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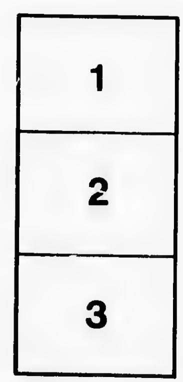
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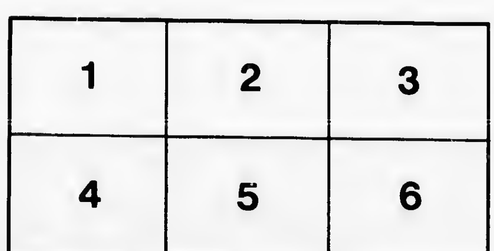
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DOMINION SERIES OF DRAWING BOOKS.

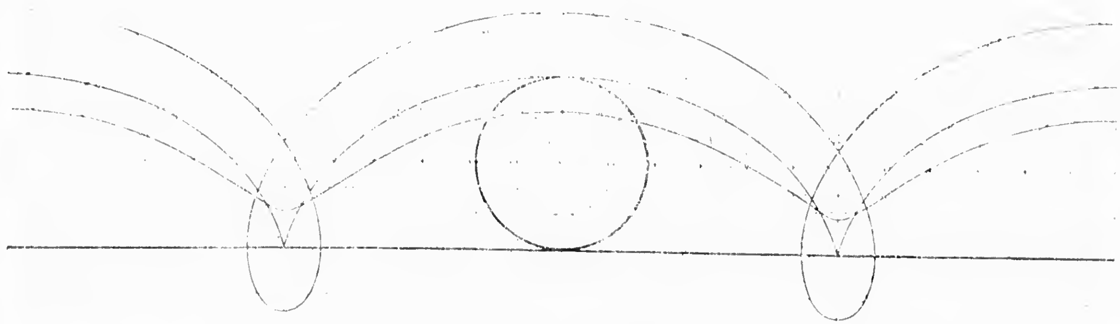
GEOMETRICAL · DRAWING

FOR THE USE OF

SCHOOLS AND COLLEGES

BY

C. H. McLEOD, Ma.E.,
Professor in the Faculty of Applied Science, McGill University, Montreal.



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PREFACE.

GEOMETRICAL Drawing consists, as the name implies, in the application of Geometry to Drawing.

The methods employed are for the greater part capable of rigid proof, and although the construction only is given in the text, the student should in all cases understand the method as well as execute the drawing.

The problems are especially selected as a preparation for the study of Descriptive Geometry and as an introduction to Kinematic Drawing and Machine Design. The dotted lines in the drawings represent construction lines. These should be replaced by the student as light but continuous pencil lines. The full lines represent either what is given or what is required. The student should make the latter somewhat heavier than the former and both much heaver than the construction lines. It is an excellent practice to "ink-in" the given and required lines, making the latter thicker than the former. The inking-in should be done with Indian ink and a drawing pen. All drawings should be made on light cartridge paper and should be executed with the greatest possible neatness and accuracy.

The instruments required are the following:—

A small drawing board and pins; compasses with pencil point; a straight-edge; a 3 H. or 4 H. drawing pencil and rubber eraser. For inking-in a drawing pen and a pen point for the compasses are required.

The drawing board should not be larger than 20″ × 26″. The Sloyd board answers the purpose excellently. The pencil point to the compasses should be of 3 H or 4 H hardness and always kept quite sharp. The steel point should be as sharp as a needle.

The edge of the straight edge should be $\frac{1}{16}$th. of an inch thick, not bevelled. It need not be more than 6″ long. After some progress has been made in the work, a small T square and a set square may be used to advantage, but should not be employed at first. A set square will, however, generally be the most convenient form of straight edge to use.

This compilation has been used in manuscript during one term in the Faculty of Applied Science, McGill University, under the direction of Prof. C. B. Smith, to whom the author's thanks are due for assistance in the selection of the problems and revision of the copy.

C. H. McLEOD.

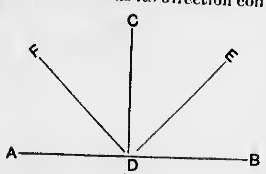
McGILL COLLEGE, MONTREAL,
August 27th, 1896.

DEFINITIONS.

A point has position in space but is without magnitude.

A line has length but is without breadth or thickness.

A straight line is the shortest distance between two points and a **curved line** has its direction continually changing.



A vertical line is perpendicular to the horizontal and would be represented by a line in the position **C D**.

A horizontal line is a level line parallel to the horizon and would be represented by a line in the position **A B**.

A perpendicular line is one which forms right angles with another but is not necessarily vertical. **C D** is perpendicular to **A B** and **E D** is perpendicular with reference to **F D**.

Parallel lines are situated in the same plane and are everywhere the same distance from one another.

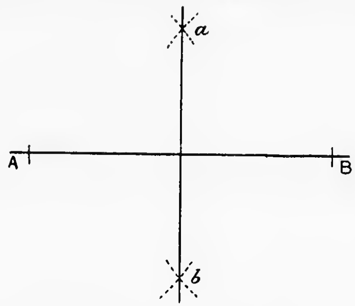
A rectilineal figure is a plane figure bounded by any number of straight lines.

The term **polygon** is usually applied to figures having more than four sides. Five-sided figures are called **pentagons**; six-sided, **hexagons**; seven-sided, **heptagons**; eight-sided, **octagons**; nine-sided, **nonagons**; ten-sided, **decagons**; eleven-sided, **undecagons**; and twelve-sided figures, **duodecagons**.

Regular polygons are figures having equal sides and equal angles.

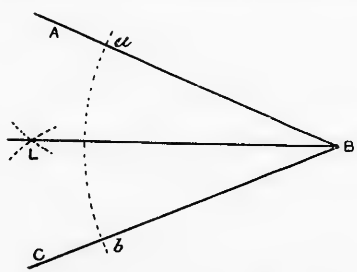
NOTE.—A large number of the usual definitions have been omitted as they are without doubt well known to all who are likely to use this book.

PROBLEM 1.—To Bisect a Given Straight Line, **A B.**



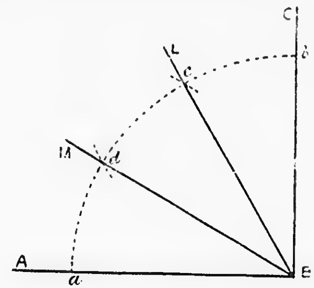
From **A**, as centre, describe an arc having a radius greater than one-half **A B.**
From **B**, as centre, describe, with the same radius, another arc cutting the former in **a** and **b.**
Join, by a straight line, the points of intersection **a, b.**
The line **a b** will bisect **A B** and will itself also be bisected at the same point.

PROBLEM 2.—To Bisect a Given Angle, **A B C.**



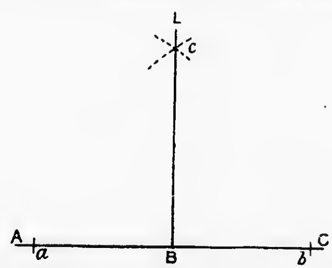
With **B** as centre, describe any arc cutting the lines **A B, B C**, in **a** and **b.**
From **a** and **b** as centres, with any length as radius, describe arcs meeting in **L.**
The straight line **L B** will bisect the given angle **A B C.**

PROBLEM 3.—To Trisect the Given Right Angle **A B C.**



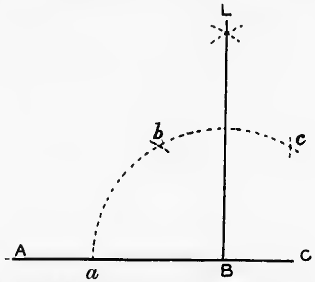
With **B** as centre, describe any arc cutting the sides **A B, B C**, in **a** and **b.**
With the same radius and the centres **a, b**, describe arcs cutting the arc **a b** in **c** and **d.**
The straight lines **B L, B M**, joining **B** to **c** and **d**, will trisect the right angle, **A B C.**

PROBLEM 4.—From a Given Point **B**, in a Straight Line **A C**, to Draw a Perpendicular to the Line.



Make **B a** equal to **B b** and from the points **a** and **b** as centres, describe equal arcs meeting in **c.** The straight line **B L**, joining **B** to **c**, will be the required perpendicular.

PROBLEM 4 (a).—FROM A GIVEN POINT **B** IN A STRAIGHT LINE **A C** AND NEAR ONE END OF IT, TO DRAW A PERPENDICULAR TO THE GIVEN LINE.

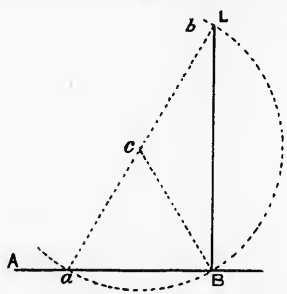


From the point **B**, describe any arc **abc** and from **a** describe with a radius **aB** an arc cutting the former arc in **b**, and from **b** with the same radius, again cut the first arc in **c**.

From **b** and **c** with any radius, describe two arcs meeting in **L**.

The straight line **B L** will be the required perpendicular.

PROBLEM 4 (b).—FROM A GIVEN POINT **B** AT THE END OF A LINE **A B**, TO ERECT A PERPENDICULAR TO THE LINE.

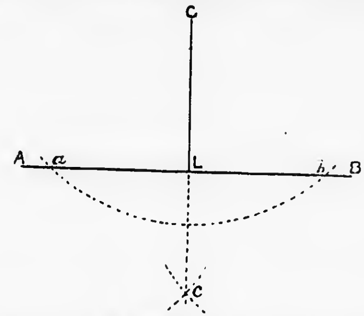


Take any convenient point **c** away from the line, and with radius **c B**, describe the arc **a B b**.

Join **a c** and produce it to cut the arc in **L**.

The line **L B** will be the required perpendicular.

PROBLEM 5.—FROM A GIVEN POINT **C** WITHOUT A GIVEN LINE **A B**, TO DRAW A PERPENDICULAR TO THE LINE.

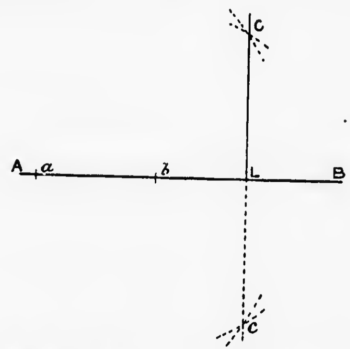


From **C** as centre, describe an arc cutting the line in the points **a** and **b**,

From **a** and **b** as centres, with a suitable radius, describe arcs cutting each other in **c**

Join **C c**, cutting **A B** in **L**. **C L** will be the required perpendicular.

PROBLEM 5 (a).—FROM A GIVEN POINT **C** WITHOUT A GIVEN LINE **A B** AND NEAR ONE END OF IT, TO DRAW A PERPENDICULAR TO THE LINE.

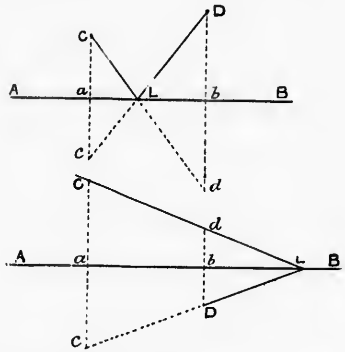


From any point **a** in the line, with a radius **a C**, describe the arc **C c**.

From any other suitable point **b**, with radius **b C**, describe a second arc cutting the former in the points **C**, **c**.

Join **C c**, cutting **A B** in **L**. **C L** will be the required perpendicular.

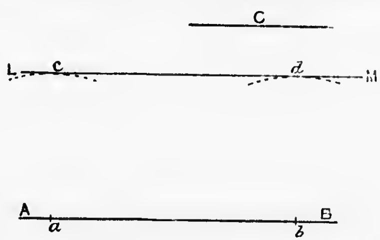
PROBLEM 6.—From Two Given Points C and D to Draw Lines Meeting in a Given Adjacent Line A B, and Making Equal Angles with it.



Draw the lines **C c** and **D d** perpendicular to **A B**, and make **c a** equal to **C a** and **d b** equal to **D b.**

Join **C d** and **D c.** These lines will meet in **L** on the line **A B** and make equal angles with it.

PROBLEM 7.—To Draw a Straight Line Parallel to a Given Straight Line A B, and at a Given Distance, C, from it.



From any two points **a** and **b**, in **A B**, describe arcs **c** and **d** with a radius equal to the given distance **C**.

The line **L M** touching the arcs will be the required parallel line.

PROBLEM 8.—Through a Given Point C to Draw a Line Parallel to a Given Line A B.

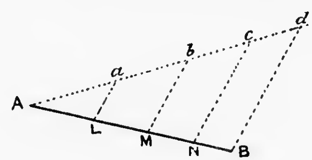
About **C** as centre describe an arc of any radius, cutting **A B** in **b.**

With the same radius and **b** as a centre, describe the arc **C c**, cutting **A B** in **c.**

From **b** measure a chord **a b** equal to the chord **C c.**

The line **C a L** will be the required parallel line.

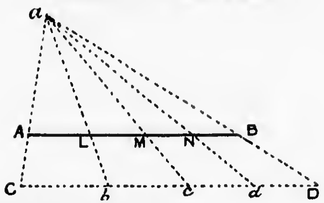
PROBLEM 9.—To Divide a Straight Line AB into Parts having any Given Ratio to Each Other.



First Method.—From **A** draw any line and measure off from **A** the parts **A a, a b,** etc., . . . **c d,** adopting any unit of measurement, so as to obtain the desired ratio of the parts.

Join **d B** and through the points **a, b,** etc., draw lines parallel to **d B**, meeting **A B** in **L, M,** etc., and dividing **A B** in the desired ratio.

PROBLEM 9.—*Second Method.*



Draw a line **C D** parallel to the given line **A B.**

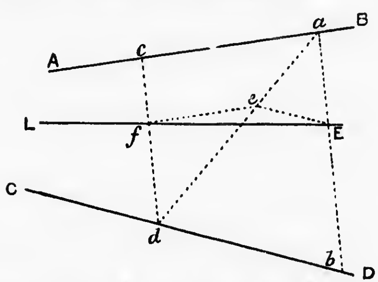
Divide **C D** in the desired ratio as in the first method. (**C D** should have either a greater or less length than **A B,** preferably greater).

Join **A** to **C** and **B** to **D** and produce these lines to meet in **a.**

Join **a b, a c, a d,** and produce them (if necessary) to cut the given line in the points **L, M, N.**

The lines **A L, L M, N B,** will be to each other in the desired ratio.

PROBLEM 10.—From a Given Point E to Draw a Straight Line, which if Produced would Pass through the Point of Intersection of Two Inclined Lines A B and C D, without Producing the Latter to Intersection.



Draw a straight line through **E,** meeting **A B** in **a** and **C B** in **b.**

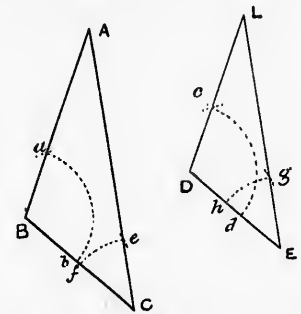
In any convenient position draw another line parallel to **a b,** meeting the two given lines in **c** and **d.**

Join **a d** and through **E** draw a line **E e** parallel to **C D,** cutting **a d** in **e.**

Through **e** draw **e f** parallel to **A B,** cutting **c d** in **f.**

The line **E f L** will be the desired line.

PROBLEM 11.—Upon a Given Straight Line D E to Construct a Triangle Similar to a Given Triangle A B C.

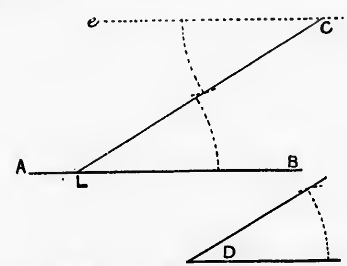


With **B** and **C** as centres and any radius, describe arcs of circles meeting the sides of the triangle in the points **a, b, e** and **f.**

With **D** and **E** as centres, describe with a radius equal to **Ba** or **Cf,** the arcs **cd, hg;** make the chord **cd** equal to the chord **a b,** and the chord **h g** equal to the chord **e f.**

Join **D c** and join **E g,** and produce these lines to meet in **L. D E L** will be the required similar triangle.

PROBLEM 12.—Through a Given External Point C to Draw a Straight Line to make a Given Angle D with a Given Line A B.



Through **C** draw a line **C e,** parallel to **A B** (Prob. 8.)

From **C** draw a line **C L,** making an angle with **C e** equal to **D,** and meeting **A B** in **L.** (Prob. 11.)

The line **C L** will make the required angle **D** with the line **A B.**

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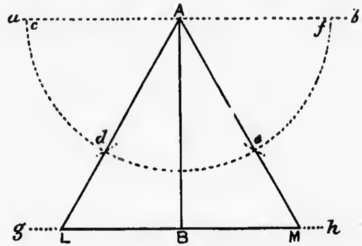
PROBLEM 13—To Construct an Equilateral Triangle Having a Given Vertical Height **A B.**



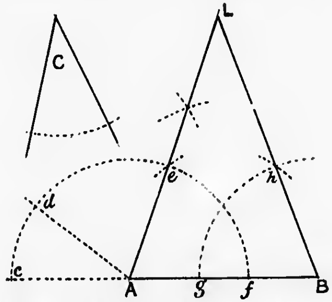
Through the points **A** and **B** draw lines **a b** and **g h** perpendicular to **A B.** (Prob. 4).

About **A** with any radius, describe a semicircle and trisect it by the chords **fe, ed, d c,** equal to radius.

Join **A d, A e,** and produce them to meet the line **g B h** in **L** and **M.**

A L M will be the required equilateral triangle.

PROBLEM 14.—To Construct an Isosceles Triangle Upon a Given Base **A B,** With a Given Vertical Angle **C.**



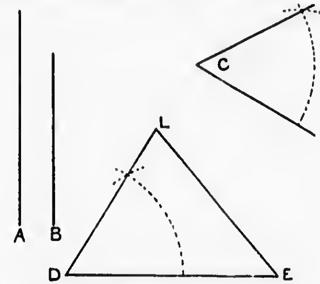
Produce the base **B A** and construct an angle **c A d** equal to the given angle **C.** (Prob. 11).

Bisect the angle **d A B** by the line **A e** (Prob. 2).

At **B** make an angle **g B h** equal to **e A f** and produce **A e, B h,** to meet in **L.**

A B L will be the required isosceles triangle.

PROBLEM 15.—To Construct a Triangle Having Two Sides Equal Respectively to the Given Lines **A** and **B,** and the Angle Between Them Equal to the Given Angle **C.**

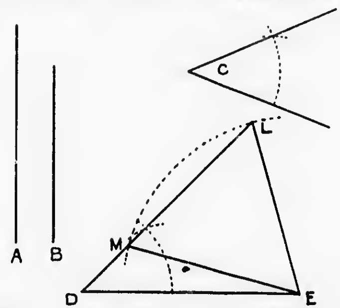


Draw the line **D E** equal to **A** and at **D** make an angle equal to **C.**

Make **D L** equal to **B** and join **L E.**

The triangle **D L E** will be the required triangle.

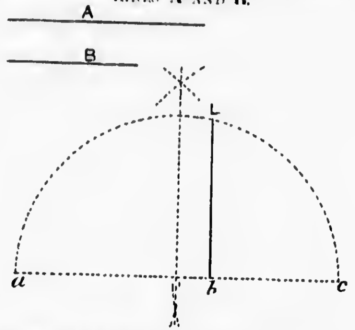
PROBLEM 16.—To Construct a Triangle Having Two Sides Equal Respectively to **A** and **B,** and an Angle Equal to the Given Angle **C,** Situated Opposite to One of the Given Sides.



Draw the line **D E** equal to **A** and at **D** make an angle **E D L** equal to the given angle **C.**

From **E** as an a centre describe an arc with a radius equal to **B.** If this arc cuts the line **D L** in two points **M** and **L,** the two triangles **D M E, D L E,** will fulfill the conditions of the problem. If the arc merely touches the line **D L,** there is but one solution, and if the radius is too short to reach the line, the case is an impossible one.

PROBLEM 17.—To Draw a Mean Proportional to Two Given Lines A and B.



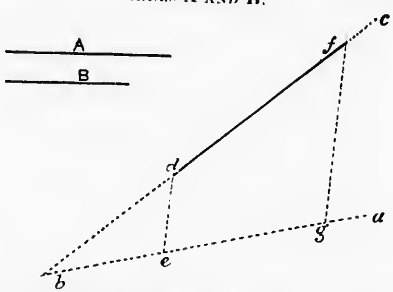
In the line a b c make a b and b c equal to A and B respectively.

On a c as a diameter describe a semicircle.

At b erect a perpendicular b L, cutting the semicircle in L. This line will be the required mean proportional.

Note.—The square on b L is equal in area to the rectangle having adjacent sides equal to A and B.

PROBLEM 18.—To Find a Third Proportional to Two Given Lines A and B.



From any point b draw two straight lines a b, b c.

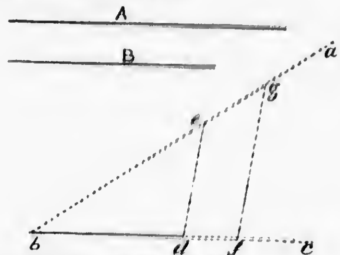
Make b d equal to A and b e equal to B. Make also e g equal to A or b d.

Join d e and through g draw a line g f parallel to d e, meeting b c in the point f.

The third proportional to A and B will be d f, and in this case d f will be greater than either A or B.

Note.—If B be taken as unity $A^2 = df$. In this way the square of any length may be obtained, the unit being given.

PROBLEM 18 (a).—To Find a Third Proportional to Two Given Lines A and B, the Required Line to be Less than Either of the Given Lines.

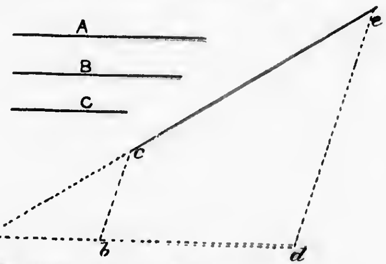


From any point b draw two straight lines a b, b c.

Make b g equal to A and b f equal to B. Make also b e equal to B or b f.

Join g f and through e draw e d parallel to g f, cutting b c in the point d. b d will be the required third proportional.

PROBLEM 19.—To Find a Fourth Proportional to Three Given Lines A, B and C, when the Required Line is to be Longer than Any of the Given Lines.



From any point a draw two straight lines a d, a e.

Make a b equal to C and a c equal to B. Make also b d equal to A.

Join b c and draw d e parallel to b c, cutting a e in the point e; c e will be the required fourth proportional.

If it is required that the fourth proportional shall be shorter than any of the given lines, make the first line a b equal to A and b d equal to C. The same construction will give a fourth proportional line similarly situated to c e and shorter than the given lines.

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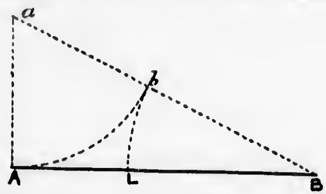
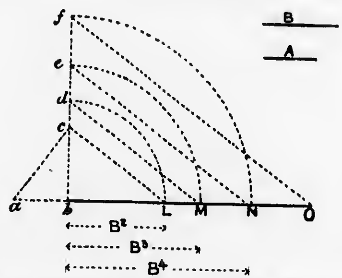
PROBLEM 20.—TO DIVIDE A STRAIGHT LINE A B INTO EXTREME AND MEAN RATIO, (*i.e.*) IN MEDIAL SECTION.



At one end of the line **A** erect a perpendicular **A a** and make it equal to one-half **A B**. Join **a B**.

Make **a b** equal to **a A** and **B L** equal to **B b**. The point **L** will divide the line **A B** in extreme and mean ratio, that is, **A L : L B : : L B : A B**.

PROBLEM 21.—TO DETERMINE ANY REQUIRED POWER OF A GIVEN LINE B, THE UNIT A BEING GIVEN.

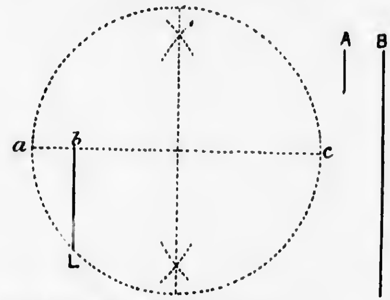


Draw two lines **a b**, **b c**, equal respectively to **A** and **B**, and at right angles to each other.

Join **a c** and draw **c L** at right angles to **a c**, meeting **a b** produced in **L**. **b L** will be equal B^2 to the unit **A**.

On **b c** produced make **b d** equal to **b L**, and draw **d M** parallel to **c L**, meeting **b L** produced in **M**. **b M** will equal B^3 to the unit **A**, and similarly **b N**, **b O**, etc., may be obtained respectively equal B^4, B^5, etc., etc.

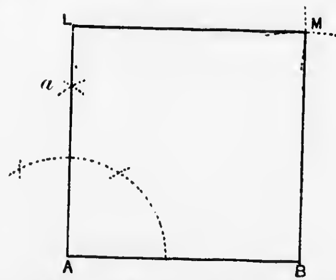
PROBLEM 22.—TO FIND THE SQUARE ROOT OF A GIVEN LINE B, REFERRED TO ANY UNIT A.



Make **a b** equal to **A** and **b c** in **a b** produced equal to **B**.

Describe a circle on **a c** as a diameter and at **b** erect a perpendicular, meeting the semicircle in **L**; **b L** will be the square root of the length **B** for the unit **A**.

PROBLEM 23.—UPON A GIVEN LINE A B TO CONSTRUCT A SQUARE.

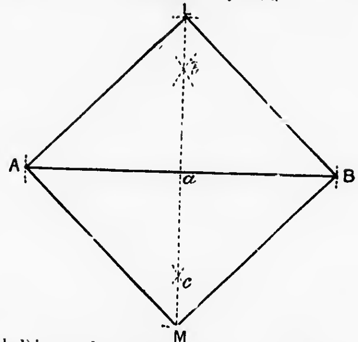


At **A** in **B A** erect a perpendicular **A a** (Prob. 4 *b*) and cut off **A L** equal to **A B**.

Similarly at **L** draw **L M** equal and perpendicular to **L A**; or parallel to **A B**, and equal to it.

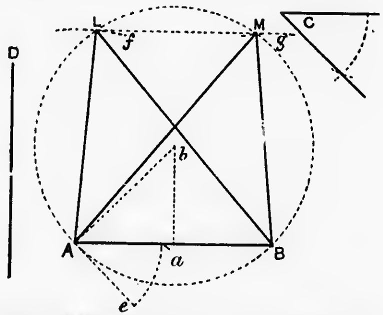
Join **M B**. **A L M B** will be the required square.

PROBLEM 24.—UPON A GIVEN LNIE **A B** AS A DIAGONAL, TO CONSTRUCT A SQUARE.



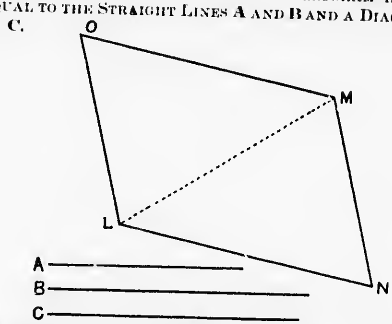
Bisect **A B** (Prob. 1) in **a**, and at **a** erect the perpendicular **b a c**.
Make **a L** and **a M** equal to **a A** and **a B**.
Join **A L, L B, B M, M A,** to form the required square **A L B M**.

PROBLEM 25.—UPON A GIVEN BASE **A B** TO CONSTRUCT A TRIANGLE HAVING A GIVEN VERTICAL ANGLE **C**, AND A GIVEN ALTITUDE **D**.



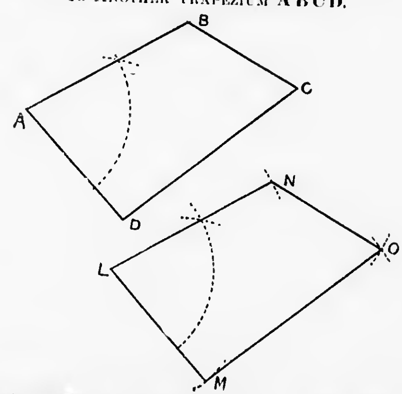
At **A** make the angle **B A e** equal to **C** (Prob. 11.) Bisect **A B** in **a**.
From the points **a** and **A** erect perpendiculars to **A B** and **A e** respectively, meeting in **b**.
From **b** as centre, with radius **b A**, describe a circle **L A B M**.
Draw the line **f g** (Prob. 7) parallel to **A B** and at the given distance **D** from it, meeting the circle in **L** and **M**.
The triangles **A B L** and **A B M** will have vertical angles equal to **C** and the required altitude **D**.

PROBLEM 26.—TO CONSTRUCT A PARALLELOGRAM HAVING SIDES EQUAL TO THE STRAIGHT LINES **A** AND **B** AND A DIAGONAL EQUAL TO **C**.



Make **L M** equal to **C**, and from **L** and **M** as centres with radii equal respectively to **A** and **B**, describe arcs meeting in **O**. Join **O L, O M**.
Then on the other side of **L M**, from the same centres with radii equal respectively to **B** and **A**, describe arcs meeting in **N**.
Join **M N, L N**, completing the required parallelogram **L O M N**.

PROBLEM 27.—TO CONSTRUCT A TRAPEZIUM SIMILAR AND EQUAL TO ANOTHER TRAPEZIUM **A B C D**.



Make an angle **N L M** equal to **B A D** (Prob. 11) and having the containing sides **N L, L M**, equal to **B A, A D**, respectively.
From the centres **N** and **M**, with radii equal respectively to **B C** and **D C**, describe arcs meeting in **O**.
Join the point **O** to **M** and **N**. **L M O N** will be the required trapezium.

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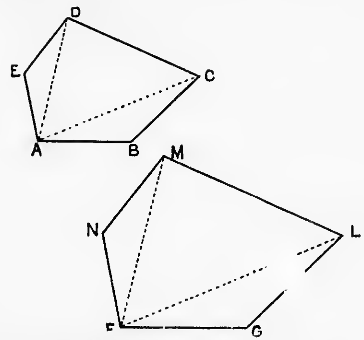
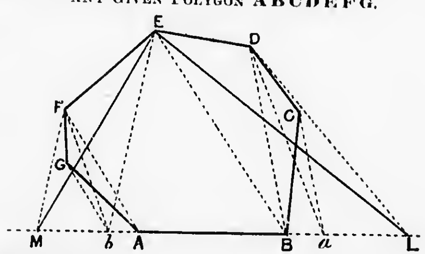
PROBLEM 28.—UPON A GIVEN STRAIGHT LINE **F G** TO CONSTRUCT A RECTILINEAR FIGURE SIMILAR TO A GIVEN RECTILINEAR FIGURE **A B C D E.**



Join **A C** and **A D** in the given figure.

Upon **FG** construct a triangle **F G L** similar to **A B C** (Prob. 11). On **F L** construct a triangle **F L M** similar to **A C D**; and on **F M** construct a triangle **F M N** similar to **A D E.** The whole figure **F G L M N** will be the required rectilinear figure.

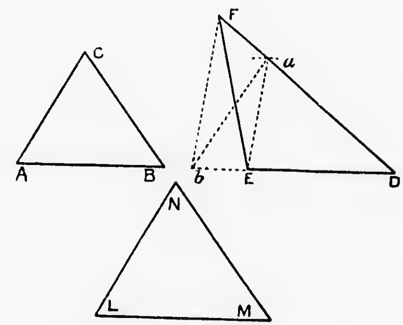
PROBLEM 29.—TO CONSTRUCT A TRIANGLE EQUAL IN AREA TO ANY GIVEN POLYGON **A B C D E F G.**



Produce any side **A B.**

Join **A F** and through **G** draw **G b** parallel to **F A.** Join **F b.** The six-sided figure **bFEDCB** will be equal in area to the given seven-sided figure. Now join **b E** and through **F** draw **F M** parallel to **E b.** Join **E M.** The five-sided figure **M E D C B** will be equal in area to the original seven-sided figure, and a four-sided figure of equal area may be obtained by joining **D M,** etc., and from that again a triangle of equal area obtained, but a better conditioned triangle will be formed by recommencing the process of reduction at **B,** joining **B D** and replacing the lines **BC, CD, D E,** by construction similar to that above described, by the line **E L.** The triangle **M E L** will thus be the required triangle.

PROBLEM 30.—TO CONSTRUCT A TRIANGLE WHICH SHALL BE EQUAL IN AREA TO ANY TRIANGLE **D E F** AND SIMILAR TO ANY OTHER TRIANGLE **A B C.**

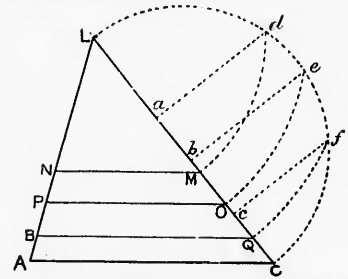


Find the point **a** in **D F** so that the altitude of the triangle **D a E** shall be the same as that of **A B C.**

Join **a E** and through **F** draw a line parallel to **a E** to meet **D E** produced in **b.** Join **a b.** The triangle **D a b** will have an altitude equal to **A B C** and an area equal to **D E F.**

Now find a mean proportional **L M** between **A B** and **D b** (Prob. 17), and on **L M** construct (Prob. 11) a triangle similar to **A B C.** **L M N** will be the required triangle

PROBLEM 31.—To Divide a Given Triangle **A L C** into any Number of Equal or Proportional Parts by Lines Drawn Parallel to One of the Sides.



Divide one of the sides of the triangle, as **L C** into the proportional parts (Prob. 9) in **a,:b** and **c.**

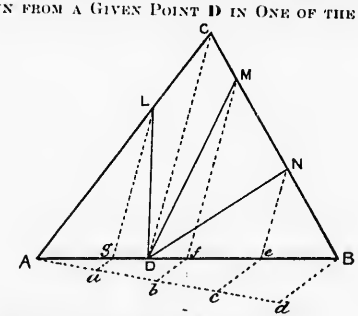
Describe a semicircle en **L C** as a diameter.

At the points **a, b** and **c.** erect the perpendiculars from the side **L C** to meet the semicircle in **d, e** and **f.**

With **L** as centre and **L d, L e, L f** as radii, describe the arcs cutting the side **A C** in **M, O** and **Q.**

From the points **M, O** and **Q.** draw lines parallel to the base **A C**, thus dividing the triangle into the required parts.

PROBLEM 32.—To Divide a Given Triangle **A B C** into any Number of Equal or Proportional Parts by Straight Lines Drawn from a Given Point **D** in One of the Sides.

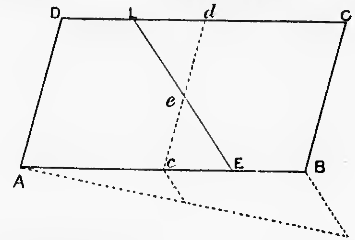


Divide the line **A B** in the required ratio in **g, f** and **e.**

Join **D C** and draw the lines **g L, f M, e N** parallel to **D C.**

The straight lines **D L, D M, D N** will divide the triangle in the given ratio **A g : g f : f e : e B.**

PROBLEM 33.—To Divide a Given Parallelogram **A B C D** into Two Parts having a Given Ratio to Each Other, by a Line Drawn from a Given Point **E** in One of the Sides.



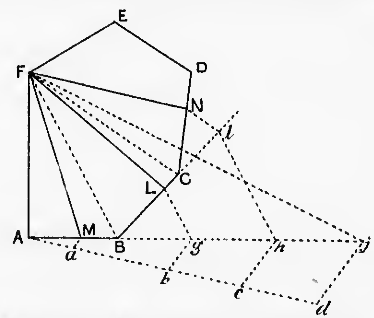
Divide **A B** in **c** in the required ratio.

Draw **c d** parellel to **A D** or **B C.**

Bisect **c d** in **e**, and draw the straight line **E L** passing through **e.**

E L will divide the parallelogram in the given ratio.

PROBLEM 34.—To Divide a Given Polygon **A B C D E F** having any Number of Equal or Proportional Parts by Straight Lines Drawn from One of the Angles **F.**



Draw the triangle **A F j** (Prob. 29) equal in area to the given polygon.

Divide the base of the triangle in the required ratie in the points **M g** and **h.**

Join **F B** and **F C.**

Through **g** and **h** draw lines parallel to **F B**, cutting **B C** and its productien at **L** and **l.**

Through **l** draw **l N** parallel to **F C**, meeting **C D** in **N.**

Join **F** to **L, M** and **N**, dividing the polygon in the desired ratio.

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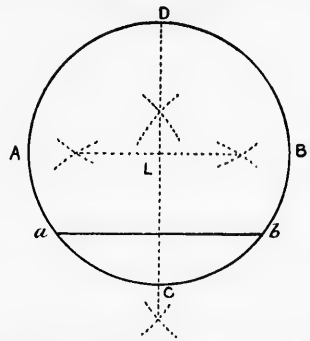
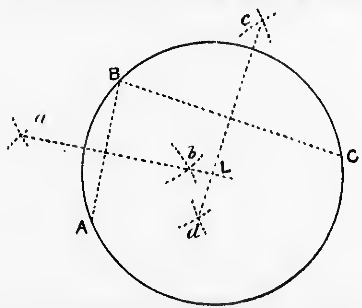
PROBLEM 35.—To Find the Centre and Radius of a Given Circle A B C D.



Join by a straight line any two points **a**, **b**, in the circumference of the circle.
Bisect **a b** and draw a line at right angles to it, through the point of bisection, meeting the circumference in **D** and **C**.
Bisect **C D** in **L**, which will be the centre of the circle.

PROBLEM 36.—To Describe a Circle the Circumference of Which Shall Pass Through Three Given Points A, B and C.

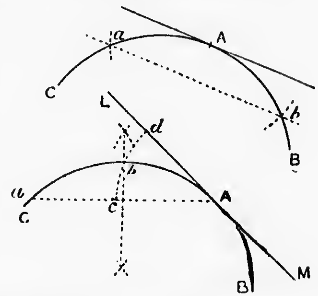


Join **A** to **B** and **B** to **C**. Bisect the lines **A B**, **B C**, by the perpendiculars **a b**, **dc**, meeting in **L**.
A circle described from **L** as a centre, with radius **L A**, will have the points **B** and **C** also contained in its circumference.

PROBLEM 37.—To Draw a Tangent at a Given Point A to a Circular Arc C A B.

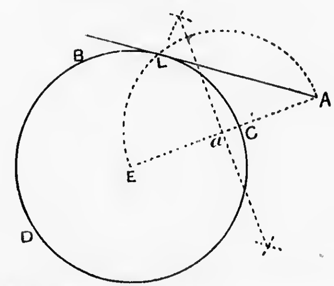
Fig. 1.

Fig. 2.



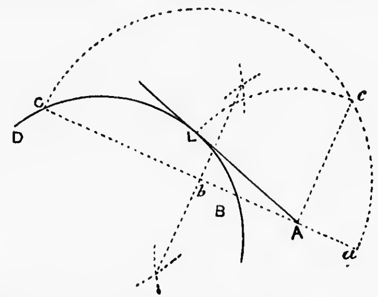
(*First Method, Fig.* 1.)—Take points **a** and **b** equally distant from **A**.
The required tangent will be a line drawn through **A** parallel to **ab**.
(*Second Method, Fig.* 2.)—Take any point **a**.
Join **A a** and bisect the arc **A a** in **b**.
With **A** as centre and **A b** as radius, describe the arc **c d**.
Make the arc **b d** equal to the arc **c b**.
The straight line **L M**, passing through **A** and **d**, will be the required tangent.

PROBLEM 38.—To Draw a Tangent to a Circle B C D from a Given Point A Without It.



Find the centre **E** and draw the straight line **E A**.
Upon **E A** as a diameter describe a semicircle **E L A**, cutting the circle in **L**.
The line joining **A** to **L** will be the required tangent.

PROBLEM 39.—To Draw a Tangent to a Given Circular Arc **DCB** from a Given External Point **A**, without Using the Centre of the Circle.



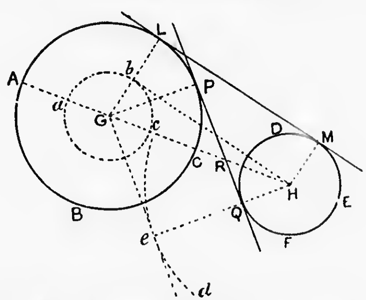
Draw a straight line through **A** cutting the arc in **B** and **C**. Produce **CA**, making **Aa** equal to **BA**. On **Ca** describe a semicircle.

At **A** draw the perpendicular **Ac** to the line **Ca**, meeting the semi-circumference in **c**.

From **A**, with **Ac** as radius, describe an arc, cutting the given arc in **L**.

The line **AL** will be the required tangent.

PROBLEM 40.—To Draw the Common Tangents to Two Given Circles **ABC**, **DEF**.



Draw any radius **GA** of the larger circle and cut off the part **Aa** equal to the radius of the smaller circle.

With **G** as centre and **Ga** as radius, describes a circle **abc**.

From the centre **H**, draw **Hb** tangent to **abc**. (Prob. 38.)

From the points **H** and **G**, draw perpendiculars to the line **Hb**, meeting the circles in **M** and **L**.

The straight line **ML** will be a tangent line, and a similar tangent line may be drawn on the opposite sides of the circles.

To Draw a Tangent Crossing the Line of Centres.

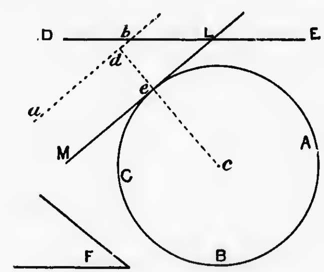
From **H** as a centre, with a radius **Hd** equal to the sum of the radii of the two circles, describe the arc **ed**.

From **G** draw a tangent to the arc **ed** (Prob. 38), and draw perpendiculars **He**, **GP**, to this line from the centres **H** and **G**, cutting the circles in **Q** and **P** respectively.

The straight line **PQ** will be the required tangent, and similarly another tangent line may be drawn on the opposite sides of the circles.

The point **R** will divide the line of centres proportionally to the radii of the circles.

PROBLEM 41.—To Draw a Line Touching a Given Circle **ABC** and Making a Given Angle **F** With a Given Line **DE**.



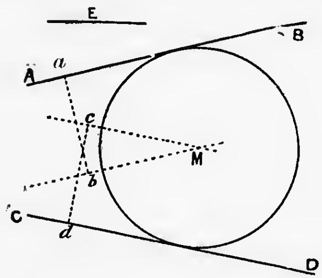
At any point **b** in the line **DE** make an angle **Dba** equal to the given angle **F**.

From the point **c** drop a perpendicular **cd** on **ba**, cutting the circumference in **e**.

At **e** draw a line **ML** perpendicular to **ce**.

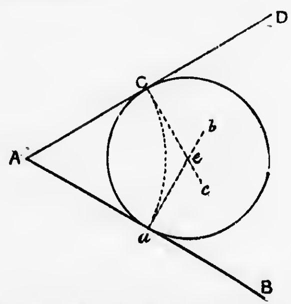
ML will be the required tangent.

PROBLEM 42.—To Describe a Circle Having a Given Radius E and Touching Two Given Straight Lines A B, C D.



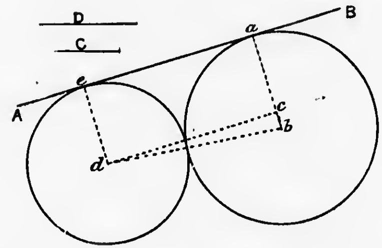
At any points **a** and **d** erect perpendiculars **a b** and **c d** to the lines **A B** and **C D** respectively, and both equal to the given radius **E**.
Through **b** and **c** draw lines parallel to **A B** and **C D**, respectively.
The circle described from the point of intersection **M** as a centre, with radius **E**, will touch the given lines **A B**, **C D**.

PROBLEM 43.—To Describe a Circle Touching Two Given Straight Lines A B, A D, and Containing a Point C in One of the Lines.



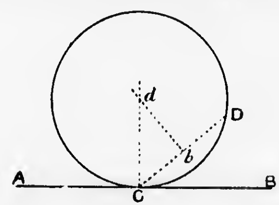
From **A** as centre with radius **A C**, describe an arc cutting **A B** in **a**.
At **C** and **a** draw perpendiculars **C e** and **a b**, to **A D** and **A B** respectively, meeting in **e**.
From the centre **e**, with the radius **e C** or **e a**, describe the required circle.

PROBLEM 44.—To Describe Two Circles with Given Radii C and D, Touching Each Other and a Given Line A B.



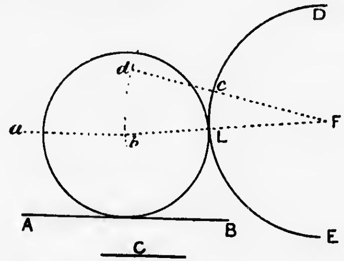
Draw any perpendicular **a c b** to the line **A B**, and make **a b** equal to **D** and **a c** equal to **C**.
Through **c** draw **c d** parallel to **A B**, and from the centre **b** with radius **bd** equal the sum of the given radii **C** and **D**, describe an arc cutting the line **cd** in **d**.
About **b** and **d** as centres, describe the required circles with radii respectively equal to **D** and **C**.

PROBLEM 45.—To Draw a Circle, the Circumference of which shall Pass Through a Given Point D and Touch a Given Straight Line A B at a Given Point C.



Join **C D** and bisect it in the point **b**.
At the points **C** and **b**, erect perpendiculars to the lines **A B** and **C D**, respectively.
The point **d**, where the perpendiculars meet, will be the centre of the circle to be described with the radius **d D** or **d C**.

PROBLEM 46.—To Describe a Circle of Given Radius **C**, whose Circumference shall Touch a Given Circle **D E L** and a Given Line **A B**.



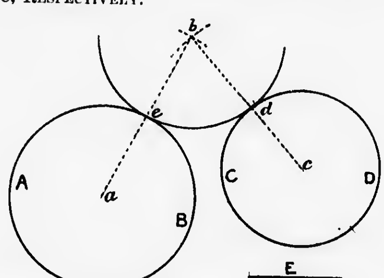
Draw a line **a b** parallel to the given line **A B** and at the distance **C** from it. (Prob. 7).

Draw any radius **F c** and produce it, making **c d** equal to **C**.

From **F** as centre with radius **F d**, describe a circle meeting the line **a b** in **b**.

Join **b F**, cutting the given circle in **L**, which will be the point of contact of the given and required circles. **b** will be the centre of the required circle and **b L** a radius.

PROBLEM 47.—To Draw a Circle of Given Radius **E** and Touching Two Given Circles **A B**, **C D**, having Centres **a** and **c**, Respectively.

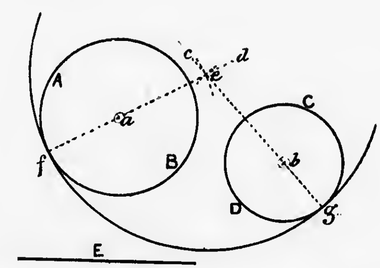


With **a** as centre and a radius equal **E** plus the radius of **A B**, describe an arc.

With **c** as centre and a radius equal to **E** plus the radius of **C D**, describe a second arc meeting the former in **b**.

Join **b a** and **b c**. The circle described with **b** as centre and **E** as radius, will be that required.

PROBLEM 48.—To Describe a Circle of Given Radius **E**, which shall Touch Two Smaller Circles **A B**, **C D**, and Include Them.



From the centre **a**, with a radius equal to the given radius **E** less the radius of the circle **A B**, describe an arc **c e**.

From the centre **b**, with a radius equal to the given radius **E** less the radius of the circle **C D**, describe an arc **e d**, meeting the former arc in **e**.

From **e** as centre with a radius equal to **E**, describe the required circle touching the given circles in **f** and **g**.

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PROBLEM 49.—To Describe a Circle Touching Two Given Circles B C, D F, and One of Them in a Given Point A.

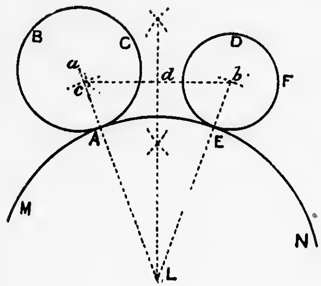
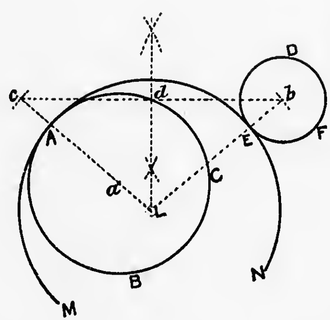


Fig. 1.

Fig. 2.

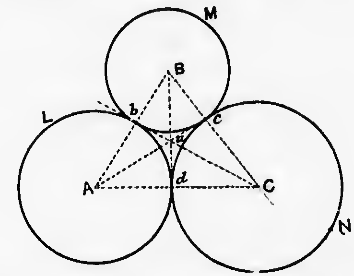


Join the point **A** to **a** the centre of the circle **C B.** In this line, or in the line produced, make **A c** equal to the radius of the other given circle **D F.**

Join **b,** the centre of the circle **D E,** to **c,** bisect **c b** in **d,** and draw the perpendicular **d L,** meeting **A a** produced in **L.**

From **L** as centre with the radius **L A,** describe the required circle.

PROBLEM 50.—From the Three Given Points A, B, C, as Centres, to Describe Three Circles Touching Each Other.



Join the given points and bisect the angles **A, B** and **C** by lines meeting in **a.**

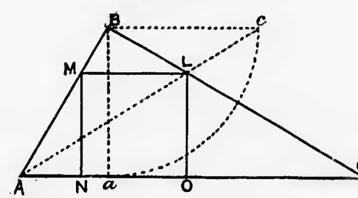
From **a** drop perpendiculars **a b, a c, a d** on the lines **A B, B C, C A,** respectively.

With **A** as centre and **A b** or **A d** as radius, describe a circle.

With **B** as centre and **B b** or **B c** as radius, describe a circle which will touch the former in **b.**

With **C** as centre and **C c** or **C d** as radius, describe a circle touching the two former circles in **d** and **c.**

PROBLEM 51.—To Inscribe a Square in any Given Triangle A B C.



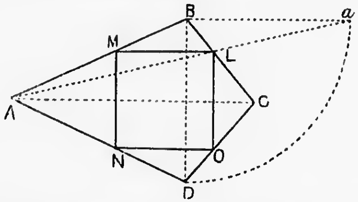
From the point **B** draw **B a** perpendicular to **A C.** Draw **B c** parallel to **A C** and equal to **B a.**

Join **A c,** cutting **B C** in **L.**

Draw **L M** parallel to **A C,** and **L O, M N** parallel to **B a.**

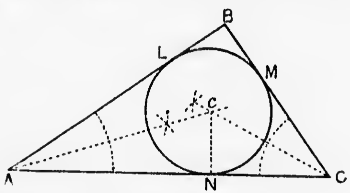
M L O N will be the required square.

PROBLEM 52.—To Inscribe a Square in a Given Trapezium **A B C D** whose Adjacent Sides **A B**, **A D** and also **B C**, **C D** are Equal to One Another.



Join **A C** and **B D**. Draw **B a** parallel to **A C** and make it equal to **B D**.
Draw **A a** cutting **B C** in **L**.
Through **L** draw a line parallel to **A C**, meeting **A B** in **M**.
Draw **L O** and **M N** parallel to **B D** and join **N O**, to complete the required square **M L O N**.

PROBLEM 53.—To Inscribe a Circle in a Given Triangle **A B C**.

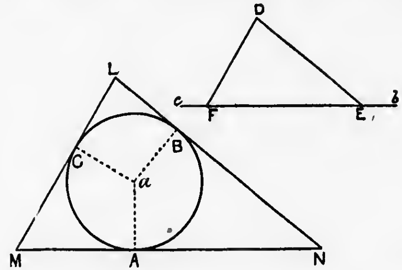


Bisect the angles **A** and **C** by the lines **A c**, **C c**, meeting in **c**.
Drop a perpendicular **c N** on **A C**, and from **c** as centre with **c N** as radius, describe the required circle, touching the sides in **L**, **M** and **N**.

PROBLEM 54.—To Describe a Circle about a Given Triangle.

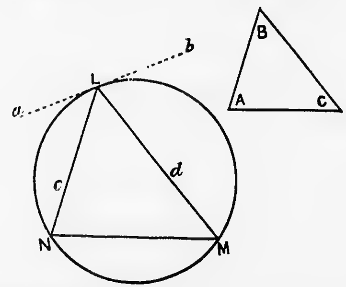
(See Problem 36).

PROBLEM 55.—To Describe a Triangle Similar to a Given Triangle **D E F** about a Given Circle **A B C**.



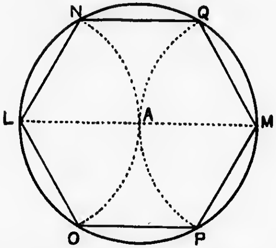
Produce one side of the given triangle **E F** and from the centre **a** of the given circle draw radii making the angles **C a A**, **A a B**, respectively equal to the angles **c F D** and **D E b**.
At the points **A**, **B** and **C**, draw tangents to the circle, meeting in **L**, **M** and **N**, and forming the required triangle.

PROBLEM 56.—In a Given Circle to Inscribe a Triangle Similar to a Given Triangle **A B C**.



Draw a straight line **a b** tangent to the circle at a point **L**.
At **L** make an angle **a L c** equal to **B A C** and an angle **b L d** equal to **B C A**.
Produce **L c** and **L d** to cut the circle in **N** and **M**.
Join **N M**, to complete the required triangle **L M N**.

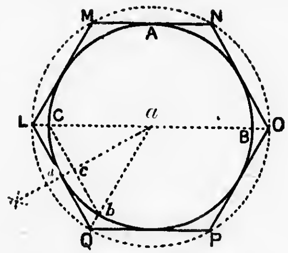
PROBLEM 57.—In a Given Circle with Centre **A**, to Inscribe a Regular Hexagon.



Draw any diameter and from its extremities **L, M**, mark off chords **L N, L O, M Q, M P**, equal to the radius of the circle.

L N Q M P O will be the required hexagon.

PROBLEM 58.—To Describe a Regular Hexagon About a Given Circle **A B C.**



Draw any diameter **C a B** and from the point **C** in the circumference of the circle set off the chord **C b** equal to the radius **C a**.

Bisect **C b** by a perpendicular, cutting the circle in **d**.

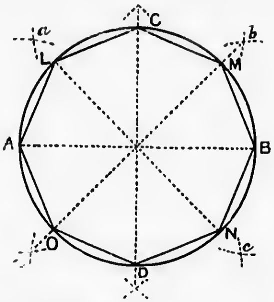
At **d** draw the tangent **L d Q**, meeting the diameter **B C** produced in **L**.

From **a** as centre with **a L** as radius, describe a circle and produce the diameter of the given circle **C B** to meet the circumference of the larger one in **O**.

From **L** and **O** as centres with **L a** as radius, mark the points **M, N, Q, P.**

The lines joining these points will be tangent to the given circle and will form the required hexagon.

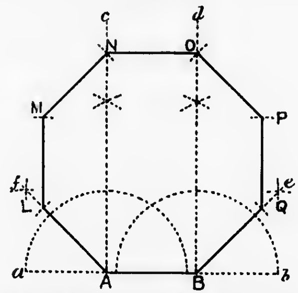
PROBLEM 59.—In a Given Circle **A C B** to Describe an Octagon.



Draw the diameters **A B, C D**, at right angles to each other. Bisect the quadrants in **L, M, N** and **O.**

Join **A L, L C**, etc., to form the required octagon.

PROBLEM 60.—Upon a Given Straight Line **A B** to Construct an Octagon.



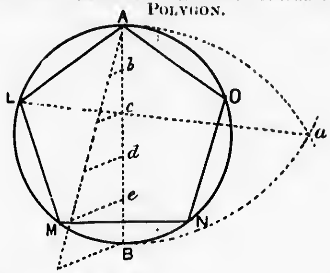
At the extremities **A** and **B** erect perpendiculars **Ac, Bd**, to the given line.

Produce **A B** to **a** and **b**, and bisect the angles **a A c, b B d** by **A f** and **B e** respectively.

Make **A L** and **B Q** equal to **A B.**

Through **L** and **Q** draw **L M, Q P**, parallel to **Ac** and equal to **A B**, and through **M** and **P** draw lines parallel to **B Q** and **A L** respectively and meeting **A c** and **B d** in **N** and **O** respectively. **M N** and **O P** will be equal to **A B** and on joining **N O** the required octagon will be completed.

PROBLEM 61.—To Inscribe in a given Circle any Regular Polygon.



Let it be required to inscribe a pentagon

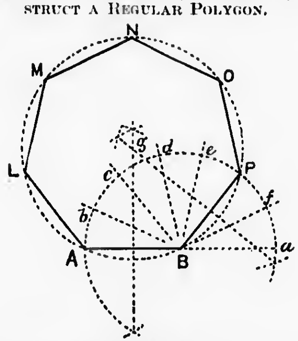
Draw any diameter **A B** and divide it into as many equal parts as the required figure has sides.

From **A** and **B** as centres, with radius **A B**, describe arcs meeting in **a**.

Join **a** to **c**, the second point from **A** on the divided diameter, and produce the line to meet the circumference in **L**.

Measure chords equal to **A L**, dividing the circle into five equal parts, and join the points.

PROBLEM 62.—Upon a Given Straight Line A B to Construct a Regular Polygon.



Let it be required to construct a regular heptagon.

Produce **A B**, making **B a** equal to **A B**, and describe a semicircle on **A a**.

Divide the semicircle into seven equal parts (Prob. 61) in **f**, **P**, **e**, etc.

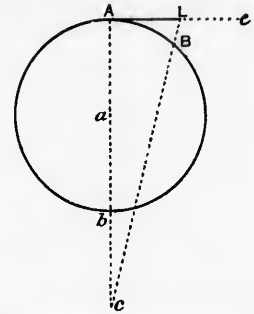
Join **B** to **P** the second point from **a**.

Find the centre of the circle passing through **A B P** and describe the circle.

Measure arcs equal to **A B** or **B P**, from **P**. These will divide the circle into seven equal parts, forming the required polygon.

The points **L**, **M**, **N**, **O**, should be on the lines drawn from **B** through **b**, **c**, **d**, and **e**, the points os division of the circle **A P a**.

PROBLEM 63.—To Draw a Straight Line which shall be Approximately Equal to any Given Arc A B of a Circle.



Find **a** the centre of the circle.

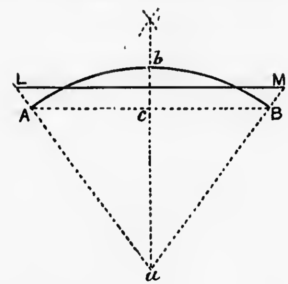
Join **A a** and produce it, making **a c** equal to twice **A a**.

Draw **A e** perpendicular to **A a**.

Join **c** to **B** and produce the line to meet **A e** in **L**.

The straight line **A L** will be approximately equal in length to the arc **A B**.

PROBLEM 63 (a).—To Draw a Straight Line which shall be Approximately Equal to any Given Arc A B (Second Method).



Find the centre **a** of the circle.

Bisect the arc in **b** and join **b** to **a**.

Join **A B**, cutting **a b** in **c**.

Bisect **b c**, and through the point of bisection draw a line parallel to **A B** and meeting **a A**, **a B**, in **L** and **M**.

M L will be the required straight line.

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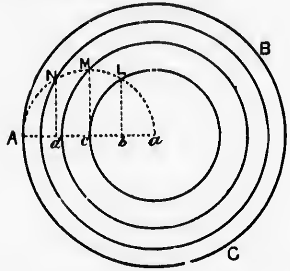
PROBLEM 64.—To Divide a Circle **ABC** into any Number of Concentric Rings, the Areas of which shall have a Given Relation to Each Other.

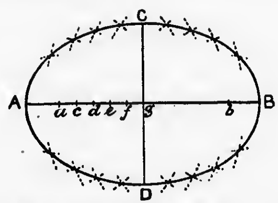


Draw the radius **aA** and divide it into parts having the desired ratio, in the points **b, c, d.**

Upon **aA** as diameter, describe a semicircle and erect perpendiculars to the line **Aa** from the points **b, c, d,** meeting the circumference in the points **L, M, N.**

From the centre **a** with radii **aL, aM, aN,** describe circles. They will divide the given circle in the desired ratio.

PROBLEM 65.—To Describe an Ellipse, having given a Major Axis, or Transverse Diameter **AB** and a Minor Axis, or Conjugate Diameter **CD.**



Draw the diameters at right angles and bisecting each other in the point **g.**

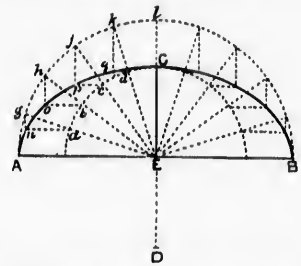
From **C** or **D** as centre, with **gA** or **gB** as radius, describe arcs cutting the major axis in the points **a** and **b.** These points will be the foci of the ellipse.

From **a** and **b** as centres, with radii **Bc, Bd, Be,** etc., describe arcs above and below the major axis.

From the same centres describe also arcs with radii **Ac, Ad, Ae,** etc., cutting the former arcs. The arcs with radius **Ac** meeting those with radius **Bc,** etc.

The points in which the arcs meet will be in the circumference of the required ellipse.

PROBLEM 65 (a).—(Second Method.)—To Construct an Ellipse having the Axes **AB, CD** Given.



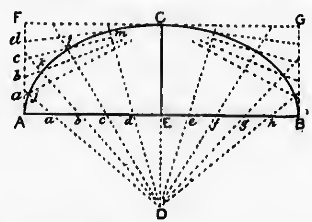
About the centre **E** describe circles with radii **EC** and **EB.**

Take any points in one of the circles as **a, b, c, d,** etc. Join these points to **E** and produce **Ea, Eb, Ec,** etc., to meet the outer circumference in **g, h, j,** etc.

Through **a, b, c, d,** etc., draw lines parallel to **AB,** and through **g, h, j,** etc., draw lines parallel to **CE,** to meet the former in **n, o, p,** etc.

These points are to be joined freehand to form the required ellipse.

PROBLEM 65 (b).—(Third Method). To Construct an Ellipse when the Axes **AB, CD,** are Given.



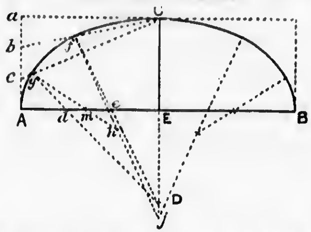
Construct the rectangle **AFCGB.**

Divide **AE, EB,** into any number of parts (for simplicity use equal parts) and divide **AF, GB** similarly.

Join **D,** the opposite extremity of the minor axis, to the points of division **a, b, c,** etc., in **AB,** and produce the lines to meet lines drawn from **C** to the points of division in **AF, GB.**

The points of intersection **j, k, l,** etc., will be in the required ellipse.

PROBLEM 65 (c).—To Construct an Ellipse Approximately by Circular Arcs.



Draw the axes **A B**, **C D**, and on **A B** construct the rectangle **B a**.

Divide **E A** into three equal parts in **e** and **d**, and also **A a** into three equal parts in **b** and **c**.

Join **C** to **b** and **c** and draw lines from **D** through **d** and **e**, cutting **C b**, **C c** in **g** and **f**.

Find the centre **j** in the line **C D** of the arc passing through **C** and **f**, and describe the arc.

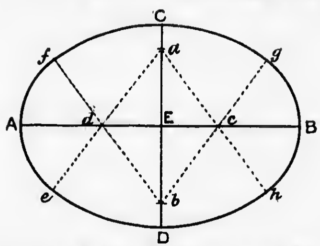
Next find the centre **h**, on the line **f j**, of the arc passing through **f** and **g**.

The point **m** where **g h** cuts **A B** will be the centre of the remaining arc **g A**.

Draw the other quadrants similarly.

A closer approximation may be obtained by dividing the lines **A a**, **A E** into four or more parts, and obtaining the centres of the arcs in a similar manner.

PROBLEM 65 (d).—To Construct an Ellipse Approximately by Means of Circular Arcs upon Given Axes AB, CD.



Make **E a** and **E b** equal to the difference between the axes **A B** and **C D**.

Make also **E d** and **E c** equal to three-fourths of this difference. (**E a = A B − C D** and **E d** = ¾ **E a**).

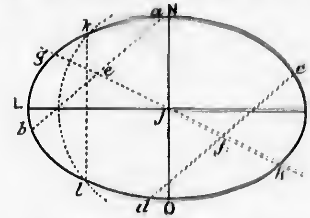
Join **a** and **b** to **d** and **c** and produce the lines.

From **d** and **c** with radii equal to **d A** or **c B** describe the arcs **e A f**, **g B h** and from **b** and **a** with radii equal to **b C**, or **a D** describe the arcs **f C g**, **h D e**.

These arcs will meet in **f**, **g**, **h** and **e** and form the required approximate ellipse.

This method is described by Mr. B. F. LaRue in *Engineering News*, vol. xxxiv., No.17.

PROBLEM 66.—An Ellipse Being Given, to Find the Axes.



Draw any two straight lines **a b**, **c d**, across the ellipse and parallel to each other.

Bisect these lines in **e** and **f**.

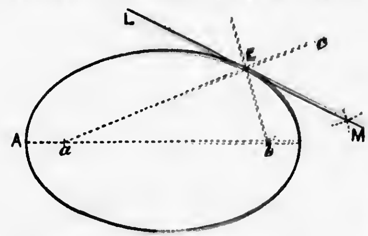
Join **e f** and produce it both ways to meet the curve in **g** and **h**.

Bisect **g h** in **j**, and from **j** as centre describe any arc cutting the curve in **k** and **l**.

Join **k l** and through **j** draw lines **N O**, **L M**, parallel and perpendicular to it.

N O and **L M** will be the required axes.

PROBLEM 67.—At a Given Point E in the Circumference of an Ellipse to Draw a Tangent to the Curve.



Find the axes and foci **a**, **b**, of the ellipse (*Probs, 66 and 65*).

Join **E** to **a** and **b**, and produce **a E** to **c**.

Bisect the angle **c E b** by the line **E M**.

L E M will be the required tangent.

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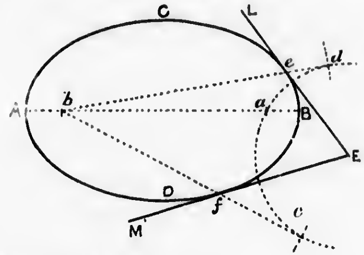
PROBLEM 68.—TO DRAW A TANGENT TO AN ELLIPSE FROM A GIVEN EXTERNAL POINT E.



Find the foci **a** and **b**, of the ellipse.

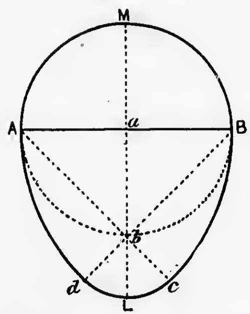
From **E** as centre with radius **Ea**, describe the arc **c a d**.

From **b** as centre with radius **AB**, describe another arc cutting the former in **d** and **c**.

Join **bd** and **bc**, cutting the curve in the joints **e** and **f**, which will be the tangent points.

Draw the tangents **EeL**, **EfM**.

PROBLEM 69.—TO CONSTRUCT AN EGG-SHAPED OVAL ON A GIVEN DIAMETER **A B**.



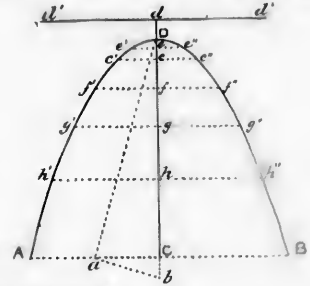
Bisect **AB** in **a**. Draw **ab** at right angles to **AB** and equal to **aA** or **aB**.

Join **Ab**, **Bb**, and produce them.

With **A** and **B** as centres and **AB** as radius, describe the arcs **Bc**, **Ad**.

With **a** as centre and **aA** as radius, describe the semicircle **AMB**. With **b** as centre and radius **bc**, describe the arc **dLc**, completing the required figure **AMBL**.

PROBLEM 70.—TO DRAW A PARABOLA ON A GIVEN AXIS **CD** AND HAVING AT THE POINT **C** THE GIVEN ORDINATES **AC**, **CB**.



Bisect one of the ordinates **AC** in **a**.

Join **aD** and draw **ab** perpendicular to **aD**, meeting **DC** produced in **b**.

Make **De**, and **Dd**, on **CD** and its production, equal to **Cb**.

The line **d d'** drawn through **d**, parallel to **A B**, is the directrix and **e** is the focus of the parabola.

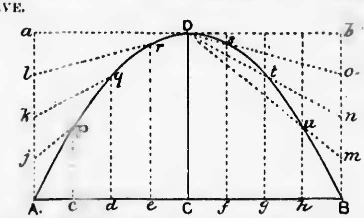
Divide the axis into any parts in the points **e**, **c**, **f**, **g**, etc., and through these draw lines parallel to **AB**.

From **e** as centre with **de** as radius, describe arcs cutting the parallel line through **e** in **e'** and **e''**.

Similarly from **e** as centre, with **de**, **df**, **dg**, etc., as radii, describe arcs cutting the parallel lines in **e'** and **e''**, **f'** and **f''**, **g'** and **g''**, etc.

The points so found will be in the curve of the parabola, which is to be drawn free-hand through them.

PROBLEM 71. THE BASE OR DOUBLE ORDINATE **AB** AND THE AXIS OR HEIGHT **CD** OF A PARABOLA BEING GIVEN TO DRAW THE CURVE.



Draw **Aa** and **Bb** parallel and equal to **CD**, and join **ba**.

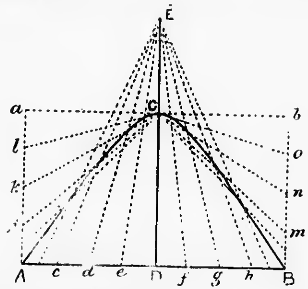
Divide **AC** and **CB** into any number of equal parts in the points **c**, **d**, etc.

Divide also **Aa** and **Bb** into the same number of equal parts in **m**, **n**, **o** and **j**, **k**, **l**.

Draw lines through **c**, **d**, etc., parallel to **CD** or perpendicular to **AB**.

Join **D** to the points **j**, **k**, **l** and **m**, **n**, **o**, meeting the vertical lines in **p**, **q**, **r**, **s**, **t**, **u**, which will be points in the required parabola.

PROBLEM 72.—To Draw an Hyperbola having Given the Double Ordinate **A B**, the Height or Abscissa **D C**, and the Vertex **E**.



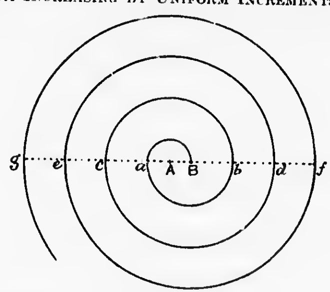
Construct the rectangular figure **A B b a**.

Divide **A B** into any number of equal parts in **c, d,** etc. Divide **A a** and **B b** into the same number of equal parts in **m, n, o** and **j, k, l**.

Join the points in **A B** to **E** and those in **A a** and **B b** to **C**.

The points of intersection of these lines will be in the required hyperbola

PROBLEM 73.—To Construct a Spiral of Arcs of Circles having Radii Increasing by Uniform Increments **A B**.



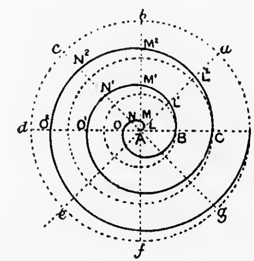
Produce **A B** both ways.

With **A** as centre and **A B** as radius, describe a semicircle meeting **B A** produced in **a**.

With **B** as centre and **B a** as radius, describe the semicircle meeting the line in **b**.

Again, with centre **A** and radius **A b**, describe another semicircle and so on, alternately using **A** and **B** as centres, with radii increasing at each change of centres, by a distance **A B**.

PROBLEM 74.—To Draw a Spiral of Any Number of Revolutions and having a Given Pitch.



Produce **A B** and measure off as many parts equal to the pitch **A B** as the required number of revolutions (in this case three).

Describe circles with centre **A** and radii **A B, A C**.

Divide the outer circle into any number of equal parts and draw the radii **A a, A b,** etc.

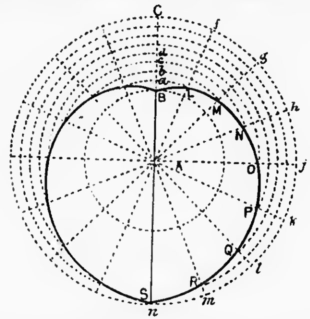
Divide **A B** and **B C** each into the same number of equal parts as the circle.

Measure along **A a** a distance from **A** equal to one of these parts; along **A b** a distance equal to two of the parts, and so on, obtaining the points **L, M, N,** etc., in the required spiral.

For the second revolution measure along **A a** from **A** a distance equal to **A B** and one of the parts, along **A b**, a distance equal to **A B** and two of the parts, and so on, obtaining points in the second revolution in **L¹, M¹, N¹**, etc.

The third revo'ution **L², M², N²**, etc., will be obtained in a similar manner.

PROBLEM 75.—To Draw a Double Spiral of 180° Forming a Heart-shaped Cam, Suitable for Communicating Uniform Motion.



Assume the axis of rotation **A**, the minimum and maximum radii **A B** and **A C**.

Describe circles with centre **A** and radii **A C, A B**.

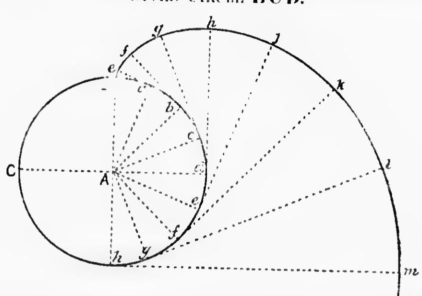
Divide the outer circumference equally in the points **f, g, h**, etc., and draw the radii **A f, A g, A h**, etc.

Divide **B C** into the same number of equal parts as the semicircle in **a, b, c**, etc.

From the centre **A** with radii **A a, A b**, etc., describe arcs to meet the radii **A f, A g, A h**, etc., in the **L, M, N**, etc., marking points in the required curve.

Join the points freehand to form the cam **B N Q S**.

PROBLEM 76.—To Describe a Portion of the Involute of a Given Circle **B C D**.

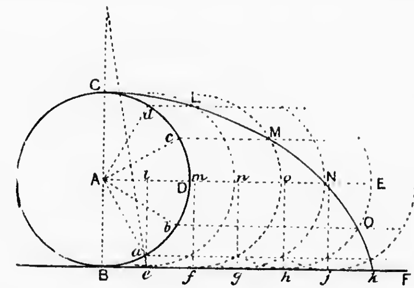


Divide the circumference into any number of equal parts in the points **a, b, c**, etc., and draw tangents **a e, b f, c g**, etc., to the circle.

Make **a e** equal to the length of the arc **a B**, **b f** equal to the arc **B b**, equal to twice **a e**, and similarly obtain the points **g, h, j, k**, etc., increasing the tangent in each case by a length equal to the rectified arc **a B** so as to equal the length of the corresponding arc.

Note:—This curve is formed by a point in a fine thread when kept taught and unwound from a circular disc. It is much used in mechanism and is one of the forms given to the teeth of wheels.

PROBLEM 77.—To Draw the Cycloid Generated by a Point **C** in the Circumference of a Given Circle **B C D**.



Draw the diameter **C A B** and through **A** and **B** draw lines **A E, B F**, at right angles to **B C**.

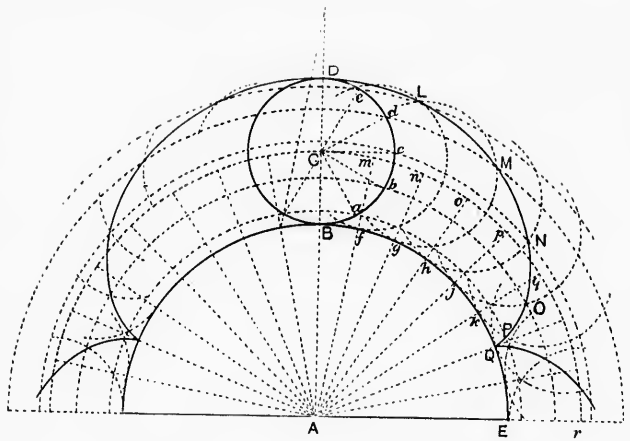
Divide the circle into any number of equal parts in the points **a, b**, etc.

Mark off on **A E** the points **l, m**, etc., at intervals equal to the rectified length of the arc **Ba**.

Draw circles with radii equal to **A C** from the centres **l, m, n**, etc.

Through **a, b, D, c, d**, draw lines parallel to **A E**, meeting the circles in the points **L, M, N**, etc., which will be in the required cycloidal curve.

Note:—This construction is equivalent to rolling the given circle along a straight line: the centre of the circle successively occupying the points **l, m, n**, etc. while the point **C** in the circumference takes the positions **L, M, N**, etc. A cycloid is described by any point in the circumference of a circle which rolls on a straight line called the **director**.



PROBLEM 78.—To Draw the Epicycloid Described by a Point in the Circumference of a Given Generating Circle **B D**, Rolling on a Given Directing Circle **B E**.

Join the centres **AC**. Divide the circle **BD** into the equal parts **Ba**, **ab**, etc.

With **A** as centre and **A C** as radius, describe an arc of a circle **Cr**.

With the same centre and radii **A a**, **A b**, **A c**, **A d** and **A e**, describe arcs of circles.

Divide the circumference **B E** into equal parts **Bf**, **fg**, etc., having the same length as the arcs **B a**, **a b**, **b c**., etc. (**a** on rolling would come into contact with **f**, **b** with **g**, **c** with **h**, etc.)

From **A** draw radii through **f**, **g**, **h**, etc., to meet the arc **C r** in the points **m**, **n**, **o**, etc.

With centre **m** and radius **CD**, describe an arc of a circle cutting the arc described with the radius **A e** in **L**. This arc **L f** represents one position of the generating circle and the point **L** the position which the generating point **D** would then have reached.

Similarly with centres **n**, **o**, etc., describe arcs meeting the arcs described, with radii **A d**, **A c**, etc., respectively in **M**, **N**, etc.

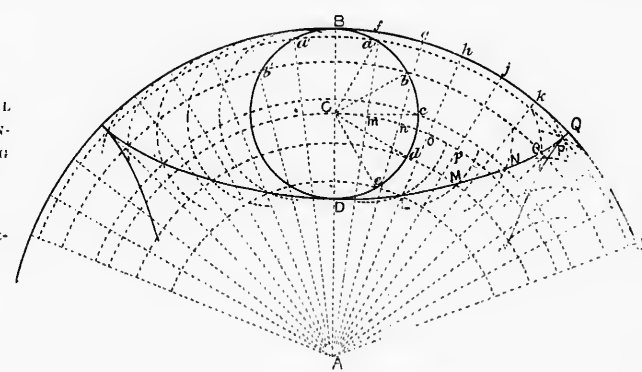
The generating point will successively occupy the positions **D**, **L**, **M**, **N**, **O**, **P**, **Q**. on the curve.

The remaining half of the loop is to be similarly formed.

PROBLEM 79.—To Draw the Hypocycloid or Internal Epicycloid Described by the Point **D** in the Generating Circle **DB**, Rolling within the Directing Circle **EB**.

The description for the external epicycloid will serve also for this curve.

When the diameter of the generating circle equals the radius of the directing circle, the curve becomes a straight line.



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PROBLEM 80.—TO TRACE THE PATH OF A POINT F WITHIN THE CIRCUMFERENCE OF A CIRCLE AD WHEN THE CIRCUMFERENCE ROLLS ON A STRAIGHT LINE AB. (Such curves are called trochoids or cycoids.)

With **E** as centre and **E F** as radius, describe a circle.

Divide the outer circle into any number of equal parts and join these to the centre, cutting the smaller circle in the points **b, c, d,** etc.

Take the rectified length of one of the parts **A a** on the rolling circle and mark off parts equal to it on the line drawn through **E** parallel to **A B.** These lengths will divide the line in the points **m, n, o,** etc.

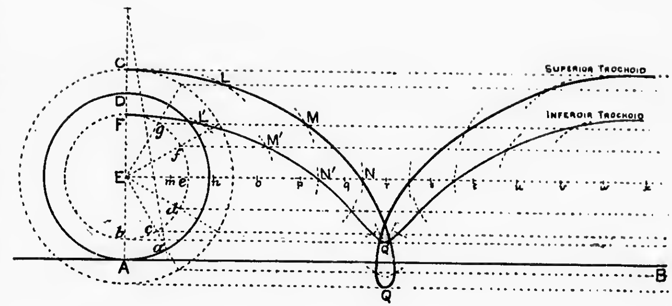
Through the points **g, f, e,** etc., draw lines parallel to **A B** and from the centres **m, n, o,** etc., with a length **E F** as radius, describe arcs (representing the successive positions of the smaller circle) to meet the parallel lines in the points **L', M', N',** etc.

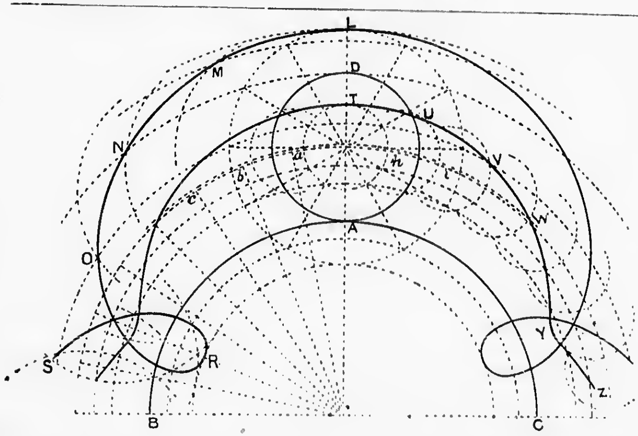
Join these points freehand to form the required curve.

PROBLEM 80 (a).—TO TRACE THE TROCHOIDAL CURVE DESCRIBED BY A POINT C SITUATED WITHOUT THE CIRCUMFERENCE OF THE ROLLING CIRCLE.

From the centre **E** with **E C** as radius, describe a circle and make a similar construction to the former.

The resulting curve **L M N Q** will form loops where it falls below the directing line. The path of the point **C** is called the **superior** or **curate trochoid.** When the point is within the rolling circle, the curve is known as an **inferior** or **prolate trochoid.**



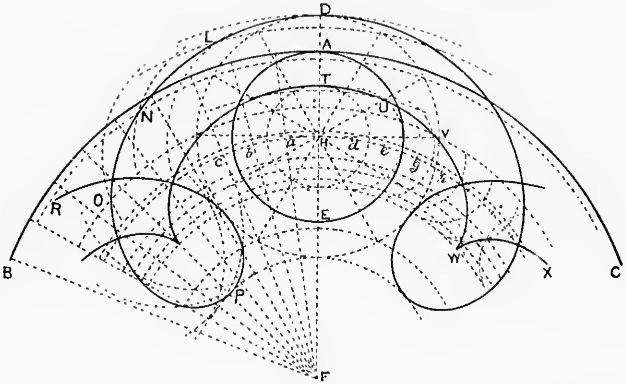


PROBLEM 81.—TO DRAW EPITROCHOIDAL CURVES.

These are described when a circle rolls upon a circle and the generating point is within or without the circumference of the rolling circle.

The former is called the **inferior** and the latter, the **superior epitrochoid.**

The construction is similar to that for the trochoidal curves. **L M N O R S** is a portion of the superior epitrochoidal curve traced by the point **L** when the circle **A D** rolls on the external circumference **B A C,** and **T U V W Y Z** is a portion of the inferior epitrochoidal curve formed by the internal point **T** under the same circumstances.



PROBLEM 82—To Draw the Hypotrochoidal Curves.

These are descrioed when the generating circle rolls within the directing circle.
The construction is similar to the foregoing. To find the curve traced by the point **D** when the circle **A E** rolls within the circumference **B A C**: Divide the circumference of the circle **A E** into any number of equal parts, and on **B A C** mark off on either side of **A** parts equal to these. Join the points so found to the centre **F** cutting the arc described through the centre **H** of the rolling circle in **a, b, c, d, e, g, etc.**

Through the points in the circumference of the circle having a radius **H D**, describe arcs from the centre **F** and from the centres **a, b, c, etc.**, describe arcs with radius equal to **H D** meeting the former in **L, N, O, etc.** This curve, the superior hypotrochoid, will form loops as **N O P R.**

The inferior hypotrochoidal curve, traced by the point **T** of which **T U V W X** is a part,, is described in a similar manner.

Involutes in General, as Involutes of an Ellipse, Cycloid, Etc.

The path of a point in a flexible inextensible thread made to unwind itself tangentially from a curve, in the plane of the curve, is an **involute.**

The curve from which the line is unwound is called the **evolute** of the involute curve. The involute curve being given, its evolute may, by a converse operation, be determined. To form an involute, tangents are drawn from the evolute, and the points of tangency are the instantaneous centres of curvature.

If from the involute normals be drawn and the centres of curvature obtained, the locus of these centres will be the evolute. (See figure and also Problem 76.)

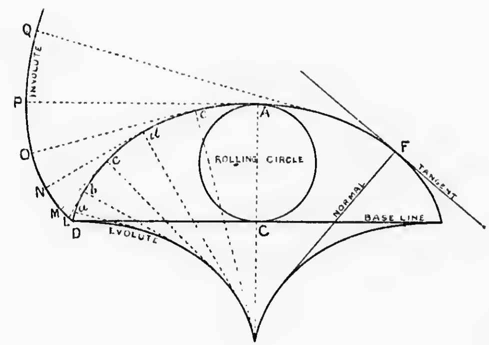
To obtain the centres of curvature when the character of the curve is not known, it is necessary to assume three points near to each other, and obtain the centre of the circle passing through them. (Prob. 36.)

As any curve may be considered an evolute from which to obtain an involute, so also it may be considered an involute from which the corresponding evolute may be obtained.

Cycloids in General.

A Cycloid is described by any point in a curve when the curve rolls on any other curve. As for example, an ellipse rolling upon an ellipse.

As an exercise, describe the cycloid generated by a point in an ellipse having axes of 3″ and 1″ when the director is a straight line.



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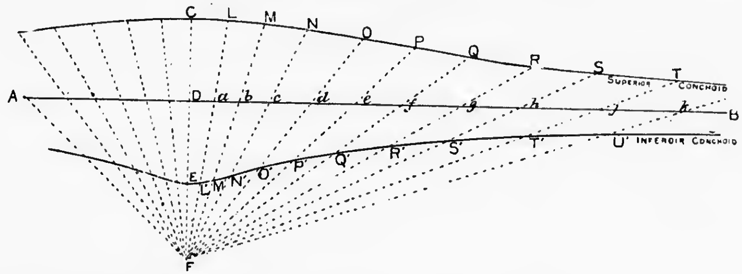
PROBLEM 83.—TO DRAW THE CONCHOID CURVES OF WHICH THE STRAIGHT LINE **A B** IS THE ASYMPTOTE, **C D** THE DIAMETER, AND **F** THE POLE.



The line **F C** is perpendicular to **A B.**

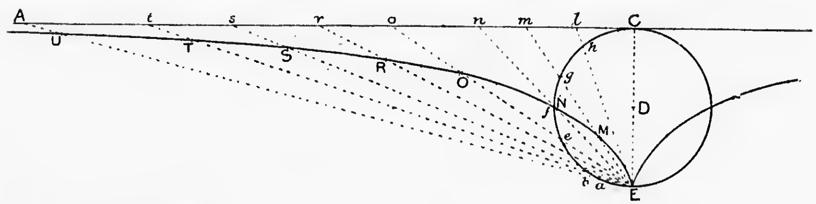
On each side of **D** mark any points **a, b, c,** etc., and from **F** draw straight lines through these points.

Make **D E** equal to **C D,** and from the points **a, b, c,** etc , measure lengths **a L, b M**, etc., **a L,' b M**, etc., each equal to **C D.**

The points **C, L, M**, are in the **superior Conchoid**, and **E, L', M'**, etc. in the **inferior Conchoid.**

A freehand curve is to be drawn through the points.

PROBLEM 84.—TO DRAW THE CISSOID CURVE FROM THE GIVEN CIRCLE, **E C.**



At **C** draw the tangent line **A C.**

From **E**, the opposite extremity of the diameter through the point **C**, draw any lines meeting the straight line **A C** in the points **l, m, n,** etc., and cutting the circle in **h, g, f,** etc.

Make **A U** equal to **a E**, **t T** equal to **b E**, etc.: **o O** equal to **e E**, **n N** equal to **f E**, etc.

The points **U** and **T**, **O** and **N** are in the required curve.

Obtain the points **S, R,** and **M,** in a similar manner and draw the curve freehand.