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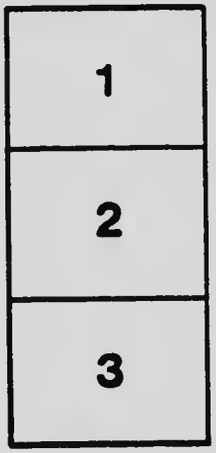

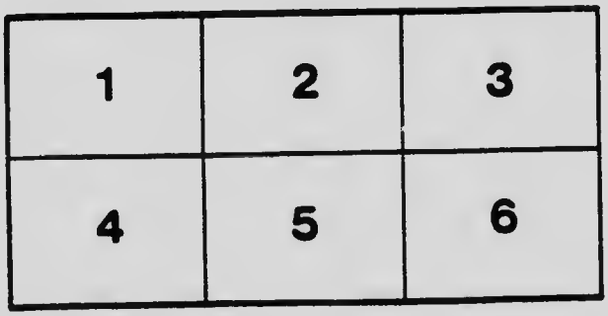

Mental Arithmetic

REVISED EDITION.

BY

AMOS O'BLENES, M. A.

INSPECTOR OF SCHOOLS, MONCTON, N. B.

PRICE 30 CENTS

BARNES & CO., LIMITED, ST. JOHN, N. B.

1912.

S. 74 a

Grade 7B

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Preface to Revised Edition

The first edition of this work has been re-arranged, and enlarged by the addition of numerous exercises. An attempt has been made to have the exercises consist of such examples as are met with in every day life. All of the examples may be solved by the application of one or more contracted methods such as are given in the various rules and the student should not be satisfied until he is sure he has found the shortest method possible.

A. O'B.

Moncton N. B.
February, 1912.

HINTS TO TEACHERS.

In order to secure accuracy and speed in mental arithmetic daily drill should be taken.

In all mental work imagine you see the figures involved. This habit can be acquired by drilling on each new rule with the figures actually before you until the rule is mastered. Then let the work be entirely oral and mental.

In the ordinary work in written arithmetic do as much as possible mentally.

Let the mental precede the written work in each new rule or exercise in arithmetic.

Do not depend alone on the examples given in any book or in all books on mental arithmetic for the necessary drill, but acquire the habit of making questions of your own.

Master the simple before attempting what is difficult.

In all cases where possible proceed from LEFT TO RIGHT in adding, subtracting or multiplying, as the separate parts in the result will then be presented to the mind in the order in which they must be expressed in giving the full result, e. g. if you wish to multiply 43 by 2, think of the two fours being eight then of the two threes being six and the result will be found in the order in which it must be expressed, that is, in the order 86.

Always look for short methods of doing work and make rules for yourself.

Do not leave any rule until your pupils can make questions for themselves.

Encourage your pupils to take a few minutes drill at home each morning before doing any other work.

Before taking up any rule with your class practise until you can do the work accurately and rapidly.

Do not allow bright pupils to answer before the others have had time to do the work in any ordinary class work.

Keep alive an interest by having match classes to test speed.

Give the pupils a few minutes each day for silent mental work on some rule which you assign them and let them make their own questions.

Before taking up the short methods given in this book be sure your pupils know the addition, subtraction, multiplication and division tables.

Many of the short methods may be taken up in grade three and some even in grade two.

MENTAL ARITHMETIC

In dealing with numbers in primary grades first use objects, then imaginary objects, then numbers in the abstract.

In using imaginary objects always have a definite place such as a table or desk on which the imaginary objects are supposed to be placed and point to them as though they were really there until the children imagine they see them.

Make questions as much as possible in line with the actual experience of the child.

Interest will increase if the pupils are permitted to question the class.

An excellent drill in addition can be had by counting by twos, threes, fours, etc., to 100, e. g., begin with 1 and count by twos to 100; thus, 1, 3, 5, 7, 9, 11, etc., to 101.

Begin with 2 and count by twos: thus, 2, 4, 6, 8, 10, etc., to 100.

In the same way count by threes from 1 to 100, from 2 to 100, from 3 to 100.

Drill in the same way with each of the digits to 9.

Continue this drill until the work can be done very rapidly.

Another excellent drill in addition can be given as follows:—

Add 2 and 2; add 12 and 2; add 12 and 12; add 22 and 2; add 22 and 12; add 22 and 22.

Add 3 and 2; add 13 and 2; add 13 and 12, etc.

It will be seen that an almost unlimited number and variety of exercises of this kind can be given.

Drill in subtraction may be had by reversing the above, e. g., count backward by twos from 100, thus, 100, 98, 96, 94, 92, etc.

Drill in the same way with all numbers from 2 to 9.

In business the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$ occur so frequently that even children in the primary grades should be taught their meaning, use and application.

After a clear understanding of the fractions mentioned has been obtained by the actual dividing of objects such as sticks, strings, apples, etc., and the handling and naming of the parts by the pupils, suitable questions should be given involving the adding, subtracting and multiplying of those fractions.

The questions should be at first solved by the pupils with real objects, then with imaginary objects and lastly with abstract numbers. Some such simple method as the following should make the work clear to the pupils:—

Find the cost of $\frac{3}{4}$ of a yard of ribbon at 20 cts. a yard.

Use a strip of paper to represent the ribbon. Have the 20 cts. in coppers. Divide the paper into four equal parts and the money into four equal piles. One pile of money pays for $\frac{1}{4}$ yard of ribbon, then three piles of money pay for $\frac{3}{4}$ yards.

The pupils should be led to do the above work by the teacher using the proper mode of questioning.

The ordinary weights and measures and pieces of money should be handled and used in the mental work for even primary grades.

Pieces of card board can be cut out to represent the various pieces of money in use, as coppers, 5 cent pieces, 10 cent pieces etc., and these should be handled in making change in the buying and selling questions which are given.

For ordinary measuring a foor-ruler and a yard stick marked off in inches will be sufficient.

The questions given to the primary classes should be graded in difficulty to suit the advancement of the class.

By following the foregoing suggestions any primary teacher should be able to make questions for her classes.

The following exercise is given as a sample. In it will be found questions suitable for primary grades.

Exercise I.

1. A boy has 3 pens in one hand and 4 in the other. How many pens has he?

2. How many are 3 and 4.

3. Tom had 8 apples and he gave John half of them. How many did he give John?

4. A boy bought a pencil for 3 cts., 2 pens for 1 ct. and an eraser for 4 cts. He gave in payment a 10 cent piece. How much change should he get back?

5. Find the half of 10.

6. Find the cost of $\frac{1}{2}$ yard cotton at 16 cts. a yard.

7. Find the cost of $\frac{3}{4}$ of a pound of butter at 24 cts. a pound.

8. How many five cent pieces make a dollar?

9. How many twenty-five cent pieces should be given for a fifty cent piece? For a dollar bill?

10. How many ten cent pieces make five dollars?

11. How many inches in one and a half feet?

12. How many inches in $\frac{1}{4}$ yard? In $\frac{1}{2}$ yard? In $\frac{3}{4}$ yard?

13. Find the cost of $2\frac{1}{2}$ lb. butter at 20 cts. a pound.

14. How many eggs in a dozen? In $\frac{1}{4}$ dozen? In $\frac{1}{2}$ dozen? In $\frac{3}{4}$ dozen?

15 Divide 3 apples equally between 2 boys.

16. How many eggs at 20 cts. a dozen will pay for 8 lb. sugar at 5 cts. a pound?

17. If a man walks 20 miles in a day how far will he walk from Monday morning to Saturday night?

18. Find the cost of 20 pens at the rate of 2 pens for 1 ct.

19. Find the cost of 30 pencils at the rate of 6 pencils for 1 ct.; at the rate of 5 pencils for 2 cts.

20. What change should be received from a dollar after paying for 9 slates at 10 cts. each?

An unlimited number of practical questions similar to the foregoing can be made by any teacher and graded to suit any class.

Rule I. To add any two numbers between 10 and 100.

Practise adding numbers that end in naught, such as 70 and 80; 50 and 90, etc. Next add two numbers one of which ends in naught, such as 67 and 80. By placing these in a column as in ordinary addition,

thus
$$\begin{array}{r} 67 \\ 80 \\ \hline 147 \end{array}$$

it will be seen that one has only to add 60 and 80 as the 7 in the sum corresponds with the 7 in the 67.

Next add any two numbers of two figures such as 67 and 86. Place as before,

$$\begin{array}{r} 67 \\ 86 \\ \hline \end{array}$$

and add the 67 and 80 as before and to the sum add the 6.

Special cases,

(a) $67 + 14 = 67 + 10 + 4 = 81$
(b) $81 + 16 = 81 + 10 + 6 = 97$
(c) $69 + 97 = 69 - 3 + 100 = 166$
(f) $79 + 93 = 79 - 7 + 100 = 172$
(g) $68 + 49 = 68 + 50 - 1 = 117$

Another method:

Add 76 and 89. Regard 76 as made up of 70 and 6 and regard 89 as made up of 80 and 9.

Place them thus,

$$\begin{array}{l} 70 \text{ and } 6 \\ 80 \text{ and } 9 \\ \hline 150 \text{ and } 15 = 165 \end{array}$$

and add, beginning at the left

It will be observed that in adding the 150 and 15 the tens figure in 150 is increased by 1 and the units figure is the same as the units in the 15.

In giving drill first place the numbers on the board in the manner shown and drill until the method is mastered and some degree of speed is gained, after which make the work oral, but get your pupils to imagine they have the numbers on the board.

Master one method before attempting the other.

Practise adding double columns of figures by one of the methods given.

Rule II. To substract any number of two figures from 100, e. g. 100 — 62.

Regard 100 as being made up of 90 and 10, and 62 as being made up of 60 and 2. Arrange thus,

90 and 10
60 and 2 and subtract beginning at the left,
30 and 8 = 38

As a preparation for the foregoing, learn to repeat the groups which make 90, namely 30 and 60; 40 and 50; 70 and 20; 80 and 10, until they can be repeated very rapidly in any order. In the same way repeat the groups which make 10, namely 9 and 1; 8 and 2; 7 and 3; 6 and 4; 5 and 5. Next drill on subtracting all numbers below 100 from 100.

As in all other rules first drill with numbers on the board until the method is mastered, then let the work be oral.

The drill should be continued until the work can be done almost without effort. Thus if you wish to subtract 37 from 100, the instant you repeat the parts of the 37 the other numbers of the groups should be in the mind, that is, as you say thirty, the sixty should be thought of, and as you say seven, the three should be thought of, and you have the result sixty-three ready in the order in which it should be given.

This rule will be found very useful in making change when a dollar is given in payment of a smaller sum.

Another method is to regard 100 as equal to 9 tens and 10 units and arrange

thus, t u
9 10 and place the number to be subtracted under it, with tens under tens and units under units; e. g., if it is required to subtract 43 from 100 arrange thus,

t u
9 10
4 3 and subtract from left to right.
5 7

The last method may be applied in subtracting numbers from 1000 or in making change out of ten dollars.

Since 1000 = h t u
9 9 10, the number to be subtracted may be placed under, and the subtraction done from left to right; e. g., 1000 — 328

Arrange thus, h t u
9 9 10
3 2 8 and subtract from left to right
6 7 2

Practise with numbers, arranged as above, on the board, until the work can be done rapidly; then do the work orally.

Apply the last method in subtracting from 500, 1500, 2000, etc., or in making change from $5.00, $15.00, $20.00, etc., e. g.

($5.00 — $2.37) = (500 — 237) = h t u / 4 9 10 — h t u / 2 3 7

Arrange thus, h t u
4 9 10
2 3 7 and subtract from left to right,
2 6 3

Another method for finding the change to be returned when $5 or $10 or any exact number of dollars is given in payment of a smaller sum, is to subtract the number of dollars of the debt from one less then the number of dollars given and subtract the cents in the debt from the one dollar, e. g. in $5.00 — $2.47, take $2 from $4 and 47 cents from the remaining dollar.

Exercise II.

1. Subtract each of the following sums of money from $10:

$4.52	$3.85	$1.49
3.27	5.76	3.77
8.64	4.39	4.52
9.22	7.14	7.39
5.36	8.62	8.13
4.22	5.73	6.15
1.58	2.43	7.24
2.75	7.52	4.52
9.63	8.41	2.81
3.27	3.64	5.26

2. Subtract each of the following sums of money from $5.00:

$2.27	$4.61	$1.64
3.41	2.49	2.38
4.28	3.13	3.24
1.62	2.17	2.93
2.75	2.81	1.54
1.84	3.72	2.63
3.52	1.67	1.79
3.76	2.16	4.36
2.64	3.19	2 11
1.48	1.42	1.88

Exercise III.

3. Subtract each of the following sums of money from $15.00:

$12.43	$13.52	$14.81
11.61	11.96	13.47
13.68	10.83	11.21
14.71	11.52	10.29
10.87	12.63	12.33
12.54	13.59	13.78
13.91	12.65	12.63
12.36	13.79	11.70
11.64	10.51	12.50
13.82	11.63	10.96

Drill in the same way in subtracting from $2, $3, $7, $11 or from any number of dollars up to $20.

Rule III. To subtract any number under 100 from any larger number.

Change the subtrahend to the nearest number ending in a naught and subtract; change the result by adding or subtracting what was added to or taken from the subtrahend to make it end in naught, e. g. 83 — 37. Change the subtrahend to 40, subtract and add 3 to the result. Thus 83 — 40 + 3 = 46. By placing the numbers thus,

$$\begin{array}{r} 83 \\ 40 \\ \hline 43 \end{array}$$

and subtracting it will be seen that to take 40 from 83 is as easy as to take 40 from 80, for since the subtrahend ends in 0 the remainder must end in 3, the same as the minuend.

Again, 71 — 34 = 71 — 30 — 4 = 37.

Make your own questions and drill on the above rule until the work can be done rapidly.

As a preparation for Rule III drill on subtracting each of the nine digits from all numbers under 100.

Rule IV. To multiply numbers under 100 by 2 proceeding from left to right.

First use numbers whose units figures are less than 5, such as 43, 81, 64, etc., e. g., 62×2. Do the work in the order twice six is 12, twice two is 4 and the result is found in the order in which it must be expressed.

It will be observed that if the units figure in the number to be multiplied by 2 is 5 or more than 5 there will be one to add to the product of the tens figure by 2. Thus in 46×2 the products are 8 and 12, but the one to be carried from the 12 changes the 8 to 9 and the result is 92.

Drill on multiplying all numbers of two figures by 2.

In the same way multiply by 3, by 4, etc., up to 9 inclusive, until the work can be done from left to right as easily as from right to left.

As the work under Rule IV is a preparation for much of the work given in the pages which follow, the drill should be most thorough, especially in multiplying by 2, by 3 and by 4.

Rule V. To multiply any whole number by 10, to the right of the number annex a 0, e. g., $46 \times 10 = 460$.

Rule VI. To multiply any whole number by 5.

Since 5 is one half of 10, five times any number will be one half of ten times the number, or ten times one half the number.

Therefore to multiply a number by 5 annex a 0 and divide the result by 2, e. g., $48 \times 5 = 48 \times 10 \div 2 = 480 \div 2 = 240$.

It will be found easier to divide the number by 2 and multiply the result by 10, e. g., $36 \times 5 = 36 \div 2 \times 10 = 18 \times 10 = 180$, or $45 \times 5 = 45 \div 2 \times 10 = 22\frac{1}{2} \times 10 = 225$.

It will be observed that if the number to be multiplied by 5 is even, the product ends in 0, but if odd it ends in 5.

EXERCISE IV.

Multiply each of the following numbers by 5:

36	76	29	128	756	129
49	85	68	246	854	764
57	67	73	532	972	875
65	93	78	764	846	621
74	68	86	352	379	359
89	82	43	249	568	762

Rule VII. To multiply any whole number by 15:—

15 = 10 + 5, therefore, 15 times any number = ten times the number + five times the number. Therefore to the number add one half itself and multiply the sum by 10.

It will be observed as in the last rule that if the number to be multiplied by 15 is even, the product will end in 0, but if odd it will end in 5, e. g.,

$34 \times 15 = (34 + 17) \times 10 = 51 \times 10 = 510.$
$47 \times 15 = (47 + 23\frac{1}{2}) \times 10 = 70\frac{1}{2} \times 10 = 705.$

Multiply all numbers under 100 by 15.

Rule VIII. To multiply any whole number by 20:—

Multiply the number by 2 (according to the method given in Rule IV) and annex a 0.

This rule together with Rule I and Rule III may be applied in the following cases:—

$36 \times 19 = 36 \times 20 - 36 = 720 - 36 = 684.$
$48 \times 18 = 48 \times 20 - (2 \times 48) = 960 - 96 = 864.$
$32 \times 21 = 32 \times 20 + 32 = 640 + 32 = 672.$
$28 \times 22 = 28 \times 20 + (2 \times 28) = 560 + 56 = 616.$

Secure drill on the above by multiplying all numbers under 100 by each of the following:—18, 19, 20, 21, 22.

Rule IX. To multiply any whole number by 25:—

Since 25 times a number = $\frac{1}{4}$ of 100 times the number or 100 times $\frac{1}{4}$ of the number; either multiply the number by 100 and divide the result by 4, or divide the number by 4 and multiply the result by 100,

e. g. $48 \times 25 = 48 \times 100 \div 4 = 4800 \div 4 = 1200.$
or $48 \times 25 = 48 \div 4 \times 100 = 12 \times 100 = 1200.$
e. g. $33 \times 25 = 33 \div 4 \times 100 = 8\frac{1}{4} \times 100 = 825.$
$34 \times 25 = 34 \div 4 \times 100 = 8\frac{1}{2} \times 100 = 850.$
$35 \times 25 = 35 \div 4 \times 100 = 8\frac{3}{4} \times 100 = 875.$

From the above examples it will be observed that when the number is divided by 4, if there is no remainder the result will end in two 0's, if there is a remainder of 1 the result ends in 25, if 2 in 50, if 3 in 75.

For drill on Rule IX multiply all numbers under 1000 by 25.

Apply Rules IX, I and III in such cases as the following:—

$36 \times 24 = 36 \times 25 - 36 = 900 - 36 = 864.$
$48 \times 23 = 48 \times 25 - (2 \times 48) = 1200 - 96 = 1104.$
$27 \times 26 = 27 \times 25 + 27 = 675 + 27 = 702.$
$34 \times 27 = 34 \times 25 + (2 \times 34) = 850 + 68 = 918.$

Multiply all whole numbers under 100 by each of the following:—23, 24, 26, 27.

By a method similar to that given in Rule VIII, numbers under 100 may be multiplied by 30, and by each of the following numbers:—28, 29, 31, 32.

The same will apply in multiplying by 40, 50, 60, 70, 80 or 90, or by any of them increased or diminished by 1 or by 2, but the work will be found more difficult.

Rule X. To multiply any whole number by $12\frac{1}{2}$:—

(a) Since $12\frac{1}{2} = \frac{1}{2}$ of 25, multiply the number by 25 according to Rule IX and take half the result.

(b) Since $12\frac{1}{2}$ = one-eighth of 100, divide the number by 8 and call the result hundreds, e. g.,

$48 \times 12\frac{1}{2} = 48 \div 8 \times 100 = 600.$
$49 \times 12\frac{1}{2} = 49 \div 8 \times 100 = 6\frac{1}{8} \times 100 = 612\frac{1}{2}.$
$34 \times 12\frac{1}{2} = 4\frac{2}{8}$ hundreds $= 425.$
$35 \times 12\frac{1}{2} = 4\frac{3}{8}$ hundreds $= 437\frac{1}{2}.$
$36 \times 12\frac{1}{2} = 4\frac{4}{8}$ hundreds $= 450.$
$45 \times 12\frac{1}{2} = 5\frac{5}{8}$ hundreds $= 562\frac{1}{2}.$
$46 \times 12\frac{1}{2} = 5\frac{6}{8}$ hundreds $= 575.$
$47 \times 12\frac{1}{2} = 5\frac{7}{8}$ hundreds $= 587\frac{1}{2}.$

If the method in (b) is used it would be well to memorize the endings for the various remainders when the number is divided by 8. Thus, if the remainder is 1 as in $57 \times 12\frac{1}{2}$ (that

is 57 ÷ 8 leaves a remainder of 1) the ending in the result is $12\frac{1}{2}$, that is $\frac{1}{8}$ of 100; if the remainder is 2 the ending is equal to $\frac{2}{8}$ or $\frac{1}{4}$ of 100, that is 25, if 3 it is equal to $\frac{3}{8}$ of 100 or $37\frac{1}{2}$, and so on with the other remainders.

(c) Since $12\frac{1}{2} = 10 + 2\frac{1}{2}$ and $2\frac{1}{2} = \frac{1}{4}$ of 10, multiply the number by 10 and to the result add $\frac{1}{4}$ of itself, e. g.,

$48 \times 12\frac{1}{2} = (48 \times 10) + \frac{1}{4}(48 \times 10) = 480 + 120 = 600.$

The nature of the question will determine which of the three methods can be most easily applied.

It will be found beneficial to do all questions by all possible methods as the extra drill will develop the power to retain in the memory the numbers used, until the work is completed, and that is the most difficult part of the work for students.

Drill by multiplying numbers of two or three figures by $12\frac{1}{2}$.

Rule XI. To multiply a number by $22\frac{1}{2}$:—

Since $22\frac{1}{2} = 20 + 2\frac{1}{2}$ and $2\frac{1}{2} = \frac{1}{8}$ of 20, multiply the number by 20 as in Rule VIII and to the result add $\frac{1}{8}$ of itself, e. g.,

$48 \times 22\frac{1}{2} = (48 \times 20) + \frac{1}{8}(48 \times 20) = 960 + 120 = 1080.$

Rule XII. To multiply a number by $37\frac{1}{2}$:—

Since $37\frac{1}{2} = 25 + 12\frac{1}{2}$ and $12\frac{1}{2} = \frac{1}{2}$ of 25, multiply the number by 25 as in Rule IX and to the result add $\frac{1}{2}$ of itself, e. g. $32 \times 37\frac{1}{2} = (32 \times 25) + \frac{1}{2}(32 \times 25) = 800 + 400 = 1200$; or multiply $\frac{3}{8}$ of the number by 100.

Rule XIII. To multiply any number by 50:—

Since fifty times a number equals $\frac{1}{2}$ of 100 times the number, or is equal to 100 times $\frac{1}{2}$ the number, divide the number by 2 and call the result hundreds, e. g.,

$48 \times 50 = 48 \div 2 \times 100 = 2400.$

$49 \times 50 = 49 \div 2 \times 100 = 24\frac{1}{2} \times 100 = 2450.$

Observe that if the number to be multiplied is even the result ends in 00, but if odd it ends in 50.

Rule XIV. To multiply a number by $62\frac{1}{2}$:—

Since $62\frac{1}{2} = 50 + 12\frac{1}{2}$ and $12\frac{1}{2} = \frac{1}{4}$ of 50, multiply the number by 50 and to the result add $\frac{1}{4}$ of itself, e. g.,

$56 \times 62\frac{1}{2} = (56 \times 50) + \frac{1}{4}(56 \times 50) = 2800 + 700 = 3500$;

or multiply $\frac{5}{8}$ of the number by 100.

Rule XV. To multiply a number by 75:—

(a) Since 75 = 3 times 25, multiply the number by 25 and multiply the result by 3, e. g.,

$72 \times 75 = (72 \times 25)\,3 = 1800 \times 3 = 5400.$

(b) Multiply $\frac{3}{4}$ of the number by 100.

(c) Since $75 = 50 + 25$ and $25 = \frac{1}{2}$ of 50, multiply the number by 50 and to the result add $\frac{1}{2}$ of itself, e. g.,

$64 \times 75 = (64 \times 50) + \frac{1}{2}(64 \times 50) = 3200 + 1600 = 4800.$

Rule XVI. To multiply a number by $87\frac{1}{2}$:—

Since $87\frac{1}{2} = 3\frac{1}{2}$ times 25, multiply the number by 25 and multiply the result by $3\frac{1}{2}$, e. g.,

$24 \times 87\frac{1}{2} = (24 \times 25)\,3\frac{1}{2} = 600 \times 3\frac{1}{2} = 2100.$

or multiply $\frac{7}{8}$ of the number by 100.

Rule XVII. To multiply a number by 125:—

(a) Since 125 times a number equals 1000 times $\frac{1}{8}$ of the number, divide the number by 8 and multiply the result by 1000.

By memorizing the endings for the seven different remainders the work may be shortened.

Thus if the remainder is	1	the ending	= $\frac{1}{8}$	of	1000	= 125
"	"	2	"	= $\frac{2}{8}$	"	" = 250
"	"	3	"	= $\frac{3}{8}$	"	" = 375
"	"	4	"	= $\frac{4}{8}$	"	" = 500
"	"	5	"	= $\frac{5}{8}$	"	" = 625
"	"	6	"	= $\frac{6}{8}$	"	" = 750
"	"	7	"	= $\frac{7}{8}$	"	" = 875

e. g. $48 \times 125 = (48 \div 8)$ thousands $= 6000.$

$49 \times 125 = (49 \div 8)$ thousands $= 6125.$

$50 \times 125 = (50 \div 8)$ thousands $= 6250.$

and so on with the other remainders,

(b) Since 125 = 100 + 25, multiply the number by 100 and add $\frac{1}{4}$ of the product, e. g.,

$72 \times 125 = (72 \times 100) + \frac{1}{4}(72 \times 100) = 7200 + 1800 = 9000$

(c) Multiply the number by 25 and that product by 5, e. g.

$84 \times 125 = (84 \times 25)\ 5 = 2100 \times 5 = 10500.$

Rule XVIII. To multiply a number by 150:—

Multiply $1\frac{1}{2}$ times the number by 100.

Similiarly multiply a number by 175 by multiplying $1\frac{3}{4}$ times the number by 100.

Apply this rule to numbers exactly divisible by 2 or by 4.

Rule XIX. To multiply a number of two figures by 99:—

EXAMPLE.

$47 \times 99 = (47 \times 100) - 47 = 4700 - 47 = 4600 + (100 - 47) = 4600 + 53 = 4653.$

Observe that the number of hundreds in the product is one less than the number which was multiplied by 99 and that the other part of the product is 100 minus the number which was multiplied by 99.

It will be seen that the only work to be done in multiplying 47 by 99 is to subtract 47 from 100.

The same method applies in multiplying any number of three figures by 999 or of four figures by 9999, etc.

Thus: $473 \times 999 = (473 \times 1000) - 473 = 473000 - 473 = 472000 + (1000 - 473) = 472527.$

Rule XX. To multiply numbers between 10 and 20, e. g., 15×18, proceed as in example,

$$\begin{array}{r} 15 \\ 18 \\ \hline 40 \\ 8 \\ 15 \\ \hline 270 \end{array}$$

By observing the method in the example it will be seen that the units figure in the product is found by multiplying the units in the multiplicand by the units in the multiplier, the tens figure in the product is the sum of the units figures in multiplicand and multiplier plus what is carried from units, and the hundreds figure in the product is 1 plus what is carried from the tens.

Thus multiply units, add units, multiply tens and add what is carried in each case.

EXERCISE V.

1. Find the cost of 17 yds. of cotton at 16 cts. per yd.; 19 yds. at 14 cts.; 18 yds. at 15cts.; 15 yds. at 19 cts.
2. If a man can walk 17 miles in a day, how far can he walk in 13 days? in 18 days? in 19 days? in 14 days? in 15 days? in 17 days?
3. Find the cost of 18 yds. of cloth at $1.60 per yd.; at $1.40 per yd., at $1.80 per yd.
4. $3 \times 5 \times 19$.
5. $7 \times 2 \times 16$.
6. $9 \times 2 \times 15$.
7. $6 \times 3 \times 13$.

Rule XXI. To find the product of two numbers when the units figure in each is 1, e. g., 41×51;

Multiply in the ordinary way and observe the result,

thus,

$$\begin{array}{r} 41 \\ 51 \\ \hline 41 \\ 205 \\ \hline 2091 \end{array}$$

Reading the result from left to right it will be observed that the 20 is the product of the tens (4 and 5), that the 9 is the sum of the tens, and that the units figure must always be 1.

Thus the rule would be, proceed from left to right and after the product of the tens write the sum of the tens and after that write 1.

In such questions as 81×61 it will be observed that the sum of the tens (14) contains two figures, and therefore only the

right hand figure, that is the 4, is written in the result, and the product of the tens is increased by the 1 to be carried from the 14.

thus,

$$\begin{array}{r} 81 \\ 61 \\ \hline 81 \\ 486 \\ \hline 4941 \end{array}$$

In such questions as 141×151 regard the 14 and the 15 as the tens in each.

Thus product of tens = 210, sum of tens = 29. Since there is 2 to carry to the 210 from the 29, the result is written 212, then 9, then 1, that is 21291.

EXERCISE VI.

1. 41×51.
2. 61×71.
3. 81×91.
4. 71×51.
5. 91×31.
6. 51×91.
7. 121×131.
8. 141×161.
9. 181×191.
10. 131×151.
11. 171×121.
12. 141×181.

Rule XXII. To square a number whose units figure is 5, e. g., $(45)^2$ that is 45×45.

EXAMPLE.

$$\begin{array}{r} 45 \\ 45 \\ \hline 25 \\ 20 \\ 20 \\ 16 \\ \hline 2025 \end{array}$$

By observing the method employed in the example, it will be found that in the partial products the numbers to be added

are placed so that units are under units, tens under tens, and so on, and that the result is the same as in the ordinary method.

It will also be observed that the right hand figures in the result must always be 25, and that the left hand figures must be the product of the tens increased by a number equal to the tens figure. The rule will therefore be:—

Multiply the figure or figures to the left of the units by the next higher integral number and to the product annex 25. e. g., $35 \times 35 = 3 \times 4$ followed by 25, that is 1225.

Exercise VII.

Square each of the following numbers:—

15	115	195
25	125	205
35	135	215
45	145	245
55	155	495
65	165	305
75	175	405
85	185	705
95	285	1005
105	505	605

Rule XXIII. To multiply any two numbers when the units are in each is 5, e. g., 45×65.

To the product of the figures to the left of the units add half their sum and to the right annex 25, e. g., $45 \times 65 = (4 \times 6) + \frac{1}{2}(4 + 6)$ followed by $25 = 24 + 5$ followed by $25 = 2925$.

When the sum of the figures to the left of the units is an odd number, add to the product one-half of the next smaller number and annex 75, e. g., $65 \times 75 = (6 \times 7) + \frac{1}{2}(12)$ followed by $75 = 42 + 6$ followed by $75 = 4875$.

Exercise VIII.

1.	25×65	**11.**	55×65	**21.**	125×145
2.	35×55	**12.**	95×45	**22.**	145×165
3.	65×75	**13.**	65×35	**23.**	165×185
4.	85×45	**14.**	85×65	**24.**	135×155
5.	75×85	**15.**	125×135	**25.**	155×175
6.	35×95	**16.**	135×145	**26.**	175×195
7.	75×45	**17.**	145×155	**27.**	15×35
8.	45×65	**18.**	155×165	**28.**	85×125
9.	55×45	**19.**	165×175	**29.**	75×135
10.	85×95	**20.**	175×185	**30.**	85×145

Rule XXIV. To multiply a number of two figures by 11, e. g., 45×11.

Example.

$$\begin{array}{r} 45 \\ 11 \\ \hline 45 \\ 45 \\ \hline 495 \end{array}$$

Observe in the example that the left hand figure in the result is the same as the left hand figure in the multiplicand, and the right hand figure in the result is the same as the right hand figure in the multiplicand, and the middle figure in the result is the sum of the two digits in the multiplicand.

The rule is therefore: Between the two figures place their sum.

If the sum of the two figures is ten or more there will be one to carry, and thus the left hand figure in the result will be one greater than the left hand figure in the multiplicand, e. g., $76 \times 11 = 836$.

Multiply all numbers of two figures by 11.

Rule XXV. To multiply any number by $33\frac{1}{3}$.

Since $33\frac{1}{3}$ times a number is equal to one-third of one hundred times the number, or one hundred times one-third of the number, divide the number by 3 and multiply the result by 100, e. g.,

$36 \times 33\frac{1}{3} = 36 \div 3 \times 100 = 1200,$
$28 \times 33\frac{1}{3} = 9\frac{1}{3} \times 100 = 933\frac{1}{3},$
$23 \times 33\frac{1}{3} = 7\frac{2}{3} \times 100 = 766\frac{2}{3}.$

Since $333\frac{1}{3}$ = one-third of 1000, apply a similar method in multiplying by $333\frac{1}{3}$.

Rule XXVI. To multiply two numbers whose tens figures are the same and whose units figures when added make ten, such as 24×26; 32×38; 41×49, etc.:—

After the product of the tens figure multiplied by a number one greater than itself, write the product of the units, e. g., $24 \times 26 = 2 \times 3$ followed by $4 \times 6 = 624$, e. g., $32 \times 38 = 3 \times 4$ followed by $2 \times 8 = 1216$.

Rule XXVII. To multiply two numbers one of which is as much greater than 30 as the other is less than 30, e. g., 26×34, that is $(30 - 4)(30 + 4) = (30)^2 - 4^2 = 900 - 16 = 884$.

The same applies with numbers above and below 40 or 50, or any other number whose units figure is 0. e. g.,
$57 \times 63 = (60 - 3)(60 + 3) = 60^2 - 3^2 = 3600 - 9 = 3591.$

This rule is founded on the algebraical formula $(a + b)(a - b) = a^2 - b^2$ that is, the product of the sum and difference of two numbers is equal to the difference of their quares.

Rule XXIX. To square any number of two figures.

Apply algebraical formula $(a + b)^2 = a^2 + 2ab + b^2$.

Thus $24^2 = (20 + 4)^2 = 20^2 + 2(20 \times 4) + 4^2 = 400 + 160 + 16 = 576$.

Special cases in multiplication:—

To multiply a number by 63, 84, 42, 105, 126, 147, 168, 189.

It will be observed that the units figure in each number is one-half the left hand figure or figures, therefore:—

Multiply the given number by the units figure in the nnulti-plier, then multiply that product by 20 and add the two results.
e. g., $32 \times 63 = (32 \times 3) + (32 \times 3)\ 20 = 96 + 1920 = 2016$
$48 \times 84 = (48 \times 4) + 20\ (48 \times 4) = 192 + 3840 = 4032$
$72 \times 105 = (72 \times 5) + 20\ (72 \times 5) = 360 + 7200 = 7560$

To multiply a number by 48. Since $48 = 40 + (\frac{1}{5}$ of $40)$ multiply the number by 40 and to the result add $\frac{1}{5}$ itself.

By applying the principle involved in the last case make rules for multiplying any number by 33, 35, 36, 44, 55.

Rule XXX. To multiply any whole number or decimal number by 10 or 100, or any other number made up of 1 followed by one or more noughts.

Remove the decimal point as many places to the right in the multiplicand as there are noughts in the multiplier.

e. g. $4.2168 \times 100 = 421.68.$
$428. \times 1000 = 428000.$
$62.31 \times 10000 = 623100.$

Rule XXXI. To divide any number whether whole or decimal by 10 or 100 or any other number made up of 1 followed by one or more noughts:

Remove the decimal point as many places to the left in the dividend as there are noughts in the divisor.

e. g., $426.75 \div 100 = 4.2685.$
$34261 \div 1000 = 34.261.$
$.2461 \div 10000 = .00002461.$

Rule XXXII. To divide a number by five.

Since any number contains 5 twice as often as it does 10, multiply the tens figure in the dividend by 2 and to the product add the number which indicates the number of fives contained in the units figure, e. g., $87 \div 5 = 2 \times 8 +$ the number of fives in the $7 = 16 + 1$ with a remainder of $2 = 17\frac{2}{5}$.

In dividing such numbers as 245 by 5, regard all the figures to the left of the units as tens, thus $245 \div 5 = 24 \times 2 + 1 = 49.$

Rule XXXIII. To divide a number by 25.

Since in any number there are four times as many twenty-fives as there are hundreds, multiply the hundreds in the dividend by four and to the product add the number which indicates the number of twenty-fives contained in the two right hand figures.

e. g., $785 \div 25 = (7 \times 4) +$ the number of twenty-fives in $85 = 28 + 3$ with a remainder of $10 = 31\frac{10}{25}$.

Rule XXXIV. To divide a number by 50.

Multiply the hundreds in the dividend by 2 and to the product add the number which indicates the number of fifties in the two right hand figures, e. g., $1965 \div 50 = (19 \times 2) +$ the number of fifties in $65 = 38 + 1$ with a remainder of $15 = 39\frac{15}{50}$.

Practise dividing numbers of two or three figures by 2, by 3, and 4, until the work can be done rapidly.

Special cases—

(a) $17\frac{1}{2} \div 2\frac{1}{2}$. Reduce both numbers to halves, thus $35 \div 5 = 7$
(b) $17\frac{1}{2} \div 2\frac{3}{4}$. Reduce both to quarters. $70 \div 11 = 6\frac{4}{11}$.
(c) $18\frac{3}{4} \div 1\frac{1}{4}$. Reduce both to twelfths, thus $224 \div 21 = 10\frac{2}{3}$.
(d) $7\frac{1}{2} \div 1\frac{2}{3}$. Reduce both to sixths, thus $45 \div 10 = 4\frac{1}{2}$.
(e) $16 \div 2\frac{1}{3} = 48 \div 7 = 6\frac{6}{7}$.
(f) $25 \div 2\frac{3}{5} = 125 \div 13 = 9\frac{8}{13}$.

FRACTIONS.

By handling and dividing such objects as strings, sticks, apples, lines on the board, etc., lead the pupils to master the below mentioned principles, facts and rules relating to fractions:

After dispensing with objects lead pupils to imagine they see the objects and parts involved in all questions.

Always have a definite place for imaginary objects and lead pupils to imagine they are handling them.

Return to the use of objects whenever an appeal to the imagination fails to make the operations with reasons perfectly clear to the pupil, but do not continue their use beyond what is actually needed.

Never give a rule to enable a pupil to do work; but by making problems and using objects real or imaginary that appeal to the pupil's experience, lead him, by requiring him to do the work involved a sufficient number of times, to make rules for himself.

The following are among the most important facts, principles and rules to be mastered and applied.

To get a fraction of any object it must be divided or marked off into two or more equal parts and one or more of these parts must be taken.

To express a fraction two numbers must be used, one to indicate the number of parts into which the object has been divided or marked off, the other to indicate the number of parts taken. Thus, if an apple has been divided into four equal parts and three of those parts have been taken the fact is expressed thus,

$$\frac{3 \text{ numerator.}}{4 \text{ denominator.}}$$

The numerator indicates the number of parts taken.

The denominator gives the name to each part, shows into how many parts the object has been divided or marked off and gives some idea of the size of the parts. To increase the denominator makes the parts smaller (Why?) and thus lessens the value of the fraction. To diminish the denominator makes the parts larger (Why?) and thus increases the value of the fraction.

To increase the numerator increases the number of parts taken and thus increases the value of the fraction. To diminish the numerator diminishes the parts taken, and thus diminishes the value of the fraction.

Therefore, to multiply a fraction by a whole number either multiply the numerator by the whole number or divide the denominator by the whole number, and to divide a fraction by a whole number either divide the numerator by the whole number or multiply the denominator by the whole number.

To multiply both numerator and denominator by the same number does not change the value of a fraction since it first multiplies the value of the fraction by the given number and then divides that result by the same number.

To divide both numerator and denominator of a fraction by the same number does not change the value of the fraction, since it first divides the fraction by the given number and then multiplies the result by the same number.

The numerator and denominator are called the terms of a fraction. When the terms of a fraction have no common factor the fraction is said to be in its lowest terms.

When the terms of a fraction have one or more common factors, other fractions may be found which have the same value as the original fractions, since the numerator and denominator of the original fraction may be divided by any of their common factors.

A fraction may be reduced to its lowest terms by dividing both numerator and denominator by their highest common factor.

Two or more fractions not having the same denominator may be changed to equivalent fractions having a common denominator in the following concrete manner: Take for example $\frac{3}{4}$ and $\frac{4}{5}$. If each quarter in any object be cut into two equal parts there will be 8 equal parts in the object and the parts will be called eighths. If each quarter be cut into three equal parts there will be 12 equal parts in the object and the parts will be called twelfths. By cutting each quarter in 4, 5, 6 equal parts you will obtain sixteenths, twentieths, twenty-fourths, etc.

In like manner by cutting each of the fifths into two, three, four, five, etc., equal parts we obtain tenths, fifteenths, twentieths, twenty-fifths, etc.

It will be seen that both fourths and fifths may be made into twentieths.

Since each fourth makes 5 twentieths, the 3 fourths makes 3 times 5 twentieths, that is makes, $\frac{15}{20}$ therefore $\frac{3}{4} = \frac{15}{20}$.

And since each fifth makes 4 twentieths the 4 fifths makes 4 times 4 twentieths, that is makes $\frac{16}{20}$ therefore $\frac{4}{5} = \frac{16}{20}$.

Many examples like the above should be solved with actual objects actually divided or cut up, and with imaginary objects divided in imagination before attempting addition and subtraction of fractions.

To reduce mixed numbers to improper fractions,—

Take for example $7\frac{3}{4}$.

We have seven undivided objects and 3 pieces of a divide object. We simply wish to divide each of the whole units int pieces just like those of the divided unit, that is, we wish to cu each of the seven objects into quarters and put those with th 3 quarters and ascertain how many quarters we will have alto gether.

Since each unit makes 4 quarters the 7 units makes 7 time 4 quarters that is 28 quarters and those put with the 3 quarter make 31 quarters, that is $7\frac{3}{4} = \frac{31}{4}$.

Solve such questions in this manner first using actual object then imaginary objects until the pupils can without assista discover a rule for such work with the reason for such rule.

To change improper fractions to whole or mixed numbers:

Take for example, $\frac{31}{4}$.

Take eight sticks each one foot long. Cut each into quarters Take 31 of these quarters. Express what you have as $\frac{31}{4}$ ft Ask pupils to stick enough quarter feet together to make a whol foot. They will find that this can be done seven times and ther will be three pieces called quarters left. This may be expressed thus, $\frac{31}{4}$ ft. $= 7\frac{3}{4}$ ft.

Proceed in the same way with real and imaginary object until pupils discover rule with reason for same.

In a similar manner all rules for working with fractions may be made, from work done with actual and imaginary objects.

Exercise IX.

1. Express 4 as halves, as quarters, as sevenths.

2. Name ten fractions each equal to 3, equal to $\frac{1}{2}$, equal to

3. Name four fractions each equal to $\frac{3}{4}$; to $\frac{4}{5}$; to $\frac{7}{8}$.

4. Reduce each of the following fractions to their equivalent fractions each in its lowest terms: $\frac{9}{12}$, $\frac{14}{18}$, $\frac{25}{125}$, $\frac{18}{27}$, $\frac{64}{96}$, $\frac{72}{84}$.

5. Express $7\frac{2}{3}$ in thirds; $9\frac{3}{4}$ in quarters,; $6\frac{5}{7}$ in sevenths $25\frac{7}{9}$ in ninths.

6. Express $5\frac{1}{2}$ in quarters; $7\frac{2}{3}$ in sixths; $8\frac{3}{4}$ in eighths; $4\frac{3}{4}$ in twelfths.

7. Reduce the following to equivalent fractions having the least common denominator: $\frac{5}{6}$ and $\frac{7}{8}$; $\frac{3}{4}$ and $\frac{4}{7}$; $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{5}{8}$; $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$.

8. Express each of the following as whole or mixed numbers: $\frac{27}{9}$, $\frac{84}{7}$, $\frac{137}{50}$, $\frac{37}{4}$, $\frac{79}{8}$, $\frac{132}{8}$.

9. Express each of the following in twelfths: $2\frac{1}{2}$, $3\frac{2}{3}$, $4\frac{3}{4}$, $7\frac{5}{6}$, $9\frac{11}{12}$.

10. Express each of the following in sixteenths: $5\frac{1}{2}$, $2\frac{1}{4}$, $5\frac{1}{4}$, $7\frac{7}{8}$, $4\frac{13}{16}$.

Rule XXXV. To add two fractions when the numerator of each is 1, e. g., $\frac{1}{5} + \frac{1}{4} = \frac{4}{20} + \frac{5}{20} = \frac{9}{20}$.

Observe that the numerator in the result is the sum of the denominators of the addends, and the denominator is their product. Hence the rule is: Add denominators for a numerator and multiply denominators for a denominator.

Similiarly in subtraction, $\frac{1}{4} - \frac{1}{5} = \frac{5}{20} - \frac{4}{20} = \frac{1}{20}$.

EXERCISE X.

1.	$\frac{1}{8} - \frac{1}{9}$.	**11.**	$\frac{1}{8} - \frac{1}{11}$.
2.	$\frac{1}{11} - \frac{1}{25}$.	**12.**	$\frac{1}{6} - \frac{1}{7}$.
3.	$\frac{1}{14} - \frac{1}{15}$.	**13.**	$\frac{1}{9} - \frac{1}{13}$.
4.	$\frac{1}{17} - \frac{1}{19}$.	**14.**	$\frac{1}{3} - \frac{1}{5}$.
5.	$\frac{1}{25} - \frac{1}{27}$.	**15.**	$\frac{1}{5} - \frac{1}{7}$.
6.	$\frac{1}{28} + \frac{1}{33}$.	**16.**	$\frac{1}{12} - \frac{1}{15}$.
7.	$\frac{1}{13} + \frac{1}{19}$.	**17.**	$\frac{1}{4} - \frac{1}{9}$.
8.	$\frac{1}{18} + \frac{1}{25}$.	**18.**	$\frac{1}{2} - \frac{1}{5}$.
9.	$\frac{1}{13} + \frac{1}{17}$.	**19.**	$\frac{1}{7} - \frac{1}{8}$.
10.	$\frac{1}{23} + \frac{1}{27}$.	**20.**	$\frac{1}{6} - \frac{1}{7}$.

Practise adding and subtracting samll fractions mentally.

EXERCISE XI.

1.	$\frac{1}{2} + \frac{1}{4} + \frac{1}{6}$.	**11.**	$\frac{3}{4} - \frac{1}{8}$.
2.	$\frac{3}{4} + \frac{5}{6} + \frac{11}{12}$.	**12.**	$\frac{6}{7} - \frac{3}{5}$.
3.	$\frac{7}{8} + \frac{5}{12}$.	**13.**	$\frac{2}{3} - \frac{5}{9}$.
4.	$\frac{2}{3} + \frac{5}{6} + \frac{3}{8}$.	**14.**	$\frac{7}{8} - \frac{5}{12}$.
5.	$\frac{3}{4} + \frac{4}{5}$.	**15.**	$\frac{5}{7} - \frac{3}{8}$.
6.	$\frac{7}{8} + \frac{3}{5}$.	**16.**	$4\frac{3}{4} - \frac{5}{8}$.
7.	$\frac{9}{10} + \frac{7}{12} + \frac{4}{15}$.	**17.**	$\frac{7}{9} - \frac{3}{5}$.
8.	$\frac{3}{5} + \frac{9}{10} + \frac{5}{8}$.	**18.**	$1\frac{1}{2} - \frac{3}{4}$.
9.	$\frac{9}{7} + \frac{7}{8}$.	**19.**	$1\frac{1}{4} - \frac{7}{8}$.
10.	$\frac{15}{16} + \frac{13}{14}$.	**20.**	$1\frac{1}{3} - \frac{8}{9}$.

Rule XXXVI. (*a*) To add two fractions whose denominato have no common divisor, e. g. $\frac{5}{7} + \frac{3}{8}$.

Find the numerators by what is known as cross multiplyin that is multiply first numerator and second denominator ar multiply second numerator by first denominator; thus,

$$\frac{5}{7} + \frac{3}{8} = \frac{40 + 21}{56} = \frac{61}{56} = 1\frac{5}{56}.$$

Exercise XII.

1. $\frac{3}{4} + \frac{5}{7}$.
2. $\frac{7}{8} + \frac{4}{9}$.
3. $\frac{5}{6} + \frac{5}{11}$.
4. $\frac{3}{7} + \frac{8}{9}$.
5. $\frac{11}{12} + \frac{4}{7}$.
6. $\frac{3}{8} + \frac{7}{11}$.
7. $\frac{14}{15} + \frac{15}{16}$.
8. $\frac{17}{18} + \frac{11}{13}$.
9. $\frac{12}{13} + \frac{9}{15}$.
10. $\frac{13}{25} + \frac{19}{21}$.

The least common denominator of several fractions will b the greatest denominator of all the fractions or same multipl of that number, e. g., Find the least common denominator o $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{11}{12}$.

The required number must be exactly divisible by 12, tha is, it must be 12 or some multiple of 12. By testing each of th multiples of 12 in order the least one that can be exactly divide by each of the denominators can be found.

Exercise XIII.

1. $\frac{3}{4} + \frac{5}{6} + \frac{7}{8} + \frac{5}{12}$.
2. $\frac{5}{6} + \frac{7}{8} + \frac{8}{9} + \frac{11}{12}$.
3. $\frac{2}{3} + \frac{3}{4} + \frac{5}{6}$.
4. $\frac{1}{2} + \frac{1}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{11}{12}$.
5. $2\frac{1}{3} + 4\frac{2}{3} + 5\frac{3}{4} + 6\frac{5}{6}$.
6. $4\frac{3}{4} + 7\frac{1}{2} + 5\frac{1}{3}$.
7. $1\frac{5}{6} + 2\frac{7}{8} + 3\frac{3}{4}$.
8. $4\frac{3}{4} + \frac{5}{9} + 2\frac{7}{12} + 3\frac{5}{18}$.
9. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \frac{1}{12} + \frac{1}{15} + \frac{1}{20}$.
10. Find the sum of $4\frac{3}{4}$, $5\frac{5}{6}$, $3\frac{7}{8}$, $4\frac{11}{12}$.

Rule XXXVII. $30 - 5\frac{1}{4} = 30 - 6 + \frac{1}{4} = 24\frac{1}{4}$.
$27 - 21\frac{3}{5} = 27 - 22 + \frac{2}{5} = 5\frac{2}{5}$.

From the minnend subtract the whole number in the subtrahend increased by 1 and to the remainder add the fraction which will make the fraction in the subtrahend equal to 1. E. g., $9\frac{1}{2} - 4\frac{5}{8} = 9\frac{1}{2} - 5 + \frac{3}{8} = 4\frac{1}{2} + \frac{3}{8} = 4\frac{7}{8}$.

EXERCISE XIV.

1. $6\frac{7}{8} - 5\frac{1}{4}$.
2. $7\frac{1}{2} - 4\frac{3}{8}$.
3. $9\frac{2}{3} - 5\frac{5}{6}$.
4. $12\frac{5}{6} - 7\frac{1}{2}$.
5. $9\frac{2}{3} - 4\frac{3}{4}$.
6. $9\frac{1}{2} - 8\frac{5}{8}$.
7. $16\frac{1}{4} - 9\frac{3}{4}$.
8. $17\frac{3}{8} - 14\frac{1}{2}$.
9. $7\frac{3}{4} - 4\frac{5}{6}$.
10. $16\frac{1}{8} - 10\frac{3}{4}$.
11. $25\frac{3}{5} - 16\frac{9}{10}$.
12. $18\frac{5}{7} - 10\frac{5}{6}$.
13. $17\frac{1}{9} - 14\frac{1}{3}$.
14. $18\frac{1}{4} - 14\frac{1}{2}$.
15. $21\frac{5}{8} - 16\frac{3}{4}$.

Rule XXXVIII. To square a mixed number whose fractional part is $\frac{1}{2}$, e. g., $8\frac{1}{2} \times 8\frac{1}{2}$.

EXAMPLE.

$8\frac{1}{2}$
$8\frac{1}{2}$
$\frac{1}{4}$
4
4
64
$72\frac{1}{4}$

By observing the work in the above example the following rule with reason may be discovered:—

Multiply the whole number by a number one greater than itself and to the product annex $\frac{1}{4}$. Thus in the example, multiply 8 by (8 + 1), that is multiply 8 by 9 and annex $\frac{1}{4}$.

EXAMPLES.

Square all numbers which end in $\frac{1}{2}$ from $1\frac{1}{2}$ to $24\frac{1}{2}$.

Rule XXXIX. To multiply mixed numbers when the fractional part in each is $\frac{1}{2}$.

To the product of the whole numbers add half their sum plus $\frac{1}{4}$. e. g.,

$7\frac{1}{2} \times 9\frac{1}{2} = (7 \times 9) + \frac{1}{2}(7 + 9) + \frac{1}{4} = 63 + 8 + \frac{1}{4} = 71\frac{1}{4}$.
$6\frac{1}{2} \times 7\frac{1}{2} = (6 \times 7) + \frac{1}{2}(6 + 7) + \frac{1}{4} = 42 + 6\frac{1}{2} + \frac{1}{4} = 48\frac{3}{4}$.

EXERCISE XV.

1. $6\frac{1}{2} \times 7\frac{1}{2}$.
2. $9\frac{1}{2} \times 10\frac{1}{2}$.
3. $14\frac{1}{2} \times 15\frac{1}{2}$.
4. $17\frac{1}{2} \times 16\frac{1}{2}$.
5. $8\frac{1}{2} \times 7\frac{1}{2}$.
6. $15\frac{1}{2} \times 19\frac{1}{2}$.
7. $14\frac{1}{2} \times 17\frac{1}{2}$.
8. $13\frac{1}{2} \times 15\frac{1}{2}$.
10. $9\frac{1}{2} \times 16\frac{1}{2}$.
11. $14\frac{1}{2} \times 16\frac{1}{2}$.
12. $18\frac{1}{2} \times 19\frac{1}{2}$.
13. $18\frac{1}{2} \times 16\frac{1}{2}$.
14. $41\frac{1}{2} \times 51\frac{1}{2}$.
15. $25\frac{1}{2} \times 35\frac{1}{2}$.
16. $26\frac{1}{2} \times 34\frac{1}{2}$.
17. $25\frac{1}{2} \times 28\frac{1}{2}$.
18. $20\frac{1}{2} \times 26\frac{1}{2}$.
19. $145\frac{1}{2} \times 145\frac{1}{2}$.
20. $161\frac{1}{2} \times 141\frac{1}{2}$.

Rule XL. To multiply mixed numbers when the fractional part in each is $\frac{1}{3}$.

To the product of the whole numbers add one-third their sum, plus $\frac{1}{9}$. e. g.,

$15\frac{1}{3} \times 18\frac{1}{3} = (15 \times 18) + \frac{1}{3}(15 + 18) + \frac{1}{9} = 1 + \frac{1}{9} = 281\frac{1}{9}$
$13\frac{1}{3} \times 16\frac{1}{3} = (13 \times 16) + \frac{1}{3}(13 + 16) + \frac{1}{9} = + 9\frac{3}{3} + \frac{1}{9} =$
$217\frac{7}{9}$.

Rule XLI. To multiply mixed numbers when the fractional part in each is $\frac{1}{4}$.

To the product of the whole numbers add one quarter their sum plus $\frac{1}{16}$, e. g.,

$8\frac{1}{4} \times 12\frac{1}{4} = (8 \times 12) + \frac{1}{4}(8 + 12) + \frac{1}{16} = 96 + 5 + \frac{1}{16} = 101\frac{1}{16}$.
$9\frac{1}{4} \times 12\frac{1}{4} = 108 + 5\frac{1}{4} + \frac{1}{16} = 113\frac{5}{16}$.

Rule XLII. To find $\frac{2}{3}$ of a number.

(a) Take twice one third the number.

(b) Take one-third of twice the number.

Use the method in (a) when the number is exactly divisible by 3, e. g., $\frac{2}{3}$ of $27 = \frac{27}{3} \times 2 = 18$.

Use the method in (b) when the number is not exactly divisible by 3, e. g., $\frac{2}{3}$ of 19 = $\frac{1}{3}$ of (2 × 19) = $\frac{1}{3}$ of 38 = $12\frac{2}{3}$.

By a method similar to that given in Rule XLII find $\frac{3}{4}$ or any other fractional part of a number.

Since much of the work in percentage can be easily done by means of fractions much practice should be given in finding fractional parts of numbers.

Exercise XVI.

Find $\frac{2}{3}$ of each of the following numbers:—45, 48, 36, 31, 63, 129, 76, 92, 99, 47, 78, 170, 120, 72.

Find $\frac{3}{4}$ of 74, 56, 49, 72, 84, 96, 62, 25.

Find $\frac{5}{8}$ of 48, 64, 72, 88, 35, 42, 73, 69.

Find $\frac{2}{5}$ of all the numbers given above.

Find $\frac{7}{8}$ of all the numbers given above.

Practise in the same way finding other fractional parts of numbers.

Rule XLIII. To multiply a fraction by a whole number:

Multiply the numerator by the whole number or divide the denominator by the whole number.

Reason: When the numerator is multiplied the number of parts taken is increased without changing the size of each part; when the denominator is divided the size of the parts is increased without changing the number of parts taken.

e. g., I. $\frac{9}{10} \times 5 = \frac{45}{10} = \frac{9}{2} = 4\frac{1}{2}$.

II. $\frac{9}{10} \times 5 = \frac{9}{2} = 4\frac{1}{2}$.

In I. five times as many tenths are taken. In II. the same number of parts are taken but they are halves instead of fifths, that is, each part is five times as large as at first.

It will be seen that to divide the denominator by the whole number where it is exactly divisible is the easier method.

Exercise XVII.

1. $\frac{15}{16} \times 4$.
2. $\frac{3}{5} \times 5$.
3. $\frac{5}{4} \times 4$.
4. $\frac{7}{9} \times 3$.
5. $\frac{19}{20} \times 10$.
6. $\frac{7}{8} \times 2$.
7. $\frac{9}{11} \times 7$.
8. $\frac{13}{14} \times 3$.
9. $\frac{17}{18} \times 6$.
10. $\frac{7}{9} \times 8$.

Rule XLIV. To multiply a mixed number by a whole number, multiply from left to right.

e. g., $6\frac{3}{4} \times 5 = 30\frac{15}{4} = 33\frac{3}{4}$.
$9\frac{5}{8} \times 4 = 36\frac{20}{8} = 38\frac{4}{2}$.

EXERCISE XVIII.

1.	$7\frac{2}{3} \times 6$.	**6.**	$13\frac{5}{8} \times 2$.
2	$19\frac{1}{6} \times 3$.	**7.**	$25\frac{7}{9} \times 12$.
3.	$6\frac{5}{7} \times 9$.	**8.**	$15\frac{9}{10} \times 5$.
4.	$9\frac{11}{12} \times 4$.	**9.**	$54\frac{3}{5} \times 4$.
5.	$8\frac{3}{4} \times 7$.	**10.**	9×7.

Rule XLV. To divide a fraction by a whole number.

Divide the numerator by the whole number when it is exactly divisible or multiply the denominator by the whole number.

Reason: To divide the numerator reduces the number of parts taken and thus divides the fraction; to multiply the denominator diminishes the size of the parts and thus divides the fraction.

e. g., I. $\frac{9}{11} \div 3 = \frac{3}{11}$.
II. $\frac{11}{12} \div 2 = \frac{11}{24}$.

EXERCISE XIX.

1.	$\frac{5}{8} \div 4$.	**6.**	$\frac{3}{5} \div 12$.
2.	$\frac{7}{9} \div 7$.	**7.**	$\frac{6}{7} \div 5$.
3.	$\frac{9}{11} \div 3$.	**8.**	$\frac{11}{15} \div 6$.
4.	$\frac{24}{25} \div 6$.	**9.**	$\frac{15}{16} \div 3$.
5.	$\frac{11}{13} \div 5$.	**10.**	$\frac{5}{14} \div 4$.

Rule XLVI. To divide a whole or a mixed number by a mixed number.

Reduce both numbers to the least common denominator and divide the numerator of the dividend by the numerator of the divisor.

e. g. $7\frac{2}{3} \div 1\frac{1}{2}$. Reduce both to sixths, thus $7\frac{2}{3} = 46$ sixths and $1\frac{1}{2} = 9$ sixths. $46 \div 9 = 5\frac{1}{9}$.

e. g., $9\frac{3}{4} \div 4$; $9\frac{3}{4} \div 4$; $9\frac{3}{4} = 39$ quarters, $4 = 16$ quarters, $39 \div 16 = 2\frac{7}{16}$.

Exercise XX.

1. $16\frac{1}{2} \div 2\frac{1}{4}$.
2. $9\frac{3}{4} \div 1\frac{1}{3}$.
3. $15\frac{2}{3} \div 2\frac{1}{2}$.
4. $9 \div 3\frac{1}{2}$.
5. $12 \div 2\frac{1}{4}$.
6. $8\frac{1}{5} \div 5\frac{1}{2}$.
7. $9\frac{2}{3} \div 1\frac{1}{3}$.
8. $14\frac{1}{2} \div 4\frac{1}{2}$.
9. $8\frac{7}{8} \div 5\frac{1}{4}$.
10. $6\frac{5}{8} \div 2\frac{1}{6}$.

Exercise XXI.

In the following exercise cancel mentally where possible.

1. $\frac{11}{15} \times \frac{16}{28}$.
2. $\frac{3}{4} \times \frac{5}{6} \times \frac{8}{9} \times \frac{11}{25}$.
3. $3\frac{1}{4} \times \frac{16}{39}$.
4. $1\frac{1}{2} \times \frac{7}{9} \times \frac{4}{7}$.
5. $\frac{15}{16} \times \frac{13}{25} \times \frac{11}{12}$.
6. $4\frac{1}{2} \times 3\frac{3}{4}$.
7. $5\frac{1}{3} \times 2\frac{1}{4}$.
8. $\frac{7}{10} \times \frac{16}{21} \times \frac{5}{8}$.
9. $14\frac{1}{2} \times 3\frac{1}{3}$.
10. $11\frac{2}{3} \times 3\frac{3}{7}$.

Exercise XXII.

Cancel mentally where possible.

1. $\frac{4}{5} \div \frac{3}{5}$.
2. $\frac{3}{4} \div \frac{7}{8}$.
3. $5\frac{1}{4} \div \frac{11}{12}$.
4. $\frac{14}{15} \div 1\frac{1}{5}$.
5. $\frac{9}{10} \div \frac{7}{12}$.
6. $\frac{2}{3} \div \frac{3}{4}$.
7. $\frac{4}{5} \div \frac{5}{7}$.
8. $\frac{8}{9} \div \frac{9}{16}$.
9. $\frac{7}{8} \div \frac{7}{8}$.
10. $\frac{5}{7} \div \frac{6}{7}$.

Rule XLVII. To reduce a complex fraction to a simple fraction. Multiply both numerator and denominator by the L. C. M. of the denominators of the fractional parts:

e. g., Reduce $\frac{4\frac{1}{2}}{5\frac{1}{3}}$ to a simple fraction. Multiply both numerator and denominator by 6 which is the L. C. M. of 2 and 3, the denominators of the fractional parts.

Thus, $\frac{4\frac{1}{2} \times 6}{5\frac{1}{3} \times 6} = \frac{27}{32}$.

e. g., Simplify $\frac{5}{7\frac{1}{2}}$. Thus $\frac{5 \times 2}{7\frac{1}{2} \times 2} = \frac{10}{15} = \frac{2}{3}$.

Exercise.

Simplify each of the following complex fractions:

$$\frac{7}{9\frac{1}{4}} \quad \frac{4\frac{1}{3}}{7} \quad \frac{3\frac{1}{2}}{4\frac{1}{3}} \quad \frac{6\frac{2}{3}}{5\frac{3}{4}} \quad \frac{4\frac{1}{4}}{5\frac{2}{5}} \quad \frac{2\frac{2}{3}}{4\frac{4}{5}} \quad \frac{1\frac{1}{2}}{2\frac{3}{4}} \quad \frac{5}{8\frac{1}{4}} \quad \frac{2\frac{1}{4}}{9} \quad \frac{5\frac{2}{3}}{8}$$

Review Exercise A.

1. A man saws $1\frac{2}{3}$ cords of wood in a day, how much will he saw in 2 days? In 3 days? In 5 days? In $2\frac{1}{2}$ days? In $3\frac{3}{4}$ days.

2. A man saws $1\frac{1}{4}$ cords of wood in a day in how many days will he saw 5 cords? $7\frac{1}{2}$ cords? $12\frac{1}{3}$ cords? 16 cords?

3. If a man walks $3\frac{1}{2}$ miles per hour, how far will he walk in 3 hours? In $2\frac{1}{2}$ hours? In 6 hours? in $4\frac{1}{3}$ hours?

4. If a man earns $\$2\frac{1}{2}$ per day, in how many days will he earn \$14? \$17? \$21.50? \$24.75? \$10.20?

5. If 7 lb. tea cost \$3, how many pounds can be bought for $\$7\frac{1}{2}$? For \$9? For $\$12\frac{1}{4}$? For $\$2\frac{1}{4}$?

6. Find the cost of 450 apples at the rate of 5 apples for 4 cents.

7. The product of two numbers is 22 and one of the numbers is $4\frac{3}{4}$. Find the other.

8. At the rate of 30 miles an hour, how far will a train run in 36 min.? In 40 min.? In 45 min.? In 48 min.?

9. $\frac{3}{4}$ is what part of 2? Of 3? Of $4\frac{1}{2}$?

10. Find the sum of $\frac{2}{3}$ of 18 and $\frac{3}{5}$ of 65.

11. When 12 bus. potatoes are given for 48 yds. cotton at 13 cts. a yard what are the potatoes worth a bushel?

Easy method for mental work,

$$\frac{48 \times 13}{12} \quad \text{cancel} \quad = 52.$$

12. A third of A's money is equal to $\frac{3}{4}$ of B's. If A has \$36, how much has B?

13. Find the number which becomes $\frac{1}{2}\frac{8}{5}$ when increased by $\frac{1}{5}$ of itself.

14. If an automobile runs 40 miles in $1\frac{1}{4}$ hours, in what time will it run 60 miles? 30 miles? 50 miles?

15. If A can do a piece of work in 5 days and B can do the same in 4 days, how much can both do together in 1 day? In how many days can they do the whole of it?

16. If A can do a piece of work in 12 days and B can do the same in 16 days, how long will it take B to finish it after A has worked 4 days.

17. A does a piece of work in 3 days. B does it in $3\frac{1}{2}$ days. How long will it take the two to do it.

18. A can do half as much work in the same time as B. If A can do a piece of work in $6\frac{3}{4}$ days, how long will it take B to do it?

19. A can do a piece of work in $2\frac{3}{4}$ days. What part of it can he do in 1 day? In what time can he do $\frac{3}{5}$ of it?

20. A and B together can do a piece of work in 4 days. A can do it alone in 6 days. What part of it can B do in 6 days?

21. A can do a piece of work in 6 days, B in 3 days and C in 12 days. How many days will it take all three working together to complete it?

22. A alone can do a piece of work in 12 days. A and B together can do it in 8 days. How long will it take B alone.

23. Divide \$40 between A and B so that A may have \$$3\frac{1}{2}$ more than B.

24. Divide \$60 between A and B so that A may have $1\frac{1}{2}$ times as much as B.

25. Divide \$90 between two people in the proportion of \$7 to \$8.

26. How many cubic yards of earth must be dug from a cellar which is 27 ft. long, 24 ft. wide, 8 ft. deep?

27. Two-thirds of 36 is $\frac{3}{4}$ of what number?

28. $14\frac{1}{2}$ is two-thirds of what number?

29. A man who owned $\frac{3}{5}$ of a building sold $\frac{3}{4}$ of his share. What part of the building did he still own?

30. Three-quarters of 60 is $\frac{3}{5}$ of what number?

31. If a man mows $2\frac{1}{2}$ acres of grain in a day, how much can he mow in $4\frac{3}{4}$ days.

32. If a man walks 25 miles in 6 hours, how far can he walk in $7\frac{1}{2}$ hours?

33. The difference between $\frac{3}{5}$ of a sum of money and $\frac{3}{4}$ of the same is \$15. Find the sum.

34. A man spent $\frac{5}{9}$ of his money and had \$48 left. How much had he at first?

35. How far can a boy wheel in 7 hours whose rate is 25 miles in 3 hours.

36. Find the cost of $9\frac{3}{4}$ yds. ribbon at 18 cts. a yard. Short method. Find the cost of 10 yds. and deduct the cost of $\frac{1}{4}$ yd.

37. Find the cost of 19 lb. 12 oz. of butter at 25 cts. a pound.

38. Find the cost of 14 doz. and 8 eggs at 26 cts. a dozen.

39. If $\frac{2}{3}$ of $\frac{3}{5}$ of a number is 28, find $\frac{5}{7}$ of the same number.

40. Four-fifths of what number equals 64.

41. What number exceeds $\frac{7}{11}$ of itself by 36?

42. A farmer sold $\frac{1}{5}$ of his farm to one man, $\frac{2}{3}$ to another and had 60 acres left. Find the size of the farm and the number of acres sold to each of the two men.

43. In a division question the divisor is $3\frac{1}{2}$, the quotient is $2\frac{1}{4}$. Find the dividend.

44. Find the total cost of all the following articles:—

$7\frac{1}{4}$ lb. butter at 25 cts. a lb.
19 doz. and 6 eggs at 18 cts. a dozen.
$24\frac{1}{4}$ lb. pork at 12 cts. a pound.

45. Find the cost of 100 lb. sugar at the rate of 16 lb. for $1.00.

46. Bought pencils at the rate of 5 for 2 cts. and sold them at 4 for 3 cts. Find the gain on 100 pencils.

47. What number taken from 4 times $\frac{3}{5}$ of 20 will leave $40\frac{3}{5}$?

48. Two-thirds of 20 is $\frac{1}{3}$ of what number?

49. From a cask containing 50 gal. of vinegar $9\frac{1}{4}$ gal. was sold at one time and $12\frac{3}{4}$ at another time. How much remained?

50. A man owned $\frac{3}{5}$ of a factory and sold $\frac{2}{3}$ of his share for $10,000. At that rate what was the whole factory worth?

DENOMINATE NUMBERS.

Exercise XIII.

1. How many inches are there in 8 ft. 4 in.? In 7 ft. 6 in.? In 12 ft. 10 in.? In 9 ft. 11 in.? In 18 ft. 9 in.? In 25 ft. 8 in.?

2. How many feet are there in 72 inches? In 100 in.? In 175 in.? In 428 in.?

3. Reduce 4 yds. 2 ft. to feet.

4. How many feet are there in 3 yd. 1 ft.? In 7 yd. 2 ft.? In $9\frac{1}{2}$ yd.? In 8 yd. 2 ft.?

5. Express each of the following in inches:—2 yd. 1 ft. 6 in.; 4 yd. 2 ft. 8 in.; 5 yd. 1 ft.; 3 yd. 1 ft. 11 in.; 8 ft.; 1 yd. 4 in.

6. Reduce 1 mile to rods; to yards; to feet; to inches. Memorize the results.

7. How many rods are there in 2 mi. 4 fur. 16 rds.? In 1 mi. 7 fur. 25 rds.? In 3 mi. 1 fur. 11 rds.? In 4 mi.? In 5 fur.?

8. Express each of the following in feet:—4 rd.; 2 rd.; 3 rd.; 2 rd. 4 yd. 2 ft.; 10 rd.; 12 rd. 2 yd.; 20 rd. 2 yd. 2 ft.

9. Find the cost of 3 yd. 1 ft. 6 in. of ribbon at 18 cts. a yard. (Express 1 ft. 6 in. as the fraction of a yard.)

10. Find the cost of 1 mi. 4 fur. of ditching at 25 cts. a rod.

11. How many steps each 2 ft. 6 in. long must be taken in walking a mile?

12. How many steps each 2 ft. long will one take in walking 250 yds.?

13. Express 306 in. in yds. ft. and in.

14. Find the cost of 20 rds. 5 yds. of ditching at 20 cts. a yard.

15. Find the cost of 3 mi. 7 fur. 20 rds. of fencing at $160 per mile.

16. Express 2 ft. 3 in. as the fraction of a yard.

17. What part of a mile is 5 fur. 20 rds.?

18. Find the cost of 2 yds. 1 ft. 6 in. of ribbon at $1\frac{1}{2}$ cts. an inch.

19. A ribbon 16 yds. 2 ft. long is cut into pieces each 3 yds. 1 ft. long. How many pieces are there?

20. At 20 cents a yard find the cost of 288 in. of ribbon.

Exercise XXIV.

1. Give area in square inches of a rectangle which is 15 in. long, 8 in. wide.

2. How many square inches in 5 sq. ft. 50 sq. in.?

3. How many square inches are there in a rectangle which is 29 in. long 25 in. wide?

4. A city lot 90 ft. by 50 ft. was sold at the rate of $50 per square yard. Find its cost.

5. A rectangular field is 160 rds. long, 70 rds. wide. How many acres does it contain?

6. Find the cost of painting the ceiling of a room which is 18 ft. long, 16 ft. 6 in. wide at the rate of 10 cts. per square yard.

7. How many mats each 2 ft. 6 in. long, 2 ft. wide will cover a floor 15 ft. long, 12 ft. wide?

8. If a roll of paper is 24 ft. long, 18 in. wide, how many rolls will cover the walls of a room 18 ft. long, 16 ft. wide, [illegible] ft. high.

9. A floor is 18 ft. long, 15 ft. wide. How many yards of carpet which is a yard wide will cover it?

(Short method. Each strip is 6 yds. long and 5 strips cover the room, thus $6 \times 5 = 30$ yds.)

10. Give difference in area of 5 sq. yds. and 5 yds. square.

11. Give area in square inches of a surface which is 1 yd. 1 ft. 2 in. long, 2 ft. 4 in. wide.

12. A field containing 5 acres is 40 rds. long. How wide is it?

13. Give area in acres of each of the following rectangular fields:—

(a) 140 rds. long, 120 rds. wide.
(b) 180 rds. long, 80 rds. wide.
(c) 200 rds. long, 80 rds. wide.
(d) 60 rds. long, 40 rds. wide.
(e) 90 rds. long, 20 rds. wide.

Solve by arranging in fractional form and cancelling: Thus,

(a) $\frac{140 \times 120}{160}$

14. A field containing one acre is 30 rds. long. How wide is it?

15. A ten acre field is 40 rds. long. Find its perimeter. How many times around it makes 10 miles?

16. Ten times round a square garden makes $\frac{1}{2}$ mile. Find its length.

17. What part of an acre does the field in Question 16 contain?

18. How many feet in a rod?

19. How many square feet in a square rod?

20. When carpet is 30 in. wide, how many square feet does a yard of it cover?

21. When carpet is 27 in. wide, how many strips will be required to cover a room 13 ft. 6 in. wide?

22. If the room in Question 21 is 15 ft. long, how many yards of carpet which is 27 inches wide will cover it?

23. How many square feet are there in a rectangle 2 rds. long, 3 yds. 1 ft. wide?

24. A rectangular field is 1 mile long, ½ mile wide. How many acres does it contain?

Exercise XXV.

1. A gallon contains how many pints?

2. At 40 cts. a gallon find the cost of 3 pints of vinegar.

3. How many half-pint bottles can be filled from 2 gal. 3 qts.?

4. At 32 cts. a gallon find the cost of oil per pint.

5. Find the cost of 4 gal. 3 qts. 1 pt. of oil at 20 cts. a gallon.

6. A milk dealer pays 20 cts. per gallon for milk and sells it for 6½ cts. a quart. Find his gain per gallon.

7. How many 1½ pt. bottles may be filled from a 3 gallon jug?

8. A cow gives 7 qts. of milk twice a day. Find the value of the milk for a week at 5 cts. a quart.

9. A man uses ½ pt. of milk at each meal. Find his milk bill per week at 6 cts. a quart.

10. A milk dealer makes a profit of 1½ cts. per quart. Find his daily profit if he sells 100 gal. per day.

Exercise XXVI.

1. Express 10 cubic yards in cubic feet.

2. In 100 cubic feet there are how many cubic yards?

3. A pile of wood is 40 ft. long, 4 ft. wide, 5 ft. 3 in. high. How many cubic feet does it contain?

4. A pile of wood is 64 ft. long, 4 ft. wide, 5 ft. high. How many cords does it contain? (128 cu. ft. = 1 cord.)

5. At 25 cts. per cu. yd. find cost of digging a cellar 36 ft. long, 24 ft. wide, 3 ft. 6 in. deep.

Solve thus: $\frac{36}{1} \times \frac{24}{1} \times \frac{7}{2} \times \frac{1}{27} \times \frac{25}{1}$ Cancel.

6. A pile of wood which is 15 ft. long, 4 ft. wide contains 300 cu. ft. How high is it?

7. How many blocks $\frac{1}{4}$ in. each way may be cut from a cubic inch?

8. If bricks are 8 in. long, 4 in. wide, 3 in. thick, how many will be required to build a wall 10 ft. long, 16 in. thick, 3 ft. 3 in. high. (Find the number of bricks long, the number wide, the number high and multiply.)

9. A stick of timber is 30 ft. long, 12 in. wide, 10 in. thick, how many cubic feet does it contain?

10. How many blocks 4 in. long, 3 in. wide, 2 in. thick, may be cut from a cubic foot?

Exercise XXVII.

1. Find the number of days from April 20th to July 8th; from September 10th to December 20th; from December 15th to February 11th.

2. How much can a man earn in 4 weeks at $1.75 per day for week days?

3. At 25 cts. per hour what can a man earn in 6 days by working 10 hr. a day?

4. Reduce 9 days to hours; to minutes.

5. How many hours are there in February in an ordinary year? In leap year?

6. How many minutes are there from 20 minutes to eleven to 15 minutes past one?

7. If May 5th falls on Sunday, on what day of the week does June 1st fall?

8. How many working days are there from Monday morning September 11th to November 10th?

10. If Christmas falls on Monday in 1911, on what day of the week will it fall in 1913?

11. At the rate of $30 per week, what does a man earn in a year?

Rule XLVIII. To reckon the value of any quantity of hay at any price per ton.

At $1.00 per ton hay is worth 5 cts. per cwt., therefore the hay is worth 5 cts. per cwt. for each dollar per ton, e. g., at $7 per ton, it is worth 7 times 5 cts., that is 35 cts. per cwt.

EXAMPLE.

Find the price of 950 lbs. of hay at $8 per ton.

At $8.00 per ton the price per cwt. is 8 × 5 or 40 cts., therefore 950 lbs. is worth $9\frac{1}{2} \times 40 = \$3.80$.

Also find price by taking fractional parts of a ton.

EXERCISE XXVIII.

Find the price of each quantity of hay given in the left hand column at each price per ton given in the right hand column?

750 lbs.	$9.00 per ton
1150 "	12.00 "
1575 "	8.00 "
1850 "	7.00 "
2500 "	11.00 "
3000 "	9.50 "
850 "	6.00 "
1360 "	10.00 "

EXERCISE XXIX.

1. What fractional part of a ton is each of the following:—200 lb.; 400 lb.; 500 lb.; 600 lb.; 800 lb.; 1000 lb.; 1200 lb.; 1400 lb.; 1500 lb.; 1600 lb.; 1800 lb.; 250 lb.; 150 lb.; 350 lb.; 450 lb.; 550 lb.; 650 lb., etc.

2. Find the cost of 3 tons 15 cwt. of hay at $8 per ton. At $10 per ton; at $12 per ton; at $9 per ton.

3. Find the cost of 75 lb. hay at $11 per ton.

4. If it requires 2 lb. flour for a loaf of bread, how many of such loaves can be made from a barrel of flour?

5. At 8 cts. per loaf what would the bread made from the barrel of flour in Question 4 be worth?

6. Find the cost of 19 lb. 10 oz. of butter at 25 cts. a pound.

7. How many ounces are there in 10 lb. 15 oz. Av.?

8. Find the cost of 24 lb. 8 oz. of beef at the rate of $10 per hundred weight.

9. Divide 9 lb. 10 oz. of candy equally among 8 boys.

10. If $5\frac{1}{2}$ lb. of beef cost 66 cts. find the cost of 9 lb. 4 oz.

11. At $2\frac{1}{2}$ cts. per ounce find the cost of 9 lb. 12 oz. of tea.

12. A pair of chickens weighs 7 lb. 7 oz. What are they worth at 16 cts. a pound? At 20 cts. a pound?

Rule XLIX. To find the superficial feet in sawed lumber which is one inch or under in thickness.

Find the number of square feet in one of the surfaces, which can be done by multiplying the numbers which represent the feet long and the inches wide, and dividing the product by 12. e. g., Find the superficial feet in a board 18 ft. long, 10 in. wide, $\frac{1}{2}$ in. thick.

$$\frac{18 \times 10}{12} = 15 \text{ ft.}$$

Rule L. To find the superficial feet in sawed lumber which is more than one inch thick. Find the product of the three numbers which represent the length in feet, the width in inches and the thickness in inches and divide the product by 12, e. g., Find the superficial feet in a stick 16 ft. long, 6 in. wide, 4 in. thick.

$$\frac{16 \times 6 \times 4}{12} = 32.$$

Clapboards are six inches wide and one-half inch thick, therefore half the number which represents the length in feet will equal the superficial feet.

In many cases the product of the width and thickness can be regarded as a fractional part of 12, and thus the work may be shortened. e. g., 16 ft. long, 3 in. by 3 in. superficial feet =

$$\frac{16 \times 3 \times 3}{12} = 16 \times \frac{3 \times 3}{12} = 16 \times \tfrac{3}{4} = 12.$$

e. g., 18 ft. long, 4 in. by 2 in. $= \frac{2}{3} \times 18 = 12$ ft.
e. g., 24 ft. long, 5 in. by 2 in. $= \frac{5}{6}$ of $24 = 20$ ft.

Exercise XXX.

Find the superficial feet in each of the following:—

1. 18 ft. long, 10 in. wide, $\frac{3}{4}$ in. thick.
2. 15 ft. " 6 in. " 2 in. "
3. 10 ft. " 4 in. " 3 in. "
4. 16 ft. " 6 in. " 4 in. "
5. 13 ft. " 8 in. " 5 in. "
6. 18 ft. " 9 in. " 2 in. "
7. 14 ft. " 5 in. " 3 in. "
8. 10 ft. " 4 in. " 4 in. "
9. 15 ft. " 6 in. " $1\frac{1}{2}$ in. "
10. 20 ft. " 8 in. " $2\frac{1}{2}$ in. "

Exercise XXXI.

1. Find the cost of 1000 pieces of timber each of which is 24 ft. long, 10 in. wide, 3 in. thick, at $32 a thousand feet.

2. Find the cost of the lumber for a floor 16 ft. long, 12 ft. 6 in. wide at $50 per M, the boards being 1 inch thick.

3. A square field containing 10 acres is fenced with a closed fence 10 ft. high, the boards being 1 inch thick. How many feet of boards are used?

4. A cubical box 2 ft. long with cover, is made of inch boards. How many feet are used?

5. How many feet of inch boards will cover the four walls of a house 30 ft. long, 25 ft. wide, 15 ft. high?

Rule LI. Express the various fractional parts of a pound sterling money in shillings and pence.

Since 20s = £1, therefore

2s 6d = $\frac{1}{8}$	of a pound.	3s 4d = $\frac{1}{6}$	of a pound
7s 6d = $\frac{3}{8}$	"	6s 8d = $\frac{1}{3}$	"
12s 6d = $\frac{5}{8}$	"	8s 4d = $\frac{5}{12}$	"
17s 6d = $\frac{7}{8}$	"	11s 8d = $\frac{7}{12}$	"
5s = $\frac{1}{4}$	"	13s 4d = $\frac{2}{3}$	"
10s = $\frac{1}{2}$	"	16s 8d = $\frac{5}{6}$	"
15s = $\frac{3}{4}$	"	18s 4d = $\frac{1}{2}$	"
1s 8d = $\frac{1}{12}$	"		

Rule. To reckon the value of any number of articles at any of the prices given in the foregoing list, take the fractional part of the articles corresponding to the price and call the result pounds.

e. g., Find the cost of 48 articles at 17s 6d each. Since 17s 6d is $\frac{7}{8}$ of a pound, 48 articles will cost $\frac{7}{8}$ of £48 or £42.

Exercise XXXII.

Find the cost of each number of articles in the left hand column at each price given in the right hand column.

48	articles	2s	6d
120	"	7	6
180	"	12	6
160	"	17	6
240	"	5	
145	"	10	
320	"	15	
96	"	1	8
40	"	3	4
72	"	6	8
84	"	8	4
24	"	11	8
36	"	13	4
140	"	16	8
480	"	18	4

By a method similar to the above express any number of ounces as the fraction of a pound, feet and inches as the fraction of a yard, rods as the fraction of a mile, square rods as the fraction of an acre, minutes as the fraction of an hour, quarts as the fraction of a bushel, individuals as the fraction of a dozen, etc., and frame and solve problems involving the various quantities.

Observe the methods for shortening the work in the following problems and apply similar methods in similar cases:—

1. Find the cost of $19\frac{3}{4}$ lb. of butter at 24 cts. a pound. Solution: Find the cost of 20 lb. and subtract the cost of $\frac{1}{4}$ lb., hat is $24 \times 20 - \frac{1}{4}$ of $24 = 480 - 6 = \$4.74$.

2. Find the cost of 160 lb. of beef at 9¼ cts. per pound. Solution: Find the cost of 160 lb. at 10 cts. a pound and subtract the cost of 160 lb. at ¼ ct. a pound, that is $160 \times 10 - \frac{1}{4}(160) = 1600 - 40 = \$15.60$.

3. How many hours are there in a year.

(a) $365 \times 24 = 365 \times 25 - 365 = 9125 - 365 = 8760$.

The work of subtracting 365 from 9125 may be done most easily by breaking up the 365 in parts, as follows:—$9125 - 365 = 9125 - 300 - 25 - 40 = 8760$.

Why is 365 broken up as above?

(b) $24 \times 365 = (24 \times 300) + (24 \times 50) + (24 \times 15) = 7200 + 1200 + 360 = 8760$.

Discover the reason for breaking up 365 as in (b).

The method given in Question 3 may be applied in many cases, but no general rule can be given as the number to be broken up will determine the way in which it may be broken up.

To multiply mixed numbers by whole numbers the work may be done from left to right as follows:—

(a) $17\frac{1}{3} \times 2 = (17 \times 2) + (\frac{1}{3} \times 2) = 34\frac{2}{3} = 35\frac{1}{3}$.

(b) $47\frac{3}{4} \times 5 = (47 \times 5) + (\frac{3}{4} \times 5) = 235 + \frac{15}{4} = 238\frac{3}{4}$.

Similarly two mixed numbers may be multiplied by breaking up into parts and working from left to right e. g., $25\frac{1}{3} \times 8\frac{1}{2} = (25 \times 8) + (\frac{1}{3} \times 8) + (25 \times \frac{1}{2}) + (\frac{1}{3} \times \frac{1}{2}) = 200 + 5\frac{1}{3} + 12\frac{1}{2} + \frac{1}{6} = 218\frac{1}{6}$.

In the above each result after the first should be added as obtained and thus save the task of carrying too many numbers in the memory.

By the application of the principles given above, an unlimited number of ways for solving questions mentally may be found.

EXERCISE XXXIII.

1. Find the cost of 10 yds. cotton at 1s. 4d. per yard.

2. Find the cost of 42 pencils at 2s. 6d. per dozen.

3. Out of £10 9s. 6d. I paid £4 6s. 8d. How much had I left?

4. At 1s. 8d. per yd. how many yards of cloth can be bought for £1?

5. I spent £2 10s. in ribbons at 8d. per yd. How many yards did I buy?

6. Find the cost of 5 gross of slate pencils at 4d. per dozen.

7. If a man earns 10s. 5d. per day, in how many days will he earn £6 5s.?

8. A farmer paid £9 6s. for six sheep. What was the average price per sheep?

9. How many farthings are there in £1?

10. Give the value of each of the following in shillings and pence:—£$\frac{1}{6}$, £$\frac{3}{8}$, £$\frac{5}{8}$, £$\frac{7}{8}$, £$\frac{1}{2}$, £$\frac{1}{4}$, £$\frac{3}{4}$.

Exercise XXXIV.

1. Find the cost of a ton of wheat at 90 cts. a bushel.

2. At 15 cts. a peck how many bus. of potatoes can be bought for $12?

3. Bought 50 bus. oats at 50 cts. a bushel, and sold them at 2 cts. a quart. Find gain.

4. A horse eats 12 qts. of oats per day. How many bushels will he eat in 32 days?

5. At a boarding stable for horses a feed of 4 qts. oats costs 10 cts. If oats are worth 55 cts. per bushel what profit does the stable make per bushel?

6. A cubical box measures one yard each way. Find cost of painting the outside including cover at 3 cts. per sq. ft.

7. A school room is 31 ft. long, 25 ft. wide, 12 ft. high. How many cu. ft. of air space is there for each of 50 pupils?

8. A box 5 ft. square holds 150 cu. ft. of water. How deep is it?

9. Find the cost of 20 lb. 10 oz. of spice at 5 cts. per ounce.

10. A box is an exact cubic yard. Give the length of all the edges in feet.

11. Shingles are sold to average 4 inches wide. If they are laid 4 inches to the weather how many will cover a sq. ft.?

12. When laid 4$\frac{1}{2}$ inches to the weather how many shingles will cover a sq. ft.?

13. How many shingles laid 4 inches to the weather will cover a wall 16 ft. long, 15 ft. high?

14. How many shingles laid 4$\frac{1}{2}$ inches to the weather will cover a space of 125 sq. ft.?

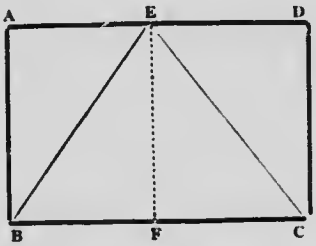
15. How many feet of inch boards will cover the four walls of a building which is 30 ft. long, 25 ft. wide, 14 ft. high?

16. How many blocks each 4 inches long, 3 inches wide, 2 inches thick, can be cut from a cu. ft.

EXERCISE XXXV.

1. At ½ ct. per sheet find the cost of 1 ream of paper.

2. The area of a triangle is equal to half the area of a rectangle on the same base and of the same altitude. (See figure below.)



Therefore area of triangle equals the product of half the base by the altitude.

3. Find the areas of the following triangles:

Base	Altitude
15 in.	20 in.
50 in.	36 in.
36 in.	16 in.
48 ft.	30 ft.
21 ft.	15 ft.
85 ft.	25 ft.

4. If a boy learns to spell 3 words in 5 minutes, how many can he learn in 2 hr. 30 min.?

5. The perimeter of a square field is 1 mile. How many acres does it contain.

6. A square field containing 10 acres is fenced with a wire fence on posts 1 rod apart. How many posts are there?

7. How many acres are there in a farm ¾ of a mile long and ½ mile wide?

8. A string 1 rd. 1 yd. 1 ft. 6 in. is cut into 7 equal pieces. What is the length of each piece?

9. A room 21 ft. long, 18 ft. wide, is covered with carpet which is 2 ft. wide and laid lengthwise. How many yards are required?

10. A room 15 ft. long, 12 ft. wide is covered with linoleum at 75 cts. per sq. yd. Find cost.

11. At the rate of 15 cts. a quire find the cost of 16 sheets of paper.

PERCENTAGES.

Exercise XXXVI.

1. What per cent. of anything does each of the following fractional parts represent?

$\frac{1}{100}$, $\frac{5}{100}$, $\frac{10}{100}$, $\frac{1}{10}$, $\frac{3}{10}$, $\frac{7}{10}$, $\frac{9}{10}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{11}{12}$, $\frac{1}{20}$, $\frac{3}{20}$, $\frac{7}{20}$, $\frac{11}{20}$, $\frac{13}{20}$, $\frac{17}{20}$, $\frac{19}{20}$, $\frac{1}{25}$, $\frac{4}{25}$, $\frac{3}{25}$, $\frac{6}{25}$, $\frac{7}{25}$, $\frac{8}{25}$, $\frac{9}{25}$, $\frac{11}{25}$, $\frac{13}{25}$, $\frac{14}{25}$, $\frac{16}{25}$, $\frac{18}{25}$, $\frac{19}{25}$, $\frac{21}{25}$, $\frac{22}{25}$, $\frac{23}{25}$, $\frac{24}{25}$.

2. Express the following as fractions in their lowest terms:—

1%, 2%, 4%, 5%, 8%, 10%, 12½%, 15%, 20%, 16%, 24%, 25%, 30%, 35%, 37½%, 40%, 45%, 48%, 36%, 50%, 55%, 60%, 65%, 62½%, 70%, 75%, 80%, 85%, 83%, 87½%, 90%, 95%.

3. Express each of the following as decimals:—

1%, 3%, 6%, 7%, 9%, ½%, ¼%, ¾%, 2½%, 1¼%, ⅛%, 1¾%.

Exercise XXXVII.

1. In a certain school 40% of the pupils are boys and there are 156 girls. How many pupils are there?

2. From 75 deduct 16%.

3. A man divided his money among his three children. To the eldest he gave 40%, to the second 32% and to the youngest $5600. How much money had he?

4. A man spent 33⅓% of his money and had $150 left. What had he at first?

5. Bought cotton at 12 cts. a yard and sold it at 15 cts. a yard. Find gain per cent.

6. A merchant bought tea at 30 cts. a pound and sold it at a profit of 20%. At what price per pound did he sell it?

7. Seventy-five is 30% of what number.

8. A house which cost $3000 is rented at an annual rate of 10% on cost. What is the rent per month?

EXERCISE XXXVIII.

Find answers:

1. $\frac{3}{5}$ of 60 days.
2. .4 of 60 minutes.
3. 40% of $60.
4. .125 of 88 pounds.
5. $\frac{1}{4}$ of 32 yards.
6. $12\frac{1}{2}$% of 72 seconds.
7. $\frac{3}{4}$ of 360 hours.
8. .75 of 160.
9. 75% of 480 inches.
10. .075 of 800 miles.
11. $\frac{1}{4}$% of 1600 books. (Find 1% and take $\frac{1}{4}$ of it.)
12. $\frac{1}{3}$% of $18000.
13. $1\frac{1}{4}$% of $4800.
14. $\frac{1}{800}$ of 640 feet.
15. 4% of $800.
16. .4 of $2500.
19. .04% of $7500.
18. $\frac{1}{25}$ of $1200.
19. 2% of $500.
20. 3% of $1500.

Rule LII. To solve many questions in percentage mentally it will be found most convenient to express the percentages in fractional form. The following will be found convenient for mental application and many others may be worked out.

$5\% = \frac{5}{100} = \frac{1}{20}$.
$15\% = \frac{15}{100} = \frac{3}{20}$.
$35\% = \frac{35}{100} = \frac{7}{20}$.
$45\% = \frac{9}{20}$.
$55\% = \frac{11}{20}$.
$65\% = \frac{13}{20}$.
$85\% = \frac{17}{20}$.
$95\% = \frac{19}{20}$.
$10\% = \frac{1}{10}$.
$30\% = \frac{3}{10}$.
$70\% = \frac{7}{10}$.
$90\% = \frac{9}{10}$.
$33\frac{1}{3}\% = \frac{1}{3}$
$66\frac{2}{3}\% = \frac{2}{3}$
$4\% = \frac{1}{25}$.
$2\% = \frac{1}{50}$

$20\% = \frac{1}{5}$.
$40\% = \frac{2}{5}$.
$60\% = \frac{3}{5}$.
$80\% = \frac{4}{5}$.
$25\% = \frac{1}{4}$.
$50\% = \frac{1}{2}$.
$75\% = \frac{3}{4}$.
$12\frac{1}{2}\% = \frac{1}{8}$.
$37\frac{1}{2}\% = \frac{3}{8}$.
$62\frac{1}{2}\% = \frac{5}{8}$.
$87\frac{1}{2}\% = \frac{7}{8}$.
$16\frac{2}{3}\% = \frac{1}{6}$.
$83\frac{1}{3}\% = \frac{5}{6}$.
$14\frac{2}{7}\% = \frac{1}{7}$
$11\frac{1}{9}\% = \frac{1}{9}$
$9\frac{1}{11}\% = \frac{1}{11}$
$8\frac{1}{3}\% = \frac{1}{12}$

Exercise XXXIX.

Find	5 %	of	each	number	in the column to the right,	40
"	10 %	"	"	"	"	60
"	12½%	"	"	"	"	70
"	15 %	"	"	"	"	80
"	20 %	"	"	"	"	50
"	25 %	"	"	"	"	30
"	30 %	"	"	"	"	20
"	35 %	"	"	"	"	90
"	37½%	"	"	"	"	100
"	40 %	"	"	"	"	48
"	45 %	"	"	"	"	120
"	50 %	"	"	"	"	640
"	55 %	"	"	"	"	720
"	60 %	"	"	"	"	360
"	62½%	"	"	"	"	540
"	33⅓%	"	"	"	"	160
"	66⅔%	"	"	"	"	180
"	70 %	"	"	"	"	25
"	75 %	"	"	"	"	36
"	80 %	"	"	"	"	96
"	83⅓%	"	"	"	"	79
"	85 %	"	"	"	"	54
"	87½%	"	"	"	"	125
"	90 %	"	"	"	"	840
"	95 %	"	"	"	"	890

Exercise XL.

1. Find 3% of $300.
2. " 4% of 700 pounds.
3. " 25% of 75 bushels.
4. " 15% of 60.
5. " 10% of 450 miles.
6. ½% of $250.
7. 66⅔% of 81 ounces.
8. 16⅔% of 72 yards.
9. ⅛% of $5600.
10. 35% of $80.

Exercise XLI.

1. What per cent of 400 is 40?
2. What per cent of 700 is 35?

3. What per cent of 600 is 100?
4. What per cent of 425 is 425?
5. What per cent. of 80 is 120?
6. What per cent. of 60 is 240?
7. What per cent. of $7\frac{1}{2}$ is 15?
8. What per cent. of 300 is $1\frac{1}{2}$?
9. What per cent. of $\frac{1}{2}$ is $\frac{1}{4}$?
10. What per cent, of $1\frac{1}{2}$ is $\frac{3}{4}$.?

Exercise XLII.

1. What number increased by 40% of itself equals $280?
2. Increase 450 by 30% of itself.
3. What number diminished by $12\frac{1}{2}\%$ of itself equals 420?
4. Diminish 320 by 45% of itself.
5. 80 is 25% of what number?
6. 25% of 80 is what number?
7. Find the numbers of which 60 is,

3%, 4%, $\frac{1}{4}\%$, $2\frac{1}{2}\%$, 60%, 8%, 80%, 9%, 90%, $33\frac{1}{3}\%$, $62\frac{1}{2}\%$, $66\frac{2}{3}\%$, $16\frac{2}{3}\%$, $83\frac{1}{3}\%$, 6%.

8. What fraction of $\frac{2}{3}$ is $\frac{3}{5}$?
9. What per cent of $\frac{2}{3}$ is $\frac{2}{5}$?
10. $4\frac{3}{4}$ is what per cent. of $9\frac{1}{2}$?

Exercise XLIII.

1. The enrollment in a school is 80. The average daily attendance is 80% of the enrollment. How many on an average are absent each day?

2. A man spends 40% of his money and has $720 left. How much does he spend?

3. A farmer bought a horse for $180 and sold it at a profit of 20%. What did he receive for it?

4. A merchant bought tea at 30 cts. a pound and sold it at 40 cts. a pound. Find his gain per cent.

5. Find $33\frac{1}{3}\%$ of $768\frac{3}{4}$ miles.

6. A merchant sold 25% of a tub of butter which weighed 48 lb. 12 oz. How much remained?

7. In a certain school 15% study Greek, 40% study Latin and 90 pupils study neigher Greek nor Latin. What is the enrollment?

8. A man pays 15% of his salary for rent which is $20 per month. Find his salary.

9. A clerk pays 35% of his salary for board, 20% of the remainder for clothes and has $620 left. Find his salary.

10. 50% of 40% is what per cent. of 30%?

11. A boy receives 10% of all the potatoes he can pick. If he receives 7½ bushels per day how many bushels does he pick in 6 days?

12. What per cent. of 50 is 75% of 80.

13. A boy who had 75 marbles gained 15. What was his gain per cent?

14. A man is taxed ¾% on his property valued at $6,500. How much are his taxes?

15. In a school district the taxes amount to 75 cts. on the $100. Find rate per cent.

16. The enrollment in a school is 480. If 24 are absent the attendance is what per cent. of the enrollment?

17. What must the selling price of tea which cost 32 cts. per pound be so as to give a profit of 25%?

PROFIT AND LOSS.

EXERCISE XLIV.

1. The selling price of cotton is 26 cts. a yard. The profit is 30%. Find cost.

2. The cost of tea is 25 cts. per pound. The profit is 20%. Find selling price.

3. The cost of lard is 15 cts. per pound. The selling price is 20 cts. per pound. Find gain per cent.

4. Goods sold at a loss of 10% bring $18. Find cost.

5. A horse which cost $180 was sold at a loss of 15%. Find selling price.

6. For what should the horse in Question 5 be sold so as to gain 10%?

7. If I sell a house for $1200 and thereby lose 20%; what fraction of $1200 do I lose? How many dollars do I lose?

8. I buy oranges at $2 per hundred and sell them at the rate of 2 oranges for 5 cts. Find gain per cent.

9. Apples are bought at the rate of 10 cts. per dozen and sold at 1½ cts. each. Find gain per cent.

10. Sugar is bought at $4.25 per 100 lb. and sold at the rate of 16 lbs. for $1.00. Find gain per cent.

11. Two horses are sold at $120 each. On one there is a profit of 25% and on the other a loss of 25%. Was there a gain or a loss on the transaction and how much?

12. A man sold a property for $1600. If 50% of the sum received is profit, what per cent. has he made?

13. A property was sold for $2400. 25% of the selling price was profit. Find gain per cent.

14. What is the gain per cent. on a dozen books bought at $15 per dozen and sold at $1.50 each? What is the gain per cent. on each book?

15. How many dollars do I gain by selling goods for $60 on which I gain 20%?

16. How many dollars do I lose by selling a carriage for $48 on which I lose 25%?

17. How much do I gain or lose by selling two houses at $2400 each, on one of which I gain 25% and on the other of which I lose 20%?

18. How much do I gain or lose by selling two farms at $3600 each on one of which I gain 33⅓% and on the other of which I lose 25%?

19. A boy buys apples at 10 cts. per dozen and sells them at the rate of 2 for 3 cts. Find gain per cent.

20. A man sells an article at three times its cost. What per cent. profit does he realize?

21. I sell an article at a reduction of 10% from the marked price and still make a profit of 26%. The marked price is what per cent. of the cost price?

22. Buying price $80, selling price $100. Find gain %.

23. Selling price 7 cts., buying price 5 cts. Find gain per cent.

24. Buying price $40, gain 15%. Find selling price.

25. Selling price $60, gain 25%. Find buying price.

26. An article is marked at 140% of cost and sold at 80% of marked price. What is the gain per cent?

27. Buying price \$75, loss 33⅓%. Find selling price.

28. Selling price \$80, gain \$20. Find gain per cent.

29. Buying price \$40, loss \$15. Find loss per cent.

30. Selling price \$260, gain 30%. Find profit.

31. Buying price \$160, gain 40%. Find profit.

32. Selling price \$30, loss 20%. Find loss.

33. Buying price \$90, loss 70%. Find loss.

34. Selling price \$180, buying price \$150. Find gain per cent.

35. Selling price \$150, buying price \$180. Find loss per cent.

36. Selling price \$288, loss 28%. Find buying price.

37. Buying price 5½ cts. Selling price 8¼ cts. Find gain per cent.

38. Sold a watch for \$46, thereby losing 20%. For how much should it have been sold so as to gain 20%?

39. A merchant marks his goods at 40% above cost but allows a discount for cash of 10%. Find gain per cent. on cash sales.

40. A man invests \$400 for two years. The first year he gains 25% which he adds to his investment and the new investment gains 20% during second year. How much is he worth at the end of the two years?

COMMERCIAL DISCOUNT.

Exercise XLV.

1. A bill of goods listed at \$600 is sold on thirty days at a discount of 25% with a further discount for cash of 10%. Find cash selling price.

2. What single discount would equal the discounts in Question 1.

3. Goods cost \$300. They are marked to be sold at 80% of cost but are sold at 80% of marked price. (a) For what are they sold? (b) For what per cent. of cost are they sold. (c) What is the percentage of discount on cost? (d) How much money is lost?

4. Find the net price of goods listed at \$480 with discounts of $33\frac{1}{3}\%$ and 20%.

5. Goods are listed at \$1000 with discounts of 30% and 20%. (a) Find selling price. (b) What per cent of the list price is the total discount? (c) What per cent of the list price is the net price?

6. (a) What per cent. is left after deducting 30%? (b) What per cent. is left after deducting 25% of the remainder?

7. What per cent. of a number is 60% of 90% of the number.

8. If a man spends $\frac{2}{3}$ of his money and $\frac{1}{4}$ of the remainder; (a) What fraction of it does he spend? (b) What per cent. of it has he left?

9. A merchant paid \$800 for an order of goods. On account of damages he marked them at 20% less than cost and sold them at 20% below marked price. (a) Find net loss. (b) What was the loss per cent.?

10. What per cent. of the list price is 80% of 90% of the list price?

11. What single discount is equal to 40% and 10%?

12. List price \$500, discounts 25% and 20%. Find net price.

13. List price 50 cts., discounts 30% and 20%. Find net price.

14. List price \$400, discounts 10% and $33\frac{1}{3}\%$. Find discount.

15. When the list price is \$100, find the net price after the deduction of the following successive discounts:—

(a) 30% and 20%.
(b) 40% and 50%.
(c) 10% and 40%.
(d) 40% and 25%.
(e) 15% and 60%.
(f) 50% and 50%.
(g) 60% and 35%.
(h) 20% and 10%.
(i) 5% and 60%.
(j) 60% and 50%.
(k) 25% and 40%.
(l) 25% and 30%.
(m) 35% and 30%.
(n) 5% and 20%.

16. Find which discount produces the smaller net price, when the list price is $100:—

(a) 20% and 40% or 40% and 20%.
(b) 40% and 40% or 50% and 30%.
(c) 40% and 25% or 60% and 5%.
(d) 60% and 30% or 50% and 40%.
(e) 30% and 10% or 20% and 20%.

17. What single discount is equal to the successive discounts of 40%, 20% and 12½%?

18. Find the single discount equal to each of the following:—

(a) 20 % and 30%.
(b) 37½% and 40%.
(c) 40 % and 5%.
(d) 60 % and 10%.
(e) 40 % and 15%.

19. The invoice price of goods is $300 and the discount is $60. Find rate of discount.

20. The list price of goods is $480 and the net price is $360. What is the rate of discount?

21. What is the net price of goods listed at $640 and subject to a discount of 40%?

22. The discount off a bill at the rate of 30% is $72. Find the amount of bill.

23. Goods are sold for $36 after allowing discounts of 20% and 10%. Find list price.

24. A man paid $150 for goods at 62½% off. Find the list price.

25. The net price of a bill of goods is $500; the rate of discount is 33⅓%. Find the full amount of the bill.

COMMISSION.

For collecting debts or selling goods an agent gets commission on all the money handled.

When money is sent to an agent to be used in buying goods commission is usually reckoned only on the actual amount used

in buying the goods and not on the part which goes to the agent as commission.

Thus if a merchant sends to his agent $105 with which to purchase goods after deducting his commission at 5% he does not take commission on the whole $105 as in that case he would be receiving commission on the money which he retained as commission as well as on the money which he invested for the merchant.

To find the amount to be invested in the case mentioned let the agent invest one dollar and pay himself five cents for the investment of the dollar. He would thus be able to invest one dollar for each $1.05 contained in the money sent, thus $105 ÷ $1.05. i. e., he would invest $100 and keep $5 as his commission on the $100 invested.

Exercise XLVI.

1. A collector collects $900 at a commission of 4%. How much does he receive?

2. An agent sells 1500 bus. grain at 59 cts. a bushel. Find his commission at 10%; at 7%; at 5%.

3. A collector received $80 for collecting a debt at 10%. Find the debt.

4. A broker sold $6000 railway stock on a commission of $\frac{2}{3}$%. Find his commission.

5. A merchant sent his agent $6240 with which to purchase goods after deducting his commission at 4%. What commission did he get? What was the cost of the goods he purchased?

6. An agent after deducting his commission at the rate of 5% on the sum invested from $7350 which was sent him, bought flour with the balance at $5 per barrel. How many barrels did he buy?

7. An acutioneer sold goods to the amount of $4800. Find his commission at 2$\frac{1}{2}$%.

8. How much money must a merchant send his agent so that he (the agent) may purchase $6400 worth of goods and pay himself a commission of 6% on the amount purchased?

9. A real estate agent sold a property for $5600. His commission was at the rate of 5%. How much did the owner of the property receive?

10. A commission merchant sold goods for $5000 and sent his principal $4600. What rate of commission did he charge?

11. How much must I send my agent so that he may purchase 1000 bus. grain at 80 cts. a bushel after deducting his commission at 6%.

12. After deducting 10% for collecting a debt an agent remits $1800 to his employer. How much has the agent collected?

13. The commission on a sale of $500 was $12.50. What was the rate of commission.

14. At $3\frac{1}{2}\%$ the commission on the sale of goods was $30. Find the sum for which the goods were sold.

15. A collector obtained 60% of his employer's bill of $500. He charged 3% commission. How much did the employer receive?

16. I sent my agent $4160 with instructions to deduct his commission at 4% and invest the remainder. Find the agent's commission.

INSURANCE.

EXERCISE XLVII.

1. A house is insured for $2000 at $\frac{3}{4}\%$. Find premium.

2. What premium must be paid for insuring a property for $3000 at $1\frac{1}{2}\%$?

3. A house which cost $6000 is insured for $\frac{3}{4}$ of its cost at $\frac{2}{3}\%$. Find premium.

4. At $1\frac{1}{4}\%$ the premium paid for insuring a house was $70. For what was the house insured?

5. Find the cost of insuring a property for $3600 at $1\frac{3}{4}\%$.

6. A house is insured for 48000 at $\frac{3}{8}\%$ and the furniture for $4000 at $\frac{1}{4}\%$. Find total premium paid.

7. A house worth $9000 is insured for $66\frac{2}{3}\%$ of its value at $\frac{3}{4}\%$. Find premium.

8. A premium of $60 is paid for insuring a house for $\frac{4}{5}$ of its value at $\frac{3}{4}\%$. Find the value of the house.

TAXES.

Exercise XLVIII.

1. A house cost $3000. It is assessed at $\frac{3}{4}$ cost. The tax on assessed value is at the rate of $\frac{1}{2}\%$. Find amount of tax. The tax amounts to what per cent. on cost?

2. In a school district valued at $25000; $268 is collected in a year. There are 18 poll taxes of $1 each. The balance of the $268 is assessed on the property. How much does a man pay on his property which is valued at $1400?

3. Find the taxes on property assessed at $1200, the rate being $\frac{3}{4}\%$.

4. A school district assessed for $16000 collects $80 taxes. What is the rate on $100? What is the rate per cent.?

5. The taxes on a property assessed for $2500 is $10. At the same rate find the tax on a property assessed for $1700.

Exercise XLIX.

1. Find interest on $850 for 1 year at 4%.

2. What is the interest on $400 for 9 mos. at 7%.

3. To what sum will $600 amount in 1 year at $4\frac{1}{2}\%$ per annum?

4. Find the interest on $1200 at 8% per annum for each of the following periods:—1 yr.; 10 mos.; 9 mos.; 8 mos.; 7 mos.; 6 mos.; 5 mos.; 4 mos.; 3 mos.; 2 mos.; 1 mo.

5. To what part of the principal does the simple interest amount in 5 years at 5%? In 4 yrs. 2 mos. at 6%? In 4 yrs. at 5%? In 7 yrs. 6 mos. at 4%? In 6 yrs. 3 mos. at 8%?

6. Find the simple interest on $40 for 5 years at 5%; on $600 for 4 yrs. 2 mos. at 6%; on $80 for 4 yrs. at 5%; on $300 for 7 yrs. 6 mos. at 4%; on $250 for 6 yrs. 3 mos. at 8%.

7. The interest on $360 for 8 months is $27. Find the rate per cent, per annum.

8. At what rate per cent. per annum will $320 give $22.40 interest in 1 year?

INTEREST.

Rule LIII. To find the interest on a sum of money at six per cent.

Since 6 cts. is gained by $1 in 12 months, then 1 ct. is gained by $1 in 2 months. Therefore multiply the principal by one-half the number of months and call the result cents.

e. g., Find interest on $480 for 10 months at 6 per cent. $480 × 5 = $24.00.

Since $1 at 6% earns 1 ct. in two months, which is about 60 days, it will earn $\frac{1}{10}$ of a cent, that is one mill, in $\frac{1}{10}$ of .60 days or in 6 days.

Therefore multiply the principal by $\frac{1}{6}$ the number of days and call the result mills, e. g., Find the interest on $4700 for 42 days at 6% = 4700 × 7 = 32900 mills = $32.90.

Exercise L.

1. Find the simple interest on $320 for 4 yrs. 2 mos. at 6%.
2. Find the simple interest on $720 for 3 yrs. 4 mos. at 6%.
3. Find the interest on $80 for 8 mos. at 6%.
4. Find the interest on $140 for 11 mos. at 6%.
5. Find the interest on $145 for 120 days at 6%.
6. Find the interest on $40.50 for 2 yrs. 6 mos. at 6%.
7. Find the interest on $95 for 240 days at 6%.

To find interest at 5% first find interest at 6%, and deduct $\frac{1}{6}$; at 7% add $\frac{1}{6}$; at 8% add $\frac{1}{3}$; at 9% add $\frac{1}{2}$; at 4% deduct $\frac{1}{3}$, etc.

Exercise LI.

1. Find the interest on $400 for 90 days at 6%; at 3%; at 9%; at7%; at 5% ; at 8%.
2. Find the interest on $640 for 16 mos. at 6%.
3. Find the interest on $80 for 1 year, 5 mos. at 6%.
4. Find the interest on $560 for 240 days at 6%; at 5%; at 4%.
5. Principal $320, amount $480, rate per cent. 5. Find time.
6. At what rate per cent. per annum will the interest on $700 be $70 in 15 mos.?

7. In how many years will \$480 give \$144 at 3% per annum?

8. In what time will the interest be $\frac{7}{50}$ of the principal at 4% per annum?

9. A certain sum of money earns \$42 interest in $1\frac{1}{2}$ yrs. at 8% per annum. Find the sum.

10. A note for \$550 bearing interest at 8% per annum is paid in 9 mos. How much money is required to pay it?

11. Find the interest on \$560 for 3 yrs. 4 mos. at 6% per annum.

12. A certain sum in 18 mos. amounts to \$560 at 8% per annum. Find the sum.

13. What principal amounts to \$530 in 9 mos. at 8% per annum.

14. For the use of \$660 for 8 mos. I pay \$22. What is the rate per cent. per annum?

15. How much money at $3\frac{1}{2}$% per annum will give an annual interest of \$1470?

INTEREST AND DISCOUNT.

Exercise LII.

Find the time in years and months from:—

1. January 4, 1907, to September 4, 1909.

2. June 6, 1905, to August 6, 1908.

3. July 10, 1908, to April 10, 1911.

Find the time in days from:—

4. June 8, 1908, to October 15, 1908.

5. December 5, 1909, to March 20, 1910.

6. August 16, 1910, to January 5, 1911.

7. Express as the fraction of a year 3 mos.; 2 mos.; 6 mos.; 8 mos.; 9 mos.; 15 mos.; 16 mos.; 18 mos.; 20 mos.; 7 mos.; 4 mos.; 21 mos.; and 5 mos.

8. Find the interest on \$1500 at 4% from:—

(a) May 10, 1909, to September 10, 1909.
(b) October 5, 1908, to March 5, 1909.
(c) November 12, 1910, to July 12, 1912.

9. Find date of maturity allowing 3 days grace of a note for 90 days drawn on:—

(a) January 10, 1908.
(b) September 15, 1909.
(c) December 20, 1907.

10. Find date of maturity allowing 3 days grace of a note for 3 months drawn on:—

(a) August 6, 1907.
(b) January 10, 1908.
(c) November 15, 1910.
(d) December 31, 1910.

11. A note drawn on May 15, 1910, for 3 months was discounted at the bank on June 24th. For how many days was the discount reckoned?

12. A note bearing interest is discounted at the bank, how is the amount found on which discount is reckoned.

13. Find the bank discount on a $600 note:—

(a) Due March 21, 1907, discounted January 21, 1907, at 6%.
(b) Due January 16, 1911, discounted September 16, 1910, at 8%.
(c) Due October 20, 1910, discounted August 20, 1910, at 7%.

REVIEW EXERCISE B.

1. Find the cost of 19 lb. 14 oz. of butter at 32 cts. a pound.
2. Find the cost of 17 doz. and 9 eggs at 16 cts. a dozen.
3. Find the cost of 25 mi. 240 rd. of ditching at $32 per mile.
4. Find the cost of 9 lb. 10 oz. of tea at 40 cts. a pound.
5. Find the interest on $80 for 90 days at 6%.
6. At $15.00 per thousand feet, find the cost of a stick of timber which is 20 ft. long, 8 in. wide, 6 in. thick.
7. A merchant bought cotton at 12½ cts. per yard and sold it at a gain of 20%. How much did he receive for 18 yards.
8. Find the cost of 41 articles at 51 cts. each.
9. How many pounds of butter at 25 cts. a pound will pay for 35 yds. of cloth at 55 cts. per yard.

10. Find the cost of 17½ lb. of beef at 13½ cts. per pound.

11. 15×24.

12. $18\frac{1}{2} \times 14$.

13. Find the cost of 1250 lb. of hay at $12 per ton.

14. Find the cost of 1600 lb. of hay at $9 per ton.

15. Find the cost of 1325 lb. of hay at $8 per ton.

16. Find the cost of 2500 lb. of hay at $8.40 per ton.

17. Find the cost of 350 lb. of hay at $10 per ton.

18. Find the cost of 24 yds. of cloth at 7s. 6d. per yard.

19. Find the cost of 36 yds. of cloth at 6s. 8d. per yard.

20. Find the cost of 25 articles at 12s. 6 d. each.

21. Find the cost of a rectangular field 80 rd. long 30 rd. wide, at $17 per acre.

22. Find the cost of 7 yd. 2 ft. 6 in. of ribbon at 12 cts. per yard.

23. Find the cost of 48 lb. of beef at 12½ cts. per pound.

24. Find the cost of 34 books at 19 cts. each, at 21 cts. each, at 18 cts. each, at 22 cts. each.

25. Find the simple interest on $85 for 2 yrs. 6 mos. at 6%.

26. Find the cost of 37½ articles at 28 cts. each.

27. How many furrows each 10 in. wide must be turned in plowing a field which is 10 rods wide.

28. A number is larger by 37 than ⅞ of it. Find the number.

29. A grocer made $18 by selling goods at a gain of 16%. Find the cost.

30. Eggs sold at 18 cts. per dozen net a profit of 20%; find the cost price per dozen.

31. At 75 cts. per yard find the cost of carpeting a room 18 ft. long, 15 ft. wide, with carpet 1 yd. wide.

32. Find the cost of 165 articles at $1.75 each.

33. Find the cost of 36 yds. of carpet at £1 6s. 8d. per yard.

34. Find the interest on $240 for 120 days at 5%. (Interest at 5% = int. at 6% $-\frac{1}{6}$ of int. at 6%.)

35. Find the interest on $160 for 180 days at 8%. (Int. at 8% = int. at 6% + ⅓ of int. at 6%).

36. If 10 men can do a piece of work in 3 days, how many men can do the same in one day? In how many days can one man do it?

37. A can do a piece of work in 5 days and B can do the same in 6 days, how long will it take them working together.

38. Find the sum of all numbers from 1 to 24 inclusive.

39. How many times does a clock strike during a day.

40. At 15 cts. per sq. yd. find the cost of painting each of the following ceilings:—

(a)	12 ft. × 15 ft.	(e)	36 ft. × 27 ft.
(b)	15 ft. × 18 ft.	(f)	24 ft. × 30 ft.
(c)	21 ft. × 18 ft.	(g)	15 ft. × 24 ft.
(d)	21 ft. × 24 ft.	(h)	9 ft. × 15 ft.

41. How many mats each 3 ft. 6 in. long, 2 ft. wide, will cover a floor which is 21 ft. long, 16 ft. wide.

42. 40 is 40% of what number?

43. 75 is 125% of what number?

44. Increase 78 by $33\frac{1}{3}\%$ of itself.

45. Cotton sold at a discount of 15% brings 17 cts. per yard. What was the first selling price?

46. Bought goods at 10% below the list price and sold them at 20% above list price. Find gain per cent.

47. Bought butter at 22 cts. per pound and sold it at 24 cts. per pound. Find gain per cent.

48. By selling books at \$1.40 each I lost 30%. Find cost.

49. By selling oats at 54 cts. per bushel I gain $12\frac{1}{2}\%$. Find cost.

50. Divide $12\frac{3}{4}$ lb. sugar into two parcels one of which will be $2\frac{1}{2}$ lb. heavier than the other.

51. What fraction must be added to the sum of $\frac{1}{3}$ and $\frac{1}{2}$ that their sum may be $\frac{11}{12}$?

52. If a horse eats 12 qts. of oats a day, in how many days will he eat 12 bushel.

53. At 12 cts. per sq. ft. find the cost of a piece of rubber which is $14\frac{1}{2}$ ft. long, $6\frac{1}{2}$ ft. wide.

54. Find the cost of 25 articles at each of the following prices per article:—42 cts., 75 cts., 60 cts., 89 cts., 149 cts., 333 cts.

55. At 5 mi. per hour how far can a man walk in 40 days of 7 hrs. each.

56. $(64 - 27) \quad (56 - 15)$.

57. If 20 men can do a piece of work in 6 dys., in how many days can 24 men do the same?

58. If A can do a piece of work in $3\frac{1}{2}$ days and B can do the same in $2\frac{1}{2}$ days, how long will it take the two together?

59. Find the cost of 67 articles at 73 cts. each.

60. Find the cost of 43 yds. of cloth at 57 cts. per yard.

61. Find the cost of 36 articles at 29 cts. each.

62. A merchant bought goods for $600. He sold $\frac{1}{3}$ of them at a gain of 15% and the rest at a loss of 25%. Find his gain or loss on the whole transaction.

63. A merchant sold tea at 54 cts. a pound thereby gaining 35%. Find the cost per pound.

64. A farmer plows a field 8 rds. wide, 40 rds. long in a day. If each furrow is 8 in. wide, how many miles does he walk?

65. If a clock ticks once each second, how often does it tick in an hour? A day? A week?

66. Bought 640 oranges at the rate of 8 for 10 cts. and sold them at the rate of 5 for 8 cts. Find gain.

67. Find the cost of 150 eggs at the rate of 22 cts. per doz. at 24 cts. per doz., at 30 cts. per doz.

68. How many eggs at 25 cts. per doz. will pay for $37\frac{1}{2}$ yds. of carpet at 48 cts. per yard.

69. A farmer raised oats on 36 acres. The crop averaged 27 bus. to the acre. How many bushels did he raise?

70. A man works 10 hrs. per day for 3 weeks (except Sundays) at 15 cts. per hour. How much does he earn?

71. A merchant bought 75 bus. oats at 48 cts. a bus. and sold them at 14 cts. a peck. How much did he gain?

72. Find the interest on $60 from May 10th to August 8th at 6%; at 3%; at 4%; at 5%; at 7%.

NOTE.—Find interest at 6% by last part of Rule LIII and for the interest at the other rates increase or diminish the interest at 6% by the required fractional part, e. g., If the interest at 6% should be $3.60, then interest at 5% would be $\$3.60 - \frac{1}{6}$ of $3.60.

73. To what will $750 amount in 1 yr. 4 mos. at 6%; at 8%; at 10%?

74. $7\frac{3}{4}$ is how much more than 5?

75. How many cubes $\frac{1}{2}$ in. each way can be cut from a cube 2 in. each way?

76. If I buy a barrel of flour for $5.85 and give in payment a ten dollar bill, now much should I get back?

77. How much would be left from $15 after paying for 48 books at 16 cts. each and $62\frac{1}{2}$ lb. sugar at 4 cts. a pound.

78. Find the superficial feet in,

(a) 24 pieces 15 ft. long 3 in. × 4 in.
(b) 21 " 14 ft. " 6 in. × 2 in.
(c) 16 " 18 ft. " 6 in. × 2 in.
(d) 8 " 12 ft. " 2½ in. × 2½ in.
(e) 15 " 18 ft. " 6 in. × 3 in.
(f) 14 " 16 ft. " 8 in. × 3 in.

79. Find the cost of 4 lb. 4 oz. of butter at 20 cts. per pound, at 23 cts. per pound, at 25 cts. per pound.

80. Find the cost of 15 eggs at 18 cts. a dozen, at 16 cts. a dozen, at 20 cts. a dozen.

81. Find the cost of 9½ yds. cotton at 8½ cts. per yard.

82. A book of 80 pages has 30 lines on each page and the average number of words in a line is 9. How many words are there in the book?

83. How many ¼ lb. packages of tea can be made from a package weighing 3½ lb.?

84. Find the cost of ¾ yd. cotton at 16 cts. per yard, at 18 cts. per yard, at 21 cts. per yard.

85. How many superficial feet of lumber which is one inch thick will be required for a floor which is,

(a) 45 ft. long 35 ft. wide?
(b) 62½ ft. " 48 ft. "
(c) 12 ft. 6 in. long 16 ft. 6 in. wide?
(d) 18 ft. " 17 ft. "
(e) 27 ft. " 26 ft. "
(f) 37 ft. 6 in. " 32 ft. "

86. How many bus. oats at 50 cts. per bus. will pay for 75 books at $1.20 each?

87. 4275 ÷ 50.

88. 3145 ÷ 25.

89. 325 ÷ 12½.

90. 7 × 7 × 8 × 7,

91. 128 — 45.

92. 164 — 86.

93. How many strips of carpet which is 30 inches wide will reach across a floor 15 ft. wide?

94. If the room in Question No. 93 is 18 ft. long, how many yards of the carpet will cover the floor?

95. How many rolls of paper 18 in. wide, 8 yds. long will cover a wall 18 ft. by 8 ft?

96. Find the cost of 99 articles at 63 cts. each.

97. 462×999.

98. Find $62\frac{1}{2}\%$ of 488.

99. Find $87\frac{1}{2}\%$ of 640.

100 A farmer insures his buildings which he values at $4500 for two-thirds their value at $\frac{3}{4}$ per cent. What premium does he pay?

Review Exercise C.

1. Subtract 7 from all numbers from 8 to 100, but not in consecutive order.

2. Deal with the other digits in the same way.

3. Find the total cost of the following articles:—

9 doz. and 6 eggs at 18 cts. per dozen.
9 lb. 8 oz. of butter at 22 cts. per lb.
7 bus. 2 pks. of oats at 40 cts. per bus.

Note.—From the cost of 10 doz. subtract the cost of $\frac{1}{2}$ doz. and you have the cost of $9\frac{1}{2}$ doz. e. g., $9\frac{1}{2}$ doz. at 18 cts $= 9\frac{1}{2} \times 18 = 10 \times 18 - \frac{1}{2}(18) = 180 - 9 = 171$. In the same way deal with the other parts of the question.

4. Find the cost of $14\frac{3}{4}$ lb. of tea at 30 cts. per pound.

5. What is the gain or loss per cent. if the cost is:—

(a) 25 cts. and the selling price $37\frac{1}{2}$ cts.
(b) $4.50 " " " $5.00
(c) $2.50 " " " $4.50
(d) $1.25 " " " $2.50
(e) $6.50 " " " $3.25
(f) $1.50 " " " $1.00

6. A merchant bought 120 bus. oats at 40 cts. a bushel and sold them at a loss of 15%. How much did he lose?

7. Bought goods for $360 and sold at a gain of 35%. Find gain.

8. Find 62½% of 720, of 640, of 136.

9. How many pupils are in a school if $\frac{3}{4}$ are boys and 28 are girls.

10. How many parcels of ¼ lb. each can be made from 7½ lb.?

11. A man paid a debt in 3 payments. At first he paid ⅓ of it, then ¼ of it, then $43. Find the debt and the amount of each of the payments.

12. A rectangular field is 60 rd. long, 40 rd. wide; how many ft. of inch boards will be required for a tight fence 5½ ft. high around it.

13. At $18 per acre find the cost of a rectangular field which is 80 rd. long, 36 rd. wide.

14. 7 fur. 20 rd. is what fraction of a mile?

15. 2 ft. 3 in. is what fraction of a yard?

16. How many inches in a rod.

17. How many seconds in a day.

18. How many hours in a year.

19. How many minutes in a week.

20. Find the cost of a foot of lumber at $8 per M, at $10 per M, at $15 per M, at $13 per M.

21. Find the cost of a pound of hay at $10 per ton, at $8 per ton, at $12 per ton.

22. How many cubic feet of air in a room 18 ft. long, 15 ft. wide, 10 ft. high?

23. How many rectangles 5 in. by 12 in. can be cut from one 180 in. by 84 in.?

24. How many times will 15¼ contain $5\frac{1}{2}$?

25. If a cubic yard of earth makes a load, how many loads must be taken from a cellar 24 ft. long, 21 ft. wide, 6 ft. deep?

26. A train travels 2 miles in 3 minutes, how far can it go in 4 hours?

27. Find the cost of 63 articles at 98 cts. each.

28. Find the cost of 36 articles at 58 cts. each.

29. Find the cost of 126 articles at 43 cts. each.

30. Find the cost of 105 articles at 79 cts. each.

31. Find the cost of 147 articles at 85 cts. each.

32. Find the cost of 189 articles at 88 cts. each.

33. Find the cost of 168 articles at 68 cts. each.

34. Find the cost of 42 articles at 96 cts. each.

35. Find the cost of 84 articles at 66 cts. each.

36. Find the cost of 33 articles at 84 cts. each.

37. Find the cost of 44 articles at 75 cts. each.

38. Find the cost of 66 articles at 72 cts. each.

39. Find the cost of 38 articles at 42 cts. each.

40. Find the cost of 47 articles at 53 cts. each.

41. Bought 150¾ lbs. butter at 20 cts. a pound. Sold 120¼ at 24 cts. a pound and the rest at 20 cts. a pound. Find the gain.

42. Find the cost of 161 articles at $1.81 each.

43. Reduce 4 rods to feet.

44. Memorize the number of rods, yards, feet, and inches in a mile.

45. A house worth $3000 is rented so as to give 5% on its value. Find the rate of rent per month?

46. How many steps each 2 ft. 6 in. long must be taken in walking a mile?

47. A field 40 rods wide contains 10 acres, how long is it?

48. A pile of wood 40 ft. long 4 ft. wide contains 3600 cu. ft. Find its height.

49. At the rate of 3 for 7 cts. find the cost of 624 oranges.

50. A man paid $\frac{2}{3}$ of his money for a horse, $\frac{1}{4}$ of the remainder for a carriage, and had $80 left. What had he at first, and what did horse and carriage cost respectively.