$$=\frac{z}{c}\,m\,.\,\frac{x}{a}\,m\,.\,\frac{y}{b}\,m \text{ where } m$$

$$=\left\{\frac{abc}{xyz}\left(\frac{x}{a}+\frac{y}{b}+\frac{z}{c}\right)\right\}^{\frac{1}{2}}$$

$$=\frac{\sqrt{1-c^2}}{c}\,.\,\frac{\sqrt{1-a^2}}{a}\,.\,\frac{\sqrt{1-b^2}}{b}.$$

4. If $yz+zx+xy=1$, show that

$$\frac{x}{1-x^2}+\frac{y}{1-y^2}+\frac{z}{1-z^2}$$

$$\frac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}.$$

4. $$\frac{x}{1-x^2}+\frac{y}{1-y^2}+\frac{z}{1-z^2}$$

$$=\frac{x(1-y^2)(1-z^2)+\ \dots}{(1-x^2)(1-y^2)(1-z^2)}$$

numerator $= x+y+z-x(y^2+z^2)$
$-y(z^2+x^2)-z(x^2+y^2)+xyz(xy+yz+zx)$
$=x+y+z+xyz-xy(x+y)$
$-yz(y+z)-zx(z+x)$
$=x+y+z+xyz-(xy+yz+zx)(x+y+z)$
$+3xyz$
$=4xyz.$

5. Show that $\left(\cos\frac{\pi}{8}\right)^8+\left(\cos\frac{3\pi}{8}\right)^8$

$$+\left(\cos\frac{5\pi}{8}\right)^8+\left(\cos\frac{7\pi}{8}\right)^8=\frac{17}{16}.$$

5. $\text{Cos}^2\frac{\pi}{8}+\sin^2\frac{\pi}{8}=1,$

$$\therefore\ \cos^4\frac{\pi}{8}+\sin^4\frac{\pi}{8}=1-\tfrac{1}{2}\,.\,4\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}$$

$$=1-\tfrac{1}{2}\sin\frac{\pi}{4}$$

$$=\tfrac{3}{4}$$

$$\cos^8\frac{\pi}{8}+\sin^8\frac{\pi}{8}=\tfrac{9}{16}-\tfrac{1}{8}\,.\,16\sin^4\frac{\pi}{8}\cos^4\frac{\pi}{8}$$

$$=\tfrac{9}{16}-\tfrac{1}{8}\sin^4\frac{\pi}{4}$$

$$=\tfrac{17}{32}.$$

Now $\left(\cos\frac{\pi}{8}\right)^8+\left(\cos\frac{3\pi}{8}\right)^8$

$$+\left(\cos\frac{5\pi}{8}\right)^8+\left(\cos\frac{7\pi}{8}\right)^8$$

$$=2\left\{\left(\cos\frac{\pi}{8}\right)^8+\left(\cos\frac{3\pi}{8}\right)^8\right\}$$

$$=2\left\{\left(\cos\frac{\pi}{8}\right)^8+\left(\sin\frac{\pi}{8}\right)^8\right\}$$

$$=\tfrac{17}{16}.$$

SQUARES AND SQUARE ROOTS.*

The tens and units figures of every perfect square will consist of one of the following 22 endings, viz :

00, 01, 04, 09, 16, 21, 24, 25, 29, 36, 41, 44, 49, 56, 61, 64, 69, 76, 81, 84, 89, 96.

Every perfect square will end with one of the following six digits, viz., 1, 4, 5, 6, 9, 0.

Any number ending in 2, 3, 7 or 8 cannot be a perfect square.

Since $(24)^2=576$, add $100=\ 676=(26)^2$
$(23)^2=529$, " $200=\ 729=(27)^2$
$(22)^2=484$, " $300=\ 784=(28)^2$
$(1)^2=\ \ 1$, " $2400=2401=(49)^2$

Rule I. (*a*) To determine the square of any number between 25 and 50, find the corresponding number below 25, and augment its square by the number of hundreds indicated by its remoteness from 25. Or more conveniently (*b*), Take the excess above 25 as hundreds, and augment by the square of what the number lacks of 50.

Example,
$(43)^2=(43-25)\times 100+(50-43)^2$
$=\ \ 1800\ \ +49=1849.$

Rule II. Conversely, to obtain the square root of any perfect square between 625 and 2500. Ascertain what square is indicated by the tens and units figures and deduct the number from 50. The remainder is the square root.

Ex. $\sqrt{1764}$ $64=(8)^2$ $50-8=42.$

Ex. $\sqrt{1024}$ $324=(18)^2$ $50-18=32.$

Rule III. To square any number from 50 to 100, take twice the excess above 50 as hundreds, and augment by the square of what the number lacks of 100.

Ex. $(89)^2=200(89-50)+(100-89)^2$
$=\ \ 7800\ \ +\ \ 121=7921.$

Ex. $\sqrt{8281}$ $81=(9)^2$ $100-9=91.$

Rule IV. To square any number from 100 to 200, take four times the excess above 100 as hundreds, and augment by the square of what the number lacks of 200.

Ex. $(180)^2=400(180-100)+(20)^2$
$=32000\ \ +400=32400.$

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* Notes of a lecture at Teachers' Convention, November, 1883, by J. H. Knight, P S. Inspector, Lindsay.