

the diff.  $C. d.$  and together with the diff.  $d. D.$ ; (thus...  $C. O.$  includes the advance of  $n.$  through  $C. d.$ , of  $M.$  through  $C. d. + \frac{C. d.}{9}$  and the distance  $d. D.$ ) But  $B.$

$C.$  and  $C. d.$  taken together equal  $B. d.$  which is equal to  $n. p.$  And  $M. N.$  is to  $n. p.$  as ten to nine. Therefore  $B. d. + C. d. + \frac{C. d.}{9}$  is to  $B. C. + C. d.$ , as ten to nine.

By the assumption  $B. O.$  contains  $(B. C. + C. O.) B. d. + C. d. + \frac{C. d.}{9} + d. D.$ , and therefore the ratio of  $B. O.$  to

$B. d.$  is greater than the ratio of  $M. N.$  to  $n. p.$  by the distance  $d. D.$  Now, it is manifestly impossible that  $B. M.$  can increase by advancing in a greater ratio of proportion to the increase of  $B. n.$  than the proportion of the sine  $M. N.$  to the sine  $n. p.$ , because  $B. M.$  and  $B. n.$  are similar arcs; therefore the distance  $C. d.$  cannot be less than the distance  $C. D.$  By similar reasoning it may be shown that the distance  $C. d.$  cannot be greater than  $C. D.$ , because then the advance of the arc  $B. M.$  would be proportional to the advance of the arc  $B. n.$  in a ratio of proportion less than the ratio of the sine  $M. N.$ , to the sine  $n. p.$ , to suppose which would be absurd. Wherefore it is demonstrated that the point of contact of the lesser arc  $B. n.$  on the straight line  $B. E.$ , indicated assumptively by  $d.$ , is the same point  $D.$  at the extremity of the perpendicular  $M. D.$

*Q. E. D.*

(5.) Again—Fig. 3 (repeats the construction of Fig. 1.) Produce  $D. M.$  through  $M.$ , and make  $F. D.$  equal to  $A. B.$  With centre  $F.$  and radius  $F. D.$  describe the arc  $D. N.$  equal and similar to the arc  $B. M.$ , produce the straight line  $E. B.$  through  $B.$  indefinitely, and upon the line so produced roll the arc  $D. N.$  from  $D.$  in the direction  $D. B.$  until the extremity  $N.$  of the arc becomes in contact upon the line at  $T.$  Now the distance  $B. T.$  is manifestly equal to the distance  $D. O.$ ,