# Examination Papers.

## UNIVERSITY OF TORONTO.

*Annual Examinations, 1886*

JUNIOR MATRICULATION—ARTS.

TRIGONOMETRY—HONOURS.

*Examiner*—J. W. REID, B.A.

1. Define the logarithm of a number.

Shew how the characteristic of the logarithm of a number to base 10 may be determined by inspection.

Given $\log 4 = .6020600$; $\log 1.04 = .0170333$:

(*a*) Find the logarithms of 2,25,83.2, $(.625)_{\frac{1}{4}}$.

(*b*) How many digits are there in the integral part of $(1.04)^{6666}$?

2. Prove the formulæ

$$\tan A = \frac{\sin A}{\cos A}, \quad 1 + \sec^2 A$$

$$\cos A \cos^4 \frac{A}{2} - \sin^4 \frac{A}{2}$$

$$\text{ver sin } A \quad \tan \frac{A}{2} \sin A$$

3. Given $\tan A = x$; find the values of the other trigonometrical ratios of $A$.

If $\tan \frac{A}{2} = 2 - \sqrt{3}$, find $\sin A$.

Find the value of tan 165°; sec 195°.

4. Prove the formulæ

(1) $\cos(A \pm B) \quad \cos A \cos B \mp \sin A \sin B$.

(2) $\sin(A + B + C) \sin B = \sin(A + B) \sin(B + C) - \sin A \sin C$.

(3) $\tan A + \tan B + \tan C = \tan A \tan B \tan C +$

$$\frac{\sin(A + B + C)}{\cos A \cos B \cos C}$$

5. In any triangle prove the following

(1) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.

(2) $\tan A \quad \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

(3) Area $= \frac{a^2 + b^2 - c^2}{4 \tan \frac{1}{2}(A + B - C)}$

$$\frac{2abc}{a+b+c} \cos \tfrac{1}{2} A \cos \tfrac{1}{2} B \cos \tfrac{1}{2} C.$$

6. If $A$, $B$, $C$, are the angles of a plain triangle prove:

(1) $\sin^2 \frac{1}{2} A + \sin^2 \frac{1}{2} B + \sin^2 \frac{1}{2} C +$
$2 \sin \frac{1}{2} A \sin \frac{1}{2} B \sin \frac{1}{2} C = 1$.

(2) $\cot A + \cot B + \cot C = \cot A \cot B \cot C +$ cosec $A$ cosec $B$ cosec $C$.

7. If in a triangle, the angles are such that $A:B:C = 2:3:4$: then will $\cos \frac{1}{2} A$

$$\frac{a+c}{2b}.$$

In any triangle the length of a perpendicular from $A$ on the opposite side

$$\frac{b^2 \sin C + c^2 \sin B}{b+c}$$

8. At 225 feet from the foot of a steeple, the elevation was exactly half what it was at 100 feet from it; find the height of the steeple.

From a station $B$ at the base of a mountain, its summit $A$ is seen at an elevation of 60°; and after walking one mile towards the summit, up a plane making an angle of 30° with the horizon, to another station $C$, the angle $BCA$ is observed to be 135°. Find, in feet, the height of the mountain, above the horizontal plane at $B$.

9. Solve the triangles:

(1) $a = 232$, $C = 345$, $A = 37\ 20'$

(2) $(a) \quad 10$, $(b) = 12$, $c = 14$.

| NUMBER. | LOG. | | LOG. |
|---|---|---|---|
| 174074 | 2407334 | sin 22° 6′ | 9.5774510 |
| 210000 | 3010300 | sin 27° 39′ | 9.6558412 |
| 230000 | 3617278 | sin 28° 33′ | 9.6795110 |
| 290,000 | 4623980 | sin 37° 55′ | 9.7882958 |
| 300000 | 4771213 | sin 39° 15′ | 9.8010300 |
| 374559 | 5733209 | sin 64° 24′ | 9.9551269 |
| 700000 | 5450980 | sin 75° 15′ | 9.9908287 |

## MEDICINE.

ARITHMETIC AND ALGEBRA—HONOURS.

*Examiner*—J. W. REID, B.A.

1. The L. C. M. of two numbers is 100793; the G. C. M. is 17, the difference of the numbers is 1224: find the numbers.

2. At what advance on cost must a merchant mark his goods, so that after allowing 10% of his sales for bad debts, 8% of the cost for expenses, and an average credit of 9 months (money being worth 4%) he may make a clear gain of 20% on the first cost of the goods?

3. Express in decimals accurately to five places the series

$$16 \tan^{-1}\left\{\frac{1}{5} - \frac{1}{3 \cdot 5^3} + \frac{1}{5 \cdot 5^5} - \frac{1}{7 \cdot 5^7} + \text{etc.}\right\}$$

$$- \frac{4}{239}.$$

4. What will be the true interest on $1000 for 6 months, it being supposed that if this interest is invested for the next six months that the whole interest for the year shall be exactly 6 per cent?

5. A merchant in London remits to Amsterdam £1000, at the rate of 1/8*d.* per guilder, directing his Amsterdam agent to remit the same to Paris at 2 francs 10 centimes per guilder, less ½ per cent. for commission; but the exchange between Amsterdam and Paris happened to be, at the time the order was received, at 2 francs 20 centimes per guilder. The merchant at London, not being appraised of this, drew upon Paris at 25 francs per pound sterling. Did he gain or lose, and how much per cent?

6. Find the factors of

(1) $a^3(c-b^2) + b^3(a-c^2) + c^3(b-a^2) + abc(abc-1)$.

(2) $(a+b)^3 + (b+c)^3 + (c+a)^3 +$
$3(a+2b+c)(b+2c+a)(c+2a+b)$.

(3) $a^4(b-c) + b^4(c-a) + c^4(a-b)$.

7. Simplify

(1) $\frac{(m-bc)(l-a)}{(n-b)(c-a)} + \frac{(m-ac)(l-b)}{(b-c)(a-b)}$,

$$\frac{(m-ab)(l-c)}{(c-a)(b-c)}.$$

(2) $$1 - \left\{\frac{a-b-(ab-1)x}{ab-1-(a-b)x}\right\}^2$$

$$\therefore \frac{\{ab-1-(a-b)x\}^2}{(1-ab)\{1+ab+(a+b)x\}^2}$$

$$(a-b)\{(a+b)+(ab+1)x\}^2.$$

8. Determine the condition necessary in order that $x^2+px+q$ and $x^2+p'x+q$ may have a common divisor.

9. Express $\frac{a+b\sqrt{-1}}{c+d\sqrt{-1}}$ in the form of $A+B\sqrt{-1}$.

Extract the square root of $-18\sqrt{-1}$.

10. Solve the equations:

(1) $\frac{x}{a+x} + \frac{a}{(a+x)^{\frac{1}{2}}} = \frac{b}{x}$

(2) $\begin{cases} x^{\frac{1}{2}} + y^{\frac{1}{2}} = 4 \\ x^{\frac{3}{2}} + y^{\frac{3}{2}} = 28. \end{cases}$

(3) $4x^2 + xy + y^2 = 40$.
$5xy - x^2 - 2y^2 = 4$.

11. A number less than 50 consists of two digits differing by 4. If the digits be inverted, the difference of the squares of the number thus formed, and the original number is 3960. Find the number.

## BOARD OF EDUCATION, MANITOBA

(Protestant Section.)

*Examination of Teachers, July 1886.*

ARITHMETIC—FIRST CLASS.

*Examiner*—D. McINTYRE.

Time—three hours.

1. A mill valued at $150,000 is insured as follows: in *A* company for ⅓ its value at ¾%; in *B* company for ⅓ its value at ¾%, in *C* company for ⅓ its value at ⅔%; in *D* company for ⅓ its value at ½%. What is the total annual premium, and in case of loss by fire to the amount of $25,000, what is due from each company?

2. $4,000. WINNIPEG, *July 1st*, 1885.

Twelve months after date for value received, I promise to pay Richard Roe or order four thousand dollars, with interest at seven per cent.

JOHN DAE.

On this note were the following endorsements: Sept. 15, 1885, $400: Dec. 20, $30; May 1, 1886, $1,970. What remained due July 3, 1886.

3. A note at 3 months, dated August 14, 1885, for $862.40, being interest at 6%, was discounted at 6% Sept 25. What were the proceeds?

4. What is the cash balance of the following account: Jan. 1, 1887, with interest at 6%?