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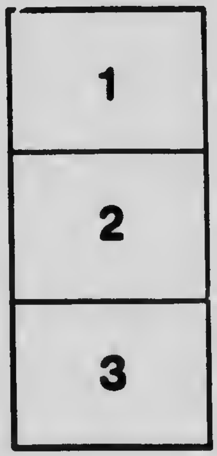

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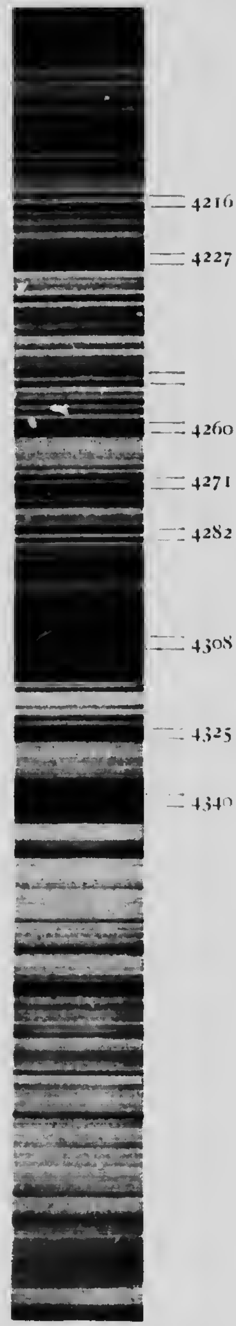
FROM THE DOMINION OBSERVATORY

THE ORBITS OF THE SPECTROSCOPIC COMPONENTS OF *d* BOOTIS

BY

W. E. HARPER

PLATE XXIV.



SPECTRUM OF d BOÖTIS, SHOWING DOUBLE LINES

Photographed 1907, April 5. Length 11 times, width 90 times the original.

THE ORBITS OF THE SPECTROSCOPIC COMPONENTS OF *d* BOOTIS

By W. E. Harper

THIS star, $\alpha = 14^h\ 05^m{\cdot}8$, $\delta = +25^\circ\ 34'$, photographic magnitude 5·3, was announced as a spectroscopic binary by Campbell and Wrig,at in 1900. Approximate measures of the four plates secured showed a range of 75 km. Their measures referred solely to one component; they made no mention of the spectrum of the other component being visible, though, no doubt, such is the case on two of their plates.

Fifty-three spectrograms of the star have been obtained at this observatory during the years 1907, 1910 and the present year, and these form the basis of the present discussion of the orbit. The first four were made with the Universal spectroscope as adapted for radial velocity v .rk, linear dispersion at H_γ 18·6 tenth-metres per millimetre, the next one with three-prism dispersion of 20·2 tenth-metres per millin ire at the same region, and the remainder of the plates with the new single prism instrument, dispersion at same region of 33·4 tenth-metres per millimetre. For number 3368 a Sigma plate was used; all the other plates were of Seed 27 Emulsion.

As intimated above both spectra are visible, (Plate XXVI.*) are quite similar and of type F 5. For considerably over half the period the spectra are well resolved and measures were made on the lines of each component. At first all the lines that were at all measurable were used; gradually those were eliminated which from their complicated nature could not be depended upon.

* NOTE. Owing to the fact that the enlargement was made by moving the negative up and down, a few spurious lines are introduced.

The final outcome was a selection of thirteen lines, given in the table below, upon which all the measures were based. When all the measures were completed a table of residuals of each line, from the mean given by the plate as a whole, was formed and new wave-lengths were derived so that the sum of the residuals equalled zero for each line. The first column gives the wave-length as assumed at the start, the second the number of times measured, the third the average residual and the last the wave-length as corrected for this star. Outside of H_γ it will be noticed that practically all the lines selected were those due to iron or blends of iron.

Lines Used in *d* Bootis

λ	Times Measured	Average Residual	Corrected λ
4549·766	34	0·0 km.	4549·766
4415·293	41	− 3·4	4415·343
4340·634	31	··8	4340·666
4325·820	43	+ 1·2	4325·812
4308·081	54	+ 2·7	4308·042
4271·760	53	− 0·9	4271·773
4200·540	50	+ 3·2	4200·495
4143·928	49	− 0·7	4143·938
4071·901	26	− 2·9	4071·940
4063·756	38	− 2·9	4063·795
4045·075	52	− 0·2	4045·078
4005·430	47	+ 5·1	4005·362

With the exception of the last line, whose wave-length was very uncertain at the commencement of the measures, none of the wave-lengths as assumed are in very great need of correction. An unpublished investigation by the writer of the effect on the elements of an orbit of the use of wave-lengths which, treated similarly, gave residuals somewhat as those above, shows that the changes are almost inappreciable. However, as the last line needed correction it was decided to use the corrected wave-lengths throughout and the measured velocities were revised accordingly.

In the table following is given a summary of the measures. The phases are reckoned from the periastron passage finally accepted, J. D. 2417679·523, using the period 9·6045 days. The maximum weight assigned a plate was 10, depending for the most part on the sum of the weights given the separate lines when the measurement was being made. The residuals given are scaled from the final curve. These appear only for those plates where the spectrum lines of the components were resolved. The other velocities are derived from measures on blends and are subject to considerable error. For some distance on either side of the intersection of the curves with the γ-line where the lines are more or less overlapping the tendency is for the measured velocities to deviate from their true position towards the γ-line and such observations can only be made use of to advantage when grouped over a large phase interval, the extent of the interval on each side of the crossing point being approximately the same.

Measures of *d* Bootis

Plate No.	Julian Day	Phase	Component I.			Component II.		
			Velocity	Wt.	O - C	Velocity	Wt.	O - C
699	2417671·769	1·851	+ 85·0	10	+ 6·0	− 59·9	9	+ 2·8
776	719·635	1·694	+ 71·3	2	− 7·5	71·5	2	− 8·7
778	720·593	2·652	+ 81·5	4	+ 12·0	− 54·7	3	2·2
798	7727·653	·108	+ 17·8	6				
3368	8759·854	4·627	+ 17·1	5				
3406	826·768	4·310	+ 7·6	7				
3475	832·702	·039	+ 35·7	5				
3512	859·591	8·319	− 54·2	10	4·5	+ 73·1	6	+ 0·7
3518	864·584	3·708	+ 26·9	10				
3524	866·581	5·705	+ 0·6	9				
3550	881·586	1·501	+ 73·4	9	− 3·9	− 61·2	9	0·0
3577	903·550	4·256	+ 16·3	7				
3594	913·542	4·643	+ 13·5	6				
3609	916·556	7·057	− 64·8	4	− 5·2	+ 69·3	3	− 11·7
3624	8923·534	5·031	+ 3·5	3				
3896	9027·967	3·814	+ 28·4	6				
3926	046·875	3·513	+ 26·9	2				
3945	053·958	·992	+ 69·0	8	0·0	− 56·6	5	7·0
3954	054·925	1·959	+ 83·2	9	+ 4·5	− 56·5	7	+ 5·8
3979	067·890	5·319	+ 8·7	4				
4044	098·881	7·497	+ 57·6	3	− 0·2	+ 78·4	2	− 2·4
4052	099·890	8·506	− 46·6	7	− 1·8	+ 69·5	5	+ 2·5
4064	102·846	1·857	+ 77·7	5	− 1·0	62·8	5	+ 4·0
4074	103·833	2·844	+ 64·8	7	− 0·4	− 60·7	5	− 12·4
4126	112·892	2·200	+ 70·6	8	+ 3·8	− 59·3	6	0·0
4142	120·824	·020	+ 49·3	3	− 1·7	39·6	2	+ 2·4
4159	129·844	·042	+ 10·3	9				
4172	134·860	5·058	+ 8·4	10				
4183	137·853	8·051	− 55·1	3	+ 0·2	+ 75·0	3	3·5
4190	138·712	8·910	29·6	4	2·0	+ 57·2	2	+ 8·2
4208	145·807	6·400	40·5	5	+ 0·2	+ 66·9	4	+ 4·1
4218	146·816	7·409	52·2	7	+ 5·0	+ 74·1	4	− 6·0
4233	148·801	9·394	+ 6·4	7				
4236	149·708	·697	+ 49·8	5	− 4·2	35·4	2	+ 1·2
4252	151·784	2·773	+ 66·6	7	0·2	− 50·6	4	− 0·9
4260	152·781	3·770	+ 22·0	4				
4278	155·812	6·801	45·3	4	+ 3·5	+ 78·0	2	+ 6·2
4304	173·687	5·467	1·0	6				
4312	176·740	8·520	40·9	4	+ 3·4	+ 71·6	2	+ 4·8
4315	178·700	·935	+ 71·3	7	+ 7·8	47·8	5	0·8
4328	183·716	5·891	17·4	2				
4338	187·705	·275	+ 11·9	5				
4344	189·606	2·236	+ 73·6	5	2·7	60·8	4	0·8
4357	196·626	9·196	0·7	7				
4382	211·606	4·968	+ 13·2	9				
4390	215·646	9·008	− 34·8	2				
4391	216·562	·340	+ 9·9	2				
4405	221·586	5·343	+ 12·4	7				
4411	222·674	6·431	37·0	2	+ 4·4	+ 57·5	1	5·8
4420	225·556	9·313	+ 6·4	8				
4425	231·603	5·756	+ 5·0	6				
4431	232·579	6·732	34·7	8	+ 12·9	+ 83·8	3	+ 13·3
4440	238·632	3·180	+ 54·6	3	2·2	− 39·9	3	0·4

For convenience of reference, the early observations of the Lick Observatory are here appended.

LICK OBSERVATIONS

Date	Julian Date	Velocity	Residual from Curve
1900 March 27	2415106·881	+ 79	+ 3·6
April 4	114·953	+ 3	− 0·3
" 9	119·905	+ 11	1·7
" 17	127·814	+ 60 ±	+ 0·6
1902 May 27*	807·715	+ 11	− 1·2

* Unpublished, but communicated through kindness of the Acting Director.

The period deduced from our own observations and the published ones of Lick, assuming their observations on the meridian, was 9·605 days. This was the period used throughout. A correction of − 0·0005 days was made to this when the G. M. T. of the Lick plates was received. With the exception of the first four all our observations are practically of one year, so that the small correction to the period will not affect the results. Likewise three out of the four 1907 plates are at the crests of the curve and no appreciable change will result from the use of the revised period. The correction, however, will accumulate to approximately 0·070 days in the interval over which our observations extend and accordingly a correction of + 0·070 days was added to the derived value of T making it 2417679·523 as given in final elements.

The observations on component I. were first grouped according to phase into thirteen normal places. The peculiar deviation effect near the intersection of the curve with the γ-line, previously referred to, was in evidence in four or five normal places, abnormal residuals for these groups being the rule.

Preliminary elements by the graphical method were obtained, which outside of the groups mentioned, satisfied the observations quite well. They were the following:

$$\left.\begin{aligned} P &= 9{\cdot}605 \text{ days} \\ e &= {\cdot}15 \\ \omega &= 280^\circ \\ K &= 68{\cdot} \text{ km.} \\ \gamma &= +\ 9{\cdot}23 \text{ km.} \\ T &= \text{J.D. } 2417679{\cdot}600 \end{aligned}\right\} \text{Preliminary elements for solution of component I.}$$

It could be seen that in a least-squares solution the large residuals, alternately above and below the curve on each side of the crossing points, would play the most important part and would cause considerable changes in the elements; nevertheless, as a matter of interest merely to see the extent of such changes, a solution was made. The period was considered determined and corrections were obtained for the other elements as follows:

$$\delta\gamma = +\ 2{\cdot}54 \text{ km.}$$
$$\delta K = -\ 2{\cdot}48 \text{ km.}$$
$$\delta e = +\ {\cdot}090$$
$$\delta\omega = -\ 13^\circ\ 11'$$
$$\delta T = -\ {\cdot}235 \text{ days,}$$

so that the first corrected set of elements for component I. are:

$$P = 9{\cdot}605 \text{ days}$$
$$e = {\cdot}240$$
$$\omega = 266^\circ\ 49'$$
$$K = 65{\cdot}52 \text{ km.}$$
$$\gamma = +\ 11{\cdot}77 \text{ km.}$$
$$T = \text{J. D. } 2417679{\cdot}365.$$

If we compare these with the values finally accepted we notice differences of considerable magnitude. The eccentricity is here considerable increased. Another marked effect is the lowering of both positive and negative maxima from that given by the final elements which latter maxima seem well substantiated by the observations at a time when the observed velocities can be relied on.

Though a second solution according to the foregoing grouping should have been carried out to satisfy the agreement between equation and ephemeris residual, yet any changes thereby deduced would have been of a vanishing order, and as the grouping at basis was faulty, no good purpose could have been served by such solution. In the new grouping which was now made all the plates whereon the component spectra were not distinctly resolved were grouped into two normal places at or near the two points of intersection of the curves. They have, owing to the number of plates involved, relatively high weights.

The same set of preliminary elements as before were used and the following corrections resulted.

$$\delta\gamma = +\ 2{\cdot}38 \text{ km.}$$
$$\delta K = -\ {\cdot}16 \text{ km.}$$
$$\delta e = +\ {\cdot}030$$
$$\delta\omega = -\ 12^\circ\ 01'$$
$$\delta T = -\ {\cdot}206 \text{ days.}$$

The corrected elements for component I. then are:

$$P = 9{\cdot}605 \text{ days}$$
$$e = {\cdot}180$$
$$\omega = 267^\circ\ 59'$$
$$K = 68{\cdot}16 \text{ km.}$$
$$\gamma = +\ 11{\cdot}61 \text{ km.}$$
$$T = \text{J. D. } 2417679{\cdot}394.$$

The sum of the squares of the residuals was reduced from 883·5 to 147·1.

Elements corresponding almost to the above corrected values were used as preliminary in a least-squares solution for component II. The maximum positive for the curve seemed to be fixed about + 79 by the observations, while the maximum negative was − 61. This with corresponding values for e and ω gave a somewhat discrepant value for γ, nevertheless the elements following were assumed as preliminary for the solution.

$$\left.\begin{array}{l} P = 9{\cdot}605 \\ e = {\cdot}180 \\ \omega = 88^\circ \\ K = 70 \text{ km.} \\ \gamma = +\ 8{\cdot}56 \text{ km.} \\ T = \text{J.D. } 2417679.394 \end{array}\right\} \text{Preliminary elements for solution of component II.}$$

The two normal places for component I. previously referred to as comprising all the plates on which the lines were blended were also used in this solution. This seemed a reasonable procedure as the blended observations refer equally to both components. The following corrections were the result of this solution.

$$\delta\gamma = +1{\cdot}07 \text{ km.}$$
$$\delta K = +2{\cdot}29 \text{ km.}$$
$$\delta e = -{\cdot}025$$
$$\delta\omega = +2^\circ\ 10'$$
$$\delta T = -{\cdot}031 \text{ day,}$$

so that the corrected value for component II. are :

$$P = 9{\cdot}605 \text{ days}$$
$$e = {\cdot}155$$
$$\omega = 90^\circ\ 10'$$
$$K = 72{\cdot}29 \text{ km.}$$
$$\gamma = +9{\cdot}63 \text{ km.}$$
$$T = \text{J. D. } 2417679{\cdot}363.$$

These values reduced Σpvv from 490·8 to 143·0.

The question now arises as to the best method of combining the results arrived at from each component to secure uniform values for the elements. For the values of γ, e and T must be identical, whilst the values for ω must differ by 180°. As we have determined them, they are :

	Component I.	Component II.
γ	+ 11·61 km.	+ 9·63 km.
e	·180	·155
ω	267° 50′	(270° 10′ − 180°)
T	J. D. 2417679·304	J. D. 2417679·303

One might combine them according to the relative weights of the observations, which in this case are 31·3 and 25·7 for components I. and II. respectively. Again one might combine according to the probable errors of the determined quantities, weighting as the inverse square of the probable errors. Both these have been performed but before giving the results a better method than either, suggested by the Director, Dr. W. F. King, will be given. It consists in combining all observations on both components into *one* set of observation equations from which, of course, only *one* set of elements result. In building up the observation equations one must be careful to remember that for ω in one case we must use $180 + \omega$ in the other.

For preliminary elements the following were assumed.

P = 9·605 days
e = ·180
ω = 268° and 88°
γ = + 8·56 km.
K_1 = 67·87 km.
K_2 = 70· km.
T = J. D. 2417679·394

} Preliminary elements for combined solution

This solution gave the following corrections:

$\delta\ \gamma$ = + 1·25 km.
$\delta\ K_1$ = + ·53 km.
$\delta\ K_2$ = + 2·05 km.
$\delta\ e$ = − ·011
$\delta\ \omega$ = + 5° 00′
$\delta\ T$ = + ·059 days

Hence the final values which are considered as definitive, with their probable errors, are the following:

P = 9·6045 days
e = ·169 ± ·011
ω_1 = 273° ± 2° 55′
ω_2 = 93° ± 2° 55′
K_1 = 68·40 km. ± 0·92 km.
K_2 = 72·05 km. ± 1·15 km.
A_1 = 69·00 km.
B_1 = 67·80 km.
A_2 = 71·41 km.
B_2 = 72·69 km.
T = J. D. 2417679·523 ± ·073
γ = + 9·80 km. ± 0·56 km.
$a_1 \sin i$ = 8,904,000 km.
$a_2 \sin i$ = 9,380,000 km.
$m_1 \sin^3 i$ = 1·36 ☉.
$m_2 \sin^3 i$ = 1·29 ☉.

} Final Values

The value of Σpvv for the normal equations was reduced from 786·5 in the case of the preliminary elements to 429·8 and as may be noted in the following table, last column, satisfactory agreement was obtained between equation and ephemeris residuals. The phases in the table are referred to the final value for T.

NORMAL PLACES, COMBINED SOLUTION

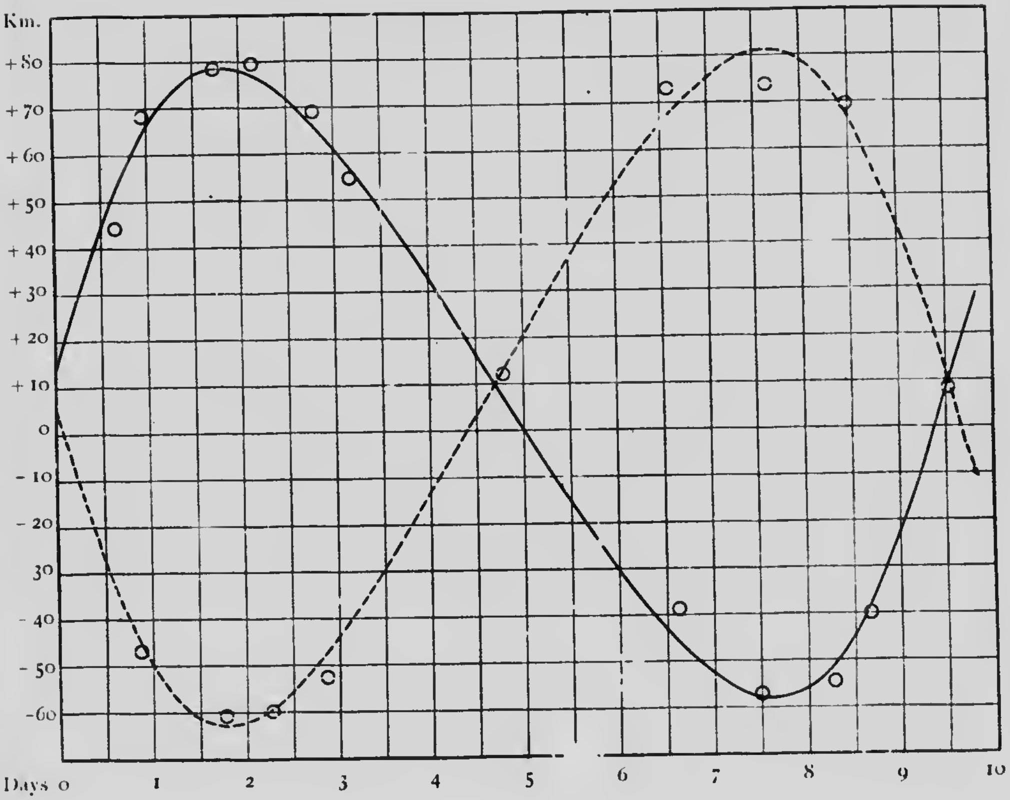
	Mean Phase	Mean Vel.	Weight	O -- C	Equation-Ephemeris
1	1·737	+ 78·53	2·5	– ·18	+ ·00
2	2·141	+ 79·71	2.	+ 2·59	+ ·09
3	2·785	+ 69·21	2·	+ 2·70	+ ·22
4	3·169	+ 54·60	·3	– 2·42	+ ·16
5	6·619	– 38·70	2·	+ 6·67	– ·08
6	7·497	– 56·96	1·5	+ ·73	– ·08
7	8·263	54·41	1·5	3·17	– ·12
8	8·657	– 39·87	2·	– 1·17	– ·14
9	·653	+ 44·26	1·5	– 7·65	– ·09
10	·955	+ 68·47	1·5	+ 4·00	+ ·05
11	1·780	– 60·70	3·5	+ 2·18	– ·02
12	2·268	– [illegible]·90	1·	– ·29	– ·05
13	2·862	– 62·65	1·5	– 4·62	– ·13
14	4·782	+ 12·08	10·	– ·14	– ·07
15	6·575	+ 73·25	1·	+ 6·35	+ ·04
16	7·643	+ 73·83	1·	– 7·38	+ ·23
17	8·487	+ 69·60	1·5	+ 2·08	– ·29
18	9·526	+ 8·41	4·5	– 2·28	+ ·13
19	·872	– 46·72	1·5	– 2·06	+ ·11

The probable error of a plate obtained from the residuals as scaled, with their corresponding weights, is ± 3·21 km. per sec. In the curves shown, which represent the final elements, the continuous curve and circles refer to component I. and the broken ones to component II.

A comparison of the elements common to both components, arrived at in the various ways, is given in the following table.

COMPARISON OF ELEMENTS

Elements	Solution Component I	Solution Component II	Combined According to Weights	Combined According to Probable Errors	One Direct Solution
γ	11·61 km.	+ 9·63 km.	+ 10·72 km.	+ 10·80 km.	+ 9·80 km.
e	·180	·155	·169	·170	·169
ω	267° 59′	90° 10′	268° 58′	269° 11′	273° and 93°
T	..... 9·394	..... 9·363	..... 9·380	..... 9·377	..... 9·453



VELOCITY CURVE OF *d* BOÖTIS

There seems no doubt to the writer that the last solution wherein all the observations on both components are grouped into one set of observation equations resulting in a uniform set of values, is the only rigid one, and, as previously stated, the suggestion for such a procedure came from the Director, Dr. W. F. King, to whom my acknowledgements are due for this and other valuable suggestions.

One detail in which, for future work, the foregoing can be improved upon. The two groupings representing the blend plates should be broken up into four, with residuals scaled from each curve, the total weight assigned the four normal places being equal to that formerly given the two; or, retaining the two normal places, adjust the corresponding residuals in the observation equations so that they represent the deviations from both curves instead of from one alone as in the present discussion.

DOMINION OBSERVATORY,
OTTAWA, CANADA,
August, 1911.