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# THE CANADIAN JOURNAL. 

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## THE PRESIDENT'S ADDRESS.

BY DANIEL WILSON, LL.D., PROMESEOR OP HISTORY AND ENGLISH LITERATURB, UNIYERSITY COLEEGE, TORONTO.

## Read before tile Canadian Institute, January 7th, 1860.

Gentlimen of the Canadian Institute,
Once more we assemble here to renew the work of another Session, as a body specially inviting its members to devote their energies to the investigation of the laws of nature, the advancement of science, and the discovery of new truths. The position in which you have placed me as your President was altogether unexpected by me. and I had anticipated a very different choice; but it would be as ungracious as unseemly for me now to cavil at a deviation from precedents indicated in the distinguished series of occupants of this chair, when I owpe to it so honorable a distinction, conferred on me in so cordial and gratifying a manner.

The functions of this Institute are of a peculiarly important kind, and claim for it a generous encouragement from all who desire the true welfare and advancement of this Province. Of its future progress I entertain no doubt; for it is impossible that Canada can attain to true greatness apart from such elements of mental and moral vigor as it is the special object of the Canadian Institute to develop. And when that greatuess has been achieved, and this young Institute has advanced with it to maturity, I doubt not that among the earnest

Vox. V.
thinkers and intellectual workers of Canada, the honors of this chair will be estecmed among the most coveted distinctions that this Province has to bestow. Meanwhile, however, I experience somewhat of the same difficulty which I believe some of my predecessors have felt, with an Annual Presidential Address to deliver, and nothing very definite to address you about.

I might enlarge upon the steady progress of the Institute, and in a special manner congratulate you, that-thanks to the zeal and wise courage of my predecessor in this cbair,-our roll has been purged of an accumulation of defaulters, mere men of buckram and straw to us, -a source of weakness instead of strength; but a very thorn in the side of our too forbearing and courteous Treasurer. It is a duty which all societies, constituted as we are, find it necessary from time to time to perform; and it is due to the neglent of this unwelcome duty by former. Councils, until forbearance had become almost culpable, that my predecessor has had the opportunity of sigualizing his close of office by a stern execution of rigorous justice, which confers no slight boon on his successor and on the Institute at large. These, however, and other facts connected with the history of our progress, have already been fully set forth in the Annual Report; and I must turn to other themes for the subject of your Annual Address.

A resume of the progress of Science and Literature in the Province would be peculiarly suitable to this occasion, but we are scarcely yet in such a condition as to furnish fresh materials for any very elaborate annual report of scientific progress. Our position as Canadians is a very peculiar one, when we consider that only sixty-two years have elapsed since Colonel Bouchette described the site of this capital of Upper Canada as a scene of dense trackless forests, where the wandering savage had constructed his ephemeral habitation beneath the luxurious foliage, while the bay and the neighbouring marshes were the haunts of such multitudes of wild-fowl as to destroy the stillness of night by their cries. But while we reflect with just pride on the changes which have been wrought on that untamed wilderness within the memory of some of our number, we are not forgetful that we are a part of the British empire, claiming our share in her greatness, and seeking to assume our part in her inherited duties; and in proof of this we can point at least to two Canadian Institutions worthy of a people sprung from the old stock that gave a Bacon and a Newton to the world.

The Provincial Geological Survey continues its valuable labours, ander the guidance of Sir William Logan, whose former occupancy of this chair reflects an honor on any one who succeeds to it; and during the past year two of the illustrated decades of Canadian Organic Remains have been issued, in a style peculiarly creditable to our young Province. The head quarters of the Geological Survey and its Provincial Museum, are established in the commercial capital of Lower Canada; while in Toronto, the Provincial Magnetic Observatory, originated under your first President, Captain Lefroy, continues in full activity, and the data of another year's magnetic and meteorological phenomena have been recorded by its director, Professor Kingston, for future publication.

Perhaps no more striking illustrations of the charges which a century has wrought on this Province could be selected, than are embodied in those two evidences of Anglo-Canadian enterprise and intellectual activity. That on the old trail of the Missassauga and the Huron, the wild forest and the swamp have given way to the busy marts and the crowded thoroughifares of an industrious and thriving city, is no trifling evidence of the healthful revolution which has been effected; and this change has all been wrought by the busy hands and the hardy endurance of the Anglo-Saxon and Celtic supplanters of the Aboriginal Indians,-by those to whom, as colonists, the well-known lauguage of Burke is still applicable: "A people but in the gristle, and not yet hardened into the bone of manhood."

That in this essentially practical age a race so thoroughly energetic and progressive as that from which the colonists of Canada have sprung, should clear the forest, drain the swamps, pave the roads, and rear costly marts and dwellings where so recently the rude birch-bark wigwam stood, is no slight triumph. Yet we scarcely need to be reminded that such material triumphs are neither the highest nor the most enduring monuments of a nation's progress. That great city: Nineveh, and the mighty Babylon, that once queened it so prouddy: on the banks of the Tigris and the Euphrates, are now but heaps of reedy clay, above which the wandering Arab feeds his flocks; while Athens lives for us still, far more by the pen of Sophocles and Socrates, Plato and Aristotle, than by the marbles of Phidias, or the columns of Callicrates and Ictinus. Even so, among the commercial marts and capitals of the civilized world, both Torouto and Montreal must still be content to claim a very secondary place; while in their relation to those two great departments of scientific. labour on which
this Province has hitherto chiefly concentrated its intellectual energies:the Geological Survey and the Magnetic Observatory, Montreal and Toronto are named with pride wherever science is cultivated and knowledge revered. These is something grand and ennobling in reflectingon the patient labors of the Magnetic, as of the Astronomical observer. In that little building which rears its modest tower in the University Park, apart from all our busy thoroughfares, on a spot so recently hewn out of the forest wilderness, observers are patiently noting, day by day, the minutest phenomena connected with the elements of terrestrial magnetic force, the laws of periodicity, the number, diversity of forms, and intensity of auroral manifestations, and the indications of a solar magnetic infiuence on the earth, dependent, as it seems, on the changes which the luminous envelope of the sun undergoes. A larger series of magnetic phenomena completes its cycle of variations from the ordinary mean within a decennial period, which coincides with a similar one observed in the solar spots; and a variation of the magnetic declination has also been traced, chiefly by means of our own Toronto observations, to luuar influence; while it has been conclusively established that the elements of the earth's magnetic force are subject to regular diurnal, annual, and decennial ranges of variation from maximum, through minimum, to maximum again. By such observed data glimpses of novel truths of the most remarkable and unexpected kind are being obtained. Through a source so unlikely as our observation of the phenomena of terrestrial magnetism, we are learning somewhat of the constitution of the central luminary of our system. Towards the close of last century, amidst an absolute ignorance of any known data to reason upon, much ingenious speculatiou was indulged in relative to the nature and constitution of the sun. In seeking to interpret observed solar phenomena, Sir William Herschell was led to the conclusion that the central body of our system is probably an opaque globe, surrounded by a luminous atmosphere, the disturbance of which he accounted for by the emission of an elastic fluid, ascending from the solid body, and producing by its currents those solar spots, to which our attention has been recently drawn by a series of interesting communications from one of our own number.

The recent ingenious application of photography by Sir John Herschell, for following up the speculations of his father, and making the sun record for us the daily changes wrought on its own luminous surface, is another means whereby materials for further philosophical
induction are being accumulated; and meanwhile the beautiful reasoning of Arago that solar light corresponds to that emitted by gaseous bodies, in being unpolarized, establishes on indisputable scientific grounds that the sun is no longer to be regarded as a solid incaadescent body.

Thus slowly, yet surely, does science widen the range of our knowledge, and also the area wherein fancy may freely speculate. The question of a plurality of inhabited worlds has engaged the inductive reasoning. as well as the fanciful speculations of eminent philosophers in recent years; and that of an inhabited central sun cannot therefore be considered as beyond the pale of such far-reaching thought. That solar luminary may inclose mithin its glowing-atmosphere a world of wondrous compass and beauty. Pure and glorious beings may dwell there, that "lie immortal in the arms of fire;" or, tempered by an intermediate cloudy vail, it may be that there, beings nobler and higher in the scale of intelligence than we are, bask in an endless summer, and a nightless day. For there is no night there, and fancy may anticipate the light which shall yet make clear to us the revelations of even greater mysteries than these.

But from such speculations I return to the fact that they have been suggested by the daily work going on in our own Provincial Magnetical Observatory. The results of such daily observations, entered in a few columns of figures, or pencilled by the sun's own rays, through the wonderful agency of photograpky, seem of little :apparent value ; yet, meanwhile at Washington, Greenwich and Kew, at Paris, Brussels, Berlin, Vienua, Rome, Christiana, Moscow, St. Petersburg, and other European cities; at Bombay, Travancore, and Mauritius, and at British Guiana, Melbourne, and other Colonial :sites, similar observations prove the simultaneous occurrence of such phenomena in the most distant parts of the earth, and thus reveal to us glimpses, at least, of the operations of an unknown force acting with corresponding results on the whole globe. Thus the space controlled and brought within the direct range of our knowledge by the records of magnetic observations comprehends not only the earth as a whole, but the distant central sun, and the bounds of the solar system. But great as is this range in space, the range in time is probably still more important. The phenomena of terrestrial magnetism take hold in many ways of other laws, and disclose irregular, or at least seemingly irregular changes, also simuitaneous in
their manifestations in the most distant parts of the earth, but embraced as a whole it may be within periods too great to have yet been comprehended in the range of time over which the longest series of magnetic observations haye extended. It may be that the full significance of the phenomena now being recorded in the Toronto Magnetic Observatory will only be understood when their normal progression completes a larger cycle, not of years but generations; and other centuries shall, by our aid, perceive the compass of greatgeneral laws. The relations already traced between magnetism, electricity, light, heat, and mechanical force, and all the singular glimpses of thermodynamics reducible to well-established laws by known mechanical principles, manifestly point to future disclosures of some comprehensive truth, as simple, yet perhaps even more wideembracing than Nerton's law : a grand lav of the universe that shall indicate long concealed relations between that vital force which is controled by mental volition and animal instincts, and the mechanical forces which control inorganic matter, and bind suns, and planets, and systems into one.

Thus do those little-heeded labors of our magnetic observers unite us as fellow-workers with the noble phalanx of intellectual toilers, whose far-reaching thoughts and speculations wander through unillumined vistas of the coming centuries, and search for revelations of truths which the angels desire to look into ; and the full significance of which, I doubt not, the spirits of just men made perfect rejoice to employ their renovated powers in mastering. But, while thus stand-ing on

> "This narrow isthmas 'twist two boundless seas, The past, the future, two cternities,
man-unconsciously stimulated by his immortal destiny:-desires to look into the unseen truths of a great future; it is also with no less characteristic zeal that he indulges in a wise retrospection; and in this also we have our indefatigable Provincial phalanx of workers. The two Decades illustrative of Canadian palæontology, issued: during the past year by the Geological Survey of Canada, minutely illustrate and describe evidences of life pertaining to formations. dating within that primary palæozoic period in which the Geologist recognises the oldest traces of organic structure, at an epoch, the remoteness of which he dimly guesses at by hundreds of thousandsof yeais. And of what use is it for us to learn of those long-perished.
crinoids and foraminifera, intombed in rocky sepulchres, grander and more lasting than the pyramids and catacombs of the Pharaobs? In this, too, Cauada is doing her appointed share in the world's search into the hidden truths of that book of nature, which is no less a divine revelation to us than the sacred volume of revalen moral truth: no less divine, thongh of inferior moment in the bearings of the truths it discloses, as revealing to us the Creator travelling in the greatness of his might through the sileuces of that infinite which lies belind us. In this, Canada claims to take her part among the world's thinkers. She will hew her lumber, raise her wheat, mine ber copper, lead the tracks of her railways ever westward, conquering the savage wilderness, and make the wilds of our vast pine forests the happy settlements of a free, industrious, and progressive people; but she aspires to sonething more than to be the mere lumberer and wheat-grower of the world; a and in so far as Canada does so, her material progress will not be the less, but greatly the more, for the intellectual vigor developed in thus claiming her place in that grand intellectual arena to which ouly the world's most gifted races find admission.
I might indeed dwell here, with justice, on the practical results of science; on the certainty that the mastery of the laws of nature increase the power of man; on the wondrous consequences that have followed from its least heeded beginuings; on the rubbed am.
 convulsions of the dead frog in the litchen of the famed Bolognese. Professor, Galvani : from whence we trace all our magnetical observatories, our new determinations of longitudes, our electric telegraphs, and the world-embracing project of our Atlautic cable. Or, again, on Newton's Apple; Jansen, the Dutch Optician's toy glasses; Watt's tea-kettle; or-apter for our present purpose,-Franklin's old key, which served him, with a silk-thread, sealing-wax, and a sheet of paper, to discover the identity of light g and electricity : these, or a thousand other germs of thought, insignificant, and barren as the sand-grains sown by the east wind, when presented to the dull common eye; but pregnant as the thousand-fold seed which the Master Sower-let fall into good ground, when they drop like the dews of summer on the fostering intellect of ripened genius. But here at least, such a defence of the sciences is umueeded. In the Cauadian Institute it may be presumed that we pursue science from the pure
delight which springs from the discovery of its secret truths; that we climb the steeps of knowledge, as the traveller asconds the mountain's unexplored cliffs, gladdened at every pause in his ascent with new grandeur and beauty in the widening horizon which opens on his delighted gaze.

But, while in thus leaving out of our present consideration the direct commercial and utilitarian results of Canadian science, our chief field of operation in Canada, and the immediate evidences of her scientific progress, are presented to us in the illustration of unknown gasterupods, crinoids, and foriminifera, discovered among the fossil forms of our older palæozoic rocks: we must not overlook the comprehensive generalizations to which the accumulation of such minate and seemingly isolated facts in ancient organic structure are leading.
With the original area of observation so immensely widened to the zoologist and naturalist by the comprehensive disclosures of palmology, all former conclusions are being subjected to revision and testing by such new evidence. The reality of the existence of very clearly discriminated specific forms, and the proofs of a continuous system of organization, development, displacement, and extinction, seem-all more evident and indisputable. Tet the immediate result appears in the removal of many old land-marks of scientific faith, whereby we witness some of those conditions of ruin, which mark all transitional and revolutionary eras,-whether of thought or action. The old has been sbaken, or thrown down, the new is still to build; and the casual and hasty observer is too apt to regard the indispensable clearing away of old and worn-out fabrics as the iudex only of ruin and desolation; while in reality it is the inevitable stage towards a higher replacement : like the ragged log-piles, the girdled-trees, and charred stumps of the pioneers of civilization in our Canadian wilderness, which are the needful precursors of the clearing, the farm-house, and the happy village homes.

In this light, I conceive, we must look upon that comprehensive question which now challenges revision in the hearing of new witnesses: What is Species? It is a question which forces us back to first principles, and equally affects the sciences of Palæontology, Zoology, and Ethnology; while it has also been made to bear in no unimportant degree on the relations of Science and Theology: involving as it dues the questions:-In what forms has creative power been manifested in the succession of organic life? and, Uuder what
conditions has man been introduced into the most diverse and widely separated provinces of the animal world? It is to the comprehensive bearings of the latter indeed, that the former owes its origin; for what is the use of entertaining the question, prematurely forced upon us: Are all men of one and the same species? while authorities in science are still so ranch at variance as to what species really is; and writers rho turn with incredulous contempt from the idea that all men are descended from Adam, can nevertheless look with counplacency on their probable descent from apes! Oue revoiutionary class of thinkers, having its representatives among the ablest men of science on this continent, incline to the belief that species is a mere logical invention of the systematiser, and that the older naturalists have converted convenient definitions and the necessary formulx of classification, into assumed realities. On the other hand, the extreme phalanx of their opponents invent a series of catastrophes, by which each geological period is closed, -the finished act, as it were, of a grand cosmic tragedy,-and all existing life is swept away, to give place to the creation of new species for the succeeding epoch of a renovated earth. This mysterious question of the origin of species is accordingly trammelled in part by that most dangerous of all hindrances to free inquiry and unbiased scientific judgment: The foregone popular conclusions relative to the supposed terms in which alone it can be answered, consistently with the inspired history of creation. Hence, on the one hand, development theories and transmutation of species; and on the other the more consistent idea not only of permanency of species, but also, along with it, of the recognition of the same great general laws which now govern the natural world having been in operation throughout all the countless ages of organic being which geology reveals to us.

Such inquiries into first principles necessarily bring about a collision between the conservative and the progressive ranks of thought; but in the conflict-whatever dust and heat arise,-the inevitable destruction of some long cherished error is of itself a clear gain. The course and tendency of thought may meanwhile be indicated to us by some of its most striking aspects:-é. $g$., by the startling propositions of Agassiz relative to supposed relations between the different types of man, and the geographical distribution and local circumscription of species in the world of inferior animal life. On the other hand, Professor Dana has produced his "Thoughts on

Species," illustrated by highly ingenious analogies, and not only suggeating clearer definitions, but also supplying some very comprehensive bases of thought. The problem, however, is nut one of easy solution. After various oscillations in the phases of expressed opinion, Protessor Baden Powell, has boldly taken up the enquiry in the whole comprehensive bearings of "The Philosophy of Creation," and in this work, among other profound questions, he gives special imnortance to that of the immutability or tramsmutation of species, as one of the most significant in relation to all the final deductions on which the disclosures of geology, and the scientific foundations of cosmo-theology, compal us to render our verdict anew.

Still more recently an eminent English Naturalist: Charles Darwin, has in his elaborate introductory trealise: "On the origin of species by means of natural selection," carried to undisguised conclusions, and with systematic details of evidence and results, some of those opinions which Professor t'owell has ouly left to be surmised. According to Mr . Darmin, the essential differences of genera are only . the product of the same powers of nature through a greatly protracted epoch, which within a less prolonged period had sufficed to produce species; and under our own limited observation are seen to give rise to permauent varieties in animals and plants. From observation of phenomena occurring within our own cognizance he has arrived at the conclusion that there is in reality no essential distinction between individual differences, varieties, and species. The well-marked variety is an incipient species; and by the operation of various simple physical causes, and comparatively slight orgànic changes, producing a tendency towards increase in one direction of variation, and arrestment, and ultimate extinction in another, that las of natural selection, as Darwin terms it, results, which leads to his "preservation of favoured races in the struggle for life." He thus establishes, as he conceives, a principle in uature, akin to that which man consciously sets in operation, when he effects chauges on domesticated animals and on plants, by altered conditions of life, and then perpetuates such as he selects by preference for his own use. The element of timeso limited in man's operations, - is for practical purposes unlimited in relation to the operation of natural causes on the development of variations in organic being in diverse directions; and as the great physical changes to which geology bears witness, supply all the means requisite for producing individual variations on a scale immensely ex-
ceeding any change observable on organic life under domestication, Mr. Darwin, conceives, and produces many illustrations in confirmation of his idea, that not only the origin of species, but the wider differences which distinguish genera, and all higher divisions of the organic kingdom may be accounted for by the same prolonged processes of variation and natural selection. Ilis "Origin of Species:" is no product of a rash theorist, but the result of the patient observation and laborious experiments of a highly giftud naturalist, extending over a period of upwards of twenty years, and-like the Reliquice Diliviance of Buckland,-it will be found to embody thoughts and facts of grast permanent value, whatever be the final decision on its special propositions. From the high authority of the writer, his well-established character as an accurate observer, and the bold and startling nature of his views, it cannot be doubted that his workwith the promised additions to the evidence now produced,-will tend. to re-open the whole question, and give courage to othor assailants. of those views of the permanency of species, which have seemed so indispensable alike to all our preconceived ideas in natural science, and to our interpretations of revealed cosmogony. Before Mr. Darwin's "Origin of Species" appeared from the press, Sir Charles. Lyell-himself no hasty or incautious doubter,-had remarked of it : " he appears to me to have succeeded by his investigations and reasonings, in throwing a flood of light on many classes of phenomena, connected with the affinities, geographical distribution, and geological succession of organic beings, for which no other hypothesis has been. able, or has even attempted to account." In relation to opiniuns advanced on questions of such profound interest and difficulty, by a. distinguished naturalist, as results of the experience and observations: of many years, our attitude ought clearly to be that of candid and. impartial jurors. We must examine for ourselves, not reject, the evidence thus honestly given. The experience of the past shows how frequently men have contended for their own blundering interpretations, while ull the while believing themselves the champions and the martyrs of truth. All truih is of God, alike in relation to the natural and the moral law, and of the former, as truly as of the latter may we say: " if this counsel or this work be of men, it will come to nought; but if it be of God, ye cannot overthrow it; lest haply ye be found even to fight against God."

But meanwhile in another, though allied direction, truth is the
gainer by this widening of the scientific horizon. In 1857 our greatest English naturalist, Prof. Owen, set forth his remarkable new system of classification of mammals, based on the form and complexity of the brain. In this novel and ingenious system he separates man, on clearly defined grounds of cerebral structure and proportions, into a distinct and crowning order of Abchenoeprata; thereby supplying by anticipation, a scientific antidote to one at least of the fallacies of Professor Powell, which may be thus stated : regarding the duration of time and the number of species as equally unlimited, he argues :-" While the number of species thus tends to become infinitely great, the extreme difference between man at one end and a zoophyte at the other end of the scale is constandly finite; hence the average difference between any two species tends to become infinitely small; multiplied by the number of species, it must still be equal to a finite quantity; and the product being finite, if the first factor be infinity the second must be zera."

It is scarcely necessary to observe that the tendency of species to an infinite multiplication of intermediate links, which is implied here, is a perfectly gratuitous assumption. The duration of time and the multiplication of species may be equally infinite; that it will be so we assuredly have no right to assume ; but in that case the analogies which palæcuiology reveals do not suggest the idea that such prolonged manifestations of the Creator's power to produce an infinite series of new forms will be exercised intermediately between those two fixed points of zoophyte and man. What if creative power should go on beyond the latter, into still higher manifestations of the divine image? Man cannot be demonstrated to he an absolute finality in organic creation. Apart, however, from any question of future creations, we look in vain among organic fossils for any such gradations of form as even to suggest a process of transmutation. Above all, in relation to man, no fossil form adds a single link to fill up the wide interval between him and the most anthropoid of inferior animals, when viewing him purely in those salient physical aspects to which the observation of the palæontologist is limited. The Archencephale of Owen stands as the crowning masterpiece of organic creation, separated from the highest type of inferior animal organization by as well defined and broad a line of demarkation as an insular kingdom from the states, republics, and confederacies of a neighbouring continent; and if the difference between man and the
inferior animals, not only in mere physical organization, but still more in all the higher attributes of animal life, be not relative but absolute, then no multiplication of intermediate links can lessen the obstacles to transmutation. One true antidote therefore to such a doctrine, and to the consequent denial of primary distinctions of species, seems to offer itself in such broad and unmistakeable lines of demarkation as Professor Owen indicates, between the cerebral structure of man and that of the most highly developed of anthropoid or other mammals.

Thus the widening range of observation is leading to other, yet related questions and discoveries of no slight importance. The whole compass of that latter one has been embraced in one aspect, in the remarkable introductory essay of Prof. Agassiz, "On Classification," which accompanies the first portion of the great American work now issuing by him under the title of "Contributions to the Natural History of the United States." Like all that comes from the gifted pen of Louis Agassiz, the Essay is bold, comprehensive, and valuable; but also it is not free from conclusions akin to those which in others of that distinguished naturalist's writings have been open to the charge of rash and hasty deductions from imaginary or defective premises. A more recent contribution to the same department of science is Prof Owen's communication to the Zoological section of the British Association, "On the Orders of Fossil and Recent Reptilia, and their distribution in time." In introducing his subject Professor Owen remarked, that, "with the exception of geology no collateral science had profited so largely from the study of organic remains as zoology. The catalogues of animal species have received immense accessions from the determination of the nature and affinities of those which have become extinct, and much deeper and clearer insight has been gained into the natural arrangement and sub-division of the classes of animals since palæoutology has expanded our surrey of them." The result of such study in the hands of the great comparative anatomist, has not.accordingly been to ignore species, but to reconsider their classifications. The boundary which modern zoological systems maintained between the classes Pisces and Reptilia is shown to be untenable, and a new group is discerned, within which extensive gradations of development link and blend together fishes, amphibia, and reptiles in one great natural series. No more important contribution has recently been made to
zoological:science ; illuminating, as it does, our knowledge of existing orders by the deeper insight acquired into forms of organic life that have long been extinct, it is a collateral contribution to scientific truth, analogous in kind, though notin degree, to that comprehensive demonstration of the typical skeleton, by which it is traced in all its details, from the highest to the lowest vertebrate forms. Such grand generalizations, bnsed, not on theory, but on laborious and exhaustive induction, reveal to us the plan of the Creator, wrought out in His unchangeable purpose, through ail the countless ages during which our planet has been the theatre of life. They tell us, moreover, in unmistakeable lauguage, that even to work out one single idea of the Divine mind, it has required the unmeasurable duration of time since that initial act in which God said let there be light, and called into being this well-ordered material world. "Lo these are parts of his ways; but how little a portion is heard of him; but the thunder of his power who can understand?"

In the fossil radiata and mollusca of our Canadian palmozoic formations, illustrater and describel in the recently published Decades of our Geological Survey, we are aided in the investigation of life as it existed in that primary geological period in which the earliest traces -of organic form appear; but an altogether different interest has been recently excited by discoveries at the very opposite end of the geological scale. It is now nearly ten years since M. Boucher de Perthes announced the discovery of the traces of human art in the same undisturbed gravel of the north of France, in which the bones of the fossil elephant and other extinct mammals are found. More recently fresh discoveries have tended to show that the statements set forth in the "Antiquités Celtiques et Antediluvicnnes" merited greater attention than, un various accounts, they received; and the testimony of Mr. Prestivick, Sir Charles Lyell, and other thoroughly trustworthy observers appears to place the fact beyond all controversy that artificially wrought weapons and implements of fint have been found both in France and Eaglaud, in such contiguity with the extinct fossil mammals of the drift, as to leave little room for question that at a period long anterior to the earliest indications of history or tradition, the north of Europe was occupied by a human population in a condition not less rude than the Indian aborigines of our own American Forests.

Purposing as I do, to take up the comprehensive inquiries to which
such discoveries point, in greater detail than could be permitted in this address, I shall only remark, meanwhile, that those who appear to be most startled with the apparent bearings of such discoveries, overlook the nearly analogous evidence we already possessed of the antiquity of the primeval colonization of the British Isles. Fully ten years since, and before the publication of $M$. Boucher de Perthes' work, in discussing the prehistoric traces of British population, I based one important line of argument for its antiquity on the discovery of artificial lances and harpoons, found beside the gigantic Balænoptere of the Scottish drift in the Carse of Stirling. These extinct fossil mammals-one of them seventy-two feet long,-lay stranded at the base of the Ochills, twenty-one feet above the present tide level, and from seven to twenty miles diatant from the nearest ocean reach. Whatever difficulties may seem to arise from the recent disclosures at Abbeville and Amiens, or the older ones at Hoxne in Suffolk, in relation to the age of man, the chronology which suffices to embrace the ancient Caledonian whaler of the valley of the Forth within the period of human history will equally answer for the more recently discovered allophylian of the French diluvium. Meanwhile it may not be unprofitable to note here also the changing phases of scientific thsology. The difficulty now is to reconcile the discovery of works of human art alongside of the fossil mammals of the drift. But when, in 1712, certain gigantic fossil bones,which would now most probably be refered to the Mastodon,-were found near Cluverach, in New England. the famous Dr. Increace Mather communicated the discovery to the Royal Society of London; and an abstract in the Philosophical Transactions duly sets forth the comforting opinion of the New England divine, of the confirmation thereby afforded to the Mosaic Narrative, that there were giants, or at least "men of very prodigious stature", in the Antediluvian world: for one of their teeth, a grinder, weighed four pounds and three-quarters, and a thigh bone measured seventeen feet long! Let it suffice for the present that geology in all its trustworthy and well established evidence still affirms that it is only in the latest posttertiary, or modern strata, that the traces of man and his arts are found : ancient indeed when compared with the times of authentic history or tradition, but only "of yesterday" when placed alongside of the Silurian organisms of our Canadian Decaucs, or even of the vertebrates of Geology's comparatively modern Tertiary formations.

From the epoch of Silurian crinoids to the ers of the drift and its included traces of human arts, is a transition as vast in point of time \&s the distances in space which the astronomer reduces to definite figures, but which the mind in vain attempts to realize. Compared with such a transition, the lapse of time from the earliest traces of human art to our modern nineteenth century is brief enough; yet the contrast seems scarcely so great between the organic forms of our lower silurian rocks, and the mammals of the drift, as that which separates the first rude evidences of human ingenuity in the latter formation, from such triumphs of mechanical skill as the "Great Eastern " of the Thames, or the "Victoria Tubular Bridge" of our own St. Lawrence. The great achievement of mechanical science and fearless enterprise ombodied in the gigantic structure which now spans the wide waters of the St. Lawrence, and has been opened for traffic since last we assembled here, is the crowning feature of that arterial system of railways which well nigh annihilates for us the impediments of time and space and is already revolutionizing our whole relations of commercial and social life.

It is impossible, however, to revert to either of those wonderful triumphs of mechanical science, without also recalling the painful coincidence that, alike in the Great Eastern Steam Ship and the Victoria Bridge, the inventive genius that had planned and directed each, throughout all the stages of its progress towards completion, was. snatched away when seemingly on the eve of realizing his most cherished hopes. The death of Robert Stephenson, at the too early ageof fifty-one, only a few weeks before the completion of that colossal creation of his genius which constitutes, not for Canada only, but for the world at large, one of the fittest memorials of the great Engineer, has already been referred to in the Annual Report of the Council: for, honored by ranking him among our Honorary Members, the Canadian Institute claims her share in the loss occasioned by the death of him whose remains have been laid amid the royal and noble dead of Westminster Abbey, with marks of distinction and tokens of public sorrow, rarely accorded but to such combinations of genius and great personal worth.

Your attention has been recalled by the interesting communication of Dr. Rae, to the latest results of Arctic discovery, which, while clearing up all mystery as to the fate of the lamented Franklin, ranks him in one sense among those whose loss we have anew mourned during.
the past year. Permit me, in thus referring to the honored name of Franklin, to couple with it that of a persona! friend, Mr. Henry Goodsir, formerly Curator of the Royal College of Surgeons of Edinburgh, who volunteered his services as Naturalist of the Franklin Expedition, and has doubtless perished, like his chief, though we lack the poor consolation of even learning his fate. I have watched with liveliest interest each detailed account of the relics of that ill-fated expedition, in the bope of recognising traces of one, not the least gifted or worthy among those whom Britain justly mourns. A young, enthusiastic, and highly gifted student of science: Henry Goodsir has fallent on a field more honorable, and striving in a nobler cause than most of those which furnish the laurels of heroes. Yet it is impossible not to revert with mournful reg-et to the ardent, sanguine votary of science, thus perishing before one desire had been accomplished, or one hope realized; going forth with the accumulated knowledge that constituted his weapons for that dread field, like the young soldier ardent for the strife:

## "And lost to life, and use, and name, and fame."

It is a duty which generally devolves on the President of a Society like this, to commemorate on such occasions, those whose loss we have to lament during the past year ; for, alas, no year passes over us, in which we have not to mourn some blank which death has made in our own numbers, or in that great Commonwealth of Science and Letters in which we claim to take our humble part. Among the ranks of our own members death has removed some who were wont to take a lively interest in our proceedings; and all of us, I doubt not, have deeply sympathised in the very painful circumstances which attended the loss of one of our number, the only son of His Excellency, Sir Edmund Head : a youth of great promise, and of rare enthusiasm in his early devotion to science. And when we look abroad on that wider circle which our sympathies embrace, we see that the Old World and the New have shared with an impartial equality in death's irrevocable bereavements. Hallam and Prescott, Brunel and Stephenson, De Quincy and Washington Irring, have, during the past year, followed one another to the grave; and it will not, I trust, be deemed an intrusion on the special duties of this occasion, if I turn aside for a moment to refer to another loss which science has recently sustained, but in which I claim a larger personal share. Death has been busy of late among Edinburgh men whom I counted my personal
friends. Dr. Samuel Brown, Professor Edward Forbes, and Hugh Miller, have followed one another to the grave within a brief period, and ere the past year drew to a close, Dr. George Wilson was added to the number of those who live only in honored memory. Dying at the early age of forty-one, when a carees full of rich promise appeared only opening before him, and his mind seemed to be ripening in many ways for a great life-work: those who knew his capacity and his genius regard all that he had accomplished as insignificant indeed when compared with what he would have doneif spared to those years in which men chiefly fulfil the promises of youth. Yet what he did accomplish, amid many and sore impediments to progress, is neither poor nor of small amount. Nor is it a light thing now to remember that one whose years of public $2 \mathrm{~L} t$ have been so few, and even these encroached on by the ever increasiryg impediments of failing health, has been laid in his grave amid demonstrations of public sorrow. such as have rarely incleed been accorded, in that native city of his, to. Edinburgh's greatest men. This was due even more to the genial kindliness and worth of a noble Christian man, than to the unwearied zeal of a popular public teacher, and an enthusiastic student of science. His loss to his university is great, but to his friends it is irreparable. In him the faith of science, and the nobler faith of the Christian, were blended into perfect harmony; for no doubt springing from halfrevealed truths of science ever marred the serene joy of his faith whilelooking at the things which are not seen. Prejudice and falsehood, ignorance and vice, were felt by him to be the common foes of both; and pardon me, if I add, that no man I have ever known carried more genially and unobtrusively, yet more thoroughly, his earnest Christian faith into all the daily business and the duties of life.

When a man of such genuine kindliness and worth is suddenly called away in his prime, with still so much of his life-work seemingly waiting its accomplishment, it is as when a brave vessel founders: in mid-ocean. The wild eddy of the troubled waters gathers around the. fatal gulf, and a cry of sympathetic sorrow rises up as the news is borne along to distant shores. But the ocean settles back to its wonted flow where that gallant bark went down, and the busy world soon, returns to, its old absorbing, occupations. But there are those to. whom that foundered bark has been the shipwreck of a life's hopes;; and to me the loss of my life-long friend and brother will make life's future - years wear a shadow they could never wear before:

But, Gentlemen, I trespass on the privileges of this chair. Let it be my apology to you thiat the event I mourn is-from accidental cir-cumstances,-peculiarly associated with this meeting and your choice of me as your President. Permit me, in closing an address already too protracted, in which I have aimed at indicating some of those lines of abstract thought whereby science is enlarging our viers and widening our sphere of knowledge, to invite you, as in a sense the selfconstituted acolytes in this temple of Canadian science, to enter with renewed energy and devotion on the work of another year : remembering, each one of us, that we know not how few our years of work may be. We may indeed-in a far more absolute and literal sense than Newton could,-say, after all our work is accomplisked, that we "seem to have been only like a boy playing on the sea-shore, and diverting himself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before us." But yet let us remember this at least, that that great ocean of truth does lie before us, and even those pebbles which our puerile labours gather on its shore, may include here and there a gem of purest ray; and meanwhile the search for truth, and even the play along the pleasant shores of its great unexplored ocean, will bring to each one of us his own exceeding great reward.

## RESOLUTION OF ALGEBRAICAL EQUATIONS.

(Continuedfrom the last Number of the Journal.)
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## Proposition VI.

If all the cognate functions (not necessarily unequal) of $f(p)$, ant integral function of $a$ variable $p$, be,

$$
\phi_{1}, \phi_{2}, \phi_{3}, \ldots . ., \phi_{m} ; \ldots . . . . . . . . . . . . . . . . . . . . . \text { (1) }
$$

and if

$$
\begin{aligned}
\mathrm{X} & =\left(x-\phi_{1}\right)\left(x-\phi_{2}\right) \ldots \ldots\left(x-\phi_{\mathrm{m}}\right) \\
& =x^{\mathrm{m}}+\mathrm{A}_{1} x^{\mathrm{m}-2}+\mathrm{A}_{2} x^{m-2}+\ldots \ldots+\mathrm{A}_{\mathrm{m}} ; \ldots \text { (2) }
\end{aligned}
$$

then the coefficients $\mathrm{A}_{1}, \mathrm{~A}_{2}$, \&c., nray be exhibited as rational expressions, that is, (see Def. 1), rational functions of $p$.

For take $\phi$, the general symbol under which are included all the particular terms in the series (1); and let the $n^{\text {th }}$ power of $\phi$, ( $n$ being a whole number), arranged so as to satisfy the conditions of Def. 8, be,

$$
\begin{equation*}
\phi_{\mathrm{n}}=a+a_{1} t_{1}+a_{2} t_{2}+\& c . ; \tag{3}
\end{equation*}
$$

where the coefficients, $a, a_{1}, \& c$., are rational ; and each of the terms, $t_{1}, t_{2}$, \&c., is either some power of an integral surd, or the continued product of several such powers. Suppose $y_{1}^{r}$ to be one of the factors of $t_{1}$; the index of the surd $y_{i}$ being $\frac{1}{\lambda}$; and let the several $\lambda^{\text {th }}$ roots of unity be, $1, z, z_{2}^{2}, \ldots \ldots, z^{\lambda-1}$. Then, from (3),

$$
\begin{gathered}
\phi_{1}^{n}=a+a_{1} v_{1}+a_{2} v_{2}+\& c, \\
\phi_{2}^{n}=a+a_{1} u_{1}+a_{2} u_{2}+\& c . \\
\cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \\
\phi_{m}^{n}=a+a_{1} w_{1}+a_{2} w_{2}+\& c . ;
\end{gathered}
$$

where $v_{1}, u_{1}, \& c$. , are what $t_{1}$ becomes in passing from $\phi$ to $\phi_{1}, \phi_{2}$, \&c.; and so of the other terms. Therefore,

$$
\begin{align*}
\Sigma\left(\phi^{\mathrm{n}}\right) & =\phi_{1}^{\mathrm{n}}+\phi_{2}^{11}+\ldots+\phi_{\mathrm{n}}^{\mathrm{n}}=\ldots+a_{1}\left(v_{1}+u_{1}+\ldots+w_{1}\right)+\& c_{.} \\
& =\ldots \ldots+a_{1} \Sigma\left(t_{1}\right)+\& c \cdot ; \ldots \ldots \ldots \ldots \ldots \ldots(4) \tag{4}
\end{align*}
$$

where, just as $\Sigma\left(\phi^{n}\right)$ represents the sum of the terms, $\phi_{1}^{n}, \phi_{2}^{n}$, $\ldots \ldots, \phi_{m}^{n}$, so $\sum_{( }\left(t_{1}\right)$ represents the sum of the terms, $v_{1}, u_{1}, \ldots, w_{1}$. Now, in the series, $v_{1}, u_{1}, \& c$., if any term $v_{1}$ be fixed upon, there are $\lambda$ terms, including $v_{1}$, of the forms,

$$
v_{1}, z v_{1}, z^{2} v_{1}, \ldots \ldots, z \lambda-1 v_{1} .
$$

The sum of these is zero. Strike these $\lambda$ terms out of $\Sigma\left(t_{1}\right)$; and then, in the same maner, whatever term among those remaining in $\Sigma\left(t_{1}\right)$ be considered, it may be demonstrated to be one of a group whose sum is zero. And so on. Therefore $\Sigma_{\text {, }}\left(t_{1}\right)$ is zero. In like manner all the terms on the right haud side of equation (4), except the first, or $m a$, must vanish. Consequently, $\Sigma\left(\phi^{n}\right)$ is rational. If now we put

$$
\begin{aligned}
& S_{1}=\phi_{1}+\phi_{2}+\ldots \ldots+\phi_{m}^{2} \\
& S_{2}=\phi_{1}^{2}+\phi_{2}^{2}+\ldots \ldots+\phi_{m}^{2} \\
& S_{3}=\phi_{1}^{3}+\phi_{2}^{3}+\ldots \ldots+\phi_{m}^{3}
\end{aligned}
$$

and so on, the expressions $S_{1}, S_{2}, \& c$., are (by what we have proved) rational. But, by Newton's Theorem for the sums of the powers of the roots of an equation, (see equation (2),

$$
\begin{array}{r}
S_{1}+A_{1}=0 . \quad \text { Therefore } A_{1} \text { is rational. } \\
S_{2}+A_{1} S_{1}+2 A_{2}=0 .
\end{array} \text { Therefore } A_{2} \text { is rational. }
$$

And in the same way all the terms $A_{1}, A_{2}, \& c$., may be exhibited as rational expressions.

Cor. 1.-Should the terms in (1) not be all unequal, let the unequal terms be,

$$
\begin{equation*}
\phi_{1}, \phi_{2}, \ldots \ldots \phi_{s} \tag{5}
\end{equation*}
$$

Then if $f(p)$ be in a simple form, and $\mathrm{X}_{1}$ be the continued product of the terms, $x-\phi_{1}, x-\phi_{a}, \ldots, x-\phi_{c}$, where $\phi_{1}, \phi_{a}, \ldots, \phi_{c}$, are a number of terms in ( 1 ), fewer than $s, \mathbf{X}_{1}$ cannot have the coefficients of the various powers of $x$ rational. For suppose, if possible, that $\mathrm{X}_{\mathbf{I}}$ has the coefflcients of the various powers of $x$ rational. Then $\phi_{1}$ is a root of the equation, $\mathrm{X}_{1}=0$. And since, by the hypothesis made in the Corollary, $f(p)$ is in a simple form, $\phi_{1}$ also (Prop. IV.) is in a simple form. Therefore (Prop. V.) all the terms in (5) are roots of the equation, $\mathrm{X}_{1}=0$; and they are all unequal: which, since the equation is of a degree lower than the $s^{\text {th }}$, is impossible. Therefore $\mathrm{X}_{1}$ cunnot have the coefficients of the powers of $x$ rational.

Cor. 2. If (See 5) we put

$$
\left(x-\phi_{2}\right)\left(x-\phi_{2}\right) \ldots \ldots\left(x-\phi_{1}\right)=x^{3}+b_{1} x^{n-1}+b_{2} x^{2-2}+\& c
$$

the coefficients, $\dot{b}_{1} \cdot, b_{2}, \& c$., may be exhibited as rational expressions; and, if $f(p)$ be in a simple form, each of the terins, $\phi_{1}, \phi_{2}, \ldots \ldots, \phi_{s}$, recurs in (1) the same number of times. For let $\phi_{1}$ occur $\lambda$ times in (1); $\phi_{2}, \beta$ times; and so on. Then
$\left(x-\phi_{1}\right)^{\lambda}\left(x-\phi_{2}\right)^{\beta} \ldots\left(x-\phi_{\mathrm{s}}\right)^{\delta}=\left(x-\phi_{1}\right)\left(x-\phi_{2}\right) \ldots\left(x-\phi_{m}\right)=\mathbf{X} \ldots \ldots$ (6)
The equation, $X=0$, has one group of $\lambda$ equal roots, another group of $\beta$ equal roots, and so on. There is therefore a common measure, $\mathrm{X}_{2}$, of X and $\frac{d \mathrm{X}}{d x}$, of the form,

$$
\begin{equation*}
\left(x-\phi_{1}\right)^{\lambda-1}\left(x-\varphi_{2}\right)^{\beta-1} \ldots \ldots\left(x-\phi_{\mathrm{s}_{-}^{\prime}}^{( }\right)^{\delta-1}=\mathbf{X}_{2} \tag{7}
\end{equation*}
$$

The expression $\mathrm{X}_{2}$ resembles X in having the coefficients of the various powers of $x$ rational; for it is the H. C. M. of X and $\frac{d \mathrm{X}}{d x}$. Hence, denoting $\frac{X}{X_{2}}$ by $X_{3}$, we have, from (6) and (7),

$$
\begin{equation*}
\left(x-\phi_{1}\right)\left(x-\phi_{2}\right) \ldots \ldots\left(x-\phi_{s}\right)=\mathrm{X}_{3}, \tag{8}
\end{equation*}
$$

where $\mathrm{X}_{3}$, being the quotient of $\boldsymbol{X}$ by $\mathrm{X}_{z}$, must have the coefficients of the various powers of $a$ rational. Hence $b_{1}, b_{2}, \& c$, may be exhibited as rational expressions. Thus the former of the two points to be proved in the Corollary is established. Next, should $f(p)$ be in a simple form, and should the numbers $\lambda, \beta, 8 c \mathrm{c}$., not be all equal to one another, let $\lambda$ be less than $\delta$, and not greater than any of the others. Then, from (8) and (6), we have, putting $\mathrm{X}_{4}$ to denote the quotient of $X$ by $X_{3}^{\lambda}$,

$$
\begin{equation*}
\left(x-\phi_{2}\right)^{\beta-\lambda \ldots \ldots\left(x-\phi_{s}\right)^{\delta-\lambda}=\mathbf{X}_{4}^{\prime}, ~} \tag{9}
\end{equation*}
$$

$\mathrm{X}_{4}$ being rational. Should the numbers, $\beta-\lambda, \delta-\lambda$, \&c., not be all equal to one another, then, exactly as we reduced equation (6) to equation ( 9 ), on the left hand side of which no power of ( $x-\phi_{1}$ ) appears as a.factor; we can reduce equation (9) to an equation bearing the samerelation to (9) that (9) bears to (6). And so on, till we arrive at an equation, such as (9), in which the indices, such as, $\beta-\lambda$, \&c., are all equal to one another. Let the result obtained when this point is reached be,

$$
\left(x-\phi_{\mathrm{x}}\right)^{l-h}\left(x-\phi_{\mathrm{G}}\right)^{k-h} \ldots \ldots\left(x-\phi_{\mathrm{S}}\right)^{\delta-h}=\mathrm{X}_{5} .
$$

From this, since the numbers, $l, k, \ldots . . ., \delta$, are equal to one another, we get, by continuing the reduction,

$$
\left(x-\phi_{\mathrm{a}}\right)\left(x-\phi_{\mathrm{c}}\right) \ldots \ldots\left(x-\phi_{\mathrm{s}}\right)=\mathrm{X}_{6} ;
$$

$X_{6}$ being a rational expression: which, since the number of its factors, $x-\phi_{a}, x-\phi_{c}, \&(c$. , is less than $s$, and since $f(p)$ is supposed to be in a simple form, is (Cor. 1) impossible. Hence $\lambda, \beta$, \&c., in (6), ave all equal to one another; and therefore each of the termes, $\phi_{1}$, $\phi_{2}, \ldots \ldots, \phi_{5}$, must recur in (1) the same number of times.

Cor. 3. $\operatorname{In} f(p)$ lett certain surds, $y_{1}, y_{2}, \& c$., (in which series of terms, as was pointed out in Def. 7, all the subordinates of any surd mentioned are included), have definite values assigned to them; and
let the cognate functions of $f(p)$, obtained without departing from such definite values, (obtained, in other words, by proceeding without reference to the surd character of $y_{1}, y_{2}, \&$ ce., be,

$$
\begin{equation*}
\phi_{1}, \phi_{2}, \ldots \ldots \ldots, \phi_{\mathrm{n}} . \tag{10}
\end{equation*}
$$

Then if

$$
\left(x-\phi_{1}\right)\left(x-\phi_{2}\right) \ldots\left(x-\phi_{n}\right)=x^{n}+B_{1} x^{n-1}+B_{2} x^{n-2}+\& c .,
$$

the coefficients, $\mathrm{B}_{1}, \mathrm{~B}_{2}$, are equal to expressions which are rational as respects all surds except $y_{1}, y_{2}$, \&c. In other words ${ }_{3}$ no surds not included in the series $y_{1}, y_{2}, \& c$., enter into these coefficients. The proof is the same as in the Proposition.

Cor. 4. In the case supposed in the preceding Corollary, it may be shown, as in Cor. 1, that, if the unequal terms in (10), (the definite valt $\lrcorner$ of $y_{1}, y_{2}, \& c$. , being understond to be adhered to), be,

$$
\phi_{1}, \phi_{2}, \ldots \ldots \ldots \ldots, \phi_{t},
$$

and if $f(p)$ be in a simple form, and we write

$$
\left(x-\phi_{1}\right)\left(x-\phi_{a}\right) \ldots \ldots \ldots\left(x-\phi_{c}\right)=\mathrm{X}_{1},
$$

where the number of terms, $\phi_{1}, \phi_{a}, \ldots . . . . ., \phi_{c}$, is less than $t$, these terims being terms in (10), $X_{1}$ cannot involve, in the coefficients of the powers of $x$, merely the surds $y_{1}, y_{2}$, \&c. For, if $X_{1}$ did int: volve merely these surds, $\phi_{1}$ would be a root of the equation, $X_{1}=0$; and therefore (Cor. Prop. V.) all the expressions, $\phi_{1}, \phi_{\dot{1}}, \ldots \ldots, \phi_{s} ;$ would be roots of that equation; the definite values given to $y_{1}, y_{2}$; \&c., being adhered to in all the expressions, $\phi_{1}, \phi_{2}, \ldots . . . . ., \phi_{1}$. But these expressions are, by hypothesis, unequal. Therefore the equation, $X_{1}=0$, has $t$ tunequal roots: which, since the equation is of a degree lower than the $t^{\text {th }}$, is impossible. Therefore $X_{1}$ cannot involve, in the coefficients of the powers of $x$, merely the surds $y_{1}, y_{2}, \& c$.

Cor. 5. In the case supposed in Cor. 3, let the unequal terms in the series (10), bej, $\phi_{1}, \phi \frac{1}{2}, \ldots . . . . . ., \phi_{t}$; and let

$$
\left(x-\phi_{1}\right)\left(x-\phi_{2}\right) \ldots \ldots\left(x-\phi_{t}\right)=x^{t}+b_{1} x^{t-1}+b_{2} x^{t-2}+\mathbb{L} c .
$$

Then the coefficients $b_{1}, b_{2}, \& c$., are equal to expressions involving no surds which do not occur in the series $y_{1}, y_{2}, \& \in$. ; and, if $f(p)$ be in a simple form, each of the unequal terms, $\phi_{1}, \phi_{3}, \ldots ., \phi_{t}$, recurs the same number of times in ( t 0 ) The proof is the same as in Cor. 2.

Cor. 6. If the equation, $F(x)=0$, be an equation in which the coefficients of the powers of $x$ are rational functions of $p$; and if $F(x)$ cannot be broken into rational factors, (by which expression we mean, factors having the coefficients of the powers of $x$ rational), then, $f(p)$, an integral function of $p$, in a simple form, being a root of the equation, $F(x)=0$, the roots of that equation are identical with the serms of the series (5), that is, with the unequal cognate functions of $f(p)$. For (Proj. V.) every term in (5) is a root of the equation, $F(x)=0$. Also (Cor. 2) the expression,

$$
\begin{equation*}
\left(x-\phi_{1}\right)\left(x-\phi_{2}\right) \ldots \ldots \ldots\left(x-\phi_{S}\right), \ldots \tag{11}
\end{equation*}
$$

when multiplied out, and arranged according to the powers of $x$, has the coefficients of the powers of $: c$ equal to rational expressions. Therefore, unless the expression (11) were identical with $F(x)$, F ( $x$ ) would have a rational factor, of less dimensions, as respects $x$, than $F(x)$ : which is contrary to supposition. Therefore the expression in (11) is identical with $F(x)$; and the roots of the equation, $\mathrm{F}(x)=0$, are the terms in the series (5).

## Proposimion VII.

Let $f(p)$ be an integral function of a variable $p$, in a simple form. Denote by $\phi_{1}, \phi_{2}, \ldots \ldots, \phi_{1}$, all the unequal cognate functions of $f(p)$, obtained by assigning definite values to certain surds, $y_{1}, y_{2}$, \&c., and proceeding (according tc Def. 7) without reference to the surd character of $y_{1}, y_{2}$. \&cc. Let

$$
\begin{aligned}
F_{1}(x) & =\left(x-\phi_{1}\right)\left(x-\phi_{2}\right) \ldots \ldots \ldots\left(x-\phi_{n}\right) \\
& =x^{n}+A_{1} x^{n-1}+A_{2} x^{n-2}+\ldots+A_{n}
\end{aligned}
$$

the coefficients $A_{1}, A_{2}$, \&c., satisfying the conditions of Def. 8, and not involving (Cor. 5, Prop. VI.) any surds not found in the series, $y_{1}, y_{2}$, \&c. Let $y_{1}$ be a surd occurring in $\mathrm{F}_{1}(x)$, that is, in the coefficients, $A_{1}, A_{2}$, \&ic., but not a subordinate of any surd in $F_{1}(x)$, its index being $\frac{1}{r}$; and, when we substitute for $y_{1}$ in $A_{1}$, $A_{2}$, \&cc., the successive values, $z y_{1}, z^{2} y_{1}, \ldots \ldots, z^{r-1} y_{1}, z$ being an $r^{\text {th }}$ root of unity, distinct from unity, let $\mathrm{F}_{\mathrm{I}}(x)$ become in succession $\mathrm{F}_{2}(x), \mathrm{F}_{3}(i), \& \mathrm{cc}$. Then, if

$$
\begin{aligned}
\mathrm{F} & =\mathrm{F}_{1}(x) \times \mathrm{F}_{2}(x) \times \mathrm{F}_{3}(x) \times \ldots \ldots \ldots \times \mathrm{F}_{r}(x) \\
& =\left(x-\phi_{1}\right)\left(x-\phi_{2}\right) \ldots \ldots\left(x-\phi_{n}\right)\left(x-\phi_{1+n}\right) \ldots\left(x-\phi_{\mathrm{n}}\right) \ldots\left(x-\phi_{n r}\right),
\end{aligned}
$$

the terms, $\phi_{1}, \phi_{2}, \ldots \ldots, \phi_{n r}$, are all the unequal cognate functions of $f(p)$, obtained by giving definite values to all the sards in $f(p)$ which are present in the coefficients of the powers of $x$ in $F$, and forming the coguate functions without reference to the surd character of the surds thus rendered definite : $F$ being understood to be generated directly by the multiplication together of the factors, $\mathrm{F}_{1}(x)$, $\mathrm{F}_{2}(x)$, \&c., and to have the coefficients of the various powers of $x$ arranged so as to satisfy the conditions of Def. 8 .
For, all the terms in the series,

$$
\begin{equation*}
\phi_{1}, \phi_{2}, \ldots \ldots \ldots, \phi_{\mathrm{n}}, \tag{1}
\end{equation*}
$$

are (by hypothesis) unequal. Suppose, if possible, that the terms,

$$
\begin{equation*}
\phi_{n+1}, \phi_{n+2}, \ldots \ldots \ldots, \phi_{2 n}, \tag{2}
\end{equation*}
$$

which are the roots of the equation, $\mathrm{F}_{2}(x)=0$, are not all unequal. Then, $\mathrm{F}_{2}(x)$, having equal factors, has a measure, $H$, of less dimensions, as respects $x$, than $F_{2}(x)$, and yet involving, in the coefficients of the powers of $x$, merely such surds as occur in $F_{2}(x)$. But the surds in $F_{2}(x)$ are identical with those in $F_{1}(x)$. [For instance, let $\mathrm{F}_{1}(x)=(1+\sqrt{ } p)^{\frac{2}{3}}$, and, $\mathrm{F}_{2}(x)=z(1+\sqrt{ } p)^{\frac{2}{3}}$, where $z$ is a third root of unity, distinct from unity. The presence of $z$ in $\mathrm{F}_{2}(x)$ does not affect the surds in the expression]. Therefore the expression H, of less dimensions as respects $x$ than $F_{1}(x)$, involves in the coefficients of the powers of $x$ merely such surds as appear in $F_{1}(x)$ : which, [since $\mathrm{F}_{1}(x)$ is the product of the terms, $x-\phi_{1}, \ldots, x-\phi_{\mathrm{n}}$, where $\phi_{1}, \varphi_{2}, \ldots \phi_{\mathrm{n}}$, are all the unequal cognate functions of $f(p)$ obtained by assigning definite values to certain surds in $f(p)$ ], is (Cor. 4, Prop. VI.) impossible. Therefore all the terms in (2) are unequal. Next suppose, if possible, that some term in (2) is equal to a term in (1). Then $F_{2}(x)$ and $F_{1}(x)$ have a common measure; and their $H$. C. M. involves only such surds as appear in $\mathrm{F}_{1}(x)$ or $F_{2}(x)$; that is, only such as appear in $F_{1}(x)$ : which, as above, is (Cor. 4, Prop. VI.) impossible, unless $\mathrm{F}_{1}(x)$ and $\mathrm{F}_{2}(x)$ are identical. Suppose then, if possible, that $\mathrm{F}_{1}(x)=\mathrm{F}_{2}(x)$. The coefficients of like powers of $x$ must be equal. Let the coefficient of a certain powers of $x$ in $\mathrm{F}_{1}(x)$, arranged according to the powers of $y_{1}$, (we choose a coefficient where $y_{1}$ occurs in some of its powers), and satisfying (as, by hypothesis, it does) the conditions of Def. S, be,

$$
b+b_{1} y_{1}+b_{2} y_{1}^{2}+\ldots \ldots+b_{r-1} y_{1}^{r-1}
$$

where $b, b_{1}$, \&c., are clear of the surd $y_{1}$. The corresponding coefficient in $\mathrm{F}_{2}(x)$ is

$$
b+b_{1} z y_{1}+b_{2} z^{2} y_{1}^{2}+\& c .
$$

Therefore, $b_{1}(z-1) y_{1}+b_{2}\left(z^{2}-1\right) y_{1}^{2}+\& c .=0$.
Since the surds present in this equation are surds occurring in $f(p)$, and $f(p)$ is in a simple form, the coefficients, $b_{1}(z-1)$, $b_{2}\left(z^{2}-1\right)$, \&c., must (Cor. 1. Def. 9) vanish separately. But, since $z$ is an $r^{\text {th }}$ root of unity, distinct from unity, $r$ being a prime number, none of the expressions, $z-1, z^{2}-1$, \&c., vanish. Therefore $b_{1}, b_{2}$, \&c., must all be zero: which is inconsistent with the assumption that the surd $y_{1}$ is present in the coefficient selected. Therefore $F_{1}(x)$ is not equal to $F_{2}(x)$; and we proved that it has no common measure with $\mathrm{F}_{2}(x)$. Therefore no term in (l) is equal to a term in (2) ; and all the terms, $\phi_{1}, \phi_{2}, \ldots ., \phi_{2 n}$, are unequal. In the same way it appears that all the terms, $\phi_{1}, \phi_{2}, \ldots \ldots$, , ar, are unequal.

The terms, $\phi_{1}, \phi_{2}, \ldots \ldots, \phi_{\mathrm{nr}}$, thus proved unequal, are the unequal cognate functious of $f(p)$, obtained by giving definite values to the surds in $F$, [which, from the manner in which $F$ was generated; are necessarily surds occurring in $f(p)$ ], and framing the cognate functions without reference to the surd character of these surds. For, in framing the cognate functions, $\phi_{1}, \phi_{2}, \ldots \ldots, \phi_{\mathrm{nr}}$, all the surds in $\mathrm{F}_{1}(x)$, except $y_{1}$, were considered as definite; and no numerical multipliers (such as $z_{1}, z_{2}$, \&c., in Def. 6) were affixed to them. If F contained all the surds in $\mathrm{F}_{1}(x)$, except $y_{1}$, our point would be easily established. It may happen, however, that $F$ does not contain all the surds in $\Gamma_{1}(x)$ except $y_{1}$. Other surds may have disappeared from it, along with $y_{1}$. Let $t$ be one of these, if there be such : and let its inder be $\frac{r}{s}$. Then, in virtue of the $s$ values that may be given to $t$, the cognate functions of $f(p)$, taken on a non-recognition of the surd character of those surds alone which appear in $F$, must include $s$ groups of such terms as

$$
\begin{equation*}
\phi_{1}, \phi_{2}, \ldots \ldots \ldots \phi_{11 r} \tag{3}
\end{equation*}
$$

In. general, if $t, t_{1}$, \&c., be the surds in $F_{1}(x)$, besides $y_{1}$, which are not in $T$; and if $\frac{1}{s}, \frac{1}{s_{1}}$, \&cc., be the indices of the surds $t, t_{1}$, \&oc.,
there will be $s s_{1}$......... groups of such terms as (3). Still further, without having respect to the surds $t, t_{1}, \&($ c., there may be (Cor. 5 , Prop. VI.: see more particularly the explanation presently to be given) $m$ distinct groups such as (3) : only (as bas been proved) the $\boldsymbol{n} \boldsymbol{\varepsilon}$ functions in (3) are the only unequal terms in all the $n$ groups On the wnole, the series of cognate functions of $f(p)$, taken on a nonrecognition of the surd character of those surds alone which are present in F , will embrace mnrs $s_{1}$......... terms, or $n s s_{1} \ldots \ldots$... lines of terms such as (3), of which the following may serve as examples:


The first of these lines is (3). The second is a cluster of terms, in addition to the $n r$ terms of the first line, obtained without having respect to $t, t_{1}$, \&c., and being a repetition of the values of the terms in the first line; for, in the mar terms, obtained without reference to $t_{5}, t_{\mathrm{I}}, \& \in$., the unequal terms which constitute the series (3) are all repeated (Cor. 5. Prop. VI.) the same number of times. The third line of (4) contains the terms in the first line, transformed by changing $t$ into $z_{1} t ; z_{1}$ being an $s^{\text {th }}$ root of unity, distinct from unity. And those in the last line contain the terms of the second line, transformed by a similar change of $t$ into $z_{1} t$. Now it can be shown that the terms of the third line are equal, in some order, to those of the first, each to each. For, since $t$, present in $\mathrm{F}_{1}(x)$, disappears from $\mathrm{F}_{3}$ it follows that the continued product of the factors of F , viz. : $\mathrm{F}_{1}(x), \mathrm{F}_{2}(x)$, \&c., remains the same when $z_{1} t$ is sulbstituted for $t$. That is, the factors,

$$
x-\phi_{1}, x-\phi_{2}, \ldots \ldots \ldots . ., x-\phi_{a r}
$$

are the same, taken in some order, with the factors,

$$
x-\phi_{1}^{\prime}, x-\phi_{2}^{\prime}, \ldots \ldots \ldots \ldots, x-\phi_{u r}^{\prime} .
$$

Hence the terms in the third line of (4) are, in some order, equal to those in the first line, each to each. In the same way it may be proved that all the mnrss $s_{1}$......... cognate functions above described, are merely repetitions of the ralues of the functions in (3). Hence the terras in (3) are all the unequal cognate functions of $f(p)$, obtain-
ed by giving definite values to the surds in $\mathbf{F}$, and taking the cognate functions without reference to these surds.

## Proposition VIII

Let an equation of the $m^{\text {th }}$ degree, whose coefficients are rational functions of a variable $p$, be, $\mathrm{X}=0$; X having no rational factors; and let an algebraical root of this equation, in a simple integral form, arranged also so as to satisfy the conditions of Def. 8, be $f(p)$. Take $u_{1}$, a surd in $f(p)$, not a subordinate of any other surd in the function, with the index $\frac{1}{n}$; and let the cognate functions of $f(p)$, obtained by successively changing $u_{1}$, wherever it occurs in $f(p)$ in any of its powers, into $u_{1}, z_{1} u_{1}, z_{1}^{2} u_{1}, \ldots \ldots, z_{1}^{n-1} u_{1}$, $z_{1}$ being an $n^{\text {th }}$ root of unity, distinct from unity, be,

$$
\phi_{1} \text { or } f(p), \phi_{2}, \phi_{3}, \ldots \ldots \ldots, \phi_{11} .
$$

Let $F_{1}(x)$ denote the continued product of the terms, $x-\phi_{1}, x-\phi_{2}$, $\ldots . . . . .,<-\phi_{n}$. The coefficients of the various powers of $x$ in $\mathrm{F}_{1}(x)$, made to satisfy the conditions of Def. S, are (Cor. 3, Prop. VI.) clear of the surd $u_{1}$; and the terms, $\phi_{1}, \phi_{2}, \ldots \ldots \ldots, \phi_{n}$, constitute (Prop. VII.) the series of the unequal cognate functions of $f(p)$, obtained by a:fixing definite values to all the surds in $\mathrm{F}_{1}(x)$, [which are necessarily surds in $f(p)]$, and taking the cognate functions without reference to the surd character of the surds so made definite. Should $\mathrm{F}_{1}(x)$, which $i .3$ clear of the surd $u_{1}$, not have the coefficients of the powers of $x$ rational, let $u_{2}$, a surd in $F_{1}(x)$, not a subordinate of any other surd in $\mathrm{F}_{1}(x)$, with the indev $\frac{1}{r}$, be successively replaced by $u_{9}$, $z_{2} u_{2}, z_{2}^{2} u_{2}, \ldots \ldots \ldots, z_{2}^{r-1} u_{2} ; z_{2}^{-}$being an $r^{\text {th }}$ root of unity, distinct from unity; and, in consequence of these alterations, let $\mathrm{F}_{1}(x)$ become successively $\mathrm{F}_{1}(x),{ }^{2} \mathrm{~F}_{1}(x),{ }^{3} \mathrm{~F}_{1}(x), \ldots \ldots \ldots,{ }^{\mathrm{r}} \mathrm{F}_{1}(x)$; the functions which are $\phi_{1}, \phi_{2}, \ldots \ldots \ldots, \phi_{n}$, in $F_{1}(x)$, becoming $\phi_{n+1}, \phi_{n+3}$, $\ldots \ldots, \phi_{2 \mathrm{n}}$, in ${ }^{2} \mathrm{~F}_{1}(x)$, and becoming $\phi_{2 \mathrm{n}+1}, \& \mathrm{c}$., in ${ }^{3} \mathrm{~F}_{1}(x)$; and so on. Denote the continued product of the terms, $\mathrm{F}_{1}(x),{ }^{2} \mathrm{~F}_{1}(x), \ldots \ldots$, ${ }^{T} F_{1}(x)$, when the result is made to satisfy the conditions of Def. 8 , by $\mathrm{F}_{2}(x)$, which is (Cor. 3. Prop. VI.) an expression clear of the surd $\mu_{2}$, and such (Prop. VII.) that the functions, $\phi_{\lambda}, \phi_{2}, \ldots \ldots, \phi_{n r}$, [the
$n=$ factors of $\mathrm{F}_{2}(x)$ being $\left.x-\phi_{1}, x-\phi_{2}, \ldots \ldots, x-\phi_{\mathrm{mr}}\right]$, constitute the series of the unequal cognate functions of $f(p)$, obtained by assigning definite values to all the surds which are found in $\mathrm{F}_{2}(x)$, [these being surds of necessity present in $f(p)$ ], and taking the cognate functions without reference to the surd character of these surds. In the same manner in which $\mathrm{F}_{1}(x)$ was derived from $f(p)$, and then $\mathrm{F}_{2}(x)$ from $F_{1}(x)$, derive $F_{3}(x)$ from $F_{2}(x)$, and $F_{4}(x)$ from $F_{3}(x)$, and so on, till an expression $F_{a}(x)$ is reached, in which the coefficients of the powers of $x$ are rational. The expression $\mathrm{F}_{\mathrm{a}}(x)$ shall be identical with X.

For, if the factors of $\mathrm{F}_{\mathrm{a}}(x)$ be $, x-\phi_{1}, x-\phi_{2}, \ldots \ldots, x-\phi_{M^{\prime}}$, then, since the cocfficients of the powers in $x$ in $F_{a}(x)$ are rational, the functions $\phi_{1}, \phi_{2}, \ldots \ldots, \phi_{\mathrm{MI}}$, constitute (Prop. VII.) the entire series of the unequal cognate functions of $f(p)$. But the entire series of the unequal cognate functions of $f(p)$ is identical (Cor.' 0. Prop. VI.) with the series of the roots of the equation, $\mathrm{X}=0$. Therefore $\mathrm{F}_{\mathrm{a}}(x)$ and X are identical.

## Proposition IX.

In the scries, in Prop. VIII.,

$$
\begin{equation*}
x-f(p), \mathrm{F}_{1}(x), \mathrm{F}_{2}(x), \ldots \ldots, \mathrm{F}_{\mathrm{a}}(x) \text { or } \mathrm{X} \tag{l}
\end{equation*}
$$

let the factors by whose continued product $\mathrm{F}_{\mathrm{c}+1}(: x)$ is generated, be,

$$
\begin{equation*}
\mathrm{F}_{\mathrm{c}}(x),{ }^{2} \mathrm{~F}_{\mathrm{c}}(x),{ }^{3} \mathrm{~F}_{\mathrm{c}}(x), \ldots \ldots,{ }^{\mathrm{s}} \mathrm{~F}_{\mathrm{c}}(x) ; \tag{2}
\end{equation*}
$$

where ${ }^{2} \mathrm{~F}_{\mathrm{c}}(x),{ }^{3} \mathrm{~F}_{\mathrm{c}}(x)$, \&c., are what $\mathrm{F}_{\mathrm{c}}(x)$ becomes, on substituting successively for $Y$, a surd in $F_{c}(x)$, not subordinate to any other surd in the function, and having the index $\frac{1}{s}$, the values $z Y, z^{2} Y$, \&c.; $z$ being an $s^{\text {th }}$ root of unity, distinct from unity. Let $U$ be a surd in $\mathrm{F}_{\mathrm{c}}(x)$, distinct from Y , and not subordinate to any other surd in $F_{c}(x)$, with the index $\frac{1}{\sigma}$. Then if the surd $U$ disappear from $F_{c+1}(x), \sigma$ is equal to $s$, and the surds $Y$ and $U$ are (as we may express it) similarly involved in the function $\mathrm{F}_{\mathrm{c}}(x)$ : by which we mean, that, when the function is arranged according to Def. 8, whenever one of them appears in the function in any of its powers, it occurs mul-
tiplied by a power of the other ; as, $\mathrm{Y}^{\lambda}$ by $\mathrm{U}^{\lambda_{1}}, \mathrm{Y}^{\beta}$ by $\mathrm{U}^{\beta_{1}}, \mathrm{Y}^{\delta}$ by $\mathrm{U}^{\delta_{1}}$, and so on; the pairs of equations,

$$
\left.\begin{array}{l}
h \lambda=k s+\beta, \\
h \lambda_{1}=q s+\beta_{1},  \tag{4}\\
H \lambda=K s+\delta, \\
H \lambda_{1}=Q s+\delta_{1},
\end{array}\right\} .
$$

and so on, subsisting ; where $h, \mathbf{H}, k, \mathrm{~K}, q, \mathrm{Q}, \mathcal{\& c}$., are whole num. bers, each less than $s$

To prevent misunderstanding, we may instance the function,
$f(p)=p^{\frac{5}{7}}(2+p)^{\frac{1}{7}}+\left\{6+p^{\frac{3}{7}}(2+p)^{\frac{2}{7}}\right\}\left\{2+\left[p+p^{\frac{1}{7}}(2+p)^{\frac{3}{7}}\right]^{\frac{1}{2}}\right\}^{\frac{7}{T^{2}}}$, as one in which the two surds $p^{\frac{2}{7}}$ and $(2+p)^{\frac{1}{7}}$ are similarly involved. For, calling the former $Y$ and the latter $U$, we have $s=\sigma=7, \lambda=5$, $\lambda_{1}=1, \beta=3, \beta_{1}=2, \delta=1$, and $\delta_{1}=3$. Consequently equations (3) and (4) become,

$$
\begin{aligned}
5 h & =7 k+3, \\
h & =7 q+2 ; \\
5 \mathrm{H} & =7 \mathrm{~K}+1, \\
\mathrm{H} & =7 \mathrm{Q}+3 ;
\end{aligned}
$$

where integral values of $h, H, \& c$, less than 7 , can be found:

$$
\begin{aligned}
& h=2, k=1, q=0, \\
& \mathrm{H}=3, \mathrm{~K}=2, \mathrm{Q}=0 .
\end{aligned}
$$

We proceed with the proof of the Proposition Let $z_{1}$ be a $\sigma^{\text {tw }}$ root of unity, distinct from unity; and when $U$ is changed into $z_{1} \mathbb{U}$, let the terms in (2) become,

$$
\begin{equation*}
f_{\mathrm{c}}(x),{ }^{2} f_{\mathrm{c}}(x),{ }^{3} f_{\mathrm{c}}(x), \ldots \ldots \ldots,{ }^{\mathrm{s}} f_{\mathrm{c}}(x) \tag{5}
\end{equation*}
$$

Since $U$ disappears from $F_{c+1}(x)$, the contiaued product of the terms in (2) is not affected when we replace $U$ by $z_{1} \mathrm{U}$. Therefore

$$
\mathbf{F}_{c}(x) \times{ }^{2} \mathbf{F}_{c}(x) \times \ldots \times{ }^{s} \mathrm{~F}_{\mathrm{c}}(x)=f_{\mathrm{c}}(x) \times{ }^{2} f_{\mathrm{c}}(x) \times \ldots \times{ }^{\mathrm{s}} f_{\mathrm{c}}(x) .
$$

Hence, either $\mathrm{F}_{\mathrm{c}}(x)$ is equal to one of the terms in (5), or it has with one of them, as ${ }^{\mathrm{a}} \mathrm{f}_{\mathrm{c}}(x)$, a common measure, of less dimensions, as respects $x$, than $\mathrm{F}_{\mathrm{c}}(x)$. Suppose, if possible, that $\mathrm{F}_{\mathrm{c}}(x)$ is not equal to any term in (5) ; and that $L$ is its H. C. M. with ${ }^{3} f_{c}(x)$. The ex-
pression L involves only such surds as occur in $\mathrm{F}_{\mathrm{c}}(x)$ or ${ }^{n} f_{\mathrm{c}}(x)$; but, since $U$ is (by hypothesis) not a subordinate of any surd in the function $\mathrm{F}_{\mathrm{c}}(x)$, the substitution of $z_{1} \mathrm{U}$ for U makes no change on the surds appearing in the function: that is, the surds in $f_{\mathrm{e}}(x)$, and therefore also those in ${ }^{a} f_{c}(x)$, are identical with those in $\mathrm{F}_{\mathrm{c}}(x)$; and consequently the surds in $L$ are all found in $\mathbf{F}_{\mathrm{c}}(x)$. Now the simple factors of $\mathrm{F}_{\mathrm{c}}(x)$ are (see Prop. VIII.) the unequal cognate functions of $f(p)$ obtained by assigning definite valucs to those surds in $f(p)$ which are also present in $\mathrm{F}_{\mathrm{c}}(x)$, and taking the cognate functions without reference to the surd character of the surds so made definite. Therefore (Cor 4, Prop. VI.) no expression such as $L$ cau involve merely such surds as appear in $\mathrm{F}_{c}(x)$. Hence $\mathrm{F}_{\mathrm{c}}(x)$ cannot but be equal to some term in (5). Let $\mathrm{F}_{\mathrm{c}}(x)={ }^{a} f_{\mathrm{c}}(x)$. This implies that the coefficients of like powers of $x$ in these expressions are equal. Let $x^{\mathrm{E}}$ be a poser of $x$ in $\mathrm{F}_{\mathrm{c}}(x)$ involving in its coefficient the surd $Y$ in one of its powers; the coefficients, $D$ and $D_{1}$, of the $E^{\text {th }}$ power of $x$ in $\mathrm{F}_{\mathrm{c}}(x)$ and ${ }^{\mathrm{a}} f_{\mathrm{c}}(x)$ respectively being,

$$
\begin{aligned}
& \mathrm{D}=\ldots \ldots+b_{1} \mathrm{Y}^{k} \mathrm{U}^{1}+\mathbb{S} c . \\
& \mathrm{D}_{1}=\ldots, \ldots+b_{1} z^{k(\mathrm{a}-1)} z_{1}^{1} \mathrm{X}^{k} U^{1}+\& c .,
\end{aligned}
$$

where such terms as $b_{1}$ are clear of the surds $Y$ and $U$, and not zero; and no two terms such as that written $\mathrm{Y}^{\mathrm{k}} \mathrm{U}^{1}$ are identical; $k$ not being zero. Since $s$ is a prime number, and $\left[\mathrm{F}_{\mathrm{c}}(x)\right.$ satisfying the conditions of Def. 8] $k$ is less than $s$, we can find whole numbers. $w$ and $w_{1}$, less than $s$, and such that

$$
w_{1} k=w s+1 \therefore \mathrm{Y}=\left(\mathbf{Y}^{s}\right)^{-w} \mathbf{Y}^{\mathrm{w}_{1} k}
$$

or, if $Y^{k}$ be represented by $V$,

$$
\mathrm{Y}=\left(\mathrm{Y}^{\mathrm{s}}\right)^{-\mathrm{w}} \mathrm{~V}^{\mathrm{w}_{1}}
$$

Now ( $\left.\mathbf{Y}^{\mathbf{s}}\right)^{-\mathrm{w}}$, when expressed as an integral function satisfying the conditions of Def. 8, involves only the subordinate surds of $Y$. Therefore, by the equation found, we can eliminate Y from $\mathrm{Fc}(x)$, introducing in its room powers of $V$, but no powers of any other surd that was not previously in the function. Let $F_{c}(x)$, as thus exhibited, be written $\mathrm{F}_{\mathrm{c}}^{\prime}(x)$. A term in $\mathrm{F}_{\mathrm{c}}^{\prime}(x)$ is $\delta_{1} \vee U^{3}$. Should $l$
be neither zero nor unity, write $V_{1}$ for $U^{1}$; and, as above, all the powers of $\bar{U}$ occurring in $F_{c}^{\prime}(x)$ can be made to disappear; powers of $\mathrm{V}_{1}$ being introduced into the function in their room, but no powers of any other surd that was not previously in the function. Let the function become, in consequence of this change, $F^{\prime \prime}{ }_{c}(x)$; and let the coefficients of the several powers of $x$ in $F_{c}^{\prime}(x)$ aud $F_{c}^{\prime \prime}(x)$ be supposed to satisfy the conditions of Def. 8, as was the case with $\mathrm{F}_{\mathrm{c}}(x)$. Then, since $V$ and $V_{1}$ are respectively powers of surds that were present in $F_{c}(x)$, but do not remain (except as implicitly involved in $V$ and $V_{1}$ ) in $\mathrm{F}_{\mathrm{c}}^{\prime \prime}(x)$; and since $\mathrm{F}_{\mathrm{c}}(x)$ is (by hypothesis) in a simple form, $\mathrm{F}_{\mathrm{c}}^{\prime}(x)$ and $\mathrm{F}_{\mathrm{c}}^{\prime \prime}(x)$ are also in a simple form. In changing $F^{\prime}{ }_{c}(x)$ into $F_{c}^{\prime \prime}(x)$, we assumed that $l$ was not zero. This may now be shown to be case. Equate the coefficients of $x^{\mathrm{E}}$ in $\mathrm{F}_{\mathrm{c}}^{\prime}(x)$ and ${ }^{\circ} f_{c}^{\prime}(x)$; this latter expression being what ${ }^{a} f_{c}(x)$ becomes when $Y$ is eliminated, and powers of V introduced in its room. Then

$$
\begin{align*}
& \ldots \ldots+b_{1} \vee U^{1}+\ldots \ldots=\ldots \ldots+b \mathrm{VU}^{1} z^{k(a-1)} z_{1}^{1}+\& c . \\
& \therefore \ldots \ldots+b_{1}\left\{1-z^{k(a-1)} z_{1}^{1}\right\} V U^{1}=\& c .=0 . \\
& \text { Therefore, by Cor. 1. Def. } 9,1-z^{k(a-1)} z_{1}^{1}=0 \ldots \ldots \text { (6) } \tag{6}
\end{align*}
$$

If $l$ were zero, this would make $z^{k(a-1)}$ equal to unity: which, since the numbers, $k, a-1$, are less than $s$, and $z$ is an $s^{\text {th }}$ root of unity, distinct from unity, is impossible. Therefore $l$ is not zero; and hence $\mathrm{F}_{\mathrm{c}}^{\prime}(x)$ cau be exhibited in the form $\mathrm{F}_{\mathrm{c}}^{\prime \prime}(x)$. Equate the coefficients of $x^{\mathrm{B}}$ in $\mathrm{F}_{\mathrm{c}}^{\prime \prime}(x)$ and ${ }^{\mathrm{a}_{\mathrm{c}}^{\prime \prime}}(x)$; this latter expression being what ${ }^{a} f_{c}(x)$ becomes when $Y$ and $V$ are eliminated, and $V$ and $V_{I}$ are introduced in their room. Then the equation (6) still holds. But such an equation implies, that, $z$ being an $s^{\text {th }}$ root of unity, and $z_{1}$ being a $\sigma^{\text {th }}$ root of unity, distinct from unity, $\sigma=s$. Again, suppose $V$ to occur in its $h^{\text {th }}$ power in any term of $F_{c}^{\prime \prime}(x)$, so that $b_{2} \nabla^{\dot{a}} \nabla_{1}^{h 2}$ is a term in the coefficient of some power of $x$. Then, by reasoning as above, we get

$$
\begin{equation*}
1-z^{\operatorname{kh}(\Omega-1)} z_{1}^{\operatorname{lh}_{1}}=0 \tag{7}
\end{equation*}
$$

Hence, by a comparison of (6) and (7),

$$
1-z_{1}^{l\left(h-h_{1}\right)}=0 \quad \therefore \quad k=h_{1} \quad \therefore \quad \nabla_{1}^{h_{1}}=V_{1}^{h} .
$$

Hence $b_{2} V^{h} V_{1}^{\mathrm{h}_{1}}$ becomes $b_{2} V^{\mathrm{h}} V_{1}^{\mathrm{h}}$ : which, again, by returning from $V$ and $V_{1}$ to $Y$ and $U$, becomes $b Y^{k h-w s} U^{l \mathrm{ll}-\mathrm{w}_{1} \mathrm{~s}}$; where ws is the greatest multiple of $s$ in $k h$, and $w_{1} s$ is the greatest multiple of $s$ in $l h ; b$ being an expression clear of the surds Y and U . Consequently $\mathrm{F}_{\mathrm{c}}(x)$ may be written,

$$
\begin{equation*}
F_{c}(x)=\Sigma\left\{B Y^{k l-w s} \sigma^{\mathrm{Lh}-w_{1} s}\right\} ; \tag{8}
\end{equation*}
$$

$B$ being an expression clear of the surds $Y$ and $U$; and the numbers, $k, l$, remaining the same in all the terms, such as $B Y^{\text {kh-ws }} U^{\text {lh-w } 1_{1}}$, included under the symbol $\Sigma$. But equation (8) implies that the surds Y and U are similarly involved in $\mathrm{F}_{\mathrm{c}}(x)$.

## Proposition X.

Let $f(p)$ be an integral function of a variable $p$, in a simple form, satisfying the conditions of Def. 8 ; end let

$$
A Y^{\lambda}=B D^{\lambda} ; \ldots \ldots \ldots \ldots(1)
$$

where $\mathbf{Y}$ is a surd in $f(p)$, with the index $\frac{1}{3}$; and $A$ is an expression, not zero, involving only surds, distinct from $Y$, which occur in $f(p)$; $\lambda$ being a whole number, not zero, and less than $s$; and the expressions $B, U$, are what $A$ and $Y$ respectively become on changing T , a chief subordinate of X , with the index $\frac{1}{\sigma}$, into $z \mathrm{~T}, z$ being a $\sigma^{\text {th }}$ root of unity, distinct from unity; $T$ not being a subordinate of any surd in the expression $A$. Then the surd $\bar{Y}$ is of the form,

$$
\begin{equation*}
\mathbf{Y}=\left(\mathrm{HT}^{\mathrm{m}}\right)^{\frac{1}{5}} \tag{2}
\end{equation*}
$$

where $H$ is an expression clear of the surd $T$; and $m$ is a whole number, less than $\sigma$. Also, $\sigma$ is not equal to $s$.

For, let $\phi$ be the general expression which includes all the cognate functions of $A Y^{\lambda}$, taken without reference to the surd character of :any of the surds in $A Y^{\lambda}$, except $T$ and $Y$; and let $\phi^{*}$, arranged so as to satisfy the conditions of Def. 8, be,

$$
\phi^{\mathrm{s}}=\mathrm{D}+\mathrm{D}_{1} z_{1} \mathrm{~T}+\mathrm{D}_{2} z_{1}^{2} \mathrm{~T}^{2}+\ldots .+\mathrm{D}_{\sigma_{-1}} z_{1}^{\sigma_{-1}} \mathrm{~T}^{\sigma_{-1}} ;
$$

where $\tilde{z}_{1}$ is an indefinite $\sigma^{\text {th }}$ root of unity ; and $D, D_{1}$, Sce, are clearof the surd T. Then

$$
\left(A Y^{\lambda}\right)^{s}=D+D_{1} T+D_{2} T^{2}+\& c ;
$$

and, $\left(\mathrm{BU}^{\lambda}\right)^{\mathrm{s}}=\mathrm{D}+\mathrm{D}_{1} \approx \mathrm{I}+\mathrm{D}_{2} z^{2} \mathrm{I}^{2}+\mathbb{E} c$.

$$
\therefore \mathrm{T} \mathrm{D}_{1}(1-z)+\mathrm{T}^{2} \mathrm{D}_{2}\left(1-z^{2}\right)+\& \mathrm{c} .=0
$$

This equation involyes only such surds as occur in $f(p)$. Therefore (Cor. 1, Def. 9) the coefficients, $\mathrm{D}_{1}(1-z), \mathrm{D}_{2}\left(1-z^{2}\right)$, \&c., vanish separately. But, since $\sigma$ is a prime number, and $\approx$ is a $\sigma^{\text {th }}$ root of unity, distinct from unity, none of the terms, $1-z, 1-z^{2}, \mathcal{S c} .$, . vanish. Therefore $D_{1}, D_{2}$, \&c., must all vanish; and

$$
A y^{\lambda}=D^{\frac{1}{s}}
$$

Raise both sides of this equation to the $r^{\text {th }}$ power; $r$ and $n$ being. whole numbers such that

$$
r \lambda=n s+1
$$

Then, $\left(A Y^{\lambda}\right)^{r}=\left(A^{r} Y^{n 5}\right) Y=\left(D^{r}\right)^{\frac{1}{s}}$,

$$
\begin{equation*}
\text { or, } P Y=Q^{\frac{I}{s}} \tag{3}
\end{equation*}
$$

where $P$ and $Q$ involve only such surds, exclusive of $Y$, as are present in $A Y^{\lambda}$; and $Q$ is clear of the surd $T$. Let the forms of $P$ and $Y$ be,

$$
\begin{align*}
& \mathbf{P}=b+b_{1} \mathrm{~T}+b_{2} \mathrm{~T}^{2}+\ldots .+b_{\sigma_{-1}} \mathrm{~T}^{\sigma-1}  \tag{4}\\
& \mathbf{Y}=\left(h+h_{1} \mathrm{~T}+h_{2} \mathrm{~T}^{2}+\ldots . .+h_{\sigma-1} \mathrm{~T}^{\sigma-1}\right)^{\frac{1}{\mathbf{s}}} \tag{5}
\end{align*}
$$

where $b, b_{1}, \& c ., h, h_{1}, \& c .$, are clear of the surd T. Suppose, if possible, that the terms $b_{1}, b_{2}$, \&c., are all zero. Then $\mathrm{P}=b$; and,

$$
b^{\mathrm{s}} h+b^{\mathrm{s}} h_{1} \mathrm{~T}+b^{\mathrm{s}} h_{2} \mathrm{~T}^{2}+\& \mathrm{c}=\mathrm{Q}
$$

But since $Q$ is clear of the surd $T$, the coefficients, $b^{\mathrm{s}} h_{1}, b^{\mathrm{s}} h_{2}$, \&cc., in this equation, must (Cor. 1, Def. 9) vanish separately. Now, A is (by hypothesis) not zero ; therefore $P$ is not zero; therefore $b^{s}$ is not zero. Therefore all the terms $h_{1}, h_{2}$, \&c., vanish: which (since $T$
is a subordinate of Y ) is impossible. Hence at least one of the terms, $b_{1}, b_{2}, \& c$. , as $b_{c}$, does not vanish. But this leads to the conclusion that all the terms,

$$
\begin{equation*}
b, b_{1}, b_{2}, \ldots \ldots, b_{\sigma-1}, \tag{6}
\end{equation*}
$$

except $b_{c}$, must vanish. For, from (3) and (4), we have,

$$
\mathrm{Y} b+\ldots .+b_{c} \mathbf{Y} \mathrm{~T}^{\mathrm{e}}+b_{\mathrm{r}} \mathrm{Y} \mathrm{~T}^{\mathrm{r}}+\& \mathrm{c}_{\iota}=Q^{\frac{1}{s}}
$$

From this equation eliminate the surd $Q^{\frac{1}{s}}$, in the same way in which $X_{1}$ was eliminated from equation (4) Prop. I. The result is,

$$
\ldots \ldots+b_{c} Y T^{c} E+b_{r} Y T^{r} E_{1}+\& c=0
$$

The conditions necessary in order that $\mathbf{E}$ and $\mathrm{E}_{1}$ may both vanish, are,

$$
\begin{aligned}
& b_{\mathrm{c}} \mathrm{YT}^{\mathbf{c}}=\tau_{\mathrm{Q}^{\frac{1}{s}}} \\
& b_{\mathrm{r}} \mathrm{Y} \mathrm{~T}^{\mathbf{r}}=q \mathrm{Q}^{\frac{1}{5}}
\end{aligned}
$$

$\varepsilon$ and $q$ being constant quantities; and these equations give us,

$$
q \partial_{\mathrm{c}} \mathrm{~T}^{\mathrm{c}}=k b_{\mathrm{r}} \mathrm{~T}^{\mathrm{r}}:
$$

which, $T^{c}$ and $T^{r}$ being distinct powers of $T$, not exceeding the $(\sigma-1)^{\text {th }}$, is (Cor. 1, Def. 9) impossible. Therefore $b_{c}$ is the only term in (6) which does not vanish ; and, from (3), (4), and (5),

$$
\begin{equation*}
b_{\mathrm{c}}^{\mathrm{s}} \lambda^{\mathrm{cs}}+\ldots .+b_{\mathrm{c}}^{\mathrm{s}} \lambda_{\mathrm{a}} \mathrm{~T}^{\mathrm{cs}+\mathrm{a}}+b_{\mathrm{c}}^{\mathrm{s}} \lambda_{\mathrm{m}} \mathrm{~T}^{\mathrm{cs}+\mathrm{m}}+\& \mathrm{c}_{.}=\mathrm{Q} \tag{7}
\end{equation*}
$$

If $h_{\mathrm{m}}$ be a term in the series, $h, h_{1}, d c$., which is not zero, all the other terms in that series vanish. For, if $h_{\mathrm{e}}$ be another term, let

$$
\begin{aligned}
c s+e & =w \sigma+\beta, \\
\text { and, } c s+m & =w_{1} \sigma+\delta ;
\end{aligned}
$$

where $\beta$ and $\delta$ are whole numbers, less than $\sigma$. Then, since $e$ ani th are not equal, and each of them is less than $\sigma, \beta$ and $\delta$ are not equal. And so likewise as regards the other terms. Therefore (Cor.1, Def. 9), all the coefficients, $b_{c} h_{a}, b^{3} h_{m}$, \&c., in (7), must vanish, except the one occurring in the term which is equal to $Q$. But $h_{\mathrm{m}}$ does not vanish. Therefore all the terms, $h_{h}, h_{1}$, Se., except $h_{\mathrm{m}}$, or (as we may write it) $H$, must ronish ; and $Y$ is reduced to the form,

$$
\mathrm{X}=\left(\mathrm{II} \mathrm{~T}^{\mathrm{m}}\right)^{\frac{1}{3}}
$$

Also, equation (7) becomes,

$$
b_{c} H T^{(s+m)}=Q:
$$

which (Cor. 1, Def. 9) is only possible if the number, es $+m$, be a uultiple of $\sigma$. Hence $\sigma$ and $s$ are not equal to one another.

Cor.-If an equation such as (3) subsist, the form of the surd $\mathbf{Y}$ is that given in (2), and $s$ and $\sigma$ are unequal.

## Proposition XI.

In the series, in Prop. VIII.,

$$
x-f(p), \mathrm{F}_{\mathrm{I}}(x), \ldots \ldots, \mathrm{F}_{\mathrm{n}}(x) \text { or } \mathrm{X}
$$

let the factors by whose continued product $F_{c+1}(\%)$ is generated, be,

$$
\begin{equation*}
F_{c}(x),{ }^{2} \mathbf{F}_{\mathrm{c}}(x),{ }^{3} \mathrm{~F}_{\mathrm{c}}(x), \ldots,{ }^{\mathrm{s}} \mathrm{~F}_{\mathrm{c}}(x) ; \tag{1}
\end{equation*}
$$

where ${ }^{2} \mathrm{~F}_{\mathrm{c}}(x),{ }^{3} \mathrm{~F}_{\mathrm{c}}(x)$, de., are what $\mathrm{F}_{\mathrm{c}}(x)$ becomes, on substituting successively for $Y$, a surd in $F_{c}(x)$, not subordinate to any other surd in the function, and having the index ${ }_{\mathrm{s}}{ }^{1}$, the values $\approx \mathrm{Y}, z^{9} \mathrm{Y}$, \&c. $; \approx$ being an $s^{\text {th }}$ root of unity, distinct from unity. Also suppose, that, besides $Y$, there are no surds in $F_{c}(x)$ which disappear from $F_{c+1}(x)$, except subordinates of $Y$. Let $T$, a chief subordinate of $Y$, with the index $\frac{1}{\sigma}$, disappear from $\mathrm{F}_{\mathrm{c}+1}(x)$, in which case T is not a subordinate of any surd in $\mathrm{F}_{\mathrm{c}}(x)$ except Y . When T is changed into $z_{1} \mathrm{~T}$, $z_{1}$ beng a $\sigma^{\text {th }}$ root of unity, distinct from unity, let the terms in (1) be transformed into,

$$
{ }_{1} X_{1},{ }_{2} X_{1},{ }_{3} X_{1}, \ldots \ldots,{ }_{s} X_{1} .
$$

Generally, if ${ }_{a} X_{b}$ be what ${ }^{2} F_{c}(x)$ becomes when $T$ is changed into $z_{1}^{\mathrm{b}} \mathrm{T}, b$ being a whole number in the series, $1,2, \ldots ., \sigma-1$, the expression, ${ }_{a} \mathrm{X}_{\mathrm{b}}$, comprehends $s(\sigma-1)$ particular forms:

Then, if $F_{c}(x)$ be equal to a term in (2), an equation,

$$
\begin{equation*}
\mathrm{Y}=\mathrm{P} \mathrm{Y}_{1}^{\lambda} \tag{3}
\end{equation*}
$$

must subsist ; where $Y_{1}$ is what $Y$ becomes when $T$ is replaced by $z_{1} T$;
and $P$ is an expression involving only surds which occur in $F_{0}(x)$, exclusive of $Y ; \lambda$ being a whole number, distinct from zero and less than $s$, satisfying the condition,

$$
\begin{equation*}
\lambda^{\sigma}=v s+1 ; \tag{4}
\end{equation*}
$$

where $w$ is a whole number. Also, if Y be not of the form shown in equation (2) Prop. $X, \lambda$ is not uuity, and $s$ is not 2.

Let us in the mean time reason on the supposition that $\mathrm{F}_{\mathrm{c}}(x)$ is equal to a term in the first horizontal line of (2). We here make an observation to which we shall have occasion subsequently to refer. When an expression is equal to some term in a series such as that constituted by the terms in the first horizontal line of (2), any one of the terms in the series may be assumed to be that to which the expression in question is equal; because any particular term in the series stands, in fact, as the representative of all the terms in the series, in consequence of the $s$ distinct values which may be given to the surd $\mathrm{Y}_{1} \quad$ Proceeding, therefore, on the supposition that $\mathrm{F}_{\mathrm{c}}(x)$ is equal to a term in the first horizontal line of (2), we may understand that $\mathrm{F}_{\mathrm{o}}(x)$ is equal to ${ }_{1} \mathrm{X}_{1}$. Take $x^{\mathrm{R}}$, a power of $x$ in $\mathrm{F}_{\mathrm{c}}(x)$ having some power of $Y$ present in its coefficient; and let the coefficient of $x^{\mathrm{B}}$ in $\mathrm{F}_{\mathrm{c}}(x)$, satisfying the conditions of Def. 8, be,

$$
D=\ldots \ldots+A_{c} Y^{c}+A_{n} Y^{n}+\& c . ;
$$

where $A_{c}, A_{n}$, \&c., none of them zero, are clear of the surd $Y$; no two powers in the series, $\mathrm{Y}^{\mathrm{c}}, \mathrm{Y}^{\mathbf{n}}$, \&c., being identical. Then, $\mathrm{D}_{1}$ being the corresponding coefficient in $X_{1}$, and $\mathrm{B}_{\mathrm{c}}, \mathrm{B}_{\mathrm{n}}$, \&c., being what $A_{c}, A_{n}$, \&c., bcome when $T$ is changed into $z_{1} T$, we have

$$
\mathrm{D}_{1}=\ldots \ldots+\mathrm{B}_{\mathrm{c}} \mathrm{Y}_{1}^{0}+\mathrm{B}_{\mathrm{n}} \mathrm{Y}_{1}^{\mathrm{n}}+\& \mathrm{c} .
$$

But $D=D_{1}$. Therefore, by Prop. II., the terms, $A_{c} Y^{c}, A Y^{n}, \& c c$., taken in some order, are equal to the terms, $B_{c} \mathbf{Y}^{0}, B_{n} \stackrel{\sharp}{Y}$, \&c., each to each Hence we may put

$$
A_{c} Y^{c}=B_{m} Y_{1}^{m}
$$

where $B_{m} Y_{1}^{m}$ is some term in the series, $B_{c} Y_{1}^{c}, B_{n} Y_{1}^{\prime \prime}$, \&c. And, since $A_{c}$ and $B_{m}$ involve only surds which occur in $F_{c}(x)$, exclusive of $Y$, this equation can easily be reduced to one of the form (3); $\lambda$ not being zero, because neither $c$ nor $m$ is zero. Now, from equation (2), the following may be derived by Prop. III. :

$$
\left.\begin{array}{l}
\mathrm{Y}_{1}=l_{1} \mathrm{P}_{1} \mathrm{X}_{2}^{\lambda}  \tag{5}\\
\mathrm{Y}_{2}=k_{2} \mathrm{P}_{2} \mathrm{Y}_{3}^{\lambda} \\
\mathrm{Y}_{3}=k_{3} \mathrm{P}_{3} \mathrm{X}_{1}^{\lambda},
\end{array}\right\}
$$

and so on; where $k_{1}$, $k_{2}$, \&c., are $s^{\text {th }}$ roots of unity ; and $P_{1}$, $P_{2}$, \&c., are what $P$ becomes when $T$ is successively changed into $z_{1} \underline{T}, z_{1}^{2} \mathrm{~T}$, \&e.; $\mathrm{Y}_{1}, \mathrm{Y}_{2}$, \&c., being what Y becomes when T is successively changed into $z_{1} T, z_{1}^{2} T$, \&c. By eliminating $Y_{1}$ betwixt equatiou (3) and the first of equations (5), $X_{1}$ and $Y_{2}$ betwixt (3) and the two first of equations (5), and so on, we get

$$
\left.\begin{array}{l}
\mathrm{Y}=\mathrm{P} \mathrm{Y}_{1}^{\lambda} ; \\
\mathrm{Y}=k_{1}^{\lambda} \mathrm{P} \mathrm{P}_{2}^{\lambda} \mathrm{Y}_{2}^{\lambda^{2}} ;  \tag{6}\\
\mathrm{I}=k_{1}^{\lambda} \lambda_{2}^{\lambda_{2}^{2}} \mathrm{PP}_{1}^{\lambda} \mathrm{P}_{2}^{\lambda^{2}} \mathrm{Y}_{3}^{\lambda^{3}} ;
\end{array}\right\}
$$

and so ou. Hence gencraliy,

$$
\begin{equation*}
Y=Q X_{n}^{\gamma^{n}} \cdot \rho \tag{7}
\end{equation*}
$$

where $n$ is any whole number whatsoever; and $\rho$ is the greatest multiple of $s$ in $\lambda^{n}$; and Q is an expression which involves ouly such surds as occur in $F_{c}(x)$, exclusive of $Y$; none of the surds which it involves having $T$ as a suburdinate. Now equation (7) has been found on the liypothesis that $F_{c}(x)$ is equal to a term in the first line of (2). But, by the same course of reasoning, an equation such as (7) may be established, should $\mathrm{F}_{\mathrm{c}}(x)$ be given equal to $a$ term in auy line of (2). And equation (7) includes the form (3). Therefore, when $F_{c}(x)$ is equal to a term in (2), whatover be the line of (2) in which that term occurs, an equation such as (3) subsists. In order to cestablish equation (4), we observe that equation ( 7 ), when $n$ is taken equal to $\sigma$, becomes,

$$
I=Q I^{\lambda^{\sigma}-\rho}
$$

Let $\lambda^{\sigma}-p=m, m$ buiug less than s. Then

$$
\mathrm{I}-\dot{Q} \mathrm{Y}^{\mathrm{m}}=0
$$

But, since $F_{c}(x)$ is a function in a siuple form, this equation
is (Cor. 1, Def. 9) impossible, unless $Y$ and $Y^{m}$ be be the same power of Y , that is, unless $m$ be unity. Therefore

$$
\lambda^{\sigma} \approx \rho+1:
$$

an equation of the form (4). Should $I$ be unity, it is plain, referring to the manner in which the first of equations (6) was obtained, that

$$
A_{c} Y^{c}=B_{c} Y_{1}^{c} ;
$$

and consequently (Prop. X.) the surd $\bar{Y}$ is of the form shown in (2) Prop. X.; so that, if $Y$ be not of that form, $\lambda$ cannot be unity. In this case, also, $s$ camnot be 2 ; for were $s$ equal to $2, \lambda$ could have no other value than unity.

Cor. 1.-Should $\mathrm{F}_{\mathrm{c}}(x)$ not be equal to a term in (2), then no such equation as (3) admits of being formed. For, since $T$ disappears from $\mathrm{F}_{\mathrm{c}+1}(x)$, the continued product of the terms in the first horizontal line of (2) is equal to that of the terms in (1) : both products being $F_{c+1}(x)$. Hence $F_{c}(x)$ has a common measure with some term in the first line of (2), which term (on the principle pointed out in the Proposition) may be assumed to be ${ }_{1} X_{1}$. Let $L$ be the H. C. M. of $F_{c}(x)$ and $X_{1}$. Since the roots of the equation, $F_{c}(x)=0$, are (Prop. VII.) the unequal cognate functions of $f(p)$, obtained by assigning definite values to those surds in $f(p)$ which are also present in $F_{c}(x)$, and taking the cognate functions without reference to the surd character of the surds so rendered definite, $L$, which is of less dimensions, as respects $x$, than $F_{c}(x)$, cannot (Cor. 4. Prop. VI.) involve, in the coeflicients of the powers of $x$, merely such surds as occur in $\mathrm{F}_{\mathrm{c}}(x)$. But the only surd not in $\mathrm{F}_{\mathrm{c}}(x)$, which can possibly appear in $L$, is $Y_{1}$; because, with the exception of $Y_{1}$, all the surds in ${ }_{1} X_{1}$ are found in $F_{c}(x)$. Hence $X_{1}$ cannot be absent from $I$. But if such an equation as (3) subsisted, all the powers of $\bar{Y}_{1}$ in L might be eliminated from L , without any surds being introduced into $I$, except such as are found in $\mathrm{F}_{\mathrm{c}}(x)$. Hence no such equation as (3) can be formed.

Cor. 2.-Should no equation such as (3) subsist, auy function involving merely such surds as are in $F_{c}(x)$, together with $\mathrm{Y}_{1}$, is in a simple form. For suppose, if possible, that $\psi(p)$ is such a function, and that it is not in a simple form. Then an equation such as (1) Prop. I. must subsist; all the surds occurring in it being found in $\psi(p)$. One of these must be $\bar{Y}_{2}$; else all the surds in the equation
would be present in $\mathrm{F}_{\mathrm{c}}(x)$ : which is inpossible. Also $\mathrm{Y}_{1}$ is not a. subordinate of any surd in the equation, because all the surds in the equation except $\mathrm{Y}_{1}$ are present in $\mathrm{F}_{\mathrm{c}}(x)$, and $\mathrm{X}_{1}$ is not in $\mathrm{F}_{\mathrm{c}}(x)$; so. that no surd to which $X_{1}$ is subordinate can appear in $F_{0}(x)$. Let then the equation, satisfying the conditions of ${ }^{r}$ f. 8, be,

$$
\mathrm{H}+\mathrm{H}_{1} \mathrm{Y}^{\lambda} \mathrm{Y}_{1}^{\lambda_{2}}+\mathrm{H}_{2} \mathrm{Y}^{\beta} \stackrel{\beta}{1}_{\beta_{1}}+\mathrm{Sc} .=0,
$$

where $\mathrm{H}, \mathrm{H}_{1} \& c$., are clear of the surds X and $\mathrm{Y}_{1}$; at least one. number in the series, $\lambda_{1}, \beta_{1}$, \&c., (say $\lambda_{1}$ ), not being zero; the corresponding coefficient $\mathrm{H}_{1}$ being at the same time distinct from zero; and no two terms in the series $Y^{\lambda} Y_{1}^{\lambda_{1}}, Y^{\beta} Y_{1}^{\beta_{1}}$, being identi-. cal. Then (Cor. 1, Prop. I.) an equation,

$$
\begin{equation*}
Y^{\lambda} Y_{1}^{\lambda_{1}}=P\left(Y^{\beta} Y_{1}^{\beta_{1}}\right)^{m} \tag{8}
\end{equation*}
$$

must subsist; where $P$ is an expression involving only such surds es occur in the expressions $\mathrm{H}, \mathrm{H}_{1}$, \&cc., or are subordinates of the surds $Y, Y_{1} ; m$ being either unity or zero: the term $Y^{\beta} X_{1}{ }^{\beta_{1}}$ standing as the type of any term in the series, $\bar{Y}^{\lambda} Y_{1}^{\lambda_{1}}, X^{\beta} Y^{\beta_{1}}$, \&c., after the first. But, should $m$ be zero, equation (8) is of the form (3): which, since $F_{c}(x)$ is not equal to a term in (2), is (Cor. 1) inadmissible. Should $m$ be unity, equation ( 8 ) becomes

$$
Y^{\lambda-\beta} Y_{1}^{\lambda_{1}-\beta_{1}}=P .
$$

Here, by hypothesis, the numbers $\lambda-\beta, \lambda_{2}-\beta_{1}$, do not both vanish. Should the latter vanish, the equation is at variance with the supposition that $F_{c}(x)$ is in a simple form. Should the former vanish, the equation is at variance with the fact that ${ }_{1} X_{1}$ is in a simple form ; which, however, it must (Prop. IV.) needs be. Should neither vanish, the equation is of the inadmissible form (3). Hence the function $\psi(p)$ caunot but be in a simple form.

Cor. 3.-Whould $\mathrm{F}_{\mathrm{c}}(3)$ not be equal to a term in (2), the equations,

$$
\left.\begin{array}{l}
\mathrm{F}_{\mathrm{c}}(x)=\mathrm{L} \times \mathrm{L}_{1} \times \mathrm{L}_{2} \times \ldots \ldots \times \mathrm{I}_{s-1},  \tag{9}\\
\mathrm{~F}_{\mathrm{c}}(x)=\mathrm{K} \times \mathrm{K}_{1} \times \mathrm{K}_{2} \times \ldots \ldots \times \mathrm{K}_{s-1},
\end{array}\right\}
$$

and so on, subsist; where $L$ is the H. C. M. of $F_{c}(x)$ and $X_{1}$;
$\mathrm{L}_{1}$, that of $\mathrm{F}_{\mathrm{c}}(x)$ and ${ }_{2} \mathrm{X}_{1} ; \mathrm{L}_{2}$, that of $\mathrm{F}_{\mathrm{c}}(x)$ and ${ }_{3} \mathrm{X}_{1}$; and so on ; and K. is the H. C. M. of $\mathrm{F}_{\mathrm{c}}(x)$ and $\mathrm{X}_{2}: \mathrm{K}_{1}$, that of $\mathrm{F}_{\mathrm{c}}(x)$ and ${ }_{2} \mathrm{X}_{2}$; and so on: all the expressions $L, K, I_{1}, K_{1}$, \&c., being of the same dimensions as respects $x$. For, since $F_{c}(x)$ necessarily has a common measure with more than one term in the first line of (2), let us (on the principle pointed out in the Proposition) take ${ }_{1} X_{1}$ to be a term in that line, such that the H. C. M. of $F_{c}(x)$ and ${ }_{1} \mathrm{X}_{1}$ is not of less dimensions, as respects $x$, than the H . C. M. of $F_{c}(x)$ and any other term in the first line of (2). Then, $X^{\prime}$ being the general symbol under which all the terms in the first line of (2) are comprehended, let the II. C. M. of $\mathrm{F}_{\mathrm{c}}(x)$ and $\mathrm{X}^{\prime}$ be sought in the ordinary method; the process being contiuued till that stage is reached, where, in the case of $F_{c}(x)$ and $X_{1}$, the operation has an end. Let the remainder $R_{\text {. }}$ [that is, in the general case of $\mathrm{F}_{\mathrm{c}}$ () and $\left.X^{\prime}\right]$, reduced to an integral function, and satisfying the conditions of Def. 8 , no two terms such as $\bar{Y}^{\lambda} Y_{1}^{\lambda_{2}}$, in the coefficient of any power of $x$ being identical, be,

$$
\mathrm{R}=\ldots \ldots+x^{\mathrm{E}}\left(\ldots \ldots+q \mathrm{Y}^{\lambda^{1}} \mathrm{X}_{1}^{\lambda_{1}}+\ldots \ldots\right)+\& \mathrm{c} .
$$

$q$ and the corresponding coefficients which are not expressed being clear of the surds $Y$ and $Y_{1}$. Then, if $R_{1}$ be what $R$ becomes in the particular case of $F_{c}(x)$ and $X_{1}$,

$$
\mathrm{R}_{1}=\ldots \ldots+x^{\mathrm{E}}\left(\ldots \ldots+q q^{\prime} \mathrm{Y}^{\lambda} \hat{\mathrm{Y}}_{1}^{\lambda_{1}}+\ldots \ldots\right)+\& \mathrm{c} .
$$

where $z^{\prime}$ is some (not definite) power of $z$. But $\mathrm{K}_{1}=0$. 'This implies that the coefficients of the different powers of $x$ vauish separately. Also (Cor. 3) any function involving merely such surds as are in $F_{c}(x)$, together with $Y_{1}$, is in a simple form. Therefore, if

$$
\varphi(p)=\ldots \ldots+q z^{\prime} \mathbf{X}^{\lambda} \bar{Y}^{\lambda_{1}}+\& c .=0
$$

$\psi(p)$ is a fuuction in a simple form. Therefore (Cor. 1, Def. 9) $q$, with all other such coeflicients, must be zero. Therefore R vanishes, as well as $\mathrm{R}_{1}$. And, in the case when $\mathrm{F}_{0}(x)$ is compared with any one in particular of the terms in the first line of (2), it is not possible for a remainder, prior to that which $m$ the general case is $R$ to vanish; because (by hypothesis) the H. C. M. of $\mathrm{F}_{\mathrm{c}}(x)$ and $\mathrm{X}_{1}$
is not of less dimensions than the H. C. M. of $\mathrm{F}_{\mathrm{c}}(x)$ and any term in the first line of (2). 'The fact, therefore, of $R$ being zero, implies that $\mathrm{F}_{\mathrm{c}}\left({ }^{(x)}\right.$ has a common measure with each of the terms in the first line of (2), and that its H. C. M. with any of these terms is of the same dimensions as its $H$. C. M. with any of the rest. Let L be the H. C. M. of $\mathrm{F}_{\mathrm{c}}(x)$ and ${ }_{1} \mathrm{X}_{1}, \mathrm{~L}_{1}$ that of $\mathrm{F}_{\mathrm{c}}(x)$ and ${ }_{2} \mathrm{X}_{1}$, and so on. The terms $\mathrm{I}_{1}, \mathrm{I}_{1}$, \&c., are all of the same dimensions; $\mathrm{L}_{1}$, in fact, being what $L$ becomes on substituting $z Y_{1}$ for $Y_{1}$; and so on. Also, since all the factors of the terms in the first line of (2), being factors of $\mathrm{F}_{\mathrm{c}+1}(x)$, are unequal, it follows that all the factors of the terms $\mathrm{L}, \mathrm{L}_{1}$ \&c., are unequal. This, taken in comnection with the fact that $\mathrm{F}_{\mathrm{c}}(x)$ is a factor of the continued product of the terms in the first line of (2), shows that $\mathrm{F}_{\mathrm{c}}(x)$ is equal to the continued product of the terms I., $L_{1}$, \&c. Thus the first of equations (9) is established. In the same manner the others can be established.

Cor. 4. -Should $\mathrm{F}_{\mathrm{c}}(x)$ not be equal to a term in (2), an equation of the form,

$$
\begin{equation*}
\underset{\mathrm{Y}}{\mathrm{Y}_{2}^{\beta}}=\mathrm{P} \mathrm{Y}_{1}^{\lambda}, \tag{10}
\end{equation*}
$$

must subsist; $\beta$ and $\lambda$ being whole numbers, distinct from zero : and $P$ an expression involving only surds which occur in $F_{c}(x)$, exclusive of $Y$; while $Y_{2}$ is what $Y$ becomes when $I$ is changed into $z_{1}^{2}$
For, let N be the $\mathrm{H} . \mathrm{C} . \mathrm{M}$. of ${ }^{2} \mathrm{~F}_{\mathrm{c}}(x)$ and ${ }_{1} \mathrm{X}_{1}, \mathrm{~N}_{1}$ that of ${ }^{2} \mathrm{~F}_{\mathrm{c}}(x)$ and ${ }_{2} \mathrm{X}_{1}$, and so on, Q being the H. C. M. of ${ }^{2} \mathrm{~F}_{\mathrm{c}}(x)$ and ${ }_{1} \mathrm{X}_{2}$, $Q_{1}$ that of ${ }^{2} F_{c}(x)$ and ${ }_{2} X_{2}$, and so on : in which case the terms, $N$, $N_{1}$, \&c., are respectively what $L, L_{1}$, \&c., (see Cor. 3), become ou changing Y into ${ }_{2} \mathrm{Y}$; and $\mathrm{Q}, \mathrm{Q}_{1}, \& c$., are what $\mathrm{K}, \mathrm{K}_{1}$, \&c., become on changing $Y$ into $z Y$. Then, in the same way in which equations (9) were found, we can establish the equations.

$$
\begin{aligned}
& { }^{2} F_{c}(x)=N \times N_{1} \times N_{2} \times \ldots \ldots \times N_{s-1}, \\
& { }^{2} F_{c}(x)=: Q \times Q_{1} \times Q_{2} \times \ldots \ldots \times Q_{s-1} .
\end{aligned}
$$

Now suppose, if possible, that such an equation as (10) carnot subsist. Then, exactly as it was shewn in Cor. 2 , [proceeding upon the hypothesis that such an equation as (3) camot subsist], that any function involving merely such surds as are in $F_{c}(x)$, together with $X_{1}$, is in a simple form, we may demonstrate [proceeding upon the hypothesis
that such an equation as (10) cannot subsist] that any function involving merely such surds as are in $\mathrm{F}_{\mathrm{c}}(x)$, together with $\mathrm{Y}_{1}$ and $\mathrm{Y}_{2}$, is in a simple form. This being premised, we remark, that, in (9), $L$ is either equal to one of the expressions $\mathrm{K}, \mathrm{K}_{1}$, \&c., or has a common measure with more than one of them. Let $K_{c}$ be a term in the series $\mathrm{K}, \mathrm{K}_{1}$, \&c., such that the H. C. M. of L and $\mathrm{K}_{\mathrm{c}}$ is not of less dimensions than the H. C. M. of L and any other term in the series. Take $\mathrm{K}^{\prime}$, the general form which includes all the terms $K, \mathrm{~K}_{1}$, $K_{s}$, and likewise all the terms $Q, Q_{1}, \ldots \ldots, Q_{s-1}$; the latter series being derived from the former by changing $Y$ into $z Y$. Perform the operation of finding the H. C. M. of L. and $K^{\prime}$, stopping at the point where, in the particular case of L and $\mathrm{K}_{\mathrm{c}}$, the process comes to an end. If at this stage the remainder be $R$, and $R_{1}$ be the corresponding remainder in the case of K and $\mathrm{K}_{\mathrm{c}}$, the forms of R and $\mathbf{R}_{1}$ are,

$$
\begin{aligned}
& \mathrm{R}=\ldots \ldots+x^{\mathrm{E}}\left(\ldots \ldots+q \mathrm{Y}^{\lambda} \mathrm{Y}_{1}^{\lambda_{1}} \mathrm{Y}_{2}^{\lambda_{2}}+\& \mathrm{c} .\right)+\mathbb{E} . \\
& \mathrm{R}_{1}=\ldots \ldots+x^{\mathrm{E}}\left(\ldots \ldots+q z^{\prime} \mathrm{Y}^{\lambda} \mathrm{X}_{1}^{\lambda_{2}} \mathrm{Y}_{2}^{\lambda_{2}}+\& \mathrm{cc} .\right)+\& \mathrm{c} . ;
\end{aligned}
$$

the expressions being similar to those in Cor. 3. But since $\Omega_{1}=0$, we find (as in Cor. 3) that $R=0$; it being kept in view that any function which involves merely such surds as occur in $F_{c}(x)$, together with $\mathrm{X}_{2}$ and $\mathrm{Y}_{1}$, is in a simple form. Hence L has a common measure with every term included under the general symbol $\mathrm{K}^{\prime}$, and therefore it is a factor of ${ }^{2} F_{c}(x)$ as well as of $F_{c}(x)$ : which, since $F_{c}(x)$ and ${ }^{2} F_{c}(x)$ have no common factors, is impossible. Therefore an equation such as (10) must subsist.

Cor. 5.-The same suppositions being made as in Cor. 4, the following equations must subsist:
and so on ; where $Y_{c}$ is what $Y$ becomes when $T$ is changed into $z_{1}^{\mathrm{c}} \mathrm{P}$; and $\mathrm{P}, \mathrm{P}_{1}, \mathrm{P}_{\mathbf{2}}$, \&c., are expressions which involve only surds, exclusive of $Y$, occurring in $F_{c}(x)$; and the whole numbers, $1, \lambda$, $\lambda_{1}, \lambda_{2}, \& c$., are such, that, if $\lambda_{a}, \lambda_{a+1}, \lambda_{a+2}$, be three consecutive terms in the series, they are related to one another by the equation,

$$
\begin{equation*}
\lambda_{\mathrm{a}+2}=\lambda \lambda_{\mathrm{a}+1}-\beta \lambda_{\mathrm{a}} \tag{12}
\end{equation*}
$$

For, the first equation in (11) subsists, by Cor. 4. From this we can deduce, by Prop. III. the following, including the first of (11):

$$
\begin{align*}
& \left.\begin{array}{c}
\mathrm{Y}_{\mathrm{Y}_{2}^{\beta}}^{\beta}=\mathrm{P} \mathrm{Y}_{1}^{\lambda}, \\
\mathrm{Y}_{1} \mathrm{Y}_{3}={ }^{1} \mathrm{P}^{\lambda} \mathrm{Y}_{2},
\end{array}\right\}  \tag{13}\\
& \mathrm{Y}_{2} \mathrm{Y} \quad{ }^{2} \mathrm{P}_{3},
\end{align*}
$$

and so on ; where ${ }^{3} P$ is the product of an $s^{\text {th }}$ root of mity by whar. $\mathbf{P}$ becomes on changing T into $z_{1} \mathrm{~T}$; ${ }^{-} \mathrm{P}$, the product of an $s^{\text {thl }}$ root of unity by what $P$ becomes on changing $T$ into $z_{1}^{2} T$; and so on. Raise the first ( $2 c-1$ ) jequations in the series (13) to the following powers respectively, viz. : the first to the first power, the second to. the $\lambda^{\text {th }}$ power, the third to the $\lambda_{1}^{\text {th }}$ power, the fourth to the $\lambda_{2}^{\text {th }}$ power $;$ $\ldots . . . . . .$. , the $(c-1)^{\text {th }}$ to the $(\lambda)^{\text {th }}$ power, the $c^{\text {th }}$ to the $\left(\lambda_{0,-2}\right)^{\text {th }}$ power, the $(c+1)^{\text {th }}$ to the $\left(\beta \lambda_{c-3}\right)^{\text {th }}$ power, the $(c+2)^{\text {th }}$ to the $\left(\beta^{2} \lambda_{c-4}\right)^{\text {th }}$ power, the $(c+3)^{\text {th }}$ to the $\left(\beta^{3} \lambda_{\mathrm{c}-5}\right)^{\text {th }}$ power, $\ldots \ldots . . .$. , the $(2 c-2)^{\text {th }}$ to the $\left(\beta^{\mathrm{c}-2} \lambda\right)^{\text {th }}$ power, and the $(2 c-1)^{\text {th }}$ to the $\left(\beta^{\mathrm{c}-1}\right)^{\text {th }}$ power. By multiplying together the results thus obtained, we get


where $P_{0-1}$ is an expression like $P,{ }^{1} P, \& c$. , involving only such surds, exclusive of $Y$, as occur in $F_{c}(x)$. But, by (12), we have.

$$
\begin{aligned}
& \lambda_{1}+\beta=\lambda \lambda, \\
& \lambda_{2}+\beta \lambda=\lambda \lambda_{1}, \\
& \cdots \cdots \cdots \cdots \\
& \lambda_{c-2}+\beta \lambda_{c-4}-\lambda \lambda_{c},
\end{aligned}
$$

and so on ; so that the equation obtained above is reduced to 0 -

$$
Y Y_{2 c}^{\beta^{c}}=P_{c-1} Y_{c}^{\lambda \lambda_{c-2}-2 \beta \lambda_{c-3}}
$$

But, by (12),

$$
\begin{aligned}
& \lambda_{c-1}-\beta \lambda_{c-3}-\lambda \lambda_{c-2}-2 \beta \lambda_{c-3} . \\
& \quad \therefore \mathrm{Y}_{2 c}^{\beta^{\mathrm{c}}}-\mathrm{P}_{\mathrm{c}-1} \mathrm{X}_{\mathrm{c}}^{\lambda_{c-1}-\beta \lambda_{c-3}}:
\end{aligned}
$$

which is the general form that includes alı the equations in the .series (11).

Cor. 6. -The $(\sigma+1)^{\text {th }}$ equation in the series (11) is,

$$
\mathrm{Y}_{\mathrm{Y}_{2}}^{\beta^{\sigma+1}}=\mathrm{P}_{\sigma} \mathrm{Y}_{1}^{\lambda_{\sigma}}-\beta \lambda_{\sigma-2} .
$$

By comparing this with the first of equations (11), we get

$$
\mathrm{Y}_{\underline{2}}^{\beta(\beta-1)}=\mathrm{P}^{-1} \mathrm{P}_{\sigma} \mathrm{Y}_{1}^{\lambda_{\sigma}-\beta \lambda_{\sigma-2}-\lambda} .
$$

But, dy Cor. 1, in connection with Prop. III., this is impossible unless

$$
\begin{equation*}
\beta^{\sigma}-\mathrm{I}=w s, . \tag{14}
\end{equation*}
$$

$w$ being a whole number. Therefore equation (14) must subsist.

## Proposition XII.

A given algebraical function of a variable $p$ can always be expressed as an integral function in a simple form; the following conditions being at the same time satisfied : First, that there shall be no surd in the function, of the form,

$$
\begin{equation*}
\mathrm{Y}=\left(\mathrm{HT}^{\mathrm{m}}\right)^{\frac{1}{s}} ; \tag{1}
\end{equation*}
$$

where T is a chief subordinate of Y , with the index $\frac{1}{\sigma}$, which is not equal to $\frac{1}{8}$; and $m$ is a whole number, not zero, and less than $\sigma$; and H is an expression clear of the surd T ; secondly, that no two surds, V and $\mathrm{V}_{1}$, principal or subordinate, shaii be similarly [see Prop. IX.] involved in the function.

For, should the given function, when rendered integral, be not in a simple form, an equation such as (I) Prop. I. must subsist; all the surds in the equation being surds which occur in the function. Substitute, then, in the function, for $\mathrm{Y}_{\mathrm{c}}$, wherever it occurs in any of its powers, its value as furnished by (l) Prop. I. Then, when the func.tion is rendered integral, the number of surds present in it, (principal
and subordinate being both reckoned), will be less, by at least one, than it was before. Agrin, should the function involve a surd $Y$, of the form shown in (1), then, since s and $\sigma$ are unequal prime numbers, we may choose $c$ and $w$, whole numbers, such that

$$
\begin{gathered}
c s+m=w \sigma \\
\therefore \mathbf{Y}=\mathrm{T}^{-c}\left(\mathrm{HT}^{w \sigma}\right)^{\frac{2}{s}}
\end{gathered}
$$

Let $\mathrm{H} \mathrm{T}^{20 \sigma}$, when made to satisfy theconditions of Def. 8 , be written K : and substitute for X , wherever it occurs in the function in any of its. powers, the value furnished by the equation,

$$
Y=T^{-c}(K)^{\frac{1}{1 / 3}} ;
$$

where it will be observed that the surd $\mathrm{K}^{\frac{2}{8}}$ has no subordinates which were not subordinates of $Y$, while it has not as a subordinate the surd $T$, which was a subordinate of $Y$. Once more, suppose that two surds, $V$ and $V_{1}$, with the common index $\frac{1}{3}$, are similarly involved in the function: that is to say, when the function has been arranged according to Def. 8, wherever one of the surds $V$ and $V_{1}$ appears in any of its powers, it occurs multiplied by a power of the other ; as $V^{\lambda}$ by $V_{1}^{\lambda_{1}}, V^{\beta}$ by $\nabla_{1}^{\beta_{1}}, V^{\delta}$ by $V_{1}^{\delta_{1}}$, and so on; the pairs of equations (3) and (4). Prop. IX, subsisting. Let $V=U^{\frac{2}{s}}$, and $V_{1}=U_{1}^{\frac{1}{s}}$; and put

$$
\begin{aligned}
V^{\lambda} V_{1}^{\lambda_{1}} & =\left(U^{\lambda} U_{1}^{\lambda_{1}}\right)^{\frac{1}{s}}=Y^{\frac{3}{s}} \\
\therefore V^{h \lambda} V_{1}^{h \lambda_{1}} & =V^{k s+\beta} V_{I}^{q s+\beta_{1}}=Y^{\frac{a}{3}} \\
\therefore V^{\beta} V_{1} \beta_{1} & =A Y^{\frac{b}{s}} ;
\end{aligned}
$$

A being put for $V^{-k s} V_{1}^{-4 s}$. Since the surds $V$ and $V_{1}$ have the common index $\frac{1}{s}$, the expression $A$ may be exhibited so as to involve only surds which are subordinates of $V$ or $V_{1}$. Let $A$ be so exhibited. In like manner,

$$
V^{\delta} V_{I}^{\delta_{1}}=B Y^{\frac{H}{S}} ;
$$

where $B$ is an expression of the same charneter with $A$. And so on. Substitute for $V^{\lambda} V_{1}^{\lambda_{1}}, V^{\beta} V_{1}^{\beta_{1}}, V^{\delta} V_{1}^{\delta_{1}}$ : the values, $Y^{\frac{1}{s}}, A Y^{\frac{h}{s}}, B Y^{\frac{K}{s}}$, \&c. Then, when the furction is rendered integral, the number of surds present in it, (principal and subordinate being both reckoned), will be less, by at least one, than it was before. Let modifica. tions of the three different kinds described continue to be made as far as possible. It is obvious that a limit will ultimately be reached; and if the function be then rendered integral, it will be an integral function in a simple form, contrining no surd such as $Y$ in (1), and having no two surds similarly involved in it.

Cor.-In $f(p)$, a function which has been made to undergo the modifications described in the Proposition, let $Y$ be a surd, not subordinate to any other in the function; its index being $\frac{3}{s}$. Also, let T and $t$ be two surds, with the common index $\frac{1}{\sigma}$, which is not equal to $\frac{1}{s}$, subordinate to $Y$, but neither of them subordinate to any other surd in $f(p)$; and suppose $t$ ? ai the form of Y is,

$$
\begin{equation*}
\mathrm{Y}=\left(\mathrm{H} \mathrm{~T}^{\left.\mathrm{m}^{m}\right)^{\frac{1}{18}}}\right. \tag{2}
\end{equation*}
$$

where $m$ is a whole number, less than $\sigma$; and $H$ is an expression in which the surds $T$ and $t$ are similarly involved. As in the Fropositicn, we can choose $c$ and $w$, whole numbers, such that

$$
\begin{gathered}
c s+m=w \sigma \\
\therefore \mathrm{Y}=\mathrm{T}^{-c}(\mathrm{~K})^{\frac{1}{s}}
\end{gathered}
$$

where K is put for $\mathrm{H} \mathrm{T}^{w \sigma}$; that is, K is the product of an expression which is clear of the surds T and $t$, by one in which T and $t$ are similarly involved. Hence again, as in the Proposition, we can eliminate the surds T and $t$ from K , introducing in their room a single new surd $V$; one of the surds $T$ and $t$, as $t$, disappearing from the function altogether. And, since $T$ and $t$ are not subordinates of any surd in $f(p)$ except Y , the function, after being subjected to this change of form, may still, if necessary, be made to satisfy the different conditions described in the Proposition. So that, upon the whole, an algebraical function of a variable $p$ may be exhibited as an integral function in a simple form, with no two surds similarly involved in it; nor with any
surd involved in it of the form (1); nor with any surd involved in it, which, while not subordinate to any other in the function, is of the form (\%).

## Proposition XIII.

Let $f^{\prime}(p)$ be an integral function of a variable $p$, in a simple form, containing no surd such as Y in (l) Prop. XII, nor any surd, which, while not subordinate to any surd in the function, is of the form shown in (2) Prop. XII ; and having no two surds similarly involved in it. Let $Y$ be a surd in $f(p)$, with the index $\frac{1}{s}$, not subordinate to any other in the function; and let the function, arranged so as to satisfy the conditions of Def. 8 , be,

$$
\begin{equation*}
f(p)=A+A_{c} Y^{c}+A_{n} Y^{n}+\& c . ; \tag{1}
\end{equation*}
$$

where $A_{c}, A_{n}$, \&c., none of them zero, are clear of the surd $\mathbf{Y}$; A also being clear of the surd $Y$; and $\mathbf{Y}^{\mathbf{c}}, \mathrm{Y}^{\mathrm{n}}$, \&c., are distinct powers of $Y$. Suppose that $T$ and $T_{1}$ are two chicf subordinates of $Y$, with the indices $\frac{1}{\sigma}$ and $\frac{1}{\sigma_{2}}$; but that neither of them is a subordinate of any other surd in the function $f(p)$. When T is changed into $z_{1} \mathrm{~T}$, $z_{1}$ being a $\sigma^{\text {th }}$ root of unity, distinct from unity, let $f(p), \mathbf{Y}, \mathbf{A}, \mathbf{A}_{\mathrm{c}}$, \&c., be transformed into $f_{1}(p), \mathrm{Y}_{1}, \mathrm{~B}, \mathrm{~B}_{\mathrm{c}}$, \&c.; and, when $\mathrm{T}_{1}$ is changed into $z_{2} \mathrm{~T}_{1}, z_{2}$ being a $\sigma_{1}^{\text {th }}$ root of unity, distinct from unity, let these same expressions become $f_{2}(p),{ }_{1} \mathbf{Y}, b, b_{c}, \& c$. ; so that

$$
\left.\begin{array}{rl}
f_{1}(p) & =\mathrm{B}+\mathrm{B}_{\mathrm{c}} \mathrm{Y}_{1}^{\mathrm{c}}+\mathrm{B}_{\mathrm{n}} \mathrm{Y}^{\mathrm{n}}+\& \mathrm{c} .,  \tag{2}\\
\text { and, } f_{2}(p) & =b+b_{\mathrm{c}}\left({ }_{1} \mathrm{Y}^{\mathrm{c}}\right)+b_{\mathrm{n}}\left({ }_{1} \mathrm{Y}^{\mathrm{n}}\right)+\& \mathrm{c} .
\end{array}\right\}
$$

Then, if $f_{\mathrm{I}}(p)=f_{2}(p)$, it can be proved by the same reasoning as in Prop. II, that the terms,

$$
\mathrm{B}_{\mathrm{c}} \mathrm{Y}_{1}^{\mathrm{c}}, \mathrm{~B}_{\mathrm{n}} \mathrm{Y}_{1}^{\mathrm{n}}, \& \mathrm{c}
$$

taken in some order, are equal to the terms,

$$
b_{c}\left({ }_{1} Y^{c}\right), b_{\mathrm{D}}\left({ }_{1} Y^{\mathrm{n}}\right), \& c .
$$

But should the numbers $\sigma$ and $\sigma_{1}$ not be both equal to $s$, the equation,

$$
\begin{equation*}
B_{c} Y_{r}^{c}=b_{c}\left({ }_{1} Y^{c}\right), \tag{3}
\end{equation*}
$$

cannot subsist.

For suppose, if possible, that equation (3) subsists. Then the reasoning of Prop II. makes it plain that $\mathrm{B}_{\mathrm{n}} \mathrm{Y}_{1}^{\mathrm{n}}$ is equal to $b_{1} \gamma_{1} \mathrm{Y}^{\mathrm{n}}$ ), and so on. We will therefore assume this. The $s^{\text {th }}$ powers of $A_{c} Y^{c}$ $B_{c} Y_{1}^{c}$, and $b_{c}\left({ }_{1} Y^{c}\right)$, arranged so as to satisfy the conditions of Def. 8, are of the forms,

$$
\left.\begin{array}{rl}
\left(A_{c} Y^{c}\right)^{c} & =D+D_{1} T^{m} T_{1}^{n}+D_{2} T^{M} T_{1}^{N}+\& c .,  \tag{4}\\
\left(B_{c} Y_{1}^{c}\right)^{2} & =D+z_{1}^{m} D_{1} T^{m} T_{1}^{n}+z_{1} D_{2} T^{N} T_{1}^{N}+\& c ., \\
\left\{Z_{c}\left(Y_{1} Y^{c}\right)\right\}^{s} & =D+z_{2}^{n} D_{1} T^{m} T_{1}^{n}+z_{2}^{N} D_{2} T^{M} T_{1}^{N}+\& c,,
\end{array}\right\}
$$

where $D, D_{1}$, \&c., are clear of the surds $T$ and $T_{1} ;$ no two terms in the series, $T^{m} T_{1}^{n}, T^{3} T$, Ec. being identical with one another. There must be at least one term in the series $D_{1}, D_{2}, \& c$. ; else $\left(A_{c} Y^{c}\right)^{3}$ would be reduced to D; in which case (Cor. Prop. X.) the surd Y would be of the inadmissible form given in (1) Prop. XII. But, from (3) and (4),

$$
T^{m} T^{n} D_{1}\left(z_{1}^{m}-z_{2}^{\mathrm{a}}\right)+T^{M} T_{1}^{N} D_{2}\left(z_{1}^{M}-z_{2}^{N}\right)+\& c .=0
$$

Hence, by Cor. 1. Lef. 9, the coefficients of $T^{m} T_{1}^{n}, r^{M} T_{1}^{3}$, \&c., vanish separately. But, since the expressions on the right hand side of (4) satisfy the conditious of Def. 8, the terms $\mathrm{D}_{1}, \mathrm{D}_{2}$, \&c., do not vanish. Therefore

$$
\begin{aligned}
& z_{1}^{m}-z_{2}^{\mathrm{n}}=0, \\
& z_{1}^{\mathrm{M}}-z_{2}^{\mathrm{N}}=0,
\end{aligned}
$$

and so on: from which it follows that $\sigma_{1}=\sigma$; and also that $z_{2}=z_{1}^{u}$; where $u$ is a whole number, less than $\sigma$, and such that

$$
\left.\begin{array}{l}
u n=w_{1} \sigma+m,  \tag{5}\\
u N=w_{2} \sigma+M,
\end{array}\right\}
$$

and so on; $w_{1}, w_{2}$, \&c., being whole numbers. Let $V$ be put for $T^{m} T_{1}{ }^{n}$. Since $\sigma_{1}$ has been proved equal to $\sigma$, the surds $T$ and $T_{1}$ have a common index; and $V$ may be considered a surd with the same index as that of $T$ and $T_{1}$. Take $W$ and $w_{3}$, whole numbers less than $\sigma$, and such that

$$
\begin{gathered}
\mathrm{W} m-w_{s} \sigma+\mathrm{M} . \\
N
\end{gathered}
$$

Vox. V.

Therefore, from (5), $\mathrm{W} n=w_{4} \sigma+\mathrm{N}$,

$$
\text { and, } \mathrm{T}^{\mathrm{Mr}} \mathrm{~T}_{1}^{\mathrm{N}}=\mathrm{H} \mathrm{~V}^{\mathrm{W}} ;
$$

$w_{4}$ being a whole number; and H , an expression clear of the surds T and $T_{1}$. And so on. Hence the form of $\left(\mathrm{A}_{c} \mathrm{Y}^{\mathrm{c}}\right)^{\text {s }}$ is,

$$
\begin{equation*}
\left(A_{c} Y^{0}\right)^{s}=D+D_{1} V+D_{8} H V^{W} \div \& c . ; \tag{6}
\end{equation*}
$$

neither of the surds, $T, T_{1}$, appearing (except as implicitly involved in $V$ ) on the right hand side of the equation. From the expressions $A_{e}$ and $Y$ let the surd $T_{1}$ be eliminated, by substituting for it, whereever it occurs in any of its powers, its value derived from the equation,

$$
\begin{equation*}
V=T^{m} \eta_{1}^{n} ; \tag{7}
\end{equation*}
$$

and, when thus modified in form, let $A_{c}$ and $Y$, satisfying the conditions of Def. 8, become respectively $P$ and $U$. Then the surd $T$ cannot (otherwise than as implicitly involved in V) be a subordinate of U. For suppose, if possible, that it is. Put $Q$ to represent the expression, $\mathrm{D}+D_{1} \mathrm{~V}+\& \mathrm{c}$. Then

$$
\begin{equation*}
P U^{c}=Q^{\frac{1}{3}} . \tag{8}
\end{equation*}
$$

Now, any function involving merely such surds as occur in equatior. (8), exclusive of $Q^{\frac{1}{s}}$, is in a simple form ; for, all the surds in the expressions, $\mathrm{P}, \mathrm{J}$, and Q , except the surd V , are found in $f(p)$; and, if an equation such as (1) Prop. I. could be formed, involving the surd $V$, that equation, irhen $V$ was replaced by $T^{m} T_{1}^{n}$, would be reduced to a corresponding equation involving only surds in $f(p)$; which, since $f(p)$ is in a simple form, is impossible. Hence, since the surd T, a chief subordinate (on the hypothesis at present made). of $U$, is not present in $Q$, it is (by Cor. Prop. X.) implied in equation (8) that the form of $U$ is

$$
U=\left(L T^{\lambda}\right)^{\frac{1}{s}}
$$

where $L$ is an expression involving merely such surds, exclusire of $U$ and $T$, as occur in the expressions $P, U$, and $Q$; and $\lambda$ is a whole number less than $s$. Restore $U$ to the form $Y$; and let the surd $V$, in $L$, be replaced by its value in (7). Then

$$
\begin{aligned}
\mathbf{Y} & =\left\{\mathrm{T}^{\lambda}\left(\ldots \ldots+\mathrm{C}_{3} \mathrm{~V}^{d_{1}}+\mathrm{C}_{2} \mathrm{~V}^{d_{2}}+\& \mathrm{c} .\right)\right\}^{\frac{1}{8}} \\
& =\left\{\mathrm{T}^{\lambda}\left(\ldots \ldots+\mathrm{H}_{1} \mathrm{~T}^{\left[d_{1} m\right]} \mathrm{T}_{1}^{[d / n]}+\mathrm{H}_{8} \mathrm{~T}^{\left[d_{\mathrm{g}} m\right]} \mathrm{T}_{1}^{\left[d_{2} n\right]}+\& \mathrm{dc} .\right)\right\}_{;}^{\frac{1}{3}} ;
\end{aligned}
$$

where $\mathrm{C}_{1}, \mathrm{C}_{9}$, \&c., are clear of the surds V and T ; and $\mathrm{H}_{1}, \mathrm{H}_{8}$, \&c., are clear of the surds ' I ' and $\mathrm{I}_{1}$; the whole numbers, $\left[d_{1} m\right],[d n]$, \&c., representing the remainders left after the greatest multiples of $\sigma$ have been rejected from $d_{1} m, d_{1} n$, \&c. But this form of $Y$ is the inadmissible form given in (2) Prop. XII. Consequently the surd $T$ is not a subordinate of $U$. This leads to the conclusion that the surd $T$ does not (except as implicitly involved in V) appear in the expression P. For suppose, if possible, that $P$ is of the form, satisfying the conditions of Def. 8,

$$
\begin{equation*}
\mathrm{P} 口 \mathrm{~L}+\mathrm{L}_{1} \mathrm{~T}^{\lambda}+. \mathrm{L}_{2} \mathrm{~T}^{\beta}+\mathbb{S c} . \tag{9}
\end{equation*}
$$

where $1, L_{1}, \& c .$, may involve the surd $V$, but are clear of $T$; and none of the terms $L_{1}, L_{2}$, \&c., are zero; and $T^{\boldsymbol{\lambda}}, T^{\beta}$, \&c., are dis. $\frac{1}{8}$
tinct powers of $T$. Then, if the form of $U$ be, $U=Q_{1}$, and if $\mathrm{QQ}_{1}^{-c}$, when rendered integral, and made to satisfy the conditions of Def. 8, be written $t$, we have, by (8) and (9),

$$
\frac{1}{t^{s}}=\mathbf{L}+\mathrm{L}_{1} \eta^{\lambda}+\mathrm{L}_{2} \eta^{\beta}+\& r_{\bullet}
$$

From this equation iet the surd $t^{\frac{1}{s}}$ be eliminated, in the same manner in which $X_{i}$ was eliminated from equation (4) Prop. I. The result is,

$$
\mathrm{LK}+\mathrm{K}_{2} \mathrm{~L}_{2} \stackrel{\eta}{ }_{\lambda}^{\lambda}+\mathrm{K}_{2} \mathrm{~L}_{2}{ }^{\prime}{ }^{\beta}+\& \mathrm{c}_{0}=0
$$

Here, since the surd $T$ does not (except as implicitly involved in $V$ ) appear in $t$, the expressions $\mathrm{K}, \mathrm{K}_{1}$, \&c., are clear of the surd T . Therefore (Cor. 1, Def. 9), the terms LK, $\mathrm{K}_{1}, \mathrm{~K}_{2}$, \&c., vanish separately. But, from the manner in which $K, K_{1}$, \&e., originated, this implies that

$$
\begin{array}{r}
\mathrm{L}=c t^{\frac{2}{s}} ; \\
\mathrm{L}_{1} \mathrm{~T}^{\lambda}=l t^{\frac{1}{s}} ; \\
\mathrm{L}_{\mathrm{g}} \mathrm{~T}^{\lambda}=q t^{\frac{1}{s}} ;
\end{array}
$$

and so on; $c, k, q, \& c .$, being constant quantities. Hence, there cannot be more than one term in the series $\mathrm{L}_{1}, \mathrm{~L}_{2}$, \&c. ; else we should have

$$
q \mathrm{~L}_{1} \mathrm{~T}^{\lambda}=k \mathrm{~L}_{2} \mathrm{~T}^{\beta}:
$$

which (Cor. 1, Dei. 9) is impossible. For a similar reason, $L$ must 1 ranish; and the form of $t^{s}$ is,

$$
t^{\frac{1}{s}}=\mathrm{L}_{\mathrm{I}} \mathrm{~T}^{\lambda}
$$

Therefore, if $\lambda s=\delta \sigma+r$, we have

$$
t=\mathrm{L}_{3} \mathrm{I}^{\top},
$$

where $\mathrm{L}_{3}$ is clear of the surd T ; and $r$ is a whole number, less than $\sigma$, but (since $s$ and $\sigma$ are unequal) not $\%$ ero. But

$$
\begin{gathered}
t=\mathrm{Q} \mathrm{Q}_{1}^{-c}=\mathrm{Q}\left(\mathrm{U}^{-c}\right) \\
\therefore \mathrm{L}_{3} \mathrm{~T}^{v}=\mathrm{Q}\left(\mathrm{U}^{-c}\right)^{3} .
\end{gathered}
$$

And, when the expression on the right hand side of this equation is rendered integral, it is clear of the surd T. Therefore, by Cor 1. Def. $9, \mathrm{~L}_{3}$ must vanisì. Hence $t$ vanishes: which implies that $\mathbf{P}$ or $A_{c}$ vanishes. But (by hypothesis) $A_{c}$ does not vanish. Therefore $T$ does not appear in the expression $P$. In like manner, if, when $T_{1}$ is eliminated from $A_{n}$ [see (1)] by substituting its value as furnished by equation ( $\overline{\text { }}$ ), $A_{n}$, made to satisfy the conditions of Def. 8, be written $P_{1}$, it may be proved, since the equation,

$$
B_{n} Y_{1}=b_{n}\left(1 Y^{n}\right)
$$

has been shown to subsist, that the surd T does not appear (except as implicitly involved in V) in $A_{n}$. Ultimately, we get

$$
\begin{equation*}
f(p)=\dot{P}^{\prime}+P U^{c}+P_{1} U^{n}+\& c \tag{10}
\end{equation*}
$$

where $P^{\prime}, P$, \&c., are what $A, A_{c}$, \&c., in (1), become on substituting
for $T_{1}$ its value as furnished by ( 7 ) ; the expression on the right hand side of (10) being clear of both the sur .s $T$ and $T_{1}$, except as these surds are implicitly ingolved in $V$. Hence the surds T and $\mathrm{T}_{1}$ are similarly involved in $f(p)$ : which is contrary to hypothesis. Therefore the equation (3) cannot subsist.

## Proposition XIV.

Let the equation, $X=U$, be an algebraical equation of the fifth degree, in which the coefficients of the powers of $x$ are rational functions of a variable $p ; \mathrm{X}$ being incapable of being broken into rational factors, that is, factors having the coefficients of the powers of $x$ rational. Then, should the roots of the equation, $\mathrm{X}=0$, admit of being represented in algebraical functions, they are all contained in the expression,

$$
\begin{align*}
f(p)= & \mathrm{A}+\left(\mathrm{A}_{1}+\mathrm{B}_{1} \sqrt{ } \mathrm{C}\right)\left(\mathrm{D}+\mathrm{D}_{1} \sqrt{\mathrm{C}}\right)^{\frac{1}{5}}+\left(\mathrm{A}_{2}+\mathrm{B}_{2} \sqrt{\mathrm{C}}\right)\left(\mathrm{D}+\mathrm{D}_{1} \sqrt{\mathrm{C}}\right)^{\frac{2}{3}} \\
& +\left(\mathrm{A}_{3}+\mathrm{B}_{3} \sqrt{ } \overline{\mathrm{C}}\right)\left(\mathrm{D}+\mathrm{D}_{1} \sqrt{ } \overline{\mathrm{C}}\right)^{\frac{3}{3}}+\left(\mathrm{A}_{4}+\mathrm{B}_{4} \sqrt{\mathrm{C}}\right)\left(\mathrm{D}+\mathrm{D}_{2} \sqrt{ } \sqrt{\frac{4}{5}}\right)^{\frac{4}{5}} ; \ldots
\end{align*}
$$

where $C, D, D_{1}, A, A_{1}, B_{1}, A_{2}, B_{2}$, \&c., are rational functions of $p$.

For let $f(p)$, a root of the given equation be reduced (Prop. XII.) to a simple integral form, containing no surd such as $\bar{Y}$ in equation (l), Prop. XII., nor any surd, which, while not subordinate to any surd in the function, is of the form shown in (2); Prop. XII., and having no two surds similarly involved in it. Take Y, a surd in $f(p)$, not subordinate to any other in the function. Then, if we consider the mamer in which the terms of the series,

$$
\begin{equation*}
x-f(p), \mathrm{F}_{1}(x), \mathrm{F}_{2}(x), \ldots \ldots, \mathrm{F}_{4}(x) \text { or } \mathrm{X}, \tag{2}
\end{equation*}
$$

in Prop. VIII, are formed, it appears, that, in an equation of the $t^{\text {d }}$ degree, the reciprocal of the index of $\bar{Y}$ is a measure of $t$. Hence, in the case before us, the index of Y is $\frac{1}{5}$; and from this it follows that the series (2) is reduced to the two terms,

$$
x-f(p), \mathrm{X}
$$

Besides Y , there can be no surd in $f(p)$, which is not a suberdinate of Y ; for, if U were a surl in $f(p)$, distinct from Y , and not subordinate to any surd in $f(p)$, then, since the coefficients of tine different powers of $x$ in X are rational, the surd U disappears from X ; consequently (Prop. MX.) the index of $\mathbf{U}$ is the same with that of Y ,
and the surds $U$ and $Y$ are similarly involved in $f(p)$ : which is not the case. Hence $f(p)$ contains no surds, except Y and its subordinates. Let $\mathbf{T}$ be a chief subordinate of $\zeta$, with the index $\frac{1}{\sigma}$. Then, since the cuefficients of the different powers of $x$ in $\mathbb{X}$ are rational, T disappears from X . Let the factors by the continued product of which X is produced be [compare (1) Prop. XI]

$$
\begin{equation*}
\{x-f(p)\} \text { or } \mathrm{F}_{\mathrm{c}}(x),{ }^{2} \mathrm{~F}_{\mathrm{c}}(x), \ldots \ldots,{ }^{5} \mathrm{~F}_{\mathrm{c}}(x): \tag{3}
\end{equation*}
$$

and, when T is changed into $z_{1} \mathrm{~T}, z_{1}$ being a $\sigma^{\text {th }}$ root of unity, distinct from unity, let the terms in (3) be transformed into

$$
\begin{equation*}
{ }_{1} X_{1},{ }_{2} X_{1}, \ldots \ldots,{ }_{5} X_{1} \tag{4}
\end{equation*}
$$

Then, since the terms in $(t)$ are the five factors of $X, F_{c}(x)$ must be equal to one of these terms, which (on the principle pointed out in Prop XI.) may be assumed to be $\mathrm{X}_{1}$. Consequently (Prop. XI.)

$$
\begin{equation*}
Y=P Y_{1}^{\lambda} \tag{5}
\end{equation*}
$$

where $Y_{1}$ is what $Y$ becomes when $T$ is changed into $\approx_{1}{ }^{\prime} \mathrm{X}$; and P is an expression involving only surds in $f(p)$, distinct from Y ; $\lambda$ being a whole number, neither zero nor unity, less than 5 , and such that

$$
\begin{equation*}
\lambda^{\sigma}=\overline{5} u+1, \tag{6}
\end{equation*}
$$

where $w$ is a whole number. Since $\sigma$ is a prime number, the only values of $\lambda$, less than 5 , which satisfy equation (6), are 1 and 4 And $\lambda$ is not unity. Therefore $\lambda=4$. Hence $\sigma=2$; and $T$ is of the form, $\mathrm{T}=\sqrt{\mathrm{C}}$. Next, suppose, if possible, that U is a chief subordinate of $Y$, distinct from $T$. By the same process of reasoning as above, it may be shown that the index of $V$ is $\frac{1}{2}$; and, if ${ }_{1} Y$ be what $Y$ becomes when $U$ is taken with the negative sign, $-U$, the equation,

$$
\mathrm{Y}=\mathrm{Q}\left(, \mathrm{Y}^{4}\right),
$$

subsists; where $Q$ is an expression such as $P$ in (5). Therefore

$$
P V_{1}^{4}=Q\left(1 Y^{4}\right)
$$

By raising both sides of this equation to the fourth power, keeping in view that the common index of $\mathrm{X}_{1}$ and ${ }_{1} \mathrm{Y}$ is $\frac{1}{5}$, we get

$$
\begin{equation*}
{ }_{1} \mathrm{Y}=\mathrm{RI}_{1} ; \tag{7}
\end{equation*}
$$

where $R$ is an expressim, like $P$ and $Q$, clear of the surds $Y_{1}$ and ${ }_{1} Y$.

When U is changed into the negative expression, -U , let the terms in (3) become,

$$
\begin{equation*}
{ }^{\mathrm{I}} \mathrm{X},{ }^{2} \mathrm{X}, \ldots \ldots . . .,{ }^{5} \mathrm{X} ; \tag{8}
\end{equation*}
$$

sad, since $\mathrm{F}_{0}(x)$ must be equal to one of the terms in (8), assume (on the principle pointed out in Prop. XI.) that $\mathrm{F}_{\mathrm{c}}(x)={ }^{1} \mathrm{X}$. Take $x^{\mathrm{m}}$, a power of $x$ in $\mathrm{F}_{\mathrm{c}}(x)$, such that some power of Y is present in its coefficient E ; and, $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$ being the corresponding coefficients of $x^{m}$ in ${ }_{1} X_{1}$ and ${ }^{1} \mathrm{X}$, let $\mathbf{E}, \mathrm{E}_{1}$, and $\mathrm{E}_{2}$, satisfying the conditions of Def. 8, be,

$$
\begin{aligned}
& \mathbf{E}=\mathrm{B}+\mathrm{B}_{\mathrm{c}} \mathrm{Y}+\mathrm{B}_{\mathrm{n}} \mathrm{Y}^{\mathrm{n}}+8 \mathrm{cc} . \\
& \mathbf{E}_{1}=b+b_{\mathrm{c}} \mathrm{Y}_{1}^{\mathrm{c}}+b_{\mathrm{n}} \mathrm{Y}_{1}^{\mathrm{n}}+\& \mathrm{c} . \\
& \mathbf{E}_{2}=\beta+\beta_{\mathrm{c}}\left({ }_{1} \mathrm{Y}^{\mathrm{c}}\right)+\beta_{\mathrm{n}}\left({ }_{1} \mathrm{Y}^{\mathrm{n}}\right)+\& c . ;
\end{aligned}
$$

where $\mathbf{B}_{\mathrm{c}}, \mathbf{B}_{\mathrm{n}}, \& \mathrm{Ec}$., none of them zero, are clear of the surd $\mathbf{Y} ; \mathbf{B}$ also being clear of Y ; and no two terms in the series, $\mathrm{Y}^{\mathrm{c}}, \mathrm{Y}^{\mathrm{n}}$, \&c., are identical with one another ; $\mathrm{Y}_{1}, b, b_{c}$, \&c., being what $\mathrm{Y}, \mathrm{B}, \mathrm{B}_{\mathrm{c}}$, \&c., become in passing from $\mathrm{F}_{\mathrm{c}}(x)$ to ${ }_{1} \mathrm{X}_{1}$; and ${ }_{1} \mathrm{X}, \beta, \beta_{\mathrm{c}}$, \&e., what these same quantities become in passing from $F_{c}(x)$ to ${ }^{1} \mathrm{X}$. Then, because ${ }_{2} \mathrm{X}_{1}$ and ${ }^{2} \mathrm{X}$ are each equal to $\mathrm{F}_{\mathrm{c}}(x)$, they are equal to one another. This implies that $E_{1}$ and $E_{2}$ are equal to one another; but (Prop. XIII.) $b_{c} Y_{1}^{c}$ is not equal to $\beta_{c}\left({ }_{1} Y^{c}\right)$. It may be shown, however, exactly as in Prop. II., that the terms, $b_{c} Y_{1}^{c}, b_{n} Y_{1}^{n}$, \&c., taken in some order, are equal to the terms, $\beta_{c}\left({ }_{1} Y^{c}\right), \beta_{\mathrm{n}}\left({ }_{1} \mathrm{Y}^{\mathrm{n}}\right), \& \mathrm{c}$., each to each; and, if the steps of the demonstration be referred to, it will be seen, that, since equation ( $\overline{\text { }}$ ) subsists, $b_{c} Y_{2}^{c}$ must be equal to to the term $\beta_{c}\left({ }_{1} Y^{c}\right)$ : which is impossible. Therefore $U$ camnot be a chief subordinate of Y ; and T is the only chief subordinate of Y . Again, suppose that $U$ is a chief subordinate of $T$, with the index $\frac{2}{\rho}$; and, when $U$ is changed into $\varepsilon_{2} U, z_{2}$ being a $\rho^{\text {th }}$ root of unity, distinct from unity, let the terms in (3) become,

$$
\begin{equation*}
{ }_{2} \mathrm{X},{ }_{2} \mathrm{X}, \ldots . . . . .,{ }_{5} \mathrm{X} ; \tag{9}
\end{equation*}
$$

the surds X and T at the same time becoming $y$ and $t$. Then if $\mathrm{E}_{3}$ be the coefficient of $x^{m}$ in ${ }_{1} \mathrm{X}$, we may put

$$
\mathbf{E}_{3}=\beta+\beta_{\mathrm{c}} y^{\mathrm{c}}+\beta_{\mathrm{n}} y^{\mathrm{a}}+\& \mathrm{c}_{\mathrm{c}}
$$

where $y, \beta, \beta_{c}$, \&c., are.what $Y, B, B_{c}$, \&c., become in passing from $F_{c}(x)$ to ${ }_{1} X$. Since $F_{c}(x)$ is equal to one of the terms in (9), we may assume (on the principle pointed out in Prop. XI.) that it is equal: to ${ }_{1} X$; in which case $E$ is equal to $E_{3}$; or,

$$
\mathrm{B}+\mathrm{B}_{\mathrm{c}} \mathrm{Y}^{\mathrm{c}}+\& \mathrm{c} .=\beta+\beta_{\mathrm{c}} y^{\mathrm{c}}+\beta_{\mathrm{n}} y^{\mathrm{n}}+\& \mathrm{c} . . \ldots(10)
$$

Therefore (Cor. Prop. I.) an equation of one or other of the forms,

$$
\begin{align*}
& \mathbf{Y}^{e}=\mathbf{L} y^{r}  \tag{11}\\
& \mathbf{Y}^{e}=\mathbf{L}, \ldots \tag{12}
\end{align*}
$$

must subsist; where L is an expression involving only such surds as. are found in the expressions, $B, \beta, B_{c}, \beta_{c}$, \&c., or are subordinates of Y or $y$; and $y^{r}$ is a term in the series $Y^{c}, Y^{n}, \& c ., y^{c}, y^{n}, \& c .$, distinct from $Y^{c}$. Let us assume that equation (12) subsists. Then $L$ cannot be clear of the surd $t$ : else all the surds in $L$ would be found in $F_{\varepsilon}(x)$ : in which case, by Def. 9, equation (12) would be impossi. ble. The expression $L$, therefore, satisfying the conditions of Def. 8 , may be written,

$$
\mathrm{L}=\mathrm{H}+\mathrm{H}_{1} t
$$

where H and $\mathrm{H}_{1}$, the latter not zero, are clear of the surd $t$. . nce

$$
\mathrm{Y}^{\mathrm{e}}=\mathrm{H}+\mathrm{H}_{1} t
$$

From this it follows (Cor. Prop. I.) that an equation of one or otherof the forms,

$$
\begin{align*}
& \mathrm{Y}^{\mathrm{c}}=\hbar \mathrm{H}, \ldots  \tag{13}\\
& \mathrm{Y}^{\mathrm{c}}=\pi \mathrm{H}_{2} t \tag{14}
\end{align*}
$$

must subsist ; where $h$ is an expression involving only surds which arefound in H or $\mathrm{F}_{1}$, or are subordinates of Y or $t$, and therefore only such surds, exclusive of Y , as occur in $\mathrm{F}_{\mathrm{c}}(x)$. But equation (13) is. impossible, by Cor. 1. Def. 9. Also, should (14) subsist, we should have, (since $\frac{1}{2}$ is the index of $t$ ),

$$
\mathrm{Y}^{2 \mathrm{c}}=\hat{h}^{n} ;
$$

where $h^{\prime}$ is an expression involving only surds which occur in $F_{e}(x)$. But this is (Cor. 1, Def. 9) impossible. Therefore neitherequation (14) nor equation (13) can subsist; and hence equation (12) cannot subsist. Therefore equation (11) must subsist; and
we may assume that $y^{r}$ in (11) is a term in the series, $y^{\text {c }}, y^{\mathrm{n}}, \& \mathrm{dc}$.; for, were it such a term as $Y^{n}$, equation (11) would be reduced to the inadmissible form (12). If then we substitute for $y$ in (10) its value derived from (11), we may [keeping in view that no such equation as (12) can subsist] demonstrate, by reasoning similar to that employed in Prop. II. and Cor. Prop. I., that $\mathcal{B}_{c} Y^{\mathbf{c}}$ is equal to some term, as $\beta_{\mathrm{a}} y^{\mathrm{a}}$, in the series $\beta_{\mathrm{c}} y^{\mathrm{c}}, \beta_{\mathrm{n}} y^{\mathrm{n}}$, \&c. Let the fifth powers of $\mathrm{B}_{\mathrm{e}} \mathrm{Y}^{\mathrm{C}}$ and $\beta_{\mathrm{a}} y^{\mathrm{a}}$, satisfying the conditions of Def. 8, be,

$$
\begin{aligned}
& \left(\mathrm{B}_{\mathrm{c}} \mathrm{Y}^{\mathrm{c}}\right)^{5}=h+k_{\mathrm{I}} \mathrm{~T} \\
& \left(\beta_{\mathrm{a}} y^{\mathrm{a}}\right)^{s}=k+k_{\mathrm{t}} t
\end{aligned}
$$

where $h$ and $h_{1}$ are clear of the surd T ; and $k$ and $k_{1}$ are clear of the surd $t$; and (Cor. Prop. X.) $h_{1}$ is not zero. Therefore, from the equation,

$$
h+h_{1} \mathrm{Y}=k+k_{1} t
$$

we have (Cor. Prop. I.) an equation of one or other of the forms,

$$
\begin{align*}
& \mathbf{T}=l, \\
& \mathbf{T}=l t, \tag{15}
\end{align*}
$$

where $l$ is an expression involving only such surds as occur in $h, h_{i}$, $K_{l}, k_{s}$, or are subordinates of T or $t$; that is, $l$ involves only surds in $F_{c}(x)$, exclusive of T. But (Def. 9 ) T camot be equal to $l$. Therefore equation (15) subsists. Since $t$ is formed from $T$ by changing $U$ into $z_{2} J$, let the forms of $T, t$, and $l$, be,

$$
\begin{aligned}
& \mathrm{T}=\left(\mathrm{R}+\mathrm{R}_{1} \mathrm{U}+\mathrm{R}_{2} \mathrm{U}^{2}+\ldots \ldots+\mathrm{R}_{\rho-1} \mathrm{U}^{\rho-1}\right)^{1} \\
& t=\left(\mathrm{R}+\mathrm{R}_{1} z_{2} \mathrm{U}+z_{2}^{2} \mathrm{R}_{2} \mathrm{U}^{2}+\& \mathrm{cc} \cdot\right)^{\frac{1}{2}} \\
& l=\mathrm{S}+\mathrm{S}_{1} \mathrm{U}+\mathrm{S}_{2} \mathrm{U}^{2}+\ldots \ldots+\mathrm{S}_{\rho-1} \mathrm{U}^{\rho-1}
\end{aligned}
$$

where $R, S, R_{1}, S_{1}, \& c$., are clear of the surd U. Then, from (15),

$$
\begin{aligned}
\left(v+v_{1} \mathrm{U}+\ldots \ldots+v_{\rho-1} \mathrm{U}^{\rho-1}\right) \mathrm{T} & =\left(v+v_{2} \mathrm{U}+\& \mathrm{c} .\right)\left(\mathrm{S}+\mathrm{S}_{2} \mathrm{U}+\& \mathrm{c} .\right) t \\
& =\left(\mathrm{V}+\mathrm{V}_{2} \mathrm{U}+\ldots \ldots+\mathrm{V}_{\rho-1} \mathrm{U}^{\rho-1}\right)_{t}
\end{aligned}
$$

where the expression, $V+V_{i} U+s c$. , is generated by the multiplication together of the two expressions, $v+v_{1} \mathrm{U}+\mathbb{S c} ., \mathrm{S}+\mathrm{S}_{2} \mathrm{U}+$ \& c, ; the coefficients, $V, V_{1}$, \&c., being clear of the surd U . Let us
determine the $\rho$ unknown quantities, $v, v_{1}, \ldots \ldots, v_{\rho-1}$, by means of the simple equations,

$$
\begin{aligned}
v & =\mathrm{V}^{\prime} \\
z_{2} v_{2} & =\mathrm{V}_{1}, \\
z_{2}^{2} v_{2} & =\mathrm{V}_{2}, \\
z_{2-1}^{\rho-1} \mathrm{~V}_{\rho-1} & =\mathrm{V}_{\rho-1} .
\end{aligned}
$$

Then equation (15) gives us

$$
\begin{aligned}
& \left(v+v_{1} \mathrm{U}+\& \mathrm{c} .\right)\left(\mathrm{R}+\mathrm{R}_{1} \mathrm{U}+\& \mathrm{c} .\right)^{\frac{1}{2}}= \\
& \left(v+v_{1} z_{2} \mathrm{U}+\& \mathrm{c} .\right)\left(\mathrm{R}+\mathrm{R}_{1} z_{2} \mathrm{U}+\& \mathrm{c} .\right)^{\frac{1}{2}}
\end{aligned}
$$

Hence, by Prop. X., one of the surds in $\mathrm{F}_{\mathrm{c}}(x)$, viz. T, is of the form of $\mathbf{Y}$ in (1) Prop. XII.: which (by hypothesis) is impossible. Hence $U$ cannot be a chief subordinate of $T$; and therefore $f(p)$ involves no surds except $Y$ and its subordinate $T$; the latter being of the form $\mathbf{T}=\sqrt{\mathbf{C}}$. Consequently $f(p)$ is of the form (1).

We may notice a particular form in which $f(p)$ admits of being expressed. By means of equation (5), we can reduce (1) to the following :

$$
\begin{aligned}
f(p)=\mathrm{A} & +\left(\mathrm{A}_{1}+\mathrm{B}_{1} \sqrt{\mathrm{C}}\right)\left(\mathrm{D}+\mathrm{D}_{1} \sqrt{\mathrm{C}}\right)^{\frac{1}{4}} \\
& +\left(\mathrm{A}_{2}+\mathrm{B}_{2} \sqrt{\mathrm{C}}\right)\left(\mathrm{D}+\mathrm{D}_{1} \sqrt{\mathrm{C}}\right)^{\frac{2}{5}} \\
& +\left(\mathrm{A}_{1}-\mathrm{B}_{1} \sqrt{\mathrm{C}}\right)\left(\mathrm{D}-\mathrm{D}_{1} \sqrt{\mathrm{C}}\right)^{\frac{1}{5}} \\
& +\left(\mathrm{A}_{2}-\mathrm{B}_{2} \sqrt{\mathrm{C}}\right)\left(\mathrm{D}-\mathrm{D}_{1} \sqrt{\mathrm{C}}\right)^{\frac{2}{5}}
\end{aligned}
$$

But, as thus exhibited, $f(p)$ is not in a simple form.
Cor. 1.-The exact resolution, in algebraical functions, of an equation of the fifth degree, is only possible when $X$ admits of being broken into rational factors, or when the roots can be reduced to the form (1). Hence, in the most general case, the exact resolution, in algebraical functions, of an equation of the fifth degree, cannot in the nature of things be effected. 4 fortiori, the exact resolution, in algebraical functions, of equations of degrees above the fifth, cannot, in the most general case, be effected.

Cor. 2.-In all the cases in which the roots of a quintic equation, whose coefficients are rational functions of a variable $p$, admit of being represented in algebraical functions, the principles which have been established enable us actually to solve the equation. For, if $X$ can be broken into rational factors, these factors may easily be found; and thus the solution of the quintic is obtained. Should X not admit of being broken into rational factors, assume $f(p)$ equal to the expression in (1). Substitute for $f(p)$ in $X$ the expression to which $x$ is thus assumed equal ; and let the result of the substitution be,

$$
\begin{aligned}
& b+\left(a_{1}+b_{1} \sqrt{ } \mathrm{C}\right)\left(\mathrm{D}+\mathrm{D}_{1} \sqrt{\mathrm{C}}\right)^{\frac{1}{5}}+\left(a_{2}+b_{3} \sqrt{\mathrm{C}}\right)\left(\mathrm{D}+\mathrm{D}_{1} \sqrt{\mathrm{C}}\right)^{\frac{2}{5}} \\
& +\left(a_{3}+b_{3} \sqrt{\mathrm{C}}\right)\left(\mathrm{D}+\mathrm{D}_{1} \sqrt{\mathrm{C}}\right)^{\frac{3}{5}}+\left(a_{4}+b_{4} \sqrt{\mathrm{C}}\right)\left(\mathrm{D}+\mathrm{D}_{1} \sqrt{\mathrm{C}}\right)^{\frac{4}{5}}
\end{aligned}
$$

where the expressions, $b, a_{1}, b_{1}, a_{2}, b_{2}, \& c$. , as well as $\mathrm{C}, \mathrm{D}, \mathrm{D}_{1}$, are to be assumed rational. Put

$$
\begin{array}{r}
a_{1}=0, a_{2}=0, a_{3}=0, a_{4}=0 \\
b=0, b_{3}=0, b_{2}=0, b_{3}=0, b_{4}=0 ;
\end{array}
$$

and these equations will emable us to find the unknown quantities in the value of $f(p)$ : it being taken for granted that the rational roots of an algebraical equation, having the coefficients rational functions of a variable, can always be found.

Note. - From what has been proved, it appears that the roots of an algebraical equation of a degree higher than the fourth do not, in the most general case, admit of being represented in finite algebraical functions; and we have seen how an equation of the fifth degree, whose coefficients are rational functions of a variable $p$, mar be actually solved, whenever, in consequence of particular relations among the coefficients, the roots are capable of being algebraically represented. It is easy to extend the conclusions which have been obtained to equations of every degree; and. from the principles established in the above Propositions, to show how an algebraical equation of any degree, whose coeficients are functions of a variable $p$, may be exactly solved, in all cases in which an exact solution in finite algebraical functions is in the nature of things possible. Thiswe propose to do in a subsequent paper.

# A POPULAR EXPOSI'ION OF THE MINERALS AND GEOLOGY OF CANADA. 

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PART II.

## Introductory Noticf.

In the first of this series of papers, published in the January Number of the Canadian Journal, we gave a brief review of the more common characters or properties employed in the determination of minerals. The present paper exhibits the practical application of these characters, in the distribution of our Canadian minerals into a small number of easily recognized groups, so arranged as to lead at once to the names of the included substances.

By referring to the heads of this arrangement or clessification,* as given below, it will be seen that there are four principal groups: $A, B, C$, and $D$ : the first two containing those minerals which exhibit a metallic aspect ; and the other two containing our glassy, stony, pearly, or earthy-looking minerals. The metallic-looking substances placed in group 4 are sufficiently hard to scratch windowglass; whilst those placed in group $B$, are too soft to effect this. In like manner, the minerals of non-metallic aspect placed in group $C$, scratch glass; whilst those placed in group $D$, are less hard than glass, and are consequently unable to scratch that substance. The term "glass," as employed in this sense, means ordinary windowglass. By these simple characters it is easy to determine in a minute, to which group a substance under examination belongs. This determined, we proceed to a consideration of the sub-groups, $1,2,3, \& c$., of the group in question. In the sub-croup or section to which the substance will thus be found to belong, there will probably be some three or four, or perhaps half-a-dozen, other minerals; but these, it will be seen, are readily distinguishable, one from another, by colour,

[^0]colour of streak, structure, or other easily determined character. In this manner we arrive, without difficulty, at the name of our mineral.

To illustrate this by example, let it be supposed that we have a piece of a red, dull, and somewhat earthy-looking substance, the name of which we wish to ascertain. By its non-metallic aspect, we see at once that it belongs either to group $C$ or to group $D$. We try if it will scratch glass. It is not sufficiently hard to do this: hence it belongs to group $D$. Turning now to the respective sub-groups or sections under $D$, we find that our mineral has no taste, and hence does not belong to D 1 . Neither does it take fire (although it blackens) when a thin splinter of it is held for a morient in the flame of a candle, or in the flame of an ignited match : and hence it does not belong to $D 2$. It has, however, a coloured streate* (red), and so belongs to the next section, $D 3$ Now in this section there are only two minerals with red streak: or only one, indeed, of undoubted Canadian occurrence-Eartly Red Iron Ore, commonly called Red Ochre; and as our mineral becomes magnetic after exposure to the flame of a match or candle, it can be nothing else than a specimen of that substance. This example will be sufficient to shew the method of procedure to be followed in order to ascertain the name, \&c., of an unknown mineral, by reference to the amnexed Tabular Distribution. In this comexion, it has been thought advisable to include a few substances of more or less common occurrence in the United States, although not yet found in Canada; and also to refer occasionally, in smaller type, to some other minerais of economic value or popular interest, so as to make the subject more complete, and render our Tables available for the examination of the small collections sometimes imported into this country for the purposes of study. Some of the substances thins noticed, may also be discovered eventually in Canada. Finally, it should be observed that the descriptions of these various minerals, given in our Tabular Distiibution, are necessarily exceedingly brief, referring only to matters of easy comprehension or general importance. When, however, the name of a mineral is once discovered, the reader, if he desire to pursue the subject further, can refer for fuller details to any of our ordinary works on Mineralogy.

[^1]A Tabular Distribution of Canidian Minerals, including; *also, a féw other Mineral Substances ofcommon occurrence.
general index.The reader is to determine, by this Index, the group and sub-groupto which his unknown mineral belongs; and he is then to refer to thedescriptions given under that sub-group in the pages immediatelyfollowing the Index.

Aspect Metallic $\left\{\begin{array}{l}\text { Hard enough to scratch glass }\end{array}\right.$ ..... A.
Aspect Non-metallic.. $\left\{\begin{array}{l}\text { Hard enough to scratch glass } \\ \text { Not herd }\end{array}\right.$ ..... $B$.
A. Aspect metallic. Hard enough to scratch glass :
Colour, Light Brass-yellow ..... AI.
Colour, Pale copper-red ..... A 2.
Colour, Tin-white, or Silver-white ..... A 3.
Colour, Steel-grey, Black, or Brown ..... A 4 .
B. Aspect metallic. Not hard enough to scratch glass :
Malleable or Ductile ..... $B 1$.
Yielding to the aail ..... $B 2$.
Not yielding to the nail ..... $B 3$.
C. Aspect non-metallic, (glassy, stony, \&c.) Hard enough to scratch glass :
Infusible. Very hard: not yielding to the knife. ..... Cl.
Infusible, or nearly so. Yielding to the knife ..... C 2.
Fusible. Not yielding water in the bulb-tube. ..... C 3.
Fusible. Yielding water in the bulb-tube (fig. 22). ..... C4.
D. Aspect non-metallic, (stony, glassy, \&c.) Not hard enough to scratch glass:
Soluble, and thus affecting the taste ..... D 1.
Taking fire when held (in thin splinters) in the flame
of a candle ..... D 2.Not exhibiting the above reactions. Streak, coloured D3.Streak, white. Not yielding water in the bulb-tube .. D 4.Streak, white. Yielding water in the bull-tube (fig.22.) D 5.

## A. Aspect Metallic. Hardness sufficient to scratch glass.

 A 1.-Colour, Light Brass-yellow.Iron Pyrites.-A substance of a pale brass-yellow colour, with greyish-black streak, occurring in amorphous, globular, and other masses, and in Monometric crystals (cubes, generally with alter-nately-striated faces, pentagonal dodecahedrons, \&c., figs. 23, 24, 25.)


Fig. 23.


Fig. 24.


Fig. 25.
H. 6.0-6.5; sp. gr. 4.8-5.1. Fusible, with sulphur fumes, into a magnetic globule. One hundred parts contain: sulphur, 53.5 ; iron, 46.7 ; but the irou is sometimes in part replaced by a little cobalt or nickel, and occasionally minute portions of gold and silver are accidentally present Iron pyrites occurs in all kinds of rocks, and is exceedingly common ; but is useless as an ore of iron. It yields copperas, or iron-vitriol, by decomposition; and it is often converted on the surface, or wholly, into hydrated brown oxide of iron. It sometimes forms the substance of organic remains, as in many of the Trilobites, \&c., of our Utica Slate. Amongst the principal Canadian localities,* we may note, more especially, the counties of Pontiac (Clarendon Township), Terrebonne, Berthier (Lanoraie Seign.), and Sherbrooke (Garthby Township), in Canada East; the vicinity of Balsam Lake, where it occurs with magnetic pyrites; and many places on the north shore of Lake Huron, Lake Superior, \&c. A nickeliferous variety occurs in D'Aillebout, Berthier Co.; and an auriferous variety in Vandreuil, Beauce Co., C. E. We have obtained some brilliant, though small, crystals from the white feldspathic trap of the Montreal Mountain ; and also from the Niagara limestone, and other fossiliferous

[^2]rocks ; but iron pyrites occurs chiefly in our Laurentian and Huronian Formations, and in the Metamor ${ }_{1}$ hic district of the Eastern Townships. The general reader will find these geological terms fully explained in some of the succeeding papers of this series.

> Radiated Pyriles, or Marcasite, also belougs to this Section, but it does not appear to have been noticed in Cauadn. It has the same composition as common Pyrit"s, but crystallizes in the Trimetric or Rhombic System. Many globular specimens, with radiated structure, sometimes referred to Mareasite, belong truly, it alould be observed, to common Pyrites.

## A. 2.-Colour; Pale, Copper `ed (usually with grey or black external tarnish.)

Arsenical Nickel - Pale copper-red, tarnishing dark-grey. Streak, brownish-black. Chiefly in small amorphous masses. H. 5.0-5.5 (it seratehes glass feebly.) Sp. gr. 7.3-7.7 (a salient character.) Fusible, with strong odour of garlic. One hundred parts contain: Arsenic, 56; Nickel, 44. This substance, often called Copper-Nickel from its copper-red colour, is the common ore of nickel; but in Canada it is very rare. It has been found in small quantities in Michipicoten Island, Lake Superior. A substance composed of sulphur, arsenic, and nickel, occurs likewise, but in very small quantities, at the Wallace Mines, Lake Huron. It is somewhat less hard than Arsenical Nickel. The Townships of Bolton and Ham, in the metamorphic district of the Eastern Townships, are also cited as localities of nickel ore. The ore is said to occur there very sparingly in Serpentine, associated with Chromic Iron Ore.

## A. 3. C'olour, Tin or Silver-white (sometimes with grey or yellowish external tarnish.)

Arsenical Pyrites (Mispickel.)-Tin or silver-white, inclining to light steel grey. Streak, greyish-black. In amorphous and granular masses, and in modified rhombic prisms (Trimetric System.) H. 5.5-6.0; Sp. gr. 6.0-6.4. Fusible, with garlic odour, into a magnetic globule. One hundred parts contain: sulphur, 20; arsenic, 46 ; iron, 34 . This mineral is of very common occurrence in many countries. It is quite useless as an ore of iron, but is em. ployed in Germany and elsewhere in the production of arsenious acid, the white arsenic of commerce. Arsenious acid is obtained also, and more abundantly, from arsenical nickel and certain cobalt ores. In Canada, arsenical pyrites occurs in small quantities with common iron
pyrites, \&c., in our azoic and metamorphic rocks more especinlly, at various localities : as at the Lake Huron Mines ; in Clarendon Township (Pontiac Co.) ; in the Chaudière Valley, \&c. It sometimes contains a little cobalt, in which case, after exposure before the blow-pipe to drive off the greater part of the arsenic and sulphur, it fuses with with borax into a rich blue glass.
The common cobalt ores (Smaltine and Cobaltine) bci Jug also to this Section, but they have not yet been discovered in Caaada.

## A 4. Colour, Steel-grey, Iron-black, or Brown. (No fumes before the Blow-pipe.)

[Principal Minerale.-Streak, dull-red: Specular Iron Ore. Strongly magnetic; streak, black: Magnetic Iron Ore. Yielding water-in the bulb-tube; streak, yellowish-brown: Brown Iron Ore.]
Specular Iron, or Red Iron Ore.—Dark steel-grey, often inclining to blucish red. Streak, dull-red, the same as the colour of the earthy varieties described in Section D 3. In rhombohedral crystals and crystalline groups, and in lamellar, micaceous, and fibrous-botryoidal masses, the latter often called Red Hæmatite. H. 5.5-6.5; sp. gr. 4.3-5.3. In thin splinters, fusible on the edges (although commonly said to be infusible.) Becomes also magnetic after exposure to the blow-pipe, and is often feebly magnetic in its normal condition. One hundred parts contain: Oxygen, 30; Iron, 70. This mineral is one of the most valuable of the Iron Ores. In Canada, it is exceedingly abundant, more especially in our Laurentian rocks, although less so than the Magnetic Iron Ore. It occurs chiefly in these rocks in the Township of MacNab, on the Ottawa, where it constitutes a vast bed, twenty-five feet thick, in crystalline limestone; and also associated with crystalline limestone at Iron Island, Lake Nipissing (Mr. Murray.) In the Huronian rocks, it is found at the Wallace Mine, Lake Huron; and it occurs likewise in metamorphic chloritic schists (altered Silurian shales of the age of the Hudson River group), associated with magnetic iron ore, dolomite, \&c., in the Eastern Townships of Sutton, Bolton, and Brome.

Ilmenite.-This substance, (normally, perhaps, a compound of the sesqui-oxides of titanium and iron, ) has an iron-black or dark steelgrey colour, with black or dark reddish-brown streak. It closely resembles and passes into Specular Iron Ore. At Baie St. Paul, C.E., a large deposit of Ilmenite, three hundred feet in length and ninety feet broad, occurs in a feldspathic rock of the Laurentian series. It is

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associated with small orange-red grains of rutile. The same substance (according to Sir W. Lognn,) occu:s also, mixed with magnetic iron ore, in a thick bed in serpentine, in Vaudreuil, Beauce Eounty, C.E.

MLagnetic Iron Ore.-Iron-black, with sub-metallic lustre and black streak. Occurs in monometric crystals (octahedrons and rhombic dodecahedrons, figs. 26 and 27), in amorphous masses of a granular or lamellar structure, and also in small grains. Strongly magnetic, often with polarity. H. 5.5-6.5; sp. gr. 4.9-5.2. Infusible, or nearly so. One hundred parts contain : Oxygen, 27.6; iron, 72.4 ; (or sesquioxide of iron, 69 ; protoxide of iron, 31.) This when pure, is the


Fig 26 . most valuable of all the iron ores. Its black streak, and strong magnetism, (and, when crystallized, its form), casily distinguish it from specular iron ore. In the Laurentian rocks of Canada, it occurs in vast beds, rendering this Province one of


Fig. 27. the richest iron-containing countries of the world. It occurs also abundantly amongst the metamorphosed Silurian strata of the Eastern Townships. Its principal "Laurentian" localities comprise: the Townships of Marmora, Belmont, and Madoc, with those of South Sherbrooke, Bedford, and Crosby, in Camada West; and the Townships of Hull aud Litchfield, on the Ottawa, in Canada East. The supply at these localitics is apparently inexhaustible. The 'Townships of Bolton and Brome, and the Chaudière Valley, may be cited amongst the localities of this ore in the metamorphic district south of the St. Lawrence. In this district, however, as remarked by Sir William Logan, its value is much lessened by admixture with titanic iron, chlorite, \&c. In the form of black magnetic sand (either alone or mixed with iserine, , this ore is also of exceedingly common occurrence on the shores of many of our lakes, islands, \&c. The black iron-sand of the 'Iorouto " Peninsula" is a well-known example.

Iserine.-This is a black titauiferous iron ore, bearing the same relation to Magnetic Iron Ore that Ilmenite bears to Specular Iron. It occurs chiefly in the form of magnetic sand, or in small granular masses, mixed with magnetic iron ore. It occurs with "iron sand" on our lake shores, \&c., and probably with magnetic iron in the Eastern Townships. It can only be distinguished from the latter mineral by a blow-pipe (or other chemical) examination. Fused on
charcoal with microcosmic salt in a reducing flame, the glass becomes, on cooling, deep red.

Chromic Iron Ore.-This substance is also closely related to Magnetic Iron Ore. It has a black colour, with sub-metallic lustre, and dark-brown streak. It occurs commonly in amorphous granular masses, and consists normally of sesqui-oxide of chromium and oxide of iron. H. 5.5; sp. gr. 4.3-4.6. Bolton and Ham, in the "metamorphic district" of the Eastern Townships (where it occurs in veins of about a foot in thickness, in serpentine) are its principal Camadian localities. It is found also in other places throughout this district, in small grains, in dolomite and magnesite rocks. When quite pure, it may be distinguished from magnetic iron ore by its brown streak and lower sp. gr.; as well as by its want of (or feeble) magnetism. Chromic Iron Ore is used for the preparation of chromium compounds, employed in dyeing, painting, \&c.

Brown Iron Ore (Iimonite).-Brown of various shades, with submetallic (or sometimes stony or silky) aspect, and yellowish-brown streak. Occurs chiefly in botryoidal masses with fibrous structure (a variety often called Brown Hematite), and also in vesicular and carthy amorphous masses (Bog Iron Ore). H. 5.0-5.5; sp.gr.3.5-4 0. Blackens before the blow-pipe, and becomes magnetic. In the bulbtube (fiy. 22) it gives off water. One hundred parts contain (if the substance be pure): Sesqui-oxide of iron, 85.6 ; water, 14.4. This is likewise a raluable ore of iron. The Bog Iron Ore rariety (in addition to yellow ochre described in Section $D$ 3) is that which chiefly occurs in Cauada. This variety is a comparatively recent product; and its formation, indeed, is still going on in places, by deposition from water in the form of carbonate of iron oxide, this being afterwards conrerted into the hydrated sesqui-oxide. It occurs in great abundance in Post-tertiary deposits in the Three Rivers District, C.E., (yielding the celebrated "St. Maurice, or Three Rivers Iron," largely employed for castings); and also in the County of Norfolk, C.W.; besides many other localities. Altogether, the following Townships and Seigniories are enumerated by Sir William Logan (Esquisse géologique du Canada) as yielding this ore: Middletown, Charlotteville, Walsingham, West Gwillimbury, Fitzroy, Eardley, March, Hull, Templeton, Vaudreuil, St. Maurice, Champlain, Batiscan, Ste. Anue, Port Ncuf, Nicolet, Stanbridge Simpson, Ireland, Lauzon, St. Vallier, \&c. These bog iron ores always contain a small amount of
phosphoric acid, which becomes reduced during the process of smelt. ing, and usually renders the iron (by the presence of phosphide or phosphuret of iron) "cold-short." Cold-short iron is more or less brittle, and, hence, as a general rule, it is only available for castings. The St. Marrice ores are said, however, to yield excellent malleable iron.
Notz. - As the minerals of this Section ( $A$ 4) present, in mavy of their varieties, a somewhat doubtful metallic aspect,* they will be referred to again, uuder Group $C$.

## B. Aspect metallic. Hardness insufficient to soratch glass.

## B l. Malleable or Ductile.

[Principal Minerals:-Colour yellow: Vative Gold Colour white, with dark tarnish. Native Siluer. Colour dark leadgrey: Sulphide of Silver. Colour copper-red: Native Copper.]
Native Gold.-Rich golden-yellow; in small granular or subcrystalline masses, scales, and dust. Sp. gr. varying from about 16.0 to 19.0. Easily fusible, but otherwise inalterable before the blowpipe. Distinguished by this latter character, and also by its high sp. gr., its malleability, \&c., from copper pyrites, iron pyrites, and other substances of a similar aspect. Another salient charaeter, applicable more especially to dust gold, is the quality of remaining unaffected by nitric acid. In Canada, native gold occurs over a wide area (in allurial sands, \&c.) in the metamorphic district south of the St. Lawrence, although not in sufficient abundance to cause the regular working of the aurifcrous sands of this district to be remumerative. The sands of the following streams and rivers, more particularly, are stated by Sir William Logan to contain gold: The Guiliaume, lessard, Bras, Touffe-des-Pins, Du Lac, Ermine, Du Loup, Metgermet, and Poser's stream; with the Chaudière and St. Francis. These, with the exception of the St. Francis, belong chiefly to Beauce Co., C.E. Sir William Logan states also, that native gold has been found in small quantities in a vein with Specular Iron Ore, in the tormship of Leeds, Megantic Co., C.E. Traces of gold have likewise been discovered in the native silver of Prince's Mine, Lake Superior. (See, also, auriferous varieties of Iron Pyrites, A1; Copper Pyrites, B3;

[^3]and Blende, B3.) The gold of the Eastern Townships contains, according to Professor Sterry Hunt, from 11 to 13 per cent. of silver. Small grains of Platinum and Irilosmium are mixed with it here and there, as in the sands of the Rivière du Loup, ©ic.

Native Silver.-Silver-white, often with dark or yellowish external tarnish. Found chiefly in crystalline arborescent groups, and in small, scaly, gramular, or wire-like masses, associated with native copper, at St. Ignace and Michipicoten Islands; and with sulphide of silver, \&c., in calcareous spar, at Prince's Mine, Spar Island, Lake Superior. Sp. gr. 10-11. Easily fusible.

Sulphide of Silver (or Silver Glance).-Dark lead-grey or black, with shining streak. Perfectly ductile. Chiefly in small masses with mative silver, sulphide of copper, galena, malachite, \&c., in a vein of quartz and calc spar, at Prince's Mine, Lake Superior. Sp. gr. about 7.2. Fusible and reducible to metallic silver per se before the blowpipe. One hundred parts contain : Sulphur, 13 ; silver, 87 . It is easily distinguished from sulphide of copper, galena, \&c., by its perfect malleability, as well as by its blow-pipe characters.

Native Copper:-Copper-red, with shining streak. Chicfly in arborescent and amorphous masses, more rarely in determinable crystal-groups (Monometric.) H. 2.5-3.0. Perfectly malleable. Sp. gr. about 8.9. Easily fusible, imparting a green colour to the flame. Native copper occurs in immense abundance on the south shore of Lake Superior, but on the Canada side of the lake it has been found in small quantities in St. Ignace and Michipicoten Islands. In the latter Island, at Maimanse and Mica Bay, accompanying copper glance and copper pyrites. It does not appear to occur at all amongst the extensive deposits of copper pyrites, \&c., on Lake Huron. In the Eastern metamorphic district, native copper is said to have been noticed at St. Henri, Dorchester County.

## B. 2. Fieldiny to the Nail

[Priacipal Minerals: Streak white, Mfica. Streak black, colour, black or darkgrey: Graphite. Strenk and Colour lead.grey; imparting a pale green tint to the blow-pipe flame: Molybdenitc.]

Mica:--In laminar or scaly masses, with a false pearly-metallic aspect. Colour, various; streak, white. See Section D. 4.

Graphite:-Chiefly in black or dark-grey foliated masses or small scales. Feels scmewhat greasy; marks on paper ; sectile, and flexible
in thin pieces; II. $1.0-2.0$; Sp. gr. about 2.0. Inalterable before the blow-pipe. It occurs in small scales disseminated more or less throughout our Laurentain formation, and more especially in the crystalline limestones of that series; but its principal Canadian localities are the townships of Gremille (Addington County,) and Fitzroy (Candeton County,) on the Ottawa. At the former locality it constitutes several veins, each of an average thickness of about five inches; and is associated with garnets, zircon, feldspar, and other minerals. Graphite when of fine granular structure and dark colour, is extensively employed, under the popular name of Plumbago or "Black-Lead," in the manufacture of the so-called black-lead pencils. It consists, however, simply of carbon (or of carbon mechanically mixed with oxide of iron,) and does not contain a trace of lead. Our Camadian graphite is unfortunately too coarse and not sufficiently intense in colour for pencils, but, according to Sir William Logan, it may be used in the manufacture of refractory crucibles. Some samples that we have seen, might be employed also when ground to powder, as a polishing material for grates and stoves.

Molybdenite :-This substance much resembles graphite, but is of a lighter colour; and whilst it leaves a black trace on paper, it makes a dull greenish streak on smooth porcelain. It occurs chiefly in small scaly masses of a lead-grey colour. Like graphite it feels somewhat greasy, and it is also flexible. H. 1.0-2.0; Sp. gr. 4.4-4.S. Infusible, but it colours the blow-pipe flame pale-green, and valatilizes very slowly, depositing a white crust of molybdic acid on the charcoal. One hundred parts consist of : sulphur 41, molybdenum 59. It is not uncommon in small quantities amongst our Laurentian rocks generally, and in the intrusive granites of that formation. As special localities, we may cite from the Reports of the Geological Survey: Jerome, C. E.; Mud Turtle Lake, north of Balsam Lake; the River Doré near Gros Cap; and a granite vein on the west side of Terrace Cove, Lake Superior. Molybdenite is the principal source of molybdenum compounds, used in porcelain painting, and as a reagent in certain chemical experiments, \&c.

To this section belong also, Pyrolusite or Blacki jifanganese Ore, and Sulphide of Antimony or Grcy Antimony Ore. The former (a compound of oxygen 36.7, manganese 63.3) oceurs chiefly in radiatiog fibrous masses of a black or dark steel-grey colour, and is quite infusible. We have received a specimen said to
have been found in the Enstern Townships or the neighbourhood, a district in which the Earthy or Bog Mangancse Ore is of not uncommon occurence (bee Section D.) Prrolusite is found also in the adjoining State of Vermont. It is a valuable ore. Sulphide of Antimony has not hitherto been recognised in Cauada. It occurs principally in fibrous masses of a lead or steel-grey colour, often with a dark or inidescent tarnish. A thin splinter will melt in the flame of a candle without the aid of the blow-pipe. It has been found in Maine, New Hampshire, \&c., in the United States.

## B. 3. Not yielding to the Nail.

(Principal Minerals:-Colour reddish; garlic-like odour before the blowpipe: Arsenical Nickel. Colour reddish, with blue or variegated tarvish; Purple Copper Pyrites. Colour, brouze-yellow; magnetic: Magnetic Pyrites. Colour, brass-yellow, often with variegated tarnish; streak,-blackish-green: Copper Pyrites. Colour, dark-grey, often with green or blue tarnish; (Sp. gr. under 5.8) Copper Glance. Colour lead.grey; breaking easily into rectangular fagments; (Sp. gr. over 7.0): Galena. Colour, dark brown or various; streak brown; Infusible: Zinc Blende.

Arsenical Nickel:-Colour light copper-red, sometimes with green-ish-white coating; exccedingly heavy; yielding an arsenical or garliclike odour before the blowpipe. Many (or most) specimens are just hard enough to scratch glass; hence, this substance is described in full under Section $A 2$, above. As a Canadiar. mineral, it is comparatively unimportant.

Maynetic Pyrites:-Colour brownish or bronze-yellow, with black streak. Chiefly in amorphous masses. Magnetic, and often exhibits polarity. IF. $3.5-1.5$; Sp. gr. 4.4-4.7. Fusible, with sulphur fumes. Easily converted, by roasting, into red oxide of iron. One hundred parts contain : sulphur 39.5, iron 60.5. This substance, like the common pyrites, is not employed as an ore of iron. It occurs in considerable veirs in St. Jerome, C. E.; also in the Chaudière Valley, where it is in part aurifcrous; and, in large quantities, about Balsam Lake, Re., C. W.

Copper Pyrites:-Brass-yellow, often with a variegated tarnish; streak, dark green or greenish-black. Chiefly in amorphous masses; sometimes in small tabular and tetrahedral crystals (Dimetric.) H. $3.5-4 \cdot 0 ; \mathrm{Sp} . \mathrm{gr} .4 .1-4.3$. Fusible with sulphur fumes into a magnetic globule. One hundred parts consist of : sulphur 35, copper 34.5 , iron 30.5 . This mineral is one of the most important of the
copper ores. It is the characteristic ore of our Huronian rocks.* It. occers abundantly in these, at the Bruce and Wallace mines, Root River, Echo Lake, \&c., on Lake Huron; and in the Michipicoten Islands, Lake Superior. It occurs likewise, but in comparatively small quantities in the Laurention formation : as in the Seigniory of Lanoraie, Berthier County, C.E.; Sc.; and it has also been found in the metamorphic district of the Castern Townships; more especially in Upton, Drummond County, (where an argentiferous variety occurs,) and in Acton, Bagot County. At the latter locality it is auriferous.

Purple Copper Pyrites, (Erubescite):-Colour pale brownish-red, but always more or less masked by a rich blue or variegated tarnish; streak, greyish-black, by which (as well as by its colour, \&c.,) this species may be easily distinguished from the variegated specimens of copper pyrites or yellow copper ore. Chiefly in amorphous or small granular masses accompanying yellow copper pyrites in quartz. Sometimes, as observed by the writer (Canadian Journal, New Series: vol. 1, page 187) in pseudomorphs, or altered (Dimetric) tetrahedrons, after the yellow ore. $\mathrm{H}=4.0$; sp. gr. 4.4-5.0. Fusible with sulphur fumes into a magnetic globule. One hundred parts contain (as a mean) : sulphur 25 , copper 60, iron 15. This mineral occurs with copper pyrites at most of the localities given in the description of that substance, above. It is found also in the townships of Inverness and Leeds, Megantic County, C.E.

Sulphuret of Copper, or Copper Glance :-Dark lead-grey often with blue or green tarnish; streak, black and slightly shining. Chiefly in amorphous masses, more rarely in small flat six-sided crystals (Trimetric.) II $2.5-3.0$; sp. gr. 5.5-5.8. Fusible with bubbling, colouring the flame green, and leaving a copper globule surrounded in

[^4][^5]gencral by a dark scoria. One hundred parts contain : sulphur 20.2, copper 79.8. This valuable ore occurs in some abundance at the Bruce Mines, Lake Huron. It is also found at Prince's Mine on Spar Island, Lake Superior, as well as in the Michipicoten Islands and in the Island of St. Iguace on that Lake, associated with copper pyrites, native copper, \&c. It occurs likewise (with purple copper pyrites, \&c.,) in the eastern metamorphic district: as in the townships of Leeds and Inverness in Megantic county. In the former of these townships it lies, according to Sir William Logan, in a ferruginous dolomite, associated with specular iron ore and a small quantity of native gold.

Galena:-Lead-grey, with black and somewhat shining streak. In amorphous masses of lamellar or granular structure, and in monome-


Fig 28. tric crystals-more especially in cubes and cubo-octahedrons, fig 28. It breaks easily, owing to its well-marked cubical cleavage, into rectangular fragments. H. 2.5 ; sp. gr. 7.2-7.7. Decrepitates before the blow-pipe and yields lead globules, with the deposition of a yellow coating on the charcoal. One hundred parts contain: sulphur 13.4, lead 86.6 ; but a portion of the sulphide of lead is generally replaced by sulphide of silver. The silver in most of the Canadian samples, however, is insufficient to meet the cost of its extraction. Galena is the source of nearly all the lead of commerce. It occurs in Canada in very many places, but nowhere, apparently, in large quantities. It is chiefly found in connection with the crystalline limestones of the Laurentian formation, associated with crystallized calc-spar and sulphate of baryta, and sometimes also with zine blende and iron pyrites. It occurs thus, occasionally forming thin veins, in the townships of Lansdowne and Bastard, (Leeds County, C.W.;) Bedford (Frontenac County, C.W.;) Fitzroy (Carleton County, C.W.;) Ramsay (Lanark Courty, C.W.;) Petite Nation (Ottawa County, C.E.;) and, in smaller quantities, in many other townships lying more especially along the southern outcrop of the Laurentian country. Galema has been met with also in the Huronian rocks of the Michipicoten and Spar Islands, Lake Superior, associated with copper ores, calc-spar, amethyst-quartz, \&c., and on the neighbouring shores. Also in the metamorphic district of Eastern Canada; more especially in the quartz veins of the Chaudicre Valley (with zine blende, common and magnetic pyrites, native gold, \&c.) as in the seigniories of Vaudreuil and St. George.

Zinc Blende:-This substanse varies in its aspect from sub-metallic to vitreo-resinous. The more metallic-looking specimens are darkbrown, black, brownish-yellow or brownish-red, with yellowish or reddish-brown streak, and high lustre. Found chiefly in lammellar and small irregular masses, and in more or less obscure crystals of the Monometric system. H $3.5-4.0$; sp. gr. 3.9-4.2. Infusible. One hundred parts contain : sulphur 33, zine 67. Zinc Blende, although so abundant in many countries, can scarcely be called an ore of zinc : the attempts to employ it for the extraction of the metal, having hitherto proved of very partial success. It may be used however, when ground to powder, as the basis of a wash or paint for frame buildings and wood-work generally. In Canada, Zine Blende occurs in some abundance at Prince's Mine on Spar Island, and at Maimanse, Lake Superior, with copper ores, galena, \&c. Also in small quantities with galena, in the townships of Lansdowne, Bedford, \&c., (see under galena, above) ; and in the eastern metamorphic district of the Chaudière Valley, The Blende of this latier locality (seigniories of Vaudreuil and St. George, Benuce Co.,) Las been shewn by Mr. Sterry Hunt of the Geological Survey, to be slightly auriferous.
(To be continued.)

# REMARKS ON THE PAPER HEADED "TEE ODAHWAH indtan language," pUblished in the cana. DIAN JOURNAL FOR NOVEMBER, 1858. 

BY F. ASSIKINACK.

Reai before the Canadian Institute, 14th January, 1860.
The paper which appeared in the Canadian Journal for November, 1858, headed "The Odahwah Indian Language," was intended to give some particulars relating to the language of the Odabwahs. Although the Odahwahs and Ojibwas may be considered to speals one common language, they, nevertheless, differ in several respects; and in many cases these distinctions are scarcely perceptible in common conversation, aud any one who is not well acquainted with
both languages may easily confound the dialect of one tribe with that of the othex. On looking over the paper alluded to above, after its publication, I discovered a few irregularities which had found their way into it previous to its going to press; and as I do not hold myself responsible for these mistakes, I will point out as briefly as possible the passages which appear to me to be inaccurate, with some additional remarks on each of the several points in question.

1. At page 482 we have "Naubegwun;" this word should be written Naubequan or Naubekwaun; it is supposed to be derived from Naubekwa, a verb intransitive of the third person singular, and which implies the act of a person running a string or thread through the eye of something, such as a needle, beads and the like. The name may have been given by the Indians to a ship, on account of its having many ropes about it. The verb from Naubekwaun is Naubekwauneka, he builds a ship.
2. On the same page, Tibahahkewenine, a land survejor, is said to be from Tibahiga, he measures. Now the substantive from this verb is Tibahigavenine, a measurer; or a man who measures, and it may mean a measure of cloth, of lumber, of grain, or anything else, and appears to be merely the verbal form of Tibahigun, a measure, which I consider the root from which all others having the idea of a measure are derived. The verb for he measures, having reference to land exclusively, is Tibahakkee, from Tibahigun, and ahke, land; and from the compound verb and ahnine, a man, is derived, in my opinion, Tibahahkewenine; simply, a measure of 'and. I may further observe Tibahkonigun siguifies a yard-stick, turned into a verb, Tibakonign, literally, he measures cloth by the yard-stick. None of the verbs above can take a noun after it, and it is somewhat doubtful whether the first syllable should begin with $T$ or $D$; but this doubt is removed when the verbs are employed as transitives which considerably change at the same time, preserving, however, under all circumstances the letter $B$, the most important part of the original word. Thus, Odibowahu and mitigoon, he measures a tree or trees, animate, Odibahahu ahke, he measures land, inanimate.
3. At page 483, "Ninakwind" and " Rinahwind" are given for we-these are Ojibwa pronouns, wot Odahwah. The latter tribe never put $D$ at the end of these words. The Odahwahs say for we, Ninahwin and Kinaluwin, with a slight nasal sound of the iast
syllable. The difference between these two forms of the first person plural is this, that when I take Ninawin I exclude the person I am spealing to, but when I make use of Kinatwin I include him, that is, he belongs to the same company or party as I do. In ordinary conversation these pronouns are not used in full; for example, Nindizhahmi and Kidizhabme, we go ; but when the speaker wishes to make his expression clear, strong, \&c., then he repeats the forms in full using particles as the same, thus, Ninahwin, so, Nindizhahmi, we ourselves are going. Kinalıwin salu, go, Kiguhizhalhni, we ourselves will or must go. When one or more of the party are selected or spoken of, there is no such change in the Odahwah as is observed in the English or in other languages, for you say, one of us, we simply have puizhig ninalwoin or ninahwin puizhig, puizhig kinalwwin tuhizhah, one of us will go. Neezh yamale nisswe kinalıvui, two or three of us; for or on account of oue of us, puizhig linahwin ondje.
4. On vage 485 , we are informed that "Mississippi is the Indian name of a large river in America." "It is composed of Missi which in composition words corresponds to Michah, and signifies very great." The first part o!' this sentence has the appearance of being too positive, and it would be better to qualify it by saying Mississippi is said to be the Indian name, \&c., for even now I could not say positively that it is an Indian name. I can only suppose that it was meant for Mashziebe by which name it is known to the Odahwabs. As regards Missi I have been unable to discover any such particle in the Odahwah, either as it stands here or in composition. I presume, therefore, it belongs to some other tribe. Michah is an Ojibwa adjective. The Odahwahs have it Mishak with S , not C , in the middle. Although they frequently make use of the former, but always in composition as an adverbial adjective, as Michahbaweze, he is a stout man; Michahkoze, it is a large tree. Mishah when used alone never varies, but when it forms part of a word it assumes different shapes as in Mashizebe Mishikaikaike, a large hawk; INisahkig, a seal, literally, a large otter, from negiz an otter. Mishah is an inanimate form, and when it refers to an animate object it is written thus, Mindido.
5. Further down on the sam^ page we have "Nibissininewug," inhabitants at Nibissing. There $\geq$ two ways of writing this word: first, when the name is written fully it stands thus, singular, Nibissiwenine; plural, Nibissiweninewug, but when the fourth syllable, we,
is omitted, the last syllable begins with Y not W , namely Nibissinine, plural, Nibissinineyug; therefore, Nibissininewuty, as given in the Canadian Journal for November, 1858, is not correct. This rule applies to many other words when contracted, for example, gataahninewuy, the ancients take away all, and we have gatanineyug.

I have now endeavoured to notice the irregularities as they occur in the article under review. I do not, of course, pretend to say that my statements are correct in every respect, but it has been my desire to convey the most correct information on the subject that it is in my power to give.

With regard to verbs, I have thought it proper to add one remark at the end of this paper in reference to the third person, as this part of the Indian verb appears to have peculiarities quite different from the first and second person, singular or plural, and which have been found rather difficult to explain by some persons. In the first place I may state that there are two words which represent he, she, his, hers, and theirs; namely, win and o. Win is always in the singular number, but $o$ is sometimes plural. Whilst the first and second invariably express the personal pronoun, it is always omitted in the third, so far as verbs which do not take a noun after them are concerned, which I suppose are to be regarded in some measure as intransitives, but when a noun can be put after them, O is added to the third person, and win, in some instances: for example, bimosa, he walks, inaindum, he thinks, nibah, he sleeps, ahkoze, he is sick; this is also the case with verbs derived from substantives, as it will be seen from the following-although they govern nouns when translated into English. Naubekwauneka, he builds a ship, from Naubekwaun, a ship; Wigivameka, he builds a house oi dwelling, from wigiwam ; Mahkolcu, he hunts after the bear, from Mahkwuh, a bear. We will take others, verbs with their substances: Naubelkwaun odozhitoon, he makes a ship; onahyahdaun owiyiwam, he leaves his house; mahkurm onisauro, he kills a bear; geizhah, he went; win go geizhah wahdi, he went then himself. Sometimes both win and o are made use of, as win go obedoon, he brings it himself.

The third person also differs in this from the first and second, that where it admits a noun after it, of the masculine or feminine gender, the noun has only one form for singular and plural; thus, mitigoon ogeeshkahwaln, he cut down a tree or trees. Here mitigoon stands for the singular as well as the plural number, as it will appear more
clearly from the following example: puizhig mitigoon ogeeshkahwahn, he cuts down one tree; ningodwahk mitigoon ogegeeshkahwainn, he cut down onc humdred trees. With nouns, however, of the neuter gender, similarly governed, there is a distinction between the singular and plural, for instance, ahnit, a spear, uhnit odahyaun, he has a spear; alniteen odahyaunun, he has spears. Strictly speaking, verbs derived from nouns denote the profession or business of a person; thus, naubekwauneka, he builds a ship,-means also his profession or business is to build a ship.
The words win and $o$ which frequently represent the personal and possessive pronouns of the third person, are likewise omitted in the event of an objective case of the first and second persons coming after a verb transitive. These appear to usurp the place of the third person, for example, Ni wahbahrnig, he sees me, Ki wahbahrnig, he sees you. Ni minig, he gives it to me. Here we see $N i$ and $K i$ are plural where win and $o$ should be; but when the verb has reference to a case of the third person, then the nominative of the third person keeps its proper place at the beginning of the verb, thus, Owahbahmahn, he sees him or them, Ominahn, he gives it to him. The pronouns of the third person are also omitted in the passive voice, namely, Wahbahmah, he is seen, Nisah, he is killed. Deponent verbs, Nisidizo, he kills himself, Kitchiahpitanimo, he thinks a good deal of himself. Ni wahbahrnig is distinguishable from the first and second persons by its final syllable $i g$, the first is Ni walbahmah, I see him. I may here observe that the first and second have invariably the same termination; but the third person does not always agree with these, as it will appear from the following:-Nindahkooz, I am sick, Kidahlkooz, you are sick, alkooze, he is sick; Ningoosah, I fear him, Kigoosah, you fear him, ogoosaun, he fears him or them, cgootaun, he fears $i t$.

The preceding observations, although constituting a mere outline of the subject, will serve to convey a general idea of the more important peculiarities regarding the third person of an Indian verb. If the subject be deemed of sufficient interest, it may perhaps be referred to more fully, in a future communication.

## REVIEWS.

North West Territory. Reports of Proyress; toyether with a preliminary and general report on the Assiniboine and Saskatchewan exploring expedition, made under instructions from the Provincial Secretary, Canada. By Henry Youle Hind, M. A. John Lovell, Toronto, 1859.
From the great interest felt by the people of Canada in the North West 'Territory, and its capacity as a scene of future occupation by industrious seetlers, the appearance of Professor Hind's Report on the Assiniboine and Saskatchewaw exploring expedition, will be welcomed with a lively curiosity as to its various contents. Nor will such curiosity fail to find much to gratify it in the volume just issued from the press. 'Topographical and Geological Maps and Sections aceotapany the letter-press, exceuted on so large a scale as readily to present to the eye the minutest features that the opportunities of the exploring party enabled them to note; and well exceuted wood-cuts illustrate the additions contributed to North American Palæontology, by the good use which Professor Hind has made of his very favourable opportunities.

Chapter XVI. introduces the "Geological Report," with a sketch of the surface geology of part of the valley of Lake Winnipeg. Notices of numerous traces of glacial action, follow, with descriptions of many indications of change in the contour of the land. Many records of former water-courses and the aspect of ancient river-valleys have also been noted. Next to the valley of the Qu'Appelle, Professor Hind remarks "the old course of the Little Souris through the depression now occupied by the Beck-fat Lakes is the most curious and imposing. Standing upon one of the most prominent of the Blue Hills of the Souris, near their southern extremity, the ancient valley can be traced as far as the first lake, which is distinctly seen by the unassisted eye, and with a good marine telescope its outline is plainly visible." In this as in other localities unheeded changes in the course of rivers have, in comparatively recent times, wrought important alterations on the contour of the region; while at other points, vast, unstable sand-hills and dunes are in constant motion and render extensive ranges of country mere barren wastes.

In the following chapter, as well as in others, exposed sections are figured and described, with their included minerals and fossils. A boldly sketched view on p. 172, exhibits a specimen of exposed cliff at

Grindstone Point, Lake Wimnipeg, composed of Chazy Limestone, illustrating the exceedingly picturesque character that generally prevails throughout that Lake Coast. Extracts from the reports and narratives of Messrs. Foster and Whitney, Sir John Richardson, Dr. Owen and other observers, accompany Professor Hind's own notes, and supply many new hints whereby to judge of the mineral resources and the prevailing geological character of this great unoccupied territory. To complete the report in its scientific bearings on the geological aspects of the various districts explored, the services of the American Palreontologist, Mr. F. B. Meek, and of Mr. E. Billings of the Canadian Geological Survey, have been called into requisition; the former describing Cretaceous fossils, and the latter those of the Silurian and Devonian formations. It was impossible, in so rapid an exploratory journey, with many other objects demanding the special attention of the observers, that anything like a complete palæontological series of illustrative specimens could have been secured; but enough has been done in this department to supply interesting materials for comparison with the Nebraska territory, and the rocks of the great basin of the Upper Missouri. Mr. Meek, whose labours in the latter district, are well known to American Geologists, remarks, after a general reference to the collections made by Professor Hind, in proof of the value of this portion of the exploratory party's work: "It is very desirable that a good series of specimens should be obtained from this remote northern locality, not only for the purfase of determining the age of the formation, but for the light they might throw upon far more interesting questions respecting the probable ch ratic zonditions in these high northern latitudes during the Secondary Period." The fossil plants, Acephala, Gasteropoda, and Cephalopoda of the Cretaccous formations are minutely described, and the new species named. From the latter, we select one or two, which we are able to iilustrate by means of the well exccuted wood engravings prepared for the report. Figs. $I$, and 2, are Lamellibranchiata procured from their matrix of soft lead gray argillaccous rock, or the Little Souris River, and named after one of members of the exploring party, Anomia Flemingi.

Plate II. is chiefly occupied by large and well executed figures of the Ammonites Barnstoni, the Ammonites Billingsi; and a beautiful varicty of the Nautilus, termed the Dekayi; but probably greater interest , I be felt by many in the following less showy Productus, supposed to be from arbonifeous limestone, and therefore viewed

with special reference to its importance as possibly indicating the existence of conl in the Red River district "There is some evidence," says Mr. Billings, "of the existence of at least a portion of the carboniferous system in this region. The fossil fiyured here, procured

from a Half-breed, who said he collected it fror. the solid rock, at some phace on the Red River, is a Productus of the group Semireiciculati, all of which appear to be confined to the carboniferous series. The specimen is nct worn, and presents all the appearance of having been freshly broken from the rock." These will suffice to indicate the character of the palæontological novelties contributed to science by Professor Hind, as the result of his recent journey. Thry furnish an interesting foretaste of the treasures in store for future scientific explorers of the territory, and of the value of the present report as the first clue to the economic characteristics of the valley of the Red River.

The volume has only reached us after a considerable portion of the present number was already in type; and we are limiter? therefore both by available time and space, in our notice of its various contents. We leave therefore to the daily press the discussion of such practical sections as those which treat of the extent and characteristics of the lake and river systems of the regions explored; of their Wooded and Prairie Land; and of the areas fit for settlement. These will doubtless receive abundant attention, and be discussed in all their bearings, by Vol. V.
those who, alike on economic and political grounds, anticipate the speedy occupation of the explored areas by Canadian and British settlers.

Turning from this practical-aspect of the expioring party's report, to the interest which attaches to the narrative of an intelligent traveller's observations in a country still mainly occupied by the wild Indian in his natural state, the volume presents many curious passages which we would gladly transfer to our pages. Bat one or two must suffice to direct our readers to the original work. Professor Hind and his party were thrown a good deal among the various tribes of IndiansCrees, Sioux, Blackfeet, and Ojibways,-who find hunting grounds, or opportunities of trading at the Hudson's Bay Company's Forts, in the territory explored. The picture he draws of the Red Man, exhibits in no very flattering aspect, the improvident, superstitious, and treacherous savage, whom civilization visits seemingly only to exterminate. Here is an account of one of the processes of reckless improvidence persisted in by the Indians on the Prairies, whereby a sterile and forbidding aspect is stamped on vast tracts of country along the courses of the Qu'Appelle and Assiniboine Bivers. A few miles west of the Souris Forks, the Qu'Appelle is only mineteen feet wide, while the great valley through which the river winds its way is still a mile broad and two hundred feet deep. Here, says Professor IIind, "we caught a glimpse of the blue outline of the Grand Coteau, with a tree1ess plain between. This afternoon we saw three fircs spring up between us and the Grond Coteau. They were Indian signs, but whether they referred to the presence of buffalo, or whether they were designed to intimate to distant bands the arrival of suspicious strangers, we could not then tell, and not knowing whether they were Crees, Assiniboines, or Blackfeet, we became more cautious. In a few days we ascertaned that the fire had been put out* by Crees, to inform their friends that they had found buffalo.
"The grandeur of a prairic on fire belongs to itself. It is like a volcano in full activity, you camnot imitate it, because it is impossible to obtain those gigantic elements from which it derives its awful splendour. Fortunately, in the present instance the wind was from the west, and drove the fires in the opposite direction, and being south of us we could contemplate the magnificent spectacle without anxicty. One object in burning the prairie at this time, was to turn

[^6]the buffalo; they had crossed the Saskatchewan in gieat numbers near the Elbow and were advancing towards us, and crossing the Qu'Appelle not far from the height of land; by burning the prairie east of their cuurse, they would be diverted to the south, and feed for a time on the Grand Coteau before they pursued their way to the Little Souris, in the country of the Sioux, south of the 49 th parallel.
"Putting out fire in the prairies is a telegraphic mode of communication frequently resorted to by Indians. Its consequences are seen in the destruction of the forests which once covered an immense area south of the Qu'Appelle and Assiniboine. The aridity of those vast prairies is yartly due to this cause. The soil, though light, derives much of its apparent sterility from the ammal fires. In low places and in shallow depressions where marshes are formed in spring, the soil is rich, much mixed with vegetable matter, and supports a very luxuriant growth of grass. If willows and aspens were permitted to grow over the prairies, they would soon be converted into humid tracts in which vegetable matter would accumulate, and a soil adapted to forest trees be formed. If a portion of prairie escapes fire for two or three years the result is seen in the g:owth of willows and aspens, first in patches, then in large areas, which in a short time become united and cover the country: thus retarding evaporation and permitting the accumulation of regetable mateer in the soil. A fire comes, destroys the young forest growth, and establishes a prairic once more. The reclamation of immense areas is not beyond human power. The extension of the prairies is evidently due to fires, and the fires are caused by Indians, chiefly for the purpose of telegraphic communication, or to divert the buffalo from the course they may be taking. These operations will cease as the Indians and buffalo diminish, events which are taking place with great rapidity."

Thus we perceive that the poor Indian learns as little as the Buffalo, $t_{0}$ profit by experience, or to adopt from the white settlers any of those simplest arts of civilization, whereby the whites, though as yet a mere handful among the Indian Tribes of that vast territory, are destined to be their supplanters; and thr apid diminution of the latter is already noted as a process in full "ctivity. The Indians, however, as will be scen, though hopelessly indifferent to all prudential regard of their own reckless proceedings, are jealously alive to the encroachments of the Whites, on their territories and hunting grounds.

After an interesting description of Sand Hill Lake, and the remark-
able drifting dunes which have invaded the Great Volley; Professor Hind narrates his visit to Shortstick, the Chief c: the Crees of that region, near the south branch of the Saskatchewan. The Crees were camping out, in the all-important occupation of their great annual Buffalo hunt, and their Chief sent on a party of mounted Crees, accompanied by his son, who informed the strangers, that they were engaged in the construction of a new Buffalo-pound; and as soon as it was ready, the visitors were invited to repair thither, and witness the capture of the Buffalo. The scene which he then witnessed was one full of interest to the traveller, and singularly illustrative of the improvident recklessness of the Indian hunters. We shall abridge somewhat, the graphic narrative, omitting some of the details, which, though vivid and truthful illustrations of this particular phase of Indian life, can be sought for in the poges of the original report, by such as desire to master all the minutio of the dreadful scene of butchery.
"We passed through the camp," says Professor Hind, " to a place which the Chief's son pointed out, and there erected our tents. The women were still employed in moving the camp, being assisted in the operation by large numbers of dogs, each dog having two poles harnessed to him, on which his load of meat, or pemican, or camp furniture was laid. After another smoke, the Chief's son asked me, through the interpreter, if I would like to see the old buffalo pound, in which they had been entrapping buffalo during the past week. With a ready compliance I accompanied the guide to a little valley between sand hills, through a lane of branches of trees, which are called 'dead men' to the gate or trap of the pound. A sight most horrible and disgusting broke upon us as we ascended a sand dune overhanging the litlle dell in which the pound was built. Within a circular fence 120 feet broad, constructed of the trunks of trees, laced with vithes together, and braced by outside supports, lay tossed in every conceivable position over two hundred dead buffalo. From old bulls to calves of three months old, animals of every age were huddled together in all the forced attitudes of violent death. * * *
"The Indians looked upon the dreadful and sickening scene with evident delight, and told how such and such a bull or cow had exhibited feats of wonderful strength in the death struggle. The flesh of many of the cows had been taken from them, and was drying in the sum on stages near the tents. At my request the Chicf's son jumped nto the pound, and with a small axe knecked off half a dozen pair
of horns, which $I$ wished to preserve in memory of this terrible slaughter. "To-morrow," said my companion, "you shall see us bring in the buffalo to the new pound."
"After the first run, ten days before our arrival, the Indians had driven about 200 buffalo into the enclosure, aud were still urging on the remainder of the herd, when one wary old bull, espying a narrow crevice which had not been closed by the robes of those on the outside, whose duty it was to conceal every orifice, made a dash and broke the fence, the whole body then ran helter skelter through the gap, and dispersing among the sand dunes, escaped, with the exception of eight who were speared or shot with arrows as they passed in their mad career. In all, 240 animals had been killed in the pound, and it was its offensive condition which led the reckless and wasteful savages to construct a new one. This was formed in a pretty dell, between sand hills, about half-a-mile from the first, and leading from it in two diverging rows, the bushes they designate dead men, and which serve to guide the buffalo when at full speed, were arranged. The dead men extended a distance of four miles into the prairie, west of and beyoud the Sand Hills. They were placed about 50 feet apart, and between the extremity of the rows might be a distance of from one and a half to two miles.
"When the skilled hunters are about to bring in a herd of buffalo from the prairie, they direct the course of the gallop of the alarmed animals by confederates stationed in hollows or small depressions, who, when the buffalo appear inclined to take a direction leading from the space marked out by the dead men, show themselves for a moment and wave their robes, immediately however hiding again. This serves to turn the buffalo slightly in another direction; and when the amimals having arrived between the rows of dead men, endeavou. to pass through them, Indians here and there stationed behind a dead man, go through the same operation, and thus keep the animals within the narrowing limits of the converging lines. At the entrance to the pound, there is a strong trunk of a tree placed about one foot from the ground, and on the inner side a shallow excavation is made, sufficiently deep, however, to prevent the buffalo from leaping back when once in the pound. As soon as the animals have taken the fatal spring they begin to gallop round and round the ring fence lonking for a chance of escape, but with the utmost silence the women and children on the outside hold their robes before every orifice until the
whole herd is brought in, they then climb to the top of the fence, and with the hunters who have followed closely in the rear of the buffalo, spear or shoot with bows and arrows or firearms at the bewildered animals rapidly becoming mad with rage and terror, within the narrow limits of the pound. It is then that a dreadful scene of confusion and slaughter begins, the oldest and strongest animals crush and toss the weaker; the shouts and screams of the excited Indians rise above the roaring of the bulls, the bellowing of the cows and tho piteous moaning of the calves. The dying struggles of so many strong, full grown animals crowded together, furnish a revolting and terrible picture, but with occasional displays of wonderful brute strength and rage; while man, in his savage, untutored and heathen state, shows both in deed and expression how little he is superior to the noble beasts he so wanionly and crueily destroys."

After witnessing this highly characteristic illustration of Indian habits, Professor Hind held a formal interveiv with the Cree Chief, the description of which furnishes a definite expression of the jealousy with: which the encroachments of the white men are regarded by the wild hunter tribes. After describing the dress of the Cree Indians, which was of the scantiest description; and the painting, as well as the scars and gashes-record of mourning for departed friends, - with which their bodies were marked, the author thus proceeds: "I encuired the age of an extremely old fellow who asked me for medicineto cure a pain in his chest; he replied he was a strong man when the two ('ompanies (the Hudson's Bay and the North West) were trading with his tribe very many summers ago. He remembers the time when his people were as numerous as the Buffalo are now, and the buffalo thick as trees in the forest. The half-breeds thought he was more than 100 years old. Shortstick accepted the presents of tea, tobacco, bullets, powder and blankets I made him, with marked satisfaction, and expressed a wish to learn the object of our visit. We held a 'talk' in my tent, during which the chief expressed himself freely on various subjects, and listened with the utmost attention to the speeches of the Indians he had summoned to attend the Council.
' All speakers objected strongly to the half-breeds' hunting buffalo during the winter in the Plain Cree country. They had no objection to trade with them or with white people, but they insisted that all strangers should purchase dried meat or pemican, and not hunt for themselves.
"They urged strong objections against the Hudson's Bay Compaay encroaching upon the prairies, and driving away the buffalo. They would be glad to see them establish as many posts as they chose on the edge of the prairic country, but they did not like to see the plains invaded. During the existence of the two companies, all went well with the Indians; they obtained excellent pay, and could sell all their meat and pemican. Since the union of the companies they had net fared half so well, had received bad pay for their provisions, and were growing poorer, and weaker, and more miserable year by year. The buffalo were fast disappearing before the encroachments of the white men, and although they acknowledge the value of fircarms they thought they were better off in old times, when they had only bows and spears, and wild animals were numerous. I asked Shortstick to name the articles he would like to have if I came into his country again. He asked for tea, a horse of English breed, a cart, a gun, a supply of powder and ball, knives, tobacco, a medal with a chain, a flag, a suit of fine clothes, and rum. The talk lasted between six and seven hours, the greater portion of time being taken up in interpreting sentence by sentence, the speeches of each man in turn. They generally commenced with the creation, giving a short history of that event in most general terms, and after a few flourishes about equality of origin, descended suddenly to buffalo, half-breeds, the Hudson Bay Company, tobacco and rum."

These extracts sufficiently illustrate the varied characteristics of the Report, which extends to upwards of 200 large denble-columned quarto pages, and embraces an Itinerary, with topographical information rendered in the concisest form ; Reports of Progress, by different members of the exploring party ; Meteorological and Geological details; and a narrative embodying descriptions of scenery, native habits, and such incidents of travel as are at once attractive to the general reader, and of value to those who are desirous of ascertaining the fitness of the region for a scene of emigration, and a future Province of British North America.

Report of the Geological Survey of the State of Iowa. By iames Hall, State Geologist, and J. D. Whitney. Iowa: 1858.
The first part of this Report-dited 1858, and embracing the general geology and the nalrontology of the State, by Protessor Hall, with its physical reography, chemistry and economic geology
by J. D. Whitney, and sundry geological details by A. H. Worthen -has been issued within the last few months by the Legislature of Iowa. The palmontological portion of the Report is bound up separately. It contains some twenty-nine or thirty stecl-plate engravings of $v \epsilon_{i} y$ superior execution, exhibiting about a thousand figures of the more characteristic or remarkable fossils collected during the prosecution of the Survey. Mere sketches of scenery, on the other hand, however pleasing in themselves, have been very properly dispensed with in this Report. Illustrations of that kind add enormously to the costs of publication, without offering, as a general rule, any compensating advantages.

The oldest recognised rock in the State of Iowa appears to be the Potsdam Sandstone, This, blended intimately with the Calciferous Sand Rock, is sparingly developed along the line of the Mississippi, in the extreme North-East corner of the State. The other subdivisions of the Silurian Series, and those of succeeding formations ap to the coal measures, follow in more or less regular gradation, with their lines of strike running in a general N.W. and S E. direction, or, as stated by Professor Hall, at right angles to the Cincinnati axis and the lines of disturbance along the Appalachian Chain. Owing to this direction of the strata, they are cut successively by the Mississippi River, and show from north-east to south-west the following sequence:-The Potsdam Sandstone and Calciferous Sand Rock; the St. Peters Sandstone; the Trenton beds; the Galena limestone (looked upon as an upper portion of the Trenton Group ;) the Hudson River Shales, showing only a narrow outcrop-band; the Leclaira Limestone (see below); the Onondaga Salt-Group, the equivalents of the Upper Helderberg Limestone, Hamilton, and Chemung groups; the Carboniferous Limestone; and the Coal Measures. Although these follow onc another regularly, here and there an underlying division is exposec. by denudation or rivercutting in some of the tributary vallies of the Mississippi. Thus, amongst other examples, the Trenton Limestone re-appears within the Galena Limestone area along the line of Turkey River; and the Carboniferous Limestone, within the sea of the Coal Measures, along the valley of the Liver Des Moines. Professor Hall remarks, that, in tracing westward such of these geological formations as are known in New York and Pennsylvatia, they are found to thin out gradually, becoming indeed, in some in-
stances, so attenuated as to be scarcely recognizable, morc especially in a district deeply covered, like that of the greater part of Iowa, with Drift and modern deposits. It is to this attenuation of the strata, as well shewn by Professor Hall, that the comparatively subdued aspect of this western country is owing. The united strata are not sufficiently thick to admit of the production of any stronglymarked features, by either denudation or ordinary disturbing forces. Where anticlinals exist, they occupy low levels; and the only real elevations of the district have been produced by denuding agencies on undisturbed or nearly horizontal strata, where these, under special conditions (as in the case of the Niagara Limestone) have presented a more than ordinary thickness to the denuding force. On comparing these results with the phenomena exhibited in the district of the Catskill Mountains and the Appalachians, where the diminished strata of the West occur in accmulations of vast thickness, our author appears inclined to refer the general production of mountain chains, more to the action of denudation, than to that of eler ating or disturbing forces. But, in this, his views are surely pushed too far. That-denudation has produced mountain masses amongst undisturbed strata, as in the Catskill district, in the old red sandstone country of the Western Highlands of Scotland, and elsewhere, all the world must admit; and equally that anticlinals often occupy comparatively low leveis, as the beds of rivers, \&c.: but when we extend our survey to the great mountain systems of the Earth-the Andes and their prolongations, which brim the eastern contour of the Pacific, the towering Himalayas, the Alps, an i other chains, it becomes manifest that elevation has been there produced by disturbing agencies of no ordinary intensity. The bare occurrence of bighly inclined and vertical (and sometimes even of reversed) strata, as seen in all these mountainous districts, the presence of stupendous volcanoes in many of them, with other well-known phenomena, point incontestibly to this fact. Denudation may have been concerned, and largely, in the excavation of valleys amongst these, in the production of lines of escarpment, and so forth; but denudation has there played the part of a mere secondary agent. The great views of Elie de Beaumont, however exaggerated and extended beyond their legitimate limits by some geologists, are still in their main features undoubtedly worthy of our reception; and these views are based essentially on the formation of mountain chains by elevating forces.

Amongst the more interesting facts discovered by Professor Halls' exploration of Iowa, may be cited, first of all, the occurrence of the Hudson River Group in that State and in Illinois; represented by more or less bituminous shaly layers, having au entire thickness of no more than sixty or seventy feet, and thus affording a remarkable exemplification of the law of decrase of sedimentary matter in the westward extension of the palæozoic beds. These Hudson River shales are the equivalents of the "Blue Limestone" of Cincinnati (formerly thought to be Trenton), and the "Blue Shale" of Wisconsin. They appear to be principally exposed around Dubuque, or in a narrow band from the vicinity of Bellevue to somewhere about the head waters of the western branches of Turkey River; growing gradually thinner and thinner, until they finally die out. Another fact of no little geological interest, brought out by this survey, is the discovery of a magnesian limestone formation lying above the Niagara Limestone, and not previously recognised in the Mississippi valley. Professor Hall has named this the Leclaire Limestone, from its development around that locality. It occupies, apparently, a considerable area, having within its limits the main portion of the Wapsipinecon river; and it forms more especially by its undulations the so-called " Opper Rapids" of the Mississippi. Professor Hall shows that these Leclaire beds occupy the same geological horizo,n as the Galt limestone of Canada West, and he is inclined to look upon the two as geologically identical. If this, on furiner exomination, provéo to be the case, it will furnish an additional argument for the separation of the Galt beds from the Onondaga salt group with which at present they are conventionally placed.

In his very interesting sketch of the physical geography of Iowa, Mr. Whitney enters on a somewhat extended description of the causes to which the absence of arboreal vegetation on the prairie lands may be considered due. He attributes the principal cause of this, and apparently on conclusive grounds, to the peculiar and highly comminuted condition of the surface soil. "Taking into consideration all the circumstances under which the peculiar vegetation of the prairie occurs, we are disposed to consider (be remarks,) the nature of the soil as the prime cause of the absence of forests and the predominance of the grasses over this widely extended region. And although chemical composition may not be without influence in bringing about this result, which is a question worthy of carefus
examination, yet we conceive that the extreme fineness of the particles of which the prairie soil is composed, is probably the principal reason why it is better adapted to the growth of its peculiar vegetation, than to the development of forests. It cannot fail to strike the careful observer that where the prairie occupies the surface, the soil and supericial material have been so comminuted as to be almost in the state of an impalpable powder. This is due partly to the peculiar nature of the underlying rocks and the facility with which they undergo decomposition, and partly to the mechanical causes which have acted during and since the accumulation of the sedimentary matter forming the prairie soil. If we go to a thickly wooded region like that of the nothern peninsula of Michigan, and examine those portions of the surface that have not been invaded by the forest, it will be observed that the beds of ancient lakes which have been filled up by the slowest possible accumulation of detrital matter, and are now perfectly dry, remain as natural prairies, and are not trespassed on by the surrounding woods. We can imagine no other reason for this than the extreme fineness of the soil which occupies these basins, and which is the result of the slow and quiet mode in which they have been filled up...... Applying these facts to the case of the prairies of larger dimensions farther south, wo infer, on what seems to be reasonable grounds, that the whole region now occupied by the prairias of the northwest was once âu inumerse lake, in whose basin sediment of almost impalpable fineness gradually accumulated; and that this basin was drained by the elevation of the whole district, but at first so slowly, that the finer particles of the superficial deposits were not washed away, but allowed to remain where they were originally deposited. After the more elevated portions of the formor basin had been laid bare, the drainage becoming concentrated in narrow channels, the current thus produced, aided perhaps by a more rapid rise of the region, acquired sufficient velocity to wear down through the finer material on the surface, wash away a portiou of it altogether, and mix the rest so effectually with the underlying drift materials, or with abraded fragments of the rocks in place, as to give rise to a different character of soil in the valleys from that of the elevated land. This valley soil being much less homogeneous in its composition, and containing a larger proportion of coarse materials than that of the uplands, seems to have been adapted to forest vegetation; and, in consequence of this, we find
such localities covered with an abundant growth of timber." Where the so-called "groves" occur upon higher levels of the prairies, there is always a partial acccumulation of drift and other coarse materials, in place of true prairie-sediment; and the same is observable, according to Mr. Whitney, wherever timber is found upon the bottom-prairies of the Mississippi and Missouri valleys south of Iowa.
In the concluding and principn] portion of Mr. Whitney's Report, numerous analyses are given of the dolomites, limestones, coals, and other economic substances, of the State; and the great lead region is described with much detail. The lead ore lies essentially in the Galena limestone, in "gash" or shrinkage veins, and in caverns, openings, or pockets, all of which are cut off at comparatively small depths beneath the surface. The whole of this portion of the Report is drawn up in an exceedingly clear and able manner, and will well repay the perusal of those interested in mining operations.

We have already alluded to the beautifully executed plates, in illustration of the Palæontology of the survey, engraved under the superintendence of Professor Hall. The letter-press to these is exceedingly copions, and contains many additional diagrams illustrative of crinoid structures. Another advantageous feature, as compared with the descriptions in the published volnmes of the Palæontology of New York, is the definition of the various genera, given in connection with the characters of their respective species. All the described forms, however, belong to the Devonian and Carboniferous formations: the Silurian fossils, met with during the survey, having been previously figured in Dr. Owen's Report on the geology of Wisconsin, and in other publications. A ferv plates of the more characteristic of these fossils would contribute nevertheless to the utility of the present work, without any very material addition to its cost ; and we trust the legislature of Iowa will afford the means of effecting this, in the volume which is yet to appear. To the enlightened legislature of this far western State-a State added but yesterday, as it were, to the Uuion-too much credit cannot be accorded for these goodly and important volumes, so usefu! to agriculture and practical art, and so liberally presented to the scientific world.
E. J.O. .

Ou!lines of Natural Theology for the use of the Canadian Stiddent. By James Bovell, M.D., Professor of Natural Theology in Trinity College, Toronto, C. W. Toronto: Printed by Rowsell and Ellis, 1859.
The accomplished author of this work is well known to the readers of the Canadian Journal, as occupying a prominent position in the scientific ranks of Canada; whilst, in the special department of physiology, his reputation has extended beyond the Province. The work now before us, unlike the general character of Dr. Bovell's writings, is strictly a compilation from various sources, put together in accordance with the author's special views: but this is fairly stated by Dr. Bovell, and is indeed in keeping with the proposed object and plan of the book: a book not intended for the critical investigation of the scientific inquirer, to whom the facts brought forward in it must necessarily be familiar, but one offered to the student of Natural Theology, as a convenient and accessible text-book, in the prosecution of his studies. This being the general intention of the work, it has been thought advisable ofo elucidate the subjects discussed in its pages, by a considerable number of wood-engravings and some lithographed geological sections. Of the engravings, chiefly restorations $0^{\prime \prime}$ extinct reptilian and other trpes, some few, perhaps, might have been judiciously omitted; and, as the work is intended mainly for Canadian students, it would have been as well-so far as regards the older rock formations-to have substituted Canadian subdivisions for the local terms and groupings more or less peculiar to the British Isles. Subordinate matters of this kind, however, can easily be rectified in a future edition; and in alluding to them here, we do so, truly, in no hypercritical spirit.

Analytically considered, the subject matter of Dr. Bovell's work, as there discussed, involves two distinct principles: the proofs of a great First Cause or Creator, and the exposition of Divine goodness and wisdom as shewn in natural objects and phenomena; and secondly, the reconciliation of geological discoverics with the statements of the Mosaic Record. Under the first division of his subject, the author refutes, with great force and skill, many of the pantheistic and other prevalent doctrines of a cognate character, that have been put forth more or less openly of late years, not only in continental Europe, but by names of distinction also in British science. The passages in which these doctrines are thus discussed, will well repay the reader's perusal. We would willingly have quoted
from them; but the necessary extracts-due regard being had to the coninuity of the argument-would he too copious for our pages; and hence, in justice to the author, we must refer the reader to the work itself. It is in this part of his treatise more especially, that the varied knowledge, eloguence, and acumen of our author are brought fully into play. We do not think he is so happy in the more purely geological portion of his book. It is to some extent a matter of opinion, but we fear he will find few geologists at the present day willing to subsuribe to his interpretation of the Mosaic Record as given in the pages before us. Following Buckland, more particularly, Dr. Bovell interprets the word $\mathrm{D}_{\Delta Y}$ in its literal sense, and looks consequently on the narrative of Moses as taking up the history of the world's creation, not from the Beginning-beyond the mere allusion to a beginning in the first verse of Genesis-but from the commencement of the present, or, what we may call, the Human Epoch. No reference, it is assumed, is made in the sacred record to the earlier creations of the globe, but those types alone are spoken of, which immediately preceded man's advent upon the scene, and which formed the parent-stocks of the fauna and flora that now people the earth and its waters. So far, perhaps, so well. But the holy writiugs record distinctly the elaboration of the world, or (according to those who adopt Buckland's theory) its regeneration, from a void or chaotic condition : and have we in the later periods of geological history any proofs of the existence of such a state? Dr. Bovell replies in the affirmative, and points to the so-called "Glacial Epoch" which marks according to his view the close of the great Tertiary age. But this is the weak point in his argument. It is a position indeed, perfectly untenable. The Glacial epoch, far from marking the close of the Tertiary age, belongs rather to the present, or forms a complete period of passage between the two epochs. Between the Tertiary Age and the Glacial Period it is absolutely impossible to draw a strict line of demarcation; and still less are we able to draw one between the latter and the existing era. Many types, both auimal and vegetable, have survived the glacial epoch; and (as so ably pointed out by Edward Forbes) it is evidently to the agency of this glacial period, as it came gradually on and gradually diminished in intensity, that the isolation of many arctic plantcolonies is due. That the Alpine plants of the Pyrennees and Scotland, for example, isolated from the surrounding vegetation, find their kindred species amongst the flora of northern Scandinavia, 一that

Uhe Alpine plants of the United States are related specifically to the flora of Labrador-depends evidently, (unless we adopt the theory of centres of creation) on a southern migration of these forms during the gradual development of this period of cold, and on their subsequent destruction in intervening districts, as the glacial forces slowly dwindled back to within their present limits. It must not be forgotten, moreover, that the results of glacial disturbance, were apparently confined to northern and extreme southern latitudes, in place of being of universal manifestation. Within the tropics for example, our true Drift deposits-the accumulations of glacial agencies on submerged areas-are properly unknorn. This fact alono, consequently, points to a very different condition of things from that indicated by the language of the sacred record. Nor can the comparatively modern uprise of large areas in South America and elsemhere, help to sustain our author's opinion; because these elevated tracts are the results of forces really still in action, and afford nowhere the slightest indication of the former existence of one grand and vast convulsion affecting equally the whole globe.

We need not carry our analysis farther; but it would be easy to shew that if we took the close of any geological period as our starting point-so far as it is possible to determine this-equal difficulties would beset the literal interpretation of the Mosaic day. But truly -and the fact becomes more and more apparent as work after work, like that norr under notice, becomes added to our stock-human science as yet is all too unprepared to undertake the investigation of these grave and apparently impenetrable mysteries. Whilst thus compelled, however, to dissent from the viers of our author, as expressed in this portion of his work, we may fairly add our testimony to the general value of the work itself. As a treatise of undoubted merit, and as a home product both of pen and press, it well deserves the attention of all interested in the progress of Canadian literature.

Book reorived:-A Course of Practical Chemistry. By Henry Groft, F.C.S., etc. Toronto : Maclear and Co, 1860.
Want of space compels us to postpone our notice of Professor Croft's useful Handbook of Analytical Chemistry, just published by Maclear and Co, until the next issue of the Journal; but, in tho mean time, we may recommend it as being especially adapted to the requirements of our Medical and University students.

# SCIENTIFIC AND LITERARY NOTES. 

## GEOLOGY AND MINERALOGY.

AGELAORINITES BILLINGSII. A NEW SPECIES.
PRELIMINARY NOTIOE OF, BY E. J. OBAPMAN.
Mr. W. M. Roger, an undergraduate of the University of Toronto. and one of our most esteemed students. lately subinitted to us a collection of fossils obtained by him from the Trenton Limestone of Peterboro', Canada West. 3 Amongst these, we discovered au undescribed species of the rare and interesting genus, Agelacriniles. We propose shortly to publish a complete description, with a figure, of this new species; but beg, in the meantime, to bestow upon it the above specific name, after the distinguished palæontologist of our Geological Survey, who has contributed so pre-eminently to our knowledge of the peculiar group of forms to which the genue Agelacrinites belongs, or to which it is elosely related. Our specimen presents a flat, circular form, exactly half-an-inch in diameter. It has five straight, or nearly straight, rays, composed of a double series of interlocking or alternating plates, and terminating in well defined rounded points, about one line from the margin of the shell or test. In the centre of the disc where the mouth is usually thought to be situated, there are five comparatively large and somewhat rhombic plates, the first ray-plates, one being common to each two adjacent rays. In the space between two of the rays, and at $\Omega$ distance of about two lines from the centre of the test. there is a well-marked "nnal-pyramid" (or "ovarian aperture') surrounded, apparently, by ten plates: five being situated in alternate positivn within the other five, exactls as in Hall's Hemicystites parasitica ( $=$ Agelacrinites parasiticus). All the olher portions of the inter-radial areas, with the margin of the test, are covered by imbricating or partiallyoverlapping and irregularly disposed plates of various sizes. At the margin there are about three or four rows of very small and exceedingly numerous plates, narrow and pointel, and succeeded by larger plates, of which the greatest diameter (unlike that of the marginal plates) lies parallel to the circumference. These are again succeeded by somewhat smaller and more pointed plates. A. Billingsii differs most obviously from A. Dicksoni, the only other Canadian species yet recognised (if we allow the Edrioaster of Billings to be a thoroughly distinct genus), by the possession of straight in place of curved rays, and by its exceedingly numerous marginal plates. It agrees much more nearly with the Niagara limestone species, A. parasiticus(Hall's Hemicystites parasitica); but from this it is distinguished essentially by the width of its rays (and by the ray-plates) being largest in the centre of the disc, and by the rays terminating in well-defined rounded points. In Hall's species, the rays are quite narrow and close together at the centre, and they broaden outwards, and, to use Professor Hall's language, " coalesce with the plates of the body;" or, in other words, are altogether undefined at their extremities. These chnracters are exactly the reverse of those which obtain in A. Billingsii. Besides which, in Hall's form there appears to-be only a single rom of small border plates, but that is probably an uncertain character. The other dis-
tinctions, however, are amply sufficient to separate the two species. Finally, it should be mentioned, that, not wishing to add to the already too copious list of unnecess.ry syeonyms, we have obtained the confirmatory opinion of Mr. Billings as to the distinctnest of our species from his $A$. Dicksoni. When we wrote to Montreal, we did not think of the apparent resemblance of the new species to A. parasiticus, but we have no doubthin. Jillings will agree with us also in placiag the two apart. In the extended notice of our species, we propose to give a general analysis of the genus Agelacrinites, with a comparative view of its structural relations and affinities ; as, on these points, we have some new suggestions to offer.

## NEW fossils frou the doal measures of nova scotia.

The following abstract is from a paper by Professor J W. Dawson, L L.D., of Montreal, read at a recent meeting (14th December, 1859) of the Geological Society of London:

On revisiting the South Joggins in the past summer, Dr. Dawson had the opportunity of examining the interior of anuabrerect tree in the same bed which had afforded the fossil stump from which the remains of Derdrerpeton headunaum und other terrestrial animals were obtained in 1851 by Sir C. Lyell and himself. This second truak was pointed out to him by Mr. Boggs, the Superintendent of the Mine. It was about 15 incles in diameter, and was much more richly stored with animal remains than that previously met with. There were here numerous specimens of the land-shell found in the tree previously discovered in this bed,-several individuals of an articulated animal, probably a Myriapod,-portions of two skeletons of Dendrerpeton,-and seven small skele-. tons belonging to another Reptilian genus, and probably to three species.

The bottom of the truak was floored with a thin layer of carbonized bark. On, this was $a$ bed of fragments of mineral charcoal (haviug Sigillaroid cell-structure), an inch thick, with a few Reptilian bones and a Sternbergia cast. Above this, the trunk was occupied, to a height of about 6 inches, with a hard black lami-. nated material, consisting of fine sand and carbonized vegetable matter, cemented by carbonate of lime. In this occurred most of the animal remains, with copro-lities, and with leaves of Noeggerathia (Poacites), Carpolithes, and Calamites,. also many small pieces of mineral charcoal showing the structures of Lepidodendron, Stigmaria, and the leafstalks of Ferns. The upper part of this carbonace-. ous mass alternated with fine grey sandstone, which filled the remainder of the trunk as far as seen. The author remarked that this tree, like other erect Sigillaric in this section, became hollow by decny, after having been more or less buried in sediment: but that, unlike most othere, it remained hollow for some tine in the soil of a forest, receiving small quantities of earthy and vegetable matter, fallng into it, or washed by rains. In this state it was probably a place of residence for the snails and myriapods and a trap and tomb for the reptiles; though the presence of coprolitic matter would seem to show that in some instances at least the latter could exist for a time in their underground prison. The occurrence of so mavy skeletovs, with a hundred or more specimens

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of land sanails and myriapods, ị a cylinder only 15 inches in diameter proves that these oreatures were by no means rare in the coal-forests; and the conditions of the tree with its air-breathing inhabitants imply that the Sigillarian forests were not so low and wet as we are apt to imagine.

The little land sbell, specimens of which with the mouth eutire have now occurred to the author, is named by him Pupa vetusta. Dr. Dawson has found entire shells of Plysa heterostropha in the stomach of Mrenobranchus lateralis and hence he supposes that the Pupa may have been the food of the little reptiles the remains of which are associated with them.
Two examples of Spirorbis carbonarius also occurred; these may have been drifted into the hollow truak whilst they were adherent to vegetable tragments, The Myriapod is named Xylobius Sigillarice, and regarded as being allied to Iulus.
The reptilian boues, scutes, and teeth referable to Dendrerpoton Acadianum bear out the supposition of its Labyrinthodont affinities. Those of the new gonus, Hylonomus, established by Dr. Dawson on the other reptilian remains, indicate a type remote from Archegosaurus and Labyrinthodon, but in many respects approaching the Lacertinas. The three sinecies determined by the author are named by him H. Lyellii, H. aciedertatus, and H. Wymani.

ON THE CLASSIFIOATION OF METEORITES-BY THE BARON VON EEIOHENBAOM.
The following distribution of Meteoric Stones and Iron Masses, in accordance with their physical characters, is condensed from a long and interesting paper (Anornung und Eintheilung der Meteoriten : von Freihern von Reichenbach) in a late number of Poggendorf's Annalen.
Section I. Iron-free Meteorites of low specific gravity and light color, with vitreous crust-

First Group-Meteorites from:
Langres, $G=3$. . 5 .
Bishopsville, $G=3.11$.
Jonsal (Transition-member to Se cond Group), $G=308$.

Second Group-Meteorites from:
Juvenas, $\mathrm{G}=3.11$.
Stannern, $G=3.07$.
Constantinople, $\mathrm{G}=317$.

Section II. Meteorites (almost iron-free) of a whitish or pale-blueish color in the mass, with disseminated pyrites; and, in general, a dull crest. Mosly cavernous, and more or less brittle:
First Group-Dark granules absent, or very sparingly scattered through the mass-
A. Whitish Meteorites from:

Macerata, G. - ; Vouille, G. 3.55; Nashville, G. 3.58; Bachmut, G. 3.42; Mauerkirchen, G. 3.45; Glasgow, G. 3.53; Kuleschofka, G. 3.49; Zaborzica, G. 3.40; Hartford, G. 3.58; Czartorya, G. -; Milena, G. - ; Yorkshire, G. 3.61; Forsyth, G. 3.44; Politz, G. 3.37 ; Aumières, G.- ; Ohandacapur, G. -; Kikina, G. -; Oesel, G. -; Charkow, G. 3.49; Ekaterinoslaw, G. 3.77; Kalova; G. -; Garz, G. -; Apt, G. 3.48; Askoe, G. 3.66.

## B. Blueish Meteorites from:

Slobodka, G. 3.47 ; Ohateau-Rebard, G. 354; Toulouse, G. 3.73; Girgenti, G. 8.76. Lissa, G. 3.50; Killeter, G.-; Oahu, G. 3.89; Cereseto, G. -- ; Favars, G.-
Second Group-Characterized by the presence of numerons enclosed globules, imparting to the mass a coarse-granular structure-

> A. Transition-members to Group I. Meteorites from:
> Sal s, G. 4.47 ; Parma, G. 3.39.
$B$. Containing dark granules. Meteorites from:
Luce, G. 8.47 : Nanjemoy, G. 3.66 ; Clarac, G. 3.50; Benares, G. 3.36; Utrecht, G. 3.57 ; Little Piney (Mo., U.S.), G. -; La Bıffe, G. 366 ; Tinochin, G. 360 ; Divina, G. 3.55 ; Horzowitz, G. 3.60 ; Richmond, G. 3.47 ; Eultown, G. 3.33.
C. Containing dark ind light granules intermixed. Moteorites from:

Siena, G. 3.39 ; Lontalax, G. 3.07 ; Nobleborough, G. 3.09 ; Bialystok, G. 3.17 ; Massing, G. 3.21.

Section III. The Meteorites of this Section present a grey colour, from finely disseminated magnetic iron ore ( $\mathrm{Fe} 0, \mathrm{Fe}^{2} \mathrm{O}^{3}$.) * They are more strongly cos herent, and contain more inon, with less pyrites, than those of the preceding sections. Their specific gravity is also higher.
A. Light-grey Meteorites from:

Sigena, G. 3.63 ; Macao, G. 3.73 ; Charsonville, G. 3.71.
B. Meteorites of a somewhat darker grey colour, from:

Esnande, G. - ; Berlanguillas, G. 3.49.
C. Meteorites of a blucish-white or grey colour, with numerous well-defined spots or flecks, from:
Liponas, G. 3.66 ; Gütersloh, G. 3.54 ; Weston, G. 3.53 ; Okaninah, G. -; Tip. perary, G. 3.64 ; Limerick, G. 3.65 ; L'Aigle, G. 3.43 ; Seres, G. 3.71 ; Madaras, G. 3.50 ; Bremervörde, G. 3.53 ; Agen, G. 3.61; Doroninsk, G. 3.63.
D. Meteorites of a dark-grey colour, from :

Lixna, G. 3.66; Cabarras, G. 3.63; Grünneberg, G. 372 ; Heredia, G. - ; Biausko, G. 3.70 ; Tabor, G. 3.65 ; Barbotan, G. 3.62 ; Wesseley, G. 3.70; Krasnoi-Ugot, G. 3.49 ; Kursk, G. 3.55 ; Tunga, G. -; Ohaba, G. 8.11 ; Borkut, G. 3.24.

Section IV. Meteorites of a green colour. From:
Ensisheim, G. 3.48 ; Simbirsk, G. 3.54 ; Wenden, G. 3.70 ; Erxleben, G. 3.64.
Section V. Dark-coloured brown or black Meteorites, containing carbonaceous matter. From:

Alais, G. 1.70; Capland, G. 2.69; Kaba, G. -; Renazzo, G. 3.26.
Section VI. Meteorites containing coarse brown patches not due to rust or oxidation. From:

$$
\text { Chantonnay, G. } 3.47 \text {; Mainz, G. 3.44. }
$$

[^7]Section VII. The Meteorites of this Section occupy a middle place between the stone and the iron meteorites. They contain a considerable amount of metallic iron.
First Group-With intermixed Olivine of the finest colour. Mean specific gravity=5.0 (3) From:-
Atacama, -; Siberia (the Pallas Meteorite), -; Saxony, -; Brabin, -; Bithurg, -.
Second Group-Mexican Meteorites, also containing Olivine, but with moremetallic iron than the Meteorites of the first group. Mean specific gravity. $=6.5$ (?) From :

Manji ; Tejupileo; Xiquipilco; Bata; Ocatitlan; Istlahuacan.
Section VIII, Iron Meteorites exhibiting "Widmannstet's Figures." From:
Seeläggen, G. 7.66; Bendego, G. 7.88; Bohumilitz, G. 7.65; Bruce, G. - ;
Union County, G. 7.07 ; Cosby, G. 7.26 ; Madoc, G. 7.85 ; Misteca, G. 7.88 ;
Burlington, G. 7.72 ; Guildford, G. 7.67 ; Durango, G. 7.88 ; St. Rosa, G. 7.30 ;
Buff, G. 7.10; Seneca, G. 7:34; Carthago, G.-; Schwetz, G. 7.77; Texas,
G. 7.82; Lockport, G.-; Red River, G. 7.82 ; Petropawlowsk, G. 7.76 ;

Caille, G. 7.64; Lenarto, G. 7.73 ; Sevier, G. - ; Elbogen, G. 7.74; Ashville.
G. 7.90; Agram, G. 7.82; Löweufluss, ©. -; Tazewell, G. 7.30; Cbarlotte, G. -; Putnam, G. 7.68.

Section IX. Iron Meteorites which do not exhibit the definite crystal markings of. those belonging to the last Section-
First Group-Transition Meteorites to Section VIII. Crystal figures partially developed. Meteorites from:
Caryiort, G.-; Zacatecas, G. 7.55.

Second Group-Containing minute points nad needles of white iron. From: The Cape of Good Hope, G. 7.50 ; Rasgata, G. 7.55 ; Salt River, G. 6.88 ; Kamtschatka, G.-.

Third Group-Containing iron in irregular masses. From:
Chester, G.-; Arva, G. 6.81; Caille, ©. 7.64.
Fourth Group-Masses of Meteoric Iron, with subordinate markinge in straight lines. From:

Tucumad, G. 7.56 ; Senegal, G. 7.72 ; Claiborne, G. 6.82.
Fifth Group-Entirely destitute of form-markings on the etched and polished surface. Meteorites from:
Tarapaca, G. 6.50; Green County, G.-; Hauptmannsdorf, G. 7.71; Smithland, $G$. -

In addition to the paper from which the above classification is abridged, the baron von Reichenbach has published in another number of Poggendorff's Annalen, an elaborate essay on the general composition of Metcoric masses. From this paper we extract the following tables-showing the mean composition, as calculated from various analyses, of stony and iron meteorites generally.


MATEEMATICS, NATURAL PHLLOSOPHY, AND ASTRONOMY. UN THE RESOLUTION OF AIGEBRAIO EQUATHONS.
While the remarkible researches of Professor G. P. Young are passing through the pages of this Journal, demonstrating the impossibility of solving algebraically the general equation of the fifth or higher order, a pamphlet has been published by the well-known analyst, Mr. Jerrard, in which he professes to demonstrate the possibility in the case of the gencral quintic. In reviewing this pamphlet, the editor of the Philosophical Magazine frankly confesses that he has not had courage to face the complicated analysis by which Mr. Jerrard attains his conelusion, and asserts his conteatment with Abel's demonstration (as modified by Wantzel) of the contrary proposition. Te validity of this demonstration has, however, been several times attacked, and, as we are not aware that it has ever been published in English, we here translate from Serret's Cours d'Algebre Supéricure the demonstration in Wantzel's words, with au abstract of the preliminary propositions, from the same author, on whish it is founded.

By a circular permutation of any letters it is implied that if the letters be arranged round a circle, each one is to be replaced by the one that precedes it in going round the circle. Thus, $a, b, c, d, c$, is circularly permuted into $b, c, d, c, a$.

By a transposition is meant an interchange of two letters only. It is readily shown that every permutation among a given set of letcers can be represented by
a set of simultaneous circular permutations; and that every circular permutation of (say) $\boldsymbol{g}$ letters is equivalent to ( $p-1$ ) successive transpositions. The following propositions are demonstrated by Cauchy:

Prop I.-If a function of given letters remains unchanged by every circular permutation of $p$ letters ( $p>8$ ), it ioill also remain unchanyed by any circolar permutation of three letters.
Prop. II.-If a function of given letters can only acquire two distinct values by any permutations of its lotters, il is changed by a single transposition; and, in general, it is or is not changed by a permutation, according as this permutation is equivalent to an odd or an even number of successive transpositions. Hence in particular:
Prop. III.-A function which has only two distinct values is not changed by circular permutations of three or of five letters.
Serret proceeds (Lesson 21st), to examine the bature of algebraic functions. $\Delta$ function of any quantities, $a, b, c, \ldots$ is algebraic when it can be obtained by performing upon them any of the following operations any finite number of times: (1) addition or subtraction; (2) multiplication; (3) division; (4) extraction of roots with prime indices. These operations, of course, include involution to integral powers, and extraction of roots with indices not prime. A function involving only the operations (2), (2), is a rational and integral function of the quatities; involving (s) also, it is rational; involving all four, it is general. If, then, A, $B, C, \ldots$ denote rational functions of $a, b, c, \ldots ; p, q, r, \ldots$, prime numbers; $f$, the operation of forming any rational function: then

$$
f(a, b, c, \ldots \ldots \ldots \sqrt{\sqrt{ } A}, q \sqrt{ } B, r \sqrt{C}, \ldots)
$$

is called a function of the first order.
If $\mathrm{A}_{1}, \mathrm{~B}_{1}, \mathrm{C}_{1}, \ldots$ denote functions of the first order ; $s, t, \ldots$ primes, then

$$
f\left(a, b, c, \ldots, p \sqrt{ } \mathrm{~A}, \ldots, \cdot \sqrt{A_{1}},{ }^{2} \mathrm{~B}_{1}, \ldots\right)
$$

is a function of the second order. Aad. generally, a function of the $\mu^{\text {th }}$ order will be of the form

$$
f(h, k, l, \ldots, \cdot \sqrt{H}, q \sqrt{ } K, \ldots)
$$

where $f$ always denotes a rational function $; \mathrm{H}, \mathrm{K}, \ldots$ are functious of the order $\mu-1 ; p, q, \ldots$ are primes; $h, k, l, \ldots$ are functions of the $(\mu-1)$ th or lower orders. From this form any radical, which can be expressed rationally in terms of the other radicals and quantities, can be eliminated; and ultimately it is shown that a function of the $\mu^{\text {th }}$ order can be thrown into the form

$$
a+p^{\frac{1}{n}}+\beta p^{\frac{2}{n}}+\ldots \ldots \ldots \ldots+\lambda p^{\frac{n-1}{n}}
$$

Where $\alpha, \beta, \ldots \lambda$ are functions of the order $\mu ; n$, a prime; $p$, a function of order ( $\mu-1$ ) whose $n$th root cannot be expressed rationally in terms of $\alpha, \beta \ldots \lambda_{\text {. }}^{*}$

[^8]If, now, the genemal value of the root $x$, of an equation $f(x)=0$ of the $m$ th degree, can be expressed in algebraic functions of the co-efficiente, let the above form be assumed for it and be substituted in the equation. The resuit will be of the form

$$
\mathrm{A}+\mathrm{B} p^{\frac{1}{n}}+\mathrm{C} p^{\frac{2}{n}}+\ldots \ldots \ldots+\mathrm{L} p^{\frac{n-1}{n}}=0
$$

where $A, B, \ldots \mathrm{~L}$, are rational functions of $p, a, \beta, \ldots \lambda$; and it is shown that this requires

$$
A=0, B=0, \ldots \ldots . L=0 ;
$$

whence it follows that the above expression for $x$ will still satisfy the equation when $p^{\frac{1}{n}}$ is replaced by $t^{\frac{1}{n}} t$ being any $n$th root of unity. We thus obtain $n$ quantities, which are roots of the proposed equation, ${ }^{*}$ and it is thence easily prored that all the quantities $p^{\frac{2}{n}}, a, \beta, \ldots \lambda$, are rational functions of the roots. By a similar investigation it follows that auy other function which enters into any of the quantities $p^{\frac{2}{n}}, \alpha, \beta, \ldots \lambda$, put under the assumei general form, ic also a rational function of the roots; and hence it is concluded generally that
Prop. IV.—If an equation is algebraically resolvible, we can give to the root such a form that all the algebraic functions of which it is composed are rational functions of the roots of the equation.

We now proceed to Abel's demonstration as modified by Wantzel, the inverted commas indicating, according to Serret, the text of Wantzel's memoir.
Let $f(x)=0$ be an equation of the $m$ th degree with arbitrary co-efficients, and iet its $m$ roots'be denoted by $x_{1}, x_{2}, \ldots x_{\mathrm{m}}$, and let us suppose them capable of being expressed as algebraic functions of the co-efficients.
"If the equation $f(x)=0$, is satisfied by the value $x_{1}$, of $x$, whatever be the co-efficients, we ought to reproduce $x_{1}$, identically by substituting in its expression the rational function [of the roots] corresponding to ench radical involved in that expression. Also, the routs being wholly arbitrary, every [apparent] relation between them must be in reality an identity, and will not cease to subsist when we exchange the ronts one among the other in any way wintever."
"Let $y$ denote the first radical, following the order of calculation [i.e., a radica of the first order with index $\frac{1}{n}, n$ prime] which caters into the value of $x_{1}$, and let $y^{n}=p$; then $p$ depends directly on the co-efficients of $f(x)=0$, and will be expressed by a symmetrical function of the roots $F\left(x_{1}, x_{2}, x_{3}, \ldots.\right)$; $y$ will be a rational function, $\phi\left(x_{1}, x_{2}, x_{3}, \ldots\right)$ also of the roots. (Prop. IV)."
"Since the fuxction $\phi$ is not symmetrical, (for if it were, the $n$th roov of $p$ would be exactly extracted), it ought to change when two of the roots, $x_{1}, x_{2}$.f.instance, are permuted; but the relation $\phi_{n}=F$ will always be satisfied. Then

[^9]the function $F$ being unchanged by this permutation, and the values of $\phi$ being the roots of $y^{n}=F$, we have
$$
\phi\left(x_{2}, x_{1} ; x_{3}, \ldots \ldots\right)=a \phi\left(x_{1}, x_{2}, x_{3}, \ldots \ldots\right)
$$
a being : [defiuite] $n$th root of unity."
"If we now interchange $x_{1}, x_{2}$, the above becomes
$$
\phi\left(x_{1}, x_{2}, x_{3}, \ldots \ldots\right)=a \phi\left(x_{3}, x_{1}, x_{3}, \ldots \ldots\right),^{*}
$$
whence, by multiplying in order, we have $\alpha^{2}=\mathrm{i}$. This result proves that the number $n$, supposed prime, is necessarily 2 , so that the first radical which presents itself in the value of the unknown must be of the second degree. This is what, in fact, happeus in those cquatious which we know how to resolve."

The fuection $\phi$ having only two values, changes by any transposition whatever, and will aot be changed by a circuiar permutation of three or five letters, for such permutations are equivalent to an even number of trauspositicns. (Prop. II., III). Let us continue the series of operations indicated to form the value $x_{1}$ of $x$.
"Combining the first radicai with the cocfficients of $f(x)=0$, (or the function $\phi$ with syametrical functions of the roow by means of the fir. t operations of algebrn, Te obtain thus a function of the roots, susceptible culy of two values, aud, consequently, invariable fer circular permutations of three letters. (Prop. III.) The succeeding tadicals may furvish more functions of the same kind if of the secoud degree. Suppose that we have come to a radical, for which the equivaleat rational function is not invariable for these permutations. Denoting it by $y=\phi\left(x_{2}, x_{2}, x_{3}, \ldots\right)$ then in the equation $y^{n}=p$, we shall still have $p=\mathrm{F}\left(x_{1}, x_{2}, x_{3}, \ldots.\right)$ but this function will no louger be symmetrical, but only invariable for circular permutotions of three letters. If in $\phi$ we replace $x_{1}, x_{2}, x_{3}$, by $x_{2}, x_{3}, x_{1}$, the relation $\phi^{n}=F$ will still subsist; anl, siuce $F$ does not change by the substitution, we shall have

$$
\phi\left(x_{2}, x_{3}, x_{2}, x_{4}, \ldots,\right)=\alpha \phi\left(x_{1}, x_{2}, x_{3}, x_{4}, \ldots\right)
$$

a being a [definite] nth root of unity."
"Performing in this equation, once and again. the circular substitution $x_{2}, x_{3}, x_{2}$, we hire

$$
\begin{aligned}
\phi\left(x_{3}, x_{1}, x_{2}, x_{4}, \ldots\right) & =a \phi\left(x_{2}, x_{3}, x_{1}, x_{4}, \ldots .\right) \\
\phi\left(x_{1}, x_{2}, x_{3}, x_{4}, .\right) & =\alpha \phi\left(x_{3}, x_{2}, x_{2}, x_{2}, \ldots . .\right)
\end{aligned}
$$

and, multiplying the three equations, we obtam $a^{3}=1$. Thus $n$ is 3 ."
"If the umber of the quantities $x_{2}, x_{2}, x_{3}, x_{4}, \ldots$ is greater than 4 , or if the equation $f(x)=0$ is of a higher degree than the fourth, we can perform

* Mr. Cockle (Phil. Mag. 1859, p. 510), remarks that this step" "taciily assumes the whole question, viz., that the suad is a quadratic. The only legitimate inference from $\phi\left(x_{2}, x_{1}, \ldots\right)=a \phi\left(x_{1}, x_{2}, \ldots\right)$ is $\phi\left(x_{1}, x_{2}, \ldots\right)=a^{-1} \phi\left(x_{2}, x_{1}, \ldots\right)$ where $a^{-1}$ is the inverse of $a_{n}$ " Mr. Cockle appears to us to have misconceived Wantzel's reasoning which recalls that "every relation among the roots must be an identity", and we are therefore entitled to permute the roots in any way ia such a relation as the one above. Mr. Cockle further alludes to some objection brought by Sir W. R. Hamilton, agaimst the validity of Abel's proof, that every radical is a rational function of the root.. We have not been able to discover where Sir W. R. Hamilton's strictures are to be found, and certaing can defect no flaw in the demonstration of the above in Serret's work.
in $\phi$ a circular substitution ci five leters, ieplaeng $x_{1}, x_{3}, x_{3}, x_{3}, x_{3}$, by $x_{3}, x_{3}, x_{4}, x_{3}, x_{1}$. The function $F$ will not change, and we shall have

$$
\phi\left(x_{2}, x_{3}, x_{4}, x_{5}, x_{1}, \ldots\right)=a \phi\left(x_{1}, x_{2}, x_{2}, x_{i}, x_{5}, \ldots\right)
$$

and, repeating the same substitution,

$$
\phi\left(x_{3}, x_{1}, x_{3}, x_{1}, x_{2}, \ldots\right)=a \phi\left(x_{2}, x_{3}, x_{1}, x_{3}, x_{1}, \ldots\right) .
$$

with three other equations similariy formed. Multiplying these together, we have $a^{5}=1$, and his requires $\alpha \approx 1$, for $a$ is a thind root of unity. Thas ure function $\phi$ is invariable for circular permutations of 5 letters," ami, cousequently, also of 3. (Prop. 1.)
"Thus, all the rudicats involved in the root of a general equation of a higher degree than the fourth, must be equal to rational functions of the roots, which remain invariable for circular permutations of three roots. Substituting these functions in the expression for $x_{1}$, we arrive at an equality of the form $x_{1}=\psi\left(x_{1}, x_{2}, x_{3}, x_{n}, x_{3}, \ldots\right)$ which ought to be an identity; but this is impossible, for the right-hand member remains invariable when we replace $x_{1}, x_{2}, x_{3}$ by $x_{2}, x_{3}, x_{1}$, while tbe left-hand member evidently chauges. It is then impossibic to resolve by radiculs a general equation of the fifth or any higher degree."
"The preceding demoustration shows at the same time that in equations of the third and fourth degrees, the first radical in the order of operations ought to be a square-root, and the second a cube-root. These circumstances, in fact, present themselve in the known formulas for these equatious."
J. B. C.

THE NEW HLANET.
31. Le Verrier enjoys the happy peculiarity that his brillimit theoretical discoveries are verified at once, and with the most complete and unexpected facilityHis audacions amouncement of the place of a planet beyond Uranus, led to the discovery of Neptuae on the very evening of its reception at Berlin, and now his still more wonderful amonnement of a planet interior to Mercury turns out to have been capable of verification before it was made. The following extract from Galignani, in default of more detailed accounts, will give some idea of this most brilliant achievement, which has at leugth shot Je Verrier far in advance ef bis rival Mr. Adams, who divided with him the honor of Neptune's discovery.
"Our readers must recollect M. Le Vervier's surprising communication to the Academy of Sciences on the 12 th of Septemter last, in which he anuouved a certain error in the secular motion of the perihelion of Mercuy, which could not be otherwise exphained, than by supposiug another planet to exist between Mercury aud the Sun. It would now seem hathi. Lee Verrier, 10 whom the world owes the nuprecedeuted prediction of the existence of the planet Neptune, has had the no less unexampled good-fortune, richly due to his scientific attainments and unceasing energy, of seeing his second prediction also verified. The intra-Mercirial planct has bcen found. Such is the astounding intelligence announced to the Academy by M. Ie Verrier himself, and, not only has it been found, but it was so several months before M. Le Vervier discovered its existence by calculation; and
stranger still, the fiuder is not an Astronomer, but a physician, Dr. Lescarbault: by uame, living at Orgeres (Eure et Lair.) The facts are simply these: on the 26th March last, the sky was overcast in many parts of France, but the Sun shonebright on the platean of Orgeres. Dr. Lescarbault, happening at the time to havea little leisure, took an observation of the Sm thr ough his telescope, and saw to his surprise a small round black spot pass over the Sum's disk. He carefully noted down the time, and afterwards calculated that the chord described by the plenet subtended an are of about 9 m .13 s . M. Le Verrier having published the result. of his calculations in September last, the Ductor wrote to hin to acquaint him with. the above fact. Shis was sufficient reasou for MI. Le Verrier to set out for Orgeres. Here he learned that Dr. Lescarbault was a man of great learuing and unisersally respected, and that he had but one fault-that of troubling himself too muchabout the stars. On arriving at Dr. Lescarbault's abode, M. Le Verrier was astovished to find a regular Observatory there, with instruments chiefly contrived by the Doctor himself, in consequence of his rery limited financial means the worthy physician not having a chronometer bas nevertheless mate himself a pendulum striking seconds, by muans of an iyory ball and a bit of string. If. Ye Verrier asked lim many questions on his observations-and his manuer of acquiring the data relating to the new planet, and received the most satisfactory answers. According to M. Le Verrier's calculations, the chord described by the planetanust have subtended an are of 9 m .17 s . So that the Doctor with his clumsy apparatus was only 4 s . wroug. The Doctor estimates the diameter of the new planet at 310 leagnes, and the inclination of its orbit to the ecliptie at 12 degrees. If such are its dimensions, it is likely that there are more plancts still in that region. The revolution round the Sun is performed in 19 days and 17 hours; in its greatest alongations, its distauce from the Sun does not exeeed 7 degrees, while Mercury's maximum elongation is 25 degrees. It was lucky that M Le Verrier had resolved upon going in person; for, the Doctor's observatory being strangely deficient in paper, his calculations were generally writen in charconl on a deal-b-ard, and whele it was full, the Doctor used to plane it down by way of eleaning it. This precious. deal-board, charged with all the calculations relating to the plauet, has been obtained by M. Le Verrier, and presented to the Academy."

The preceding will make a capital page iv the Romance of Science. Mr. Scott, the Chamberlain of the City of London, writes to the Times, claining that he also saw this (or another) planet about Midsummer, 1847, crossing the sun's disk at sumset, seeming, by a basty calculation, to be of the same size as Venus, or about 4,000 miles in diameter. No transit of Venus or Mercury occurred at the time; and Mr. Seott thinks he could not have mistaken a spot of the sun, as the image was "perfectly circular, and its outine was as sharply defined as a blot of the darkest ink on the whitest of paper," and, moreover, it had disappeared at sunrise; whereas, had it been a spot, its position would have required six or seven days before it rould have been carried out of sight. Mr. Scott communicated his observation to several persons, but had not sufficient confidence to publish so startling a fact. Dr. Dick, horerer, on becoming aware of it, referred him to a passage iu one of his works, where, with the doctor's well-known facility of conjecture, he had speculated on the possibility of the existence of such planets,
and asserts roundly that such an object was actually seen " by Mr. Lloft and others, on the Gth of Jauuary, 1818," but who Mr. Lluft-and who the others were-seems not to be known to the present generation of astronomers. Mr. Scott is also of ojinion that his planet cannot be Doctor Lescarbault's, on account of the difference in size. Another correspondent of the Times writes, that a conjunction of Lescarbault's planet, must have occurred at the dates of both Mr. Lloft's and Mr. Scott's observations, but Mr. Hind comes to the conclusion, that at neither of them could this planet have been visible. Trausits this year may be expected between the 25th March and 10th April, and between 28th September and 13th October; each transit not exceeding four hours and a half in duration. It is to be hoped that astronomers will eatch the wanderer at one of these periods. Meanwhile it will be curious to ascertain whether this planet, or one of its group may not explain the puzzling observations by Cassini and others, of a supposed satellite of Venus.
J. B. C.

## CANADIAN INSTITUTE.

Session-]S59.00.
first ordinary mefting-3rd December, 1859.
Hon. G. W. Allas; M.L.C., President, in the Chair.
I. The Report of the Council relative to the change of name of the Institute vas read and laid on the Table.
(Copy.)
Final Report of Committee on proposed change of name and netr charter.
To the Council of the Canadian Institute, - Your Committee beg to report that, in furtherance of the directions remitted to them relative to the proposed change of name of the Institution and the suggested application for a Royal Charter with extended powers-

They obtained an interview with His Excellency, who was pleased to express his willinguess to exercise his full official infuence for the accomplishment of the Wishes of the Institute.

At the same time His Excellency expressed some doubt as to the expediency of conferring the exclusive right to such a name as The Royal Society of Canada, on an Institution hasing its head quarters permanently established in Upper Canada; and reeerved his final decision on that point."

From the information, however, which your committee received, as to the probable cost of a Royal Charter, added to the difficulties suggested in reference to the proposed designation of the Institute, they recommend that the Council delay for the present taking any steps for procuring such new charter. At the same time your committee feel bound to represent to the Council that their conviction of the injuries to be apprehended from the confusion of the Canadian Institute with Mechanica' Institutes and other societies of a merely local character, consequent on the correspondence in name, is in no degree abated. In the debates in the Legishative Assembly during the past Seesion, which led to the withdrawal
of many of the aunual grants heretofore made to Hechanice' Institutes, the majority of the speeches referred to the Canadiau Institute under the belief that it was the local Institute of Toronto : corresponding in all ways, and especially in respect to its claims for the continuance of its annual grant, to the ordinary Mechanics Institutes of the province.-
It is for the Council to consider whether a chnuge of name would suffice to prevent such confusion in future; or what other means is best calculated to give the requsite promineuce to the eseatially provincial character of the Iustitute, and to secure for it the continuation of those funds which have been expended by it on objects of a purely public and Provincial character; and in no degree for the promotion of individual or local interests, these being amply provided for by the annual income derived from members' subscriptions.

Should the Council on further consideration, revert to a change of name for the Canadian Institute, as the means best calculated to ward off the dangers arising from confusion with mechanios' and other local Institutes, your committee have reason to belicve that a new proviucial charter with such changes of name and constitu tion, as, after mature deliberation, shouid seem most expedient, will be readily accorded to the Iustitute by His Excelleney.

All of which is respectfully reported,
(Sigued, G. W. ALLAN,
Canadian Institute, Niov, 29th, 1859.
Convener.
II. The following gentlemen, who we:• proposed at the last meeting of the Institute for membership, and those elected previously by the Council during the recess, were balloted for and declared duly ciected members:

William Ince, Esq., Toronto.
A. E. Williamson, Esq., Toronto.
W. C. Campbell, Esq., Toronto.
J. A. Cattanach, Esq., Toronto.
W. Tassie, Esq., Galt.
G. H. Wilson, Esq., Toronto.
III. The following papers were read.

1. By the Rev. J. McCanl, LL.D :
"On Ancient Shields."
2 By Professor Chapman:
"On Canadian Miuerals."
second ordisary ametria-10th December, 18059.
Hon. G. W. Allar, M.L.C., President, ia the Chnir.
I. The following gentlemen were clected mombers :

John Paterson, Esq., Turonio.
Herbert F. Tuck, Esq., Toronto.
II. The follooing donation for the Library was announced, and the thanks of the Institute voled to the donor.
From T. C. Wallbridge, Esq.
The Poetical Works of James Haskins, A.B., M.B., Trinity College, Dublia. Ed. by Henry Baldwin, A.M., Osguode Hall, U.C., Barrister at Law. One Vol.
III. The folloning paper was read.

By Doctor Rae.
"On the Search for Franklin."
IV. The requisite nominations for the election of ofice-bearers for the ensuing year, were made; and the President announced the anuual general meeting to be held on the 17th inst., to reccive the Report of the Council. to elect office.bearers and members of Council for the ensuing year, and for other business.

$$
\text { annual general meeting-17h December, } 1859 .
$$

Hon. G. W. Allas, M.L.C., President, in the Chair.
I. The following gentlemen vere elected members:

Doctor James Ross, Toronto.
Doctor John Wanless, Toronto. W. T. Thomas, Esq., Toronto. Doctor-Lizars, Torouto. Rev. A. Wickson, M.A., Toronto. Rev. E. Hatch, B.A., Toronto. Doctor H. H. Wright, Toronto.
II. A ballot haviug been taken for officers of the ' stitute, for the ensuing year, the following gentlemen were declared duly electe ${ }^{-1}$ iz:
President
Professor D. Wilson, J.
1st Vice President
Professor H. Croft, D.
2nd do ........ Rev. Professor W. Hineks. F.LS.
Srd do ........ Professor J. Bovell, MLD.
Treasurer .................. D. Crawford, Esq.
Corresponding Secretary ... Professor J. B. Cherriman, M.A.
Recording do ... Patrick Freeland, Esq.
Librarian . . . . . . . . . . . . . . Professor H. Y. Hind, M.A.
Curator .................. J. F. Smith, Jun., Esq.
Council. ................. . . Hon. (. W. Allan, M.L.C. Professor E. J. Chapman.
Sandford Fleming. Esq. W. Hay, Esq. G. R. R. Coskburo, M.A.
III. The report of the Council for the year 1858-59, was then read and adopted on mation of Professor Hind, seconderl by Doctor Morris.
IV. The President brought before the Meeting a recommeadation from the Council that the Institute should unte its sense of tho valuable and zealous services rendered by Doctor Wilson as chief Editor of the Canadian Journal, by a mark of recognition similar to that made two years ago.

It was then moved by G. A. Pyper, Esq,, seconded by Captain Dick.
That the Council be empowered to carry into effect the recommendation made by them thre" rh the President to the Institute, in reference to some recognition of the valual.c services of Doctor Wilson as Editor of the Journal. Carried.

## V. The following paper was read.

By Professor Chapman:<br>"On the Geology of Belleville and surrounding district."

VI. Moved by S. Fleming, Esq., seconded by G. A. Pyper, Esq., that the President do now leave the chair and that Prof, Hincks be called thereto. Carried.
VII. Moved by P. Freeland, Esq., seconded by S. Spreull, Esq., that the cordial thanks of the Institute be given to the Hon. G. W. Allan for his valuable services during the past year as President of the Institute. Carried.

## ANNUAL REPORT OF THE COUNCIL FOR 1859.

The Council of the Canadian Institute, at the expiration of their term of office, have the honor to lay before the Members, the usual yearly report of the proceedings and progress of the Society. Since the last annual report, 35 new names have been added to the list of Members; but the total number constituting the Society, has undergone a considerable reduction, as will appear by the following statement:

Members at commencement of Session 1858-59 ........... ............ 650
New Members elected, Session 1858-59 .................................... 30
By Council, during recess ............. ..... ................................ 8
Total ................................ 683
Deduct-Deaths . .............. ........................................... 8
Left the Province .......................................... . 11
Withdiawn ....................................................... 34
Struck off for non-payment of Subseription, per
Committee's report .................................. 129
Athenæum Members, who have never paid their
subscriptions since the amalgamation of the
societies .............................................................

Total on 30th November, 1859 ............... 467
Composed of-Honorary Members............................................. 4
Life Members .................................. ............... 36
Corresponding Members .................................. 5
Members ................................... .................... 410
Junior Members ................................................ 12
Total .... ....................... 467
This decrease in the total number of Members, compared with last year has been occasioned, (irrespective of losses by death or remoral), by striking off the names of a large number from the roll, who have persisted, for an unreasonable length of time, in ignoring the Treasurer's clains upon them, and neither paid their subscriptions nor sent in their resigaations.

The necessity for removing from the roll these merely nomiual members, most of whom had been defaulters for several successive years, had long been strongly felt by the Council; and before the close of the last Session, a Specinl Committee was appointed to revise the list, with view to striking of those who appeared, from the Treasurer's returns, to be hopeless defaulters. This uecessary, but unwelcome duty, bas been very carefully and considerately performed, and none but those who have proved themselves so long unmindful of the claims of the Society as to leave no hope of their amendment, have had their names removed from the roll.

In reality, thercfore, this temporary decrease in the uumerical strength of the Instiute, has neither diminishod its pecuniary resources as compared with past years, nor should it be considered as any indication of decreasing interest in the 'Society's operations, or of less hearty en-operation or support on the part of its friends generally, as the Members now cut off bave long been such ouly in naine, and were rather a source of weakuess thau of strength.
Since the last Session, a blank has necurred in the list of Honorary Hembers, by the death of one whose name conferred honor on the Society, and whose memory will long be perpetuated in Cauada, by the enduring memorial of one of the noblest productions of his genius. But a few short weeks before the com. pletion of the Victoria Bridge, at Montreal, Robert Stephenson closed his earthly career. Struck down at the age of fifty-one, while in the full maturity of bis great and vigorous intellect, be has passed away-if not full of years, yet full of howors-leaving behind him a name which will loug survive, not only in the grateful memory of his countrymen, but which will be cherished and honored in every part of the world, where exalted genius and practical energy and worth, are honored and appreciated.

Just six years ago, when this Society was still iu its infancy, Mrr. Stephenson; then on a visit to this country, honored the Institute by allowing his name to be enrolled on the list of Honorary Members; and ia answer to the address which was presented to him on that occasion by the Cuncil, he expressed his strong sense of the benefits which might accrue to the cause of science in this country through the instrumentality of such associations as the Canadian Institute-not only by the publication of its Journal aud the communications read at its meettings, but more especially by discusions on the subjects of the various papers brought under the consideration of the Society.

The Council would fain hope that this expressiou of opinion on the part of one whose experience so well qualified him to judge, may not be forgotten, but have its due weight with every Member of the Association, inducing more active and zealous co-operation in furthering the objects of the Institute, and rendering it a powerful and efficient agent in advancing the scientific progress of the couutry.

In one very important particular,-the extension of its Library,-the Instituto has continued to make satisfactory progress, and the Council have pleasure in reporting, that many valuable additions have been made to it during the past year, -chiefly through the liberality of various donors, to several of whom the Institute tas before been very largely indebted.
The following is a hast of the varions books aded to the Library, by purchase or otherwise, during the year:
BOOKS PURCHASED.
Books marked thus (*) are in parts, or unhound. vols.
Orania Britannica. Decade lII ..... 1*
Rawlingou's Liferodotus. Vols. 2 nud 3 , ..... 2
Canadian Almanac for 1859 ..... 1*
Dietrichsen and Hanuay's Royal Almume for 1859 ..... $2^{*}$
Encyclopredia Britannica. 8th edition. Vols 17 and 18 ..... 2
Wanderings of an Artist among the Indians of North America. By Paul Kane ..... 1
Miller's Popular Geology ..... 1
Substance of a Journal during a residence at the Red River Colony, British North America, from 1820-23. 2nd edition, enlarged ..... 1
Chronological History of North Enstern Vogages of Discovery, and of the Eastern Navigations of the Russians. By Capt. J. Burney, F.R.S ..... 1
Cavendish, Debates on the Quebec Bill, 1774 ..... 1
Bopp's Comparative Grammar. Vols. 1, 2 and 3 ..... 3
Total ..... 14
BOOKS BOUND FROM PERIODICALS RECEIVED.
Illustrate:l London News. J uly to December, 1858 ..... 1
Mining Journal, 1858 ..... 1
Builder, 1858 ..... 1
Journal of the Society of Arts. Vols, 3 and 6 ..... 2
Athenæum, July to December, 1858 ..... 1
Artizan, 1858 ..... 1
Canadian Merchnnts' Magazine, 1858 ..... 2
Journal of the Franklin Institute, 1858 ..... 2
Civil Engineers and Architects Journal, 1858 ..... 1
Journal of Education, Upper Canada, 1858 ..... 1
Journal de l'Instruction Publique, 1858 ..... 1
Art Journal, 1858 ..... 1
DONATIONS OF BOOKS TO THE LIBRARF.
From Office of Routine and Records.
Appendix to Vol. 16 of the Journal of Legislative Assembly, 1858. 1-2, No. 1; 2 to 4, No. 2; 3-4-5-1§, Ň. 3; 13 to 20, No. 5; 26 to 29 , No. 6; 29 to 43. No. 7; 29-43, No. 8; 43-65, No. 9 ..... 7
Appendix to Vol. 17 of the Journals of the Legislature, 1859. 1-5, No. 1 ; 5-9, No. 2; 9-36, No. 3 ..... 3.
Trade and Navigation Reports, 1858 ..... 1
Journals Legislative Assembly. Vol. 17, 1859 ..... $1^{*}$
Report of Progress Geological Survey of Canada, 1857 ..... 1
From Hon. J. R. Brodnead, Washington. vols.
Patent Office Reporte, U. S., 1857:-Agriculture, 1 Vol.; Mechanics, 3 Vols ..... 4 ..... 1
Explorations and Survey for a Railroad route from the Mississippi River to the Pacific Ocean, 1853-56. Vols. 8 and 9 ..... 2
From the United Statrs Patent Office, Washington.
Patent Office Reports, 1856 :-Agriculture, Vol. 1 ; Mechamics, Vols. 1, 2, 8.. 4
Do 1857 Do Vol.1; Do Vols. 1, 2, 3.. 4
From the Regents of the Unidersity, Ex-Officio I'rustees of the State Library in behalf of the State of New York.
Documents relative to the Colonial History of the State of New York, pro- cured in Holland: England, and France. By John Romeyn Brodhead, Esq. Vol. 2 ..... 1
Catalogue of the Books on Bibliography, Topography, and Eugraving, in the New Yorb State Library, 1858 ..... 1
Annual Report of the Trustees of the New York State Library made to the Legislature, February, 1858 ..... 1
Seventy-first Annual Report of the Regeats of the University of the State of New York, made to the Legislature, January 08th, 1858. ..... $1 *$
Eleveuth Annual Report of the Regents of the University of the State of New York ou the condition of the State Cabinet of Natural History, and the Bistorical and Autiquarian Collection, 16 th 3arch, 1858 ..... 1*
Firom Col. J. H. Lefroy, Rl. Aryillery, F. R.S., Ko., London.
Mortality of the British Army, at home aud abroad, aud during the RussianWar, as compared with the mortality of the Civil Population in Eugland.Illustrated by Tables and Diagrams from Report of the Royal Commis-sion, 18581*
Contribution to the Sanitary History of the British Army during the late war with Russia, \&c., 1859 ..... 1
Geology of North America. By Jules Mareou. With three Geological Maps and seven,plates of Fossils, 1858 ..... 1
Ordnance Trigonometrical Survey of Great Britain and Ireland. Published by order of the Master Geueral and Board of Ordannce, 1858 ..... 1
Account of the Principal Triangulation Plates. Ordnance Survey ..... 1
Military sketch of the Island of St. Helena. By Capt. E. Palner, Rnyal Ar-tillery, F.R.G.S. 1850-52. Lit'd. at the Top. Department of the WarOfice. Col. James, R.E., F.R.S., M.R.I.A., de., Director1.1
From Henry J. Boyn, Esq, York Street, Covent Gerden, Loondor.
Diary and Correspondence of Samuel Pepys, F.R.S, Secretary to the Admir-alty in the reigus of Oharles II. and James II, \&c. \&c. By Lord Bray-brooke. 6th Edit. Vols. 1, 2, 3, 44
The Pretenders and their adherents. By John Henage Jesse. New Editiou. Complete in one volume, \&ic. ..... 1
Vox. V.
vOLS.
Life and Letters of Johe Locke, with extracts from his Journal and Common place Books. By Lord King. New Edit., with a genernl Index ..... 1
Letters on the History of Christian Dogmas. By Di: Augustus Neander. Edited by Dr. J. L. Jacobi. Translated from the German by J. E. Ry- land, M.A. In two Vols. Vol. 1 and 2 ..... 2
General Fistory of the Christian Religion and Chmech. Transhted from the German of Dr. Augustus Neander by J. Torrey, \&c, Part 1, Vol. 9. Part 2, Vol. $G$ ..... 2
Bibliographers' Manual of English Literature, \&c. By W. T. Lowndes. New Edition, Revised, corrected, and enlarged, by F. G. Bohn. Vol. 1, Part 2, Vol. 2, Part : ..... 2
The Orlando Furioso. Translated into Engli,h verse from the Italian of Ludovico Ariosto. With Notes by Will. Stewart Rose. Vols. I. and II. ..... 2
Holbein's Dance of Death. Exhibited in elegant Engravings on Wood, dc. By Francis Douce, Esq., F.A.S. Also Holbein's Bible Cuts, \&e. In- troduction by Thos. F. Dibdin ..... 17
Parables of Frederic Adolphus Krummacher. Translated from the German. Tth Ed., \&c. ..... 1
A Book for a Corner, \&e. By Leigh Hunt. Two vols. in one ..... 17
Noble Deeds of Women, or Examples of Temale Courage and Virtue. By Elizabeth Starling ..... 1
Pope's Poetical Works. Vol. II. By R. Carruthers ..... 1
Elements of Botany. By M. Adrien de Jussieu. Translated with consider- able additions by James Hewetson Wilson, F.Is S., \&c. de. \&e. ..... 1
Humbolt's Cosmos. Vol. V. ..... 1
Medals of Creation, or First Lessons in Geology. By Fideon Algernon Mantel, L.L.D., \&sc., in two volumes. Vols. I and II. ..... 2
Vegetable Physiology and Systematic Botany. By W. B. Carpenter, M.D., \&c. Edited by Edwin Eankester, M.D., \&c. ..... 1
Anecdotes of Dogs. By Ed. Jesse, Esq ..... 1
From Prof. James Hali, Albany, New York.
Report on the Geological Survey of Iowa. Vol. 1. Parts 1 and 2. ..... 2
From Educational Department of Lower Canada,
Rapport du Surintendant de l'Education dans le Bas-Canada pour l'année 1857. French ..... $I^{*}$
Do do do do English ..... 1*
Rapport du Surintendant de l'Education dans le Bas-Canada pour l'anuée 1858 ..... 1*
From R. S. M. Boucaette, Esq.
British Dominions in North America, or a Topegraphical and Statistical Des-cription of the Provinces of Upper and Lower Canada, \&c. \&cc. By Jos.Bouchette, Esq., Surveyor General of Lower Canada, de. \&c. London,1832. Vols. 1 and 22
Bouchette's Topographical Dictionary of Lower Ganada. London, 1882 ..... 1
From E. B. O'Callaghan, LL.D., Albany, United States Patent Office. Reports for 1857. vols.
Agriculture ..... 1
Mechanics. Vol. II. ..... 1
From Royal Dubln Society.
Journal of the Society. Vol. Y., 1856.7 ..... 1
From Smmisonian Suorety, Wushington.Smithsonian Contributions to Knowledge. Vol. X.1
From T. D. Harmigaton, Esq.
Teneriffe, an Astronomer's Experiments, or Specialities of a Resideuce above the Clouds. By C. Piazzi Smith, de. ..... 1
From Historioas. Society of Prenneylyania.
Publications of the Society. Contributions to American History. Memoirs. Vol. VI. ..... 1
From Hon. East India Coaipany.
Geological Survey of Iudia, Geological Museum. Vol. i. Part 2. Publish- ed by order of the Right Efon. the Governor in Council ..... 1
From Unitied States Coast Sunvey Ofrioe, Washington, woith compliments of Prof. Á. D. Bacie.
Report of the Superintendent of United States Coast Survey for 1856 \& 1857 ..... 2
DONATION OF PAMPHLET:
From Prog. G. Ianvon, Kingston.
Transactions of the Scottish Arboricultural Society. Vol. I, Parts 1, 2, 3 ..... 3*
Remarks on Lepas Anatifern, Líus ..... 1*
On the occurrence of Cinchonaceous glands in Galiaceæ, \&c. ..... 1*
On the structure of the Victoria Regia ..... 1*
Report on Musci and Desmideæ, \&c. ..... 1*
Papers read to Botanical Society of Edinburgh ..... 2*
Bemerkungen von Gilbert J. French ..... I*
From Prof. Kendall, M.A., T. College.
Theory and Experiment, a lecture delivered before the Board of Arts andManufactures for Lower Canada, 20th December, 1858 . ................... 1*Defence of Dr. Gould, by the Scientifie Council of the Dudley Observatory.. l**From Bernard Quaritoa, London, England.
Catalogue Raisonne of rare, valuable and curious books, Januar $\mathfrak{j}, 1859$ ..... 1)

| Do | do | do | February, | do ..... | 1* |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Do | do | do | April, | do ...... | 1* |
| Do | do | do | May, | do ...... | 1** |
| Do | do | do | June, | do ...... | 1* |
| ${ }^{\text {D }}$ | do | do | July, | do ...... | 1* |
| Do | do | do | August, | do | 1* |
| $D_{0}$ | do | do | Sept. | do | 1* |
| Do | do | do | October, | do. |  |

From Universitar Regia Firedericana, Christiania. vols.
Plyrikalske Meddelelser Ved A-Arndsten, 1858 ..... 1*
Asaf-den Helliges Saga, Universitets Program for Andet Semestir, 1853 ..... 1*
Oord lak Boltsebog, Jl 1832-1849 ..... 1*
Moı phologie Végetale, J. M. Norman, 1857
Sur les Phenoménes d'érosion ..... 1*
Inversio Vesicæ Urinariæ, L. Vnss... ..... 1*
Aubert Lateinischen Grammatik, 1850 ..... 1*
Zulu-Sproget Grammatik, 1850 ..... 1*
Symbolæ ad Historiam Antiquiorem Rerum Norvegicarum, P. A. Munch, His. Prof. ..... $1^{*}$
Graptolitherne ..... 1*
Forhandlinger ved de Skandinaviske Naturforskeres Syvende mode 1, Christi- ania Den 12-18, Juli, 1856 ..... 1*
Statistiske Tabeller for Kongeriget Norge, 1857 ..... $2^{*}$
Udtog of Norges-Riges Historic. Christiania, 1834 ..... 1*
From O.fice of Routine and Records.
Report of the Postmaster General for year ending 30th September, 1858 ..... $1 *$
Report of the Crown Land Department, for the year 1858 ..... $1 *$
Public Accounts,-Province of Canada, year 1858 ..... 1*
Summary of Proceedings, Legislative Assembly, 2nd Session, 6th Parliament, 1859 ..... 1*
Report of Committee on Banking and Currency ..... 1*
Third Report on Public Accounts ..... 1*
And Sheets, Proceedings of Legislative Assem!ly, Bills, Reports or Select Committees, \&c.
Receiscd from Society of Antiquaries of the North, Copenhagen, Denmark.
:Hemoirs of Northern intiquaries. 1840-1844 ..... 1*
Do do 1845-1849 ..... 1*
Do 1852 ..... 1*
Runcindskrift I. Piræeus, Inscription Runique du Pirée ..... I*
:Saga J'àtvardar Konungs Ilins Helga, \&c., 1852 ..... I*
Cabinet d'Antiquités A méricaines à Copenhague, 1858 ..... $2^{*}$
Sheets.
Mémoire sur la découverte de l'Amerique au dixième siècle, par Charles $C$. Rafn. Second Tirage ..... 1*
Société Royale des Antiquaires du Nord, le premier Janvier, 1858 ..... 1*
Discovery of America by the Northmen. ..... 2*
Antiquités de l'Orient, Monuments Runographiques Interprétés, par C. C. Rafu, \&c. ..... 1*
Sur la construction des salles dites des Géants par S. M. le Roi Frédéric VII. de Danemark. ..... $1 *$
Saga Jàt vardar Kouungs Hins Helga, \&c. 1852 ..... 1*
Société Royale des Antiquaires du Nord, 1858. Sheets, 8 pages. Duplicate. ..... 1*
Discovery of America by Northmen. Duplicate ..... 2*
vOLS.
Oonnection of the North men with the East ..... $2^{*}$
Books recently published by the Royal Society of Northern Antiquaries ..... $2^{*}$
Oritical Opinions on works recently published by Royal Soc. N. Antiqun.Société Royale des Antiquaires du Nord.Sénnce Annuelle du 29 Janvier, 18 as2*
Do du 26 do 1837 ..... 1*
Do duler do tās ..... 2*
Do du Cabinet d'Autiquités Amerimanes ..... $2^{*}$
Do Antiquitates Amerirane ..... 2*From J. W. Dawson, L.L D., F:G.S., Principal of MrGill College, Montreal.
Additional on the Post Pliocene Deposits of the St. Latwreuce Valley ..... 2*
Catalogue of Camadiau Plants in the Holmes Herbarium in the Cabinet of the University of JIeGill College. Prepared by the late Prof. James Barns- ton, M.D. ..... 1*
On the Luwer Coni Measures, as developed in British America by J. W. Daw- son, LL.D., F.G.S., Principal of McAill College, Montreal, Proceediags of Geological Society, April 28, 1858, pages 61-t7 ..... 1*
From Histomoal Societr, Chicago.
First Annual Statement of the Trale and Commerce of Chicayo, ending 31st December, 1858 ..... 1
Sketches of the listory of Oglo County, Illinuis, and the early settlement of the Northwest, written for the Polo Advertiser ..... 1*
First Circular of the Latw School of the University of Chicago, year 1859-60 ..... I*
Seventh National Exhibition by the Western States Agricaltural Society, t.o be held at Chicago, September 12, 13, 14, 15, 16, and 17, 15059 . $\$ 20,000$ offered in preusiums ..... 1*
From the Historioas. Society, Montrall.
Mémoires et Documents relatif a l'Histoire du Canada, putliés par la Société Historique de Montréal. ..... 1*
From the Royar. Grooraphioal Suomety of London, per Mr. Allàn. I'Ifometh 3Ir. Todn, Librarian, Legislative Assembly.
Proceedings of -Vol. III, No. 1, 1859 ; No. 2, 1859; No. $\overline{\text {, }} 1859$ ..... 8*Fr-om G. D. Gibb, M.D., M.A., F.G.S., \&c., de., London.
A Ohapter on Fossil Lightuing, by Doctor Gibb ..... 1"
From Hartard University
Oatalogue of Offeers and Students, year 1859-60. First ter a ..... 1*
Calalogues from Mr. Allev, Pablisher, London.
Allen's Bibliotieca it nericana. ..... 1"
Do Books relating to America ..... 1*
Do (lld Books relating to Ainericn ..... 1*
Mrom Lh Hexpen, Esq., Junior.
Inauguration of the Dudley Observatory. at Albany, 28th Augnst, 1856 ..... 1 *
Proceediugs of the American Association for the alvancenent of science, loch Mecting held at ilbany, N. Y., August, 1856. ..... 1*
From Captain Meade, U. S. A. vols.
Conrisillustrative of the Uuited States Topographical Survey of the North American Lakes ..... 1
From Societies, in exchange for Journa:
Journal of Education for Upper Camada, 1859
" Franklin Institute, 1859
" Education for Lower Canada, 1859
" de liInstruction Publique, 1859
" the Socicty of Arts (duplicale), 1859
Artizan, Lundon, 1859
Silliman's Journal, 1859
Canadian Naturalist and Geologist, 1859
Queen's Bench Reports, 1559 (three Numbers)
Proceediugs of the Buston Naturai History Society. Vol. 6, pages 401-432;Vol. 7, pares 1-128
The Atlantis, Dublin, 1859
Transactions of the Academy of Scieuce, St. Louis. Vol. 1, No. 3, 1859 ..... 1*
Bulletin de la Société Géologique de Frauce, for 1857, four Nos.; 1858, four Nos. ..... 8*
Anuales des Mines. Nos. for 1856, one; 1857, six; and 1858, six ..... 13*
Historical Recollections of the Essex Institute. Vol. 1, 1839. Nos. 1-4.. ..... 4*
Transactions of the Royal Scottish Society of Arts. Vol. 5. Part II. 1859 ..... $1^{*}$
Fifteenth Semi-Anusual Report of the Water Commissioners of the City of Chicaro ..... 1*
Charter aud By-Laws of the Chicago Historical Society, and list of officers. ..... l*
Journal of the dmerican Geographical and Statistical Society. Vol. 1. Nos. 1, 2, 3, 4, for January, February, March, and April, 1859 ..... 4*
Annals of the Lyceum of Natural History of New York. Vol. 6. February, 1858, Nos. 8 and 9, 1 ; September, 10-13, 1. Vol. 7. December, 1858; March, 1859, Nos. 1-3, 1 ..... 3*
From Masor Lachlas:
Christy's Teiters on Geolory, 1848 ..... 1*
Report of the Nantabala Land and Mineral Company ..... I*
From Hon. G. W. Allan, M.L.C.
Monogram of the Trochilide. Parts 15 and if ..... 2*
DONATIONS FOR THE MUSEUM.
From T. C. Whlibridge, Ese. ..... nos.
Geological Specimens. ..... 2
From His Exoeliency the Govenon Gexeral.Specimeus of the Coinare of Canada-in a Case.
2
Silver Truenty Cent pieces
2
Ten do.2
Copper One do. ..... 2

From Rev. V. Clementi, B. A, Peterborough, C. W. nos. Skull of a Beaver (Castor Fiber) from Stouey Lake, C. W. Trapped by John
Naugun, an Indian of the Mississauga, 185s ............. .............. 1
Specimens of Fossils ..... 11
Skins of the star nosed Mole. ..... 2
Specimen of Columnaria Alveoluta from Indian River, Dummer, Peterborough ..... 1From C. J. Bernune, Ese., Trinity College, Toronto.
Box of Fossils. Specimens ..... 57
From Prof. Dawson, L.L.D, F.G.S., Montreal, per Prof. Geapman, Toronto.Specimens of the Devouian Plants of Gaspé. Numbering..................... 12From Jajes Wrigrt, Esq., Toronto.
An Indian Pipe found in Meaford, C. W ..... 1
From G. B. Wyluie, EsQ., Toronto.
Canadian Lymx suffed ..... 1
From T. D. Harringron, Toronto.
Bag of Pebbles and Indiau Earthenware from Lake Hurou ..... 1
One old Coin. Ob. Head Apollo. Reverse Tripod ..... 1
From Major Tachlan.
Geological Specimens of the Mount Auburn Rocks, Ohio Numberiag ..... 35
From W. Hax, Ese.
Brainstove (Meandrina Cerebreformis) from the Bermudas ..... 1
From J. Gould, Esc., L.ondon.
Box of birds, skius-Specimens Trochilida ..... 50
The list of Papers read during the Session of 185S-9 will be found to containvery many communications of great iuterest. Amongst them are several valuablecontributions to the uatural bistory of the country, and many others also bearingdirectly on the mincral aud other cionomic resources of the Province; both ofthem a class of subjects to which the Council would especially invite their membersto assist in contributing.
COMMUNICATIONE.

The subjoined list contains he titles of the various Papers read at the ordinary meetings of the Session 1858-9:
S Prof. E. J. Chapman-" On the alleged discovery of a Couns in the drift of Western Canada." 4th December, 1858.

Prof. Dr. D. Wilson.-"Notices of the Beaver in Europe and Canada." 4th December, 1858.

Prof. Rev. W. Hincke, F.I.S.-"On Canadian Ornithology. 11th December, 1558.
F. Assikinack, Esq.-"On the Grammatical Construction of the Odahwah Language" Read by Prof Wilson, 11 th December, 1858.

Dr. Oille.-"Un Parasites." 11 th December, 1858.
Prof. E. J. Chapman.-" On a new species of Asaphus," 18th December, 1858.
W. Weir, Esq.-"On the Manufactures of Canada." 18th December, 18 of8.

Dr. Harvey.-"On the Increase and Decline of Mnlarious Disenses in the Valley of the Grand River." Read by Prof. Croft. Sth January, 1859.

Major Lachlata.-"Sketch of the Geolory of Ohio." accompanying a series of specimens illustrative of the same. Read by Prof. E J. Chapman. 1Eth January, 1859.

Prof. D. Wilson.-"Notice of an aucient Stone Axי, inser: bed in unknown characters, recently tarned up by the plough in New Jersey." Ijth Jamary, 1859.

Jos. Bunchette, Esq.-"Report upon Explomations in the Nonth West." Read by Andrew liussell, lisq. 29nl Jamuary, 18 ā9.

Pros. H. Croft, D.C.L —"On Dust Storms." Send January, 1853
Dr. Rae.-"(1) "On the Fomsation of Iecbergs," and (2) "On the Pransportation of Boulders." "9th January. 1859.

Dr. Morris. --" On a Species of Intestinal Worm found in the White Fish.' 29th Jamuary, 1554 .

Prof. G. T. Kingston, M..1.-" Meteorclogical report for year 1S5S." 'Toronto, 5th February, 1 S59

Prof. D. Wilson, T, $:$ D.-"On the Pre-Coiumbian Discovery of America." 6th February, 1859.
T. G. Cottle. M.D.-"On the Crames of Ganada." Read by Rev. Prof. Hincks F.L.S. I2th February, 1859.

Prof. E. J. Chmpan.-"Remarks on cert.in specimens of Camadian Sarble." 12th February, 1859.

Prof. D. Wilson, LL.D.-"Remarks on the Quirrich, an ancient Scattish Relic." 12th February. 1859.
E. Billings, F.G.S.--" On the Fossil Corals of the Devonian Rocks of Canada." Read by Prof E. J. Cnapman. 19th February, 1859.

Rev. Prof. G. P. Young, M. A.-"The exact solution of General Algebraical Equations of uvery degree, in all cases where the roots or any number of them admit of being Algcbraically represented." 19th February, 1859.

Rev. J. McGaul, LL.D.-"On some Mint Marks of the Lower Empire." 19th February, 1859.

Rev. Prof. W. Hiucks, F.L.S.-"The Sensational Philosophy respecting the Humav Mind and its operatiots, the treatment it has met with, and its real character and pretensions." 26 th February, 1859.
W. G. Tomkins, Esq., C.E.-"On Comparative Tabular Meteorological observations in Canada, England, and Russia." Read by S. M. Jarvis, Eiq. 26th Feb. ruary, 1859.
W. Hay, Esq., Architect.-"Some Remarks on Iron Construction as applied to Street Architecture." 5th March, 1859.
T. G. Cottle, Esq.. M.D.-" On two Rare Birds observed in Canada." Read by Rev. Prof. Hincks, F.L.S. ith March, is53.

Prof. E. J. Chapman.-"Remarks on some Specimens of Fossil Plants from the Devonian Rocks of Gepsi, presented to the Institute by Prof. J. W. Dawson F.G:S." 5th March, 1859.
B. R. Morris, Esq, M. D--" On the luminous appearance of the sea, commonly. ealied phosphoresceut." 12th March, 1859.

Prof. H. Y. Hind, M. A.-" On che Qu'Appelle or Calling River, and the diver: sion of the waters of the south branch of the Saskatchewan. down the Qu'A ppella
valley, to the Assinitioine River, and past Fort Garry into Red River, with a view to the estnblishment of direct steam communication from the Red River to the foot of the Rocky Mountains, in a line nenly west from Fort Garry." 19th March. 1859.

Rev. Prof. W. Hiucks, F. I. S.-"On the Canadian species of Lynx." 19th March, 1859.

Prof. H. Croft, D. C. L.-"Some experiments with Ruhmkorf"s Induction Coil." 19th March, 1859.
Rev. C. Dade.-"On the Law of Storms." read by Prof. Cherriman, M.A. 26tb March, 1859.
Rev. J. McCaul, LL.D.—"New Reading: of Oll Inseriptions." 26th Mareh 1859.
J. F. Smith, Esq - "Notes nin some of the more characteristic Fossils of the Hudson River Group of Western Canada." Read by Prof. E. J. Chapmau. 2nd April, 1859.

Prof. D. Wilson, LL,D.-" Nutes on the Development of New Varieties among the Inurusive Populations of America." and April, 1859.

John Langton, M.A.-" On the Agc of Trees." 9th April, 1859.
C. Smallwood, M.D.-"On the I!eteorological Phenmmena of Lower Canada for 1857-8." Read by Prof. Cherriman, M. A. 9th april, 1850.
E. M. Hodder, M.D.-" On the influence of the Storms during the winter of 1858-9 on the Peninsula, and the probable effects on the Esplanade and Harbor." 16 th April, 1859.
S. Fleming, Esq., C. E.-"On the Settlement of Wild Land." 16 th April, 1859.

In submitting the Report laid before them by the Etiting Commitce, the Council would take the opportmity of expressing their deep sense of the zeal and efficiency with which the late greneral Editor, Dr. Wilson, has discharged the arduous duties connected with the editorial superintendence of the Journal. Under his able management, and with the valuable assistance of the other members of the Editiug Committee, the Journal has continued to maintain the high character which it has so long enjoyed, and through its instrumeatality the Society is not only becoming more widely and favorably known in this Province, but is slso rapidiy extending its intercourse with the scientific bodies both of Europe and America. Having contiuned his editorial superintendence for a period of four years, often at a cunsiderable sacrifice of persomal conveuience, and to the interruption of other pursuits, Dr. Wilson has now expressed his desire to be released from his duties, aud the Council are glad to have it in cheir power to congratulate the Institute on lanving secured the services of so able and efficient a succebsor as Professor Chapman, who has coneented to fill the chair of Geveral Editor.

## REPORT OF THE EDITING COMMITTEE.

The Editing Committec beg lave to subnit their Ammal Report to the Coun. cil, on completing the fourth volume of the new series of the Canadian Journal.

Bearing in view the objects of the Institute as a society designed to promote the development of a native Canadian Science and Literature, the committeo havo
continued to aim at the acquisition of such communications as are calculated, along with the critical Reviews and Scientific and Literary Notes, to maintain the special character which the Journal' is designed to bear among the periodicals of the Province. Among the contributions to the present volume, special thanks are due to Mr. F. Billings, of the Canadian Geological Survey, for his valuable paper on the Fossil Corals of the Devonian Rocks of Canada West, as well as for the carefully executed illustrations which added so largely to its interest. While, however, the utility of the Canadian Journal is acknowledged, alike as a provincial mediumfor the interchange of communications on exclusively scientific and literary subjects, and also as a means of intercourse with men of science both in Europe and America; the editors have also anxiously desired to bear in view the aims and interests of the members at large. They hare accordingly deemed it perfectly compatible with the objects of such a Journal, to introduce occasionally, especially in the departments of criticism and literary notes, subject of a more general and popular interest than can be supposed to attach to strictly scientific contributions. By such means it is hoped that the Journal has accomplished purposes equivalent to the printed proceedings of the older and more exclusive scientific societies of Europe : serving not ouly to diffuse valuable scientific and literary infcrmation, but also to constitute a bond of mutual interest and union among a body of members seattered throughout the Province.

During the past year the Editing Committee have added the following societiea and learned foreign bodies to the free list furnished in former reports. From the increasing value of the exchanges which they continue to receive, and the direct intercourse tisereby established with the principal scientific societics of Europe and America, they ferljustified in regarding this as one of the most important functions of the Iustitute as a provincial society:

> Geological Survey of India, Calculta.
> Royal Dublin Society.
> American Geographical and Statistical Society, New York.
> American Antiquarian Society, Boston.
> Historical Society of Pensylvania.
> Harvard University, Cambridge, Massachusetts.
> Natural History Society, Montreal.
> Literary Society, Quebec.
> Hamilton Association, Canada West.

The Committee continue to receive gratifying evidence of the favourable reception of the printed proceedings of the Institute, as shown in reference to them, and still more in the re publication of extracts, and ceven of whole papers from their pages, in British and Foreign Jourmals In addition to this, one of the learned societies of Europe: the Royal Suciety of Northern Antiquaries, of Copenhagen, of which His Majesty, the King of Denmark, is President, in ackuowledging the receipt of the Journal during the past year; through their distinguished Secretary, C. C. Rafn communicated the desire of the Society to elect the President of the Canadian Institute, and the Editor of iss Jourual, on their rauk of Honorary Members.

The Jouroal has been conducted, since the establishment of the New Series, under the editorial superintendence of Dr. Wilson, with the aid and oo-operation
of the members of the Editing Committee; and with such results as to justify the course adopted, in rendering this series strictly a periodical issue of original papers, embodying the printed proceedings of the Institute. This object having been secured, and the present Geacral Editor having expressed his desire to be rolicved of duties he has now fulfilled for four yeare, the Editing Committee have much pleasure in reporting, that the Geveral Editorship has been undertaken by Professor Chapman; whose frequent contributions to the Journal in past years, as well as his high standing in a department of science so important in its practical bearing on the development of the mineral and other economic resonrees of the Province, render him peculiarly fitted for the responsible duties thus devolved on him.

An unusually large space has been devoted to the department of reviews during the past year; while at the same time they are fewer in number than iu former volumes; the object aimed at having been to transfer mere notices of books to the notes, and to give to the department more of the character of review articles. With a larger body of contributors, this section might be extended with great advantage, and the Journal increased in size, and rendered acceptable to a much wider circle of readers; while the opportunity thereby afforded for the discussion of important questions in science and literature could in no degree detract from the legitimate characteristics of such a periodical. The Committee, however, cannot overlook the fact, that a large portion of the materials hitherto contributed to this department have been the worls of two or three members, on whom, accordingly, an amount of labour has been imposed, which, though freely rendered, must be felt to be an undue tax on the voluntary services of so small $\Omega$ number, in a society of some hundred members, including many well qualified to share in such labours.

It was the intention of the Committee to have aimed at giving increased interest te the Journal during the past year, by means of illustrations, but a series of disappointments by the artist engaged on the work, involving much trouble and anxiety to the editor, ended in the abandonment of the scheme for the present. The Committee however, have pleasure in calling attention to the beautiful lithograph of the Quigrich, which as a specimen of art, executed in Toronto, cannot but be regarded as highly satisfactory proof of progress, when it is bome in remembrauce that similar illustrations for a former volume had to be procured from New York.

In conclusion, the Commitice have to express their carnest hope that the new General Editor may be able to secure such an amount of varied and hearty co-operation, as, while materially lessening his own labours, shall contribute fresh attraotions both to the publications and the meetings of the Iustitute.

Toronto, 3rd December, 1859.
Daniel Wilson, Convenet.
On referring to the details given in the Treasurer's Report submitted below, it will be seen that the general funds of the Institute are, upon the whole, in asatisfactory condition. The Buildiug Fund has been slightly increased by the accumulation of interest, but the Council deemitright to call the attention of the Members to the fact, that the subscription list for that special purpose still remains uncol-
leoted, and unless some stops are taken during the ensuing year to call in at least a per ecntage on the amounts subseribed, much of what appears to the eredit of tho Fund on paper, may be lost altogether.

The Comeil have hitherto reftaned from urging the pryment of their subsoriptions upon the contributors to the Fund, is in consequence of tho pressure of the times, it was not deomed expedient to proceed with tha building, amd it therefore seemed uncensonable to press for subseriptions which were not immediately required. But as there is overy reason to hope that the time is not far distant when returoing prosperity and the improved financial condition of the comntry, will justify the Council in proceeding with the work, it seems to them most desirable that some efforts should in tho mean time bo male, to place the fund on a more eatisfactory footing. Until the Institute is installed in a permanent honse of its own it cam seareely bo expected that any satisfactory progress will be made in carrying out one of the special objects for which the Suciety was instituted-the furmation of a Museum illustrative of the Natural History, the Geological and Mineral products, and the economic andindustrial resourees of the Province. It is true that many valuable contributions have already been made to our eollection, but with our present limited accommodation, and the uncertainty as to future arraugements, it is impossible to render this department of any practical utility or interest either to the members genmallys or to the public at latere. The Council therefore venture to hope that the libernlity of the friends of the fustitute and the exertions of its Members, will mable their sucecssors to place the Building Fund on a more satisfactory footing. and that the impediments which have hitherto interposed themselves to the commencement of the building itself, will speedily be removed.



## AUDITORS' REPORT, 1859.

Tononto, 9th December, 1869.
Examined Youchers with Cash Book. Balance ia bands of Trensurer six hundred and two pounds four shillings and five peuce, correct, and securities for one thousand four hundred and twenty five pounds exlibited.
> $\left.\begin{array}{l}\text { Sasurl Sprable, } \\ \text { Gogrge R. I. Cockbuen, }\end{array}\right\}$ Auditors

Before drawing their report to a close, the Comacil desire $\begin{gathered}\text { brielly to refer to a }\end{gathered}$ question which was brought umler the notice of the Institute, during its Session of $1858-59$, and on which a final report from a Committee of Cruncil was submitted to the Members at their last General Meetmg. The suinject of this report was the inconvenience which had been found to arise from the present appellation of the Snciety. - the name of Institute having led to its beiug confounded with other associations of a purely local character: insomuch that when the withdrawal of the grants to Mechanics' and other local Institutes was under discussion in the Legislative Assembly, during their last Session, the Canadian Institute, from its similarity of name, was classed with the others. and had it not been for the timely exertions of some friends of the Association, would probnbly have been deprived of its anmual grant.

As the report alluded to, has been entered upon the minutes of the Society, the Council do not deem it veceasary to refer to the subject beyond expressing
their conviction of its importance, and their hope that the matter will not be allowed to drop, but that further consideration will be given to it during the course of the ensuing Session.

In couclusion, the Council would remind the Members of the Iustitute, that even should it hereafter be deemed expedient to seck for a change of name, it must still rest with the Members of'the Institute themselves, by their active cooperation and diligent exerious, to stamp the Association with such an umistakeable character of vitality and usefulness,--to rencer it so truly a representation of the science and intellect of the Province, -as in a great measure to prevent the danger of its being confounded, (except by tisose who will not take the trouble to inform themselves, ) with asscciations of less extended or inferior aims.

> G. W. ALLAN, Prosident.
thind ondinaty meeting-7th January, 1860.
Hon. G. W. Allav, M. L. C. President, in the Chair.
I. The following Gentiemen were elecled Members:

Hon. W. Caydey, M. P. P, Toronto.
Henry Cawthra, Esq., Toronto.
II. The following donations to the Library were annonnced, and the thanks of the Institute voled to the doners.
From the Publishers, B. Dawson w Son.
Archaia, or Studies of the Cosmogony and Natural History of the Hebrew Scriptures: by J. W. Dawson, LL.D., FG.S. One Vol.

Fiom Tie Curef Superintendent of Education for Upper Canada.
Annual Report of the Normal, Model, Grammar and Common Schools in Upper Canada for the year i 558 , with Appendix. By the Chief Superintendent of Education for Upper Cauada. Two Vols. Unbound.

From Dr. Hayden of Albany (through Prof. Chapman.)
Two Maps of Nebraskia Territory.
III. An address presented by the Council to Alex. M. Ross and James Hodges, Civil Engineers, congratulating them upon the opening of the Victoria Bridge, and the reply of those Gentlemen to that address, was read by the Secretary.

Toronto, 19th Dec. 1859.
To Alexander Mackenzie Ross and James Hodges, Esqrs., the Engineer and the Builder of the Victoria Bridge :
Gentlemen,-At a special meeting of the Council of the Canadian Institute, convened this day, the followiug resolution, congratalating you on the completion of the magnificent work with which your names are so intimately connected, was adopted unavimously.

Resolved.-That the Victoria Bridge at Montreal having this day been opened for public traffic, the Council of the Canadian Institute deem it a fitting opportunity to congratulate Messrs Alexander Mackenzie Ross and James Hodges, Civil Engineers, on the completion of that great and voble work.

The Institute have watched with a double interest the progress of the Victoria Bridge, not only as a work of the highest national-importance, but also as closely associnted with the name of Robert Stephenson: that renowned and much lamented engineet, whom the Institute had the high honor to enroll amongst its members on the occasion of his visit to Canada, priol to the commeneement of the great undertaking which has just been brought to so successful a completion.

In the Victoria Bridge, Canadians must not only feel that they possess one of the noblest monuments of engineeriug skill and scieuce existing on this continent; but that also, by the completion of this magnificent structure, a great highway has been opened, over which the trade and commerce not only of Canads, but of the furthest west, may at all times flow: uninterrupted by the natural obstacles which have heretofore opposed themselves for a large period of the year, to a free communication with the sea-board.

To the gentlemen whose vames are so closely connected with this great work, the Council of the Institute desire now to express their sincere congratulations on the successful termination of their labours; and they desire also by this resolution, to record in the archives of this Society (expressly established for the promotion of Science aud Industry) the completion of the noble mouument of Science and Mechanical skill which has this day been opened to the traffic of the Province.

The Conncil further resolved that copies of the above resolution should be engrossed and transmitted to Messrs. Ross and Hodges.
(Signed,) G. W. ALLAN, President.
Reply. Copy.
Dontreal, 27 th December, 1859.
To the President and Council of the Canadian Institute, Toronto.
Gentlemen,-We have the honor to acknowledge receipt of copies of Resolution passed at a special meeting of your Council, convened on the 19ih instant, in which you congratulate us, as Engineer and Builder, on the completion of the Victoria Bridge, that day opened for public traffic.

In returning you our thanks for the notice you have taken of ourselves in connexion with the termination of our labors, a notice, which to us is more valuable, emanating from a Suciety established for the promotion of Science and Iudustry, and numbering so many respected names amongst its members, we rejoice to think that the work with which our names have been counected, is one which is so lighly calculated to assist in developing the interests of a country for the prosperity of which our best wishes can never cease to be formed.

And it is not ouly our present hope, but our confident belief, that the sacrifices which this Province has made with such enlightened foresight in order to establish a great aud ever open highway of conmunication betwixt the rising territories of the farthest west and the Atlantic se: board, will in due time find a return corresponding to the spirit in which that great enterprise was conceived, and the perseverance with which the mean= have been found for bringing it to a successful completion.

Amidst so much that is calculatel to afford satisfaction to all concerned, our pleasure is yet damped by the melancholy reflection, that the distinguished man to
whom you allude, with whose name this undertaking is so closely associated, has beev p:evented from witnessing its completion by a too early death.

Asain begring to lender you our grateful and respectful acknowledgements,
We have the honor to be,
Gentlemen,
Your very fnithful Servants,
(Signed)
:

ALEXANDER M. ROSS. JAMES HODGES.

## V. The following papers were read:

1. By the President, Prof. Dauiel Wilson, LL.D.:

The Annual Address.
2. By Prof. H. Y. Hind, M.A. :
"On the distribution of Clay Iron Stone in the Carbonaceous rocks of Rupert's Land, or the North Western 'rerritory, and its value as a source of Irou in that Country."

> fourta ondinaby meetra-14th January, 1860. Prof. D. Wilson, LL D., President, in the Cbair. I. The following Gentlemen werc elected Members: Alex. M. Ross, Esq., Civil Engineer, Montreal. Hunorary Membor. Jases Hodaes, Esq., Civil Engineer, Montreal, Corresponding Member. Jonn H. Hont, Esq., M.D. Army Medical Staff, Torouto. Wardinary O'Hara, Esq., Toronto. II. The folloing papers were read:

1. By F. Assikinack, Esq.:
"On some peculiarities of the Odahwah language."
2. By Rev. Prof. W. Hincks, F.L.S.:

Specimens of a Canadian Elora.
fipth ordinary meeting-21st January, 1860.
Professor Wilsos, LL.D., President, in the Chair.
I. The following papers werc read:

1. By the Hon. G. W. Allam, M.L.C.:
"On the Topography of the Roman Forum, Illustrated by a series of Photographic views."
2. By the President, Professor Wilson, LLL.D :
"Observations on the skull of a Circassian Lady, brought from Kertch in the Crimea."
sixth ordinary meeting-28th January, 1860.
Prof. Daitel Wilson, LL.D., President, in the Chair.
I. The following Gentlemen veere elected Members:

Alvio DeGrassie, Esq., To:outo, Ordinary Member.


## II. The following papers were read:

1. By Prof. H. Y. Hind, M.A:
" Remarks on Indian Art, illustrated by a collection of Indian relics, obtained during the Assiniboine and Saskatcherrau expedition.
2. Dr. Bovell made some observations on the akull of an infant Indian found with many others in a pit near Weston.
sefentif ordinaty meeting-4th Februayy, 1860.
Professor D. Wilson, LL.D., President, in the Chair.
I. The following Gentlemen wore elected Mfembers:

The Fou. Mr. Justice Eagabty, Toronto.
Rev. E Coofer, M. A.
II. The following donation to the Institute was announced, and thantes voted to the donors: .
A copy in chromo-Lithography of the picture by Paul Kane, Esq., of the denth of a Blackfoot Cbief. By Miessrs. Fuller \& Beucke.
III. The following Paper was Read:

1. By Professor Bovell, M.D.
"Notes of a visit to Barbadoes in 1859."

## ERRATA.

Tl.e following orrata occur in the first part of the paper "On the Resolution of Equations," which appeared in the January Number of the Journal :

Page 22, line 10 , for said, read surd.
Page 23, line 29, for $z_{2}$ in the second term of the value of $\phi(p)$, read $z_{2}^{2}$.
Page 27, line 13, for $U^{m}$, read $U_{m}$.
Page 27 , line 21, and page 28 , line 13 , for same, read some.
Page 31, line 7, delete the comma before the word "having."
Page 34, line 10, for same, read some; also, in the last line of the same page, for $A_{c} Y, \operatorname{read} A_{c} Y^{c}$.
Page 35, line 16, insert the siga + before $\mathrm{Y}^{\mathrm{c}}$.
Page 38, line 22, instead of $Y$, after the word "expressions," read $Y_{c}$.
Vol. V.

## METEOROLOGY.

## mean meteorological resulit at toronto, for the year - 1859.

BY PROFEBSOR TINGSTON, M. A., DIRECTOR OF THE PROVINCLAL MAGNETIC OBSERVATORF.

> (Read boforo the Cunadian Instituci, February 11th, 1860.)

The mean temperature of the year 1859 was $44^{\circ} .19$, which differs only $0^{\circ} .08$ in excess fiom the average of 20 years.

The mean temperatures of the several months were in six instances above and six below their respective averages. As shown by the table, the warmest month absolutely though relatively a cold one, was July, and the month that was absolutely coldest, though it was relatively warm, was Februarg. The warmest month relatively, was March, being $6^{\circ} .27$ above the average, aud the relatively coldest month December, which was lower than the average by $8^{\circ} .08$. December was the coldest December on record, being $3^{\circ} .2$ colder than the coldest December previously recorded.

The warmest day was July 12th, with a mean temperature 79. 88 , and the coldest January l0th, with a mean temperature $-8^{\circ} .65$.

The highest temperature of the year was $88^{\circ} 0$ being $2^{\circ} .5$ below the average. It occurred on July 12th, already mentioned as the warmest day. The lowest temperature of the year, occuring on January 10th, (also the coldest day in the year;) was $-26^{\circ} .5$ being $14^{\circ} .7$ below the average, and the lowest ever recorded at the observatory. The absolute amual range thus amounted to $114^{\circ} .5$.

Fumidity.-The meau humidity of the year was .74 , being nearly identical with that of 1558. The annual march, as exhibited in the monthly means, corresponded in its alternate iucrease and diminution, very accurately with that of the preceding year, and in most casea showed nearly exactly the same numbers.

Clouds.-The extent of sky clouded, on the average of the year, was nearly $\frac{3}{5}$ of the hemisphere, and for nine months the sky was on the average at least half overcast. This accords with the experience of previous years, but in the distribu: tion of cloudiness nonong the different months, a want of parallelism is apparent.

Wind.-The resultant direction of the wind, was $\mathrm{N} ~ 61^{\circ} \mathrm{W}$. 'The mean velocity of the year was 8.17 miles per hour, which was 1.60 miles above the average, and slows an increase on the tro preceding years. The most windy month was April, with a mean velocity 10.79 miles, and the least windy montn May, with a mean velocity, 5.70 miles. The most windy day, was March 19 th, when the menn velocity was 31.16 miles, the greatest recorded; and the calmest day September 23 rd .

The most windy hour on the average of the year, was from 1 PM. to 2 P.M. with a mean velocity 11.00 miles; and the calnest hour, from midnighe to 1 A.M.
when the moan velocity was 6.64 miles. These statements agree very nearly with those made in the precoding year, when the most windy hour was from 2 to 3 P.M. and the calmest hour, from miduight to 1 A. ''.

Ratn and Snou.-The depth of min 33.274 inches, shows an increase of more than 5 inches on that of the year 1858, and was 2.415 above the avernge. The depth of snow shows also an increase of 9 inches on that of the preceding yen. This however, was principally due to the heavy falls in December, as the amount that fell in other months was below the average in every case but in January, when it exceeded it ouly by about 3 inches. The total depth of rain and melted snow oxceeded the average by 2.724 inches.

November was the most rainy month with respect to the amount of rain, and June with respect to its frequency. The smallest amount of rain fell in February, and the fewest rainy days occurred in December.

The heaviest rain occurred on August 23 d , when it fell to the depth of 1.655 inches, and the heaviest fall of snow on December I'sh, when the depth was estimated at 6 inches.

The fall of rain was distributed over 127 days, and the fall of snow over 87 days, including 23 diass which occurred in December alone; and there were 169 days only, or less than half the year, without either rain or snow.

The rain occupied about 514 hours and the snow about 380 hours in its fall, making thus a total of about 894 hours, or $37 \frac{1}{5}$ days, during which either rain or snow was falling; a result it is to be remarked differing only by about one day from that of last year.

The hour at which rain or snow was most frequent, was between 2 P.M. and 3 P.M. and the hour most free from rain and snow, ou the average of the year, was between 1 A.M. and 9 A. MI.

Thunderstorns.-There were 30 thunderstorms, reckoning as such those eases in which thunder or lightning occurred accomparied by rain or hail, besides 16 instances in which the thunder or lightning occurred singly or together, but without vain or bail.

Auroras.- The auroras in 1859 were not quite so numerous as in 1858, but there was au increase in the number of days in which those of the first class were observed. The ayrom of August 28 th, and the following days, was probably one of the most rem $n$ kahle efer recorded, when considered with respect to its brilliancy, its duration, and the extent of the earth's surface at which it was visible. It was accompanied by an extraordinary magnetie disturbance. The magnets were deflected from their normal positions to the extent of about 207 ' in the delination and $2^{\circ} 20^{\prime}$ in the dip; and in the horizontal and vertical components of the force, there was a departure from their normals, of about .08 and .006 of their respertive normal absohnte values. The magnitude of these deviations will be better appreciated when it is remembered that a disturbance is reckoned large when the declination differs 5 ', the dip $1^{\prime}$, the horizontal forve .0012 , and the vertical force .00026 , from their respective normals.

The following is the general Meteorolngical abstract for the year 1859, deduced from the observations taken at the Provincial Observatury:

## GENERAL METEOROLOGICAL

## Provincial Magnetical Observatory，

Latitude， $43^{\circ} 39^{\prime} .4$ North．Lovgitede， 5 h .17 m .33 s ．West．Eeevation adove

|  | Jan． | Fcb． | March． | April． | May． |
| :---: | :---: | :---: | :---: | :---: | :---: |
| MeanTemperature $\qquad$ Dillerence from averane（ 20 years） | 26.43 | 20.04 | 36.34 | $3{ }^{19} .53$ | 85.16 |
|  | $+2.72$ | $+3.21$ | ＋6．27 | $-1.47$ | ＋3．78 |
| Thermic Anvmaiy（Lit． $43^{\circ} 40^{\circ} \mathrm{N}$ ）．．．．．．．．．．．．．．．．．．．．． | －6．30 | －8．66 | －3．76 | －10．67 | －2．91 |
| Hiphest Temperature | 43.2 | 18.2 | 54.2 | 64.8 | 79.6 |
| Lowest Temperature |  | 2.1 | 9.8 | 22.6 | 39.5 |
| Monthly and Anmual Ranges | 69.7 | 4.1 | 41.4 | 42.2 | 40.2 |
| Mean Maximum Temperature | 30.48 | 31.85 | 12.10 | 46.51 | 63.40 |
| Mean Minimmm Temperature | 18．53， | 79.71 | 30.45 | 32.92 | 47.13 |
| Mlean Daily Range ．．．．．．． | 11.91 | 12．15 | 31.62 | 13.62 | 16.26 |
| Greatest Daily Range | 39.5 | 21.9 | 20.3 | 27.2 | 25.4 |
| Mean Meight of Barometer Dillerence from average（12 years）$\qquad$$\qquad$ | 29.6770 | 29.6321 | 29.4125 | 29.5350 | 29.6595 |
|  | ＋．0472 | $+.0190$ | －2159 | －． 0721 | $+.0763$ |
| Highest l3arometer | 30.312 | 30.002 | 30.255 | 30.016 | 29．956 |
| Lowest Barometer | 25.934 | 23.577 | 28.25 | 25.993 | 29.224 |
| Monthly and Amual Ranges | 1.377 | 1.125 | 1.969 | 1.053 | 0．76： |
| Mrean Humidity of the Air | ． 51 | ． 79 | ．75 | ． 63 | ． 67 |
| Mean Elasticity of Aqueous Var | 126 | ． 114 | ITS | 151 | ． 298 |
| Mean of Cloudincss ．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．．． | 0.72 | 0.74 | 0.65 | 0.59 | 0.41 |
| Resultant Direction of the Wind Resultant Velocity of the Wind $\qquad$ Mean Velocity（Afiles jer hour） $\qquad$ Differenec from average（12 years） $\qquad$ | Sl St | N 54 W | 20）w | 20 36 | ズフ2 |
|  | 3.17 | 2.72 | 1.96 | 2.33 | 1.59 |
|  | 8.76 | 5.50 | 10.59 | 10．79 | 5.70 |
|  | ＋1．12 | ＋0．69 | ＋2．28 | ＋3．21 | $-0.60$ |
|  |  |  | 4.031 | 2.59 |  |
|  |  | －0．58s | ＋2．501 | $\div$ | ＋0．105 |
|  |  |  |  |  | 11 |
| Total Amount or Snow（in inches）． Difference from averase（ 17 years） $\qquad$ <br> Number of Days Snow． $\qquad$ $\qquad$ | 16.4 | 8.3 | 1.0 | 1.2 | 0.0 |
|  | 2.59 |  | S．25 | 1.18 | 0.08 |
|  | $19$ | 1． | 8 | S | 0 |
| Number of Fair Days | 10 | 9 | 10 | 15 | 20 |
| Number of Auroras ohserved Possible to see Aurora（No．of Nights） | 18 | 3 | $\stackrel{3}{7}$ | 7 |  |
|  |  | 11 | 17 | 17 | 22 |
| Yumber of Thunder | 0 | 1 | 2 | 0 | 5 |

## 241

REGISTER FOR THE YEAR IS59.
Toronto, Cancada West.
Lafe Ontabio, 109 feet. Approximate Elbfation adove tme Sed, 342 feet.


Vox. V.
24.2


जSEI| नल๓ーज
REMARLS ON TORONTO METEOROLOGIOAL REGISTER FOR DECEMBER.

les. which was the and al and absolutedy the 5reatest recorded here in l)ecember. The Mean Yelocity of the Wind was 2.75 miles per hom above the average of 12 years. The mean Yelocity of the lyind was 2.5 milos per honr above the averake of 12 years. unusual depth or suov.
COMPARATIVE TABLE FOR DECEMBER.

2.43

## Itighest Barometor Lowest Barometer

 $60^{\circ} 3$
range

会号 Mean maximum Temperature
 OE (Teast daily range.................

 Maximum Solar. Aurora olserved on 5 nights, viz. on $13 t h, 14 t h$, 1sth, 23 ru, and $30 t h$. Possible to sec Auroma 11 nights; impossible on 20 nigits. Shaning on 3 days,- -depth 1.03 j inches ; duration of fall 23.5 hours. Then ot cloudiness = 0.73 $10 \mathrm{p} . \mathrm{ml}$, meath, $=0.6 \mathrm{G}$.
Sums of the components of the itmospheric Cubront, expressell in miles.
1631.23 421G.0s.
Resnltant dircetion N. $53^{\circ}$ V. ; Resultant Felocity

Most wind day........................ 17th...Mean velucity 20.49 miles per hour.
 1st. Fos 3 to $4 \mathrm{p} . \mathrm{m}$. Very mihk day:

Gth. Dense Fos 10.15 a. m. to 7 j , m. Very mild day. 132h. Beautiful display of Anroral Light and Streamors tingel with parple and 17th and 18th. Stormy days. Wind very high, with heavy fall of snow, 2ath and 20th. Yery colk, storny days; higli wind, and heavy fall or snow.


## 244



## 245

30,142 at $8 \mathrm{n} . \mathrm{m}$ ．on 2nd， 3 fonthly range $=\|$ The mean Temperature for the month of January， 8860 ，differed but little from tive perage of 21 years．The Rain and Snow，were both less than the average；the former，by 0.703 －the latter，by $4.5 \boldsymbol{L}^{2}$ inches．The velocity of the Wind，was 1.62 miles per hour above the average of 13 years．The month was therefore
COMPARATLVETABLE FOR JANUARY．
$\xrightarrow{\left[\begin{array}{l}\text { Mean } \\ \text { Velocity }\end{array}\right.}$
 miles per hour above the average of 13 years．The month was therefore dry，and willdy，and was characterized by some very rapid changes of
Resultant


7.75

0 ositex 0
：：：：：：：©
을 $1 \vdots \vdots \vdots$


 traspraturus．
－
3．


$$
\left.\because \frac{29083}{47_{5}^{\circ}}\right\} \text { Mrean daily range }=12025 .
$$

$\qquad$
首： Warmest day
Coldest dny Maximum \｛ Solar
点定 Ifinimum temperature ．． ．

$$
T \mathrm{~cm}
$$


Aurora on 16 nights；implessible on 15 nights． 50.5 hours．
South．
1848.85
응웅

$$
\begin{aligned}
& \text { mperature } \\
& \text { mperature }
\end{aligned}
$$

 cloudy hom observed，midnight，wean $=0.47$ ．
sums of the compononts of the Atmosphoric Current，expressed in Miles．
I eny volocity of the wind 0.37 miles per hour．
ITost windy day．：soth－ircan velocits， 20.31 miles per hour．$\}$ Difference

| Most windy hour， 1 to 2 p ．m．－Mlean velocity， |
| :--- | cast wingy how， 7 ，

2nd．Imperfect Lunar Halo，from 9.30 p．m．－6th．Perfect Lunar Malo，from 11.30
m．－
 mild day：－sist．Woll deflned Lumar Halo，from 6.40 to $7.50 \mathrm{p}, \mathrm{m}$ ．
Great change of temperature from 30 th to 31 st ．Nean temperature， $30 h_{1}=23^{\circ} 35$ $31^{\circ} 0$ ．Jininume on 31 st，at midnight，$-5^{\circ} 1$ ．Range in 32 hours $=39^{\circ} 1$ ．
Tho Resultant Direction and Velocity of the Wind for the month of January，from
18 is to 8800 inclusive，were respectively N． $76^{\circ}$ W．，and L .09 milos．

## 246

MONTELY METEOROLOGICAL REGISTER, ST. HARTIN, ISLE JESUS, CANADA EAST—DECEMBER, 1859. (NINB MILES WEST OF MONTREAL,


## 247

MONTHLY METEOROLOGICAL REGISTER,

REMARIS ON THE ST. MARTIN, ISLE JESUS, MEMEOROLOGICAL REGISTER FOR DECEMBER, 1859.
Barometer ...... $\left\{\begin{array}{l}\text { Irowest, the } 20 \text { th day } \\ \text { Monthly Drean ......... }\end{array}\right.$
(Highest, the 3rd day ..... 30.720 ..... 29.410
\{ Monthly Irean ..... 29.971
IIfighest, the 1st day ..... $42^{\circ} 1$
Thermometer Lowest, the $29 t h$ day ..... $32^{\circ} 6$
$\left\{\begin{array}{l}\text { Monthly Mean. } \\ \text { Monthly Rango }\end{array}\right.$ ..... 8093 ..... $39^{\circ} 0$
Grentest intensity or the sun'sial ..... $36^{\circ} 6$
Mean of Ifumidity ..... 808
Rain tell on 3 days, amounting to 1.450 inches; it was raining 26 hours 20 minutes.
Snow fell on 14 days, amounting to 23.87 inches; it was snowing 150 hours 30 minutes
Least prevalent wind, S.
Most windy day, the 4th day; mean miles per hour, 15.30
Least windy day, the 2sth day ; mean miles per hour. 0.14.
Aurora Borealis visible on 3 nights.
Iunar Halo visible on 1 night.
zodiacal Light visible on 1 night.
Ozone was present in large quantities.
The Electrical state of the atmosphere has indicated high tension.
remarks on the st. martin, isle jesus, meteorological register FOR JANUARY, 1860.
Barometer ...... $\left\{\begin{array}{l}\text { Lowest, the } 21 \text { st day }\end{array}\right.$ ..... 30.455 $\left\{\begin{array}{l}\text { Lowest, the } 21 \\ \text { Monthly Mean }\end{array}\right.$ ..... 29.314
Mronthy mean ..... 29.861 ..... 29.861
(Highest, the 1otil uay ..... 1.141 ..... $46^{\circ} 4$
Towest, the 3rd day
Towest, the 3rd day Thermometer. ..... $13^{\circ} 15$
Monthly Mrean ..... $71^{\circ} 8$
Greatest Intensity of the Sun's Rays ..... $66^{\circ} 4$
Lowest point of 'Ierrestrial Radiation ..... $-27^{\circ} 1$Mean of Humidity786Most prevalent wind, tho W. by S
Least prevalent wind, E.
Most windy day, the 25 th day; mean miles per hour, 24.S8.
Least windy day, tho ist day; mean miles per hour, 0.21 .
Aurora Borealis visible on 1 night.
Lunar Halo visible on 1 night.
Solar Halo visible on 1 day.The elentrical state of the atmosphere has indicated moderate intensity.

MEAN RESULTS OF METEOROLOGICAL OBSERVATIONS AT HAMILTON, C.W., FOR THE YEAR 1859.

| 1859. | TMERSOMETER. |  |  |  | BAROMETER. |  |  | DA |  | TEARS. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Months. |  |  |  |  | ITean. | 容 |  |  | $\stackrel{\text { ¢ }}{\text { ¢ }}$ | Mcan Temperature of |
| January | 26.45 | 27.9 | 27.175 | 49 -29 | 29.707 | 30.20 | 23.96 | 5 |  | 1847...48.163 |
| February ......... | 27.7 | 29.5 | 28.6 | $54-3$ | . 614 | 29.34 | . 34 | 11 | 17 | 1848...49.295 |
| March ........... | 37.096 | 38.526 | 37.811 | 6317 | . 437 | 30.17 | . 41 | 4 | 19 | 1849...48.105 |
| April. | 41.11 | 40.8 | 41.11 | $65 \quad 20$ | . 512 | . 00 | 29.00 | 2 | 21 | 1850...48.732 |
| May | 58.77 | 57.13 | 57.95 | 86 | . 688 | . 00 | . 36 | 3 | 23 | 1551...48.756 |
| June . | 64.33 | 61.73 | 63.03 | 9237 | . 631 | . 00 | . 37 | 1 | 23 | 1852...48.248 |
| July ............... | 72.51 | 70.65 | 71.58 | ${ }^{97} 50$ | . 518 | . 12 | . 35 | 2 | 20 | 1853..49.474 |
| August............... | 70.00 | 69.66 | 69.83 | 874 | . 672 | 29.55 | . 69 | 1 | 23 | 1854...43.102 |
| September ...... | 58.36 | 58.73 | 5 S .66 | 7932 | . 698 | 30.06 | . 16 | 2 | 20 | 1855...47.316 |
| October........... | 45.58 | 46.29 | 45.935 | $76 \quad 25$ | . 66 | 29.96 | - 28 | 4 | 21 | 1856...44.888 |
| November .. | 39.53 | 41.60 | 40.57 | 73.24 | -679 | 30.20 | . 00 | 3 | 21 | 1857...45.868 |
| December. | 22.36 | 22.26 | 21.81 | 60-6 | . 674 | . 17 | 10 | 5 | 13 | 1858...43.142 |
| Mean Temp... | . $\cdot \cdot$ | ...... | 46.936 | .... | 29.637 | . | .. | 33 | 240 | ... |


[^0]:    - The general reader should understand that this classification is a murely artificial oue, intended solely to lead to the recoguition of minerals by weans of their more obvious or easily determined characters-somewhat on the principle of the Linnean elassification of plants.

[^1]:    * For an explanation of these characters, technical terms, \&c., see Part I.

[^2]:    *For the localities mentioned in these descriptions, we are very largely indebted to the publications of the Canadian Geological Survey, and especcially to the Esquisse Géologiquo du Canada, by Sir W. E. Logan and T. Sterry Cfunt. We shail be greatiy obliged to our readers for any information respecting localities of Canadian minerals; and more especially, if a small fragment of tho substance referred to in the information, be furnished at the same time. 1 piece no larger than our ordinary pea will bo of sufficient size. Although we are constantly receiving specimens of different kinds for examination, the exact localities of these are generally kept secret by the senders, in the belief that something has been discovered of more than usual value.

[^3]:    "The term "aspect," as here employed, refers not merely to the " lustre" of the substance, but to its geneml apyearance and characters, taken together. Thus but few, if any, snecimens of 30 g Iron Ore exhibit a metalliclustre properly so-called; and yet most persous, on takins up one of these syecineus, would refer it at once to the metallic sroup, or, in other words, weuld consider il to be a metalic substance of some kind.

[^4]:    - The following Table shows (in a descending order) the positions of the rock-groups recognised in Canada. These groups, with their various subdivisions, \&c., will be discussed in detail in one of the sueceeding Parts of this serios of papers, but the present Table may prove useful in the mean time.

    > Modera or Post-Tertiary Deposits.
    > The true Drift Formation.
    > (Here a great jreak occurs in the geolegical scale as ropresented in Canada.) Carboniferous Formation (developed in part only in Gaspe.)
    > Devonian Pormation,
    > Silurian Formation,*
    > IIuronian Pormation,
    > Laurontian Formation.

[^5]:    * The great fossiliferous formation of Canada. Metamorphosed or rendered crystalline in part, in the so-called "metamornhic district" of the Eastern Townships and surrounding region.

[^6]:    - This native expression: 'put out fire,' signifies to set the prairie on fire.

[^7]:    * These magnetic iron grains may very probably contain a portion of the magnesia found in all the Meteorites of this Section. The verification of this idea, after Mammelsberg's dis--covery of magnesis in somo of the Vesnvian iron ores, would be of no little interest.-E, J.O.

[^8]:    *Serret makes a forther distinotion among functions of the same order as being of different degrees, but his definition is strangely obscure, and this distinction does not appear to have any effect whatever on the subsequent reasoning. His use of the term degree is also inconsistent with the sense in which the word is employed in Trantrel's memoir.

[^9]:    * Serret remarks that all these roots are cifferint, but his proof of this is curiously erroneous; still it is otherwise easy to see that such must be the case. He adds, however "Au surplus, cette remarque n'est pas indispensable pour ce qui va suivre."

