

we have
$$\left(\frac{b^3 - c^3}{a^3} + \frac{c^3 - a^3}{b^3} + \frac{a^3 - b^3}{c^3} \right) =$$

$$= \frac{(b^3 - c^3)(c^3 - a^3)(a^3 - b^3)}{a^3 b^3 c^3}.$$

Given expression

$$= \frac{1}{a^3 b^3 c^3} [a^3(a^3 - b^3)(a^3 - c^3) + \text{etc.}]$$

$$= \frac{a^9 + b^9 + c^9 - a^6(b^3 + c^3) - b^6(c^3 + a^3) - c^6(a^3 + b^3)}{a^3 b^3 c^3}$$

$$= \frac{c^3(a^3 + b^3) + 3a^3 b^3 c^3}{a^3 b^3 c^3}$$

$$= (a^3 + b^3 + c^3)^3 - 4a^3(b^3 + c^3) - 4b^3(c^3 + a^3) - 4c^3(a^3 + b^3) - 3a^3 b^3 c^3,$$

introducing the condition $a^3 + b^3 + c^3 = 3abc$, this becomes $24 - 4a^3(b^3 + c^3) - 4b^3(c^3 + a^3) - 4c^3(a^3 + b^3) = 36 - 4(a^3 + b^3 + c^3)(a^3 + b^3 + c^3)$.

2. If $(z + x - y)(x + y - z) = ayz$

$$(x + y - z)(y + z - x) = bzx$$

$$(y + z - x)(z + x - y) = cxy$$

prove $(abc)^{\frac{1}{2}} + a + b + c = 4$.

Let $lx = y + z - x$, $my = z - x$, $nz = x - y$, (1)

we get $mn = a$, $nl = b$, $lm = c$,

$\therefore al = bm = cn = (abc)^{\frac{1}{2}}$.

Eliminating xyz from (1) we have

$$\begin{vmatrix} l + 1, -1, -1 \\ -1, m + 1, -1 \\ -1, -1, n + 1 \end{vmatrix} = 0,$$

substituting for l, m and n in the determinant $(abc)^{\frac{1}{2}} + a + b + c = 4$.

3. Solve the equations

$$(z + x - y)(x + y - z) = ax$$

$$(x + y - z)(y + z - x) = by$$

$$(y + z - x)(z + x - y) = cz.$$

Let $p = y + z - x$, $q = \text{etc.}$, $r = \text{etc.}$, then $2qr = a(q + r)$ etc.,

whence $\frac{1}{p} = \frac{1}{b} + \frac{1}{c} - \frac{1}{a}$, $\frac{1}{q} = \text{etc.}$, $\frac{1}{r} = \text{etc.}$,

$\therefore x = \frac{q + r}{z} = \frac{1}{a} \div \left(\frac{1}{a} + \frac{1}{b} - \frac{1}{c} \right)$

$$\left(\frac{1}{a} + \frac{1}{c} - \frac{1}{b} \right), \text{ etc.}$$

SEE FEBRUARY NO., 1882.

Answers by Wilbur Grant, T. C. I.

1. $1\frac{2}{3}\frac{2}{3}$.

2. 494'550.

3. $4\frac{1}{2}$.

4. 504 grammes.

5. 9, 9, 9, 9, 9, 9.

6. (a) $(x \pm 1)(x^2 \pm x + 1)$;

(b) $\frac{x+1}{x+1} \cdot \frac{x+2}{x+2} \cdot \frac{x+3}{x+3} \cdot \frac{x+4}{x+4}$.

7. (a) $\frac{x+b}{bx+1}$;

(b) $\frac{x+y+z}{(x^2+y^2+z^2) + (xy+yz+zx)}$.

8. Ratio of 1 : 4 or 4 : 1.

9. The problem as stated is incorrect.

10. £30; £40; £24; £6.

SELECTED PROBLEMS.

1. If O be the centre of the circle described about the triangle ABC and AO, BO, CO , be produced to meet the opposite sides in D, E, F , the circle in D', E', F' , respectively,

prove that $\frac{DD'}{AD} + \frac{EE'}{BE} + \frac{FF'}{CF} = 1$.

2. If $x + y + z = x^{-1} + y^{-1} + z^{-1} = 0$,

prove $\frac{x^3 + y^3 + z^3}{x^3 + y^3 + z^3} = \frac{x^9 + y^9 + z^9}{x^6 + y^6 + z^6} = xyz$,

and $x^3 + y^3 + z^3 = 0$.

3. If $x = bz + cy$

$$y = cx + az$$

$$z = ay + bx,$$

prove $\frac{x^2}{1-a^2} = \frac{y^2}{1-b^2} = \frac{z^2}{1-c^2}$,

$$\sqrt{\frac{1-a^2}{a}} + \sqrt{\frac{1-b^2}{b}} + \sqrt{\frac{1-c^2}{c}}$$

$$= \frac{\sqrt{1-a^2} \cdot \sqrt{1-b^2} \cdot \sqrt{1-c^2}}{abc}.$$

4. If $yz + zx + xy = 1$, show that

$$\frac{x}{1-x^2} + \frac{y}{1-y^2} + \frac{z}{1-z^2}$$

$$= \frac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}.$$

5. Show that $\left(\cos \frac{\pi}{8} \right)^8 + \left(\cos \frac{3\pi}{8} \right)^8$

$$+ \left(\cos \frac{5\pi}{8} \right)^8 + \left(\cos \frac{7\pi}{8} \right)^8 = \frac{17}{16}.$$