and $a = \frac{1}{xyz} \therefore ab = 1$

and substituting these values in the original equation, each member reduces to zero.

4. If $\frac{ax-by}{z} = \frac{ay-bz}{x} = \frac{az-bx}{y}$ prove that $x = y = z$; each fract. $= a-b$ and by equating them separately to $a-b$, we obtain

$$-\frac{b}{a} = \frac{x-y}{y-z} = \frac{y-z}{z-x} = \frac{z-x}{x-y}$$

$\therefore (y-z)^2 = (z-x)(x-y), (z-x)^2 = (x-y)(y-z)$

eliminate $x-y$ and we have $(y-z)^3 = (z-x)^3$ whence $2z = x+y$, similarly $2x = y+z$. eliminate y from these and we get $x = z$, &c.

5. Prove that

$$3^n = 1 + n2^n + \frac{n(n-1)(n-2)}{\lfloor 3} 2^{n-2} + \frac{n(n-1)(n-2)(n-3)(n-4)}{\lfloor 5} 2^{n-4} + \&c.$$

$$(2+1)^n - (2-1)^n = 2^n + n2^{n-1} + \frac{n(n-1)}{\lfloor 2} 2^{n-2} + \frac{n(n-1)(n-2)}{\lfloor 3} 2^{n-3} + \&c.$$

$$-2^n + n2^{n-1} - \frac{n(n-1)}{\lfloor 2} 2^{n-2} + \frac{n(n-1)(n-2)}{\lfloor 3} 2^{n-3} - \&c.$$

$$= n2^n + \frac{n(n-1)(n-2)}{\lfloor 3} 2^{n-2} + \&c.$$

$$\therefore 3^n = 1 + n.2^n + \frac{n(n-1)(n-2)}{\lfloor 3} 2^{n-2} + \&c$$

6. If A, B, C, are the angles of a triangle, then $\sin(A-B)\sin C + \sin(B-C)\sin A + \sin(C-A)\sin B = O$

This expression $= \sin(A-B)\sin(A+B) + \&c. = \sin^2 A - \sin^2 B + \sin^2 B - \sin^2 C + \sin^2 C - \sin^2 A.$

7. If $\sin l = \frac{a-b}{a+b}$, $\sin m = \frac{b-c}{b+c}$, $\sin n = \frac{c-a}{c+a}$

prove that

$\sec^2 l + \sec^2 m + \sec^2 n = 2 \sec l \sec m \sec n + 1$

$\sec^2 l = \frac{(a+b)^2}{4ab}$, $\sec^2 m = \frac{(b+c)^2}{4bc}$, $\sec^2 n = \frac{(c+a)^2}{4ac}$

$\therefore \sec^2 l + \sec^2 m + \sec^2 n =$

$$\frac{c(a+b)^2 + b(a+c)^2 + a(b+c)^2}{4abc}$$

$$= 2\frac{(a+b)}{2\sqrt{ab}} \times \frac{(b+c)}{2\sqrt{bc}} \times \frac{(a+c)}{2\sqrt{ac}} + 1 = \&c.$$

8. A started from Ottawa at 9 a.m. to walk to Chelsea. After he had walked $1\frac{1}{6}$ miles. B started and overtook A half way there. A then increased his pace one fifth and B decreased his one-ninth, and they reached Chelsea together at $11.28\frac{1}{2}$ a m Find the distance to Chelsea.

Let x=A's rate y=B's and $z=\frac{1}{2}$ distance. Then from the second part of the question,

$$\frac{6}{5}x = \frac{8}{9}y \quad \therefore x = \frac{20}{27}y$$

and from the first part

$$\frac{z-1\frac{1}{6}}{x} = \frac{z}{y} \therefore 2z = 9$$

9. O is the point in OA perpendicular to the straight line ABC, from which BC appears the longest; prove that

$$\tan COB = \frac{BC}{2\,AO}$$

The point in AO from which BC appears the longest is the pt. where $AO^2 = AB, AC$ or where the circle described through B & C touches O.

Now $\tan. COB = \tan. (AOC - AOB)$

$$\frac{\tan(AOC) - \tan AOB}{1 + \tan AOB, \tan AOC} = \frac{BC}{2AO}$$

10. An object is observed at three pts. A,B,C, lying in a horizontal line which passes directly underneath the object; the angles of elevation at A,B,C, are m, $2m$, $3m$, and $AB=a$, $BC=b$; prove that the height of the object is

$$\frac{a}{2b}\sqrt{(3a-b)(a+b)}$$

Let the perpendicular from the object meet the one in D and let h denote the height of the object.

$\tan. m = \frac{h}{a+b+CD}$ $\tan. 2m = \frac{h}{b+CD}$