

5. And the triangle ABC is the half of the parallelogram GBCA, because the diameter AB bisects it. (*Prop 34, Book I.*)

6. And the triangle DEF is the half of the parallelogram DEFH, because the diameter DF bisects it. (*Prop. 34, Book I.*)

7. But the halves of equal things are equal; therefore the triangle ABC is equal to the triangle DEF. (*Axiom 7.*)

CONCLUSION.—Wherefore triangles upon equal bases, &c. (*See Enunciation.*) Which was to be shewn.

PROPOSITION 39.—THEOREM.

*Equal triangles upon the same base, and upon the same side of it, are between the same parallels.*

HYPOTHESIS.—Let the equal triangles, ABC, DBC, be upon the same base BC, and upon the same side of it.

SEQUENCE.—They shall be between the same parallels; or, in other words—Join AD, then AD shall be parallel to BC.

CONSTRUCTION.—1. For if AD is not parallel to BC, through the point A draw AE parallel to BC. (*Prop. 31, Book I.*)

2. Join EC.

DEMONSTRATION.—1. The triangle ABC is equal to the triangle EBC, because they are upon the same base BC, and between the same parallels, BC, AE. (*Prop. 37, Book I.*)

2. But the triangle ABC is equal to the triangle DBC. (*Hypothesis.*)

3. Therefore the triangle DBC is equal to the triangle EBC (*Axiom 1*), the greater equal to the less, which is impossible.

4. Therefore AE is not parallel to BC.

5. In the same manner it may be demonstrated that no other line but AD is parallel to BC.

