(b) $\left(\frac{3x}{x+y}\right)^{\frac{1}{2}} + \left(\frac{x+y}{3x}\right)^{\frac{1}{2}} = 2$; $xy-(x+y)=54$.

Put $y=vx$. Then the first equation gives $\left(\frac{3}{1+v}\right)^{\frac{1}{2}} + \left(\frac{1+v}{3}\right)^{\frac{1}{2}} = 2$; and squaring and reducing $(1+v)^2 - 6(1+v)+9=0$: and $1+v=3$, or $v=2$.

Then from second equation,

$$2x^2-3x=54$$

Whence $x=6$ or $-4\frac{1}{2}$

and $y=12$ or -9.

7. Given that the roots of the equation $ax^2+bx+c=0$ are p and q, and those of $a_{,}x^2+b_{,}x+c_{,}=0$ are $p_{,}$ and $q_{,}$; also that $\frac{p}{q}=\frac{p_{,}}{q_{,}}$, prove that $\frac{a_{,}c_{,}}{ac}=\left(\frac{b_{,}}{b}\right)^2$.

Let $q=mp$ Then $q_{,}=mp_{,}$.

But $p+q=(1+m)p=-\frac{b}{a}$; and $p_{,}+q_{,}=(1+m)p_{,}=-\frac{b}{a_{,}}$.

Also $pq=mp^2=+\frac{c}{a}$; and $p_{,}q_{,}=mp_{,}^2=+\frac{c_{,}}{a_{,}}$. from the theory of the quadratic.

From these four equations we must eliminate p, p, and m.

Now by division we get the two relations—

$$\frac{p}{p_{,}}=\frac{b}{a}\div\frac{b_{,}}{a_{,}}=\frac{a_{,}b}{ab_{,}}\ ;\ \frac{p^2}{p_{,}^2}=\frac{c}{a}\div\frac{c_{,}}{a_{,}}=\frac{ca_{,}}{c_{,}a}$$

$$\therefore \frac{a_{,}^2b^2}{a^2b_{,}^2}=\frac{ca_{,}}{c_{,}a}\ ;\ \text{whence}\ \frac{a_{,}c_{,}}{ac}=\frac{b_{,}^2}{b^2}\ ;\ \text{q.e.d.},$$

8 Two vehicles start at the same moment from two towns, A and B respectively, and travel towards each other. They meet after $10\frac{1}{2}$ hours, one taking $\frac{1}{12}$ hour more to the mile than the other. If the distance between the towns is 105 miles, what are the rates at which the vehicles travel?

Let x be the time it takes the first carriage to go a mile. Then $x+\frac{1}{12}$ is the time in hours taken by the second carriage in going a mile.

$\therefore \frac{1}{x}$ is the rate of the first carriage, and $\frac{1}{x+\frac{1}{12}}$ is the rate of the second.

And $\left(\frac{1}{x}+\frac{1}{x+\frac{1}{12}}\right)10\frac{1}{2}=105$.

From which we get $x=\frac{1}{6}$.

$\therefore$ The first carriage goes 6 miles an hour,

And the second goes 4 miles an hour.

9. If a carriage wheel $16\frac{1}{2}$ feet in circumference took one second more to revolve, the rate of the carriage would be $1\frac{7}{8}$ miles less. At what rate is the carriage travelling?

Let the carriage wheel revolve once in t secs. Then the carriage goes $\frac{16\frac{1}{2}}{t}$ feet per sec. or $\frac{3600}{5280}\cdot\frac{16\frac{1}{2}}{t}$ miles per hour. Similarly under the second supposition the carriage goes $\frac{3600}{5280}\cdot\frac{16\frac{1}{2}}{t+1}$ miles per hour.

$$\text{And}\ \frac{3600\times16\frac{1}{2}}{5280}\left\{\frac{1}{t}+\frac{1}{t+1}\right\}=1\frac{7}{8}.$$

Whence we readily find $t^2+t=6$; and $t=2$ or -3. Then the velocity of the carriage is $\frac{3600}{5280}\cdot\frac{16\frac{1}{2}}{2}$ m. per hour, or $5\frac{5}{8}$ miles per hour.

The second value of t, (-3), has also a meaning, but I doubt if many of the candidates could make much out of it.

Upon going over this paper I am not astonished that it created great dissatisfaction in the schools, and that the committee found itself constrained to pass men who made 20 or 25% on it ; for a considerable portion of it is beyond the state of efficiency possessed by the average Junior Leaving candidate.

Paper for 1898. N F. DUPUIS.