

Taking any two directions at right angles to each other, let each force be resolved into its components in these directions. Let the algebraic sum of these resolved parts in the one direction be X , and in the other Y .

Then the whole set of Forces is equivalent to the two X, Y .

Hence, if R be the Resultant of the whole set, and therefore also of the two X, Y , and θ the angle it makes with the direction of X , the equations in § 22 give

$$R^2 = X^2 + Y^2, \tan \theta = \frac{Y}{X},$$

which determine the Resultant in magnitude and direction. We have also the equivalent relations

$$X = R \cos \theta, Y = R \sin \theta.$$

Conditions of
equilibrium.

25. *To find the conditions of equilibrium when any Forces act at a point, their directions being in one plane.*

Retaining the notation and method of the last article, since the only condition, in order that the point acted on by the Forces may be kept at rest, is (§ 15) that the Resultant of the Forces must be zero; that is, $R = 0$, we have

$$X = 0, Y = 0.$$

And, conversely, if $X = 0$, and $Y = 0$, then we also have $R = 0$, and the point will be kept at rest; hence the necessary and sufficient conditions of equilibrium are that

The algebraic sums of the Forces resolved into two perpendicular directions shall separately vanish.

This principle will be cited under the name of "The vanishing of the Resultant."

26. The process might be readily extended to forces not all acting in one plane.

Thus, if three forces not in one plane act at a point, and three lines be drawn representing them in magnitude and direction; then, if the parallelepiped, of which these lines are adjacent edges, be completed,