

§22. *Second Example.*—As an illustrative example, let

$$x^5 - 10x^3 - 20x^2 - 1505x - 7412 = 0.$$

Here $g = -\frac{p_2}{10} = 1$, and $k = -\frac{p_3}{20} = 1$. Therefore the equations (37) and (38) become

$$50y^2t + y(8t^3 + 8t^2 + 528t + 7412) + 596t - 6216 = 0,$$

and

$$25y^3 - (16t^4 + 56t^3 - 64t - 1771)y^2 - (2384t^3 - 89384)y - 88804 = 0.$$

The commensurable values of y and t which satisfy these equations are $y = 2$, $t = -4$. Then, we can get the values of B and $B'\sqrt{z}$ from the equations (34), keeping in view that azA' is known by (35). As far as B is concerned, it is simpler to make use of the second of equations (6), keeping in view that $acz = ty = -8$. Therefore

$$4B = 7412 - 320, \\ \therefore B = 1773.$$

In order to obtain the value of $B'\sqrt{z}$, we must find P , Q and azA' . By (33) and (35),

$$P = -62 + 9 = -53, \\ Q = 248 + 5 = 253,$$

and

$$azA' = -77 + 31 = -46.$$

Therefore

$$B'\sqrt{z} = 653a\sqrt{z} + 2A'\sqrt{z} \\ = 653a\sqrt{z} - \frac{26(azA')a\sqrt{z}}{a^2z} \\ = a\sqrt{z}(653 + 598) = 1251\sqrt{z}, \\ \therefore B + B'\sqrt{z} = 9(197 + 139\sqrt{2}).$$

Hence, since $u_1u_4 = g + a\sqrt{z} = 1 + \sqrt{2}$,

$$x = 9^{\frac{1}{4}} \left[(197 + 139\sqrt{2}) + \sqrt{\{(197 + 139\sqrt{2})^2 - \frac{1}{81}(1 + \sqrt{2})^5\}}^{\frac{1}{2}} \right. \\ + 9^{\frac{1}{4}} \left[(197 + 139\sqrt{2}) - \sqrt{\{(197 + 139\sqrt{2})^2 - \frac{1}{81}(1 + \sqrt{2})^5\}}^{\frac{1}{2}} \right. \\ + 9^{\frac{1}{4}} \left[(197 - 139\sqrt{2}) + \sqrt{\{(197 - 139\sqrt{2})^2 - \frac{1}{81}(1 - \sqrt{2})^5\}}^{\frac{1}{2}} \right. \\ + 9^{\frac{1}{4}} \left[(197 - 139\sqrt{2}) - \sqrt{\{(197 - 139\sqrt{2})^2 - \frac{1}{81}(1 - \sqrt{2})^5\}}^{\frac{1}{2}} \right].$$