GEOMETRY.

PROP. 31. To find a point on a given straight line, the sum of whose distances from two given points is the least possible.

1. If the points be on opposite sides of the given line, the point required is evidently the intersection of the given line with the straight line joining the points.

2. Let the points be on the same side of the given line.



Let A, B be the two given points, and CD the given line. From either of the points, say A, draw AE \perp r to CD. Produce AE to F, making EF = AE.

Join BF, eutting CD in P. Join AP.

Then AP + PB is less than the sum of any other lines drawn from A and B to a point on CD.

Let Q be any other point on CD.

Then points on CD are equidistant from A and F;

 $\therefore AQ + BQ = FQ + BQ;$ >FB, >FP + PB, >AP + PB;

i.e., AP + PB is a minimum.

It will be noted that when AP + PB is a minimum, AP and BP make equal angles with CD.

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