$$\begin{vmatrix} (b+c)^2-a^2, & 0, & 2a^2+(b+c)^2 \\ b^2-(c+a)^2, & (c+a)^2-b^2, & 2b^2+(c+a)^2 \\ 0, & c^2-(a+b)^2, & 2c^2+(a+b)^2 \end{vmatrix}$$

$$= 3 \times 2abc\,(a+b+c)^3$$

and dividing the 1st and 2nd columns by $(a+b+c)$

$$\begin{vmatrix} b+c-a, & 0, & 2a^2+(b+c)^2 \\ b-c-a, & c+a-b & 2b^2+(c+a)^2 \\ 0, & c-a-b, & 2c^2+(a+b)^2 \end{vmatrix}$$

The determinant now vanishes when $a=0$, since it reduces to

$$\begin{vmatrix} b+c & 0 & (b+c)^2 \\ b-c & c-b & 2b^2+c^2 \\ 0 & c-b & 2c^2+b^2 \end{vmatrix}$$

Subtract the third row from the second, for a new first row.

$$\begin{vmatrix} b-c & 0 & b^2-c^2 \\ b+c & 0 & (b+c)^2 \\ 0 & c-b & 2c^2+b^2 \end{vmatrix} = (b^2-c^2)\times$$

$$\begin{vmatrix} 1 & 0 & b+c \\ 1 & 0 & b+c \\ 0 & c-b & 2c^2+b^2 \end{vmatrix} = (b^2-c^2)\times 0$$

Similarly the determinant vanishes when $b=0$, and when $c=0$, and since it is symmetrical and of four dimensions the only other factor must be $k.(a+b+c)$.

To determine k put $a=b=c=1$.

$$\begin{vmatrix} 1 & 0 & 6 \\ -1 & 1 & 6 \\ 0 & -1 & 6 \end{vmatrix} = 3k.$$

$$6\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & 1 \end{vmatrix} = 3k.$$

$$2\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = k = 6$$

$$\therefore \begin{vmatrix} \frac{b+c}{a} & \frac{a}{b+c} & \frac{a}{b+c} \\ \frac{b}{c+a} & \frac{c+a}{b} & \frac{b}{c+a} \\ \frac{c}{a+b} & \frac{c}{a+b} & \frac{a+b}{c} \end{vmatrix} = \frac{2\,(a+b+c)^3}{(a+b)\,(b+c)\,(c+a)}.$$

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PROBLEMS.

140. Shew, using only propositions of Book I., how, by means of a string of unlimited length, a straight line may be drawn through a given point, parallel to a given straight line.

Let ABC be a triangle right-angled at A, and having the angle at B, double the angle at C. Bisect the angle B by the line BD, meeting AC in D, and draw AE, DF, perpendicular to BC. Shew, by Geometry, that $DC = BD$, that $AE : AD :: 3 : 2$, and that triangle $DEF = \frac{1}{4}$ triangle ABC.

141. Shew, n being a positive and even integer, that

$$1 + \frac{n+1}{\underline{|2}} + \frac{(n+2)\ (n+1)}{\underline{|3}} + \frac{(n+3)\ (n+2)\ (n+1)}{\underline{|4}} + \text{etc...}$$

$$= \frac{2}{\underline{|n}}\left\{\frac{\underline{|n}}{\underline{|1}} + \frac{\underline{|n+2}}{\underline{|3}} + \frac{\underline{|n+4}}{\underline{|5}} + \ldots\right\}$$

$$\times\left\{1 + n + \frac{n\,(n-1)}{\underline{|2}} + \frac{n\,(n-1)\,(n-2)}{\underline{|3}} + \ldots \text{to } (n+1) \text{ terms.}\right\}$$

142. Resolve into elementary factors,

$6x^3 + x^2 - 64x + 21$. $x^4 - 9x^3 + 5x^2 + 93x - 90$. $x^4 + 8x^3 + 20x^2 + 16x - 21$.

143. If $a : b : c :: x : y : z$, and

$$\frac{a^2+5b^2}{7a^2x^2+9b^2y^2} = \frac{5b^2+6c^2}{9b^2y^2+11c^2z^2} = \frac{6c^2+a^2}{11c^2z^2+7a^2x^2};$$

then each of the latter fractions,

$$= \frac{x^2+5y^2+6z^2}{7x^4+9y^4+11z^4}.$$

144. If A, B, C, be the angles of a plane triangle, a, b, c, the sides respectively opposite to them, then

$$\left(\frac{a}{\cos A} + \frac{b}{\cos B} + \frac{c}{\cos C}\right)^{-1}$$

$$= \frac{2\cos A\cos B\cos C}{a\cos A + b\cos B + c\cos C}.$$

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