Mathematical Department.

JUNIOR MATRICULATION, JUNE, 1881

UNIVERSITY OF TORONTO.

PASS MATHEMATICS.

Examiner: A. K. Blackadar, B.A.

1. Simplify

$$\frac{\cdot 8}{\cdot 3} - \frac{45}{8}\left(\frac{16}{25} - \cdot 1\right) + \frac{\frac{4}{5}(\cdot 64 - \frac{1}{4}) \times \cdot 390625}{\cdot 11\,(\frac{1}{3} + \frac{1}{2})}.$$

2. Find the square root of 5 to five decimal places, and reduce the values of

$$\frac{1}{\sqrt{5}},\quad \sqrt{\cdot 002},\quad \frac{5+\sqrt{5}}{5-\sqrt{5}},\text{ and }\sqrt{6+2\sqrt{5}}.$$

3. (*a*) How much will $1000 amount in $2\frac{1}{2}$ years, compound interest, 4 per cent. per annum, payable half-yearly?

(*b*) A person pays $292.50 for $300 due three months hence. What rate per cent. interest does he receive?

4. What is meant by the expression, "Sterling Exchange, $9\frac{1}{2}$ p. c. premium"?

A person pays $181.50 for £37 : 10s., stg. What per cent. premium is Sterling Exchange?

5. Multiply $b^2+(a-b)(b-c)$ by $c^2+(b-c)(c-a)$.

Shew that your answer is correct by substituting $a=2$, $b=0$, $c=-3$.

6. Simplify (1) $\dfrac{a^2b\ c^{-2}}{a^{-1}b^2c^{-3}}$.

(2) $\dfrac{x^2-2+x^{-2}}{x^2-x^{-2}} - \dfrac{x^3}{x^2+1}$.

7. Resolve into factors.

a^2-b^2, $ab+bc+ca+b^2$,

$a(b+c)^2+b(c+a)^2+c(a+b)^2-4abc$.

$(a+b)^2-2b\dfrac{a^3-b^3}{a-b}+c(a^2-b^2)-2ab^2$.

Find the Greatest Common Measure, and the Least Common Multiple of these *four* quantities.

8. Solve the equations

(1) $ax+b=bx+a$.

(2) $\dfrac{1}{x^2+3x+2}+\dfrac{1}{x^2+5x+6}=\dfrac{1}{x^2+x-2}$.

(3) $\begin{cases}\dfrac{2}{x}-\dfrac{3}{y}=4.\\ 2x-3y=2xy.\end{cases}$

(4) $\begin{cases}xy-yz=18.\\ x^2+z^2=4y^2+2xz.\\ x^2-8=2xy+2xz.\end{cases}$

9. There are two vessels, *A* and *B*, each containing a mixture of water and wine, *A* in the ratio of 2 : 3, *B* in the ratio of 3 : 7. What quantity must be taken from each in order to form a third mixture which shall contain 5 gallons of water and 11 of wine?

10. Describe a triangle, of which the sides shall be equal to three given straight lines, any two of which are together greater than the third.

A straight line *AD* is divided into three equal parts by the points *B* and *C*; on *AB*, *BC*, *CD* are described equilateral triangles *AEB*, *BFC*, *CGD* respectively; shew that the three straight lines *AE*, *AF*, *AG*, can form a triangle equal in area to the equilateral triangle *A*

11. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts shall be equal to the square on the other part.

12. Define the terms, *circle*, *tangent to a circle*, and *segment of a circle*.

The angles in the same segment of a circle are equal to one another.

SOLUTIONS.

1. Expression $= \frac{6}{25} - \frac{45}{8}\left(\frac{16}{25} - \frac{1}{9}\right) + \frac{54}{5} \times \frac{4}{5}\left(\frac{16}{25} - \frac{1}{9}\right)\frac{25}{64}$

$= \frac{6}{25} - \frac{45}{8} \times \frac{17}{15} \times \frac{7}{15} + \frac{27}{8} \times \frac{17}{15} \times \frac{7}{15}$

$= \frac{6}{25} - \frac{9}{4} \times \frac{119}{225} = \frac{6}{25} - \frac{119}{100} = -\frac{95}{100}$.

2. See Hamblin Smith's Alg., p. 315. and Hamblin Smith's Arith. App. I. (Can. Ed.)

We use the Binomial Theorem for larger numbers and the latter method for small numbers.

We see by inspection, ordinary method $\sqrt{5}=2\cdot 2+$

Substitute 5 for N and $2\cdot 2$ for a in the formula

$\sqrt{N} = a.\dfrac{3N+a^2}{N+3a^2}$ and $\sqrt{5} = 2\cdot 2 \times \dfrac{15+4\cdot 84}{5+3(4\cdot 84)} = \dfrac{1\cdot 1\times 1\cdot 24}{\cdot 61} = 2\cdot 23606.$

N.B.—The general formula is

$$\sqrt[n]{N} = a.\frac{(n+1)N+(n-1)a^n}{(n-1)N+(n+1)a^n} \text{ approximately.}$$

$$\frac{1}{\sqrt{5}} = \frac{1}{5}\sqrt{5} = \cdot 44721$$

$$\sqrt{\cdot 002} = \frac{1}{10}\cdot\frac{1}{\sqrt{5}} = \cdot 044721$$

$$\frac{5+\sqrt{5}}{5-\sqrt{5}} = \frac{(5+\sqrt{5})(5+\sqrt{5})}{(5-\sqrt{5})(5+\sqrt{5})} = \tfrac{1}{2}(3+\sqrt{5}) = \tfrac{1}{2}\times 5\cdot 23606 = 2\cdot 61803$$

$\sqrt{6+2\sqrt{5}} = \sqrt{x}+\sqrt{y}$. See H. Smith's Alg., p. 226, theorem II.

$\therefore\ 6 = x+y$ and $2\sqrt{5} = 2\sqrt{xy}$

$36 = x^2+4+y+y^2$ and $20 = 4xy$

$\therefore\ 16 = x^2-2xy+y^2$ or $\pm 4 = x-y$

$\therefore\ \sqrt{x}+\sqrt{y} = 1+\sqrt{5} = 3\cdot 23606.$

3. (*a*) Taking 4% per annum = 2% half-yearly.

$$A = 1000\left(1+\frac{2}{100}\right)^6$$

$$= 1000\left\{1+5\left(\frac{2}{100}\right)+10\left(\frac{2}{100}\right)^2+10\left(\frac{2}{100}\right)^3+\text{etc.}\right\}$$

$$= 1000\left\{1+\cdot 1+\cdot 004+\cdot 00008+\text{etc.}\right\}$$

$$= 1000\times 1\cdot 10408 = \$1104.08.$$

(*b*) Discount $= \frac{7\frac{1}{2}}{300}$ debt $= \frac{1}{40}$ $\therefore$ interest $= \frac{1}{39}$ per quarter

$= \frac{4}{39} = 10\cdot 25+\%$ per annum.

N.B.—When interest $= \left(\frac{a}{b}\right)$ principal, disc't $= \frac{a}{a+b}$ (debt).

See McLellan's Exam. Papers, p. 220, and Key.

4. Book-work.

$$£37\tfrac{1}{2} \times \frac{40}{9} \times \frac{x}{100} = \$181\tfrac{1}{2}.$$

$$\therefore\ x = 108\frac{9}{10}\text{ premium.}$$

5. Expression $\{bc+a(b-c)\}\{bc-a(b-c)\} = b^2c^2-a^2(b-c)^2$.

and $-36 = -36$.

6. (2) Expression $= \dfrac{x^4-2x^2+1}{x^4-1} - \dfrac{x^2}{x^2+1}$

$$= \frac{x^2-1}{x^2+1} - \frac{x^2}{x^2+1} = -\frac{1}{x^2+1}.$$

7. $(a+b)(a-b)$

$(a+b)(b+c)$ are the factors of the first two expressions

2nd Exp. $= a(b-c)^2+b(c+a)^2+c(a+b)^2 = (a+b)(b+c)(c+a)$.

3rd Exp. $= (a+b)^3-2b(a+b)^2+c(a^2-b^2)$

$= (a+b)\{(a^2-b^2)+c(a-b)\} = (a+b)(a-b)(a+b+c$

$\therefore$ G.C.M. $= (a+b)$ and L.C.M. $= (a+b)(b+c)(c+a)(a-b)(a+b+c)$.

8. (1) $x=1$.

(2) $\dfrac{1}{(x+3)(x+2)}+\dfrac{1}{(x+1)(x+2)}-\dfrac{1}{(x+2)(x-1)}=0$

Divide through by $x+2$ $\therefore\ x=-2$,

also $x^2-2x+-7=0$, whence $x=1\pm 2\sqrt{2}$.