4. Solve the equations—

$$\left.\begin{aligned} y^2+z^2+yz&=a^2\\ z^2+x^2+zx&=b^2\\ x^2+y^2+xy&=c^2 \end{aligned}\right\}$$

4. In these equations it will be found that the solution can be made to depend on r, where $r^2=3(a+b+c)(a-b+c)(a+b-c)$
$(b+c-a)$

x will have the value

$$\frac{3(b^2+c^2-a^2)+r}{3\sqrt{2(b^2+c^2+a^2+r)}}$$

and y and z analogous values.

7. Solve the equation—tan (cot x)=cot (tan x).

7. $\tan(\cot x)=\cot(\tan x)$

$$\therefore \tan(\cot x)=\tan\left(\frac{\pi}{2}-\tan x\right)$$

the general solution of which is

$$\cot x=n\pi+\left(\frac{\pi}{2}-\tan x\right)$$

$$\therefore \frac{1}{\tan x}=\frac{(2n+1)\pi}{2}-\tan x$$

$$\therefore \tan x+\frac{1}{\tan x}=(2n+1)\frac{\pi}{2}$$

$$\therefore \frac{1+\tan^2 x}{\tan x}=(2n+1)\frac{\pi}{2}$$

or $$\frac{1}{\sin x\cos x}=(2n+1)\frac{\pi}{2}$$

$$\therefore \sin 2x=\frac{2}{(2n+1)\frac{\pi}{2}}$$

$$x=\tfrac{1}{2}\sin^{-1}\left\{\frac{2}{(2n+1)\frac{\pi}{2}}\right\}$$

$$\text{or } x=\tfrac{1}{2}\sin^{-1}\left(\frac{4}{(2n+1)\pi}\right).$$

8. If $x\cos(\phi+\theta)+y\sin(\phi+\theta)=a\sin 2\phi$ and $y\cos(\phi+\theta)-x\sin(\phi+\theta)=2a\cos 2\phi$, then $(x\sin\theta-y\cos\theta)^{\frac{2}{3}}+(y\sin\theta+x\cos\theta)^{\frac{2}{3}}$ $=(2a)^{\frac{2}{3}}$.

8. $\therefore (x\cos\theta+y\sin\theta)\cos\phi-(x\sin\theta-y\cos\theta)\sin\phi-a\sin 2\phi=0.$

$(x\cos\theta+y\sin\theta)\sin\phi+(x\sin\theta-y\cos\theta)\cos\phi+2a\cos 2\phi=0.$

$$\therefore \frac{x\cos\theta+y\sin\theta}{2\sin^3\phi}=\frac{x\sin\theta-y\cos\theta}{2\cos^3\phi}=\frac{a}{1}$$

$$\therefore x\cos\theta+y\sin\theta=2a\sin^3\phi$$
$$x\sin\theta+y\cos\theta=2a\cos^3\phi$$

$$\therefore (A)^{\frac{2}{3}}+(B)^{\frac{2}{3}}=(2a)^{\frac{2}{3}}(\sin^2\phi+\cos^2\phi=1)$$
$$=(2a)^{\frac{2}{3}}.$$

10. If a, b, c, the sides of a triangle, be in *H. P.*, then

$$\frac{\sin\frac{A}{2}}{\sin\frac{C}{2}}=\sqrt{\frac{\cos B-\cos A}{\cos C-\cos B}}$$

10. $$\frac{\cos B-\cos A}{\cos C-\cos B}$$

$$=\frac{\dfrac{c^2+a^2-b^2}{2ac}-\dfrac{b^2+c^2-a^2}{2bc}}{\dfrac{a^2+b^2-c^2}{2ab}-\dfrac{c^2+a^2-b^2}{2ac}}$$

$$=\frac{(a-b)(a+b+c)(a+b-c)}{(b-c)(b+c+a)(b+c-a)}$$

$$=\frac{a-b}{b-c}\cdot\frac{a+b-c}{b+c-a}=\frac{a-b}{b-c}\cdot\frac{s-c}{s-a}.$$

But $$\frac{\sin^2\frac{A}{2}}{\sin^2\frac{C}{2}}=\frac{\dfrac{s-b\,.\,s-c}{bc}}{\dfrac{s-b\,.\,s-a}{ab}}=\frac{a}{c}\cdot\frac{s-a}{s-c}$$

and since a, b, c, are in *H. P.*, $\frac{a}{c}=\frac{a-c}{b-c}$

$\therefore$ etc.

11. Find for what values of a and c the expression $(a+c^{-1})^{\frac{1}{2}}+(c+a^{-1})^{\frac{1}{2}}>2^{\frac{3}{2}}$.

11. $$a+\frac{1}{c}>2\sqrt{\frac{a}{c}},\quad c+\frac{1}{a}>2\sqrt{\frac{c}{a}},$$

$$\therefore (a+c^{-1})^{\frac{1}{2}}+(c+a^{-1})^{\frac{1}{2}}>2^{\frac{1}{2}}\left(\sqrt[4]{\frac{a}{c}}+\sqrt[4]{\frac{c}{a}}\right)$$

$$\text{and }\therefore >2^{\frac{1}{2}}\,2\,\sqrt[8]{\frac{a}{c}}\cdot\sqrt[8]{\frac{c}{a}}$$

$$>2^{\frac{1}{2}}.\,2>2^{\frac{3}{2}}.$$

$\therefore (a+c^{-1})^{\frac{1}{2}}+(c+a^{-1})^{\frac{1}{2}}>2^{\frac{3}{2}}$ for all values of a and c.

13. Show that the square described about a circle is $\frac{4}{3}$ of the inscribed duodecagon.

13. The area of one of the twelve equal triangles into which the duodecagon may be divided by lines drawn from the angular points to the centre of the circle will evi-