IMAGE EVALUATION TEST TARGET (MT-3)

6"

Photographic Sciences Corporation

23 WEST MAIN STREET
WEBSTER, N.Y. 14580
(716) 872-4503

Technical and Bibliographic Notes/Notes techniques et bibliographiques

The Institute has attempted to obtain the best original copy available for filming. Features of this copy which may be bibliographically unique, which may alter any of the images in the reproduction, or which may significantly change the usual method of filming, are checked below.

- [x] Coloured covers/ Couverture de couleur
- [ ] Covers damaged/ Couverture endommagée
- [ ] Covers restored and/or laminated/ Couverture restaurée et/ou pelliculée
- [ ] Cover title missing/ Le titre de couverture manque
- [ ] Coloured maps/ Cartes géographiques en couleur
- [ ] Coloured ink (i.e. other than blue or black)/ Encre de couleur (i.e. autre que bleue ou noire)
- [ ] Coloured plates and/or illustrations/ Planches et/ou illustrations en couleur
- [ ] Bound with other material/ Relié avec d'autres documents
- [ ] Tight binding may cause shadows or distortion along interior margin/ La reliure serrée peut causer de l'ombre ou de la distorsion le long de la marge intérieure
- [ ] Blank leaves added during restoration may appear within the text. Whenever possible, these have been omitted from filming/ Il se peut que certaines pages blanches ajoutées lors d'une restauration apparaissent dans le texte, mais, lorsque cela était possible, ces pages n'ont pas été filmées.
- [ ] Additional comments:/ Commentaires supplémentaires:

L'Institut a microfilmé le meilleur exemplaire qu'il lui a été possible de se procurer. Les détails de cet exemplaire qui sont peut-être uniques du point de vue bibliographique, qui peuvent modifier une image reproduite, ou qui peuvent exiger une modification dans la méthode normale de filmage sont indiqués ci-dessous.

- [ ] Coloured pages/ Pages de couleur
- [ ] Pages damaged/ Pages endommagées
- [ ] Pages restored and/or laminated/ Pages restaurées et/ou pelliculées
- [x] Pages discoloured, stained or foxed/ Pages décolorées, tachetées ou piquées
- [ ] Pages detached/ Pages détachées
- [x] Showthrough/ Transparence
- [ ] Quality of print varies/ Qualité inégale de l'impression
- [ ] Includes supplementary material/ Comprend du matériel supplémentaire
- [ ] Only edition available/ Seule édition disponible
- [ ] Pages wholly or partially obscured by errata slips, tissues, etc., have been refilmed to ensure the best possible image/ Les pages totalement ou partiellement obscurcies par un feuillet d'errata, une pelure, etc., ont été filmées à nouveau de façon à obtenir la meilleure image possible.

This item is filmed at the reduction ratio checked below/
Ce document est filmé au taux de réduction indiqué ci-dessous.

10X	12X	14X	16X	18X	20X	22X	24X	26X	28X	30X	32X
				✓							

The copy filmed here has been reproduced thanks to the generosity of:

Bibliothèque nationale du Québec

The images appearing here are the best quality possible considering the condition and legibility of the original copy and in keeping with the filming contract specifications.

Original copies in printed paper covers are filmed beginning with the front cover and ending on the last page with a printed or illustrated impression, or the back cover when appropriate. All other original copies are filmed beginning on the first page with a printed or illustrated impression, and ending on the last page with a printed or illustrated impression.

The last recorded frame on each microfiche shall contain the symbol —► (meaning "CONTINUED"), or the symbol ▼ (meaning "END"), whichever applies.

Maps, plates, charts, etc., may be filmed at different reduction ratios. Those too large to be entirely included in one exposure are filmed beginning in the upper left hand corner, left to right and top to bottom, as many frames as required. The following diagrams illustrate the method:

L'exemplaire filmé fut reproduit grâce à la générosité de:

Bibliothèque nationale du Québec

Les images suivantes ont été reproduites avec le plus grand soin, compte tenu de la condition et de la netteté de l'exemplaire filmé, et en conformité avec les conditions du contrat de filmage.

Les exemplaires originaux dont la couverture en papier est imprimée sont filmés en commençant par le premier plat et en terminant soit par la dernière page qui comporte une empreinte d'impression ou d'illustration, soit par le second plat, selon le cas. Tous les autres exemplaires originaux sont filmés en commençant par la première page qui comporte une empreinte d'impression ou d'illustration et en terminant par la dernière page qui comporte une telle empreinte.

Un des symboles suivants apparaîtra sur la dernière image de chaque microfiche, selon le cas: le symbole —► signifie "A SUIVRE", le symbole ▼ signifie "FIN".

Les cartes, planches, tableaux, etc., peuvent être filmés à des taux de réduction différents. Lorsque le document est trop grand pour être reproduit en un seul cliché, il est filmé à partir de l'angle supérieur gauche, de gauche à droite, et de haut en bas, en prenant le nombre d'images nécessaire. Les diagrammes suivants illustrent la méthode.

1	2	3

1
2
3

1	2	3
4	5	6

ALGEBRA

APPLIED TO

Practical Geometry,

OR

Algebraic Solutions of Geometrical Problems,

FOR

BEGINNERS:

BY REV. L. P. PAQUIN, O.M.I.,
Professor of Civil Engineering in the College of Ottawa.

OTTAWA, CANADA:
PRINTED BY JOSEPH LOVEDAY.
1876.

Th
to
Al
an
tic
res
pla
the
cas
lan
T
exp
ces
cor
the
Fo
Inv
I
the
un
fig
spe
C
cn
hop
tio

PREFACE.

THE present little work is intended to have a twofold object—to furnish the young Student with useful exercises in applying Algebra to Geometry, and to put within his reach an easy and ready means of obtaining the Solution of the most Practical Problems of Elementary Geometry, especially of those respecting the construction, measurement and division of plane figures. A good many of the Formulas presented in the following pages, may be used advantageously in ordinary cases of Land Surveying, as the laying out, the dividing of lands, and the calculating of their content.

The Author having in mind not to present an extensive and expensive book on the subject, has omitted to insert the process of operations which brought him to the General Formulas contained therein. His doing so affords moreover the Student the advantage of verifying by himself the correctness of these Formulas, and therefore of exercising himself in Algebraic Investigations.

No Geometrical Figures are drawn; but the Problems and their Solutions are given with sufficient clearness to be plainly understood by the Student, who may very easily construct the figure corresponding to the given Problems, according to the special cases.

On account of the practical nature of this little book on the one hand, and its cheapness on the other, the Author is in hopes that it will be received and found convenient in Institutions where Mathemathical Sciences are taught.

L.P.P.

W
lin
th
Bo
ar
ve
bo

(

B

An
phi

Algebra Applied To Practical Geometry.

INTRODUCTION.

I.

When the relative positions of points connected by straight lines are to be indicated, then the connecting lines are, for the sake of convenience, represented by two letters, AD, BC . . . ; but when the relations of the magnitude of lines are only under consideration, then these lines are more conveniently represented by a single letter, or an algebraic symbol, a, b, c . . .

Let a and b represent two given lines, then

$a + b$ expresses their sum,

$a - b$ expresses their difference,

a^2 expresses a square constructed on the line a,

$(a + b)^2$ expresses a square constructed on the sum of these two lines,

ab expresses a rectangle whose a is the base, and b the height, or *vice versâ*,

$\frac{ab}{2}$ expresses a triangle, whose a is the base, and b the height.

$\frac{p(m+n)}{2}$ expresses a trapezoid, whose p is the height; m and n the two bases.

II.

Any linear problem may be solved either numerically or graphically; and in both cases an algebraic expression of the quantity sought furnishes the student with an easy and ready means of obtaining the solution, inasmuch as it indicates either the arithmetical operations, or the graphical construction to be performed.

When a problem is to be solved numerically, any expression, in which the quantity sought is put in terms of known quantities, will suit; should it be solved graphically, then the expression must be reduced to one of the five following formulas:—

(1) $x = a + b - c$ ..... *Sum and difference of lines.*

(2) $x = \frac{bc}{a}$ .... Fourth proportional.

(3) $x = \sqrt{ab}$ .... Mean proportional.

(4) $x = \sqrt{a^2 + b^2}$ ...... Hypotenuse.

(5) $x = \sqrt{a^2 - b^2}$ ...... Side of a right angle.

III.

The first formula is evident by itself.

When $x = \frac{bc}{a}$, then it is a fourth proportional to a, b, c, because if both members of the equation are multiplied by a, we obtain

$$a x = b c$$

$$\text{Hence,} \quad a : b :: c : x$$

Hence, in general, any fraction containing two factors of the first degree in the numerator, and one in the denominator, will express a *fourth proportional.*

When $x = \sqrt{ab}$, it is a mean proportional between a and b, because, if both members of the equation are raised to the second power, we obtain

$$x^2 = a b$$

$$\text{Hence,} \quad a : x :: x : b$$

Hence, in general, any expression, being a radical of the second degree including two factors of the first degree, will represent a *mean proportional.*

When $x = \sqrt{a^2 + b^2}$, an hypotenuse is to be drawn on a and b put at right angle, because, if both members of the equation are raised to the second power, we obtain

$$x^2 = a^2 + b^2$$

The fifth formula is proved in the same way.

N.B.—The student is supposed to know how to draw a fourth proportional to three given lines, a mean proportional between two given lines, etc.

IV.

Let it be illustrated by a few examples, that any expression may easily be reduced to one of the five preceding formulas.

Ex. 1. Let $x = \dfrac{a\,b\,c}{r\,s}$

then $x = \dfrac{a\,b}{r} \times \dfrac{c}{s}$ by decomposing.

Let the *fourth proportional* expressed by $\dfrac{a\,b}{r}$ be drawn and represented by y, then

$$x = y \times \frac{c}{s} = \frac{y\,c}{s} \text{ (2nd formula).}$$

Ex. 2. Let $x = \dfrac{a^3 + b^2\,c + d\,h\,m}{p^2 + q^2}$

Let an hypotenuse be drawn on p and q, and represented by y, then

$$x = \frac{a^3 + b^2\,c + d\,h\,m}{y^2} =$$

$$= \frac{a^3}{y^2} + \frac{b^2\,c}{y^2} + \frac{dhm}{y^2} = \frac{a\,a\,a}{y\,y} + \frac{b\,b\,c}{y\,y} + \frac{d\,h\,m}{y\,y}$$

Let each of these last three terms be reduced as in the Ex. 1, and the three lines found be represented respectively by u, v, z, then

$$x = u + v + z \text{ (1st formula).}$$

Ex. 3. Let $x = \sqrt{3\,a^2}$

then $x = \sqrt{3\,a \times a}$ (3rd formula).

Ex. 4. Let $x = \sqrt{\frac{3\,a^2}{5}}$

then $x = \sqrt{a \times \frac{3\,a}{5}}$ (3rd formula).

It is evident that in this last case, the line a should be divided into five equal parts, and a mean proportional drawn between the whole line and three of the equal parts.

Ex. 5. Let $x = \sqrt{\frac{a\,b\,c}{e}}$

then $x = \sqrt{\frac{a\,b}{e} \times c}$ by decomposing.

making $\frac{a\,b}{e} = y$ (by formula 2nd).

then $x = \sqrt{y\,c}$ (3rd formula).

Ex. 6. Let $x = \sqrt{a^2 + b^2 + c^2 + d^2}$

Let an hypotenuse y be drawn on a and b, and substituted in the expression, then

$$x = \sqrt{y^2 + c^2 + d^2}$$

making $y^2 + c^2 = z^2$, in the same way, then

$$x = \sqrt{z^2 + d^2} \text{ (4th formula).}$$

ALGEBRAIC SOLUTIONS OF PROBLEMS.

[In the following pages, the letters N. S., put after a formula, stand for *Numerical Solution*; and the letters G. S. stand for *Graphical Solution*.]

PROBLEM I.

To find the side x of a square x^2, whose area shall be equivalent to twice that of a given square a^2.

$$x = \sqrt{2\,a^2} \qquad \text{N. S.}$$
$$= \sqrt{2\,a \times a} \qquad \text{G. S.}$$

PROBLEM II.

To find the side x of a square x^2, whose area shall be equivalent to five times that of a given square a^2.

$$x = \sqrt{5\,a^2} \qquad \text{N. S.}$$
$$= \sqrt{5\,a \times a} \qquad \text{G. S.}$$

PROBLEM III.

To find the side x of a square x^2, whose area shall be equivalent to one-fifth of that of a given square a^2.

$$x = \sqrt{\frac{a^2}{5}} \qquad \text{N. S.}$$
$$= \sqrt{a \times \frac{a}{5}} \qquad \text{G. S.}$$

PROBLEM IV.

To find the side x of a square x^2, whose area exceeds that of a given square a^2, by the three-eighths of it.

$$x = \sqrt{\frac{11\,a^2}{8}} \qquad \text{N. S.}$$
$$= \sqrt{\frac{11\,a}{8} \times a} \qquad \text{G. S.}$$

PROBLEM V.

To find the side x of a square x^2, whose area shall be equivalent to that of a given square a^2 less by the two-sevenths of it.

$$x = \sqrt{\frac{5\ a^2}{7}} \qquad \text{N. S.}$$

$$= \sqrt{\frac{5\ a}{7} \times a} \qquad \text{G. S.}$$

PROBLEM VI.

To find the side x of a square x^2, whose area shall be to that of a given square a^2, in the ratio of $m : n$.

$$x = \sqrt{\frac{m\ a^2}{n}} \qquad \text{N. S.}$$

$$= \sqrt{\frac{m\ a}{n} \times a}$$

$$= \sqrt{y\ a} \quad \text{G. S. (See Intro. IV., Ex. 5.)}$$

PROBLEM VII.

To find the side x of a square x^2, equivalent to a given rectangle $a\,b$.

$$x = \sqrt{a\ b} \qquad \text{N. and G. S.}$$

PROBLEM VIII.

To find the side x of a square x^2, equivalent to a given triangle $\frac{a\ h}{2}$

$$x = \sqrt{\frac{a\ h}{2}} \qquad \text{N. S.}$$

$$= \sqrt{a \times \frac{h}{2}} \qquad \text{G. S.}$$

PROBLEM IX.

To find the the side x of a square x^2, equivalent to a given trapezoid $\frac{p\ (m + n)}{2}$

$$x = \sqrt{\frac{p\,(m+n)}{2}} \quad \text{N. S.}$$

$$= \sqrt{p \times \frac{m+n}{2}}$$

make $\frac{m+n}{2} = y$ by drawing a line parallel to the bases, at equal distance from each; then

$$x = \sqrt{p\,y} \qquad \text{G. S.}$$

PROBLEM X.

To find the side x of a square x^2, equivalent to a given regular polygon.

Let $c =$ the side of the polygon; $a =$ the number of sides; and $r =$ the apothem; then

$$x = \sqrt{\frac{a\,c\,r}{2}} \qquad \text{N. S.}$$

$$= \sqrt{\frac{a}{2}\,c \times r} \qquad \text{G. S.}$$

N.B.—a being a given number, then $\frac{a}{2}\,c$ is only one literal factor, and we have the 3rd formula.

PROBLEM XI.

To find the side x of a square x^2, equivalent to a given triangle $\frac{a\,h}{2}$, rectangle $b\,d$, trapezoid $\frac{p\,(m+n)}{2}$, and regular pentagon $\frac{5\,c\,r}{2}$

$$x = \sqrt{\frac{a\,h}{2} + b\,d + p\,\frac{(m+n)}{2} + \frac{5\,c\,r}{2}} \quad \text{N. S.}$$

Transform the four expressions which are under the radical, into equivalent squares, y^2, y'^2, y''^2, y'''^2, by the Problems VII., VIII., IX., X., then

$$x = \sqrt{y^2 + y'^2 + y''^2 + y'''^2}$$

make $y^2 + y'^2 = z^2$ (See Intro. IV., Ex. 6) then

$$x = \sqrt{z^2 + y''^2 + y'''^2}$$

making $z^2 + y''^2 = z'^2$, then

$$x = \sqrt{z'^2 + y'''^2} \qquad \text{G. S.}$$

PROBLEM XII.

To find the side x of a square x^2, equivalent to any irregular polygon.

Divide the polygon into triangles by drawing diagonals; let these triangular parts be calculated separately, and represented respectively by P, Q, R, S......... ; then

$$x = \sqrt{P + Q + R + S...} \quad \text{N. S.}$$
$$= \sqrt{y^2 + y'^2 + y''^2 + y'''^2} \quad \text{(See Prob. XI.)}$$
$$= \sqrt{z^2 + y'' + y'''^2}$$
$$= \sqrt{z'^2 + y'''^2} \quad \text{G. S.}$$

PROBLEM XIII.

To find the side x of a square x^2, equivalent to a given circle.

$$x = \sqrt{\pi R^2} \quad \text{N. S.}$$
$$= \sqrt{3 R \times R} \quad \text{G. S.}$$

N.B.—If a very exact graphical construction is required, the line R should be taken 3.14 times, instead of 3 times.

PROBLEM XIV.

To find the side x of a square x^2, which is a mean proportional between any two given polygons P and R.

$$x = \sqrt[4]{P \times R}$$

transforming the two polygons P and R into equivalent squares, y^2, z^2 then

$$x = \sqrt[4]{y^2, \times z^2} = \sqrt{y z} \quad \text{N. and G. S.}$$

PROBLEM XV.

To construct a triangle $\frac{c x}{2}$ on a given base c, equivalent to a given square a^2.

$$\text{Altitude } x = \frac{2 a^2}{c} \quad \text{N. S.}$$
$$= \frac{2 a \times a}{c} \quad \text{G. S.}$$

PROBLEM XVI.

To construct a rectangle $c\,x$ on a given base c, equivalent to a given square a^2.

$$\text{Altitude } x = \frac{a^2}{c} \qquad \text{N. S.}$$

$$= \frac{a \times a}{c} \qquad \text{G. S.}$$

PROBLEM XVII.

To construct a trapezoid $\frac{p\,(m+n)}{2}$ equivalent to a given square a^2, having given the two bases, m and n.

$$\text{Altitude } p = \frac{a^2}{\frac{1}{2}\,(m+n)} \qquad \text{N. S.}$$

$$= \frac{a^2}{y} \qquad \text{(See Problem IX.)}$$

$$= \frac{a \times a}{y} \qquad \text{G. S.}$$

PROBLEM XVIII.

To construct a trapezoid $\frac{p\,(m+n)}{2}$ equivalent to a given square a^2, having given one of the bases m, and the altitude p.

$$\text{the other base } n = \frac{a^2 - \frac{1}{2}\,p\,m}{\frac{1}{2}\,p} \qquad \text{N. S.}$$

make $\frac{1}{2}\,p\,m = y^2$ by Problem VIII., then

$$n = \frac{a^2 - y^2}{\frac{1}{2}p}$$

make $a^2 - y^2 = z^2$ by the 5th formula, then

$$n = \frac{z^2}{\frac{1}{2}p} = \frac{z \times z}{\frac{1}{2}p} \qquad \text{G. S.}$$

PROBLEM XIX.

To construct a regular polygon $\frac{b\,c\,r}{2}$, equivalent to a given square a^2, having given the side c, and the number of sides b.

$$\text{Apothem } r = \frac{2\,a^2}{b\,c} \qquad \text{N. S.}$$

$$= \frac{2\,a \times a}{b\,c} \qquad \text{G. S.}$$

N.B.—The factor b being a number, then $b\,c$ is only one literal factor, viz., the line c taken b times. The radius of the circumscribed circle is evidently

$$R = \sqrt{r^2 + \frac{c^2}{4}} \qquad \text{N. and G. S.}$$

PROBLEM XX.

To construct a circle equivalent to a given square a^2.

$$\text{Radius} = \sqrt{\frac{a^2}{\pi}} \qquad \text{N. S.}$$

$$= \sqrt{\frac{a}{3} \times a} \qquad \text{G. S.} \left\{ \begin{array}{l} \text{(See N. B.} \\ \text{Prob. XIII.)} \end{array} \right.$$

PROBLEM XXI.

To transform a triangle $\frac{a\,h}{2}$ into a rectangle $b\,x$, having given the base b.

$$\text{Altitude } x = \frac{a\,h}{2\,b} \qquad \text{N. and G. S.}$$

PROBLEM XXII.

To transform an irregular polygon into a triangle $\frac{a\,h}{2}$ having given the base a.

Let P, Q, R, S...... be the triangular parts of the polygon, and x^2 a square equivalent to their sum; find x by the Problem XII., and then

$$\text{Altitude } h = \frac{2\,x^2}{a} \qquad \text{N. S.}$$

$$= \frac{2\,x \times x}{a} \qquad \text{G. S.}$$

By a similar process, any polygon or any number of different polygons may be easily and readily transformed into an equivalent one of any kind.

PROBLEM XXIII.

To construct a square x^2, having given the excess a, of the diagonal over the side x.

$$x = a + \sqrt{2\,a^2} \qquad \text{N. S.}$$

$$= a + \sqrt{2\,a \times a}$$

making $\sqrt{2\,a \times a} = y$ by the 3rd formula, then

$$x = a + y \qquad \text{G. S.}$$

PROBLEM XXIV.

To construct a rectangle $x\,y$, having given the perimeter $b = 2\,x + 2\,y$, and the surface $S = x\,y$.

Let $a =$ the side of an equivalent square, then

$$\text{the base } x = \frac{b}{4} + \sqrt{\frac{b^2}{16} - a^2} \qquad \text{N. S.}$$

making $\frac{b^2}{16} - a^2 = z^2$ by the 5th formula, then

$$x = \frac{b}{4} + \sqrt{z^2} = \frac{b}{4} + z \quad \text{G. S.}$$

It is evident that the height $y = \dfrac{b - 2\,x}{2}$

PROBLEM XXV.

To construct a rectangle, having given the surface $S = x\,y$, and the difference of the adjacent sides $x - y = d$.

Let $a =$ the side of an equivalent square; then

$$\text{the base } x = \frac{d}{2} + \sqrt{a^2 + \frac{d^2}{4}} \qquad \text{N. S.}$$

$$= \frac{d}{2} + \sqrt{z^2} \qquad \left\{ \text{(See Problem XXIV.)} \right.$$

$$= \frac{d}{2} + z \qquad \text{G. S.}$$

$$\text{the height } y = -\frac{d}{2} + \sqrt{a^2 + \frac{d^2}{4}}$$

PROBLEM XXVI.

To construct a parallelogram, having given the adjacent sides a, b, and the difference d of the two diagonals.

Let the greater diagonal $= x$

$$x = \frac{d}{2} + \sqrt{a^2 + b^2 - \frac{d^2}{4}} \quad \text{N. S.}$$

$$= \frac{d}{2} + \sqrt{y^2 - \frac{d^2}{4}} \qquad \text{(by formula 4)}$$

$$= \frac{d}{2} + z \ \text{(by formula 5)} \quad \text{G. S.}$$

PROBLEM XXVII.

To determine a Right-Angled Triangle, having given the hypotenuse a, and the sum of the two sides $b + c$.

$$b = \frac{b+c}{2} \pm \sqrt{\frac{a^2}{2} - \frac{(b+c)^2}{4}}$$

PROBLEM XXVIII.

To determine a Right-Angled Triangle, having given the base b, and the sum of the hypotenuse, and the other side $a + c$.

$$\text{side } c = \frac{(a+c)^2 - b^2}{2\,(a+c)}$$

PROBLEM XXIX.

To determine a Right-Angled Triangle, having given the base b, and the difference d between the hypotenuse a and the other side c; so that $a - c = d$.

$$c = \frac{b^2 - d^2}{2\,d} \qquad \text{N. S.}$$

make $b^2 - d^2 = y^2$ (by formula 5),

$$\text{then } c = \frac{y^2}{2\,d} = \frac{y \times y}{2\,d} \qquad \text{G. S.}$$

and then $a = d + c$.

PROBLEM XXX.

To determine a Right-Angled Triangle, having given the hypotenuse a, and the difference d between the two other sides b and c; so that $b - c = d$.

$$c = -\frac{d}{2} + \tfrac{1}{2}\sqrt{2\,a^2 - d^2} \qquad \text{N. S.}$$

make $\frac{1}{2}\sqrt{2\,a^2 - d^2} = x$ (by formula 5),

$$\text{then } c = x - \frac{d}{2} \qquad \text{G. S.}$$

and then $b = c + d$

PROBLEM XXXI.

To determine a triangle, having given the base b, the perpendicular h to the base from vertical angle, and the difference d between the two other sides a and c; so that $a - c = d$.

Let x = one of the segments determined on the base by the perpendicular; then

$$x = b + \sqrt{\frac{4\,h^2\,d^2}{b^2 - d^2} + d^2} \quad \text{N. S.}$$

make $b^2 - d^2 = y^2$ (by formula 5),

then $x = b + \sqrt{\frac{4\,h^2\,d^2}{y^2} + d^2} = b + \sqrt{\frac{4\,h \times h}{y} \times \frac{d \times d}{y} + d^2}$

make $\frac{4\,h \times h}{y} = t$; and $\frac{d \times d}{y} = s$ (by formula 2),

then $x = b + \sqrt{t\,s + d^2}$

$= b + \sqrt{r^2 + d^2}$ (See Problem VII.)

$= b + v$ (by formula 4). G. S.

N.B.—In this case, because the Graphical Solution requires many transformations, the numerical solution is much preferable.

PROBLEM XXXII.

Having given the three sides a, b, c, of a triangle, to find the radius r of the inscribed circle.

$$r = \frac{a\sqrt{b^2 - \frac{(a^2 + b^2 - c^2)^2}{4\,a^2}}}{a + b + c} \quad \text{N. S.}$$

PROBLEM XXXIII.

To determine a Right-Angled Triangle, having given the hypotenuse a, and the radius r of the inscribed circle.

Let b and c be the two other sides.

$$b = r + \frac{a}{2} + \tfrac{1}{2}\sqrt{a^2 - 4\,a\,r - 4\,r^2} \quad \text{N. S.}$$

$$c = r + \frac{a}{2} - \tfrac{1}{2}\sqrt{a^2 - 4\,a\,r - 4\,r^2} \quad \text{N. S.}$$

PROBLEM XXXIV.

Having given the two equal sides a of an Isosceles triangle, and the base b, to find the radius r of the inscribed circle.

$$r = \frac{\frac{b}{2}\sqrt{a^2 - \frac{b^2}{4}}}{a \times \frac{1}{2}\,b} \quad \text{N. S.}$$

PROBLEM XXXV.

Let the radius of the circle circumscribed to a polygon to be represented by R; the radius of the inscribed circle to be represented by r; and the surface of the polygon to be expressed by S.—Then, having given the three sides a, b, c of a triangle, and S, to find R and r.

$$R = \frac{a\,b\,c}{4S} \qquad \text{N. S.}$$

$$r = \frac{2\,S}{a+b+c} \qquad \text{N. S.}$$

PROBLEM XXXVI.

Having given the side a of an equilateral triangle, to find R, r and S (See Problem XXXV.).

$$R = \sqrt{\frac{a^2}{3}}$$

$$r = \sqrt{\frac{a^2}{3} - \frac{a^2}{4}}$$

$$S = \frac{a^2\sqrt{3}}{4}$$

PROBLEM XXXVII.

Having given the side a of a regular hexagon, find R, r and S (See Problem XXXV.).

$$R = a$$

$$r = \sqrt{a^2 - \frac{a^2}{4}}$$

$$S = \frac{3\,a^2\sqrt{3}}{2}$$

PROBLEM XXXVIII.

Having given the side a of a square, to find R and r (See XXXV.)

$$R = \sqrt{\frac{2\,a^2}{4}} \qquad \text{N. S.}$$

$$= \sqrt{a \times \frac{2\,a}{4}} \qquad \text{G. S.}$$

$$r = \sqrt{\frac{a^2}{4}} \qquad \text{N. S.}$$

$$= \sqrt{a \times \frac{a}{4}} \qquad \text{G. S.}$$

PROBLEM XXXIX.

Having given the side a of a regular pentagon, to find R, r and S (See Problem XXXV.).

$$R = \sqrt{\frac{a^2}{1.3816}}$$

$$r = \sqrt{\frac{a^2}{1.3816} - \frac{a^2}{4}}$$

$$S = \frac{5\ a}{2} \sqrt{\frac{a^2}{1.3816} - \frac{a^2}{4}}$$

PROBLEM XL.

Having given the side a of a regular octagon, to find R, r and S (See Problem XXXV.).

$$R = \sqrt{\frac{a^2}{.5858}}$$

$$r = \sqrt{\frac{a^2}{.5858} - \frac{a^2}{4}}$$

$$S = 4\ a \sqrt{\frac{a^2}{.5858} - \frac{a^2}{4}}$$

PROBLEM XLI.

Having given the side a of a regular decagon, to find R, r and S (See Problem XXXV.).

$$R = \frac{a}{2} + \sqrt{a^2 + \frac{a^2}{4}}$$

$$r = \sqrt{R^2 - \frac{a^2}{4}}$$

$$S = 5\ a \sqrt{R^2 - \frac{a^2}{4}}$$

N.B.—In the last two formulas the value of R, when found, is to be substituted for R.

PROBLEM XLII.

Having given the side a of a regular dodecagon, to find R, r and S (See Problem XXXV.).

$$R = \sqrt{\frac{4\,a^2}{1.072}}$$

$$r = \sqrt{\frac{4\,a^2}{1.072} - \frac{a^2}{4}}$$

$$S = 6\,a\sqrt{\frac{4\,a^2}{1.072} - \frac{a^2}{4}}$$

PROBLEM XLIII.

To express the side of the following regular polygons in terms of the radius R of the circumscribed circle.

side of the triangle...... $= R \times 1.732$
side of the square ...... $= R \times 1.414$
side of the pentagon ... $= R \times 1.176$
side of the hexagon...... $= R$
side of the octagon ...... $= R \times .7653$
side of the decagon...... $= R \times .618$
side of the dodecagon... $= R \times .5176$

PROBLEM XLIV.

Having given the hypotenuse a and one side b, of a right-angled triangle, to find the surface S.

$$S = \frac{b}{2}\sqrt{a^2 - b^2}$$

PROBLEM XLV.

Having given the base b of an isosceles triangle, and the oblique side a, to find the surface.

$$S = \frac{b}{4}\sqrt{4\,a^2 - b^2}$$

PROBLEM XLVI.

Having given the three sides a, b, c, of any triangle, to find the surface.

$$S = \sqrt{\frac{(a+b+c)}{2}\;\frac{(a+b-c)}{2}\;\frac{(a+c-b)}{2}\;\frac{(b+c-a)}{a}}$$

make, for the sake of abbreviation, $\frac{a+b+c}{2} = p$

then $S = \sqrt{p\,(p-c)\,(p-b)\,(p-a)}$

PROBLEM XLVII.

Having given the surface S of a regular polygon inscribed in a circle, to find the surface s of a similar circumscribed polygon (See Problem XXXV.).

$$s = \frac{S\,R^2}{r^2}$$

PROBLEM XLVIII.

To express the surface of a circle in terms of the radius R.

$$S = \pi R^2$$

PROBLEM XLIX.

Having given the difference d between the circumferences of two circles, and the ratio of the radii, so that $r : R :: m : n$, to find the circumference and the radii.

$$\text{Circ. of the greater} = \frac{m\,d}{m - n}$$

$$\text{Circ. of the lesser} = \frac{m\,n\,d}{m^2 - m\,n}$$

$$\text{Rad. of the greater} = \frac{m\,d}{2\,\pi\,(m - n)}$$

$$\text{Rad, of the lesser} = \frac{m\,n\,d}{2\,\pi\,m^2 - m\,n}$$

PROBLEM L.

Let the base of a triangle be $B\,C = b$, the vertical angle be A, and then the sides $A\,B$ and $A\,C$ be respectively a and c. To divide this triangle into two equivalent parts by a straight line passing through a given point D on the side $A\,C$. Let the distance $A\,D$ be $= d$. The Problem is solved as soon as another point H on the side $A\,B$, is found, through which the dividing line is to pass. Let the distance $A\,H$ be denoted by x; then

$$x = \frac{c\,a}{2\,d} \qquad \text{N. S.}$$

$$= \frac{c \times \frac{1}{2}\,a}{d} \qquad \text{G. S.}$$

PROBLEM LI.

To divide the same triangle into three equivalent parts by two lines passing through the same point D. It is evident that two points H and H' are to be found on the side $A\,B$. Let the distance $A\,H = x$, and the distance $A\,H' = x'$.

$$x = \frac{c\,a}{3\,d} \qquad \text{N. S.}$$

$$= \frac{c \times \frac{1}{3}\,a}{d} \qquad \text{G. S.}$$

$$x' = \frac{2\,c\,a}{3\,d} \qquad \text{N. S.}$$

$$= \frac{c \times \frac{2}{3}\,a}{d} \qquad \text{G. S.}$$

PROBLEM LII.

To divide the same triangle (see Problem L.) into two equivalent parts by a line parallel to the base. It is evident that the problem is solved as soon as one point D is found on one side $A\,C$, through which the dividing line is to pass.

Let the distance $A\,D = x$; then

$$x = \sqrt{\frac{c^2}{2}} \qquad \text{N. S.}$$

$$= \sqrt{c \times \frac{c}{2}} \qquad \text{G. S.}$$

PROBLEM LIII.

To divide the same triangle (see Problem L.) into five equivalent parts by lines parallel to the base.

Then 4 points D, D', D'', D''' are to be found on the same side $A\,C$.

Let $A\,D = x$; $A\,D' = x'$; $A\,D'' = x''$; $A\,D''' = x'''$.

N. S.		G. S.
$x = \sqrt{\frac{a^2}{5}}$	$=$	$\sqrt{a \times \frac{a}{5}}$
$x' = \sqrt{\frac{2\,a^2}{5}}$	$=$	$\sqrt{a \times \frac{2\,a}{5}}$
$x'' = \sqrt{\frac{3\,a^2}{5}}$	$=$	$\sqrt{a \times \frac{3\,a}{5}}$
$x''' = \sqrt{\frac{4\,a^2}{5}}$	$=$	$\sqrt{a \times \frac{4\,a}{5}}$

PROBLEM LIV.

To divide the same triangle (see Problem L.) into two parts which are in the ratio $m : n$.

$$x = \sqrt{\frac{m\ a^2}{m+n}} \qquad \text{N. S.}$$

$$= \sqrt{a \times \frac{m\ a}{m+n}}$$

$$\text{make } \frac{m\ a}{m+n} = y \text{ (by formula 2).}$$

$$\text{then } x = \sqrt{a\ y} \qquad \text{G. S.}$$

PROBLEM LV.

To divide a trapezoid $\frac{p\ (m+n)}{2}$ (see Intro. I.), into two equivalent parts by a line x parallel to the bases m, n.

$$x = \sqrt{\frac{m^2 + n^2}{2}} \qquad \text{N. S.}$$

$$\text{make } m^2 + n^2 = y^2 \text{ (by formula 4).}$$

$$\text{then } x = \sqrt{\frac{y^2}{2}}$$

$$= \sqrt{\frac{y \times y}{2}} \qquad \text{G. S.}$$

N.B.—The length of the dividing line being known, its position in the figure will be easily found.

PROBLEM LVI.

Having given a circle, whose radius $= R$, to find the radius r of another circle, whose area shall be to that of the first circle in the ratio of $m : n$.

$$r = \sqrt{\frac{m\ R^2}{n}} \qquad \text{N. S.}$$

$$= \sqrt{R \times \frac{m\ R}{n}} \qquad \text{G. S.}$$

PROBLEM LVII.

Having a given circle πR^2, to find the radius r of a concentric circle, so that the circular ring may be equal to the three-fifths of the smaller circle.

$$r = \sqrt{\frac{5\ R^2}{8}} \qquad \text{N. S.}$$

$$= \sqrt{R \times \frac{5\ R}{8}} \qquad \text{G. S.}$$

PROBLEM LVIII.

To make the same construction (see Problem LVII.) so that the circular ring may be a mean proportional between the two circles.

$$r = -\frac{R}{2} \pm \sqrt{R^2 + \frac{R^2}{4}}$$

PROBLEM LIX.

To make the same construction so that the interior circle may be a mean proportional between the exterior circle and the circular ring.

$$r = \pm \sqrt{-\frac{R^2}{2} \pm \sqrt{R^4 \pm \frac{R^4}{4}}}$$

Joseph Loveday, Printer, Ottawa, Canada.