Langevin has carried out four of the remaining integrations in such a way as to obtain the result

$$\nu=\frac{2\pi^3k^2}{hm}\sqrt{\frac{\pi}{2hm}}\int_0^\sigma bdb\int_0^\infty g^3e^{-\frac{1}{2}hmg^2}dg,$$

where σ is the radius of the sphere of action of the molecule and

$$k=n\sqrt{\frac{h^3m^3}{\pi^3}}.$$

If now we let θ (fig. 4 *b*) be the angle between g and the normal to the sphere of action, then v the velocity normal to this sphere is equal to $g\cos\theta$ and $b=\sigma\sin\theta$. Making these substitutions in the above integral, we obtain for the total number of collisions per c.c. per second for which the relative velocity normal to the sphere of action is less than a certain v,

$$\begin{aligned}
\mathrm{L}&=\frac{2\pi^3k^2\sigma^2}{hm}\sqrt{\frac{\pi}{2hm}}\int_0^v v^3dv\int_0^{\frac{\pi}{2}}\frac{\sin\theta}{\cos^3\theta}e^{-\frac{hmv^2}{2\cos^2\theta}}d\theta\\
&=\frac{2\pi^3k^2\sigma^2}{h^2m^2}\sqrt{\frac{\pi}{2hm}}\int_0^v ve^{-\frac{1}{2}hmv^2}dv\\
&=\frac{2\pi^3k^2\sigma^2}{h^3m^3}\sqrt{\frac{\pi}{2hm}}(1-e^{-\frac{1}{2}hmv^2})\\
&=n^2\sigma^2\sqrt{\frac{2\pi}{hm}}(1-e^{-\frac{1}{2}hmv^2}).
\end{aligned}$$

To obtain the number of collisions per c.c. per second for which the relative velocity tangential to the sphere of collision is greater than a certain u, put $u=g\sin\theta$, and $b=\sigma\sin\theta$ in the expression for ν above. Then the number of such collisions is

$$\begin{aligned}
\mathrm{M}&=\frac{2\pi^3k^2\sigma^2}{hm}\sqrt{\frac{\pi}{2hm}}\int_u^\infty u^3du\int_0^{\frac{\pi}{2}}\frac{\cos\theta}{\sin^3\theta}e^{-\frac{hmu^2}{2\sin^2\theta}}d\theta\\
&=\frac{2\pi^3k^2\sigma^2}{h^2m^2}\sqrt{\frac{\pi}{2hm}}\int_u^\infty ue^{-\frac{1}{2}hmu^2}du\\
&=\frac{2\pi^3k^2\sigma^2}{h^3m^3}\sqrt{\frac{\pi}{2hm}}e^{-\frac{1}{2}hmu^2}\\
&=n^2\sigma^2\sqrt{\frac{2\pi}{hm}}e^{-\frac{1}{2}hmu^2}.
\end{aligned}$$