free end. The beam then becomes a statically indeterminate beam fixed at one end and simply supported at the other. Neglecting the effect of the redundant simple support, let the deflection of the free end point A produced by the load P concentrated at any point, O, be $\Delta_{AO} = P\delta_{AO}$, δ_{AO} being the deflection at A under unit load at O. The reaction at A must be just great enough to produce an equal and opposite deflection of the point A or Δ_{AA} must be equal to Δ_{AO}

or
$$\Delta_{AA} = R\delta_{AA} = P\delta_{AO}$$

and
$$R = P\frac{\delta_{AO}}{\delta_{AA}}$$

According to Maxwell's theorem of reciprocal displacements the deflection at O produced by a unit load at A is equal to the deflection at A under unit load at O or

$$\delta_{AO} = \delta_{OA}$$

then
$$R = P\frac{\delta_{OA}}{\delta_{AA}}$$

The preceding example is of theoretical interest only and is given to explain the method. In order to make the method clearer it will now be applied to the case of a centre-bearing girder swing-bridge which is in the form of a two-span continuous beam. The complete solution, including typical influence lines for shear and moments, is given in Fig. 3.

The girder is statically indeterminate to the first degree and the centre bearing may be considered as a redundant support. Let the centre support be removed and replaced by a unit vertical force. The moment diagram for the resulting simple beam may then be drawn. The girder should be divided into about twenty parts making $\frac{\Delta s}{EI}$ constant. The centres of these parts are then projected down in parallel lines. The ordinates of the moment diagram under these centre points are laid off to scale on the force polygon with a pole at Q and pole length p. The funicular polygon is then drawn with its sides parallel

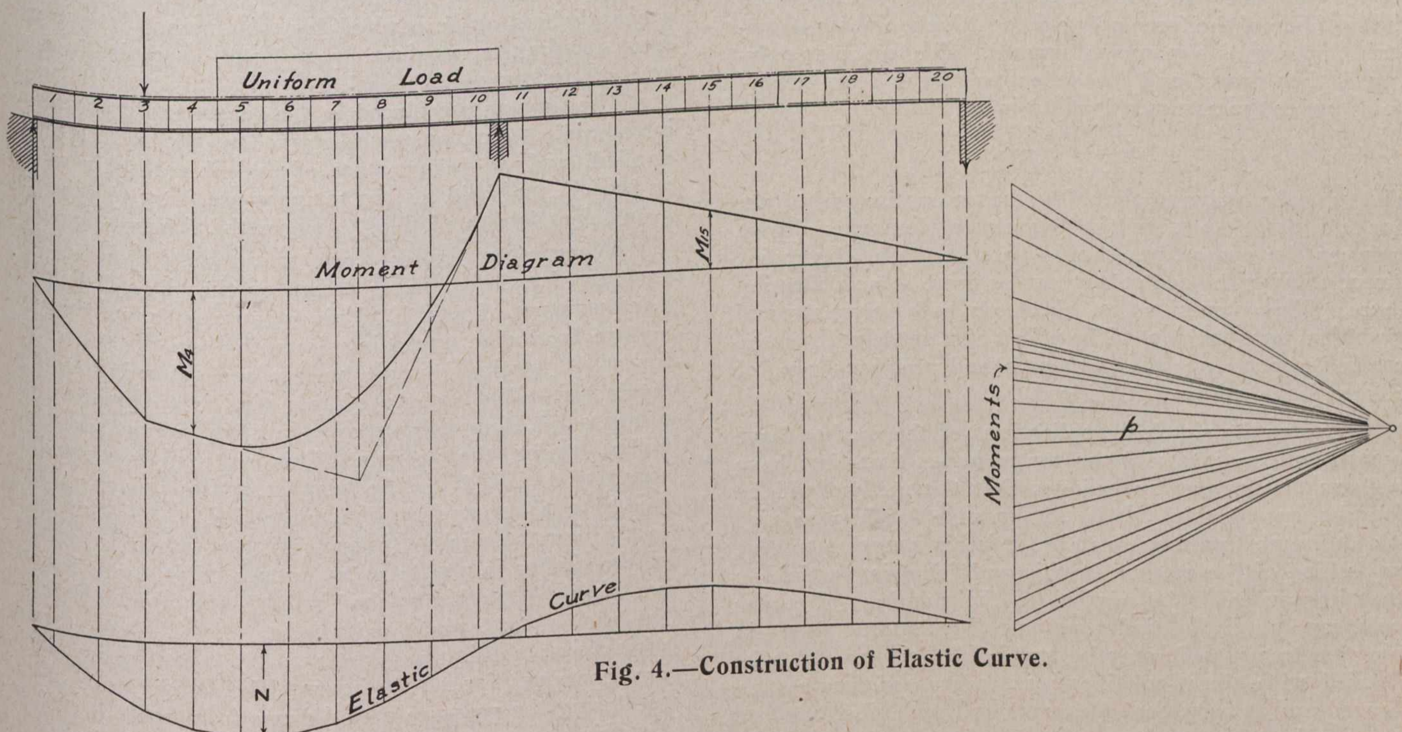

Fig. 4.—Construction of Elastic Curve.

In Fig. 2, the elastic curve of the simple cantilever under a load at A is already found so that when P is unity

$$\delta_{OA} = z_O\, p\frac{\Delta s}{EI} \text{ and } \delta_{AA} = z_A\, p\frac{\Delta s}{EI}$$

or
$$R = P\frac{\delta_{OA}}{\delta_{AA}} = P\frac{z_O}{z_A}$$

from which the reaction at A under a load P at any point may be easily found. The elastic curve previously determined then becomes the influence line from the reaction R since R equals the constant $\frac{P}{z_A}$ times the ordinate of the curve under the point of load.

It should be noted that the solution may be made perfectly general and also that the effect of a displacement of the support may be considered. If the support at A settled an amount d, the reaction would then only have to be great enough to produce the deflection ($\delta_{OA} - d$) at A and the formula for R becomes,

$$R = P\frac{\delta_{OA} - d}{\delta_{AA}} = P\frac{z_O - \frac{dEI}{p\Delta s}}{z_A}$$

to the rays and the closing line of the polygon is a check on the accuracy of its construction. As before, the curve tangent to the polygon is the elastic curve and the deflection of the beam under central load is $\frac{zp\Delta s}{EI}$. Since the centre is so supported that there can be no deflection there, the deflection of the centre point produced by the centre reaction must be equal and opposite to that produced by the superimposed load or for a load P at any point O

$$R\delta_{AA} = P\delta_{AO} = P\delta_{OA}$$

and
$$R = P\frac{\delta_{OA}}{\delta_{AA}} = P\frac{z_O}{z_A}$$

The curve tangent to the funicular polygon, the elastic curve, is therefore the influence line of the centre reaction. The different values of z can be readily scaled off and tabulated as desired. When the centre reaction influence line has been constructed, influence lines for moment and shear at any section may readily be constructed and the position of load for maximum stresses determined without the use of involved formulae. When the load is on the right of the section considered, the moment at the section is equal to