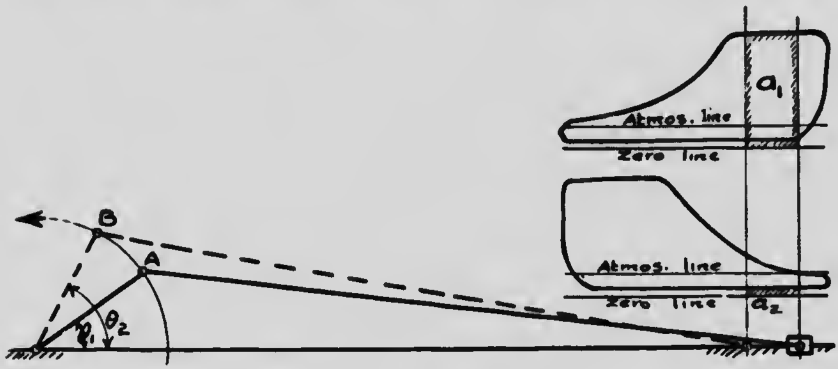


Fig. 126

turning moment, and in this case, the work done by the engine will evidently be $\frac{\theta_2 - \theta_1}{360}$ of the total work done per revolution or the ... ne by the engine is in direct proportion to the angle passed ... y the crank. The effect of friction will be neglected.

... W be the work done per revolution, as shown by the indi... iagrams, and let some numerical value of $\theta_2 - \theta_1$ be chosen for convenience, say 18°, then the work done by the engine while the crank moves from A to B will be $\frac{18}{360} \times W = \frac{W}{20}$. Now let A_1 be the area of the head end of the cylinder and A_2 that of the crank end in sq. in., also let L be the stroke of the piston in ft., and l be the length of the diagram in in., the scale of the diagram being s pds. per. sq. in. = 1 in. Then each square inch on the diagram will represent $s\,A_1\,\frac{L}{l}$ and $s\,A_2\,\frac{L}{l}$ ft. pds., for the head and crank ends respectively. Let the area of the head end diagram, reckoned above the zero line of pressures, swept out during the motion of the crank under consideration, be a_1 sq. in., the corresponding area for the crank and diagram being a_2 sq in., both of which are shown hatched on the diagram Fig. 126.

Now the total energy delivered to the machine by the steam during the interval under consideration will be $a_1 s A_1 \frac{L}{l} - a_2 s A_2 \frac{L}{l}$ ft. pds., while the energy delivered by the machine to the pump will be $\frac{W}{20}$ ft. pds. (Note that the total work W must be the same as that represented by the sum of the two indicator diagrams, and is the

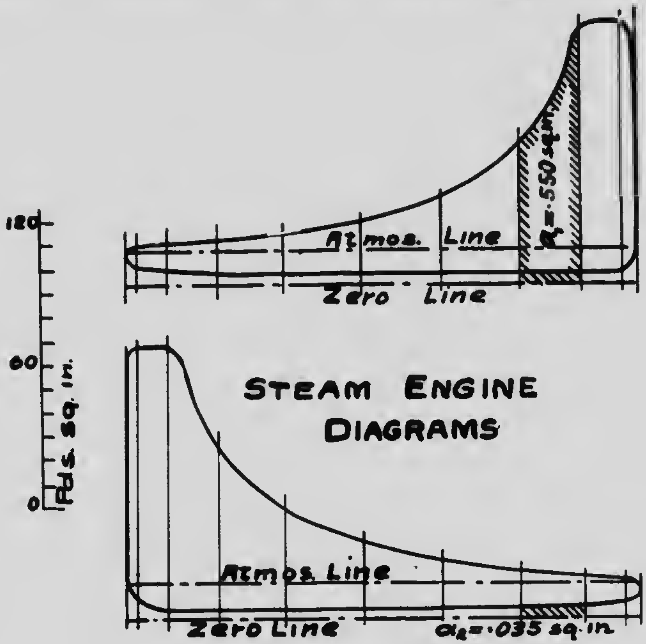


Fig. 127

work corresponding to the areas of these two diagrams, per revolution). Hence we have an unbalanced amount of work $a_1 s A_1 \frac{L}{l} - a_2 s A_2 \frac{L}{l} - \frac{W}{20}$ and this amount of work must be stored up in the moving parts of the machine during this interval, or if E_1 represents the kinetic energy of the machine when the crank is at A and E_2 the corresponding energy when the crank is at B, then

$E_2 - E_1 = a_1 s A_1 \frac{L}{l} - a_2 s A_2 \frac{L}{l} - \frac{W}{20}$ the numerical value of which will thus be known. But $E_1 = \frac{1}{2} J_1 \omega_1^2$ and $E_2 = \frac{1}{2} J_2 \omega_2^2$ and J_1 and J_2 are to be found according to the method already explained, so that $\delta J = J_2 - J_1$. The change in speed is then found from the formula $\delta \omega = \frac{\delta E - \frac{1}{2} \omega^2 \delta J}{J \omega}$ where δE equals $E_2 - E_1$, ω is the mean speed of rotation and $J = \frac{J_1 + J_2}{2}$. The change in speed $\delta \omega$, may be positive or negative, and in fact, for the whole revolution must change from one to the other, otherwise the engine would continually increase in speed.

The complete determination of these quantities is given for an engine with a cylinder 12 1-16 in. dia. × 30 in. stroke, running at 86 revs. per min. The connecting rod is 90 in. centre to centre, and weighs 175 lbs., the radius of gyration of the rod about its centre of gravity is 31.2 in. The piston, crosshead, etc., weigh 250 lbs., while the fly-wheel has a weight of 5820 lbs., and a moment of inertia about the axis of rotation of 2400.

Taking the data in this problem gives $a = 1.25$ feet, $b = 7.5$ ft., $\omega = 9$ radians per sec., $m_a = 181$, $m_b = 5.44$, $m_c = 7.78$, $I_a = 2400$, $k_b = 2.60$ ft. The units are the ft., pd., sq. in., unless otherwise stated.

The indicator diagrams for the engine are given in Fig. 127, and the complete calculation for all of the quantities in the ta[illegible] while the crank is turning from $\theta = 36°$ to $\theta = 54°$ is give[illegible] [illegible]elow. The drawings, Figs. 127 and 128, show the different quantities on the diagram for one position of the mechanism and also the areas on the indicator diagrams for the positions stated above. It is assumed that the engine is driving a dynamo at constant load so that the resisting torque due to the load will be constant.

The following quantities were measured directly from the drawing in feet.

θ degrees	b' ft.	OH' ft.	k'_b ft.	K_b ft.	OQ' ft.
36	1.017	.9[illegible]5	.352	1.00	.84
54	.741	1.123	.257	1.15	1.11

from which there are at once obtained the following results for the two crank angles.

θ deg.	$I'_h = m_h K_h^2$	$I'_c = m_c . OQ'^2$	I_a	$J = I_a + I'_b + I'_c$	δJ
36	5.442	5.488	2400	2410.9	+ 5.9
54	7.200	9.578	2400	2416.8	

Thus during this part of the revolution there is a gain in the reduced inertia of amount 5.9, while in some other parts of the revolution value of δJ is negative.

Measurements were then made on the indicator diagrams, which were taken with a 60 spring, after computing the values of A_1 = 114.28 sq. in., and A_2 = 111.52 sq.in., the piston rod being 1 7-8 in. dia. The lengths of l of the diagrams were 3.55 in., and 3.58 in., the stroke of the piston L being 2.5 ft., so that

$$s A_1 \frac{L}{l} = 60 \times 114.28 \times \frac{2.5}{3.55} = 4829 \text{ ft. pds. and}$$

$$s A_2 \frac{L}{l} = 60 \times 111.52 \times \frac{2.5}{3.58} = 4673 \text{ ft. pds. per sq. in. of diagram}$$

area on the head and crank ends respectively. The results are set down in the following table:

θ degrees	Diagram Areas: Head a_1 sq. in.	Diagram Areas: Crank a_2 sq. in.	Work Done in ft. pds.: Head	Work Done in ft. pds.: Crank	Work Done in ft. pds.: Total	Work Done on Dynamo ft. pds.	Net work producing change of kinetic energy ft. pds.
36 to 54	.550	.035	2656	163	2493	1079	1414

The diagram area is measured directly as indicated and the work computed as above thus $a_1 \times 4829 = .550 \times 4829 = 2656$ pds. and $a_2 \times 4673 = .035 \times 4673 = 163$ ft. pds. The work done on the dynamo is $\frac{18}{360}$ of the total work represented by the two diagrams together, and the net work producing the increase of kinetic energy is 2493 — 1079 = 1414 ft. pds., so that the gain in energy of the machine during the interval is also 1414 ft. pds., or $\delta E = + 1414$ ft. pds.

The gain in angular velocity is now readily obtained, thus the average value of $J = \frac{1}{2}(2410.9 + 2416.8) = 2413.8$, and hence, $J\omega = 2413.8 \times 9 = 21724.6$, also $\frac{1}{2}\omega^2 \delta J = \frac{1}{2} \times 9^2 \times 5.9 = 238.9$, and thus $\delta\omega = \frac{\delta E - \frac{1}{2}\omega^2 \delta J}{J\omega} =$

$$\frac{1414 - 238.9}{21724.6} = .054 \text{ radians per sec.},$$

which gives the gain in velocity during the time the crank is turning through the 18° considered.

Having obtained these values of $\delta\omega$, they are then plotted with a straight line base, Fig. 129, which has been divided into 20 equal parts to represent the corresponding crank angles. If the variation in speed of the engine is small then no serious error will be made by assuming that these crank angles are passed through in equal intervals of time, and hence, that the base line of the diagram on which the values of $\delta\omega$ is plotted is also a time base, equal distances along which represent equal intervals of time. If desired, the equal angle base may be corrected for the variations of angular velocity by using the values of $\delta\omega$ as already found, but the author does not think it is worth the labor and had made no correction of this kind on the diagram shown.

Now, since the space traversed is the product of the velocity and the time, we may find the space variation $\delta\theta$ by multi-

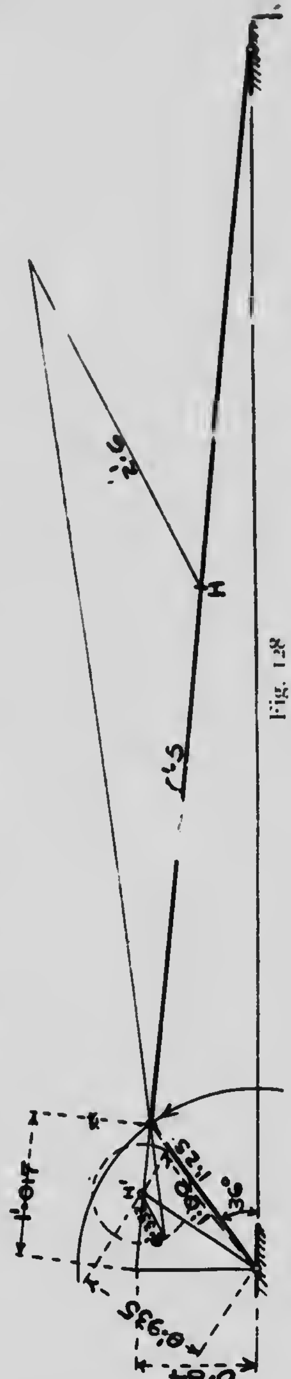


Fig. 128

TABLE OF RESULTS FOR 12 1-6 IN x 30 IN. ENGINE FOR DETERMINING SPEED FLUCTUATIONS.

θ deg.	K_b ft.	$I'_b = m_b k_b^2$	OQ' ft.	$I'_c = m_c \times OQ'^2$	I_a	$J = I_a + I_b + I_c$	δJ	Work done by steam on piston: Positive Work ft. pds	Work done by steam on piston: Negative Work ft. pds	Work done by steam on piston: Net Work ft. pds	Work Output to Generator ft. pds.	Net Work producing change of kinetic energy δE ft. pds.	$\frac{1}{2}\omega^2 \delta J$	$\delta E - \frac{1}{2}\omega^2 \delta J$	Angular Velocity Variation $\delta\omega$ Radians per seconc +	Angular Velocity Variation $\delta\omega$ Radians per seconc —
0	.76	3.15	0.00	0.00	2400	2403.2										
18	.84	3.84	.45	1.58	2400	2405.4	+ 2.2	893	47	846	In each interval $= \frac{21584}{20} = 1079$	— 233	+ 93.1	— 326.1		.0151
36	1.00	5.44	.84	5.49	2400	2410 9	+ 5.5	2583	117	2466		+ 1387	+ 222.7	+ 1164.3	.0536	
54	1.15	7.20	1.11	9.58	2400	2416.8	+ 5.9	2656	163	2493		+ 1414	+ 238.9	+ 1175.1	.0541	
72	1.24	8.37	1.25	12.15	2400	2420.5	+ 3.7	1956	178	1778		+ 699	+ 149.8	+ 549.2	.0252	
90	1.25	8.51	1.25	12.15	2400	2420.7	+ 0.2	1424	159	1265		+ 186	+ 8.1	+ 177.9	.0082	
108	1.18	7.58	1.13	9.93	2400	2418.5	— 2.2	1038	150	888		— 191	— 89.1	— 101.9		.0047
126	1.06	6.12	.92	6.58	2400	2412.7	— 5.8	749	131	618		— 461	— 234.9	— 226.1		.0104
144	.93	4.71	.64	3.19	2400	2407.9	— 4.8	531	98	433		— 646	— 194.4	— 451.6		.0208
162	.81	3.57	.33	.85	2400	2404.4	— 2.5	314	93	221		— 858	— 101.2	— 756.8		.0350
180	.76	3.15	0.00	0.00	2400	2403.2	— 1.2	72	70	2		— 1077	— 48.6	— 1028.4		.0475
198	.81	3.57	.33	.85	2400	2404.4	+ 1.2	607	48	559		— 520	+ 48.6	— 568.6		.0263
216	.93	4.71	.64	3.19	2400	2407.9	+ 2.5	1878	121	1757		+ 678	+ 101.2	+ 576.8	.0266	
234	1.06	6.12	.92	6.58	2400	2412.7	+ 4.8	2584	145	2439		+ 1360	+ 194.4	+ 1165.6	.0537	
252	1.18	7.58	1.13	9.93	2400	2418.5	+ 5.8	1986	169	1817		+ 738	+ 234.9	+ 503.1	.0231	
270	1.25	8.51	1.25	12.15	2400	2420.7	+ 2.2	1542	169	1373		+ 294	+ 89.1	+ 204.9	.0094	
288	1.24	8.37	1.25	12.15	2400	2420.5	— 0.2	1215	169	1046		— 33	— 8.1	— 24.9		.0011
306	1.15	7.20	1.11	9.58	2400	2416.8	— 3.7	916	169	747		— 332	— 149.8	— 182.2		.0084
324	1.00	5.44	.84	5.49	2400	2410.9	— 5.9	654	121	533		— 546	— 238.9	— 307.1		.0141
342	.84	3.84	.45	1.58	2400	2405.4	— 5.5	378	96	282		— 797	— 222.7	— 574.3		.0265
360	.76	3.15	0.00	0.00	2400	2403.2	— 2.2	93	72	21		— 1058	— 93.1	— 964.9		.0446
							Totals			21584		0.00	0.00		.2539	.2545

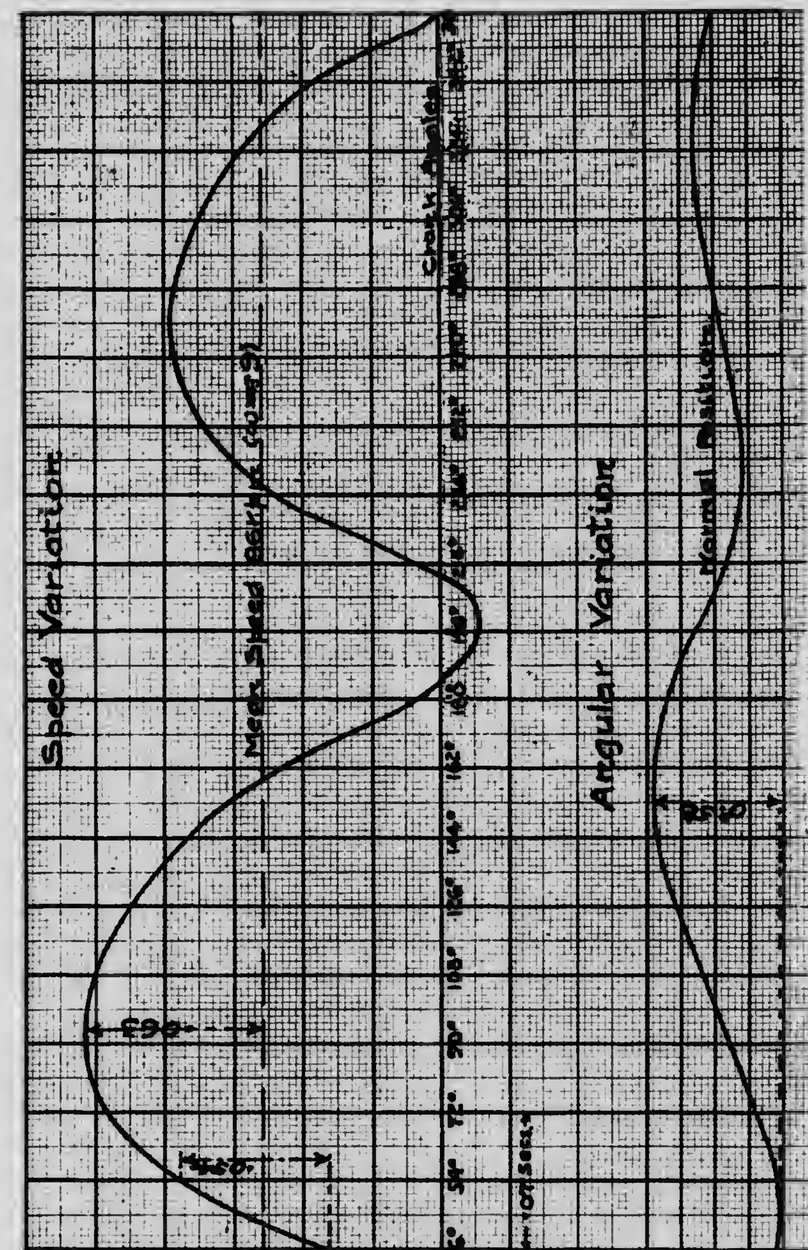

Speed Variation
Angular Variation
Crank Angles
Radians per sec.

Fig. 129

plying the corresponding $\delta\,\omega$ by the time t required to turn the crank 18° or $\delta\,\theta = t\,.\,\delta\,\omega$, and in this way the integration of the $\delta\,\omega - t$ curve gives the $\delta\,\theta - t$ curve, which shows the number of radians or degrees which the fly-wheel swings back and forth from its mean position This is a very important matter for alternators running in parallel.

The results for the complete revolution of the crank are given in the table on page 196.

The reader should notice that the result of the calculation gives the *gain* in velocity and angular position, so that in plotting some arbitrary zero line is assumed, and the results are laid off in succession, not from the base line but from the end of the curve in each case. The line of mean speed is in such a position that the sums of the positive and negative areas between this line and the curve are equal. In the engine discussed, the minimum speed was 8.922 radians per sec., while the maximum was 9.063 radians per sec., a variation .141 radians or 1.57 %.

The angular space variation had a maximum value of 0.58 degrees as measured from the curves drawn.

The complete results for this engine have been given here in the hope that it will make the method clear, and that the student will understand the procedure in any other case. The process is not very lengthy, and results may be obtained very quickly by the use of the slide rule and the drafting board.

In the case of other machines or other arrangements of steam or gas engines, the method is precisely the same. In some machines the variation of angular velocity is all that is required while in others it is necessary to determine the space variation, as in the case of alternators in parallel, when there is a definite limit set to the number of degrees of oscillation of the rotor about its mean position.

CHAPTER XIV.

THE PROPER WEIGHT OF FLY WHEELS

In the preceding chapter a complete discussion has been given as to the causes of speed fluctuations in machinery and the method of determining the amount of such fluctuation. In very many cases a certain machine is on hand and it is the province of the designer to find out whether it will satisfy certain conditions which are laid down. This being the case the problem is to be solved in the manner already discussed, i.e., the speed fluctuation corresponding to this machine and its methods of loading are to be determined.

Most frequently, however, the converse problem is given. It is required to design a machine which will conform to certain definite conditions, thus a steam engine may be required for driving a certain machine at a given mean speed but it is also stipulated that the variation in speed during a revolution must not exceed a certain amount. In any such case the weights and dimensions of the piston, crosshead, etc., are fixed by constructional conditions and are independent of the speed condition.

Thus the diameter of the piston depends upon the power, pressure, mean speed, and stroke of the piston. Having determined the diameter, the thickness, and hence the weight is fixed from a consideration of the strength, so also with the crosshead, connecting rod and crank, the dimensions of all of these parts being fixed without regard to the speed fluctuation. The dimensions of the fly-wheel are, however, independent of the conditions of power, and this wheel may be light or heavy, large or small, just as required, some machines having no fly-wheel at all, others having very heavy and very large ones.

Under ordinary circumstances, the fly-wheel is designed to prevent undue fluctuations in speed, being of large diameter, and having a heavy rim, in general, if the fluctuations are small, and vice versa. Or again, the conditions may be satisfied by using a small wheel running at high speed, if such is permissible, and it is to the discussion of this very important problem that the present chapter is devoted. The problem will be to determine the proper dimensions of a fly-wheel to satisfy given conditions at a given mean speed.

Referring to the preceding chapter, the equation giving the kinetic energy of a machine is $E = \frac{1}{2} J \omega^2$, where J is the reduced inertia, and by re-arranging there is found $\frac{1}{2}\,\omega^2 = \frac{E}{J}$ which gives the speed at any instant at which E and J are known. The method of obtaining E has been explained in Chapter XIII., the value of E being obtained in any case from the input and load conditions, for example, in a steam engine, by a consideration of the indicator and load diagrams, J being determined from the dimensions of the machine.

For the purpose of presenting the subject in the clearest possible way, the whole discussion will be taken up and applied to one particular machine. The machine selected will again be the reciprocating steam engine, partly because of the general nature of the

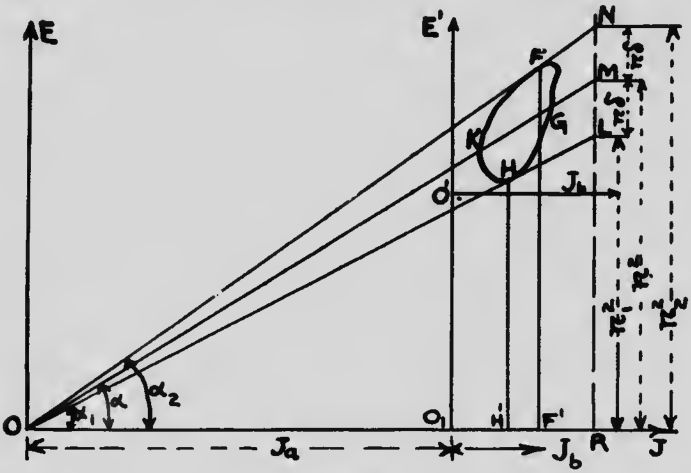


Fig. 130

discussion as applied to such a machine, and partly because the data in such a case may very readily be assumed.

For the present purpose, it will be convenient to divide J into two parts (a) that due to the rotating parts, crank, fly-wheel, etc., alone, which may be called J_a, and (b) that due to the connecting rod, piston, etc., which is called J_b. The former of these will be the same for all positions of the crank, and the latter will vary with the crank angle, both of these will, however, be independent of the speed of the engine, simply depending on the masses of the parts and the distribution of the masses about their centres of gravity.

Suppose now that for any machine the values of J be plotted on a diagram along the x axis, the ordinates of which diagram represent the corresponding value of the energy E, this will give a diagram as shown at Fig. 130, where the curve represents J for the corresponding value of E shown on the vertical line.

Having now obtained the figure $KFGHK$, it is evident that its width depends on the value of J at the instant and this value of J is independent of the speed. Also, the height of this figure depends on the difference between the works put into the machine and the work delivered by the machine during given intervals, that is, it will depend on the shapes of the indicator and load curves. The shape of the indicator diagrams within certain limits depends on whether the engine is run by gas or steam, and on whether it is simple or compound, etc., but for a given engine this is also independent of the speed: the load cur will, of course, depend on what is being driven, whether it is dynamo, compressor, etc.. so that the height of the curve is also independent of the speed.

It will further be noted that the shape of the figure does not depend on J_a, which is constant for given wheel, but only on the values of J_b, so that the shape of this figure will be independent of the weight of the fly-wheel and speed, in so far as the indicator and load curves are independent of the speed, depending solely on the reciprocating masses, the connecting rod, the indicator diagrams and the load curves.

Now draw from O the two tangents, OF and OH, to $KFGH$, touching it at F and H respectively, then for OH we have $E_1 = H H'$, and $J_1 = O H'$ and $\frac{1}{2} \omega_1^2 = \frac{E_1}{J_1} = \tan \alpha_1$, and since α_1 is the least value such an angle can have it is evident that ω_1 is the minimum speed of the engine. Similarly, $E_2 = F F'$ and $J_2 = O F'$, and $\frac{1}{2} \omega_2^2 = \frac{E_2}{J_2} = \tan \alpha_2$ and hence, ω_2 would be the maximum speed of the engine.

If now it is desired to design a fly-wheel, determine beforehand the allowable values of ω_1 and ω_2 and also the mean speed $\omega = \frac{\omega_1 + \omega_2}{2}$. The allowable variation in speed $\omega_2 - \omega_1$, as has already been explained, is fixed by the class of service for which the machine is designed, thus in driving alternators $\omega_2 - \omega_1$ must be a very small proportion of ω, whereas, in plunger pumps, much larger variations may be allowed. Next from assumed or known indicator diagrams and from the load curve as well as the dimensions of the parts of the machine, except the fly-wheel, draw the $E - J_b$ diagram $K\ F\ G\ H$. Observe that the exact position of the figure $K\ F\ G\ H$ with regard to the origin and the axes of E and J cannot be found without previously knowing the value of J_a, i.e., the weight of the fly-wheel. A little consideration will show, however, that a new axis $E'\ O_1$ may be assumed where the distance OO_1 represents J_a, and then from the axis $O_1\ E'$ the values of J_b may be laid off.

Again the shape of the figure does not depend upon the absolute value of E but only upon the changes in the latter. Thus an arbitrary axis $O'J_b$ may be assumed and starting with any arbitrary initial value of E the figure may be plotted. In fact, for a given machine with given load and indicator diagrams, the weight of whose fly-wheel is to be determined, the figure $FGHK$ and the position and direction of the axis $E'\ O_1$ are known, but the position of the origin O and thus of the axes of E and J will depend entirely upon J_a and the speed of the engine.

Having settled ω_1 and ω_2, two lines may be drawn tangent to the figure at H and F and making the angles α_1 and α_2 respectively, with the direction $O'\ J_b$ where $\tan\ \alpha_1 = \frac{1}{2}\,\omega_1^2$ and $\tan\ \alpha_2 = \frac{1}{2}\,\omega_2^2$. The intersection of these two lines gives O and hence the axis $E\ O$, so that the required moment of inertia of the wheel may be scaled from the figure, thus $J_a = OO_1$. It should, however, be pointed out that if the position of the axis of E is known, it is not possible to choose ω_1 and ω_2 at will. for the selection of either one will determine the position of O. In making a design it is usual to select ω and ω_1 and ω_2, and from the chosen values to determine the position of O and hence the axes of E and J. The mean speed ω corresponds with the angle α.

Draw a line $N\ M\ L\ R$ perpendicular to $O\ J$, in any convenient position. Then $\frac{LR}{OR} = \tan\ \alpha_1$, $\frac{NR}{OR} = \tan\ \alpha_2$ and $\frac{MR}{OR} = \tan\ \alpha$, so that on some scale which may be found, $N\ R$ represents ω_1^2, or the square of the speed n_1 in revs. per min., $L\ R$ represents n_2^2 and $M\ R$ represents the square of the mean speed n all on the same scale. As in engines the difference between n_1 and n_2 is never large it is fairly safe to assume $2n^2 = n_2^2 + n_1^2$ or that M is midway between N and L.

Using now δ to denote the *coefficient of speed fluctuation*, then δ

is defined by the relation $\delta = \frac{n_2 - n_1}{n}$

$$\therefore \delta = \frac{n_2 - n_1}{n} = \frac{n_2 - n_1}{\frac{n_2 + n_1}{2}} = 2\frac{n_2^2 - n_1^2}{(n_2 + n_1)^2} = 2\frac{n_2^2 - n_1^2}{(2n)^2}$$

$$\text{or } 2\delta = \frac{n_2^2 - n_1^2}{n^2}$$

but $\frac{1}{2}\,\omega_1^2 = \frac{E_1}{J_1} = \tan \alpha_1$ and as $\omega = \frac{2\pi n}{60}$

then $\frac{4\pi^2 n_1^2}{2 \times 60^2} = \tan \alpha_1$ or $n_1^2 = C' \tan \alpha_1$ similarly $n_2^2 = C' \tan \alpha_2$

and $n^2 = C' \tan \alpha$ where C' is constant and equal to $\frac{2 \times 60^2}{4\pi^2} = 182.3$

Now, since in the figure all angles have been measured with the common base OR, therefore, $RL = OR \tan \alpha_1 = OR \frac{n_1^2}{C'} = C n_1^2$

where $C = \frac{OR}{C_1}$

Also $RM = C n^2$ and $RN = C n_2^2$.

$$\text{Hence, } 2\delta = \frac{n_2^2 - n_1^2}{n^2} = \frac{\frac{RN}{C} - \frac{RL}{C}}{\frac{RM}{C}} = \frac{RN - RL}{RM} = \frac{NL}{RM}$$

Thus $NL = 2\,RM\,.\,\delta$ In general, $\alpha_2 - \alpha_1$ is a small angle in which case M will usually nearly bisect NL, the error introduced by assuming this to be the case being small in general. Hence $NM = ML = n\,\delta$ nearly.

The reader will see that the shape of the curve on the $E - J$ diagram has a very important effect on the best speed and the best weight of a fly-wheel to suit given conditions. Thus suppose this curve were long and flat, as shown in Fig. 131, then it will be seen that there is one certain speed which will give the smallest fluctuation δ. Thus, if the origin be located along the line passing through the long diameter of the figure the case would correspond to a very small fluctuation in speed, even where O were moderately close to the figure, because $\delta = \frac{NM}{MR}$ would be very small. If now the speed be lowered, keeping the same fly-wheel, then this raises O vertically to O_1, and the speed fluctuation will be $\frac{N_1M_1}{R_1M_1}$ which is very much greater than before, while an increase in speed lowers O to O_2, and gives a greater fluctuation in speed.

On the other hand, increasing the inertia, and hence the weight, of the fly-wheel without changing the speed, moves O out to O_3 and an examination of the figure also shows a greater speed fluctuation than in the original case.

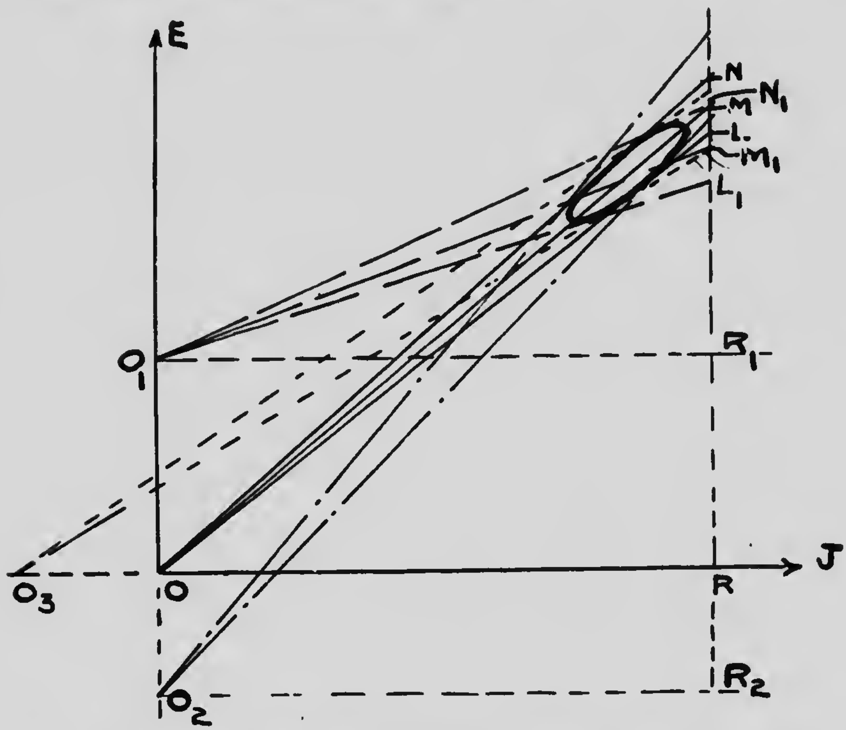


Fig. 131

For a given case there is in general a certain weight of fly-wheel and speed of rotation which gives minimum speed fluctuation, and an increase or decrease in fly-wheel weight or speed will cause an increase in the fluctuation. This is much more marked in the case where the E-J curve is elongated and narrow and less marked where its boundaries come nearest to touching an enclosing circle.

If the axis of J were to cut the $E - J$ curve the machine would not work, no matter how heavy a fly-wheel it possessed, because for the part of the curve below the axis of J it could have no speed. The limiting case is where the curve touches the axis of J, in which

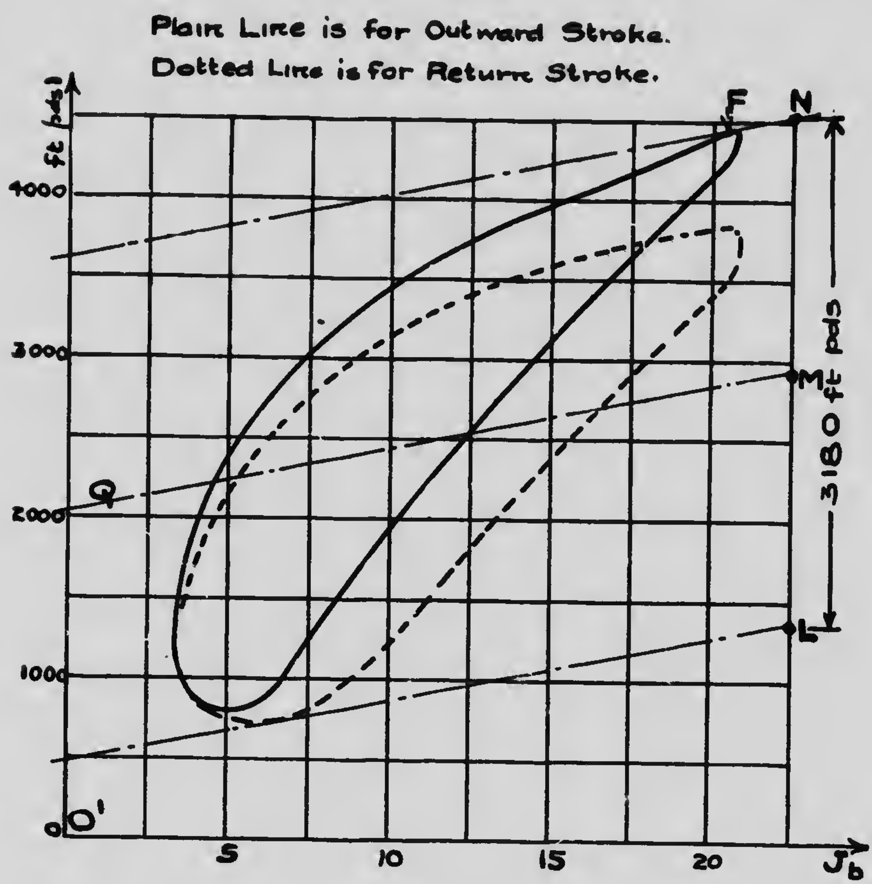


Fig. 132

case the engine would stop at this point where contact occurs which is not usually the dead centre.

The matter will now be illustrated by a practical example, and the case chosen will be the simple slow-speed engine discussed in the last chapter. This engine having a cylinder 12 1-16 in. dia., 30 in. stroke, and a mean speed of 87 revs. per min. In the table appearing in Chapter XIII., the value of J is put down for each 18° of crank angle, and also the work input and output corresponding to the various angles, and hence the corresponding gain in E for each 18°. In table given herewith, there is set down for convenience, the corresponding values of J, and also the gain in E for the angles given, these being directly copied from table referred to in Chapter XIII.

TABLE OF VALUES OF J AND E FOR 12 1-16 IN. × 30 IN. ENGINE

θ degrees	J_a	J_b	$J_a + J_b$	δE
0	2400	3.2	2403.2	— 233
18	"	5.4	2405.4	+ 1387
36	"	10.9	2410.9	+ 1414
54	"	16.8	2416.8	+ 699
72	"	20.5	2420.5	+ 186
90	"	20.7	2420.7	— 191
108	"	18.5	2418.5	— 461
126	"	12.7	2412.7	— 646
144	"	7.9	2407.9	— 858
162	"	4.4	2404.4	— 1077
180	"	3.2	2403.2	— 520
198	"	4.4	2404.4	+ 678
216	"	7.9	2407.9	+ 1360
234	"	12.7	2412.7	+ 738
252	"	18.5	2418.5	+ 294
270	"	20.7	2420.7	— 35
288	"	20.5	2420.5	— 332
306	"	16.8	2416.8	— 546
324	"	10.9	2410.9	— 797
342	"	5.4	2405.4	— 1058
360	"	3.2	2403.2	

The values in this table are plotted on Fig. 132, where a scale of 5 has been used for J, and of 1000 ft. pds. = 1 in. for E, the axis of E' being placed at J_a = 2400 (the moment of inertia of the crank and fly-wheel), in order to prevent undue length of the figure. The axis of J' is chosen arbitrarily for the present, its location being found later, and of the two diagrams plotted, the plain one is for

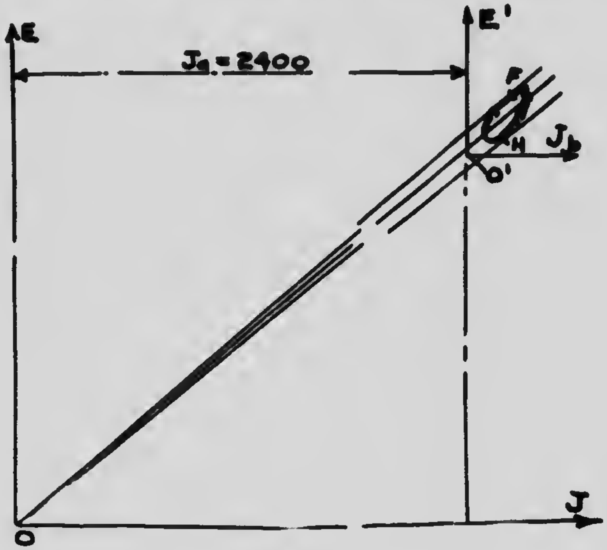


Fig. 133

the out-stroke and the dotted one for the return stroke of the piston.

Having now obtained these figures, the axis of J is determined by remembering that $\frac{1}{2}\omega^2 = \frac{E}{J} = \tan\alpha$ and hence $\tan\alpha = \frac{1}{2}(9)^2 = 40.5. = \frac{E}{J}$. Let the height e in inches on the diagram represent the energy E, so that $E = 1000\,e$ since the scale is 1000 ft. pds. per inch. and using a similar notation, $J = 5j$. Thus $\frac{E}{J} = 40.5 = \frac{1000\,e}{5\,j}$ where e and j are a number of inches as measured on the diagram already constructed, hence $\frac{e}{j} = \frac{5}{1000} \times 40.5 = .202$. Now lay off a distance to the left of $O'\,E' = 2400$ to represent the value J_a and we thus get the axis of E. Then drawing as nearly as possible through the centre of the figure a line having a tangent .202, which corresponds to the mean speed of rotation $\omega = 9$, gives at once the origin O, and the axis of J is a horizontal line through O. See Fig. 133.

Now through the origin O draw tangents OF and OH to the $E - J$ curve, then OF corresponds to the maximum speed of rotation n_2, and OH to the minimum speed n_1. In drawing the tangents and locating O usually a smaller scale will have to be adopted than that used in plotting the $E - J$ curve, but this gives no real trouble and the smaller scale may in general be avoided if desired. Thus, QM, Fig. 132, may be drawn through the centre of the figure to represent n, it being inclined to the axis of J at an angle whose tangent is .202, and for the variations ordinarily occurring, FN and HL, the tangents at F and H may have the same angle if the vertical line NML is close to the figure. Now, if R be the point where NML cuts the axis of J, then $n_2 = \text{const.} \sqrt{NR}$, $n_1 = \text{const.} \sqrt{LR}$ and $n = \text{const.} \sqrt{RM}$ and $2\delta = \dfrac{NL}{MR}$. Now MR corresponds to the mean speed n for which $\omega = 9$, and since $E = \dfrac{1}{2} J\omega^2$, and J has a mean value in this case of approximately 2410, the value of E will be $E = \dfrac{1}{2} \times 2410 \times 9^2 = 97605$ ft. pds., which is represented by RM. Now from measurements on Fig. 132, there is obtained $NL = 3180$ ft. pds., hence $2\delta = \dfrac{3180}{97605} = .0312$ or $\delta = .0156$ or the total variation in speed will be 1.56%. Compare this with result given on page 198.

This method shows at once the effect of changing the weight of fly-wheel and also of changing the speed of the engine.

(1) Let the speed of the engine be kept constant at 87 revs. per min., then the *direction* of the line OQM is fixed for in the case of this line $\tan \alpha = \dfrac{1}{2}\omega^2$, which depends on ω only, and also the *position* of the line is fixed because it must pass through the centre of the figure. So long as the speed remains constant, therefore, the origin O must lie on a fixed line OQM. If the moment of inertia of the fly-wheel be decreased, the point O moves toward M, the tangents OF and OH make wider angles with one another, raising

N and lowering L and increasing NL, and hence the speed fluctuation, since RM decreases at the same time.

Suppose, for example, the moment of inertia (or what is approximately the same thing, the weight of the rim) of the fly-wheel is reduced to $\frac{1}{2}$ its present value, making it 1200 instead of 2400. Then RM represents $E = \frac{1}{2} \times 1210 \times 9^2 = 49005$ ft. pds., and NL will also increase slightly, but the change in it will be small and it will be considered constant. Then $2\delta = \frac{3180}{49000} = .065$ or $\delta = 032$ or the variation in speed will be 3.2% or double what it was before.

(2) Let the moment of inertia of the fly-wheel be kept constant but let the speed of the engine be increased, then the origin O will

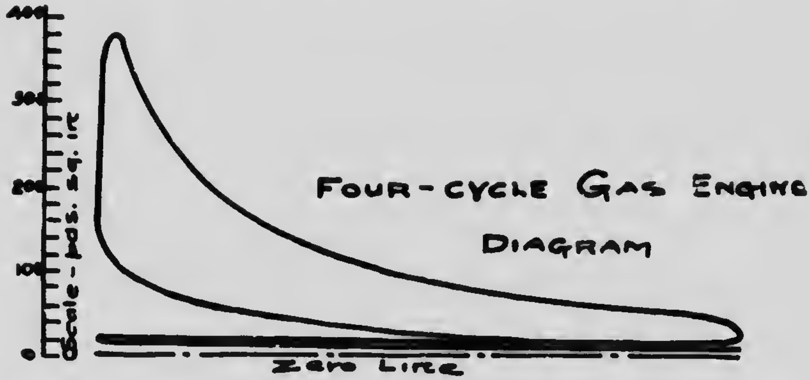


Fig. 134

travel down the vertical line through O. Let the speed be increased $\frac{1}{9} = 11.1\%$ so as to bring n up to 97 revs. per min. ($\omega = 10$), then $\tan\alpha = \frac{1}{2}\omega^2 = 50$, and hence $\frac{E}{J} = 50$ or since $J = 2410$ as before, $E = 50 \times 2410 = 120500$ ft. pds., which is represented by MR. The fluctuation in speed is now $2\delta = \frac{NL}{MR} = \frac{2960}{120500} = .0245$ or $\delta = .0123$, or the variation in speed is reduced to 1.23% as against 1.56% in the original case.

The graphical method described here is very useful and instructive, but it is much more helpful to use a combination of this method with the arithmetical process described in the last chapter.

Another illustration may be given in concluding the chapter, this being the case of a four-cycle gas engine which was built for

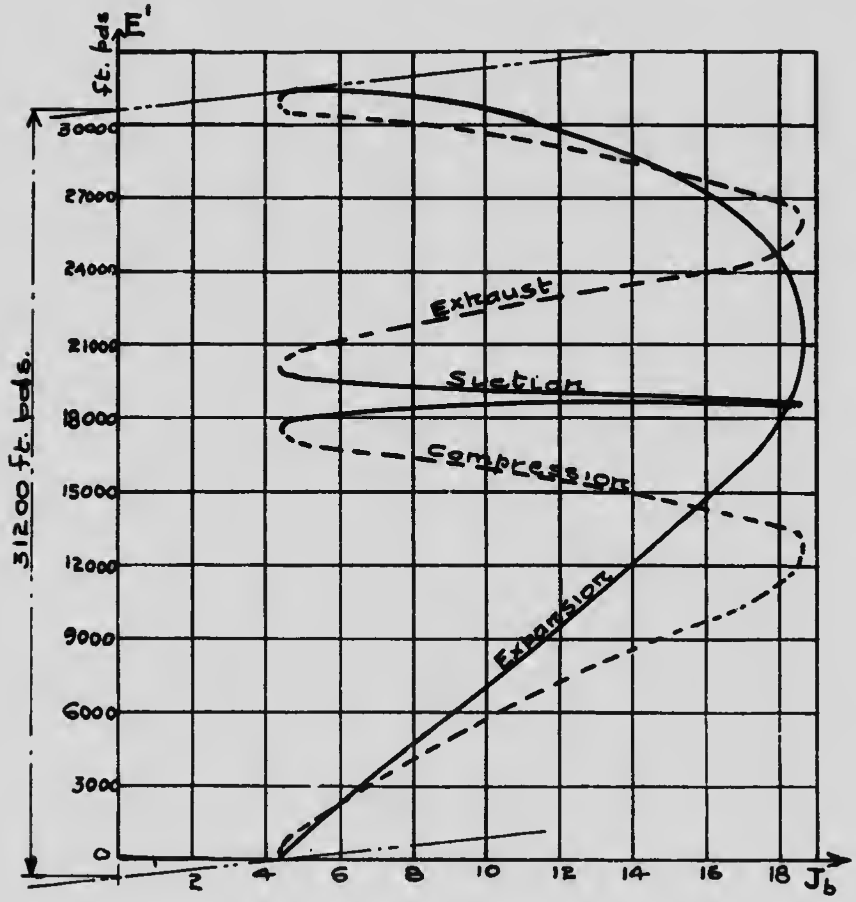


Fig. 135

direct connection to an electric generator. The engine was single-acting having a cylinder $14\frac{1}{2}$ in. diam. and 22 in. stroke, the piston, etc., weighed 360 lbs., the connecting rod weighed 332 lbs., and had a radius of gyration about the centre of gravity of 1.97 ft., the centre of gravity of the rod was 24.3 in. from the centre of the crank pin, while the length of the rod was 55 in. centre to centre.

The engine had two fly-wheels which had a combined weight of 7000 lb. and a combined moment of inertia of 1600 (ft. pd. units). No allowance was made for the rotating part of the generator which was small in diameter, and would produce very little effect as far as steadiness of motion was concerned.

The mean speed of the engine was 172 revs. per min. The indicator diagram for this engine is given at Fig. 134, and the $E-J$ diagram at Fig. 135, the axis of E being chosen at the point where $J = 1600$, the moment of inertia of the fly-wheels, so that the figure represents only J_b.

The figure in this case differs very materially in appearance from that for the steam engine. For this diagram $\tan \alpha = \frac{1}{2}\omega^2 = \frac{E}{J} = 162$ since $\omega = 18$. Further, $E = 6000e$ and $J = 4j$ so that $\tan \alpha = 162 = \frac{6000e}{4j}$, hence $\frac{e}{j} = .108$ which gives the actual slope of the mean speed line on the paper. The mean value of E for $J = 1600$ is $E = 162\ J = 259200$ ft. pds. and hence the speed variation is

$$\delta = \frac{1}{2} \times \frac{31200}{259200} = .0602 = 6.02\%$$

It is needless to say that the engine was absolutely unfitted for its purpose, and the student will do well to compute the necessary moment of inertia of the wheels to reduce the variation to say 2%, first at the speed of 172 r.p.m., and also if the speed were increased to 210 r.p.m.

The writer believes that the E -- J diagram is due to Wittenbauer, see "Zeitschrift des Vereines deutscher Ingenieure" for 1905. The method is fully discussed in "Die Regelung der Kraftmaschinen" by M. Tolle, (Springer, Berlin) which is recommended to those interested.

CHAPTER XV.

ACCELERATIONS IN MACHINERY AND THE FORCES DUE TO THE INERTIA OF PARTS

It is frequently necessary to determine the acceleration of the different parts of a machine, e.g., in the steam or gasoline engine it is desirable to know the pressure necessary at any time to accelerate the piston and cross-head, in order to determine the turning effect on the crank shaft, or in the case of valve gears of gas engines the acceleration of the valve is necessary in order to determine the proper design of the different parts, for the force necessary to move

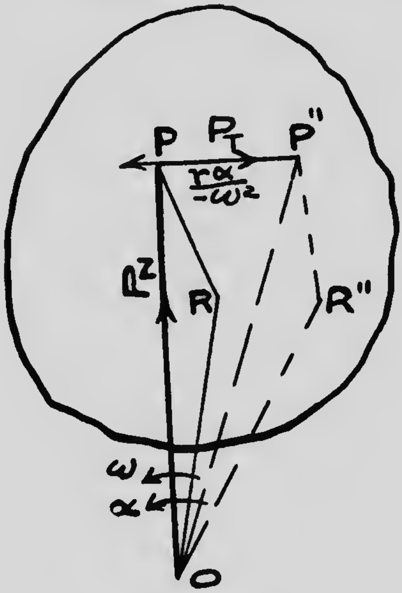


Fig. 136

any link at any time depends upon its acceleration and the resistance acting upon it. The method here described shows how the acceleration of the parts of a machine may be found in a simple and direct way.

Only motion in one plane is being considered, which will cover most cases accurring in practice. Suppose a body of weight w lbs. and mass $m = \frac{w}{g}$ is moving in a plane at any instant, then by the principle of the virtual centre, it is known that its motion is equivalent to that of rotation, for the instant about some point; if this point is at an infinite distance the motion is simply one of translation. Let Fig. 136 be the body under consideration, which is moving in the plane of the paper, and for the instant of its motion let it be rotating about the centre O, then any point in the body, such as P will travel in a direction normal to OP and the sense will be as indicated, where the angular velocity is in the sense shown. This point P has an acceleration toward O of amount $OP \cdot \omega^2$ or $r\omega^2$, and the force necessary to produce this acceleration is $m\ r\ \omega^2$ in the radial direction, the force balancing this is usually called the centrifugal force.

If now the body is rotating about O with varying velocity then the point P has also acceleration or change of velocity in the direction of motion.

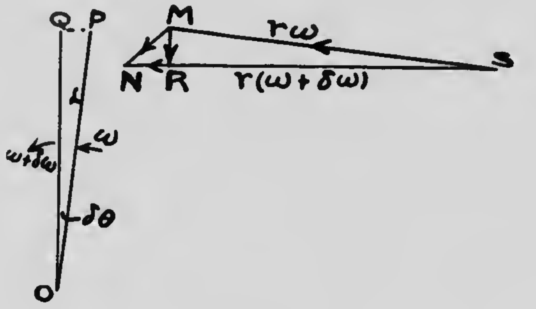


Fig 137

Let OP, Fig. 137, represent the radius to the point P, the angular velocity being ω and let OQ represent the position of this radius at time δt later, when the angular velocity is $\omega + \delta\omega$, the gain in angular velocity in time δt being $\delta\omega$, or the angular acceleration $\alpha = \dfrac{\delta\omega}{\delta t}$. Draw SM to represent the linear velocity of P, i.e., make $SM = OP \cdot \omega = r\omega$ and draw SN at angle $\delta\theta$ from SM to represent the velocity after the time δt when OP has reached OQ or $SN = r(\omega + \delta\omega)$, then the gain in velocity of P in time δt is MN, and the normal and tangential components of this gain in velocity are respectively MR and RN.

Now the normal gain in velocity in time δt is $MR = r\omega \cdot \delta\theta$, and therefore, the normal acceleration is $\frac{MR}{\delta t} = \frac{r\omega\ \delta\theta}{\delta t} = r\omega^2$ while the tangential acceleration is evidently

$$\frac{RN}{\delta t} = \frac{SN - SR}{\delta t} = \frac{r(\omega + \delta\omega) - r\omega}{\delta t} = r\frac{\delta\omega}{\delta t} = r\alpha.$$

The sense of the tangential acceleration $r\alpha$ is determined by the sense of α and the normal acceleration MR is toward the centre of rotation. Evidently $r\alpha = 0$ if $\alpha = 0$, but $r\omega^2$ is never zero if the body is rotating.

Returning now to Fig. 136, since the normal acceleration of P or P_N is $r\omega^2$ toward O, take the length OP to represent this quantity adopting the scale of $-\omega^2$: 1; this is negative since the line OP represents the acceleration $r\omega^2$ in the direction and sense PO.

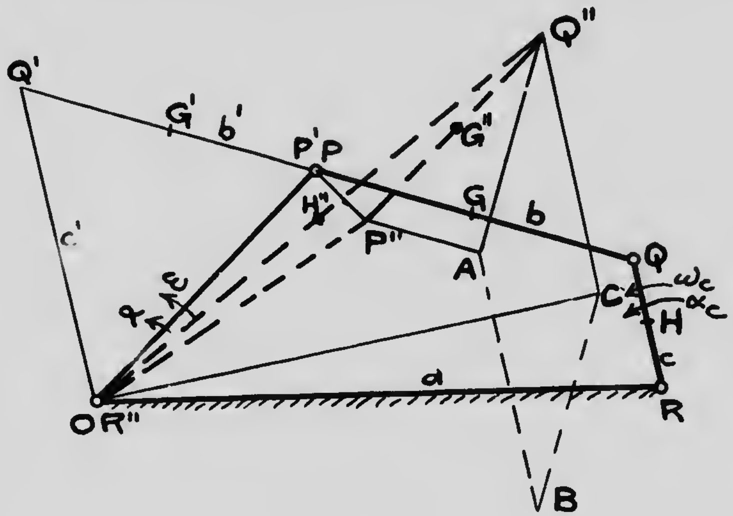


Fig. 138

Then also the tangential acceleration P_T may be represented by a line normal to OP, its length will be $\frac{r\alpha}{\omega^2}$ since the scale is ω^2. 1, and its sense is to the right, since the scale is negative, hence draw $PP'' = \frac{r\alpha}{-\omega^2}$. Now if OP'' be drawn, then OP'' = vector sum $OP + PP''$ or $OP'' = P_N + P_T$ which will therefore give the total acceleration of P, or the total acceleration of P is $P''O\ .\ \omega^2$ in the direction and sense $P''O$. It may very easily be shown that in order to find the acceleration of any other point R on this body at the given instant it will only be necessary to locate a point R'' bearing the same relation to OP'' that R does to OP, the acceleration of R, which is represented by OR'', being $R''O \cdot \omega^2$ and its direction and sense $R''O$.

These ideas may now be applied to machines and the first case considered will be as general as possible, the machine being one of four links with four turning pairs, Fig. 138. Let the angular velocity ω and the angular acceleration α of the primary link be known, it is required to find the angular accelerations of the other links as well as the linear accelerations of different points in them. From the phorograph, Chapter IV, the angular velocities of the links b and c are $\omega_b = \frac{b'}{b}\,\omega$ and $\omega_c = \frac{c'}{c}\,\omega$, and from the foregoing propositions $P_N = a\,\omega^2$; $P_T = a\,\alpha$; $Q_N = b\,\omega_b^2$; $Q_T = b\,\alpha_b$ also $R_N = c\,\omega_c^2$ and $R_T = c\,\alpha_c$.

Using the principle of vector addition the total acceleration of R with regard to O is the vector sum of the accelerations of R with regard to Q, of Q with regard to P and of P with regard to O. But as R and O are stationary, the total acceleration of R with regard to O is zero. Hence, the sum of the above three accelerations is zero, or $R_T + R_N + Q_T + Q_N + P_T + P_N = O$, i.e., the vector polygon made up with these accelerations as its sides must close, or if the polygon be started at O it will close at O also.

The point P'' may be located according to the method previously given, the scale being $-\,\omega^2$ to 1, and in order to locate Q'', giving the total acceleration of Q, proceed from P'' O by means of the vectors $Q_N + Q_T + R_N + R_T$. The direct d sense of both Q_N and R_N are known, they are respectively Q nd RQ, further, the direction, but not the sense of Q_T and R_T is known, in each case, it is normal to the link itself, or Q_T is normal to b and R_T is normal to c,

Again $Q_N = b\,\omega_b^2$ toward P and $\omega_b = \frac{b'}{b} \cdot \omega$, therefore,

$$Q_N = b\left(\frac{b'\,\omega}{b}\right)^2 = \frac{b'^2}{b} \cdot \omega^2,$$ similarly $R_N = \frac{c'^2}{c} \cdot \omega^2$, and since the scale is $-\,\omega^2$ to 1, draw $P''A = \frac{Q_N}{\omega^2} = \frac{b'^2}{b}$, and further,

$AB = \frac{R_N}{\omega^2} = \frac{c'^2}{c}$. The polygon from B to O may now be completed by adding the vectors Q_T and R_T, and as the directions of these are known, the process is evidently to draw from O the line OC in the direction R_T, i.e., normal and from B the line $B\ C$ normal to b, which is in the direction of Q_T, these lines intersecting at the point C. Then it is evident that BC represents Q_T on the scale $-\omega^2$ to 1, and that OC represents R_T on the same scale, so that in the diagram $OPP''\ A\ B\ C\ Q''\ O$ it is evident that $OP = P_N$, $PP'' = P_T$, $P''\ A = Q_N$, $AB = R_N$, $BC = Q_T$ and $CO = R_T$, all on the scale $-\omega^2$ to 1. By completing the parallelogram CA evidently $OP'' = P_N + P_T$, $P''Q'' = Q_N + Q_T$ and $Q''O = R_N + R_T$, and therefore, the vector triangle $OP''Q''R$ gives the vector acceleration diagram of all links on the machine.

The angular accelerations of the links may be found as follows. Since $Q_T = AQ'' \times -\omega^2 = b\,\alpha_b$, then $b\,\alpha_b = -AQ''\,.\,\omega^2$ or $-\alpha_b = AQ''\,.\,\frac{\omega^2}{b}$ so that the length AQ'' represents α_b, the angular acceleration of the link b, and similarly CO represents the angular acceleration α_c of c or $\alpha_c = -CO\,\frac{\omega^2}{c}$. The sense of these angular accelerations may be found by noticing the way one turns to them in going from the corresponding normal acceleration line, thus, in going from P_N to P_T one turns to the right, in going from Q_N ($P''\ A$) to Q_T (AQ'') the turn is to the left and hence α_b is in opposite sense to α, and by a similar process of reasoning α_c is in the same sense as α. Thus, in the position shown in the diagram, Fig. 138, all of the angular velocities are increasing.

The linear acceleration of any point such as G on b is readily shown to be represented by OG'' and to be equal to $G''O\,.\,\omega^2$, where the point G'' divides $P''Q''$ in the same way that G divides PQ, the direction and sense of the acceleration of G is $G''O$. Similarly, the acceleration of H in c is $H''O\,.\,\omega^2$ in magnitude, direction and sense where H'' divides $OQ''(R''Q'')$ in the same way as H divides RQ.

THE FORCES ACTING ON THE MACHINE PARTS

It is often necessary to find the force which must be exerted upon any link to balance the inertia of the link, and the determinations of the above accelerations enable this to be done. Let $OP''Q''O$, Fig. 139, be the vector acceleration diagram for the machine, the phorograph being $OP'Q'O$, and let it be required to find the force which must be exerted on the link b to produce its motion in the given position. Let G be the centre of gravity of the link and let $I_b = m_b k_b^2$ represent the moment of inertia of the link about G, k_b being the radius of gyration, and m_b the mass, i.e., the weight divided by g. Let a_b be found as already described, also the acceleration of G is $G''O \cdot \omega^2$, as already explained.

Now in order to produce the acceleration of the centre of gravity of the link it is necessary to apply a force F acting through G and in the sense and direction $G''O$ of the acceleration. The magni-

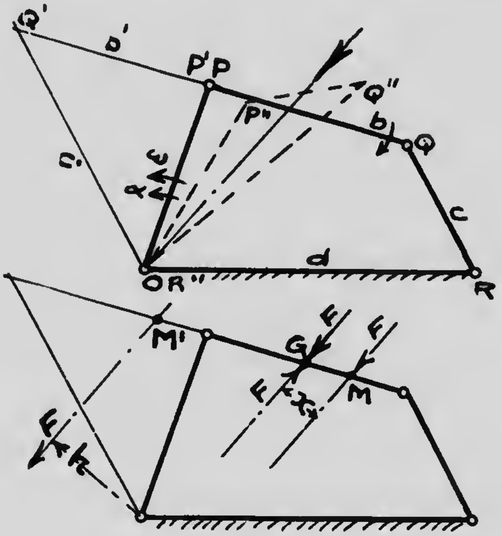


Fig. 139

tude of this force is the mass of the link multiplied by the acceleration of the centre of gravity, or $F = m_b \cdot G''O \cdot \omega^2$, a force which is given completely in magnitude, direction, sense and position. Again, in order to produce the angular acceleration a_b which the body has, there must be applied a torque T equal to the moment of inertia of the link about G multiplied by the angular acceleration of the link, and the sense of T must be the same as a_b, thus $T = I_b\ a_b = m_b\ k_b^2\ a_b$. This torque may be produced by a couple, consisting of two parallel forces, and the forces composing the couple may have any magnitude so long as the distance apart is made sufficient to give the torque T. Choose therefore, two parallel forces in opposite sense each equal to F and let the distance between them be x ft., then must $Fx = T$.

Now, as this couple may act in any position on the link b let it be so placed that one of the forces passes through G and let the forces have the same direction as the acceleration of G. Further, let the force passing through G be the one which acts in opposite sense to the accelerating force F, this is shown on Fig. 139. Now the accelerating force F and one of the forces F composing the couple act through G and neutralize one another and thus the accelerating force and the couple producing the torque reduce to a single force F whose magnitude is $m_b . G''O . \omega^2$, whose direction and sense are the same as the acceleration of the centre of gravity G of b, and which acts at a distance x from G, (x being determined by the relation $T = Fx$). and on that side of G which makes the torque act in the same sense as the angular acceleration α.

The distance x of the force F from G may be found as follows: Since $Q_T = b\,\alpha_b = Q''A\ \omega^2$, Fig. 138, then $\alpha_b = Q''A \cdot \dfrac{\omega^2}{b}$, because the line AQ'' represents Q_T on a scale — $\omega^2 : 1$.

Also $T = I_b\ \alpha_b = m_b\, k_b^2\ \dfrac{Q''A}{b} \cdot \omega^2$

and $F = m_b \,.\, G''O \,.\, \omega^2$

$$\text{therefore } x = \frac{T}{F} = \frac{m_b\, k_b^2\ \dfrac{Q''A}{b} . \omega^2}{m_b \,.\, G''O \,.\, \omega^2} = \frac{k_b^2}{b} . \frac{Q''A}{G''O} \text{ where } \frac{k_b^2}{b} \text{ is a con-}$$

stant, so that $x = \text{const.} \times \dfrac{Q''A}{G''O}$ which ratio can readily be found for any position of the mechanism. This gives the line of action of the single force F and, having found the position of the force, let M be its point of intersection with the axis of link b. Now find M' the image of M and move the force from M to its image M', then the turning moment necessary on the link a to accelerate the link b is Fh, where h is the shortest distance from O to the direction of F, Fig. 139.

This completes the problem, giving the force acting on the link and also the turning moment at the link a necessary to produce this force. The same construction may be applied to each of the other links, such as c and a, and thus the turning moment on a necessary to accelerate the different links may be found.

DETERMINATION OF THE STRESSES IN THE PARTS DUE TO THEIR INERTIA

The method just described may be used to find the bending moment produced in any link at any instant due to its inertia. Any part such as the connecting rod of an engine is subject to stresses due to the transmission of the pressure from the piston to the crank pin, but in addition to this the rod is continually being accelerated

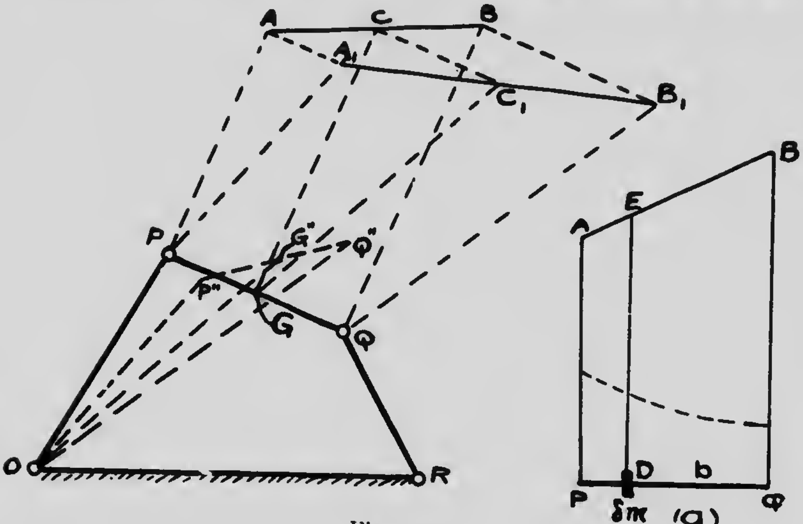


Fig. 140

and retarded, these changes of velocity producing bending stresses in the rod and these latter stresses may now be determined.

To make the case as general as possible, let $OPQR$, Fig. 140, represent a machine for which the vector acceleration diagram is $OP''Q''O$, it is required to find the bending moment in the rod b due to its inertia. Lay off at each point on b the acceleration of that point, thus make PA_1, GC_1, QB_1 etc., equal and parallel respectively to OP'', OG'', OQ'' etc., obtaining in this way the curve $A_1C_1B_1$.

Now resolve the accelerations at each point in b into two parts, one normal to b and the other parallel to the link. Thus PA is the acceleration of P normal to b, and GC and QB are the corresponding accelerations for the points G and Q respectively. In this way a second curve ACB may be drawn, and the perpendicular to b drawn from any point in it to the line ACB represents the acceleration at the given point in b in the direction normal to the axis of the latter, the scale in all cases being $-\omega^2 : 1$. Thus the acceleration of P normal to b is $AP \cdot \omega^2$, and so for other points.

The bending moment in the rod is due to the acceleration normal to the axis of the latter and hence is proportional to the distances from b to the curve ACB. Further, the bending moment is proportional to the distribution of the mass of the rod; thus it is proportional to the mass and the acceleration. In this discussion the actual bending moment is not determined, simply the *load curve*.

In Fig. 140 (a), the rod is shown along with the curve AB. Divide the rod up into a series of elementary parts, each of weight δw and of mass $\delta m = \frac{\delta w}{g}$, one of these masses being shown at D, at which place the acceleration is $ED \cdot \omega^2$, and hence the force acting at this point is $\delta m \cdot ED \cdot \omega^2$. In this way a *load curve* may be drawn for the rod and for the load curve thus obtained, the corresponding bending moment curve may be found by the ordinary methods of mechanics.

If the rod is of uniform diameter, then ACB is also a load curve on a scale which may readily be determined, whereas if the rod tapers uniformly from P to Q the load curve will take a form similar to that shown dotted above b. Usually the rod varies in shape and cross-section from end to end, frequently being larger in the centre than at the ends, in which case the process is rather more tedious but may be carried out to any degree of accuracy desired by the designer.

It may be mentioned that the method is essentially one for the drafting-board and the methods of the calculus are usually too cumbersome to be adopted except in the most simple shapes.

APPLICATION TO THE STEAM ENGINE

This construction and the determination of the accelerations and forces has a very useful application in the case of the reciprocating engine and this machine will now be taken up. Fig. 141 represents an engine in which O is the crank shaft, P the crank pin and Q the wrist pin, the block c representing the crosshead, piston and piston rod. Let the crank turn with angular velocity ω and have an acceleration α in the sense shown, and let G be the centre of gravity of the connecting ro ' To get the vecter acceleration diagram find P'' exactly as in the former construction, OP representing the acceleration $PO \cdot \omega^2$ and PP'' the acceleration $a\,\alpha$, both on the scale — ω^2 to 1.

Now the motion of Q is one of sliding and thus Q has only tangential acceleration, or acceleration in the direction of sliding, in this case QS, the sense being determined later. Hence, the total acceleration of Q must be represented by a line through O in the direction QS so that Q'' lies on a line through the centre of the crank shaft, and the diagram is reduced to a simpler form than in the more general case. Having found P'', draw $P''A$ parallel to b, of length $\frac{b''^2}{b}$ to represent Q_N, and also draw AQ'', normal to $P''A$, to meet the line $Q''O$ (which is parallel to QS)

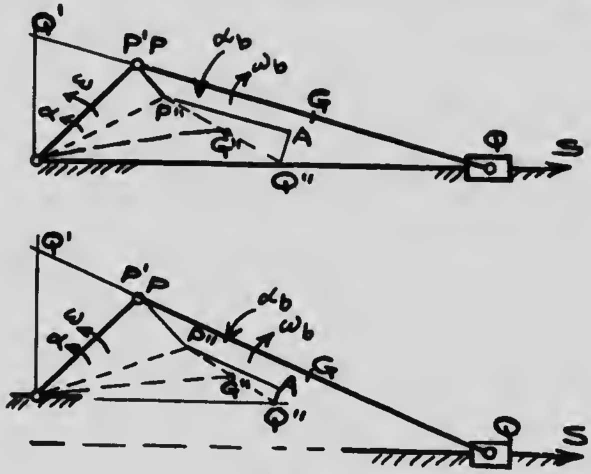


Fig. 141

in Q''. Then will AQ'' represent the value of the angular acceleration of the rod b, since $b\alpha_b = Q''A \cdot \omega^2$ or $\alpha_b = Q''A \cdot \frac{\omega^2}{b}$, and since AQ'' lies on the same side of $P''A$ that PP'' does of OP, therefore α_b is in the same sense as α; thus since ω_b is opposite to ω, the angular velocity of the rod is decreasing, or the rod is being retarded.

The acceleration of the centre of gravity of b is represented by OG'' and is equal to $G''O \cdot \omega^2$, and similarly the acceleration of the end Q of the rod is represented by OQ'' and is equal to $Q''O \cdot \omega^2$, this being also the acceleration of the piston.

It will be observed that all of these accelerations increase as the square of the number of revolutions per minute of the crank shaft, so that while in si ~ed engines the inertia forces may not produce any very serious troubles, yet in high speed engines they are very important and in the case of such engines as are used on automobiles, which run at speeds of 1500 revs. per min., these accelerations are very large and the forces necessary to produce them cause considerable disturbances. Take the piston for example, the force required to move it will depend on the product of its weight and its acceleration so that if an engine ran normally at 750 revs. per min. and then it was afterwards decided to speed it up to 1500 revs. per min., the force

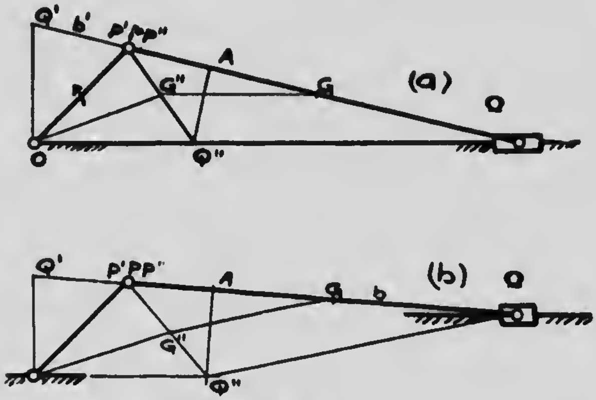


Fig. 142

required to move the piston in any position in the latter case would be four times as great as in the former case.

In the actual ~f the steam engine, the calculations may be very much simp: to certain limitations which are imposed on all designs of en ng other machinery, these limitations being briefly that the in velocity of the fly-wheel must be comparatively small, i.e., the angular acceleration of the fly-wheel must not be great, and in fact, on engines the fly-wheels are made so heavy that α cannot be large.

To get a definite idea on this subject a case was worked out for a 10 in. × 10 in. steam engine, running at 310 revs. per min., and the maximum angular acceleration of the crank was found to be slightly less than 7 radians per sec., per sec. For this case the normal acceleration of P is $r\omega^2 = \frac{5}{12} \times 1100 = 458$ ft. per sec., per sec., while the tangential acceleration is $r\alpha = \frac{5}{12} \times 7 = 5.8$ ft. per sec. per sec., which is very small compared with 458 ft. per sec. per sec., so that on any ordinary drawing the point P'' would be very close to P. Thus without serious error $r\alpha$ may be neglected compared with $r\omega^2$ and thus we may take P''at P in the case of the steam engine.

With the foregoing modification for the steam engine, the complete acceleration diagram is shown at Fig. 142, the length PA representing $\frac{b'^2}{b}$, AQ'' being normal to b, thus $P''Q''$ is the acceleration diagram for the connecting rod and OQ'' resents the acceleration of the piston on the scale — ω^2 to 1. Tv cases are shown (a) for the ordinary construction and (b) for the off-set cylinder. The acceleration of any such point as G is found by finding G'', making the line GG'' parallel to QQ'', the accelerations then is $G''O \cdot \omega^2$. Dealing only with the case shown in figure (a) it is seen that when the crank is vertical, b' is zero, and hence A is at P, or Q'' lies to the left of O, so that the piston is being retarded. The numerical value of the acceleration may be found in this case by remembering that $Q''Q$ may be taken as the diameter of a circle which will pass through P and hence $Q''O \cdot OQ = OP^2$ or $OQ'' = \frac{OP^2}{OQ} = \frac{a^2}{\sqrt{b^2-a^2}}$

so that the acceleration of the piston is $OQ'' \cdot \omega^2 = \frac{a^2}{\sqrt{b^2-a^2}}\,\omega^2$

At both the head and crank ends $b' = a$ hence $P''A = \frac{b'^2}{b} = \frac{a^2}{b}$,

so that for the head end $Q''O = a + \frac{a^2}{b}$ and the piston has its maximum acceleration at this point, which is $\left(a + \frac{a^2}{b}\right)\omega^2$ toward O, while for the crank end $Q''O = a - \frac{a^2}{b}$ and the acceleration is $\left(a - \frac{a^2}{b}\right)\omega^2$ toward O, so that the piston is being retarded.

Example: Let an engine with 7 in. stroke and a connecting rod 18 in. long run at 525 revs. per min. Then $a = \frac{3\frac{1}{2}}{12} = .29$ ft., $b = \frac{18}{12} = 1.5$ ft. and $\omega = 55$ radians per sec.

At the head end the acceleration of the piston would be

$$\left(a + \frac{a^2}{b}\right)\omega^2 = \left(.29 + \frac{.29^2}{1.5}\right) \times 55^2 = 931 \text{ ft. per sec.,}$$

per sec.

At the crank end the acceleration would be:

$$\left(a - \frac{a^2}{b}\right)\omega^2 = \left(.29 - \frac{.29^2}{1.5}\right) \times 55^2 = 623 \text{ ft. per sec.,}$$

per sec.

At the time when the crank is vertical the result is:

$$\frac{a^2}{\sqrt{b^2 - a^2}}\omega^2 = \left[\frac{.29^2}{\sqrt{1.5^2 - .29^2}}\right] \times 55^2 = 173 \text{ ft. per sec., per sec.}$$

The angular acceleration of the rod, being determined by the length AQ'', is zero at each of the dead points but when the crank is vertical this velocity has nearly its maximum value, its exact value being $Q''A \cdot \frac{\omega^2}{b}$. When the crank is vertical a diagram will show that $Q''A = \frac{a\,b}{\sqrt{b^2 - a^2}}$, and the acceleration will be $\alpha_b = \frac{a}{\sqrt{b^2 - a^2}}\omega^2$.

For the engine already examined $\alpha_b = \left[\frac{.29}{\sqrt{1.5^2 - .29^2}}\right] 55^2$ $= 596$ radians per sec. per sec.

APPROXIMATE GRAPHICAL SOLUTION FOR THE STEAM ENGINE

In the approximate method already described, in which the angular acceleration of the crank shaft is neglected and P'' is assumed to coincide with P, it will be noticed that length $P''A = \frac{b'^2}{b}$, is laid off along the connecting rod, the length $P'Q'$ representing b', and PQ the length b, and then AQ'' is drawn perpendicular to PQ. This may be carried out by a very simple graphical method as follows: With centre P and radius $P'Q' = b'$ describe a circle, Fig. 143, then describe a second circle, having the connecting rod b as its diameter cutting the first circle at M and N and join MN, where MN cuts b locates the point A and where it cuts the line through O in the direction of motion of Q gives Q''

The proof is that PMG being the angle in a semicircle is a right angle, also MN a chord in the circle $MPNQ$, is normal to PQ by construction, so that MN is bisected at A. Thus in the circle $MPNQ$, there are two chords PQ and MN intersecting at A, hence

$$PA \,.\, AQ = MA^2$$

hence $PA\,(PQ - PA) = b'^2 - PA^2$

that is $PA \,.\, PQ - PA^2 = b'^2 - PA^2$ from which $PA \,.\, PQ = b'^2$

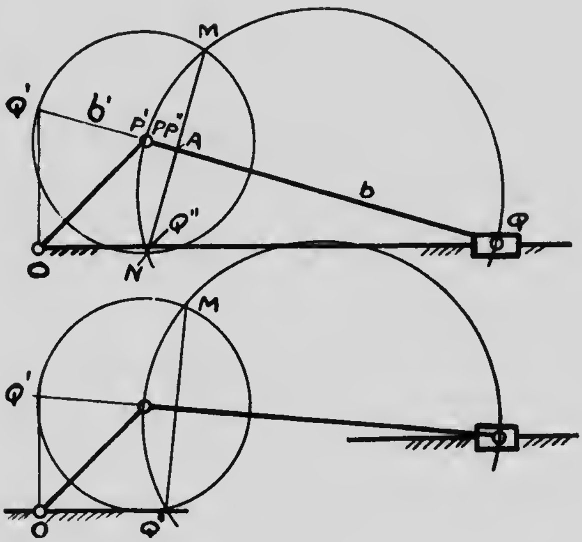


Fig. 143

or $PA \,.\, b = b'^2$ or $PA = \dfrac{b'^2}{b}$ which proves that the construction is correct.

THE EFFECTS OF THE ACCELERATIONS OF THE PARTS UPON THE FORCES ACTING AT THE CRANK SHAFT OF AN ENGINE.

In order to accelerate or retard the various parts of the engine, some torque must be required or will be produced at the crank shaft, and a study of this will now be taken up in detail.

(a) *The effect produced by the piston.*

By the construction already described the acceleration of the piston is readily found and it will be seen that Q'' lies first on the cylinder side of O and then on the opposite side. When Q'' lies between O and Q, Fig. 144, then since the acceleration is $Q''O \cdot \omega^2$, the acceleration of the piston is in the same sense as the motion of the piston, or the piston is being accelerated. Conversely, when Q'' lies on QO produced the acceleration being in the opposite sense to the motion of the piston, the latter is being retarded. If now the accelerations for the different piston positions on the forward stroke be plotted, the diagram EJH will be obtained, Fig. 144, where the part of the diagram EJ represents accelerations of the piston, and the part JH negative accelerations, or retardations. The corresponding diagram for the return stroke of the piston is omitted to avoid complexity.

Let the combined weight of the piston, piston rod and cross-head be w_c lbs., the corresponding mass being $m_c = \frac{w_c}{g}$, and let f represent the acceleration of the piston at any instant, then the force

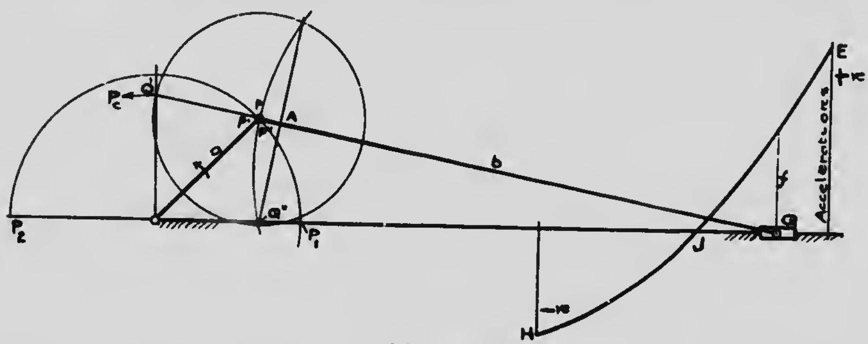


Fig. 144

P_c necessary to produce this acceleration will be $P_c = m_c \cdot f$. This force will be positive if f is positive and vice versa, i.e., if f is positive a force must be exerted on the piston in its direction of motion and if it is negative the force must be opposed to the motion. In the first case energy must be supplied by the fly-wheel, or steam, or gas pressure, to speed up the piston, whereas, in the latter case, energy will be given up to the fly-wheel due to the decreasing velocity of the piston, but it is to be remembered that since no net energy is received during the operation, therefore, the work done on the piston in accelerating it must be equal to that done by the piston while it is being retarded.

Two methods are employed for finding the turning effect of this force, P_c : (a) to reduce it to an equivalent amount per square inch of piston area by the formula $p_c = \frac{P_c}{A}$ where A is the area of the piston, and then to correct the corresponding pressures as shown by the indicator diagram by this amount. In this way a reduced indicator diagram for each end is found, as shown for a steam engine in Fig. 145, where the dotted diagram is the reduced diagram found by subtracting the quantity p_c from the upper line on each diagram. The remaining area is the part effective in producing a turning moment on the crank shaft.

(b) The second method is to find directly the turning effect necessary on the crank shaft to overcome the force P_c, and from the

Fig. 145

principles of the phorograph this torque is evidently $T_c = P_c \times OQ' = m_c \cdot f \cdot OQ'$. In the position shown in Fig. 144, P_c would act as shown, and a torque acting in the same sense as the motion of a would have to be applied.

The first method is very instructive in that it shows that the force necessary to accelerate the piston at the beginning of the stroke in very high speed engines may be greater than that produced by the steam of gas pressure, and hence, that in such cases the connecting rod may be in tension at the beginning of the stroke, but, of course, before the stroke has very much proceeded it is in compression again. This change in the condition of stress in the rod frequently causes "pounding" due to the slight slackness allowed at the various pins.

(b) *The effect produced by the connecting rod.*

This effect is rather more difficult to deal with on account of the nature of the motion of the rod. The resultant force acting may, however, be found by the method described earlier in the chapter, but in the case of the steam engine, the construction may be much simplified, and on account of the importance of the problem the simpler method will be described here. It consists in dividing the rod up into two equivalent concentrated masses, one at the crosshead pin the other at a point to be determined.

Referring to Fig. 146, the rod is represented on the acceleration diagrams by $P''Q''$ and the acceleration of any point on it or the angular acceleration of the rod may be found at once by processes already explained. Let I_b be the moment of inertia of the rod about its centre of gravity, k_b being the corresponding radius of gyration and m_b the mass, so that $I_b = m_b k_b^2$, and let the centre of gravity lie on PQ at distance r_1 from Q. Instead of considering the actual rod it is possible to substitute for it two masses m_1 and m_2, which,

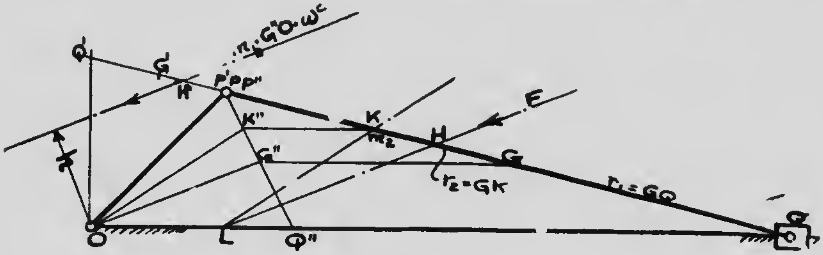


Fig. 146

if properly placed, and if of proper weight, will have the same inertia and weight as the original rod. Let these masses be m_1 and m_2 where $m_1 = \frac{w_1}{g}$ and $m_2 = \frac{w_2}{g}$, w_1 and w_2 being the weights of the masses in lbs. Further, let mass m_1 be concentrated at Q, it is required to find the weights w_1 and w_2 and the position of the weight w_2. Let r_2 be the distance from the centre of gravity of the rod to mass m_2.

These masses are determined by the following three conditions:

(1) The sum of the weights of the two masses must be equal to the weight of the rod, or $w_1 + w_2 = w_b$, or $m_1 + m_2 = m_b$.

(2) The two masses m_1 and m_2, must have their combined centre of gravity in the same place as before, or $m_1 r_1 = m_2 r_2$

(3) The two masses must have the same moment of inertia

about their combined centre of gravity as the original rod has about the same point, or $m_1 r_1^2 + m_2 r_2^2 = m_b k_b^2$.

For convenience we shall assemble these together.

$$m_1 + m_2 = m_b \qquad (1)$$
$$m_1 r_1 = m_2 r_2 \qquad (2)$$
$$m_1 r_1^2 + m_2 r_2^2 = m_b k_b^2 \qquad (3)$$

Solving these gives $m_1 = m_b \dfrac{r_2}{r_1 + r_2}$ and $m_2 = m_b \dfrac{r_1}{r_1 + r_2}$ and $r_1 r_2 = k_b^2$ or $r_2 = \dfrac{k_b^2}{r_1}$.

Thus, for the purposes of our problem the whole rod may be replaced by the two masses m_1 and m_2 placed as shown in Fig. 146. The one mass m_1 merely has the same effect as an increase in the weight of the piston and the method of finding the force required to accelerate it has already been described. Turning then to the mass m_2, which is at a fixed distance r_2 from G; the centre of gravity of m_2 is K and the acceleration of K is evidently $K''O \cdot \omega^2$, $K''K$ being parallel to $G''G$. The direction of the force acting on m_2 is the same as that of the acceleration of its centre of gravity and is therefore through K parallel to $K''O$, and the magnitude of this force is $m_2 \cdot K''O \cdot \omega^2$. The force acts through K, its line of action being KL parallel to $K''O$.

The whole rod has now been replaced by the two masses m_1 and m_2, the force acting on the former being $m_1 \cdot Q''O \cdot \omega^2$ through Q parallel to $Q''O$, i.e., this force is in the direction of motion of Q and passes through L on $Q''O$. The force on the mass m_2 is $m_2 \cdot K''O . \omega^2$, which also passes through L, so that the resultant force F acting on the rod must also pass through L. Thus the construction just described gives a convenient graphical method for locating one point L on the line of action of the resultant force F acting on the connecting rod.

Having found the point L the direction of the force F has been already shown to be parallel to $G''O$ and its magnitude is $m_b \cdot G''O . \omega^2$. Let F intersect the axis of the rod at H, find the image H' of H, and transfer F to H', the moment required to produce the acceleration of the rod is then Fh.

A number of trials on different forms and proportions of engines have shown that the point L remains in the same position for all crank angles, and hence if this is determined once for a given mechanism it will only be necessary to determine $G''O$ for the different crank positions.

In the position of the machine shown in Fig. 146, let P be the total pressure acting on the piston due to the steam or gas pressure then the turning moment acting on the crank shaft is $P \,.\, OQ' - [m_c \,.\, Q''O \,.\, \omega^2 \,.\, OQ' + m_b \,.\, G''O \,.\, \omega^2 \,.\, h]$ after making allowance for the effect of the inertia of the connecting rod and piston, and this turning moment will produce an acceleration of the fly-wheel if it exceeds that necessary to produce the, power and a retardation of the wheel if the turning moment required fo the power is in excess.

THE FORCES ACTING AT THE BEARINGS

The principles already discussed enable a determination to be made of the forces acting at the bearings in engines of different types and as the accelerations have been discussed very fully the reader is left to work out this important problem by himself.

In high speed engines the pressures on the bearings due to the inertia of the parts become very high indeed and may readily exceed the pressures due to the working fluid. Take, for example, a four cycle automobile engine, running at a speed of possibly 1600 r.p.m., it has already been shown that the inertia forces vary as the square of the speed and in such a machine they will be found very high. Due to the working fluid the connecting rod would be in compression in all but the suction stroke, however, when the inertia forces are considered, the rod may easily be in tension in all strokes and in

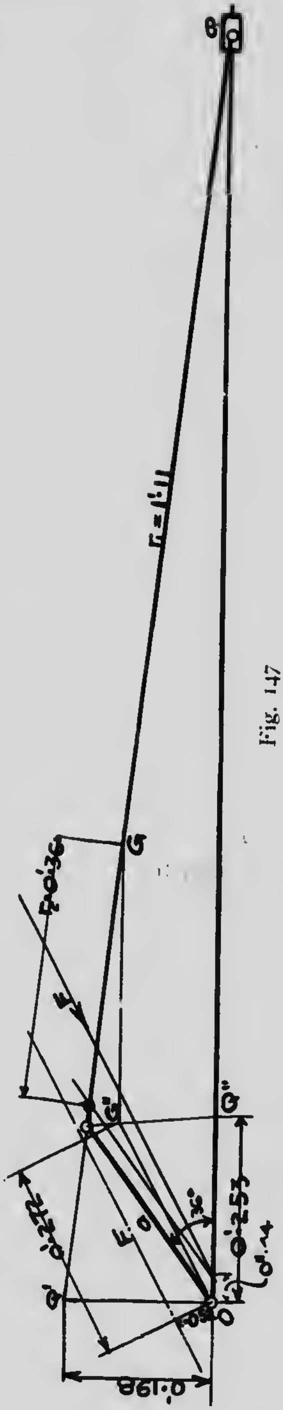


Fig. 147

TABLE SHOWING THE EFFECT DUE TO THE INERTIA OF THE PARTS OF AN 11 IN. X 7 IN. STEAM ENGINE RUNNING AT 525 R. P. M.

Crank Angle	Piston, Crosshead, etc.					Connecting Rod					Total turning moment required at crank to move all parts ft. pds.	
θ	$Q''O$ ft.	Accel. of piston ft. sec.[2]	Force to accel. piston, pds.	$Q'O$ ft.	Turning moment on crank ft. pds.	$G''O$ ft.	Acceln. of G ft. sec.[2]	Force to accel. rod, pds.	h ft.	Turning mom. on crank ft. pds.	+	—
0	+ .348	+ 1053	+ 5262	0	0	.349	1056	1542	0	0	0	
18	.325	983	4916	.107	+ 526	.297	898	1311	.033	43	569	
36	.253	765	3327	.198	758	.272	823	1201	.052	62	820	
54	.153	463	2314	.264	611	.242	732	1069	.050	53	664	
72	+ .047	+ 142	+ 711	.295	+ 210	.217	656	958	.023	22	232	
90	— .060	— 181	— 908	.292	— 265	.215	650	950	.023	22		287
108	.137	414	2072	.260	539	.228	690	1007	.045	45		584
126	.187	566	2829	.208	588	.248	750	1095	.048	48		636
144	.217	656	3282	.143	469	.267	808	1179	.035	41		510
162	— .232	— 702	— 3509	.073	— 256	.273	824	1205	.017	20		276
180	.235	711	3555	0	0	.235	711	1038	0	0		0
198	+ .232	+ 702	+ 3509	.073	+ 256	.273	824	1205	.017	20	276	
216	.217	656	3282	.143	469	.267	808	1179	.035	41	510	
234	.187	566	2829	.208	588	.248	750	1095	.048	48	636	
252	.137	414	2072	.260	539	.228	690	1007	.045	45	584	
270	+ .060	+ 181	+ 908	.292	+ 265	.215	650	950	.023	22	287	
288	— .047	— 142	— 711	.295	— 210	.217	656	958	.023	22		232
306	.153	463	2314	.264	611	.242	732	1069	.050	53		664
324	.253	765	3327	.198	758	.272	823	1201	.052	62		820
342	.325	983	4916	.107	— 526	.297	898	1311	.033	43		569
360	— .348	— 1053	— 5262	0	0	.349	1056	1542	0	0		0

case the engine does not explode at any time the forces acting on the rods and pins may be very much greater than during an explosion, because of the great forces required to move the parts.

Evidently, in such machines much damage might very easily be done by allowing the speed to become unduly high, and although the engine were doing no work it might easily be destroyed at this high speed.

The matter is well worthy of the careful study of the student.

COMPUTATION FOR AN ACTUAL ENGINE

This chapter will now be ended by a computation on an actual steam engine, partly for the purpose of explaining the method more fully and partly to give an idea of the magnitude of the various forces.

The engine selected is the high-pressure side of a vertical, compound, high-speed engine of about 125 h. p. The engine has a cylinder 11 in. dia. and 7 in. stroke and runs at 525 revs. per min., the piston, piston rod and crosshead weigh 161 lbs. The connecting rod is 18 in. long centre to centre, weighs 47 lbs. and has a radius of gyration about its centre of gravity of 7.56 in. The centre of gravity of the rod is 4.7 in. from the centre of the crank pin.

Taking the above data gives $\omega = 55$ radians per sec., the mass of the piston, etc., $m_c = 5$. For the connecting rod $m_b = 1.46$, $k_b = .63$ ft., $r_1 = 1.11$ ft., $r_2 = \dfrac{.63^2}{1.11} = .36$ ft. $m_1 = 1.46 \times \dfrac{.36}{1.11 + .36} = .35$, $m_2 = 1.11$.

The complete construction for the crank angle 36°, is shown in Fig. 147, where all the quantities have been clearly marked. In this case L remained fixed for all crank angles, being at a distance .44 in. from O on the same side as the piston. The results for this engine are set down in the accompanying table in which it is observed that at the head end of the stroke a force of 5262 pds. would be required to move the piston which would mean a net pressure on the piston area of over 55 pds. per sq. in., in other words, if the engine were driven with an effective steam pressure of less than 55 pds. at the beginning of the down stroke, then the piston rod would be in tension instead of compression for this position.

It is further to be noted that the disturbing effect of the connecting rod is much less marked than that of the crank and as far as its effect on the turning moment is concerned the connecting rod might be neglected. The total effect of all the moving parts, as given in the last column, is evidently very decided.

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