8. Prove that $\frac{a-b}{1+ab}+\frac{b-c}{1+bc}+\frac{c-a}{1+ca}$

$$=\frac{a-b}{1+ab}\cdot\frac{b-c}{1+bc}\cdot\frac{c-a}{1+ca}.$$

8. Transposing and simplifying

$$\frac{a-b}{1+ab}\left(1-\frac{(b-c)(c-a)}{(1+bc)(1+ca)}\right)$$

$$+\frac{b-c}{1+bc}+\frac{c-a}{1+ca}=0.$$

$$\frac{a-b}{1+ab}\cdot\frac{(1+c^2)(1+ab)}{(1+bc)(1+ca)}$$

$$+\frac{(1+c^2)(b-a)}{(1+bc)(1+ca)}=0.$$

if $1-1=0$. Q.E.D.

9. Solve the equations

(1) $x^3+y^3=a.$

$xy(x+y)=b.$

(2) $(x^3+x^2y+xy^2+y^3)(x+y)=a.$

$(x^3-x^2y+xy^2-y^3)(x-y)=b.$

(3) $\sqrt[3]{x+\sqrt{2}}+\sqrt[3]{x-\sqrt{2}}=\sqrt{2}.$

9. Let $y=vx$, then substituting for y and dividing we have $\frac{1-v+v^2}{v}=\frac{a}{b}=c$ say whence $v=1$ etc. $-c$, $y=x$ etc. $-cx$, whence x and y are readily obtained.

(2) We have $(x^2+y^2)(x+y)^2=a.$

$(x^2+y^2)(x-y)^2=b.$

$\therefore\ \frac{x+y}{x-y}=\pm\sqrt{\frac{a}{b}}=\pm c$ say, whence x and y.

10. Show that if the arithmetical and geometrical means of two quantities be given, the quantities themselves may be found, and give expressions for them.

(1) Sum the series

$$1-\frac{2}{m}+\frac{1}{m^2}-\frac{2}{m^3}+\frac{1}{m^4}-\text{etc., } ad\ inf.$$

(2) Show that the sum of n terms of the series $1+3+7+15+\ldots\ldots+(2^n-1)$ is $2^{n+1}-(n+1)$.

(3) Write down four terms of the series whose n^{th} term is $\frac{4n^2-1}{4n^2+1}$.

10. Let a and b be the two quantities, a and b their arithmetic and geometric means respectively, then

$x+y=2a.$

$\sqrt{xy}=b.$

$\therefore\ x-y=\pm 2\sqrt{a^2-b^2}$, whence x and y.

(1) $S=1+\frac{1}{m^2}+\frac{1}{m^4}+\ldots\ldots$

$$-\frac{2}{m}\left(1+\frac{1}{m^2}+\ldots\ldots\right)=\frac{1-\frac{2}{m}}{1-\frac{1}{m^2}}$$

(2) $S=2+2^2+\ldots\ldots+2^n-n$

$$=\frac{2^{n+1}-2}{2-1}-n=2^{n+1}-(n+)2.$$

(3) $\frac{3}{5},\ \frac{15}{17},\ \frac{35}{37},\ \frac{63}{65}.$

11. The number of combinations of $n+1$ things 4 together is 9 times the number of combinations of n things 2 together; find n.

11. $n=11$.

12. Show that there are only $n+1$ terms in the expansion of $(1+x)^n$ when n is a positive integer.

(1) Write down the 5^{th} term of $(1-x)^{-\frac{2}{3}}$.

(2) Write down the middle term of $(1+x)^{2n}$.

12. Bookwork.

(1) 5^{th} term of $(1-x)^{-\frac{2}{3}}$

$$=\frac{\frac{2}{3}\left(\frac{2}{3}+1\right)\left(\frac{2}{3}+2\left(\frac{2}{3}+3\right)\right.}{\lfloor 4}x^4.$$

(2) Middle term of

$$(1+x)^{2n}=\frac{\lfloor 2n}{(\lfloor n)^2}x^n.$$

CLASSICS.

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I.

Translate:

His mihi rebus, ... re experti probare possitis.

—CICERO, *Cato Major.*