therefore, by (70),
$$R_e^{\frac{1}{n}} = A_e(\phi_\sigma^\sigma \psi_\tau^\tau \ldots . X_\delta^\delta F_\beta^\beta)^{\frac{1}{n}}.$$
Therefore
$$R_1 = A_1^n(\phi_\sigma^\sigma \psi_\tau^\tau \ldots . X_\delta^\delta F_\beta^\beta). \quad (85)$$

Thus the form of the fundamental element in (72) is established. Also, it was necessary to take $R_0^{\frac{1}{n}}$ with its rational value, because, by § 5, $nR_0^{\frac{1}{n}}$ is the sum of the roots of the equation $f(x)=0$. And equation (85) is identical with (76), which establishes the necessity of the forms assigned to all those expressions which are contained under $R_e^{\frac{1}{n}}$. It remains to prove that the expressions contained under $R_{ev}^{\frac{1}{n}}$, $\frac{n}{v}$ or y being a term in the series (75) distinct from n, have the forms assigned to them in (77).

§ 44. Since $yv=n$, and y is not equal to n, y is the continued product of some of the prime factors of n, but not of them all. Let s, t, etc., be the factors of n that are factors of y, while b, d, etc., are not factors of y. Because $yv=n=b\beta$, and b is not a factor of y, b is a factor of v. Let $v=ab$; then $v\beta=an$. Therefore $w^{ev\beta}=w^{ean}=w^0$. Therefore $F_{ev\beta}=F_0$. In like manner $X_{ev\delta}=X_0$. And so on as regards all those terms of the type $F_{ev\beta}$ in which $\frac{n}{\beta}$ or b is not a measure of y. Hence, putting ev for z in the second of equations (74), and separating those factors of $R_{ev}^{\frac{1}{n}}$ that are of the type $F_{ev\beta}^{\frac{1}{n}}$ from those that are not,
$$R_{ev}^{\frac{1}{n}} = w'' A_{ev}(F_0^\beta X_0^\delta \ldots .)^{\frac{1}{n}}(\phi_{ev\sigma}^\sigma \psi_{ev\tau}^\tau \ldots .)^{\frac{1}{n}}, \quad (86)$$
w'' being an n^{th} root of unity. We understand that $F_0^{\frac{\beta}{n}}$, $X_0^{\frac{\delta}{n}}$, etc., are here taken with the rational values which it has been proved that they admit. The continued product of these expressions may be called Q, which gives us
$$R_{ev}^{\frac{1}{n}} = w'' A_{ev} Q(\phi_{ev\sigma}^\sigma \psi_{ev\tau}^\tau \ldots .)^{\frac{1}{n}}.$$
When e is taken with the particular value c, let w'' become w^r, and when e has the value unity, let w'' become w^a. Then
$$\left.\begin{aligned} R_{cv}^{\frac{1}{n}} &= w^r A_{cv} Q(\phi_{cv\sigma}^\sigma \psi_{cv\tau}^\tau \ldots .)^{\frac{1}{n}} \\ \text{and} \quad R_v^{\frac{1}{n}} &= w^a A_v Q(\phi_{v\sigma}^\sigma \psi_{v\tau}^\tau \ldots .)^{\frac{1}{n}} \end{aligned}\right\} \quad (87)$$
Because equations (3) and (5) subsist together, and w^c is included under w^e,
$$\left.\begin{aligned} R_v^{\frac{1}{n}} &= k_1 R_1^{\frac{v}{n}} \\ \text{and} \quad R_{cv}^{\frac{1}{n}} &= k_c R_c^{\frac{v}{n}} \end{aligned}\right\} \quad (88)$$