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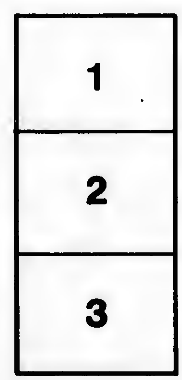
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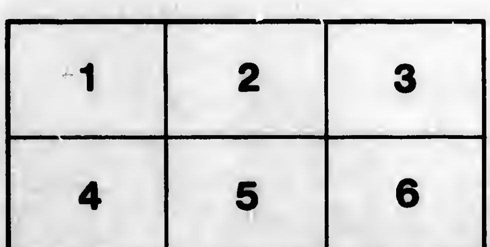
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SIMPLE EXERCISES

IN

MENSURATION;

DESIGNED FOR THE USE OF

CANADIAN COMMON AND GRAMMAR SCHOOLS

BY

JOHN HERBERT SANGSTER, M.A., M.D.,

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MONTREAL:

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AND FOR SALE AT THE BOOKSTORES.

1872.

PREFACE.

THIS little book is not intended to supersede the more elaborate text-books upon the same subject, in use in our Schools, but rather to serve as an introduction to one or other of them. The great mass of Common and Grammar School pupils have not time, amid the many other important studies claiming their attention, to devote to any lengthened course of instruction upon Mensuration. All that the teacher can ordinarily hope, under existing circumstances, to accomplish in this department, is to make his scholars capable of readily computing the area of regular surfaces and the volume or capacity of regular solids. Where more is attempted, it is, as a general thing, done at the expense of other important branches of instruction. Those who are intended for professions which require an intimate knowledge of Land Surveying, Astronomy, Gauging, &c., may, of course, profitably devote one or more entire years to the study of the various departments of mensuration, but for general purposes—for the farmer, the mechanic, the merchant, a knowledge of the mensuration of ordinary surfaces and solids is amply sufficient, and it is for such that the following pages have been thrown together.

The rules are given in the form of formulas, because it is believed that they are thus much more readily and lastingly remembered, and a very little effort on the part of the teacher will enable the pupil both to understand the dependence of the rules upon one another, and the interpretation and application of the formulas.

Toronto, October, 1867.

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MENSURATION.

DEFINITIONS.

☞ *The teacher is expected to draw on the blackboard or slate, figures illustrating those definitions.*

1 A *figure* is that which is enclosed by one or more boundaries.

2. A *plane figure* is enclosed by one or more lines, which lie on the same plane or flat surface.

3. A *solid body* is that which is contained or bounded by one or more surfaces.

4. A plane figure or *surface* is said to have two dimensions, viz: length and breadth; a solid is said to have three dimensions, viz: length, breadth and thickness.

5. The *area* of a plane figure is the number of square units of measurement contained within its bounding line or lines; the *volume* of a *solid* body is the number of cubic units of measurement contained within its bounding surface or surfaces.

6. *Mensuration* consists in the determination of the areas of surfaces and the volume of solids from their linear dimensions.

7. A *plane rectilineal angle* is the mutual inclination of two straight lines towards one another—which meet but are not in the same straight line.

NOTE.—The magnitude of the angle depends upon the rate of divergence of the lines—not upon their length.

8. When one straight line, standing upon another, makes the adjacent angles equal, each of them is called a *right angle;* and the line which stands upon the other is called a *perpendicular* to it.

9. An angle less than a right angle is called an *acute angle;* an angle greater than a right angle is called an *obtuse angle.*

10. *Parallel straight lines* are those that lie in the same plane and which have the same direction, so that being produced ever so far, both ways, they never meet.

11. A *triangle* is a figure contained by three straight lines.

12. An *equilateral triangle* has all of its sides equal; an *isosceles triangle* has two of its sides equal; and a *scalene triangle* has all of its sides unequal.

13. A *right-angled triangle* has one of its angles a right angle; an *obtuse-angled triangle* has one of its angles an obtuse angle; and an *acute-angled triangle* has all three of its angles acute angles. The two latter are often called *oblique-angled triangles.*

14. A *quadrilateral* figure is that which is enclosed by four straight lines.

15. A *trapezium* is a quadrilateral figure having no two of its sides parallel.

16. A *trapezoid* is a quadrilateral figure having one pair of opposite sides parallel.

17. A *parallelogram* is a quadrilateral figure having each pair of opposite sides parallel.

18. A *rectangle* or *oblong* is a parallelogram whose angles are all right angles, but its adjacent sides are not equal, *i. e.*, its length is greater than its breadth.

19. A *square* is a parallelogram whose angles are all right angles and its sides are all equal.

20. The *diagonal* of a quadrilateral figure is a straight line joining its opposite angles.

21. A *polygon* or *multilateral figure* is a figure contained by more than four straight lines.

22. A *regular polygon* is one whose sides are all equal to one another, as also are its angles.

23. Polygons are named from the number of their sides — thus a five-sided polygon is called a *pentagon;* a six-sides polygon, is called a *hexagon;* a seven-sided polygon is called a *heptagon;* an eight-sided polygon is called an *octagon,* &c.

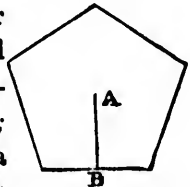


Fig. 1—Pentagon.

24. The *apothem* of a regular polygon is a perpendicular from its centre on any of its sides; as AB, Fig. 1 or 2.

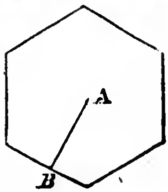


Fig. 2—Hexagon.

25. A *circle* is a plane figure, bounded by one line, called the circumference, and is such that every part of the circumference is equally distant from a point within, called the centre.

NOTE.—The circumference is the bounding line—the circle the space inclosed.

26. The *diameter* of a circle is a straight line passing through the centre, and terminated both ways in the circumference.

27. A *semicircle* is the figure contained by the diameter and the part of the circumference cut off by the diameter.

28. The *radius* of a circle is half the diameter, or is a straight line joining the centre of the circle with the circumference.

29. An *arc* of a circle is any part of the circumference.

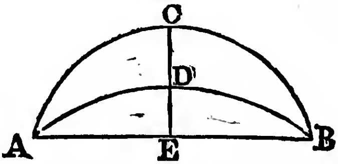
30. A *chord* of a circle is any straight line joining the extremities of an arc.

31. A *segment* of a circle is the figure contained by a chord and the arc of the circumference cut off by the chord.

32. A *sector* of a circle is the figure contained by an arc of the circumference, and the two radii joining its extremities with the centre of the circle.

33. A *lune* is the figure contained between the circular arcs of two dissimilar circular segments which have a common chord.

Fig. 3.



Thus in Fig. 3 AB is a chord, ACB and ADB are two dissimilar circular segments, ACBD is a lune.

34. A *degree* is the 360th part of the circumference of a circle.

NOTE.—The length of the degree depends upon the magnitude of the circle.

35. *Concentric circles* are such as have a common centre.

36. A *circular annulus* is the figure inclosed between the circumferences of two concentric circles.

37. The *perimeter* or *periphery* of any figure is its circumference, or the aggregate length of all its boundaries.

38. A *polyhedron* is any solid contained by planes, which planes are called its *sides* or *faces*. The lines bounding its sides are called its *edges*.

39. A *regular polyhedron* is one whose sides are equal and regular figures of the same kind, and whose solid angles are equal.

40. There are only five regular polyhedrons, viz.:—

The *tetrahedron* contained by four equilateral triangles. Fig. 4.

The *hexahedron* contained by six squares. Fig. 5.

The *octahedron* contained by eight equilateral triangles. Fig. 6.

The *dodecahedron* contained by twelve pentagons. Fig. 7.

The *icosahedron* contained by twenty equilateral triangles. Fig. 8.

Fig. 4.

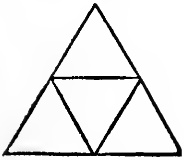


Fig. 5.

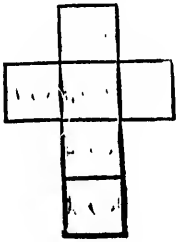


Fig. 6.

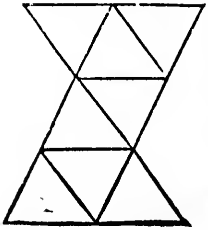


Fig. 7.

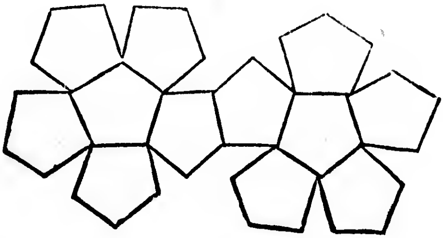
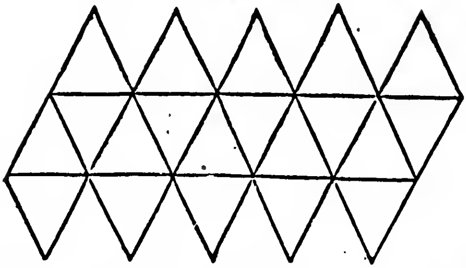


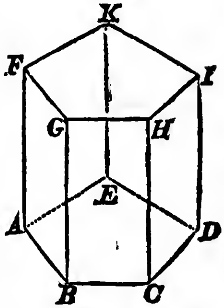
Fig. 8.



41. A *prism* is a solid contained by plane figures, of which two are equal, similar and opposite; with their sides parallel each to each, and the other sides are parallelograms.

42. The *ends* or terminating planes of the prism are the two similar sides, and the edges of these are called *terminating edges* to distinguish them from the lateral sides and edges. The prism is *triangular*, *rectangular*, *square* or *polygonal*, according as its terminating planes or ends are triangles, rectangles, squares or polygons. When the lateral edges are perpendicular to the end, the prism is called a *right prism*, when otherwise, an *oblique prism*. The line joining the centres of the terminating planes of a prism is called its *axis*.

Fig. 9.

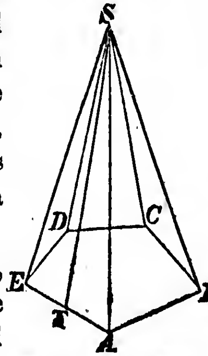


A Polygonal Right Prism; ABCDE and FGHIK its ends. AB, BC, CD, &c., terminating edges. AF, GB, CH, &c., its lateral edges.

43. A *parallelopiped* is a prism having parallelograms for its terminating planes or ends.

44. A *cube* is a solid contained by six equal squares.

Fig. 10.

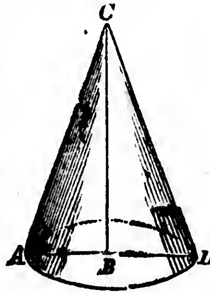


A Regular Pyramid ABCDE its base. BCS, ACS, &c., its sides.

45. A *pyramid* is a solid having any rectilineal figure for its base; and for its other sides triangles, which have a common vertex. The pyramid is triangular, square, rectangular, &c., according as its base is a triangle, a square, a rectangle, &c.

46. When the base is a regular figure, a line joining its centre with the vertex or the pyramid is called the axis of the pyramid. When the axis is at right angles to the base, the pyramid is called a *regular pyramid*.

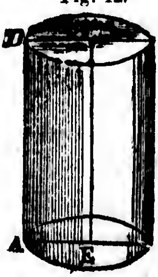
Fig. 11.



47. A *cone* is a round pyramid having a circle for its base, and is conceived to be produced by the revolution of a right-angled triangle about its perpendicular side which remains fixed. The line joining the vertex of the cone with the centre of the base is called the axis of the cone. Fig. 11.

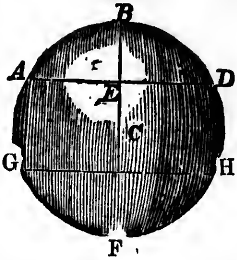
48. A *right cone* is one in which the axis is perpendicular to the base—all other cones are called *oblique*.

Fig. 12.



49. A *cylinder* is a prism having circles for its ends or terminating planes, and is conceived to be produced by the revolution of a rectangle about one of its sides, which remains fixed. Fig. 12.

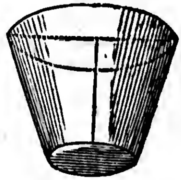
Fig. 13.



50. A *sphere* or *globe* is a solid body which may be supposed to be produced by the revolution of a semi-circle about its diameter which remains fixed. Fig. 13.

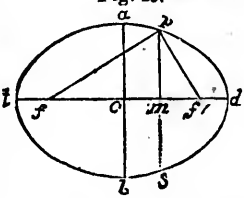
51. A *segment* of a sphere is a part of it cut off by a plane; a segment of a pyramid, cone, cylinder or other solid, with a plane base is a portion cut off from the top by a plane parallel to the base.

Fig. 14.



52. A *frustum* of a solid is the portion contained between the base and a plane parallel to the base as in fig. 14; the *frustum* or *zone* of a sphere is the portion cut off by two parallel planes as ADGH in Fig. 13.

Fig. 15.



53. An *ellipse* or *oval* (Fig. 15) is a plane figure bounded by a curved line such that the sum of the distances of any point in its circumference from two given points in it is constant, *i. e.*, is equal to a given straight line.

Thus Fig. 15 *atbd* is an ellipse because $pf + pf'$ is constant: f and f' are the foci, c the centre, td the transverse and ab the conjugate diameter or axis, sm is an ordinate and sp a double ordinate, tm and md are the abscisses to the ordinate sm.

54. The two given points are called the *foci* of the ellipse, and the middle of the line joining them is called the *centre* of the ellipse. The distance of either focus from the centre is called the *eccentricity* of the ellipse.

55. The *major* or *long axis* or *transverse diameter* of an ellipse, is a line through both foci, and terminating in the bounding curve.

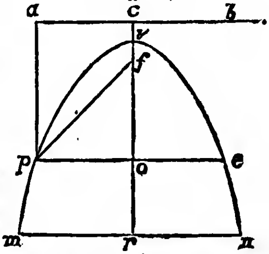
56. The *minor* or *short axis* or *conjugate diameter*, is a line passing through the centre, at right angles to the major axis, and terminating both ways in the bounding curve.

57. An *ordinate* to either axis is a line drawn from any point in the curve perpendicular to the axis; when it is continued to meet the curve on the other side, it is called a *double ordinate.*

58. Each of the segments into which the ordinate divides the axis is called an *absciss.*

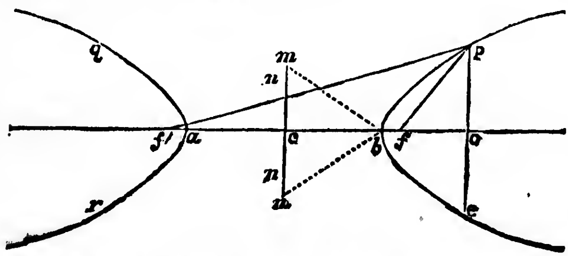
59. A *parabola* is a curve such that any point of it is equally distant from a given point within the curve, and a given line without it.

Fig. 16,



Thus if the curve *mvn* is such that any point *p* in it is equally distant from the point *f*, and the line *ab*, that is, if *pa* is equal to *pf*, than the curve *mvn* is a parabola. Also *f* is the *focus*; *po* is the *ordinate*, and *pe* the *double ordinate* or *base*. *v* is the *vertex*, and *ov* is the *absciss*. The double ordinate through *f* is called the *parameter*.

60. An *hyperbola* is a curve, such that the difference between the distances of any point in it from two given points, one within, and the other without the curve, is equal to a given line.



Thus if any point *p* in the curve *pbe* is such that $pf'-pf=ab$, a given line, then the curve *pbe* is an hyperbola. Also *f* and *f'* are the *foci*, *ab* is the *transverse axis*, e is the *centre*; mn is the conjugate axis, the points m and n being distant from a or b by cf or qf, *i. e.*, by the eccentricity; po is the *ordinate*; and pe the double ordinate or *base*; ab is the smaller *absciss*; and oa the greater *absciss*.

61. A *paraboloid* or *parabolic conoid* is a solid generated by the revolution of a parabola about its axis, which remains fixed.

NOTE.—A frustum of a paraboloid is a portion contained between two parallel planes perpendicular to its axis.

62. A *spheroid* is a solid generated by the revolution of an ellipse about one of its axis which remains fixed.

63. A *spheroid* is said to be *oblate* or *prolate*, according as it is the conjugate or the transverse axis that is fixed.

NOTE.—The fixed axis is called the *polar* axis, and the revolving axis the *equatorial* axis.

64. A *segment* of a spheroid is a portion cut off by a plane perpendicular to one of its axes.

NOTE.—When the plane is perpendicular to the fixed axis, the base is a circle, and the segment is said to be circular; when the plane is perpendicular to the revolving axis, the segment is called an elliptical one, because the base is an ellipse.

65. The *middle zone* of a spheroid or of a sphere is a portion contained between two parallel planes perpendicular to an axis and equally distant from the centre.

66. An *hyperboloid* or *hyperbolic conoid* is a solid generated by an hyperbola about its axis which remains fixed.

NOTE—A frustum of an hyperboloid is a portion of it contained between two parallel planes perpendicular to its axis.

MENSURATION OF SURFACES.

SYNOPSIS OF FORMULAS.

Let A = area, b = base, p = perpendicular or altitude, d = diagonal.

SQUARE. $A = b^2$ (I) $\therefore b = \sqrt{A}$ (II). Also $A - \frac{1}{2}d^2$ (III) $\therefore d = \sqrt{2A}$ (IV).

RECTANGLE or PARALLELOGRAM. $A = bp$ (V) $\therefore b = \frac{A}{p}$ (VI) and $p = \frac{A}{b}$ (VII).

RECTANGLE. $A = b\sqrt{(d+b)\,(d-b)}$ (VIII).

RIGHT-ANGLED TRIANGLE. Let h = the hypothenuse, p' = the perpendicular from the right angle on the hypothenuse, and s and s' = the segments into which this divides the hypothenuse, s being that adjacent to the base of the triangle. Then $h = \sqrt{b^2 + p^2}$ (IX); $b = \sqrt{h^2 - p^2}$ (X); $p = \sqrt{h^2 - b^2}$ (XI); $s = \frac{p^2}{h}$ (XII); $s' = \frac{b^2}{h}$ (XIII); and $p' = \sqrt{ss'}$ (XIV).

TRIANGLE. $A = \frac{1}{2}bp$ (XV); $\therefore b = \frac{2A}{p}$ (XVI); and $p = \frac{2A}{b}$ (XVII). Also, if a, b, c be the three sides and $s = \frac{1}{2}(a+b+c)$ then $A = \sqrt{s(s-a)(s-b)(s-c)}$ (XVIII). In the case of an equilateral triangle this formula becomes $A = \sqrt{\frac{3b}{2} \times \frac{b}{2} \times \frac{b}{2} \times \frac{b}{2}} = \sqrt{3 \times \left(\frac{b}{2}\right)^4} = \sqrt{3} \times \frac{b^2}{4} = \cdot 433b^2$ (XIX).

Trapezoid. Let b and b' be the parallel sides, then $A = \frac{1}{2}(b + b')\,p$ (xx).

Quadrilateral. Let d = diagonal, and p and p' the perpendiculars from the diagonal to the opposite angles, then $A = \frac{1}{2}(p - p')\,d$ (xxi).

Quadrilateral in a Circle—*i. e.*, that may be inscribed in a circle. Let a, b, c, d be the four sides, and let $s = \frac{1}{2}(a + b + c + d)$ then we have the formula $A = \sqrt{(s - a)(s - b)(s - c)(s - d)}$ (xxii).

Regular Polygon. Let a = apothem when side is = 1, s = a side, and n = number of sides. Then $A = \frac{1}{2}ans$ (xxiii); $\therefore s = \frac{2A}{an}$ (xxiv); and $n = \frac{2A}{as}$ (xxv).

Circle. Let d = diameter, r = radius, c = circumference; and $\pi = 3{\cdot}1416$.

$c = \pi d$ (xxvi); $\therefore$ d $= \frac{c}{\pi} = c \times \frac{1}{3.1416} = c \times 3183$ that is $d = {\cdot}3183c$ (xxvii).

$A = \frac{1}{4}cd$ (xxviii); $\therefore d = \frac{4A}{c}$ (xxix); and $c = \frac{4A}{d}$ (xxx).

$A = \pi r^2$ (xxxi) $\therefore r = \sqrt{\frac{A}{\pi}} = \sqrt{A \times {\cdot}3183}$ (xxxii).

$A = d^2 \times 7854$ (xxxiii), since $d^2 = 4r^2$, and $3{\cdot}1416 \div 4 = {\cdot}7854$

$A = {\cdot}0796c^2$ (xxxiv), since $d = {\cdot}3184c$, and $\therefore d^2 = ({\cdot}3183)^2c^2$

Circular Annulus. Let d = diameter of the greater, and d' = that of smaller circle, and let c and c' = their respective circumferences.

$A = \frac{\pi}{4}(d + d')(d - d')$ (xxxv).

$A = {\cdot}0796\,(c + c')(c - c')$ (xxxvi).

$A = \frac{1}{4}(c + c')(d - d')$ (xxxvii.)

LENGTH OF CIRCULAR ARC. Let n = number of degrees in the arc, d = diameter of circle, and l = length of arc, k = chord of whole arc, $k_{,}$ = chord of half the arc; a = apothem or perpendicular from centre on the chord, and consequently $r - a = h$ = height of the segment. Then

$$l = \frac{\pi n d}{360} \text{ that is } l = \cdot 008726 nd \text{ (XXXVIII).}$$

$$k = 2\sqrt{r^2 - a^2} \text{ (XXXIX).} \quad a = \tfrac{1}{2}\sqrt{4r^2 - h^2} \text{ (XL).}$$

$$k_{,} = \sqrt{2r(b - a)} \text{ (XLI), and } r = \frac{k_{,}^{2}}{2h} \text{ (XLII).}$$

SECTOR. Let l = length of circular arc, and r = radius of circle. Then $A = \frac{1}{2}lr$ (XLIII.) Also from (XXXVII and XLIII.) $A = \cdot 008726 nr^2$ (XLIV).

SEGMENT. A = area of corresponding sector $\pm$ area of included triangle (XLV).

LUNE. A = area of greater segment, *minus* area of smaller segment (XLVI).

ELLIPSE. Let C = circumference, t = transverse axis, c = conjugate axis, a = absciss, and o = ordinate.

$$C = \pi\sqrt{\tfrac{1}{2}\,(t^2 + c^2)} \text{ (XLVII);} \quad A = \tfrac{1}{4}\pi tc \text{ (XLVIII);}$$

$$o = \frac{c}{t}\sqrt{(t - a)a} \text{ (XLIX);} \quad a = \frac{t}{2} \pm \text{d, where}$$

$$d = \frac{t}{c}\sqrt{\tfrac{1}{4}c^2 - o^2} \text{ (L);} \quad t = \frac{ca}{o^2}\left\{\tfrac{1}{2}c + \sqrt{\tfrac{1}{4}c^2 - o^2}\right\} \text{ (LI);}$$

$$c = \frac{ot}{\sqrt{(t - a)a}} \text{ (LII).}$$

PARABOLA.—Let p = parameter, a and a' = any two abscisses, o and o' = the corresponding ordinates,

b = base or double ordinate, and l = length of parabolic curve.

$p = \frac{o^2}{a}$ (LIII); $o' = o\sqrt{\frac{a'}{a}}$ (LIV); $a' = a\left(\frac{o'}{o}\right)^2$ (LV);

$l = 2\sqrt{o^2 + \frac{3}{4}a^2}$ (LVI); $A = \frac{2}{3}ab$ (LVII). For parabolic zone $A = \frac{2}{3}h\left(b' + \frac{b^2}{b + b'}\right)$ (LVIII), where h = height of zone, and b, b' = bases or double ordinates.

HYPERBOLA. Symbols same as in ellipse and parabola.

$o = \frac{c}{t}\sqrt{(t+a)a}$ (LIX); $a = d \pm \frac{t}{a}$ where

$d = \frac{t}{c}\sqrt{\frac{1}{4}c^2 + o^2}$ (LX); $c = \frac{ot}{\sqrt{(t+a)a}}$ (LXI);

$t = \frac{ca}{o}\left\{\frac{c}{2} \pm \sqrt{\frac{1}{4}c^2 + o^2}\right\}$ (LXII), $\pm$ according as the smaller or greater absciss is given.

$A = \frac{4ca}{75t}\{3\sqrt{7a(7t+5a)} + 4\sqrt{ta}\}$ (LXIII).

MENSURATION OF SOLIDS.

SYNOPSIS OF FORMULAS.

REGULAR SOLIDS. Let s = surface, and v = volume or solid contents, and let e = one edge.

TETRAHEDRON OR REGULAR TRIANGULAR PYRAMID. $s = e^2\sqrt{3} = 1{\cdot}732e$ (LXIV), and $v = \frac{1}{12}e^3\sqrt{2} = {\cdot}11785e$ (LXV).

HEXAHEDRON OR CUBE. $s = 6e^2$ (LXVI); $v = e^3$ (LXVII).

OCTAHEDRON. $s = e^2\sqrt{3} = 3{\cdot}464e^2$ (XVIII); $v = \frac{1}{3}e^3\sqrt{2}$ $= {\cdot}471405e^3$ (LXIX).

DODECAHEDRON. $s = 15e^2\sqrt{\frac{1}{5}(5 + 2\sqrt{5})} = 20{\cdot}645775e^2$ (LXX);

$v = 5e^3\sqrt{\frac{1}{40}(47 + 21\sqrt{5})} = 7{\cdot}6631e^3$ (LXXI).

ICOSAHEDRON. $s = 5e^2\sqrt{3} = 8{\cdot}66e^2$ (LXXII);

$v = \frac{5}{6}e^3\sqrt{(7 + 3\sqrt{5})} = 2.18169e^3$ (LXXIII).

PARALLELOPIPED; PRISM; CYLINDER. Let a = area of base or end, p = perimeter of base, and p' = perimeter of section perpendicular to one of the edges of the solid; also let h = the height, and s = the whole surface.

$v = ah$ (LXXIV), $s = hp + 2a$ (LXXV), when the solid is right, and $s = hp' + 2a$ (LXXVI), when the solid is oblique.

REGULAR PYRAMID AND CONE. Let p = perimeter of base, l = length of slant side, h = height, *i. e.*, perpendicular height of vertex above the base, and a = area of base.

$v = \frac{1}{3}ah$ (LXXVII); $s = \frac{1}{2}pl + a$ (LXXVIII).

FRUSTUM OF PYRAMID. Let a and a' = areas of the two ends, h = height, e and e' = the edges of the ends, and let p and p' = perimeters of ends.

$v = \frac{1}{3}h(a + a' + \sqrt{aa'})$ (LXXIX), $v = \frac{1}{3}h\left(\frac{ae - a'e'}{e - e'}\right)$ (LXXX), $s = \frac{1}{2}(p + p')l + a + a'$ (LXXXI).

FRUSTUM OF CONE. Symbols as in frustum of pyramid also d and $d_{,}$ = diameters of ends.

$v = \frac{1}{3}h(a + a_{,} + \sqrt{aa'})$ (LXXXII).

Also $v = {\cdot}7854d(d^2 + d_{,}^2 + dd_{,})\frac{h}{3} = {\cdot}2618h(d^2 + d_{,}^2 + dd_{,})$ (LXXXIII), since $a = {\cdot}7854d^2$ and $a' = {\cdot}7854d_{,}^2$ and $\sqrt{aa'} = \sqrt{({\cdot}7854d^2 + {\cdot}7854d_{,}^2)}$, $s = \frac{1}{2}(p + p')l + a + a'$ (LXXXIV).

WEDGE. Let l and b = length and breadth of back, e = length of edge, h = height. Then $v = \frac{1}{6}bh(e+2l)$ (LXXXV).

SPHERE. $v = \cdot 5236d^3$ (LXXXVI), $s = \pi d^2 =$ (LXXXVII).

SPHERICAL SEGMENT. Let r = radius of base, d = diameter of sphere, h = height, and s = convex surface, $v = \cdot 5236h\,(3r^2 + h^2)$ (LXXXVIII); $v = \cdot 5236h^2(3d - 2h)$ (LXXXIX), $s = \pi dh$ (XC).

SPHERICAL ZONE. $v = \dfrac{h}{2}(r^2 + r_{,}^2 + \frac{1}{3}h^2)$ (XCI), where r and $r_{,}$ are the radii of the ends. For middle zone $v = \dfrac{\pi h}{4}(d^2 + \frac{2}{3}h^2)$ (XCII), $v = \dfrac{h}{4}(d_{,}^2 - \frac{1}{3}h^2)$ (XCIII), where d is the diameter of the end of the zone and d′ is the diameter of the sphere, $s = \pi d_{,}h$ (XCIV) where s = convex surface.

PARABOLOID. $v = \frac{1}{2}ah = \dfrac{\pi d^2h}{8} = \cdot 3927d^2h$ (XCV).

FRUSTUM OF PARABOLOID. Let a and a' = areas of ends; d and $d_{,}$ their diameters, and h = height, then $v = \frac{1}{2}h(a + a')$ (XCVI); $v = \frac{1}{8}\pi h(d^2 + d_{,}^2) = \cdot 3927h$ $(d^2 + d_{,}^2)$ (XCVII).

SPHEROID. Let t = transverse, and c = conjugate axes. Then $v = \cdot 5236ct^2$ (XCVIII) for oblate spheroid. $v = \cdot 5236c^2t$ (XCIX) for prolate spheroid.

CIRCULAR SEGMENT OF SPHEROID.

Oblate $v = \cdot 5236(3c - 2h)\dfrac{t^2h^2}{c^2}$ (C),

Prolate $v = \cdot 5236(3t - 2h)\dfrac{c^2h^2}{t^2}$ (CI).

Elliptical Segment of Spheroid.

Oblate $v = \cdot 5236(3t - 2h)\dfrac{ch^2}{t}$ (CII),

Prolate $v = \cdot 5236\,(3c - 2h)\dfrac{th^2}{c}$ (CIII).

Middle Frustum of Spheroid.

(Circular) oblate $v = \cdot 2618(2t^2 + d^2)l$ (CIV),

prolate $v = \cdot 2318(2c^2 + d^2)l$ (CV),

(Elliptical $v = \cdot 2618(2tc + dd_{\prime})l$ (CVI) for either oblate or prolate, where $l =$ length of frustum, $d =$ diameter; and, in (CVI), d and $d_{\prime} =$ the greater and less diameters of one end.

Hyperboloid. $v = \cdot 5236(r^2 + d^2)h$ (CVII), where $r =$ radius of base, $d =$ diameter half way between the base and vertex, and $h =$ height.

Frustum of Hyperboloid. $v = \cdot 5236(r^2 + r_{\prime}^2 + d^2)h$ (CVIII), where r and $r_{\prime} =$ radii of ends, and $d =$ diameter half way between the ends.

ILLUSTRATIONS AND EXERCISES.

SQUARE.

Formulæ. $A = b^2$ (I); $b = \sqrt{A}$ (II); $A = \frac{1}{2}d^2$ (III), $d = \sqrt{2A}$ (IV).

Ex. 1. Find the area of a square whose base is 40 chains.

Solution.

Here $b = 40$; then $A = b^2 = 40^2 = 1600$ chains $=$ Ans. 160 acres.

Ex. 2. Find the diagonal of a square whose area is 91347 yards.

Solution.

Here $A = 91347$; then $d = \sqrt{2 \times 91347} = \sqrt{182694} = 427{\cdot}42$ yards.

Exercise I.

1. Find the area of a square whose base is 916 yards.
Ans. 173 a. 1 rood 17 per.

2 Find the area of a square whose diagonal is 107.
Ans. $5724\frac{1}{2}$.

3. Find the base of a square whose area is $2\frac{1}{2}$ acres.
Ans. 110 yards.

4. Required the diagonal or square whose area is 208 yards.
Ans. 20·39 yards.

RECTANGLE OR PARALLELOGRAM.

Formulæ. $A = bp$ (V) ; $b = \frac{A}{p}$(VI,) and $p = \frac{A}{b}$(VII).

Also, for rectangle, $A = b\sqrt{(d+b)\ (d-b)}$ (VIII).

Ex. 1. Find the area of a rectangular field whose adjacent sides are 600 and 800 links.

Solution.

Here $p = 600$ links, and $b = 800$ links; then $A = bp = 600 \times 800 = 480000$ links, and this divided by 100000, the square links in an acre, we get 4·8 acres = 4 a. 3 roods 8 per.

Ex. 1. Find the base of a parallelogram whose area is 5 a 16 per., the perpendicular distance between the sides being 200 yards.

Solution.

Here $A = 5a\ 16p = 24684$ yards, and $p = 200$ yards; then $b = \frac{A}{p}$. $= \frac{24684}{200} = 123{\cdot}42$ yards.

Ex. 3. Find the area of a rectangular field whose base is 90 yards and diagonal 160 yards.

Solution.

Here $d = 160$, and $b = 90$; then by formula VIII, $A = b\sqrt{(d+b)(d-b)}$ $= 90 \times \sqrt{(160+90)(160-90)} = 90 \times \sqrt{250 \times 70} = 90 \times \sqrt{17500} = 90 \times 132{\cdot}28$ yards $= 11905{\cdot}2$ square yards.

Exercise II.

1. Find the area of a rectangle whose base is 11 and side 16.
   Ans. 176.

2. Find the area of a rectangle whose base is 28 and diagonal 30.
   Ans. 301·56.

3. Required the area of a field in the form of a parallelogram whose base is 760 links, and altitude 250 links.
   Ans. 1 a. 3 r. 24 per.

4. Required the base of a parallelogram whose area is 2 a. 3 r. 17 per., and perpendicular altitude 120 links.
   Ans. 2380·208 links.

5. Find the diagonal of a rectangle whose area is 200, and base 50.
   Ans. 50·159.

6. Find the distance between the sides of a parallelogram whose base is 900 yards and area 6 acres 2 r. 28 per. 17 yards.
   Ans. 35·915 yards.

RIGHT-ANGLED TRIANGLE.

Formulæ. Let b = base, p = perpendicular, h = hypothenuse, p' = perpendicular from right angle on the hypothenuse, s and s' = the segments into which this divides the hypothenuse, s being that adjacent to the base of the triangle; then $h = \sqrt{b^2 + p^2}$ (IX); $b = \sqrt{h^2 - p^2}$ (X); $p = \sqrt{h^2 - b^2}$ (XI); $s = \frac{p^2}{h^2}$ (XII); $s = \frac{b^2}{h}$ (XIII); and $p' = \sqrt{ss'}$ (XIV).

Ex. 1. Find the hypothenuse of a right-angled triangle whose base is 10 and perpendicular 15.

Solution.

Here $b = 10$, and $p = 15$; then by formula (IX), $h = \sqrt{b^2 + p^2} = \sqrt{100 + 225}$ $= \sqrt{325} = 18·0277$.

Ex. 2. Find the base of a right-angled triangle whose hypothenuse is 605 and perpendicular 20.

Solution.

Here $h = 605$, and $p = 20$; then by formula (X), $b = \sqrt{h^2 - p^2} = \sqrt{3600 - 400}$ $= \sqrt{3200} = 56·568$.

Ex. 3. Find the perpendicular let fall from the right angle of right-angled triangle to the hypothenuse, and also the segment of the hypothenuse—the base and perpendicular of the given triangle being 20 and 25 yards.

Solution.

First $h = \sqrt{b^2 + p^2} = \sqrt{400 + 625} = \sqrt{1025} = 32·0156$ yards.

Then $s = \frac{p}{h} = \frac{625}{32·0156} = 19·52$ yards; and $s' = h - s = 32·0156 - 1952 =$ $12·4956$.

Lastly $p = \sqrt{ss'} = \sqrt{19·52 \times 12·49} = \sqrt{243·8048} = 15·61$ yards.

Exercise III.

1. Find the perpendicular of a right-angled triangle whose base is 9 and hypothenuse 30. Ans. 28·618.
2. Find the hypothenuse of a right-angled triangle whose base is 11 and perpendicular 17. Ans. 20·248.

3. Required the base of a right-angled triangle whose hypothenuse is 40 and perpendicular 20. Ans. 34·641.
4. Find the perpendicular on the hypothenuse, and also find the segments of the hypothenuse of a right-angled triangle, whose base and perpendicular are 50 and 60.

Ans. Perpendicular = 38·4.
Greater segment = 46·093;
Smaller segment = 32·009.

5. Find the perpendicular on the hypothenuse and also the segments of the hypothenuse of a right-angled triangle whose perpendicular and base are 30 and 35.

Ans. Greater segment = 26·574;
Smaller segment = 19·5237;
Perpen. on hypo. = 22·77.

TRIANGLE.

FORMULÆ. $A = \frac{1}{2}bp$ (XV); $b = \frac{2A}{p}$ (XVI); $p = \frac{2A}{b}$ (XVII); $A = \sqrt{s(s-a)\ (s-b\ (s-c)}$ (XVIII), where a, b, and c are the sides, and $s = \frac{1}{2}(a+b+c)$. Also, for equilateral triangle, $A = 433b^2$ (XIX).

Ex. 1. Find the area of a triangle whose base is 91 and altitude 24 chains.

Solution.

Here $b = 91$, and $p = 24$; [then by formula (XV), $A = \frac{1}{2}bp = \frac{1}{2} \times 91 \times 24 = 1092$ chains $= 109·2$ acres $= 109$ acres 0 r. 32 per.

Ex. 2. Required the area of a triangle whose three sides are 100, 120, and 140 links.

Solution.

Here $a = 100$, $b = 120$, and $c = 140$; then $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(100+120+140) = 180$.

Then by formula (XVIII) $A = \sqrt{180 \times (180-100)(180-120)(180-140)}$ $= \sqrt{180 \times 80 \times 60 \times 40} = \sqrt{34560000} = 5878·7$ sq. links $= ·058787$ acres $= 0$ a. 0 r. 9 sq. per. 12 sq. yards.

Ex. 3. Find the area of an equilateral triangle whose base is 1000 yards.

Solution.

Here $b = 1000$, then by formula (XIX) $A = 433b^2 = \cdot433 \times 1000^2 = \cdot433 \times 100000 = 433000$ sq. yards $= 89$ a. 1 r. 34 per. $1\frac{1}{2}$ yards.

Ex. 4. Find the length of a side of an equilateral garden which contains 4 a. 3 r. 30 per. $19\frac{1}{2}$ yards.

Solution.

Here $A = 4$ a. 3r. 30 per. $19\frac{1}{2}$ yards $= 23917$ yards.

Then by formula (XIX) $A = \cdot433b^2$ $\therefore b^2 = \dfrac{A}{\cdot433}$ and $\therefore b = \sqrt{\dfrac{A}{\cdot433}}$

$$= \sqrt{\frac{23917}{\cdot433}} = \sqrt{52926} = 230\cdot05 \text{ yards.}$$

Exercise IV.

1. Find the area of a triangle whose base is 9 and altitude 11. Ans. $49\frac{1}{2}$.
2. What is the perpendicular altitude of a triangle whose base is 750 chains and area 500 acres? Ans. $13\frac{1}{3}$ chains.
3. Find the area of a triangle whose three sides are 40, 60 and 80 yards. Ans. 1161·8 yards.
4. What is the area of a triangle whose three sides are 420, 480 and 700 links? Ans. 3 r. 37 per. 24 yards.
5. The area of an equilateral triangle is 9134 square yards, what is its base? Ans. 145·2 yards.
6. Find the altitude of a triangle whose area is 7196 square feet and base 120 feet. Ans. 119·93 feet.

TRAPEZOID; TRAPEZIUM; QUADRILATERAL INSCRIBED IN A CIRCLE.

Formulæ. *Trapezoid*, $A = \frac{1}{2}p(b + b')$ (xx) where b and b' are the parallel sides.

Trapezium, $A = \frac{1}{2}d\,(p+p')$ (XXI) where p and p' are the perpendiculars from opposite angles to diagonal.

Quadrilateral in Circle, $A = \sqrt{(s-a()s-b)(s-c(s-d)}$ (XXII) where a, b, c, d, are the four sides, and $c = \frac{1}{2}(a+b+c+d)$.

Ex. 1. What is the area of a trapezoid whose parallel sides are 19 and 25 chains, and the perpendicular distance between them 13 chains?

Solution.

Here $p = 13$, $b = 19$ and $b' = 25$. Then by formula (XX), $A = \frac{1}{2}p\,(b \times b') = \frac{1}{2} \times 13 + (19 + 25) = \frac{1}{2} \times 13 \times 44 = 286$ chains $= 28$ a. 2 r. 16 per.

Ex. 2. What is the area of a trapezium whose diagonal is 700 yards and the perpendiculars from it to the opposite angles, 120 and 80 yards?

Solution.

Here $d = 700$, $p = (120$ and $p) = 80$, then by formula (XXI), $A = \frac{1}{2}d\,(p + p\ )$ $= \frac{1}{2} \times 700 \times (120 + 80) = \frac{1}{2} \times 700 \times 200 = 70000$ square yards $= 14$ a. 1 r. 85 per. 1 yard. Ans.

Ex. 3. Find the area of a field in the form of a quadrilateral whose opposite angles are equal to two right angles; *i. e.*, a quadrilateral which may be inscribed in a circle, whose four sides are 9, 11, 20 and 8 chains respectively.

Solution.

Here $a = 9$, $b = 11$, $c = 20$ and $d = 8$ chains $\therefore s = \frac{1}{2}(9 + 11 \times 20 + 8) =$ 24 chains.

Then by formula (XXXII), $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$ $= \sqrt{(24-9)(24-11)(24-20)(24-8)} = \sqrt{15 \times 13 \times 4 \times 16} = 111{\cdot}7139$ chains $= 11{\cdot}17139$ acres $= 11$ a. 0 r. 27 per. $12{\cdot}7$ yards.

Exercise V.

1. Find the area of a field in the form of a quadrilateral, which may be inscribed in a circle, its four sides being 40, 50, 60 and 70 yards. Ans. 2 r. 15 per. 24 yds.

2. Find the area of a quadrilateral field whose diagonal is 640 links and the perpendiculars on it from the opposite angles 240 links and 300 links. Ans. 1 a. 2 r. 36 per. 14 yds.
3. What is the area of a park in the form of a trapezoid, whose parallel sides are 90 and 110, and the perpendicular distance between them 60? Ans. 6000.

REGULAR POLYGON.

FORMULÆ. $A = \frac{1}{2}ans$ (XXIII;) $s = \frac{2A}{an}$ (XXIV); and $n = \frac{2A}{as}$ (XXV,) where a = apothem or perpendicular from the centre on a side, n = number of sides, and s = length of a side.

Ex. 1. Find the area of a regular pentagon whose side is 20 feet.

Solution.

Here, from table of apothems, it appears that for pentagon whose side is 20 feet, the apothem $= 0{\cdot}68819 \times 20 = 13{\cdot}7638$ feet. Then by formula (XXIII), $A = \frac{1}{2} \times 13{\cdot}7638 \times 5 \times 20 = 688{\cdot}19$ square feet.

Ex. 2. Find the length of the side of a regular octagon whose area is 3 a. 2 r. 14·56 per., and apothem 72·42 yards.

Solution.

Here A = 3 a. 2 r. 14·56 per. = 17380·44 sq. yds., $n = 8$, and $a = 72{\cdot}42$.

Then by formula (XXIV), $s = \frac{2A}{an} = \frac{17380{\cdot}44 \times 2}{72{\cdot}42 \times 8} = 59{\cdot}998$, *i. e.*, say 60 yds.

EXERCISE VI.

1. What is the area of a regular undecagon whose side is 20? Ans. 3746·248.
2. What is the area of a regular heptagon whose side is 60 yards? Ans. 13082.076 square yards.

3. Find the number of sides in a regular polygon whose area is 123·1072 square yards, its side being 4 yards and apothem 6·15536 yards. Ans. 10 sides, a decagon.

4. Find the length of each side of a regular hexagon whose area is 4156·915 square yards, and apothem 34·6408.

Ans. 40 yards.

CIRCLE.

FORMULÆ. Let $d =$ diameter, $r =$ radius, $c =$ circumference, and $\pi = 3{\cdot}1416$.

Then $c = \pi d$ (XXVI); $d = \dfrac{c}{\pi} = {\cdot}3183c$ (XXVII);

$A = \frac{1}{4}cd$ (XXVIII); $d = \dfrac{9A}{c}$ (XXIX); $c = \dfrac{4A}{d}$ (XXX);

$A = \pi r^2$ (XXXI); $r = \sqrt{\dfrac{A}{\pi}} = \sqrt{3183A}$ (XXXII);

$A = {\cdot}7854d^2$ (XXXIII); and $A = {\cdot}796c^2$ (XXXIV).

Ex. 1. Find the diameter and circumference of a circular garden which contains as much ground as an equilateral triangle whose side is 600 links.

Solution.

Area of equilateral triangle by formula (XIX) $= {\cdot}433b^2 = {\cdot}433 \times 360000 = 155880$ links.

Then by formula (XXXII), $d = 2r = 2 \times \sqrt{3183 \times 155880} = 2\sqrt{49616064} = 2 \times 222{\cdot}74 = 445{\cdot}48$ links.

Also by formula (XXVI), $= c\pi d = 3{\cdot}1416 \times 445{\cdot}48 = 1399{\cdot}52$ links.

Ex. 2. Find the area of a circle whose diameter is 200 yards.

Solution.

Here $d = 200 \therefore r = 100$; then by formula (XXXI), $A = \pi r_2 = 3{\cdot}1416 \times 100^2 = 3{\cdot}1416 \times 10000 = 31416$ sq. yds; or, by formula (XXXIII), $A = {\cdot}7854d^2 = {\cdot}7854 \times 40000 = 31416$ square yards.

EXERCISE VII.

1. Find the area of a circle whose circumference is 91. Ans. 659·1676.
2. Find the area of a circle whose circumference is 100 perches. Ans. 4 acres 3 roods 36 perches.
3. What is the diameter of a circle whose area is 1256 64 square yards? Ans. 40 yards.
4. What is the circumference of the earth, the mean diameter being 7921 miles? Ans. 24884·6136 miles.
5. What is the diameter of a circle whose circumference is 6850? Ans. 2180·4176.
6. A man has a circular meadow of which the diameter is 875 yards and wishes to exchange it for a square one of equal size: what must be the side of the square? Ans. 775·425.

CIRCULAR ANNULUS.

FORMULÆ. $A = \frac{\pi}{4}(d+d')(d-d')$ (XXXV) where d and d' are the diameters.

$A = {\cdot}0796(c+c')(c-c')$ (XXXVI) where c and c' are the two circumferences.

$A = \frac{1}{4}(c+c')(d-d')$ (XXXVII).

Ex. 1. Find the area of the annulus contained between two concentric circles whose diameters are 12 and 8 feet.

Solution.

By formula (XXXV), $A = \frac{3{\cdot}1416}{4} \times (12 \times 8)(12 - 8) = \frac{3{\cdot}1416 \times 20 \times 4}{4}$
$= 3{\cdot}1416 \times 20 = 62{\cdot}832$ square feet.

Ex. 2. What is the area of a circular annulus, the circumferences of the circles being 60 and 40 feet?

Solution.

By formula (XXXVI), $A = {\cdot}0796 \times (60 \times 40) \times (60 - 40) = {\cdot}0796 \times 100 \times 20$
$= 159{\cdot}2$ square feet.

Exercise VIII.

1. Find the area of an annulus contained between two concentric circles whose circumferences are 20 and 50 feet.
Ans. 167·16 square feet.

2. Find the area of an annulus contained between two concentric circles whose diameters are 30 and 20 yards.
Ans. 392·7 square yards.

3. What is the area of a circular annulus, the diameters of the circle being 20 and 50 and the circumferences 62·832 and 157·08? Ans. 1649·34.

LENGTH OF CIRCULAR ARC; CHORD OF ARC; CHORD OF HALF THE ARC.

FORMULÆ. $l = \frac{\pi n d}{360} = \cdot 008726nd$ (XXXVIII) where n = number of degrees in the arc and d = the diameter of the circle.

$k = 2\sqrt{r^2 - a^2}$ (XXXIX); $a = \frac{1}{2}\sqrt{4r^2 - k^2}$ (XL);

$k' = \sqrt{2r(r-a)}$ (XLI); and $r = \frac{k^2}{2h}$ (XLII).

Where k = chord of whole arc, k' = chord of half the arc, r = radius, a = apothem or perpendicular from centre on the chord, and consequently $r - a = h$ = height of the segment.

Ex. 1. Find the length of a circular arc of 140°, the diameter of the circle being 80 yards.

Solution.

By formula (XXXVIII), $l = \cdot 008726 \times 140 \times 80 = 97\cdot 7312$ yards.

Ex. 2. Find the chord of the arc whose apothem is 12 and radius 15·205.

Solution.

By formula (XXXIX), $k = 2\sqrt{r^2 - a^2} = 2\sqrt{15\cdot 6205^2 - 12^2} = 2\sqrt{244 - 144}$
$= 2\sqrt{100} = 2 \times 10 = 20.$

Ex. 3. Find the chord of half the arc whose height is 6 and radius 18·75.

Solution.

By formula (XLI), $k, = \sqrt{2r(r-a)} = \sqrt{2rh}$, since $h = (r-a) = \sqrt{2 \times 18{\cdot}75 \times 6}$ $= \sqrt{225} = 15$.

Exercise IX.

1. The radius of a circle is 30·8058 yards, the chord of an arc thereof is 36 yards, required its apothem and the chord of half the arc. Ans. 25 yards; 18·91 yards.
2. What is the chord of an arc whose height is 4, the radius of the circle being $56\frac{1}{4}$? Ans. 41·66.
3. Find the length of a circular arc of 108°, the radius of the circle being 75 feet. Ans. 141·3612 feet.
4. Find the chord of an arc whose apothem is 20 and the radius of the circle 40 yards. Ans. 69·282 yards.
5. What is the apothem of an arc whose chord is 90 and radius 70 feet? Ans. 53·619 feet.

SECTOR OF CIRCLE; SEGMENT; LUNE.

FORMULÆ. $A = \frac{1}{2}lr$ (XLIII). Also, from (XXXVIII) and (XLIII), $A = {\cdot}008726nr^2$ (XLIV) where l = length of arc, n = number of degrees it contains, and r = radius.

SEGMENT. Let A = area of corresponding sector and A' = area of associate triangle; then area $= A + A'$ (XLV) according as the segment is greater or less than a semicircle.

LUNE. Let A = area of greater segment and A' = area of smaller; then area $= A - A'$ (XLVI).

Ex. 1. Find the area of a sector of a circle whose radius is 90 yards, the arc of the sector being 80 yards in length.

Solution.

By formula (XLIII), $A = \frac{1}{2} \times 80 \times 90 = 3600$ square yards.

Ex. 1. Find the area of a circular sector whose arc contains 120°, the radius of the circle being 40 feet.

Solution.

By formula (XLIV), $A = \cdot 008726 \times 120 \times 40^2 = 1675\cdot 392$ square feet.

Ex 3. Find the area of a circular segment whose arc contains 150°, the diameter of the circle being 60 yards and the apothem of arc 6 yards.

Solution.

Here length of arc, by formula (XXXVIII), $= \cdot 008726 \times 150 \times 60 = 78\cdot 534$.
Area of sector $= \frac{1}{2}lr = \frac{1}{2} \times 78\cdot 534 \times 30 = 1178\cdot 01$ square yards.
Area of triangle whose base is $58\cdot 787$ and altitude (apothem), 6
$= \frac{1}{2} \times 58\cdot 787 \times 6 = 176\cdot 36$.
Hence area of segment $= 1178\cdot 01 - 176\cdot 361 = 1001\cdot 649$ square yards.

NOTE.—We subtract because the segment is less than a semicircle.

Ex. 4. Find the area of a lune the outer arc containing 240° and the inner one 32°, the radius of the smaller circle being 23 feet and of the larger 80 feet, the common chord being 38.

Solution.

Here by formula (XI), apothem of larger arc $= \frac{1}{2} \times \sqrt{4 \times 23^2 - 38^2}$
$= \frac{1}{2} \times \sqrt{2116 - 1444} = \frac{1}{2} \times \sqrt{672} = \frac{1}{2} \times 25\cdot 9229 = 12\cdot 96$ square feet, and length of arc $= 95\ 225$.
Similarly apothem of smaller arc $= 77\cdot 7$ feet and length of arc $= 44\cdot 68$.
Of smaller circle area of sector $= A\ \frac{1}{2}lr = \frac{1}{2} \times 96\cdot 225 \times 23 = 1106\cdot 587$ sq. feet, and area of triangle $= \frac{1}{2} \times 38 \times 12\cdot 96 = 246\cdot 24$ sq. ft. Then since segment is greater than semicircle; A of segment $= 1106\cdot 587 + 246\cdot 24$ $= 1352\cdot 827$ square feet $=$ area of greater segment.
Of greater circle, area of sector $= A = \frac{1}{2}lr = \frac{1}{2} \times 44\cdot 68 \times 80 = 1787\cdot 2$ sq. ft. and of associate triangle, area $= \frac{1}{2}bp = \frac{1}{2} \times 38 \times 77\cdot 7 = 1476\cdot 3$ square feet $\therefore$ A of smaller segment $= 1787\cdot 2 - 1476\cdot 3 = 310\cdot 9$ square feet.
Hence area of lune $= A - A' = 1352\cdot 827 - 310\cdot 9 = 1041\cdot 927$ square feet.

EXERCISE X.

1. What is the area of a sector whose arc contains 36° and whose radius is 3 feet? Ans. $2\cdot 8272$.

2. What is the area of a circular sector whose arc is 650 feet in length and whose radius is 325 feet? Ans. 105625 sq. feet.
3. Find the area of a segment of a circle, the arc containing 280°, the radius being 5 feet and apothem 3 feet.

Ans. 73·082.

ELLIPSE.

FORMULÆ. Let C = circumference, t = transverse axis; c = conjugate axis, a = absciss, o = ordinate.

Then $C = \pi\sqrt{\dfrac{t^2+c^2}{2}}$ (XLVII) $A = \dfrac{\pi tc}{4} = \cdot 7854tc$ (XLVIII);

$o = \dfrac{c}{t}\sqrt{(t-a)a}$ (XLIX); $a = \dfrac{t}{2} \pm d$ and d'

$= \dfrac{t}{c}\sqrt{\left(\dfrac{c}{2}+o\right)\left(\dfrac{c}{2}-o\right)}$ (L); $t = \dfrac{ca}{o^2}\left\{\dfrac{c}{2}+\sqrt{\dfrac{c^2}{4}-o^2}\right\}$ (LI;)

$c = \dfrac{ot}{\sqrt{(t-a)a}}$ (LII).

Ex. 1. Find the transverse axis of an ellipse whose conjugate axis is 15, an ordinate 6 and the smaller absciss 9.

Solution.

By formula (LI), $t = \dfrac{ca}{o^2}\left\{\dfrac{c}{2}+\sqrt{\dfrac{c^2}{4}-o^2}\right\} = \dfrac{15\times 9}{6^2}\left\{\dfrac{15}{2}+\sqrt{\dfrac{225}{4}-36}\right\}$
$= \frac{135}{36}\{\frac{15}{2}+\sqrt{\frac{81}{4}}\} = \frac{15}{4}\left(\frac{15}{2}+\frac{9}{2}\right) = \frac{15}{4}\times 12 = 45.$

Ex. 2. Find the ordinate of an ellipse whose axes are 45 and 15 and one absciss 9.

Solution.

By formula (XLIX), $o = \frac{c}{t}\sqrt{(t-a)a} = \frac{15}{45}\times\sqrt{(45-9)\times 9} = \frac{1}{3}\sqrt{36\times 9}$
$= \frac{1}{3}\times\sqrt{324} = \frac{1}{3}\times 18 = 6.$

Ex. 3. Find the area of an ellipse whose axes are 30 and 40.

Solution.

By formula (XLVIII(, $A=\frac{\pi t c}{4}=\cdot7854\times30\times40=942\cdot48$.

EXERCISE XI.

1. Find the circumference of an ellipse whose axes are 20 and 16. Ans. 56·8943.
2. What are the abscisses of an ellipse whose axes are 80 and 120 and an ordinate 25? Ans. 106·836 and 13·163.
3. What is the area of an ellipse whose axes are 28 and 20 chains? Ans. 43 a. 3 r. 37 per. 5 yds.
4. What is the ordinate of an ellipse of which the axes are $25\frac{1}{4}$ and $18\frac{1}{4}$ and one absciss $7\frac{1}{2}$? Ans. 8·429.
5. What is the area of an elliptical park of which the conjugate axis is 1800 links, an ordinate 400 links, and the smaller absciss 600 links? Ans. 162 a. 3 r. 10 per. 28 yds.
6. What is the area of an ellipse whose transverse axis is 100, an ordinate being 20 and the greater absciss 75? Ans. 3627·44.

PARABOLA.

FORMULÆ. Let $p=$ parameter, a and $a'=$ any two abscisses, o and o' their corresponding ordinates, $b=$ base or double ordinate, and $l=$ length of parabolic curve.

Then $p=\frac{o^2}{a}$ (LIII); $o'=o\sqrt{\left(\frac{a'}{a}\right)}$ (LIV); $a'=a\left(\frac{o'}{o}\right)^2$ (LV);

$l=2\sqrt{(o^2+\frac{4}{3}a^2)}$ (LVI); $A=\frac{2}{3}ab$ (LVII).

For parabolic zone, $A=\frac{2}{3}h\left(b'+\frac{b^2}{b+b'}\right)$ (LVIII) where h $=$ height of zone and b and $b'=$ bases or double ordinates.

Ex. 1. Find the parameter of a parabola whose ordinate is 25 and absciss 12.

Solution.

By formula (LIII), $p = \dfrac{o^2}{a} = \dfrac{25^2}{12} = \dfrac{625}{12} = 52\tfrac{1}{12}$.

Ex. 2. Find the area of a parabola whose base or double ordinate is 30 and height 22.

Solution.

By formula (LVIII), $A = \frac{2}{3}ab = \frac{2}{3} \times 30 \times 22 = 440$.

Ex. 3. Find the length of a parabolic curve of which the ordinate and absciss are respectively 30 and 8.

Solution.

By formula (LVI), $l = 2\sqrt{o^2 + \frac{4}{3}a^2} = 2\sqrt{900 + \frac{4}{3} \times 64} = 2\sqrt{900 + \frac{256}{3}}$
$= 2\sqrt{\frac{2956}{3}} = \frac{2}{3}\sqrt{8868} = \frac{2}{3} \times 94{\cdot}17 = 62{\cdot}78$.

EXERCISE XII.

1. Given an ordinate of a parabola, 60 and its absciss 42, find, the parameter. Ans. 85·7.
2. Two ordinates are 40 and 30 and the absciss of the former 21, find that of the latter. Ans. 11·8125.
3. Find the area of a parabola whose base is 75 and height 48 chains. Ans. 240 acres.
4. Find the area of parabolic zone whose parallel ends are 12 and 16 and height 8. Ans. 112·76.
5. Find the length of a parabolic curve whose absciss is 12 and ordinate 15. Ans. 40·841.
6. What is the ordinate of a parabola whose absciss is 20; a second absciss and ordinate being 6 and 4 respectively. Ans. 7·302.

HYPERBOLA.

Symbols same as for ellipse and parabola.

FORMULÆ $o = \frac{c}{t}\sqrt{(t+a)a}$ (LIX); $a = d \pm \frac{t}{2}$ and

$$d = \frac{t}{c}\sqrt{\frac{c^2}{4}+o^2} \text{ (LX)}; \quad c = \frac{ot}{\sqrt{(t+a)a}} \text{ (LXI)};$$

$$t = \frac{ca}{o^2}\left\{\frac{c}{2} \pm \sqrt{\frac{c^2}{4} \div o^2}\right\} \text{ (LXII)};$$

$$A = \frac{4ca}{75t}\{3\sqrt{7a}(7t+5a)+4\sqrt{ta}\} \text{ (LXIII)}.$$

1. What is the ordinate of an hyperbola of which the axes are 30 and 15, and the smaller absciss 10?

Solution.

By formula (LIX), $o = \frac{c}{t}\sqrt{(t+a)x} = \frac{15}{30}\sqrt{(30+10)\times 10} = \frac{1}{2}\sqrt{400} = 10.$

Ex. 2. What is the transverse axis of an hyperbola whose conjugate axis is 36, ordinate 12 and smaller absciss 20?

Solution.

By formula (LXII), $t = \frac{ca}{o^2}\left\{\frac{c}{2} \pm \sqrt{\frac{c^2}{4}+o^2}\right\} \frac{36\times 20}{144}\left\{\frac{36}{2}+\sqrt{\frac{1296}{4}+144}\right\}$

$$= 5\times\left(18+\sqrt{\frac{1872}{4}}\right) = 5\times\left(18+\frac{43\cdot 26}{2}\right) = 5\times\frac{36+43\cdot 2}{2} = 198\cdot 16.$$

Ex. 3. Find the area of an hyperbola whose axes are 60 and 45, the smaller absciss being $7\frac{1}{2}$.

Solution.

By formula (LXIII), $A = \frac{4ca}{75t}\{3\sqrt{7a(7t+5a)}+4\sqrt{ta}\} = \frac{4\times 45\times 7\cdot 5}{75\times 60}$

$$\times\{3\sqrt{7\times 7\cdot 5(7\times 60+5\times 7\cdot 5)}+4\sqrt{60\times 7.5}\}$$

$$= \tfrac{3}{10}\{3\sqrt{52\cdot 5\times(420-37\cdot 5)}+4\sqrt{450}\} = \tfrac{3}{10}\times(3\times 141\cdot 708+3\times 28\cdot 284$$

$$= \tfrac{9}{10}(141\cdot 708+28\cdot 284) = 152\cdot 99.$$

EXERCISE XIII.

1. Find the transverse axis of an hyperbola whose ordinate is 20, smaller absciss $10\frac{2}{3}$, and conjugate axis 30. Ans. 50.
2. What are the abscisses of an hyperbola whose axes are 30 and 25 and the ordinate 16? Ans. 39·36 and 9·36
3. What is the area of an hyperbola whose axes are 45 and 90, the smaller absciss being 30?
4. The conjugate axis of an hyperbola is 45, the ordinate 90, and the smaller absciss $7\frac{1}{2}$, find the transverse axis. Ans. 22·5.
5. The axes of an hyperbola are 15 and 20, and an ordinate 10, find the abscisses. Ans. $26\frac{2}{3}$ and $6\frac{2}{3}$.
6. Find the area of an hyperbola whose axes are 55 and 33 chains, and smaller absciss $18\frac{1}{3}$ chains.

Ans. 50 a. 3. r. 37 per.

MENSURATION OF SOLIDS.

The Five Regular Solids.

Let s = surface, v = volume or solid contents, and e = one of the edges.

TETRAHEDRON OR REGULAR TRIANGULAR PYRAMID.

$s = e^2\sqrt{3} = 1{\cdot}732e$ (LXIV); and

$v = \frac{1}{2}e^3\sqrt{2} = {\cdot}11785e^3$ (LXV).

HEXAHEDRON OR CUBE. $s = 6e^2$ (LXVI); $v = e^2$ (LXVII).

OCTAHEDRON.

$s = 2e^2\sqrt{3} = 3{\cdot}464e^2$ (LXVIII);

$v = \frac{1}{2}e^3\sqrt{2} = {\cdot}471405e^3$ (LXIX).

DODECAHEDRON.

$s = 15e^2\sqrt{\frac{1}{5}(8 + 2\sqrt{5})} = 20{\cdot}645775e^2$ (LXX);

$v = 5e^3\sqrt{\frac{45 + 21\sqrt{5}}{40}} = 7{\cdot}6631e^3$ (LXXI).

ICOSAHEDRON.

$s = 5e^2\sqrt{3} = 8{\cdot}66e^2$ (LXXII);

$v = \frac{5}{6}e^3\sqrt{\frac{1}{2}(7+3\sqrt{5})} = 2{\cdot}18169e^3$ (LXXIII).

Ex. 1. Find the surface and volume of a tetrahedron whose edge is 20 feet.

Solution.

By formula (LXIV), $s = 1{\cdot}732e^2 = 1{\cdot}732 \times 20^2 = 692{\cdot}8$ square feet.
By formula (LXV), $v = {\cdot}11785e^3 = {\cdot}11785 \times 20^3 = 942{\cdot}8$ cubic feet.

Ex. 2. Find the surface and solidity of a hexahedron or cube whose edge is 9 feet.

Solution.

By formulas (LXVI) and (LXVII), $s = 6e^2 = 6 \times 9^2 = 486$ square feet, and $v = e^3 = 9^3 = 729$ cubic feet.

Ex. 3. Find the surface and cubic contents of a dodecahedron whose edge is 4 feet 2 inches.

Solution.

By formula (LXX), $s = 20{\cdot}645775e^2 = 20{\cdot}645785 \times 50^2$ (since 4 ft. 5 in. = 50 in.) $= 51614{\cdot}4375$ square inches $= 358{\cdot}483$ square feet.
By formula (LXXI), $v = 5{\cdot}6631e^3 = 7{\cdot}6631 \times 50^3 = 957913{\cdot}75$ cubic inches $= 554{\cdot}849$ cubic feet.

EXERCISE XIV.

1. Find the surface and cubic contents of an icosahedron whose edge is 4. Ans. 138·56 and 139·628.
2. Find the surface and solidity of a cube or hexahedron whose edge is 20. Ans. 2400 and 8000.
3. Find the surface and volume of a tetrahedron whose edge is 8. Ans. 110·848 and 60·339.
4. Find the surface and solid contents of a dodecahedron whose edge is 10. Ans. 2064·5775 and 7663·1.
5. Find the surface and volume of an octahedron whose edge is 11. Ans. 419·144 and 627·44.
6. Find the surface and cubic contents of an icosahedron whose edge is 5 yds. Ans. 216·506 sq. yds. and 272·711 cub. yds.

RIGHT AND OBLIQUE PARALLELOPIPEDS, PRISMS, CYLINDERS.

Let a = area of base or end, p = perimeter of base, and p' = the perimeter of a section perpendicular to one of the edges of the solid; also let h = the height, s = the whole surface.

Then $v = ah$ (LXXIV), $s = hp + 2a$ (LXXV) when the solid is right, $s = hp' + 2a$ (LXXVI) when the solid is oblique.

Ex. 1. What are the surface and volume of a prism whose height is 20 feet and base an equilateral triangle, each side of which is 4 feet?

Solution.

By formula (XIX), $a = \cdot 433 b^2 = \cdot 433 \times 4^2 = 6\cdot 928$ square feet = area of base and perimeter $= 4 \times 3 = 12$.

Then by formula (LXXV), $s = hp + 2a = 20 \times 12 + 2 \times 6\cdot 928 = 240 + 13\cdot 856 = 253\cdot 856$ square feet = surface.

Also by formula (LXXIV), $v = ah = 6\cdot 928 \times 20 = 138\cdot 56$ cubic feet.

Ex. 2. Find the surface and solid contents of an oblique prism whose base is a regular hexagon, with edge = 10 inches, the lateral edges of the prism being 40 feet long and the perimeter of a section perpendicular to them $4\frac{1}{2}$ feet.

Solution.

By formula (XXIII), area of base $= A = \frac{1}{2}ans = \frac{1}{2} \times 8\cdot 66025 \times 6 \times 10 = 259\cdot 8075$ square inches $= 1\cdot 8042$ square feet.

Then $s = hp' + 2a = 40 \times 4\frac{1}{2} + 2 \times 1\cdot 8042 = 180 + 3\cdot 6084 = 183\cdot 6084$ sq. ft.

Also $v = ah = 1\cdot 8042 \times 40 = 72\cdot 168$ cubic feet.

Ex. 3. Find the surface and solidity of a right cylinder whose height is 20 feet and diameter 12 feet.

Solution.

By formula (XXXI), area of the base $= A = \pi r^2 = 3\cdot 1416 \times 6^2 = 113\cdot 0976$ sq. ft.

Also $c = 12 \times 3\cdot 1416 = 37\cdot 6992$ feet.

Then by formula (LXXV), $s = hp + 2a = 20 \times 37{\cdot}6992 + 2 \times 113{\cdot}0976 = 753{\cdot}984 + 226{\cdot}1952 = 980{\cdot}1732$ square feet.
Also $v = ah = 113{\cdot}0976 \times 20 = 2261{\cdot}952$ cubic feet.

Ex. 4. How many gallons of water will a cylindrical cistern contain, whose diameter is 6 feet and depth 7 feet?

Solution.

Area of base $= 3{\cdot}1416 \times 3^2 = 37{\cdot}2744$ square feet.
Hence volume $= ah = 37{\cdot}2744 \times 7 = 260.9208$ cubic feet.
Then since each cubic foot contains $6\frac{1}{4}$ gallons $\times 6\frac{1}{4} = 1628{\cdot}88$ gallons.

Exercise XV.

1. Find the surface and cubic contents of a rectangular parallelopiped whose height is 25 feet, its base being 4 feet wide and 5 feet long. Ans. 490 sq. ft.; 500 cub. ft.
2. How many gallons of water are contained in a circular cistern whose diameter is 12 feet and depth 10 feet?
Ans. 7068·6 gallons.
3. Find surface and solidity of a right prism whose base is an octagon, each side of which is 2 feet, the edges of the prism being each 18 feet long.
Ans. 326·6274 sq. ft. and 347·6448 cub. ft.
4. Find the surface and solidity of an oblique prism, each end being a regular nonagon whose side is 20 inches, the edges of the prism being 14 feet long and the perimeter of a section perpendicular to them being 13 feet.
Ans. 216·3434 sq. ft.; 240·4038 cub. ft.
5. How many pails of water are contained in a pentagonal cistern whose depth is 15 feet, each edge of the bottom being 6 feet?
Ans. 2322·6446 pails.

Note. A pail holds 10 quarts.

6. Find the surface and solidity of a right cylinder whose height is 42 feet and circumference 22 inches.
Ans. 77·535 sq. ft. and 11·2368 cub. ft.

REGULAR PYRAMID OR CONE.

Formulæ. Let p = perimeter of base, l = length of slant side, h = height of vertex above the base, and a = area of the base.

Then $v = \frac{1}{3}ah$ (LXXVII); and $s = \frac{1}{2}pl + a$ (LXXVIII.)

Ex. 1. Find the surface and solidity of a regular cone whose slant side is 20 feet and the diameter of the base 10 feet.

Solution.

By formula (XI), $h = \sqrt{20^2 - 5^2} = \sqrt{400 - 25} = \sqrt{375} = 19\cdot3649$.

By formula (XXVI), $c = \pi d = 3\cdot1416 \times 10 = 31\cdot416$.

By formula (XXXI), $a = \pi r^2 = 3\cdot1416 \times 5^2 = 3\cdot1416 \times 25 = 78\cdot54$.

By formula (LXXVII), $v = \frac{1}{3}ah = \frac{1}{3} \times 78.54 \times 19\cdot3649 = 1520\cdot9192$ cub. ft.

By formula (LXXVIII), $s = \frac{1}{2}pl + a = \frac{1}{2} \times 31\cdot416 \times 20 + 78\cdot54 = 392\cdot7$ sq. ft.

Ex. 2. Find the solidity and surface of a right pyramid whose slant side is 24 feet, the base being a pentagon whose diameter is 10 feet.

Solution.

By formula (XXIII), $a = \frac{1}{2}ans = \frac{1}{2} \times 0\cdot68819 \times 5 \times 10 = 17\cdot204$.
Also perimeter $= 5 \times 10 = 50$ feet.

By formula (XI), $h = \sqrt{24^2 - 6\cdot88^2} = \sqrt{576 - 47\cdot33} = \sqrt{528\cdot66} = 22\cdot99$.

By formula (LXXVII), $v = \frac{1}{3}ah = \frac{1}{3} \times 17\cdot204 \times 22\cdot99 = 131\cdot839$ cub. ft.

By formula (LXXVIII),) $= \frac{1}{2}pl + a = \frac{1}{2} \times 50 \times 24 + 17\cdot204 = 617\cdot204$ sq. ft.

Exercise XVI.

1. Find the solidity and surface of a right cone whose slant side is 10 feet and the diameter of the base 6 feet.
   Ans. 122·552 sq. ft. ; 89·905 cub. ft.
2. Find the surface and solidity of a regular square pyramid whose slant side is 20 feet and area of the base 576 sq. ft.
   Ans. 1536 sq. ft. ; 3072 cub. ft.
3. Find the surface and solidity of a right cone whose base has a diameter of 14 feet and whose height is 10 feet.
   Ans. 422·3629 sq. ft. ; 513·139 cub. ft.
4. Find the surface and solidity of a right pyramid whose height is 18 feet, its base being a regular hexagon whose side is 9 feet. Ans. 500·955 sq. ft. ; 1262·664 cub. ft.

FRUSTUM OF PYRAMID OR CONE.

FORMULÆ. Let a and a' = areas of the ends, h = height, e and e' = edges of ends of pyramid, p and p' = perimeters of ends; and, in case of cone, d and d' $\sqrt{aa'}$ = diameters of ends.

$v = \frac{1}{3}h(a + a' + \sqrt{aa'})$ (LXXIX);

$v = \frac{1}{3}h\left(\frac{ae - a'e'}{e - e'}\right)$ (LXXX);

$v = \cdot2618h(d^2 + d_{,}^2 + dd_{,})$ (LXXXIII);

$s = \frac{1}{2}(p + p')l + a + a'$ (LXXXI).

Ex. 1. Find the surface and solidity of a right cone, whose slant side is 20 feet, the diameters of the end being 4 and 2 feet.

Solution.

Here the radii are 1 and 2 feet and their difference is 1 foot.

Hence height $= \sqrt{20^2 - 1^2} = \sqrt{400 - 1} = \sqrt{399} = 19\cdot9749$.

By Formula (LXXXIV), $v = \cdot2618h(d^2 + d_{,}^2 + dd_{,})$
$= \cdot2618 \times 19\cdot9749 \times (2^2 + 4^2 + 2 \times 4) = \cdot2618 \times 19\cdot9749 \times 28$
$= 146\cdot424$ cub. feet.

By formula (LXXXI) $s = \frac{1}{2}(p + p')l + a + a_{,}$
$= \frac{1}{2} \times 18\cdot8496 \times 20 + 12\cdot5664 + 3\cdot1416 = 188\cdot496 + 15\cdot708 = 204\cdot204$ sq. ft

Ex. 2. Required the surface and solidity of a frustum of a regular hexagonal pyramid, the sides of its ends being 6 and 4 feet respectively, and its length 24 feet.

Solution.

By formula (XXIII), areas of ends $= \frac{1}{2}ans = \frac{1}{2} \times 5\cdot19615 \times 6 \times 6 = 93\cdot5307$; and $\frac{1}{2} \times 3\cdot4641 \times 6 \times 4 = 41\cdot5692$.

Difference of apothems of hexagons whose sides are 6 and 4 feet $= 5\cdot1961$ and $3\cdot4641 = 1\cdot732$.

Hence by formula (XI), height of frustum $= \sqrt{24^2 - 1\cdot732^2}$
$= \sqrt{576 - 3 \text{ (nearly)}} = \sqrt{573} = 23\cdot937$.

Then by formula (LXXIX), $v = \frac{1}{3} \times h \times (a + a_{,} + \sqrt{aa_{,}})$
$= \frac{1}{3} \times 23\cdot937 \times (93\cdot5307 + 41\cdot5692 + \sqrt{93\cdot5307 \times 41\cdot5692})$
$= 7\cdot979 \times (135\cdot0999 = \sqrt{3887\cdot996374}) = 7.979 + 135\cdot0999 + 62\cdot353)$
$= 7\cdot979 \times 197\cdot4529 = 1575\cdot476$ cub. ft.

By formula (LXXXI), $s = \frac{1}{2}(36 + 24) \times 24 + 93\cdot5307 + 41\cdot5692$,
$= 30 \times 24 + 135\cdot0999 = 720 + 135\cdot0999 = 855\cdot0999$ sq. ft.

WEDGE.

FORMULÆ. Let l = length of back and b = breadth of back, e = length of edge, and h = height.

Then $v = \frac{1}{6}\,bh(e + 2l)$ (LXXXV).

Ex. 1. The length and breadth of the base of a wedge are 70 inches and 15 inches, the edge is 110 inches in length, and the height is 17·145 inches; what are its solid contents?

Solution.

Here $b = 15$ in $= 1{\cdot}25$ ft.; $h = 17{\cdot}145$ in $= 1{\cdot}42875$ ft.; $e = 110$ in $= 9\frac{1}{6}$ ft. and $l = 70$ in $= 5\frac{5}{6}$ ft.

By formula (LXXXV). $v = \frac{1}{6}bh(e + 2l(= \frac{1}{6} \times 1{\cdot}25 \times 1{\cdot}42875) \times 9\frac{1}{6} + 5\frac{5}{6} \times 2)$

$= \frac{1}{6} \times 1{\cdot}25 \times 1{\cdot}42875 \times 20\frac{5}{6} = 5{\cdot}767$ cub. ft.

Ex. 2. Find the solidity of a wedge whose base is 6 inches long and 4 wide, its edge being 16 inches in length and height 15·8745 inches.

Solution.

By formula (LXXXV), $v = \frac{1}{6}bh(e + 2l) = \frac{1}{6} \times 4 \times 15{\cdot}8745 \times (16 + 2 \times 6$

$= \frac{1}{6} \times 4 \times 15{\cdot}8745 \times 28 = \frac{1}{2} \times 15{\cdot}8745 \times 56 = 56 \times 5{\cdot}2915 = 286{\cdot}324$ cub. in

EXERCISE XVII.

1. Find the solid contents of a wedge whose length is 64 inches, the edge being 42 inches long, the base 9 inches broad, and the height of the wedge 28 inches.

   Ans. 4 cub. ft 288 cub. in.

2. Find the solid contents of a wedge whose height is 20 inches, the base 12 inches wide and 15 inches long, the edge being 24 inches.

   Ans. 1 cub. ft. 432 cub. in.

3. Find the solidity of a wedge whose edge is 2·7 feet long, and back 3·2 feet long, the breadth of the back being40 inches and the height of the wedge 4 feet.

   Ans. 20 cub. ft. 384 cub. in.

SPHERE, SPHERICAL SEGMENT.

FORMULÆ. Let d = diameter. Then

FOR SPHERE; $v = \cdot 5236d^3$ (LXXXVI); and $s = \pi d^2$ (LXXXVII).

FOR SPHERICAL SEGMENT, let r = radius of base, and h = height of segment, d = diameter of sphere and s = convex surface.

$v = \cdot 5236h\ (3r^2 + h^2)$ (LXXXVIII);

$v = \cdot 5226h^2\ ({}^3d - {}^2h$ (LXXXIX); and $s = \pi dh$ (XC).

Ex. 1. The diameter of a sphere is 50 inches, required its solidity and surface.

Solution.

By formula (LXXXVI), $v = \cdot 5236d^3 = \cdot 5236 \times 50^3 = \cdot 5236 \times 125000 = 523 \cdot 6 \times 125 = 65450$ cub. in.

By formula (LXXXVII), $s = \pi d^2 = 3 \cdot 1416 \times 50^2 = 3 \cdot 1416 \times 2500 = 314 \cdot 16 \times 25 = 7854$ sq. inches.

Ex. 2. Find the convex surface and the solidity of a spherical segment whose height is 2 feet, the diameter of the sphere being 5 feet.

Solution.

By formula (LXXXIX), $v = \cdot 5236h^2\ (3d - 2h) = \cdot 5236 \times 2^2 \times (15 - 4) = \cdot 5236 \times 4 \times 11 = 23 \cdot 0384$ cub. ft.

By formula (XC), $s = \pi dh = 3 \cdot 1416 \times 5 \times 2 = 31 \cdot 416$ square feet for convex surface.

EXERCISE XVIII.

1. Find the solidity of a sphere whose diameter is 16 feet.
   Ans. 2144·6656 cub. ft.
2. Find the surface of a globe whose diameter is 24 feet.
   Ans. 1809 ft. 80·87 sq. in.
3. What is the solidity of a sphere whose diameter is 6 feet?
   Ans. 113·0976 cub. ft.

4. What is the surface of a sphere whose diameter is 16 inches?
Ans. 5 sq. ft. 84·24 sq. in.

5. Find the solidity and surface of a sphere whose diameter is 12 feet. Ans. 409·7808 cub. ft.; 452·3904 sq. ft.

6. Find the solidity and surface of a sphere whose diameter is 15 feet. Ans. 1767·15 cub. ft.; 706·86 sq. ft.

7. What is the solidity of a spherical segment, the height being 4 feet and diameter of the base 14 feet? Ans. 341·387 cub. ft.

8. What is the solidity of a spherical segment whose height is 2 feet, the diameter of the sphere being 8 feet?
Ans. 41·888 cub. ft.

9. Find the solidity and surface of a spherical segment whose height is 5 feet, the diameter of the sphere being 12 feet.
Ans 340·34 cub. ft.; 188·496 sq. ft.

10. What are the solid contents and convex surface of a spherical segment whose height is 4 feet, the diameter of the sphere being 16 feet? Ans. 335·104 cub. ft.; 201·0624 sq. ft.

SPHERICAL ZONE.

Let r and $r_{,}$ = radii of the ends, d = diameter of end of zone, $d_{,}$ = diameter of sphere, and s = convex surface.

Then $v=\frac{\pi h}{2}(r^2+r_{,}^2+\frac{1}{3}h^2)$ (XCI); for other than middle zone.

For middle zone $v=\frac{\pi h}{4}(d^2+\frac{2}{3}h^2)$ (XCII);

$$v=\frac{\pi h}{4}(d_{,}^2-\frac{1}{3}h^2) \text{ (XCIII), and } s=\pi d_{,}h \text{ (XCIV).}$$

Ex. 1. Find the solidity of the middle zone of a sphere, the height of which is 5 inches, the diameter of the end being 25 inches.

Solution.

By formula (XCII), solidity $=\frac{\pi h}{4}(d+\frac{2}{3}h^2)=\frac{3·1416\times 5}{4}(25^2+\frac{2}{3}\times 5^2$

$=·7854\times 5(625+16\frac{2}{3})=3·928\times 641\frac{2}{3}=2520·466$ cub. in.

Ex. 2. Find the convex surface of a spherical zone whose height is 5 inches, the diameter of the sphere being 25 inches.

Solution.

By formula (XCIV), surface $= \pi dh = 3{\cdot}1416 \times 25 \times 5 = 392{\cdot}7$ inches.

Ex. 3. Find the solidity of a spherical zone whose height is 3 feet, the diameters of the ends being 4 feet and 5 feet.

Solution.

By formula (XCI), $v = \frac{\pi h}{2}(r^2 + r_1^2 + \frac{1}{3}h^2) = \frac{3{\cdot}1416 \times 3}{2}(4^2 + 5\ \ + \frac{1}{3} \text{ of } 3^2)$

$1{\cdot}5708 \times 3 \times (16 + 25 + 3) = 1{\cdot}5708 \times 3 \times 44 = 207{\cdot}3456$ cub. feet.

Exercise XIX.

1. Find the convex surface of a spherical zone whose height is 3 feet, the diameter of the sphere being 9 feet.
   Ans. 84·8232 sq. ft.
2. Find the solidity of a spherical zone whose height is 4 feet, the radii of its ends being 7 feet and 9 feet.
   Ans. 850·3264 cub. ft.
3. Find the solidity of a middle spherical zone whose height is 5 feet, the diameter of the end being 8 feet.
   Ans. 316·778 cub. ft.
4. Find the convex surface of a spherical zone whose height is 4 inches, the diameter of the sphere being 25 inches.
   Ans. 314·16 sq. in.
5. Find the solidity of the middle zone of a sphere whose height is 2 feet 8 inches, the diameter of the ends being 2 feet.
   Ans. 24·1242 cub. ft.
6. Find the volume of a spherical zone whose height is 4 feet, the diameter of its ends being 6 feet. Ans. 146·608.
7. Find the solidity of the middle zone of a sphere whose height is 7 feet, the diameter of the sphere being 12 feet.
   Ans. 701·8858 cub. ft.

PARABOLOID; FRUSTUM OF PARABOLOID.

Let a = area of base of paraboloid and h = height; also let a and a' = areas of ends of frustum, and d and $d_{\prime}$ their diameters.

Then for PARABOLOID, $v = \frac{1}{2}ah = \frac{\pi d^2 h}{8} = \cdot 3927\, d^2 h$ (XCV).

" FRUSTUM OF PARABOLOID, $v = \frac{1}{2}h(a + a')$ (XCVI),

$$\text{or } v = \frac{\pi h}{8}(d^2 + d_{\prime}^2) = \cdot 3927\, h(d^2 + d_{\prime}^2) \quad \text{(XCVII)}.$$

Ex. 1. Find the solidity of a paraboloid whose height is 10 feet, the diameter of the base being 20 feet.

Solution.

By formula (XCV), $v = \cdot 3927 d^2 h = \cdot 3927 \times 20^2 \times 10 = \cdot 3927 \times 400 \times 10 = 1570\cdot 8$ cubic feet.

Ex. 2. What is the volume of the frustum of a paraboloid whose end diameters are 30 and 24 inches, the height of the frustum being 9 inches?

Solution.

By formula (XCVII), $v = \cdot 3927 h(d^2 + d'^2) = \cdot 3927 \times 9 \times (900 + 576) = \cdot 3927 \times 9 \times 1476 = 5216\cdot 626$ cub. in.

EXERCISE XX.

1. Find the solidity of a paraboloid whose end diameter is 12 and height 15 feet. Ans. 848·232 cub. ft.
2. Find the solid contents of a paraboloid whose height is 12 inches, the end diameter being 10 inches. Ans. 471·24 cub. in.
3. What is the volume of the frustum of a paraboloid whose end diameters are 20 and 28, the height of the frustum being 14? Ans. 6509·3952.
4. What is the volume of the frustum of a paraboloid whose end diameters are 10 and 12 inches, the height of the frustum being 6 inches? Ans. 574·9128 cub. in.

SPHEROID; SEGMENT OF SPHEROID.

Let t = transverse and c = conjugate axis; h = height of segment. Then $v = {\cdot}5236ct^2$ (XCVIII) for oblate, and

$v = {\cdot}5236tc^2$ (XCIX) for prolate spheroid;

$v = {\cdot}5236(3c - 2h)\dfrac{t^2h^2}{c^2}$ (C) for circular segment of oblate spheroid, and

$v = {\cdot}5236(3t - 2h)\dfrac{c^2h^2}{t}$ (CI) for circular segment of prolate spheroid;

$v = {\cdot}5236(3t - 2h)\dfrac{ch^2}{t}$ (CII) for elliptical segment of oblate, and

$v = {\cdot}5236(3c - 2h)\dfrac{th^2}{c}$ (CIII) for elliptical segment of prolate spheroid.

Ex. 1. Find the solidity of a prolate spheroid whose transverse axis is 7 feet and conjugate axis 5 feet.

Solution.

By formula (XCIX), $v = {\cdot}5236tc^2 + {\cdot}5236 \times 7 \times 25 = 91{\cdot}63$ cub. ft.

Ex. 2. Find the solid contents of a circular segment of an oblate spheroid, the height of the segment being 3 inches and the axes of the spheroid 25 and 15 inches.

Solution.

By formula (C), $v = {\cdot}5236(3c\ 2h)\dfrac{t^2h^2}{c^2} = {\cdot}5236(3 \times 15 - 2 \times 3) \times \dfrac{625 \times 9}{225}$

$= {\cdot}5236 \times 39 \times 25 = 510{\cdot}51$ cub. in.

Ex. 3. Find the solidity of an elliptical segment of a prolate spheroid whose height is 10, the axes being 100 and 60.

Solution.

By formula (CIII), $v = {\cdot}5236(3c - 2h)\dfrac{th^2}{c} = {\cdot}5136(3 \times 60 - 2\ 10) \times \dfrac{100 \times 102}{60}$

$= {\cdot}5236 \times 160 \times \frac{1000}{6} = 13962\frac{2}{3}$.

Exercise XXI.

1. Find the solid contents of a prolate spheroid whose diameters are 12 and 16 feet. Ans. 1206·3744 cub. ft.
2. Find the solid contents of a circular segment of a prolate spheroid whose diameters are 24 and 40 inches, the height of the segment being 4 inches. Ans. 337·7748.
3. Find the solidity of an elliptical segment of an oblate spheroid whose diameters are 20 and 24, the height of the segment being 5 inches. Ans. 2028·95 cub. in.
4. Find the solidity of an oblate spheroid whose diameters are 16 and 26 inches. Ans. 5663·2576 cub. in.
5. What is the volume of a circular segment of an oblate spheroid whose diameters are 10 and 16 inches, the height of the segment being 4 inches. Ans. 471·8264 cub. in.
6. What is the volume of an elliptical segment of a prolate spheroid whose diameters are 11 and 15 feet, the height of the segment being 6 feet? Ans. 539·884 cub. ft.

MIDDLE FRUSTUM OF SPHEROID.

Let l = length of frustum, and d = end diameter.

Then for circular frustum of oblate spheroid,

$v = {\cdot}2618l(2t^2 + d^2)$ (CIV);

For prolate spheroid $v = {\cdot}2618l(2c^2 + d^2)$ (CV.)

For elliptical frustum let d' and $d_{,} =$ diameters of ends, then whether the frustum is a portion of an oblate or prolate spheroid, $v = {\cdot}2618l(tc + dd_{,})$ (CVI.)

Ex. 1. Find the solid contents of the middle circular frustum of an oblate spheroid, the axis of the spheroid being 25 inches, the end diameters 20 and the length 9 inches.

Solution.

By formula (CIV), $v = \cdot 2618l(2t^2 + d^2) = \cdot 2618(2 \times 25^2 + 20^2) \times 9$
$= \cdot 2618(1250 + 400) + 9 = \cdot 2618 \times 1650 \times 9 = 3887 \cdot 73$ cub. in.

Ex. 2. Find the solid contents of an elliptic middle frustum of a prolate spheroid whose axes are 24 and 30, the end diameters being 16 and 20, and the length 10 inches.

Solution.

By formula (CVI), $v = \cdot 2618(2tc + dd')l = \cdot 2618(2 \times 30 \times 24 + 16 \times 20) \times$
$10 = \cdot 2618(1440 + 320) \times 10 = \cdot 2618 \times 1760 \times 10 = 4607 \cdot 68$ cub. in.

EXERCISE XXII.

1. Find the solid contents of a circular middle frustum of an oblate spheroid whose middle axis (i. e. the transverse axis) is 20, the diameter of the end being 14 and the height 10.
Ans. 2607·528.
2. Find the solid contents of an elliptical middle frustum of a spheroid whose axes are 30 and 50 inches, the end diameters of the frustum being 18 and 30 inches and its length 40 inches.
Ans. 21 cub. ft. 782·88 cub. in.
3. Find the cubic contents of a circular middle frustum of a prolate spheroid whose middle or conjugate axis is 20 inches, the end diameter of the frustum being 15 and its length 30 inches.
Ans. 8050·35 cub. in.

HYPERBOLOID; FRUSTUM OF HYPERBOLOID.

Let r = radius of base, and d = diameter half way between base and vertex, and h = height. Also for frustum let r and $r_{,}$ = radii of ends, and d = diameter of section half way between the ends. Then for

Hyperboloid, $v = \cdot 5236(r^2 + d^2)h$ (CVII).

Frustum of hyperboloid, $v = \cdot 5236(r^2 + r_{,}^2 + d^2)h$ (CVIII).

Ex. 1. Find the solidity of a hyperboloid whose altitude is 4 feet 2 inches, the diameter of the base 8 feet 8 inches, and the middle diameter 5 feet 8 inches.

Solution.

Here $h = 50$ inches, $r = \frac{1}{2}$ of $104 = 52$ inches, and $d = 68$ inches.
By formula (CVII), $v = \cdot5236(r^2 + d^2)h = \cdot5236(52^2 + 68^2) \times 50 = \cdot5236(2704 + 4624) \times 50 = \cdot5236 \times 7348 \times 50$ $191847\cdot04$ cub. in. $= 111$ cub. ft. $39\cdot04$ cub. in.

Ex. 2. Find the solid contents of a frustum of hyperboloid, the diameters of the ends being 16 and 24 and the middle diameter 22, the height of the frustum being 20.

Solution.

By formula (CVIII), $v = \cdot5236(r^2 + r_{,}^2 + d^2)h = \cdot5236(8^2 + 12^2 + 22^2) \times 20 = \cdot5236(64 + 144 + 484) \times 20 = \cdot5236 \times 692 \times 20 = 7246\cdot624$.

Exercise XXIII.

1. Find the solid contents of a hyperboloid whose middle diameter is 30, end diameter 50, and altitude 24. Ans. 19163·76.
2. Find the solidity of a frustum of hyperboloid whose end diameters are 16 and 30, middle diameter 26, and altitude 20. Ans. 10105·48.
3. Find the solid contents of a hyperboloid whose middle diameter is 15, end diameter 24, and height 20. Ans. 3864·168.
4. Find the solid contents of a frustum of a hyperboloid whose middle diameter is 40, end diameters 20 and 50, and altitude 42. Ans. 51129·54.

Miscellaneous Exercises.

1. How many acres are there in a square field whose side contains 809 links? Ans 6 a. 2 r. $7\frac{1}{6}$ per.
2. What is the side of a square whose area is 3025 yards? Ans. 55 yards.
3. How many square feet of carpet are required for a square room whose diagonal is 31 feet? Ans. $408\frac{1}{2}$ feet.

4. Required the diagonal of a square table whose area is 16 square feet. Ans. 5 ft. 7·8822 in.

5. Find the number of square inches in a sheet of paper whose length is 11 inches and breadth $8\frac{1}{3}$ inches. Ans. $93\frac{1}{2}$ sq. in.

6. A rectangle whose end is 11 yards long, contains 2112 square yards, what is the length of its base? Ans. 192 yds.

7. The area of a rectangular pond is 43,750 square yards—one side is 350 yards, what is the length of the other? Ans. 125 yards.

8. Find the area of a rectangle whose base is 21 and diagonal 35 yards. Ans. 588 sq. yds.

9 Find the area of a parallelogram whose base is 90 and perpendicular altitude $12\frac{1}{2}$ feet in length. Ans. 1125 sq. ft.

10. Required the area of a triangle whose base is 81 feet and altitude 46 feet in length. Ans. 1863 sq. feet.

11. What is the length of the base of a triangle whose area is 2560 square feet and altitude 40 feet? Ans. 128 feet.

12. Required the altitude of a triangle which contains 117·5625 square yards—its base being 49 feet 6 inches in length. Ans. 42 feet 9 in.

13. Find the area of a triangular field whose sides are respectively 1200, 1800, 2400 links in length. Ans. 10 a. 1 r. 33 per.

14. Find the area of an equilateral flower bed whose side is 25 yards long. Ans. 270 sq. yards; 5·625 sq. feet.

15. The four sides of a quadrilateral, inscribed in a circle, are 75, 40, 60, and 55 chains, what is its area? Ans. 314 a. 2 r. 22 per. 26 yards.

16. Find the area of a park in the form of an octagon whose side is 12 chains and apothem 14·485 chains. Ans. 69 a. 2 r. 4·6 per.

17. What is the circumference of a circle whose diameter is 44 feet? Ans. 138·23 feet.

18. Required the diameter of a circle whose circumference is 78·54 yards? Ans. 25 yards.

19. What is the area of a circle whose diameter is 80 feet? Ans. 5026·56 feet.

20. Find an area of a circular garden whose diameter is 200 yards and circumference 628·32 yards.

Ans. 6 a. 1 r. 38 per. 16 yards.

21. Find the area of a circle whose circumference is 200 feet.

Ans. 3184 sq. feet.

22. What is the area of a sector of a circle whose radius is 50 feet, the arc of the sector being 30 feet in length?

Ans. 750 sq. feet.

23. Find the area of a sector whose arc contains 40° — the diameter of the circle being 60 feet. Ans. 314·16 sq. feet.

24. Find the area of a circular annulus—the circumferences of the containing circles being 90 and 60. Ans. 358·2.

25. The diameters of two concentric circles are 50 and 30 feet—find the area of the included annulus. Ans. 1256·64.

26. What is the area of a triangle whose basis is $12\frac{1}{4}$ chains and altitude $8\frac{2}{5}$ chains? Ans. 5 a. 0 r. 33 per.

27. What is the area of a trapezoid whose parallel sides are $7\frac{1}{2}$ chains and $12\frac{1}{4}$ chains, the perpendicular distance between them being $15\frac{1}{4}$ chains. Ans. 15 a. 0 r. 33 per. 6 yards.

28. The circumference of a circular fish pond is 400 yards—what is the side of a square pond of equal area?

Ans. 112·85 yards.

29. What is the area of a triangle whose sides are 24, 36 and 48 yards respectively? Ans. 418·282 sq. yards.

30. Find the area of a square field whose side is 19 chains.

Ans. 36 a. 0 r. 16 per.

31. Find the area of a triangular field whose three sides are respectively 120, 140, and 160 yards.

Ans. 1 a. 2 r. 28 per. 26 yards.

32. Required the area of a field in the form of a rectangle whose adjacent sides are 740 yards and 180 yards.

Ans. 27 a. 2 r. 3 per. 9 yards.

33. What is the area of a circle whose circumference is 92?

Ans. 673·734.

34. Find the area of a quadrilateral inscribed in a circle—its four sides being 400, 360, 300, and 280 links.

Ans. 1 a. 15 per. 25 yards.

35. Find the area of an equilateral triangle whose base is 20. Ans. 173·2.

36. Find the area of a circle whose radius is 35. Ans. 3848·46.

37. Find the area of a quadrilateral whose diagonal is 80 chains and perpendiculars from it to the opposite angles 29 chains and 23 chains respectively. Ans. 208 acres.

38. Find the area of a trapezoid whose parallel sides are 750 and 600 links, and the perpendicular distance between them 240 links. Ans. 1 a. 2 r. 19 per. 6 yards.

39. Find the area of a triangle whose sides are respectively 90, 70, and 60 chains in length. Ans. 209 a. 3 r. 1 per. 6 yds.

40. A circular garden is to be formed so as to contain as much land as an equilateral triangle whose side is 56 chains. Required the diameter of the circular garden and also its area. Ans. 914·76; 135 a. 3 r. 6 per. 6 yds.

41. Find the area of a circular annulus contained between two circles whose diameters are respectively 100 and 160. Ans. 12252·24.

42. Required the length of a circular arc of 68°, the diameter of the circle being 250 feet. Ans. 148 34.

43. Find the area of the sector of a circle whose radius is 50 feet, the arc of the sector containing 70°. Ans. 1527·05.

44. Find the area of the segment of a circle whose diameter is 60 chains, the circular arc containing 130°, and its chord being 52 chains in length. Ans. 63 a. 0 r. 29 per. 6½ yds.

45. Find the area of a regular decagon whose side is 11 and apothem 9·526279. Ans. 523 9453.

46. Find the area of the sector of a circle whose radius is 60 yards, the arc of the sector being 280 yards in length. Ans. 1 a. 2 r. 37 per. 20 yds. 6 ft.

47. Find the area of a field whose opposite sides are parallel, the base beimg 620 yards, and the perpendicular altitude 108 yards. Ans. 13 a. 3 r. 13 per. 16 yards.

48. What is the length of an arc of 197½° of a circle whose diameter is 240 yards? Ans. 413 6124 yards.

49. Required the circumference of an ellipse whose diameters are 600 and 400. Ans. 1570·8.

50. A field containing 7 a. 3 r. 21 per. 17 yds. is divided into two parts, the one forming a circle whose diameter is 80 yards, what must be the dimensions of an equilateral triangle whose area shall be equal to the remainder?
Ans. 276·63 yards.

51. Find the area of a circular annulus contained between two circles whose circumferences are 360 and 240 chains.
Ans. 573 a. 0 r. 19 per. 6 yards.

52. What is the area of an ellipse whose diameters are 5 and 10?
Ans. 39·27.

53. The axes of an ellipse are 30 and 10, and one absciss is 24; what is the ordinate? Ans. 4.

54. The axes of an ellipse are 70 and 50, and an ordinate 20; what are the abscisses? Ans. 56 and 14.

55. The conjugate axis of an ellipse is 10, the smaller absciss 6, and the ordinate 4; what is the transverse axis? Ans. 30.

56. The transverse axis is 280, and ordinate 80, and one absciss 56; what is the conjugate axis? Ans. 200.

57. If an ordinate of a parabola is 20 and its absciss 36, what is the parameter? Ans. 11·1.

58. The two abscisses are 9 and 16, and the ordinate of the former is 6; find that of the latter. Ans. 8.

59. Given the two ordinates 6 and 8, and the absciss of the former 9, to find that of the latter. Ans. 16.

60. Find the area of a parabola whose base or double ordinate is 15, and height or absciss 22. Ans. 220.

61. Required the length of a parabolic curve whose absciss is 6 and ordinate 12. Ans. 27·71.

62. The transverse axis of an hyperbola is 15, the conjugate axis 9, the smaller absciss 5, required the ordinate.
Ans. 6.

63. The transverse and conjugate axes of an hyperbola are 60 and 45, and one ordinate is 30; what are the abscisses?
Ans. $67\frac{1}{2}$ and $7\frac{1}{2}$.

64. The transverse axis of an hyperbola is 60, an ordinate 24, and the smaller absciss 20; what is the conjugate axis?
Ans. 36.

65. The conjugate axis of an hyperbola is 45, the smaller absciss 30, and the ordinate 30; what is the transverse axis? Ans. 90.

66. What is the area of an hyperbola whose axes are 15 and 9, and the smaller absciss 52? Ans. 37·919.

67. Find the volume and surface of a tetrahedron whose edge is 8. Ans. 60·3 and 110·85.

68. Find the volume and surface of a hexahedron whose edge is 11. Ans. 1331 and 726.

69. Find the volume and surface of an octahedron whose edge is 10. Ans. 471·4 and 346·4.

70. Find the volume and surface of a dodecahedron whose edge is 4. Ans. 490·44 and 330·33.

71. Find the volume and surface of an icosahedron whose edge is 6. Ans. 471·245 and 311·76.

72. What is the surface of a right cylinder whose length is 20 and circumference 6? Ans. 125·73.

73. What is the surface of a regular pentagonal pyramid, each side of its base being $1\frac{2}{3}$ feet and its slant side 10 feet in length? Ans. 46·4456.

74. Find the surface of a frustum of a right cone, its length being 31, and the circumference of its two ends being 62·832 and 37-6992. Ans. 1985·49.

75. What is the surface of a sphere whose diameter is 800 inches. Ans. 2010624 sq. inches.

76. Find the surface of a globe whose diameter is 13 and circumference 37·6992. Ans. 452·39.

77. Find the surface of a spherical segment whose height is 2, the diameter of the sphere being 10. Ans. 62·832.

78. What is the volume of a prism whose length is 18 feet, its base being a regular hexagon whose side is 16 inches and apothem 13·8564 inches? Ans. 83·138 cub. feet.

79. If the volume of a triangular prism is 7·656 and its length is $10\frac{1}{2}$; what is the area of its base? Ans. 729.

80. Required the volume of a frustum of a square pyramid, the side of the greater base being 16, of the lesser 10, and its length 18. Ans. 37·152.

81. What is the solidity of a cone whose altitude is 12 feet, the diameter of its base being 10 feet? Ans. 314·16.

82. Find the area of the base of a cone whose volume is 282·74 and altitude 30. Ans. 28·274.

83. What is the solidity of a sphere whose diameter is 30? Ans. 14137·2.

84. What is the diameter of a sphere whose volume is equal to 65449·85 feet? Ans. 50 feet.

85. What is the solidity of a segment of a sphere, the height of the segment being 2, the diameter of the sphere 10? Ans. 54·4544.

86. What is the volume of a spherical segment, whose height is 10, and the diameter of its base 20? Ans. 2094·4.

87. Find the volume of a spherical zone, the diameter of its end being 10 and 12, and its height 2. Ans. 195·9159.

88. Required the solidity of the middle zone of a sphere, its height being 32 feet, and the diameter of the sphere 40. Ans. 31633·8.

89. Find the volume of the middle zone of a sphere, its height being 8, and end diameters 6. Ans. 494·278.

90. Find the solidity of an oblate spheroid whose axes are 20 and 12. Ans. 2513·28.

91. What is the volume of a prolate spheroid, its polar axis being 7, and equitorial axis 5? Ans. 91·63.

92. Find the area of the segment of a circle whose radius is 40 yards—the arc containing 136°, the chord being 60 yards in length. Ans. 1105·0523 yds.

93. What is the transverse axis of an ellipse whose conjugate axis is 90 and area is equal to that of an equilateral triangle, whose side is 70 and a circle whose circumference is 240? Ans. 94·87.

94. Find the area of an equilateral triangle whose side is 90. Ans. 3507·3.

95. What is the area of a triangle whose sides are 48, 54, and 60 respectively? Ans. 1231·09.

96. Find the area of an ellipse whose diameters are 40 and 48. Ans. 1507·968.

97. Find the length of a rectangular field whose breadth is 220 yards, and which contains as much ground as an ellipse whose axes are 900 and 1100 yards. Ans. 3534·3 yards.

98. Find the diameter of a circle whose area is 5 acres, 3 roods, 27 per., 20 yards. Ans. 191·04.

99. Find the altitude of a parallelogram whose base is 500 yards, and area equal to the combined areas of a circle whose circumference is 200 yards, and a circular sector whose arc contains 200° and whose radius is 40 yards.
Aus. 11·95 yards.

100. Find the area of a sector, a circle whose radius is 300 links, the arc being 500 links in length. Ans. 3 roods.

101. Find the solidity and surface of a hexahedron whose edge is 7. Ans. 343 and 294.

102. Find the surface and volume of a dodecahedron whose edge is 4. Ans. 330·3324 and 490·4384.

103. Find the surface and solidity of a cone whose height is 20 feet, and diameter of base 10 feet. Ans. 402·36 and 523·6.

104. Required the surface and solidity of a right prism whose base is a regular heptagon, having each of its sides 8 feet, the edges of the prism being each $2\frac{1}{2}$ feet in length.
Ans. 661·14 and 813·9963

105. How many pails of water (each containing 10 qts.) may be contained in a circular cistern whose diameter is 7 feet, and depth 11 feet? Ans. 1058·3265 pails.

106. What must be the depth of a pentagonal cistern which contains as much water as a circular cistern 8 ft. in diameter and $4\frac{1}{2}$ feet deep, and a rectangular tank 7 feet long, 5 feet wide and $3\frac{1}{2}$ feet deep—one side of the pentagonal cistern being 5 feet. Ans. 12·667 feet.

107. Find the surfaee and solidity of a pyramid whose height is 9 feet—the base being a regular hexagon whose edge is 3 feet. Ans. 107·69 and 70·148.

108. Find the surface and solidity of an oblique prism whose base is a pentagon with each edge 4 feet—the edges of the prism being each 10 feet long, and the perimeter of a section perpendicular or to them 18 feet.
Ans. 235·055 sq. ft. 275·276 cub. ft.

109. Find the area of triangular field whose sides are 8, 12 and 14 chains. Ans. 4 a. 3 r. 6 per. 15 yds.

110. Find the surface and solidity of an icosahedron whose edge is 3 feet. Ans. 77·94 and 58·90563.

111. Find the solidity and surface of the frustum of a right cone whose slant side is 60 feet—the diameters of the ends being 10 and 20 feet.

Ans. 10957·11 cub. feet and 3220·14 sq. ft.

112. Find the solidity and surface of the frustum of a right pyramid whose ends are squares with edges 10 and 12 feet respectively and height 8 feet. Ans. 970·66 and 508·653.

113. How many cubic feet are there in a squared stick of timber whose end edges are respectively 28 and 20 inches, th length of the stick being 42 feet? Ans. 169 cub. feet.

114. What is the solidity and surface of a hexagonal frustum whose height is 6 feet, the edges of the ends being respectively 2 feet and $1\frac{1}{2}$ feet?

Ans. 48·054 cub. feet; 63·147 sq. feet.

115. Find the area of a triangular park whose three sides are 900, 1100 and 1300 links respectively.

Ans. 4 a. 3 r. 20 per. 27 yds.

116. Find the diameter of a circle which shall contain as much ground as a quadrilateral inscribed in a circle, whose four sides are 900, 1000, 600 and 800 yards respectively.

Ans. 916·48 yards.

117. Find the area of a square whose diagonal is 44.

Ans. 968 sq. yds.

118. Find the area of an annulus inclosed between two concentric circles whose circumferences are 180 and 225 yards respectively. Ans. 1450·71 sq. yds.

119. Find the area of an elliptical field whose diameters are 980 and 1250 links. Ans. 9 a. 2 r. 19 per. 11·6 yds.

120. Find the area of a parabolic zone whose height is 25 yards, its double ordinates being respectively 90 and 70 yards.

Ans. 2010·416 sq. yds.

121. Find the surface and solidity of an icosahedron whose edge is $8\frac{1}{2}$ feet. Ans. 625·685 sq. ft.; 1339·83 cub. ft.

122. Find the solidity of an oblique triangular prism, the edges of the ends being 10, 16, and 24 feet, the height 20 feet.
Ans. 1161·894 cub. feet.

123. Find the surface and solidity of a right cone whose height is 20 feet—the diameter of the end being 12 feet.
Ans. 506·68 sq. ft.; 753·984 cub. ft.

124. Find the solidity of a prolate spheroid, whose axes are 11 and 7 respectively. Ans. 282·22.

125. Find the solidity of an oblate spheroid, whose axes are 20 and 15 respectively. Ans. 3141.6.

126. Find the surface and solidity of a sphere, whose diameter is 26 feet. Ans. 2123·7216 sq. ft.; 9202·7936 cub. ft.

127. Find the surface and solidity of a spherical segment, whose height is 2 inches, the diameter of the sphere being 5 inches. Ans. 31·416 sq. in.; 23·0384 cub. in.

128. Find the solidity of the middle zone of a sphere, the diameters of its ends being 7 feet and its height 6½ feet.
Ans. 393·943 cub. ft.

129. Find the convex surface of a spherical segment, whose height is 9 inches, the diameter of the sphere being 3 feet 6 inches. Ans. 1187·52 sq. in.

130. Find the surface and solidity of a frustum of a right cone whose height is 9 feet, the diameters of the ends being 10 feet and 6 feet. Ans. 461·815 cub. ft.; 461·81 sq. ft.

131. Find the solidity and surface of an octagonal pyramid whose height is 8 feet, each edge of the base being 5 feet.
Ans. 321·89 cub. ft.; 321·13 sq. ft.

132. Find the area of a field in the form of a circle, having a diameter of 11 chains, 64 links.
Ans. 10 a. 2 r. 26 per. 19·23 yds.

133. Find the area of a triangle whose three sides are respectively 79, 80, and 90 yards long.
Ans. 2 r. 8 per. 21 yds. 1·8 ft.

134. Required the area of a quadrilateral field whose diagonal is 29 chains, the perpendiculars upon it from the opposite angles being 9 and 17 chains respectively.
Ans. 37 a. 2 r. 32 per.

135. What is the area of a regular nonagon whose side is 13 yards? Ans. 1 r. 37 per. 2 yds. 1·6 ft.

136. Find the solidity and surface of a right cone whose height is 12 feet, the circumference of the base being 31·416 feet. Ans. 314·16 cub. ft.; 382·74 sq. ft.

137. Find the solidity and surface of a hexagonal pyramid whose height is 24 feet, each edge of the base being 7 feet. Ans. 1018·44 cub. ft.; 647·13 sq. ft.

138. What must be the diameter of a circular garden to contain as much ground as a field in the form of an equilateral triangle, whose side is 250 yards long? Ans. 185·6 yds.

139. Find the area of an annulus contained between two concentric circles, whose diameters are 12 and 15 feet. Ans. 63·6174 sq. ft.

140. Find the area of a circular sector whose arc contains 40 degrees, the diameter of the circle being 20 yards. Ans. 34·905 sq. yds.

141. Find the volume of a spherical zone, the diameters of its ends being 20 and 28 inches, and its height $7\frac{1}{2}$ inches. Ans. 3708·0697 cub. in.

142. Find the area of a sector whose arc is 500 links long, the diameter of the circle being 500 links. Ans. 2 roods, 20 per.

143. Required the solidity of a cone whose height is 12 feet, the circumference of the base being 50 feet. Ans. 796 cub. ft.

144. Required the solidity of an oblique octagonal prism, each side of the base being 9 feet, the height of the prism being 12 feet and each edge 20 feet, the perimeter of a section perpendicular to the edges being 60 feet. Ans. 6008·7 cub. ft.

145. Find the surface and solidity of a sphere whose diameter is 30 feet. Ans. 14137·2 cub. ft.; 2827·44 sq. ft.

146. Find the surface of a spherical segment whose height is 4 inches, the diameter of the sphere being 6 feet. Ans. 904·78 sq. ft.

147. Find the solid contents of a hyperboloid, whose middle diameter is 30, end diameter 40, and altitude 24. Ans. 25132·8.

148. Find the solidity and surface of a regular pyramid, whose base is a square, each side being 6 feet, the apothem or perpendicular on the side of the pyramid being 40 feet.
Ans. Surface = 516 sq. ft.

149. Find the volume of a middle zone of a sphere, whose height is 8 feet, the diameters of the ends being 6 feet. Ans. 494·278.

150. Find the contents of a regular hexagonal frustum, whose altitude is 6 feet, the side of the greater end 18 inches, and of the smaller end 12 inches. Ans. 24·6817 cub. ft.

Problems for Practice.

1. Find the area of a square, whose side is 13 chains.
2. Find the side of a square, whose area is 3 acres, 14 per., 18 yards.
3. What is the area of a square, whose diagonal is 260 yards?
4. The area of a square is 7 acres, 1 rood, 30 per., required the length of its diagonal in yards.
5. (*a*) Find the area of a rectangle, whose length is 700 and breadth 500 links.
   (*b*) Find the area of a parallelogram, whose base is 600 and perpendicular 250 yards. Answer in acres, roods, &c.
6. (*a*) The area of a rectangle is 700, its breadth is 35, what is its length?
   (*b*) The area of a parallelogram is 4 acres, 3 roods, 16 yards, its breadth is 120 yards, what is its length?
7. (*a*) The area of a rectangle is 17 acres, 1 rood, 16 per., its length is 1600 links, what is its breadth?
   (*b*) The area of a parallelogram is 1600, its length is 240, what is the perpendicular distance between its sides?
8. Find the area of a rectangle, whose diagonal is 500 links, and breadth 300 links.
9. Find the hypothenuse of a right angled triangle, whose base is 75 yards and perpendicular 48 yards.
10. The hypothenuse of a right angled triangle is 600, the perpendicular is 230, what is the base?
11. The hypothenuse of a right angled triangle is 73, the base is 29, what is the perpendicular?

12. In a right angled triangle the hypothenuse is 50, the perpendicular 20, what are the segments into which a perpendicular from the right angle cuts the hypothenuse?

13. In a right angled triangle the hypothenuse is 40, the base 15, into what two segments does a perpendicular from the right angle cut the hypothenuse?

14. In a right angled triangle the segments into which a perpendicular from the right angle cuts the hypothenuse are 40 and 30, what is the perpendicular distance of the right angle from the hypothenuse?

15. Find the area of a triangle, whose base is 750 and altitude 340 links.

16. The area of a triangle is 17 acres, 4 per., 16 yards, the altitude is 570 yards, what is the length of the base?

17. The area of a triangle is 640, the base is 120, what is the altitude?

18. Find the area of a triangle, whose three sides are respectively 640, 320, 480 links.

19. What is the area of an equilateral triangle, whose base is 160 yards long?

20. Find the area of a trapezoid, whose parallel sides are 500 and 300 links, the perpendicular distance between them being 120 links.

21. Find the area of a quadrilateral field, whose diagonal is 420 yards, the perpendiculars or the diagonal from the opposite angle being 70 and 130 yards.

22. Find the area of a quadrilateral which may be inscribed in a circle, its four sides being respectively 80, 90, 100 and 120 yards long.

23. Find the area of a regular octagon, whose side is 13 feet. (See Table I. for apothem, page 72).

24. The area of a regular heptagon is 5 acres, 1 rood, 27½ per., find the length of a side. (See Table I., page 72).

25. The area of a regular polygon is 4278·4 square feet, each side is 20 feet long and the apothem 37·32 feet, how many sides has the polygon?

26. Find the circumference of a circle whose diameter is 12¼.

27. Find the diameter of a circle whose circumference is 180 yds.

28. Find the area of a circle whose diameter is 14 and circumference 43 9824 feet.

29. Required the diameter of a circle whose area is 490·875 square yards, and circumference 78 54 yards.

30. Required the circumference of a circle, whose area is 1256·64 square feet and diameter 20·40 feet.

31. Find the area of a circle, whose diameter is 840 links.

32. The area of a circle is 15 acres, 2 roods, 16 per., 20 yards, what is its diameter in links?

33. Find the area of a circle whose diameter is 220 yards.

34. Find the area of a circle whose circumference is 330 links.

35. What is the area of a circular annulus, the diameters of the concentric circles being 70 and 50 feet?

36. What is the area of a circular annulus, the circumferences of containing circles being 80 and 280?

37. Required the area of a circular annulus, the containing circles having circumferences 251·328 and 439·824 links, and diameters 80 and 140 links.

38. The diameter of a circle is 60 feet, what is the length of an arc of $87\frac{1}{2}°$ of the circumference?

39. What is the length of the chord of a circle, the diameter of the circle being 61·6116 and apothem 25?

40. Find the apothem on the chord of a circle, whose diameter is 24·4, the length of the chord being 24.

41. Find the chord of half the arc of a circle, whose radius is 18·75, the height of the whole arc being 6.

42. Find the radius of a circle, the height of the arc being 4, and the chord of half the arc being 20.

43. Find the area of the segment of a circle, whose diameter is 45·3 feet, the arc containing 2409, the chord being 40 feet and apothem 10 feet.

44. Find the area of a sector of a circle, whose radius is 50 yards, the length of the arc being 90 yards.

45. Find the area of the sector of a circle, whose radius is 80 feet, the circular arc containing 72 degrees.

46. Find the area of a lune, whose common chord is 40 feet, the length of the outer arc is 94·876 and of the inner one 60 feet; the apothem of the outer smaller circle being 12·65 feet, and

its diameter 45·3 feet, the apothem of the larger circle being 48 feet and its diameter 200 feet.

47. Find the circumference of an ellipse whose axes are 16 and 28.
48. Find the area of an ellipse whose diameters are 20 and 14.
49. Required the ordinate of an ellipse, whose diameters are 30 and 10, and one absciss 24.
50. Find the abscisses of an ellipse, whose axes are 60 and 80, and an ordinate 30.
51. What is the transverse axis of an ellipse, whose conjugate axis is 70, an ordinate 28, and one absciss 168?
52. What is the conjugate axis of an ellipse, whose major axis is 70, an ordinate 20, and the smaller absciss 14?
53. Find the parameter of a parabola, the ordinate and one absciss being 12 and 28.
54. Two abscisses of a parabola are 9 and 16, the ordinate of the former is 6, find that of the latter.
55. An absciss of a parabola is 32 and its ordinate 24, a second ordinate is 18; what is its absciss?
56. Find the length of the arc of a parabola, whose absciss and ordinate are 3 and 5.
57. Find the area of a parabola, whose base and height are 20 and 28.
58. Find the area of a parabola zone, whose bases are 7 and 10 and height 6 feet.
59. In an hyperbola the axes are 90 and 45, the less absciss is 30; find the ordinate.
60. The axes are 15 and $7\frac{1}{2}$, the ordinate 5; what are the abscisses of the hyperbola?
61. What is the solidity of a tetrahedron, whose edge is 5 inches?
62. In an hyperbola, the transverse axis is 25, the less absciss $8\frac{1}{2}$, and its ordinate 10; required the conjugate axis.
63. The conjugate axis is $31\frac{1}{2}$, the smaller absciss 12, the ordinate 21; what is the transverse axis of the hyperbola?
64. What is the area of an hyperbola, whose axes are 15 and 9, the smaller absciss being 5?
65. What is the surface of a regular triangular pyramid, whose edge is 7 feet?
66. What is the aggregate surface of a cube whose edge is $8\frac{1}{2}$ feet?

67. What are the solid contents of a hexahedron whose edge is $1\frac{1}{2}$ feet long?

68. Find the surface of an octahedron whose edge is $4\frac{1}{2}$ feet?

69. Required the cubic contents of an octahedron whose edge is 15 inches.

70. The edge of a dodecahedron is 6 inches, what is its entire surface?

71. Find the volume of a dodecahedron whose edge is $1\frac{3}{4}$ feet.

72. Required the surface of an icosahedron whose edge is $10\frac{1}{2}$ inches.

73. What are the cubic contents of an icosahedron, whose edge is 20 inches?

74. (*a*) What is the volume of a right rectangular parallelopiped, whose length is 20 feet, breadth $4\frac{1}{2}$ feet, and height 18 feet?

(*b*) What is the volume of an oblique triangular prism, the edges of the end being 7, 9 and 11 inches, and the length of the prism 45 inches?

(*c*) What is the volume of a hexagonal prism, whose length is 22 feet, each edge of the end being 20 inches.

75. (*a*) Find the surface of a right rectangular parallelopiped, whose base is 16 inches by 9 inches, the height of the solid being 4 feet.

(*b*) Find the surface of a right octagonal prism, whose height is 12 feet, each edge of the end being $2\frac{1}{2}$ feet.

76. Required the solidity of the frustum of a cone, whose height is 5 feet and end diameters 4 and 2 feet.

77. Find the surface of an oblique pentagonal prism, whose length is $4\frac{1}{2}$ feet and edge 3 feet, the perimeter of a section perpendicular to one of the lateral edges being 16 feet.

78. (*a*) Find the volume of a regular pyramid, whose height is 10 feet, the base being a heptagon, whose side is 2 feet.

(*b*.) Find the volume of a regular cone, whose base has a diameter of 7 feet, the height of the cone being 9 feet.

79. (*a*) Find the entire surface of a regular octagonal pyramid, whose height is 11 feet, each edge of the base being 5 feet.

(*b*.) Find the entire surface of a right cone, whose height is 14 feet, the diameter of the base being $7\frac{1}{2}$ feet.

80. Find the solidity of a frustum of a pyramid, whose height is 5 feet, the areas of the two ends being 12 and 18 square feet.

81. Find the volume of the frustum of a right pentagonal pyramid, the upper end edges being $3\frac{1}{2}$ feet and the lower 5 feet each, and the height of the frustum 7 feet.

82. Find the volume of the frustum of a hexagonal pyramid, the edge of the bottom being 4 feet and of the top $2\frac{1}{2}$ feet, while the height of the frustum is 6 feet.

83. What are the solid contents of the frustum of a cone, whose height is 10 feet, the end diameters being 5 and 3 feet?

84. What is the whole surface of the frustum of a cone, whose end diameters are 4 and 8 feet, the slant side being $6\frac{1}{2}$ ft. long?

85. Find the volume of a wedge, whose edge is 12 inches and back 10 inches long, the breadth of the back being $4\frac{1}{2}$ inches and the length of the wedge 2 feet.

86. Find the solidity of a spherical segment, whose height is 6 feet, the radius of the base being 2 feet.

87. What are the solid contents of a spherical segment, whose height is 7 inches, the diameter of the sphere being 10 inches?

88. Find the convex surface of a spherical segment, whose height is 10 inches, the diameter of the sphere being 4 feet, 2 inches.

89. Find the volume of a sphere whose diameter is $8\frac{1}{2}$ feet.

90. Find the surface of a sphere whose diameter is 7924 miles.

91. Find the volume of a spherical zone, whose height is 2 feet, the radii of the ends being 3 feet and 4 feet.

92. Find the volume of the middle zone of a sphere, the height of the zone being 4 feet and the diameter of either end 3 feet.

93. What is the volume of the middle zone of a sphere, the height of the zone being 6 feet and the diameter of the sphere 10 feet?

94. Find the convex surface of the middle zone of a sphere, the height of the zone being 7 feet and the diameter of the sphere 20 feet.

95. Find the volume of a paraboloid, whose height is 10 feet and diameter 8 feet.

96. Find the volume of the frustum of a paraboloid, whose height is 5 feet, the areas of the ends being 7 and 9 square feet.

97. What is the volume of the frustum of a paraboloid, whose height is 8 feet and end diameters 10 and 4 feet?

98. Find the volume of an oblate spheroid, whose diameters are 12 and 17 feet.

99. Find the volume of a prolate spheroid, whose axes are 7 and 11 feet.

100. What is the volume of a circular segment of an oblate spheroid, the axes being 18 and 12 feet, and the height 4 feet?

101. What is the volume of a circular segment of a prolate spheroid, the axes being 10 and 15 and the height 6 feet.

102. Find the volume of an elliptical segment of an oblate spheroid, whose height is 4 feet, the axes being 12 and 16 feet.

103. Find the volume of an elliptical segment of a prolate spheroid, whose height is 6 feet, the axes being 18 and 20 feet.

104. Find the solid contents of a circular middle frustum of a prolate spheroid, whose conjugate axis is 12 feet, the diameter of the frustum being 6 feet and its height 7 feet.

105. Find the volume of a circular middle frustum of an oblate spheroid, the diameter of the frustum being 6 feet, the transverse axis 20 feet, and the length of the frustum 8 feet.

106. Find the cubic contents of an elliptical middle segment of a spheroid, whose axes are 16 and 24, the length of the frustum being 12 feet, and the greater and less diameters of either end 6 and 4 feet.

107. Find the solidity of an hyperboloid, whose base has a diameter of 10 feet, the diameter half way between the base and vertex is 6 feet, and the height of the hyperboloid 8 feet.

108. Find the solidity of a frustum of an hyperboloid, whose height is 6 feet, the radii of the ends being 3 and 7 feet, and the diameter half way between the ends 12 feet.

TABLE I.,

SHOWING APOTHEM AND AREA OF POLYGONS.

Name of Polygon.	No. of Sides.	Apothem when side = 1.	Area when side = .
Triangle..................	3	0·2886751	0·4330127
Square.....................	4	0·5	1·
Pentagon.................	5	0·6881910	1·7204774
Hexagon..................	6	0·8660254	2·5980762
Heptagon................	7	1·0382607	3·6339124
Octagon..................	8	1·2071068	4·8284271
Nonagon..................	9	1·3737387	6· ·818242
Decagon..................	10	1·5388418	7·6942088
Undecagon	11	1·7028436	9·3656399
Dodecagon...............	12	1·8660254	11·1961524

TABLE II.

A gallon of water weighs 10 lbs. avoir.
A cubic foot of water weighs 62½ lbs. = 1000 oz.
A pail of water = 2½ gallons = 25 lbs.
A gallon is equal to 277·274 cubic inches.

TABLE III.

LAND MEASURE.

7·92 inches = 1 link.
100 links = 1 chain.
80 chains = 1 mile.
10000 square links = 1 square chain.
10 square chains = or 100,000 square links = 1 acre.
1 chain = 4 rods.
1 acre = 160 square rods = 4840 square yards.

NOTE.—If we desire to compute area of polygon by tabular area, we must remember that similar polygons are to each other as squares of homologous sides; hence 1^2: side^2: : tabular area : required area.

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