Mathematics.

All communications intended for this column should be sent before the 20th of each month to C. Clarkson, B.A., Seaforth, Ont.

A SLIGHT omission was made in the heading of the first part of this column in the issue of Feb. 1st. The first eleven solutions belonged to the Entrance Examination Arithmetic given on page 283.

ARITHMETIC.

1. Find a number which leaves remainders 1, 2, and 3 respectively when divided by 7, 8, and 9; and the sum of the three quotients = 570.

Solution :—$\frac{1}{7}$ of the No. = 1st Quotient + $\frac{1}{7}$
$\frac{1}{8}$ " " = 2nd " + $\frac{2}{8}$
$\frac{1}{9}$ " " = 3rd " + $\frac{3}{9}$
$\therefore (\frac{1}{7}+\frac{1}{8}+\frac{1}{9})$ of No. $= 570 + (\frac{1}{7}+\frac{2}{8}+\frac{3}{9})$
$\frac{72+63+56}{504}$ of No. $= 570 + \frac{72+126+168}{504}$
$\therefore$ 191 times No. $= 570 \times 504 + 366 = 287646$.
$\therefore$ No $= 287646 \div 191 = 1506$.

Remark :—In this solution we have a fair example of the *Arithmetical Equation*, in the use of which pupils in the second and third classes should be carefully drilled. Sometimes one hears the strange objection that such solutions are algebra in disguise; and the implied statement seems to be that every problem involving the use of the equation must be algebraical. The fact is quite the reverse, algebra is only generalised arithmetic, and algebraical equations are only a higher kind of arithmetical equations. The empirical, disorderly solutions too often seen, even in text-books, are really arithmetical equations so disguised that they travel *incog*.

2. A might have got home in $\frac{4}{5}$ of the time he actually took, if he had only walked half a mile an hour faster than he did. Had he, however, gone half a mile an hour slower than he did he would have been $2\frac{1}{2}$ hours longer on the road than he really was. How for had he to walk?

Solution :—

1st Case.—Time $= \frac{4}{5}$ actual time; $\therefore$ rate would have been $= \frac{5}{4}$ actual rate; *i.e.*, increase on actual rate would have been $= \frac{1}{4}$ actual rate $= \frac{1}{2}$ mile per hour.

Therefore actual rate must have been = 2 miles per hour.

2nd Case.—Decrease of rate would have been $= \frac{1}{2}$ mile on a rate of 2 miles per hour; $\therefore$ decrease of speed $= \frac{1}{4}$ actual rate.

$\therefore$ Decrease speed $= \frac{3}{4}$ actual rate; therefore increased time on the road $= \frac{4}{3}$ actual time; in other words, the increase of time $= \frac{1}{3}$ actual time $= 2\frac{1}{2}$ hours.
$\therefore$ actual time $= 7\frac{1}{2}$ hours at 2 miles per hour;
$\therefore$ distance traveled = 15 miles.

3 An hour after starting a train breaks down, and spends another hour on repairs. Afterwards it runs at three-fifths of its former rate and arrives three hours behind time. The conductor observes that if the mishap had occurred fifty miles nearer the terminus, he would have got his train in an hour and twenty minutes sooner. Find the length of the trip.

Solution :—Decreased rate $= \frac{3}{5}$ regular rate; therefore increased time $= \frac{5}{3}$ regular time; that is the time lost $= \frac{2}{3}$ regular time $= \frac{4}{3}$ hours on 50 miles;
$\therefore$ schedule time = 2 hours on 50 miles;
i.e. schedule rate = 25 miles an hour; consequently distance run *before* accident happened = 25 miles.

After the accident.—Loss of time $= \frac{2}{3}$ regular time = 2 hours;
$\therefore$ regular time = 3 hours; and regular rate = 25 miles per hour, $\therefore$ distance after mishap = 75 miles.
$\therefore$ whole trip = 25 + 75 miles = 100 miles.

Remark.—These two solutions illustrate the application of the important principle of *Inverse Ratio.* For any given distance, every increase of speed produces a decrease of time; and the fractions expressing the rate and the time are mutually reciprocal.

4. I have two debts, one of $400 due in two years, the other of $2,100 due in eight years, both without interest. I wish to give a mortgage without interest for the whole $2,500. For what length of time should the instrument be drawn, supposing money is worth 5% per annum simple interest?

Solution.—Interest $= \frac{5}{100}$ principal $= \frac{1}{20}$ principal.

Therefore discount $= \frac{1}{21}$ principal. Thus the discount on $2,100 for a year = $100; and the interest on $400 for a year = $20. Now the interest must be so arranged as to cancel the discount. Hence we see by inspection that *one* way of accomplishing this end is to pay the $2,100 a year *before* it is due, and the $400 five years after it is due. And this arrangement will exactly coincide with the whole time, viz., eight years. Hence the mortgage may be drawn for $2,500 and allowed to run seven years without interest.

5. There are not quite 200 oranges in a case. They can be exactly counted by twos, threes, fours, fives, or sixes at a time. If counted by sevens, there will, however, be five left. Find the number in the case.

Solution.—L.C.M. of 2, 3, 4, 5, 6 = 60. Therefore number in the case is a multiple of 60, and less than 200; *i.e.* it must be 60, 120, or 180. And on dividing these three numbers by 7 we get remainders 4, 1 and 5 respectively.

$\therefore$ 180 must be the number in the case.

The two following problems were omitted from the February number.

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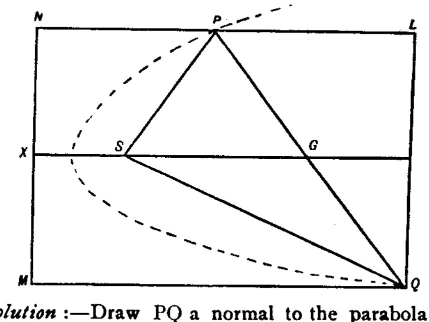
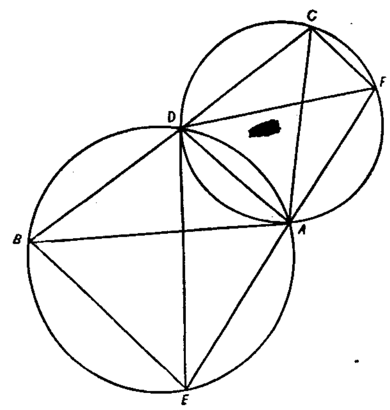
(2) The normal chord subtending a right angle at the focus of a parabola is divided by the axis in the ratio 2:3.



Solution :—Draw PQ a normal to the parabola at P, and QL perpendicular to the diameter thro. P. Then $SP = PL = \frac{1}{2}NL = \frac{1}{2}SQ$.

But SG = SP, $\therefore$ we have $QG.QP = SQ^2 - SP^2 = 3SP^2$.
$QP^2 = SQ^2 + SP^2 = 5SP^2$.
$\therefore QG:QP = 3:5$, *i.e.*, $PG:GQ = 2:3$.

(5.) ABC is a right-angled triangle. A point D is taken in the hyp., BC, and circles are described about ABD, ACD. If E, F are the middle points of the arcs AB, AC, remote from D, prove that EAF is a straight line.



Solution.—E is the middle point of BA
$\therefore$ Angle BDE = angle EDA, and $\therefore$ angle BAE = angle ABE.

Also angle FDA = angle FDC = angle CAF = angle ACF. But angle EAD = angle BAD + angle BAE = angle BED + angle BAE.

And angle DAF = 180° − angle ADF − angle AFD
= 90° − angle CAF + 90° − angle ACD
= angle BAE or angle ABE + angle ABD or angle AED.
$\therefore$ Angle EAD + angle DAF = angle ABE + angle BED + angle DEA + angle EAB.
= 2 right angles.
$\therefore$ E, A, F are collinear.

CORRESPONDENCE.

DEAR SIR,—In the January number I notice a solution of the "bankrupt problem," on page 217. I think Mr. Flaherty's solution may be proved incorrect. Thus:— Bankrupt's liabilities = $\$22857\frac{1}{7}$, and his apparent assets = 80% of this sum = $\$18285\frac{5}{7}$. The problem says that on $20,000 of his apparent assets he recovers only 80c. on the dollar. Now, where will this $20,000 come from, if his total assets are only $\$18285\frac{5}{7}$? Yours truly,

JESSIE C. GERRIE.

INGERSOLL, *Jan. 6th.*

[Perhaps he will apply for the appointment of a new liquidator *a la* Central Bank, or perhaps visit his cousins in the States. Will some of our correspondents examine this bankrupt and decide whether he is really "a fraud" or not.]

MR. WM. MCKAY, Rodgerville, contributed the following solution. We repeat the problem to which a solution was formerly given:—A and B put in $3,400 into business; A's money was in 12 months, and B's 16. On settlement A received $2,070 as his share, and B $1,920. What capital did each invest?

Let S = A's stock invested.
Then 3400 − S = B's.
S for 12 mo. = 12 S. for 1 mo.
3400 − S for 16 mo. = 54400 − 16 S for 1 mo.
54400 − 4 S. = total capital for 1 mo.
(2070 + 1920) − 3400 = 590 gain.

$$\frac{12 S.}{54400 - 4 S.} \text{ or } \frac{3 S.}{13600 - S.} \text{ of } \frac{590}{1} = \text{A's gain} = 2070 - S.$$

$$\therefore \frac{3 S.}{13600 - S.} \text{ of } \frac{590}{1} = \frac{2070 - S.}{1}$$

$$\text{or } \frac{1770 S.}{13600 - S.} = \frac{2070 - S.}{1}$$

$S^2 - 17440 S. + 28152000 = 0.$
$(S - 1800)(S - 15640) = 0.$
$\therefore$ S = $1800 or $15640 or A's share.

and $\$3400 - \left\{\begin{matrix}1800\\15640\end{matrix}\right\} = \left\{\begin{matrix}1600\\-12240\end{matrix}\right\}$ B's share of stock.

S may also be found by a similar treatment of B's share and gain.

MONO, Marnoch, Ont., sends for solution the following problems:—

1. A cistern has two supplying pipes A and B, and a tap C. When the cistern is empty A and B are turned on, and it is filled in four hours. Then B is shut and C turned on, and the cistern is quite emptied in forty hours when, lastly, A is shut and B turned on, and in sixty hours afterwards the cistern is again filled. In what time could the cistern be filled by each of the pipes A and B singly?

2. A speculator borrowed $5,000, which he immediately invested in land. Six months afterwards he sold the land for $7,500, on a credit of twelve months, with interest. Money was at 6%. what is the speculator's profit at the end of twelve months, at which time he pays $5,000.

3. How much may be gained by hiring money at 5% to pay a debt of $6,400 due in 8 months, allowing the present worth of this debt to be reckoned by deducting 5% per annum discount.

The EDITOR has answered a number of private inquiries after the plan of the English and the American Correspondence University. Their method of giving instruction by mail in science, mathematics, and languages seems to be growing in favor with the large numbers of students who are unable to take a regular college course. Will Mr. Wm. Linton kindly repeat his problem which has been mislaid? All our patrons will do us a favor by making their special wants known. Occasionally we get hints that this column is pitched at too high a level, and sometimes hints of the opposite character. We must be guided by the actual necessities of our subscribers. Friends keep us posted; and always send the questions as well as references to text-books.

NEGATIVE RESULTS.

THE moral sensibilities of pupils may be blunted or destroyed by unwise action on the part of teachers. An unmerciful punishment may inflict an injury for life. Dr. Carpenter says:—"Not thing tends so much to prevent the healthful development of the moral sense as the infliction of punishment which the child feels to be unjust; and nothing retards the acquirement of the power of directing the intellectual processes so much as the emotional disturbance which the feeling of injustice provokes." A pupil accustomed to see others treated brutally becomes hardened and loses that acute sympathy with suffering which is the impelling force to service when such duty is demanded. In cases where brutality is very frequent, children may learn even to take delight in suffering, thus nullifying moral culture, reversing the moral law, and developing a demoniac rather than a moral character. Denunciation, sarcastic remarks, calculated to wound the sensibilities, scoldings, uncharitableness, exhibitions of favoritism, unnecessary rules and commands, and all forms of caprice upon the part of the teacher, have a tendency to produce these negative moral results in the minds of the pupils. By a careless discipline and a slip-shod administration of justice in school, children grow up with little idea of self-control, with their regulative faculties entirely undeveloped, and they often pass through life intent upon the gratification of personal desires, but entirely insensible to the welfare of others.—*Johonnot.*