12. The sum of the first series $= \frac{n}{2}(n+1)$

" " second " $= \frac{n}{2}(3n+1)$

" " third " $= \frac{n}{2}(5n+1)$

$\therefore$ sum of all series

$= \frac{n}{2}(n+1+3n+1+5n+1+ \&c.)$

$= \frac{n}{2}(p+n\,\overline{1+3+5+\ldots})$

$= \frac{n}{2}(p+np^2) = \frac{1}{2}\,np\,(1+np)$

$= \frac{1}{2}\,p\,(n+pn^2)$

= sum of n such series each continued to p terms.

Toronto University, Pass Algebra, 1st Year, 1861.

1. When are magnitudes in Algebra said to be *positive* or *negative*, and in what sense is a negative quantity said to be less than zero?

If $n > a+b$, then $\frac{1}{n} < \frac{1}{a} + \frac{1}{b}$;

but if $n < a+b$, it cannot be inferred that

$\frac{1}{n} > \frac{1}{a} + \frac{1}{b}$.

2. Investigate Horner's method of division and apply it to divide $5x^5 - 3x^2 + 1$, by $x^2 - 2x + 3$, exhibiting both the complete remainder and the continuation of the quotient in descending powers of x.

3. If in $f(x)$, a rational and integral function of x, there be substituted $px+q$ for x^2, wherever x^2 occurs either singly or as a factor in some higher power of x, till the expression is reduced to the form $Ax+B$, in which A and B do not involve x, then $Ax+B$ will be the remainder after the division of $f(x)$ by $x^2 - px - q$.

Shew that

$(x^2-1)(x^2-2)+(x^2-x+1)^2+(x^2+x+1)^2$ leaves the same remainder whether divided by x^2-x+1, or by x^2+x+1.

4. Investigate a rule for finding the highest common divisor of two algebraic quantities.

Find that of

$x^2y+xy^2-3x^2+3y^2-9x+9y-2y^3$

$x^2y+2xy^2+x^2+4xy-5y^2+2x-2y-3y^3$

and examine the case of $y=1$.

5. Prove that if n be 0, 1 or 2 the following have a common divisor:

$x^n(a-y)(b-y)-y^n(a-x)(b-x)$;

$a^n(x-b)(y-b)-b^n(x-a)(y-a)$.

6. Shew what changes may be made in the numerator and denominator of a fraction without altering its value.

If $\frac{a}{b} = \frac{c}{d}$ then will $\frac{a+c}{b+d} = \sqrt{\frac{ac}{bd}}$.

7. If $\frac{a}{b}$ and $\frac{c}{d}$ be not equal, then $\frac{a+c}{b+d}$ and $\sqrt{\frac{ac}{bd}}$ lie between them in magnitude, and $\frac{a+c}{b+d}$ will be greater than $\sqrt{\frac{ac}{bd}}$ except when $\frac{a}{c}$ lies between $\frac{b}{d}$ and $\frac{d}{b}$; a, b, c, d being positive quantities.

8. "If three magnitudes are continued proportionals, the ratio which the first has to the third is said to be the duplicate of that which the first has to the second."—(Euclid B. V.) From this definition shew how to express algebraically the duplicate ratio of two quantities.

Of the ratios $a:b, c:d, e:f, \ldots\ldots y:z$, each is the duplicate of that which follows; find the relation connecting the magnitudes a, b, y, z.

9. When is one quantity said to vary as another? If a varies as $\sqrt{b}$ and c^2 varies as b^3 then will ac vary as b^2?

10. What is meant by a root of an equation? Prove that a quadratic equation can only have