ALGEBRA. FORM III., 1898.

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1. (*a*) $\dfrac{z}{\dfrac{1}{x}-\dfrac{1}{y}}+\dfrac{y}{1-\dfrac{y}{x}}-\dfrac{x}{1-\dfrac{x}{y}}=\dfrac{xyz}{y-x}+\dfrac{xy}{x-y}-\dfrac{xy}{y-x}$

$$=\frac{xyz}{y-x}-\frac{2xy}{y-x}=\frac{xy(z-2)}{y-x}\text{ or }\frac{xy(2-z)}{x-y}.$$

(*b*) $\dfrac{1}{1-x}+\dfrac{1}{1+x}+\dfrac{2}{1+x^2}+\dfrac{4}{1+x^4}=\dfrac{2}{1-x^2}+\dfrac{2}{1+x^2}+\dfrac{4}{1+x^4}$

$$=\frac{4}{1-x^4}+\frac{4}{1+x^4}=\frac{8}{(1-x^4)(1+x^4)}=\frac{8}{(1-x^8)}.$$

2. Since $x^2=x-1$, $x^2-x+1=0$; $\therefore$ $x^3+1=0$, being $=$ to $(x+1)(x^2-x+1)$.
Now, $x^6-3x^5+2x^4-x^3-3x^2+2x-2=x^6-1-3x^5-3x^2+2x^4+2x-x^3-1$
$=(x^3-1)(x^3+1)-3x^2(x^3+1)+2x(x^3+1)-(x^3+1)=0.$

Or, substituting $x^6=(x^2)^3=\overline{x-1}^3$, etc., reduce the expression.

3. (*a*) $(a^2+b^2-c^2)^2-4(ab)^2=(a^2+b^2-c^2+2ab)(a^2+b^2-c^2-2ab)$
$=(a+b+c)(a+b-c)(a-b+c)(a-b-c).$

(*b*) $x^5+x^3-x^2-1=x^3(x^2+1)-(x^2+1)=(x^2+1)(x^3-1)$

$$=(x+\sqrt{-1})\,(x-\sqrt{-1})(x-1)(x^2+x+1)$$

$$=(x+\sqrt{-1})\,(x-\sqrt{-1})\,(x-1)\left(x+\frac{1-\sqrt{-3}}{2}\right)\left(x+\frac{1+\sqrt{-3}}{2}\right)$$

(*c*) $2x^2-y^2-2z^2+3yz-xy=(2x+y-z)(x-y+2z).$

4. $i^3=i.i^2=-i$; $i^5=i.i^4=+i$, etc.
$\therefore$ $(1+i)(1+i^3)\ldots\ldots$ to n factors $=(1+i)(1-i)(1+i)(1-i)\ldots\ldots$ to n factors

$$=\{(1+i)(1-i)\}^{\frac{n}{2}}=2^{\frac{n}{2}}=(\sqrt{2})^n.$$

5. Assuming that $(+a)\times(+b)$ is $+ab$, or $+a$ taken b times additively, then $(-a)\times(+b)$ must be equal $(-a)$ taken b times additively
$=(-a)+(-a)+(-a)\ldots\ldots$ to b terms $=b(-a)=-ab$
And $(-a)\times(-b)$ must be $(-a)$ taken b times subtractively or $-(-a)-(-a)\ldots$ to b terms, or $-(-ab)=+ab$.

6. (*a*) and (*b*) Book-work.

(*c*)

1	$1+6z+10z^2-2z^3-15z^4$	$1+5z+z^2-13z^3+6z^4$	
	$1+5z+z^2-13z^3+6z^4$	$1+9z+11z^2-21z^3$	
1	$z(1+9z+11z^2-21z^3)$	$-4z-10z^2+8z^3+6z^4$	$-4z$
	$1+2z-3z^2$	$-4z-36z^2-44z^3+84z^4$	
$+7z$	$7z+14z^2-21z^3$	$26z^2(1+2z-3z^2$	
	$7z+14z^2-21z^3$		

7. Let $7x=$ A's money, $8x=$ B's money. $\therefore$ $\dfrac{7x+18}{8x+18}=\dfrac{17}{19}$

$x\,(7.19-8.17)=18.17-19.18=18(17-19)=-36$
$-3x=-36$, $x=12$; $7x=84=$ A's money, $8x=96=$ B's money.