character of this motion and the motion of the smallest masses into which matter can practically be divided. The hypothesis of vibration in the fourth dimension merely suggests the possibility that this kind of motion may mark what is essentially different from the motion of masses. Of course, such an hypothesis as this is not to be put forward as a theory. It must be worked out with mathematical rigor, and shown to actually explain phenomena before we assign it to any such rank.

I cannot but fear that some confusion on this subject is caused by the tendency among both geometers and psychologists to talk of space as an entity in itself. As I have already said, a fourth dimension in space is nothing more than the addition of a fourth possibility of motion to material bodies. The laws of space are only laws of relative position. Certain fundamental axioms are derived from experience, not alone individual experience, perhaps, but the experience of the race, giving rise to hereditary conceptions born in the mind and corresponding to the facts of individual experience. A tree confined to one spot, even if it had eyes to see and a brain to think, could never have a conception of space. For us the limits of space are simply the limits to which we can suppose a body to move. Hence when space itself is spoken of as having possible curvatures, hills and hollows it seems to me that this should be regarded only as a curvature, if I may use the term, of the laws of position of material bodies in space. Clifford has set forth, with great acuteness and plausibility, that the minute spaces occupied by the ultimate atoms of matter may, in this respect, have properties different from the larger space which alone makes itself known to our conceptions. If so, we should only regard this as expressive of some different law of motion, or, since motion is only change of position, of some different law of position among the molecules of bodies.

This consideration leads us to a possible form of space relations distinct from those of our Euclidean geometry, and from the hypothesis of space of more than three dimensions, I refer to what is commonly known as 'curved space.' The history of this conception is now so well known to mathematicians that I shall mention it only so far as is necessary to bring it to your minds. The question whether Euclid's axioms of parallels is really an independent axiom, underivable from the other axioms of geometry, is one which has occupied the attention of mathematicians for centuries. Perhaps the simplest form of this axiom is that through a point in a plane one straight line and no more can be drawn which shall be parallel to a given straight line in the plane. Here we must understand that parallel lines mean those which never meet. The axiom, therefore, asserts that through such a point we can draw one line which shall never meet the other line in either direction, but that if we give this one line the slightest motion around the point in the plane it will meet the other in one direction or the opposite. Thus stated, the proposition seems to be an axiom, but it is an axiom that does not grow out of any other axioms of geometry. The question thus arising was attacked by Lobatchevsky in this very conclusive manner. If this axiom is independent of the other axioms of geometry then we should be able to construct a self-consistent geometrical system, in conformity to the other axioms, in which this axiom no longer held. The axiom of parallels may be deviated from in two directions. In the one it is supposed that every two lines in the plane must meet; no line parallel to another can be drawn through the same point in the plane. Deviating in the other direction we have several lines drawn