H_2 = Vertical distance from the water surface $n-n$ in the forebay to the centre of gravity of the entrance to the surge tank.

Q, Q_1, Q_2 = Volumes of water, corresponding to time t, the time during steady flow, and for time $t = o$ (beginning).

h, h_1 and h_2 = Friction heads corresponding to the pipe dimensions and velocities v, v_1 and v_2.

z = Vertical distance from the elevation $n-n$, of the water surface in the surge tank at the time t, taken positive above $n-n$ and negative below $n-n$.

A = Sectional area of surge tank in square feet for the elevation determined by z, so that in general A is a function of z.

s = Velocity of the water surface elevation in the surge tank, positive when rising, negative when falling, as an average value assumed constant in the section A.

q = Volume of discharge through the penstock at the time t, in cubic feet per second.

$c = \frac{q}{A}$ = the discharge velocity of the volume q, in feet per second, with respect to A.

More notations will be introduced during the course of the investigations.

II. Derivation of Principal Equations.—Referring to Fig. 2, the distance between two adjacent cross-sections of the main conduit $= dl$, and therefore $\frac{w.a.dl}{g}$ = mass of the water between the two sections (w = weight of cubic unit of water, g = acceleration due to gravity).

At the left side cross-section at the time t, there exists the pressure p in pounds per square foot, an average value, assumed constant for the entire cross-section. In the right, the pressure is p' at the same time t. The whole of p' is in general different from that of p, by the amount dp. The pressure p is a function of the location of the cross-section, that is to say, depends upon l; (l = the distance of the left side cross-section from the entrance of the main conduit) and depends further upon the time t (the flow varies with the time).

$$dp = \frac{\delta p}{\delta l} dl + \frac{\delta p}{\delta t}.dt \qquad (1)$$

Since p and p' occur at the same time, p' is different from p by the difference due to the distance between the cross-sections, and therefore in the foregoing formula the differential dt of the time $= o$. Therefore, $dp = \frac{\delta p}{\delta l}.dl$ (2). The following forces act upon an element of unit mass, in the direction of flow, that is, in the direction of v:

1st. The weight component $P_1 = w.a.dl \sin\alpha$ (3) where α = the inclination of the axis of the main conduit from the horizontal, $dl.\sin\alpha = dH$ = vertical distance between the centres of gravity of the sections $a\,b$ and $a_1\,b_1$.

$$P_1 = w.a.dH \qquad (4)$$

2nd. The difference between the reactions due to the pressure p and $p + \frac{\delta p}{\delta l}.dl$ which is

$$P_2 = p.a - (p + \frac{\delta p}{\delta l}.dl)\,a = -a\frac{\delta p}{\delta l}dl \qquad (5)$$

Contrary to the direction of motion the friction exerts a force. Let K be the amount of the friction in terms of the velocity v and we get $P = -wa.dl.K$ (6) where K represents a pure number with respect to its dimension.

According to the general fundamental law: mass × acceleration = acting force, it results (as $\frac{dv}{dt}$ is the acceleration with respect to the velocity v at the time t) that,

$$m.\frac{dv}{dt} = P_1 + P_2 - P \qquad (7)$$

$$\frac{w.a.dl}{g} = w.a.dH - a\frac{\delta p}{\delta l}dl - w.a\,dl.K \qquad (8)$$

$$\frac{dl}{g}.\frac{dv}{dt} = dH - \frac{1}{w}\frac{\delta p}{\delta l}dl - K\,dl \qquad (9)$$

The velocity v has the same value at the time t in the whole length of the main conduit. The case is therefore the same for $\frac{dv}{dt}$ and K. Therefore, if we integrate between the limiting values $l = o$ and $l = L$ or $H = H_1$ and $H = H_2$ relative to a motion from the intake to the main conduit, we get the following:

$$\frac{L}{g}.\frac{dv}{dt} = H_2 - H_1 - \frac{1}{w}\int_0^L \frac{\delta p}{\delta l} dl - KL \qquad (10)$$

In the integral $\int_0^L \frac{\delta p}{\delta l}.dl$, p is, as demonstrated, a function of t and l. But as the integration relates to the condition at a certain time t, t is to be considered as a constant and therefore

$$\int_0^L \frac{\delta p}{\delta l} dl = p_2 - p_1 \qquad (11)$$

where p_2 = pressure at the entrance to the surge tank p_1 at the entrance to the main conduit, KL is nothing other than the friction head h for the entire main conduit at the time t. Now, we may say that

$$\frac{p_2}{w} = H_2 + z + \frac{p_0}{w};\quad \frac{p_1}{w} = H_1 + \frac{p_0}{w} \qquad (12)$$

in which $\frac{p_0}{w}$ equals the water column equivalent to the atmospheric pressure, and the following equation results:

$$\frac{L}{g}.\frac{dv}{dt} + z + h = o \qquad (13)$$

Having assumed a flow in the main conduit from the intake to the surge tank, we consider now the flow as reversed (from the surge tank to the intake) and keep the direction for the measurement of the length l the same, then we must consider that the friction now acts in the