Examination Questions.

JULY EXAMINATIONS, 1881.

FIRST CLASS TEACHERS.—Grades A and B.

(Continued from last month.)

ELEMENTARY GEOMETRICAL OPTICS.

Time—Two Hours.

Examiner—J. C. Glashan.

Note—*Questions marked with Roman Numerals are to be answered on separate sheets, folded separately from the rest, though enclosed in the same envelope.*

1. Find the geometrical focus of a pencil after direct refraction at a plane surface.

A point within a solid cube of glass is viewed directly through each of the faces. Show that the six apparent positions of the point from an octahedron whose volume is $\frac{1}{6}\left(1-\frac{1}{\mu}\right)^3$ of the cube, where μ is the refractive index from air into the glass.

2. Show how to determine the apparent position of a luminous point under water to an eye above the water.

To a person standing in water of uniform depth the bottom appears to approach the surface in all directions from him. Give a drawing showing clearly how to determine the shape of the bottom.

3. Determine the deviation of a ray of light reflected through a prism.

Show that if the ray be composed of different kinds of light, with various refractive indices, they will be separated in passing through the prism, selecting for convenience the case in which the angles of incidence and emergence are small, and show that in this case the deviation is independent of the angle of incidence.

4. Find the distance from the centre of a sphere of the geometrical forms of a pencil of rays after direct refraction at the surface.

If a pencil of parallel rays be incident directly on a solid sphere, be turned back by reflection at the opposite face, and after a second refraction pass from the sphere, it can never finally deviate from the centre, the foci being supposed all within the sphere.

5. Find the geometrical focus of a pencil after direct refraction through a thin lens.

Determine the kind of lenses suitable for long and short sighted persons.

Will the difficulties of seeing be increased or diminished to a short-sighted person under water? Explain clearly.

6. Explain how vision takes place through an opera glass, tracing the course of a pencil of rays through it; and explain fully how the eye is assisted by such an instrument.

Determine the magnifying power, having given the focal length of each lens.

VII. Describe the construction of a spectroscope, and show by diagram the relative position of its optical parts.

Describe the different kinds of spectra.

VIII. Explain the method of determining the illuminating power of a source of light by means of Bunsen's photometer.

ANALYTICAL GEOMETRY.

Time—Two Hours and a Half.

Examiner—J. C. Glashan.

1. Find the angle between two lines whose equations are given.

If a, b be the differences of the intercepts of the two lines on the axes of x and y respectively, and θ, the angle between the lines, be constant, then $\frac{x}{a}-\frac{y}{b}=\cot\,\theta$ is the locus of their point of intersection.

2. The equation $f(x, y)=0$ represents a curve, and cannot represent any part of a plane.

Interpret the equations

$$f(\theta)=0,\ \frac{\sin\theta}{\rho}=0,\ \rho\sin n\,\theta=0,$$
$$x^6-y^6=0,\ x^2-4xy+5y^2=0.$$

3. Find the conditions to which the constants in the equation $ax^2+by^2+2cxy+2dx+2ey+f=0$ are subject in order that the axes may be

(1.) A tangent and normal to the curve.
(2.) Two tangents.
(3.) A tangent and line from point of contact through the centre.
(4.) Two lines through the centre.

4. The equation to a conic is $4y^2+3xy+4x^2-8x-8y=0$; find its centre and the magnitudes and positions of its principal axes.

5. Determine the curve represented by the equations $x=a\sec\alpha$, $y=b\tan\alpha$, where x, y, α are variables, and interpret geometrically the angle α.

If a tangent be drawn from any point on the curve to the circle on its transverse axis, its length is $ae\tan\alpha$.

6. The equation to the tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point (x', y') is $\frac{xx'}{a^2}+\frac{yy'}{b^2}=1$.

By transferring the origin to the point (2, 3), find the tangent to the curve $3x^2-xy+2y^2-6x-4y=0$ at this point.

7. A is a point without a conic, and a chord is drawn cutting the conic in B and C: find the locus of the intersection of tangents at B and C, as ABC revolves about A.

The angle subtended at the focus by any chord is bisected by the line joining the focus to its pole.

8. Show that the polar equation to a conic section is $\frac{l}{\rho}=1+e\cos\theta$; also that the polar equation to a tangent at the point $\left(\alpha,\frac{l}{1+e\cos\alpha}\right)$ is $\frac{l}{\rho}=\cos(\theta-\alpha)+e\cos\theta$.

9. In an ellipse if P, Q be the ends of conjugate diameters
$$CP^2+CQ^2=a^2+b^2;$$
and if they be the ends of lines drawn from the centre at right angles to each other $\frac{1}{CP^2}+\frac{1}{CQ^2}=\frac{1}{a^2}+\frac{1}{b^2}$.

DYNAMICS.

Time—Two Hours and a Half.

Examiner—Alfred Baker, M.A.

1. Given an acceleration and velocity with certain units of space and time, show how to express them when the units of space and time are changed.

If the unit of acceleration be the acceleration due to gravity, and the unit of velocity be that of a point which describes 1 foot in 32 seconds, find the units of space and time.

2. Obtain the formula $s=ut+\frac{1}{2}ft^2$.

If a particle move from rest under the influence of two given uniform accelerations, making an angle θ with one another, obtain expressions for the position of the particle and for the resultant velocity at the end of any time.

How would you require to proceed to determine the space described?

3. When a particle is moving in a plane curve obtain an expression for the normal acceleration.

A 3lb. weight is attached to one end of a string 4 ft. long, and is swung round in a horizontal circle, the string being held at the other end. If one revolution be made each second, find in pounds the tension of the string.

4. Explain the illustrations of the laws of motion furnished by Atwood's machine.

If A and B be the weights ($B>A$), and if as A rises it receives at a platform an additional weight $\frac{B^2-A^2}{A}$, and as it descends deposits it again on the platform, and so continues to oscillate above and below the platform, receiving and depositing the weight, the whole time above the platform before the system is reduced to rest is equal to the whole time below it.

5. The time of descent down any chord of a vertical circle from the highest point is constant.

If A be the highest point of the circle, and B the lowest, and C any other point on the circle, and friction be supposed to act, the time of descent from A to C is always greater than that from C to B, provided AC be less than CB.

6. Determine the amount of kinetic energy lost in the following