Thence eliminating x as in preceding,

$$\left(\frac{c}{a-\alpha}-\beta\right)^2-\frac{b-\beta}{a-\alpha}\left(\frac{c}{a-\alpha}-\beta\right)\left(\frac{b-\beta}{a-\alpha}-\alpha\right)+$$
$$\frac{c}{a-\alpha}\left(\frac{b-\beta}{a-\alpha}-\alpha\right)^2=0.$$

9. (1) Book-work.

(2) $\dfrac{x+a}{\sqrt{x^2+a^2}} \gtrless \dfrac{x+b}{\sqrt{x^2+b^2}}$

accd. as $\dfrac{x^2+2ax+a^2}{x^2+a^2}-1 \gtrless \dfrac{x^2+2bx+b^2}{x^2+b^2}-1$

accd: as $\dfrac{ax}{x^2+a^2} \gtrless \dfrac{bx}{x^2+b^2}$

as $ax^2+ab^2 \gtrless bx^2+a^2b$

as $(a-b)x^2 \gtrless ab(a-b)$ "

as $x^2 \gtrless ab$. If $a-b$ be negative, it will be according as $ab \gtrless x^2$.

FIRST-CLASS PAPER.

1. (1) Todhunter, § 501.

(2) It may be easily shewn by geometry that if a circle can be described through any four consecutive points of a polygon, any one of such circles passes through all the angular points. Hence the answer required is one circle. The question is intended as a "catch" on the geometrical problem.

2. (1) $x(1-x)^{-2}=x+2x^2+3x^3+\ldots\ldots+nx^n+(n+1)x^{n+1}+\ldots$

$$(1-x)^n=1-nx+\frac{n(n-1)}{|\underline{2}}x^2-\ldots\ldots$$

$\therefore$ Multiplying these identities, coefficient of x^{n+1} on right-hand side is

$$n+1-n.n+(n-1).\frac{n(n-1}{|\underline{2}}-\ \ldots\ldots\quad (1)$$

While coefficient of x^{n+1} on left-hand side is coefficient of x^{n+1} in $x(1-x)^{n-2}$, which is zero. Hence (1) is zero.

(2) $\dfrac{-\frac{2}{3}.-\frac{5}{3}}{|\underline{0}\ |\underline{1}\ |\underline{1}}(3)(-4)+\dfrac{-\frac{2}{3}.-\frac{5}{3}.-\frac{8}{3}}{|\underline{2}\ |\underline{0}\ |\underline{1}}(-2)^2(-4)+\dfrac{-\frac{2}{3}.-\frac{5}{3}.-\frac{8}{3}}{|\underline{1}\ |\underline{2}\ |\underline{0}}$

$(-2)(3)^2+\dfrac{-\frac{2}{3}\ldots\ldots-\frac{11}{3}}{|\underline{3}\ |\underline{1}\ |\underline{0}}(-2)^3(3)+\dfrac{-\frac{2}{3}\ldots\ldots-\frac{14}{3}}{|\underline{5}\ |\underline{0}\ |\underline{0}}(-2)^5.$

3. (1) In Gross's Algebra the Exponential Theorem is established, and $\log_e(1+x)$ is then deduced. In Todhunter $\log(1+x)$ arises in the course of the investigation which establishes the Exponential Theorem.

(2) In $2\log(1-3x)$ the coeffi. of x^n is $2\left(-\dfrac{1}{n}3^n\right)=-\dfrac{1}{n}\cdot\dfrac{6^n}{2^{n-1}}$

Also $\log\{1-6x(1-\frac{3}{2}x)\}$

$$=-\left[6x(1-\tfrac{3}{2}x)+\tfrac{1}{2}\{6x(1-\tfrac{3}{2}x)\}^2+\ldots+\frac{1}{n}\{6x(1-\tfrac{3}{2}x)\}^n+\ldots\right]$$

In which the coefficient of x^n is

$$-\left\{\frac{1}{n}6^n-\frac{1}{n-1}6^{n-1}.(n-1).\tfrac{3}{2}+\right.$$
$$\left.\frac{1}{n-2}6^{n-2}.\frac{(n-2)(n-3)}{|\underline{2}}\left(\frac{3}{2}\right)^2-\ldots\right\}$$

and equating these coefficients of x^n the required identity is obtained.

4. (1) Book-work.

(2) $1000=I\{(1\cdot04)^{-\frac{1}{6}}+(1\cdot04)^{-\frac{2}{6}}+\ldots\ldots+(1\cdot04)^{-\frac{120}{6}}\}.$

$$=I(1\cdot04)^{-\frac{1}{6}}.\frac{\{(1\cdot04)^{-\frac{1}{6}}\}^{120}-1}{(1\cdot04)^{-\frac{1}{6}}-1}=I\,\frac{(1\cdot04)^{-20}-1}{1-(1.04)^{\frac{1}{6}}}$$

$$\therefore\ I=1000\frac{(1\cdot04)^{\frac{1}{6}}-1}{1-(1\cdot04)^{-20}}\text{ Ans.}$$

5. The value of the mortgage two months from this, when the first instalment is to be paid, is

$$100+100\{(1\cdot04)^{-\frac{1}{2}}+(1\cdot04)^{-\frac{2}{2}}+\ldots\ 25\text{ terms.}\}$$

And the present value of this for the two months is found by multiplying it by $(1\cdot04)^{-\frac{1}{6}}$, giving

$$\left\{100\ \frac{(1\cdot04)^{-13}-1}{(1\cdot04)^{-\frac{1}{2}}-1}\right\}(1\cdot04)^{-\frac{1}{6}}.$$

6. Book-work.

7. (1) Todhunter, §§ 631, 630.

(2) Let z be the number, and x and y the quotients; then $13x+7=z=14y+2$. Thence the general value of x is $5+14t$, and the succession of values is 5, 19, ... 61, 75, ... 75 is the value of x which makes z nearest to 1000; and $z=982$.

8. (1) The scale of relation is $1-5x+4x^2$. Whence series

$$\text{equals }\frac{1}{(1-x)(1-4x)}=-\frac{1}{3}\cdot\frac{1}{1-x}+\frac{4}{3}\cdot\frac{1}{1-4x}$$
$$=-\tfrac{1}{3}\{1+x+x^2+\ldots\}+\tfrac{4}{3}\{1+4x+(4x)^2+\ldots\ldots\}$$

Hence general term $=\frac{1}{3}x^n(2^{2n+2}-1)$.

The rates of the $(n+2)$th to this is $x\dfrac{2^{2n+4}-1}{2^{2n+2}-1}=x\dfrac{2^2-\frac{1}{2^{2n+2}}}{1-\frac{1}{2^{2n+2}}}$

$=4x$ ultimately. Hence that the series may be convergent x must be less than $\frac{1}{4}$, (Todhunter, § 559).

(2) Revert the series, *i.e.* assume

$x=A+By+Cy^2+Dy^3+\ldots\ldots$

$\therefore\ x=A+B(x+2x^2+3x^3)+C(x+2x^2+3x^3)^2+\ldots$

Equate coeffs. of x;

$A=0,\ B=1,\ 2B+C=0,\ 3B+4C+D=0$, &c.

$\therefore\ A=0,\ B=1,\ C=-2,\ D=5$, &c., and

$x=y-2y^2+5y^3+\ldots$

9. $u_0=\dfrac{1}{a+a^{-1}},\ u_1=\dfrac{1}{a+a^{-1}-\dfrac{1}{a+a^{-1}}}$, and the law may be shewn to hold for these terms, for $u_0=\dfrac{a-a^{-1}}{a^2-a^{-2}},\ u_1=\dfrac{a^2-a^{-2}}{a^3-a^{-3}}$.

Assume that it holds for the n^{th} term, then

$$u^{n+1}=\frac{1}{a+a^{-1}-\dfrac{a^{n+1}-a^{-(n+1)}}{a^{n+2}-a^{-(n+2)}}}$$
$$=\frac{a^{n+2}-a^{-(n+2)}}{a^{n+3}-a^{-(n+3)}}$$

Hence if the law holds for the n^{th}, it holds for the $(n+1)^{th}$; but it has been shewn that it holds for the second; hence it holds for the third; hence for the fourth, &c.; and hence generally.

PROBLEMS FOR SOLUTION.

1. Three gamblers A, B, C began to play with \$6.00 altogether in their possession. They played three games and each of them had \$2.00 at the close. In the first game A and B lost and the winner doubled his money; in the second A and C lost and the winner doubled his money; in the third B and C lost and