

$$\frac{a-b-c+d}{a-b+c-d} = \frac{ad-bc}{ac-bd}$$

$$\therefore \frac{ac-bd}{a-b+c-d} = \frac{ad-bc}{a-b-c+d}$$

12. The interest on a sum of money for one year at 5 per cent. per annum is $\frac{1}{20}$ of the sum. Hence we require to find the time for which the discount is $\frac{1}{20}$ of the sum. Now, when the discount is $\frac{1}{20}$, the interest is $\frac{1}{20}$. If, therefore, the interest becomes $\frac{1}{20}$ in one year it will become $\frac{1}{20}$ in $1 \frac{1}{20}$ yr. Ans

13. Let a denote the remainder in each case, then the full quotients are, respectively :

$$1.11 + \frac{a}{9.009} \text{ and } 9.009 + \frac{a}{1.11}$$

and since $\frac{a}{9.009}$ is less than $\frac{a}{1.11}$ it follows that 1.11 is more nearly correct than 9.009.

14. Pressure = WAL where W is the weight of a cubic unit of the fluid ($= 62\frac{1}{2}$ lbs), A the area of the surface against which the fluid presses ($= 484 \times 9$ sq. ft.) and L the average depth ($= 16\frac{1}{2}$ ft.) Hence the pressure is $62\frac{1}{2} \times 484 \times 9 \times 16\frac{1}{2}$ lbs.

15. Multiply up and the equations become

$$x^{x+y+1} = y^{x+y+1}, x^{x+y-1} = y^{x-y-1}$$

$$\therefore x = \frac{y^{x+y+1}}{y^{x+y-1}}, x = \frac{y^{x-y-1}}{y^{x+y-1}}$$

$$\therefore \frac{x+y+1}{x+y-1} = \frac{y-x-y}{x+y-1}$$

whence $x+y = 2$ or -2 (2)

$\therefore$ (1) becomes $x^3 = y^6 \therefore x = y^2$

$\therefore$ from (2) we have $y = 1, -2, \&c.$

16. Were it not for emigration the population at the end of each year would be 1.05 as great as at the beginning of that year; but since $\frac{1}{2}$ per cent. is annually carried off the population at the end of each year will be only .995 of 1.05 as great as at the beginning. Therefore in five years the pop. becomes $(.995 \times 1.05)^5$ as great as at first; therefore the increase per cent. in five years is $(.995 \times 1.05)^5 - 1$.

PROBLEMS.

17. (a) Every common multiple of a and b is a multiple of their least common multiple.

(b) Every common measure of a and b is a measure of their greatest common measure.

(c) The greatest common measure of a and b is the least common multiple of all their common measures.

(d) The least common multiple of a and b is the greatest common measure of all their common multiples.

18. If the G. C. M. of a and b contain n simple factors, then the whole numbers of common factors of a and b (leaving out unity) is $2^n - 1$.

19. If x is real the value of the expression

$$\frac{ax^2 + bx + c}{1 + x^2} \text{ must lie between}$$

$$\frac{a+c+\sqrt{[b^2+(a-c)^2]}}{2}, \text{ and } \frac{a+c-\sqrt{[b^2+(a-c)^2]}}{2}$$

20. If x, y, z be real numbers, prove that $a^2 x - y)(x - z) + b^2 (y - z)(y - x) + c^2 (z - x)(z - y)$ will always be positive, provided that any two of the quantities a, b, c are together greater than the third.

21. Prove that

$$1 + 3n + \frac{3 \cdot 4}{1 \cdot 2} \frac{n(n-1)}{1 \cdot 2} + \frac{4 \cdot 5}{1 \cdot 2} \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} + \&c.$$

$$= 2^{n-3}(n^2 + 7n + 8)$$

22. The united salaries of two persons amounts to 4 400 dollars. The one spends two-thirds of his salary and the other three-fourths. They save between them 1,310 dollars; find the salary of each. (By arithmetic.)

23. What is the price of eggs per dozen when two less in the shilling's worth raises the price one penny per dozen? (By arith.)

24. The distance between the earth and moon being expressed by 59 9643 with reference to the earth's radius as unit, and this radius being 3962.8 miles, each of these numbers being exact to the nearest decimal, what can be known of the moon's distance from the earth in miles?

25. Two travellers, A and B, set out from two places, P and Q at the same time; A