Applying the equation of "momentum" or "impact" as expressed by (10a) and considering the same prism of water as before, we obtain the following result:—

$$\frac{Q\gamma(v_2 - v_1)}{g} = \frac{F_1 v_1 \gamma (v_2 - v_1)}{g} = F_1 p_1 + (F_2 - F_1) p_2 - F_2 p_2$$

where F_1p_1 is the total pressure on the small end of the prism, $(F_2 - F_1)p_2$ is the total pressure on the annular surface of the prism at the enlargement in section and F_2p_2 is the opposing pressure on the large end of the prism. This equation reduces to

$$\frac{p_1 - p_2}{\gamma} = \frac{v_1(v_2 - v_1)}{g} \quad . \quad . \quad (12)$$

which is the universally accepted formula for this case, which is a case of true "direct central impact" since the centre of gravity of the prism moves continuously forward in the direction of motion of the prism, or looked at in another way, the line, joining the centres of gravity of the prisms impinging at the change of section, is in the direction of motion of the prisms.

We will next consider the problem of the standing wave. Here we do not have a case of true "direct central impact" and hence the analysis by the impact method is not exactly rigorous. By the method of "work and energy," (Equation 9a), we have, referring to Fig. 4,

$$\frac{bh_1^2\gamma l_1}{2} - \frac{b(h_1 + x)^2\gamma l_2}{2} - \frac{bh_1\gamma l_1 x}{2} = \frac{bh_1 l_1\gamma}{g} \cdot \frac{(v_2^2 - v_1^2)}{2}$$

where the first two terms of the left-hand member represent the work of the end pressures on the prism and the third, the work of raising the centre of gravity of the entire prism an amount $\frac{x}{2}$.

Since $bh_1l_1 = bh_2l_2 = b(h_1 + x)l_2$ (inasmuch as we have under consideration the prism CDE^1E during the time it takes it to come into position FEE^1GH), we easily find

$$x = \frac{v_1^2 - v_2^2}{2g} = \frac{v_1^2}{2g}\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2gh}{v_1^2}}\right) - h_1, \quad (13)$$

which, just as in the previous case, indicates no loss due to shock and is therefore incorrect. If we apply the impact method (Equation 10a) to the same prism of water over the same period of time, we have

$$\frac{bh_1^2\gamma}{2} - \frac{b(h_1 + x)^2\gamma}{2} = \frac{bh_1\gamma v_1(v_2 - v_1)}{g}$$

Putting $h_1 + x = h_2$ and remembering that $bh_1v_1 = bh_2v_2$, we find

$$h_2 - h_1 = x = \frac{2v_1v_2(v_1 - v_2)}{g(v_1 + v_2)} \quad (14)$$

Except for the error in assuming this to be a case of direct central impact, this is the correct formula for the standing wave. This error can be expressed in another way, by saying that the end pressures $\frac{bh_1^2\gamma}{2}$ and $\frac{b(h_1 + x)^2\gamma}{2}$ do not act in the same straight line and hence cannot strictly be used in the equation derived from Newton's second law. A more convenient and useful form of this equation for the standing wave is

$$x = -\frac{3}{2}h_1 + \sqrt{\frac{h_1^2}{4} + \frac{2h_1v_1^2}{g}} = h_1\left(-\frac{3}{2} + \sqrt{\frac{1}{4} + \frac{2v_1^2}{gh_1}}\right), \quad (14a)$$

which is the form given by Gibson and others. Despite its acknowledged defect, it seems to be the most rational formula yet derived for this problem and certainly is based upon sounder reasoning than the formula given in Merriman's "Hydraulics" and in the American Civil Engineer's Handbook.

There still remains for solution the problem of the rise in the water surface in the flume due to instantaneous closure of a gate or valve at its lower end. Referring to Fig. 5, we will consider a prism of water of original length l_1 and depth h_1. Then, after the expiration of some length of time "t", we assume the prism will have taken the position shown by the dotted lines and that the section at A has just reached A^1 and stopped. We further assume that the upper surface of the water already raised remains practically horizontal and in a fixed position.

Evidently from the equation of continuity

$$xl_2 = h_1(l_1 - l_2)$$

$$\text{or } l_1 - l_2 = \frac{xl_1}{h_1 + x} = \frac{xl_1}{h_2}$$

and hence we may write $t = \frac{l_1 - l_2}{v_1} = \frac{xl_1}{h_2v_1}$.

If we apply the equation of work and energy (Equation 9a) to the mass under consideration for the time "t" and neglect the loss due to sudden change or shock, we have

$$\frac{bh_1^2\gamma(l_1 - l_2)}{2} - \frac{bh_1l_1\gamma x}{2} = \frac{bh_1l_1\gamma}{g} \cdot \frac{(-v_1^2)}{2}$$

Solving for x, the rise in height, we find

$$x = \frac{v_1^2}{2g}\left(1 + \sqrt{1 + \frac{4gh_1}{v_1^2}}\right) \quad . \quad . \quad (15)$$

If we call the velocity of propagation of the "bore" or wave front "C", then since it travels the distance l_2 while the water travels the distance $(l_1 - l_2)$,

$$\frac{C}{v_1} = \frac{l_2}{l_1 - l_2} = \frac{h_1}{x}$$

and

$$C = \frac{h_1v_1}{x} \quad . \quad . \quad . \quad . \quad (16)$$

We will now apply the equation of impact (Equation 10a) to this problem although, as in the case of the standing wave, it is not a case of true direct central impact,

$$\frac{bh_1^2\gamma}{2} - \frac{b(h_1 + x)^2\gamma}{2} = -\frac{bh_1v_1\gamma(-v_1)}{g}$$

Putting $h_1 + x = h_2$ and solving for $x = h_2 - h_1$, we find

$$x = h_1\left(-1 + \sqrt{1 + \frac{2v_1^2}{gh_1}}\right) \quad . \quad . \quad (17)$$

This would seem to be the most rational formula for this case.

For the reasons stated, the writer believes the formulas for the standing wave and "bore" derived by the "momentum" or "impact" method to be the most rational yet proposed. It seems quite probable that they can be revised to take account of the fact that the impact is not "central" and thus be made entirely rigorous.

The question of the "bore" in a flume arose several years ago in connection with a certain hydro-electric development where the nature of the load was such that it was liable to be thrown off almost instantaneously. Nothing could be found on the subject in English, but after making independent studies in collaboration with Mr. W. E. Germer and arriving at the formulas here given, a number of experiments on the subject undertaken by Darcy and Bazin were called to our attention by Prof. I. P. Church. A study of this data led us to the conclusion that for all practical purposes the impact formula for this case is sufficiently accurate.

The six-mile Selkirk tunnel will be completed in the fall, according to the C.P.R. officials. The cost will be about $12,000,000. That is about the only really big work in which the C.P.R. has been engaged lately, but it is interesting to recall that in the years before the war the company spent between $25,000,000 and $35,000,000 per annum in the development of the West, and since its inception, has spent over $200,000,000 for that purpose.