$4)$12(1......1st partial quotient—a *multiplier.*
$4
1st Rem'r. $8(+ 1....2nd partial quotient—a *multiplier*
$4
2nd Rem'r.$4(+ 1...3rd partial quotient—a *multiplier.*
$4

The complete quotient is therefore 1 + 1 + 1 = THREE: NOT three dollars, or "three, four dollars," or three *anything else* outside of the world of *Mind*

(*b*) Divide $12 by 4, or find the number of dollars which, with four as a multiplier, will make $12. Or, using the idea of partial quotients, the problem is: $12 = ($1 + $1 + ...) × 4, find the number within the brackets.

4)$12($1......1st partial quotient—a *multiplicand.*
$4
$8(+ $1...2nd partial quotient—a *multiplicand.*
$4
$4(+ $1...3rd partial quotient—a *multiplicand.*
$4

The complete quotient is therefore $1 + $1 + $1 = $3—the required multiplicand, which must be used in recalculating the dividend. In (*a*) we had to use the preliminary ("apperceiving") knowledge that *once* $4 is $4; in (*b*) the preliminary knowledge that *four times* $1 is $4.

Since the operations of the Common Rules may be performed by counting, the foregoing examples may be illustrated by simple intuitions. (*a*) Divide $12 by $4, i.e. find the number of $4 groups in $12—Representing the $12 by dots, the result is obtained directly by counting:—

. . . | . ...1st 4 dollar group
. . . | . ...2nd " " "
. . . | . ...3rd " " "

Here each of the rows is not simply *four*, but four *dollars*, the absolute value of the unit-group. While the vertical column of three dots may be taken to *represent* the *pure number three*, i.e. the *times* the unit-group occurs. It corresponds to the 1 + 1 + 1 of the operation (*a*) with symbols.

Now take the other case (*b*). Divide $12 by 4, i.e. find the group of dollars which multiplied by 4 will make $12.

. . . | . .....four groups of $1 each
. . . | . ..... " " " " "
. . . | . ..... " " " " "

Counting the 1st row we get *four* groups of $1 each; then the 2nd row, etc. Therefore, in all, four groups of ($1 + $1 + $1 = $3)each. Where the right hand column of dots no longer represents a *pure number*, but the absolute magnitude of a quantity, i.e. three—*dollars*. Now, once more, the operations here indicated are *not* "widely and radically different;" they are correlative; the column cannot be had without the row, nor the row without the column: The *difference* is in the *meaning* of the result.

From the foregoing it appears plain that, as to preliminary knowledge:—

1. The idea of *times* must be clealy grasped since it is essential to number-conception.

2. The multiplier *must* be abstract—it denotes simply *times*.

3. In multiplication the factors may be interchanged, i.e. $a \times b = b \times a$; or in words, *b* groups of *a* units each is identical with *a* groups of *b* units each.

As to division:—

1. The operation in division is the inverse of that in multiplication. That is the problem of division is: Given the product and one of two factors which produce it, to find the other factor.

2. Therefore there may, "in a sense," be two kinds of division, corresponding respectively to the search for the *multipiler*, and the search for the multiplicann—i.e.for the *number* of unit *groups*, and for the number of units in each group.